

# Computer algebra independent integration tests

6-Hyperbolic-functions/6.1-Hyperbolic-sine/6.1.1-c+d-x^m-a+b-sinh^n

Nasser M. Abbasi

May 24, 2020

Compiled on May 24, 2020 at 11:38am

## Contents

<b>1</b>	<b>Introduction</b>	<b>21</b>
1.1	Listing of CAS systems tested . . . . .	21
1.2	Results . . . . .	22
1.3	Performance . . . . .	25
1.4	list of integrals that has no closed form antiderivative . . . . .	26
1.5	list of integrals solved by CAS but has no known antiderivative . . . . .	26
1.6	list of integrals solved by CAS but failed verification . . . . .	26
1.7	Timing . . . . .	27
1.8	Verification . . . . .	27
1.9	Important notes about some of the results . . . . .	27
1.10	Design of the test system . . . . .	29
<b>2</b>	<b>detailed summary tables of results</b>	<b>31</b>
2.1	List of integrals sorted by grade for each CAS . . . . .	31
2.2	Detailed conclusion table per each integral for all CAS systems . . . . .	36
2.3	Detailed conclusion table specific for Rubi results . . . . .	137
<b>3</b>	<b>Listing of integrals</b>	<b>153</b>
3.1	$\int (c + dx)^4 \sinh(a + bx) dx$ . . . . .	153
3.2	$\int (c + dx)^3 \sinh(a + bx) dx$ . . . . .	158
3.3	$\int (c + dx)^2 \sinh(a + bx) dx$ . . . . .	162
3.4	$\int (c + dx) \sinh(a + bx) dx$ . . . . .	166

3.5	$\int \frac{\sinh(a+bx)}{c+dx} dx$	169
3.6	$\int \frac{\sinh(a+bx)}{(c+dx)^2} dx$	173
3.7	$\int \frac{\sinh(a+bx)}{(c+dx)^3} dx$	177
3.8	$\int (c+dx)^4 \sinh^2(a+bx) dx$	181
3.9	$\int (c+dx)^3 \sinh^2(a+bx) dx$	186
3.10	$\int (c+dx)^2 \sinh^2(a+bx) dx$	191
3.11	$\int (c+dx) \sinh^2(a+bx) dx$	195
3.12	$\int \frac{\sinh^2(a+bx)}{c+dx} dx$	198
3.13	$\int \frac{\sinh^2(a+bx)}{(c+dx)^2} dx$	202
3.14	$\int \frac{\sinh^2(a+bx)}{(c+dx)^3} dx$	206
3.15	$\int \frac{\sinh^2(a+bx)}{(c+dx)^4} dx$	211
3.16	$\int (c+dx)^4 \sinh^3(a+bx) dx$	216
3.17	$\int (c+dx)^3 \sinh^3(a+bx) dx$	222
3.18	$\int (c+dx)^2 \sinh^3(a+bx) dx$	227
3.19	$\int (c+dx) \sinh^3(a+bx) dx$	232
3.20	$\int \frac{\sinh^3(a+bx)}{c+dx} dx$	236
3.21	$\int \frac{\sinh^3(a+bx)}{(c+dx)^2} dx$	240
3.22	$\int \frac{\sinh^3(a+bx)}{(c+dx)^3} dx$	245
3.23	$\int (c+dx)^3 \operatorname{csch}(a+bx) dx$	250
3.24	$\int (c+dx)^2 \operatorname{csch}(a+bx) dx$	255
3.25	$\int (c+dx) \operatorname{csch}(a+bx) dx$	259
3.26	$\int \frac{\operatorname{csch}(a+bx)}{c+dx} dx$	263
3.27	$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} dx$	266
3.28	$\int (c+dx)^3 \operatorname{csch}^2(a+bx) dx$	269
3.29	$\int (c+dx)^2 \operatorname{csch}^2(a+bx) dx$	275
3.30	$\int (c+dx) \operatorname{csch}^2(a+bx) dx$	280
3.31	$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx} dx$	283
3.32	$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2} dx$	286
3.33	$\int (c+dx)^3 \operatorname{csch}^3(a+bx) dx$	289
3.34	$\int (c+dx)^2 \operatorname{csch}^3(a+bx) dx$	297
3.35	$\int (c+dx) \operatorname{csch}^3(a+bx) dx$	303
3.36	$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx} dx$	308
3.37	$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} dx$	311

3.38	$\int (c + dx)^{5/2} \sinh(a + bx) dx$	314
3.39	$\int (c + dx)^{3/2} \sinh(a + bx) dx$	319
3.40	$\int \sqrt{c + dx} \sinh(a + bx) dx$	324
3.41	$\int \frac{\sinh(a+bx)}{\sqrt{c+dx}} dx$	328
3.42	$\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}} dx$	332
3.43	$\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}} dx$	336
3.44	$\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}} dx$	341
3.45	$\int (c + dx)^{5/2} \sinh^2(a + bx) dx$	346
3.46	$\int (c + dx)^{3/2} \sinh^2(a + bx) dx$	351
3.47	$\int \sqrt{c + dx} \sinh^2(a + bx) dx$	356
3.48	$\int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}} dx$	361
3.49	$\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}} dx$	365
3.50	$\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}} dx$	370
3.51	$\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}} dx$	375
3.52	$\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}} dx$	381
3.53	$\int (c + dx)^{5/2} \sinh^3(a + bx) dx$	387
3.54	$\int (c + dx)^{3/2} \sinh^3(a + bx) dx$	393
3.55	$\int \sqrt{c + dx} \sinh^3(a + bx) dx$	399
3.56	$\int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}} dx$	405
3.57	$\int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}} dx$	409
3.58	$\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}} dx$	415
3.59	$\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}} dx$	421
3.60	$\int (dx)^{3/2} \sinh(fx) dx$	429
3.61	$\int \sqrt{dx} \sinh(fx) dx$	434
3.62	$\int \frac{\sinh(fx)}{\sqrt{dx}} dx$	438
3.63	$\int \frac{\sinh(fx)}{(dx)^{3/2}} dx$	442
3.64	$\int \frac{\sinh(fx)}{(dx)^{5/2}} dx$	446
3.65	$\int \sqrt{c + dx} \operatorname{csch}(a + bx) dx$	451
3.66	$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}} dx$	454
3.67	$\int \frac{\sinh^2(x)}{x^3} dx$	457
3.68	$\int \left( \frac{x}{\sinh^2(x)} - x\sqrt{\sinh(x)} \right) dx$	460

3.69	$\int \left( \frac{x}{\sinh^2(x)} + \frac{x}{3\sqrt{\sinh(x)}} \right) dx$	463
3.70	$\int \left( \frac{x}{\sinh^2(x)} + \frac{3}{5}x\sqrt{\sinh(x)} \right) dx$	466
3.71	$\int \left( \frac{x^2}{\sinh^2(x)} - x^2\sqrt{\sinh(x)} \right) dx$	469
3.72	$\int (c + dx)^m (b \sinh(e + fx))^n dx$	473
3.73	$\int (c + dx)^m \sinh^3(a + bx) dx$	476
3.74	$\int (c + dx)^m \sinh^2(a + bx) dx$	480
3.75	$\int (c + dx)^m \sinh(a + bx) dx$	484
3.76	$\int (c + dx)^m \operatorname{csch}(a + bx) dx$	488
3.77	$\int (c + dx)^m \operatorname{csch}^2(a + bx) dx$	491
3.78	$\int x^{3+m} \sinh(a + bx) dx$	494
3.79	$\int x^{2+m} \sinh(a + bx) dx$	497
3.80	$\int x^{1+m} \sinh(a + bx) dx$	500
3.81	$\int x^m \sinh(a + bx) dx$	503
3.82	$\int x^{-1+m} \sinh(a + bx) dx$	506
3.83	$\int x^{-2+m} \sinh(a + bx) dx$	509
3.84	$\int x^{-3+m} \sinh(a + bx) dx$	512
3.85	$\int x^{3+m} \sinh^2(a + bx) dx$	515
3.86	$\int x^{2+m} \sinh^2(a + bx) dx$	519
3.87	$\int x^{1+m} \sinh^2(a + bx) dx$	523
3.88	$\int x^m \sinh^2(a + bx) dx$	527
3.89	$\int x^{-1+m} \sinh^2(a + bx) dx$	531
3.90	$\int x^{-2+m} \sinh^2(a + bx) dx$	535
3.91	$\int x^{-3+m} \sinh^2(a + bx) dx$	539
3.92	$\int \left( \frac{x}{\operatorname{csch}^2(x)} + \frac{1}{3}x\sqrt{\operatorname{csch}(x)} \right) dx$	543
3.93	$\int \left( \frac{x}{\operatorname{csch}^2(x)} + \frac{3x}{5\sqrt{\operatorname{csch}(x)}} \right) dx$	546
3.94	$\int \left( \frac{x}{\operatorname{csch}^2(x)} - \frac{5}{21}x\sqrt{\operatorname{csch}(x)} \right) dx$	549
3.95	$\int \left( \frac{x^2}{\operatorname{csch}^2(x)} + \frac{1}{3}x^2\sqrt{\operatorname{csch}(x)} \right) dx$	552
3.96	$\int (c + dx)^3 (a + ia \sinh(e + fx)) dx$	556
3.97	$\int (c + dx)^2 (a + ia \sinh(e + fx)) dx$	561
3.98	$\int (c + dx) (a + ia \sinh(e + fx)) dx$	565
3.99	$\int \frac{a + ia \sinh(e + fx)}{c + dx} dx$	569

3.100	$\int \frac{a+ia \sinh(e+fx)}{(c+dx)^2} dx$	573
3.101	$\int \frac{a+ia \sinh(e+fx)}{(c+dx)^3} dx$	577
3.102	$\int (c+dx)^3 (a+ia \sinh(e+fx))^2 dx$	582
3.103	$\int (c+dx)^2 (a+ia \sinh(e+fx))^2 dx$	589
3.104	$\int (c+dx)(a+ia \sinh(e+fx))^2 dx$	595
3.105	$\int \frac{(a+ia \sinh(e+fx))^2}{c+dx} dx$	599
3.106	$\int \frac{(a+ia \sinh(e+fx))^2}{(c+dx)^2} dx$	603
3.107	$\int \frac{(a+ia \sinh(e+fx))^2}{(c+dx)^3} dx$	608
3.108	$\int \frac{(c+dx)^3}{a+ia \sinh(e+fx)} dx$	614
3.109	$\int \frac{(c+dx)^2}{a+ia \sinh(e+fx)} dx$	619
3.110	$\int \frac{c+dx}{a+ia \sinh(e+fx)} dx$	624
3.111	$\int \frac{1}{(c+dx)(a+ia \sinh(e+fx))} dx$	628
3.112	$\int \frac{1}{(c+dx)^2(a+ia \sinh(e+fx))} dx$	631
3.113	$\int \frac{(c+dx)^3}{(a+ia \sinh(e+fx))^2} dx$	634
3.114	$\int \frac{(c+dx)^2}{(a+ia \sinh(e+fx))^2} dx$	641
3.115	$\int \frac{c+dx}{(a+ia \sinh(e+fx))^2} dx$	647
3.116	$\int \frac{1}{(c+dx)(a+ia \sinh(e+fx))^2} dx$	652
3.117	$\int \frac{1}{(c+dx)^2(a+ia \sinh(e+fx))^2} dx$	656
3.118	$\int x^4 \sqrt{a+ia \sinh(e+fx)} dx$	660
3.119	$\int x^3 \sqrt{a+ia \sinh(e+fx)} dx$	664
3.120	$\int x^2 \sqrt{a+ia \sinh(e+fx)} dx$	668
3.121	$\int x \sqrt{a+ia \sinh(e+fx)} dx$	672
3.122	$\int \frac{\sqrt{a+ia \sinh(e+fx)}}{x} dx$	676
3.123	$\int \frac{\sqrt{a+ia \sinh(e+fx)}}{x^2} dx$	680
3.124	$\int \frac{\sqrt{a+ia \sinh(e+fx)}}{x^3} dx$	684
3.125	$\int x^3 (a+ia \sinh(e+fx))^{3/2} dx$	688
3.126	$\int x^2 (a+ia \sinh(e+fx))^{3/2} dx$	693
3.127	$\int x (a+ia \sinh(e+fx))^{3/2} dx$	698
3.128	$\int \frac{(a+ia \sinh(e+fx))^{3/2}}{x} dx$	702
3.129	$\int \frac{(a+ia \sinh(e+fx))^{3/2}}{x^2} dx$	706
3.130	$\int x^3 (a+ia \sinh(c+dx))^{5/2} dx$	711
3.131	$\int x^2 (a+ia \sinh(c+dx))^{5/2} dx$	717
3.132	$\int x (a+ia \sinh(c+dx))^{5/2} dx$	722

3.133	$\int \frac{(a+ia \sinh(c+dx))^{5/2}}{x} dx$	726
3.134	$\int \frac{(a+ia \sinh(c+dx))^{5/2}}{x^2} dx$	731
3.135	$\int \frac{(a+ia \sinh(c+dx))^{5/2}}{x^3} dx$	736
3.136	$\int \frac{x^2}{\sqrt{a+ia \sinh(e+fx)}} dx$	743
3.137	$\int \frac{x}{\sqrt{a+ia \sinh(e+fx)}} dx$	748
3.138	$\int \frac{1}{\sqrt{a+ia \sinh(e+fx)}} dx$	753
3.139	$\int \frac{1}{x\sqrt{a+ia \sinh(e+fx)}} dx$	757
3.140	$\int \frac{1}{x^2\sqrt{a+ia \sinh(e+fx)}} dx$	760
3.141	$\int \frac{x^3}{(a+ia \sinh(e+fx))^{3/2}} dx$	763
3.142	$\int \frac{x^2}{(a+ia \sinh(e+fx))^{3/2}} dx$	769
3.143	$\int \frac{x}{(a+ia \sinh(e+fx))^{3/2}} dx$	775
3.144	$\int \frac{1}{x(a+ia \sinh(e+fx))^{3/2}} dx$	780
3.145	$\int \frac{1}{x^2(a+ia \sinh(e+fx))^{3/2}} dx$	783
3.146	$\int \frac{x^3}{(a+ia \sinh(c+dx))^{5/2}} dx$	786
3.147	$\int \frac{x^2}{(a+ia \sinh(c+dx))^{5/2}} dx$	793
3.148	$\int \frac{x}{(a+ia \sinh(c+dx))^{5/2}} dx$	799
3.149	$\int \frac{1}{x(a+ia \sinh(c+dx))^{5/2}} dx$	804
3.150	$\int \frac{\sqrt[3]{a+ia \sinh(e+fx)}}{x} dx$	807
3.151	$\int (c+dx)^m (a+ia \sinh(e+fx))^n dx$	810
3.152	$\int (c+dx)^m (a+ia \sinh(e+fx))^3 dx$	813
3.153	$\int (c+dx)^m (a+ia \sinh(e+fx))^2 dx$	818
3.154	$\int (c+dx)^m (a+ia \sinh(e+fx)) dx$	822
3.155	$\int \frac{(c+dx)^m}{a+ia \sinh(e+fx)} dx$	826
3.156	$\int \frac{(c+dx)^m}{(a+ia \sinh(e+fx))^2} dx$	829
3.157	$\int (c+dx)^3 (a+b \sinh(e+fx)) dx$	832
3.158	$\int (c+dx)^2 (a+b \sinh(e+fx)) dx$	836
3.159	$\int (c+dx) (a+b \sinh(e+fx)) dx$	840
3.160	$\int \frac{a+b \sinh(e+fx)}{c+dx} dx$	844
3.161	$\int \frac{a+b \sinh(e+fx)}{(c+dx)^2} dx$	848
3.162	$\int \frac{a+b \sinh(e+fx)}{(c+dx)^3} dx$	852
3.163	$\int (c+dx)^3 (a+b \sinh(e+fx))^2 dx$	857
3.164	$\int (c+dx)^2 (a+b \sinh(e+fx))^2 dx$	863

3.165	$\int (c + dx)(a + b \sinh(e + fx))^2 dx$	869
3.166	$\int \frac{(a+b \sinh(e+fx))^2}{c+dx} dx$	873
3.167	$\int \frac{(a+b \sinh(e+fx))^2}{(c+dx)^2} dx$	878
3.168	$\int \frac{(a+b \sinh(e+fx))^2}{(c+dx)^3} dx$	883
3.169	$\int \frac{(c+dx)^3}{a+b \sinh(e+fx)} dx$	889
3.170	$\int \frac{(c+dx)^2}{a+b \sinh(e+fx)} dx$	895
3.171	$\int \frac{c+dx}{a+b \sinh(e+fx)} dx$	900
3.172	$\int \frac{1}{(c+dx)(a+b \sinh(e+fx))} dx$	905
3.173	$\int \frac{1}{(c+dx)^2(a+b \sinh(e+fx))} dx$	908
3.174	$\int \frac{(c+dx)^2}{(a+b \sinh(e+fx))^2} dx$	911
3.175	$\int \frac{c+dx}{(a+b \sinh(e+fx))^2} dx$	919
3.176	$\int \frac{1}{(c+dx)(a+b \sinh(e+fx))^2} dx$	925
3.177	$\int \frac{1}{(c+dx)^2(a+b \sinh(e+fx))^2} dx$	928
3.178	$\int \frac{e+fx}{(a+b \sinh(c+dx))^3} dx$	931
3.179	$\int \frac{1}{(e+fx)(a+b \sinh(c+dx))^3} dx$	941
3.180	$\int \frac{1}{(e+fx)^2(a+b \sinh(c+dx))^3} dx$	945
3.181	$\int (c + dx)^m (a + b \sinh(e + fx))^n dx$	949
3.182	$\int (c + dx)^m (a + b \sinh(e + fx))^3 dx$	952
3.183	$\int (c + dx)^m (a + b \sinh(e + fx))^2 dx$	957
3.184	$\int (c + dx)^m (a + b \sinh(e + fx)) dx$	962
3.185	$\int \frac{(c+dx)^m}{a+b \sinh(e+fx)} dx$	966
3.186	$\int \frac{(c+dx)^m}{(a+b \sinh(e+fx))^2} dx$	969
3.187	$\int \frac{(e+fx)^3 \sinh(c+dx)}{a+ia \sinh(c+dx)} dx$	972
3.188	$\int \frac{(e+fx)^2 \sinh(c+dx)}{a+ia \sinh(c+dx)} dx$	978
3.189	$\int \frac{(e+fx) \sinh(c+dx)}{a+ia \sinh(c+dx)} dx$	983
3.190	$\int \frac{\sinh(c+dx)}{a+ia \sinh(c+dx)} dx$	987
3.191	$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$	990
3.192	$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$	993
3.193	$\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia \sinh(c+dx)} dx$	996
3.194	$\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+ia \sinh(c+dx)} dx$	1003
3.195	$\int \frac{(e+fx) \sinh^2(c+dx)}{a+ia \sinh(c+dx)} dx$	1009

3.196	$\int \frac{\sinh^2(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1014
3.197	$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$	. . . . .	1018
3.198	$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$	. . . . .	1021
3.199	$\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1024
3.200	$\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1031
3.201	$\int \frac{(e+fx) \sinh^3(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1038
3.202	$\int \frac{\sinh^3(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1043
3.203	$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$	. . . . .	1047
3.204	$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$	. . . . .	1050
3.205	$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1053
3.206	$\int \frac{(e+fx)^2 \operatorname{csch}(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1060
3.207	$\int \frac{(e+fx) \operatorname{csch}(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1067
3.208	$\int \frac{\operatorname{csch}(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1072
3.209	$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$	. . . . .	1076
3.210	$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$	. . . . .	1079
3.211	$\int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1082
3.212	$\int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1091
3.213	$\int \frac{(e+fx) \operatorname{csch}^2(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1098
3.214	$\int \frac{\operatorname{csch}^2(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1103
3.215	$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$	. . . . .	1107
3.216	$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$	. . . . .	1110
3.217	$\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1113
3.218	$\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1125
3.219	$\int \frac{(e+fx) \operatorname{csch}^3(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1134
3.220	$\int \frac{\operatorname{csch}^3(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1140
3.221	$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$	. . . . .	1144
3.222	$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$	. . . . .	1148



3.223	$\int \frac{(e+fx)^3 \sinh(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1152
3.224	$\int \frac{(e+fx)^2 \sinh(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1159
3.225	$\int \frac{(e+fx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1165
3.226	$\int \frac{\sinh(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1170
3.227	$\int \frac{\sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	1175
3.228	$\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1178
3.229	$\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1186
3.230	$\int \frac{(e+fx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1193
3.231	$\int \frac{\sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1199
3.232	$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	1204
3.233	$\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1207
3.234	$\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1217
3.235	$\int \frac{(e+fx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1225
3.236	$\int \frac{\sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1231
3.237	$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	1236
3.238	$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1239
3.239	$\int \frac{(e+fx)^2 \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1246
3.240	$\int \frac{(e+fx) \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1252
3.241	$\int \frac{\operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1257
3.242	$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	1261
3.243	$\int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1264
3.244	$\int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1274
3.245	$\int \frac{(e+fx) \operatorname{csch}^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1282
3.246	$\int \frac{\operatorname{csch}^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1288
3.247	$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	1293
3.248	$\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1296
3.249	$\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1304
3.250	$\int \frac{(e+fx) \operatorname{csch}^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1316

3.251	$\int \frac{\operatorname{csch}^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1325
3.252	$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	1331
3.253	$\int \frac{(e+fx)^3 \cosh(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1334
3.254	$\int \frac{(e+fx)^2 \cosh(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1339
3.255	$\int \frac{(e+fx) \cosh(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1344
3.256	$\int \frac{\cosh(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1348
3.257	$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$	. . . . .	1351
3.258	$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$	. . . . .	1354
3.259	$\int \frac{(e+fx)^3 \cosh^2(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1357
3.260	$\int \frac{(e+fx)^2 \cosh^2(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1362
3.261	$\int \frac{(e+fx) \cosh^2(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1367
3.262	$\int \frac{\cosh^2(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1371
3.263	$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$	. . . . .	1374
3.264	$\int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$	. . . . .	1378
3.265	$\int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1382
3.266	$\int \frac{(e+fx)^2 \cosh^3(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1388
3.267	$\int \frac{(e+fx) \cosh^3(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1393
3.268	$\int \frac{\cosh^3(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1397
3.269	$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$	. . . . .	1400
3.270	$\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$	. . . . .	1405
3.271	$\int \frac{(e+fx)^3 \operatorname{sech}(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1410
3.272	$\int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1418
3.273	$\int \frac{(e+fx) \operatorname{sech}(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1424
3.274	$\int \frac{\operatorname{sech}(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1429
3.275	$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$	. . . . .	1433
3.276	$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$	. . . . .	1436
3.277	$\int \frac{(e+fx)^3 \operatorname{sech}^2(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1439
3.278	$\int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	1447

3.279	$\int \frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	.1454
3.280	$\int \frac{\operatorname{sech}^2(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	.1459
3.281	$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$	. . . . .	.1463
3.282	$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$	. . . . .	.1467
3.283	$\int \frac{(e+fx)^3 \operatorname{sech}^3(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	.1471
3.284	$\int \frac{(e+fx)^2 \operatorname{sech}^3(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	.1482
3.285	$\int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	.1490
3.286	$\int \frac{\operatorname{sech}^3(c+dx)}{a+ia \sinh(c+dx)} dx$	. . . . .	.1496
3.287	$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$	. . . . .	.1500
3.288	$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$	. . . . .	.1505
3.289	$\int \frac{(e+fx)^3 \cosh(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.1510
3.290	$\int \frac{(e+fx)^2 \cosh(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.1515
3.291	$\int \frac{(e+fx) \cosh(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.1520
3.292	$\int \frac{\cosh(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.1525
3.293	$\int \frac{\cosh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	.1528
3.294	$\int \frac{(e+fx)^3 \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.1531
3.295	$\int \frac{(e+fx)^2 \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.1538
3.296	$\int \frac{(e+fx) \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.1545
3.297	$\int \frac{\cosh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.1551
3.298	$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	.1556
3.299	$\int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.1559
3.300	$\int \frac{(e+fx)^2 \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.1568
3.301	$\int \frac{(e+fx) \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.1577
3.302	$\int \frac{\cosh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.1584
3.303	$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	.1588
3.304	$\int \frac{(e+fx)^3 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.1591
3.305	$\int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.1598

3.306	$\int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	1605
3.307	$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	1611
3.308	$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$	. . . . .	1615
3.309	$\int \frac{(e+fx)^3 \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	1618
3.310	$\int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	1629
3.311	$\int \frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	1638
3.312	$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	1645
3.313	$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$	. . . . .	1650
3.314	$\int \frac{(e+fx)^2 \operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	1653
3.315	$\int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	1668
3.316	$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	1678
3.317	$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$	. . . . .	1684
3.318	$\int \frac{x^m \cosh^3(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	1687
3.319	$\int \frac{x^m \cosh^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	1690
3.320	$\int \frac{x^m \cosh(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	1693
3.321	$\int \frac{(e+fx) \cosh(c+dx)}{(a+b\sinh(c+dx))^2} dx$	. . . . .	1696
3.322	$\int \frac{(e+fx)^2 \cosh(c+dx)}{(a+b\sinh(c+dx))^2} dx$	. . . . .	1700
3.323	$\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^2} dx$	. . . . .	1706
3.324	$\int \frac{(e+fx) \cosh(c+dx)}{(a+b\sinh(c+dx))^2} dx$	. . . . .	1712
3.325	$\int \frac{(e+fx)^2 \cosh(c+dx)}{(a+b\sinh(c+dx))^2} dx$	. . . . .	1716
3.326	$\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^2} dx$	. . . . .	1722
3.327	$\int \frac{(e+fx) \cosh(c+dx)}{(a+b\sinh(c+dx))^3} dx$	. . . . .	1728
3.328	$\int \frac{(e+fx)^2 \cosh(c+dx)}{(a+b\sinh(c+dx))^3} dx$	. . . . .	1733
3.329	$\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^3} dx$	. . . . .	1741
3.330	$\int \frac{(e+fx) \cosh(c+dx)}{(a+b\sinh(c+dx))^3} dx$	. . . . .	1752
3.331	$\int \frac{(e+fx)^2 \cosh(c+dx)}{(a+b\sinh(c+dx))^3} dx$	. . . . .	1757
3.332	$\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b\sinh(c+dx))^3} dx$	. . . . .	1765
3.333	$\int \frac{(e+fx)^3 \cosh(c+dx) \sinh(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	1776

3.334	$\int \frac{(e+fx)^2 \cosh(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$	.1783
3.335	$\int \frac{(e+fx) \cosh(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$	.1790
3.336	$\int \frac{\cosh(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$	.1795
3.337	$\int \frac{\cosh(c+dx) \sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	.1799
3.338	$\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$	.1802
3.339	$\int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$	.1812
3.340	$\int \frac{(e+fx) \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$	.1821
3.341	$\int \frac{\cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$	.1829
3.342	$\int \frac{\cosh^2(c+dx) \sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	.1834
3.343	$\int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$	.1837
3.344	$\int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$	.1848
3.345	$\int \frac{(e+fx) \cosh^3(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$	.1860
3.346	$\int \frac{\cosh^3(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$	.1868
3.347	$\int \frac{\cosh^3(c+dx) \sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	.1872
3.348	$\int \frac{(e+fx)^3 \tanh(c+dx)}{a+b \sinh(c+dx)} dx$	.1875
3.349	$\int \frac{(e+fx)^2 \tanh(c+dx)}{a+b \sinh(c+dx)} dx$	.1884
3.350	$\int \frac{(e+fx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$	.1892
3.351	$\int \frac{\tanh(c+dx)}{a+b \sinh(c+dx)} dx$	.1899
3.352	$\int \frac{\tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	.1903
3.353	$\int \frac{(e+fx)^3 \operatorname{sech}(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$	.1906
3.354	$\int \frac{(e+fx)^2 \operatorname{sech}(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$	.1917
3.355	$\int \frac{(e+fx) \operatorname{sech}(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$	.1927
3.356	$\int \frac{\operatorname{sech}(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$	.1935
3.357	$\int \frac{\operatorname{sech}(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	.1940
3.358	$\int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$	.1943
3.359	$\int \frac{(e+fx) \operatorname{sech}^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$	.1959
3.360	$\int \frac{\operatorname{sech}^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$	.1970
3.361	$\int \frac{\operatorname{sech}^2(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	.1976

3.362	$\int \frac{(e+fx)^3 \cosh(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1980
3.363	$\int \frac{(e+fx)^2 \cosh(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1991
3.364	$\int \frac{(e+fx) \cosh(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	1999
3.365	$\int \frac{\cosh(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	2005
3.366	$\int \frac{\cosh(c+dx) \sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	2009
3.367	$\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	2012
3.368	$\int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	2023
3.369	$\int \frac{(e+fx) \cosh^2(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	2033
3.370	$\int \frac{\cosh^2(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	2041
3.371	$\int \frac{\cosh^2(c+dx) \sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	2047
3.372	$\int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	2050
3.373	$\int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	2065
3.374	$\int \frac{(e+fx) \cosh^3(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	2076
3.375	$\int \frac{\cosh^3(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	2085
3.376	$\int \frac{\cosh^3(c+dx) \sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	2090
3.377	$\int \frac{(e+fx)^3 \sinh(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	2093
3.378	$\int \frac{(e+fx)^2 \sinh(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	2103
3.379	$\int \frac{(e+fx) \sinh(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	2112
3.380	$\int \frac{\sinh(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	2120
3.381	$\int \frac{\sinh(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	2125
3.382	$\int \frac{(e+fx)^3 \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	2128
3.383	$\int \frac{(e+fx)^2 \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	2140
3.384	$\int \frac{(e+fx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	2150
3.385	$\int \frac{\tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	2158
3.386	$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	2163
3.387	$\int \frac{(e+fx)^2 \operatorname{sech}(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	2166
3.388	$\int \frac{(e+fx) \operatorname{sech}(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	2182

3.389	$\int \frac{\operatorname{sech}(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2193
3.390	$\int \frac{\operatorname{sech}(c+dx) \tanh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . . 2199
3.391	$\int \frac{(e+fx)^3 \cosh(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2203
3.392	$\int \frac{(e+fx)^2 \cosh(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2214
3.393	$\int \frac{(e+fx) \cosh(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2224
3.394	$\int \frac{\cosh(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2232
3.395	$\int \frac{\cosh(c+dx) \sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . . 2236
3.396	$\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2239
3.397	$\int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2252
3.398	$\int \frac{(e+fx) \cosh^2(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2262
3.399	$\int \frac{\cosh^2(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2272
3.400	$\int \frac{\cosh^2(c+dx) \sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . . 2278
3.401	$\int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2281
3.402	$\int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2290
3.403	$\int \frac{(e+fx) \cosh^3(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2305
3.404	$\int \frac{\cosh^3(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2316
3.405	$\int \frac{\cosh^3(c+dx) \sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . . 2321
3.406	$\int \frac{(e+fx)^3 \sinh^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2324
3.407	$\int \frac{(e+fx)^2 \sinh^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2336
3.408	$\int \frac{(e+fx) \sinh^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2347
3.409	$\int \frac{\sinh^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2357
3.410	$\int \frac{\sinh^2(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . . 2362
3.411	$\int \frac{(e+fx)^3 \sinh(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2365
3.412	$\int \frac{(e+fx)^2 \sinh(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2378
3.413	$\int \frac{(e+fx) \sinh(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2389
3.414	$\int \frac{\sinh(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . . 2398
3.415	$\int \frac{\sinh(c+dx) \tanh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . . 2403

3.416	$\int \frac{(e+fx)^2 \tanh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2406
3.417	$\int \frac{(e+fx) \tanh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2422
3.418	$\int \frac{\tanh^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2433
3.419	$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	.2439
3.420	$\int \frac{(e+fx)^3 \coth(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2442
3.421	$\int \frac{(e+fx)^2 \coth(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2449
3.422	$\int \frac{(e+fx) \coth(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2455
3.423	$\int \frac{\coth(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2460
3.424	$\int \frac{\coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	.2464
3.425	$\int \frac{(e+fx)^3 \cosh(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2467
3.426	$\int \frac{(e+fx)^2 \cosh(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2475
3.427	$\int \frac{(e+fx) \cosh(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2482
3.428	$\int \frac{\cosh(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2489
3.429	$\int \frac{\cosh(c+dx) \coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	.2494
3.430	$\int \frac{(e+fx)^3 \cosh^2(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2497
3.431	$\int \frac{(e+fx)^2 \cosh^2(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2507
3.432	$\int \frac{(e+fx) \cosh^2(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2515
3.433	$\int \frac{\cosh^2(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2522
3.434	$\int \frac{\cosh^2(c+dx) \coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	.2526
3.435	$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2529
3.436	$\int \frac{(e+fx)^2 \operatorname{csch}(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2539
3.437	$\int \frac{(e+fx) \operatorname{csch}(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2549
3.438	$\int \frac{\operatorname{csch}(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2557
3.439	$\int \frac{\operatorname{csch}(c+dx) \operatorname{sech}(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	.2562
3.440	$\int \frac{(e+fx)^3 \operatorname{csch}(c+dx) \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2565
3.441	$\int \frac{(e+fx)^2 \operatorname{csch}(c+dx) \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2580
3.442	$\int \frac{(e+fx) \operatorname{csch}(c+dx) \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2593
3.443	$\int \frac{\operatorname{csch}(c+dx) \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.2602



3.444	$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$	. . . . .	2608
3.445	$\int \frac{(e+fx)^2\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2611
3.446	$\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2623
3.447	$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2637
3.448	$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$	. . . . .	2643
3.449	$\int \frac{(e+fx)^3\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2647
3.450	$\int \frac{(e+fx)^2\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2656
3.451	$\int \frac{(e+fx)\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2664
3.452	$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2670
3.453	$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$	. . . . .	2674
3.454	$\int \frac{(e+fx)^3\coth^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2677
3.455	$\int \frac{(e+fx)^2\coth^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2687
3.456	$\int \frac{(e+fx)\coth^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2696
3.457	$\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2703
3.458	$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$	. . . . .	2708
3.459	$\int \frac{(e+fx)^3\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2711
3.460	$\int \frac{(e+fx)^2\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2722
3.461	$\int \frac{(e+fx)\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2732
3.462	$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2740
3.463	$\int \frac{\cosh(c+dx)\coth^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$	. . . . .	2744
3.464	$\int \frac{(e+fx)^3\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2747
3.465	$\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2761
3.466	$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2773
3.467	$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2783
3.468	$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$	. . . . .	2788
3.469	$\int \frac{(e+fx)^2\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2791
3.470	$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2807

3.471	$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2818
3.472	$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$	. . . . .	2825
3.473	$\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2828
3.474	$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2840
3.475	$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$	. . . . .	2847
3.476	$\int \frac{(e+fx)^3 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2851
3.477	$\int \frac{(e+fx)^2 \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2864
3.478	$\int \frac{(e+fx) \coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2875
3.479	$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2883
3.480	$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$	. . . . .	2887
3.481	$\int \frac{(e+fx)^3 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2890
3.482	$\int \frac{(e+fx)^2 \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2907
3.483	$\int \frac{(e+fx) \coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2920
3.484	$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2930
3.485	$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$	. . . . .	2936
3.486	$\int \frac{(e+fx)^3 \coth^3(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2939
3.487	$\int \frac{(e+fx)^2 \coth^3(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2949
3.488	$\int \frac{(e+fx) \coth^3(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2962
3.489	$\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2972
3.490	$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$	. . . . .	2976
3.491	$\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2979
3.492	$\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	2990
3.493	$\int \frac{(e+fx) \operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	3008
3.494	$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	3020
3.495	$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$	. . . . .	3026
3.496	$\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	3029
3.497	$\int \frac{(e+fx) \operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx$	. . . . .	3042

3.498	$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.3057
3.499	$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	.3065
3.500	$\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.3069
3.501	$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b \sinh(c+dx)} dx$	. . . . .	.3082
3.502	$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$	. . . . .	.3089

#### 4 Listing of Grading functions

3093



# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 502 ]. This is test number [ 160 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. ( 502 )	% 0. ( 0 )
Mathematica	% 92.63 ( 465 )	% 7.37 ( 37 )
Maple	% 67.93 ( 341 )	% 32.07 ( 161 )
Maxima	% 51.39 ( 258 )	% 48.61 ( 244 )
Fricas	% 90.44 ( 454 )	% 9.56 ( 48 )
Sympy	% 18.73 ( 94 )	% 81.27 ( 408 )
Giac	% 39.04 ( 196 )	% 60.96 ( 306 )

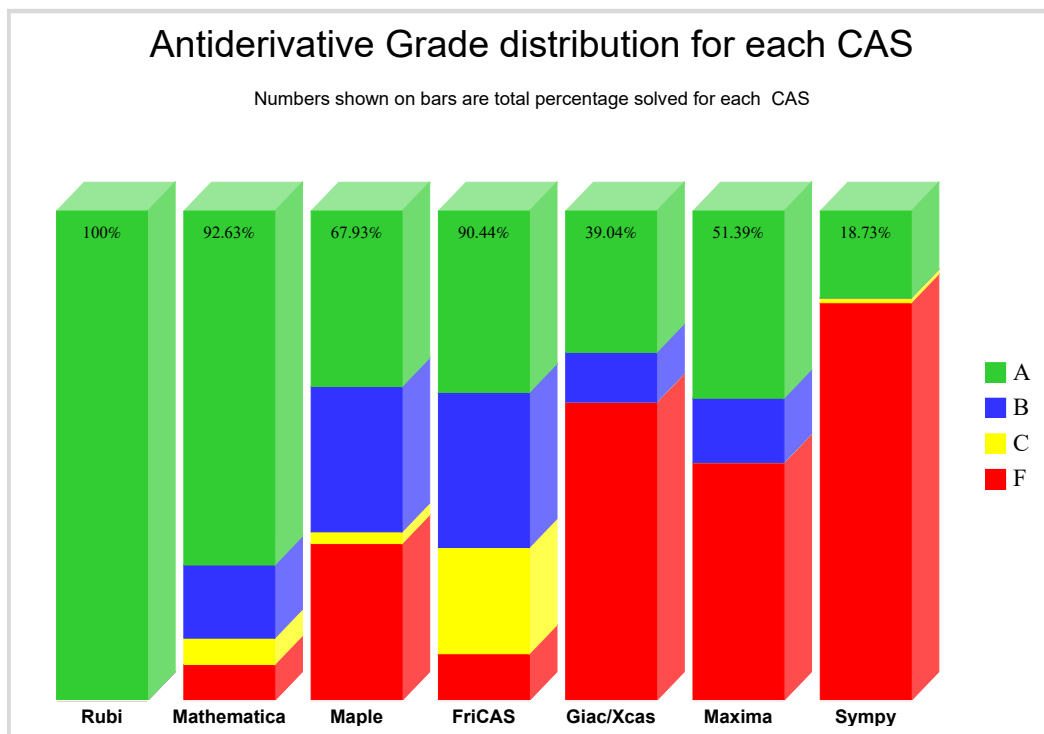
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

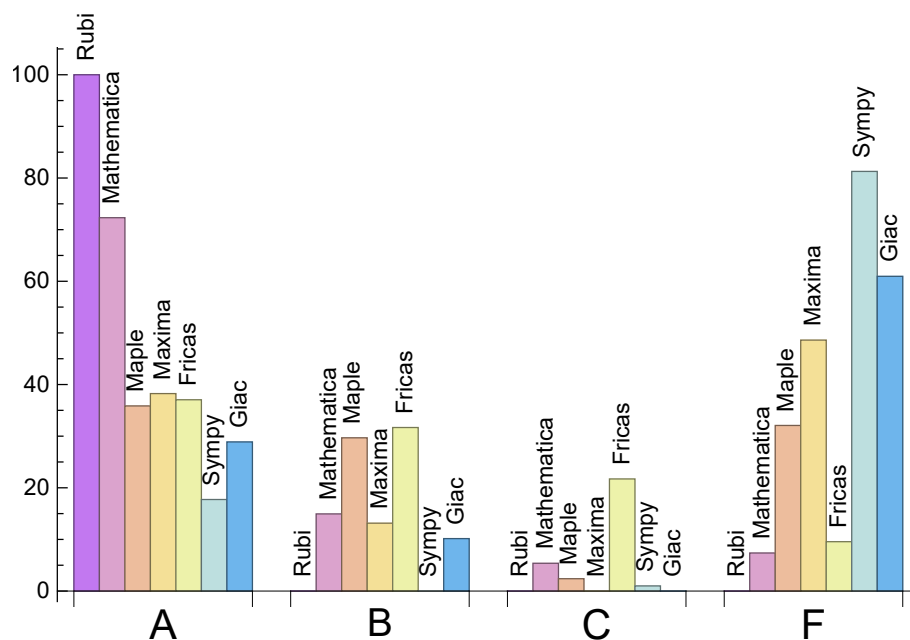
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	72.31	14.94	5.38	7.37
Maple	35.86	29.68	2.39	32.07
Maxima	38.25	13.15	0.	48.61
Fricas	37.05	31.67	21.71	9.56
Sympy	17.73	0.	1.	81.27
Giac	28.88	10.16	0.	60.96

The following is a Bar chart illustration of the data in the above table.





The figure below compares the CAS systems for each grade level.



### 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.46	267.92	0.79	139.	1.
Mathematica	12.22	787.06	1.65	173.	0.93
Maple	0.26	387.95	1.7	150.	1.61
Maxima	0.89	183.91	1.31	105.	1.21
Fricas	2.28	3413.25	8.39	881.5	5.6
Sympy	6.65	156.88	1.46	32.5	0.9
Giac	0.86	187.32	1.69	128.	1.82

## 1.4 list of integrals that has no closed form antiderivative

{26, 27, 31, 32, 36, 37, 65, 66, 67, 72, 76, 77, 111, 112, 116, 117, 139, 140, 144, 145, 149, 150, 151, 155, 156, 172, 173, 176, 177, 179, 180, 181, 185, 186, 191, 192, 197, 198, 203, 204, 209, 210, 215, 216, 221, 222, 227, 232, 237, 242, 247, 252, 257, 258, 275, 276, 281, 282, 287, 288, 293, 298, 303, 308, 313, 317, 318, 319, 320, 337, 342, 347, 352, 357, 361, 366, 371, 376, 381, 386, 390, 395, 400, 405, 410, 415, 419, 424, 429, 434, 439, 444, 448, 453, 458, 463, 468, 472, 475, 480, 485, 490, 495, 499, 502}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {23, 29, 45, 205, 217, 238, 248, 271, 277, 283, 284, 300, 306, 310, 314, 328, 329, 332, 338, 339, 340, 343, 344, 350, 354, 358, 362, 363, 372, 373, 379, 383, 387, 391, 392, 396, 397, 398, 401, 402, 407, 408, 412, 416, 425, 430, 435, 436, 440, 441, 445, 449, 450, 459, 460, 464, 465, 466, 469, 470, 476, 477, 481, 486, 487, 491, 492, 493, 496, 497, 500}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail with error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and X-CAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via `sagemath`) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

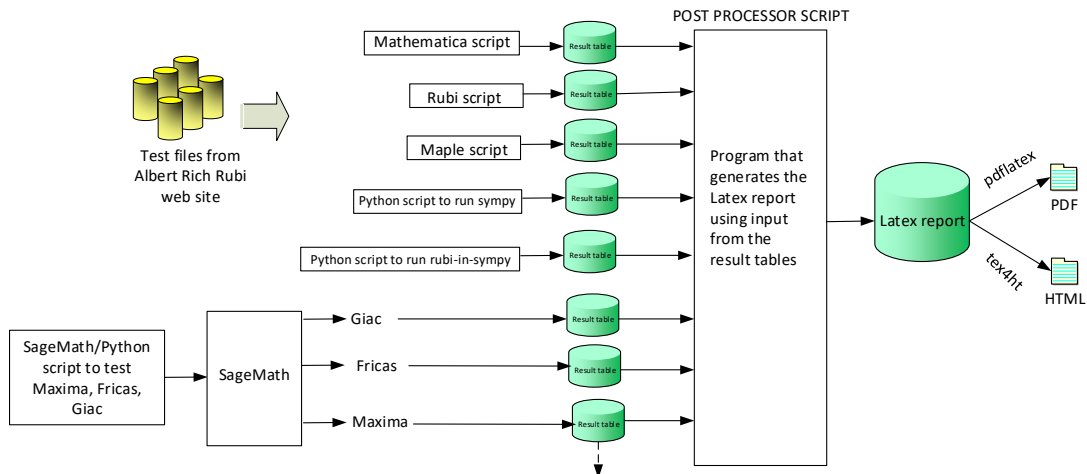
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

### High level overview of the CAS independent integration test build system

# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502 }

B grade: { }

C grade: { }

F grade: { }

## 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 52, 53, 54, 55, 56, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 194, 195, 196, 199, 201, 202, 205, 206, 208, 209, 210, 214, 215, 220, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 251, 253, 254, 255, 256, 257, 258, 259, 260, 261, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 274, 275, 278, 279, 280, 281, 286, 287, 289, 290, 291, 292, 293, 294, 295, 296, 298, 301, 302, 303, 306, 307, 308, 309, 310, 311, 312, 313, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 330, 335, 336, 337, 341, 345, 346, 350, 351, 352, 353, 354, 355, 356, 357, 359, 360, 361, 364, 365, 367, 368, 369, 370, 374, 375, 379, 382, 383, 384, 385, 388, 393, 394, 399, 403, 404, 406, 408, 411, 412, 413, 414, 417, 422, 423, 424, 425, 426, 427, 428, 429, 432, 433, 439, 440, 441, 442, 443, 446, 447, 448, 451, 452, 454, 455, 456, 457, 461, 462, 466, 467, 468, 471, 472, 473, 478, 479, 484, 488, 489, 493, 494, 495, 498 }

B grade: { 34, 49, 51, 57, 59, 110, 130, 135, 189, 193, 200, 207, 211, 212, 213, 217, 218, 219, 248, 249, 262, 273, 277, 283, 284, 285, 299, 300, 304, 305, 314, 328, 329, 331, 332, 333, 334, 343, 344, 348, 349, 358, 362, 363, 372, 373, 377, 378, 387, 391, 392, 401, 402, 407, 416, 420, 421, 430, 431, 435, 436, 437, 445, 450, 460, 464, 465, 469, 477, 482, 487, 491, 492, 496, 500 }

C grade: { 25, 29, 35, 71, 250, 297, 338, 339, 340, 380, 389, 396, 397, 398, 409, 418, 438, 449, 459, 470, 474, 476, 481, 483, 486, 497, 501 }

F grade: { 197, 198, 203, 204, 216, 221, 222, 237, 252, 276, 282, 288, 342, 347, 366, 371, 376, 381, 386, 390, 395, 400, 405, 410, 415, 419, 434, 444, 453, 458, 463, 475, 480, 485, 490, 499, 502 }

## 2.1.3 Maple

A grade: { 4, 5, 6, 12, 13, 19, 20, 21, 25, 26, 27, 30, 31, 32, 36, 37, 65, 66, 67, 72, 76, 77, 99, 100, 104, 105, 106, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 139, 140, 144, 145, 149, 150, 151, 155, 156, 160, 161, 165, 166, 167, 172, 173, 176, 177, 179, 180, 181, 185, 186, 189, 190, 191, 192, 195, 197, 198, 200, 201, 203, 204, 207, 208, 209, 210, 214, 215, 216, 220, 221, 222, 226, 227, 231, 232, 235, 237, 241, 242, 246, 247, 251, 252, 256, 257, 258, 261, 263, 264, 266, 267, 268, 269, 270, 273, 275, 276, 278, 279, 280, 281, 282, 287, 288, 292, 293, 298, 303, 307, 308, 312, 313, 317, 318, 319, 320, 336, 337, 342, 347, 351, 352, 356, 357, 361, 365, 366, 371, 376, 381, 385, 386, 390, 394, 395, 400, 405, 410, 414, 415,



419, 423, 424, 429, 434, 438, 439, 443, 444, 448, 452, 453, 458, 463, 467, 468, 471, 472, 475, 479, 480, 484, 485, 490, 494, 495, 498, 499, 502 }

B grade: { 1, 2, 3, 7, 8, 9, 10, 11, 14, 15, 16, 17, 18, 22, 23, 24, 28, 29, 33, 34, 35, 96, 97, 98, 101, 102, 103, 107, 108, 109, 113, 157, 158, 159, 162, 163, 164, 168, 171, 175, 178, 187, 188, 193, 194, 196, 199, 202, 205, 206, 211, 212, 213, 217, 218, 219, 225, 230, 236, 240, 245, 250, 253, 254, 255, 259, 260, 262, 265, 271, 272, 274, 277, 283, 284, 285, 286, 291, 296, 297, 301, 302, 306, 311, 315, 316, 321, 322, 324, 325, 327, 328, 330, 331, 335, 340, 341, 345, 346, 350, 355, 359, 360, 364, 369, 370, 374, 375, 379, 380, 384, 388, 389, 393, 398, 399, 403, 404, 408, 409, 413, 417, 418, 422, 427, 428, 432, 433, 437, 442, 446, 447, 451, 456, 457, 461, 462, 466, 470, 473, 474, 478, 483, 488, 489, 493, 497, 500, 501 }

C grade: { 60, 61, 62, 63, 64, 78, 79, 80, 81, 82, 83, 84 }

F grade: { 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 68, 69, 70, 71, 73, 74, 75, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 169, 170, 174, 182, 183, 184, 223, 224, 228, 229, 233, 234, 238, 239, 243, 244, 248, 249, 289, 290, 294, 295, 299, 300, 304, 305, 309, 310, 314, 323, 326, 329, 332, 333, 334, 338, 339, 343, 344, 348, 349, 353, 354, 358, 362, 363, 367, 368, 372, 373, 377, 378, 382, 383, 387, 391, 392, 396, 397, 401, 402, 406, 407, 411, 412, 416, 420, 421, 425, 426, 430, 431, 435, 436, 440, 441, 445, 449, 450, 454, 455, 459, 460, 464, 465, 469, 476, 477, 481, 482, 486, 487, 491, 492, 496 }

## 2.1.4 Maxima

A grade: { 5, 6, 7, 10, 11, 12, 13, 14, 15, 20, 21, 22, 26, 27, 31, 32, 36, 37, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 63, 64, 65, 66, 67, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 98, 99, 100, 101, 104, 105, 106, 107, 110, 111, 112, 116, 117, 139, 140, 144, 145, 149, 150, 151, 155, 156, 159, 160, 161, 162, 164, 165, 166, 167, 168, 172, 173, 176, 177, 179, 180, 181, 185, 186, 189, 190, 191, 192, 196, 197, 198, 202, 206, 208, 209, 210, 215, 216, 220, 221, 222, 227, 232, 237, 242, 247, 252, 254, 256, 257, 258, 263, 264, 268, 271, 272, 275, 276, 277, 279, 281, 282, 287, 288, 292, 293, 298, 303, 307, 308, 313, 316, 317, 318, 319, 320, 337, 342, 347, 351, 352, 357, 360, 361, 366, 371, 376, 380, 381, 386, 389, 390, 395, 400, 405, 409, 410, 415, 418, 419, 424, 429, 434, 438, 439, 444, 447, 448, 453, 458, 463, 467, 468, 472, 474, 475, 480, 485, 490, 494, 495, 499, 502 }

B grade: { 1, 2, 3, 4, 8, 9, 16, 17, 18, 19, 23, 24, 28, 30, 33, 34, 38, 39, 40, 41, 60, 61, 62, 96, 97, 102, 103, 108, 113, 115, 157, 158, 163, 187, 193, 195, 205, 211, 212, 214, 217, 218, 253, 259, 260, 261, 262, 274, 280, 283, 284, 286, 302, 336, 346, 365, 375, 394, 404, 423, 433, 452, 462, 479, 489, 501 }

C grade: { }

F grade: { 25, 29, 35, 68, 69, 70, 71, 74, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 109, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 169, 170, 171, 174, 175, 178, 182, 183, 184, 188, 194, 199, 200, 201, 203, 204, 207, 213, 219, 223, 224, 225, 226, 228, 229, 230, 231, 233, 234, 235, 236, 238, 239, 240, 241, 243, 244, 245, 246, 248, 249, 250, 251, 255, 265, 266, 267, 269, 270, 273, 278, 285, 289, 290, 291, 294, 295, 296, 297, 299, 300, 301, 304, 305, 306, 309, 310, 311, 312, 314, 315, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 338, 339, 340, 341, 343, 344, 345, 348, 349, 350, 353, 354, }

355, 356, 358, 359, 362, 363, 364, 367, 368, 369, 370, 372, 373, 374, 377, 378, 379, 382, 383, 384, 385, 387, 388, 391, 392, 393, 396, 397, 398, 399, 401, 402, 403, 406, 407, 408, 411, 412, 413, 414, 416, 417, 420, 421, 422, 425, 426, 427, 428, 430, 431, 432, 435, 436, 437, 440, 441, 442, 443, 445, 446, 449, 450, 451, 454, 455, 456, 457, 459, 460, 461, 464, 465, 466, 469, 470, 471, 473, 476, 477, 478, 481, 482, 483, 484, 486, 487, 488, 491, 492, 493, 496, 497, 498, 500 }

## 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 9, 10, 11, 12, 18, 19, 20, 26, 27, 31, 32, 36, 37, 41, 48, 56, 62, 65, 66, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 101, 104, 105, 106, 110, 111, 112, 115, 116, 117, 139, 140, 144, 145, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 172, 173, 176, 177, 179, 180, 181, 182, 183, 184, 185, 186, 189, 190, 191, 192, 195, 196, 197, 198, 201, 202, 203, 204, 208, 209, 210, 215, 216, 221, 222, 227, 232, 237, 242, 247, 252, 255, 256, 257, 258, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 275, 276, 279, 280, 281, 282, 287, 288, 293, 298, 303, 306, 307, 308, 313, 317, 318, 319, 320, 337, 342, 347, 350, 351, 352, 357, 361, 366, 371, 376, 379, 380, 381, 386, 390, 395, 400, 405, 410, 415, 419, 423, 424, 429, 434, 438, 439, 444, 453, 458, 463, 468, 472, 480, 485, 490, 499 }

B grade: { 6, 7, 8, 13, 14, 15, 16, 17, 21, 22, 25, 29, 30, 35, 38, 39, 40, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 63, 64, 69, 96, 97, 102, 103, 107, 109, 114, 162, 168, 171, 175, 178, 188, 194, 200, 207, 213, 214, 219, 220, 225, 226, 230, 231, 235, 236, 240, 241, 245, 246, 250, 251, 259, 260, 273, 274, 278, 285, 286, 291, 292, 296, 297, 301, 302, 311, 312, 315, 316, 321, 322, 324, 325, 327, 328, 330, 331, 335, 336, 340, 341, 345, 346, 355, 356, 359, 360, 364, 365, 369, 370, 374, 375, 384, 385, 388, 389, 393, 394, 398, 399, 403, 404, 408, 409, 413, 414, 417, 418, 422, 427, 428, 432, 433, 437, 442, 443, 446, 447, 451, 452, 456, 457, 461, 462, 466, 467, 470, 471, 474, 478, 479, 483, 484, 488, 489, 493, 494, 497, 498, 501 }

C grade: { 23, 24, 28, 33, 34, 108, 113, 169, 170, 174, 187, 193, 199, 205, 206, 211, 212, 217, 218, 223, 224, 228, 229, 233, 234, 238, 239, 243, 244, 249, 253, 254, 271, 272, 277, 283, 284, 289, 290, 294, 295, 299, 300, 304, 305, 309, 310, 314, 323, 326, 329, 332, 333, 334, 338, 339, 343, 344, 348, 349, 353, 354, 358, 362, 363, 367, 368, 372, 373, 377, 378, 382, 383, 387, 391, 392, 396, 397, 402, 406, 407, 411, 412, 416, 420, 421, 425, 426, 430, 431, 435, 436, 440, 441, 449, 450, 454, 455, 459, 460, 464, 465, 469, 476, 477, 481, 482, 487, 492 }

F grade: { 67, 68, 70, 71, 92, 93, 94, 95, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 150, 248, 401, 445, 448, 473, 475, 486, 491, 495, 496, 500, 502 }

## 2.1.6 SymPy

A grade: { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 19, 26, 27, 31, 32, 36, 37, 65, 66, 67, 72, 76, 77, 96, 97, 98, 102, 103, 104, 110, 115, 139, 140, 144, 145, 150, 157, 158, 159, 163, 164, 165, 185, 186, 190, 195, 196, 202, 226, 242, 247, 256, 259, 260, 261, 262, 265, 266, 267, 268, 275, 281, 292, 308, 313, 317, 318, 319, 320, 336, 352, 357, 361, 365, 381, 386, 390, 394, 410, 415, 419, 424, 429, 434, 453, 458, 463, 490 }

B grade: { }

C grade: { 60, 61, 62, 63, 64 }

F grade: { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 28, 29, 30, 33, 34, 35, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 68, 69, 70, 71, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 105, 106, 107, 108, 109, 111, 112, 113, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 160, 161, 162, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 187, 188, 189, 191, 192, 193, 194, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 263, 264, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309, 310, 311, 312, 314, 315, 316, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 358, 359, 360, 362, 363, 364, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 382, 383, 384, 385, 387, 388, 389, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 411, 412, 413, 414, 416, 417, 418, 420, 421, 422, 423, 425, 426, 427, 428, 430, 431, 432, 433, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 454, 455, 456, 457, 459, 460, 461, 462, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502 }

## 2.1.7 Giac

A grade: { 4, 5, 10, 11, 12, 19, 20, 26, 27, 31, 32, 36, 37, 38, 39, 40, 41, 60, 61, 62, 65, 66, 67, 72, 76, 77, 98, 99, 100, 104, 105, 110, 111, 112, 115, 116, 117, 139, 140, 144, 145, 149, 150, 151, 155, 156, 159, 160, 161, 165, 166, 167, 172, 173, 176, 177, 179, 180, 181, 185, 186, 190, 191, 192, 196, 197, 198, 202, 203, 204, 208, 209, 214, 220, 226, 227, 231, 232, 236, 237, 241, 246, 251, 256, 257, 258, 262, 263, 268, 269, 275, 276, 279, 280, 281, 292, 293, 297, 298, 302, 303, 307, 308, 312, 318, 319, 320, 336, 337, 341, 342, 346, 347, 351, 356, 360, 365, 366, 370, 371, 376, 380, 385, 389, 394, 395, 399, 400, 405, 409, 414, 418, 423, 424, 433, 434, 438, 439, 443, 452, 462, 471, 479, 490, 498 }

B grade: { 1, 2, 3, 6, 7, 8, 9, 14, 15, 16, 17, 18, 21, 22, 30, 96, 97, 101, 102, 103, 106, 107, 157, 158, 162, 163, 164, 168, 189, 195, 201, 259, 260, 261, 265, 266, 267, 274, 286, 316, 375, 404, 428, 447, 457, 467, 474, 484, 489, 494, 501 }

C grade: { }

F grade: { 13, 23, 24, 25, 28, 29, 33, 34, 35, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 63, 64, 68, 69, 70, 71, 73, 74, 75, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 108, 109, 113, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 141, 142, 143, 146, 147, 148, 152, 153, 154, 169, 170, 171, 174, 175, 178, 182, 183, 184, 187, 188, 193, 194, 199, 200, 205, 206, 207, 210, 211, 212, 213, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 228, 229, 230, 233, 234, 235, 238, 239, 240, 242, 243, 244, 245, 247, 248, 249,

250, 252, 253, 254, 255, 264, 270, 271, 272, 273, 277, 278, 282, 283, 284, 285, 287, 288, 289, 290, 291, 294, 295, 296, 299, 300, 301, 304, 305, 306, 309, 310, 311, 313, 314, 315, 317, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 338, 339, 340, 343, 344, 345, 348, 349, 350, 352, 353, 354, 355, 357, 358, 359, 361, 362, 363, 364, 367, 368, 369, 372, 373, 374, 377, 378, 379, 381, 382, 383, 384, 386, 387, 388, 390, 391, 392, 393, 396, 397, 398, 401, 402, 403, 406, 407, 408, 410, 411, 412, 413, 415, 416, 417, 419, 420, 421, 422, 425, 426, 427, 429, 430, 431, 432, 435, 436, 437, 440, 441, 442, 444, 445, 446, 448, 449, 450, 451, 453, 454, 455, 456, 458, 459, 460, 461, 463, 464, 465, 466, 468, 469, 470, 472, 473, 475, 476, 477, 478, 480, 481, 482, 483, 485, 486, 487, 488, 491, 492, 493, 495, 496, 497, 499, 500, 502 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	76	547	440	350	311	437
normalized size	1	1.	0.84	6.01	4.84	3.85	3.42	4.8
time (sec)	N/A	0.12	0.347	0.033	1.123	2.599	3.237	1.168

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	61	308	300	231	202	275
normalized size	1	1.	0.87	4.4	4.29	3.3	2.89	3.93
time (sec)	N/A	0.079	0.22	0.007	1.059	2.537	1.502	1.2

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	147	181	139	112	151
normalized size	1	1.	0.9	3.	3.69	2.84	2.29	3.08
time (sec)	N/A	0.05	0.157	0.006	1.118	2.561	0.722	1.167

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	53	92	72	46	62
normalized size	1	1.	0.96	1.89	3.29	2.57	1.64	2.21
time (sec)	N/A	0.02	0.063	0.004	1.088	2.529	0.265	1.148

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	49	82	77	193	0	77
normalized size	1	1.	0.96	1.61	1.51	3.78	0.	1.51
time (sec)	N/A	0.109	0.085	0.059	1.289	2.58	0.	1.157

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	65	133	108	315	0	197
normalized size	1	1.	0.92	1.87	1.52	4.44	0.	2.77
time (sec)	N/A	0.127	0.238	0.033	1.386	2.678	0.	1.275

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	88	277	127	518	0	406
normalized size	1	1.	0.85	2.66	1.22	4.98	0.	3.9
time (sec)	N/A	0.166	0.558	0.039	1.384	2.61	0.	1.23

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	132	910	516	663	660	505
normalized size	1	1.	0.81	5.62	3.19	4.09	4.07	3.12
time (sec)	N/A	0.105	0.681	0.016	1.257	2.683	7.344	1.24

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	104	523	355	459	456	328
normalized size	1	1.	0.78	3.9	2.65	3.43	3.4	2.45
time (sec)	N/A	0.074	0.403	0.007	1.146	2.702	3.8	1.258

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	75	262	223	288	264	184
normalized size	1	1.	0.79	2.76	2.35	3.03	2.78	1.94
time (sec)	N/A	0.054	0.311	0.007	1.14	2.693	1.771	1.18

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	52	103	119	165	126	85
normalized size	1	1.	0.95	1.87	2.16	3.	2.29	1.55
time (sec)	N/A	0.026	0.162	0.006	1.218	2.626	0.8	1.219

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	66	97	97	232	0	92
normalized size	1	1.	0.85	1.24	1.24	2.97	0.	1.18
time (sec)	N/A	0.165	0.122	0.074	1.384	2.64	0.	1.2

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	75	152	119	365	0	0
normalized size	1	1.	0.93	1.88	1.47	4.51	0.	0.
time (sec)	N/A	0.155	0.444	0.072	1.389	2.721	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	102	299	134	597	0	446
normalized size	1	1.	0.91	2.67	1.2	5.33	0.	3.98
time (sec)	N/A	0.195	0.916	0.076	1.477	2.714	0.	1.158

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	123	555	149	857	0	725
normalized size	1	1.	0.76	3.43	0.92	5.29	0.	4.48
time (sec)	N/A	0.187	0.888	0.118	1.515	2.739	0.	1.156

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	225	225	150	1217	863	1141	772	883
normalized size	1	1.	0.67	5.41	3.84	5.07	3.43	3.92
time (sec)	N/A	0.36	1.019	0.156	1.354	2.822	12.508	1.208

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	127	676	587	759	495	559
normalized size	1	1.	0.73	3.86	3.35	4.34	2.83	3.19
time (sec)	N/A	0.226	0.992	0.007	1.377	2.578	6.796	1.23

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	86	320	363	466	284	311
normalized size	1	1.	0.7	2.6	2.95	3.79	2.31	2.53
time (sec)	N/A	0.131	0.404	0.007	1.357	2.625	3.839	1.183

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	59	115	190	255	126	132
normalized size	1	1.	0.79	1.53	2.53	3.4	1.68	1.76
time (sec)	N/A	0.058	0.183	0.007	1.227	2.625	1.82	1.186

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	102	166	158	400	0	153
normalized size	1	1.	0.84	1.37	1.31	3.31	0.	1.26
time (sec)	N/A	0.282	0.239	0.083	1.393	2.531	0.	1.201

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	160	271	196	662	0	400
normalized size	1	1.	1.1	1.87	1.35	4.57	0.	2.76
time (sec)	N/A	0.262	1.341	0.091	1.387	2.696	0.	1.246

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	220	562	196	1123	0	811
normalized size	1	1.	1.2	3.05	1.07	6.1	0.	4.41
time (sec)	N/A	0.425	0.843	0.089	1.6	2.665	0.	1.186



Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	191	541	450	986	0	0
normalized size	1	1.	1.28	3.63	3.02	6.62	0.	0.
time (sec)	N/A	0.136	2.94	0.072	1.507	2.788	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	118	306	263	635	0	0
normalized size	1	1.	1.19	3.09	2.66	6.41	0.	0.
time (sec)	N/A	0.09	2.299	0.03	1.549	2.644	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	174	60	0	340	0	0
normalized size	1	1.	3.48	1.2	0.	6.8	0.	0.
time (sec)	N/A	0.046	0.079	0.004	0.	2.635	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	11.29	0.051	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.022	11.125	0.034	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	185	473	432	2689	0	0
normalized size	1	1.	1.8	4.59	4.19	26.11	0.	0.
time (sec)	N/A	0.227	2.298	0.059	2.	2.853	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	198	240	0	1521	0	0
normalized size	1	1.	2.68	3.24	0.	20.55	0.	0.
time (sec)	N/A	0.148	5.399	0.03	0.	2.79	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	52	56	123	431	0	108
normalized size	1	1.	1.79	1.93	4.24	14.86	0.	3.72
time (sec)	N/A	0.03	0.099	0.02	1.143	2.577	0.	1.317

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	17.713	0.061	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	18.673	0.072	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	440	876	817	8979	0	0
normalized size	1	1.	1.72	3.42	3.19	35.07	0.	0.
time (sec)	N/A	0.274	10.083	0.06	1.974	3.475	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	420	444	531	5252	0	0
normalized size	1	1.	2.73	2.88	3.45	34.1	0.	0.
time (sec)	N/A	0.166	10.747	0.044	1.843	3.169	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	313	197	0	2677	0	0
normalized size	1	1.	3.4	2.14	0.	29.1	0.	0.
time (sec)	N/A	0.081	2.674	0.033	0.	2.838	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	74.269	0.484	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	79.6	0.81	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	108	0	416	1179	0	313
normalized size	1	1.	0.63	0.	2.43	6.89	0.	1.83
time (sec)	N/A	0.37	0.059	0.033	1.226	2.752	0.	1.372

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	106	0	362	894	0	273
normalized size	1	1.	0.73	0.	2.48	6.12	0.	1.87
time (sec)	N/A	0.248	0.095	0.026	1.193	2.643	0.	1.335

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	104	0	311	718	0	227
normalized size	1	1.	0.85	0.	2.53	5.84	0.	1.85
time (sec)	N/A	0.187	0.078	0.023	1.156	2.734	0.	1.275

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	244	273	0	124
normalized size	1	1.	1.	0.	2.35	2.62	0.	1.19
time (sec)	N/A	0.135	0.038	0.03	1.151	2.629	0.	1.217

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	120	0	139	814	0	0
normalized size	1	1.	1.02	0.	1.18	6.9	0.	0.
time (sec)	N/A	0.2	0.149	0.023	1.111	2.768	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	161	0	154	1226	0	0
normalized size	1	1.	1.08	0.	1.03	8.23	0.	0.
time (sec)	N/A	0.251	0.754	0.025	1.336	2.787	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	168	0	154	1820	0	0
normalized size	1	1.	0.97	0.	0.89	10.46	0.	0.
time (sec)	N/A	0.308	0.572	0.026	1.345	2.955	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	239	239	190	0	379	2319	0	0
normalized size	1	1.	0.79	0.	1.59	9.7	0.	0.
time (sec)	N/A	0.449	6.591	0.062	1.749	3.198	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	163	0	323	1789	0	0
normalized size	1	1.	0.77	0.	1.53	8.48	0.	0.
time (sec)	N/A	0.321	2.297	0.058	1.723	3.315	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	129	0	255	1438	0	0
normalized size	1	1.	0.78	0.	1.54	8.66	0.	0.
time (sec)	N/A	0.296	0.579	0.055	1.629	2.912	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	142	0	144	369	0	0
normalized size	1	1.	1.02	0.	1.04	2.65	0.	0.
time (sec)	N/A	0.241	0.13	0.062	1.681	2.824	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	570	0	157	1442	0	0
normalized size	1	1.	4.01	0.	1.11	10.15	0.	0.
time (sec)	N/A	0.253	4.858	0.066	1.3	2.856	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	156	0	159	2053	0	0
normalized size	1	1.	0.9	0.	0.91	11.8	0.	0.
time (sec)	N/A	0.321	1.215	0.066	1.309	3.016	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	825	0	159	2974	0	0
normalized size	1	1.	3.75	0.	0.72	13.52	0.	0.
time (sec)	N/A	0.322	9.187	0.067	1.316	3.084	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	222	0	159	3951	0	0
normalized size	1	1.	0.88	0.	0.63	15.74	0.	0.
time (sec)	N/A	0.395	0.563	0.069	1.294	3.576	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	243	0	693	4817	0	0
normalized size	1	1.	0.64	0.	1.82	12.64	0.	0.
time (sec)	N/A	1.066	9.446	0.082	1.826	3.426	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	243	0	581	3622	0	0
normalized size	1	1.	0.75	0.	1.79	11.14	0.	0.
time (sec)	N/A	0.799	3.891	0.078	1.793	3.301	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	209	0	450	2966	0	0
normalized size	1	1.	0.76	0.	1.64	10.79	0.	0.
time (sec)	N/A	0.522	0.291	0.078	1.886	2.998	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	191	0	240	595	0	0
normalized size	1	1.	0.84	0.	1.05	2.61	0.	0.
time (sec)	N/A	0.401	0.194	0.095	1.701	2.725	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	2058	0	266	3312	0	0
normalized size	1	1.	8.37	0.	1.08	13.46	0.	0.
time (sec)	N/A	0.454	10.265	0.088	1.473	3.014	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	253	0	265	4826	0	0
normalized size	1	1.	0.91	0.	0.96	17.42	0.	0.
time (sec)	N/A	0.673	3.061	0.084	1.428	3.153	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	3211	0	266	7120	0	0
normalized size	1	1.	9.7	0.	0.8	21.51	0.	0.
time (sec)	N/A	0.775	18.188	0.089	1.425	3.811	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	50	132	236	467	133	194
normalized size	1	1.	0.45	1.19	2.13	4.21	1.2	1.75
time (sec)	N/A	0.159	0.013	0.052	1.151	2.752	152.835	1.36

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	49	120	201	356	99	136
normalized size	1	1.	0.53	1.3	2.18	3.87	1.08	1.48
time (sec)	N/A	0.11	0.016	0.024	1.225	2.735	3.05	1.404

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	47	71	157	138	70	82
normalized size	1	1.	0.61	0.92	2.04	1.79	0.91	1.06
time (sec)	N/A	0.076	0.009	0.022	1.218	2.636	1.477	1.314



Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	49	120	100	355	94	0
normalized size	1	1.	0.56	1.38	1.15	4.08	1.08	0.
time (sec)	N/A	0.112	0.02	0.022	1.136	2.699	7.424	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	84	132	77	454	129	0
normalized size	1	1.	0.74	1.16	0.68	3.98	1.13	0.
time (sec)	N/A	0.149	0.083	0.024	1.187	2.648	175.807	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	23.482	0.042	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.03	21.786	0.043	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	61	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	6.015	0.033	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	17	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.113	0.075	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	0	0	375	0	0
normalized size	1	1.	0.92	0.	0.	15.62	0.	0.
time (sec)	N/A	0.06	0.071	0.069	0.	2.621	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	33	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.117	0.092	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	68	0	0	0	0	0
normalized size	1	1.	1.17	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	1.242	0.073	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	20	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	2.994	0.063	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	206	0	217	799	0	0
normalized size	1	1.	0.87	0.	0.92	3.37	0.	0.
time (sec)	N/A	0.318	0.198	0.089	1.546	2.921	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	131	0	0	597	0	0
normalized size	1	1.	0.91	0.	0.	4.15	0.	0.
time (sec)	N/A	0.197	0.168	0.074	0.	2.692	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	101	0	107	377	0	0
normalized size	1	1.	0.92	0.	0.97	3.43	0.	0.
time (sec)	N/A	0.093	0.055	0.043	1.282	2.779	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	6.427	0.029	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	3.779	0.033	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	74	258	0	0
normalized size	1	1.	0.92	1.24	1.25	4.37	0.	0.
time (sec)	N/A	0.078	0.026	0.042	1.231	2.79	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	53	73	74	258	0	0
normalized size	1	1.	0.9	1.24	1.25	4.37	0.	0.
time (sec)	N/A	0.073	0.019	0.048	1.34	2.733	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	74	258	0	0
normalized size	1	1.	0.92	1.24	1.25	4.37	0.	0.
time (sec)	N/A	0.074	0.022	0.04	1.299	2.744	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	53	73	74	225	0	0
normalized size	1	1.	0.9	1.24	1.25	3.81	0.	0.
time (sec)	N/A	0.07	0.017	0.034	1.319	2.705	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	67	58	236	0	0
normalized size	1	1.	1.	1.37	1.18	4.82	0.	0.
time (sec)	N/A	0.069	0.022	0.041	1.217	2.693	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	51	67	74	258	0	0
normalized size	1	1.	0.93	1.22	1.35	4.69	0.	0.
time (sec)	N/A	0.069	0.018	0.046	1.285	2.688	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	71	74	258	0	0
normalized size	1	1.	0.92	1.2	1.25	4.37	0.	0.
time (sec)	N/A	0.071	0.022	0.03	1.325	2.698	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	0	427	0	0
normalized size	1	1.	0.92	0.	0.	4.97	0.	0.
time (sec)	N/A	0.159	0.127	0.046	0.	2.748	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	78	0	0	427	0	0
normalized size	1	1.	0.92	0.	0.	5.02	0.	0.
time (sec)	N/A	0.136	0.123	0.044	0.	2.759	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	0	427	0	0
normalized size	1	1.	0.92	0.	0.	4.97	0.	0.
time (sec)	N/A	0.14	0.13	0.076	0.	2.773	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	76	0	0	375	0	0
normalized size	1	1.	0.89	0.	0.	4.41	0.	0.
time (sec)	N/A	0.127	0.105	0.053	0.	2.704	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	63	0	0	365	0	0
normalized size	1	1.	0.88	0.	0.	5.07	0.	0.
time (sec)	N/A	0.127	0.076	0.069	0.	2.754	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	72	0	0	424	0	0
normalized size	1	1.	0.87	0.	0.	5.11	0.	0.
time (sec)	N/A	0.137	0.1	0.046	0.	2.796	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	77	0	0	427	0	0
normalized size	1	1.	0.92	0.	0.	5.08	0.	0.
time (sec)	N/A	0.143	0.112	0.049	0.	2.781	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	0.094	0.077	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.118	0.124	0.073	0.	0.	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	45	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.13	0.11	0.081	0.	0.	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	63	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.207	0.144	0.071	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	128	494	317	656	573	356
normalized size	1	1.	1.31	5.04	3.23	6.69	5.85	3.63
time (sec)	N/A	0.14	0.832	0.01	1.099	2.787	2.23	1.297

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	88	249	190	402	350	205
normalized size	1	1.	1.19	3.36	2.57	5.43	4.73	2.77
time (sec)	N/A	0.097	0.472	0.01	1.065	2.648	1.532	1.263

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	48	96	89	192	177	96
normalized size	1	1.	0.96	1.92	1.78	3.84	3.54	1.92
time (sec)	N/A	0.053	0.084	0.009	1.043	2.632	0.944	1.297

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	60	96	96	162	0	96
normalized size	1	1.	0.86	1.37	1.37	2.31	0.	1.37
time (sec)	N/A	0.161	0.295	0.045	1.208	2.773	0.	1.291

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	83	153	119	290	0	225
normalized size	1	1.	0.87	1.61	1.25	3.05	0.	2.37
time (sec)	N/A	0.189	0.488	0.053	1.221	2.788	0.	1.284

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	109	303	134	471	0	451
normalized size	1	1.	0.83	2.31	1.02	3.6	0.	3.44
time (sec)	N/A	0.232	0.669	0.06	1.22	2.73	0.	1.348

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	220	1082	709	1283	1153	788
normalized size	1	1.	0.9	4.42	2.89	5.24	4.71	3.22
time (sec)	N/A	0.287	1.49	0.018	1.152	3.273	4.628	1.346



Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	189	550	440	768	706	455
normalized size	1	1.	1.09	3.16	2.53	4.41	4.06	2.61
time (sec)	N/A	0.197	0.713	0.014	1.269	3.045	3.206	1.26

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	86	215	225	389	364	215
normalized size	1	1.	0.7	1.76	1.84	3.19	2.98	1.76
time (sec)	N/A	0.105	1.209	0.015	1.186	2.956	2.064	1.24

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	117	193	203	309	0	188
normalized size	1	1.	0.79	1.3	1.36	2.07	0.	1.26
time (sec)	N/A	0.352	0.395	0.119	1.388	2.811	0.	1.334

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	214	313	247	581	0	485
normalized size	1	1.	1.26	1.84	1.45	3.42	0.	2.85
time (sec)	N/A	0.34	0.661	0.141	1.395	2.708	0.	2.11

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	198	625	277	959	0	953
normalized size	1	1.	0.84	2.65	1.17	4.06	0.	4.04
time (sec)	N/A	0.53	2.355	0.154	1.397	2.905	0.	1.401

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	206	435	320	863	0	0
normalized size	1	1.	1.56	3.3	2.42	6.54	0.	0.
time (sec)	N/A	0.302	2.894	0.112	1.683	2.727	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	150	227	0	483	0	0
normalized size	1	1.	1.49	2.25	0.	4.78	0.	0.
time (sec)	N/A	0.219	2.219	0.056	0.	2.697	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	185	66	101	151	56	97
normalized size	1	1.	2.94	1.05	1.6	2.4	0.89	1.54
time (sec)	N/A	0.074	0.451	0.056	1.068	3.194	0.69	1.217

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	20.624	0.098	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	21.087	0.164	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	443	723	856	2147	0	0
normalized size	1	1.	1.45	2.37	2.81	7.04	0.	0.
time (sec)	N/A	0.398	6.247	0.141	1.999	2.604	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	269	374	0	1162	0	0
normalized size	1	1.	1.12	1.55	0.	4.82	0.	0.
time (sec)	N/A	0.278	3.557	0.113	0.	2.504	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	241	113	346	398	173	285
normalized size	1	1.	1.53	0.72	2.19	2.52	1.09	1.8
time (sec)	N/A	0.109	1.081	0.103	1.196	2.539	8.021	1.325

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	36.176	0.823	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	37.907	1.103	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	141	174	0	0	0	0
normalized size	1	1.	0.78	0.96	0.	0.	0.	0.
time (sec)	N/A	0.212	0.236	0.087	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	125	151	0	0	0	0
normalized size	1	1.	0.92	1.11	0.	0.	0.	0.
time (sec)	N/A	0.172	0.3	0.062	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	105	128	0	0	0	0
normalized size	1	1.	0.95	1.15	0.	0.	0.	0.
time (sec)	N/A	0.141	0.224	0.06	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	87	105	0	0	0	0
normalized size	1	1.	1.32	1.59	0.	0.	0.	0.
time (sec)	N/A	0.075	0.164	0.057	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	96	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	0.159	0.305	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	133	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.175	0.254	0.062	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	170	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.198	0.34	0.059	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	377	269	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.351	1.403	0.046	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	173	0	0	0	0	0
normalized size	1	1.	0.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.251	1.125	0.046	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	138	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	0.776	0.043	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	146	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.273	0.785	0.045	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	243	0	0	0	0	0
normalized size	1	1.	0.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.29	0.974	0.046	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	638	638	2918	0	0	0	0	0
normalized size	1	1.	4.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.64	7.439	0.063	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	506	506	300	0	0	0	0	0
normalized size	1	1.	0.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.39	1.807	0.046	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	218	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.208	1.387	0.046	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	242	0	0	0	0	0
normalized size	1	1.	0.6	0.	0.	0.	0.	0.
time (sec)	N/A	0.418	1.497	0.046	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	444	444	347	0	0	0	0	0
normalized size	1	1.	0.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.437	2.434	0.046	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-2)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	536	536	4751	0	0	0	0	0
normalized size	1	1.	8.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.638	7.686	0.044	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	493	493	331	0	0	0	0	0
normalized size	1	1.	0.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.261	1.27	0.201	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	276	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.202	0.974	0.069	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	221	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.505	0.066	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	3.843	0.052	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	3.952	0.053	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	807	807	546	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.438	3.239	0.047	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	506	506	384	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.311	1.764	0.047	0.	0.	0.	0.



Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	332	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.169	0.853	0.046	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	21.625	0.046	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	24.086	0.044	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1016	1016	1200	0	0	0	0	0
normalized size	1	1.	1.18	0.	0.	0.	0.	0.
time (sec)	N/A	0.677	4.532	0.046	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	689	689	482	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.46	2.284	0.048	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	411	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.243	1.577	0.045	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	35.33	0.046	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	3.972	0.073	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	3.916	0.048	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	339	0	0	857	0	0
normalized size	1	1.	0.83	0.	0.	2.09	0.	0.
time (sec)	N/A	0.602	1.585	0.12	0.	2.832	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	229	0	0	581	0	0
normalized size	1	1.	0.85	0.	0.	2.17	0.	0.
time (sec)	N/A	0.361	0.799	0.101	0.	2.617	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	207	0	0	301	0	0
normalized size	1	1.	1.53	0.	0.	2.23	0.	0.
time (sec)	N/A	0.154	0.56	0.056	0.	2.542	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	4.278	0.034	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	15.745	0.058	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	123	482	316	365	264	351
normalized size	1	1.	1.38	5.42	3.55	4.1	2.97	3.94
time (sec)	N/A	0.143	0.451	0.01	1.312	2.424	1.933	1.267

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	83	240	188	231	151	200
normalized size	1	1.	1.24	3.58	2.81	3.45	2.25	2.99
time (sec)	N/A	0.096	0.309	0.007	1.186	2.456	0.88	1.328

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	43	91	88	128	68	89
normalized size	1	1.	0.96	2.02	1.96	2.84	1.51	1.98
time (sec)	N/A	0.048	0.111	0.007	1.302	2.46	0.365	1.281

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	57	94	96	230	0	95
normalized size	1	1.	0.89	1.47	1.5	3.59	0.	1.48
time (sec)	N/A	0.124	0.144	0.023	1.425	2.494	0.	1.249

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	71	149	119	351	0	225
normalized size	1	1.	0.82	1.71	1.37	4.03	0.	2.59
time (sec)	N/A	0.169	0.366	0.022	1.455	2.471	0.	1.307

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	95	296	134	572	0	441
normalized size	1	1.	0.77	2.41	1.09	4.65	0.	3.59
time (sec)	N/A	0.205	0.639	0.024	1.466	2.458	0.	1.247

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	235	1061	702	880	779	813
normalized size	1	1.	0.94	4.24	2.81	3.52	3.12	3.25
time (sec)	N/A	0.29	1.383	0.014	1.334	2.495	5.313	1.263

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	249	535	435	541	456	470
normalized size	1	1.	1.37	2.94	2.39	2.97	2.51	2.58
time (sec)	N/A	0.202	0.753	0.016	1.271	2.479	2.357	1.296

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	98	208	221	294	219	220
normalized size	1	1.	0.84	1.79	1.91	2.53	1.89	1.9
time (sec)	N/A	0.106	0.745	0.014	1.255	2.3	0.935	1.237

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	134	201	200	483	0	200
normalized size	1	1.	0.86	1.29	1.28	3.1	0.	1.28
time (sec)	N/A	0.325	0.298	0.108	1.593	2.507	0.	1.294

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	232	319	244	778	0	487
normalized size	1	1.	1.27	1.74	1.33	4.25	0.	2.66
time (sec)	N/A	0.35	0.642	0.128	1.502	2.564	0.	2.087

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	395	626	274	1237	0	948
normalized size	1	1.	1.63	2.59	1.13	5.11	0.	3.92
time (sec)	N/A	0.447	0.96	0.135	1.587	2.47	0.	1.31

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	404	404	318	0	0	2412	0	0
normalized size	1	1.	0.79	0.	0.	5.97	0.	0.
time (sec)	N/A	0.826	0.261	0.168	0.	2.721	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	233	0	0	1733	0	0
normalized size	1	1.	0.79	0.	0.	5.85	0.	0.
time (sec)	N/A	0.669	0.156	0.12	0.	2.52	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	142	393	0	1127	0	0
normalized size	1	1.	0.76	2.1	0.	6.03	0.	0.
time (sec)	N/A	0.369	0.037	0.054	0.	2.545	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.944	0.036	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	0.939	0.036	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	549	549	428	0	0	8910	0	0
normalized size	1	1.	0.78	0.	0.	16.23	0.	0.
time (sec)	N/A	1.037	1.935	0.29	0.	3.528	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	194	519	0	4073	0	0
normalized size	1	1.	0.76	2.04	0.	16.04	0.	0.
time (sec)	N/A	0.442	1.075	0.103	0.	3.115	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	53.355	0.283	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	55.645	0.47	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	544	544	836	1232	0	13728	0	0
normalized size	1	1.	1.54	2.26	0.	25.24	0.	0.
time (sec)	N/A	2.081	9.854	0.192	0.	3.711	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	104.981	0.61	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	98.068	0.79	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	4.448	0.042	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	543	448	0	0	1885	0	0
normalized size	1	1.	0.83	0.	0.	3.47	0.	0.
time (sec)	N/A	0.807	1.747	0.108	0.	2.967	0.	0.



Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	254	0	0	1191	0	0
normalized size	1	1.	0.9	0.	0.	4.24	0.	0.
time (sec)	N/A	0.375	0.764	0.089	0.	2.493	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	118	0	0	585	0	0
normalized size	1	1.	0.9	0.	0.	4.47	0.	0.
time (sec)	N/A	0.148	0.192	0.046	0.	2.731	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	1.194	0.03	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	5.302	0.068	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	232	501	428	1096	0	0
normalized size	1	1.	1.42	3.07	2.63	6.72	0.	0.
time (sec)	N/A	0.356	3.276	0.103	1.731	2.483	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	188	269	0	649	0	0
normalized size	1	1.	1.45	2.07	0.	4.99	0.	0.
time (sec)	N/A	0.272	2.455	0.069	0.	2.464	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	239	86	146	235	0	180
normalized size	1	1.	2.66	0.96	1.62	2.61	0.	2.
time (sec)	N/A	0.112	0.637	0.079	1.267	2.538	0.	1.215

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	61	67	49	78	24	46
normalized size	1	1.	1.74	1.91	1.4	2.23	0.69	1.31
time (sec)	N/A	0.044	0.207	0.026	1.152	2.322	0.353	1.235

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	48.143	0.091	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.051	41.715	0.102	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	857	688	906	1904	0	0
normalized size	1	1.	3.56	2.85	3.76	7.9	0.	0.
time (sec)	N/A	0.526	6.595	0.183	1.983	2.628	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	260	374	0	1123	0	0
normalized size	1	1.	1.41	2.03	0.	6.1	0.	0.
time (sec)	N/A	0.391	3.46	0.13	0.	2.723	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	238	134	325	416	449	367
normalized size	1	1.	2.	1.13	2.73	3.5	3.77	3.08
time (sec)	N/A	0.177	1.089	0.106	1.341	2.666	2.36	1.528

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	59	107	100	178	102	89
normalized size	1	1.	1.13	2.06	1.92	3.42	1.96	1.71
time (sec)	N/A	0.089	0.221	0.036	1.009	2.463	0.748	1.411

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	180.001	0.128	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	180.002	0.218	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	393	376	928	0	2449	0	0
normalized size	1	1.	0.96	2.36	0.	6.23	0.	0.
time (sec)	N/A	0.702	7.311	0.155	0.	2.782	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	1661	508	0	1419	0	0
normalized size	1	1.	5.79	1.77	0.	4.94	0.	0.
time (sec)	N/A	0.547	5.136	0.128	0.	2.678	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	325	197	0	586	0	479
normalized size	1	1.	1.86	1.13	0.	3.35	0.	2.74
time (sec)	N/A	0.263	1.802	0.109	0.	2.556	0.	1.437

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	109	196	132	243	180	127
normalized size	1	1.	1.31	2.36	1.59	2.93	2.17	1.53
time (sec)	N/A	0.08	0.164	0.041	1.029	2.6	1.391	1.47

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	180.003	0.137	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	180.002	0.157	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	313	313	363	1034	783	2367	0	0
normalized size	1	1.	1.16	3.3	2.5	7.56	0.	0.
time (sec)	N/A	0.482	5.599	0.268	1.969	2.824	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	275	573	468	1368	0	0
normalized size	1	1.	1.23	2.56	2.09	6.11	0.	0.
time (sec)	N/A	0.346	4.153	0.131	1.856	2.71	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	345	211	0	548	0	0
normalized size	1	1.	2.74	1.67	0.	4.35	0.	0.
time (sec)	N/A	0.147	1.252	0.132	0.	2.625	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	52	42	84	154	0	72
normalized size	1	1.	1.27	1.02	2.05	3.76	0.	1.76
time (sec)	N/A	0.06	0.064	0.037	1.086	2.496	0.	1.243

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	46.109	0.742	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	57.749	1.383	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	419	419	1042	1535	1268	5860	0	0
normalized size	1	1.	2.49	3.66	3.03	13.99	0.	0.
time (sec)	N/A	0.805	17.279	0.266	2.224	3.037	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	296	715	847	814	3237	0	0
normalized size	1	1.	2.42	2.86	2.75	10.94	0.	0.
time (sec)	N/A	0.583	13.087	0.151	2.081	2.763	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	454	316	0	1272	0	0
normalized size	1	1.	2.79	1.94	0.	7.8	0.	0.
time (sec)	N/A	0.228	5.436	0.151	0.	2.592	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	61	79	149	378	0	128
normalized size	1	1.	1.07	1.39	2.61	6.63	0.	2.25
time (sec)	N/A	0.086	0.205	0.04	1.072	2.566	0.	1.228

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	131.951	0.571	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	180.001	1.329	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	546	546	2479	2058	1777	10116	0	0
normalized size	1	1.	4.54	3.77	3.25	18.53	0.	0.
time (sec)	N/A	1.212	68.67	0.256	2.749	3.725	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	368	368	1378	1107	1165	5378	0	0
normalized size	1	1.	3.74	3.01	3.17	14.61	0.	0.
time (sec)	N/A	0.856	17.044	0.192	2.444	2.959	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	541	423	0	2163	0	0
normalized size	1	1.	2.53	1.98	0.	10.11	0.	0.
time (sec)	N/A	0.367	2.887	0.184	0.	2.772	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	90	119	213	640	0	142
normalized size	1	1.	1.03	1.37	2.45	7.36	0.	1.63
time (sec)	N/A	0.123	0.419	0.049	1.152	2.603	0.	1.215

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	180.023	1.332	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	180.047	2.257	0.	0.	0.	0.



Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	453	453	607	0	0	2677	0	0
normalized size	1	1.	1.34	0.	0.	5.91	0.	0.
time (sec)	N/A	0.794	2.312	0.241	0.	3.122	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	366	0	0	1913	0	0
normalized size	1	1.	1.09	0.	0.	5.68	0.	0.
time (sec)	N/A	0.709	1.691	0.179	0.	2.742	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	163	440	0	1243	0	0
normalized size	1	1.	0.74	2.	0.	5.65	0.	0.
time (sec)	N/A	0.413	0.699	0.069	0.	2.53	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	64	87	0	473	371	115
normalized size	1	1.	1.19	1.61	0.	8.76	6.87	2.13
time (sec)	N/A	0.075	0.106	0.003	0.	2.473	169.228	1.296

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	9.225	0.065	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	551	551	979	0	0	5847	0	0
normalized size	1	1.	1.78	0.	0.	10.61	0.	0.
time (sec)	N/A	1.027	2.97	0.105	0.	3.192	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	407	453	0	0	3897	0	0
normalized size	1	1.	1.11	0.	0.	9.57	0.	0.
time (sec)	N/A	0.853	3.032	0.088	0.	2.811	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	299	510	0	2272	0	0
normalized size	1	1.	1.13	1.93	0.	8.61	0.	0.
time (sec)	N/A	0.487	2.186	0.071	0.	2.661	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	74	132	0	822	0	161
normalized size	1	1.	1.04	1.86	0.	11.58	0.	2.27
time (sec)	N/A	0.128	0.275	0.03	0.	2.521	0.	1.168

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	136.201	0.088	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	712	712	1407	0	0	11113	0	0
normalized size	1	1.	1.98	0.	0.	15.61	0.	0.
time (sec)	N/A	1.235	4.584	0.099	0.	4.027	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	522	522	740	0	0	7156	0	0
normalized size	1	1.	1.42	0.	0.	13.71	0.	0.
time (sec)	N/A	1.044	4.36	0.086	0.	3.252	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	307	589	0	3992	0	0
normalized size	1	1.	0.92	1.76	0.	11.92	0.	0.
time (sec)	N/A	0.593	2.687	0.067	0.	2.91	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	107	101	262	0	1434	0	217
normalized size	1	1.	0.94	2.45	0.	13.4	0.	2.03
time (sec)	N/A	0.221	0.305	0.03	0.	2.428	0.	1.203

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	180.002	0.091	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	605	605	757	0	0	3903	0	0
normalized size	1	1.	1.25	0.	0.	6.45	0.	0.
time (sec)	N/A	0.965	3.12	0.438	0.	3.052	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	433	433	454	0	0	2674	0	0
normalized size	1	1.	1.05	0.	0.	6.18	0.	0.
time (sec)	N/A	0.814	2.12	0.334	0.	2.682	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	306	532	0	1621	0	0
normalized size	1	1.	1.17	2.04	0.	6.21	0.	0.
time (sec)	N/A	0.46	1.95	0.112	0.	2.547	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	69	65	0	591	0	144
normalized size	1	1.	1.08	1.02	0.	9.23	0.	2.25
time (sec)	N/A	0.088	0.082	0.003	0.	2.662	0.	1.587

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	5.925	0.065	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	745	745	1353	0	0	13974	0	0
normalized size	1	1.	1.82	0.	0.	18.76	0.	0.
time (sec)	N/A	1.311	18.596	0.692	0.	4.129	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	535	535	795	0	0	8581	0	0
normalized size	1	1.	1.49	0.	0.	16.04	0.	0.
time (sec)	N/A	1.08	16.147	0.743	0.	3.497	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	405	626	0	4370	0	0
normalized size	1	1.	1.32	2.05	0.	14.28	0.	0.
time (sec)	N/A	0.565	5.601	0.105	0.	2.892	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	100	105	0	1200	0	177
normalized size	1	1.	1.25	1.31	0.	15.	0.	2.21
time (sec)	N/A	0.147	0.734	0.003	0.	2.869	0.	1.63

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	117.797	0.879	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1053	1053	2800	0	0	0	0	0
normalized size	1	1.	2.66	0.	0.	0.	0.	0.
time (sec)	N/A	1.752	45.331	0.806	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	725	725	1531	0	0	22066	0	0
normalized size	1	1.	2.11	0.	0.	30.44	0.	0.
time (sec)	N/A	1.372	25.65	0.655	0.	4.142	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	420	736	861	0	10647	0	0
normalized size	1	1.	1.75	2.05	0.	25.35	0.	0.
time (sec)	N/A	0.726	7.984	0.162	0.	2.998	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	145	164	0	2844	0	248
normalized size	1	1.	1.28	1.45	0.	25.17	0.	2.19
time (sec)	N/A	0.404	1.944	0.003	0.	3.025	0.	1.231

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	180.001	1.941	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	118	635	356	752	0	0
normalized size	1	1.	0.85	4.57	2.56	5.41	0.	0.
time (sec)	N/A	0.213	0.078	0.118	1.641	2.173	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	94	393	221	482	0	0
normalized size	1	1.	0.89	3.71	2.08	4.55	0.	0.
time (sec)	N/A	0.184	0.05	0.073	1.562	2.229	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	66	188	0	252	0	0
normalized size	1	1.	0.9	2.58	0.	3.45	0.	0.
time (sec)	N/A	0.111	0.026	0.073	0.	2.188	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	23	27	57	22	45
normalized size	1	1.	1.	1.	1.17	2.48	0.96	1.96
time (sec)	N/A	0.027	0.015	0.013	1.124	2.184	4.537	1.236

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	27.507	0.088	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	32.427	0.09	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	106	448	504	605	610	1084
normalized size	1	1.	0.98	4.15	4.67	5.6	5.65	10.04
time (sec)	N/A	0.164	0.779	0.045	2.022	2.218	2.666	1.346

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	78	223	366	373	376	648
normalized size	1	1.	0.95	2.72	4.46	4.55	4.59	7.9
time (sec)	N/A	0.126	0.484	0.042	1.598	2.165	1.947	1.261

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	57	84	254	178	194	312
normalized size	1	1.	1.02	1.5	4.54	3.18	3.46	5.57
time (sec)	N/A	0.072	0.638	0.04	1.509	2.151	1.102	1.163

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	139	85	59	92	82	47
normalized size	1	1.	6.32	3.86	2.68	4.18	3.73	2.14
time (sec)	N/A	0.043	0.145	0.044	1.062	2.113	0.434	1.224



Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	62	103	103	159	0	109
normalized size	1	1.	0.82	1.36	1.36	2.09	0.	1.43
time (sec)	N/A	0.204	0.321	0.09	1.515	2.19	0.	1.189

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	85	164	124	278	0	0
normalized size	1	1.	0.83	1.59	1.2	2.7	0.	0.
time (sec)	N/A	0.221	0.528	0.095	1.657	2.166	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	134	429	0	906	1061	1357
normalized size	1	1.	0.58	1.86	0.	3.92	4.59	5.87
time (sec)	N/A	0.261	1.3	0.16	0.	2.22	4.774	1.287

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	99	241	0	520	644	764
normalized size	1	1.	0.58	1.41	0.	3.04	3.77	4.47
time (sec)	N/A	0.186	0.911	0.138	0.	2.13	3.36	1.222

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	60	113	0	242	330	332
normalized size	1	1.	0.61	1.15	0.	2.47	3.37	3.39
time (sec)	N/A	0.1	1.133	0.121	0.	2.145	2.076	1.195

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	29	81	120	136	65
normalized size	1	1.	0.82	0.85	2.38	3.53	4.	1.91
time (sec)	N/A	0.048	0.049	0.015	1.163	2.089	0.766	1.159

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	112	180	0	258	0	208
normalized size	1	1.	0.85	1.37	0.	1.97	0.	1.59
time (sec)	N/A	0.324	0.419	0.114	0.	2.122	0.	1.187

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	212	299	0	510	0	0
normalized size	1	1.	1.18	1.66	0.	2.83	0.	0.
time (sec)	N/A	0.392	0.713	0.125	0.	2.213	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	463	463	767	1152	923	3418	0	0
normalized size	1	1.	1.66	2.49	1.99	7.38	0.	0.
time (sec)	N/A	0.483	11.38	0.244	2.029	2.454	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	501	613	522	1958	0	0
normalized size	1	1.	1.87	2.29	1.95	7.31	0.	0.
time (sec)	N/A	0.264	11.362	0.132	1.995	2.388	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	707	268	0	902	0	0
normalized size	1	1.	4.39	1.66	0.	5.6	0.	0.
time (sec)	N/A	0.142	2.691	0.151	0.	2.289	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	30	91	117	267	0	147
normalized size	1	1.	0.71	2.17	2.79	6.36	0.	3.5
time (sec)	N/A	0.057	0.048	0.046	1.168	2.12	0.	1.153

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	65.483	0.714	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	180.007	1.079	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	450	450	1049	1001	984	3393	0	0
normalized size	1	1.	2.33	2.22	2.19	7.54	0.	0.
time (sec)	N/A	0.589	12.276	0.18	2.502	2.413	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	564	509	0	1760	0	0
normalized size	1	1.	1.74	1.57	0.	5.42	0.	0.
time (sec)	N/A	0.387	8.262	0.154	0.	2.246	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	194	143	339	527	0	352
normalized size	1	1.	1.23	0.91	2.15	3.34	0.	2.23
time (sec)	N/A	0.158	1.134	0.164	1.339	2.298	0.	1.219

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	75	140	144	0	81
normalized size	1	1.	1.	1.6	2.98	3.06	0.	1.72
time (sec)	N/A	0.055	0.046	0.049	1.182	2.071	0.	1.159

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	122.852	0.248	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	180.02	1.753	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	667	667	1804	2026	1800	9095	0	0
normalized size	1	1.	2.7	3.04	2.7	13.64	0.	0.
time (sec)	N/A	0.711	13.394	0.259	6.253	2.896	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	423	423	1284	1044	1081	5145	0	0
normalized size	1	1.	3.04	2.47	2.56	12.16	0.	0.
time (sec)	N/A	0.398	12.986	0.208	3.734	2.77	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	1290	445	0	2423	0	0
normalized size	1	1.	5.54	1.91	0.	10.4	0.	0.
time (sec)	N/A	0.19	6.668	0.213	0.	2.445	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	101	180	243	763	0	250
normalized size	1	1.	1.11	1.98	2.67	8.38	0.	2.75
time (sec)	N/A	0.081	0.105	0.056	1.149	2.104	0.	1.191

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	115.857	1.718	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	180.034	2.361	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	329	0	0	2128	0	0
normalized size	1	1.	0.92	0.	0.	5.98	0.	0.
time (sec)	N/A	0.483	0.149	0.259	0.	2.475	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	244	0	0	1504	0	0
normalized size	1	1.	0.92	0.	0.	5.7	0.	0.
time (sec)	N/A	0.411	0.146	0.204	0.	2.311	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	157	412	0	964	0	0
normalized size	1	1.	0.92	2.42	0.	5.67	0.	0.
time (sec)	N/A	0.234	0.035	0.077	0.	2.162	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	104	41	45
normalized size	1	1.	1.	1.06	1.33	5.78	2.28	2.5
time (sec)	N/A	0.027	0.007	0.002	1.157	2.043	1.097	1.161

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	12.865	0.197	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	527	527	933	0	0	4691	0	0
normalized size	1	1.	1.77	0.	0.	8.9	0.	0.
time (sec)	N/A	0.908	3.284	0.269	0.	3.064	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	389	447	0	0	3163	0	0
normalized size	1	1.	1.15	0.	0.	8.13	0.	0.
time (sec)	N/A	0.785	2.919	0.23	0.	2.75	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	258	901	0	1872	0	0
normalized size	1	1.	1.02	3.58	0.	7.43	0.	0.
time (sec)	N/A	0.443	1.958	0.089	0.	2.325	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	492	174	0	684	0	157
normalized size	1	1.	7.24	2.56	0.	10.06	0.	2.31
time (sec)	N/A	0.117	2.086	0.039	0.	2.166	0.	1.188

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	25.094	0.111	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	642	642	10263	0	0	9852	0	0
normalized size	1	1.	15.99	0.	0.	15.35	0.	0.
time (sec)	N/A	0.785	23.902	0.28	0.	3.065	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	477	477	3021	0	0	6356	0	0
normalized size	1	1.	6.33	0.	0.	13.32	0.	0.
time (sec)	N/A	0.642	17.629	0.234	0.	2.647	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	251	975	0	3507	0	0
normalized size	1	1.	0.84	3.27	0.	11.77	0.	0.
time (sec)	N/A	0.358	1.366	0.11	0.	2.369	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	53	291	171	848	0	128
normalized size	1	1.	0.9	4.93	2.9	14.37	0.	2.17
time (sec)	N/A	0.069	0.051	0.043	1.16	2.271	0.	1.163



Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	81.998	0.234	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	786	786	10644	0	0	4177	0	0
normalized size	1	1.	13.54	0.	0.	5.31	0.	0.
time (sec)	N/A	1.457	28.679	0.351	0.	3.035	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	558	558	1639	0	0	2745	0	0
normalized size	1	1.	2.94	0.	0.	4.92	0.	0.
time (sec)	N/A	1.034	21.549	0.267	0.	2.617	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	334	334	439	954	0	1547	0	0
normalized size	1	1.	1.31	2.86	0.	4.63	0.	0.
time (sec)	N/A	0.596	2.733	0.128	0.	2.419	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	114	100	128	247	0	171
normalized size	1	1.	1.65	1.45	1.86	3.58	0.	2.48
time (sec)	N/A	0.07	0.096	0.004	1.67	2.156	0.	1.149

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.049	18.473	0.074	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	780	780	1143	0	0	14519	0	0
normalized size	1	1.	1.47	0.	0.	18.61	0.	0.
time (sec)	N/A	1.688	13.884	0.705	0.	4.216	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	548	548	905	0	0	8370	0	0
normalized size	1	1.	1.65	0.	0.	15.27	0.	0.
time (sec)	N/A	1.295	8.36	0.539	0.	3.29	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	284	1928	0	3164	0	0
normalized size	1	1.	0.96	6.54	0.	10.73	0.	0.
time (sec)	N/A	0.717	3.162	0.169	0.	2.48	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	104	90	0	884	0	165
normalized size	1	1.	1.35	1.17	0.	11.48	0.	2.14
time (sec)	N/A	0.1	0.173	0.003	0.	2.158	0.	1.234

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	68.394	0.567	0.	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	928	928	3368	0	0	24035	0	0
normalized size	1	1.	3.63	0.	0.	25.9	0.	0.
time (sec)	N/A	1.772	31.523	0.477	0.	5.246	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	560	560	588	2051	0	11385	0	0
normalized size	1	1.	1.05	3.66	0.	20.33	0.	0.
time (sec)	N/A	0.938	7.076	0.176	0.	3.515	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	104	468	292	2215	0	400
normalized size	1	1.	0.87	3.93	2.45	18.61	0.	3.36
time (sec)	N/A	0.144	0.188	0.003	1.963	2.437	0.	1.367

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	133.342	1.058	0.	0.	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	11.348	0.072	0.	0.	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	8.58	0.115	0.	0.	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.038	5.523	0.068	0.	0.	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	78	164	0	1015	0	0
normalized size	1	1.	1.05	2.22	0.	13.72	0.	0.
time (sec)	N/A	0.073	0.503	0.214	0.	2.118	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	175	491	0	3195	0	0
normalized size	1	1.	0.75	2.1	0.	13.65	0.	0.
time (sec)	N/A	0.44	1.378	0.19	0.	2.401	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	368	0	0	5462	0	0
normalized size	1	1.	1.06	0.	0.	15.7	0.	0.
time (sec)	N/A	0.74	2.567	0.32	0.	2.787	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	78	164	0	1015	0	0
normalized size	1	1.	1.05	2.22	0.	13.72	0.	0.
time (sec)	N/A	0.07	0.396	0.	0.	2.609	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	234	175	491	0	3195	0	0
normalized size	1	1.	0.75	2.1	0.	13.65	0.	0.
time (sec)	N/A	0.44	0.551	0.001	0.	2.932	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	368	0	0	5462	0	0
normalized size	1	1.	1.06	0.	0.	15.7	0.	0.
time (sec)	N/A	0.731	0.388	0.	0.	2.899	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	112	308	0	2811	0	0
normalized size	1	1.	1.	2.75	0.	25.1	0.	0.
time (sec)	N/A	0.098	1.162	0.319	0.	2.314	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	306	306	623	805	0	11416	0	0
normalized size	1	1.	2.04	2.63	0.	37.31	0.	0.
time (sec)	N/A	0.52	16.412	0.3	0.	3.171	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	631	631	5753	0	0	24868	0	0
normalized size	1	1.	9.12	0.	0.	39.41	0.	0.
time (sec)	N/A	1.095	24.68	0.43	0.	4.171	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	112	308	0	2811	0	0
normalized size	1	1.	1.	2.75	0.	25.1	0.	0.
time (sec)	N/A	0.095	0.374	0.	0.	2.219	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	623	805	0	11416	0	0
normalized size	1	1.	2.04	2.63	0.	37.31	0.	0.
time (sec)	N/A	0.521	7.106	0.	0.	3.044	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	631	631	5753	0	0	24868	0	0
normalized size	1	1.	9.12	0.	0.	39.41	0.	0.
time (sec)	N/A	1.087	7.562	0.	0.	4.705	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	448	10378	0	0	4582	0	0
normalized size	1	1.	23.17	0.	0.	10.23	0.	0.
time (sec)	N/A	0.647	20.959	0.198	0.	2.771	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	1301	0	0	3051	0	0
normalized size	1	1.	3.94	0.	0.	9.25	0.	0.
time (sec)	N/A	0.55	14.217	0.126	0.	2.475	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	206	483	0	1760	0	0
normalized size	1	1.	0.97	2.28	0.	8.3	0.	0.
time (sec)	N/A	0.312	1.057	0.064	0.	2.279	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	33	35	112	354	65	84
normalized size	1	1.	0.97	1.03	3.29	10.41	1.91	2.47
time (sec)	N/A	0.057	0.039	0.013	1.075	2.123	1.476	1.149

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.057	73.846	0.118	0.	0.	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	696	696	2961	0	0	8699	0	0
normalized size	1	1.	4.25	0.	0.	12.5	0.	0.
time (sec)	N/A	1.13	14.694	0.191	0.	3.104	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	510	510	2170	0	0	5658	0	0
normalized size	1	1.	4.25	0.	0.	11.09	0.	0.
time (sec)	N/A	0.962	10.612	0.145	0.	2.716	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	327	327	1549	1012	0	3200	0	0
normalized size	1	1.	4.74	3.09	0.	9.79	0.	0.
time (sec)	N/A	0.55	4.619	0.082	0.	2.382	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	109	260	0	1150	0	223
normalized size	1	1.	1.15	2.74	0.	12.11	0.	2.35
time (sec)	N/A	0.181	0.428	0.035	0.	2.279	0.	1.152

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	180.001	0.109	0.	0.	0.	0.



Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	864	864	7460	0	0	17256	0	0
normalized size	1	1.	8.63	0.	0.	19.97	0.	0.
time (sec)	N/A	1.117	51.077	0.24	0.	3.589	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	636	636	3509	0	0	10886	0	0
normalized size	1	1.	5.52	0.	0.	17.12	0.	0.
time (sec)	N/A	0.875	16.722	0.191	0.	3.049	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	400	551	1102	0	5828	0	0
normalized size	1	1.	1.38	2.76	0.	14.57	0.	0.
time (sec)	N/A	0.485	3.435	0.099	0.	2.569	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	75	428	247	1613	0	215
normalized size	1	1.	0.88	5.04	2.91	18.98	0.	2.53
time (sec)	N/A	0.122	0.164	0.039	1.067	2.207	0.	1.152

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.084	180.001	0.155	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1021	1021	2333	0	0	4177	0	0
normalized size	1	1.	2.29	0.	0.	4.09	0.	0.
time (sec)	N/A	1.33	25.527	0.353	0.	3.12	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	716	716	1640	0	0	2743	0	0
normalized size	1	1.	2.29	0.	0.	3.83	0.	0.
time (sec)	N/A	1.067	19.14	0.285	0.	2.701	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	421	421	438	1287	0	1544	0	0
normalized size	1	1.	1.04	3.06	0.	3.67	0.	0.
time (sec)	N/A	0.597	2.631	0.154	0.	2.467	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	51	113	128	247	0	115
normalized size	1	1.	0.74	1.64	1.86	3.58	0.	1.67
time (sec)	N/A	0.077	0.077	0.002	1.605	2.139	0.	1.446

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	16.827	0.244	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	917	917	1143	0	0	14924	0	0
normalized size	1	1.	1.25	0.	0.	16.27	0.	0.
time (sec)	N/A	1.712	13.295	0.728	0.	4.408	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	648	648	906	0	0	8627	0	0
normalized size	1	1.	1.4	0.	0.	13.31	0.	0.
time (sec)	N/A	1.321	8.188	0.931	0.	3.445	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	335	285	1858	0	3272	0	0
normalized size	1	1.	0.85	5.55	0.	9.77	0.	0.
time (sec)	N/A	0.663	3.106	0.168	0.	2.509	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	104	100	0	884	0	158
normalized size	1	1.	1.33	1.28	0.	11.33	0.	2.03
time (sec)	N/A	0.112	0.193	0.001	0.	2.157	0.	1.277

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	60.499	0.559	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1176	1176	3390	0	0	24520	0	0
normalized size	1	1.	2.88	0.	0.	20.85	0.	0.
time (sec)	N/A	1.696	31.477	0.454	0.	5.548	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	711	711	587	2074	0	11628	0	0
normalized size	1	1.	0.83	2.92	0.	16.35	0.	0.
time (sec)	N/A	0.993	8.545	0.19	0.	3.697	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	105	474	294	2240	0	304
normalized size	1	1.	0.86	3.89	2.41	18.36	0.	2.49
time (sec)	N/A	0.199	0.208	0.003	1.588	2.469	0.	1.359

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	142.055	1.159	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	606	606	2872	0	0	8556	0	0
normalized size	1	1.	4.74	0.	0.	14.12	0.	0.
time (sec)	N/A	0.879	28.481	0.216	0.	2.986	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	449	449	1496	0	0	5513	0	0
normalized size	1	1.	3.33	0.	0.	12.28	0.	0.
time (sec)	N/A	0.717	11.766	0.302	0.	2.646	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	423	565	0	3054	0	0
normalized size	1	1.	1.52	2.03	0.	10.99	0.	0.
time (sec)	N/A	0.418	1.087	0.116	0.	2.387	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	54	161	784	87	123
normalized size	1	1.	0.89	0.98	2.93	14.25	1.58	2.24
time (sec)	N/A	0.081	0.071	0.017	1.051	2.211	3.597	1.299

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	180.002	0.26	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	897	897	1667	0	0	15351	0	0
normalized size	1	1.	1.86	0.	0.	17.11	0.	0.
time (sec)	N/A	1.473	7.795	0.217	0.	3.591	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	649	649	966	0	0	9708	0	0
normalized size	1	1.	1.49	0.	0.	14.96	0.	0.
time (sec)	N/A	1.202	4.989	0.228	0.	3.493	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	676	1128	0	5268	0	0
normalized size	1	1.	1.68	2.8	0.	13.07	0.	0.
time (sec)	N/A	0.69	3.058	0.106	0.	2.809	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	123	398	0	1882	0	309
normalized size	1	1.	0.87	2.82	0.	13.35	0.	2.19
time (sec)	N/A	0.498	0.392	0.037	0.	2.138	0.	1.169

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	180.001	0.135	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1123	1123	8734	0	0	26953	0	0
normalized size	1	1.	7.78	0.	0.	24.	0.	0.
time (sec)	N/A	1.524	42.185	0.273	0.	4.358	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	819	819	5198	0	0	16764	0	0
normalized size	1	1.	6.35	0.	0.	20.47	0.	0.
time (sec)	N/A	1.171	19.325	0.243	0.	3.447	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	499	499	853	1217	0	8818	0	0
normalized size	1	1.	1.71	2.44	0.	17.67	0.	0.
time (sec)	N/A	0.675	3.21	0.118	0.	2.758	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	98	614	316	2631	0	298
normalized size	1	1.	0.87	5.43	2.8	23.28	0.	2.64
time (sec)	N/A	0.163	0.302	0.046	1.05	2.238	0.	1.202

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	180.001	0.194	0.	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1218	1218	3387	0	0	4721	0	0
normalized size	1	1.	2.78	0.	0.	3.88	0.	0.
time (sec)	N/A	1.694	26.838	0.652	0.	3.275	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	861	861	1759	0	0	3077	0	0
normalized size	1	1.	2.04	0.	0.	3.57	0.	0.
time (sec)	N/A	1.366	19.743	0.49	0.	2.841	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	516	516	438	3882	0	1767	0	0
normalized size	1	1.	0.85	7.52	0.	3.42	0.	0.
time (sec)	N/A	0.776	2.875	0.225	0.	2.503	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	78	153	149	284	0	128
normalized size	1	1.	1.05	2.07	2.01	3.84	0.	1.73
time (sec)	N/A	0.16	0.086	0.001	1.628	2.309	0.	1.524

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.06	180.001	0.421	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1118	1118	1147	0	0	14864	0	0
normalized size	1	1.	1.03	0.	0.	13.3	0.	0.
time (sec)	N/A	1.98	13.669	0.732	0.	4.223	0.	0.



Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	772	772	910	0	0	8585	0	0
normalized size	1	1.	1.18	0.	0.	11.12	0.	0.
time (sec)	N/A	1.533	8.489	0.576	0.	3.41	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	284	1928	0	3274	0	0
normalized size	1	1.	0.74	5.01	0.	8.5	0.	0.
time (sec)	N/A	0.758	3.052	0.162	0.	2.585	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	106	103	0	883	0	163
normalized size	1	1.	1.18	1.14	0.	9.81	0.	1.81
time (sec)	N/A	0.106	0.196	0.001	0.	2.124	0.	1.378

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	180.009	0.487	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1256	1256	3390	0	0	24496	0	0
normalized size	1	1.	2.7	0.	0.	19.5	0.	0.
time (sec)	N/A	1.968	31.353	0.448	0.	5.43	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	760	760	588	2068	0	11621	0	0
normalized size	1	1.	0.77	2.72	0.	15.29	0.	0.
time (sec)	N/A	1.148	9.5	0.174	0.	3.666	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	130	475	296	2241	0	305
normalized size	1	1.	1.07	3.93	2.45	18.52	0.	2.52
time (sec)	N/A	0.229	0.327	0.003	1.771	2.417	0.	1.261

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	180.001	1.098	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	792	792	7460	0	0	14880	0	0
normalized size	1	1.	9.42	0.	0.	18.79	0.	0.
time (sec)	N/A	1.197	29.333	0.268	0.	3.389	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	578	578	3510	0	0	9353	0	0
normalized size	1	1.	6.07	0.	0.	16.18	0.	0.
time (sec)	N/A	0.928	14.951	0.208	0.	2.858	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	447	671	0	5007	0	0
normalized size	1	1.	1.28	1.93	0.	14.39	0.	0.
time (sec)	N/A	0.526	1.73	0.1	0.	2.532	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	66	73	231	1484	105	171
normalized size	1	1.	0.87	0.96	3.04	19.53	1.38	2.25
time (sec)	N/A	0.097	0.154	0.016	1.122	2.497	2.837	1.257

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	180.003	0.162	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1038	1038	7520	0	0	22876	0	0
normalized size	1	1.	7.24	0.	0.	22.04	0.	0.
time (sec)	N/A	1.827	29.101	0.238	0.	4.206	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	755	755	5115	0	0	14272	0	0
normalized size	1	1.	6.77	0.	0.	18.9	0.	0.
time (sec)	N/A	1.514	19.522	0.198	0.	3.298	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	474	474	2915	1213	0	7611	0	0
normalized size	1	1.	6.15	2.56	0.	16.06	0.	0.
time (sec)	N/A	0.865	11.889	0.106	0.	2.798	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	153	624	0	2803	0	379
normalized size	1	1.	0.83	3.39	0.	15.23	0.	2.06
time (sec)	N/A	0.793	1.989	0.049	0.	2.295	0.	1.203

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	180.001	0.123	0.	0.	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1443	1443	5157	0	0	0	0	0
normalized size	1	1.	3.57	0.	0.	0.	0.	0.
time (sec)	N/A	2.187	21.873	0.23	0.	0.	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1049	1049	3179	0	0	24563	0	0
normalized size	1	1.	3.03	0.	0.	23.42	0.	0.
time (sec)	N/A	1.623	14.96	0.211	0.	4.609	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	641	641	958	1363	0	12709	0	0
normalized size	1	1.	1.49	2.13	0.	19.83	0.	0.
time (sec)	N/A	0.944	4.34	0.115	0.	4.196	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	123	804	405	4018	0	383
normalized size	1	1.	0.87	5.7	2.87	28.5	0.	2.72
time (sec)	N/A	0.223	0.388	0.052	1.318	2.587	0.	1.198

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	180.002	0.223	0.	0.	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1519	1519	2861	0	0	10368	0	0
normalized size	1	1.	1.88	0.	0.	6.83	0.	0.
time (sec)	N/A	2.154	26.213	1.115	0.	4.307	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1067	1067	3418	0	0	6611	0	0
normalized size	1	1.	3.2	0.	0.	6.2	0.	0.
time (sec)	N/A	1.691	19.677	0.853	0.	3.659	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	631	631	481	4066	0	3537	0	0
normalized size	1	1.	0.76	6.44	0.	5.61	0.	0.
time (sec)	N/A	0.957	4.787	0.309	0.	3.115	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	91	196	198	733	0	169
normalized size	1	1.	1.02	2.2	2.22	8.24	0.	1.9
time (sec)	N/A	0.196	0.176	0.06	1.566	2.723	0.	1.414

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	180.002	0.934	0.	0.	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1294	1294	1111	0	0	16386	0	0
normalized size	1	1.	0.86	0.	0.	12.66	0.	0.
time (sec)	N/A	2.455	13.038	0.785	0.	5.391	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	904	904	937	0	0	9671	0	0
normalized size	1	1.	1.04	0.	0.	10.7	0.	0.
time (sec)	N/A	1.821	8.541	0.651	0.	4.138	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	454	317	1897	0	3753	0	0
normalized size	1	1.	0.7	4.18	0.	8.27	0.	0.
time (sec)	N/A	0.892	4.543	0.214	0.	3.014	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	96	158	0	1126	0	182
normalized size	1	1.	0.79	1.31	0.	9.31	0.	1.5
time (sec)	N/A	0.209	0.502	0.003	0.	2.539	0.	1.432

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	180.003	0.753	0.	0.	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1479	1479	3368	0	0	24035	0	0
normalized size	1	1.	2.28	0.	0.	16.25	0.	0.
time (sec)	N/A	2.453	30.882	0.447	0.	5.787	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	894	894	588	2284	0	11386	0	0
normalized size	1	1.	0.66	2.55	0.	12.74	0.	0.
time (sec)	N/A	1.415	7.864	0.202	0.	4.002	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	152	472	293	2217	0	301
normalized size	1	1.	1.27	3.93	2.44	18.48	0.	2.51
time (sec)	N/A	0.201	0.402	0.002	1.675	2.906	0.	1.449

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	180.001	0.905	0.	0.	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	451	1924	0	0	2996	0	0
normalized size	1	1.	4.27	0.	0.	6.64	0.	0.
time (sec)	N/A	0.77	19.046	0.495	0.	2.858	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	1296	0	0	2053	0	0
normalized size	1	1.	3.99	0.	0.	6.32	0.	0.
time (sec)	N/A	0.654	14.156	0.39	0.	2.617	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	236	451	0	1243	0	0
normalized size	1	1.	1.15	2.2	0.	6.06	0.	0.
time (sec)	N/A	0.38	0.946	0.134	0.	2.486	0.	0.



Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	35	101	170	0	85
normalized size	1	1.	0.82	1.03	2.97	5.	0.	2.5
time (sec)	N/A	0.047	0.021	0.001	1.083	2.392	0.	1.324

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.05	29.08	0.379	0.	0.	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	638	638	802	0	0	3588	0	0
normalized size	1	1.	1.26	0.	0.	5.62	0.	0.
time (sec)	N/A	1.275	2.476	0.734	0.	3.418	0.	0.

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	462	462	489	0	0	2483	0	0
normalized size	1	1.	1.06	0.	0.	5.37	0.	0.
time (sec)	N/A	1.04	1.871	0.594	0.	2.734	0.	0.

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	339	970	0	1544	0	0
normalized size	1	1.	1.19	3.39	0.	5.4	0.	0.
time (sec)	N/A	0.579	1.835	0.187	0.	2.725	0.	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	80	150	0	559	0	188
normalized size	1	1.	1.13	2.11	0.	7.87	0.	2.65
time (sec)	N/A	0.213	0.146	0.002	0.	2.581	0.	2.397

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	57.994	0.826	0.	0.	0.	0.

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	656	656	2574	0	0	7715	0	0
normalized size	1	1.	3.92	0.	0.	11.76	0.	0.
time (sec)	N/A	1.23	18.452	1.22	0.	3.658	0.	0.

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	486	486	1521	0	0	5030	0	0
normalized size	1	1.	3.13	0.	0.	10.35	0.	0.
time (sec)	N/A	1.025	14.301	1.063	0.	2.862	0.	0.

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	296	932	0	2799	0	0
normalized size	1	1.	0.92	2.89	0.	8.69	0.	0.
time (sec)	N/A	0.592	1.925	0.255	0.	2.998	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	48	178	176	533	0	146
normalized size	1	1.	0.84	3.12	3.09	9.35	0.	2.56
time (sec)	N/A	0.128	0.082	0.078	1.088	3.2	0.	1.548

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	180.002	1.079	0.	0.	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1049	1049	3000	0	0	5839	0	0
normalized size	1	1.	2.86	0.	0.	5.57	0.	0.
time (sec)	N/A	1.341	32.847	0.756	0.	3.698	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	734	734	3268	0	0	3794	0	0
normalized size	1	1.	4.45	0.	0.	5.17	0.	0.
time (sec)	N/A	1.097	33.66	0.58	0.	3.056	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	439	439	1540	1065	0	2091	0	0
normalized size	1	1.	3.51	2.43	0.	4.76	0.	0.
time (sec)	N/A	0.637	2.69	0.171	0.	2.522	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	92	123	186	350	0	211
normalized size	1	1.	1.02	1.37	2.07	3.89	0.	2.34
time (sec)	N/A	0.17	0.134	0.003	1.726	2.695	0.	1.272

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	27.58	0.098	0.	0.	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1164	1164	1467	0	0	21639	0	0
normalized size	1	1.	1.26	0.	0.	18.59	0.	0.
time (sec)	N/A	2.199	16.941	1.674	0.	5.446	0.	0.

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	795	795	1245	0	0	12713	0	0
normalized size	1	1.	1.57	0.	0.	15.99	0.	0.
time (sec)	N/A	1.614	11.537	2.283	0.	3.992	0.	0.

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	442	442	459	1815	0	5233	0	0
normalized size	1	1.	1.04	4.11	0.	11.84	0.	0.
time (sec)	N/A	0.806	7.246	0.279	0.	2.639	0.	0.

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	171	136	0	1455	0	209
normalized size	1	1.	1.51	1.2	0.	12.88	0.	1.85
time (sec)	N/A	0.26	0.265	0.001	0.	3.117	0.	1.219

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	180.001	1.864	0.	0.	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1185	1185	4072	0	0	0	0	0
normalized size	1	1.	3.44	0.	0.	0.	0.	0.
time (sec)	N/A	2.225	36.451	1.372	0.	0.	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	746	746	886	2580	0	17816	0	0
normalized size	1	1.	1.19	3.46	0.	23.88	0.	0.
time (sec)	N/A	1.059	10.841	0.244	0.	4.229	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	196	530	358	3066	0	485
normalized size	1	1.	1.22	3.31	2.24	19.16	0.	3.03
time (sec)	N/A	0.271	0.793	0.003	1.814	4.251	0.	1.766

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.089	174.592	3.421	0.	0.	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	601	601	5638	0	0	9829	0	0
normalized size	1	1.	9.38	0.	0.	16.35	0.	0.
time (sec)	N/A	1.003	48.702	0.909	0.	3.623	0.	0.

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	419	419	1595	0	0	6053	0	0
normalized size	1	1.	3.81	0.	0.	14.45	0.	0.
time (sec)	N/A	0.818	20.236	0.668	0.	2.939	0.	0.

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	416	528	0	3152	0	0
normalized size	1	1.	1.71	2.17	0.	12.97	0.	0.
time (sec)	N/A	0.464	1.894	0.138	0.	2.409	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	35	149	554	0	132
normalized size	1	1.	1.	0.7	2.98	11.08	0.	2.64
time (sec)	N/A	0.075	0.041	0.001	1.179	2.146	0.	1.337

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	180.001	0.916	0.	0.	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	721	721	1350	0	0	10496	0	0
normalized size	1	1.	1.87	0.	0.	14.56	0.	0.
time (sec)	N/A	1.636	8.687	0.827	0.	3.238	0.	0.

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	C	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	517	517	792	0	0	6512	0	0
normalized size	1	1.	1.53	0.	0.	12.6	0.	0.
time (sec)	N/A	1.296	8.037	0.713	0.	2.817	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	364	1017	0	3430	0	0
normalized size	1	1.	1.24	3.46	0.	11.67	0.	0.
time (sec)	N/A	0.688	3.669	0.213	0.	2.356	0.	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	98	147	0	975	0	201
normalized size	1	1.	1.27	1.91	0.	12.66	0.	2.61
time (sec)	N/A	0.266	0.441	0.003	0.	2.255	0.	1.84

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	180.	0.999	0.	0.	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	718	718	10534	0	0	13009	0	0
normalized size	1	1.	14.67	0.	0.	18.12	0.	0.
time (sec)	N/A	1.645	13.512	0.986	0.	3.424	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	518	518	1720	0	0	8101	0	0
normalized size	1	1.	3.32	0.	0.	15.64	0.	0.
time (sec)	N/A	1.287	14.832	0.81	0.	2.945	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	324	324	313	938	0	4255	0	0
normalized size	1	1.	0.97	2.9	0.	13.13	0.	0.
time (sec)	N/A	0.743	2.585	0.23	0.	2.478	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	52	172	177	751	0	153
normalized size	1	1.	0.88	2.92	3.	12.73	0.	2.59
time (sec)	N/A	0.123	0.086	0.001	1.226	2.292	0.	1.388



Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	180.002	0.944	0.	0.	0.	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1428	1428	7234	0	0	21464	0	0
normalized size	1	1.	5.07	0.	0.	15.03	0.	0.
time (sec)	N/A	2.284	14.016	1.48	0.	5.387	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	982	982	2107	0	0	12925	0	0
normalized size	1	1.	2.15	0.	0.	13.16	0.	0.
time (sec)	N/A	1.668	13.796	1.095	0.	4.04	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	591	591	535	1529	0	6273	0	0
normalized size	1	1.	0.91	2.59	0.	10.61	0.	0.
time (sec)	N/A	0.9	6.528	0.315	0.	3.039	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	160	159	234	1098	0	284
normalized size	1	1.	1.54	1.53	2.25	10.56	0.	2.73
time (sec)	N/A	0.17	0.594	0.003	1.743	2.819	0.	1.319

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	76.395	1.513	0.	0.	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	914	914	2677	0	0	23058	0	0
normalized size	1	1.	2.93	0.	0.	25.23	0.	0.
time (sec)	N/A	2.037	28.63	1.658	0.	5.03	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	499	499	1862	1771	0	9453	0	0
normalized size	1	1.	3.73	3.55	0.	18.94	0.	0.
time (sec)	N/A	0.978	8.95	0.268	0.	3.129	0.	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	135	174	0	2483	0	262
normalized size	1	1.	0.94	1.21	0.	17.24	0.	1.82
time (sec)	N/A	0.311	2.645	0.003	0.	3.115	0.	1.724

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	154.862	1.777	0.	0.	0.	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	978	978	1337	3280	0	0	0	0
normalized size	1	1.	1.37	3.35	0.	0.	0.	0.
time (sec)	N/A	1.411	10.954	0.273	0.	0.	0.	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	227	478	473	5906	0	637
normalized size	1	1.	1.26	2.66	2.63	32.81	0.	3.54
time (sec)	N/A	0.259	0.93	0.004	1.885	4.706	0.	1.367

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	F(-1)	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	180.002	3.345	0.	0.	0.	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	752	752	6441	0	0	24935	0	0
normalized size	1	1.	8.57	0.	0.	33.16	0.	0.
time (sec)	N/A	1.353	65.606	0.936	0.	4.926	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	502	502	1816	0	0	14549	0	0
normalized size	1	1.	3.62	0.	0.	28.98	0.	0.
time (sec)	N/A	1.004	30.406	0.615	0.	3.814	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	376	649	0	7004	0	0
normalized size	1	1.	1.26	2.18	0.	23.5	0.	0.
time (sec)	N/A	0.57	6.872	0.143	0.	2.604	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	60	73	217	1370	0	181
normalized size	1	1.	0.83	1.01	3.01	19.03	0.	2.51
time (sec)	N/A	0.111	0.101	0.001	1.269	2.193	0.	1.436

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	180.001	1.971	0.	0.	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1038	1038	2384	0	0	29336	0	0
normalized size	1	1.	2.3	0.	0.	28.26	0.	0.
time (sec)	N/A	2.241	43.631	0.889	0.	4.754	0.	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	C	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	714	714	1529	0	0	17416	0	0
normalized size	1	1.	2.14	0.	0.	24.39	0.	0.
time (sec)	N/A	1.728	26.417	0.684	0.	3.626	0.	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	413	413	734	1284	0	8606	0	0
normalized size	1	1.	1.78	3.11	0.	20.84	0.	0.
time (sec)	N/A	0.929	8.171	0.233	0.	2.719	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	145	162	0	2272	0	298
normalized size	1	1.	1.31	1.46	0.	20.47	0.	2.68
time (sec)	N/A	0.569	1.281	0.003	0.	2.463	0.	1.993

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	180.008	2.181	0.	0.	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	972	972	11848	0	0	0	0	0
normalized size	1	1.	12.19	0.	0.	0.	0.	0.
time (sec)	N/A	2.203	73.768	0.872	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	689	689	2277	0	0	18048	0	0
normalized size	1	1.	3.3	0.	0.	26.19	0.	0.
time (sec)	N/A	1.708	42.209	0.671	0.	2.996	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	455	1098	0	8753	0	0
normalized size	1	1.	1.05	2.52	0.	20.12	0.	0.
time (sec)	N/A	0.976	4.727	0.171	0.	2.389	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	64	194	234	1554	0	224
normalized size	1	1.	0.8	2.42	2.92	19.42	0.	2.8
time (sec)	N/A	0.105	0.118	0.001	1.294	1.848	0.	1.417

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	180.001	0.499	0.	0.	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1795	1795	9045	0	0	0	0	0
normalized size	1	1.	5.04	0.	0.	0.	0.	0.
time (sec)	N/A	3.272	87.99	2.194	0.	0.	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	C	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1219	1219	2784	0	0	29429	0	0
normalized size	1	1.	2.28	0.	0.	24.14	0.	0.
time (sec)	N/A	2.222	42.449	1.398	0.	6.108	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	762	762	913	1478	0	13535	0	0
normalized size	1	1.	1.2	1.94	0.	17.76	0.	0.
time (sec)	N/A	1.138	7.784	0.204	0.	3.736	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	164	219	319	2503	0	370
normalized size	1	1.	1.26	1.68	2.45	19.25	0.	2.85
time (sec)	N/A	0.235	0.364	0.003	1.691	3.375	0.	1.346

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	F(-1)	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	158.407	3.241	0.	0.	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1245	1245	2574	0	0	0	0	0
normalized size	1	1.	2.07	0.	0.	0.	0.	0.
time (sec)	N/A	3.437	27.558	1.557	0.	0.	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	699	699	863	2767	0	24195	0	0
normalized size	1	1.	1.23	3.96	0.	34.61	0.	0.
time (sec)	N/A	1.356	8.417	0.276	0.	4.714	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	185	233	0	5921	0	317
normalized size	1	1.	0.9	1.13	0.	28.74	0.	1.54
time (sec)	N/A	0.42	2.471	0.002	0.	4.718	0.	1.77

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	A	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.134	180.001	3.536	0.	0.	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1122	1122	2868	3563	0	0	0	0
normalized size	1	1.	2.56	3.18	0.	0.	0.	0.
time (sec)	N/A	1.807	10.2	0.289	0.	0.	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	237	539	564	7170	0	652
normalized size	1	1.	1.12	2.55	2.67	33.98	0.	3.09
time (sec)	N/A	0.367	0.808	0.003	1.764	7.085	0.	1.775

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	F	A	A	F(-1)	F(-1)	F(-1)
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	38	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.134	180.001	3.319	0.	0.	0.	0.



## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [496] had the largest ratio of [ 0.9167 ]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	2	1.	14	0.143
2	A	4	2	1.	14	0.143
3	A	3	2	1.	14	0.143
4	A	2	2	1.	12	0.167
5	A	3	3	1.	14	0.214
6	A	4	4	1.	14	0.286
7	A	5	4	1.	14	0.286
8	A	6	4	1.	16	0.25
9	A	4	3	1.	16	0.188
10	A	4	4	1.	16	0.25
11	A	2	1	1.	14	0.071
12	A	5	4	1.	16	0.25
13	A	5	5	1.	16	0.312
14	A	7	6	1.	16	0.375
15	A	7	7	1.	16	0.438
16	A	12	4	1.	16	0.25
17	A	8	4	1.	16	0.25
18	A	6	4	1.	16	0.25
19	A	3	3	1.	14	0.214
20	A	8	4	1.	16	0.25
21	A	8	4	1.	16	0.25
22	A	12	5	1.	16	0.312
23	A	9	5	1.	14	0.357

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
24	A	7	4	1.	14	0.286
25	A	5	3	1.	12	0.25
26	A	0	0	0.	0	0.
27	A	0	0	0.	0	0.
28	A	6	6	1.	16	0.375
29	A	5	5	1.	16	0.312
30	A	2	2	1.	14	0.143
31	A	0	0	0.	0	0.
32	A	0	0	0.	0	0.
33	A	15	8	1.	16	0.5
34	A	9	6	1.	16	0.375
35	A	6	4	1.	14	0.286
36	A	0	0	0.	0	0.
37	A	0	0	0.	0	0.
38	A	8	5	1.	16	0.312
39	A	7	5	1.	16	0.312
40	A	6	5	1.	16	0.312
41	A	5	4	1.	16	0.25
42	A	6	5	1.	16	0.312
43	A	7	5	1.	16	0.312
44	A	8	5	1.	16	0.312
45	A	10	8	1.	18	0.444
46	A	9	7	1.	18	0.389
47	A	8	6	1.	18	0.333
48	A	7	5	1.	18	0.278
49	A	7	6	1.	18	0.333
50	A	9	7	1.	18	0.389
51	A	9	8	1.	18	0.444
52	A	11	7	1.	18	0.389
53	A	23	7	1.	18	0.389
54	A	20	7	1.	18	0.389
55	A	14	6	1.	18	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	12	5	1.	18	0.278
57	A	12	5	1.	18	0.278
58	A	18	6	1.	18	0.333
59	A	19	7	1.	18	0.389
60	A	7	5	1.	12	0.417
61	A	6	5	1.	12	0.417
62	A	5	4	1.	12	0.333
63	A	6	5	1.	12	0.417
64	A	7	5	1.	12	0.417
65	A	0	0	0.	0	0.
66	A	0	0	0.	0	0.
67	A	0	0	0.	0	0.
68	A	2	1	1.	18	0.056
69	A	2	1	1.	20	0.05
70	A	3	1	1.	20	0.05
71	A	4	3	1.	22	0.136
72	A	0	0	0.	0	0.
73	A	8	3	1.	16	0.188
74	A	5	3	1.	16	0.188
75	A	3	2	1.	14	0.143
76	A	0	0	0.	0	0.
77	A	0	0	0.	0	0.
78	A	3	2	1.	12	0.167
79	A	3	2	1.	12	0.167
80	A	3	2	1.	12	0.167
81	A	3	2	1.	10	0.2
82	A	3	2	1.	12	0.167
83	A	3	2	1.	12	0.167
84	A	3	2	1.	12	0.167
85	A	5	3	1.	14	0.214
86	A	5	3	1.	14	0.214
87	A	5	3	1.	14	0.214

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	5	3	1.	12	0.25
89	A	5	3	1.	14	0.214
90	A	5	3	1.	14	0.214
91	A	5	3	1.	14	0.214
92	A	4	2	1.	20	0.1
93	A	4	2	1.	20	0.1
94	A	5	2	1.	20	0.1
95	A	7	5	1.	24	0.208
96	A	6	3	1.	21	0.143
97	A	5	3	1.	21	0.143
98	A	4	3	1.	19	0.158
99	A	5	4	1.	21	0.19
100	A	6	5	1.	21	0.238
101	A	7	5	1.	21	0.238
102	A	10	6	1.	23	0.261
103	A	9	7	1.	23	0.304
104	A	6	4	1.	21	0.19
105	A	9	5	1.	23	0.217
106	A	9	5	1.	23	0.217
107	A	15	6	1.	23	0.261
108	A	7	7	1.	23	0.304
109	A	6	6	1.	23	0.261
110	A	3	3	1.	21	0.143
111	A	0	0	0.	0	0.
112	A	0	0	0.	0	0.
113	A	10	9	1.	23	0.391
114	A	9	9	1.	23	0.391
115	A	4	4	1.	21	0.19
116	A	0	0	0.	0	0.
117	A	0	0	0.	0	0.
118	A	6	3	1.	21	0.143
119	A	5	3	1.	21	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
120	A	4	3	1.	21	0.143
121	A	3	3	1.	19	0.158
122	A	4	4	1.	21	0.19
123	A	5	5	1.	21	0.238
124	A	6	5	1.	21	0.238
125	A	9	5	1.	21	0.238
126	A	7	5	1.	21	0.238
127	A	4	4	1.	19	0.21
128	A	9	5	1.	21	0.238
129	A	9	5	1.	21	0.238
130	A	14	5	1.	21	0.238
131	A	10	5	1.	21	0.238
132	A	5	4	1.	19	0.21
133	A	12	5	1.	21	0.238
134	A	12	5	1.	21	0.238
135	A	21	6	1.	21	0.286
136	A	10	6	1.	21	0.286
137	A	8	5	1.	21	0.238
138	A	6	4	1.	19	0.21
139	A	0	0	0.	0	0.
140	A	0	0	0.	0	0.
141	A	16	9	1.	21	0.429
142	A	10	7	1.	21	0.333
143	A	7	5	1.	19	0.263
144	A	0	0	0.	0	0.
145	A	0	0	0.	0	0.
146	A	23	10	1.	21	0.476
147	A	13	8	1.	21	0.381
148	A	8	5	1.	19	0.263
149	A	0	0	0.	0	0.
150	A	0	0	0.	0	0.
151	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
152	A	12	5	1.	23	0.217
153	A	9	5	1.	23	0.217
154	A	5	3	1.	21	0.143
155	A	0	0	0.	0	0.
156	A	0	0	0.	0	0.
157	A	6	3	1.	18	0.167
158	A	5	3	1.	18	0.167
159	A	4	3	1.	16	0.188
160	A	5	4	1.	18	0.222
161	A	6	5	1.	18	0.278
162	A	7	5	1.	18	0.278
163	A	10	6	1.	20	0.3
164	A	9	7	1.	20	0.35
165	A	6	4	1.	18	0.222
166	A	10	5	1.	20	0.25
167	A	11	7	1.	20	0.35
168	A	14	8	1.	20	0.4
169	A	12	7	1.	20	0.35
170	A	10	6	1.	20	0.3
171	A	8	5	1.	18	0.278
172	A	0	0	0.	0	0.
173	A	0	0	0.	0	0.
174	A	18	10	1.	20	0.5
175	A	11	8	1.	18	0.444
176	A	0	0	0.	0	0.
177	A	0	0	0.	0	0.
178	A	35	11	1.	18	0.611
179	A	0	0	0.	0	0.
180	A	0	0	0.	0	0.
181	A	0	0	0.	0	0.
182	A	18	5	1.	20	0.25
183	A	10	5	1.	20	0.25

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	5	3	1.	18	0.167
185	A	0	0	0.	0	0.
186	A	0	0	0.	0	0.
187	A	9	9	1.	29	0.31
188	A	8	8	1.	29	0.276
189	A	5	4	1.	27	0.148
190	A	2	2	1.	22	0.091
191	A	0	0	0.	0	0.
192	A	0	0	0.	0	0.
193	A	14	11	1.	31	0.355
194	A	12	10	1.	31	0.323
195	A	8	6	1.	29	0.207
196	A	4	4	1.	24	0.167
197	A	0	0	0.	0	0.
198	A	0	0	0.	0	0.
199	A	19	13	1.	31	0.419
200	A	17	13	1.	31	0.419
201	A	11	7	1.	29	0.241
202	A	2	2	1.	24	0.083
203	A	0	0	0.	0	0.
204	A	0	0	0.	0	0.
205	A	17	10	1.	29	0.345
206	A	14	11	1.	29	0.379
207	A	9	7	1.	27	0.259
208	A	3	3	1.	22	0.136
209	A	0	0	0.	0	0.
210	A	0	0	0.	0	0.
211	A	24	10	1.	31	0.323
212	A	20	11	1.	31	0.355
213	A	12	7	1.	29	0.241
214	A	5	5	1.	24	0.208
215	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
216	A	0	0	0.	0	0.
217	A	40	13	1.	31	0.419
218	A	30	13	1.	31	0.419
219	A	19	8	1.	29	0.276
220	A	6	6	1.	24	0.25
221	A	0	0	0.	0	0.
222	A	0	0	0.	0	0.
223	A	14	9	1.	26	0.346
224	A	12	8	1.	26	0.308
225	A	10	6	1.	24	0.25
226	A	4	4	1.	19	0.21
227	A	0	0	0.	0	0.
228	A	19	11	1.	28	0.393
229	A	16	10	1.	28	0.357
230	A	13	8	1.	26	0.308
231	A	6	6	1.	21	0.286
232	A	0	0	0.	0	0.
233	A	24	13	1.	28	0.464
234	A	21	13	1.	28	0.464
235	A	16	9	1.	26	0.346
236	A	6	6	1.	21	0.286
237	A	0	0	0.	0	0.
238	A	22	9	1.	26	0.346
239	A	18	8	1.	26	0.308
240	A	14	7	1.	24	0.292
241	A	5	5	1.	19	0.263
242	A	0	0	0.	0	0.
243	A	29	11	1.	28	0.393
244	A	24	12	1.	28	0.429
245	A	17	9	1.	26	0.346
246	A	7	7	1.	21	0.333
247	A	0	0	0.	0	0.

Continued on next page



Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
248	A	45	14	1.	28	0.5
249	A	34	14	1.	28	0.5
250	A	24	10	1.	26	0.385
251	A	7	7	1.	21	0.333
252	A	0	0	0.	0	0.
253	A	6	6	1.	29	0.207
254	A	5	5	1.	29	0.172
255	A	4	4	1.	27	0.148
256	A	2	2	1.	22	0.091
257	A	0	0	0.	0	0.
258	A	0	0	0.	0	0.
259	A	6	4	1.	31	0.129
260	A	5	4	1.	31	0.129
261	A	4	3	1.	29	0.103
262	A	2	2	1.	24	0.083
263	A	5	5	1.	31	0.161
264	A	6	6	1.	31	0.194
265	A	10	8	1.	31	0.258
266	A	7	5	1.	31	0.161
267	A	6	6	1.	29	0.207
268	A	2	1	1.	24	0.042
269	A	9	6	1.	31	0.194
270	A	11	7	1.	31	0.226
271	A	22	13	1.	29	0.448
272	A	13	10	1.	29	0.345
273	A	10	8	1.	27	0.296
274	A	4	3	1.	22	0.136
275	A	0	0	0.	0	0.
276	A	0	0	0.	0	0.
277	A	20	12	1.	31	0.387
278	A	16	12	1.	31	0.387
279	A	7	7	1.	29	0.241

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
280	A	3	3	1.	24	0.125
281	A	0	0	0.	0	0.
282	A	0	0	0.	0	0.
283	A	32	16	1.	31	0.516
284	A	17	12	1.	31	0.387
285	A	11	7	1.	29	0.241
286	A	4	3	1.	24	0.125
287	A	0	0	0.	0	0.
288	A	0	0	0.	0	0.
289	A	11	6	1.	26	0.231
290	A	9	5	1.	26	0.192
291	A	7	4	1.	24	0.167
292	A	2	2	1.	19	0.105
293	A	0	0	0.	0	0.
294	A	18	11	1.	28	0.393
295	A	15	10	1.	28	0.357
296	A	12	8	1.	26	0.308
297	A	5	5	1.	21	0.238
298	A	0	0	0.	0	0.
299	A	21	14	1.	28	0.5
300	A	16	10	1.	28	0.357
301	A	13	10	1.	26	0.385
302	A	3	2	1.	21	0.095
303	A	0	0	0.	0	0.
304	A	29	10	1.	26	0.385
305	A	24	9	1.	26	0.346
306	A	19	8	1.	24	0.333
307	A	6	6	1.	19	0.316
308	A	0	0	0.	0	0.
309	A	29	13	1.	28	0.464
310	A	24	14	1.	28	0.5
311	A	15	11	1.	26	0.423

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
312	A	5	5	1.	21	0.238
313	A	0	0	0.	0	0.
314	A	39	14	1.	28	0.5
315	A	31	12	1.	26	0.462
316	A	7	6	1.	21	0.286
317	A	0	0	0.	0	0.
318	A	0	0	0.	0	0.
319	A	0	0	0.	0	0.
320	A	0	0	0.	0	0.
321	A	4	4	1.	24	0.167
322	A	9	6	1.	26	0.231
323	A	11	7	1.	26	0.269
324	A	4	4	1.	24	0.167
325	A	9	6	1.	26	0.231
326	A	11	7	1.	26	0.269
327	A	6	6	1.	24	0.25
328	A	12	9	1.	26	0.346
329	A	19	11	1.	26	0.423
330	A	6	6	1.	24	0.25
331	A	12	9	1.	26	0.346
332	A	19	11	1.	26	0.423
333	A	16	9	1.	32	0.281
334	A	13	8	1.	32	0.25
335	A	10	7	1.	30	0.233
336	A	4	3	1.	25	0.12
337	A	0	0	0.	0	0.
338	A	23	14	1.	34	0.412
339	A	20	14	1.	34	0.412
340	A	15	10	1.	32	0.312
341	A	5	5	1.	27	0.185
342	A	0	0	0.	0	0.
343	A	30	16	1.	34	0.471

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
344	A	23	13	1.	34	0.382
345	A	17	12	1.	32	0.375
346	A	4	3	1.	27	0.111
347	A	0	0	0.	0	0.
348	A	39	11	1.	26	0.423
349	A	32	10	1.	26	0.385
350	A	25	9	1.	24	0.375
351	A	6	5	1.	19	0.263
352	A	0	0	0.	0	0.
353	A	36	14	1.	32	0.438
354	A	30	15	1.	32	0.469
355	A	18	12	1.	30	0.4
356	A	5	5	1.	25	0.2
357	A	0	0	0.	0	0.
358	A	49	15	1.	34	0.441
359	A	38	13	1.	32	0.406
360	A	8	7	1.	27	0.259
361	A	0	0	0.	0	0.
362	A	22	14	1.	34	0.412
363	A	17	10	1.	34	0.294
364	A	14	10	1.	32	0.312
365	A	4	3	1.	27	0.111
366	A	0	0	0.	0	0.
367	A	31	16	1.	36	0.444
368	A	25	16	1.	36	0.444
369	A	19	12	1.	34	0.353
370	A	8	8	1.	29	0.276
371	A	0	0	0.	0	0.
372	A	40	17	1.	36	0.472
373	A	28	14	1.	36	0.389
374	A	22	13	1.	34	0.382
375	A	4	3	1.	29	0.103

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	0	0	0.	0	0.
377	A	46	12	1.	32	0.375
378	A	38	11	1.	32	0.344
379	A	30	10	1.	30	0.333
380	A	7	6	1.	25	0.24
381	A	0	0	0.	0	0.
382	A	45	15	1.	28	0.536
383	A	37	16	1.	28	0.571
384	A	21	13	1.	26	0.5
385	A	8	7	1.	21	0.333
386	A	0	0	0.	0	0.
387	A	53	15	1.	34	0.441
388	A	42	13	1.	32	0.406
389	A	8	7	1.	27	0.259
390	A	0	0	0.	0	0.
391	A	30	15	1.	34	0.441
392	A	22	10	1.	34	0.294
393	A	18	11	1.	32	0.344
394	A	4	3	1.	27	0.111
395	A	0	0	0.	0	0.
396	A	38	18	1.	36	0.5
397	A	31	18	1.	36	0.5
398	A	24	14	1.	34	0.412
399	A	9	8	1.	29	0.276
400	A	0	0	0.	0	0.
401	A	55	18	1.	36	0.5
402	A	40	15	1.	36	0.417
403	A	31	14	1.	34	0.412
404	A	4	3	1.	29	0.103
405	A	0	0	0.	0	0.
406	A	61	15	1.	34	0.441
407	A	50	14	1.	34	0.412

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
408	A	39	13	1.	32	0.406
409	A	7	6	1.	27	0.222
410	A	0	0	0.	0	0.
411	A	53	18	1.	34	0.529
412	A	44	19	1.	34	0.559
413	A	25	15	1.	32	0.469
414	A	9	8	1.	27	0.296
415	A	0	0	0.	0	0.
416	A	71	17	1.	28	0.607
417	A	55	15	1.	26	0.577
418	A	7	6	1.	21	0.286
419	A	0	0	0.	0	0.
420	A	18	8	1.	26	0.308
421	A	15	7	1.	26	0.269
422	A	12	6	1.	24	0.25
423	A	4	4	1.	19	0.21
424	A	0	0	0.	0	0.
425	A	33	14	1.	32	0.438
426	A	27	13	1.	32	0.406
427	A	21	11	1.	30	0.367
428	A	6	6	1.	25	0.24
429	A	0	0	0.	0	0.
430	A	34	17	1.	34	0.5
431	A	26	13	1.	34	0.382
432	A	22	13	1.	32	0.406
433	A	4	3	1.	27	0.111
434	A	0	0	0.	0	0.
435	A	40	13	1.	32	0.406
436	A	33	12	1.	32	0.375
437	A	26	11	1.	30	0.367
438	A	7	6	1.	25	0.24
439	A	0	0	0.	0	0.

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
440	A	53	22	1.	34	0.647
441	A	44	23	1.	34	0.676
442	A	26	19	1.	32	0.594
443	A	10	9	1.	27	0.333
444	A	0	0	0.	0	0.
445	A	57	23	1.	34	0.676
446	A	43	20	1.	32	0.625
447	A	9	7	1.	27	0.259
448	A	0	0	0.	0	0.
449	A	27	11	1.	32	0.344
450	A	22	12	1.	32	0.375
451	A	15	9	1.	30	0.3
452	A	4	3	1.	25	0.12
453	A	0	0	0.	0	0.
454	A	41	17	1.	28	0.607
455	A	34	18	1.	28	0.643
456	A	25	14	1.	26	0.538
457	A	7	7	1.	21	0.333
458	A	0	0	0.	0	0.
459	A	48	19	1.	34	0.559
460	A	37	17	1.	34	0.5
461	A	28	15	1.	32	0.469
462	A	4	3	1.	27	0.111
463	A	0	0	0.	0	0.
464	A	64	20	1.	34	0.588
465	A	53	21	1.	34	0.618
466	A	37	18	1.	32	0.562
467	A	7	6	1.	27	0.222
468	A	0	0	0.	0	0.
469	A	51	25	1.	36	0.694
470	A	30	20	1.	34	0.588
471	A	13	11	1.	29	0.379

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
472	A	0	0	0.	0	0.
473	A	57	27	1.	34	0.794
474	A	9	7	1.	29	0.241
475	A	0	0	0.	0	0.
476	A	34	14	1.	34	0.412
477	A	26	14	1.	34	0.412
478	A	19	11	1.	32	0.344
479	A	4	3	1.	27	0.111
480	A	0	0	0.	0	0.
481	A	67	22	1.	34	0.647
482	A	52	22	1.	34	0.647
483	A	38	17	1.	32	0.531
484	A	8	8	1.	27	0.296
485	A	0	0	0.	0	0.
486	A	62	23	1.	28	0.821
487	A	47	20	1.	28	0.714
488	A	36	18	1.	26	0.692
489	A	3	2	1.	21	0.095
490	A	0	0	0.	0	0.
491	A	87	28	1.	34	0.824
492	A	71	26	1.	34	0.765
493	A	49	23	1.	32	0.719
494	A	7	6	1.	27	0.222
495	A	0	0	0.	0	0.
496	A	88	33	1.	36	0.917
497	A	44	22	1.	34	0.647
498	A	17	12	1.	29	0.414
499	A	0	0	0.	0	0.
500	A	65	28	1.	34	0.824
501	A	9	7	1.	29	0.241
502	A	0	0	0.	0	0.



# Chapter 3

## Listing of integrals

### 3.1 $\int (c + dx)^4 \sinh(a + bx) dx$

**Optimal.** Leaf size=91

$$-\frac{24d^3(c + dx) \sinh(a + bx)}{b^4} + \frac{12d^2(c + dx)^2 \cosh(a + bx)}{b^3} - \frac{4d(c + dx)^3 \sinh(a + bx)}{b^2} + \frac{24d^4 \cosh(a + bx)}{b^5} + \frac{(c + dx)^4}{b}$$

[Out]  $(24*d^4*Cosh[a + b*x])/b^5 + (12*d^2*(c + d*x)^2*Cosh[a + b*x])/b^3 + ((c + d*x)^4*Cosh[a + b*x])/b - (24*d^3*(c + d*x)*Sinh[a + b*x])/b^4 - (4*d*(c + d*x)^3*Sinh[a + b*x])/b^2$

---

**Rubi [A]** time = 0.120453, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3296, 2638}

$$-\frac{24d^3(c + dx) \sinh(a + bx)}{b^4} + \frac{12d^2(c + dx)^2 \cosh(a + bx)}{b^3} - \frac{4d(c + dx)^3 \sinh(a + bx)}{b^2} + \frac{24d^4 \cosh(a + bx)}{b^5} + \frac{(c + dx)^4}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^4\*Sinh[a + b\*x], x]

[Out]  $(24*d^4*Cosh[a + b*x])/b^5 + (12*d^2*(c + d*x)^2*Cosh[a + b*x])/b^3 + ((c + d*x)^4*Cosh[a + b*x])/b - (24*d^3*(c + d*x)*Sinh[a + b*x])/b^4 - (4*d*(c + d*x)^3*Sinh[a + b*x])/b^2$

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int (c + dx)^4 \sinh(a + bx) dx &= \frac{(c + dx)^4 \cosh(a + bx)}{b} - \frac{(4d) \int (c + dx)^3 \cosh(a + bx) dx}{b} \\
&= \frac{(c + dx)^4 \cosh(a + bx)}{b} - \frac{4d(c + dx)^3 \sinh(a + bx)}{b^2} + \frac{(12d^2) \int (c + dx)^2 \sinh(a + bx) dx}{b^2} \\
&= \frac{12d^2(c + dx)^2 \cosh(a + bx)}{b^3} + \frac{(c + dx)^4 \cosh(a + bx)}{b} - \frac{4d(c + dx)^3 \sinh(a + bx)}{b^2} - \frac{(24d^3)}{b^2} \\
&= \frac{12d^2(c + dx)^2 \cosh(a + bx)}{b^3} + \frac{(c + dx)^4 \cosh(a + bx)}{b} - \frac{24d^3(c + dx) \sinh(a + bx)}{b^4} - \frac{4d(c + dx)^3 \sinh(a + bx)}{b^2} \\
&= \frac{24d^4 \cosh(a + bx)}{b^5} + \frac{12d^2(c + dx)^2 \cosh(a + bx)}{b^3} + \frac{(c + dx)^4 \cosh(a + bx)}{b} - \frac{24d^3(c + dx) \sinh(a + bx)}{b^4} - \frac{4d(c + dx)^3 \sinh(a + bx)}{b^2}
\end{aligned}$$

**Mathematica [A]** time = 0.347484, size = 76, normalized size = 0.84

$$\frac{\cosh(a + bx) (12b^2 d^2 (c + dx)^2 + b^4 (c + dx)^4 + 24d^4) - 4bd(c + dx) \sinh(a + bx) (b^2 (c + dx)^2 + 6d^2)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sinh[a + b*x],x]
```

```
[Out] ((24*d^4 + 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cosh[a + b*x] - 4*b*d*(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Sinh[a + b*x])/b^5
```

**Maple [B]** time = 0.033, size = 547, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^4\*sinh(b\*x+a),x)

[Out]  $\frac{1}{b}*(c^4*\cosh(b*x+a)+1/b^4*d^4*((b*x+a)^4*\cosh(b*x+a)-4*(b*x+a)^3*\sinh(b*x+a)+12*(b*x+a)^2*\cosh(b*x+a)-24*(b*x+a)*\sinh(b*x+a)+24*\cosh(b*x+a))+1/b^4*d^4*a^4*\cosh(b*x+a)-4/b^4*d^4*a*((b*x+a)^3*\cosh(b*x+a)-3*(b*x+a)^2*\sinh(b*x+a)+6*(b*x+a)*\cosh(b*x+a)-6*\sinh(b*x+a))+4/b^3*d^3*c*((b*x+a)^3*\cosh(b*x+a)-3*(b*x+a)^2*\sinh(b*x+a)+6*(b*x+a)*\cosh(b*x+a)-6*\sinh(b*x+a))+6/b^4*d^4*a^2*((b*x+a)^2*\cosh(b*x+a)-2*(b*x+a)*\sinh(b*x+a)+2*\cosh(b*x+a))+6/b^2*d^2*c^2*((b*x+a)^2*\cosh(b*x+a)-2*(b*x+a)*\sinh(b*x+a)+2*\cosh(b*x+a))-4/b^4*d^4*a^3*((b*x+a)*\cosh(b*x+a)-\sinh(b*x+a))+4/b*d*c^3*((b*x+a)*\cosh(b*x+a)-\sinh(b*x+a))-4/b^3*d^3*a^3*c*\cosh(b*x+a)+6/b^2*d^2*a^2*c^2*\cosh(b*x+a)-4/b*d*a*c^3*\cosh(b*x+a)-12/b^3*d^3*a*c*((b*x+a)^2*\cosh(b*x+a)-2*(b*x+a)*\sinh(b*x+a)+2*\cosh(b*x+a))+12/b^3*d^3*a^2*c*((b*x+a)*\cosh(b*x+a)-\sinh(b*x+a))-12/b^2*d^2*a*c^2*((b*x+a)*\cosh(b*x+a)-\sinh(b*x+a))$

**Maxima [B]** time = 1.1229, size = 440, normalized size = 4.84

$$\frac{c^4 e^{(bx+a)}}{2b} + \frac{2(bxe^a - e^a)c^3 de^{(bx)}}{b^2} + \frac{c^4 e^{(-bx-a)}}{2b} + \frac{2(bx+1)c^3 de^{(-bx-a)}}{b^2} + \frac{3(b^2x^2e^a - 2bx e^a + 2e^a)c^2 d^2 e^{(bx)}}{b^3} + \frac{3(b^2x^2 + 2bx e^a - 2e^a)c^2 d^2 e^{(-bx-a)}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^4\*sinh(b\*x+a),x, algorithm="maxima")

[Out]  $\frac{1}{2}*c^4*e^{(b*x + a)}/b + 2*(b*x*e^a - e^a)*c^3*d*e^{(b*x)}/b^2 + 1/2*c^4*e^{(-b*x - a)}/b + 2*(b*x + 1)*c^3*d*e^{(-b*x - a)}/b^2 + 3*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*c^2*d^2*e^{(b*x)}/b^3 + 3*(b^2*x^2 + 2*b*x + 2)*c^2*d^2*e^{(-b*x - a)}/b^3 + 2*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*c*d^3*e^{(b*x)}/b^4 + 2*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*c*d^3*e^{(-b*x - a)}/b^4 + 1/2*(b^4*x^4*e^a - 4*b^3*x^3*e^a + 12*b^2*x^2*e^a - 24*b*x*e^a + 24*e^a)*d^4*e^{(b*x)}/b^5 + 1/2*(b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*d^4*e^{(-b*x - a)}/b^5$

**Fricas [A]** time = 2.59911, size = 350, normalized size = 3.85

$$\frac{(b^4 d^4 x^4 + 4 b^4 c d^3 x^3 + b^4 c^4 + 12 b^2 c^2 d^2 + 24 d^4 + 6 (b^4 c^2 d^2 + 2 b^2 d^4) x^2 + 4 (b^4 c^3 d + 6 b^2 c d^3) x) \cosh(bx + a) - 4 (b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + b^3 c^4 + 12 b^2 c^2 d^2 + 24 d^4 + 6 (b^4 c^2 d^2 + 2 b^2 d^4) x^2 + 4 (b^4 c^3 d + 6 b^2 c d^3) x) \sinh(bx + a)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^4\*sinh(b\*x+a),x, algorithm="fricas")

[Out] ((b^4\*d^4\*x^4 + 4\*b^4\*c\*d^3\*x^3 + b^4\*c^4 + 12\*b^2\*c^2\*d^2 + 24\*d^4 + 6\*(b^4\*c^2\*d^2 + 2\*b^2\*d^4)\*x^2 + 4\*(b^4\*c^3\*d + 6\*b^2\*c\*d^3)\*x)\*cosh(b\*x + a) - 4\*(b^3\*d^4\*x^3 + 3\*b^3\*c\*d^3\*x^2 + b^3\*c^3\*d + 6\*b\*c\*d^3 + 3\*(b^3\*c^2\*d^2 + 2\*b\*d^4)\*x)\*sinh(b\*x + a))/b^5

**Sympy [A]** time = 3.23735, size = 311, normalized size = 3.42

$$\left\{ \frac{c^4 \cosh(ax+bx)}{b} + \frac{4c^3 dx \cosh(ax+bx)}{b} + \frac{6c^2 d^2 x^2 \cosh(ax+bx)}{b} + \frac{4cd^3 x^3 \cosh(ax+bx)}{b} + \frac{d^4 x^4 \cosh(ax+bx)}{b} - \frac{4c^3 d \sinh(ax+bx)}{b^2} - \frac{12c^2 d^2 x \sinh(ax+bx)}{b^2} - \frac{12cd^3 x^2 \sinh(ax+bx)}{b^2} - \frac{d^4 x^3 \sinh(ax+bx)}{b^2} \right\} \sinh(a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*4\*sinh(b\*x+a),x)

[Out] Piecewise((c\*\*4\*cosh(a + b\*x)/b + 4\*c\*\*3\*d\*x\*cosh(a + b\*x)/b + 6\*c\*\*2\*d\*\*2\*x\*\*2\*cosh(a + b\*x)/b + 4\*c\*d\*\*3\*x\*\*3\*cosh(a + b\*x)/b + d\*\*4\*x\*\*4\*cosh(a + b\*x)/b - 4\*c\*\*3\*d\*sinh(a + b\*x)/b\*\*2 - 12\*c\*\*2\*d\*\*2\*x\*sinh(a + b\*x)/b\*\*2 - 12\*c\*d\*\*3\*x\*\*2\*sinh(a + b\*x)/b\*\*2 - 4\*d\*\*4\*x\*\*3\*sinh(a + b\*x)/b\*\*2 + 12\*c\*\*2\*d\*\*2\*cosh(a + b\*x)/b\*\*3 + 24\*c\*d\*\*3\*x\*cosh(a + b\*x)/b\*\*3 + 12\*d\*\*4\*x\*\*2\*cosh(a + b\*x)/b\*\*3 - 24\*c\*d\*\*3\*sinh(a + b\*x)/b\*\*4 - 24\*d\*\*4\*x\*sinh(a + b\*x)/b\*\*4 + 24\*d\*\*4\*cosh(a + b\*x)/b\*\*5, Ne(b, 0)), ((c\*\*4\*x + 2\*c\*\*3\*d\*x\*\*2 + 2\*c\*\*2\*d\*\*2\*x\*\*3 + c\*d\*\*3\*x\*\*4 + d\*\*4\*x\*\*5/5)\*sinh(a), True))

**Giac [B]** time = 1.16781, size = 437, normalized size = 4.8

$$\frac{(b^4 d^4 x^4 + 4 b^4 c d^3 x^3 + 6 b^4 c^2 d^2 x^2 - 4 b^3 d^4 x^3 + 4 b^4 c^3 d x - 12 b^3 c d^3 x^2 + b^4 c^4 - 12 b^3 c^2 d^2 x + 12 b^2 d^4 x^2 - 4 b^3 c^3 d + 24 b^2 c d^3 x - 12 b^2 c^2 d^2 x + 12 b c^3 d - 4 b^2 c^4 + 12 b c^2 d^2 x - 4 b c^3 d + 12 c^4) \sinh(a)}{2 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^4\*sinh(b\*x+a),x, algorithm="giac")

[Out] 1/2\*(b^4\*d^4\*x^4 + 4\*b^4\*c\*d^3\*x^3 + 6\*b^4\*c^2\*d^2\*x^2 - 4\*b^3\*d^4\*x^3 + 4\*b^4\*c^3\*d\*x - 12\*b^3\*c\*d^3\*x^2 + b^4\*c^4 - 12\*b^3\*c^2\*d^2\*x + 12\*b^2\*d^4\*x^2 - 4\*b^3\*c^3\*d + 24\*b^2\*c\*d^3\*x + 12\*b^2\*c^2\*d^2 - 24\*b\*d^4\*x - 24\*b\*c\*d^3 + 24\*d^4)\*e^(b\*x + a)/b^5 + 1/2\*(b^4\*d^4\*x^4 + 4\*b^4\*c\*d^3\*x^3 + 6\*b^4\*c^2\*d^2\*x^2 - 4\*b^3\*d^4\*x^3 + 4\*b^4\*c^3\*d\*x - 12\*b^3\*c\*d^3\*x^2 + b^4\*c^4 - 12\*b^3\*c^2\*d^2\*x + 12\*b^2\*d^4\*x^2 - 4\*b^3\*c^3\*d + 24\*b^2\*c\*d^3\*x + 12\*b^2\*c^2\*d^2 - 24\*b\*d^4\*x - 24\*b\*c\*d^3 + 24\*d^4)\*e^(b\*x + a)/b^5

$$\begin{aligned} & *d^2*x^2 + 4*b^3*d^4*x^3 + 4*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + b^4*c^4 + 12* \\ & b^3*c^2*d^2*x + 12*b^2*d^4*x^2 + 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2* \\ & d^2 + 24*b*d^4*x + 24*b*c*d^3 + 24*d^4)*e^{(-b*x - a)}/b^5 \end{aligned}$$

## 3.2 $\int (c + dx)^3 \sinh(a + bx) dx$

**Optimal.** Leaf size=70

$$\frac{6d^2(c + dx) \cosh(a + bx)}{b^3} - \frac{3d(c + dx)^2 \sinh(a + bx)}{b^2} - \frac{6d^3 \sinh(a + bx)}{b^4} + \frac{(c + dx)^3 \cosh(a + bx)}{b}$$

[Out]  $(6*d^2*(c + d*x)*Cosh[a + b*x])/b^3 + ((c + d*x)^3*Cosh[a + b*x])/b - (6*d^3*\sinh[a + b*x])/b^4 - (3*d*(c + d*x)^2*\sinh[a + b*x])/b^2$

**Rubi [A]** time = 0.0790137, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3296, 2637}

$$\frac{6d^2(c + dx) \cosh(a + bx)}{b^3} - \frac{3d(c + dx)^2 \sinh(a + bx)}{b^2} - \frac{6d^3 \sinh(a + bx)}{b^4} + \frac{(c + dx)^3 \cosh(a + bx)}{b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^3*\sinh[a + b*x], x]$

[Out]  $(6*d^2*(c + d*x)*Cosh[a + b*x])/b^3 + ((c + d*x)^3*Cosh[a + b*x])/b - (6*d^3*\sinh[a + b*x])/b^4 - (3*d*(c + d*x)^2*\sinh[a + b*x])/b^2$

### Rule 3296

$\text{Int}[(c + d*x)^m*\sin[e + f*x], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\cos[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{m-1}*\cos[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f, x\} \&\& \text{GtQ}[m, 0]$

### Rule 2637

$\text{Int}[\sin[\pi/2 + (c + d*x)], x\_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /; \text{FreeQ}\{c, d, x\}$

### Rubi steps

$$\begin{aligned}
\int (c + dx)^3 \sinh(a + bx) dx &= \frac{(c + dx)^3 \cosh(a + bx)}{b} - \frac{(3d) \int (c + dx)^2 \cosh(a + bx) dx}{b} \\
&= \frac{(c + dx)^3 \cosh(a + bx)}{b} - \frac{3d(c + dx)^2 \sinh(a + bx)}{b^2} + \frac{(6d^2) \int (c + dx) \sinh(a + bx) dx}{b^2} \\
&= \frac{6d^2(c + dx) \cosh(a + bx)}{b^3} + \frac{(c + dx)^3 \cosh(a + bx)}{b} - \frac{3d(c + dx)^2 \sinh(a + bx)}{b^2} - \frac{(6d^3) \int \sinh(a + bx) dx}{b^2} \\
&= \frac{6d^2(c + dx) \cosh(a + bx)}{b^3} + \frac{(c + dx)^3 \cosh(a + bx)}{b} - \frac{6d^3 \sinh(a + bx)}{b^4} - \frac{3d(c + dx)^2 \sinh(a + bx)}{b^2}
\end{aligned}$$

**Mathematica [A]** time = 0.21966, size = 61, normalized size = 0.87

$$\frac{b(c + dx) \cosh(a + bx) (b^2(c + dx)^2 + 6d^2) - 3d \sinh(a + bx) (b^2(c + dx)^2 + 2d^2)}{b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3\*Sinh[a + b\*x], x]

[Out] (b\*(c + d\*x)\*(6\*d^2 + b^2\*(c + d\*x)^2)\*Cosh[a + b\*x] - 3\*d\*(2\*d^2 + b^2\*(c + d\*x)^2)\*Sinh[a + b\*x])/b^4

**Maple [B]** time = 0.007, size = 308, normalized size = 4.4

$$\frac{1}{b} \left( \frac{d^3 \left( (bx + a)^3 \cosh(bx + a) - 3(bx + a)^2 \sinh(bx + a) + 6(bx + a) \cosh(bx + a) - 6 \sinh(bx + a) \right)}{b^3} - 3 \frac{d^3 a \left( (bx + a) \cosh(bx + a) - \sinh(bx + a) \right)}{b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3\*sinh(b\*x+a), x)

[Out] 1/b\*(1/b^3\*d^3\*((b\*x+a)^3\*cosh(b\*x+a)-3\*(b\*x+a)^2\*sinh(b\*x+a)+6\*(b\*x+a)\*cosh(b\*x+a)-6\*sinh(b\*x+a))-3/b^3\*d^3\*a\*((b\*x+a)^2\*cosh(b\*x+a)-2\*(b\*x+a)\*sinh(b\*x+a)+2\*cosh(b\*x+a))+3/b^2\*d^2\*c\*((b\*x+a)^2\*cosh(b\*x+a)-2\*(b\*x+a)\*sinh(b\*x+a)+2\*cosh(b\*x+a))+3/b^3\*d^3\*a^2\*((b\*x+a)\*cosh(b\*x+a)-sinh(b\*x+a))-6/b^2\*d^2\*a\*c\*((b\*x+a)\*cosh(b\*x+a)-sinh(b\*x+a))+3/b\*d\*c^2\*((b\*x+a)\*cosh(b\*x+a)-sinh(b\*x+a))-1/b^3\*d^3\*a^3\*cosh(b\*x+a)+3/b^2\*d^2\*a^2\*c\*cosh(b\*x+a)-3/b\*d\*a\*c^2\*cosh(b\*x+a)+c^3\*cosh(b\*x+a))

---

**Maxima [B]** time = 1.0586, size = 300, normalized size = 4.29

$$\frac{c^3 e^{(bx+a)}}{2b} + \frac{3(bxe^a - e^a)c^2 de^{(bx)}}{2b^2} + \frac{c^3 e^{(-bx-a)}}{2b} + \frac{3(bx+1)c^2 de^{(-bx-a)}}{2b^2} + \frac{3(b^2x^2e^a - 2bx e^a + 2e^a)cd^2 e^{(bx)}}{2b^3} + \frac{3(b^2x^2 + 2bx - 2e^a)cd^2 e^{(-bx-a)}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*sinh(b\*x+a),x, algorithm="maxima")

[Out]  $\frac{1}{2}c^3e^{(bx+a)}/b + \frac{3}{2}(bx+1)c^2de^{(bx)}/b^2 + \frac{1}{2}c^3e^{(-bx-a)}/b + \frac{3}{2}(bx+1)c^2de^{(-bx-a)}/b^2 + \frac{3}{2}(b^2x^2e^a - 2bx e^a + 2e^a)cd^2e^{(bx)}/b^3 + \frac{3}{2}(b^2x^2e^a - 2bx e^a + 2e^a)cd^2e^{(-bx-a)}/b^3 + \frac{1}{2}(b^3x^3e^a - 3b^2x^2e^a + 6bx e^a - 6e^a)d^3e^{(bx)}/b^4 + \frac{1}{2}(b^3x^3e^a - 3b^2x^2e^a + 6bx e^a - 6e^a)d^3e^{(-bx-a)}/b^4$

---

**Fricas [A]** time = 2.53654, size = 231, normalized size = 3.3

$$\frac{(b^3d^3x^3 + 3b^3cd^2x^2 + b^3c^3 + 6bcd^2 + 3(b^3c^2d + 2bd^3)x) \cosh(bx+a) - 3(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d + 2d^3) \sinh(bx+a)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*sinh(b\*x+a),x, algorithm="fricas")

[Out]  $((b^3d^3x^3 + 3b^3cd^2x^2 + b^3c^3 + 6b^3cd^2 + 3(b^3c^2d + 2b^3cd^2)x) \cosh(bx+a) - 3(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d + 2d^3) \sinh(bx+a))/b^4$

---

**Sympy [A]** time = 1.50237, size = 202, normalized size = 2.89

$$\left\{ \frac{c^3 \cosh(a+bx)}{b} + \frac{3c^2 dx \cosh(a+bx)}{b} + \frac{3cd^2x^2 \cosh(a+bx)}{b} + \frac{d^3x^3 \cosh(a+bx)}{b} - \frac{3c^2d \sinh(a+bx)}{b^2} - \frac{6cd^2x \sinh(a+bx)}{b^2} - \frac{3d^3x^2 \sinh(a+bx)}{b^2} + \frac{6cd^2}{b^2} \right\} \sinh(a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*3\*sinh(b\*x+a),x)



```
[Out] Piecewise((c**3*cosh(a + b*x)/b + 3*c**2*d*x*cosh(a + b*x)/b + 3*c*d**2*x**2*cosh(a + b*x)/b + d**3*x**3*cosh(a + b*x)/b - 3*c**2*d*sinh(a + b*x)/b**2 - 6*c*d**2*x*sinh(a + b*x)/b**2 - 3*d**3*x**2*sinh(a + b*x)/b**2 + 6*c*d**2*cosh(a + b*x)/b**3 + 6*d**3*x*cosh(a + b*x)/b**3 - 6*d**3*sinh(a + b*x)/b**4, Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sinh(a), True))
```

**Giac [B]** time = 1.19993, size = 275, normalized size = 3.93

$$\frac{(b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x - 3 b^2 d^3 x^2 + b^3 c^3 - 6 b^2 c d^2 x - 3 b^2 c^2 d + 6 b d^3 x + 6 b c d^2 - 6 d^3) e^{(b x + a)}}{2 b^4} + \frac{(b^3 d^3 x^3 + 3 b^3 c d^2 x^2 + 3 b^3 c^2 d x - 3 b^2 d^3 x^2 + b^3 c^3 - 6 b^2 c d^2 x - 3 b^2 c^2 d + 6 b d^3 x + 6 b c d^2 - 6 d^3) e^{(-b x - a)}}{2 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x - 3*b^2*d^3*x^2 + b^3*c^3 - 6*b^2*c*d^2*x - 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 6*d^3)*e^(b*x + a)/b^4 + 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*b^2*d^3*x^2 + b^3*c^3 + 6*b^2*c*d^2*x + 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 6*d^3)*e^(-b*x - a)/b^4
```

### 3.3 $\int (c + dx)^2 \sinh(a + bx) dx$

**Optimal.** Leaf size=49

$$-\frac{2d(c + dx) \sinh(a + bx)}{b^2} + \frac{2d^2 \cosh(a + bx)}{b^3} + \frac{(c + dx)^2 \cosh(a + bx)}{b}$$

[Out]  $(2*d^2*Cosh[a + b*x])/b^3 + ((c + d*x)^2*Cosh[a + b*x])/b - (2*d*(c + d*x)*Sinh[a + b*x])/b^2$

**Rubi [A]** time = 0.0496901, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3296, 2638}

$$-\frac{2d(c + dx) \sinh(a + bx)}{b^2} + \frac{2d^2 \cosh(a + bx)}{b^3} + \frac{(c + dx)^2 \cosh(a + bx)}{b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^2*\text{Sinh}[a + b*x], x]$

[Out]  $(2*d^2*Cosh[a + b*x])/b^3 + ((c + d*x)^2*Cosh[a + b*x])/b - (2*d*(c + d*x)*Sinh[a + b*x])/b^2$

#### Rule 3296

$\text{Int}[(c + d*x)^m \sin(e + f*x), x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m \cos(e + f*x)/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{m-1} \cos(e + f*x), x], x] /; \text{FreeQ}\{c, d, e, f, x\} \&\& \text{GtQ}[m, 0]$

#### Rule 2638

$\text{Int}[\sin(c + d*x), x\_Symbol] \rightarrow -\text{Simp}[\cos[c + d*x]/d, x] /; \text{FreeQ}\{c, d, x\}$

#### Rubi steps

$$\begin{aligned}
\int (c + dx)^2 \sinh(a + bx) dx &= \frac{(c + dx)^2 \cosh(a + bx)}{b} - \frac{(2d) \int (c + dx) \cosh(a + bx) dx}{b} \\
&= \frac{(c + dx)^2 \cosh(a + bx)}{b} - \frac{2d(c + dx) \sinh(a + bx)}{b^2} + \frac{(2d^2) \int \sinh(a + bx) dx}{b^2} \\
&= \frac{2d^2 \cosh(a + bx)}{b^3} + \frac{(c + dx)^2 \cosh(a + bx)}{b} - \frac{2d(c + dx) \sinh(a + bx)}{b^2}
\end{aligned}$$

**Mathematica [A]** time = 0.157426, size = 44, normalized size = 0.9

$$\frac{\cosh(a + bx) (b^2(c + dx)^2 + 2d^2) - 2bd(c + dx) \sinh(a + bx)}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2\*Sinh[a + b\*x],x]

[Out] ((2\*d^2 + b^2\*(c + d\*x)^2)\*Cosh[a + b\*x] - 2\*b\*d\*(c + d\*x)\*Sinh[a + b\*x])/b^3

**Maple [B]** time = 0.006, size = 147, normalized size = 3.

$$\frac{1}{b} \left( \frac{d^2 ((bx + a)^2 \cosh(bx + a) - 2(bx + a) \sinh(bx + a) + 2 \cosh(bx + a))}{b^2} - 2 \frac{d^2 a ((bx + a) \cosh(bx + a) - \sinh(bx + a))}{b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2\*sinh(b\*x+a),x)

[Out] 1/b\*(1/b^2\*d^2\*((b\*x+a)^2\*cosh(b\*x+a)-2\*(b\*x+a)\*sinh(b\*x+a)+2\*cosh(b\*x+a))-2/b^2\*d^2\*a\*((b\*x+a)\*cosh(b\*x+a)-sinh(b\*x+a))+2/b\*d\*c\*((b\*x+a)\*cosh(b\*x+a)-sinh(b\*x+a))+1/b^2\*d^2\*a^2\*cosh(b\*x+a)-2/b\*d\*a\*c\*cosh(b\*x+a)+c^2\*cosh(b\*x+a))

**Maxima [B]** time = 1.1176, size = 181, normalized size = 3.69

$$\frac{c^2 e^{(bx+a)}}{2b} + \frac{(bx e^a - e^a) c d e^{(bx)}}{b^2} + \frac{c^2 e^{(-bx-a)}}{2b} + \frac{(bx+1) c d e^{(-bx-a)}}{b^2} + \frac{(b^2 x^2 e^a - 2 b x e^a + 2 e^a) d^2 e^{(bx)}}{2b^3} + \frac{(b^2 x^2 + 2 b x + 2) d^2 e^{(bx)}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*sinh(b\*x+a),x, algorithm="maxima")

[Out]  $\frac{1}{2}c^2e^{(bx+a)}/b + (bx*e^a - e^a)*c*d*e^{(bx)}/b^2 + \frac{1}{2}c^2e^{(-bx-a)}/b + (bx+1)*c*d*e^{(-bx-a)}/b^2 + \frac{1}{2}(b^2x^2e^a - 2bx*e^a + 2e^a)*d^2e^{(bx)}/b^3 + \frac{1}{2}(b^2x^2 + 2bx + 2)*d^2e^{(-bx-a)}/b^3$

**Fricas [A]** time = 2.56058, size = 139, normalized size = 2.84

$$\frac{(b^2d^2x^2 + 2b^2cdx + b^2c^2 + 2d^2) \cosh(bx + a) - 2(bd^2x + bcd) \sinh(bx + a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*sinh(b\*x+a),x, algorithm="fricas")

[Out]  $((b^2d^2x^2 + 2b^2c*d*x + b^2c^2 + 2d^2)*\cosh(b*x + a) - 2*(b*d^2*x + b*c*d)*\sinh(b*x + a))/b^3$

**Sympy [A]** time = 0.721562, size = 112, normalized size = 2.29

$$\begin{cases} \frac{c^2 \cosh(ax+bx)}{b} + \frac{2cdx \cosh(ax+bx)}{b} + \frac{d^2x^2 \cosh(ax+bx)}{b} - \frac{2cd \sinh(ax+bx)}{b^2} - \frac{2d^2x \sinh(ax+bx)}{b^2} + \frac{2d^2 \cosh(ax+bx)}{b^3} & \text{for } b \neq 0 \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \sinh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2\*sinh(b\*x+a),x)

[Out] Piecewise((c\*\*2\*cosh(a + b\*x)/b + 2\*c\*d\*x\*cosh(a + b\*x)/b + d\*\*2\*x\*\*2\*cosh(a + b\*x)/b - 2\*c\*d\*sinh(a + b\*x)/b\*\*2 - 2\*d\*\*2\*x\*sinh(a + b\*x)/b\*\*2 + 2\*d\*\*2\*cosh(a + b\*x)/b\*\*3, Ne(b, 0)), ((c\*\*2\*x + c\*d\*x\*\*2 + d\*\*2\*x\*\*3/3)\*sinh(a), True))

**Giac [B]** time = 1.16676, size = 151, normalized size = 3.08

$$\frac{(b^2d^2x^2 + 2b^2cdx + b^2c^2 - 2bd^2x - 2bcd + 2d^2)e^{(bx+a)}}{2b^3} + \frac{(b^2d^2x^2 + 2b^2cdx + b^2c^2 + 2bd^2x + 2bcd + 2d^2)e^{(-bx-a)}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*b*d^2*x - 2*b*c*d + 2*d^2)*e^(
b*x + a)/b^3 + 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*b*d^2*x + 2*b*c
*d + 2*d^2)*e^(-b*x - a)/b^3
```

### 3.4 $\int (c + dx) \sinh(a + bx) dx$

**Optimal.** Leaf size=28

$$\frac{(c + dx) \cosh(a + bx)}{b} - \frac{d \sinh(a + bx)}{b^2}$$

[Out]  $((c + d*x)*Cosh[a + b*x])/b - (d*Sinh[a + b*x])/b^2$

**Rubi [A]** time = 0.0199479, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3296, 2637}

$$\frac{(c + dx) \cosh(a + bx)}{b} - \frac{d \sinh(a + bx)}{b^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)*Sinh[a + b*x], x]$

[Out]  $((c + d*x)*Cosh[a + b*x])/b - (d*Sinh[a + b*x])/b^2$

#### Rule 3296

$\text{Int}[(c + d*x)^m \cos[e + f*x], x] \rightarrow -\text{Simp}[(c + d*x)^m \cos[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{m-1} \cos[e + f*x], x], x] /;$   $\text{FreeQ}\{c, d, e, f, x\} \ \&\& \ \text{GtQ}[m, 0]$

#### Rule 2637

$\text{Int}[\sin[\pi/2 + (c + d*x)], x] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$   $\text{FreeQ}\{c, d, x\}$

#### Rubi steps

$$\begin{aligned} \int (c + dx) \sinh(a + bx) dx &= \frac{(c + dx) \cosh(a + bx)}{b} - \frac{d \int \cosh(a + bx) dx}{b} \\ &= \frac{(c + dx) \cosh(a + bx)}{b} - \frac{d \sinh(a + bx)}{b^2} \end{aligned}$$

**Mathematica [A]** time = 0.0625134, size = 27, normalized size = 0.96

$$\frac{b(c + dx) \cosh(a + bx) - d \sinh(a + bx)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)\*Sinh[a + b\*x],x]

[Out] (b\*(c + d\*x)\*Cosh[a + b\*x] - d\*Sinh[a + b\*x])/b^2

**Maple [A]** time = 0.004, size = 53, normalized size = 1.9

$$\frac{1}{b} \left( \frac{d((bx + a) \cosh(bx + a) - \sinh(bx + a))}{b} - \frac{da \cosh(bx + a)}{b} + c \cosh(bx + a) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)\*sinh(b\*x+a),x)

[Out] 1/b\*(1/b\*d\*((b\*x+a)\*cosh(b\*x+a)-sinh(b\*x+a))-1/b\*d\*a\*cosh(b\*x+a)+c\*cosh(b\*x+a))

**Maxima [B]** time = 1.08766, size = 92, normalized size = 3.29

$$\frac{ce^{(bx+a)}}{2b} + \frac{(bx e^a - e^a)de^{(bx)}}{2b^2} + \frac{ce^{(-bx-a)}}{2b} + \frac{(bx + 1)de^{(-bx-a)}}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sinh(b\*x+a),x, algorithm="maxima")

[Out] 1/2\*c\*e^(b\*x + a)/b + 1/2\*(b\*x\*e^a - e^a)\*d\*e^(b\*x)/b^2 + 1/2\*c\*e^(-b\*x - a)/b + 1/2\*(b\*x + 1)\*d\*e^(-b\*x - a)/b^2

**Fricas [A]** time = 2.52931, size = 72, normalized size = 2.57

$$\frac{(bdx + bc) \cosh(bx + a) - d \sinh(bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sinh(b\*x+a),x, algorithm="fricas")

[Out] ((b\*d\*x + b\*c)\*cosh(b\*x + a) - d\*sinh(b\*x + a))/b^2

**Sympy [A]** time = 0.265362, size = 46, normalized size = 1.64

$$\begin{cases} \frac{c \cosh(ax+bx)}{b} + \frac{dx \cosh(ax+bx)}{b} - \frac{d \sinh(ax+bx)}{b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right) \sinh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sinh(b\*x+a),x)

[Out] Piecewise((c\*cosh(a + b\*x)/b + d\*x\*cosh(a + b\*x)/b - d\*sinh(a + b\*x)/b\*\*2, Ne(b, 0)), ((c\*x + d\*x\*\*2/2)\*sinh(a), True))

**Giac [A]** time = 1.14771, size = 62, normalized size = 2.21

$$\frac{(bdx + bc - d)e^{(bx+a)}}{2b^2} + \frac{(bdx + bc + d)e^{(-bx-a)}}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sinh(b\*x+a),x, algorithm="giac")

[Out] 1/2\*(b\*d\*x + b\*c - d)\*e^(b\*x + a)/b^2 + 1/2\*(b\*d\*x + b\*c + d)\*e^(-b\*x - a)/b^2



$$3.5 \quad \int \frac{\sinh(a+bx)}{c+dx} dx$$

**Optimal.** Leaf size=51

$$\frac{\sinh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{d} + \frac{\cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{d}$$

[Out] (CoshIntegral[(b\*c)/d + b\*x]\*Sinh[a - (b\*c)/d])/d + (Cosh[a - (b\*c)/d]\*SinhIntegral[(b\*c)/d + b\*x])/d

**Rubi [A]** time = 0.109284, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3303, 3298, 3301}

$$\frac{\sinh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{d} + \frac{\cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[a + b\*x]/(c + d\*x), x]

[Out] (CoshIntegral[(b\*c)/d + b\*x]\*Sinh[a - (b\*c)/d])/d + (Cosh[a - (b\*c)/d]\*SinhIntegral[(b\*c)/d + b\*x])/d

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}

} , x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\sinh(a + bx)}{c + dx} dx &= \cosh\left(a - \frac{bc}{d}\right) \int \frac{\sinh\left(\frac{bc}{d} + bx\right)}{c + dx} dx + \sinh\left(a - \frac{bc}{d}\right) \int \frac{\cosh\left(\frac{bc}{d} + bx\right)}{c + dx} dx \\ &= \frac{\text{Chi}\left(\frac{bc}{d} + bx\right) \sinh\left(a - \frac{bc}{d}\right)}{d} + \frac{\cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{d} \end{aligned}$$

**Mathematica [A]** time = 0.0848559, size = 49, normalized size = 0.96

$$\frac{\sinh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right) + \cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[a + b\*x]/(c + d\*x),x]

[Out] (CoshIntegral[(b\*c)/d + b\*x]\*Sinh[a - (b\*c)/d] + Cosh[a - (b\*c)/d]\*SinhIntegral[(b\*c)/d + b\*x])/d

**Maple [A]** time = 0.059, size = 82, normalized size = 1.6

$$\frac{1}{2d} e^{-\frac{da-cb}{d}} \text{Ei}\left(1, bx + a - \frac{da-cb}{d}\right) - \frac{1}{2d} e^{\frac{da-cb}{d}} \text{Ei}\left(1, -bx - a - \frac{-da+cb}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(b\*x+a)/(d\*x+c),x)

[Out] 1/2/d\*exp(-(a\*d-b\*c)/d)\*Ei(1,b\*x+a-(a\*d-b\*c)/d)-1/2/d\*exp((a\*d-b\*c)/d)\*Ei(1,-b\*x-a-(-a\*d+b\*c)/d)

**Maxima [A]** time = 1.2888, size = 77, normalized size = 1.51

$$\frac{e^{\left(-a+\frac{bc}{d}\right)} E_1\left(\frac{(dx+c)b}{d}\right)}{2d} - \frac{e^{\left(a-\frac{bc}{d}\right)} E_1\left(-\frac{(dx+c)b}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)/(d\*x+c),x, algorithm="maxima")

[Out] 1/2\*e^(-a + b\*c/d)\*exp\_integral\_e(1, (d\*x + c)\*b/d)/d - 1/2\*e^(a - b\*c/d)\*exp\_integral\_e(1, -(d\*x + c)\*b/d)/d

**Fricas [A]** time = 2.58023, size = 193, normalized size = 3.78

$$\frac{\left(Ei\left(\frac{bdx+bc}{d}\right) - Ei\left(-\frac{bdx+bc}{d}\right)\right) \cosh\left(-\frac{bc-ad}{d}\right) + \left(Ei\left(\frac{bdx+bc}{d}\right) + Ei\left(-\frac{bdx+bc}{d}\right)\right) \sinh\left(-\frac{bc-ad}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)/(d\*x+c),x, algorithm="fricas")

[Out] 1/2\*((Ei((b\*d\*x + b\*c)/d) - Ei(-(b\*d\*x + b\*c)/d))\*cosh(-(b\*c - a\*d)/d) + (Ei((b\*d\*x + b\*c)/d) + Ei(-(b\*d\*x + b\*c)/d))\*sinh(-(b\*c - a\*d)/d))/d

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)/(d\*x+c),x)

[Out] Integral(sinh(a + b\*x)/(c + d\*x), x)

**Giac [A]** time = 1.15718, size = 77, normalized size = 1.51

$$\frac{\operatorname{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)} - \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)/(d\*x+c),x, algorithm="giac")

[Out] 1/2\*(Ei((b\*d\*x + b\*c)/d)\*e^(a - b\*c/d) - Ei(-(b\*d\*x + b\*c)/d)\*e^(-a + b\*c/d))/d

$$3.6 \quad \int \frac{\sinh(a+bx)}{(c+dx)^2} dx$$

**Optimal.** Leaf size=71

$$\frac{b \cosh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{d^2} + \frac{b \sinh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{d^2} - \frac{\sinh(a + bx)}{d(c + dx)}$$

[Out] (b\*Cosh[a - (b\*c)/d]\*CoshIntegral[(b\*c)/d + b\*x])/d^2 - Sinh[a + b\*x]/(d\*(c + d\*x)) + (b\*Sinh[a - (b\*c)/d]\*SinhIntegral[(b\*c)/d + b\*x])/d^2

**Rubi [A]** time = 0.126858, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3297, 3303, 3298, 3301}

$$\frac{b \cosh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{d^2} + \frac{b \sinh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{d^2} - \frac{\sinh(a + bx)}{d(c + dx)}$$

Antiderivative was successfully verified.

[In] Int[Sinh[a + b\*x]/(c + d\*x)^2,x]

[Out] (b\*Cosh[a - (b\*c)/d]\*CoshIntegral[(b\*c)/d + b\*x])/d^2 - Sinh[a + b\*x]/(d\*(c + d\*x)) + (b\*Sinh[a - (b\*c)/d]\*SinhIntegral[(b\*c)/d + b\*x])/d^2

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\sinh(a+bx)}{(c+dx)^2} dx &= -\frac{\sinh(a+bx)}{d(c+dx)} + \frac{b \int \frac{\cosh(a+bx)}{c+dx} dx}{d} \\ &= -\frac{\sinh(a+bx)}{d(c+dx)} + \frac{\left(b \cosh\left(a - \frac{bc}{d}\right)\right) \int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx} dx}{d} + \frac{\left(b \sinh\left(a - \frac{bc}{d}\right)\right) \int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx} dx}{d} \\ &= \frac{b \cosh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d}+bx\right)}{d^2} - \frac{\sinh(a+bx)}{d(c+dx)} + \frac{b \sinh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d}+bx\right)}{d^2} \end{aligned}$$

**Mathematica [A]** time = 0.238479, size = 65, normalized size = 0.92

$$\frac{b \cosh\left(a - \frac{bc}{d}\right) \text{Chi}\left(b\left(\frac{c}{d} + x\right)\right) + b \sinh\left(a - \frac{bc}{d}\right) \text{Shi}\left(b\left(\frac{c}{d} + x\right)\right) - \frac{d \sinh(a+bx)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]/(c + d*x)^2, x]
```

```
[Out] (b*Cosh[a - (b*c)/d]*CoshIntegral[b*(c/d + x)] - (d*Sinh[a + b*x])/(c + d*x)
) + b*Sinh[a - (b*c)/d]*SinhIntegral[b*(c/d + x)]/d^2
```

**Maple [A]** time = 0.033, size = 133, normalized size = 1.9

$$\frac{be^{-bx-a}}{2d(bdx+cb)} - \frac{b}{2d^2} e^{-\frac{da-cb}{d}} \text{Ei}\left(1, bx+a - \frac{da-cb}{d}\right) - \frac{be^{bx+a}}{2d^2} \left(\frac{cb}{d} + bx\right)^{-1} - \frac{b}{2d^2} e^{\frac{da-cb}{d}} \text{Ei}\left(1, -bx-a - \frac{-da+cb}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(b*x+a)/(d*x+c)^2,x)`

[Out]  $\frac{1}{2}b \exp(-b*x-a)/d/(b*d*x+b*c) - \frac{1}{2}b/d^2 \exp(-(a*d-b*c)/d) * Ei(1, b*x+a-(a*d-b*c)/d) - \frac{1}{2}b/d^2 \exp(b*x+a)/(b*c/d+b*x) - \frac{1}{2}b/d^2 \exp((a*d-b*c)/d) * Ei(1, -b*x-a-(-a*d+b*c)/d)$

**Maxima [A]** time = 1.38578, size = 108, normalized size = 1.52

$$-\frac{b \left( \frac{e^{\left(-a+\frac{bc}{d}\right)} Ei\left(\frac{(dx+c)b}{d}\right) + e^{\left(\frac{a-bc}{d}\right)} Ei\left(-\frac{(dx+c)b}{d}\right) \right)}{2d} - \frac{\sinh(bx+a)}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)/(d*x+c)^2,x, algorithm="maxima")`

[Out]  $-1/2*b*(e^{(-a+b*c/d)} \exp\_integral\_e(1, (d*x+c)*b/d)/d + e^{(a-b*c/d)} \exp\_integral\_e(1, -(d*x+c)*b/d)/d) / d - \sinh(b*x+a)/((d*x+c)*d)$

**Fricas [B]** time = 2.67794, size = 315, normalized size = 4.44

$$\frac{\left( (bdx+bc) Ei\left(\frac{bdx+bc}{d}\right) + (bdx+bc) Ei\left(-\frac{bdx+bc}{d}\right) \right) \cosh\left(-\frac{bc-ad}{d}\right) - 2d \sinh(bx+a) + \left( (bdx+bc) Ei\left(\frac{bdx+bc}{d}\right) - (bdx+bc) Ei\left(-\frac{bdx+bc}{d}\right) \right) \sinh\left(-\frac{bc-ad}{d}\right)}{2(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)/(d*x+c)^2,x, algorithm="fricas")`

[Out]  $\frac{1}{2} * (((b*d*x + b*c) * Ei((b*d*x + b*c)/d) + (b*d*x + b*c) * Ei(-(b*d*x + b*c)/d)) * \cosh(-(b*c - a*d)/d) - 2*d*\sinh(b*x + a) + ((b*d*x + b*c) * Ei((b*d*x + b*c)/d) - (b*d*x + b*c) * Ei(-(b*d*x + b*c)/d)) * \sinh(-(b*c - a*d)/d)) / (d^3*x + c*d^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)/(d\*x+c)\*\*2,x)

[Out] Timed out

**Giac [B]** time = 1.27503, size = 197, normalized size = 2.77

$$\frac{bdx\text{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)} + bdx\text{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)} + bc\text{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)} + bc\text{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)} - de^{(bx+a)} + de^{(-bx-a)}}{2(d^3x + cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)/(d\*x+c)^2,x, algorithm="giac")

[Out] 1/2\*(b\*d\*x\*Ei((b\*d\*x + b\*c)/d)\*e^(a - b\*c/d) + b\*d\*x\*Ei(-(b\*d\*x + b\*c)/d)\*e^(-a + b\*c/d) + b\*c\*Ei((b\*d\*x + b\*c)/d)\*e^(a - b\*c/d) + b\*c\*Ei(-(b\*d\*x + b\*c)/d)\*e^(-a + b\*c/d) - d\*e^(b\*x + a) + d\*e^(-b\*x - a))/(d^3\*x + c\*d^2)



### 3.7 $\int \frac{\sinh(ax+bx)}{(c+dx)^3} dx$

**Optimal.** Leaf size=104

$$\frac{b^2 \sinh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{2d^3} + \frac{b^2 \cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{2d^3} - \frac{b \cosh(a + bx)}{2d^2(c + dx)} - \frac{\sinh(a + bx)}{2d(c + dx)^2}$$

[Out]  $-(b*\text{Cosh}[a + b*x])/(2*d^2*(c + d*x)) + (b^2*\text{CoshIntegral}[(b*c)/d + b*x]*\text{Sin}h[a - (b*c)/d])/(2*d^3) - \text{Sinh}[a + b*x]/(2*d*(c + d*x)^2) + (b^2*\text{Cosh}[a - (b*c)/d]*\text{SinhIntegral}[(b*c)/d + b*x])/(2*d^3)$

**Rubi [A]** time = 0.166022, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3297, 3303, 3298, 3301}

$$\frac{b^2 \sinh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{2d^3} + \frac{b^2 \cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{2d^3} - \frac{b \cosh(a + bx)}{2d^2(c + dx)} - \frac{\sinh(a + bx)}{2d(c + dx)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sinh}[a + b*x]/(c + d*x)^3, x]$

[Out]  $-(b*\text{Cosh}[a + b*x])/(2*d^2*(c + d*x)) + (b^2*\text{CoshIntegral}[(b*c)/d + b*x]*\text{Sin}h[a - (b*c)/d])/(2*d^3) - \text{Sinh}[a + b*x]/(2*d*(c + d*x)^2) + (b^2*\text{Cosh}[a - (b*c)/d]*\text{SinhIntegral}[(b*c)/d + b*x])/(2*d^3)$

#### Rule 3297

$\text{Int}[\frac{(c + d*x)^m \sin[e + f*x]}{(c + d*x)^{m+1}}, x] := \text{Simp}[\frac{\sin[e + f*x]}{d}, \text{Int}[\frac{\cos[e + f*x]}{c + d*x}, x]] - \text{Dist}[\frac{f}{d}, \text{Int}[\frac{\cos[e + f*x]}{c + d*x}, x]]$  ; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

#### Rule 3303

$\text{Int}[\frac{\sin[e + f*x]}{(c + d*x)}, x] := \text{Dist}[\frac{\cos[e + f*x]}{d}, \text{Int}[\frac{\sin[e + f*x]}{c + d*x}, x]] + \text{Dist}[\frac{\sin[e + f*x]}{d}, \text{Int}[\frac{\cos[e + f*x]}{c + d*x}, x]]$  ; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sinh(a+bx)}{(c+dx)^3} dx &= -\frac{\sinh(a+bx)}{2d(c+dx)^2} + \frac{b \int \frac{\cosh(a+bx)}{(c+dx)^2} dx}{2d} \\ &= -\frac{b \cosh(a+bx)}{2d^2(c+dx)} - \frac{\sinh(a+bx)}{2d(c+dx)^2} + \frac{b^2 \int \frac{\sinh(a+bx)}{c+dx} dx}{2d^2} \\ &= -\frac{b \cosh(a+bx)}{2d^2(c+dx)} - \frac{\sinh(a+bx)}{2d(c+dx)^2} + \frac{\left(b^2 \cosh\left(a - \frac{bc}{d}\right)\right) \int \frac{\sinh\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{2d^2} + \frac{\left(b^2 \sinh\left(a - \frac{bc}{d}\right)\right) \int \frac{\cosh\left(\frac{bc}{d} + bx\right)}{c+dx} dx}{2d^2} \\ &= -\frac{b \cosh(a+bx)}{2d^2(c+dx)} + \frac{b^2 \text{Chi}\left(\frac{bc}{d} + bx\right) \sinh\left(a - \frac{bc}{d}\right)}{2d^3} - \frac{\sinh(a+bx)}{2d(c+dx)^2} + \frac{b^2 \cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{2d^3} \end{aligned}$$

**Mathematica [A]** time = 0.558116, size = 88, normalized size = 0.85

$$\frac{b^2 \sinh\left(a - \frac{bc}{d}\right) \text{Chi}\left(b\left(\frac{c}{d} + x\right)\right) + b^2 \cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(b\left(\frac{c}{d} + x\right)\right) - \frac{d(b(c+dx) \cosh(a+bx) + d \sinh(a+bx))}{(c+dx)^2}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]/(c + d*x)^3, x]
```

```
[Out] (b^2*CoshIntegral[b*(c/d + x)]*Sinh[a - (b*c)/d] - (d*(b*(c + d*x)*Cosh[a + b*x] + d*Sinh[a + b*x]))/(c + d*x)^2 + b^2*Cosh[a - (b*c)/d]*SinhIntegral[b*(c/d + x)]/(2*d^3)
```

**Maple [B]** time = 0.039, size = 277, normalized size = 2.7

$$\frac{b^3 e^{-bx-a} x}{4d(b^2 d^2 x^2 + 2b^2 cdx + c^2 b^2)} - \frac{b^3 e^{-bx-a} c}{4d^2(b^2 d^2 x^2 + 2b^2 cdx + c^2 b^2)} + \frac{b^2 e^{-bx-a}}{4d(b^2 d^2 x^2 + 2b^2 cdx + c^2 b^2)} + \frac{b^2}{4d^3} e^{-\frac{da-cb}{d}} \text{Ei}\left(1, bx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(b*x+a)/(d*x+c)^3,x)`

[Out] 
$$-1/4*b^3*\exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*x-1/4*b^3*\exp(-b*x-a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c+1/4*b^2*\exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)+1/4*b^2/d^3*\exp(-(a*d-b*c)/d)*\text{Ei}(1,b*x+a-(a*d-b*c)/d)-1/4*b^2/d^3*\exp(b*x+a)/(b*c/d+b*x)^2-1/4*b^2/d^3*\exp(b*x+a)/(b*c/d+b*x)-1/4*b^2/d^3*\exp((a*d-b*c)/d)*\text{Ei}(1,-b*x-a-(-a*d+b*c)/d)$$

**Maxima [A]** time = 1.38369, size = 127, normalized size = 1.22

$$b \left( \frac{e^{(-a+\frac{bc}{d})} E_2\left(\frac{(dx+c)b}{d}\right)}{(dx+c)d} + \frac{e^{(a-\frac{bc}{d})} E_2\left(-\frac{(dx+c)b}{d}\right)}{(dx+c)d} \right) - \frac{\sinh(bx+a)}{2(dx+c)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)/(d*x+c)^3,x, algorithm="maxima")`

[Out] 
$$-1/4*b*(e^{(-a+b*c/d)}*\exp\_integral\_e(2,(d*x+c)*b/d)/((d*x+c)*d)+e^{(a-b*c/d)}*\exp\_integral\_e(2,-(d*x+c)*b/d)/((d*x+c)*d)/d-1/2*\sinh(b*x+a)/((d*x+c)^2*d)$$

**Fricas [B]** time = 2.60996, size = 518, normalized size = 4.98

$$\frac{2d^2 \sinh(bx+a) + 2(bd^2x + bcd) \cosh(bx+a) - \left( (b^2d^2x^2 + 2b^2cdx + b^2c^2) \text{Ei}\left(\frac{bdx+bc}{d}\right) - (b^2d^2x^2 + 2b^2cdx + b^2c^2) \right)}{4(d^5x^2 - \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)/(d*x+c)^3,x, algorithm="fricas")`

[Out]  $-1/4*(2*d^2*\sinh(b*x + a) + 2*(b*d^2*x + b*c*d)*\cosh(b*x + a) - ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\text{Ei}((b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\text{Ei}(-(b*d*x + b*c)/d))*\cosh(-(b*c - a*d)/d) - ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\text{Ei}((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*\text{Ei}(-(b*d*x + b*c)/d))*\sinh(-(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)/(d*x+c)**3,x)`

[Out] Timed out

**Giac [B]** time = 1.23032, size = 406, normalized size = 3.9

$$\frac{b^2 d^2 x^2 \text{Ei}\left(\frac{bdx+bc}{d}\right) e^{\left(a-\frac{bc}{d}\right)} - b^2 d^2 x^2 \text{Ei}\left(-\frac{bdx+bc}{d}\right) e^{\left(-a+\frac{bc}{d}\right)} + 2 b^2 c d x \text{Ei}\left(\frac{bdx+bc}{d}\right) e^{\left(a-\frac{bc}{d}\right)} - 2 b^2 c d x \text{Ei}\left(-\frac{bdx+bc}{d}\right) e^{\left(-a+\frac{bc}{d}\right)} + b^2 c^2}{4(d^5 x^2 + 2c d^4 x + c^2 d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)/(d*x+c)^3,x, algorithm="giac")`

[Out]  $1/4*(b^2*d^2*x^2*\text{Ei}((b*d*x + b*c)/d)*e^{(a - b*c/d)} - b^2*d^2*x^2*\text{Ei}(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 2*b^2*c*d*x*\text{Ei}((b*d*x + b*c)/d)*e^{(a - b*c/d)} - 2*b^2*c*d*x*\text{Ei}(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + b^2*c^2*\text{Ei}((b*d*x + b*c)/d)*e^{(a - b*c/d)} - b^2*c^2*\text{Ei}(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - b*d^2*x*e^{(b*x + a)} - b*d^2*x*e^{(-b*x - a)} - b*c*d*e^{(b*x + a)} - b*c*d*e^{(-b*x - a)} - d^2*e^{(b*x + a)} + d^2*e^{(-b*x - a)})/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)$

### 3.8 $\int (c + dx)^4 \sinh^2(a + bx) dx$

**Optimal.** Leaf size=162

$$-\frac{3d^3(c + dx) \sinh^2(a + bx)}{2b^4} + \frac{3d^2(c + dx)^2 \sinh(a + bx) \cosh(a + bx)}{2b^3} - \frac{d(c + dx)^3 \sinh^2(a + bx)}{b^2} + \frac{3d^4 \sinh(a + bx) \cosh(a + bx)}{4b^5}$$

[Out]  $(-3*d^4*x)/(4*b^4) - (d*(c + d*x)^3)/(2*b^2) - (c + d*x)^5/(10*d) + (3*d^4*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^5) + (3*d^2*(c + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x])/(2*b^3) + ((c + d*x)^4*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (3*d^3*(c + d*x)*Sinh[a + b*x]^2)/(2*b^4) - (d*(c + d*x)^3*Sinh[a + b*x]^2)/b^2$

**Rubi [A]** time = 0.104895, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3311, 32, 2635, 8}

$$-\frac{3d^3(c + dx) \sinh^2(a + bx)}{2b^4} + \frac{3d^2(c + dx)^2 \sinh(a + bx) \cosh(a + bx)}{2b^3} - \frac{d(c + dx)^3 \sinh^2(a + bx)}{b^2} + \frac{3d^4 \sinh(a + bx) \cosh(a + bx)}{4b^5}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^4\*Sinh[a + b\*x]^2,x]

[Out]  $(-3*d^4*x)/(4*b^4) - (d*(c + d*x)^3)/(2*b^2) - (c + d*x)^5/(10*d) + (3*d^4*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^5) + (3*d^2*(c + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x])/(2*b^3) + ((c + d*x)^4*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (3*d^3*(c + d*x)*Sinh[a + b*x]^2)/(2*b^4) - (d*(c + d*x)^3*Sinh[a + b*x]^2)/b^2$

#### Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sinh[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sinh[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sinh[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cosh[e + f*x]*(b*Sinh[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

#### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

### Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

### Rubi steps

$$\begin{aligned} \int (c + dx)^4 \sinh^2(a + bx) dx &= \frac{(c + dx)^4 \cosh(a + bx) \sinh(a + bx)}{2b} - \frac{d(c + dx)^3 \sinh^2(a + bx)}{b^2} - \frac{1}{2} \int (c + dx)^4 dx + \frac{(3d^2(c + dx)^2 \cosh(a + bx) \sinh(a + bx))}{2b} \\ &= -\frac{(c + dx)^5}{10d} + \frac{3d^2(c + dx)^2 \cosh(a + bx) \sinh(a + bx)}{2b^3} + \frac{(c + dx)^4 \cosh(a + bx) \sinh(a + bx)}{2b} \\ &= -\frac{d(c + dx)^3}{2b^2} - \frac{(c + dx)^5}{10d} + \frac{3d^4 \cosh(a + bx) \sinh(a + bx)}{4b^5} + \frac{3d^2(c + dx)^2 \cosh(a + bx) \sinh(a + bx)}{2b^3} \\ &= -\frac{3d^4 x}{4b^4} - \frac{d(c + dx)^3}{2b^2} - \frac{(c + dx)^5}{10d} + \frac{3d^4 \cosh(a + bx) \sinh(a + bx)}{4b^5} + \frac{3d^2(c + dx)^2 \cosh(a + bx) \sinh(a + bx)}{2b^3} \end{aligned}$$

**Mathematica [A]** time = 0.681253, size = 132, normalized size = 0.81

$$\frac{10 \sinh(2(a + bx)) (6b^2 d^2 (c + dx)^2 + 2b^4 (c + dx)^4 + 3d^4) - 20bd(c + dx) \cosh(2(a + bx)) (2b^2 (c + dx)^2 + 3d^2) - 8b^5 x (10d^2 (c + dx)^2 + 2b^4 (c + dx)^4 + 3d^4)}{80b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sinh[a + b*x]^2,x]
```

```
[Out] (-8*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) - 20*b*d*(c + d*x)*(3*d^2 + 2*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)] + 10*(3*d^4 + 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sinh[2*(a + b*x)]/(80*b^5)
```

**Maple [B]** time = 0.016, size = 910, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^4*sinh(b*x+a)^2,x)`

[Out] 
$$\frac{1}{b} \left( \frac{12}{b^3} d^3 c^3 a^2 \left( \frac{1}{2} (b*x+a) \cosh(b*x+a) \sinh(b*x+a) - \frac{1}{4} (b*x+a)^2 - \frac{1}{4} \cosh(b*x+a)^2 \right) - \frac{12}{b^3} d^3 c^3 a \left( \frac{1}{2} (b*x+a)^2 \cosh(b*x+a) \sinh(b*x+a) - \frac{1}{6} (b*x+a)^3 - \frac{1}{2} (b*x+a) \cosh(b*x+a)^2 + \frac{1}{4} \cosh(b*x+a) \sinh(b*x+a) + \frac{1}{4} b*x + \frac{1}{4} a \right) + \frac{6}{b^4} d^4 a^2 \left( \frac{1}{2} (b*x+a)^2 \cosh(b*x+a) \sinh(b*x+a) - \frac{1}{6} (b*x+a)^3 - \frac{1}{2} (b*x+a) \cosh(b*x+a)^2 + \frac{1}{4} \cosh(b*x+a) \sinh(b*x+a) + \frac{1}{4} b*x + \frac{1}{4} a \right) - \frac{4}{b^4} d^4 a^3 \left( \frac{1}{2} (b*x+a) \cosh(b*x+a) \sinh(b*x+a) - \frac{1}{4} (b*x+a)^2 - \frac{1}{4} \cosh(b*x+a)^2 \right) + \frac{4}{b^3} d^3 c^3 \left( \frac{1}{2} (b*x+a)^3 \cosh(b*x+a) \sinh(b*x+a) - \frac{1}{8} (b*x+a)^4 - \frac{3}{4} (b*x+a)^2 \cosh(b*x+a)^2 + \frac{3}{4} (b*x+a) \cosh(b*x+a) \sinh(b*x+a) + \frac{3}{8} (b*x+a)^2 - \frac{3}{8} \cosh(b*x+a)^2 \right) + \frac{6}{b^2} d^2 c^2 \left( \frac{1}{2} (b*x+a)^2 \cosh(b*x+a) \sinh(b*x+a) - \frac{1}{6} (b*x+a)^3 - \frac{1}{2} (b*x+a) \cosh(b*x+a)^2 + \frac{1}{4} \cosh(b*x+a) \sinh(b*x+a) + \frac{1}{4} b*x + \frac{1}{4} a \right) + \frac{4}{b} d c^3 \left( \frac{1}{2} (b*x+a) \cosh(b*x+a) \sinh(b*x+a) - \frac{1}{4} (b*x+a)^2 - \frac{1}{4} \cosh(b*x+a)^2 \right) - \frac{4}{b^3} d^3 a^3 c^3 \left( \frac{1}{2} \cosh(b*x+a) \sinh(b*x+a) - \frac{1}{2} b*x - \frac{1}{2} a \right) + \frac{6}{b^2} d^2 a^2 c^2 \left( \frac{1}{2} \cosh(b*x+a) \sinh(b*x+a) - \frac{1}{2} b*x - \frac{1}{2} a \right) - \frac{4}{b} d a c^3 \left( \frac{1}{2} \cosh(b*x+a) \sinh(b*x+a) - \frac{1}{2} b*x - \frac{1}{2} a \right) - \frac{4}{b^4} d^4 a^4 \left( \frac{1}{2} (b*x+a)^3 \cosh(b*x+a) \sinh(b*x+a) - \frac{1}{8} (b*x+a)^4 - \frac{3}{4} (b*x+a)^2 \cosh(b*x+a)^2 + \frac{3}{4} (b*x+a) \cosh(b*x+a) \sinh(b*x+a) + \frac{3}{8} (b*x+a)^2 - \frac{3}{8} \cosh(b*x+a)^2 \right) + \frac{1}{b^4} d^4 \left( \frac{1}{2} (b*x+a)^4 \cosh(b*x+a) \sinh(b*x+a) - \frac{1}{10} (b*x+a)^5 - (b*x+a)^3 \cosh(b*x+a)^2 + \frac{3}{2} (b*x+a)^2 \cosh(b*x+a) \sinh(b*x+a) + \frac{1}{2} (b*x+a)^3 - \frac{3}{2} (b*x+a) \cosh(b*x+a)^2 + \frac{3}{4} \cosh(b*x+a) \sinh(b*x+a) + \frac{3}{4} b*x + \frac{3}{4} a \right) + \frac{1}{b^4} d^4 a^4 \left( \frac{1}{2} \cosh(b*x+a) \sinh(b*x+a) - \frac{1}{2} b*x - \frac{1}{2} a \right) - \frac{12}{b^2} d^2 c^2 a \left( \frac{1}{2} (b*x+a) \cosh(b*x+a) \sinh(b*x+a) - \frac{1}{4} (b*x+a)^2 - \frac{1}{4} \cosh(b*x+a)^2 \right) + c^4 \left( \frac{1}{2} \cosh(b*x+a) \sinh(b*x+a) - \frac{1}{2} b*x - \frac{1}{2} a \right) \right)$$

**Maxima [B]** time = 1.2571, size = 516, normalized size = 3.19

$$-\frac{1}{4} \left( 4x^2 - \frac{(2bx e^{2a}) - e^{2a}}{b^2} e^{2bx} + \frac{(2bx + 1)e^{(-2bx - 2a)}}{b^2} \right) c^3 d - \frac{1}{8} \left( 8x^3 - \frac{3(2b^2 x^2 e^{2a}) - 2bx e^{2a} + e^{2a}}{b^3} e^{2bx} + \frac{3(2bx - 2a)}{b^2} e^{(-2bx - 2a)} \right) c^3 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^4*sinh(b*x+a)^2,x, algorithm="maxima")`

[Out] 
$$-\frac{1}{4} (4x^2 - (2bx e^{2a}) - e^{2a}) e^{2bx} / b^2 + (2bx + 1) e^{(-2bx - 2a)} / b^2 * c^3 d - \frac{1}{8} (8x^3 - 3(2b^2 x^2 e^{2a}) - 2bx e^{2a} - 2bx e^{2a} +$$

$$e^{(2*a)}*e^{(2*b*x)}/b^3 + 3*(2*b^2*x^2 + 2*b*x + 1)*e^{(-2*b*x - 2*a)}/b^3*c^2*d^2 - 1/8*(4*x^4 - (4*b^3*x^3*e^{(2*a)} - 6*b^2*x^2*e^{(2*a)} + 6*b*x*e^{(2*a)} - 3*e^{(2*a)})*e^{(2*b*x)}/b^4 + (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^{(-2*b*x - 2*a)}/b^4)*c*d^3 - 1/80*(8*x^5 - 5*(2*b^4*x^4*e^{(2*a)} - 4*b^3*x^3*e^{(2*a)} + 6*b^2*x^2*e^{(2*a)} - 6*b*x*e^{(2*a)} + 3*e^{(2*a)})*e^{(2*b*x)}/b^5 + 5*(2*b^4*x^4 + 4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^{(-2*b*x - 2*a)}/b^5)*d^4 - 1/8*c^4*(4*x - e^{(2*b*x + 2*a)}/b + e^{(-2*b*x - 2*a)}/b)$$

**Fricas [B]** time = 2.68257, size = 663, normalized size = 4.09

$$2b^5d^4x^5 + 10b^5cd^3x^4 + 20b^5c^2d^2x^3 + 20b^5c^3dx^2 + 10b^5c^4x + 5(2b^3d^4x^3 + 6b^3cd^3x^2 + 2b^3c^3d + 3bcd^3 + 3(2b^3c^2d^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^4\*sinh(b\*x+a)^2,x, algorithm="fricas")

[Out]  $-1/20*(2*b^5*d^4*x^5 + 10*b^5*c*d^3*x^4 + 20*b^5*c^2*d^2*x^3 + 20*b^5*c^3*d*x^2 + 10*b^5*c^4*x + 5*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d + 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 + b*d^4)*x)*\cosh(b*x + a)^2 - 5*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^2*d^2 + 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 + b^2*d^4)*x^2 + 4*(2*b^4*c^3*d + 3*b^2*c*d^3)*x)*\cosh(b*x + a)*\sinh(b*x + a) + 5*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d + 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 + b*d^4)*x)*\sinh(b*x + a)^2)/b^5$

**Sympy [A]** time = 7.34425, size = 660, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*4\*sinh(b\*x+a)\*\*2,x)

[Out]  $\text{Piecewise}((c**4*x*\sinh(a + b*x)**2/2 - c**4*x*\cosh(a + b*x)**2/2 + c**3*d*x**2*\sinh(a + b*x)**2 - c**3*d*x**2*\cosh(a + b*x)**2 + c**2*d**2*x**3*\sinh(a + b*x)**2 - c**2*d**2*x**3*\cosh(a + b*x)**2 + c*d**3*x**4*\sinh(a + b*x)**2/2 - c*d**3*x**4*\cosh(a + b*x)**2/2 + d**4*x**5*\sinh(a + b*x)**2/10 - d**4*x**5*\cosh(a + b*x)**2/10 + c**4*\sinh(a + b*x)*\cosh(a + b*x)/(2*b) + 2*c**3*d*x*\sinh(a + b*x)*\cosh(a + b*x)/b + 3*c**2*d**2*x**2*\sinh(a + b*x)*\cosh(a +$



```

b*x)/b + 2*c*d**3*x**3*sinh(a + b*x)*cosh(a + b*x)/b + d**4*x**4*sinh(a +
b*x)*cosh(a + b*x)/(2*b) - c**3*d*sinh(a + b*x)**2/b**2 - 3*c**2*d**2*x*sin
h(a + b*x)**2/(2*b**2) - 3*c**2*d**2*x*cosh(a + b*x)**2/(2*b**2) - 3*c*d**3
*x**2*sinh(a + b*x)**2/(2*b**2) - 3*c*d**3*x**2*cosh(a + b*x)**2/(2*b**2) -
d**4*x**3*sinh(a + b*x)**2/(2*b**2) - d**4*x**3*cosh(a + b*x)**2/(2*b**2)
+ 3*c**2*d**2*sinh(a + b*x)*cosh(a + b*x)/(2*b**3) + 3*c*d**3*x*sinh(a + b*
x)*cosh(a + b*x)/b**3 + 3*d**4*x**2*sinh(a + b*x)*cosh(a + b*x)/(2*b**3) -
3*c*d**3*sinh(a + b*x)**2/(2*b**4) - 3*d**4*x*sinh(a + b*x)**2/(4*b**4) - 3
*d**4*x*cosh(a + b*x)**2/(4*b**4) + 3*d**4*x*sinh(a + b*x)*cosh(a + b*x)/(4*b
**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4
+ d**4*x**5/5)*sinh(a)**2, True))

```

**Giac [B]** time = 1.24048, size = 505, normalized size = 3.12

$$-\frac{1}{10}d^4x^5 - \frac{1}{2}cd^3x^4 - c^2d^2x^3 - c^3dx^2 - \frac{1}{2}c^4x + \frac{(2b^4d^4x^4 + 8b^4cd^3x^3 + 12b^4c^2d^2x^2 - 4b^3d^4x^3 + 8b^4c^3dx - 12b^3cd^3x^2)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^4\*sinh(b\*x+a)^2,x, algorithm="giac")

```

[Out] -1/10*d^4*x^5 - 1/2*c*d^3*x^4 - c^2*d^2*x^3 - c^3*d*x^2 - 1/2*c^4*x + 1/16*
(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 8*b
^4*c^3*d*x - 12*b^3*c*d^3*x^2 + 2*b^4*c^4 - 12*b^3*c^2*d^2*x + 6*b^2*d^4*x^
2 - 4*b^3*c^3*d + 12*b^2*c*d^3*x + 6*b^2*c^2*d^2 - 6*b*d^4*x - 6*b*c*d^3 +
3*d^4)*e^(2*b*x + 2*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4
*c^2*d^2*x^2 + 4*b^3*d^4*x^3 + 8*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + 2*b^4*c^4
+ 12*b^3*c^2*d^2*x + 6*b^2*d^4*x^2 + 4*b^3*c^3*d + 12*b^2*c*d^3*x + 6*b^2*
c^2*d^2 + 6*b*d^4*x + 6*b*c*d^3 + 3*d^4)*e^(-2*b*x - 2*a)/b^5

```

### 3.9 $\int (c + dx)^3 \sinh^2(a + bx) dx$

**Optimal.** Leaf size=134

$$\frac{3d^2(c + dx) \sinh(a + bx) \cosh(a + bx)}{4b^3} - \frac{3d(c + dx)^2 \sinh^2(a + bx)}{4b^2} - \frac{3d^3 \sinh^2(a + bx)}{8b^4} + \frac{(c + dx)^3 \sinh(a + bx) \cosh(a + bx)}{2b}$$

[Out]  $(-3*c*d^2*x)/(4*b^2) - (3*d^3*x^2)/(8*b^2) - (c + d*x)^4/(8*d) + (3*d^2*(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^3) + ((c + d*x)^3*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (3*d^3*Sinh[a + b*x]^2)/(8*b^4) - (3*d*(c + d*x)^2*Sinh[a + b*x]^2)/(4*b^2)$

**Rubi [A]** time = 0.0742048, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {3311, 32, 3310}

$$\frac{3d^2(c + dx) \sinh(a + bx) \cosh(a + bx)}{4b^3} - \frac{3d(c + dx)^2 \sinh^2(a + bx)}{4b^2} - \frac{3d^3 \sinh^2(a + bx)}{8b^4} + \frac{(c + dx)^3 \sinh(a + bx) \cosh(a + bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^3\*Sinh[a + b\*x]^2,x]

[Out]  $(-3*c*d^2*x)/(4*b^2) - (3*d^3*x^2)/(8*b^2) - (c + d*x)^4/(8*d) + (3*d^2*(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^3) + ((c + d*x)^3*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (3*d^3*Sinh[a + b*x]^2)/(8*b^4) - (3*d*(c + d*x)^2*Sinh[a + b*x]^2)/(4*b^2)$

#### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*Sinh[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*Sinh[e + f\*x])^(n - 2), x], x] - Dist[(d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*Sinh[e + f\*x])^n, x], x] - Simp[(b\*(c + d\*x)^m\*Cosh[e + f\*x]\*(b\*Sinh[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

#### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
  Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^3 \sinh^2(a + bx) dx &= \frac{(c + dx)^3 \cosh(a + bx) \sinh(a + bx)}{2b} - \frac{3d(c + dx)^2 \sinh^2(a + bx)}{4b^2} - \frac{1}{2} \int (c + dx)^3 dx + \dots \\ &= -\frac{(c + dx)^4}{8d} + \frac{3d^2(c + dx) \cosh(a + bx) \sinh(a + bx)}{4b^3} + \frac{(c + dx)^3 \cosh(a + bx) \sinh(a + bx)}{2b} \\ &= -\frac{3cd^2x}{4b^2} - \frac{3d^3x^2}{8b^2} - \frac{(c + dx)^4}{8d} + \frac{3d^2(c + dx) \cosh(a + bx) \sinh(a + bx)}{4b^3} + \frac{(c + dx)^3 \cosh(a + bx) \sinh(a + bx)}{2b} \end{aligned}$$

**Mathematica [A]** time = 0.403, size = 104, normalized size = 0.78

$$\frac{2b(c + dx) \sinh(2(a + bx)) (2b^2(c + dx)^2 + 3d^2) - 3d \cosh(2(a + bx)) (2b^2(c + dx)^2 + d^2) - 2b^4x (6c^2dx + 4c^3 + 4cd^2x^2)}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sinh[a + b*x]^2,x]
```

```
[Out] (-2*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 3*d*(d^2 + 2*b^2*(c
+ d*x)^2)*Cosh[2*(a + b*x)] + 2*b*(c + d*x)*(3*d^2 + 2*b^2*(c + d*x)^2)*Si
nh[2*(a + b*x)])/(16*b^4)
```

**Maple [B]** time = 0.007, size = 523, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sinh(b*x+a)^2,x)
```

[Out]  $\frac{1}{b} \left( \frac{1}{b^3 d^3} \left( \frac{1}{2} (bx+a)^3 \cosh(bx+a) \sinh(bx+a) - \frac{1}{8} (bx+a)^4 - \frac{3}{4} (bx+a)^2 \cosh(bx+a)^2 + \frac{3}{4} (bx+a) \cosh(bx+a) \sinh(bx+a) + \frac{3}{8} (bx+a)^2 - \frac{3}{8} \cosh(bx+a)^2 \right) - \frac{3}{b^3 d^3 a} \left( \frac{1}{2} (bx+a)^2 \cosh(bx+a) \sinh(bx+a) - \frac{1}{6} (bx+a)^3 - \frac{1}{2} (bx+a) \cosh(bx+a)^2 + \frac{1}{4} \cosh(bx+a) \sinh(bx+a) + \frac{1}{4} bx + \frac{1}{4} a \right) + \frac{3}{b^3 d^3 a^2} \left( \frac{1}{2} (bx+a) \cosh(bx+a) \sinh(bx+a) - \frac{1}{4} (bx+a)^2 - \frac{1}{4} \cosh(bx+a)^2 \right) - \frac{1}{b^3 d^3 a^3} \left( \frac{1}{2} \cosh(bx+a) \sinh(bx+a) - \frac{1}{2} bx - \frac{1}{2} a \right) + \frac{3}{b^2 c d^2} \left( \frac{1}{2} (bx+a)^2 \cosh(bx+a) \sinh(bx+a) - \frac{1}{6} (bx+a)^3 - \frac{1}{2} (bx+a) \cosh(bx+a)^2 + \frac{1}{4} \cosh(bx+a) \sinh(bx+a) + \frac{1}{4} bx + \frac{1}{4} a \right) - \frac{6}{b^2 c d^2 a} \left( \frac{1}{2} (bx+a) \cosh(bx+a) \sinh(bx+a) - \frac{1}{4} (bx+a)^2 - \frac{1}{4} \cosh(bx+a)^2 \right) + \frac{3}{b^2 c d^2 a^2} \left( \frac{1}{2} \cosh(bx+a) \sinh(bx+a) - \frac{1}{2} bx - \frac{1}{2} a \right) + \frac{3}{b c^2 d} \left( \frac{1}{2} (bx+a) \cosh(bx+a) \sinh(bx+a) - \frac{1}{4} (bx+a)^2 - \frac{1}{4} \cosh(bx+a)^2 \right) - \frac{3}{b c^2 d a} \left( \frac{1}{2} \cosh(bx+a) \sinh(bx+a) - \frac{1}{2} bx - \frac{1}{2} a \right) + c^3 \left( \frac{1}{2} \cosh(bx+a) \sinh(bx+a) - \frac{1}{2} bx - \frac{1}{2} a \right) \right)$

**Maxima [B]** time = 1.14597, size = 355, normalized size = 2.65

$$-\frac{3}{16} \left( 4x^2 - \frac{(2bx e^{(2a)} - e^{(2a)}) e^{(2bx)}}{b^2} + \frac{(2bx+1) e^{(-2bx-2a)}}{b^2} \right) c^2 d - \frac{1}{16} \left( 8x^3 - \frac{3(2b^2 x^2 e^{(2a)} - 2bx e^{(2a)} + e^{(2a)}) e^{(2bx)}}{b^3} + \frac{3(2bx+1) e^{(-2bx-2a)}}{b^3} \right) c^2 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*sinh(b\*x+a)^2,x, algorithm="maxima")

[Out]  $-3/16 * (4*x^2 - (2*b*x*e^{(2*a)} - e^{(2*a)}) * e^{(2*b*x)} / b^2 + (2*b*x + 1) * e^{(-2*b*x - 2*a)} / b^2) * c^2 * d - 1/16 * (8*x^3 - 3 * (2*b^2*x^2*e^{(2*a)} - 2*b*x*e^{(2*a)} + e^{(2*a)}) * e^{(2*b*x)} / b^3 + 3 * (2*b^2*x^2 + 2*b*x + 1) * e^{(-2*b*x - 2*a)} / b^3) * c^2 * d - 1/32 * (4*x^4 - (4*b^3*x^3*e^{(2*a)} - 6*b^2*x^2*e^{(2*a)} + 6*b*x*e^{(2*a)} - 3*e^{(2*a)}) * e^{(2*b*x)} / b^4 + (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3) * e^{(-2*b*x - 2*a)} / b^4) * d^3 - 1/8 * c^3 * (4*x - e^{(2*b*x + 2*a)} / b + e^{(-2*b*x - 2*a)} / b)$

**Fricas [A]** time = 2.70187, size = 459, normalized size = 3.43

$$\frac{2b^4 d^3 x^4 + 8b^4 c d^2 x^3 + 12b^4 c^2 d x^2 + 8b^4 c^3 x + 3(2b^2 d^3 x^2 + 4b^2 c d^2 x + 2b^2 c^2 d + d^3) \cosh(bx+a)^2 - 4(2b^3 d^3 x^3 + 6b^3 c d^2 x^2 + 6b^3 c^2 d x + 6b^3 c^3) \sinh(bx+a)^2}{b^4 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*sinh(b\*x+a)^2,x, algorithm="fricas")

```
[Out] -1/16*(2*b^4*d^3*x^4 + 8*b^4*c*d^2*x^3 + 12*b^4*c^2*d*x^2 + 8*b^4*c^3*x + 3
*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*cosh(b*x + a)^2 - 4*(2
*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^2*d + 3*b*c*d^2 + 3*(2*b^3*c^2*d + b
*d^3)*x)*cosh(b*x + a)*sinh(b*x + a) + 3*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2
*b^2*c^2*d + d^3)*sinh(b*x + a)^2)/b^4
```

**Sympy [A]** time = 3.79951, size = 456, normalized size = 3.4

$$\left\{ \frac{c^3 x \sinh^2(a+bx)}{2} - \frac{c^3 x \cosh^2(a+bx)}{2} + \frac{3c^2 dx^2 \sinh^2(a+bx)}{4} - \frac{3c^2 dx^2 \cosh^2(a+bx)}{4} + \frac{cd^2 x^3 \sinh^2(a+bx)}{2} - \frac{cd^2 x^3 \cosh^2(a+bx)}{2} + \frac{d^3 x^4 \sinh^2(a+bx)}{8} \right. \\ \left. \left( c^3 x + \frac{3c^2 dx^2}{2} + cd^2 x^3 + \frac{d^3 x^4}{4} \right) \sinh^2(a) \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sinh(b*x+a)**2,x)
```

```
[Out] Piecewise((c**3*x*sinh(a + b*x)**2/2 - c**3*x*cosh(a + b*x)**2/2 + 3*c**2*d
*x**2*sinh(a + b*x)**2/4 - 3*c**2*d*x**2*cosh(a + b*x)**2/4 + c*d**2*x**3*s
inh(a + b*x)**2/2 - c*d**2*x**3*cosh(a + b*x)**2/2 + d**3*x**4*sinh(a + b*x
)**2/8 - d**3*x**4*cosh(a + b*x)**2/8 + c**3*sinh(a + b*x)*cosh(a + b*x)/(2
*b) + 3*c**2*d*x*sinh(a + b*x)*cosh(a + b*x)/(2*b) + 3*c*d**2*x**2*sinh(a +
b*x)*cosh(a + b*x)/(2*b) + d**3*x**3*sinh(a + b*x)*cosh(a + b*x)/(2*b) - 3
*c**2*d*sinh(a + b*x)**2/(4*b**2) - 3*c*d**2*x*sinh(a + b*x)**2/(4*b**2) -
3*c*d**2*x*cosh(a + b*x)**2/(4*b**2) - 3*d**3*x**2*sinh(a + b*x)**2/(8*b**2
) - 3*d**3*x**2*cosh(a + b*x)**2/(8*b**2) + 3*c*d**2*sinh(a + b*x)*cosh(a +
b*x)/(4*b**3) + 3*d**3*x*sinh(a + b*x)*cosh(a + b*x)/(4*b**3) - 3*d**3*sin
h(a + b*x)**2/(8*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3
+ d**3*x**4/4)*sinh(a)**2, True))
```

**Giac [B]** time = 1.2579, size = 328, normalized size = 2.45

$$-\frac{1}{8}d^3x^4 - \frac{1}{2}cd^2x^3 - \frac{3}{4}c^2dx^2 - \frac{1}{2}c^3x + \frac{(4b^3d^3x^3 + 12b^3cd^2x^2 + 12b^3c^2dx - 6b^2d^3x^2 + 4b^3c^3 - 12b^2cd^2x - 6b^2c^2d + c^3d^3)}{32b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*d^3*x^4 - 1/2*c*d^2*x^3 - 3/4*c^2*d*x^2 - 1/2*c^3*x + 1/32*(4*b^3*d^3*
x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x - 6*b^2*d^3*x^2 + 4*b^3*c^3 - 12*b^2
```

$$\begin{aligned} & 2*c*d^2*x - 6*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 3*d^3)*e^{(2*b*x + 2*a)/b^4} \\ & - 1/32*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 6*b^2*d^3*x^2 \\ & + 4*b^3*c^3 + 12*b^2*c*d^2*x + 6*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 3*d^3) \\ & )*e^{(-2*b*x - 2*a)/b^4} \end{aligned}$$

### 3.10 $\int (c + dx)^2 \sinh^2(a + bx) dx$

**Optimal.** Leaf size=95

$$-\frac{d(c + dx) \sinh^2(a + bx)}{2b^2} + \frac{d^2 \sinh(a + bx) \cosh(a + bx)}{4b^3} + \frac{(c + dx)^2 \sinh(a + bx) \cosh(a + bx)}{2b} - \frac{d^2 x}{4b^2} - \frac{(c + dx)^3}{6d}$$

[Out]  $-(d^2*x)/(4*b^2) - (c + d*x)^3/(6*d) + (d^2*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^3) + ((c + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (d*(c + d*x)*Sinh[a + b*x]^2)/(2*b^2)$

**Rubi [A]** time = 0.0543412, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3311, 32, 2635, 8}

$$-\frac{d(c + dx) \sinh^2(a + bx)}{2b^2} + \frac{d^2 \sinh(a + bx) \cosh(a + bx)}{4b^3} + \frac{(c + dx)^2 \sinh(a + bx) \cosh(a + bx)}{2b} - \frac{d^2 x}{4b^2} - \frac{(c + dx)^3}{6d}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^2\*Sinh[a + b\*x]^2,x]

[Out]  $-(d^2*x)/(4*b^2) - (c + d*x)^3/(6*d) + (d^2*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^3) + ((c + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (d*(c + d*x)*Sinh[a + b*x]^2)/(2*b^2)$

#### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*Sinh[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*Sinh[e + f\*x])^(n - 2), x], x] - Dist[(d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*Sinh[e + f\*x])^n, x], x] - Simp[(b\*(c + d\*x)^m\*Cosh[e + f\*x]\*(b\*Sinh[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

#### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

#### Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

### Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

### Rubi steps

$$\begin{aligned} \int (c + dx)^2 \sinh^2(a + bx) dx &= \frac{(c + dx)^2 \cosh(a + bx) \sinh(a + bx)}{2b} - \frac{d(c + dx) \sinh^2(a + bx)}{2b^2} - \frac{1}{2} \int (c + dx)^2 dx + \frac{d^2}{2} \int (c + dx) dx \\ &= -\frac{(c + dx)^3}{6d} + \frac{d^2 \cosh(a + bx) \sinh(a + bx)}{4b^3} + \frac{(c + dx)^2 \cosh(a + bx) \sinh(a + bx)}{2b} - \frac{d(c + dx)^2}{2} \\ &= -\frac{d^2 x}{4b^2} - \frac{(c + dx)^3}{6d} + \frac{d^2 \cosh(a + bx) \sinh(a + bx)}{4b^3} + \frac{(c + dx)^2 \cosh(a + bx) \sinh(a + bx)}{2b} \end{aligned}$$

**Mathematica [A]** time = 0.310909, size = 75, normalized size = 0.79

$$\frac{3 \sinh(2(a + bx)) (2b^2(c + dx)^2 + d^2) - 6bd(c + dx) \cosh(2(a + bx)) - 4b^3 x (3c^2 + 3cdx + d^2 x^2)}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sinh[a + b*x]^2,x]
```

```
[Out] (-4*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) - 6*b*d*(c + d*x)*Cosh[2*(a + b*x)] + 3*(d^2 + 2*b^2*(c + d*x)^2)*Sinh[2*(a + b*x)])/(24*b^3)
```

**Maple [B]** time = 0.007, size = 262, normalized size = 2.8

$$\frac{1}{b} \left( \frac{d^2}{b^2} \left( \frac{(bx + a)^2 \cosh(bx + a) \sinh(bx + a)}{2} - \frac{(bx + a)^3}{6} - \frac{(bx + a) (\cosh(bx + a))^2}{2} + \frac{\cosh(bx + a) \sinh(bx + a)}{4} + \frac{bx}{4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sinh(b*x+a)^2,x)
```



[Out]  $1/b*(1/b^2*d^2*(1/2*(b*x+a)^2*\cosh(b*x+a)*\sinh(b*x+a)-1/6*(b*x+a)^3-1/2*(b*x+a)*\cosh(b*x+a)^2+1/4*\cosh(b*x+a)*\sinh(b*x+a)+1/4*b*x+1/4*a)-2/b^2*d^2*a*(1/2*(b*x+a)*\cosh(b*x+a)*\sinh(b*x+a)-1/4*(b*x+a)^2-1/4*\cosh(b*x+a)^2)+1/b^2*d^2*a^2*(1/2*\cosh(b*x+a)*\sinh(b*x+a)-1/2*b*x-1/2*a)+2/b*c*d*(1/2*(b*x+a)*\cosh(b*x+a)*\sinh(b*x+a)-1/4*(b*x+a)^2-1/4*\cosh(b*x+a)^2)-2/b*c*d*a*(1/2*\cosh(b*x+a)*\sinh(b*x+a)-1/2*b*x-1/2*a)+c^2*(1/2*\cosh(b*x+a)*\sinh(b*x+a)-1/2*b*x-1/2*a))$

**Maxima [A]** time = 1.14048, size = 223, normalized size = 2.35

$$-\frac{1}{8} \left( 4x^2 - \frac{(2bx e^{(2a)} - e^{(2a)})e^{(2bx)}}{b^2} + \frac{(2bx + 1)e^{(-2bx - 2a)}}{b^2} \right) cd - \frac{1}{48} \left( 8x^3 - \frac{3(2b^2x^2e^{(2a)} - 2bx e^{(2a)} + e^{(2a)})e^{(2bx)}}{b^3} + \frac{3(2bx + 1)e^{(-2bx - 2a)}}{b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*sinh(b\*x+a)^2,x, algorithm="maxima")

[Out]  $-1/8*(4*x^2 - (2*b*x*e^{(2*a)} - e^{(2*a)})*e^{(2*b*x)}/b^2 + (2*b*x + 1)*e^{(-2*b*x - 2*a)}/b^2)*c*d - 1/48*(8*x^3 - 3*(2*b^2*x^2*e^{(2*a)} - 2*b*x*e^{(2*a)} + e^{(2*a)})*e^{(2*b*x)}/b^3 + 3*(2*b^2*x^2 + 2*b*x + 1)*e^{(-2*b*x - 2*a)}/b^3)*d^2 - 1/8*c^2*(4*x - e^{(2*b*x + 2*a)}/b + e^{(-2*b*x - 2*a)}/b)$

**Fricas [A]** time = 2.69329, size = 288, normalized size = 3.03

$$\frac{2b^3d^2x^3 + 6b^3cdx^2 + 6b^3c^2x + 3(bd^2x + bcd) \cosh(bx + a)^2 - 3(2b^2d^2x^2 + 4b^2cdx + 2b^2c^2 + d^2) \cosh(bx + a) \sinh(bx + a) + 3(bd^2x + bcd) \sinh(bx + a)^2}{12b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*sinh(b\*x+a)^2,x, algorithm="fricas")

[Out]  $-1/12*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 + 6*b^3*c^2*x + 3*(b*d^2*x + b*c*d)*\cosh(b*x + a)^2 - 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*\cosh(b*x + a)*\sinh(b*x + a) + 3*(b*d^2*x + b*c*d)*\sinh(b*x + a)^2)/b^3$

**Sympy [A]** time = 1.77054, size = 264, normalized size = 2.78

$$\left\{ \begin{array}{l} \frac{c^2 x \sinh^2(a+bx)}{2} - \frac{c^2 x \cosh^2(a+bx)}{2} + \frac{cdx^2 \sinh^2(a+bx)}{2} - \frac{cdx^2 \cosh^2(a+bx)}{2} + \frac{d^2 x^3 \sinh^2(a+bx)}{6} - \frac{d^2 x^3 \cosh^2(a+bx)}{6} + \frac{c^2 \sinh(a+bx) \cosh(a+bx)}{2b} \\ \left( c^2 x + cdx^2 + \frac{d^2 x^3}{3} \right) \sinh^2(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2\*sinh(b\*x+a)\*\*2,x)

[Out] Piecewise((c\*\*2\*x\*sinh(a + b\*x)\*\*2/2 - c\*\*2\*x\*cosh(a + b\*x)\*\*2/2 + c\*d\*x\*\*2\*sinh(a + b\*x)\*\*2/2 - c\*d\*x\*\*2\*cosh(a + b\*x)\*\*2/2 + d\*\*2\*x\*\*3\*sinh(a + b\*x)\*\*2/6 - d\*\*2\*x\*\*3\*cosh(a + b\*x)\*\*2/6 + c\*\*2\*sinh(a + b\*x)\*cosh(a + b\*x)/(2\*b) + c\*d\*x\*sinh(a + b\*x)\*cosh(a + b\*x)/b + d\*\*2\*x\*\*2\*sinh(a + b\*x)\*cosh(a + b\*x)/(2\*b) - c\*d\*sinh(a + b\*x)\*\*2/(2\*b\*\*2) - d\*\*2\*x\*sinh(a + b\*x)\*\*2/(4\*b\*\*2) - d\*\*2\*x\*cosh(a + b\*x)\*\*2/(4\*b\*\*2) + d\*\*2\*sinh(a + b\*x)\*cosh(a + b\*x)/(4\*b\*\*3), Ne(b, 0)), ((c\*\*2\*x + c\*d\*x\*\*2 + d\*\*2\*x\*\*3/3)\*sinh(a)\*\*2, True))

**Giac [A]** time = 1.17973, size = 184, normalized size = 1.94

$$-\frac{1}{6}d^2x^3 - \frac{1}{2}cdx^2 - \frac{1}{2}c^2x + \frac{(2b^2d^2x^2 + 4b^2cdx + 2b^2c^2 - 2bd^2x - 2bcd + d^2)e^{(2bx+2a)}}{16b^3} - \frac{(2b^2d^2x^2 + 4b^2cdx + 2b^2c^2 + 2bd^2x - 2bcd + d^2)e^{(-2bx-2a)}}{16b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*sinh(b\*x+a)^2,x, algorithm="giac")

[Out] -1/6\*d^2\*x^3 - 1/2\*c\*d\*x^2 - 1/2\*c^2\*x + 1/16\*(2\*b^2\*d^2\*x^2 + 4\*b^2\*c\*d\*x + 2\*b^2\*c^2 - 2\*b\*d^2\*x - 2\*b\*c\*d + d^2)\*e^(2\*b\*x + 2\*a)/b^3 - 1/16\*(2\*b^2\*d^2\*x^2 + 4\*b^2\*c\*d\*x + 2\*b^2\*c^2 + 2\*b\*d^2\*x + 2\*b\*c\*d + d^2)\*e^(-2\*b\*x - 2\*a)/b^3

### 3.11 $\int (c + dx) \sinh^2(a + bx) dx$

**Optimal.** Leaf size=55

$$-\frac{d \sinh^2(a + bx)}{4b^2} + \frac{(c + dx) \sinh(a + bx) \cosh(a + bx)}{2b} - \frac{cx}{2} - \frac{dx^2}{4}$$

[Out]  $-(c*x)/2 - (d*x^2)/4 + ((c + d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (d*Sinh[a + b*x]^2)/(4*b^2)$

**Rubi [A]** time = 0.0262664, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {3310}

$$-\frac{d \sinh^2(a + bx)}{4b^2} + \frac{(c + dx) \sinh(a + bx) \cosh(a + bx)}{2b} - \frac{cx}{2} - \frac{dx^2}{4}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)\*Sinh[a + b\*x]^2,x]

[Out]  $-(c*x)/2 - (d*x^2)/4 + ((c + d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) - (d*Sinh[a + b*x]^2)/(4*b^2)$

#### Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
  Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

#### Rubi steps

$$\begin{aligned} \int (c + dx) \sinh^2(a + bx) dx &= \frac{(c + dx) \cosh(a + bx) \sinh(a + bx)}{2b} - \frac{d \sinh^2(a + bx)}{4b^2} - \frac{1}{2} \int (c + dx) dx \\ &= -\frac{cx}{2} - \frac{dx^2}{4} + \frac{(c + dx) \cosh(a + bx) \sinh(a + bx)}{2b} - \frac{d \sinh^2(a + bx)}{4b^2} \end{aligned}$$

**Mathematica [A]** time = 0.161799, size = 52, normalized size = 0.95

$$\frac{2b((c + dx) \sinh(2(a + bx)) - 2ac - bx(2c + dx)) - d \cosh(2(a + bx))}{8b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)\*Sinh[a + b\*x]^2,x]

[Out]  $(-(d*\text{Cosh}[2*(a + b*x)]) + 2*b*(-2*a*c - b*x*(2*c + d*x) + (c + d*x)*\text{Sinh}[2*(a + b*x)]))/(8*b^2)$

**Maple [B]** time = 0.006, size = 103, normalized size = 1.9

$$\frac{1}{b} \left( \frac{d}{b} \left( \frac{(bx + a) \cosh(bx + a) \sinh(bx + a)}{2} - \frac{(bx + a)^2}{4} - \frac{(\cosh(bx + a))^2}{4} \right) - \frac{da}{b} \left( \frac{\cosh(bx + a) \sinh(bx + a)}{2} - \frac{bx}{2} - \frac{a}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)\*sinh(b\*x+a)^2,x)

[Out]  $1/b*(1/b*d*(1/2*(b*x+a)*\cosh(b*x+a)*\sinh(b*x+a)-1/4*(b*x+a)^2-1/4*\cosh(b*x+a)^2)-1/b*d*a*(1/2*\cosh(b*x+a)*\sinh(b*x+a)-1/2*b*x-1/2*a)+c*(1/2*\cosh(b*x+a)*\sinh(b*x+a)-1/2*b*x-1/2*a)$

**Maxima [A]** time = 1.21832, size = 119, normalized size = 2.16

$$-\frac{1}{16} \left( 4x^2 - \frac{(2bx e^{2a} - e^{2a})e^{2bx}}{b^2} + \frac{(2bx + 1)e^{(-2bx - 2a)}}{b^2} \right) d - \frac{1}{8} c \left( 4x - \frac{e^{(2bx + 2a)}}{b} + \frac{e^{(-2bx - 2a)}}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sinh(b\*x+a)^2,x, algorithm="maxima")

[Out]  $-1/16*(4*x^2 - (2*b*x*e^{(2*a)} - e^{(2*a)})*e^{(2*b*x)}/b^2 + (2*b*x + 1)*e^{(-2*b*x - 2*a)}/b^2)*d - 1/8*c*(4*x - e^{(2*b*x + 2*a)}/b + e^{(-2*b*x - 2*a)}/b)$

**Fricas [A]** time = 2.62637, size = 165, normalized size = 3.

$$\frac{2b^2dx^2 + 4b^2cx + d \cosh(bx + a)^2 - 4(bdx + bc) \cosh(bx + a) \sinh(bx + a) + d \sinh(bx + a)^2}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sinh(b\*x+a)^2,x, algorithm="fricas")

[Out]  $-1/8*(2*b^2*d*x^2 + 4*b^2*c*x + d*\cosh(b*x + a)^2 - 4*(b*d*x + b*c)*\cosh(b*x + a)*\sinh(b*x + a) + d*\sinh(b*x + a)^2)/b^2$

**Sympy [A]** time = 0.800126, size = 126, normalized size = 2.29

$$\left\{ \begin{array}{l} \frac{cx \sinh^2(a+bx)}{2} - \frac{cx \cosh^2(a+bx)}{2} + \frac{dx^2 \sinh^2(a+bx)}{4} - \frac{dx^2 \cosh^2(a+bx)}{4} + \frac{c \sinh(a+bx) \cosh(a+bx)}{2b} + \frac{dx \sinh(a+bx) \cosh(a+bx)}{2b} - \frac{d \sinh^2(a+bx)}{4b^2} \\ \left( cx + \frac{dx^2}{2} \right) \sinh^2(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sinh(b\*x+a)\*\*2,x)

[Out] Piecewise((c\*x\*sinh(a + b\*x)\*\*2/2 - c\*x\*cosh(a + b\*x)\*\*2/2 + d\*x\*\*2\*sinh(a + b\*x)\*\*2/4 - d\*x\*\*2\*cosh(a + b\*x)\*\*2/4 + c\*sinh(a + b\*x)\*cosh(a + b\*x)/(2\*b) + d\*x\*sinh(a + b\*x)\*cosh(a + b\*x)/(2\*b) - d\*sinh(a + b\*x)\*\*2/(4\*b\*\*2), N e(b, 0)), ((c\*x + d\*x\*\*2/2)\*sinh(a)\*\*2, True))

**Giac [A]** time = 1.21919, size = 85, normalized size = 1.55

$$-\frac{1}{4}dx^2 - \frac{1}{2}cx + \frac{(2bdx + 2bc - d)e^{(2bx+2a)}}{16b^2} - \frac{(2bdx + 2bc + d)e^{(-2bx-2a)}}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sinh(b\*x+a)^2,x, algorithm="giac")

[Out]  $-1/4*d*x^2 - 1/2*c*x + 1/16*(2*b*d*x + 2*b*c - d)*e^{(2*b*x + 2*a)}/b^2 - 1/16*(2*b*d*x + 2*b*c + d)*e^{(-2*b*x - 2*a)}/b^2$

### 3.12 $\int \frac{\sinh^2(a+bx)}{c+dx} dx$

**Optimal.** Leaf size=78

$$\frac{\cosh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2bc}{d} + 2bx\right)}{2d} + \frac{\sinh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{2d} - \frac{\log(c+dx)}{2d}$$

[Out] (Cosh[2\*a - (2\*b\*c)/d]\*CoshIntegral[(2\*b\*c)/d + 2\*b\*x])/(2\*d) - Log[c + d\*x]/(2\*d) + (Sinh[2\*a - (2\*b\*c)/d]\*SinhIntegral[(2\*b\*c)/d + 2\*b\*x])/(2\*d)

**Rubi [A]** time = 0.16549, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3312, 3303, 3298, 3301}

$$\frac{\cosh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2bc}{d} + 2bx\right)}{2d} + \frac{\sinh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{2d} - \frac{\log(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[a + b\*x]^2/(c + d\*x), x]

[Out] (Cosh[2\*a - (2\*b\*c)/d]\*CoshIntegral[(2\*b\*c)/d + 2\*b\*x])/(2\*d) - Log[c + d\*x]/(2\*d) + (Sinh[2\*a - (2\*b\*c)/d]\*SinhIntegral[(2\*b\*c)/d + 2\*b\*x])/(2\*d)

#### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

#### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

#### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f}

, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\sinh^2(a+bx)}{c+dx} dx &= - \int \left( \frac{1}{2(c+dx)} - \frac{\cosh(2a+2bx)}{2(c+dx)} \right) dx \\ &= -\frac{\log(c+dx)}{2d} + \frac{1}{2} \int \frac{\cosh(2a+2bx)}{c+dx} dx \\ &= -\frac{\log(c+dx)}{2d} + \frac{1}{2} \cosh\left(2a - \frac{2bc}{d}\right) \int \frac{\cosh\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx + \frac{1}{2} \sinh\left(2a - \frac{2bc}{d}\right) \int \frac{\sinh\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx \\ &= \frac{\cosh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2bc}{d} + 2bx\right)}{2d} - \frac{\log(c+dx)}{2d} + \frac{\sinh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{2d} \end{aligned}$$

**Mathematica [A]** time = 0.121673, size = 66, normalized size = 0.85

$$\frac{\cosh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2b(c+dx)}{d}\right) + \sinh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2b(c+dx)}{d}\right) - \log(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[a + b\*x]^2/(c + d\*x), x]

[Out] (Cosh[2\*a - (2\*b\*c)/d]\*CoshIntegral[(2\*b\*(c + d\*x))/d] - Log[c + d\*x] + Sinh[2\*a - (2\*b\*c)/d]\*SinhIntegral[(2\*b\*(c + d\*x))/d])/(2\*d)

**Maple [A]** time = 0.074, size = 97, normalized size = 1.2

$$-\frac{\ln(dx+c)}{2d} - \frac{1}{4d} e^{-2\frac{da-cb}{d}} \text{Ei}\left(1, 2bx + 2a - 2\frac{da-cb}{d}\right) - \frac{1}{4d} e^{2\frac{da-cb}{d}} \text{Ei}\left(1, -2bx - 2a - 2\frac{-da+cb}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(b*x+a)^2/(d*x+c),x)`

[Out]  $-1/2*\ln(d*x+c)/d-1/4/d*\exp(-2*(a*d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/4/d*\exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)$

**Maxima [A]** time = 1.38425, size = 97, normalized size = 1.24

$$-\frac{e^{\left(-2a+\frac{2bc}{d}\right)}E_1\left(\frac{2(dx+c)b}{d}\right)}{4d}-\frac{e^{\left(2a-\frac{2bc}{d}\right)}E_1\left(-\frac{2(dx+c)b}{d}\right)}{4d}-\frac{\log(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)^2/(d*x+c),x, algorithm="maxima")`

[Out]  $-1/4*e^{(-2*a + 2*b*c/d)}*\exp\_integral\_e(1, 2*(d*x + c)*b/d)/d - 1/4*e^{(2*a - 2*b*c/d)}*\exp\_integral\_e(1, -2*(d*x + c)*b/d)/d - 1/2*\log(d*x + c)/d$

**Fricas [A]** time = 2.63968, size = 232, normalized size = 2.97

$$\frac{\left(Ei\left(\frac{2(bdx+bc)}{d}\right) + Ei\left(-\frac{2(bdx+bc)}{d}\right)\right) \cosh\left(-\frac{2(bc-ad)}{d}\right) + \left(Ei\left(\frac{2(bdx+bc)}{d}\right) - Ei\left(-\frac{2(bdx+bc)}{d}\right)\right) \sinh\left(-\frac{2(bc-ad)}{d}\right) - 2 \log(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)^2/(d*x+c),x, algorithm="fricas")`

[Out]  $1/4*((Ei(2*(b*d*x + b*c)/d) + Ei(-2*(b*d*x + b*c)/d))*\cosh(-2*(b*c - a*d)/d) + (Ei(2*(b*d*x + b*c)/d) - Ei(-2*(b*d*x + b*c)/d))*\sinh(-2*(b*c - a*d)/d) - 2*\log(d*x + c))/d$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(sinh(b\*x+a)\*\*2/(d\*x+c),x)

[Out] Integral(sinh(a + b\*x)\*\*2/(c + d\*x), x)

**Giac [A]** time = 1.19975, size = 92, normalized size = 1.18

$$\frac{\operatorname{Ei}\left(\frac{2(bdx+bc)}{d}\right)e^{\left(2a-\frac{2bc}{d}\right)} + \operatorname{Ei}\left(-\frac{2(bdx+bc)}{d}\right)e^{\left(-2a+\frac{2bc}{d}\right)} - 2 \log(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2/(d\*x+c),x, algorithm="giac")

[Out] 1/4\*(Ei(2\*(b\*d\*x + b\*c)/d)\*e^(2\*a - 2\*b\*c/d) + Ei(-2\*(b\*d\*x + b\*c)/d)\*e^(-2\*a + 2\*b\*c/d) - 2\*log(d\*x + c))/d

### 3.13 $\int \frac{\sinh^2(a+bx)}{(c+dx)^2} dx$

**Optimal.** Leaf size=81

$$\frac{b \sinh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2bc}{d} + 2bx\right)}{d^2} + \frac{b \cosh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{d^2} - \frac{\sinh^2(a+bx)}{d(c+dx)}$$

[Out] (b\*CoshIntegral[(2\*b\*c)/d + 2\*b\*x]\*Sinh[2\*a - (2\*b\*c)/d])/d^2 - Sinh[a + b\*x]^2/(d\*(c + d\*x)) + (b\*Cosh[2\*a - (2\*b\*c)/d]\*SinhIntegral[(2\*b\*c)/d + 2\*b\*x])/d^2

**Rubi [A]** time = 0.154983, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3313, 12, 3303, 3298, 3301}

$$\frac{b \sinh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2bc}{d} + 2bx\right)}{d^2} + \frac{b \cosh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{d^2} - \frac{\sinh^2(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[Sinh[a + b\*x]^2/(c + d\*x)^2,x]

[Out] (b\*CoshIntegral[(2\*b\*c)/d + 2\*b\*x]\*Sinh[2\*a - (2\*b\*c)/d])/d^2 - Sinh[a + b\*x]^2/(d\*(c + d\*x)) + (b\*Cosh[2\*a - (2\*b\*c)/d]\*SinhIntegral[(2\*b\*c)/d + 2\*b\*x])/d^2

#### Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
1), x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
LtQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sinh^2(a+bx)}{(c+dx)^2} dx &= -\frac{\sinh^2(a+bx)}{d(c+dx)} - \frac{(2ib) \int \frac{i \sinh(2a+2bx)}{2(c+dx)} dx}{d} \\ &= -\frac{\sinh^2(a+bx)}{d(c+dx)} + \frac{b \int \frac{\sinh(2a+2bx)}{c+dx} dx}{d} \\ &= -\frac{\sinh^2(a+bx)}{d(c+dx)} + \frac{\left(b \cosh\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\sinh\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx}{d} + \frac{\left(b \sinh\left(2a - \frac{2bc}{d}\right)\right) \int \frac{\cosh\left(\frac{2bc}{d} + 2bx\right)}{c+dx} dx}{d} \\ &= \frac{b \operatorname{Chi}\left(\frac{2bc}{d} + 2bx\right) \sinh\left(2a - \frac{2bc}{d}\right)}{d^2} - \frac{\sinh^2(a+bx)}{d(c+dx)} + \frac{b \cosh\left(2a - \frac{2bc}{d}\right) \operatorname{Shi}\left(\frac{2bc}{d} + 2bx\right)}{d^2} \end{aligned}$$

**Mathematica [A]** time = 0.44441, size = 75, normalized size = 0.93

$$\frac{b \sinh\left(2a - \frac{2bc}{d}\right) \operatorname{Chi}\left(\frac{2b(c+dx)}{d}\right) + b \cosh\left(2a - \frac{2bc}{d}\right) \operatorname{Shi}\left(\frac{2b(c+dx)}{d}\right) - \frac{d \sinh^2(a+bx)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^2/(c + d*x)^2, x]
```

[Out]  $(b \cdot \text{CoshIntegral}[(2 \cdot b \cdot (c + d \cdot x))/d] \cdot \text{Sinh}[2 \cdot a - (2 \cdot b \cdot c)/d] - (d \cdot \text{Sinh}[a + b \cdot x]^2)/(c + d \cdot x) + b \cdot \text{Cosh}[2 \cdot a - (2 \cdot b \cdot c)/d] \cdot \text{SinhIntegral}[(2 \cdot b \cdot (c + d \cdot x))/d])/d^2$

**Maple [A]** time = 0.072, size = 152, normalized size = 1.9

$$\frac{1}{2d(dx+c)} - \frac{be^{-2bx-2a}}{(4bdx+4cb)d} + \frac{b}{2d^2} e^{-2\frac{da-cb}{d}} \text{Ei}\left(1, 2bx+2a-2\frac{da-cb}{d}\right) - \frac{be^{2bx+2a}}{4d^2} \left(\frac{cb}{d} + bx\right)^{-1} - \frac{b}{2d^2} e^{2\frac{da-cb}{d}} \text{Ei}\left(1, -2\frac{da-cb}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(b*x+a)^2/(d*x+c)^2,x)`

[Out]  $1/2/d/(d*x+c) - 1/4*b*\exp(-2*b*x-2*a)/(b*d*x+b*c)/d + 1/2*b/d^2*\exp(-2*(a*d-b*c)/d)*\text{Ei}(1, 2*b*x+2*a-2*(a*d-b*c)/d) - 1/4*b/d^2*\exp(2*b*x+2*a)/(b*c/d+b*x) - 1/2*b/d^2*\exp(2*(a*d-b*c)/d)*\text{Ei}(1, -2*b*x-2*a-2*(-a*d+b*c)/d)$

**Maxima [A]** time = 1.38888, size = 119, normalized size = 1.47

$$-\frac{e^{\left(-2a+\frac{2bc}{d}\right)} E_2\left(\frac{2(dx+c)b}{d}\right)}{4(dx+c)d} - \frac{e^{\left(2a-\frac{2bc}{d}\right)} E_2\left(-\frac{2(dx+c)b}{d}\right)}{4(dx+c)d} + \frac{1}{2(d^2x+cd)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")`

[Out]  $-1/4*e^{(-2*a + 2*b*c/d)}*\exp\_integral\_e(2, 2*(d*x + c)*b/d)/((d*x + c)*d) - 1/4*e^{(2*a - 2*b*c/d)}*\exp\_integral\_e(2, -2*(d*x + c)*b/d)/((d*x + c)*d) + 1/2/(d^2*x + c*d)$

**Fricas [B]** time = 2.72063, size = 365, normalized size = 4.51

$$\frac{d \cosh (bx+a)^2 + d \sinh (bx+a)^2 - \left( (bdx+bc) \text{Ei}\left(\frac{2(bdx+bc)}{d}\right) - (bdx+bc) \text{Ei}\left(-\frac{2(bdx+bc)}{d}\right) \right) \cosh\left(-\frac{2(bc-ad)}{d}\right) - \left( (bdx+bc) \text{Ei}\left(\frac{2(bc-ad)}{d}\right) - (bdx+bc) \text{Ei}\left(-\frac{2(bc-ad)}{d}\right) \right) \cosh\left(\frac{2(bdx+bc)}{d}\right)}{2(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2/(d\*x+c)^2,x, algorithm="fricas")

[Out] 
$$-1/2*(d*\cosh(b*x + a)^2 + d*\sinh(b*x + a)^2 - ((b*d*x + b*c)*\text{Ei}(2*(b*d*x + b*c)/d) - (b*d*x + b*c)*\text{Ei}(-2*(b*d*x + b*c)/d))*\cosh(-2*(b*c - a*d)/d) - ((b*d*x + b*c)*\text{Ei}(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*\text{Ei}(-2*(b*d*x + b*c)/d))*\sinh(-2*(b*c - a*d)/d) - d)/(d^3*x + c*d^2)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)\*\*2/(d\*x+c)\*\*2,x)

[Out] Integral(sinh(a + b\*x)\*\*2/(c + d\*x)\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx + a)^2}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2/(d\*x+c)^2,x, algorithm="giac")

[Out] integrate(sinh(b\*x + a)^2/(d\*x + c)^2, x)

$$3.14 \quad \int \frac{\sinh^2(a+bx)}{(c+dx)^3} dx$$

**Optimal.** Leaf size=112

$$\frac{b^2 \cosh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2bc}{d} + 2bx\right)}{d^3} + \frac{b^2 \sinh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{b \sinh(a+bx) \cosh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2}$$

[Out] (b^2\*Cosh[2\*a - (2\*b\*c)/d]\*CoshIntegral[(2\*b\*c)/d + 2\*b\*x])/d^3 - (b\*Cosh[a + b\*x]\*Sinh[a + b\*x])/(d^2\*(c + d\*x)) - Sinh[a + b\*x]^2/(2\*d\*(c + d\*x)^2) + (b^2\*Sinh[2\*a - (2\*b\*c)/d]\*SinhIntegral[(2\*b\*c)/d + 2\*b\*x])/d^3

**Rubi [A]** time = 0.195081, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {3314, 31, 3312, 3303, 3298, 3301}

$$\frac{b^2 \cosh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2bc}{d} + 2bx\right)}{d^3} + \frac{b^2 \sinh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{d^3} - \frac{b \sinh(a+bx) \cosh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Int[Sinh[a + b\*x]^2/(c + d\*x)^3,x]

[Out] (b^2\*Cosh[2\*a - (2\*b\*c)/d]\*CoshIntegral[(2\*b\*c)/d + 2\*b\*x])/d^3 - (b\*Cosh[a + b\*x]\*Sinh[a + b\*x])/(d^2\*(c + d\*x)) - Sinh[a + b\*x]^2/(2\*d\*(c + d\*x)^2) + (b^2\*Sinh[2\*a - (2\*b\*c)/d]\*SinhIntegral[(2\*b\*c)/d + 2\*b\*x])/d^3

### Rule 3314

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*(b\*Sinh[e + f\*x])^n)/(d\*(m + 1)), x] + (Dist[(b^2\*f^2\*n\*(n - 1))/(d^2\*(m + 1)\*(m + 2)), Int[(c + d\*x)^(m + 2)\*(b\*Sinh[e + f\*x])^(n - 2), x], x] - Dist[(f^2\*n^2)/(d^2\*(m + 1)\*(m + 2)), Int[(c + d\*x)^(m + 2)\*(b\*Sinh[e + f\*x])^n, x], x] - Simp[(b\*f\*n\*(c + d\*x)^(m + 2)\*Cos[e + f\*x]\*(b\*Sinh[e + f\*x])^(n - 1))/(d^2\*(m + 1)\*(m + 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]

### Rule 31

Int[((a\_.) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(a+bx)}{(c+dx)^3} dx &= -\frac{b \cosh(a+bx) \sinh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} + \frac{b^2 \int \frac{1}{c+dx} dx}{d^2} + \frac{(2b^2) \int \frac{\sinh^2(a+bx)}{c+dx} dx}{d^2} \\
&= \frac{b^2 \log(c+dx)}{d^3} - \frac{b \cosh(a+bx) \sinh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} - \frac{(2b^2) \int \left( \frac{1}{2(c+dx)} - \frac{\cosh(2a+2bx)}{2(c+dx)} \right) dx}{d^2} \\
&= -\frac{b \cosh(a+bx) \sinh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} + \frac{b^2 \int \frac{\cosh(2a+2bx)}{c+dx} dx}{d^2} \\
&= -\frac{b \cosh(a+bx) \sinh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} + \frac{\left( b^2 \cosh \left( 2a - \frac{2bc}{d} \right) \right) \int \frac{\cosh \left( \frac{2bc}{d} + 2bx \right)}{c+dx} dx}{d^2} + \frac{(b^2 \sinh(2a+2bx))}{d^2} \\
&= \frac{b^2 \cosh \left( 2a - \frac{2bc}{d} \right) \text{Chi} \left( \frac{2bc}{d} + 2bx \right)}{d^3} - \frac{b \cosh(a+bx) \sinh(a+bx)}{d^2(c+dx)} - \frac{\sinh^2(a+bx)}{2d(c+dx)^2} + \frac{b^2 \sinh(2a+2bx)}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.916274, size = 102, normalized size = 0.91

$$\frac{2b^2 \cosh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2b(c+dx)}{d}\right) + 2b^2 \sinh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2b(c+dx)}{d}\right) - \frac{d(b(c+dx) \sinh(2(a+bx)) + d \sinh^2(a+bx))}{(c+dx)^2}}{2d^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[a + b\*x]^2/(c + d\*x)^3,x]

[Out] (2\*b^2\*Cosh[2\*a - (2\*b\*c)/d]\*CoshIntegral[(2\*b\*(c + d\*x))/d] - (d\*(d\*Sinh[a + b\*x]^2 + b\*(c + d\*x)\*Sinh[2\*(a + b\*x)]))/(c + d\*x)^2 + 2\*b^2\*Sinh[2\*a - (2\*b\*c)/d]\*SinhIntegral[(2\*b\*(c + d\*x))/d])/(2\*d^3)

**Maple [B]** time = 0.076, size = 299, normalized size = 2.7

$$\frac{1}{4d(dx+c)^2} + \frac{b^3 e^{-2bx-2a} x}{4d(b^2 d^2 x^2 + 2b^2 cdx + c^2 b^2)} + \frac{b^3 e^{-2bx-2a} c}{4d^2(b^2 d^2 x^2 + 2b^2 cdx + c^2 b^2)} - \frac{b^2 e^{-2bx-2a}}{8d(b^2 d^2 x^2 + 2b^2 cdx + c^2 b^2)} - \frac{b^2}{2d^3} e^{-2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(b\*x+a)^2/(d\*x+c)^3,x)

[Out] 1/4/d/(d\*x+c)^2+1/4\*b^3\*exp(-2\*b\*x-2\*a)/d/(b^2\*d^2\*x^2+2\*b^2\*c\*d\*x+b^2\*c^2)\*x+1/4\*b^3\*exp(-2\*b\*x-2\*a)/d^2/(b^2\*d^2\*x^2+2\*b^2\*c\*d\*x+b^2\*c^2)\*c-1/8\*b^2\*exp(-2\*b\*x-2\*a)/d/(b^2\*d^2\*x^2+2\*b^2\*c\*d\*x+b^2\*c^2)-1/2\*b^2/d^3\*exp(-2\*(a\*d-b\*c)/d)\*Ei(1,2\*b\*x+2\*a-2\*(a\*d-b\*c)/d)-1/8\*b^2/d^3\*exp(2\*b\*x+2\*a)/(b\*c/d+b\*x)^2-1/4\*b^2/d^3\*exp(2\*b\*x+2\*a)/(b\*c/d+b\*x)-1/2\*b^2/d^3\*exp(2\*(a\*d-b\*c)/d)\*Ei(1,-2\*b\*x-2\*a-2\*(-a\*d+b\*c)/d)

**Maxima [A]** time = 1.47718, size = 134, normalized size = 1.2

$$\frac{1}{4(d^3 x^2 + 2cd^2 x + c^2 d)} - \frac{e^{\left(-2a + \frac{2bc}{d}\right)} E_3\left(\frac{2(dx+c)b}{d}\right)}{4(dx+c)^2 d} - \frac{e^{\left(2a - \frac{2bc}{d}\right)} E_3\left(-\frac{2(dx+c)b}{d}\right)}{4(dx+c)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2/(d\*x+c)^3,x, algorithm="maxima")



```
[Out] 1/4/(d^3*x^2 + 2*c*d^2*x + c^2*d) - 1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(3
, 2*(d*x + c)*b/d)/((d*x + c)^2*d) - 1/4*e^(2*a - 2*b*c/d)*exp_integral_e(3
, -2*(d*x + c)*b/d)/((d*x + c)^2*d)
```

**Fricas [B]** time = 2.71413, size = 597, normalized size = 5.33

$$d^2 \cosh(bx + a)^2 + d^2 \sinh(bx + a)^2 + 4(bd^2x + bcd) \cosh(bx + a) \sinh(bx + a) - d^2 - 2\left((b^2d^2x^2 + 2b^2cdx + b^2c^2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(d^2*cosh(b*x + a)^2 + d^2*sinh(b*x + a)^2 + 4*(b*d^2*x + b*c*d)*cosh(
b*x + a)*sinh(b*x + a) - d^2 - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(
2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-2*(b*d*x + b
*c)/d))*cosh(-2*(b*c - a*d)/d) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*E
i(2*(b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-2*(b*d*x +
b*c)/d))*sinh(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a + bx)}{(c + dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(sinh(a + b*x)**2/(c + d*x)**3, x)
```

**Giac [B]** time = 1.15751, size = 446, normalized size = 3.98

$$4b^2d^2x^2\text{Ei}\left(\frac{2(bdx+bc)}{d}\right)e^{\left(2a-\frac{2bc}{d}\right)} + 4b^2d^2x^2\text{Ei}\left(-\frac{2(bdx+bc)}{d}\right)e^{\left(-2a+\frac{2bc}{d}\right)} + 8b^2cdx\text{Ei}\left(\frac{2(bdx+bc)}{d}\right)e^{\left(2a-\frac{2bc}{d}\right)} + 8b^2cdx\text{Ei}\left(-\frac{2(bdx+bc)}{d}\right)e^{\left(-2a+\frac{2bc}{d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2/(d\*x+c)^3,x, algorithm="giac")

[Out]  $\frac{1}{8}(4b^2d^2x^2\text{Ei}(2(bdx + bc)/d)e^{(2a - 2bc/d)} + 4b^2d^2x^2\text{Ei}(-2(bdx + bc)/d)e^{(-2a + 2bc/d)} + 8b^2cdx\text{Ei}(2(bdx + bc)/d)e^{(2a - 2bc/d)} + 8b^2cdx\text{Ei}(-2(bdx + bc)/d)e^{(-2a + 2bc/d)} + 4b^2c^2\text{Ei}(2(bdx + bc)/d)e^{(2a - 2bc/d)} + 4b^2c^2\text{Ei}(-2(bdx + bc)/d)e^{(-2a + 2bc/d)} - 2bd^2xe^{(2bx + 2a)} + 2bd^2xe^{(-2bx - 2a)} - 2bcd^2e^{(2bx + 2a)} + 2bcd^2e^{(-2bx - 2a)} - d^2e^{(2bx + 2a)} - d^2e^{(-2bx - 2a)} + 2d^2)/(d^5x^2 + 2cd^4x + c^2d^3)$

$$3.15 \quad \int \frac{\sinh^2(a+bx)}{(c+dx)^4} dx$$

**Optimal.** Leaf size=162

$$\frac{2b^3 \sinh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} + \frac{2b^3 \cosh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} - \frac{2b^2 \sinh^2(a+bx)}{3d^3(c+dx)} - \frac{b \sinh(a+bx) \cosh(a+bx)}{3d^2(c+dx)^2}$$

[Out]  $-b^2/(3*d^3*(c + d*x)) + (2*b^3*CoshIntegral[(2*b*c)/d + 2*b*x]*Sinh[2*a - (2*b*c)/d])/(3*d^4) - (b*Cosh[a + b*x]*Sinh[a + b*x])/(3*d^2*(c + d*x)^2) - Sinh[a + b*x]^2/(3*d*(c + d*x)^3) - (2*b^2*Sinh[a + b*x]^2)/(3*d^3*(c + d*x)) + (2*b^3*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(3*d^4)$

**Rubi [A]** time = 0.18726, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {3314, 32, 3313, 12, 3303, 3298, 3301}

$$\frac{2b^3 \sinh\left(2a - \frac{2bc}{d}\right) \text{Chi}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} + \frac{2b^3 \cosh\left(2a - \frac{2bc}{d}\right) \text{Shi}\left(\frac{2bc}{d} + 2bx\right)}{3d^4} - \frac{2b^2 \sinh^2(a+bx)}{3d^3(c+dx)} - \frac{b \sinh(a+bx) \cosh(a+bx)}{3d^2(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Int[Sinh[a + b\*x]^2/(c + d\*x)^4, x]

[Out]  $-b^2/(3*d^3*(c + d*x)) + (2*b^3*CoshIntegral[(2*b*c)/d + 2*b*x]*Sinh[2*a - (2*b*c)/d])/(3*d^4) - (b*Cosh[a + b*x]*Sinh[a + b*x])/(3*d^2*(c + d*x)^2) - Sinh[a + b*x]^2/(3*d*(c + d*x)^3) - (2*b^2*Sinh[a + b*x]^2)/(3*d^3*(c + d*x)) + (2*b^3*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(3*d^4)$

#### Rule 3314

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*(b\*Sinh[e + f\*x])^n)/(d\*(m + 1)), x] + (Dist[(b^2\*f^2\*n\*(n - 1))/(d^2\*(m + 1)\*(m + 2)), Int[(c + d\*x)^(m + 2)\*(b\*Sinh[e + f\*x])^(n - 2), x], x] - Dist[(f^2\*n^2)/(d^2\*(m + 1)\*(m + 2)), Int[(c + d\*x)^(m + 2)\*(b\*Sinh[e + f\*x])^n, x], x] - Simp[(b\*f\*n\*(c + d\*x)^(m + 2)\*Cos[e + f\*x]\*(b\*Sinh[e + f\*x])^(n - 1))/(d^2\*(m + 1)\*(m + 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]

#### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m + 1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n - 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(a+bx)}{(c+dx)^4} dx &= -\frac{b \cosh(a+bx) \sinh(a+bx)}{3d^2(c+dx)^2} - \frac{\sinh^2(a+bx)}{3d(c+dx)^3} + \frac{b^2 \int \frac{1}{(c+dx)^2} dx}{3d^2} + \frac{(2b^2) \int \frac{\sinh^2(a+bx)}{(c+dx)^2} dx}{3d^2} \\
&= -\frac{b^2}{3d^3(c+dx)} - \frac{b \cosh(a+bx) \sinh(a+bx)}{3d^2(c+dx)^2} - \frac{\sinh^2(a+bx)}{3d(c+dx)^3} - \frac{2b^2 \sinh^2(a+bx)}{3d^3(c+dx)} - \frac{(4ib^3) \int \frac{i \sinh^2(a+bx)}{(c+dx)^2} dx}{3d^2} \\
&= -\frac{b^2}{3d^3(c+dx)} - \frac{b \cosh(a+bx) \sinh(a+bx)}{3d^2(c+dx)^2} - \frac{\sinh^2(a+bx)}{3d(c+dx)^3} - \frac{2b^2 \sinh^2(a+bx)}{3d^3(c+dx)} + \frac{(2b^3) \int \frac{\sinh^2(a+bx)}{(c+dx)^2} dx}{3d^2} \\
&= -\frac{b^2}{3d^3(c+dx)} - \frac{b \cosh(a+bx) \sinh(a+bx)}{3d^2(c+dx)^2} - \frac{\sinh^2(a+bx)}{3d(c+dx)^3} - \frac{2b^2 \sinh^2(a+bx)}{3d^3(c+dx)} + \frac{(2b^3 \cosh(a+bx) \sinh(a+bx)) \int \frac{\sinh(a+bx)}{(c+dx)^2} dx}{3d^2} \\
&= -\frac{b^2}{3d^3(c+dx)} + \frac{2b^3 \operatorname{Chi}\left(\frac{2bc}{d} + 2bx\right) \sinh\left(2a - \frac{2bc}{d}\right)}{3d^4} - \frac{b \cosh(a+bx) \sinh(a+bx)}{3d^2(c+dx)^2} - \frac{\sinh^2(a+bx)}{3d(c+dx)^3}
\end{aligned}$$

**Mathematica [A]** time = 0.888442, size = 123, normalized size = 0.76

$$\frac{4b^3 \sinh\left(2a - \frac{2bc}{d}\right) \operatorname{Chi}\left(\frac{2b(c+dx)}{d}\right) - \frac{d(\cosh(2(a+bx))(2b^2(c+dx)^2+d^2)+d(b(c+dx) \sinh(2(a+bx))-d))}{(c+dx)^3} + 4b^3 \cosh\left(2a - \frac{2bc}{d}\right) \operatorname{Shi}\left(\frac{2b(c+dx)}{d}\right)}{6d^4}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[a + b\*x]^2/(c + d\*x)^4,x]

[Out] (4\*b^3\*CoshIntegral[(2\*b\*(c + d\*x))/d]\*Sinh[2\*a - (2\*b\*c)/d] - (d\*((d^2 + 2\*b^2\*(c + d\*x)^2)\*Cosh[2\*(a + b\*x)] + d\*(-d + b\*(c + d\*x)\*Sinh[2\*(a + b\*x)])))/(c + d\*x)^3 + 4\*b^3\*Cosh[2\*a - (2\*b\*c)/d]\*SinhIntegral[(2\*b\*(c + d\*x))/d])/(6\*d^4)

**Maple [B]** time = 0.118, size = 555, normalized size = 3.4

$$\frac{1}{6d(dx+c)^3} - \frac{b^5 e^{-2bx-2ax^2}}{6d(b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 dx + c^3 b^3)} - \frac{b^5 e^{-2bx-2ax^2}}{3d^2(b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 dx + c^3 b^3)} - \frac{b^5 e^{-2bx-2ax^2}}{6d^3(b^3 d^3 x^3 + 3b^3 c d^2 x^2 + 3b^3 c^2 dx + c^3 b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(b\*x+a)^2/(d\*x+c)^4,x)

[Out]  $\frac{1}{6} \frac{d}{d} \frac{1}{(d*x+c)^3} - \frac{1}{6} \frac{b^5 \exp(-2*b*x-2*a)}{d} \frac{1}{(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*x^2} - \frac{1}{3} \frac{b^5 \exp(-2*b*x-2*a)}{d^2} \frac{1}{(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c*x} - \frac{1}{6} \frac{b^5 \exp(-2*b*x-2*a)}{d^3} \frac{1}{(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c^2} + \frac{1}{12} \frac{b^4 \exp(-2*b*x-2*a)}{d} \frac{1}{(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*x} + \frac{1}{12} \frac{b^4 \exp(-2*b*x-2*a)}{d^2} \frac{1}{(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c} - \frac{1}{12} \frac{b^3 \exp(-2*b*x-2*a)}{d} \frac{1}{(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)} + \frac{1}{3} \frac{b^3}{d^4} \frac{\exp(-2*(a*d-b*c)/d) * Ei(1, 2*b*x+2*a-2*(a*d-b*c)/d) - 1/12*b^3/d^4*\exp(2*b*x+2*a)/(b*c/d+b*x)^3 - 1/12*b^3/d^4*\exp(2*b*x+2*a)/(b*c/d+b*x)^2 - 1/6*b^3/d^4*\exp(2*b*x+2*a)/(b*c/d+b*x) - 1/3*b^3/d^4*\exp(2*(a*d-b*c)/d) * Ei(1, -2*b*x-2*a-2*(-a*d+b*c)/d)}{d}$

**Maxima [A]** time = 1.51533, size = 149, normalized size = 0.92

$$\frac{1}{6(d^4x^3 + 3cd^3x^2 + 3c^2d^2x + c^3d)} - \frac{e^{(-2a + \frac{2bc}{d})} E_4\left(\frac{2(dx+c)b}{d}\right)}{4(dx+c)^3d} - \frac{e^{(2a - \frac{2bc}{d})} E_4\left(-\frac{2(dx+c)b}{d}\right)}{4(dx+c)^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2/(d\*x+c)^4,x, algorithm="maxima")

[Out]  $\frac{1}{6} \frac{1}{(d^4*x^3 + 3*c*d^3*x^2 + 3*c^2*d^2*x + c^3*d)} - \frac{1}{4} \frac{e^{(-2*a + 2*b*c/d)} \exp\_integral\_e(4, 2*(d*x + c)*b/d)/((d*x + c)^3*d)}{d} - \frac{1}{4} \frac{e^{(2*a - 2*b*c/d)} \exp\_integral\_e(4, -2*(d*x + c)*b/d)/((d*x + c)^3*d)}{d}$

**Fricas [B]** time = 2.73919, size = 857, normalized size = 5.29

$$d^3 - (2b^2d^3x^2 + 4b^2cd^2x + 2b^2c^2d + d^3) \cosh(bx + a)^2 - 2(bd^3x + bcd^2) \cosh(bx + a) \sinh(bx + a) - (2b^2d^3x^2 + 4b^2c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2/(d\*x+c)^4,x, algorithm="fricas")

[Out]  $\frac{1}{6} \frac{1}{(d^3 - (2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*\cosh(b*x + a)^2 - 2*(b*d^3*x + b*c*d^2)*\cosh(b*x + a)*\sinh(b*x + a) - (2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*\sinh(b*x + a)^2 + 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*Ei(2*(b*d*x + b*c)/d) - (b^3*d^3*x^3$

$$+ 3b^3cd^2x^2 + 3b^3c^2dx + b^3c^3) \operatorname{Ei}(-2(bdx + bc)/d) \cosh(-2(bc - ad)/d) + 2((b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + b^3c^3) \operatorname{Ei}(2(bdx + bc)/d) + (b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + b^3c^3) \operatorname{Ei}(-2(bdx + bc)/d)) \sinh(-2(bc - ad)/d) / (d^7x^3 + 3cd^6x^2 + 3c^2d^5x + c^3d^4)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a + bx)}{(c + dx)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)\*\*2/(d\*x+c)\*\*4, x)

[Out] Integral(sinh(a + b\*x)\*\*2/(c + d\*x)\*\*4, x)

**Giac [B]** time = 1.15627, size = 725, normalized size = 4.48

$$4b^3d^3x^3 \operatorname{Ei}\left(\frac{2(bdx+bc)}{d}\right) e^{\left(2a-\frac{2bc}{d}\right)} - 4b^3d^3x^3 \operatorname{Ei}\left(-\frac{2(bdx+bc)}{d}\right) e^{\left(-2a+\frac{2bc}{d}\right)} + 12b^3cd^2x^2 \operatorname{Ei}\left(\frac{2(bdx+bc)}{d}\right) e^{\left(2a-\frac{2bc}{d}\right)} - 12b^3cd^2x^2 \operatorname{Ei}\left(-\frac{2(bdx+bc)}{d}\right) e^{\left(-2a+\frac{2bc}{d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2/(d\*x+c)^4, x, algorithm="giac")

[Out] 1/12\*(4\*b^3\*d^3\*x^3\*Ei(2\*(b\*d\*x + b\*c)/d)\*e^(2\*a - 2\*b\*c/d) - 4\*b^3\*d^3\*x^3\*Ei(-2\*(b\*d\*x + b\*c)/d)\*e^(-2\*a + 2\*b\*c/d) + 12\*b^3\*c\*d^2\*x^2\*Ei(2\*(b\*d\*x + b\*c)/d)\*e^(2\*a - 2\*b\*c/d) - 12\*b^3\*c\*d^2\*x^2\*Ei(-2\*(b\*d\*x + b\*c)/d)\*e^(-2\*a + 2\*b\*c/d) + 12\*b^3\*c^2\*d\*x\*Ei(2\*(b\*d\*x + b\*c)/d)\*e^(2\*a - 2\*b\*c/d) - 12\*b^3\*c^2\*d\*x\*Ei(-2\*(b\*d\*x + b\*c)/d)\*e^(-2\*a + 2\*b\*c/d) - 2\*b^2\*d^3\*x^2\*e^(2\*b\*x + 2\*a) - 2\*b^2\*d^3\*x^2\*e^(-2\*b\*x - 2\*a) + 4\*b^3\*c^3\*Ei(2\*(b\*d\*x + b\*c)/d)\*e^(2\*a - 2\*b\*c/d) - 4\*b^3\*c^3\*Ei(-2\*(b\*d\*x + b\*c)/d)\*e^(-2\*a + 2\*b\*c/d) - 4\*b^2\*c\*d^2\*x\*e^(2\*b\*x + 2\*a) - 4\*b^2\*c\*d^2\*x\*e^(-2\*b\*x - 2\*a) - 2\*b^2\*c^2\*d\*e^(2\*b\*x + 2\*a) - b\*d^3\*x\*e^(2\*b\*x + 2\*a) - 2\*b^2\*c^2\*d\*e^(-2\*b\*x - 2\*a) + b\*d^3\*x\*e^(-2\*b\*x - 2\*a) - b\*c\*d^2\*e^(2\*b\*x + 2\*a) + b\*c\*d^2\*e^(-2\*b\*x - 2\*a) - d^3\*e^(2\*b\*x + 2\*a) - d^3\*e^(-2\*b\*x - 2\*a) + 2\*d^3)/(d^7\*x^3 + 3\*c\*d^6\*x^2 + 3\*c^2\*d^5\*x + c^3\*d^4)

### 3.16 $\int (c + dx)^4 \sinh^3(a + bx) dx$

**Optimal.** Leaf size=225

$$-\frac{8d^3(c + dx)\sinh^3(a + bx)}{27b^4} + \frac{160d^3(c + dx)\sinh(a + bx)}{9b^4} - \frac{80d^2(c + dx)^2 \cosh(a + bx)}{9b^3} + \frac{4d^2(c + dx)^2 \sinh^2(a + bx) \cosh(a + bx)}{9b^3}$$

[Out]  $(-488*d^4*Cosh[a + b*x])/(27*b^5) - (80*d^2*(c + d*x)^2*Cosh[a + b*x])/(9*b^3) - (2*(c + d*x)^4*Cosh[a + b*x])/(3*b) + (8*d^4*Cosh[a + b*x]^3)/(81*b^5) + (160*d^3*(c + d*x)*Sinh[a + b*x])/(9*b^4) + (8*d*(c + d*x)^3*Sinh[a + b*x])/(3*b^2) + (4*d^2*(c + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x]^2)/(9*b^3) + ((c + d*x)^4*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (8*d^3*(c + d*x)*Sinh[a + b*x]^3)/(27*b^4) - (4*d*(c + d*x)^3*Sinh[a + b*x]^3)/(9*b^2)$

**Rubi [A]** time = 0.359891, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3311, 3296, 2638, 2633}

$$-\frac{8d^3(c + dx)\sinh^3(a + bx)}{27b^4} + \frac{160d^3(c + dx)\sinh(a + bx)}{9b^4} - \frac{80d^2(c + dx)^2 \cosh(a + bx)}{9b^3} + \frac{4d^2(c + dx)^2 \sinh^2(a + bx) \cosh(a + bx)}{9b^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^4\*Sinh[a + b\*x]^3,x]

[Out]  $(-488*d^4*Cosh[a + b*x])/(27*b^5) - (80*d^2*(c + d*x)^2*Cosh[a + b*x])/(9*b^3) - (2*(c + d*x)^4*Cosh[a + b*x])/(3*b) + (8*d^4*Cosh[a + b*x]^3)/(81*b^5) + (160*d^3*(c + d*x)*Sinh[a + b*x])/(9*b^4) + (8*d*(c + d*x)^3*Sinh[a + b*x])/(3*b^2) + (4*d^2*(c + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x]^2)/(9*b^3) + ((c + d*x)^4*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (8*d^3*(c + d*x)*Sinh[a + b*x]^3)/(27*b^4) - (4*d*(c + d*x)^3*Sinh[a + b*x]^3)/(9*b^2)$

#### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*Sinh[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*Sinh[e + f\*x])^(n - 2), x], x] - Dist[(d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*Sinh[e + f\*x])^n, x], x] - Simp[(b\*(c + d\*x)^m\*Cos[e + f\*x]\*(b\*Sinh[e + f\*x])^(n - 1))/(f\*n), x]) /;

FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

#### Rule 3296



```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

### Rubi steps

$$\begin{aligned}
\int (c + dx)^4 \sinh^3(a + bx) dx &= \frac{(c + dx)^4 \cosh(a + bx) \sinh^2(a + bx)}{3b} - \frac{4d(c + dx)^3 \sinh^3(a + bx)}{9b^2} - \frac{2}{3} \int (c + dx)^4 \sinh(a + bx) dx \\
&= -\frac{2(c + dx)^4 \cosh(a + bx)}{3b} + \frac{4d^2(c + dx)^2 \cosh(a + bx) \sinh^2(a + bx)}{9b^3} + \frac{(c + dx)^4 \cosh(a + bx)}{3b} \\
&= -\frac{8d^2(c + dx)^2 \cosh(a + bx)}{9b^3} - \frac{2(c + dx)^4 \cosh(a + bx)}{3b} + \frac{8d(c + dx)^3 \sinh(a + bx)}{3b^2} + \frac{4d^4 \cosh(a + bx)}{27b^5} \\
&= -\frac{8d^4 \cosh(a + bx)}{27b^5} - \frac{80d^2(c + dx)^2 \cosh(a + bx)}{9b^3} - \frac{2(c + dx)^4 \cosh(a + bx)}{3b} + \frac{8d^4 \cosh(a + bx)}{81b^5} \\
&= -\frac{56d^4 \cosh(a + bx)}{27b^5} - \frac{80d^2(c + dx)^2 \cosh(a + bx)}{9b^3} - \frac{2(c + dx)^4 \cosh(a + bx)}{3b} + \frac{8d^4 \cosh(a + bx)}{81b^5} \\
&= -\frac{488d^4 \cosh(a + bx)}{27b^5} - \frac{80d^2(c + dx)^2 \cosh(a + bx)}{9b^3} - \frac{2(c + dx)^4 \cosh(a + bx)}{3b} + \frac{8d^4 \cosh(a + bx)}{81b^5}
\end{aligned}$$

**Mathematica [A]** time = 1.01925, size = 150, normalized size = 0.67

$$\frac{-243 \cosh(a + bx) (12b^2 d^2 (c + dx)^2 + b^4 (c + dx)^4 + 24d^4) + \cosh(3(a + bx)) (36b^2 d^2 (c + dx)^2 + 27b^4 (c + dx)^4 + 8d^4)}{324b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sinh[a + b*x]^3,x]
```

```
[Out] (-243*(24*d^4 + 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cosh[a + b*x] + (
8*d^4 + 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cosh[3*(a + b*x)] - 24
*b*d*(c + d*x)*(-242*d^2 - 39*b^2*(c + d*x)^2 + (2*d^2 + 3*b^2*(c + d*x)^2)
*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(324*b^5)
```

**Maple [B]** time = 0.156, size = 1217, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sinh(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^4*d^4*(-2/3*(b*x+a)^4*cosh(b*x+a)+1/3*(b*x+a)^4*cosh(b*x+a)*sinh(b
*x+a)^2+28/9*(b*x+a)^3*sinh(b*x+a)-80/9*(b*x+a)^2*cosh(b*x+a)+488/27*(b*x+a
)*sinh(b*x+a)-1456/81*cosh(b*x+a)-4/9*(b*x+a)^3*sinh(b*x+a)*cosh(b*x+a)^2+4
/9*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)-8/27*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)
^2+8/81*sinh(b*x+a)^2*cosh(b*x+a))-4/b^4*d^4*a*(-2/3*(b*x+a)^3*cosh(b*x+a)+
1/3*(b*x+a)^3*cosh(b*x+a)*sinh(b*x+a)^2+7/3*(b*x+a)^2*sinh(b*x+a)-40/9*(b*x
+a)*cosh(b*x+a)+122/27*sinh(b*x+a)-1/3*(b*x+a)^2*sinh(b*x+a)*cosh(b*x+a)^2+
2/9*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)-2/27*sinh(b*x+a)*cosh(b*x+a)^2)+6/b^4
*d^4*a^2*(1/3*(b*x+a)^2*sinh(b*x+a)^2*cosh(b*x+a)-2/3*(b*x+a)^2*cosh(b*x+a)
-2/9*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2+14/9*(b*x+a)*sinh(b*x+a)+2/27*sinh(b
*x+a)^2*cosh(b*x+a)-40/27*cosh(b*x+a))-4/b^4*d^4*a^3*(-2/3*(b*x+a)*cosh(b*x
+a)+1/3*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)+7/9*sinh(b*x+a)-1/9*sinh(b*x+a)*c
osh(b*x+a)^2)+1/b^4*d^4*a^4*(-2/3+1/3*sinh(b*x+a)^2)*cosh(b*x+a)+4/b^3*c*d^
3*(-2/3*(b*x+a)^3*cosh(b*x+a)+1/3*(b*x+a)^3*cosh(b*x+a)*sinh(b*x+a)^2+7/3*(
b*x+a)^2*sinh(b*x+a)-40/9*(b*x+a)*cosh(b*x+a)+122/27*sinh(b*x+a)-1/3*(b*x+a
)^2*sinh(b*x+a)*cosh(b*x+a)^2+2/9*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)-2/27*si
nh(b*x+a)*cosh(b*x+a)^2)-12/b^3*c*d^3*a*(1/3*(b*x+a)^2*sinh(b*x+a)^2*cosh(b
*x+a)-2/3*(b*x+a)^2*cosh(b*x+a)-2/9*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2+14/9*
(b*x+a)*sinh(b*x+a)+2/27*sinh(b*x+a)^2*cosh(b*x+a)-40/27*cosh(b*x+a))+12/b^
3*c*d^3*a^2*(-2/3*(b*x+a)*cosh(b*x+a)+1/3*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)
+7/9*sinh(b*x+a)-1/9*sinh(b*x+a)*cosh(b*x+a)^2)-4/b^3*c*d^3*a^3*(-2/3+1/3*si
nh(b*x+a)^2)*cosh(b*x+a)+6/b^2*c^2*d^2*(1/3*(b*x+a)^2*sinh(b*x+a)^2*cosh(b
*x+a)-2/3*(b*x+a)^2*cosh(b*x+a)-2/9*(b*x+a)*sinh(b*x+a)*cosh(b*x+a)^2+14/9*
(b*x+a)*sinh(b*x+a)+2/27*sinh(b*x+a)^2*cosh(b*x+a)-40/27*cosh(b*x+a))-12/b^
2*c^2*d^2*a*(-2/3*(b*x+a)*cosh(b*x+a)+1/3*(b*x+a)*sinh(b*x+a)^2*cosh(b*x+a)
+7/9*sinh(b*x+a)-1/9*sinh(b*x+a)*cosh(b*x+a)^2)+6/b^2*c^2*d^2*a^2*(-2/3+1/3
*sinh(b*x+a)^2)*cosh(b*x+a)+4/b*c^3*d*(-2/3*(b*x+a)*cosh(b*x+a)+1/3*(b*x+a)
*sinh(b*x+a)^2*cosh(b*x+a)+7/9*sinh(b*x+a)-1/9*sinh(b*x+a)*cosh(b*x+a)^2)-4
/b*c^3*d*a*(-2/3+1/3*sinh(b*x+a)^2)*cosh(b*x+a)+c^4*(-2/3+1/3*sinh(b*x+a)^2
```

) \* cosh(b\*x+a))

**Maxima [B]** time = 1.35361, size = 863, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^4\*sinh(b\*x+a)^3,x, algorithm="maxima")

[Out]  $\frac{1}{18}c^3d((3bx^3e^{3a} - e^{3a})e^{3bx}/b^2 - 27(bxe^a - e^a)e^{bx}/b^2 - 27(bx + 1)e^{-bx - a}/b^2 + (3bx + 1)e^{-3bx - 3a}/b^2) + \frac{1}{24}c^4(e^{3bx + 3a}/b - 9e^{(bx + a)}/b - 9e^{-(bx - a)}/b + e^{(-3bx - 3a)}/b) + \frac{1}{36}c^2d^2((9b^2x^2e^{3a} - 6bx^2e^{3a} + 2e^{3a})e^{3bx}/b^3 - 81(b^2x^2e^a - 2bx^2e^a + 2e^a)e^{bx}/b^3 - 81(b^2x^2 + 2bx + 2)e^{-bx - a}/b^3 + (9b^2x^2 + 6bx + 2)e^{-3bx - 3a}/b^3) + \frac{1}{54}cd^3((9b^3x^3e^{3a} - 9b^2x^3e^{3a} + 6bx^3e^{3a} - 2e^{3a})e^{3bx}/b^4 - 81(b^3x^3e^a - 3b^2x^3e^a + 6bx^3e^a - 6e^a)e^{bx}/b^4 - 81(b^3x^3 + 3b^2x^2 + 6bx + 6)e^{-bx - a}/b^4 + (9b^3x^3 + 9b^2x^2 + 6bx + 2)e^{-3bx - 3a}/b^4) + \frac{1}{648}d^4((27b^4x^4e^{3a} - 36b^3x^4e^{3a} + 36b^2x^4e^{3a} - 24bx^4e^{3a} + 8e^{3a})e^{3bx}/b^5 - 243(b^4x^4e^a - 4b^3x^4e^a + 12b^2x^4e^a - 24bx^4e^a + 24e^a)e^{bx}/b^5 - 243(b^4x^4 + 4b^3x^3 + 12b^2x^2 + 24bx + 24)e^{-bx - a}/b^5 + (27b^4x^4 + 36b^3x^3 + 36b^2x^2 + 24bx + 8)e^{-3bx - 3a}/b^5)$

**Fricas [B]** time = 2.82189, size = 1141, normalized size = 5.07

$(27b^4d^4x^4 + 108b^4cd^3x^3 + 27b^4c^4 + 36b^2c^2d^2 + 8d^4 + 18(9b^4c^2d^2 + 2b^2d^4)x^2 + 36(3b^4c^3d + 2b^2cd^3)x) \cosh(bx + a)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^4\*sinh(b\*x+a)^3,x, algorithm="fricas")

[Out]  $\frac{1}{324}((27b^4d^4x^4 + 108b^4cd^3x^3 + 27b^4c^4 + 36b^2c^2d^2 + 8d^4 + 18(9b^4c^2d^2 + 2b^2d^4)x^2 + 36(3b^4c^3d + 2b^2cd^3)x) \cosh(bx + a)^3 + 3(27b^4d^4x^4 + 108b^4cd^3x^3 + 27b^4c^4 + 36b^2c^2d^2 + 8d^4 + 18(9b^4c^2d^2 + 2b^2d^4)x^2 + 36(3b^4c^3d + 2b^2cd^3)x) \cosh(bx + a) \sinh(bx + a)^2 - 12(3b^3d^4x^3 + 9b^2cd^3x^2 + 3b^2c^2d^2 + 8d^4 + 18(9b^4c^2d^2 + 2b^2d^4)x^2 + 36(3b^4c^3d + 2b^2cd^3)x) \cosh(bx + a) \sinh(bx + a) - 12(3b^3d^4x^3 + 9b^2cd^3x^2 + 3b^2c^2d^2 + 8d^4 + 18(9b^4c^2d^2 + 2b^2d^4)x^2 + 36(3b^4c^3d + 2b^2cd^3)x) \sinh(bx + a)^2)$

$$\frac{b^3 c d^3 x^2 + 3 b^3 c^3 d + 2 b c d^3 + (9 b^3 c^2 d^2 + 2 b d^4) x \sinh(b x + a)^3 - 243 (b^4 d^4 x^4 + 4 b^4 c d^3 x^3 + b^4 c^4 + 12 b^2 c^2 d^2 + 24 d^4 + 6 (b^4 c^2 d^2 + 2 b^2 d^4) x^2 + 4 (b^4 c^3 d + 6 b^2 c d^3) x) \cosh(b x + a) + 36 (27 b^3 d^4 x^3 + 81 b^3 c d^3 x^2 + 27 b^3 c^3 d + 16 2 b c d^3 - (3 b^3 d^4 x^3 + 9 b^3 c d^3 x^2 + 3 b^3 c^3 d + 2 b c d^3 + (9 b^3 c^2 d^2 + 2 b d^4) x) \cosh(b x + a)^2 + 81 (b^3 c^2 d^2 + 2 b d^4) x \sinh(b x + a)) / b^5$$

**Sympy [A]** time = 12.5076, size = 772, normalized size = 3.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*4\*sinh(b\*x+a)\*\*3,x)

[Out] Piecewise((c\*\*4\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/b - 2\*c\*\*4\*cosh(a + b\*x)\*\*3/(3\*b) + 4\*c\*\*3\*d\*x\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/b - 8\*c\*\*3\*d\*x\*cosh(a + b\*x)\*\*3/(3\*b) + 6\*c\*\*2\*d\*\*2\*x\*\*2\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/b - 4\*c\*\*2\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*\*3/b + 4\*c\*d\*\*3\*x\*\*3\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/b - 8\*c\*d\*\*3\*x\*\*3\*cosh(a + b\*x)\*\*3/(3\*b) + d\*\*4\*x\*\*4\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/b - 2\*d\*\*4\*x\*\*4\*cosh(a + b\*x)\*\*3/(3\*b) - 28\*c\*\*3\*d\*sinh(a + b\*x)\*\*3/(9\*b\*\*2) + 8\*c\*\*3\*d\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/(3\*b\*\*2) - 28\*c\*\*2\*d\*\*2\*x\*sinh(a + b\*x)\*\*3/(3\*b\*\*2) + 8\*c\*\*2\*d\*\*2\*x\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/b\*\*2 - 28\*c\*d\*\*3\*x\*\*2\*sinh(a + b\*x)\*\*3/(3\*b\*\*2) + 8\*c\*d\*\*3\*x\*\*2\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/b\*\*2 - 28\*d\*\*4\*x\*\*3\*sinh(a + b\*x)\*\*3/(9\*b\*\*2) + 8\*d\*\*4\*x\*\*3\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/(3\*b\*\*2) + 28\*c\*\*2\*d\*\*2\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/(3\*b\*\*3) - 80\*c\*\*2\*d\*\*2\*cosh(a + b\*x)\*\*3/(9\*b\*\*3) + 56\*c\*d\*\*3\*x\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/(3\*b\*\*3) - 160\*c\*d\*\*3\*x\*cosh(a + b\*x)\*\*3/(9\*b\*\*3) + 28\*d\*\*4\*x\*\*2\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/(3\*b\*\*3) - 80\*d\*\*4\*x\*\*2\*cosh(a + b\*x)\*\*3/(9\*b\*\*3) - 488\*c\*d\*\*3\*sinh(a + b\*x)\*\*3/(27\*b\*\*4) + 160\*c\*d\*\*3\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/(9\*b\*\*4) - 488\*d\*\*4\*x\*sinh(a + b\*x)\*\*3/(27\*b\*\*4) + 160\*d\*\*4\*x\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/(9\*b\*\*4) + 488\*d\*\*4\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/(27\*b\*\*5) - 1456\*d\*\*4\*cosh(a + b\*x)\*\*3/(81\*b\*\*5), Ne(b, 0)), ((c\*\*4\*x + 2\*c\*\*3\*d\*x\*\*2 + 2\*c\*\*2\*d\*\*2\*x\*\*3 + c\*d\*\*3\*x\*\*4 + d\*\*4\*x\*\*5/5)\*sinh(a)\*\*3, True))

**Giac [B]** time = 1.2083, size = 883, normalized size = 3.92

$$\frac{(27 b^4 d^4 x^4 + 108 b^4 c d^3 x^3 + 162 b^4 c^2 d^2 x^2 - 36 b^3 d^4 x^3 + 108 b^4 c^3 d x - 108 b^3 c d^3 x^2 + 27 b^4 c^4 - 108 b^3 c^2 d^2 x + 36 b^2 d^4 x^2 - 3648 b^5)}{648 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^4\*sinh(b\*x+a)^3,x, algorithm="giac")

[Out]  $\frac{1}{648} \cdot (27 \cdot b^4 \cdot d^4 \cdot x^4 + 108 \cdot b^4 \cdot c \cdot d^3 \cdot x^3 + 162 \cdot b^4 \cdot c^2 \cdot d^2 \cdot x^2 - 36 \cdot b^3 \cdot d^4 \cdot x^3 + 108 \cdot b^4 \cdot c^3 \cdot d \cdot x - 108 \cdot b^3 \cdot c \cdot d^3 \cdot x^2 + 27 \cdot b^4 \cdot c^4 - 108 \cdot b^3 \cdot c^2 \cdot d^2 \cdot x + 36 \cdot b^2 \cdot d^4 \cdot x^2 - 36 \cdot b^3 \cdot c^3 \cdot d + 72 \cdot b^2 \cdot c \cdot d^3 \cdot x + 36 \cdot b^2 \cdot c^2 \cdot d^2 - 24 \cdot b \cdot d^4 \cdot x - 24 \cdot b \cdot c \cdot d^3 + 8 \cdot d^4) \cdot e^{(3 \cdot b \cdot x + 3 \cdot a)} / b^5 - \frac{3}{8} \cdot (b^4 \cdot d^4 \cdot x^4 + 4 \cdot b^4 \cdot c \cdot d^3 \cdot x^3 + 6 \cdot b^4 \cdot c^2 \cdot d^2 \cdot x^2 - 4 \cdot b^3 \cdot d^4 \cdot x^3 + 4 \cdot b^4 \cdot c^3 \cdot d \cdot x - 12 \cdot b^3 \cdot c \cdot d^3 \cdot x^2 + b^4 \cdot c^4 - 12 \cdot b^3 \cdot c^2 \cdot d^2 \cdot x + 12 \cdot b^2 \cdot d^4 \cdot x^2 - 4 \cdot b^3 \cdot c^3 \cdot d + 24 \cdot b^2 \cdot c \cdot d^3 \cdot x + 12 \cdot b^2 \cdot c^2 \cdot d^2 - 24 \cdot b \cdot d^4 \cdot x - 24 \cdot b \cdot c \cdot d^3 + 24 \cdot d^4) \cdot e^{(b \cdot x + a)} / b^5 - \frac{3}{8} \cdot (b^4 \cdot d^4 \cdot x^4 + 4 \cdot b^4 \cdot c \cdot d^3 \cdot x^3 + 6 \cdot b^4 \cdot c^2 \cdot d^2 \cdot x^2 + 4 \cdot b^3 \cdot d^4 \cdot x^3 + 4 \cdot b^4 \cdot c^3 \cdot d \cdot x + 12 \cdot b^3 \cdot c \cdot d^3 \cdot x^2 + b^4 \cdot c^4 + 12 \cdot b^3 \cdot c^2 \cdot d^2 \cdot x + 12 \cdot b^2 \cdot d^4 \cdot x^2 + 4 \cdot b^3 \cdot c^3 \cdot d + 24 \cdot b^2 \cdot c \cdot d^3 \cdot x + 12 \cdot b^2 \cdot c^2 \cdot d^2 + 24 \cdot b \cdot d^4 \cdot x + 24 \cdot b \cdot c \cdot d^3 + 24 \cdot d^4) \cdot e^{(-b \cdot x - a)} / b^5 + \frac{1}{648} \cdot (27 \cdot b^4 \cdot d^4 \cdot x^4 + 108 \cdot b^4 \cdot c \cdot d^3 \cdot x^3 + 162 \cdot b^4 \cdot c^2 \cdot d^2 \cdot x^2 + 36 \cdot b^3 \cdot d^4 \cdot x^3 + 108 \cdot b^4 \cdot c^3 \cdot d \cdot x + 108 \cdot b^3 \cdot c \cdot d^3 \cdot x^2 + 27 \cdot b^4 \cdot c^4 + 108 \cdot b^3 \cdot c^2 \cdot d^2 \cdot x + 36 \cdot b^2 \cdot d^4 \cdot x^2 + 36 \cdot b^3 \cdot c^3 \cdot d + 72 \cdot b^2 \cdot c \cdot d^3 \cdot x + 36 \cdot b^2 \cdot c^2 \cdot d^2 + 24 \cdot b \cdot d^4 \cdot x + 24 \cdot b \cdot c \cdot d^3 + 8 \cdot d^4) \cdot e^{(-3 \cdot b \cdot x - 3 \cdot a)} / b^5$

### 3.17 $\int (c + dx)^3 \sinh^3(a + bx) dx$

**Optimal.** Leaf size=175

$$-\frac{40d^2(c + dx) \cosh(a + bx)}{9b^3} + \frac{2d^2(c + dx) \sinh^2(a + bx) \cosh(a + bx)}{9b^3} - \frac{d(c + dx)^2 \sinh^3(a + bx)}{3b^2} + \frac{2d(c + dx)^2 \sinh(a + bx)}{b^2}$$

[Out]  $(-40*d^2*(c + d*x)*Cosh[a + b*x])/(9*b^3) - (2*(c + d*x)^3*Cosh[a + b*x])/(3*b) + (40*d^3*Sinh[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*Sinh[a + b*x])/b^2 + (2*d^2*(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^2)/(9*b^3) + ((c + d*x)^3*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (2*d^3*Sinh[a + b*x]^3)/(27*b^4) - (d*(c + d*x)^2*Sinh[a + b*x]^3)/(3*b^2)$

**Rubi [A]** time = 0.226485, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3311, 3296, 2637, 3310}

$$-\frac{40d^2(c + dx) \cosh(a + bx)}{9b^3} + \frac{2d^2(c + dx) \sinh^2(a + bx) \cosh(a + bx)}{9b^3} - \frac{d(c + dx)^2 \sinh^3(a + bx)}{3b^2} + \frac{2d(c + dx)^2 \sinh(a + bx)}{b^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^3\*Sinh[a + b\*x]^3,x]

[Out]  $(-40*d^2*(c + d*x)*Cosh[a + b*x])/(9*b^3) - (2*(c + d*x)^3*Cosh[a + b*x])/(3*b) + (40*d^3*Sinh[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*Sinh[a + b*x])/b^2 + (2*d^2*(c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^2)/(9*b^3) + ((c + d*x)^3*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (2*d^3*Sinh[a + b*x]^3)/(27*b^4) - (d*(c + d*x)^2*Sinh[a + b*x]^3)/(3*b^2)$

#### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*Sinh[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*Sinh[e + f\*x])^(n - 2), x], x] - Dist[(d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*Sinh[e + f\*x])^n, x], x] - Simp[(b\*(c + d\*x)^m\*Cosh[e + f\*x]\*(b\*Sinh[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> -Simp[(c + d\*x)^m\*Cosh[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[

$e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

### Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x\_Symbol] \text{ :> } \text{Simp}[\sin[c + d*x]/d, x] /;$   
 $\text{FreeQ}\{c, d\}, x]$

### Rule 3310

$\text{Int}[((c_.) + (d_.)*(x_.))*((b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \text{ :>}$   
 $\text{Simp}[(d*(b*\sin[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n - 1))/n, \text{Int}[(c$   
 $+ d*x)*(b*\sin[e + f*x])^{(n - 2)}, x], x] - \text{Simp}[(b*(c + d*x)*\cos[e + f*x]*(b$   
 $*\sin[e + f*x])^{(n - 1)}/(f*n), x]) /; \text{FreeQ}\{b, c, d, e, f\}, x] \ \&\& \ \text{GtQ}[n, 1$   
 $]$

### Rubi steps

$$\begin{aligned} \int (c + dx)^3 \sinh^3(a + bx) dx &= \frac{(c + dx)^3 \cosh(a + bx) \sinh^2(a + bx)}{3b} - \frac{d(c + dx)^2 \sinh^3(a + bx)}{3b^2} - \frac{2}{3} \int (c + dx)^3 \sinh(a + bx) dx \\ &= -\frac{2(c + dx)^3 \cosh(a + bx)}{3b} + \frac{2d^2(c + dx) \cosh(a + bx) \sinh^2(a + bx)}{9b^3} + \frac{(c + dx)^3 \cosh(a + bx)}{3b} \\ &= -\frac{4d^2(c + dx) \cosh(a + bx)}{9b^3} - \frac{2(c + dx)^3 \cosh(a + bx)}{3b} + \frac{2d(c + dx)^2 \sinh(a + bx)}{b^2} + \frac{2d^2(c + dx) \sinh^3(a + bx)}{3b^3} \\ &= -\frac{40d^2(c + dx) \cosh(a + bx)}{9b^3} - \frac{2(c + dx)^3 \cosh(a + bx)}{3b} + \frac{4d^3 \sinh(a + bx)}{9b^4} + \frac{2d(c + dx) \sinh^3(a + bx)}{3b^3} \\ &= -\frac{40d^2(c + dx) \cosh(a + bx)}{9b^3} - \frac{2(c + dx)^3 \cosh(a + bx)}{3b} + \frac{40d^3 \sinh(a + bx)}{9b^4} + \frac{2d(c + dx) \sinh^3(a + bx)}{3b^3} \end{aligned}$$

**Mathematica [A]** time = 0.992243, size = 127, normalized size = 0.73

$$\frac{-162b(c + dx) \cosh(a + bx) (b^2(c + dx)^2 + 6d^2) + 6b(c + dx) \cosh(3(a + bx)) (3b^2(c + dx)^2 + 2d^2) - 4d \sinh(a + bx) (c + dx)^3}{216b^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3\*Sinh[a + b\*x]^3,x]

[Out] (-162\*b\*(c + d\*x)\*(6\*d^2 + b^2\*(c + d\*x)^2)\*Cosh[a + b\*x] + 6\*b\*(c + d\*x)\*(2\*d^2 + 3\*b^2\*(c + d\*x)^2)\*Cosh[3\*(a + b\*x)] - 4\*d\*(-242\*d^2 - 117\*b^2\*(c + d\*x)^2 + (2\*d^2 + 9\*b^2\*(c + d\*x)^2)\*Cosh[2\*(a + b\*x)])\*Sinh[a + b\*x]/(216\*b^4)

$6*b^4)$

**Maple [B]** time = 0.007, size = 676, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^3*sinh(b*x+a)^3,x)`

[Out]  $\frac{1}{b} \left( \frac{1}{b^3 d^3} (-\frac{2}{3} (b*x+a)^3 \cosh(b*x+a) + \frac{1}{3} (b*x+a)^3 \cosh(b*x+a) \sinh(b*x+a)^2 + \frac{7}{3} (b*x+a)^2 \sinh(b*x+a) - \frac{40}{9} (b*x+a) \cosh(b*x+a) + \frac{122}{27} \sinh(b*x+a) - \frac{1}{3} (b*x+a)^2 \sinh(b*x+a) \cosh(b*x+a)^2 + \frac{2}{9} (b*x+a) \sinh(b*x+a)^2 \cosh(b*x+a) - \frac{2}{27} \sinh(b*x+a) \cosh(b*x+a)^2 - \frac{3}{b^3 d^3 a} (\frac{1}{3} (b*x+a)^2 \sinh(b*x+a)^2 \cosh(b*x+a) - \frac{2}{3} (b*x+a)^2 \cosh(b*x+a) - \frac{2}{9} (b*x+a) \sinh(b*x+a) \cosh(b*x+a)^2 + \frac{14}{9} (b*x+a) \sinh(b*x+a) + \frac{2}{27} \sinh(b*x+a)^2 \cosh(b*x+a) - \frac{40}{27} \cosh(b*x+a)) + \frac{3}{b^3 d^3 a^2} (-\frac{2}{3} (b*x+a) \cosh(b*x+a) + \frac{1}{3} (b*x+a) \sinh(b*x+a)^2 \cosh(b*x+a) + \frac{7}{9} \sinh(b*x+a) - \frac{1}{9} \sinh(b*x+a) \cosh(b*x+a)^2) - \frac{1}{b^3 d^3 a^3} (-\frac{2}{3} + \frac{1}{3} \sinh(b*x+a)^2) \cosh(b*x+a) + \frac{3}{b^2 c d^2} (\frac{1}{3} (b*x+a)^2 \sinh(b*x+a)^2 \cosh(b*x+a) - \frac{2}{3} (b*x+a)^2 \cosh(b*x+a) - \frac{2}{9} (b*x+a) \sinh(b*x+a) \cosh(b*x+a)^2 + \frac{14}{9} (b*x+a) \sinh(b*x+a) + \frac{2}{27} \sinh(b*x+a)^2 \cosh(b*x+a) - \frac{40}{27} \cosh(b*x+a)) - \frac{6}{b^2 c d^2 a} (-\frac{2}{3} (b*x+a) \cosh(b*x+a) + \frac{1}{3} (b*x+a) \sinh(b*x+a)^2 \cosh(b*x+a) + \frac{7}{9} \sinh(b*x+a) - \frac{1}{9} \sinh(b*x+a) \cosh(b*x+a)^2) + \frac{3}{b^2 c d^2 a^2} (-\frac{2}{3} + \frac{1}{3} \sinh(b*x+a)^2) \cosh(b*x+a) + \frac{3}{b c^2 d} (-\frac{2}{3} (b*x+a) \cosh(b*x+a) + \frac{1}{3} (b*x+a) \sinh(b*x+a)^2 \cosh(b*x+a) + \frac{7}{9} \sinh(b*x+a) - \frac{1}{9} \sinh(b*x+a) \cosh(b*x+a)^2) - \frac{3}{b c^2 d a} (-\frac{2}{3} + \frac{1}{3} \sinh(b*x+a)^2) \cosh(b*x+a) + c^3 (-\frac{2}{3} + \frac{1}{3} \sinh(b*x+a)^2) \cosh(b*x+a) \right)$

**Maxima [B]** time = 1.37749, size = 587, normalized size = 3.35

$\frac{1}{24} c^2 d \left( \frac{(3bx e^{(3a)} - e^{(3a)}) e^{(3bx)}}{b^2} - \frac{27 (bx e^a - e^a) e^{(bx)}}{b^2} - \frac{27 (bx + 1) e^{(-bx-a)}}{b^2} + \frac{(3bx + 1) e^{(-3bx-3a)}}{b^2} \right) + \frac{1}{24} c^3 \left( \frac{e^{(3bx+3a)}}{b} - \frac{9 e^{(3bx+3a)}}{b^2} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^3*sinh(b*x+a)^3,x, algorithm="maxima")`

[Out]  $\frac{1}{24} c^2 d \left( \frac{(3bx e^{(3a)} - e^{(3a)}) e^{(3bx)}}{b^2} - \frac{27 (bx e^a - e^a) e^{(bx)}}{b^2} - \frac{27 (bx + 1) e^{(-bx-a)}}{b^2} + \frac{(3bx + 1) e^{(-3bx-3a)}}{b^2} \right) + \frac{1}{24} c^3 \left( \frac{e^{(3bx+3a)}}{b} - \frac{9 e^{(3bx+3a)}}{b^2} \right)$



2) + 1/24\*c^3\*(e^(3\*b\*x + 3\*a)/b - 9\*e^(b\*x + a)/b - 9\*e^(-b\*x - a)/b + e^(-3\*b\*x - 3\*a)/b) + 1/72\*c\*d^2\*((9\*b^2\*x^2\*e^(3\*a) - 6\*b\*x\*e^(3\*a) + 2\*e^(3\*a))\*e^(3\*b\*x)/b^3 - 81\*(b^2\*x^2\*e^a - 2\*b\*x\*e^a + 2\*e^a)\*e^(b\*x)/b^3 - 81\*(b^2\*x^2 + 2\*b\*x + 2)\*e^(-b\*x - a)/b^3 + (9\*b^2\*x^2 + 6\*b\*x + 2)\*e^(-3\*b\*x - 3\*a)/b^3) + 1/216\*d^3\*((9\*b^3\*x^3\*e^(3\*a) - 9\*b^2\*x^2\*e^(3\*a) + 6\*b\*x\*e^(3\*a) - 2\*e^(3\*a))\*e^(3\*b\*x)/b^4 - 81\*(b^3\*x^3\*e^a - 3\*b^2\*x^2\*e^a + 6\*b\*x\*e^a - 6\*e^a)\*e^(b\*x)/b^4 - 81\*(b^3\*x^3 + 3\*b^2\*x^2 + 6\*b\*x + 6)\*e^(-b\*x - a)/b^4 + (9\*b^3\*x^3 + 9\*b^2\*x^2 + 6\*b\*x + 2)\*e^(-3\*b\*x - 3\*a)/b^4)

**Fricas [B]** time = 2.57785, size = 759, normalized size = 4.34

$$3(3b^3d^3x^3 + 9b^3cd^2x^2 + 3b^3c^3 + 2bcd^2 + (9b^3c^2d + 2bd^3)x) \cosh(bx + a)^3 + 9(3b^3d^3x^3 + 9b^3cd^2x^2 + 3b^3c^3 + 2bcd^2 + (9b^3c^2d + 2bd^3)x) \cosh(bx + a)^2 \sinh(bx + a) + 9(3b^3d^3x^3 + 9b^3cd^2x^2 + 3b^3c^3 + 2bcd^2 + (9b^3c^2d + 2bd^3)x) \cosh(bx + a) \sinh^2(bx + a) + 9(3b^3d^3x^3 + 9b^3cd^2x^2 + 3b^3c^3 + 2bcd^2 + (9b^3c^2d + 2bd^3)x) \sinh^3(bx + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*sinh(b\*x+a)^3,x, algorithm="fricas")

[Out] 1/108\*(3\*(3\*b^3\*d^3\*x^3 + 9\*b^3\*c\*d^2\*x^2 + 3\*b^3\*c^3 + 2\*b\*c\*d^2 + (9\*b^3\*c^2\*d + 2\*b\*d^3)\*x)\*cosh(b\*x + a)^3 + 9\*(3\*b^3\*d^3\*x^3 + 9\*b^3\*c\*d^2\*x^2 + 3\*b^3\*c^3 + 2\*b\*c\*d^2 + (9\*b^3\*c^2\*d + 2\*b\*d^3)\*x)\*cosh(b\*x + a)\*sinh(b\*x + a)^2 - (9\*b^2\*d^3\*x^2 + 18\*b^2\*c\*d^2\*x + 9\*b^2\*c^2\*d + 2\*d^3)\*sinh(b\*x + a)^3 - 81\*(b^3\*d^3\*x^3 + 3\*b^3\*c\*d^2\*x^2 + b^3\*c^3 + 6\*b\*c\*d^2 + 3\*(b^3\*c^2\*d + 2\*b\*d^3)\*x)\*cosh(b\*x + a) + 3\*(81\*b^2\*d^3\*x^2 + 162\*b^2\*c\*d^2\*x + 81\*b^2\*c^2\*d + 162\*d^3 - (9\*b^2\*d^3\*x^2 + 18\*b^2\*c\*d^2\*x + 9\*b^2\*c^2\*d + 2\*d^3)\*cosh(b\*x + a)^2)\*sinh(b\*x + a))/b^4

**Sympy [A]** time = 6.79592, size = 495, normalized size = 2.83

$$\left\{ \begin{array}{l} \frac{c^3 \sinh^2(a+bx) \cosh(a+bx)}{3b} - \frac{2c^3 \cosh^3(a+bx)}{3b} + \frac{3c^2 dx \sinh^2(a+bx) \cosh(a+bx)}{b} - \frac{2c^2 dx \cosh^3(a+bx)}{b} + \frac{3cd^2 x^2 \sinh^2(a+bx) \cosh(a+bx)}{b} - \frac{2cd^2 x^2}{b} \\ \left( c^3 x + \frac{3c^2 dx^2}{2} + cd^2 x^3 + \frac{d^3 x^4}{4} \right) \sinh^3(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*3\*sinh(b\*x+a)\*\*3,x)

[Out] Piecewise((c\*\*3\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/b - 2\*c\*\*3\*cosh(a + b\*x)\*\*3/(3\*b) + 3\*c\*\*2\*d\*x\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/b - 2\*c\*\*2\*d\*x\*cosh(a + b\*x)\*sinh(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^2(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^3(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^4(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^5(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^6(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^7(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^8(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^9(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^10(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^11(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^12(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^13(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^14(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^15(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^16(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^17(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^18(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^19(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^20(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^21(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^22(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^23(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^24(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^25(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^26(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^27(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^28(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^29(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^30(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^31(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^32(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^33(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^34(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^35(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^36(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^37(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^38(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^39(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^40(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^41(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^42(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^43(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^44(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^45(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^46(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^47(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^48(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^49(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^50(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^51(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^52(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^53(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^54(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^55(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^56(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^57(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^58(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^59(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^60(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^61(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^62(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^63(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^64(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^65(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^66(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^67(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^68(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^69(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^70(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^71(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^72(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^73(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^74(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^75(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^76(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^77(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^78(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^79(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^80(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^81(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^82(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^83(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^84(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^85(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^86(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^87(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^88(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^89(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^90(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^91(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^92(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^93(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^94(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^95(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^96(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^97(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^98(a + b\*x) + 3\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^99(a + b\*x) - 2\*c\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*sinh^100(a + b\*x)

```

*x)**3/b + 3*c*d**2*x**2*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*c*d**2*x**2*c
osh(a + b*x)**3/b + d**3*x**3*sinh(a + b*x)**2*cosh(a + b*x)/b - 2*d**3*x**
3*cosh(a + b*x)**3/(3*b) - 7*c**2*d*sinh(a + b*x)**3/(3*b**2) + 2*c**2*d*si
nh(a + b*x)*cosh(a + b*x)**2/b**2 - 14*c*d**2*x*sinh(a + b*x)**3/(3*b**2) +
  4*c*d**2*x*sinh(a + b*x)*cosh(a + b*x)**2/b**2 - 7*d**3*x**2*sinh(a + b*x)
**3/(3*b**2) + 2*d**3*x**2*sinh(a + b*x)*cosh(a + b*x)**2/b**2 + 14*c*d**2*
sinh(a + b*x)**2*cosh(a + b*x)/(3*b**3) - 40*c*d**2*cosh(a + b*x)**3/(9*b**
3) + 14*d**3*x*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**3) - 40*d**3*x*cosh(a +
  b*x)**3/(9*b**3) - 122*d**3*sinh(a + b*x)**3/(27*b**4) + 40*d**3*sinh(a +
  b*x)*cosh(a + b*x)**2/(9*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d
**2*x**3 + d**3*x**4/4)*sinh(a)**3, True))

```

**Giac [B]** time = 1.23016, size = 559, normalized size = 3.19

$$\frac{(9b^3d^3x^3 + 27b^3cd^2x^2 + 27b^3c^2dx - 9b^2d^3x^2 + 9b^3c^3 - 18b^2cd^2x - 9b^2c^2d + 6bd^3x + 6bcd^2 - 2d^3)e^{(3bx+3a)}}{216b^4} - \frac{3(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx - 3b^2d^3x^2 + b^3c^3 - 6b^2cd^2x - 3b^2c^2d + 6bd^3x + 6bcd^2 - 6d^3)e^{(bx+a)}}{b^4} - \frac{3(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx - 3b^2d^3x^2 + b^3c^3 + 6b^2cd^2x + 3b^2c^2d + 6bd^3x + 6b^2cd^2 + 6d^3)e^{(-bx-a)}}{b^4} + \frac{1}{216} \frac{(9b^3d^3x^3 + 27b^3cd^2x^2 + 27b^3c^2dx + 9b^2d^3x^2 + 9b^3c^3 + 18b^2cd^2x + 9b^2c^2d + 6bd^3x + 6bcd^2 + 2d^3)e^{(-3bx-3a)}}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x^2 + 27*b^3*c^2*d*x - 9*b^2*d^3*x^2 +
9*b^3*c^3 - 18*b^2*c*d^2*x - 9*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 2*d^3)*e
^(3*b*x + 3*a)/b^4 - 3/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x - 3
*b^2*d^3*x^2 + b^3*c^3 - 6*b^2*c*d^2*x - 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^
2 - 6*d^3)*e^(b*x + a)/b^4 - 3/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2
*d*x + 3*b^2*d^3*x^2 + b^3*c^3 + 6*b^2*c*d^2*x + 3*b^2*c^2*d + 6*b*d^3*x +
6*b*c*d^2 + 6*d^3)*e^(-b*x - a)/b^4 + 1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x
^2 + 27*b^3*c^2*d*x + 9*b^2*d^3*x^2 + 9*b^3*c^3 + 18*b^2*c*d^2*x + 9*b^2*c^
2*d + 6*b*d^3*x + 6*b*c*d^2 + 2*d^3)*e^(-3*b*x - 3*a)/b^4
```

### 3.18 $\int (c + dx)^2 \sinh^3(a + bx) dx$

**Optimal.** Leaf size=123

$$-\frac{2d(c + dx) \sinh^3(a + bx)}{9b^2} + \frac{4d(c + dx) \sinh(a + bx)}{3b^2} + \frac{2d^2 \cosh^3(a + bx)}{27b^3} - \frac{14d^2 \cosh(a + bx)}{9b^3} - \frac{2(c + dx)^2 \cosh(a + bx)}{3b}$$

[Out]  $(-14*d^2*Cosh[a + b*x])/(9*b^3) - (2*(c + d*x)^2*Cosh[a + b*x])/(3*b) + (2*d^2*Cosh[a + b*x]^3)/(27*b^3) + (4*d*(c + d*x)*Sinh[a + b*x])/(3*b^2) + ((c + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (2*d*(c + d*x)*Sinh[a + b*x]^3)/(9*b^2)$

**Rubi [A]** time = 0.131249, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3311, 3296, 2638, 2633}

$$-\frac{2d(c + dx) \sinh^3(a + bx)}{9b^2} + \frac{4d(c + dx) \sinh(a + bx)}{3b^2} + \frac{2d^2 \cosh^3(a + bx)}{27b^3} - \frac{14d^2 \cosh(a + bx)}{9b^3} - \frac{2(c + dx)^2 \cosh(a + bx)}{3b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^2\*Sinh[a + b\*x]^3,x]

[Out]  $(-14*d^2*Cosh[a + b*x])/(9*b^3) - (2*(c + d*x)^2*Cosh[a + b*x])/(3*b) + (2*d^2*Cosh[a + b*x]^3)/(27*b^3) + (4*d*(c + d*x)*Sinh[a + b*x])/(3*b^2) + ((c + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (2*d*(c + d*x)*Sinh[a + b*x]^3)/(9*b^2)$

#### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*Sinh[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*Sinh[e + f\*x])^(n - 2), x], x] - Dist[(d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*Sinh[e + f\*x])^n, x], x] - Simp[(b\*(c + d\*x)^m\*Cosh[e + f\*x]\*(b\*Sinh[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[(c + d\*x)^m\*Cosh[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[

$e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

### Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x\_Symbol] \text{ :> } -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

### Rule 2633

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \text{ :> } -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{((n - 1)/2)}, x], x], x, \text{Cos}[c + d*x]], x] /; \text{FreeQ}[\{c, d\}, x] \ \&\& \ \text{IGtQ}[(n - 1)/2, 0]$

### Rubi steps

$$\begin{aligned} \int (c + dx)^2 \sinh^3(a + bx) dx &= \frac{(c + dx)^2 \cosh(a + bx) \sinh^2(a + bx)}{3b} - \frac{2d(c + dx) \sinh^3(a + bx)}{9b^2} - \frac{2}{3} \int (c + dx)^2 \sinh(a + bx) dx \\ &= -\frac{2(c + dx)^2 \cosh(a + bx)}{3b} + \frac{(c + dx)^2 \cosh(a + bx) \sinh^2(a + bx)}{3b} - \frac{2d(c + dx) \sinh^3(a + bx)}{9b^2} \\ &= -\frac{2d^2 \cosh(a + bx)}{9b^3} - \frac{2(c + dx)^2 \cosh(a + bx)}{3b} + \frac{2d^2 \cosh^3(a + bx)}{27b^3} + \frac{4d(c + dx) \sinh(a + bx)}{3b^2} \\ &= -\frac{14d^2 \cosh(a + bx)}{9b^3} - \frac{2(c + dx)^2 \cosh(a + bx)}{3b} + \frac{2d^2 \cosh^3(a + bx)}{27b^3} + \frac{4d(c + dx) \sinh(a + bx)}{3b^2} \end{aligned}$$

**Mathematica [A]** time = 0.403865, size = 86, normalized size = 0.7

$$\frac{-81 \cosh(a + bx) (b^2(c + dx)^2 + 2d^2) + \cosh(3(a + bx)) (9b^2(c + dx)^2 + 2d^2) - 6bd(c + dx)(\sinh(3(a + bx)) - 27 \sinh(a + bx))}{108b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2\*Sinh[a + b\*x]^3,x]

[Out] (-81\*(2\*d^2 + b^2\*(c + d\*x)^2)\*Cosh[a + b\*x] + (2\*d^2 + 9\*b^2\*(c + d\*x)^2)\*Cosh[3\*(a + b\*x)] - 6\*b\*d\*(c + d\*x)\*(-27\*Sinh[a + b\*x] + Sinh[3\*(a + b\*x)])/(108\*b^3)

**Maple [B]** time = 0.007, size = 320, normalized size = 2.6

$$\frac{1}{b} \left( \frac{d^2}{b^2} \left( \frac{(bx+a)^2 (\sinh(bx+a))^2 \cosh(bx+a)}{3} - \frac{2(bx+a)^2 \cosh(bx+a)}{3} - \frac{(2bx+2a) \sinh(bx+a) (\cosh(bx+a))^2}{9} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2\*sinh(b\*x+a)^3,x)

[Out]  $\frac{1}{b} \left( \frac{1}{b^2 d^2} \left( \frac{1}{3} (bx+a)^2 \sinh(bx+a)^2 \cosh(bx+a) - \frac{2}{3} (bx+a)^2 \cosh(bx+a) - \frac{2}{9} (bx+a) \sinh(bx+a) \cosh(bx+a)^2 + \frac{14}{9} (bx+a) \sinh(bx+a) + \frac{2}{27} \sinh(bx+a)^2 \cosh(bx+a) - \frac{40}{27} \cosh(bx+a) \right) - \frac{2}{b^2 d^2} \left( \frac{1}{3} (bx+a) \cosh(bx+a) + \frac{1}{3} (bx+a) \sinh(bx+a)^2 \cosh(bx+a) + \frac{7}{9} \sinh(bx+a) - \frac{1}{9} \sinh(bx+a) \cosh(bx+a)^2 \right) + \frac{1}{b^2 d^2} \left( \frac{1}{3} \sinh(bx+a)^2 \cosh(bx+a) + \frac{2}{b^2 c} d \left( -\frac{2}{3} (bx+a) \cosh(bx+a) + \frac{1}{3} (bx+a) \sinh(bx+a)^2 \cosh(bx+a) + \frac{7}{9} \sinh(bx+a) - \frac{1}{9} \sinh(bx+a) \cosh(bx+a)^2 \right) - \frac{2}{b^2 c} d a \left( -\frac{2}{3} + \frac{1}{3} \sinh(bx+a)^2 \right) \cosh(bx+a) + c^2 \left( -\frac{2}{3} + \frac{1}{3} \sinh(bx+a)^2 \right) \cosh(bx+a) \right) \right)$

**Maxima [B]** time = 1.35653, size = 363, normalized size = 2.95

$$\frac{1}{36} cd \left( \frac{(3bx e^{3a} - e^{3a}) e^{3bx}}{b^2} - \frac{27(bx e^a - e^a) e^{bx}}{b^2} - \frac{27(bx+1) e^{-bx-a}}{b^2} + \frac{(3bx+1) e^{-3bx-3a}}{b^2} \right) + \frac{1}{24} c^2 \left( \frac{e^{3bx+3a}}{b} - \frac{9e^{-bx-a}}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*sinh(b\*x+a)^3,x, algorithm="maxima")

[Out]  $\frac{1}{36} c d \left( \frac{(3bx e^{3a} - e^{3a}) e^{3bx}}{b^2} - \frac{27(bx e^a - e^a) e^{bx}}{b^2} - \frac{27(bx+1) e^{-bx-a}}{b^2} + \frac{(3bx+1) e^{-3bx-3a}}{b^2} \right) + \frac{1}{24} c^2 \left( \frac{e^{3bx+3a}}{b} - \frac{9e^{-bx-a}}{b} \right) + \frac{1}{216} d^2 \left( \frac{(9b^2 x^2 e^{3a} - 6bx e^{3a} + 2e^{3a}) e^{3bx}}{b^3} - \frac{81(b^2 x^2 e^a - 2bx e^a + 2e^a) e^{bx}}{b^3} - \frac{81(b^2 x^2 + 2bx + 2) e^{-bx-a}}{b^3} + \frac{(9b^2 x^2 + 6bx + 2) e^{-3bx-3a}}{b^3} \right)$

**Fricas [A]** time = 2.62494, size = 466, normalized size = 3.79

$$\frac{(9b^2 d^2 x^2 + 18b^2 c dx + 9b^2 c^2 + 2d^2) \cosh(bx+a)^3 + 3(9b^2 d^2 x^2 + 18b^2 c dx + 9b^2 c^2 + 2d^2) \cosh(bx+a) \sinh(bx+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*sinh(b\*x+a)^3,x, algorithm="fricas")

[Out]  $\frac{1}{108} * ((9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 2*d^2) * \cosh(b*x + a)^3 + 3*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 2*d^2) * \cosh(b*x + a) * \sinh(b*x + a)^2 - 6*(b*d^2*x + b*c*d) * \sinh(b*x + a)^3 - 81*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*d^2) * \cosh(b*x + a) + 18*(9*b*d^2*x + 9*b*c*d - (b*d^2*x + b*c*d) * \cosh(b*x + a)^2) * \sinh(b*x + a)) / b^3$

**Sympy [A]** time = 3.83939, size = 284, normalized size = 2.31

$$\left\{ \begin{array}{l} \frac{c^2 \sinh^2(a+bx) \cosh(a+bx)}{b} - \frac{2c^2 \cosh^3(a+bx)}{3b} + \frac{2cdx \sinh^2(a+bx) \cosh(a+bx)}{b} - \frac{4cdx \cosh^3(a+bx)}{3b} + \frac{d^2x^2 \sinh^2(a+bx) \cosh(a+bx)}{b} - \frac{2d^2x^2 \cosh^3(a+bx)}{3b} \\ \left( c^2x + cdx^2 + \frac{d^2x^3}{3} \right) \sinh^3(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2\*sinh(b\*x+a)\*\*3,x)

[Out] Piecewise(((c\*\*2\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/b - 2\*c\*\*2\*cosh(a + b\*x)\*\*3/(3\*b) + 2\*c\*d\*x\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/b - 4\*c\*d\*x\*cosh(a + b\*x)\*\*3/(3\*b) + d\*\*2\*x\*\*2\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/b - 2\*d\*\*2\*x\*\*2\*cosh(a + b\*x)\*\*3/(3\*b) - 14\*c\*d\*sinh(a + b\*x)\*\*3/(9\*b\*\*2) + 4\*c\*d\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/(3\*b\*\*2) - 14\*d\*\*2\*x\*sinh(a + b\*x)\*\*3/(9\*b\*\*2) + 4\*d\*\*2\*x\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/(3\*b\*\*2) + 14\*d\*\*2\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/(9\*b\*\*3) - 40\*d\*\*2\*cosh(a + b\*x)\*\*3/(27\*b\*\*3), Ne(b, 0)), ((c\*\*2\*x + c\*d\*x\*\*2 + d\*\*2\*x\*\*3/3)\*sinh(a)\*\*3, True))

**Giac [B]** time = 1.18285, size = 311, normalized size = 2.53

$$\frac{(9b^2d^2x^2 + 18b^2cdx + 9b^2c^2 - 6bd^2x - 6bcd + 2d^2)e^{(3bx+3a)}}{216b^3} - \frac{3(b^2d^2x^2 + 2b^2cdx + b^2c^2 - 2bd^2x - 2bcd + 2d^2)e^{(bx+a)}}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*sinh(b\*x+a)^3,x, algorithm="giac")

[Out]  $\frac{1}{216} * (9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 6*b*d^2*x - 6*b*c*d + 2*d^2) * e^{(3*b*x + 3*a)} / b^3 - \frac{3}{8} * (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*b*d^2*x - 2*b*c*d + 2*d^2) * e^{(b*x + a)} / b^3$

$$2*x - 2*b*c*d + 2*d^2)*e^{(b*x + a)/b^3} - 3/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*b*d^2*x + 2*b*c*d + 2*d^2)*e^{(-b*x - a)/b^3} + 1/216*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 6*b*d^2*x + 6*b*c*d + 2*d^2)*e^{(-3*b*x - 3*a)/b^3}$$

### 3.19 $\int (c + dx) \sinh^3(a + bx) dx$

**Optimal.** Leaf size=75

$$-\frac{d \sinh^3(a + bx)}{9b^2} + \frac{2d \sinh(a + bx)}{3b^2} - \frac{2(c + dx) \cosh(a + bx)}{3b} + \frac{(c + dx) \sinh^2(a + bx) \cosh(a + bx)}{3b}$$

[Out]  $(-2*(c + d*x)*Cosh[a + b*x])/(3*b) + (2*d*Sinh[a + b*x])/(3*b^2) + ((c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (d*Sinh[a + b*x]^3)/(9*b^2)$

**Rubi [A]** time = 0.0576496, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3310, 3296, 2637}

$$-\frac{d \sinh^3(a + bx)}{9b^2} + \frac{2d \sinh(a + bx)}{3b^2} - \frac{2(c + dx) \cosh(a + bx)}{3b} + \frac{(c + dx) \sinh^2(a + bx) \cosh(a + bx)}{3b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)\*Sinh[a + b\*x]^3,x]

[Out]  $(-2*(c + d*x)*Cosh[a + b*x])/(3*b) + (2*d*Sinh[a + b*x])/(3*b^2) + ((c + d*x)*Cosh[a + b*x]*Sinh[a + b*x]^2)/(3*b) - (d*Sinh[a + b*x]^3)/(9*b^2)$

#### Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
  Simp[(d*(b*Sinh[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sinh[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sinh[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

#### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

#### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```



Rubi steps

$$\begin{aligned} \int (c + dx) \sinh^3(a + bx) dx &= \frac{(c + dx) \cosh(a + bx) \sinh^2(a + bx)}{3b} - \frac{d \sinh^3(a + bx)}{9b^2} - \frac{2}{3} \int (c + dx) \sinh(a + bx) dx \\ &= -\frac{2(c + dx) \cosh(a + bx)}{3b} + \frac{(c + dx) \cosh(a + bx) \sinh^2(a + bx)}{3b} - \frac{d \sinh^3(a + bx)}{9b^2} + \frac{(2d)}{3} \int \sinh(a + bx) dx \\ &= -\frac{2(c + dx) \cosh(a + bx)}{3b} + \frac{2d \sinh(a + bx)}{3b^2} + \frac{(c + dx) \cosh(a + bx) \sinh^2(a + bx)}{3b} - \frac{d \sinh^3(a + bx)}{9b^2} \end{aligned}$$

**Mathematica [A]** time = 0.183453, size = 59, normalized size = 0.79

$$\frac{-27b(c + dx) \cosh(a + bx) + 3b(c + dx) \cosh(3(a + bx)) + d(27 \sinh(a + bx) - \sinh(3(a + bx)))}{36b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)\*Sinh[a + b\*x]^3,x]

[Out] (-27\*b\*(c + d\*x)\*Cosh[a + b\*x] + 3\*b\*(c + d\*x)\*Cosh[3\*(a + b\*x)] + d\*(27\*Sinh[a + b\*x] - Sinh[3\*(a + b\*x)]))/(36\*b^2)

**Maple [A]** time = 0.007, size = 115, normalized size = 1.5

$$\frac{1}{b} \left( \frac{d}{b} \left( -\frac{(2bx + 2a) \cosh(bx + a)}{3} + \frac{(bx + a) (\sinh(bx + a))^2 \cosh(bx + a)}{3} + \frac{7 \sinh(bx + a)}{9} - \frac{\sinh(bx + a) (\cosh(bx + a))^2}{9} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)\*sinh(b\*x+a)^3,x)

[Out] 1/b\*(1/b\*d\*(-2/3\*(b\*x+a)\*cosh(b\*x+a)+1/3\*(b\*x+a)\*sinh(b\*x+a)^2\*cosh(b\*x+a)+7/9\*sinh(b\*x+a)-1/9\*sinh(b\*x+a)\*cosh(b\*x+a)^2)-1/b\*d\*a\*(-2/3+1/3\*sinh(b\*x+a)^2)\*cosh(b\*x+a)+c\*(-2/3+1/3\*sinh(b\*x+a)^2)\*cosh(b\*x+a))

**Maxima [B]** time = 1.22695, size = 190, normalized size = 2.53

$$\frac{1}{72} d \left( \frac{(3bx e^{3a} - e^{3a}) e^{3bx}}{b^2} - \frac{27(bx e^a - e^a) e^{bx}}{b^2} - \frac{27(bx + 1) e^{-bx-a}}{b^2} + \frac{(3bx + 1) e^{-3bx-3a}}{b^2} \right) + \frac{1}{24} c \left( \frac{e^{3bx+3a}}{b} - \frac{9e^{bx+a}}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sinh(b\*x+a)^3,x, algorithm="maxima")

[Out]  $\frac{1}{72}d*((3bx+e^{3a}) - e^{3a})e^{3bx}/b^2 - 27*(bx+e^a - e^a)e^{bx}/b^2 - 27*(bx+1)e^{-bx-a}/b^2 + (3bx+1)e^{-3bx-3a}/b^2 + 1/24*c*(e^{3bx+3a}/b - 9e^{bx+a}/b - 9e^{-bx-a}/b + e^{-3bx-3a}/b)$

**Fricas [A]** time = 2.62549, size = 255, normalized size = 3.4

$$\frac{3(bdx+bc)\cosh(bx+a)^3 + 9(bdx+bc)\cosh(bx+a)\sinh(bx+a)^2 - d\sinh(bx+a)^3 - 27(bdx+bc)\cosh(bx+a) - 3d\sinh(bx+a)^2 - 9d\sinh(bx+a)}{36b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sinh(b\*x+a)^3,x, algorithm="fricas")

[Out]  $\frac{1}{36}*(3*(b*d*x + b*c)*\cosh(b*x + a)^3 + 9*(b*d*x + b*c)*\cosh(b*x + a)*\sinh(b*x + a)^2 - d*\sinh(b*x + a)^3 - 27*(b*d*x + b*c)*\cosh(b*x + a) - 3*(d*\cosh(b*x + a)^2 - 9*d)*\sinh(b*x + a))/b^2$

**Sympy [A]** time = 1.8197, size = 126, normalized size = 1.68

$$\left\{ \begin{array}{l} \frac{c \sinh^2(a+bx) \cosh(a+bx)}{3b} - \frac{2c \cosh^3(a+bx)}{3b} + \frac{dx \sinh^2(a+bx) \cosh(a+bx)}{b} - \frac{2dx \cosh^3(a+bx)}{3b} - \frac{7d \sinh^3(a+bx)}{9b^2} + \frac{2d \sinh(a+bx) \cosh^2(a+bx)}{3b^2} \\ \left( cx + \frac{dx^2}{2} \right) \sinh^3(a) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sinh(b\*x+a)\*\*3,x)

[Out] Piecewise((c\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/b - 2\*c\*cosh(a + b\*x)\*\*3/(3\*b) + d\*x\*sinh(a + b\*x)\*\*2\*cosh(a + b\*x)/b - 2\*d\*x\*cosh(a + b\*x)\*\*3/(3\*b) - 7\*d\*sinh(a + b\*x)\*\*3/(9\*b\*\*2) + 2\*d\*sinh(a + b\*x)\*cosh(a + b\*x)\*\*2/(3\*b\*\*2), N e(b, 0)), ((c\*x + d\*x\*\*2/2)\*sinh(a)\*\*3, True))

**Giac [A]** time = 1.18635, size = 132, normalized size = 1.76

$$\frac{(3bdx + 3bc - d)e^{(3bx+3a)}}{72b^2} - \frac{3(bdx + bc - d)e^{(bx+a)}}{8b^2} - \frac{3(bdx + bc + d)e^{(-bx-a)}}{8b^2} + \frac{(3bdx + 3bc + d)e^{(-3bx-3a)}}{72b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*sinh(b\*x+a)^3,x, algorithm="giac")

[Out] 1/72\*(3\*b\*d\*x + 3\*b\*c - d)\*e^(3\*b\*x + 3\*a)/b^2 - 3/8\*(b\*d\*x + b\*c - d)\*e^(b\*x + a)/b^2 - 3/8\*(b\*d\*x + b\*c + d)\*e^(-b\*x - a)/b^2 + 1/72\*(3\*b\*d\*x + 3\*b\*c + d)\*e^(-3\*b\*x - 3\*a)/b^2

### 3.20 $\int \frac{\sinh^3(a+bx)}{c+dx} dx$

**Optimal.** Leaf size=121

$$\frac{\sinh\left(3a - \frac{3bc}{d}\right) \text{Chi}\left(\frac{3bc}{d} + 3bx\right)}{4d} - \frac{3 \sinh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{4d} - \frac{3 \cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{4d} + \frac{\cosh\left(3a - \frac{3bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{4d}$$

[Out] (CoshIntegral[(3\*b\*c)/d + 3\*b\*x]\*Sinh[3\*a - (3\*b\*c)/d])/(4\*d) - (3\*CoshIntegral[(b\*c)/d + b\*x]\*Sinh[a - (b\*c)/d])/(4\*d) - (3\*Cosh[a - (b\*c)/d]\*SinhIntegral[(b\*c)/d + b\*x])/(4\*d) + (Cosh[3\*a - (3\*b\*c)/d]\*SinhIntegral[(3\*b\*c)/d + 3\*b\*x])/(4\*d)

**Rubi [A]** time = 0.282192, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3312, 3303, 3298, 3301}

$$\frac{\sinh\left(3a - \frac{3bc}{d}\right) \text{Chi}\left(\frac{3bc}{d} + 3bx\right)}{4d} - \frac{3 \sinh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{4d} - \frac{3 \cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{4d} + \frac{\cosh\left(3a - \frac{3bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{4d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[a + b\*x]^3/(c + d\*x), x]

[Out] (CoshIntegral[(3\*b\*c)/d + 3\*b\*x]\*Sinh[3\*a - (3\*b\*c)/d])/(4\*d) - (3\*CoshIntegral[(b\*c)/d + b\*x]\*Sinh[a - (b\*c)/d])/(4\*d) - (3\*Cosh[a - (b\*c)/d]\*SinhIntegral[(b\*c)/d + b\*x])/(4\*d) + (Cosh[3\*a - (3\*b\*c)/d]\*SinhIntegral[(3\*b\*c)/d + 3\*b\*x])/(4\*d)

#### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

#### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sinh^3(a+bx)}{c+dx} dx &= i \int \left( \frac{3i \sinh(a+bx)}{4(c+dx)} - \frac{i \sinh(3a+3bx)}{4(c+dx)} \right) dx \\ &= \frac{1}{4} \int \frac{\sinh(3a+3bx)}{c+dx} dx - \frac{3}{4} \int \frac{\sinh(a+bx)}{c+dx} dx \\ &= \frac{1}{4} \cosh\left(3a - \frac{3bc}{d}\right) \int \frac{\sinh\left(\frac{3bc}{d} + 3bx\right)}{c+dx} dx - \frac{1}{4} \left(3 \cosh\left(a - \frac{bc}{d}\right)\right) \int \frac{\sinh\left(\frac{bc}{d} + bx\right)}{c+dx} dx + \frac{1}{4} \sinh\left(3a - \frac{3bc}{d}\right) \\ &= \frac{\text{Chi}\left(\frac{3bc}{d} + 3bx\right) \sinh\left(3a - \frac{3bc}{d}\right)}{4d} - \frac{3 \text{Chi}\left(\frac{bc}{d} + bx\right) \sinh\left(a - \frac{bc}{d}\right)}{4d} - \frac{3 \cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{4d} \end{aligned}$$

**Mathematica [A]** time = 0.239217, size = 102, normalized size = 0.84

$$\frac{\sinh\left(3a - \frac{3bc}{d}\right) \text{Chi}\left(\frac{3b(c+dx)}{d}\right) - 3 \sinh\left(a - \frac{bc}{d}\right) \text{Chi}\left(b\left(\frac{c}{d} + x\right)\right) - 3 \cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(b\left(\frac{c}{d} + x\right)\right) + \cosh\left(3a - \frac{3bc}{d}\right) \text{Shi}\left(\frac{3b(c+dx)}{d}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]^3/(c + d*x), x]
```

```
[Out] (CoshIntegral[(3*b*(c + d*x))/d]*Sinh[3*a - (3*b*c)/d] - 3*CoshIntegral[b*(c/d + x)]*Sinh[a - (b*c)/d] - 3*Cosh[a - (b*c)/d]*SinhIntegral[b*(c/d + x)] + Cosh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*(c + d*x))/d])/(4*d)
```

**Maple [A]** time = 0.083, size = 166, normalized size = 1.4

$$\frac{1}{8d} e^{-3 \frac{da-cb}{d}} \operatorname{Ei}\left(1, 3bx + 3a - 3 \frac{da-cb}{d}\right) - \frac{3}{8d} e^{-\frac{da-cb}{d}} \operatorname{Ei}\left(1, bx + a - \frac{da-cb}{d}\right) + \frac{3}{8d} e^{\frac{da-cb}{d}} \operatorname{Ei}\left(1, -bx - a - \frac{-da+cb}{d}\right) - \frac{1}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(b*x+a)^3/(d*x+c),x)`

[Out]  $\frac{1}{8d} \exp(-3(a*d-b*c)/d) \operatorname{Ei}(1, 3*b*x+3*a-3*(a*d-b*c)/d) - \frac{3}{8d} \exp(-(a*d-b*c)/d) \operatorname{Ei}(1, b*x+a-(a*d-b*c)/d) + \frac{3}{8d} \exp((a*d-b*c)/d) \operatorname{Ei}(1, -b*x-a-(-a*d+b*c)/d) - \frac{1}{8d} \exp(3*(a*d-b*c)/d) \operatorname{Ei}(1, -3*b*x-3*a-3*(-a*d+b*c)/d)$

**Maxima [A]** time = 1.39345, size = 158, normalized size = 1.31

$$\frac{e^{\left(-3a+\frac{3bc}{d}\right)} E_1\left(\frac{3(dx+c)b}{d}\right)}{8d} - \frac{3e^{\left(-a+\frac{bc}{d}\right)} E_1\left(\frac{(dx+c)b}{d}\right)}{8d} + \frac{3e^{\left(a-\frac{bc}{d}\right)} E_1\left(-\frac{(dx+c)b}{d}\right)}{8d} - \frac{e^{\left(3a-\frac{3bc}{d}\right)} E_1\left(-\frac{3(dx+c)b}{d}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)^3/(d*x+c),x, algorithm="maxima")`

[Out]  $\frac{1}{8} e^{(-3a + 3bc/d)} \exp\_integral\_e(1, 3*(d*x + c)*b/d)/d - \frac{3}{8} e^{(-a + bc/d)} \exp\_integral\_e(1, (d*x + c)*b/d)/d + \frac{3}{8} e^{(a - bc/d)} \exp\_integral\_e(1, -(d*x + c)*b/d)/d - \frac{1}{8} e^{(3a - 3bc/d)} \exp\_integral\_e(1, -3*(d*x + c)*b/d)/d$

**Fricas [A]** time = 2.53058, size = 400, normalized size = 3.31

$$\frac{3 \left( \operatorname{Ei}\left(\frac{bdx+bc}{d}\right) - \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right) \right) \cosh\left(-\frac{bc-ad}{d}\right) - \left( \operatorname{Ei}\left(\frac{3(bdx+bc)}{d}\right) - \operatorname{Ei}\left(-\frac{3(bdx+bc)}{d}\right) \right) \cosh\left(-\frac{3(bc-ad)}{d}\right) + 3 \left( \operatorname{Ei}\left(\frac{bdx+bc}{d}\right) + \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right) \right) \cosh\left(-\frac{bc-ad}{d}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)^3/(d*x+c),x, algorithm="fricas")`

[Out]  $-\frac{1}{8} * (3 * (\operatorname{Ei}((b*d*x + b*c)/d) - \operatorname{Ei}(-(b*d*x + b*c)/d)) * \cosh(-(b*c - a*d)/d) - (\operatorname{Ei}(3*(b*d*x + b*c)/d) - \operatorname{Ei}(-3*(b*d*x + b*c)/d)) * \cosh(-3*(b*c - a*d)/d) +$

$$3*(\text{Ei}((b*d*x + b*c)/d) + \text{Ei}(-(b*d*x + b*c)/d))*\sinh(-(b*c - a*d)/d) - (\text{Ei}(3*(b*d*x + b*c)/d) + \text{Ei}(-3*(b*d*x + b*c)/d))*\sinh(-3*(b*c - a*d)/d))/d$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)\*\*3/(d\*x+c), x)

[Out] Timed out

**Giac [A]** time = 1.20068, size = 153, normalized size = 1.26

$$\frac{\text{Ei}\left(\frac{3(bdx+bc)}{d}\right)e^{\left(3a-\frac{3bc}{d}\right)} - 3\text{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)} + 3\text{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)} - \text{Ei}\left(-\frac{3(bdx+bc)}{d}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^3/(d\*x+c), x, algorithm="giac")

[Out]  $\frac{1}{8}*(\text{Ei}(3*(b*d*x + b*c)/d)*e^{(3*a - 3*b*c/d)} - 3*\text{Ei}((b*d*x + b*c)/d)*e^{(a - b*c/d)} + 3*\text{Ei}(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} - \text{Ei}(-3*(b*d*x + b*c)/d)*e^{(-3*a + 3*b*c/d)})/d$

### 3.21 $\int \frac{\sinh^3(a+bx)}{(c+dx)^2} dx$

**Optimal.** Leaf size=145

$$-\frac{3b \cosh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{4d^2} + \frac{3b \cosh\left(3a - \frac{3bc}{d}\right) \text{Chi}\left(\frac{3bc}{d} + 3bx\right)}{4d^2} - \frac{3b \sinh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{4d^2} + \frac{3b \sinh\left(3a - \frac{3bc}{d}\right) \text{Shi}\left(\frac{3bc}{d} + 3bx\right)}{4d^2}$$

[Out]  $(-3*b*Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*Cosh[3*a - (3*b*c)/d]*CoshIntegral[(3*b*c)/d + 3*b*x])/(4*d^2) - Sinh[a + b*x]^3/(d*(c + d*x)) - (3*b*Sinh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d + 3*b*x])/(4*d^2)$

**Rubi [A]** time = 0.262214, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3313, 3303, 3298, 3301}

$$-\frac{3b \cosh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{4d^2} + \frac{3b \cosh\left(3a - \frac{3bc}{d}\right) \text{Chi}\left(\frac{3bc}{d} + 3bx\right)}{4d^2} - \frac{3b \sinh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{4d^2} + \frac{3b \sinh\left(3a - \frac{3bc}{d}\right) \text{Shi}\left(\frac{3bc}{d} + 3bx\right)}{4d^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[Sinh[a + b*x]^3/(c + d*x)^2, x]$

[Out]  $(-3*b*Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*Cosh[3*a - (3*b*c)/d]*CoshIntegral[(3*b*c)/d + 3*b*x])/(4*d^2) - Sinh[a + b*x]^3/(d*(c + d*x)) - (3*b*Sinh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d + 3*b*x])/(4*d^2)$

#### Rule 3313

$\text{Int}[\frac{(c + d*x)^m \sin[e + f*x]^n}{(c + d*x)^{m+1}}, x] - \text{Dist}[\frac{f^n}{d*(m+1)}, \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^{m+1}, \text{Cos}[e + f*x]*\text{Sin}[e + f*x]^{n-1}], x], x] /;$  FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]

#### Rule 3303

$\text{Int}[\frac{\sin[e + f*x]}{(c + d*x)}, x] - \text{Dist}[\frac{\cos[e + f*x]}{d}, \text{Int}[\frac{\sin[c*f]}{d + f*x}, x], x] + \text{Dist}[\frac{\sin[d*e - c*f]}{d}, \text{Int}[\frac{\cos[c*f]}{d + f*x}, x], x]$



)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] &&  
NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\sinh^3(a+bx)}{(c+dx)^2} dx &= -\frac{\sinh^3(a+bx)}{d(c+dx)} - \frac{(3b) \int \left( \frac{\cosh(a+bx)}{4(c+dx)} - \frac{\cosh(3a+3bx)}{4(c+dx)} \right) dx}{d} \\ &= -\frac{\sinh^3(a+bx)}{d(c+dx)} - \frac{(3b) \int \frac{\cosh(a+bx)}{c+dx} dx}{4d} + \frac{(3b) \int \frac{\cosh(3a+3bx)}{c+dx} dx}{4d} \\ &= -\frac{\sinh^3(a+bx)}{d(c+dx)} + \frac{\left( 3b \cosh \left( 3a - \frac{3bc}{d} \right) \right) \int \frac{\cosh \left( \frac{3bc}{d} + 3bx \right)}{c+dx} dx}{4d} - \frac{\left( 3b \cosh \left( a - \frac{bc}{d} \right) \right) \int \frac{\cosh \left( \frac{bc}{d} + bx \right)}{c+dx} dx}{4d} \\ &= -\frac{3b \cosh \left( a - \frac{bc}{d} \right) \text{Chi} \left( \frac{bc}{d} + bx \right)}{4d^2} + \frac{3b \cosh \left( 3a - \frac{3bc}{d} \right) \text{Chi} \left( \frac{3bc}{d} + 3bx \right)}{4d^2} - \frac{\sinh^3(a+bx)}{d(c+dx)} - \frac{3b \sinh^3(a+bx)}{8d^2(c+dx)} \end{aligned}$$

**Mathematica [A]** time = 1.34078, size = 160, normalized size = 1.1

$$\frac{6b(c+dx) \left( -\cosh \left( a - \frac{bc}{d} \right) \text{Chi} \left( b \left( \frac{c}{d} + x \right) \right) + \cosh \left( 3a - \frac{3bc}{d} \right) \text{Chi} \left( \frac{3b(c+dx)}{d} \right) - \sinh \left( a - \frac{bc}{d} \right) \text{Shi} \left( b \left( \frac{c}{d} + x \right) \right) + \sinh \left( 3a - \frac{3bc}{d} \right) \text{Shi} \left( \frac{3b(c+dx)}{d} \right) \right)}{8d^2(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[a + b\*x]^3/(c + d\*x)^2,x]

[Out] (6\*d\*Cosh[b\*x]\*Sinh[a] - 2\*d\*Cosh[3\*b\*x]\*Sinh[3\*a] + 6\*d\*Cosh[a]\*Sinh[b\*x] - 2\*d\*Cosh[3\*a]\*Sinh[3\*b\*x] + 6\*b\*(c + d\*x)\*(-(Cosh[a - (b\*c)/d]\*CoshIntegr

al[b\*(c/d + x)]) + Cosh[3\*a - (3\*b\*c)/d]\*CoshIntegral[(3\*b\*(c + d\*x))/d] - Sinh[a - (b\*c)/d]\*SinhIntegral[b\*(c/d + x)] + Sinh[3\*a - (3\*b\*c)/d]\*SinhIntegral[(3\*b\*(c + d\*x))/d])/(8\*d^2\*(c + d\*x))

**Maple [A]** time = 0.091, size = 271, normalized size = 1.9

$$\frac{be^{-3bx-3a}}{(8bdx+8cb)d} - \frac{3b}{8d^2}e^{-3\frac{da-cb}{d}}\text{Ei}\left(1, 3bx+3a-3\frac{da-cb}{d}\right) - \frac{3be^{-bx-a}}{8d(bdx+cb)} + \frac{3b}{8d^2}e^{-\frac{da-cb}{d}}\text{Ei}\left(1, bx+a-\frac{da-cb}{d}\right) + \frac{3be^{-bx-a}}{8d(bdx+cb)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(b\*x+a)^3/(d\*x+c)^2,x)

[Out] 1/8\*b\*exp(-3\*b\*x-3\*a)/(b\*d\*x+b\*c)/d-3/8\*b/d^2\*exp(-3\*(a\*d-b\*c)/d)\*Ei(1,3\*b\*x+3\*a-3\*(a\*d-b\*c)/d)-3/8\*b\*exp(-b\*x-a)/d/(b\*d\*x+b\*c)+3/8\*b/d^2\*exp(-(a\*d-b\*c)/d)\*Ei(1,b\*x+a-(a\*d-b\*c)/d)+3/8\*b/d^2\*exp(b\*x+a)/(b\*c/d+b\*x)+3/8\*b/d^2\*exp((a\*d-b\*c)/d)\*Ei(1,-b\*x-a-(-a\*d+b\*c)/d)-1/8\*b/d^2\*exp(3\*b\*x+3\*a)/(b\*c/d+b\*x)-3/8\*b/d^2\*exp(3\*(a\*d-b\*c)/d)\*Ei(1,-3\*b\*x-3\*a-3\*(-a\*d+b\*c)/d)

**Maxima [A]** time = 1.38717, size = 196, normalized size = 1.35

$$\frac{e^{\left(-3a+\frac{3bc}{d}\right)}E_2\left(\frac{3(dx+c)b}{d}\right)}{8(dx+c)d} - \frac{3e^{\left(-a+\frac{bc}{d}\right)}E_2\left(\frac{(dx+c)b}{d}\right)}{8(dx+c)d} + \frac{3e^{\left(a-\frac{bc}{d}\right)}E_2\left(-\frac{(dx+c)b}{d}\right)}{8(dx+c)d} - \frac{e^{\left(3a-\frac{3bc}{d}\right)}E_2\left(-\frac{3(dx+c)b}{d}\right)}{8(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^3/(d\*x+c)^2,x, algorithm="maxima")

[Out] 1/8\*e^(-3\*a + 3\*b\*c/d)\*exp\_integral\_e(2, 3\*(d\*x + c)\*b/d)/((d\*x + c)\*d) - 3/8\*e^(-a + b\*c/d)\*exp\_integral\_e(2, (d\*x + c)\*b/d)/((d\*x + c)\*d) + 3/8\*e^(-a - b\*c/d)\*exp\_integral\_e(2, -(d\*x + c)\*b/d)/((d\*x + c)\*d) - 1/8\*e^(3\*a - 3\*b\*c/d)\*exp\_integral\_e(2, -3\*(d\*x + c)\*b/d)/((d\*x + c)\*d)

**Fricas [B]** time = 2.69591, size = 662, normalized size = 4.57

$$2d \sinh(bx+a)^3 + 3\left((bdx+bc)\text{Ei}\left(\frac{bdx+bc}{d}\right) + (bdx+bc)\text{Ei}\left(-\frac{bdx+bc}{d}\right)\right) \cosh\left(-\frac{bc-ad}{d}\right) - 3\left((bdx+bc)\text{Ei}\left(\frac{3(bdx+bc)}{d}\right) + (bdx+bc)\text{Ei}\left(-\frac{3(bdx+bc)}{d}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^3/(d\*x+c)^2,x, algorithm="fricas")

[Out] 
$$-1/8*(2*d*\sinh(b*x + a)^3 + 3*((b*d*x + b*c)*\text{Ei}((b*d*x + b*c)/d) + (b*d*x + b*c)*\text{Ei}(-(b*d*x + b*c)/d))*\cosh(-(b*c - a*d)/d) - 3*((b*d*x + b*c)*\text{Ei}(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*\text{Ei}(-3*(b*d*x + b*c)/d))*\cosh(-3*(b*c - a*d)/d) + 6*(d*\cosh(b*x + a)^2 - d)*\sinh(b*x + a) + 3*((b*d*x + b*c)*\text{Ei}((b*d*x + b*c)/d) - (b*d*x + b*c)*\text{Ei}(-(b*d*x + b*c)/d))*\sinh(-(b*c - a*d)/d) - 3*((b*d*x + b*c)*\text{Ei}(3*(b*d*x + b*c)/d) - (b*d*x + b*c)*\text{Ei}(-3*(b*d*x + b*c)/d))*\sinh(-3*(b*c - a*d)/d)/(d^3*x + c*d^2)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)\*\*3/(d\*x+c)\*\*2,x)

[Out] Integral(sinh(a + b\*x)\*\*3/(c + d\*x)\*\*2, x)

**Giac [B]** time = 1.24623, size = 400, normalized size = 2.76

$$3 b d x \text{Ei}\left(\frac{3(b d x+b c)}{d}\right) e^{\left(3 a-\frac{3 b c}{d}\right)} - 3 b d x \text{Ei}\left(\frac{b d x+b c}{d}\right) e^{\left(a-\frac{b c}{d}\right)} - 3 b d x \text{Ei}\left(-\frac{b d x+b c}{d}\right) e^{\left(-a+\frac{b c}{d}\right)} + 3 b d x \text{Ei}\left(-\frac{3(b d x+b c)}{d}\right) e^{\left(-3 a+\frac{3 b c}{d}\right)} + 3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^3/(d\*x+c)^2,x, algorithm="giac")

[Out] 
$$1/8*(3*b*d*x*\text{Ei}(3*(b*d*x + b*c)/d)*e^{(3*a - 3*b*c/d)} - 3*b*d*x*\text{Ei}((b*d*x + b*c)/d)*e^{(a - b*c/d)} - 3*b*d*x*\text{Ei}(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 3*b*d*x*\text{Ei}(-3*(b*d*x + b*c)/d)*e^{(-3*a + 3*b*c/d)} + 3*b*c*\text{Ei}(3*(b*d*x + b*c)/d)*e^{(3*a - 3*b*c/d)} - 3*b*c*\text{Ei}((b*d*x + b*c)/d)*e^{(a - b*c/d)} - 3*b*c*\text{Ei}(-(b*d*x + b*c)/d)*e^{(-a + b*c/d)} + 3*b*c*\text{Ei}(-3*(b*d*x + b*c)/d)*e^{(-3*a + 3*b*c/d)} - d*e^{(3*b*x + 3*a)} + 3*d*e^{(b*x + a)} - 3*d*e^{(-b*x - a)} + d*e^{(-3*b*x$$

$$- 3*a))/(d^3*x + c*d^2)$$

$$3.22 \quad \int \frac{\sinh^3(a+bx)}{(c+dx)^3} dx$$

**Optimal.** Leaf size=184

$$\frac{9b^2 \sinh\left(3a - \frac{3bc}{d}\right) \text{Chi}\left(\frac{3bc}{d} + 3bx\right)}{8d^3} - \frac{3b^2 \sinh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{8d^3} - \frac{3b^2 \cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{8d^3} + \frac{9b^2 \cosh\left(3a - \frac{3bc}{d}\right)}{8d^3}$$

[Out]  $(9*b^2*\text{CoshIntegral}[(3*b*c)/d + 3*b*x]*\text{Sinh}[3*a - (3*b*c)/d])/(8*d^3) - (3*b^2*\text{CoshIntegral}[(b*c)/d + b*x]*\text{Sinh}[a - (b*c)/d])/(8*d^3) - (3*b*\text{Cosh}[a + b*x]*\text{Sinh}[a + b*x]^2)/(2*d^2*(c + d*x)) - \text{Sinh}[a + b*x]^3/(2*d*(c + d*x)^2) - (3*b^2*\text{Cosh}[a - (b*c)/d]*\text{SinhIntegral}[(b*c)/d + b*x])/(8*d^3) + (9*b^2*\text{Cosh}[3*a - (3*b*c)/d]*\text{SinhIntegral}[(3*b*c)/d + 3*b*x])/(8*d^3)$

**Rubi [A]** time = 0.424843, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3314, 3303, 3298, 3301, 3312}

$$\frac{9b^2 \sinh\left(3a - \frac{3bc}{d}\right) \text{Chi}\left(\frac{3bc}{d} + 3bx\right)}{8d^3} - \frac{3b^2 \sinh\left(a - \frac{bc}{d}\right) \text{Chi}\left(\frac{bc}{d} + bx\right)}{8d^3} - \frac{3b^2 \cosh\left(a - \frac{bc}{d}\right) \text{Shi}\left(\frac{bc}{d} + bx\right)}{8d^3} + \frac{9b^2 \cosh\left(3a - \frac{3bc}{d}\right)}{8d^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sinh}[a + b*x]^3/(c + d*x)^3, x]$

[Out]  $(9*b^2*\text{CoshIntegral}[(3*b*c)/d + 3*b*x]*\text{Sinh}[3*a - (3*b*c)/d])/(8*d^3) - (3*b^2*\text{CoshIntegral}[(b*c)/d + b*x]*\text{Sinh}[a - (b*c)/d])/(8*d^3) - (3*b*\text{Cosh}[a + b*x]*\text{Sinh}[a + b*x]^2)/(2*d^2*(c + d*x)) - \text{Sinh}[a + b*x]^3/(2*d*(c + d*x)^2) - (3*b^2*\text{Cosh}[a - (b*c)/d]*\text{SinhIntegral}[(b*c)/d + b*x])/(8*d^3) + (9*b^2*\text{Cosh}[3*a - (3*b*c)/d]*\text{SinhIntegral}[(3*b*c)/d + 3*b*x])/(8*d^3)$

### Rule 3314

$\text{Int}[(c + d*x)^m * (b + f*x)^n, x] \rightarrow \text{Simp}[(c + d*x)^{m+1} * (b + f*x)^n / (d*(m+1)), x] + (\text{Dist}[b^2 * f^{2*n} * (n-1) / (d^2 * (m+1) * (m+2)), \text{Int}[(c + d*x)^{m+2} * (b + f*x)^{n-2}, x], x] - \text{Dist}[f^{2*n} * n^2 / (d^2 * (m+1) * (m+2)), \text{Int}[(c + d*x)^{m+2} * (b + f*x)^n, x], x] - \text{Simp}[(b*f^n * (c + d*x)^{m+2} * \text{Cos}[e + f*x] * (b + f*x)^{n-1}) / (d^2 * (m+1) * (m+2)), x]) /;$  FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(a+bx)}{(c+dx)^3} dx &= -\frac{3b \cosh(a+bx) \sinh^2(a+bx)}{2d^2(c+dx)} - \frac{\sinh^3(a+bx)}{2d(c+dx)^2} + \frac{(3b^2) \int \frac{\sinh(a+bx)}{c+dx} dx}{d^2} + \frac{(9b^2) \int \frac{\sinh^3(a+bx)}{c+dx} dx}{2d^2} \\
&= -\frac{3b \cosh(a+bx) \sinh^2(a+bx)}{2d^2(c+dx)} - \frac{\sinh^3(a+bx)}{2d(c+dx)^2} + \frac{(9ib^2) \int \left( \frac{3i \sinh(a+bx)}{4(c+dx)} - \frac{i \sinh(3a+3bx)}{4(c+dx)} \right) dx}{2d^2} + \frac{(3b^2) \int \frac{\sinh^3(a+bx)}{c+dx} dx}{2d^2} \\
&= \frac{3b^2 \operatorname{Chi}\left(\frac{bc}{d} + bx\right) \sinh\left(a - \frac{bc}{d}\right)}{d^3} - \frac{3b \cosh(a+bx) \sinh^2(a+bx)}{2d^2(c+dx)} - \frac{\sinh^3(a+bx)}{2d(c+dx)^2} + \frac{3b^2 \cosh\left(a - \frac{bc}{d}\right)}{2d^2} \\
&= \frac{3b^2 \operatorname{Chi}\left(\frac{bc}{d} + bx\right) \sinh\left(a - \frac{bc}{d}\right)}{d^3} - \frac{3b \cosh(a+bx) \sinh^2(a+bx)}{2d^2(c+dx)} - \frac{\sinh^3(a+bx)}{2d(c+dx)^2} + \frac{3b^2 \cosh\left(a - \frac{bc}{d}\right)}{2d^2} \\
&= \frac{9b^2 \operatorname{Chi}\left(\frac{3bc}{d} + 3bx\right) \sinh\left(3a - \frac{3bc}{d}\right)}{8d^3} - \frac{3b^2 \operatorname{Chi}\left(\frac{bc}{d} + bx\right) \sinh\left(a - \frac{bc}{d}\right)}{8d^3} - \frac{3b \cosh(a+bx) \sinh^2(a+bx)}{2d^2(c+dx)} + \frac{3b^2 \cosh\left(a - \frac{bc}{d}\right)}{2d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.842618, size = 220, normalized size = 1.2

$$6b^2(c+dx)^2 \left( 3 \sinh \left( 3a - \frac{3bc}{d} \right) \text{Chi} \left( \frac{3b(c+dx)}{d} \right) - \sinh \left( a - \frac{bc}{d} \right) \text{Chi} \left( b \left( \frac{c}{d} + x \right) \right) - \cosh \left( a - \frac{bc}{d} \right) \text{Shi} \left( b \left( \frac{c}{d} + x \right) \right) + 3 \cosh \left( a - \frac{bc}{d} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[a + b\*x]^3/(c + d\*x)^3,x]

[Out] (6\*d\*Cosh[b\*x]\*(b\*(c + d\*x)\*Cosh[a] + d\*Sinh[a]) - 2\*d\*Cosh[3\*b\*x]\*(3\*b\*(c + d\*x)\*Cosh[3\*a] + d\*Sinh[3\*a]) + 6\*d\*(d\*Cosh[a] + b\*(c + d\*x)\*Sinh[a])\*Sinh[b\*x] - 2\*d\*(d\*Cosh[3\*a] + 3\*b\*(c + d\*x)\*Sinh[3\*a])\*Sinh[3\*b\*x] + 6\*b^2\*(c + d\*x)^2\*(3\*CoshIntegral[(3\*b\*(c + d\*x))/d]\*Sinh[3\*a - (3\*b\*c)/d] - CoshIntegral[b\*(c/d + x)]\*Sinh[a - (b\*c)/d] - Cosh[a - (b\*c)/d]\*SinhIntegral[b\*(c/d + x)] + 3\*Cosh[3\*a - (3\*b\*c)/d]\*SinhIntegral[(3\*b\*(c + d\*x))/d]))/(16\*d^3\*(c + d\*x)^2)

**Maple [B]** time = 0.089, size = 562, normalized size = 3.1

$$\frac{3b^3e^{-3bx-3a}x}{16d(b^2d^2x^2 + 2b^2cdx + c^2b^2)} - \frac{3b^3e^{-3bx-3a}c}{16d^2(b^2d^2x^2 + 2b^2cdx + c^2b^2)} + \frac{b^2e^{-3bx-3a}}{16d(b^2d^2x^2 + 2b^2cdx + c^2b^2)} + \frac{9b^2}{16d^3}e^{-3\frac{da-cb}{d}}\text{Ei}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(b\*x+a)^3/(d\*x+c)^3,x)

[Out] -3/16\*b^3\*exp(-3\*b\*x-3\*a)/d/(b^2\*d^2\*x^2+2\*b^2\*c\*d\*x+b^2\*c^2)\*x-3/16\*b^3\*exp(-3\*b\*x-3\*a)/d^2/(b^2\*d^2\*x^2+2\*b^2\*c\*d\*x+b^2\*c^2)\*c+1/16\*b^2\*exp(-3\*b\*x-3\*a)/d/(b^2\*d^2\*x^2+2\*b^2\*c\*d\*x+b^2\*c^2)+9/16\*b^2/d^3\*exp(-3\*(a\*d-b\*c)/d)\*Ei(1,3\*b\*x+3\*a-3\*(a\*d-b\*c)/d)+3/16\*b^3\*exp(-b\*x-a)/d/(b^2\*d^2\*x^2+2\*b^2\*c\*d\*x+b^2\*c^2)\*x+3/16\*b^3\*exp(-b\*x-a)/d^2/(b^2\*d^2\*x^2+2\*b^2\*c\*d\*x+b^2\*c^2)\*c-3/16\*b^2\*exp(-b\*x-a)/d/(b^2\*d^2\*x^2+2\*b^2\*c\*d\*x+b^2\*c^2)-3/16\*b^2/d^3\*exp(-(a\*d-b\*c)/d)\*Ei(1,b\*x+a-(a\*d-b\*c)/d)+3/16\*b^2/d^3\*exp(b\*x+a)/(b\*c/d+b\*x)^2+3/16\*b^2/d^3\*exp(b\*x+a)/(b\*c/d+b\*x)+3/16\*b^2/d^3\*exp((a\*d-b\*c)/d)\*Ei(1,-b\*x-a-(-a\*d+b\*c)/d)-1/16\*b^2/d^3\*exp(3\*b\*x+3\*a)/(b\*c/d+b\*x)^2-3/16\*b^2/d^3\*exp(3\*b\*x+3\*a)/(b\*c/d+b\*x)-9/16\*b^2/d^3\*exp(3\*(a\*d-b\*c)/d)\*Ei(1,-3\*b\*x-3\*a-3\*(a\*d+b\*c)/d)

**Maxima [A]** time = 1.60042, size = 196, normalized size = 1.07

$$\frac{e^{\left(-3a + \frac{3bc}{d}\right)} E_3\left(\frac{3(dx+c)b}{d}\right)}{8(dx+c)^2d} - \frac{3e^{\left(-a + \frac{bc}{d}\right)} E_3\left(\frac{(dx+c)b}{d}\right)}{8(dx+c)^2d} + \frac{3e^{\left(a - \frac{bc}{d}\right)} E_3\left(-\frac{(dx+c)b}{d}\right)}{8(dx+c)^2d} - \frac{e^{\left(3a - \frac{3bc}{d}\right)} E_3\left(-\frac{3(dx+c)b}{d}\right)}{8(dx+c)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^3/(d\*x+c)^3,x, algorithm="maxima")

[Out]  $\frac{1}{8}e^{(-3a + 3bc/d)} \exp\_integral\_e(3, 3(dx+c)b/d) / ((dx+c)^{2d}) - \frac{3}{8}e^{(-a + bc/d)} \exp\_integral\_e(3, (dx+c)b/d) / ((dx+c)^{2d}) + \frac{3}{8}e^{(a - bc/d)} \exp\_integral\_e(3, -(dx+c)b/d) / ((dx+c)^{2d}) - \frac{1}{8}e^{(3a - 3bc/d)} \exp\_integral\_e(3, -3(dx+c)b/d) / ((dx+c)^{2d})$

**Fricas [B]** time = 2.66529, size = 1123, normalized size = 6.1

$$2d^2 \sinh(bx+a)^3 + 6(bd^2x + bcd) \cosh(bx+a)^3 + 18(bd^2x + bcd) \cosh(bx+a) \sinh(bx+a)^2 - 6(bd^2x + bcd) \cosh$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^3/(d\*x+c)^3,x, algorithm="fricas")

[Out]  $-1/16*(2*d^2*\sinh(b*x + a)^3 + 6*(b*d^2*x + b*c*d)*\cosh(b*x + a)^3 + 18*(b*d^2*x + b*c*d)*\cosh(b*x + a)*\sinh(b*x + a)^2 - 6*(b*d^2*x + b*c*d)*\cosh(b*x + a) + 3*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-(b*d*x + b*c)/d))*\cosh(-(b*c - a*d)/d) - 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(3*(b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-3*(b*d*x + b*c)/d))*\cosh(-3*(b*c - a*d)/d) + 6*(d^2*\cosh(b*x + a)^2 - d^2)*\sinh(b*x + a) + 3*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-(b*d*x + b*c)/d))*\sinh(-(b*c - a*d)/d) - 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-3*(b*d*x + b*c)/d))*\sinh(-3*(b*c - a*d)/d)) / (d^5*x^2 + 2*c*d^4*x + c^2*d^3)$



**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(a + bx)}{(c + dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)\*\*3/(d\*x+c)\*\*3,x)

[Out] Integral(sinh(a + b\*x)\*\*3/(c + d\*x)\*\*3, x)

**Giac [B]** time = 1.18637, size = 811, normalized size = 4.41

$$9 b^2 d^2 x^2 \operatorname{Ei}\left(\frac{3(bdx+bc)}{d}\right) e^{\left(3a-\frac{3bc}{d}\right)} - 3 b^2 d^2 x^2 \operatorname{Ei}\left(\frac{bdx+bc}{d}\right) e^{\left(a-\frac{bc}{d}\right)} + 3 b^2 d^2 x^2 \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right) e^{\left(-a+\frac{bc}{d}\right)} - 9 b^2 d^2 x^2 \operatorname{Ei}\left(-\frac{3(bdx+bc)}{d}\right) e^{\left(-3a+\frac{3bc}{d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^3/(d\*x+c)^3,x, algorithm="giac")

[Out] 
$$\frac{1}{16} \left( 9 b^2 d^2 x^2 \operatorname{Ei}\left(\frac{3(bdx+bc)}{d}\right) e^{\left(3a-\frac{3bc}{d}\right)} - 3 b^2 d^2 x^2 \operatorname{Ei}\left(\frac{bdx+bc}{d}\right) e^{\left(a-\frac{bc}{d}\right)} + 3 b^2 d^2 x^2 \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right) e^{\left(-a+\frac{bc}{d}\right)} - 9 b^2 d^2 x^2 \operatorname{Ei}\left(-\frac{3(bdx+bc)}{d}\right) e^{\left(-3a+\frac{3bc}{d}\right)} + 18 b^2 c d x \operatorname{Ei}\left(\frac{3(bdx+bc)}{d}\right) e^{\left(3a-\frac{3bc}{d}\right)} - 6 b^2 c d x \operatorname{Ei}\left(\frac{bdx+bc}{d}\right) e^{\left(a-\frac{bc}{d}\right)} + 6 b^2 c d x \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right) e^{\left(-a+\frac{bc}{d}\right)} - 18 b^2 c d x \operatorname{Ei}\left(-\frac{3(bdx+bc)}{d}\right) e^{\left(-3a+\frac{3bc}{d}\right)} + 9 b^2 c^2 \operatorname{Ei}\left(\frac{3(bdx+bc)}{d}\right) e^{\left(3a-\frac{3bc}{d}\right)} - 3 b^2 c^2 \operatorname{Ei}\left(\frac{bdx+bc}{d}\right) e^{\left(a-\frac{bc}{d}\right)} + 3 b^2 c^2 \operatorname{Ei}\left(-\frac{bdx+bc}{d}\right) e^{\left(-a+\frac{bc}{d}\right)} - 9 b^2 c^2 \operatorname{Ei}\left(-\frac{3(bdx+bc)}{d}\right) e^{\left(-3a+\frac{3bc}{d}\right)} - 3 b^2 d^2 x e^{\left(3bx+3a\right)} + 3 b^2 d^2 x e^{\left(bx+a\right)} + 3 b^2 d^2 x e^{\left(-bx-a\right)} - 3 b^2 d^2 x e^{\left(-3bx-3a\right)} - 3 b^2 c d e^{\left(3bx+3a\right)} + 3 b^2 c d e^{\left(bx+a\right)} + 3 b^2 c d e^{\left(-bx-a\right)} - 3 b^2 c d e^{\left(-3bx-3a\right)} - d^2 e^{\left(3bx+3a\right)} + 3 d^2 e^{\left(bx+a\right)} - 3 d^2 e^{\left(-bx-a\right)} + d^2 e^{\left(-3bx-3a\right)} \right) / (d^5 x^2 + 2 c d^4 x + c^2 d^3)$$

### 3.23 $\int (c + dx)^3 \operatorname{csch}(a + bx) dx$

**Optimal.** Leaf size=149

$$\frac{6d^2(c + dx)\operatorname{PolyLog}(3, -e^{a+bx})}{b^3} - \frac{6d^2(c + dx)\operatorname{PolyLog}(3, e^{a+bx})}{b^3} - \frac{3d(c + dx)^2\operatorname{PolyLog}(2, -e^{a+bx})}{b^2} + \frac{3d(c + dx)^2\operatorname{PolyLog}(2, e^{a+bx})}{b^2}$$

[Out]  $(-2*(c + d*x)^3*\operatorname{ArcTanh}[E^{(a + b*x)}])/b - (3*d*(c + d*x)^2*\operatorname{PolyLog}[2, -E^{(a + b*x)}])/b^2 + (3*d*(c + d*x)^2*\operatorname{PolyLog}[2, E^{(a + b*x)}])/b^2 + (6*d^2*(c + d*x)*\operatorname{PolyLog}[3, -E^{(a + b*x)}])/b^3 - (6*d^2*(c + d*x)*\operatorname{PolyLog}[3, E^{(a + b*x)}])/b^3 - (6*d^3*\operatorname{PolyLog}[4, -E^{(a + b*x)}])/b^4 + (6*d^3*\operatorname{PolyLog}[4, E^{(a + b*x)}])/b^4$

**Rubi [A]** time = 0.136457, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {4182, 2531, 6609, 2282, 6589}

$$\frac{6d^2(c + dx)\operatorname{PolyLog}(3, -e^{a+bx})}{b^3} - \frac{6d^2(c + dx)\operatorname{PolyLog}(3, e^{a+bx})}{b^3} - \frac{3d(c + dx)^2\operatorname{PolyLog}(2, -e^{a+bx})}{b^2} + \frac{3d(c + dx)^2\operatorname{PolyLog}(2, e^{a+bx})}{b^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^3*\operatorname{Csch}[a + b*x], x]$

[Out]  $(-2*(c + d*x)^3*\operatorname{ArcTanh}[E^{(a + b*x)}])/b - (3*d*(c + d*x)^2*\operatorname{PolyLog}[2, -E^{(a + b*x)}])/b^2 + (3*d*(c + d*x)^2*\operatorname{PolyLog}[2, E^{(a + b*x)}])/b^2 + (6*d^2*(c + d*x)*\operatorname{PolyLog}[3, -E^{(a + b*x)}])/b^3 - (6*d^2*(c + d*x)*\operatorname{PolyLog}[3, E^{(a + b*x)}])/b^3 - (6*d^3*\operatorname{PolyLog}[4, -E^{(a + b*x)}])/b^4 + (6*d^3*\operatorname{PolyLog}[4, E^{(a + b*x)}])/b^4$

#### Rule 4182

$\operatorname{Int}[\operatorname{csc}[(e_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^{(m_.)}, x, x\_Symbol] \rightarrow \operatorname{Simp}[(-2*(c + d*x)^m*\operatorname{ArcTanh}[E^{-(I*e) + f*fz*x}])/(f*fz*I), x] + (-\operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 - E^{-(I*e) + f*fz*x}], x], x] + \operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 + E^{-(I*e) + f*fz*x}], x], x]) /; \operatorname{FreeQ}\{c, d, e, f, fz, x\} \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2531

$\operatorname{Int}[\operatorname{Log}[1 + (e_.)*((F_)^{((c_.)*((a_.) + (b_.)*(x_)))})^{(n_.)}]*((f_.) + (g_.)*(x_))^{(m_.)}, x, x\_Symbol] \rightarrow -\operatorname{Simp}[(f + g*x)^m*\operatorname{PolyLog}[2, -(e*(F^{(c*(a + b*x)})))]]$

)))^n]]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(p\_.)], x\_Symbol] := Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\* (F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int (c + dx)^3 \operatorname{csch}(a + bx) dx &= -\frac{2(c + dx)^3 \tanh^{-1}(e^{a+bx})}{b} - \frac{(3d) \int (c + dx)^2 \log(1 - e^{a+bx}) dx}{b} + \frac{(3d) \int (c + dx)^2 \log(1 + e^{a+bx}) dx}{b} \\
 &= -\frac{2(c + dx)^3 \tanh^{-1}(e^{a+bx})}{b} - \frac{3d(c + dx)^2 \operatorname{Li}_2(-e^{a+bx})}{b^2} + \frac{3d(c + dx)^2 \operatorname{Li}_2(e^{a+bx})}{b^2} + \frac{(6d^2) \int (c + dx) \log(1 - e^{a+bx}) dx}{b} \\
 &= -\frac{2(c + dx)^3 \tanh^{-1}(e^{a+bx})}{b} - \frac{3d(c + dx)^2 \operatorname{Li}_2(-e^{a+bx})}{b^2} + \frac{3d(c + dx)^2 \operatorname{Li}_2(e^{a+bx})}{b^2} + \frac{6d^2(c + dx) \log(1 - e^{a+bx})}{b} \\
 &= -\frac{2(c + dx)^3 \tanh^{-1}(e^{a+bx})}{b} - \frac{3d(c + dx)^2 \operatorname{Li}_2(-e^{a+bx})}{b^2} + \frac{3d(c + dx)^2 \operatorname{Li}_2(e^{a+bx})}{b^2} + \frac{6d^2(c + dx) \log(1 - e^{a+bx})}{b} \\
 &= -\frac{2(c + dx)^3 \tanh^{-1}(e^{a+bx})}{b} - \frac{3d(c + dx)^2 \operatorname{Li}_2(-e^{a+bx})}{b^2} + \frac{3d(c + dx)^2 \operatorname{Li}_2(e^{a+bx})}{b^2} + \frac{6d^2(c + dx) \log(1 - e^{a+bx})}{b}
 \end{aligned}$$

**Mathematica [A]** time = 2.94047, size = 191, normalized size = 1.28

$$-3d(b^2(c+dx)^2 \text{PolyLog}(2, -\sinh(a+bx) - \cosh(a+bx)) - 2bd(c+dx) \text{PolyLog}(3, -\sinh(a+bx) - \cosh(a+bx)) + \dots$$


---

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x)^3\*Csch[a + b\*x], x]

[Out]  $(-2*b^3*(c + d*x)^3*\text{ArcTanh}[\text{Cosh}[a + b*x] + \text{Sinh}[a + b*x]] - 3*d*(b^2*(c + d*x)^2*\text{PolyLog}[2, -\text{Cosh}[a + b*x] - \text{Sinh}[a + b*x]] - 2*b*d*(c + d*x)*\text{PolyLog}[3, -\text{Cosh}[a + b*x] - \text{Sinh}[a + b*x]] + 2*d^2*\text{PolyLog}[4, -\text{Cosh}[a + b*x] - \text{Sinh}[a + b*x]]) + 3*d*(b^2*(c + d*x)^2*\text{PolyLog}[2, \text{Cosh}[a + b*x] + \text{Sinh}[a + b*x]] - 2*b*d*(c + d*x)*\text{PolyLog}[3, \text{Cosh}[a + b*x] + \text{Sinh}[a + b*x]] + 2*d^2*\text{PolyLog}[4, \text{Cosh}[a + b*x] + \text{Sinh}[a + b*x]])/b^4$

---

**Maple [B]** time = 0.072, size = 541, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3\*csch(b\*x+a), x)

[Out]  $-6*d^3*\text{polylog}(4, -\exp(b*x+a))/b^4 + 6*d^3*\text{polylog}(4, \exp(b*x+a))/b^4 + 6/b^2*c*d^2*\text{polylog}(2, \exp(b*x+a))*x - 3/b*c^2*d*\ln(1+\exp(b*x+a))*x - 3/b^2*c^2*d*\ln(1+\exp(b*x+a))*a + 3/b*c^2*d*\ln(1-\exp(b*x+a))*x + 3/b^2*c^2*d*\ln(1-\exp(b*x+a))*a + 6/b^2*c^2*d*a*\text{arctanh}(\exp(b*x+a)) - 6/b^3*c*d^2*a^2*\text{arctanh}(\exp(b*x+a)) - 3/b*c*d^2*\ln(1+\exp(b*x+a))*x^2 + 3/b^3*c*d^2*\ln(1+\exp(b*x+a))*a^2 - 6/b^2*c*d^2*\text{polylog}(2, -\exp(b*x+a))*x + 3/b*c*d^2*\ln(1-\exp(b*x+a))*x^2 - 3/b^3*c*d^2*\ln(1-\exp(b*x+a))*a^2 + 6/b^3*d^3*\text{polylog}(3, -\exp(b*x+a))*x + 2/b^4*d^3*a^3*\text{arctanh}(\exp(b*x+a)) - 3/b^2*c^2*d*\text{polylog}(2, -\exp(b*x+a)) + 3/b^2*c^2*d*\text{polylog}(2, \exp(b*x+a)) + 6/b^3*c*d^2*\text{polylog}(3, -\exp(b*x+a)) - 6/b^3*c*d^2*\text{polylog}(3, \exp(b*x+a)) - 1/b^4*d^3*a^3*\ln(1+\exp(b*x+a)) + 1/b^4*d^3*a^3*\ln(1-\exp(b*x+a)) + 1/b*d^3*\ln(1-\exp(b*x+a))*x^3 + 3/b^2*d^3*\text{polylog}(2, \exp(b*x+a))*x^2 - 6/b^3*d^3*\text{polylog}(3, \exp(b*x+a))*x - 1/b*d^3*\ln(1+\exp(b*x+a))*x^3 - 3/b^2*d^3*\text{polylog}(2, -\exp(b*x+a))*x^2 - 2/b*c^3*a*\text{rctanh}(\exp(b*x+a))$

---

**Maxima [B]** time = 1.50737, size = 450, normalized size = 3.02

$$-c^3 \left( \frac{\log(e^{-bx-a} + 1)}{b} - \frac{\log(e^{-bx-a} - 1)}{b} \right) - \frac{3 \left( bx \log(e^{(bx+a)} + 1) + \text{Li}_2(-e^{(bx+a)}) \right) c^2 d}{b^2} + \frac{3 \left( bx \log(-e^{(bx+a)} + 1) + \text{Li}_2(-e^{(bx+a)}) \right) c^2 d}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*csh(b\*x+a),x, algorithm="maxima")

[Out]  $-c^3 \left( \frac{\log(e^{-bx-a} + 1)}{b} - \frac{\log(e^{-bx-a} - 1)}{b} \right) - 3 \left( bx \log(e^{(bx+a)} + 1) + \text{Li}_2(-e^{(bx+a)}) \right) \frac{c^2 d}{b^2} + 3 \left( bx \log(-e^{(bx+a)} + 1) + \text{Li}_2(-e^{(bx+a)}) \right) \frac{c^2 d}{b^2}$

**Fricas [C]** time = 2.78795, size = 986, normalized size = 6.62

$$6d^3 \text{polylog}(4, \cosh(bx+a) + \sinh(bx+a)) - 6d^3 \text{polylog}(4, -\cosh(bx+a) - \sinh(bx+a)) + 3(b^2 d^3 x^2 + 2b^2 cd^2 x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*csh(b\*x+a),x, algorithm="fricas")

[Out]  $(6d^3 \text{polylog}(4, \cosh(bx+a) + \sinh(bx+a)) - 6d^3 \text{polylog}(4, -\cosh(bx+a) - \sinh(bx+a)) + 3(b^2 d^3 x^2 + 2b^2 cd^2 x) + 3(b^2 d^3 x^2 + 2b^2 cd^2 x) \text{dilog}(\cosh(bx+a) + \sinh(bx+a)) - 3(b^2 d^3 x^2 + 2b^2 cd^2 x) \text{dilog}(-\cosh(bx+a) - \sinh(bx+a)) - (b^3 d^3 x^3 + 3b^3 cd^2 x^2 + 3b^3 c^2 d x + b^3 c^3) \log(\cosh(bx+a) + \sinh(bx+a) + 1) + (b^3 c^3 - 3a b^2 c^2 d + 3a^2 b c d^2 - a^3 d^3) \log(\cosh(bx+a) + \sinh(bx+a) - 1) + (b^3 d^3 x^3 + 3b^3 cd^2 x^2 + 3b^3 c^2 d x + 3a b^2 c^2 d - 3a^2 b c d^2 + a^3 d^3) \log(-\cosh(bx+a) - \sinh(bx+a) + 1) - 6(b^3 d^3 x + b^3 c d^2) \text{polylog}(3, \cosh(bx+a) + \sinh(bx+a)) + 6(b^3 d^3 x + b^3 c d^2) \text{polylog}(3, -\cosh(bx+a) - \sinh(bx+a)))/b^4$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \operatorname{csch}(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*3\*csch(b\*x+a),x)

[Out] Integral((c + d\*x)\*\*3\*csch(a + b\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \operatorname{csch}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*csch(b\*x+a),x, algorithm="giac")

[Out] integrate((d\*x + c)^3\*csch(b\*x + a), x)

### 3.24 $\int (c + dx)^2 \operatorname{csch}(a + bx) dx$

**Optimal.** Leaf size=99

$$-\frac{2d(c + dx)\operatorname{PolyLog}(2, -e^{a+bx})}{b^2} + \frac{2d(c + dx)\operatorname{PolyLog}(2, e^{a+bx})}{b^2} + \frac{2d^2\operatorname{PolyLog}(3, -e^{a+bx})}{b^3} - \frac{2d^2\operatorname{PolyLog}(3, e^{a+bx})}{b^3}$$

[Out]  $(-2*(c + d*x)^2*\operatorname{ArcTanh}[E^{(a + b*x)}])/b - (2*d*(c + d*x)*\operatorname{PolyLog}[2, -E^{(a + b*x)}])/b^2 + (2*d*(c + d*x)*\operatorname{PolyLog}[2, E^{(a + b*x)}])/b^2 + (2*d^2*\operatorname{PolyLog}[3, -E^{(a + b*x)}])/b^3 - (2*d^2*\operatorname{PolyLog}[3, E^{(a + b*x)}])/b^3$

**Rubi [A]** time = 0.0903547, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {4182, 2531, 2282, 6589}

$$-\frac{2d(c + dx)\operatorname{PolyLog}(2, -e^{a+bx})}{b^2} + \frac{2d(c + dx)\operatorname{PolyLog}(2, e^{a+bx})}{b^2} + \frac{2d^2\operatorname{PolyLog}(3, -e^{a+bx})}{b^3} - \frac{2d^2\operatorname{PolyLog}(3, e^{a+bx})}{b^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^2*\operatorname{Csch}[a + b*x], x]$

[Out]  $(-2*(c + d*x)^2*\operatorname{ArcTanh}[E^{(a + b*x)}])/b - (2*d*(c + d*x)*\operatorname{PolyLog}[2, -E^{(a + b*x)}])/b^2 + (2*d*(c + d*x)*\operatorname{PolyLog}[2, E^{(a + b*x)}])/b^2 + (2*d^2*\operatorname{PolyLog}[3, -E^{(a + b*x)}])/b^3 - (2*d^2*\operatorname{PolyLog}[3, E^{(a + b*x)}])/b^3$

#### Rule 4182

$\operatorname{Int}[\operatorname{csc}[(e_{.}) + (\operatorname{Complex}[0, fz_{.}])*(f_{.})*(x_{.})]*((c_{.}) + (d_{.})*(x_{.}))^{(m_{.})}, x\_Symbol] :> \operatorname{Simp}[(-2*(c + d*x)^m*\operatorname{ArcTanh}[E^{-(I*e) + f*fz*x}])/(f*fz*I), x] + (-\operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 - E^{-(I*e) + f*fz*x}], x], x] + \operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 + E^{-(I*e) + f*fz*x}], x], x]) /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x \ \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2531

$\operatorname{Int}[\operatorname{Log}[1 + (e_{.})*((F_{.})^{((c_{.})*((a_{.}) + (b_{.})*(x_{.})))^{(n_{.})})*((f_{.}) + (g_{.})*(x_{.}))^{(m_{.})}, x\_Symbol] :> -\operatorname{Simp}[(f + g*x)^m*\operatorname{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n])/ (b*c*n*\operatorname{Log}[F]), x] + \operatorname{Dist}[(g*m)/(b*c*n*\operatorname{Log}[F]), \operatorname{Int}[(f + g*x)^{(m-1)}*\operatorname{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n]), x], x] /; \operatorname{FreeQ}\{F, a, b, c, e, f, g, n\}, x \ \&\& \operatorname{GtQ}[m, 0]$

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^2 \operatorname{csch}(a + bx) dx &= -\frac{2(c + dx)^2 \tanh^{-1}(e^{a+bx})}{b} - \frac{(2d) \int (c + dx) \log(1 - e^{a+bx}) dx}{b} + \frac{(2d) \int (c + dx) \log(1 + e^{a+bx}) dx}{b} \\ &= -\frac{2(c + dx)^2 \tanh^{-1}(e^{a+bx})}{b} - \frac{2d(c + dx) \operatorname{Li}_2(-e^{a+bx})}{b^2} + \frac{2d(c + dx) \operatorname{Li}_2(e^{a+bx})}{b^2} + \frac{(2d^2) \int \operatorname{Li}_2(e^{a+bx}) dx}{b^2} \\ &= -\frac{2(c + dx)^2 \tanh^{-1}(e^{a+bx})}{b} - \frac{2d(c + dx) \operatorname{Li}_2(-e^{a+bx})}{b^2} + \frac{2d(c + dx) \operatorname{Li}_2(e^{a+bx})}{b^2} + \frac{(2d^2) \operatorname{Subst}[\operatorname{Li}_2(e^u), u, a + bx]}{b^2} \\ &= -\frac{2(c + dx)^2 \tanh^{-1}(e^{a+bx})}{b} - \frac{2d(c + dx) \operatorname{Li}_2(-e^{a+bx})}{b^2} + \frac{2d(c + dx) \operatorname{Li}_2(e^{a+bx})}{b^2} + \frac{2d^2 \operatorname{Li}_3(-e^{a+bx})}{b^3} \end{aligned}$$

**Mathematica [A]** time = 2.29854, size = 118, normalized size = 1.19

$$\frac{\frac{2d(b(c+dx)\operatorname{PolyLog}(2,-e^{a+bx})-d\operatorname{PolyLog}(3,-e^{a+bx}))}{b^2} + \frac{2d(b(c+dx)\operatorname{PolyLog}(2,e^{a+bx})-d\operatorname{PolyLog}(3,e^{a+bx}))}{b^2}}{b} + (c + dx)^2 \log(1 - e^{a+bx}) - (c + dx)^2 \log(1 + e^{a+bx})$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csch[a + b*x],x]
```

```
[Out] ((c + d*x)^2*Log[1 - E^(a + b*x)] - (c + d*x)^2*Log[1 + E^(a + b*x)] - (2*d
*(b*(c + d*x)*PolyLog[2, -E^(a + b*x)] - d*PolyLog[3, -E^(a + b*x)]))/b^2 +
(2*d*(b*(c + d*x)*PolyLog[2, E^(a + b*x)] - d*PolyLog[3, E^(a + b*x)]))/b^
2)/b
```



**Maple [B]** time = 0.03, size = 306, normalized size = 3.1

$$-2 \frac{c^2 \operatorname{Arctanh}(e^{bx+a})}{b} + 2 \frac{d^2 \operatorname{polylog}(3, -e^{bx+a})}{b^3} - 2 \frac{d^2 \operatorname{polylog}(3, e^{bx+a})}{b^3} + 2 \frac{cd \ln(1 - e^{bx+a}) a}{b^2} - 2 \frac{cd \ln(1 + e^{bx+a}) x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^2*cscsch(b*x+a),x)`

[Out]  $-2/b*c^2*\operatorname{arctanh}(\exp(b*x+a))+2*d^2*\operatorname{polylog}(3,-\exp(b*x+a))/b^3-2*d^2*\operatorname{polylog}(3,\exp(b*x+a))/b^3+2/b^2*c*d*\ln(1-\exp(b*x+a))*a-2/b*c*d*\ln(1+\exp(b*x+a))*x-2/b^2*c*d*\ln(1+\exp(b*x+a))*a+2/b*c*d*\ln(1-\exp(b*x+a))*x+4/b^2*c*d*a*\operatorname{arctanh}(\exp(b*x+a))+1/b*d^2*\ln(1-\exp(b*x+a))*x^2-1/b^3*d^2*\ln(1-\exp(b*x+a))*a^2+2/b^2*d^2*\operatorname{polylog}(2,\exp(b*x+a))*x-1/b*d^2*\ln(1+\exp(b*x+a))*x^2+1/b^3*d^2*\ln(1+\exp(b*x+a))*a^2-2/b^2*d^2*\operatorname{polylog}(2,-\exp(b*x+a))*x-2/b^3*d^2*a^2*\operatorname{arctanh}(\exp(b*x+a))-2/b^2*c*d*\operatorname{polylog}(2,-\exp(b*x+a))+2/b^2*c*d*\operatorname{polylog}(2,\exp(b*x+a))$

**Maxima [B]** time = 1.54884, size = 263, normalized size = 2.66

$$-c^2 \left( \frac{\log(e^{(-bx-a)} + 1)}{b} - \frac{\log(e^{(-bx-a)} - 1)}{b} \right) - \frac{2(bx \log(e^{(bx+a)} + 1) + \operatorname{Li}_2(-e^{(bx+a)}))cd}{b^2} + \frac{2(bx \log(-e^{(bx+a)} + 1) + \operatorname{Li}_2(-e^{(bx+a)}))cd}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^2*cscsch(b*x+a),x, algorithm="maxima")`

[Out]  $-c^2*(\log(e^{(-b*x - a)} + 1)/b - \log(e^{(-b*x - a)} - 1)/b) - 2*(b*x*\log(e^{(b*x + a)} + 1) + \operatorname{dilog}(-e^{(b*x + a)}))*c*d/b^2 + 2*(b*x*\log(-e^{(b*x + a)} + 1) + \operatorname{dilog}(e^{(b*x + a)}))*c*d/b^2 - (b^2*x^2*\log(e^{(b*x + a)} + 1) + 2*b*x*\operatorname{dilog}(-e^{(b*x + a)}) - 2*\operatorname{polylog}(3, -e^{(b*x + a)}))*d^2/b^3 + (b^2*x^2*\log(-e^{(b*x + a)} + 1) + 2*b*x*\operatorname{dilog}(e^{(b*x + a)}) - 2*\operatorname{polylog}(3, e^{(b*x + a)}))*d^2/b^3$

**Fricas [C]** time = 2.64391, size = 635, normalized size = 6.41

$$2d^2 \operatorname{polylog}(3, \cosh(bx + a) + \sinh(bx + a)) - 2d^2 \operatorname{polylog}(3, -\cosh(bx + a) - \sinh(bx + a)) - 2(bd^2x + bcd)\operatorname{Li}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*d^2*polylog(3, cosh(b*x + a) + sinh(b*x + a)) - 2*d^2*polylog(3, -cosh(
b*x + a) - sinh(b*x + a)) - 2*(b*d^2*x + b*c*d)*dilog(cosh(b*x + a) + sinh(
b*x + a)) + 2*(b*d^2*x + b*c*d)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b^
2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) -
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) - (
b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cosh(b*x + a) - sinh(
b*x + a) + 1))/b^3
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \operatorname{csch}(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csch(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*csch(a + b*x), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \operatorname{csch}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csch(b*x + a), x)
```

### 3.25 $\int (c + dx) \operatorname{csch}(a + bx) dx$

**Optimal.** Leaf size=50

$$-\frac{d\operatorname{PolyLog}(2, -e^{a+bx})}{b^2} + \frac{d\operatorname{PolyLog}(2, e^{a+bx})}{b^2} - \frac{2(c + dx) \tanh^{-1}(e^{a+bx})}{b}$$

[Out]  $(-2*(c + d*x)*\operatorname{ArcTanh}[E^{(a + b*x)}])/b - (d*\operatorname{PolyLog}[2, -E^{(a + b*x)}])/b^2 + (d*\operatorname{PolyLog}[2, E^{(a + b*x)}])/b^2$

**Rubi [A]** time = 0.04626, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {4182, 2279, 2391}

$$-\frac{d\operatorname{PolyLog}(2, -e^{a+bx})}{b^2} + \frac{d\operatorname{PolyLog}(2, e^{a+bx})}{b^2} - \frac{2(c + dx) \tanh^{-1}(e^{a+bx})}{b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)*\operatorname{Csch}[a + b*x], x]$

[Out]  $(-2*(c + d*x)*\operatorname{ArcTanh}[E^{(a + b*x)}])/b - (d*\operatorname{PolyLog}[2, -E^{(a + b*x)}])/b^2 + (d*\operatorname{PolyLog}[2, E^{(a + b*x)}])/b^2$

#### Rule 4182

$\operatorname{Int}[\operatorname{csc}[(e_{.}) + (\operatorname{Complex}[0, fz_{.}])*(f_{.})*(x_{.})]*((c_{.}) + (d_{.})*(x_{.}))^{(m_{.})}, x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(-2*(c + d*x)^m*\operatorname{ArcTanh}[E^{-(I*e) + f*fz*x}])/(f*fz*I), x] + (-\operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 - E^{-(I*e) + f*fz*x}], x], x] + \operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 + E^{-(I*e) + f*fz*x}], x], x]) /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x\} \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2279

$\operatorname{Int}[\operatorname{Log}[(a_{.}) + (b_{.})*((F_{.})^{((e_{.})*((c_{.}) + (d_{.})*(x_{.})))^{(n_{.})}], x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[1/(d*e*n*\operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x\} \&\& \operatorname{GtQ}[a, 0]$

#### Rule 2391

$\operatorname{Int}[\operatorname{Log}[(c_{.})*((d_{.}) + (e_{.})*(x_{.}))^{(n_{.})}]/(x_{.}), x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[\operatorname{PolyLog}[2, -(c*e*x^n)]/n, x] /; \operatorname{FreeQ}\{c, d, e, n\}, x\} \&\& \operatorname{EqQ}[c*d, 1]$

Rubi steps

$$\begin{aligned}
\int (c + dx) \operatorname{csch}(a + bx) dx &= -\frac{2(c + dx) \tanh^{-1}(e^{a+bx})}{b} - \frac{d \int \log(1 - e^{a+bx}) dx}{b} + \frac{d \int \log(1 + e^{a+bx}) dx}{b} \\
&= -\frac{2(c + dx) \tanh^{-1}(e^{a+bx})}{b} - \frac{d \operatorname{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{a+bx}\right)}{b^2} + \frac{d \operatorname{Subst}\left(\int \frac{\log(1+x)}{x} dx, x, e^{a+bx}\right)}{b^2} \\
&= -\frac{2(c + dx) \tanh^{-1}(e^{a+bx})}{b} - \frac{d \operatorname{Li}_2(-e^{a+bx})}{b^2} + \frac{d \operatorname{Li}_2(e^{a+bx})}{b^2}
\end{aligned}$$

**Mathematica [C]** time = 0.0785852, size = 174, normalized size = 3.48

$$\frac{d\left(-a \log\left(\tanh\left(\frac{1}{2}(a + bx)\right)\right) - i\left(i\left(\operatorname{PolyLog}\left(2, -e^{i(a+ibx)}\right) - \operatorname{PolyLog}\left(2, e^{i(a+ibx)}\right)\right) + (ia + ibx)\left(\log\left(1 - e^{i(a+ibx)}\right) - \log\left(1 + e^{i(a+ibx)}\right)\right)\right)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)\*Csch[a + b\*x], x]

[Out] -((c\*Log[Cosh[a/2 + (b\*x)/2]])/b) + (c\*Log[Sinh[a/2 + (b\*x)/2]])/b + (d\*(-(a\*Log[Tanh[(a + b\*x)/2]]) - I\*((I\*a + I\*b\*x)\*(Log[1 - E^(I\*(I\*a + I\*b\*x))]] - Log[1 + E^(I\*(I\*a + I\*b\*x))])) + I\*(PolyLog[2, -E^(I\*(I\*a + I\*b\*x))] - PolyLog[2, E^(I\*(I\*a + I\*b\*x))])))/b^2

**Maple [A]** time = 0.004, size = 60, normalized size = 1.2

$$\frac{1}{b} \left( \frac{d}{b} \left( 2 \operatorname{dilog}(e^{-bx-a}) - \frac{\operatorname{dilog}(e^{-2bx-2a})}{2} \right) + 2 \frac{da \operatorname{Artanh}(e^{bx+a})}{b} - 2c \operatorname{Artanh}(e^{bx+a}) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)\*csch(b\*x+a), x)

[Out] 1/b\*(1/b\*d\*(2\*dilog(exp(-b\*x-a))-1/2\*dilog(exp(-2\*b\*x-2\*a)))+2/b\*d\*a\*arctanh(exp(b\*x+a))-2\*c\*arctanh(exp(b\*x+a)))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-c \left( \frac{\log(e^{(-bx-a)} + 1)}{b} - \frac{\log(e^{(-bx-a)} - 1)}{b} \right) + 2d \left( \int \frac{x}{2(e^{(bx+a)} + 1)} dx + \int \frac{x}{2(e^{(bx+a)} - 1)} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*csch(b\*x+a),x, algorithm="maxima")

[Out] -c\*(log(e^(-b\*x - a) + 1)/b - log(e^(-b\*x - a) - 1)/b) + 2\*d\*(integrate(1/2\*x/(e^(b\*x + a) + 1), x) + integrate(1/2\*x/(e^(b\*x + a) - 1), x))

**Fricas [B]** time = 2.63454, size = 340, normalized size = 6.8

$$\frac{d\text{Li}_2(\cosh(bx + a) + \sinh(bx + a)) - d\text{Li}_2(-\cosh(bx + a) - \sinh(bx + a)) - (bdx + bc) \log(\cosh(bx + a) + \sinh(bx + a))}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*csch(b\*x+a),x, algorithm="fricas")

[Out] (d\*dilog(cosh(b\*x + a) + sinh(b\*x + a)) - d\*dilog(-cosh(b\*x + a) - sinh(b\*x + a)) - (b\*d\*x + b\*c)\*log(cosh(b\*x + a) + sinh(b\*x + a) + 1) + (b\*c - a\*d)\*log(cosh(b\*x + a) + sinh(b\*x + a) - 1) + (b\*d\*x + a\*d)\*log(-cosh(b\*x + a) - sinh(b\*x + a) + 1))/b^2

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \operatorname{csch}(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*csch(b\*x+a),x)

[Out] Integral((c + d\*x)\*csch(a + b\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \operatorname{csch}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csch(b*x + a), x)
```

$$3.26 \quad \int \frac{\mathbf{csch}(a+bx)}{c+dx} dx$$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{\mathbf{csch}(a+bx)}{c+dx}, x\right)$$

[Out] Unintegrable[Csch[a + b\*x]/(c + d\*x), x]

**Rubi [A]** time = 0.0235079, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\mathbf{csch}(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[Csch[a + b\*x]/(c + d\*x), x]

[Out] Defer[Int][Csch[a + b\*x]/(c + d\*x), x]

Rubi steps

$$\int \frac{\mathbf{csch}(a+bx)}{c+dx} dx = \int \frac{\mathbf{csch}(a+bx)}{c+dx} dx$$

**Mathematica [A]** time = 11.29, size = 0, normalized size = 0.

$$\int \frac{\mathbf{csch}(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[Csch[a + b\*x]/(c + d\*x), x]

[Out] Integrate[Csch[a + b\*x]/(c + d\*x), x]

---

**Maple [A]** time = 0.051, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(b\*x+a)/(d\*x+c),x)

[Out] int(csch(b\*x+a)/(d\*x+c),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(b\*x+a)/(d\*x+c),x, algorithm="maxima")

[Out] integrate(csch(b\*x + a)/(d\*x + c), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx + a)}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(b\*x+a)/(d\*x+c),x, algorithm="fricas")

[Out] integral(csch(b\*x + a)/(d\*x + c), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a + bx)}{c + dx} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(csch(a + b*x)/(c + d*x), x)
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx + a)}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)/(d*x + c), x)
```

$$3.27 \quad \int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} dx$$

**Optimal.** Leaf size=16

$$\text{Unintegrable}\left(\frac{\operatorname{csch}(a+bx)}{(c+dx)^2}, x\right)$$

[Out] Unintegrable[Csch[a + b\*x]/(c + d\*x)^2, x]

**Rubi [A]** time = 0.0220008, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[Csch[a + b\*x]/(c + d\*x)^2, x]

[Out] Defer[Int][Csch[a + b\*x]/(c + d\*x)^2, x]

Rubi steps

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} dx = \int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} dx$$

**Mathematica [A]** time = 11.1254, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Csch[a + b\*x]/(c + d\*x)^2, x]

[Out] Integrate[Csch[a + b\*x]/(c + d\*x)^2, x]

---

**Maple [A]** time = 0.034, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(b\*x+a)/(d\*x+c)^2,x)

[Out] int(csch(b\*x+a)/(d\*x+c)^2,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(b\*x+a)/(d\*x+c)^2,x, algorithm="maxima")

[Out] integrate(csch(b\*x + a)/(d\*x + c)^2, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx + a)}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(b\*x+a)/(d\*x+c)^2,x, algorithm="fricas")

[Out] integral(csch(b\*x + a)/(d^2\*x^2 + 2\*c\*d\*x + c^2), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(csch(a + b*x)/(c + d*x)**2, x)
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx + a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)/(d*x + c)^2, x)
```

### 3.28 $\int (c + dx)^3 \operatorname{csch}^2(a + bx) dx$

**Optimal.** Leaf size=103

$$\frac{3d^2(c + dx)\operatorname{PolyLog}(2, e^{2(a+bx)})}{b^3} - \frac{3d^3\operatorname{PolyLog}(3, e^{2(a+bx)})}{2b^4} + \frac{3d(c + dx)^2 \log(1 - e^{2(a+bx)})}{b^2} - \frac{(c + dx)^3 \operatorname{coth}(a + bx)}{b}$$

[Out]  $-\frac{(c + dx)^3}{b} - \frac{(c + dx)^3 \operatorname{Coth}[a + bx]}{b} + \frac{3d(c + dx)^2 \operatorname{Log}[1 - E^{(2(a + bx))}]}{b^2} + \frac{3d^2(c + dx) \operatorname{PolyLog}[2, E^{(2(a + bx))}]}{b^3} - \frac{3d^3 \operatorname{PolyLog}[3, E^{(2(a + bx))}]}{2b^4}$

**Rubi [A]** time = 0.227086, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4184, 3716, 2190, 2531, 2282, 6589}

$$\frac{3d^2(c + dx)\operatorname{PolyLog}(2, e^{2(a+bx)})}{b^3} - \frac{3d^3\operatorname{PolyLog}(3, e^{2(a+bx)})}{2b^4} + \frac{3d(c + dx)^2 \log(1 - e^{2(a+bx)})}{b^2} - \frac{(c + dx)^3 \operatorname{coth}(a + bx)}{b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + dx)^3 \operatorname{Csch}[a + bx]^2, x]$

[Out]  $-\frac{(c + dx)^3}{b} - \frac{(c + dx)^3 \operatorname{Coth}[a + bx]}{b} + \frac{3d(c + dx)^2 \operatorname{Log}[1 - E^{(2(a + bx))}]}{b^2} + \frac{3d^2(c + dx) \operatorname{PolyLog}[2, E^{(2(a + bx))}]}{b^3} - \frac{3d^3 \operatorname{PolyLog}[3, E^{(2(a + bx))}]}{2b^4}$

#### Rule 4184

$\operatorname{Int}[\operatorname{csc}[(e_{.}) + (f_{.})(x_{.})]^{2*((c_{.}) + (d_{.})(x_{.}))^{(m_{.})}}, x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[\frac{(c + dx)^m \operatorname{Cot}[e + fx]}{f}, x] + \operatorname{Dist}[\frac{d^m}{f}, \operatorname{Int}[(c + dx)^{m-1} \operatorname{Cot}[e + fx], x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x] \ \&\& \ \operatorname{GtQ}[m, 0]$

#### Rule 3716

$\operatorname{Int}[\frac{(c_{.}) + (d_{.})(x_{.})^{(m_{.})} \tan[(e_{.}) + \operatorname{Pi}(k_{.}) + (\operatorname{Complex}[0, fz_{.}])*(f_{.})(x_{.})]}{x_{\text{Symbol}}}, x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[\frac{I(c + dx)^{m+1}}{d(m+1)}, x] + \operatorname{Dist}[2 * I, \operatorname{Int}[\frac{(c + dx)^m E^{(2*(-Ie) + f*fz*x))}}{(E^{(2*I*k*Pi)}*(1 + E^{(2*(-Ie) + f*fz*x))})/E^{(2*I*k*Pi)})}, x], x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x] \ \&\& \ \operatorname{IntegerQ}[4*k] \ \&\& \ \operatorname{IGtQ}[m, 0]$

#### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int (c+dx)^3 \operatorname{csch}^2(a+bx) dx &= -\frac{(c+dx)^3 \operatorname{coth}(a+bx)}{b} + \frac{(3d) \int (c+dx)^2 \operatorname{coth}(a+bx) dx}{b} \\
&= -\frac{(c+dx)^3}{b} - \frac{(c+dx)^3 \operatorname{coth}(a+bx)}{b} - \frac{(6d) \int \frac{e^{2(a+bx)}(c+dx)^2}{1-e^{2(a+bx)}} dx}{b} \\
&= -\frac{(c+dx)^3}{b} - \frac{(c+dx)^3 \operatorname{coth}(a+bx)}{b} + \frac{3d(c+dx)^2 \log(1-e^{2(a+bx)})}{b^2} - \frac{(6d^2) \int (c+dx) dx}{b^2} \\
&= -\frac{(c+dx)^3}{b} - \frac{(c+dx)^3 \operatorname{coth}(a+bx)}{b} + \frac{3d(c+dx)^2 \log(1-e^{2(a+bx)})}{b^2} + \frac{3d^2(c+dx) \operatorname{Li}_2(e^{-2(a+bx)})}{b^3} \\
&= -\frac{(c+dx)^3}{b} - \frac{(c+dx)^3 \operatorname{coth}(a+bx)}{b} + \frac{3d(c+dx)^2 \log(1-e^{2(a+bx)})}{b^2} + \frac{3d^2(c+dx) \operatorname{Li}_2(e^{-2(a+bx)})}{b^3} \\
&= -\frac{(c+dx)^3}{b} - \frac{(c+dx)^3 \operatorname{coth}(a+bx)}{b} + \frac{3d(c+dx)^2 \log(1-e^{2(a+bx)})}{b^2} + \frac{3d^2(c+dx) \operatorname{Li}_2(e^{-2(a+bx)})}{b^3}
\end{aligned}$$

**Mathematica [A]** time = 2.29771, size = 185, normalized size = 1.8

---


$$-6d^2 (b(c+dx) \operatorname{PolyLog}(2, -e^{-a-bx}) + d \operatorname{PolyLog}(3, -e^{-a-bx})) - 6d^2 (b(c+dx) \operatorname{PolyLog}(2, e^{-a-bx}) + d \operatorname{PolyLog}(3, e^{-a-bx}))$$


---

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3\*Csch[a + b\*x]^2,x]

[Out]  $((-2*b^3*(c+d*x)^3)/(-1+E^{2*a})) + 3*b^2*d*(c+d*x)^2*\log[1-E^{-(a-b*x)}] + 3*b^2*d*(c+d*x)^2*\log[1+E^{-(a-b*x)}] - 6*d^2*(b*(c+d*x)*\operatorname{PolyLog}[2,-E^{-(a-b*x)}] + d*\operatorname{PolyLog}[3,-E^{-(a-b*x)}]) - 6*d^2*(b*(c+d*x)*\operatorname{PolyLog}[2,E^{-(a-b*x)}] + d*\operatorname{PolyLog}[3,E^{-(a-b*x)}]) + b^3*(c+d*x)^3*\operatorname{Csch}[a+b*x]*\operatorname{Sinh}[b*x])/b^4$

**Maple [B]** time = 0.059, size = 473, normalized size = 4.6

$$12 \frac{cd^2 a \ln(e^{bx+a})}{b^3} + 6 \frac{cd^2 \ln(1-e^{bx+a}) x}{b^2} + 6 \frac{cd^2 \ln(1-e^{bx+a}) a}{b^3} + 6 \frac{cd^2 \ln(1+e^{bx+a}) x}{b^2} - 6 \frac{cd^2 a \ln(e^{bx+a}-1)}{b^3} - 12 \frac{cd^2 a \ln(e^{bx+a})}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3\*csch(b\*x+a)^2,x)

```
[Out] 12*d^2/b^3*c*a*ln(exp(b*x+a))+6*d^2/b^2*c*ln(1-exp(b*x+a))*x+6*d^2/b^3*c*ln
(1-exp(b*x+a))*a+6*d^2/b^2*c*ln(1+exp(b*x+a))*x-6*d^2/b^3*c*a*ln(exp(b*x+a)
-1)-12*d^2/b^2*c*a*x+6*d^3/b^3*a^2*x-6*d^2/b*c*x^2-6*d^2/b^3*c*a^2-6*d/b^2*
c^2*ln(exp(b*x+a))+3*d/b^2*c^2*ln(1+exp(b*x+a))+3*d/b^2*c^2*ln(exp(b*x+a)-1
)+3*d^3/b^2*ln(1-exp(b*x+a))*x^2+6*d^3/b^3*polylog(2,exp(b*x+a))*x+3*d^3/b^
2*ln(1+exp(b*x+a))*x^2+6*d^3/b^3*polylog(2,-exp(b*x+a))*x-3*d^3/b^4*a^2*ln(
1-exp(b*x+a))-6*d^3/b^4*a^2*ln(exp(b*x+a))+3*d^3/b^4*a^2*ln(exp(b*x+a)-1)+6
*d^2/b^3*c*polylog(2,-exp(b*x+a))+6*d^2/b^3*c*polylog(2,exp(b*x+a))+4*d^3/b
^4*a^3-2*d^3/b*x^3-6*d^3/b^4*polylog(3,-exp(b*x+a))-6*d^3/b^4*polylog(3,exp
(b*x+a))-2/b*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/(exp(2*b*x+2*a)-1)
```

**Maxima [B]** time = 1.99967, size = 432, normalized size = 4.19

$$-3c^2d \left( \frac{2xe^{2bx+2a}}{be^{2bx+2a}-b} - \frac{\log\left(\left(e^{(bx+a)}+1\right)e^{-a}\right)}{b^2} - \frac{\log\left(\left(e^{(bx+a)}-1\right)e^{-a}\right)}{b^2} \right) + \frac{6\left(bx \log\left(e^{(bx+a)}+1\right) + \text{Li}_2\left(-e^{(bx+a)}\right)\right)cd^2}{b^3} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*c*sch(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -3*c^2*d*(2*x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) - b) - log((e^(b*x + a) +
1)*e^(-a))/b^2 - log((e^(b*x + a) - 1)*e^(-a))/b^2) + 6*(b*x*log(e^(b*x + a)
+ 1) + dilog(-e^(b*x + a)))*c*d^2/b^3 + 6*(b*x*log(-e^(b*x + a) + 1) + di
log(e^(b*x + a)))*c*d^2/b^3 + 2*c^3/(b*(e^(-2*b*x - 2*a) - 1)) - 2*(d^3*x^3
+ 3*c*d^2*x^2)/(b*e^(2*b*x + 2*a) - b) + 3*(b^2*x^2*log(e^(b*x + a) + 1) +
2*b*x*dilog(-e^(b*x + a)) - 2*polylog(3, -e^(b*x + a)))*d^3/b^4 + 3*(b^2*x
^2*log(-e^(b*x + a) + 1) + 2*b*x*dilog(e^(b*x + a)) - 2*polylog(3, e^(b*x +
a)))*d^3/b^4 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2)/b^4
```

**Fricas [C]** time = 2.85253, size = 2689, normalized size = 26.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*c*sch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 2*a^3*d^3 + 2*(b^3*d^3*x^3 +
3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*
```



$$\begin{aligned} & \cosh(b*x + a)^2 + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*\cosh(b*x + a)*\sinh(b*x + a) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*\sinh(b*x + a)^2 + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*\cosh(b*x + a))^2 - 2*(b*d^3*x + b*c*d^2)*\cosh(b*x + a)*\sinh(b*x + a) - (b*d^3*x + b*c*d^2)*\sinh(b*x + a)^2*\operatorname{dilog}(\cosh(b*x + a) + \sinh(b*x + a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*\cosh(b*x + a))^2 - 2*(b*d^3*x + b*c*d^2)*\cosh(b*x + a)*\sinh(b*x + a) - (b*d^3*x + b*c*d^2)*\sinh(b*x + a)^2*\operatorname{dilog}(-\cosh(b*x + a) - \sinh(b*x + a)) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*\cosh(b*x + a))^2 - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*\cosh(b*x + a)*\sinh(b*x + a) - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*\sinh(b*x + a)^2*\log(\cosh(b*x + a) + \sinh(b*x + a) + 1) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3 - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*\cosh(b*x + a))^2 - 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*\cosh(b*x + a)*\sinh(b*x + a) - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*\sinh(b*x + a)^2*\log(\cosh(b*x + a) + \sinh(b*x + a) - 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3 - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*\cosh(b*x + a))^2 - 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*\cosh(b*x + a)*\sinh(b*x + a) - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*\sinh(b*x + a)^2*\log(-\cosh(b*x + a) - \sinh(b*x + a) + 1) + 6*(d^3*\cosh(b*x + a))^2 + 2*d^3*\cosh(b*x + a)*\sinh(b*x + a) + d^3*\sinh(b*x + a)^2 - d^3*\operatorname{polylog}(3, \cosh(b*x + a) + \sinh(b*x + a)) + 6*(d^3*\cosh(b*x + a))^2 + 2*d^3*\cosh(b*x + a)*\sinh(b*x + a) + d^3*\sinh(b*x + a)^2 - d^3*\operatorname{polylog}(3, -\cosh(b*x + a) - \sinh(b*x + a)))/(b^4*\cosh(b*x + a)^2 + 2*b^4*\cosh(b*x + a)*\sinh(b*x + a) + b^4*\sinh(b*x + a)^2 - b^4) \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \operatorname{csch}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*3\*csch(b\*x+a)\*\*2,x)

[Out] Integral((c + d\*x)\*\*3\*csch(a + b\*x)\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \operatorname{csch}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csh(b*x + a)^2, x)
```

### 3.29 $\int (c + dx)^2 \operatorname{csch}^2(a + bx) dx$

**Optimal.** Leaf size=74

$$\frac{d^2 \operatorname{PolyLog}(2, e^{2(a+bx)})}{b^3} + \frac{2d(c+dx) \log(1 - e^{2(a+bx)})}{b^2} - \frac{(c+dx)^2 \operatorname{coth}(a+bx)}{b} - \frac{(c+dx)^2}{b}$$

[Out]  $-\left(\frac{(c+dx)^2}{b}\right) - \left(\frac{(c+dx)^2 \operatorname{Coth}[a+bx]}{b}\right) + \frac{2d(c+dx) \operatorname{Log}[1 - E^{2(a+bx)}]}{b^2} + \frac{d^2 \operatorname{PolyLog}[2, E^{2(a+bx)}]}{b^3}$

**Rubi [A]** time = 0.147543, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {4184, 3716, 2190, 2279, 2391}

$$\frac{d^2 \operatorname{PolyLog}(2, e^{2(a+bx)})}{b^3} + \frac{2d(c+dx) \log(1 - e^{2(a+bx)})}{b^2} - \frac{(c+dx)^2 \operatorname{coth}(a+bx)}{b} - \frac{(c+dx)^2}{b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c+dx)^2 \operatorname{Csch}[a+bx]^2, x]$

[Out]  $-\left(\frac{(c+dx)^2}{b}\right) - \left(\frac{(c+dx)^2 \operatorname{Coth}[a+bx]}{b}\right) + \frac{2d(c+dx) \operatorname{Log}[1 - E^{2(a+bx)}]}{b^2} + \frac{d^2 \operatorname{PolyLog}[2, E^{2(a+bx)}]}{b^3}$

#### Rule 4184

$\operatorname{Int}[\operatorname{csc}[(e_.) + (f_.)(x_.)]^2 \left( (c_.) + (d_.)(x_.)^{m_.} \right), x\_Symbol] \rightarrow -\operatorname{Simp}\left[\frac{(c+dx)^m \operatorname{Cot}[e+fx]}{f}, x\right] + \operatorname{Dist}\left[\frac{d^m}{f}, \operatorname{Int}[(c+dx)^{m-1} \operatorname{Cot}[e+fx], x], x\right] /; \operatorname{FreeQ}\{c, d, e, f\}, x \ \&\& \operatorname{GtQ}[m, 0]$

#### Rule 3716

$\operatorname{Int}\left[\left( (c_.) + (d_.)(x_.)^{m_.} \right) \tan\left[ (e_.) + \operatorname{Pi}(k_.) + \operatorname{Complex}[0, fz\_](f_.)(x_.) \right], x\_Symbol\right] \rightarrow -\operatorname{Simp}\left[\frac{(c+dx)^{m+1}}{d(m+1)}, x\right] + \operatorname{Dist}\left[2 * I, \operatorname{Int}\left[\frac{(c+dx)^m E^{2(-Ie + f * fz * x)}}{E^{2Ik * Pi} (1 + E^{2(-Ie + f * fz * x)}) / E^{2Ik * Pi}}\right], x\right] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x \ \&\& \operatorname{IntegerQ}[4 * k] \ \&\& \operatorname{IGtQ}[m, 0]$

#### Rule 2190

$\operatorname{Int}\left[\frac{\left( (F_.)^{\left( (g_.) \left( (e_.) + (f_.)(x_.) \right) \right)} \right)^{n_.} \left( (c_.) + (d_.)(x_.)^{m_.} \right)}{\left( (a_.) + (b_.) \left( (F_.)^{\left( (g_.) \left( (e_.) + (f_.)(x_.) \right) \right)} \right)^{n_.} \right)}, x\_Symbol] \rightarrow \operatorname{Simp}$

```

[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

### Rule 2279

```

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

```

### Rule 2391

```

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

```

### Rubi steps

$$\begin{aligned}
\int (c + dx)^2 \operatorname{csch}^2(a + bx) dx &= -\frac{(c + dx)^2 \operatorname{coth}(a + bx)}{b} + \frac{(2d) \int (c + dx) \operatorname{coth}(a + bx) dx}{b} \\
&= -\frac{(c + dx)^2}{b} - \frac{(c + dx)^2 \operatorname{coth}(a + bx)}{b} - \frac{(4d) \int \frac{e^{2(a+bx)}(c+dx)}{1-e^{2(a+bx)}} dx}{b} \\
&= -\frac{(c + dx)^2}{b} - \frac{(c + dx)^2 \operatorname{coth}(a + bx)}{b} + \frac{2d(c + dx) \log(1 - e^{2(a+bx)})}{b^2} - \frac{(2d^2) \int \log(1 - e^{2(a+bx)})}{b^2} \\
&= -\frac{(c + dx)^2}{b} - \frac{(c + dx)^2 \operatorname{coth}(a + bx)}{b} + \frac{2d(c + dx) \log(1 - e^{2(a+bx)})}{b^2} - \frac{d^2 \operatorname{Subst}\left(\int \frac{\log(1-x)}{x}\right)}{b^3} \\
&= -\frac{(c + dx)^2}{b} - \frac{(c + dx)^2 \operatorname{coth}(a + bx)}{b} + \frac{2d(c + dx) \log(1 - e^{2(a+bx)})}{b^2} + \frac{d^2 \operatorname{Li}_2(e^{2(a+bx)})}{b^3}
\end{aligned}$$

**Mathematica [C]** time = 5.39859, size = 198, normalized size = 2.68

$$\operatorname{csch}(a) \left( d^2 \left( -\sinh(a) \operatorname{PolyLog}\left(2, e^{-2(\tanh^{-1}(\tanh(a))+bx)}\right) - b^2 x^2 \cosh(a) e^{-\tanh^{-1}(\tanh(a))} \sqrt{\operatorname{sech}^2(a)} + i\pi b x \sinh(a) - i\pi \right) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csch[a + b*x]^2,x]
```

```
[Out] (Csch[a]*(-2*b*c*d*(b*x*Cosh[a] - Log[Sinh[a + b*x]]*Sinh[a]) + d^2*(-((b^2
*x^2*Cosh[a]*Sqrt[Sech[a]^2])/E^ArcTanh[Tanh[a]])) + I*b*Pi*x*Sinh[a] - I*Pi
```

\*Log[1 + E^(2\*b\*x)]\*Sinh[a] + 2\*b\*x\*Log[1 - E^(-2\*(b\*x + ArcTanh[Tanh[a]]))]  
 ]\*Sinh[a] + I\*Pi\*Log[Cosh[b\*x]]\*Sinh[a] + 2\*ArcTanh[Tanh[a]]\*(b\*x + Log[1 -  
 E^(-2\*(b\*x + ArcTanh[Tanh[a]])]) - Log[I\*Sinh[b\*x + ArcTanh[Tanh[a]]]])\*Si  
 nh[a] - PolyLog[2, E^(-2\*(b\*x + ArcTanh[Tanh[a]]))]\*Sinh[a] + b^2\*(c + d\*x  
 )^2\*Csch[a + b\*x]\*Sinh[b\*x])/b^3

**Maple [B]** time = 0.03, size = 240, normalized size = 3.2

$$-2 \frac{d^2 x^2 + 2 c d x + c^2}{b (e^{2 b x + 2 a} - 1)} - 4 \frac{c d \ln (e^{b x + a})}{b^2} + 2 \frac{c d \ln (1 + e^{b x + a})}{b^2} + 2 \frac{c d \ln (e^{b x + a} - 1)}{b^2} - 2 \frac{d^2 x^2}{b} - 4 \frac{a d^2 x}{b^2} - 2 \frac{a^2 d^2}{b^3} + 2 \frac{d^2 \ln ($$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2\*cscsch(b\*x+a)^2,x)

[Out] -2/b\*(d^2\*x^2+2\*c\*d\*x+c^2)/(exp(2\*b\*x+2\*a)-1)-4\*d/b^2\*c\*ln(exp(b\*x+a))+2\*d/  
 b^2\*c\*ln(1+exp(b\*x+a))+2\*d/b^2\*c\*ln(exp(b\*x+a)-1)-2\*d^2/b\*x^2-4\*d^2/b^2\*a\*x  
 -2\*d^2/b^3\*a^2+2\*d^2/b^2\*ln(1+exp(b\*x+a))\*x+2\*d^2/b^3\*polylog(2,-exp(b\*x+a)  
 )+2\*d^2/b^2\*ln(1-exp(b\*x+a))\*x+2\*d^2/b^3\*ln(1-exp(b\*x+a))\*a+2\*d^2/b^3\*polyl  
 og(2,exp(b\*x+a))+4\*d^2/b^3\*a\*ln(exp(b\*x+a))-2\*d^2/b^3\*a\*ln(exp(b\*x+a)-1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-2 d^2 \left( \frac{x^2}{b e^{(2 b x + 2 a)} - b} + 2 \int \frac{x}{2 (b e^{(b x + a)} + b)} dx - 2 \int \frac{x}{2 (b e^{(b x + a)} - b)} dx \right) - 2 c d \left( \frac{2 x e^{(2 b x + 2 a)}}{b e^{(2 b x + 2 a)} - b} - \frac{\log \left( (e^{(b x + a)} + 1) e^{(-a)} \right)}{b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*cscsch(b\*x+a)^2,x, algorithm="maxima")

[Out] -2\*d^2\*(x^2/(b\*e^(2\*b\*x + 2\*a) - b) + 2\*integrate(1/2\*x/(b\*e^(b\*x + a) + b)  
 , x) - 2\*integrate(1/2\*x/(b\*e^(b\*x + a) - b), x)) - 2\*c\*d\*(2\*x\*e^(2\*b\*x + 2  
 \*a)/(b\*e^(2\*b\*x + 2\*a) - b) - log((e^(b\*x + a) + 1)\*e^(-a))/b^2 - log((e^(b  
 \*x + a) - 1)\*e^(-a))/b^2) + 2\*c^2/(b\*(e^(-2\*b\*x - 2\*a) - 1))

**Fricas [B]** time = 2.78976, size = 1521, normalized size = 20.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cosh(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cosh(b*x + a)*sinh(b*x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*sinh(b*x + a)^2 - (d^2*cosh(b*x + a)^2 + 2*d^2*cosh(b*x + a)*sinh(b*x + a) + d^2*sinh(b*x + a)^2 - d^2)*dilog(cosh(b*x + a) + sinh(b*x + a)) - (d^2*cosh(b*x + a)^2 + 2*d^2*cosh(b*x + a)*sinh(b*x + a) + d^2*sinh(b*x + a)^2 - d^2)*dilog(-cosh(b*x + a) - sinh(b*x + a)) + (b*d^2*x + b*c*d - (b*d^2*x + b*c*d)*cosh(b*x + a)^2 - 2*(b*d^2*x + b*c*d)*cosh(b*x + a)*sinh(b*x + a) - (b*d^2*x + b*c*d)*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + 1) + (b*c*d - a*d^2 - (b*c*d - a*d^2)*cosh(b*x + a)^2 - 2*(b*c*d - a*d^2)*cosh(b*x + a)*sinh(b*x + a) - (b*c*d - a*d^2)*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) - 1) + (b*d^2*x + a*d^2 - (b*d^2*x + a*d^2)*cosh(b*x + a)^2 - 2*(b*d^2*x + a*d^2)*cosh(b*x + a)*sinh(b*x + a) - (b*d^2*x + a*d^2)*sinh(b*x + a)^2)*log(-cosh(b*x + a) - sinh(b*x + a) + 1))/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2 - b^3)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \operatorname{csch}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csch(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*csch(a + b*x)**2, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \operatorname{csch}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csch(b*x + a)^2, x)
```

### 3.30 $\int (c + dx) \operatorname{csch}^2(a + bx) dx$

**Optimal.** Leaf size=29

$$\frac{d \log(\sinh(a + bx))}{b^2} - \frac{(c + dx) \operatorname{coth}(a + bx)}{b}$$

[Out] -(((c + d\*x)\*Coth[a + b\*x])/b) + (d\*Log[Sinh[a + b\*x]])/b^2

**Rubi [A]** time = 0.0303038, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {4184, 3475}

$$\frac{d \log(\sinh(a + bx))}{b^2} - \frac{(c + dx) \operatorname{coth}(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)\*Csch[a + b\*x]^2,x]

[Out] -(((c + d\*x)\*Coth[a + b\*x])/b) + (d\*Log[Sinh[a + b\*x]])/b^2

#### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_.)]^2\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] := -Simp[p[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned} \int (c + dx) \operatorname{csch}^2(a + bx) dx &= -\frac{(c + dx) \operatorname{coth}(a + bx)}{b} + \frac{d \int \operatorname{coth}(a + bx) dx}{b} \\ &= -\frac{(c + dx) \operatorname{coth}(a + bx)}{b} + \frac{d \log(\sinh(a + bx))}{b^2} \end{aligned}$$



**Mathematica [A]** time = 0.099239, size = 52, normalized size = 1.79

$$\frac{d \log(\sinh(a + bx))}{b^2} - \frac{c \coth(a + bx)}{b} - \frac{dx \coth(a)}{b} + \frac{dx \operatorname{csch}(a) \sinh(bx) \operatorname{csch}(a + bx)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)\*Csch[a + b\*x]^2,x]

[Out] -((d\*x\*Coth[a])/b) - (c\*Coth[a + b\*x])/b + (d\*Log[Sinh[a + b\*x]])/b^2 + (d\*x\*Csch[a]\*Csch[a + b\*x]\*Sinh[b\*x])/b

**Maple [A]** time = 0.02, size = 56, normalized size = 1.9

$$-2 \frac{dx}{b} - 2 \frac{da}{b^2} - 2 \frac{dx + c}{b(e^{2bx+2a} - 1)} + \frac{d \ln(e^{2bx+2a} - 1)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)\*csch(b\*x+a)^2,x)

[Out] -2\*d/b\*x-2\*d/b^2\*a-2/b\*(d\*x+c)/(exp(2\*b\*x+2\*a)-1)+d/b^2\*ln(exp(2\*b\*x+2\*a)-1)

**Maxima [B]** time = 1.14333, size = 123, normalized size = 4.24

$$-d \left( \frac{2xe^{2bx+2a}}{be^{2bx+2a}-b} - \frac{\log((e^{bx+a}+1)e^{-a})}{b^2} - \frac{\log((e^{bx+a}-1)e^{-a})}{b^2} \right) + \frac{2c}{b(e^{-2bx-2a}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*csch(b\*x+a)^2,x, algorithm="maxima")

[Out] -d\*(2\*x\*e^(2\*b\*x + 2\*a)/(b\*e^(2\*b\*x + 2\*a) - b) - log((e^(b\*x + a) + 1)\*e^(-a)))/b^2 - log((e^(b\*x + a) - 1)\*e^(-a))/b^2 + 2\*c/(b\*(e^(-2\*b\*x - 2\*a) - 1))

**Fricas [B]** time = 2.57725, size = 431, normalized size = 14.86

$$\frac{2 b d x \cosh (b x+a)^2+4 b d x \cosh (b x+a) \sinh (b x+a)+2 b d x \sinh (b x+a)^2+2 b c-\left(d \cosh (b x+a)^2+2 d \cosh (b x+a)\right)}{b^2 \cosh (b x+a)^2+2 b^2 \cosh (b x+a) \sinh (b x+a)+b^2 \sinh (b x+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*csch(b\*x+a)^2,x, algorithm="fricas")

[Out]  $-(2*b*d*x*cosh(b*x + a)^2 + 4*b*d*x*cosh(b*x + a)*sinh(b*x + a) + 2*b*d*x*sinh(b*x + a)^2 + 2*b*c - (d*cosh(b*x + a)^2 + 2*d*cosh(b*x + a)*sinh(b*x + a) + d*sinh(b*x + a)^2 - d*log(2*sinh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a))))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*x + a) + b^2*sinh(b*x + a)^2 - b^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \operatorname{csch}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*csch(b\*x+a)\*\*2,x)

[Out] Integral((c + d\*x)\*csch(a + b\*x)\*\*2, x)

**Giac [B]** time = 1.31742, size = 108, normalized size = 3.72

$$\frac{2 b d x e^{(2 b x+2 a)}-d e^{(2 b x+2 a)} \log \left(e^{(2 b x+2 a)}-1\right)+2 b c+d \log \left(e^{(2 b x+2 a)}-1\right)}{b^2 e^{(2 b x+2 a)}-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*csch(b\*x+a)^2,x, algorithm="giac")

[Out]  $-(2*b*d*x*e^{(2*b*x + 2*a)} - d*e^{(2*b*x + 2*a)}*log(e^{(2*b*x + 2*a)} - 1) + 2*b*c + d*log(e^{(2*b*x + 2*a)} - 1))/(b^2*e^{(2*b*x + 2*a)} - b^2)$

$$3.31 \quad \int \frac{\operatorname{csch}^2(a+bx)}{c+dx} dx$$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^2(a+bx)}{c+dx}, x\right)$$

[Out] Unintegrable[Csch[a + b\*x]^2/(c + d\*x), x]

**Rubi [A]** time = 0.0403233, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[Csch[a + b\*x]^2/(c + d\*x), x]

[Out] Defer[Int][Csch[a + b\*x]^2/(c + d\*x), x]

Rubi steps

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx} dx = \int \frac{\operatorname{csch}^2(a+bx)}{c+dx} dx$$

**Mathematica [A]** time = 17.713, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[Csch[a + b\*x]^2/(c + d\*x), x]

[Out] Integrate[Csch[a + b\*x]^2/(c + d\*x), x]

---

**Maple [A]** time = 0.061, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(bx + a))^2}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(b*x+a)^2/(d*x+c),x)`

[Out] `int(csch(b*x+a)^2/(d*x+c),x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$4d \int \frac{1}{4(bd^2x^2 + 2bcdx + bc^2 + (bd^2x^2e^a + 2bcdxe^a + bc^2e^a)e^{(bx)})} dx - 4d \int -\frac{1}{4(bd^2x^2 + 2bcdx + bc^2 - (bd^2x^2e^a + 2bcdxe^a + bc^2e^a)e^{(bx)})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(b*x+a)^2/(d*x+c),x, algorithm="maxima")`

[Out] `4*d*integrate(1/4/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2*e^a + 2*b*c*d*x*e^a + b*c^2*e^a)*e^(b*x)), x) - 4*d*integrate(-1/4/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 - (b*d^2*x^2*e^a + 2*b*c*d*x*e^a + b*c^2*e^a)*e^(b*x)), x) + 2/(b*d*x + b*c - (b*d*x*e^(2*a) + b*c*e^(2*a))*e^(2*b*x))`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx + a)^2}{dx + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(b*x+a)^2/(d*x+c),x, algorithm="fricas")`

[Out] `integral(csch(b*x + a)^2/(d*x + c), x)`

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(b\*x+a)\*\*2/(d\*x+c), x)

[Out] Integral(csch(a + b\*x)\*\*2/(c + d\*x), x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx + a)^2}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(b\*x+a)^2/(d\*x+c), x, algorithm="giac")

[Out] integrate(csch(b\*x + a)^2/(d\*x + c), x)

$$3.32 \quad \int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2} dx$$

**Optimal.** Leaf size=18

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2}, x\right)$$

[Out] Unintegrable[Csch[a + b\*x]^2/(c + d\*x)^2, x]

**Rubi [A]** time = 0.0378597, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[Csch[a + b\*x]^2/(c + d\*x)^2, x]

[Out] Defer[Int][Csch[a + b\*x]^2/(c + d\*x)^2, x]

Rubi steps

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2} dx = \int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2} dx$$

**Mathematica [A]** time = 18.6732, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Csch[a + b\*x]^2/(c + d\*x)^2, x]

[Out] Integrate[Csch[a + b\*x]^2/(c + d\*x)^2, x]

---

**Maple [A]** time = 0.072, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(bx+a))^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(b\*x+a)^2/(d\*x+c)^2,x)

[Out] int(csch(b\*x+a)^2/(d\*x+c)^2,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$4d \int \frac{1}{2(bd^3x^3 + 3bcd^2x^2 + 3bc^2dx + bc^3 + (bd^3x^3e^a + 3bcd^2x^2e^a + 3bc^2dxe^a + bc^3e^a)e^{bx})} dx - 4d \int -\frac{1}{2(bd^3x^3 + 3bcd^2x^2 + 3bc^2dx + bc^3 + (bd^3x^3e^a + 3bcd^2x^2e^a + 3bc^2dxe^a + bc^3e^a)e^{bx})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(b\*x+a)^2/(d\*x+c)^2,x, algorithm="maxima")

[Out] 4\*d\*integrate(1/2/(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3 + (b\*d^3\*x^3\*e^a + 3\*b\*c\*d^2\*x^2\*e^a + 3\*b\*c^2\*d\*x\*e^a + b\*c^3\*e^a)\*e^(b\*x)), x) - 4\*d\*integrate(-1/2/(b\*d^3\*x^3 + 3\*b\*c\*d^2\*x^2 + 3\*b\*c^2\*d\*x + b\*c^3 - (b\*d^3\*x^3\*e^a + 3\*b\*c\*d^2\*x^2\*e^a + 3\*b\*c^2\*d\*x\*e^a + b\*c^3\*e^a)\*e^(b\*x)), x) + 2/(b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2 - (b\*d^2\*x^2\*e^(2\*a) + 2\*b\*c\*d\*x\*e^(2\*a) + b\*c^2\*e^(2\*a))\*e^(2\*b\*x))

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)^2}{d^2x^2 + 2cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(b\*x+a)^2/(d\*x+c)^2,x, algorithm="fricas")

[Out] integral(csch(b\*x + a)^2/(d^2\*x^2 + 2\*c\*d\*x + c^2), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(b\*x+a)\*\*2/(d\*x+c)\*\*2,x)

[Out] Integral(csch(a + b\*x)\*\*2/(c + d\*x)\*\*2, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx + a)^2}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(b\*x+a)^2/(d\*x+c)^2,x, algorithm="giac")

[Out] integrate(csch(b\*x + a)^2/(d\*x + c)^2, x)



### 3.33 $\int (c + dx)^3 \operatorname{csch}^3(a + bx) dx$

**Optimal.** Leaf size=256

$$-\frac{3d^2(c + dx)\operatorname{PolyLog}(3, -e^{a+bx})}{b^3} + \frac{3d^2(c + dx)\operatorname{PolyLog}(3, e^{a+bx})}{b^3} + \frac{3d(c + dx)^2\operatorname{PolyLog}(2, -e^{a+bx})}{2b^2} - \frac{3d(c + dx)^2\operatorname{PolyLog}(2, e^{a+bx})}{2b^2}$$

[Out]  $(-6*d^2*(c + d*x)*\operatorname{ArcTanh}[E^{(a + b*x)}])/b^3 + ((c + d*x)^3*\operatorname{ArcTanh}[E^{(a + b*x)}])/b - (3*d*(c + d*x)^2*\operatorname{Csch}[a + b*x])/(2*b^2) - ((c + d*x)^3*\operatorname{Coth}[a + b*x]*\operatorname{Csch}[a + b*x])/(2*b) - (3*d^3*\operatorname{PolyLog}[2, -E^{(a + b*x)}])/b^4 + (3*d*(c + d*x)^2*\operatorname{PolyLog}[2, -E^{(a + b*x)}])/(2*b^2) + (3*d^3*\operatorname{PolyLog}[2, E^{(a + b*x)}])/b^4 - (3*d*(c + d*x)^2*\operatorname{PolyLog}[2, E^{(a + b*x)}])/(2*b^2) - (3*d^2*(c + d*x)*\operatorname{PolyLog}[3, -E^{(a + b*x)}])/b^3 + (3*d^2*(c + d*x)*\operatorname{PolyLog}[3, E^{(a + b*x)}])/b^3 + (3*d^3*\operatorname{PolyLog}[4, -E^{(a + b*x)}])/b^4 - (3*d^3*\operatorname{PolyLog}[4, E^{(a + b*x)}])/b^4$

**Rubi [A]** time = 0.273616, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {4186, 4182, 2279, 2391, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c + dx)\operatorname{PolyLog}(3, -e^{a+bx})}{b^3} + \frac{3d^2(c + dx)\operatorname{PolyLog}(3, e^{a+bx})}{b^3} + \frac{3d(c + dx)^2\operatorname{PolyLog}(2, -e^{a+bx})}{2b^2} - \frac{3d(c + dx)^2\operatorname{PolyLog}(2, e^{a+bx})}{2b^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^3*\operatorname{Csch}[a + b*x]^3, x]$

[Out]  $(-6*d^2*(c + d*x)*\operatorname{ArcTanh}[E^{(a + b*x)}])/b^3 + ((c + d*x)^3*\operatorname{ArcTanh}[E^{(a + b*x)}])/b - (3*d*(c + d*x)^2*\operatorname{Csch}[a + b*x])/(2*b^2) - ((c + d*x)^3*\operatorname{Coth}[a + b*x]*\operatorname{Csch}[a + b*x])/(2*b) - (3*d^3*\operatorname{PolyLog}[2, -E^{(a + b*x)}])/b^4 + (3*d*(c + d*x)^2*\operatorname{PolyLog}[2, -E^{(a + b*x)}])/(2*b^2) + (3*d^3*\operatorname{PolyLog}[2, E^{(a + b*x)}])/b^4 - (3*d*(c + d*x)^2*\operatorname{PolyLog}[2, E^{(a + b*x)}])/(2*b^2) - (3*d^2*(c + d*x)*\operatorname{PolyLog}[3, -E^{(a + b*x)}])/b^3 + (3*d^2*(c + d*x)*\operatorname{PolyLog}[3, E^{(a + b*x)}])/b^3 + (3*d^3*\operatorname{PolyLog}[4, -E^{(a + b*x)}])/b^4 - (3*d^3*\operatorname{PolyLog}[4, E^{(a + b*x)}])/b^4$

#### Rule 4186

$\operatorname{Int}[(\operatorname{csc}[e_.] + (f_.)*(x_.))*(b_.)^{(n_.)*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] := -\operatorname{Simp}[(b^2*(c + d*x)^m*\operatorname{Cot}[e + f*x]*(b*\operatorname{Csc}[e + f*x])^{(n-2)})/(f*(n-1)), x] + (\operatorname{Dist}[(b^2*d^2*m*(m-1))/(f^2*(n-1)*(n-2)), \operatorname{Int}[(c + d*x)^{(m-2)}*(b*\operatorname{Csc}[e + f*x])^{(n-2)}, x], x] + \operatorname{Dist}[(b^2*(n-2))/(n-1), \operatorname{Int}[(c + d*x)^m*(b*\operatorname{Csc}[e + f*x])^{(n-2)}, x], x]$

```
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]] /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_.)*((a_.)*(v_)^(n_.))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned} \int (c + dx)^3 \operatorname{csch}^3(a + bx) dx &= -\frac{3d(c + dx)^2 \operatorname{csch}(a + bx)}{2b^2} - \frac{(c + dx)^3 \operatorname{coth}(a + bx) \operatorname{csch}(a + bx)}{2b} - \frac{1}{2} \int (c + dx)^3 \operatorname{csch}(a + bx) dx \\ &= -\frac{6d^2(c + dx) \tanh^{-1}(e^{a+bx})}{b^3} + \frac{(c + dx)^3 \tanh^{-1}(e^{a+bx})}{b} - \frac{3d(c + dx)^2 \operatorname{csch}(a + bx)}{2b^2} - \frac{(c + dx)^3 \operatorname{csch}(a + bx)}{2b} \\ &= -\frac{6d^2(c + dx) \tanh^{-1}(e^{a+bx})}{b^3} + \frac{(c + dx)^3 \tanh^{-1}(e^{a+bx})}{b} - \frac{3d(c + dx)^2 \operatorname{csch}(a + bx)}{2b^2} - \frac{(c + dx)^3 \operatorname{csch}(a + bx)}{2b} \\ &= -\frac{6d^2(c + dx) \tanh^{-1}(e^{a+bx})}{b^3} + \frac{(c + dx)^3 \tanh^{-1}(e^{a+bx})}{b} - \frac{3d(c + dx)^2 \operatorname{csch}(a + bx)}{2b^2} - \frac{(c + dx)^3 \operatorname{csch}(a + bx)}{2b} \\ &= -\frac{6d^2(c + dx) \tanh^{-1}(e^{a+bx})}{b^3} + \frac{(c + dx)^3 \tanh^{-1}(e^{a+bx})}{b} - \frac{3d(c + dx)^2 \operatorname{csch}(a + bx)}{2b^2} - \frac{(c + dx)^3 \operatorname{csch}(a + bx)}{2b} \\ &= -\frac{6d^2(c + dx) \tanh^{-1}(e^{a+bx})}{b^3} + \frac{(c + dx)^3 \tanh^{-1}(e^{a+bx})}{b} - \frac{3d(c + dx)^2 \operatorname{csch}(a + bx)}{2b^2} - \frac{(c + dx)^3 \operatorname{csch}(a + bx)}{2b} \end{aligned}$$

**Mathematica [A]** time = 10.0834, size = 440, normalized size = 1.72

$$\frac{-3d(b^2(c + dx)^2 - 2d^2) \operatorname{PolyLog}(2, -e^{a+bx}) + 3d(b^2(c + dx)^2 - 2d^2) \operatorname{PolyLog}(2, e^{a+bx}) + 6bcd^2 \operatorname{PolyLog}(3, -e^{a+bx})}{1}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3\*Csch[a + b\*x]^3,x]

[Out]  $-(b^2(c + dx)^2(3d + b(c + dx))\operatorname{Coth}[a + bx])\operatorname{Csch}[a + bx] + b^3c^3 \operatorname{Log}[1 - E^{(a + b*x)}] - 6*b*c*d^2*\operatorname{Log}[1 - E^{(a + b*x)}] + 3*b^3*c^2*d*x*\operatorname{Log}[1 - E^{(a + b*x)}] - 6*b*d^3*x*\operatorname{Log}[1 - E^{(a + b*x)}] + 3*b^3*c*d^2*x^2*\operatorname{Log}[1 - E^{(a + b*x)}] + b^3*d^3*x^3*\operatorname{Log}[1 - E^{(a + b*x)}] - b^3*c^3*\operatorname{Log}[1 + E^{(a + b*x)}] + 6*b*c*d^2*\operatorname{Log}[1 + E^{(a + b*x)}] - 3*b^3*c^2*d*x*\operatorname{Log}[1 + E^{(a + b*x)}] + 6*b*d^3*x*\operatorname{Log}[1 + E^{(a + b*x)}] - 3*b^3*c*d^2*x^2*\operatorname{Log}[1 + E^{(a + b*x)}] - b^3*d^3*x^3*\operatorname{Log}[1 + E^{(a + b*x)}] - 3*d*(-2*d^2 + b^2*(c + dx)^2)*\operatorname{PolyLog}[2,$

$$-E^{(a + b*x)} + 3*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, E^{(a + b*x)}] + 6*b*c*d^2*PolyLog[3, -E^{(a + b*x)}] + 6*b*d^3*x*PolyLog[3, -E^{(a + b*x)}] - 6*b*c*d^2*PolyLog[3, E^{(a + b*x)}] - 6*b*d^3*x*PolyLog[3, E^{(a + b*x)}] - 6*d^3*PolyLog[4, -E^{(a + b*x)}] + 6*d^3*PolyLog[4, E^{(a + b*x)}]/(2*b^4)$$

**Maple [B]** time = 0.06, size = 876, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3\*c\*sch(b\*x+a)^3,x)

[Out]  $3*d^3*\text{polylog}(4, -\exp(b*x+a))/b^4 - 3*d^3*\text{polylog}(4, \exp(b*x+a))/b^4 - 3*d^3*\text{polylog}(2, -\exp(b*x+a))/b^4 + 3*d^3*\text{polylog}(2, \exp(b*x+a))/b^4 - 3/b^2*c*d^2*\text{polylog}(2, \exp(b*x+a))*x + 3/2/b*c^2*d*\ln(1+\exp(b*x+a))*x + 3/2/b^2*c^2*d*\ln(1+\exp(b*x+a))*a - 3/2/b*c^2*d*\ln(1-\exp(b*x+a))*x - 3/2/b^2*c^2*d*\ln(1-\exp(b*x+a))*a - 3/b^2*c^2*d*a*\text{arctanh}(\exp(b*x+a)) + 3/b^3*c*d^2*a^2*\text{arctanh}(\exp(b*x+a)) + 3/2/b*c*d^2*\ln(1+\exp(b*x+a))*x^2 - 3/2/b^3*c*d^2*\ln(1+\exp(b*x+a))*a^2 + 3/b^2*c*d^2*\text{polylog}(2, -\exp(b*x+a))*x - 3/2/b*c*d^2*\ln(1-\exp(b*x+a))*x^2 + 3/2/b^3*c*d^2*\ln(1-\exp(b*x+a))*a^2 + 6/b^4*d^3*a*\text{arctanh}(\exp(b*x+a)) - 6/b^3*c*d^2*\text{arctanh}(\exp(b*x+a)) - 3/b^4*d^3*a*\ln(1+\exp(b*x+a)) + 3/b^4*d^3*a*\ln(1-\exp(b*x+a)) - 3/b^3*d^3*\ln(1+\exp(b*x+a))*x + 3/b^3*d^3*\ln(1-\exp(b*x+a))*x - \exp(b*x+a)*(b*d^3*x^3*\exp(2*b*x+2*a) + 3*b*c*d^2*x^2*\exp(2*b*x+2*a) + 3*b*c^2*d*x*\exp(2*b*x+2*a) + b*d^3*x^3 + 3*d^3*x^2*\exp(2*b*x+2*a) + b*c^3*\exp(2*b*x+2*a) + 3*b*c*d^2*x^2 + 6*c*d^2*x*\exp(2*b*x+2*a) + 3*b*c^2*d*x + 3*c^2*d*\exp(2*b*x+2*a) - 3*d^3*x^2 + b*c^3 - 6*c*d^2*x - 3*c^2*d) / b^2 / (\exp(2*b*x+2*a) - 1)^2 - 3/b^3*d^3*\text{polylog}(3, -\exp(b*x+a))*x - 1/b^4*d^3*a^3*a*\text{arctanh}(\exp(b*x+a)) + 3/2/b^2*c^2*d*\text{polylog}(2, -\exp(b*x+a)) - 3/2/b^2*c^2*d*\text{polylog}(2, \exp(b*x+a)) - 3/b^3*c*d^2*\text{polylog}(3, -\exp(b*x+a)) + 3/b^3*c*d^2*\text{polylog}(3, \exp(b*x+a)) + 1/2/b^4*d^3*a^3*\ln(1+\exp(b*x+a)) - 1/2/b^4*d^3*a^3*\ln(1-\exp(b*x+a)) - 1/2/b*d^3*\ln(1-\exp(b*x+a))*x^3 - 3/2/b^2*d^3*\text{polylog}(2, \exp(b*x+a))*x^2 + 3/b^3*d^3*\text{polylog}(3, \exp(b*x+a))*x + 1/2/b*d^3*\ln(1+\exp(b*x+a))*x^3 + 3/2/b^2*d^3*\text{polylog}(2, -\exp(b*x+a))*x^2 + 1/b*c^3*\text{arctanh}(\exp(b*x+a))$

**Maxima [B]** time = 1.97381, size = 817, normalized size = 3.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*csh(b\*x+a)^3,x, algorithm="maxima")

[Out]  $\frac{1}{2}c^3(\log(e^{-bx-a}) + 1)/b - \log(e^{-bx-a} - 1)/b + 2(e^{-bx-a} + e^{-3bx-3a})/(b(2e^{-2bx-2a} - e^{-4bx-4a} - 1))) + 3/2(b^2x^2\log(e^{bx+a}) + 1) + 2bx\text{dilog}(-e^{bx+a}) - 2\text{polylog}(3, -e^{bx+a}))cd^2/b^3 - 3/2(b^2x^2\log(-e^{bx+a}) + 1) + 2bx\text{dilog}(e^{bx+a}) - 2\text{polylog}(3, e^{bx+a}))cd^2/b^3 - 3cd^2\log(e^{bx+a} + 1)/b^3 + 3cd^2\log(e^{bx+a} - 1)/b^3 - ((bd^3x^3e^{3a} + 3c^2de^{3a} + 3(bcd^2 + d^3)x^2e^{3a} + 3(bc^2d + 2cd^2)xe^{3a}))e^{3bx} + (bd^3xe^a - 3c^2de^a + 3(bcd^2 - d^3)x^2e^a + 3(bc^2d - 2cd^2)xe^a)e^{bx})/(b^2e^{(4bx+4a)} - 2b^2e^{(2bx+2a)} + b^2) + 1/2(b^3x^3\log(e^{bx+a}) + 1) + 3b^2x^2\text{dilog}(-e^{bx+a}) - 6bx\text{polylog}(3, -e^{bx+a}) + 6\text{polylog}(4, -e^{bx+a}))d^3/b^4 - 1/2(b^3x^3\log(-e^{bx+a}) + 1) + 3b^2x^2\text{dilog}(e^{bx+a}) - 6bx\text{polylog}(3, e^{bx+a}) + 6\text{polylog}(4, e^{bx+a}))d^3/b^4 + 3/2(b^2c^2d - 2d^3)(bx\log(e^{bx+a}) + 1) + \text{dilog}(-e^{bx+a}))/b^4 - 3/2(b^2c^2d - 2d^3)(bx\log(-e^{bx+a}) + 1) + \text{dilog}(e^{bx+a}))/b^4$

---

**Fricas [C]** time = 3.47511, size = 8979, normalized size = 35.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*csh(b\*x+a)^3,x, algorithm="fricas")

[Out]  $-1/2(2(b^3d^3x^3 + b^3c^3 + 3b^2c^2d + 3(b^3cd^2 + b^2d^3))x^2 + 3(b^3c^2d + 2b^2cd^2)x)\cosh(bx+a)^3 + 6(b^3d^3x^3 + b^3c^3 + 3b^2c^2d + 3(b^3cd^2 + b^2d^3))x^2 + 3(b^3c^2d + 2b^2cd^2)x)\cosh(bx+a)\sinh(bx+a)^2 + 2(b^3d^3x^3 + b^3c^3 + 3b^2c^2d + 3(b^3cd^2 + b^2d^3))x^2 + 3(b^3c^2d + 2b^2cd^2)x)\sinh(bx+a)^3 + 2(b^3d^3x^3 + b^3c^3 - 3b^2c^2d + 3(b^3cd^2 - b^2d^3))x^2 + 3(b^3c^2d - 2b^2cd^2)x)\cosh(bx+a) + 3(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3)\cosh(bx+a)^4 + 4(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3)\cosh(bx+a)\sinh(bx+a)^3 + (b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3)\sinh(bx+a)^4 - 2d^3 - 2(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3)\cosh(bx+a)^2 - 2(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3 - 3(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3))\cosh(bx+a)^2)\sinh(bx+a)^2 + 4((b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3)\cosh(bx+a)^3 - (b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3)\cosh(bx+a))\sinh(bx+a)\text{dilog}(\cosh(bx+a) + \sinh(bx+a)) - 3(b^2d^3x^2 + 2b^2cd^2x$

$$\begin{aligned}
& x + b^2c^2d + (b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3)\cosh(bx + a)^4 + 4(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3)\cosh(bx + a) \\
& \sinh(bx + a)^3 + (b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3)\sinh(bx + a)^4 - 2d^3 - 2(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3)\cosh \\
& (bx + a)^2 - 2(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3) - 3(b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3)\cosh(bx + a)^2*\sinh(bx + a)^ \\
& 2 + 4*((b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3)\cosh(bx + a)^3 - (b^2d^3x^2 + 2b^2cd^2x + b^2c^2d - 2d^3)\cosh(bx + a))*\sinh(bx + a) \\
& ))*\operatorname{dilog}(-\cosh(bx + a) - \sinh(bx + a)) - (b^3d^3x^3 + 3b^3cd^2x^2 + b^3c^3 + (b^3d^3x^3 + 3b^3cd^2x^2 + b^3c^3 - 6b^3cd^2 + 3(b^3c^2d - 2b^3d^3)x)*\cosh(bx + a)^4 + 4(b^3d^3x^3 + 3b^3cd^2x^2 + b^3c^3 - 6b^3cd^2 + 3(b^3c^2d - 2b^3d^3)x)*\cosh(bx + a)*\sinh(bx + a)^3 + (b^3d^3x^3 + 3b^3cd^2x^2 + b^3c^3 - 6b^3cd^2 + 3(b^3c^2d - 2b^3d^3)x)*\sinh(bx + a)^4 - 6b^3cd^2 - 2(b^3d^3x^3 + 3b^3cd^2x^2 + b^3c^3 - 6b^3cd^2 + 3(b^3c^2d - 2b^3d^3)x)*\cosh(bx + a)^2 - 2(b^3d^3x^3 + 3b^3cd^2x^2 + b^3c^3 - 6b^3cd^2 - 3(b^3d^3x^3 + 3b^3cd^2x^2 + b^3c^3 - 6b^3cd^2 + 3(b^3c^2d - 2b^3d^3)x)*\cosh(bx + a)^2 + 3(b^3c^2d - 2b^3d^3)x)*\sinh(bx + a)^2 + 3(b^3c^2d - 2b^3d^3)x + 4*((b^3d^3x^3 + 3b^3cd^2x^2 + b^3c^3 - 6b^3cd^2 + 3(b^3c^2d - 2b^3d^3)x)*\cosh(bx + a)^3 - (b^3d^3x^3 + 3b^3cd^2x^2 + b^3c^3 - 6b^3cd^2 + 3(b^3c^2d - 2b^3d^3)x)*\cosh(bx + a))*\sinh(bx + a))*\log(\cosh(bx + a) + \sinh(bx + a) + 1) + (b^3c^3 - 3ab^2c^2d + 3(a^2 - 2)b^3cd^2 + (b^3c^3 - 3ab^2c^2d + 3(a^2 - 2)b^3cd^2 - (a^3 - 6a)d^3)*\cosh(bx + a)^4 + 4(b^3c^3 - 3ab^2c^2d + 3(a^2 - 2)b^3cd^2 - (a^3 - 6a)d^3)*\cosh(bx + a)*\sinh(bx + a)^3 + (b^3c^3 - 3ab^2c^2d + 3(a^2 - 2)b^3cd^2 - (a^3 - 6a)d^3)*\sinh(bx + a)^4 - (a^3 - 6a)d^3 - 2(b^3c^3 - 3ab^2c^2d + 3(a^2 - 2)b^3cd^2 - (a^3 - 6a)d^3)*\cosh(bx + a)^2 - 2(b^3c^3 - 3ab^2c^2d + 3(a^2 - 2)b^3cd^2 - (a^3 - 6a)d^3 - 3(b^3c^3 - 3ab^2c^2d + 3(a^2 - 2)b^3cd^2 - (a^3 - 6a)d^3)*\cosh(bx + a)^2*\sinh(bx + a)^2 + 4*((b^3c^3 - 3ab^2c^2d + 3(a^2 - 2)b^3cd^2 - (a^3 - 6a)d^3)*\cosh(bx + a)^3 - (b^3c^3 - 3ab^2c^2d + 3(a^2 - 2)b^3cd^2 - (a^3 - 6a)d^3)*\cosh(bx + a))*\sinh(bx + a))*\log(\cosh(bx + a) + \sinh(bx + a) - 1) + (b^3d^3x^3 + 3b^3cd^2x^2 + 3ab^2c^2d - 3a^2b^3cd^2 + (b^3d^3x^3 + 3b^3cd^2x^2 + 3ab^2c^2d - 3a^2b^3cd^2 + (a^3 - 6a)d^3 + 3(b^3c^2d - 2b^3d^3)x)*\cosh(bx + a)*\sinh(bx + a)^3 + (b^3d^3x^3 + 3b^3cd^2x^2 + 3ab^2c^2d - 3a^2b^3cd^2 + (a^3 - 6a)d^3 + 3(b^3c^2d - 2b^3d^3)x)*\sinh(bx + a)^4 + (a^3 - 6a)d^3 - 2(b^3d^3x^3 + 3b^3cd^2x^2 + 3ab^2c^2d - 3a^2b^3cd^2 + (a^3 - 6a)d^3 + 3(b^3c^2d - 2b^3d^3)x)*\cosh(bx + a)^2 - 2(b^3d^3x^3 + 3b^3cd^2x^2 + 3ab^2c^2d - 3a^2b^3cd^2 + (a^3 - 6a)d^3 + 3(b^3c^2d - 2b^3d^3)x)*\cosh(bx + a)^2 + 3(b^3c^2d - 2b^3d^3)x)*\sinh(bx + a)^2 + 3(b^3c^2d - 2b^3d^3)x + 4*((b^3d^3x^3 + 3b^3cd^2x^2 + 3ab^2c^2d -
\end{aligned}$$

```

3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)
^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3
- 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a))*sinh(b*x + a))*log(-
cosh(b*x + a) - sinh(b*x + a) + 1) + 6*(d^3*cosh(b*x + a)^4 + 4*d^3*cosh(b*
x + a)*sinh(b*x + a)^3 + d^3*sinh(b*x + a)^4 - 2*d^3*cosh(b*x + a)^2 + d^3
+ 2*(3*d^3*cosh(b*x + a)^2 - d^3)*sinh(b*x + a)^2 + 4*(d^3*cosh(b*x + a)^3
- d^3*cosh(b*x + a))*sinh(b*x + a))*polylog(4, cosh(b*x + a) + sinh(b*x + a
)) - 6*(d^3*cosh(b*x + a)^4 + 4*d^3*cosh(b*x + a)*sinh(b*x + a)^3 + d^3*sin
h(b*x + a)^4 - 2*d^3*cosh(b*x + a)^2 + d^3 + 2*(3*d^3*cosh(b*x + a)^2 - d^3
)*sinh(b*x + a)^2 + 4*(d^3*cosh(b*x + a)^3 - d^3*cosh(b*x + a))*sinh(b*x +
a))*polylog(4, -cosh(b*x + a) - sinh(b*x + a)) - 6*(b*d^3*x + (b*d^3*x + b*
c*d^2)*cosh(b*x + a)^4 + 4*(b*d^3*x + b*c*d^2)*cosh(b*x + a)*sinh(b*x + a)^
3 + (b*d^3*x + b*c*d^2)*sinh(b*x + a)^4 + b*c*d^2 - 2*(b*d^3*x + b*c*d^2)*c
osh(b*x + a)^2 - 2*(b*d^3*x + b*c*d^2 - 3*(b*d^3*x + b*c*d^2)*cosh(b*x + a)
^2)*sinh(b*x + a)^2 + 4*((b*d^3*x + b*c*d^2)*cosh(b*x + a)^3 - (b*d^3*x + b
*c*d^2)*cosh(b*x + a))*sinh(b*x + a))*polylog(3, cosh(b*x + a) + sinh(b*x +
a)) + 6*(b*d^3*x + (b*d^3*x + b*c*d^2)*cosh(b*x + a)^4 + 4*(b*d^3*x + b*c*
d^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*d^3*x + b*c*d^2)*sinh(b*x + a)^4 +
b*c*d^2 - 2*(b*d^3*x + b*c*d^2)*cosh(b*x + a)^2 - 2*(b*d^3*x + b*c*d^2 - 3*
(b*d^3*x + b*c*d^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*((b*d^3*x + b*c*d^
2)*cosh(b*x + a)^3 - (b*d^3*x + b*c*d^2)*cosh(b*x + a))*sinh(b*x + a))*poly
log(3, -cosh(b*x + a) - sinh(b*x + a)) + 2*(b^3*d^3*x^3 + b^3*c^3 - 3*b^2*c
^2*d + 3*(b^3*c*d^2 - b^2*d^3)*x^2 + 3*(b^3*d^3*x^3 + b^3*c^3 + 3*b^2*c^2*d
+ 3*(b^3*c*d^2 + b^2*d^3)*x^2 + 3*(b^3*c^2*d + 2*b^2*c*d^2)*x)*cosh(b*x +
a)^2 + 3*(b^3*c^2*d - 2*b^2*c*d^2)*x)*sinh(b*x + a))/(b^4*cosh(b*x + a)^4 +
4*b^4*cosh(b*x + a)*sinh(b*x + a)^3 + b^4*sinh(b*x + a)^4 - 2*b^4*cosh(b*x
+ a)^2 + b^4 + 2*(3*b^4*cosh(b*x + a)^2 - b^4)*sinh(b*x + a)^2 + 4*(b^4*co
sh(b*x + a)^3 - b^4*cosh(b*x + a))*sinh(b*x + a))

```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^3 \operatorname{csch}^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*3\*csch(b\*x+a)\*\*3,x)

[Out] Integral((c + d\*x)\*\*3\*csch(a + b\*x)\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^3 \operatorname{csch}(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csch(b*x + a)^3, x)
```



### 3.34 $\int (c + dx)^2 \operatorname{csch}^3(a + bx) dx$

**Optimal.** Leaf size=154

$$\frac{d(c + dx)\operatorname{PolyLog}(2, -e^{a+bx})}{b^2} - \frac{d(c + dx)\operatorname{PolyLog}(2, e^{a+bx})}{b^2} - \frac{d^2\operatorname{PolyLog}(3, -e^{a+bx})}{b^3} + \frac{d^2\operatorname{PolyLog}(3, e^{a+bx})}{b^3} - \frac{d(c + dx)}{b^2}$$

[Out]  $((c + d*x)^2*\operatorname{ArcTanh}[E^{(a + b*x)}])/b - (d^2*\operatorname{ArcTanh}[\operatorname{Cosh}[a + b*x]])/b^3 - (d*(c + d*x)*\operatorname{Csch}[a + b*x])/b^2 - ((c + d*x)^2*\operatorname{Coth}[a + b*x]*\operatorname{Csch}[a + b*x])/(2*b) + (d*(c + d*x)*\operatorname{PolyLog}[2, -E^{(a + b*x)}])/b^2 - (d*(c + d*x)*\operatorname{PolyLog}[2, E^{(a + b*x)}])/b^2 - (d^2*\operatorname{PolyLog}[3, -E^{(a + b*x)}])/b^3 + (d^2*\operatorname{PolyLog}[3, E^{(a + b*x)}])/b^3$

**Rubi [A]** time = 0.165645, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {4186, 3770, 4182, 2531, 2282, 6589}

$$\frac{d(c + dx)\operatorname{PolyLog}(2, -e^{a+bx})}{b^2} - \frac{d(c + dx)\operatorname{PolyLog}(2, e^{a+bx})}{b^2} - \frac{d^2\operatorname{PolyLog}(3, -e^{a+bx})}{b^3} + \frac{d^2\operatorname{PolyLog}(3, e^{a+bx})}{b^3} - \frac{d(c + dx)}{b^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^2*\operatorname{Csch}[a + b*x]^3, x]$

[Out]  $((c + d*x)^2*\operatorname{ArcTanh}[E^{(a + b*x)}])/b - (d^2*\operatorname{ArcTanh}[\operatorname{Cosh}[a + b*x]])/b^3 - (d*(c + d*x)*\operatorname{Csch}[a + b*x])/b^2 - ((c + d*x)^2*\operatorname{Coth}[a + b*x]*\operatorname{Csch}[a + b*x])/(2*b) + (d*(c + d*x)*\operatorname{PolyLog}[2, -E^{(a + b*x)}])/b^2 - (d*(c + d*x)*\operatorname{PolyLog}[2, E^{(a + b*x)}])/b^2 - (d^2*\operatorname{PolyLog}[3, -E^{(a + b*x)}])/b^3 + (d^2*\operatorname{PolyLog}[3, E^{(a + b*x)}])/b^3$

#### Rule 4186

$\operatorname{Int}[(\operatorname{csc}[e_.] + (f_.)*(x_.))*(b_.)^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow -\operatorname{Simp}[(b^2*(c + d*x)^m*\operatorname{Cot}[e + f*x]*(b*\operatorname{Csc}[e + f*x])^{(n-2)})/(f*(n-1)), x] + (\operatorname{Dist}[(b^2*d^2*m*(m-1))/(f^2*(n-1)*(n-2)), \operatorname{Int}[(c + d*x)^{(m-2)}*(b*\operatorname{Csc}[e + f*x])^{(n-2)}, x], x] + \operatorname{Dist}[(b^2*(n-2))/(n-1), \operatorname{Int}[(c + d*x)^m*(b*\operatorname{Csc}[e + f*x])^{(n-2)}, x], x] - \operatorname{Simp}[(b^2*d*m*(c + d*x)^{(m-1)}*(b*\operatorname{Csc}[e + f*x])^{(n-2)})/(f^2*(n-1)*(n-2)), x]) /; \operatorname{FreeQ}\{b, c, d, e, f\}, x] \&\& \operatorname{GtQ}[n, 1] \&\& \operatorname{NeQ}[n, 2] \&\& \operatorname{GtQ}[m, 1]$

#### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x)) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.))]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int (c + dx)^2 \operatorname{csch}^3(a + bx) dx &= -\frac{d(c + dx) \operatorname{csch}(a + bx)}{b^2} - \frac{(c + dx)^2 \coth(a + bx) \operatorname{csch}(a + bx)}{2b} - \frac{1}{2} \int (c + dx)^2 \operatorname{csch}(a + bx) dx \\
&= \frac{(c + dx)^2 \tanh^{-1}(e^{a+bx})}{b} - \frac{d^2 \tanh^{-1}(\cosh(a + bx))}{b^3} - \frac{d(c + dx) \operatorname{csch}(a + bx)}{b^2} - \frac{(c + dx)^2 \operatorname{csch}(a + bx)}{2b} \\
&= \frac{(c + dx)^2 \tanh^{-1}(e^{a+bx})}{b} - \frac{d^2 \tanh^{-1}(\cosh(a + bx))}{b^3} - \frac{d(c + dx) \operatorname{csch}(a + bx)}{b^2} - \frac{(c + dx)^2 \operatorname{csch}(a + bx)}{2b} \\
&= \frac{(c + dx)^2 \tanh^{-1}(e^{a+bx})}{b} - \frac{d^2 \tanh^{-1}(\cosh(a + bx))}{b^3} - \frac{d(c + dx) \operatorname{csch}(a + bx)}{b^2} - \frac{(c + dx)^2 \operatorname{csch}(a + bx)}{2b} \\
&= \frac{(c + dx)^2 \tanh^{-1}(e^{a+bx})}{b} - \frac{d^2 \tanh^{-1}(\cosh(a + bx))}{b^3} - \frac{d(c + dx) \operatorname{csch}(a + bx)}{b^2} - \frac{(c + dx)^2 \operatorname{csch}(a + bx)}{2b}
\end{aligned}$$

**Mathematica [B]** time = 10.7472, size = 420, normalized size = 2.73

$$\frac{2bd(c + dx)\operatorname{PolyLog}(2, -e^{a+bx}) - 2bd(c + dx)\operatorname{PolyLog}(2, e^{a+bx}) - 2d^2\operatorname{PolyLog}(3, -e^{a+bx}) + 2d^2\operatorname{PolyLog}(3, e^{a+bx})}{b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2\*Csch[a + b\*x]^3,x]

[Out]  $-\frac{d(c + dx)\operatorname{csch}(a)}{b^2} + \frac{(-c^2 - 2cdx - d^2x^2)\operatorname{csch}(a/2 + (bx)/2)^2}{8b} + \frac{(-b^2c^2\operatorname{Log}[1 - E^{(a + bx)}]) + 2d^2\operatorname{Log}[1 - E^{(a + bx)}] - 2b^2cdx\operatorname{Log}[1 - E^{(a + bx)}] - b^2d^2x^2\operatorname{Log}[1 - E^{(a + bx)}] + b^2c^2\operatorname{Log}[1 + E^{(a + bx)}] - 2d^2\operatorname{Log}[1 + E^{(a + bx)}] + 2b^2cdx\operatorname{Log}[1 + E^{(a + bx)}] + b^2d^2x^2\operatorname{Log}[1 + E^{(a + bx)}] + 2bd(c + dx)\operatorname{PolyLog}[2, -E^{(a + bx)}] - 2bd(c + dx)\operatorname{PolyLog}[2, E^{(a + bx)}] - 2d^2\operatorname{PolyLog}[3, -E^{(a + bx)}] + 2d^2\operatorname{PolyLog}[3, E^{(a + bx)}]}{(2b^3)} + \frac{(-c^2 - 2cdx - d^2x^2)\operatorname{sech}(a/2 + (bx)/2)^2}{8b} + \frac{(\operatorname{csch}(a/2)\operatorname{csch}(a/2 + (bx)/2)(cd\sinh((bx)/2) + d^2x\sinh((bx)/2))}{(2b^2)} + \frac{(\operatorname{sech}(a/2)\operatorname{sech}(a/2 + (bx)/2)(cd\sinh((bx)/2) + d^2x\sinh((bx)/2))}{(2b^2)}$

**Maple [B]** time = 0.044, size = 444, normalized size = 2.9

$$\frac{e^{bx+a} (bd^2x^2e^{2bx+2a} + 2bcdxe^{2bx+2a} + bc^2e^{2bx+2a} + bd^2x^2 + 2d^2xe^{2bx+2a} + 2bcdx + 2cde^{2bx+2a} + bc^2 - 2d^2x - 2cd)}{b^2(e^{2bx+2a} - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2\*csch(b\*x+a)^3,x)

[Out]  $-\exp(b*x+a)*(b*d^2*x^2*\exp(2*b*x+2*a)+2*b*c*d*x*\exp(2*b*x+2*a)+b*c^2*\exp(2*b*x+2*a)+b*d^2*x^2+2*d^2*x*\exp(2*b*x+2*a)+2*b*c*d*x+2*c*d*\exp(2*b*x+2*a)+b*c^2-2*d^2*x-2*c*d)/b^2/(\exp(2*b*x+2*a)-1)^2-d^2*\text{polylog}(3,-\exp(b*x+a))/b^3+d^2*\text{polylog}(3,\exp(b*x+a))/b^3-2/b^3*d^2*\text{arctanh}(\exp(b*x+a))+1/b*c*d*\ln(1+\exp(b*x+a))*x+1/b^2*c*d*\ln(1+\exp(b*x+a))*a-1/b*c*d*\ln(1-\exp(b*x+a))*x-1/b^2*c*d*\ln(1-\exp(b*x+a))*a-2/b^2*c*d*a*\text{arctanh}(\exp(b*x+a))+1/2/b*d^2*\ln(1+\exp(b*x+a))*x^2+1/b^2*d^2*\text{polylog}(2,-\exp(b*x+a))*x-1/2/b*d^2*\ln(1-\exp(b*x+a))*x^2-1/b^2*d^2*\text{polylog}(2,\exp(b*x+a))*x+1/b^2*c*d*\text{polylog}(2,-\exp(b*x+a))-1/b^2*c*d*\text{polylog}(2,\exp(b*x+a))+1/b^3*d^2*a^2*\text{arctanh}(\exp(b*x+a))+1/b*c^2*\text{arctanh}(\exp(b*x+a))-1/2/b^3*d^2*\ln(1+\exp(b*x+a))*a^2+1/2/b^3*d^2*\ln(1-\exp(b*x+a))*a^2$

**Maxima [B]** time = 1.84349, size = 531, normalized size = 3.45

$$\frac{1}{2}c^2\left(\frac{\log(e^{(-bx-a)}+1)}{b}-\frac{\log(e^{(-bx-a)}-1)}{b}+\frac{2(e^{(-bx-a)}+e^{(-3bx-3a)})}{b(2e^{(-2bx-2a)}-e^{(-4bx-4a)}-1)}\right)+\frac{(bx\log(e^{(bx+a)}+1)+\text{Li}_2(-e^{(bx+a)}))cd}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*csch(b\*x+a)^3,x, algorithm="maxima")

[Out]  $1/2*c^2*(\log(e^{(-b*x-a)}+1)/b-\log(e^{(-b*x-a)}-1)/b+2*(e^{(-b*x-a)}+e^{(-3*b*x-3*a)})/(b*(2*e^{(-2*b*x-2*a)}-e^{(-4*b*x-4*a)}-1)))+(b*x*\log(e^{(b*x+a)}+1)+\text{dilog}(-e^{(b*x+a)}))*c*d/b^2-(b*x*\log(-e^{(b*x+a)}+1)+\text{dilog}(e^{(b*x+a)}))*c*d/b^2-((b*d^2*x^2*e^{(3*a)}+2*c*d*e^{(3*a)}+2*(b*c*d+d^2)*x*e^{(3*a)})*e^{(3*b*x)}+(b*d^2*x^2*e^a-2*c*d*e^a+2*(b*c*d-d^2)*x*e^a)*e^{(b*x)})/(b^2*e^{(4*b*x+4*a)}-2*b^2*e^{(2*b*x+2*a)}+b^2)+1/2*(b^2*x^2*\log(e^{(b*x+a)}+1)+2*b*x*\text{dilog}(-e^{(b*x+a)}))-2*\text{polylog}(3,-e^{(b*x+a)})*d^2/b^3-1/2*(b^2*x^2*\log(-e^{(b*x+a)}+1)+2*b*x*\text{dilog}(e^{(b*x+a)}))-2*\text{polylog}(3,e^{(b*x+a)})*d^2/b^3-d^2*\log(e^{(b*x+a)}+1)/b^3+d^2*\log(e^{(b*x+a)}-1)/b^3$

**Fricas [C]** time = 3.16852, size = 5252, normalized size = 34.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*csch(b\*x+a)^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/2*(2*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + b*d^2)*x)*\cosh(b*x \\ & + a)^3 + 6*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + b*d^2)*x)*\cosh(b \\ & *x + a)*\sinh(b*x + a)^2 + 2*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + \\ & b*d^2)*x)*\sinh(b*x + a)^3 + 2*(b^2*d^2*x^2 + b^2*c^2 - 2*b*c*d + 2*(b^2*c*d \\ & d - b*d^2)*x)*\cosh(b*x + a) + 2*((b*d^2*x + b*c*d)*\cosh(b*x + a)^4 + 4*(b*d \\ & ^2*x + b*c*d)*\cosh(b*x + a)*\sinh(b*x + a)^3 + (b*d^2*x + b*c*d)*\sinh(b*x + \\ & a)^4 + b*d^2*x + b*c*d - 2*(b*d^2*x + b*c*d)*\cosh(b*x + a)^2 - 2*(b*d^2*x + \\ & b*c*d - 3*(b*d^2*x + b*c*d)*\cosh(b*x + a)^2)*\sinh(b*x + a)^2 + 4*((b*d^2*x \\ & + b*c*d)*\cosh(b*x + a)^3 - (b*d^2*x + b*c*d)*\cosh(b*x + a))*\sinh(b*x + a)) \\ & *dilog(\cosh(b*x + a) + \sinh(b*x + a)) - 2*((b*d^2*x + b*c*d)*\cosh(b*x + a)^4 \\ & + 4*(b*d^2*x + b*c*d)*\cosh(b*x + a)*\sinh(b*x + a)^3 + (b*d^2*x + b*c*d)*\sinh(b*x + a)^4 \\ & + b*d^2*x + b*c*d - 2*(b*d^2*x + b*c*d)*\cosh(b*x + a)^2 - 2*(b*d^2*x + b*c*d - \\ & 3*(b*d^2*x + b*c*d)*\cosh(b*x + a)^2)*\sinh(b*x + a)^2 + 4*((b*d^2*x + b*c*d)*\cosh(b*x + a)^3 \\ & - (b*d^2*x + b*c*d)*\cosh(b*x + a))*\sinh(b*x + a))*dilog(-\cosh(b*x + a) - \sinh(b*x + a)) - \\ & (b^2*d^2*x^2 + 2*b^2*c*d*x + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*\cosh(b*x + a)^4 + 4*(b^2 \\ & *d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*\cosh(b*x + a)*\sinh(b*x + a)^3 + ( \\ & b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*\sinh(b*x + a)^4 + b^2*c^2 - 2* \\ & (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*\cosh(b*x + a)^2 - 2*(b^2*d^2*x \\ & x^2 + 2*b^2*c*d*x + b^2*c^2 - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2) \\ & )*\cosh(b*x + a)^2 - 2*d^2)*\sinh(b*x + a)^2 - 2*d^2 + 4*((b^2*d^2*x^2 + 2*b \\ & ^2*c*d*x + b^2*c^2 - 2*d^2)*\cosh(b*x + a)^3 - (b^2*d^2*x^2 + 2*b^2*c*d*x + \\ & b^2*c^2 - 2*d^2)*\cosh(b*x + a))*\sinh(b*x + a))*log(\cosh(b*x + a) + \sinh(b*x \\ & + a) + 1) + ((b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*\cosh(b*x + a)^4 + 4*(b^2 \\ & *c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*\cosh(b*x + a)*\sinh(b*x + a)^3 + (b^2*c^2 \\ & - 2*a*b*c*d + (a^2 - 2)*d^2)*\sinh(b*x + a)^4 + b^2*c^2 - 2*a*b*c*d + (a^2 \\ & - 2)*d^2 - 2*(b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*\cosh(b*x + a)^2 - 2*(b^2 \\ & *c^2 - 2*a*b*c*d + (a^2 - 2)*d^2 - 3*(b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)* \\ & \cosh(b*x + a)^2)*\sinh(b*x + a)^2 + 4*((b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2) \\ & *\cosh(b*x + a)^3 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*\cosh(b*x + a))*\sinh \\ & (b*x + a))*log(\cosh(b*x + a) + \sinh(b*x + a) - 1) + (b^2*d^2*x^2 + 2*b^2*c \\ & *d*x + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\cosh(b*x + a)^4 + \\ & 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\cosh(b*x + a)*\sinh(b*x \\ & + a)^3 + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\sinh(b*x + a)^4 \\ & + 2*a*b*c*d - a^2*d^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2) \\ & *\cosh(b*x + a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - 3*( \\ & b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\cosh(b*x + a)^2)*\sinh(b*x \\ & + a)^2 + 4*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\cosh(b*x + a) \\ & ^3 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*\cosh(b*x + a))*\sinh( \\ & b*x + a))*log(-\cosh(b*x + a) - \sinh(b*x + a) + 1) - 2*(d^2*\cosh(b*x + a)^4 \\ & + 4*d^2*\cosh(b*x + a)*\sinh(b*x + a)^3 + d^2*\sinh(b*x + a)^4 - 2*d^2*\cosh(b* \\ & x + a)^2 + 2*(3*d^2*\cosh(b*x + a)^2 - d^2)*\sinh(b*x + a)^2 + d^2 + 4*(d^2*c \end{aligned}$$

$$\begin{aligned} & \text{osh}(b*x + a)^3 - d^2*\text{cosh}(b*x + a)*\sinh(b*x + a))*\text{polylog}(3, \text{cosh}(b*x + a) \\ & + \sinh(b*x + a)) + 2*(d^2*\text{cosh}(b*x + a)^4 + 4*d^2*\text{cosh}(b*x + a)*\sinh(b*x + \\ & a)^3 + d^2*\sinh(b*x + a)^4 - 2*d^2*\text{cosh}(b*x + a)^2 + 2*(3*d^2*\text{cosh}(b*x + a) \\ & )^2 - d^2)*\sinh(b*x + a)^2 + d^2 + 4*(d^2*\text{cosh}(b*x + a)^3 - d^2*\text{cosh}(b*x + \\ & a))*\sinh(b*x + a))*\text{polylog}(3, -\text{cosh}(b*x + a) - \sinh(b*x + a)) + 2*(b^2*d^2* \\ & x^2 + b^2*c^2 - 2*b*c*d + 3*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + \\ & b*d^2)*x)*\text{cosh}(b*x + a)^2 + 2*(b^2*c*d - b*d^2)*x)*\sinh(b*x + a))/(b^3*\text{cos} \\ & h(b*x + a)^4 + 4*b^3*\text{cosh}(b*x + a)*\sinh(b*x + a)^3 + b^3*\sinh(b*x + a)^4 - \\ & 2*b^3*\text{cosh}(b*x + a)^2 + b^3 + 2*(3*b^3*\text{cosh}(b*x + a)^2 - b^3)*\sinh(b*x + a) \\ & ^2 + 4*(b^3*\text{cosh}(b*x + a)^3 - b^3*\text{cosh}(b*x + a))*\sinh(b*x + a)) \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^2 \operatorname{csch}^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2\*csch(b\*x+a)\*\*3,x)

[Out] Integral((c + d\*x)\*\*2\*csch(a + b\*x)\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^2 \operatorname{csch}(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*csch(b\*x+a)^3,x, algorithm="giac")

[Out] integrate((d\*x + c)^2\*csch(b\*x + a)^3, x)

### 3.35 $\int (c + dx) \operatorname{csch}^3(a + bx) dx$

**Optimal.** Leaf size=92

$$\frac{d\operatorname{PolyLog}(2, -e^{a+bx})}{2b^2} - \frac{d\operatorname{PolyLog}(2, e^{a+bx})}{2b^2} - \frac{d\operatorname{csch}(a + bx)}{2b^2} + \frac{(c + dx) \tanh^{-1}(e^{a+bx})}{b} - \frac{(c + dx) \operatorname{coth}(a + bx) \operatorname{csch}(a + bx)}{2b}$$

[Out] ((c + d\*x)\*ArcTanh[E^(a + b\*x)])/b - (d\*Csch[a + b\*x])/(2\*b^2) - ((c + d\*x)\*Coth[a + b\*x]\*Csch[a + b\*x])/(2\*b) + (d\*PolyLog[2, -E^(a + b\*x)])/(2\*b^2) - (d\*PolyLog[2, E^(a + b\*x)])/(2\*b^2)

**Rubi [A]** time = 0.0813309, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {4185, 4182, 2279, 2391}

$$\frac{d\operatorname{PolyLog}(2, -e^{a+bx})}{2b^2} - \frac{d\operatorname{PolyLog}(2, e^{a+bx})}{2b^2} - \frac{d\operatorname{csch}(a + bx)}{2b^2} + \frac{(c + dx) \tanh^{-1}(e^{a+bx})}{b} - \frac{(c + dx) \operatorname{coth}(a + bx) \operatorname{csch}(a + bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)\*Csch[a + b\*x]^3, x]

[Out] ((c + d\*x)\*ArcTanh[E^(a + b\*x)])/b - (d\*Csch[a + b\*x])/(2\*b^2) - ((c + d\*x)\*Coth[a + b\*x]\*Csch[a + b\*x])/(2\*b) + (d\*PolyLog[2, -E^(a + b\*x)])/(2\*b^2) - (d\*PolyLog[2, E^(a + b\*x)])/(2\*b^2)

#### Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_.)), x_Symbol] :>
-Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x]
+ (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x]
- Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

#### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] :>
Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]]/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x]
+ Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)], x], x]) /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int (c + dx) \operatorname{csch}^3(a + bx) dx &= -\frac{d \operatorname{csch}(a + bx)}{2b^2} - \frac{(c + dx) \operatorname{coth}(a + bx) \operatorname{csch}(a + bx)}{2b} - \frac{1}{2} \int (c + dx) \operatorname{csch}(a + bx) dx \\ &= \frac{(c + dx) \tanh^{-1}(e^{a+bx})}{b} - \frac{d \operatorname{csch}(a + bx)}{2b^2} - \frac{(c + dx) \operatorname{coth}(a + bx) \operatorname{csch}(a + bx)}{2b} + \frac{d \int \log(1 - e^{-2(a+bx)})}{2b^2} \\ &= \frac{(c + dx) \tanh^{-1}(e^{a+bx})}{b} - \frac{d \operatorname{csch}(a + bx)}{2b^2} - \frac{(c + dx) \operatorname{coth}(a + bx) \operatorname{csch}(a + bx)}{2b} + \frac{d \operatorname{Subst}\left(\int \log(1 - e^{-2u}) du\right)}{2b^2} \\ &= \frac{(c + dx) \tanh^{-1}(e^{a+bx})}{b} - \frac{d \operatorname{csch}(a + bx)}{2b^2} - \frac{(c + dx) \operatorname{coth}(a + bx) \operatorname{csch}(a + bx)}{2b} + \frac{d \operatorname{Li}_2(-e^{-2(a+bx)})}{2b^2} \end{aligned}$$

**Mathematica [C]** time = 2.67384, size = 313, normalized size = 3.4

$$\frac{d \left( -a \log \left( \tanh \left( \frac{1}{2}(a + bx) \right) \right) - i \left( i \left( \operatorname{PolyLog} \left( 2, -e^{i(a+ibx)} \right) - \operatorname{PolyLog} \left( 2, e^{i(a+ibx)} \right) \right) + (ia + ibx) \left( \log \left( 1 - e^{i(a+ibx)} \right) - \log \left( 1 - e^{-i(a+ibx)} \right) \right) \right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csch[a + b*x]^3,x]
```

```
[Out] -(d*x*Csch[a/2 + (b*x)/2]^2)/(8*b) - (c*Csch[(a + b*x)/2]^2)/(8*b) - (c*Log
[Tanh[(a + b*x)/2]])/(2*b) - (d*(-(a*Log[Tanh[(a + b*x)/2]]) - I*((I*a + I*
b*x)*(Log[1 - E^(I*(I*a + I*b*x))] - Log[1 + E^(I*(I*a + I*b*x))] + I*(Pol
yLog[2, -E^(I*(I*a + I*b*x))] - PolyLog[2, E^(I*(I*a + I*b*x)])))))/(2*b^2)
- (d*x*Sech[a/2 + (b*x)/2]^2)/(8*b) - (c*Sech[(a + b*x)/2]^2)/(8*b) + (d*C
sch[a/2]*Csch[a/2 + (b*x)/2]*Sinh[(b*x)/2])/(4*b^2) + (d*Sech[a/2]*Sech[a/2
+ (b*x)/2]*Sinh[(b*x)/2])/(4*b^2)
```



**Maple [B]** time = 0.033, size = 197, normalized size = 2.1

$$\frac{e^{bx+a} (bdxe^{2bx+2a} + bce^{2bx+2a} + bdx + de^{2bx+2a} + cb - d)}{b^2 (e^{2bx+2a} - 1)^2} + \frac{c \operatorname{Artanh}(e^{bx+a})}{b} + \frac{d \ln(1 + e^{bx+a}) x}{2b} + \frac{d \ln(1 + e^{bx+a})}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)*csch(b*x+a)^3,x)`

[Out]  $-\exp(bx+a) * (b*d*x*\exp(2*b*x+2*a) + b*c*\exp(2*b*x+2*a) + b*d*x + d*\exp(2*b*x+2*a) + c*b - d) / b^2 / (\exp(2*b*x+2*a) - 1)^2 + 1/b*c*\operatorname{arctanh}(\exp(b*x+a)) + 1/2/b*d*\ln(1 + \exp(b*x+a)) * x + 1/2/b^2*d*\ln(1 + \exp(b*x+a)) * a + 1/2*d*\operatorname{polylog}(2, -\exp(b*x+a)) / b^2 - 1/2/b*d*\ln(1 - \exp(b*x+a)) * x - 1/2/b^2*d*\ln(1 - \exp(b*x+a)) * a - 1/2*d*\operatorname{polylog}(2, \exp(b*x+a)) / b^2 - 1/b^2*d*a*\operatorname{arctanh}(\exp(b*x+a))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-d \left( \frac{(bx e^{3a} + e^{3a}) e^{3bx} + (bx e^a - e^a) e^{bx}}{b^2 e^{4bx+4a} - 2b^2 e^{2bx+2a} + b^2} + 8 \int \frac{x}{16(e^{bx+a} + 1)} dx + 8 \int \frac{x}{16(e^{bx+a} - 1)} dx \right) + \frac{1}{2} c \left( \frac{\log(e^{-bx-a} + 1)}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*csch(b*x+a)^3,x, algorithm="maxima")`

[Out]  $-d * (((b*x*e^{(3*a)} + e^{(3*a)}) * e^{(3*b*x)} + (b*x*e^a - e^a) * e^{(b*x)}) / (b^2 * e^{(4*b*x + 4*a)} - 2*b^2 * e^{(2*b*x + 2*a)} + b^2) + 8 * \operatorname{integrate}(1/16*x / (e^{(b*x + a)} + 1), x) + 8 * \operatorname{integrate}(1/16*x / (e^{(b*x + a)} - 1), x) + 1/2 * c * (\log(e^{-b*x - a} + 1) / b - \log(e^{-b*x - a} - 1) / b + 2 * (e^{-b*x - a} + e^{-3*b*x - 3*a}) / (b * (2 * e^{-2*b*x - 2*a} - e^{-4*b*x - 4*a} - 1))))$

**Fricas [B]** time = 2.83789, size = 2677, normalized size = 29.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*csch(b*x+a)^3,x, algorithm="fricas")`

```
[Out] -1/2*(2*(b*d*x + b*c + d)*cosh(b*x + a)^3 + 6*(b*d*x + b*c + d)*cosh(b*x +
a)*sinh(b*x + a)^2 + 2*(b*d*x + b*c + d)*sinh(b*x + a)^3 + 2*(b*d*x + b*c -
d)*cosh(b*x + a) + (d*cosh(b*x + a)^4 + 4*d*cosh(b*x + a)*sinh(b*x + a)^3
+ d*sinh(b*x + a)^4 - 2*d*cosh(b*x + a)^2 + 2*(3*d*cosh(b*x + a)^2 - d)*sin
h(b*x + a)^2 + 4*(d*cosh(b*x + a)^3 - d*cosh(b*x + a))*sinh(b*x + a) + d)*d
ilog(cosh(b*x + a) + sinh(b*x + a)) - (d*cosh(b*x + a)^4 + 4*d*cosh(b*x + a
)*sinh(b*x + a)^3 + d*sinh(b*x + a)^4 - 2*d*cosh(b*x + a)^2 + 2*(3*d*cosh(b
*x + a)^2 - d)*sinh(b*x + a)^2 + 4*(d*cosh(b*x + a)^3 - d*cosh(b*x + a))*si
nh(b*x + a) + d)*dilog(-cosh(b*x + a) - sinh(b*x + a)) - ((b*d*x + b*c)*cos
h(b*x + a)^4 + 4*(b*d*x + b*c)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*d*x + b*c
)*sinh(b*x + a)^4 + b*d*x - 2*(b*d*x + b*c)*cosh(b*x + a)^2 - 2*(b*d*x - 3*
(b*d*x + b*c)*cosh(b*x + a)^2 + b*c)*sinh(b*x + a)^2 + b*c + 4*((b*d*x + b*
c)*cosh(b*x + a)^3 - (b*d*x + b*c)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b
*x + a) + sinh(b*x + a) + 1) + ((b*c - a*d)*cosh(b*x + a)^4 + 4*(b*c - a*d)
*cosh(b*x + a)*sinh(b*x + a)^3 + (b*c - a*d)*sinh(b*x + a)^4 - 2*(b*c - a*d
)*cosh(b*x + a)^2 + 2*(3*(b*c - a*d)*cosh(b*x + a)^2 - b*c + a*d)*sinh(b*x
+ a)^2 + b*c - a*d + 4*((b*c - a*d)*cosh(b*x + a)^3 - (b*c - a*d)*cosh(b*x
+ a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - 1) + ((b*d*x + a*d
)*cosh(b*x + a)^4 + 4*(b*d*x + a*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (b*d*x
+ a*d)*sinh(b*x + a)^4 + b*d*x - 2*(b*d*x + a*d)*cosh(b*x + a)^2 - 2*(b*d*x
- 3*(b*d*x + a*d)*cosh(b*x + a)^2 + a*d)*sinh(b*x + a)^2 + a*d + 4*((b*d*x
+ a*d)*cosh(b*x + a)^3 - (b*d*x + a*d)*cosh(b*x + a))*sinh(b*x + a))*log(-
cosh(b*x + a) - sinh(b*x + a) + 1) + 2*(b*d*x + 3*(b*d*x + b*c + d)*cosh(b*
x + a)^2 + b*c - d)*sinh(b*x + a))/(b^2*cosh(b*x + a)^4 + 4*b^2*cosh(b*x +
a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 - 2*b^2*cosh(b*x + a)^2 + 2*(3*b^2
*cosh(b*x + a)^2 - b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2*cosh(b*x + a)^3 - b^
2*cosh(b*x + a))*sinh(b*x + a))
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (c + dx) \operatorname{csch}^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*csch(a + b*x)**3, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c) \operatorname{csch}(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csch(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csch(b*x + a)^3, x)
```

$$3.36 \quad \int \frac{\operatorname{csch}^3(a+bx)}{c+dx} dx$$

**Optimal.** Leaf size=18

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^3(a+bx)}{c+dx}, x\right)$$

[Out] Unintegrable[Csch[a + b\*x]^3/(c + d\*x), x]

**Rubi [A]** time = 0.0396201, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Int[Csch[a + b\*x]^3/(c + d\*x), x]

[Out] Defer[Int][Csch[a + b\*x]^3/(c + d\*x), x]

Rubi steps

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx} dx = \int \frac{\operatorname{csch}^3(a+bx)}{c+dx} dx$$

**Mathematica [A]** time = 74.2687, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(a+bx)}{c+dx} dx$$

Verification is Not applicable to the result.

[In] Integrate[Csch[a + b\*x]^3/(c + d\*x), x]

[Out] Integrate[Csch[a + b\*x]^3/(c + d\*x), x]

---

**Maple [A]** time = 0.484, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(bx+a))^3}{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(b\*x+a)^3/(d\*x+c),x)

[Out] int(csch(b\*x+a)^3/(d\*x+c),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{(bdxe^{(3a)} + (bc - d)e^{(3a)})e^{(3bx)} + (bdxe^a + (bc + d)e^a)e^{(bx)}}{b^2d^2x^2 + 2b^2cdx + b^2c^2 + (b^2d^2x^2e^{(4a)} + 2b^2cdxe^{(4a)} + b^2c^2e^{(4a)})e^{(4bx)} - 2(b^2d^2x^2e^{(2a)} + 2b^2cdxe^{(2a)} + b^2c^2e^{(2a)})e^{(2bx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(b\*x+a)^3/(d\*x+c),x, algorithm="maxima")

[Out]  $-\left(\left(bdx e^{(3a)} + (bc - d)e^{(3a)}\right)e^{(3bx)} + (bdxe^a + (bc + d)e^a)e^{(bx)}\right) / \left(b^2d^2x^2 + 2b^2cdx + b^2c^2 + (b^2d^2x^2e^{(4a)} + 2b^2cdxe^{(4a)} + b^2c^2e^{(4a)})e^{(4bx)} - 2(b^2d^2x^2e^{(2a)} + 2b^2cdxe^{(2a)} + b^2c^2e^{(2a)})e^{(2bx)}\right) - 8 \int \frac{1}{16(b^2d^2x^2 + 2b^2cdx + b^2c^2 - 2d^2)} / (b^2d^3x^3 + 3b^2cd^2x^2 + 3b^2c^2dx + b^2c^3 + (b^2d^3x^3e^a + 3b^2cd^2x^2e^a + 3b^2c^2dx e^a + b^2c^3e^a)e^{(bx)}), x) - 8 \int \frac{-1}{16(b^2d^2x^2 + 2b^2cdx + b^2c^2 - 2d^2)} / (b^2d^3x^3 + 3b^2cd^2x^2 + 3b^2c^2dx + b^2c^3 - (b^2d^3x^3e^a + 3b^2cd^2x^2e^a + 3b^2c^2dx e^a + b^2c^3e^a)e^{(bx)}), x)$

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)^3}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^3/(d*x + c), x)
```

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(a + bx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(csch(a + b*x)**3/(c + d*x), x)
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx + a)^3}{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3/(d*x + c), x)
```

$$3.37 \quad \int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} dx$$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2}, x\right)$$

[Out] Unintegrable[Csch[a + b\*x]^3/(c + d\*x)^2, x]

**Rubi [A]** time = 0.0372557, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Int[Csch[a + b\*x]^3/(c + d\*x)^2, x]

[Out] Defer[Int][Csch[a + b\*x]^3/(c + d\*x)^2, x]

Rubi steps

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} dx = \int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} dx$$

**Mathematica [A]** time = 79.6001, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[Csch[a + b\*x]^3/(c + d\*x)^2, x]

[Out] Integrate[Csch[a + b\*x]^3/(c + d\*x)^2, x]

---

**Maple [A]** time = 0.81, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(bx+a))^3}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(b\*x+a)^3/(d\*x+c)^2,x)

[Out] int(csch(b\*x+a)^3/(d\*x+c)^2,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{(bdxe^{3a} + (bc - 2d)e^{3a})e^{3bx} + (bdxe^a + (bc + 2d)e^a)e^{bx}}{b^2d^3x^3 + 3b^2cd^2x^2 + 3b^2c^2dx + b^2c^3 + (b^2d^3x^3e^{4a} + 3b^2cd^2x^2e^{4a} + 3b^2c^2dxe^{4a} + b^2c^3e^{4a})e^{4bx} - 2(b^2d^3x^3e^{2a} + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(b\*x+a)^3/(d\*x+c)^2,x, algorithm="maxima")

[Out] -((b\*d\*x\*e^(3\*a) + (b\*c - 2\*d)\*e^(3\*a))\*e^(3\*b\*x) + (b\*d\*x\*e^a + (b\*c + 2\*d)\*e^a)\*e^(b\*x))/(b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3 + (b^2\*d^3\*x^3\*e^(4\*a) + 3\*b^2\*c\*d^2\*x^2\*e^(4\*a) + 3\*b^2\*c^2\*d\*x\*e^(4\*a) + b^2\*c^3\*e^(4\*a))\*e^(4\*b\*x) - 2\*(b^2\*d^3\*x^3\*e^(2\*a) + 3\*b^2\*c\*d^2\*x^2\*e^(2\*a) + 3\*b^2\*c^2\*d\*x\*e^(2\*a) + b^2\*c^3\*e^(2\*a))\*e^(2\*b\*x)) - 8\*integrate(1/16\*(b^2\*d^2\*x^2 + 2\*b^2\*c\*d\*x + b^2\*c^2 - 6\*d^2)/(b^2\*d^4\*x^4 + 4\*b^2\*c\*d^3\*x^3 + 6\*b^2\*c^2\*d^2\*x^2 + 4\*b^2\*c^3\*d\*x + b^2\*c^4 + (b^2\*d^4\*x^4\*e^a + 4\*b^2\*c\*d^3\*x^3\*e^a + 6\*b^2\*c^2\*d^2\*x^2\*e^a + 4\*b^2\*c^3\*d\*x\*e^a + b^2\*c^4\*e^a)\*e^(b\*x)), x) - 8\*integrate(-1/16\*(b^2\*d^2\*x^2 + 2\*b^2\*c\*d\*x + b^2\*c^2 - 6\*d^2)/(b^2\*d^4\*x^4 + 4\*b^2\*c\*d^3\*x^3 + 6\*b^2\*c^2\*d^2\*x^2 + 4\*b^2\*c^3\*d\*x + b^2\*c^4 - (b^2\*d^4\*x^4\*e^a + 4\*b^2\*c\*d^3\*x^3\*e^a + 6\*b^2\*c^2\*d^2\*x^2\*e^a + 4\*b^2\*c^3\*d\*x\*e^a + b^2\*c^4\*e^a)\*e^(b\*x)), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx+a)^3}{d^2x^2 + 2cdx + c^2}, x\right)$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csch(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(a + bx)}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(csch(a + b*x)**3/(c + d*x)**2, x)
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx + a)^3}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)^3/(d*x + c)^2, x)
```

### 3.38 $\int (c + dx)^{5/2} \sinh(ax + bx) dx$

**Optimal.** Leaf size=171

$$-\frac{15\sqrt{\pi}d^{5/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} - \frac{15\sqrt{\pi}d^{5/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} + \frac{15d^2\sqrt{c+dx}\cosh(ax+bx)}{4b^3} - \frac{5d(c+dx)^{3/2}\sinh(ax+bx)}{2b^2}$$

[Out]  $(15*d^2*\sqrt{c+d*x}*Cosh[a+b*x])/(4*b^3) + ((c+d*x)^{(5/2)}*Cosh[a+b*x])/b - (15*d^{(5/2)}*E^{-a+(b*c)/d}*\sqrt{\pi}*\operatorname{Erf}[(\sqrt{b}*\sqrt{c+d*x})/\sqrt{d}])/(16*b^{(7/2)}) - (15*d^{(5/2)}*E^{a-(b*c)/d}*\sqrt{\pi}*\operatorname{Erfi}[(\sqrt{b}*\sqrt{c+d*x})/\sqrt{d}])/(16*b^{(7/2)}) - (5*d*(c+d*x)^{(3/2)}*\sinh[a+b*x])/(2*b^2)$

**Rubi [A]** time = 0.370048, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3296, 3307, 2180, 2204, 2205}

$$-\frac{15\sqrt{\pi}d^{5/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} - \frac{15\sqrt{\pi}d^{5/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} + \frac{15d^2\sqrt{c+dx}\cosh(ax+bx)}{4b^3} - \frac{5d(c+dx)^{3/2}\sinh(ax+bx)}{2b^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c+d*x)^{(5/2)}*\sinh[a+b*x],x]$

[Out]  $(15*d^2*\sqrt{c+d*x}*Cosh[a+b*x])/(4*b^3) + ((c+d*x)^{(5/2)}*Cosh[a+b*x])/b - (15*d^{(5/2)}*E^{-a+(b*c)/d}*\sqrt{\pi}*\operatorname{Erf}[(\sqrt{b}*\sqrt{c+d*x})/\sqrt{d}])/(16*b^{(7/2)}) - (15*d^{(5/2)}*E^{a-(b*c)/d}*\sqrt{\pi}*\operatorname{Erfi}[(\sqrt{b}*\sqrt{c+d*x})/\sqrt{d}])/(16*b^{(7/2)}) - (5*d*(c+d*x)^{(3/2)}*\sinh[a+b*x])/(2*b^2)$

#### Rule 3296

$\operatorname{Int}[(c + d*x)^m * \sin(e + f*x), x\_Symbol] \rightarrow -\operatorname{Simp}[(c + d*x)^m * \cos(e + f*x)/f, x] + \operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{m-1} * \cos(e + f*x), x], x] /;$   $\operatorname{FreeQ}\{c, d, e, f\}, x \&\& \operatorname{GtQ}[m, 0]$

#### Rule 3307

$\operatorname{Int}[(c + d*x)^m * \sin(e + \pi*k + f*x), x\_Symbol] \rightarrow \operatorname{Dist}[I/2, \operatorname{Int}[(c + d*x)^m / (E^{I*k*\pi} * E^{I*(e + f*x)}), x], x] - \operatorname{Dist}[I/2, \operatorname{Int}[(c + d*x)^m * E^{I*k*\pi} * E^{I*(e + f*x)}, x], x] /;$   $\operatorname{FreeQ}\{c, d, e,$

f, m}, x] && IntegerQ[2\*k]

### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

### Rubi steps

$$\begin{aligned}
 \int (c + dx)^{5/2} \sinh(a + bx) dx &= \frac{(c + dx)^{5/2} \cosh(a + bx)}{b} - \frac{(5d) \int (c + dx)^{3/2} \cosh(a + bx) dx}{2b} \\
 &= \frac{(c + dx)^{5/2} \cosh(a + bx)}{b} - \frac{5d(c + dx)^{3/2} \sinh(a + bx)}{2b^2} + \frac{(15d^2) \int \sqrt{c + dx} \sinh(a + bx)}{4b^2} \\
 &= \frac{15d^2 \sqrt{c + dx} \cosh(a + bx)}{4b^3} + \frac{(c + dx)^{5/2} \cosh(a + bx)}{b} - \frac{5d(c + dx)^{3/2} \sinh(a + bx)}{2b^2} - \frac{(15d^2) \int \sqrt{c + dx} \cosh(a + bx)}{4b^2} \\
 &= \frac{15d^2 \sqrt{c + dx} \cosh(a + bx)}{4b^3} + \frac{(c + dx)^{5/2} \cosh(a + bx)}{b} - \frac{5d(c + dx)^{3/2} \sinh(a + bx)}{2b^2} - \frac{(15d^2) \int \sqrt{c + dx} \sinh(a + bx)}{4b^2} \\
 &= \frac{15d^2 \sqrt{c + dx} \cosh(a + bx)}{4b^3} + \frac{(c + dx)^{5/2} \cosh(a + bx)}{b} - \frac{5d(c + dx)^{3/2} \sinh(a + bx)}{2b^2} - \frac{(15d^2) \int \sqrt{c + dx} \cosh(a + bx)}{4b^2} \\
 &= \frac{15d^2 \sqrt{c + dx} \cosh(a + bx)}{4b^3} + \frac{(c + dx)^{5/2} \cosh(a + bx)}{b} - \frac{15d^{5/2} e^{-a + \frac{bc}{d}} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{c + dx}}{\sqrt{d}}\right)}{16b^{7/2}}
 \end{aligned}$$

**Mathematica [A]** time = 0.058734, size = 108, normalized size = 0.63

$$\frac{d^3 e^{-a - \frac{bc}{d}} \left( e^{\frac{2bc}{d}} \sqrt{\frac{b(c+dx)}{d}} \text{Gamma}\left(\frac{7}{2}, \frac{b(c+dx)}{d}\right) - e^{2a} \sqrt{-\frac{b(c+dx)}{d}} \text{Gamma}\left(\frac{7}{2}, -\frac{b(c+dx)}{d}\right) \right)}{2b^4 \sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^(5/2)\*Sinh[a + b\*x], x]

[Out] (d^3 \* E^(-a - (b\*c)/d) \* (-E^(2\*a) \* Sqrt[-((b\*(c + d\*x))/d)] \* Gamma[7/2, -((b\*(c + d\*x))/d)]) + E^((2\*b\*c)/d) \* Sqrt[(b\*(c + d\*x))/d] \* Gamma[7/2, (b\*(c + d\*x))/d]) / (2\*b^4 \* Sqrt[c + d\*x])

**Maple [F]** time = 0.033, size = 0, normalized size = 0.

$$\int (dx + c)^{\frac{5}{2}} \sinh(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^(5/2)\*sinh(b\*x+a), x)

[Out] int((d\*x+c)^(5/2)\*sinh(b\*x+a), x)

**Maxima [B]** time = 1.22586, size = 416, normalized size = 2.43

$$32(dx + c)^{\frac{7}{2}} \sinh(bx + a) - \frac{\left( \frac{105 \sqrt{\pi} d^4 \operatorname{erf}\left(\sqrt{dx+c} \sqrt{-\frac{b}{d}}\right) e^{\left(a - \frac{bc}{d}\right)}}{b^4 \sqrt{-\frac{b}{d}}} + \frac{105 \sqrt{\pi} d^4 \operatorname{erf}\left(\sqrt{dx+c} \sqrt{\frac{b}{d}}\right) e^{\left(-a + \frac{bc}{d}\right)}}{b^4 \sqrt{\frac{b}{d}}} - 2 \left( 8(dx+c)^{\frac{7}{2}} b^3 d e^{\left(\frac{bc}{d}\right)} + 28(dx+c)^{\frac{5}{2}} b^2 d^2 e^{\left(\frac{bc}{d}\right)} + 70(dx+c)^{\frac{3}{2}} b d^3 e^{\left(\frac{bc}{d}\right)} \right) \right)}{112 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(5/2)\*sinh(b\*x+a), x, algorithm="maxima")

[Out] 1/112\*(32\*(d\*x + c)^(7/2)\*sinh(b\*x + a) - (105\*sqrt(pi)\*d^4\*erf(sqrt(d\*x + c)\*sqrt(-b/d))\*e^(a - b\*c/d)/(b^4\*sqrt(-b/d)) + 105\*sqrt(pi)\*d^4\*erf(sqrt(d

$$\begin{aligned} & *x + c) * \sqrt{b/d}) * e^{(-a + b*c/d)/(b^4 * \sqrt{b/d})} - 2 * (8 * (d*x + c)^{(7/2)} * b^3 * d * e^{(b*c/d)} + 28 * (d*x + c)^{(5/2)} * b^2 * d^2 * e^{(b*c/d)} + 70 * (d*x + c)^{(3/2)} * b * d^3 * e^{(b*c/d)} + 105 * \sqrt{d*x + c} * d^4 * e^{(b*c/d)}) * e^{(-a - (d*x + c)*b/d)/b^4} \\ & + 2 * (8 * (d*x + c)^{(7/2)} * b^3 * d * e^a - 28 * (d*x + c)^{(5/2)} * b^2 * d^2 * e^a + 70 * (d*x + c)^{(3/2)} * b * d^3 * e^a - 105 * \sqrt{d*x + c} * d^4 * e^a) * e^{((d*x + c)*b/d - b*c/d)/b^4} * b/d)/d \end{aligned}$$

**Fricas [B]** time = 2.75247, size = 1179, normalized size = 6.89

$$15 \sqrt{\pi} \left( d^3 \cosh(bx + a) \cosh\left(-\frac{bc-ad}{d}\right) - d^3 \cosh(bx + a) \sinh\left(-\frac{bc-ad}{d}\right) + \left( d^3 \cosh\left(-\frac{bc-ad}{d}\right) - d^3 \sinh\left(-\frac{bc-ad}{d}\right) \right) \sinh\left(-\frac{bc-ad}{d}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(5/2)\*sinh(b\*x+a),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/16 * (15 * \sqrt{\pi} * (d^3 * \cosh(b*x + a) * \cosh(-(b*c - a*d)/d) - d^3 * \cosh(b*x + a) * \sinh(-(b*c - a*d)/d) + (d^3 * \cosh(-(b*c - a*d)/d) - d^3 * \sinh(-(b*c - a*d)/d)) * \sinh(b*x + a)) * \sqrt{b/d} * \operatorname{erf}(\sqrt{d*x + c} * \sqrt{b/d}) - 15 * \sqrt{\pi} * (d^3 * \cosh(b*x + a) * \cosh(-(b*c - a*d)/d) + d^3 * \cosh(b*x + a) * \sinh(-(b*c - a*d)/d) + (d^3 * \cosh(-(b*c - a*d)/d) + d^3 * \sinh(-(b*c - a*d)/d)) * \sinh(b*x + a)) * \sqrt{-b/d} * \operatorname{erf}(\sqrt{d*x + c} * \sqrt{-b/d}) - 2 * (4 * b^3 * d^2 * x^2 + 4 * b^3 * c^2 + 10 * b^2 * c * d + 15 * b * d^2 + (4 * b^3 * d^2 * x^2 + 4 * b^3 * c^2 - 10 * b^2 * c * d + 15 * b * d^2 + 2 * (4 * b^3 * c * d - 5 * b^2 * d^2) * x) * \cosh(b*x + a)^2 + 2 * (4 * b^3 * d^2 * x^2 + 4 * b^3 * c^2 - 10 * b^2 * c * d + 15 * b * d^2 + 2 * (4 * b^3 * c * d - 5 * b^2 * d^2) * x) * \cosh(b*x + a) * \sinh(b*x + a) + (4 * b^3 * d^2 * x^2 + 4 * b^3 * c^2 - 10 * b^2 * c * d + 15 * b * d^2 + 2 * (4 * b^3 * c * d - 5 * b^2 * d^2) * x) * \sinh(b*x + a)^2 + 2 * (4 * b^3 * c * d + 5 * b^2 * d^2) * x) * \sqrt{d*x + c}) / (b^4 * \cosh(b*x + a) + b^4 * \sinh(b*x + a)) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*(5/2)\*sinh(b\*x+a),x)

[Out] Timed out

**Giac [A]** time = 1.37161, size = 313, normalized size = 1.83

$$\frac{15\sqrt{\pi}d^4 \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bdb^3}} + \frac{15\sqrt{\pi}d^4 \operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bdb^3}} + \frac{2\left(4(dx+c)^{\frac{5}{2}}b^2d-10(dx+c)^{\frac{3}{2}}bd^2+15\sqrt{dx+cd^3}\right)e^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b^3} + \frac{2\left(4(dx+c)^{\frac{5}{2}}b^2d-10(dx+c)^{\frac{3}{2}}bd^2+15\sqrt{dx+cd^3}\right)e^{\left(-\frac{(dx+c)b-bc+ad}{d}\right)}}{b^3}$$


---

$16d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(5/2)\*sinh(b\*x+a),x, algorithm="giac")

[Out]  $\frac{1}{16} \cdot \left( \frac{15 \sqrt{\pi} d^4 \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right) e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bdb^3}} + \frac{15 \sqrt{\pi} d^4 \operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right) e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bdb^3}} + \frac{2 \left(4(dx+c)^{\frac{5}{2}}b^2d - 10(dx+c)^{\frac{3}{2}}bd^2 + 15\sqrt{dx+cd^3}\right) e^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b^3} + \frac{2 \left(4(dx+c)^{\frac{5}{2}}b^2d - 10(dx+c)^{\frac{3}{2}}bd^2 + 15\sqrt{dx+cd^3}\right) e^{\left(-\frac{(dx+c)b-bc+ad}{d}\right)}}{b^3} \right) / d$

### 3.39 $\int (c + dx)^{3/2} \sinh(a + bx) dx$

**Optimal.** Leaf size=146

$$-\frac{3\sqrt{\pi}d^{3/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} + \frac{3\sqrt{\pi}d^{3/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} - \frac{3d\sqrt{c+dx}\sinh(a+bx)}{2b^2} + \frac{(c+dx)^{3/2}\cosh(a+bx)}{b}$$

[Out]  $((c + d*x)^{(3/2)}*\operatorname{Cosh}[a + b*x])/b - (3*d^{(3/2)}*E^{(-a + (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])]/(8*b^{(5/2)})) + (3*d^{(3/2)}*E^{(a - (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])]/(8*b^{(5/2)})) - (3*d*\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[a + b*x])/(2*b^2)$

**Rubi [A]** time = 0.248011, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3296, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}d^{3/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} + \frac{3\sqrt{\pi}d^{3/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} - \frac{3d\sqrt{c+dx}\sinh(a+bx)}{2b^2} + \frac{(c+dx)^{3/2}\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^{(3/2)}*\operatorname{Sinh}[a + b*x], x]$

[Out]  $((c + d*x)^{(3/2)}*\operatorname{Cosh}[a + b*x])/b - (3*d^{(3/2)}*E^{(-a + (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])]/(8*b^{(5/2)})) + (3*d^{(3/2)}*E^{(a - (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])]/(8*b^{(5/2)})) - (3*d*\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[a + b*x])/(2*b^2)$

#### Rule 3296

$\operatorname{Int}[(c + d*x)^m*\operatorname{Cos}[e + f*x], x] \rightarrow -\operatorname{Simp}[(c + d*x)^m*\operatorname{Cos}[e + f*x]/f, x] + \operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{m-1}*\operatorname{Cos}[e + f*x], x], x] /;$   $\operatorname{FreeQ}\{c, d, e, f\}, x \ \&\& \ \operatorname{GtQ}[m, 0]$

#### Rule 3308

$\operatorname{Int}[(c + d*x)^m*\operatorname{Sin}[e + f*x], x] \rightarrow \operatorname{Dist}[I/2, \operatorname{Int}[(c + d*x)^m/E^{(I*(e + f*x))}, x], x] - \operatorname{Dist}[I/2, \operatorname{Int}[(c + d*x)^m*E^{(I*(e + f*x))}, x], x] /;$   $\operatorname{FreeQ}\{c, d, e, f, m\}, x$

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{3/2} \sinh(a + bx) dx &= \frac{(c + dx)^{3/2} \cosh(a + bx)}{b} - \frac{(3d) \int \sqrt{c + dx} \cosh(a + bx) dx}{2b} \\
&= \frac{(c + dx)^{3/2} \cosh(a + bx)}{b} - \frac{3d\sqrt{c + dx} \sinh(a + bx)}{2b^2} + \frac{(3d^2) \int \frac{\sinh(a+bx)}{\sqrt{c+dx}} dx}{4b^2} \\
&= \frac{(c + dx)^{3/2} \cosh(a + bx)}{b} - \frac{3d\sqrt{c + dx} \sinh(a + bx)}{2b^2} + \frac{(3d^2) \int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}} dx}{8b^2} - \frac{(3d^2) \int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}} dx}{8b^2} \\
&= \frac{(c + dx)^{3/2} \cosh(a + bx)}{b} - \frac{3d\sqrt{c + dx} \sinh(a + bx)}{2b^2} - \frac{(3d) \text{Subst} \left( \int e^{i \left( ia - \frac{ibc}{d} \right) - \frac{bx^2}{d}} dx, x, \sqrt{c + dx} \right)}{4b^2} \\
&= \frac{(c + dx)^{3/2} \cosh(a + bx)}{b} - \frac{3d^{3/2} e^{-a + \frac{bc}{d}} \sqrt{\pi} \operatorname{erf} \left( \frac{\sqrt{b} \sqrt{c + dx}}{\sqrt{d}} \right)}{8b^{5/2}} + \frac{3d^{3/2} e^{a - \frac{bc}{d}} \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{b} \sqrt{c + dx}}{\sqrt{d}} \right)}{8b^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0952787, size = 106, normalized size = 0.73

$$\frac{d\sqrt{c + dx} e^{-a - \frac{bc}{d}} \left( \frac{e^{\frac{2bc}{d}} \operatorname{Gamma} \left( \frac{5}{2}, \frac{b(c+dx)}{d} \right)}{\sqrt{\frac{b(c+dx)}{d}}} - \frac{e^{2a} \operatorname{Gamma} \left( \frac{5}{2}, -\frac{b(c+dx)}{d} \right)}{\sqrt{-\frac{b(c+dx)}{d}}} \right)}{2b^2}$$

Antiderivative was successfully verified.



[In] Integrate[(c + d\*x)^(3/2)\*Sinh[a + b\*x],x]

[Out] (d\*E^(-a - (b\*c)/d)\*Sqrt[c + d\*x]\*(-(E^(2\*a)\*Gamma[5/2, -((b\*(c + d\*x))/d)])/Sqrt[-((b\*(c + d\*x))/d)]) + (E^((2\*b\*c)/d)\*Gamma[5/2, (b\*(c + d\*x))/d])/Sqrt[(b\*(c + d\*x))/d))/(2\*b^2)

**Maple [F]** time = 0.026, size = 0, normalized size = 0.

$$\int (dx + c)^{\frac{3}{2}} \sinh(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^(3/2)\*sinh(b\*x+a),x)

[Out] int((d\*x+c)^(3/2)\*sinh(b\*x+a),x)

**Maxima [B]** time = 1.19344, size = 362, normalized size = 2.48

$$16(dx + c)^{\frac{5}{2}} \sinh(bx + a) + \frac{\left( \frac{15\sqrt{\pi}d^3 \operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}} - \frac{15\sqrt{\pi}d^3 \operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}} + \frac{2\left(4(dx+c)^{\frac{5}{2}}b^2d^2e^{\left(\frac{bc}{d}\right)} + 10(dx+c)^{\frac{3}{2}}bd^2e^{\left(\frac{bc}{d}\right)} + 15\sqrt{dx+cd^3}e^{\left(\frac{bc}{d}\right)}\right)}{b^3} \right)}{40d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(3/2)\*sinh(b\*x+a),x, algorithm="maxima")

[Out] 1/40\*(16\*(d\*x + c)^(5/2)\*sinh(b\*x + a) + (15\*sqrt(pi)\*d^3\*erf(sqrt(d\*x + c)\*sqrt(-b/d))\*e^(a - b\*c/d)/(b^3\*sqrt(-b/d)) - 15\*sqrt(pi)\*d^3\*erf(sqrt(d\*x + c)\*sqrt(b/d))\*e^(-a + b\*c/d)/(b^3\*sqrt(b/d)) + 2\*(4\*(d\*x + c)^(5/2)\*b^2\*d\*e^(b\*c/d) + 10\*(d\*x + c)^(3/2)\*b\*d^2\*e^(b\*c/d) + 15\*sqrt(d\*x + c)\*d^3\*e^(b\*c/d))\*e^(-a - (d\*x + c)\*b/d)/b^3 - 2\*(4\*(d\*x + c)^(5/2)\*b^2\*d\*e^a - 10\*(d\*x + c)^(3/2)\*b\*d^2\*e^a + 15\*sqrt(d\*x + c)\*d^3\*e^a)\*e^((d\*x + c)\*b/d - b\*c/d)/b^3)\*b/d/d

**Fricas [B]** time = 2.64267, size = 894, normalized size = 6.12

$$3\sqrt{\pi}\left(d^2 \cosh(bx+a) \cosh\left(-\frac{bc-ad}{d}\right) - d^2 \cosh(bx+a) \sinh\left(-\frac{bc-ad}{d}\right) + \left(d^2 \cosh\left(-\frac{bc-ad}{d}\right) - d^2 \sinh\left(-\frac{bc-ad}{d}\right)\right) \sinh(bx)$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(3/2)\*sinh(b\*x+a),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/8*(3*\sqrt{\pi}*(d^2*\cosh(b*x+a)*\cosh(-(b*c-a*d)/d) - d^2*\cosh(b*x+a) \\ & )*\sinh(-(b*c-a*d)/d) + (d^2*\cosh(-(b*c-a*d)/d) - d^2*\sinh(-(b*c-a*d)/ \\ & d))*\sinh(b*x+a))*\sqrt{b/d}*\operatorname{erf}(\sqrt{d*x+c}*\sqrt{b/d}) + 3*\sqrt{\pi}*(d^2 \\ & *\cosh(b*x+a)*\cosh(-(b*c-a*d)/d) + d^2*\cosh(b*x+a)*\sinh(-(b*c-a*d)/d \\ & ) + (d^2*\cosh(-(b*c-a*d)/d) + d^2*\sinh(-(b*c-a*d)/d))*\sinh(b*x+a))*\sqrt{ \\ & -b/d}*\operatorname{erf}(\sqrt{d*x+c}*\sqrt{-b/d}) - 2*(2*b^2*d*x + 2*b^2*c + (2*b^2*d*x \\ & + 2*b^2*c - 3*b*d)*\cosh(b*x+a)^2 + 2*(2*b^2*d*x + 2*b^2*c - 3*b*d)*\cosh \\ & (b*x+a)*\sinh(b*x+a) + (2*b^2*d*x + 2*b^2*c - 3*b*d)*\sinh(b*x+a)^2 + 3 \\ & *b*d)*\sqrt{d*x+c})/(b^3*\cosh(b*x+a) + b^3*\sinh(b*x+a)) \end{aligned}$$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (c+dx)^{\frac{3}{2}} \sinh(a+bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*(3/2)\*sinh(b\*x+a),x)

[Out] Integral((c + d\*x)\*\*(3/2)\*sinh(a + b\*x), x)

---

**Giac [A]** time = 1.33518, size = 273, normalized size = 1.87

$$\frac{3\sqrt{\pi}d^3 \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right) e^{\left(\frac{bc-ad}{d}\right)} - 3\sqrt{\pi}d^3 \operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right) e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{bd}b^2} + \frac{2\left(2(dx+c)^{\frac{3}{2}}bd-3\sqrt{dx+cd^2}\right) e^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b^2} + \frac{2\left(2(dx+c)^{\frac{3}{2}}bd+3\sqrt{dx+cd^2}\right) e^{\left(-\frac{(dx+c)b-bc+ad}{d}\right)}}{b^2}$$

$8d$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/8*(3*sqrt(pi)*d^3*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(sqrt
(b*d)*b^2) - 3*sqrt(pi)*d^3*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - a*d
)/d)/(sqrt(-b*d)*b^2) + 2*(2*(d*x + c)^(3/2)*b*d - 3*sqrt(d*x + c)*d^2)*e^(
((d*x + c)*b - b*c + a*d)/d)/b^2 + 2*(2*(d*x + c)^(3/2)*b*d + 3*sqrt(d*x +
c)*d^2)*e^(-((d*x + c)*b - b*c + a*d)/d)/b^2/d
```

### 3.40 $\int \sqrt{c + dx} \sinh(a + bx) dx$

**Optimal.** Leaf size=123

$$-\frac{\sqrt{\pi}\sqrt{d}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{\pi}\sqrt{d}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{c+dx}\cosh(a+bx)}{b}$$

[Out] (Sqrt[c + d\*x]\*Cosh[a + b\*x])/b - (Sqrt[d]\*E^(-a + (b\*c)/d)\*Sqrt[Pi]\*Erf[(Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(4\*b^(3/2)) - (Sqrt[d]\*E^(a - (b\*c)/d)\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(4\*b^(3/2))

**Rubi [A]** time = 0.187407, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3296, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{d}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{\pi}\sqrt{d}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{c+dx}\cosh(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[c + d\*x]\*Sinh[a + b\*x], x]

[Out] (Sqrt[c + d\*x]\*Cosh[a + b\*x])/b - (Sqrt[d]\*E^(-a + (b\*c)/d)\*Sqrt[Pi]\*Erf[(Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(4\*b^(3/2)) - (Sqrt[d]\*E^(a - (b\*c)/d)\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(4\*b^(3/2))

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3307

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Dist[I/2, Int[(c + d\*x)^m/(E^(I\*k\*Pi)\*E^(I\*(e + f\*x))), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(I\*k\*Pi)\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

#### Rule 2180

```
Int[(F_)^((g_.)*(e_.) + (f_.)*(x_))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

### Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]
```

### Rubi steps

$$\begin{aligned}
\int \sqrt{c+dx} \sinh(ax+bx) dx &= \frac{\sqrt{c+dx} \cosh(ax+bx)}{b} - \frac{d \int \frac{\cosh(ax+bx)}{\sqrt{c+dx}} dx}{2b} \\
&= \frac{\sqrt{c+dx} \cosh(ax+bx)}{b} - \frac{d \int \frac{e^{-i(i+ibx)}}{\sqrt{c+dx}} dx}{4b} - \frac{d \int \frac{e^{i(i+ibx)}}{\sqrt{c+dx}} dx}{4b} \\
&= \frac{\sqrt{c+dx} \cosh(ax+bx)}{b} - \frac{\text{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}} dx, x, \sqrt{c+dx}\right)}{2b} - \frac{\text{Subst}\left(\int e^{-i\left(ia-\frac{ibc}{d}\right)+\frac{bx^2}{d}} dx, x, \sqrt{c+dx}\right)}{2b} \\
&= \frac{\sqrt{c+dx} \cosh(ax+bx)}{b} - \frac{\sqrt{d} e^{-a+\frac{bc}{d}} \sqrt{\pi} \text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{d} e^{a-\frac{bc}{d}} \sqrt{\pi} \text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0779187, size = 104, normalized size = 0.85

$$\frac{\sqrt{c+dx} e^{-a-\frac{bc}{d}} \left( \frac{e^{2a} \text{Gamma}\left(\frac{3}{2}, -\frac{b(c+dx)}{d}\right)}{\sqrt{-\frac{b(c+dx)}{d}}} + \frac{e^{\frac{2bc}{d}} \text{Gamma}\left(\frac{3}{2}, \frac{b(c+dx)}{d}\right)}{\sqrt{\frac{b(c+dx)}{d}}} \right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Sinh[a + b*x], x]
```

[Out]  $(E^{-(a - (b*c)/d)}*\text{Sqrt}[c + d*x]*((E^{(2*a)}*\text{Gamma}[3/2, -((b*(c + d*x))/d)])/\text{Sqrt}[-((b*(c + d*x))/d)] + (E^{((2*b*c)/d)}*\text{Gamma}[3/2, (b*(c + d*x))/d])/(\text{Sqrt}[(b*(c + d*x))/d]))/(2*b)$

**Maple [F]** time = 0.023, size = 0, normalized size = 0.

$$\int \sinh(bx + a) \sqrt{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(b*x+a)*(d*x+c)^(1/2), x)`

[Out] `int(sinh(b*x+a)*(d*x+c)^(1/2), x)`

**Maxima [B]** time = 1.15584, size = 311, normalized size = 2.53

$$8(dx + c)^{\frac{3}{2}} \sinh(bx + a) - \frac{\left( \frac{3\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(\frac{a-bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} + \frac{3\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} - \frac{2\left(2(dx+c)^{\frac{3}{2}}bde^{\left(\frac{bc}{d}\right)} + 3\sqrt{dx+cd}e^{\left(\frac{bc}{d}\right)}\right)e^{\left(-a-\frac{(dx+c)b}{d}\right)}}{b^2} + \frac{2\left(2(dx+c)^{\frac{3}{2}}\right)}{b^2} \right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)*(d*x+c)^(1/2), x, algorithm="maxima")`

[Out]  $\frac{1}{12}*(8*(d*x + c)^{(3/2)}*\sinh(b*x + a) - (3*\text{sqrt}(\pi)*d^2*\text{erf}(\text{sqrt}(d*x + c)*\text{sqrt}(-b/d))*e^{(a - b*c/d)}/(b^2*\text{sqrt}(-b/d)) + 3*\text{sqrt}(\pi)*d^2*\text{erf}(\text{sqrt}(d*x + c)*\text{sqrt}(b/d))*e^{(-a + b*c/d)}/(b^2*\text{sqrt}(b/d)) - 2*(2*(d*x + c)^{(3/2)}*b*d*e^{(b*c/d)} + 3*\text{sqrt}(d*x + c)*d^2*e^{(b*c/d)})*e^{(-a - (d*x + c)*b/d)}/b^2 + 2*(2*(d*x + c)^{(3/2)}*b*d*e^a - 3*\text{sqrt}(d*x + c)*d^2*e^a)*e^{((d*x + c)*b/d - b*c/d)}/b^2)*b/d)/d$

**Fricas [B]** time = 2.73369, size = 718, normalized size = 5.84

$$\sqrt{\pi} \left( d \cosh(bx + a) \cosh\left(-\frac{bc-ad}{d}\right) - d \cosh(bx + a) \sinh\left(-\frac{bc-ad}{d}\right) + \left( d \cosh\left(-\frac{bc-ad}{d}\right) - d \sinh\left(-\frac{bc-ad}{d}\right) \right) \sinh(bx + a) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)\*(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] 
$$-1/4*(\sqrt{\pi}*(d*\cosh(b*x + a)*\cosh(-(b*c - a*d)/d) - d*\cosh(b*x + a)*\sinh(-(b*c - a*d)/d) + (d*\cosh(-(b*c - a*d)/d) - d*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{b/d}*\operatorname{erf}(\sqrt{d*x + c}*\sqrt{b/d}) - \sqrt{\pi}*(d*\cosh(b*x + a)*\cosh(-(b*c - a*d)/d) + d*\cosh(b*x + a)*\sinh(-(b*c - a*d)/d) + (d*\cosh(-(b*c - a*d)/d) + d*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{-b/d}*\operatorname{erf}(\sqrt{d*x + c}*\sqrt{-b/d}) - 2*(b*\cosh(b*x + a)^2 + 2*b*\cosh(b*x + a)*\sinh(b*x + a) + b*\sinh(b*x + a)^2 + b)*\sqrt{d*x + c})/(b^2*\cosh(b*x + a) + b^2*\sinh(b*x + a))$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c + dx} \sinh(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)\*(d\*x+c)\*\*(1/2),x)

[Out] Integral(sqrt(c + d\*x)\*sinh(a + b\*x), x)

**Giac [A]** time = 1.27523, size = 227, normalized size = 1.85

$$\frac{\frac{\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bdb}} + \frac{\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bdb}}}{4d} + \frac{2\sqrt{dx+c}d e^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b} + \frac{2\sqrt{dx+c}d e^{\left(-\frac{(dx+c)b-bc+ad}{d}\right)}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)\*(d\*x+c)^(1/2),x, algorithm="giac")

[Out] 
$$1/4*(\sqrt{\pi}*d^2*\operatorname{erf}(-\sqrt{b*d}*\sqrt{d*x + c}/d)*e^{((b*c - a*d)/d)/(\sqrt{b*d}*b)} + \sqrt{\pi}*d^2*\operatorname{erf}(-\sqrt{-b*d}*\sqrt{d*x + c}/d)*e^{(-(b*c - a*d)/d)/(\sqrt{-b*d}*b)} + 2*\sqrt{d*x + c}*d*e^{(((d*x + c)*b - b*c + a*d)/d)/b} + 2*\sqrt{d*x + c}*d*e^{(-(d*x + c)*b - b*c + a*d)/d)/b)/d$$

### 3.41 $\int \frac{\sinh(ax+bx)}{\sqrt{c+dx}} dx$

**Optimal.** Leaf size=104

$$\frac{\sqrt{\pi} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} - \frac{\sqrt{\pi} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

[Out]  $-(E^{-a + (b*c)/d} * \operatorname{Sqrt}[Pi] * \operatorname{Erf}[(\operatorname{Sqrt}[b] * \operatorname{Sqrt}[c + d*x]) / \operatorname{Sqrt}[d]]) / (2 * \operatorname{Sqrt}[b] * \operatorname{Sqrt}[d]) + (E^{a - (b*c)/d} * \operatorname{Sqrt}[Pi] * \operatorname{Erfi}[(\operatorname{Sqrt}[b] * \operatorname{Sqrt}[c + d*x]) / \operatorname{Sqrt}[d]]) / (2 * \operatorname{Sqrt}[b] * \operatorname{Sqrt}[d])$

**Rubi [A]** time = 0.134614, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} - \frac{\sqrt{\pi} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sinh}[a + b*x] / \operatorname{Sqrt}[c + d*x], x]$

[Out]  $-(E^{-a + (b*c)/d} * \operatorname{Sqrt}[Pi] * \operatorname{Erf}[(\operatorname{Sqrt}[b] * \operatorname{Sqrt}[c + d*x]) / \operatorname{Sqrt}[d]]) / (2 * \operatorname{Sqrt}[b] * \operatorname{Sqrt}[d]) + (E^{a - (b*c)/d} * \operatorname{Sqrt}[Pi] * \operatorname{Erfi}[(\operatorname{Sqrt}[b] * \operatorname{Sqrt}[c + d*x]) / \operatorname{Sqrt}[d]]) / (2 * \operatorname{Sqrt}[b] * \operatorname{Sqrt}[d])$

#### Rule 3308

$\operatorname{Int}[(c_.) + (d_.) * (x_.)^{(m_.)} * \sin[(e_.) + (f_.) * (x_.)], x\_Symbol] :> \operatorname{Dist}[I/2, \operatorname{Int}[(c + d*x)^m / E^{I*(e + f*x)}, x], x] - \operatorname{Dist}[I/2, \operatorname{Int}[(c + d*x)^m * E^{I*(e + f*x)}, x], x] /;$   $\operatorname{FreeQ}\{c, d, e, f, m\}, x]$

#### Rule 2180

$\operatorname{Int}[(F_.)^{(g_.)} * ((e_.) + (f_.) * (x_.))) / \operatorname{Sqrt}[(c_.) + (d_.) * (x_.)], x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{g*(e - (c*f)/d) + (f*g*x^2)/d}, x], x, \operatorname{Sqrt}[c + d*x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\$UseGamma == True$

#### Rule 2204



```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

### Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F])], 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

### Rubi steps

$$\begin{aligned} \int \frac{\sinh(a + bx)}{\sqrt{c + dx}} dx &= \frac{1}{2} \int \frac{e^{-i(i a + i b x)}}{\sqrt{c + dx}} dx - \frac{1}{2} \int \frac{e^{i(i a + i b x)}}{\sqrt{c + dx}} dx \\ &= -\frac{\text{Subst}\left(\int e^{i\left(i a - \frac{i b c}{d}\right) - \frac{b x^2}{d}} dx, x, \sqrt{c + dx}\right)}{d} + \frac{\text{Subst}\left(\int e^{-i\left(i a - \frac{i b c}{d}\right) + \frac{b x^2}{d}} dx, x, \sqrt{c + dx}\right)}{d} \\ &= -\frac{e^{-a + \frac{bc}{d}} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{b} \sqrt{c + dx}}{\sqrt{d}}\right)}{2\sqrt{b} \sqrt{d}} + \frac{e^{a - \frac{bc}{d}} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b} \sqrt{c + dx}}{\sqrt{d}}\right)}{2\sqrt{b} \sqrt{d}} \end{aligned}$$

**Mathematica [A]** time = 0.0376037, size = 104, normalized size = 1.

$$\frac{e^{-a - \frac{bc}{d}} \left( e^{2a} \sqrt{-\frac{b(c+dx)}{d}} \operatorname{Gamma}\left(\frac{1}{2}, -\frac{b(c+dx)}{d}\right) + e^{\frac{2bc}{d}} \sqrt{\frac{b(c+dx)}{d}} \operatorname{Gamma}\left(\frac{1}{2}, \frac{b(c+dx)}{d}\right) \right)}{2b\sqrt{c + dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[a + b*x]/Sqrt[c + d*x], x]
```

```
[Out] (E^(-a - (b*c)/d)*(E^(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*
x))/d)] + E^((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d]))
/(2*b*Sqrt[c + d*x])
```

**Maple [F]** time = 0.03, size = 0, normalized size = 0.

$$\int \sinh(bx + a) \frac{1}{\sqrt{dx + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(b*x+a)/(d*x+c)^(1/2),x)`

[Out] `int(sinh(b*x+a)/(d*x+c)^(1/2),x)`

**Maxima [B]** time = 1.15108, size = 244, normalized size = 2.35

$$4\sqrt{dx+c}\sinh(bx+a) + \frac{\left( \frac{\sqrt{\pi d} \operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right) e^{\left(a-\frac{bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}} - \frac{\sqrt{\pi d} \operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right) e^{\left(-a+\frac{bc}{d}\right)}}{b\sqrt{\frac{b}{d}}} - \frac{2\sqrt{dx+c}e^{\left(a+\frac{(dx+c)b-bc}{d}\right)}}{b} + \frac{2\sqrt{dx+c}e^{\left(-a-\frac{(dx+c)b+bc}{d}\right)}}{b} \right) b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")`

[Out] `1/2*(4*sqrt(d*x + c)*sinh(b*x + a) + (sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/(b*sqrt(-b/d)) - sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b*sqrt(b/d)) - 2*sqrt(d*x + c)*d*e^(a + (d*x + c)*b/d - b*c/d)/b + 2*sqrt(d*x + c)*d*e^(-a - (d*x + c)*b/d + b*c/d)/b)*b/d`

**Fricas [A]** time = 2.62937, size = 273, normalized size = 2.62

$$\frac{\sqrt{\pi}\sqrt{\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right) - \sinh\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right) + \sqrt{\pi}\sqrt{-\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right) + \sinh\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")`

[Out] `-1/2*(sqrt(pi)*sqrt(b/d)*(cosh(-(b*c - a*d)/d) - sinh(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(b/d)) + sqrt(pi)*sqrt(-b/d)*(cosh(-(b*c - a*d)/d) + sinh(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-b/d))/b`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(a + bx)}{\sqrt{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)/(d\*x+c)\*\*(1/2), x)

[Out] Integral(sinh(a + b\*x)/sqrt(c + d\*x), x)

**Giac [A]** time = 1.21713, size = 124, normalized size = 1.19

$$\frac{\frac{\sqrt{\pi d} \operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right) e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}} - \frac{\sqrt{\pi d} \operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right) e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)/(d\*x+c)^(1/2), x, algorithm="giac")

[Out] 1/2\*(sqrt(pi)\*d\*erf(-sqrt(b\*d)\*sqrt(d\*x + c)/d)\*e^((b\*c - a\*d)/d)/sqrt(b\*d) - sqrt(pi)\*d\*erf(-sqrt(-b\*d)\*sqrt(d\*x + c)/d)\*e^(-(b\*c - a\*d)/d)/sqrt(-b\*d))/d

### 3.42 $\int \frac{\sinh(ax+bx)}{(c+dx)^{3/2}} dx$

**Optimal.** Leaf size=118

$$\frac{\sqrt{\pi}\sqrt{b}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{\pi}\sqrt{b}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sinh(ax+bx)}{d\sqrt{c+dx}}$$

[Out] (Sqrt[b]\*E^(-a + (b\*c)/d)\*Sqrt[Pi]\*Erf[(Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/d^(3/2) + (Sqrt[b]\*E^(a - (b\*c)/d)\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/d^(3/2) - (2\*Sinh[a + b\*x])/(d\*Sqrt[c + d\*x])

**Rubi [A]** time = 0.200403, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3297, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\sqrt{b}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{\pi}\sqrt{b}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sinh(ax+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[a + b\*x]/(c + d\*x)^(3/2), x]

[Out] (Sqrt[b]\*E^(-a + (b\*c)/d)\*Sqrt[Pi]\*Erf[(Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/d^(3/2) + (Sqrt[b]\*E^(a - (b\*c)/d)\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/d^(3/2) - (2\*Sinh[a + b\*x])/(d\*Sqrt[c + d\*x])

#### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

#### Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_)*((e_) + (f_)*(x_)))/Sqrt[(c_) + (d_)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_) + (b_)*((c_) + (d_)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_) + (b_)*((c_) + (d_)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}} dx &= -\frac{2\sinh(a+bx)}{d\sqrt{c+dx}} + \frac{(2b) \int \frac{\cosh(a+bx)}{\sqrt{c+dx}} dx}{d} \\
&= -\frac{2\sinh(a+bx)}{d\sqrt{c+dx}} + \frac{b \int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}} dx}{d} + \frac{b \int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}} dx}{d} \\
&= -\frac{2\sinh(a+bx)}{d\sqrt{c+dx}} + \frac{(2b) \text{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}-\frac{bx^2}{d}\right)} dx, x, \sqrt{c+dx}\right)}{d^2} + \frac{(2b) \text{Subst}\left(\int e^{-i\left(ia-\frac{ibc}{d}+\frac{bx^2}{d}\right)} dx, x, \sqrt{c+dx}\right)}{d^2} \\
&= \frac{\sqrt{b}e^{-a+\frac{bc}{d}} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{b}e^{a-\frac{bc}{d}} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sinh(a+bx)}{d\sqrt{c+dx}}
\end{aligned}$$

**Mathematica [A]** time = 0.148855, size = 120, normalized size = 1.02

$$\frac{e^{-a-\frac{bc}{d}} \left( e^{2a} \sqrt{-\frac{b(c+dx)}{d}} \operatorname{Gamma}\left(\frac{1}{2}, -\frac{b(c+dx)}{d}\right) - e^{\frac{2bc}{d}} \sqrt{\frac{b(c+dx)}{d}} \operatorname{Gamma}\left(\frac{1}{2}, \frac{b(c+dx)}{d}\right) - 2e^{a+\frac{bc}{d}} \sinh(a+bx) \right)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[a + b\*x]/(c + d\*x)^(3/2), x]

[Out]  $(E^{-(a - (b*c)/d)}*(E^{(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*x))/d)] - E^{((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]}*Gamma[1/2, (b*(c + d*x))/d] - 2*E^{(a + (b*c)/d)*Sinh[a + b*x]}))/(d*Sqrt[c + d*x])$

**Maple [F]** time = 0.023, size = 0, normalized size = 0.

$$\int \sinh(bx + a)(dx + c)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(b*x+a)/(d*x+c)^(3/2),x)`

[Out] `int(sinh(b*x+a)/(d*x+c)^(3/2),x)`

**Maxima [A]** time = 1.11129, size = 139, normalized size = 1.18

$$\frac{\left( \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{\sqrt{-\frac{b}{d}}} + \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{\sqrt{\frac{b}{d}}} \right) b}{d} - \frac{2 \sinh(bx+a)}{\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")`

[Out]  $((\sqrt{\pi})*\operatorname{erf}(\sqrt{d*x + c})*\sqrt{-b/d})*e^{(a - b*c/d)}/\sqrt{-b/d} + \sqrt{\pi})*\operatorname{erf}(\sqrt{d*x + c})*\sqrt{b/d})*e^{(-a + b*c/d)}/\sqrt{b/d})*b/d - 2*\sinh(b*x + a)/\sqrt{d*x + c}))/d$

**Fricas [B]** time = 2.76847, size = 814, normalized size = 6.9

$$\sqrt{\pi} \left( (dx + c) \cosh(bx + a) \cosh\left(-\frac{bc-ad}{d}\right) - (dx + c) \cosh(bx + a) \sinh\left(-\frac{bc-ad}{d}\right) \right) + \left( (dx + c) \cosh\left(-\frac{bc-ad}{d}\right) - (dx + c) \sinh\left(-\frac{bc-ad}{d}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)/(d\*x+c)^(3/2),x, algorithm="fricas")

[Out] (sqrt(pi)\*((d\*x + c)\*cosh(b\*x + a)\*cosh(-(b\*c - a\*d)/d) - (d\*x + c)\*cosh(b\*x + a)\*sinh(-(b\*c - a\*d)/d) + ((d\*x + c)\*cosh(-(b\*c - a\*d)/d) - (d\*x + c)\*sinh(-(b\*c - a\*d)/d))\*sinh(b\*x + a)\*sqrt(b/d)\*erf(sqrt(d\*x + c)\*sqrt(b/d)) - sqrt(pi)\*((d\*x + c)\*cosh(b\*x + a)\*cosh(-(b\*c - a\*d)/d) + (d\*x + c)\*cosh(b\*x + a)\*sinh(-(b\*c - a\*d)/d) + ((d\*x + c)\*cosh(-(b\*c - a\*d)/d) + (d\*x + c)\*sinh(-(b\*c - a\*d)/d))\*sinh(b\*x + a))\*sqrt(-b/d)\*erf(sqrt(d\*x + c)\*sqrt(-b/d)) - sqrt(d\*x + c)\*(cosh(b\*x + a)^2 + 2\*cosh(b\*x + a)\*sinh(b\*x + a) + sinh(b\*x + a)^2 - 1))/((d^2\*x + c\*d)\*cosh(b\*x + a) + (d^2\*x + c\*d)\*sinh(b\*x + a))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(a + bx)}{(c + dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)/(d\*x+c)\*\*(3/2),x)

[Out] Integral(sinh(a + b\*x)/(c + d\*x)\*\*(3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx + a)}{(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)/(d\*x+c)^(3/2),x, algorithm="giac")

[Out] integrate(sinh(b\*x + a)/(d\*x + c)^(3/2), x)

### 3.43 $\int \frac{\sinh(ax+bx)}{(c+dx)^{5/2}} dx$

**Optimal.** Leaf size=149

$$-\frac{2\sqrt{\pi}b^{3/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2\sqrt{\pi}b^{3/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{4b\cosh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}}$$

[Out]  $(-4*b*\operatorname{Cosh}[a + b*x])/(3*d^2*\operatorname{Sqrt}[c + d*x]) - (2*b^{(3/2)}*E^{(-a + (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/\operatorname{Sqrt}[d]])/(3*d^{(5/2)}) + (2*b^{(3/2)}*E^{(a - (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/\operatorname{Sqrt}[d]])/(3*d^{(5/2)}) - (2*\operatorname{Sinh}[a + b*x])/(3*d*(c + d*x)^{(3/2)})$

**Rubi [A]** time = 0.250753, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3297, 3308, 2180, 2204, 2205}

$$-\frac{2\sqrt{\pi}b^{3/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2\sqrt{\pi}b^{3/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{4b\cosh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sinh(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sinh}[a + b*x]/(c + d*x)^{(5/2)}, x]$

[Out]  $(-4*b*\operatorname{Cosh}[a + b*x])/(3*d^2*\operatorname{Sqrt}[c + d*x]) - (2*b^{(3/2)}*E^{(-a + (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/\operatorname{Sqrt}[d]])/(3*d^{(5/2)}) + (2*b^{(3/2)}*E^{(a - (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/\operatorname{Sqrt}[d]])/(3*d^{(5/2)}) - (2*\operatorname{Sinh}[a + b*x])/(3*d*(c + d*x)^{(3/2)})$

#### Rule 3297

$\operatorname{Int}[(c + d*x)^m \sin(e + f*x), x] := \operatorname{Simp}[(c + d*x)^{m+1} \operatorname{Sin}[e + f*x] / (d*(m+1)), x] - \operatorname{Dist}[f / (d*(m+1)), \operatorname{Int}[(c + d*x)^m \operatorname{Cos}[e + f*x], x], x] /;$  FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

#### Rule 3308

$\operatorname{Int}[(c + d*x)^m \sin(e + f*x), x] := \operatorname{Dist}[I/2, \operatorname{Int}[(c + d*x)^m / E^{I*(e + f*x)}, x], x] - \operatorname{Dist}[I/2, \operatorname{Int}[(c + d*x)^m * E^{I*(e + f*x)}, x], x]$



$I*(e + f*x)), x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x]$

### Rule 2180

$\text{Int}[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/\text{Sqrt}[(c_.) + (d_.)*(x_)], x\_Symbol] :> \text{Dist}[2/d, \text{Subst}[\text{Int}[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{F, c, d, e, f, g\}, x] \&\& \text{!}\$UseGamma == \text{True}$

### Rule 2204

$\text{Int}[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2}), x\_Symbol] :> \text{Simp}[(F^a*\text{Sqrt}[\text{Pi}]*\text{Erfi}[(c + d*x)*\text{Rt}[b*\text{Log}[F], 2]])/(2*d*\text{Rt}[b*\text{Log}[F], 2]), x] /; \text{FreeQ}\{F, a, b, c, d\}, x] \&\& \text{PosQ}[b]$

### Rule 2205

$\text{Int}[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2}), x\_Symbol] :> \text{Simp}[(F^a*\text{Sqrt}[\text{Pi}]*\text{Erf}[(c + d*x)*\text{Rt}[-(b*\text{Log}[F]), 2]])/(2*d*\text{Rt}[-(b*\text{Log}[F]), 2]), x] /; \text{FreeQ}\{F, a, b, c, d\}, x] \&\& \text{NegQ}[b]$

### Rubi steps

$$\begin{aligned} \int \frac{\sinh(a + bx)}{(c + dx)^{5/2}} dx &= -\frac{2 \sinh(a + bx)}{3d(c + dx)^{3/2}} + \frac{(2b) \int \frac{\cosh(a + bx)}{(c + dx)^{3/2}} dx}{3d} \\ &= -\frac{4b \cosh(a + bx)}{3d^2 \sqrt{c + dx}} - \frac{2 \sinh(a + bx)}{3d(c + dx)^{3/2}} + \frac{(4b^2) \int \frac{\sinh(a + bx)}{\sqrt{c + dx}} dx}{3d^2} \\ &= -\frac{4b \cosh(a + bx)}{3d^2 \sqrt{c + dx}} - \frac{2 \sinh(a + bx)}{3d(c + dx)^{3/2}} + \frac{(2b^2) \int \frac{e^{-i(ia + ibx)}}{\sqrt{c + dx}} dx}{3d^2} - \frac{(2b^2) \int \frac{e^{i(ia + ibx)}}{\sqrt{c + dx}} dx}{3d^2} \\ &= -\frac{4b \cosh(a + bx)}{3d^2 \sqrt{c + dx}} - \frac{2 \sinh(a + bx)}{3d(c + dx)^{3/2}} - \frac{(4b^2) \text{Subst} \left( \int e^{i \left( ia - \frac{ibc}{d} \right) - \frac{bx^2}{d}} dx, x, \sqrt{c + dx} \right)}{3d^3} + \frac{(4b^2) \text{Subst} \left( \int e^{i \left( ia + \frac{ibc}{d} \right) - \frac{bx^2}{d}} dx, x, \sqrt{c + dx} \right)}{3d^3} \\ &= -\frac{4b \cosh(a + bx)}{3d^2 \sqrt{c + dx}} - \frac{2b^{3/2} e^{-a + \frac{bc}{d}} \sqrt{\pi} \text{erf} \left( \frac{\sqrt{b} \sqrt{c + dx}}{\sqrt{d}} \right)}{3d^{5/2}} + \frac{2b^{3/2} e^{a - \frac{bc}{d}} \sqrt{\pi} \text{erfi} \left( \frac{\sqrt{b} \sqrt{c + dx}}{\sqrt{d}} \right)}{3d^{5/2}} - \frac{2 \sinh(a + bx)}{3d(c + dx)^{3/2}} \end{aligned}$$

**Mathematica [A]** time = 0.754084, size = 161, normalized size = 1.08

$$\frac{2b \left( \frac{e^a \left( e^{-\frac{bc}{d}} \sqrt{\frac{b(c+dx)}{d}} \Gamma\left(\frac{1}{2}, -\frac{b(c+dx)}{d}\right) - e^{bx} \right)}{d\sqrt{c+dx}} + \frac{e^{-a-bx} \left( e^{b\left(\frac{c}{d}+x\right)} \sqrt{\frac{b(c+dx)}{d}} \Gamma\left(\frac{1}{2}, \frac{b(c+dx)}{d}\right) - 1 \right)}{d\sqrt{c+dx}} \right)}{3d} - \frac{2 \sinh(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[a + b\*x]/(c + d\*x)^(5/2), x]

[Out] (2\*b\*((E^a\*(-E^(b\*x) + (Sqrt[-((b\*(c + d\*x))/d)]\*Gamma[1/2, -((b\*(c + d\*x))/d)]) / E^((b\*c)/d)))/(d\*Sqrt[c + d\*x]) + (E^(-a - b\*x)\*(-1 + E^(b\*(c/d + x)) \* Sqrt[(b\*(c + d\*x))/d]\*Gamma[1/2, (b\*(c + d\*x))/d]))/(d\*Sqrt[c + d\*x]))/(3\*d) - (2\*Sinh[a + b\*x])/(3\*d\*(c + d\*x)^(3/2))

**Maple [F]** time = 0.025, size = 0, normalized size = 0.

$$\int \sinh(bx + a) (dx + c)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(b\*x+a)/(d\*x+c)^(5/2), x)

[Out] int(sinh(b\*x+a)/(d\*x+c)^(5/2), x)

**Maxima [A]** time = 1.33558, size = 154, normalized size = 1.03

$$\frac{\left( \frac{\sqrt{\frac{(dx+c)b}{d}} e^{\left(-a+\frac{bc}{d}\right)} \Gamma\left(-\frac{1}{2}, \frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}} + \frac{\sqrt{-\frac{(dx+c)b}{d}} e^{\left(a-\frac{bc}{d}\right)} \Gamma\left(-\frac{1}{2}, -\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}} \right) b}{d} + \frac{2 \sinh(bx+a)}{(dx+c)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)/(d\*x+c)^(5/2), x, algorithm="maxima")

```
[Out] -1/3*((sqrt((d*x + c)*b/d)*e^(-a + b*c/d)*gamma(-1/2, (d*x + c)*b/d)/sqrt(d
*x + c) + sqrt(-(d*x + c)*b/d)*e^(a - b*c/d)*gamma(-1/2, -(d*x + c)*b/d)/sq
rt(d*x + c))*b/d + 2*sinh(b*x + a)/(d*x + c)^(3/2))/d
```

**Fricas [B]** time = 2.7866, size = 1226, normalized size = 8.23

$$2\sqrt{\pi}\left((bd^2x^2 + 2bcdx + bc^2)\cosh(bx + a)\cosh\left(-\frac{bc-ad}{d}\right) - (bd^2x^2 + 2bcdx + bc^2)\cosh(bx + a)\sinh\left(-\frac{bc-ad}{d}\right) + \left(ba\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(2*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-(b*c
- a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-(b*c - a*d)
/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c - a*d)/d) - (b*d^2*x^2 +
2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(
d*x + c)*sqrt(b/d)) + 2*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x
+ a)*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*s
inh(-(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c - a*d)/d)
+ (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqr
t(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) + (2*b*d*x + (2*b*d*x + 2*b*c + d)*co
sh(b*x + a)^2 + 2*(2*b*d*x + 2*b*c + d)*cosh(b*x + a)*sinh(b*x + a) + (2*b*
d*x + 2*b*c + d)*sinh(b*x + a)^2 + 2*b*c - d)*sqrt(d*x + c))/((d^4*x^2 + 2*
c*d^3*x + c^2*d^2)*cosh(b*x + a) + (d^4*x^2 + 2*c*d^3*x + c^2*d^2)*sinh(b*x
+ a))
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(a + bx)}{(c + dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)**(5/2),x)
```

```
[Out] Integral(sinh(a + b*x)/(c + d*x)**(5/2), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx + a)}{(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(b*x + a)/(d*x + c)^(5/2), x)
```

$$3.44 \quad \int \frac{\sinh(ax+bx)}{(c+dx)^{7/2}} dx$$

**Optimal.** Leaf size=174

$$\frac{4\sqrt{\pi}b^{5/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{4\sqrt{\pi}b^{5/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} - \frac{8b^2\sinh(ax+bx)}{15d^3\sqrt{c+dx}} - \frac{4b\cosh(ax+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh(ax+bx)}{5d(c+dx)^{5/2}}$$

```
[Out] (-4*b*Cosh[a + b*x])/(15*d^2*(c + d*x)^(3/2)) + (4*b^(5/2)*E^(-a + (b*c)/d)
*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]/(15*d^(7/2)) + (4*b^(5/2)*E
^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]/(15*d^(7/2))
- (2*Sinh[a + b*x])/(5*d*(c + d*x)^(5/2)) - (8*b^2*Sinh[a + b*x])/(15*d^3*
Sqrt[c + d*x])
```

**Rubi [A]** time = 0.308154, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {3297, 3307, 2180, 2204, 2205}

$$\frac{4\sqrt{\pi}b^{5/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{4\sqrt{\pi}b^{5/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} - \frac{8b^2\sinh(ax+bx)}{15d^3\sqrt{c+dx}} - \frac{4b\cosh(ax+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh(ax+bx)}{5d(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]/(c + d*x)^(7/2), x]
```

```
[Out] (-4*b*Cosh[a + b*x])/(15*d^2*(c + d*x)^(3/2)) + (4*b^(5/2)*E^(-a + (b*c)/d)
*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]/(15*d^(7/2)) + (4*b^(5/2)*E
^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]/(15*d^(7/2))
- (2*Sinh[a + b*x])/(5*d*(c + d*x)^(5/2)) - (8*b^2*Sinh[a + b*x])/(15*d^3*
Sqrt[c + d*x])
```

#### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

#### Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
]:> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(2)), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

### Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(2)), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sinh(a+bx)}{(c+dx)^{7/2}} dx &= -\frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}} + \frac{(2b) \int \frac{\cosh(a+bx)}{(c+dx)^{5/2}} dx}{5d} \\
&= -\frac{4b \cosh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}} + \frac{(4b^2) \int \frac{\sinh(a+bx)}{(c+dx)^{3/2}} dx}{15d^2} \\
&= -\frac{4b \cosh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2 \sinh(a+bx)}{15d^3\sqrt{c+dx}} + \frac{(8b^3) \int \frac{\cosh(a+bx)}{\sqrt{c+dx}} dx}{15d^3} \\
&= -\frac{4b \cosh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2 \sinh(a+bx)}{15d^3\sqrt{c+dx}} + \frac{(4b^3) \int \frac{e^{-i(i a + i b x)}}{\sqrt{c+dx}} dx}{15d^3} + \frac{(4b^3) \int \frac{e^{i(i a + i b x)}}{\sqrt{c+dx}} dx}{15d^3} \\
&= -\frac{4b \cosh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2 \sinh(a+bx)}{15d^3\sqrt{c+dx}} + \frac{(8b^3) \text{Subst} \left( \int e^{i \left( i a - \frac{i b c}{d} \right) - \frac{b x^2}{d}} dx, x, \sqrt{c+dx} \right)}{15d^4} \\
&= -\frac{4b \cosh(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{4b^{5/2} e^{-a + \frac{bc}{d}} \sqrt{\pi} \operatorname{erf} \left( \frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{15d^{7/2}} + \frac{4b^{5/2} e^{-a - \frac{bc}{d}} \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{15d^{7/2}} - \frac{2\sinh(a+bx)}{5d(c+dx)^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.572398, size = 168, normalized size = 0.97

$$\frac{2 \left( -b(c+dx) \left( e^{a - \frac{bc}{d}} \left( 2d \left( -\frac{b(c+dx)}{d} \right)^{3/2} \operatorname{Gamma} \left( \frac{1}{2}, -\frac{b(c+dx)}{d} \right) + e^{b \left( \frac{c}{d} + x \right)} (2b(c+dx) + d) \right) + e^{-a-bx} \left( 2d e^{b \left( \frac{c}{d} + x \right)} \left( \frac{b(c+dx)}{d} \right)^{3/2} \operatorname{Gamma} \left( \frac{1}{2}, \frac{b(c+dx)}{d} \right) \right) \right)}{15d^3(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[a + b\*x]/(c + d\*x)^(7/2), x]

[Out] (2\*(-(b\*(c + d\*x))\*(E^(a - (b\*c)/d))\*(E^(b\*(c/d + x))\*(d + 2\*b\*(c + d\*x))) + 2\*d\*(-((b\*(c + d\*x))/d)^(3/2)\*Gamma[1/2, -((b\*(c + d\*x))/d)]) + E^(-a - b\*x)\*(d - 2\*b\*(c + d\*x) + 2\*d\*E^(b\*(c/d + x))\*((b\*(c + d\*x))/d)^(3/2)\*Gamma[1/2, (b\*(c + d\*x))/d])) - 3\*d^2\*Sinh[a + b\*x])/(15\*d^3\*(c + d\*x)^(5/2))

**Maple [F]** time = 0.026, size = 0, normalized size = 0.

$$\int \sinh(bx + a) (dx + c)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(b*x+a)/(d*x+c)^(7/2),x)`

[Out] `int(sinh(b*x+a)/(d*x+c)^(7/2),x)`

**Maxima [A]** time = 1.34525, size = 154, normalized size = 0.89

$$\frac{\left( \frac{\left( \frac{dx+c}{d} \right)^{\frac{3}{2}} e^{-a+\frac{bc}{d}} \Gamma\left(-\frac{3}{2}, \frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}} + \frac{\left( -\frac{dx+c}{d} \right)^{\frac{3}{2}} e^{a-\frac{bc}{d}} \Gamma\left(-\frac{3}{2}, -\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}} \right) b}{d} + \frac{2 \sinh(bx+a)}{(dx+c)^{\frac{5}{2}}}$$


---


$$5d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)/(d*x+c)^(7/2),x, algorithm="maxima")`

[Out]  $-1/5 * (((d*x + c)*b/d)^{(3/2)} * e^{(-a + b*c/d)} * \text{gamma}(-3/2, (d*x + c)*b/d) / (d*x + c)^{(3/2)} + (-d*x + c)*b/d)^{(3/2)} * e^{(a - b*c/d)} * \text{gamma}(-3/2, -(d*x + c)*b/d) / (d*x + c)^{(3/2))} * b/d + 2 * \sinh(b*x + a) / (d*x + c)^{(5/2)} / d$

**Fricas [B]** time = 2.9549, size = 1820, normalized size = 10.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)/(d*x+c)^(7/2),x, algorithm="fricas")`

[Out]  $1/15 * (4 * \text{sqrt}(\pi) * ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3) * \cosh(b*x + a) * \cosh(-(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3) * \cosh(b*x + a) * \sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3) * \cosh(-(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3) * \sinh(-(b*c - a*d)/d)) * \sinh(b*x + a) * \text{sqrt}(b/d) * \text{erf}(\text{sqrt}(d*x + c) * \text{sqrt}(b/d)) - 4 * \text{sqrt}(\pi) * ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3) * \cosh(b*x + a) * \cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3) * \cosh(b*x + a) * \sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3) * \cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3) * \sinh(-(b*c - a*d)/d)) * \sinh(b*x + a) * \text{sqrt}(-b/d) * \text{erf}(\text{sqrt}(\text{sqrt}(d*x + c) * \text{sqrt}(-b/d))))$



$$d*x + c)*\sqrt{-b/d)) + (4*b^2*d^2*x^2 + 4*b^2*c^2 - 2*b*c*d - (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*\cosh(b*x + a)^2 - 2*(4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*\cosh(b*x + a)*\sinh(b*x + a) - (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*\sinh(b*x + a)^2 + 3*d^2 + 2*(4*b^2*c*d - b*d^2)*x)*\sqrt{d*x + c))/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*\cosh(b*x + a) + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*\sinh(b*x + a))$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)/(d\*x+c)\*\*(7/2), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx + a)}{(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)/(d\*x+c)^(7/2), x, algorithm="giac")

[Out] integrate(sinh(b\*x + a)/(d\*x + c)^(7/2), x)

### 3.45 $\int (c + dx)^{5/2} \sinh^2(a + bx) dx$

**Optimal.** Leaf size=239

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}} - \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}} + \frac{15d^2\sqrt{c+dx}\sinh(2a+2bx)}{64b^3} - \frac{5d(c+dx)^{3/2}\sinh(2a+2bx)}{8b^2}$$

[Out]  $(-5*d*(c + d*x)^{(3/2)})/(16*b^2) - (c + d*x)^{(7/2)}/(7*d) + (15*d^{(5/2)}*E^{(-2*a + (2*b*c)/d)*\operatorname{Sqrt}[Pi/2]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(256*b^{(7/2)}) - (15*d^{(5/2)}*E^{(2*a - (2*b*c)/d)*\operatorname{Sqrt}[Pi/2]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(256*b^{(7/2)}) + ((c + d*x)^{(5/2)}*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x])/(2*b) - (5*d*(c + d*x)^{(3/2)}*\operatorname{Sinh}[a + b*x]^2)/(8*b^2) + (15*d^2*\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[2*a + 2*b*x])/(64*b^3)$

**Rubi [A]** time = 0.448625, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {3311, 32, 3312, 3296, 3308, 2180, 2204, 2205}

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}} - \frac{15\sqrt{\frac{\pi}{2}}d^{5/2}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}} + \frac{15d^2\sqrt{c+dx}\sinh(2a+2bx)}{64b^3} - \frac{5d(c+dx)^{3/2}\sinh(2a+2bx)}{8b^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^{(5/2)}*\operatorname{Sinh}[a + b*x]^2, x]$

[Out]  $(-5*d*(c + d*x)^{(3/2)})/(16*b^2) - (c + d*x)^{(7/2)}/(7*d) + (15*d^{(5/2)}*E^{(-2*a + (2*b*c)/d)*\operatorname{Sqrt}[Pi/2]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(256*b^{(7/2)}) - (15*d^{(5/2)}*E^{(2*a - (2*b*c)/d)*\operatorname{Sqrt}[Pi/2]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(256*b^{(7/2)}) + ((c + d*x)^{(5/2)}*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x])/(2*b) - (5*d*(c + d*x)^{(3/2)}*\operatorname{Sinh}[a + b*x]^2)/(8*b^2) + (15*d^2*\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[2*a + 2*b*x])/(64*b^3)$

#### Rule 3311

$\operatorname{Int}[(c + d*x)^m*(b*\sin[e + f*x])^n, x] \rightarrow \operatorname{Simp}[(d*m*(c + d*x)^{(m-1)}*(b*\sin[e + f*x])^n)/(f^{2*n^2}), x] + (\operatorname{Dist}[(b^{2*(n-1)})/n, \operatorname{Int}[(c + d*x)^m*(b*\sin[e + f*x])^{n-2}, x], x] - \operatorname{Dist}[(d^{2*m*(m-1)})/(f^{2*n^2}), \operatorname{Int}[(c + d*x)^{(m-2)}*(b*\sin[e + f*x])^n, x], x] - \operatorname{Simp}[(b*(c + d*x)^m*\cos[e + f*x]*(b*\sin[e + f*x])^{n-1})/(f*n), x]) /;$   
 $\operatorname{FreeQ}\{b, c, d, e, f, x\} \ \&\& \ \operatorname{GtQ}[n, 1] \ \&\& \ \operatorname{GtQ}[m, 1]$

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^{5/2} \sinh^2(a + bx) dx &= \frac{(c + dx)^{5/2} \cosh(a + bx) \sinh(a + bx)}{2b} - \frac{5d(c + dx)^{3/2} \sinh^2(a + bx)}{8b^2} - \frac{1}{2} \int (c + dx)^{5/2} dx \\
&= -\frac{(c + dx)^{7/2}}{7d} + \frac{(c + dx)^{5/2} \cosh(a + bx) \sinh(a + bx)}{2b} - \frac{5d(c + dx)^{3/2} \sinh^2(a + bx)}{8b^2} - \frac{1}{2} \int (c + dx)^{5/2} dx \\
&= -\frac{5d(c + dx)^{3/2}}{16b^2} - \frac{(c + dx)^{7/2}}{7d} + \frac{(c + dx)^{5/2} \cosh(a + bx) \sinh(a + bx)}{2b} - \frac{5d(c + dx)^{3/2} \sinh^2(a + bx)}{8b^2} \\
&= -\frac{5d(c + dx)^{3/2}}{16b^2} - \frac{(c + dx)^{7/2}}{7d} + \frac{(c + dx)^{5/2} \cosh(a + bx) \sinh(a + bx)}{2b} - \frac{5d(c + dx)^{3/2} \sinh^2(a + bx)}{8b^2} \\
&= -\frac{5d(c + dx)^{3/2}}{16b^2} - \frac{(c + dx)^{7/2}}{7d} + \frac{(c + dx)^{5/2} \cosh(a + bx) \sinh(a + bx)}{2b} - \frac{5d(c + dx)^{3/2} \sinh^2(a + bx)}{8b^2} \\
&= -\frac{5d(c + dx)^{3/2}}{16b^2} - \frac{(c + dx)^{7/2}}{7d} + \frac{(c + dx)^{5/2} \cosh(a + bx) \sinh(a + bx)}{2b} - \frac{5d(c + dx)^{3/2} \sinh^2(a + bx)}{8b^2} \\
&= -\frac{5d(c + dx)^{3/2}}{16b^2} - \frac{(c + dx)^{7/2}}{7d} + \frac{15d^{5/2} e^{-2a + \frac{2bc}{d}} \sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}} - \frac{15d^{5/2} e^{2a - \frac{2bc}{d}} \sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}}
\end{aligned}$$

**Mathematica [A]** time = 6.59064, size = 190, normalized size = 0.79

$$\frac{\sqrt{c + dx} \left( -7\sqrt{2}d^4 \sqrt{-\frac{b^2(c+dx)^2}{d^2}} \operatorname{Gamma}\left(\frac{7}{2}, -\frac{2b(c+dx)}{d}\right) \left( \sinh\left(2a - \frac{2bc}{d}\right) + \cosh\left(2a - \frac{2bc}{d}\right) \right) - b(c + dx) \left( 7\sqrt{2}d^3 \operatorname{Gamma}\left(\frac{7}{2}, -\frac{2b(c+dx)}{d}\right) \right) \right)}{448b^3d^2 \left( \frac{b(c+dx)}{d} \right)^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x)^(5/2)\*Sinh[a + b\*x]^2,x]

[Out] (Sqrt[c + d\*x]\*(-(b\*(c + d\*x)\*(64\*b^3\*(c + d\*x)^3\*Sqrt[(b\*(c + d\*x))/d] + 7\*Sqrt[2]\*d^3\*Gamma[7/2, (2\*b\*(c + d\*x))/d]\*(Cosh[2\*a - (2\*b\*c)/d] - Sinh[2\*a - (2\*b\*c)/d]))) - 7\*Sqrt[2]\*d^4\*Sqrt[-((b^2\*(c + d\*x)^2)/d^2)]\*Gamma[7/2, (-2\*b\*(c + d\*x))/d]\*(Cosh[2\*a - (2\*b\*c)/d] + Sinh[2\*a - (2\*b\*c)/d])))/(448\*b^3\*d^2\*((b\*(c + d\*x))/d)^(3/2))

**Maple [F]** time = 0.062, size = 0, normalized size = 0.

$$\int (dx + c)^{\frac{5}{2}} (\sinh(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^(5/2)*sinh(b*x+a)^2,x)`

[Out] `int((d*x+c)^(5/2)*sinh(b*x+a)^2,x)`

**Maxima [A]** time = 1.74935, size = 379, normalized size = 1.59

$$512(dx+c)^{\frac{7}{2}} + \frac{105\sqrt{2}\sqrt{\pi}d^3 \operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2a-\frac{2bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}} - \frac{105\sqrt{2}\sqrt{\pi}d^3 \operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2a+\frac{2bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}} + \frac{28\left(16(dx+c)^{\frac{5}{2}}b^2de^{\left(\frac{2bc}{d}\right)}+20(dx+c)^{\frac{3}{2}}b\right)}{3584d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^(5/2)*sinh(b*x+a)^2,x, algorithm="maxima")`

[Out] `-1/3584*(512*(d*x + c)^(7/2) + 105*sqrt(2)*sqrt(pi)*d^3*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))*e^(2*a - 2*b*c/d)/(b^3*sqrt(-b/d)) - 105*sqrt(2)*sqrt(pi)*d^3*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2*a + 2*b*c/d)/(b^3*sqrt(b/d)) + 28*(16*(d*x + c)^(5/2)*b^2*d*e^(2*b*c/d) + 20*(d*x + c)^(3/2)*b*d^2*e^(2*b*c/d) + 15*sqrt(d*x + c)*d^3*e^(2*b*c/d))*e^(-2*a - 2*(d*x + c)*b/d)/b^3 - 28*(16*(d*x + c)^(5/2)*b^2*d*e^(2*a) - 20*(d*x + c)^(3/2)*b*d^2*e^(2*a) + 15*sqrt(d*x + c)*d^3*e^(2*a))*e^(2*(d*x + c)*b/d - 2*b*c/d)/b^3)/d`

**Fricas [B]** time = 3.19787, size = 2319, normalized size = 9.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^(5/2)*sinh(b*x+a)^2,x, algorithm="fricas")`

[Out] `1/3584*(105*sqrt(2)*sqrt(pi)*(d^4*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - d^4*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^4*cosh(-2*(b*c - a*d)/d) - d^4*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^4*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) - d^4*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) + 105*sqrt(2)*sqrt(pi)*(d^4*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + d^4*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) - d^4*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) - d^4*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d) + 28*(16*(d*x + c)^(5/2)*b^2*d*e^(2*b*c/d) + 20*(d*x + c)^(3/2)*b*d^2*e^(2*b*c/d) + 15*sqrt(d*x + c)*d^3*e^(2*b*c/d))*e^(-2*a - 2*(d*x + c)*b/d)/b^3 - 28*(16*(d*x + c)^(5/2)*b^2*d*e^(2*a) - 20*(d*x + c)^(3/2)*b*d^2*e^(2*a) + 15*sqrt(d*x + c)*d^3*e^(2*a))*e^(2*(d*x + c)*b/d - 2*b*c/d)/b^3)/d`

$$\begin{aligned}
& a*d)/d) + (d^4*\cosh(-2*(b*c - a*d)/d) + d^4*\sinh(-2*(b*c - a*d)/d))*\sinh(b*x + a)^2 + 2*(d^4*\cosh(b*x + a)*\cosh(-2*(b*c - a*d)/d) + d^4*\cosh(b*x + a)*\sinh(-2*(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{-b/d}*\operatorname{erf}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{-b/d})) - 4*(112*b^3*d^3*x^2 + 112*b^3*c^2*d + 140*b^2*c*d^2 - 7*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d^3)*x))*\cosh(b*x + a)^4 - 28*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d^3)*x))*\cosh(b*x + a)*\sinh(b*x + a)^3 - 7*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d^3)*x))*\sinh(b*x + a)^4 + 105*b*d^3 + 128*(b^4*d^3*x^3 + 3*b^4*c*d^2*x^2 + 3*b^4*c^2*d*x + b^4*c^3)*\cosh(b*x + a)^2 + 2*(64*b^4*d^3*x^3 + 192*b^4*c*d^2*x^2 + 192*b^4*c^2*d*x + 64*b^4*c^3 - 21*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d^3)*x))*\cosh(b*x + a)^2)*\sinh(b*x + a)^2 + 28*(8*b^3*c*d^2 + 5*b^2*d^3)*x - 4*(7*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d^3)*x))*\cosh(b*x + a)^3 - 64*(b^4*d^3*x^3 + 3*b^4*c*d^2*x^2 + 3*b^4*c^2*d*x + b^4*c^3)*\cosh(b*x + a))*\sinh(b*x + a))*\sqrt{d*x + c}))/ (b^4*d*\cosh(b*x + a)^2 + 2*b^4*d*\cosh(b*x + a)*\sinh(b*x + a) + b^4*d*\sinh(b*x + a)^2)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*(5/2)\*sinh(b\*x+a)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{\frac{5}{2}} \sinh(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(5/2)\*sinh(b\*x+a)^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^(5/2)\*sinh(b\*x + a)^2, x)

### 3.46 $\int (c + dx)^{3/2} \sinh^2(a + bx) dx$

**Optimal.** Leaf size=211

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}} + \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}} - \frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\sinh(a+bx)}{2b}$$

[Out]  $(-3*d*\operatorname{Sqrt}[c + d*x])/(16*b^2) - (c + d*x)^{(5/2)}/(5*d) + (3*d^{(3/2)}*E^{(-2*a + (2*b*c)/d)*\operatorname{Sqrt}[\pi/2]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(64*b^{(5/2)}) + (3*d^{(3/2)}*E^{(2*a - (2*b*c)/d)*\operatorname{Sqrt}[\pi/2]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(64*b^{(5/2)}) + ((c + d*x)^{(3/2)}*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x])/(2*b) - (3*d*\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[a + b*x]^2)/(8*b^2)$

**Rubi [A]** time = 0.320699, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {3311, 32, 3312, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}} + \frac{3\sqrt{\frac{\pi}{2}}d^{3/2}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}} - \frac{3d\sqrt{c+dx}\sinh^2(a+bx)}{8b^2} + \frac{(c+dx)^{3/2}\sinh(a+bx)}{2b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^{(3/2)}*\operatorname{Sinh}[a + b*x]^2, x]$

[Out]  $(-3*d*\operatorname{Sqrt}[c + d*x])/(16*b^2) - (c + d*x)^{(5/2)}/(5*d) + (3*d^{(3/2)}*E^{(-2*a + (2*b*c)/d)*\operatorname{Sqrt}[\pi/2]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(64*b^{(5/2)}) + (3*d^{(3/2)}*E^{(2*a - (2*b*c)/d)*\operatorname{Sqrt}[\pi/2]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(64*b^{(5/2)}) + ((c + d*x)^{(3/2)}*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x])/(2*b) - (3*d*\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[a + b*x]^2)/(8*b^2)$

#### Rule 3311

$\operatorname{Int}[(c + d*x)^m*(b*\sin[e + f*x])^n, x] \rightarrow \operatorname{Simp}[d*m*(c + d*x)^{m-1}*(b*\sin[e + f*x])^n/(f^2*n^2), x] + (\operatorname{Dist}[(b^2*(n-1))/n, \operatorname{Int}[(c + d*x)^m*(b*\sin[e + f*x])^{n-2}, x], x] - \operatorname{Dist}[(d^2*m*(m-1))/(f^2*n^2), \operatorname{Int}[(c + d*x)^{m-2}*(b*\sin[e + f*x])^n, x], x] - \operatorname{Simp}[(b*(c + d*x)^m*\cos[e + f*x]*(b*\sin[e + f*x])^{n-1})/(f*n), x]) /;$   
 $\operatorname{FreeQ}\{b, c, d, e, f\}, x \ \&\& \operatorname{GtQ}[n, 1] \ \&\& \operatorname{GtQ}[m, 1]$

#### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

### Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]
```

### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

### Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]
```

### Rubi steps



$$\begin{aligned}
\int (c + dx)^{3/2} \sinh^2(a + bx) dx &= \frac{(c + dx)^{3/2} \cosh(a + bx) \sinh(a + bx)}{2b} - \frac{3d\sqrt{c + dx} \sinh^2(a + bx)}{8b^2} - \frac{1}{2} \int (c + dx)^{3/2} dx \\
&= -\frac{(c + dx)^{5/2}}{5d} + \frac{(c + dx)^{3/2} \cosh(a + bx) \sinh(a + bx)}{2b} - \frac{3d\sqrt{c + dx} \sinh^2(a + bx)}{8b^2} - \frac{1}{2} \int (c + dx)^{3/2} dx \\
&= -\frac{3d\sqrt{c + dx}}{16b^2} - \frac{(c + dx)^{5/2}}{5d} + \frac{(c + dx)^{3/2} \cosh(a + bx) \sinh(a + bx)}{2b} - \frac{3d\sqrt{c + dx} \sinh^2(a + bx)}{8b^2} \\
&= -\frac{3d\sqrt{c + dx}}{16b^2} - \frac{(c + dx)^{5/2}}{5d} + \frac{(c + dx)^{3/2} \cosh(a + bx) \sinh(a + bx)}{2b} - \frac{3d\sqrt{c + dx} \sinh^2(a + bx)}{8b^2} \\
&= -\frac{3d\sqrt{c + dx}}{16b^2} - \frac{(c + dx)^{5/2}}{5d} + \frac{(c + dx)^{3/2} \cosh(a + bx) \sinh(a + bx)}{2b} - \frac{3d\sqrt{c + dx} \sinh^2(a + bx)}{8b^2} \\
&= -\frac{3d\sqrt{c + dx}}{16b^2} - \frac{(c + dx)^{5/2}}{5d} + \frac{3d^{3/2} e^{-2a + \frac{2bc}{d}} \sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}} + \frac{3d^{3/2} e^{2a - \frac{2bc}{d}} \sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 2.29712, size = 163, normalized size = 0.77

$$\frac{5\sqrt{2}d^3 \sqrt{\frac{b(c+dx)}{d}} \operatorname{Gamma}\left(\frac{5}{2}, \frac{2b(c+dx)}{d}\right) \left(\sinh\left(2a - \frac{2bc}{d}\right) - \cosh\left(2a - \frac{2bc}{d}\right)\right) + 5\sqrt{2}d^3 \sqrt{-\frac{b(c+dx)}{d}} \operatorname{Gamma}\left(\frac{5}{2}, -\frac{2b(c+dx)}{d}\right) \left(\sinh\left(2a - \frac{2bc}{d}\right) + \cosh\left(2a - \frac{2bc}{d}\right)\right)}{160b^3 d \sqrt{c + dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^(3/2)\*Sinh[a + b\*x]^2,x]

[Out] (-32\*b^3\*(c + d\*x)^3 + 5\*Sqrt[2]\*d^3\*Sqrt[(b\*(c + d\*x))/d]\*Gamma[5/2, (2\*b\*(c + d\*x))/d]\*(-Cosh[2\*a - (2\*b\*c)/d] + Sinh[2\*a - (2\*b\*c)/d]) + 5\*Sqrt[2]\*d^3\*Sqrt[-(b\*(c + d\*x))/d]\*Gamma[5/2, (-2\*b\*(c + d\*x))/d]\*(Cosh[2\*a - (2\*b\*c)/d] + Sinh[2\*a - (2\*b\*c)/d]))/(160\*b^3\*d\*Sqrt[c + d\*x])

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int (dx + c)^{\frac{3}{2}} (\sinh(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^(3/2)*sinh(b*x+a)^2,x)`

[Out] `int((d*x+c)^(3/2)*sinh(b*x+a)^2,x)`

**Maxima [A]** time = 1.72278, size = 323, normalized size = 1.53

$$\frac{128(dx+c)^{\frac{5}{2}} - \frac{15\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2a-\frac{2bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} - \frac{15\sqrt{2}\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2a+\frac{2bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} + \frac{20\left(4(dx+c)^{\frac{3}{2}}bde^{\left(\frac{2bc}{d}\right)}+3\sqrt{dx+cd^2}e^{\left(\frac{2bc}{d}\right)}\right)}{b^2}}{640d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^(3/2)*sinh(b*x+a)^2,x, algorithm="maxima")`

[Out] `-1/640*(128*(d*x + c)^(5/2) - 15*sqrt(2)*sqrt(pi)*d^2*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))*e^(2*a - 2*b*c/d)/(b^2*sqrt(-b/d)) - 15*sqrt(2)*sqrt(pi)*d^2*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2*a + 2*b*c/d)/(b^2*sqrt(b/d)) + 20*(4*(d*x + c)^(3/2)*b*d*e^(2*b*c/d) + 3*sqrt(d*x + c)*d^2*e^(2*b*c/d))*e^(-2*a - 2*(d*x + c)*b/d)/b^2 - 20*(4*(d*x + c)^(3/2)*b*d*e^(2*a) - 3*sqrt(d*x + c)*d^2*e^(2*a))*e^(2*(d*x + c)*b/d - 2*b*c/d)/b^2)/d`

**Fricas [B]** time = 3.31488, size = 1789, normalized size = 8.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^(3/2)*sinh(b*x+a)^2,x, algorithm="fricas")`

[Out] `1/640*(15*sqrt(2)*sqrt(pi)*(d^3*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - d^3*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^3*cosh(-2*(b*c - a*d)/d) - d^3*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^3*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) - d^3*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 15*sqrt(2)*sqrt(pi)*(d^3*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + d^3*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^3*cosh(-2*(b*c - a*d)/d) + d^3*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^3*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^3*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sq`

```

rt(-b/d)) - 4*(20*b^2*d^2*x - 5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*cosh(b*
x + a)^4 - 20*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*cosh(b*x + a)*sinh(b*x +
a)^3 - 5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*sinh(b*x + a)^4 + 20*b^2*c*d +
15*b*d^2 + 32*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*cosh(b*x + a)^2 + 2*(1
6*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 - 15*(4*b^2*d^2*x + 4*b^2*c*d - 3
*b*d^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 4*(5*(4*b^2*d^2*x + 4*b^2*c*d -
3*b*d^2)*cosh(b*x + a)^3 - 16*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*cosh(b*
x + a))*sinh(b*x + a))*sqrt(d*x + c)/(b^3*d*cosh(b*x + a)^2 + 2*b^3*d*cosh
(b*x + a)*sinh(b*x + a) + b^3*d*sinh(b*x + a)^2)

```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^{\frac{3}{2}} \sinh^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*(3/2)\*sinh(b\*x+a)\*\*2,x)

[Out] Integral((c + d\*x)\*\*(3/2)\*sinh(a + b\*x)\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{\frac{3}{2}} \sinh^2(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(3/2)\*sinh(b\*x+a)^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^(3/2)\*sinh(b\*x + a)^2, x)

### 3.47 $\int \sqrt{c + dx} \sinh^2(a + bx) dx$

**Optimal.** Leaf size=166

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{de}^{\frac{2bc}{d}-2a} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{de}^{2a-\frac{2bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} + \frac{\sqrt{c+dx} \sinh(2a+2bx)}{4b} - \frac{(c+dx)^{3/2}}{3d}$$

[Out]  $-(c + d*x)^{(3/2)}/(3*d) + (\operatorname{Sqrt}[d]*E^{(-2*a + (2*b*c)/d)}*\operatorname{Sqrt}[\pi/2]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(16*b^{(3/2)}) - (\operatorname{Sqrt}[d]*E^{(2*a - (2*b*c)/d)}*\operatorname{Sqrt}[\pi/2]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(16*b^{(3/2)}) + (\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[2*a + 2*b*x])/(4*b)$

**Rubi [A]** time = 0.295711, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3312, 3296, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}} \sqrt{de}^{\frac{2bc}{d}-2a} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} \sqrt{de}^{2a-\frac{2bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} + \frac{\sqrt{c+dx} \sinh(2a+2bx)}{4b} - \frac{(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[a + b*x]^2, x]$

[Out]  $-(c + d*x)^{(3/2)}/(3*d) + (\operatorname{Sqrt}[d]*E^{(-2*a + (2*b*c)/d)}*\operatorname{Sqrt}[\pi/2]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(16*b^{(3/2)}) - (\operatorname{Sqrt}[d]*E^{(2*a - (2*b*c)/d)}*\operatorname{Sqrt}[\pi/2]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(16*b^{(3/2)}) + (\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[2*a + 2*b*x])/(4*b)$

#### Rule 3312

$\operatorname{Int}[(c + d*x)^m \sin(e + f*x)^n, x] \rightarrow \operatorname{Int}[\operatorname{ExpandTrigReduce}[(c + d*x)^m \sin(e + f*x)^n, x], x] /; \operatorname{FreeQ}\{c, d, e, f, m\}, x \ \&\& \ \operatorname{IGtQ}[n, 1] \ \&\& \ (\operatorname{!RationalQ}[m] \ \|\ (\operatorname{GeQ}[m, -1] \ \&\& \ \operatorname{LtQ}[m, 1]))$

#### Rule 3296

$\operatorname{Int}[(c + d*x)^m \cos(e + f*x), x] \rightarrow -\operatorname{Simp}[(c + d*x)^m \operatorname{Cos}[e + f*x]/f, x] + \operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{m-1} \operatorname{Cos}[e + f*x], x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x \ \&\& \ \operatorname{GtQ}[m, 0]$

Rule 3308

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Dist[I/2, Int[(c + d\*x)^m/E^(I\*(e + f\*x)), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^2), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rubi steps

$$\begin{aligned}
 \int \sqrt{c+dx} \sinh^2(a+bx) dx &= - \int \left( \frac{1}{2} \sqrt{c+dx} - \frac{1}{2} \sqrt{c+dx} \cosh(2a+2bx) \right) dx \\
 &= - \frac{(c+dx)^{3/2}}{3d} + \frac{1}{2} \int \sqrt{c+dx} \cosh(2a+2bx) dx \\
 &= - \frac{(c+dx)^{3/2}}{3d} + \frac{\sqrt{c+dx} \sinh(2a+2bx)}{4b} - \frac{d \int \frac{\sinh(2a+2bx)}{\sqrt{c+dx}} dx}{8b} \\
 &= - \frac{(c+dx)^{3/2}}{3d} + \frac{\sqrt{c+dx} \sinh(2a+2bx)}{4b} - \frac{d \int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}} dx}{16b} + \frac{d \int \frac{e^{i(2ia+2ibx)}}{\sqrt{c+dx}} dx}{16b} \\
 &= - \frac{(c+dx)^{3/2}}{3d} + \frac{\sqrt{c+dx} \sinh(2a+2bx)}{4b} + \frac{\text{Subst} \left( \int e^{i \left( 2ia - \frac{2ibc}{d} \right) - \frac{2bx^2}{d}} dx, x, \sqrt{c+dx} \right)}{8b} - \frac{\text{Subst} \left( \int e^{-i \left( 2ia - \frac{2ibc}{d} \right) - \frac{2bx^2}{d}} dx, x, \sqrt{c+dx} \right)}{8b} \\
 &= - \frac{(c+dx)^{3/2}}{3d} + \frac{\sqrt{d} e^{-2a + \frac{2bc}{d}} \sqrt{\frac{\pi}{2}} \operatorname{erf} \left( \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{16b^{3/2}} - \frac{\sqrt{d} e^{2a - \frac{2bc}{d}} \sqrt{\frac{\pi}{2}} \operatorname{erfi} \left( \frac{\sqrt{2} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{16b^{3/2}} + \frac{\sqrt{c+dx}}{2d}
 \end{aligned}$$

**Mathematica [A]** time = 0.578671, size = 129, normalized size = 0.78

$$\frac{1}{48} \sqrt{c+dx} \left( \frac{3\sqrt{2}e^{2a-\frac{2bc}{d}} \text{Gamma}\left(\frac{3}{2}, -\frac{2b(c+dx)}{d}\right)}{b\sqrt{-\frac{b(c+dx)}{d}}} - \frac{3\sqrt{2}e^{\frac{2bc}{d}-2a} \text{Gamma}\left(\frac{3}{2}, \frac{2b(c+dx)}{d}\right)}{b\sqrt{\frac{b(c+dx)}{d}}} - \frac{16(c+dx)}{d} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d\*x]\*Sinh[a + b\*x]^2,x]

[Out] (Sqrt[c + d\*x]\*((-16\*(c + d\*x))/d + (3\*Sqrt[2]\*E^(2\*a - (2\*b\*c)/d)\*Gamma[3/2, (-2\*b\*(c + d\*x))/d])/(b\*Sqrt[-((b\*(c + d\*x))/d)]) - (3\*Sqrt[2]\*E^(-2\*a + (2\*b\*c)/d)\*Gamma[3/2, (2\*b\*(c + d\*x))/d])/(b\*Sqrt[(b\*(c + d\*x))/d]))/48

**Maple [F]** time = 0.055, size = 0, normalized size = 0.

$$\int (\sinh(bx + a))^2 \sqrt{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(b\*x+a)^2\*(d\*x+c)^(1/2),x)

[Out] int(sinh(b\*x+a)^2\*(d\*x+c)^(1/2),x)

**Maxima [A]** time = 1.62864, size = 255, normalized size = 1.54

$$\frac{3\sqrt{2}\sqrt{\pi d} \operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{2a-\frac{2bc}{d}}}{b\sqrt{-\frac{b}{d}}} - \frac{3\sqrt{2}\sqrt{\pi d} \operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{-2a+\frac{2bc}{d}}}{b\sqrt{\frac{b}{d}}} + 32(dx+c)^{\frac{3}{2}} - \frac{12\sqrt{dx+c}e^{2a+\frac{2(dx+c)b}{d}-\frac{2bc}{d}}}{b} + \frac{12\sqrt{dx+c}e^{-2a-\frac{2(dx+c)b}{d}+\frac{2bc}{d}}}{b}$$

96 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2\*(d\*x+c)^(1/2),x, algorithm="maxima")

[Out] -1/96\*(3\*sqrt(2)\*sqrt(pi)\*d\*erf(sqrt(2)\*sqrt(d\*x + c)\*sqrt(-b/d))\*e^(2\*a - 2\*b\*c/d)/(b\*sqrt(-b/d)) - 3\*sqrt(2)\*sqrt(pi)\*d\*erf(sqrt(2)\*sqrt(d\*x + c)\*sqrt(b/d))\*e^(-2\*a + 2\*b\*c/d)/(b\*sqrt(b/d)) + 32\*(d\*x + c)^(3/2) - 12\*sqrt(d\*

$$x + c) * d * e^{(2*a + 2*(d*x + c)*b/d - 2*b*c/d)/b} + 12 * \sqrt{d*x + c} * d * e^{(-2*a - 2*(d*x + c)*b/d + 2*b*c/d)/b} / d$$

**Fricas [B]** time = 2.91236, size = 1438, normalized size = 8.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2\*(d\*x+c)^(1/2),x, algorithm="fricas")

[Out]  $\frac{1}{96} * (3 * \sqrt{2} * \sqrt{\pi}) * (d^2 * \cosh(b*x + a)^2 * \cosh(-2*(b*c - a*d)/d) - d^2 * \cosh(b*x + a)^2 * \sinh(-2*(b*c - a*d)/d) + (d^2 * \cosh(-2*(b*c - a*d)/d) - d^2 * \sinh(-2*(b*c - a*d)/d)) * \sinh(b*x + a)^2 + 2 * (d^2 * \cosh(b*x + a) * \cosh(-2*(b*c - a*d)/d) - d^2 * \cosh(b*x + a) * \sinh(-2*(b*c - a*d)/d)) * \sinh(b*x + a) * \sqrt{b/d} * \operatorname{erf}(\sqrt{2} * \sqrt{d*x + c} * \sqrt{b/d}) + 3 * \sqrt{2} * \sqrt{\pi} * (d^2 * \cosh(b*x + a)^2 * \cosh(-2*(b*c - a*d)/d) + d^2 * \cosh(b*x + a)^2 * \sinh(-2*(b*c - a*d)/d) + (d^2 * \cosh(-2*(b*c - a*d)/d) + d^2 * \sinh(-2*(b*c - a*d)/d)) * \sinh(b*x + a)^2 + 2 * (d^2 * \cosh(b*x + a) * \cosh(-2*(b*c - a*d)/d) + d^2 * \cosh(b*x + a) * \sinh(-2*(b*c - a*d)/d)) * \sinh(b*x + a) * \sqrt{-b/d} * \operatorname{erf}(\sqrt{2} * \sqrt{d*x + c} * \sqrt{-b/d}) + 4 * (3 * b * d * \cosh(b*x + a)^4 + 12 * b * d * \cosh(b*x + a) * \sinh(b*x + a)^3 + 3 * b * d * \sinh(b*x + a)^4 - 8 * (b^2 * d * x + b^2 * c) * \cosh(b*x + a)^2 - 2 * (4 * b^2 * d * x - 9 * b * d * \cosh(b*x + a)^2 + 4 * b^2 * c) * \sinh(b*x + a)^2 - 3 * b * d + 4 * (3 * b * d * \cosh(b*x + a)^3 - 4 * (b^2 * d * x + b^2 * c) * \cosh(b*x + a)) * \sinh(b*x + a) * \sqrt{d*x + c}) / (b^2 * d * \cosh(b*x + a)^2 + 2 * b^2 * d * \cosh(b*x + a) * \sinh(b*x + a) + b^2 * d * \sinh(b*x + a)^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c + dx} \sinh^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)\*\*2\*(d\*x+c)\*\*(1/2),x)

[Out] Integral(sqrt(c + d\*x)\*sinh(a + b\*x)\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx + c} \sinh(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^2*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x + c)*sinh(b*x + a)^2, x)
```



$$3.48 \quad \int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}} dx$$

**Optimal.** Leaf size=139

$$\frac{\sqrt{\frac{\pi}{2}} e^{\frac{2bc}{d}-2a} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}} + \frac{\sqrt{\frac{\pi}{2}} e^{2a-\frac{2bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}} - \frac{\sqrt{c+dx}}{d}$$

[Out]  $-(\operatorname{Sqrt}[c + d*x]/d) + (E^{(-2*a + (2*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(4*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d]) + (E^{(2*a - (2*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(4*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d])$

**Rubi [A]** time = 0.241308, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3312, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}} e^{\frac{2bc}{d}-2a} \operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}} + \frac{\sqrt{\frac{\pi}{2}} e^{2a-\frac{2bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}} - \frac{\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

[In] `Int[Sinh[a + b*x]^2/Sqrt[c + d*x], x]`

[Out]  $-(\operatorname{Sqrt}[c + d*x]/d) + (E^{(-2*a + (2*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(4*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d]) + (E^{(2*a - (2*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(4*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[d])$

### Rule 3312

`Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))`

### Rule 3307

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]`

Rule 2180

```
Int[(F_)^((g_.)*(e_.) + (f_.)*(x_))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(a + bx)}{\sqrt{c + dx}} dx &= - \int \left( \frac{1}{2\sqrt{c + dx}} - \frac{\cosh(2a + 2bx)}{2\sqrt{c + dx}} \right) dx \\
&= -\frac{\sqrt{c + dx}}{d} + \frac{1}{2} \int \frac{\cosh(2a + 2bx)}{\sqrt{c + dx}} dx \\
&= -\frac{\sqrt{c + dx}}{d} + \frac{1}{4} \int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c + dx}} dx + \frac{1}{4} \int \frac{e^{i(2ia+2ibx)}}{\sqrt{c + dx}} dx \\
&= -\frac{\sqrt{c + dx}}{d} + \frac{\text{Subst}\left(\int e^{i\left(2ia-\frac{2ibc}{d}\right)-\frac{2bx^2}{d}} dx, x, \sqrt{c + dx}\right)}{2d} + \frac{\text{Subst}\left(\int e^{-i\left(2ia-\frac{2ibc}{d}\right)+\frac{2bx^2}{d}} dx, x, \sqrt{c + dx}\right)}{2d} \\
&= -\frac{\sqrt{c + dx}}{d} + \frac{e^{-2a+\frac{2bc}{d}} \sqrt{\frac{\pi}{2}} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}} + \frac{e^{2a-\frac{2bc}{d}} \sqrt{\frac{\pi}{2}} \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}
\end{aligned}$$

**Mathematica [A]** time = 0.130485, size = 142, normalized size = 1.02

$$\frac{e^{2a-\frac{2bc}{d}} \sqrt{-\frac{b(c+dx)}{d}} \operatorname{Gamma}\left(\frac{1}{2}, -\frac{2b(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{c + dx}} - \frac{e^{\frac{2bc}{d}-2a} \sqrt{\frac{b(c+dx)}{d}} \operatorname{Gamma}\left(\frac{1}{2}, \frac{2b(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{c + dx}} - \frac{\sqrt{c + dx}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[a + b\*x]^2/Sqrt[c + d\*x],x]

[Out]  $-(\text{Sqrt}[c + d*x]/d) + (E^{(2*a - (2*b*c)/d)}*\text{Sqrt}[-((b*(c + d*x))/d)]*\text{Gamma}[1/2, (-2*b*(c + d*x))/d])/(4*\text{Sqrt}[2]*b*\text{Sqrt}[c + d*x]) - (E^{(-2*a + (2*b*c)/d)}*\text{Sqrt}[(b*(c + d*x))/d]*\text{Gamma}[1/2, (2*b*(c + d*x))/d])/(4*\text{Sqrt}[2]*b*\text{Sqrt}[c + d*x])$

**Maple [F]** time = 0.062, size = 0, normalized size = 0.

$$\int (\sinh(bx + a))^2 \frac{1}{\sqrt{dx + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(b\*x+a)^2/(d\*x+c)^(1/2),x)

[Out] int(sinh(b\*x+a)^2/(d\*x+c)^(1/2),x)

**Maxima [A]** time = 1.68104, size = 144, normalized size = 1.04

$$\frac{\frac{\sqrt{2}\sqrt{\pi} \operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2a-\frac{2bc}{d}\right)}}{\sqrt{-\frac{b}{d}}} + \frac{\sqrt{2}\sqrt{\pi} \operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2a+\frac{2bc}{d}\right)}}{\sqrt{\frac{b}{d}}}}{8d} - 8\sqrt{dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2/(d\*x+c)^(1/2),x, algorithm="maxima")

[Out]  $1/8*(\text{sqrt}(2)*\text{sqrt}(\pi)*\text{erf}(\text{sqrt}(2)*\text{sqrt}(d*x + c)*\text{sqrt}(-b/d))*e^{(2*a - 2*b*c/d)}/\text{sqrt}(-b/d) + \text{sqrt}(2)*\text{sqrt}(\pi)*\text{erf}(\text{sqrt}(2)*\text{sqrt}(d*x + c)*\text{sqrt}(b/d))*e^{(-2*a + 2*b*c/d)}/\text{sqrt}(b/d) - 8*\text{sqrt}(d*x + c))/d$

**Fricas [A]** time = 2.82359, size = 369, normalized size = 2.65

$$\frac{\sqrt{2}\sqrt{\pi}\left(d \cosh\left(-\frac{2(bc-ad)}{d}\right) - d \sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sqrt{\frac{b}{d}} \operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right) - \sqrt{2}\sqrt{\pi}\left(d \cosh\left(-\frac{2(bc-ad)}{d}\right) + d \sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sqrt{\frac{b}{d}}}{8bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2/(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] 1/8\*(sqrt(2)\*sqrt(pi)\*(d\*cosh(-2\*(b\*c - a\*d)/d) - d\*sinh(-2\*(b\*c - a\*d)/d))  
\*sqrt(b/d)\*erf(sqrt(2)\*sqrt(d\*x + c)\*sqrt(b/d)) - sqrt(2)\*sqrt(pi)\*(d\*cosh(-2\*(b\*c - a\*d)/d) + d\*sinh(-2\*(b\*c - a\*d)/d))\*sqrt(-b/d)\*erf(sqrt(2)\*sqrt(d\*x + c)\*sqrt(-b/d)) - 8\*sqrt(d\*x + c)\*b/(b\*d)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a + bx)}{\sqrt{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)\*\*2/(d\*x+c)\*\*(1/2),x)

[Out] Integral(sinh(a + b\*x)\*\*2/sqrt(c + d\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(bx + a)}{\sqrt{dx + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2/(d\*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(sinh(b\*x + a)^2/sqrt(d\*x + c), x)

$$3.49 \quad \int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}} dx$$

**Optimal.** Leaf size=142

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}$$

[Out] -((Sqrt[b]\*E^(-2\*a + (2\*b\*c)/d)\*Sqrt[Pi/2]\*Erf[(Sqrt[2]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/d^(3/2)) + (Sqrt[b]\*E^(2\*a - (2\*b\*c)/d)\*Sqrt[Pi/2]\*Erfi[(Sqrt[2]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/d^(3/2) - (2\*Sinh[a + b\*x]^2)/(d\*Sqrt[c + d\*x])

**Rubi [A]** time = 0.253019, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3313, 12, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{b}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[a + b\*x]^2/(c + d\*x)^(3/2), x]

[Out] -((Sqrt[b]\*E^(-2\*a + (2\*b\*c)/d)\*Sqrt[Pi/2]\*Erf[(Sqrt[2]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/d^(3/2)) + (Sqrt[b]\*E^(2\*a - (2\*b\*c)/d)\*Sqrt[Pi/2]\*Erfi[(Sqrt[2]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/d^(3/2) - (2\*Sinh[a + b\*x]^2)/(d\*Sqrt[c + d\*x])

### Rule 3313

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x]^n)/(d\*(m + 1)), x] - Dist[(f\*n)/(d\*(m + 1)), Int[ExpandTrigReduce[(c + d\*x)^(m + 1), Cos[e + f\*x]\*Sin[e + f\*x]^(n - 1), x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

### Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}} dx &= -\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}} - \frac{(4ib) \int \frac{i\sinh(2a+2bx)}{2\sqrt{c+dx}} dx}{d} \\
&= -\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}} + \frac{(2b) \int \frac{\sinh(2a+2bx)}{\sqrt{c+dx}} dx}{d} \\
&= -\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}} + \frac{b \int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}} dx}{d} - \frac{b \int \frac{e^{i(2ia+2ibx)}}{\sqrt{c+dx}} dx}{d} \\
&= -\frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}} - \frac{(2b) \text{Subst} \left( \int e^{i\left(2ia-\frac{2ibc}{d}\right)-\frac{2bx^2}{d}} dx, x, \sqrt{c+dx} \right)}{d^2} + \frac{(2b) \text{Subst} \left( \int e^{-i\left(2ia-\frac{2ibc}{d}\right)+\frac{2bx^2}{d}} dx, x, \sqrt{c+dx} \right)}{d^2} \\
&= -\frac{\sqrt{be}^{-2a+\frac{2bc}{d}} \sqrt{\frac{\pi}{2}} \operatorname{erf} \left( \frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}} \right)}{d^{3/2}} + \frac{\sqrt{be}^{2a-\frac{2bc}{d}} \sqrt{\frac{\pi}{2}} \operatorname{erfi} \left( \frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}} \right)}{d^{3/2}} - \frac{2\sinh^2(a+bx)}{d\sqrt{c+dx}}
\end{aligned}$$

**Mathematica [B]** time = 4.85797, size = 570, normalized size = 4.01

$$e^{-\frac{2b(c+dx)}{d}} \left( \sqrt{2}\sqrt{d} e^{\frac{2b(c+dx)}{d}} \sqrt{-\frac{b(c+dx)}{d}} \operatorname{Gamma} \left( \frac{1}{2}, -\frac{2b(c+dx)}{d} \right) \left( \cosh \left( 2a - \frac{2bc}{d} \right) + \sinh(2a) \cosh \left( \frac{2bc}{d} \right) \right) + \sqrt{2}\sqrt{d} e^{\frac{2b(c+dx)}{d}} \sqrt{\frac{b(c+dx)}{d}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[a + b\*x]^2/(c + d\*x)^(3/2), x]

[Out] (2\*Sqrt[d]\*E^((2\*b\*(c + d\*x))/d) - Sqrt[d]\*Cosh[2\*a]\*Cosh[(2\*b\*c)/d] - Sqrt[d]\*E^((4\*b\*(c + d\*x))/d)\*Cosh[2\*a]\*Cosh[(2\*b\*c)/d] + Sqrt[d]\*Cosh[(2\*b\*c)/d]\*Sinh[2\*a] - Sqrt[d]\*E^((4\*b\*(c + d\*x))/d)\*Cosh[(2\*b\*c)/d]\*Sinh[2\*a] + Sqrt[2]\*Sqrt[d]\*E^((2\*b\*(c + d\*x))/d)\*Sqrt[-((b\*(c + d\*x))/d)]\*Gamma[1/2, (-2\*b\*(c + d\*x))/d]\*(Cosh[2\*a - (2\*b\*c)/d] + Cosh[(2\*b\*c)/d]\*Sinh[2\*a]) - Sqrt[d]\*Cosh[2\*a]\*Sinh[(2\*b\*c)/d] + Sqrt[d]\*E^((4\*b\*(c + d\*x))/d)\*Cosh[2\*a]\*Sinh[(2\*b\*c)/d] - Sqrt[b]\*E^((2\*b\*(c + d\*x))/d)\*Sqrt[2\*Pi]\*Sqrt[c + d\*x]\*Cosh[2\*a]\*Erf[(Sqrt[2]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]]\*Sinh[(2\*b\*c)/d] - Sqrt[b]\*E^((2\*b\*(c + d\*x))/d)\*Sqrt[2\*Pi]\*Sqrt[c + d\*x]\*Cosh[2\*a]\*Erfi[(Sqrt[2]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]]\*Sinh[(2\*b\*c)/d] + Sqrt[d]\*Sinh[2\*a]\*Sinh[(2\*b\*c)/d] + Sqrt[d]\*E^((4\*b\*(c + d\*x))/d)\*Sinh[2\*a]\*Sinh[(2\*b\*c)/d] + Sqrt[2]\*Sqrt[d]\*E^((2\*b\*(c + d\*x))/d)\*Sqrt[(b\*(c + d\*x))/d]\*Gamma[1/2, (2\*b\*(c + d\*x))/d]\*(Cosh[2\*a]\*Cosh[(2\*b\*c)/d] - Sinh[2\*a]\*(Cosh[(2\*b\*c)/d] + Sinh[(2\*b\*c)/d]))/(2\*d^(3/2)\*E^((2\*b\*(c + d\*x))/d)\*Sqrt[c + d\*x])

---

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int (\sinh (bx + a))^2 (dx + c)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(b\*x+a)^2/(d\*x+c)^(3/2),x)

[Out] int(sinh(b\*x+a)^2/(d\*x+c)^(3/2),x)

---

**Maxima [A]** time = 1.29964, size = 157, normalized size = 1.11

$$\frac{\frac{\sqrt{2}\sqrt{\frac{(dx+c)b}{d}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{2(dx+c)b}{d}\right)}{\sqrt{dx+c}} + \frac{\sqrt{2}\sqrt{-\frac{(dx+c)b}{d}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{2(dx+c)b}{d}\right)}{\sqrt{dx+c}} - \frac{4}{\sqrt{dx+c}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2/(d\*x+c)^(3/2),x, algorithm="maxima")

[Out] -1/4\*(sqrt(2)\*sqrt((d\*x + c)\*b/d)\*e^(2\*(b\*c - a\*d)/d)\*gamma(-1/2, 2\*(d\*x + c)\*b/d)/sqrt(d\*x + c) + sqrt(2)\*sqrt(-(d\*x + c)\*b/d)\*e^(-2\*(b\*c - a\*d)/d)\*gamma(-1/2, -2\*(d\*x + c)\*b/d)/sqrt(d\*x + c) - 4/sqrt(d\*x + c)/d

---

**Fricas [B]** time = 2.85584, size = 1442, normalized size = 10.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2/(d\*x+c)^(3/2),x, algorithm="fricas")

[Out] -1/2\*(sqrt(2)\*sqrt(pi))\*((d\*x + c)\*cosh(b\*x + a)^2\*cosh(-2\*(b\*c - a\*d)/d) - (d\*x + c)\*cosh(b\*x + a)^2\*sinh(-2\*(b\*c - a\*d)/d) + ((d\*x + c)\*cosh(-2\*(b\*c - a\*d)/d) - (d\*x + c)\*sinh(-2\*(b\*c - a\*d)/d))\*sinh(b\*x + a)^2 + 2\*((d\*x + c)\*cosh(b\*x + a)\*cosh(-2\*(b\*c - a\*d)/d) - (d\*x + c)\*cosh(b\*x + a)\*sinh(-2\*(b\*c - a\*d)/d))\*sinh(b\*x + a)\*sqrt(b/d)\*erf(sqrt(2)\*sqrt(d\*x + c)\*sqrt(b/d))



+ sqrt(2)\*sqrt(pi)\*((d\*x + c)\*cosh(b\*x + a)^2\*cosh(-2\*(b\*c - a\*d)/d) + (d\*x + c)\*cosh(b\*x + a)^2\*sinh(-2\*(b\*c - a\*d)/d) + ((d\*x + c)\*cosh(-2\*(b\*c - a\*d)/d) + (d\*x + c)\*sinh(-2\*(b\*c - a\*d)/d))\*sinh(b\*x + a)^2 + 2\*((d\*x + c)\*cosh(b\*x + a)\*cosh(-2\*(b\*c - a\*d)/d) + (d\*x + c)\*cosh(b\*x + a)\*sinh(-2\*(b\*c - a\*d)/d))\*sinh(b\*x + a)\*sqrt(-b/d)\*erf(sqrt(2)\*sqrt(d\*x + c)\*sqrt(-b/d)) + (cosh(b\*x + a)^4 + 4\*cosh(b\*x + a)\*sinh(b\*x + a)^3 + sinh(b\*x + a)^4 + 2\*(3\*cosh(b\*x + a)^2 - 1)\*sinh(b\*x + a)^2 - 2\*cosh(b\*x + a)^2 + 4\*(cosh(b\*x + a)^3 - cosh(b\*x + a))\*sinh(b\*x + a) + 1)\*sqrt(d\*x + c))/((d^2\*x + c\*d)\*cosh(b\*x + a)^2 + 2\*(d^2\*x + c\*d)\*cosh(b\*x + a)\*sinh(b\*x + a) + (d^2\*x + c\*d)\*sinh(b\*x + a)^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a + bx)}{(c + dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)\*\*2/(d\*x+c)\*\*(3/2),x)

[Out] Integral(sinh(a + b\*x)\*\*2/(c + d\*x)\*\*(3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx + a)^2}{(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2/(d\*x+c)^(3/2),x, algorithm="giac")

[Out] integrate(sinh(b\*x + a)^2/(d\*x + c)^(3/2), x)

$$3.50 \quad \int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}} dx$$

**Optimal.** Leaf size=174

$$\frac{2\sqrt{2\pi}b^{3/2}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2\sqrt{2\pi}b^{3/2}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}$$

[Out] (2\*b^(3/2)\*E^(-2\*a + (2\*b\*c)/d)\*Sqrt[2\*Pi]\*Erf[(Sqrt[2]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]]/(3\*d^(5/2)) + (2\*b^(3/2)\*E^(2\*a - (2\*b\*c)/d)\*Sqrt[2\*Pi]\*Erfi[(Sqrt[2]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]]/(3\*d^(5/2)) - (8\*b\*Cosh[a + b\*x]\*Sinh[a + b\*x])/(3\*d^2\*Sqrt[c + d\*x]) - (2\*Sinh[a + b\*x]^2)/(3\*d\*(c + d\*x)^(3/2))

**Rubi [A]** time = 0.321269, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {3314, 32, 3312, 3307, 2180, 2204, 2205}

$$\frac{2\sqrt{2\pi}b^{3/2}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2\sqrt{2\pi}b^{3/2}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{2\sinh^2(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[a + b\*x]^2/(c + d\*x)^(5/2), x]

[Out] (2\*b^(3/2)\*E^(-2\*a + (2\*b\*c)/d)\*Sqrt[2\*Pi]\*Erf[(Sqrt[2]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]]/(3\*d^(5/2)) + (2\*b^(3/2)\*E^(2\*a - (2\*b\*c)/d)\*Sqrt[2\*Pi]\*Erfi[(Sqrt[2]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]]/(3\*d^(5/2)) - (8\*b\*Cosh[a + b\*x]\*Sinh[a + b\*x])/(3\*d^2\*Sqrt[c + d\*x]) - (2\*Sinh[a + b\*x]^2)/(3\*d\*(c + d\*x)^(3/2))

### Rule 3314

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*(b\*Ssin[e + f\*x])^n)/(d\*(m + 1)), x] + (Dist[(b^2\*f^2\*n\*(n - 1))/(d^2\*(m + 1)\*(m + 2)), Int[(c + d\*x)^(m + 2)\*(b\*Ssin[e + f\*x])^(n - 2), x], x] - Dist[(f^2\*n^2)/(d^2\*(m + 1)\*(m + 2)), Int[(c + d\*x)^(m + 2)\*(b\*Ssin[e + f\*x])^n, x], x] - Simp[(b\*f\*n\*(c + d\*x)^(m + 2)\*Cos[e + f\*x]\*(b\*Ssin[e + f\*x])^(n - 1))/(d^2\*(m + 1)\*(m + 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}} dx &= -\frac{8b \cosh(a+bx) \sinh(a+bx)}{3d^2 \sqrt{c+dx}} - \frac{2 \sinh^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{(8b^2) \int \frac{1}{\sqrt{c+dx}} dx}{3d^2} + \frac{(16b^2) \int \frac{\sinh^2(a+bx)}{\sqrt{c+dx}} dx}{3d^2} \\
&= \frac{16b^2 \sqrt{c+dx}}{3d^3} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{3d^2 \sqrt{c+dx}} - \frac{2 \sinh^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{(16b^2) \int \left( \frac{1}{2\sqrt{c+dx}} - \frac{\cosh(2a+2bx)}{2\sqrt{c+dx}} \right) dx}{3d^2} \\
&= -\frac{8b \cosh(a+bx) \sinh(a+bx)}{3d^2 \sqrt{c+dx}} - \frac{2 \sinh^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{(8b^2) \int \frac{\cosh(2a+2bx)}{\sqrt{c+dx}} dx}{3d^2} \\
&= -\frac{8b \cosh(a+bx) \sinh(a+bx)}{3d^2 \sqrt{c+dx}} - \frac{2 \sinh^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{(4b^2) \int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}} dx}{3d^2} + \frac{(4b^2) \int \frac{e^{i(2ia+2ibx)}}{\sqrt{c+dx}} dx}{3d^2} \\
&= -\frac{8b \cosh(a+bx) \sinh(a+bx)}{3d^2 \sqrt{c+dx}} - \frac{2 \sinh^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{(8b^2) \text{Subst} \left( \int e^{i(2ia-\frac{2ibc}{d})-\frac{2bx^2}{d}} dx, x, \sqrt{c+dx} \right)}{3d^3} \\
&= \frac{2b^{3/2} e^{-2a+\frac{2bc}{d}} \sqrt{2\pi} \operatorname{erf} \left( \frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}} \right)}{3d^{5/2}} + \frac{2b^{3/2} e^{2a-\frac{2bc}{d}} \sqrt{2\pi} \operatorname{erfi} \left( \frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}} \right)}{3d^{5/2}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{3d^2 \sqrt{c+dx}}
\end{aligned}$$

**Mathematica [A]** time = 1.21472, size = 156, normalized size = 0.9

$$\frac{2e^{-2\left(a+\frac{bc}{d}\right)} \left( \sqrt{2}e^{4a}d \left( -\frac{b(c+dx)}{d} \right)^{3/2} \operatorname{Gamma} \left( \frac{1}{2}, -\frac{2b(c+dx)}{d} \right) + \sqrt{2}de^{\frac{4bc}{d}} \left( \frac{b(c+dx)}{d} \right)^{3/2} \operatorname{Gamma} \left( \frac{1}{2}, \frac{2b(c+dx)}{d} \right) + e^{2\left(a+\frac{bc}{d}\right)} (2b(c+dx) \right)}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[a + b\*x]^2/(c + d\*x)^(5/2), x]

[Out] (-2\*(Sqrt[2]\*d\*E^(4\*a)\*(-(b\*(c + d\*x))/d))^(3/2)\*Gamma[1/2, (-2\*b\*(c + d\*x))/d] + Sqrt[2]\*d\*E^((4\*b\*c)/d)\*((b\*(c + d\*x))/d)^(3/2)\*Gamma[1/2, (2\*b\*(c + d\*x))/d] + E^(2\*(a + (b\*c)/d))\*(d\*Sinh[a + b\*x]^2 + 2\*b\*(c + d\*x)\*Sinh[2\*(a + b\*x)]))/(3\*d^2\*E^(2\*(a + (b\*c)/d))\*(c + d\*x)^(3/2))

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int (\sinh(bx + a))^2 (dx + c)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(b*x+a)^2/(d*x+c)^(5/2),x)`

[Out] `int(sinh(b*x+a)^2/(d*x+c)^(5/2),x)`

**Maxima [A]** time = 1.30908, size = 159, normalized size = 0.91

$$\frac{3\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}} + \frac{3\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}} - \frac{2}{(dx+c)^{\frac{3}{2}}}$$

$6d$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")`

[Out] 
$$-1/6*(3*\sqrt{2})*((d*x + c)*b/d)^{(3/2)}*e^{(2*(b*c - a*d)/d)}*\gamma(-3/2, 2*(d*x + c)*b/d)/(d*x + c)^{(3/2)} + 3*\sqrt{2}*(-(d*x + c)*b/d)^{(3/2)}*e^{(-2*(b*c - a*d)/d)}*\gamma(-3/2, -2*(d*x + c)*b/d)/(d*x + c)^{(3/2)} - 2/(d*x + c)^{(3/2)}/d$$

**Fricas [B]** time = 3.01618, size = 2053, normalized size = 11.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")`

[Out] 
$$1/6*(4*\sqrt{2})*\sqrt{\pi}*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)^2*\cosh(-2*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)^2*\sinh(-2*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(-2*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\sinh(-2*(b*c - a*d)/d))*\sinh(b*x + a)^2 + 2*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)*\cosh(-2*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)*\sinh(-2*(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{b/d}*\operatorname{erf}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/d}) - 4*\sqrt{2}*\sqrt{\pi}*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)^2*\cosh(-2*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)^2*\sinh(-2*(b*c - a*d)/d)$$

+ ((b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*cosh(-2\*(b\*c - a\*d)/d) + (b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*sinh(-2\*(b\*c - a\*d)/d))\*sinh(b\*x + a)^2 + 2\*((b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*cosh(b\*x + a)\*cosh(-2\*(b\*c - a\*d)/d) + (b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*cosh(b\*x + a)\*sinh(-2\*(b\*c - a\*d)/d))\*sinh(b\*x + a))\*sqrt(-b/d)\*erf(sqrt(2)\*sqrt(d\*x + c)\*sqrt(-b/d)) - ((4\*b\*d\*x + 4\*b\*c + d)\*cosh(b\*x + a)^4 + 4\*(4\*b\*d\*x + 4\*b\*c + d)\*cosh(b\*x + a)\*sinh(b\*x + a)^3 + (4\*b\*d\*x + 4\*b\*c + d)\*sinh(b\*x + a)^4 - 4\*b\*d\*x - 2\*d\*cosh(b\*x + a)^2 + 2\*(3\*(4\*b\*d\*x + 4\*b\*c + d)\*cosh(b\*x + a)^2 - d)\*sinh(b\*x + a)^2 - 4\*b\*c + 4\*((4\*b\*d\*x + 4\*b\*c + d)\*cosh(b\*x + a)^3 - d\*cosh(b\*x + a))\*sinh(b\*x + a) + d)\*sqrt(d\*x + c))/((d^4\*x^2 + 2\*c\*d^3\*x + c^2\*d^2)\*cosh(b\*x + a)^2 + 2\*(d^4\*x^2 + 2\*c\*d^3\*x + c^2\*d^2)\*cosh(b\*x + a)\*sinh(b\*x + a) + (d^4\*x^2 + 2\*c\*d^3\*x + c^2\*d^2)\*sinh(b\*x + a)^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(a + bx)}{(c + dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)\*\*2/(d\*x+c)\*\*(5/2),x)

[Out] Integral(sinh(a + b\*x)\*\*2/(c + d\*x)\*\*(5/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx + a)^2}{(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2/(d\*x+c)^(5/2),x, algorithm="giac")

[Out] integrate(sinh(b\*x + a)^2/(d\*x + c)^(5/2), x)

$$3.51 \quad \int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}} dx$$

**Optimal.** Leaf size=220

$$-\frac{8\sqrt{2\pi}b^{5/2}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{8\sqrt{2\pi}b^{5/2}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} - \frac{32b^2\sinh^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}$$

[Out]  $(-16*b^2)/(15*d^3*\operatorname{Sqrt}[c + d*x]) - (8*b^{(5/2)}*E^{(-2*a + (2*b*c)/d)}*\operatorname{Sqrt}[2*Pi]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(15*d^{(7/2)}) + (8*b^{(5/2)}*E^{(2*a - (2*b*c)/d)}*\operatorname{Sqrt}[2*Pi]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(15*d^{(7/2)}) - (8*b*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x])/(15*d^2*(c + d*x)^{(3/2)}) - (2*\operatorname{Sinh}[a + b*x]^2)/(5*d*(c + d*x)^{(5/2)}) - (32*b^2*\operatorname{Sinh}[a + b*x]^2)/(15*d^3*\operatorname{Sqrt}[c + d*x])$

**Rubi [A]** time = 0.321564, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {3314, 32, 3313, 12, 3308, 2180, 2204, 2205}

$$-\frac{8\sqrt{2\pi}b^{5/2}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{8\sqrt{2\pi}b^{5/2}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} - \frac{32b^2\sinh^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sinh}[a + b*x]^2/(c + d*x)^{(7/2)}, x]$

[Out]  $(-16*b^2)/(15*d^3*\operatorname{Sqrt}[c + d*x]) - (8*b^{(5/2)}*E^{(-2*a + (2*b*c)/d)}*\operatorname{Sqrt}[2*Pi]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(15*d^{(7/2)}) + (8*b^{(5/2)}*E^{(2*a - (2*b*c)/d)}*\operatorname{Sqrt}[2*Pi]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/(\operatorname{Sqrt}[d])])/(15*d^{(7/2)}) - (8*b*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x])/(15*d^2*(c + d*x)^{(3/2)}) - (2*\operatorname{Sinh}[a + b*x]^2)/(5*d*(c + d*x)^{(5/2)}) - (32*b^2*\operatorname{Sinh}[a + b*x]^2)/(15*d^3*\operatorname{Sqrt}[c + d*x])$

### Rule 3314

$\operatorname{Int}[(c_0 + d_0*x)^{m_0}*(b_0*\sin[e_0 + f_0*x])^{n_0}, x] \operatorname{Symbol} := \operatorname{Simp}[(c + d*x)^{(m + 1)}*(b*\sin[e + f*x])^n/(d*(m + 1)), x] + (\operatorname{Dist}[b^2*f^2*n*(n - 1)/(d^2*(m + 1)*(m + 2)), \operatorname{Int}[(c + d*x)^{(m + 2)}*(b*\sin[e + f*x])^{(n - 2)}, x], x] - \operatorname{Dist}[f^2*n^2/(d^2*(m + 1)*(m + 2)), \operatorname{Int}[(c + d*x)^{(m + 2)}*(b*\sin[e + f*x])^n, x], x] - \operatorname{Simp}[(b*f*n*(c + d*x)^{(m + 2)}*\cos[e +$

$f*x*(b*\sin[e + f*x])^{(n - 1)}/(d^2*(m + 1)*(m + 2)), x) /;$  FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]

### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

### Rule 3313

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x]^n)/(d\*(m + 1)), x] - Dist[(f\*n)/(d\*(m + 1)), Int[ExpandTrigReduce[(c + d\*x)^(m + 1), Cos[e + f\*x]\*Sin[e + f\*x]^(n - 1), x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 3308

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Dist[I/2, Int[(c + d\*x)^m/E^(I\*(e + f\*x)), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^(2)), x\_Symbol] := Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]



Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(a+bx)}{(c+dx)^{7/2}} dx &= -\frac{8b \cosh(a+bx) \sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2 \sinh^2(a+bx)}{5d(c+dx)^{5/2}} + \frac{(8b^2) \int \frac{1}{(c+dx)^{3/2}} dx}{15d^2} + \frac{(16b^2) \int \frac{\sinh^2(a+bx)}{(c+dx)^{3/2}} dx}{15d^2} \\
&= -\frac{16b^2}{15d^3 \sqrt{c+dx}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2 \sinh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2 \sinh^2(a+bx)}{15d^3 \sqrt{c+dx}} - \frac{(64b^3)}{15d^3 \sqrt{c+dx}} \\
&= -\frac{16b^2}{15d^3 \sqrt{c+dx}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2 \sinh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2 \sinh^2(a+bx)}{15d^3 \sqrt{c+dx}} + \frac{(32b^3)}{15d^3 \sqrt{c+dx}} \\
&= -\frac{16b^2}{15d^3 \sqrt{c+dx}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2 \sinh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2 \sinh^2(a+bx)}{15d^3 \sqrt{c+dx}} + \frac{(16b^3)}{15d^3 \sqrt{c+dx}} \\
&= -\frac{16b^2}{15d^3 \sqrt{c+dx}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{2 \sinh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2 \sinh^2(a+bx)}{15d^3 \sqrt{c+dx}} - \frac{(32b^3)}{15d^3 \sqrt{c+dx}} \\
&= -\frac{16b^2}{15d^3 \sqrt{c+dx}} - \frac{8b^{5/2} e^{-2a + \frac{2bc}{d}} \sqrt{2\pi} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{8b^{5/2} e^{2a - \frac{2bc}{d}} \sqrt{2\pi} \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} - \frac{8b \cosh(2a - \frac{2bc}{d})}{15d^3 \sqrt{c+dx}}
\end{aligned}$$

**Mathematica [B]** time = 9.18661, size = 825, normalized size = 3.75

$$e^{-\frac{2b(c+dx)}{d}} \left( 16\sqrt{2}d^2 e^{\frac{2b(c+dx)}{d}} \operatorname{Gamma}\left(\frac{1}{2}, -\frac{2b(c+dx)}{d}\right) \left( \cosh\left(2a - \frac{2bc}{d}\right) + \sinh\left(2a - \frac{2bc}{d}\right) \right) \left(-\frac{b(c+dx)}{d}\right)^{5/2} + 6d^2 e^{\frac{2b(c+dx)}{d}} - 16b^2 c^2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[a + b\*x]^2/(c + d\*x)^(7/2), x]

[Out] (6\*d^2\*E^((2\*b\*(c + d\*x))/d) - 16\*b^2\*c^2\*Cosh[2\*a - (2\*b\*c)/d] + 4\*b\*c\*d\*Cosh[2\*a - (2\*b\*c)/d] - 3\*d^2\*Cosh[2\*a - (2\*b\*c)/d] - 16\*b^2\*c^2\*E^((4\*b\*(c + d\*x))/d)\*Cosh[2\*a - (2\*b\*c)/d] - 4\*b\*c\*d\*E^((4\*b\*(c + d\*x))/d)\*Cosh[2\*a - (2\*b\*c)/d] - 3\*d^2\*E^((4\*b\*(c + d\*x))/d)\*Cosh[2\*a - (2\*b\*c)/d] - 32\*b^2\*c\*d\*x\*Cosh[2\*a - (2\*b\*c)/d] + 4\*b\*d^2\*x\*Cosh[2\*a - (2\*b\*c)/d] - 32\*b^2\*c\*d\*E^((4\*b\*(c + d\*x))/d)\*x\*Cosh[2\*a - (2\*b\*c)/d] - 4\*b\*d^2\*E^((4\*b\*(c + d\*x))/d)\*x\*Cosh[2\*a - (2\*b\*c)/d] - 16\*b^2\*d^2\*x^2\*Cosh[2\*a - (2\*b\*c)/d] - 16\*b^2\*d^2\*E^((4\*b\*(c + d\*x))/d)\*x^2\*Cosh[2\*a - (2\*b\*c)/d] + 16\*sqrt[2]\*d^2\*E^((2\*b\*(c + d\*x))/d)\*((b\*(c + d\*x))/d)^(5/2)\*Gamma[1/2, (2\*b\*(c + d\*x))/d]\*(Cosh[2\*a - (2\*b\*c)/d] - Sinh[2\*a - (2\*b\*c)/d]) + 16\*b^2\*c^2\*Sinh[2\*a - (2\*b\*c)/d]

$$\begin{aligned}
& - 4*b*c*d*\text{Sinh}[2*a - (2*b*c)/d] + 3*d^2*\text{Sinh}[2*a - (2*b*c)/d] - 16*b^2*c^2 \\
& *E^{((4*b*(c + d*x))/d)*\text{Sinh}[2*a - (2*b*c)/d]} - 4*b*c*d*E^{((4*b*(c + d*x))/d)} \\
& *\text{Sinh}[2*a - (2*b*c)/d] - 3*d^2*E^{((4*b*(c + d*x))/d)*\text{Sinh}[2*a - (2*b*c)/d]} \\
& + 32*b^2*c*d*x*\text{Sinh}[2*a - (2*b*c)/d] - 4*b*d^2*x*\text{Sinh}[2*a - (2*b*c)/d] - 3 \\
& 2*b^2*c*d*E^{((4*b*(c + d*x))/d)*x*\text{Sinh}[2*a - (2*b*c)/d]} - 4*b*d^2*E^{((4*b*(c + d*x))/d)*x*\text{Sinh}[2*a - (2*b*c)/d]} \\
& + 16*b^2*d^2*x^2*\text{Sinh}[2*a - (2*b*c)/d] - 16*b^2*d^2*E^{((4*b*(c + d*x))/d)*x^2*\text{Sinh}[2*a - (2*b*c)/d]} \\
& + 16*\text{Sqrt}[2]*d^2*E^{((2*b*(c + d*x))/d)*(-(b*(c + d*x))/d)}^{(5/2)}*\text{Gamma}[1/2, (-2*b*(c + d*x))/d] \\
& *(Cosh[2*a - (2*b*c)/d] + \text{Sinh}[2*a - (2*b*c)/d])/(30*d^3*E^{((2*b*(c + d*x))/d)*(c + d*x)^{(5/2)}})
\end{aligned}$$

**Maple [F]** time = 0.067, size = 0, normalized size = 0.

$$\int (\sinh(bx + a))^2 (dx + c)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(b\*x+a)^2/(d\*x+c)^(7/2), x)

[Out] int(sinh(b\*x+a)^2/(d\*x+c)^(7/2), x)

**Maxima [A]** time = 1.31562, size = 159, normalized size = 0.72

$$\frac{5\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2}, \frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}} + \frac{5\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2}, -\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}} - \frac{1}{(dx+c)^{\frac{5}{2}}}$$

$5d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2/(d\*x+c)^(7/2), x, algorithm="maxima")

[Out]  $-1/5*(5*\text{sqrt}(2)*((d*x + c)*b/d)^{(5/2)}*e^{(2*(b*c - a*d)/d)}*\text{gamma}(-5/2, 2*(d*x + c)*b/d)/(d*x + c)^{(5/2)} + 5*\text{sqrt}(2)*(-(d*x + c)*b/d)^{(5/2)}*e^{(-2*(b*c - a*d)/d)}*\text{gamma}(-5/2, -2*(d*x + c)*b/d)/(d*x + c)^{(5/2)} - 1/(d*x + c)^{(5/2)}/d$

**Fricas [B]** time = 3.08432, size = 2974, normalized size = 13.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2/(d\*x+c)^(7/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/30*(16*\sqrt{2}*\sqrt{\pi})*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x \\ & + b^2*c^3)*\cosh(b*x + a)^2*\cosh(-2*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c* \\ & d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cosh(b*x + a)^2*\sinh(-2*(b*c - a*d)/d) + \\ & ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cosh(-2*(b*c - \\ & a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\sinh(-2 \\ & *(b*c - a*d)/d))*\sinh(b*x + a)^2 + 2*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^ \\ & 2*c^2*d*x + b^2*c^3)*\cosh(b*x + a)*\cosh(-2*(b*c - a*d)/d) - (b^2*d^3*x^3 + \\ & 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cosh(b*x + a)*\sinh(-2*(b*c - a*d \\ & )/d))*\sinh(b*x + a))*\sqrt{b/d}*\operatorname{erf}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{b/d}) + 16*\sqrt{ \\ & 2}*\sqrt{\pi}*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\c \\ & osh(b*x + a)^2*\cosh(-2*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3* \\ & b^2*c^2*d*x + b^2*c^3)*\cosh(b*x + a)^2*\sinh(-2*(b*c - a*d)/d) + ((b^2*d^3*x \\ & ^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cosh(-2*(b*c - a*d)/d) + (b \\ & ^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\sinh(-2*(b*c - a*d) \\ & /d))*\sinh(b*x + a)^2 + 2*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + \\ & b^2*c^3)*\cosh(b*x + a)*\cosh(-2*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2* \\ & x^2 + 3*b^2*c^2*d*x + b^2*c^3)*\cosh(b*x + a)*\sinh(-2*(b*c - a*d)/d))*\sinh(b \\ & *x + a))*\sqrt{-b/d}*\operatorname{erf}(\sqrt{2}*\sqrt{d*x + c}*\sqrt{-b/d}) + (16*b^2*d^2*x^2 \\ & + (16*b^2*d^2*x^2 + 16*b^2*c^2 + 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x \\ & )*\cosh(b*x + a)^4 + 4*(16*b^2*d^2*x^2 + 16*b^2*c^2 + 4*b*c*d + 3*d^2 + 4*(8 \\ & *b^2*c*d + b*d^2)*x)*\cosh(b*x + a)*\sinh(b*x + a)^3 + (16*b^2*d^2*x^2 + 16*b \\ & ^2*c^2 + 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x)*\sinh(b*x + a)^4 + 16*b^ \\ & 2*c^2 - 6*d^2*\cosh(b*x + a)^2 - 4*b*c*d + 6*((16*b^2*d^2*x^2 + 16*b^2*c^2 + \\ & 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x)*\cosh(b*x + a)^2 - d^2)*\sinh(b*x \\ & + a)^2 + 3*d^2 + 4*(8*b^2*c*d - b*d^2)*x + 4*((16*b^2*d^2*x^2 + 16*b^2*c^2 \\ & + 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x)*\cosh(b*x + a)^3 - 3*d^2*\cosh( \\ & b*x + a))*\sinh(b*x + a))*\sqrt{d*x + c})/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4 \\ & *x + c^3*d^3)*\cosh(b*x + a)^2 + 2*(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^ \\ & 3*d^3)*\cosh(b*x + a)*\sinh(b*x + a) + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + \\ & c^3*d^3)*\sinh(b*x + a)^2) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)**2/(d*x+c)**(7/2),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(bx + a)}{(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="giac")`

[Out] `integrate(sinh(b*x + a)^2/(d*x + c)^(7/2), x)`

$$3.52 \quad \int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}} dx$$

**Optimal.** Leaf size=251

$$\frac{32\sqrt{2\pi}b^{7/2}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}} + \frac{32\sqrt{2\pi}b^{7/2}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}} - \frac{32b^2\sinh^2(a+bx)}{105d^3(c+dx)^{3/2}} - \frac{128b^3\sinh(a+bx)\cos}{105d^4\sqrt{c+dx}}$$

[Out]  $(-16*b^2)/(105*d^3*(c + d*x)^{(3/2)}) + (32*b^{(7/2)}*E^{(-2*a + (2*b*c)/d)}*\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(105*d^{(9/2)}) + (32*b^{(7/2)}*E^{(2*a - (2*b*c)/d)}*\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(105*d^{(9/2)}) - (8*b*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x])/(35*d^2*(c + d*x)^{(5/2)}) - (128*b^3*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x])/(105*d^4*\operatorname{Sqrt}[c + d*x]) - (2*\operatorname{Sinh}[a + b*x]^2)/(7*d*(c + d*x)^{(7/2)}) - (32*b^2*\operatorname{Sinh}[a + b*x]^2)/(105*d^3*(c + d*x)^{(3/2)})$

**Rubi [A]** time = 0.394982, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {3314, 32, 3312, 3307, 2180, 2204, 2205}

$$\frac{32\sqrt{2\pi}b^{7/2}e^{\frac{2bc}{d}-2a}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}} + \frac{32\sqrt{2\pi}b^{7/2}e^{2a-\frac{2bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}} - \frac{32b^2\sinh^2(a+bx)}{105d^3(c+dx)^{3/2}} - \frac{128b^3\sinh(a+bx)\cos}{105d^4\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sinh}[a + b*x]^2/(c + d*x)^{(9/2)}, x]$

[Out]  $(-16*b^2)/(105*d^3*(c + d*x)^{(3/2)}) + (32*b^{(7/2)}*E^{(-2*a + (2*b*c)/d)}*\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(105*d^{(9/2)}) + (32*b^{(7/2)}*E^{(2*a - (2*b*c)/d)}*\operatorname{Sqrt}[2*\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(105*d^{(9/2)}) - (8*b*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x])/(35*d^2*(c + d*x)^{(5/2)}) - (128*b^3*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x])/(105*d^4*\operatorname{Sqrt}[c + d*x]) - (2*\operatorname{Sinh}[a + b*x]^2)/(7*d*(c + d*x)^{(7/2)}) - (32*b^2*\operatorname{Sinh}[a + b*x]^2)/(105*d^3*(c + d*x)^{(3/2)})$

### Rule 3314

$\operatorname{Int}[(c + d*x)^m*(b*\sin[e + f*x])^n, x] := \operatorname{Simp}[(c + d*x)^{m+1}*(b*\sin[e + f*x])^n/(d*(m+1)), x] + (\operatorname{Dist}[b^2*f^2*n*(n-1)/(d^2*(m+1)*(m+2)), \operatorname{Int}[(c + d*x)^{m+2}*(b*\sin[e +$

$f*x]^{(n-2)}, x], x] - \text{Dist}[(f^2*n^2)/(d^2*(m+1)*(m+2)), \text{Int}[(c+d*x)^{(m+2)}*(b*\text{Sin}[e+f*x])^n, x], x] - \text{Simp}[(b*f*n*(c+d*x)^{(m+2)}*\text{Cos}[e+f*x]*(b*\text{Sin}[e+f*x])^{(n-1)})/(d^2*(m+1)*(m+2)), x]) /; \text{FreeQ}\{b, c, d, e, f\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{LtQ}[m, -2]$

### Rule 32

$\text{Int}[(a + b*x)^{(m+1)}, x\_Symbol] := \text{Simp}[(a + b*x)^{(m+1)}/(b*(m+1)), x] /; \text{FreeQ}\{a, b, m\}, x] \&\& \text{NeQ}[m, -1]$

### Rule 3312

$\text{Int}[(c + d*x)^m * \text{sin}[e + f*x]^n, x\_Symbol] := \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[e + f*x]^n, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x] \&\& \text{IGtQ}[n, 1] \&\& (!\text{RationalQ}[m] || (\text{GeQ}[m, -1] \&\& \text{LtQ}[m, 1]))$

### Rule 3307

$\text{Int}[(c + d*x)^m * \text{sin}[e + \text{Pi}*(k) + f*x], x\_Symbol] := \text{Dist}[I/2, \text{Int}[(c + d*x)^m / (\text{E}^{I*k*\text{Pi}} * \text{E}^{I*(e+f*x)}), x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m * \text{E}^{I*k*\text{Pi}} * \text{E}^{I*(e+f*x)}, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x] \&\& \text{IntegerQ}[2*k]$

### Rule 2180

$\text{Int}[(F + g*(e + f*x))/\text{Sqrt}[c + d*x], x\_Symbol] := \text{Dist}[2/d, \text{Subst}[\text{Int}[F^{g*(e - (c*f)/d) + (f*g*x^2)/d}, x], x, \text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{F, c, d, e, f, g\}, x] \&\& !\$UseGamma == True$

### Rule 2204

$\text{Int}[(F + b*(c + d*x))^2, x\_Symbol] := \text{Simp}[(F^a * \text{Sqrt}[\text{Pi}] * \text{Erfi}[(c + d*x) * \text{Rt}[b * \text{Log}[F], 2]]) / (2 * d * \text{Rt}[b * \text{Log}[F], 2]), x] /; \text{FreeQ}\{F, a, b, c, d\}, x] \&\& \text{PosQ}[b]$

### Rule 2205

$\text{Int}[(F + b*(c + d*x))^2, x\_Symbol] := \text{Simp}[(F^a * \text{Sqrt}[\text{Pi}] * \text{Erf}[(c + d*x) * \text{Rt}[-(b * \text{Log}[F]), 2]]) / (2 * d * \text{Rt}[-(b * \text{Log}[F]), 2]), x] /; \text{FreeQ}\{F, a, b, c, d\}, x] \&\& \text{NegQ}[b]$

### Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(a+bx)}{(c+dx)^{9/2}} dx &= -\frac{8b \cosh(a+bx) \sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{2 \sinh^2(a+bx)}{7d(c+dx)^{7/2}} + \frac{(8b^2) \int \frac{1}{(c+dx)^{5/2}} dx}{35d^2} + \frac{(16b^2) \int \frac{\sinh^2(a+bx)}{(c+dx)^{5/2}} dx}{35d^2} \\
&= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3 \cosh(a+bx) \sinh(a+bx)}{105d^4 \sqrt{c+dx}} - \frac{2 \sinh^2}{7d(c+dx)^{7/2}} \\
&= -\frac{16b^2}{105d^3(c+dx)^{3/2}} + \frac{256b^4 \sqrt{c+dx}}{105d^5} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3 \cosh(a+bx) \sinh(a+bx)}{105d^4 \sqrt{c+dx}} \\
&= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3 \cosh(a+bx) \sinh(a+bx)}{105d^4 \sqrt{c+dx}} - \frac{2 \sinh^2}{7d(c+dx)^{7/2}} \\
&= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3 \cosh(a+bx) \sinh(a+bx)}{105d^4 \sqrt{c+dx}} - \frac{2 \sinh^2}{7d(c+dx)^{7/2}} \\
&= -\frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{8b \cosh(a+bx) \sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3 \cosh(a+bx) \sinh(a+bx)}{105d^4 \sqrt{c+dx}} - \frac{2 \sinh^2}{7d(c+dx)^{7/2}} \\
&= -\frac{16b^2}{105d^3(c+dx)^{3/2}} + \frac{32b^{7/2} e^{-2a + \frac{2bc}{d}} \sqrt{2\pi} \operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}} + \frac{32b^{7/2} e^{2a - \frac{2bc}{d}} \sqrt{2\pi} \operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}} - \dots
\end{aligned}$$

**Mathematica [A]** time = 0.562512, size = 222, normalized size = 0.88

$$\frac{2 \left( 16\sqrt{2}b^3(c+dx)^3 e^{2a - \frac{2bc}{d}} \sqrt{-\frac{b(c+dx)}{d}} \operatorname{Gamma}\left(\frac{1}{2}, -\frac{2b(c+dx)}{d}\right) - 16\sqrt{2}b^3(c+dx)^3 e^{\frac{2bc}{d} - 2a} \sqrt{\frac{b(c+dx)}{d}} \operatorname{Gamma}\left(\frac{1}{2}, \frac{2b(c+dx)}{d}\right) - 16\sqrt{2}b^3(c+dx)^3 \right)}{105d^9}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[a + b\*x]^2/(c + d\*x)^(9/2), x]

[Out] (2\*(-8\*b^2\*d\*(c + d\*x)^2 + 16\*Sqrt[2]\*b^3\*E^(2\*a - (2\*b\*c)/d)\*(c + d\*x)^3\*Sqrt[-((b\*(c + d\*x))/d)]\*Gamma[1/2, (-2\*b\*(c + d\*x))/d] - 16\*Sqrt[2]\*b^3\*E^(-2\*a + (2\*b\*c)/d)\*(c + d\*x)^3\*Sqrt[(b\*(c + d\*x))/d]\*Gamma[1/2, (2\*b\*(c + d\*x))/d] - 15\*d^3\*Sinh[a + b\*x]^2 - 16\*b^2\*d\*(c + d\*x)^2\*Sinh[a + b\*x]^2 - 6\*b\*d^2\*(c + d\*x)\*Sinh[2\*(a + b\*x)] - 32\*b^3\*(c + d\*x)^3\*Sinh[2\*(a + b\*x)])/(105\*d^4\*(c + d\*x)^(7/2))

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int (\sinh(bx + a))^2 (dx + c)^{-\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(b*x+a)^2/(d*x+c)^(9/2),x)`

[Out] `int(sinh(b*x+a)^2/(d*x+c)^(9/2),x)`

**Maxima [A]** time = 1.29399, size = 159, normalized size = 0.63

$$\frac{14\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{7}{2}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{7}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{7}{2}}} + \frac{14\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{7}{2}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{7}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{7}{2}}} - \frac{1}{(dx+c)^{\frac{7}{2}}}$$

$7d$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="maxima")`

[Out]  $-1/7*(14*\sqrt{2}*((d*x + c)*b/d)^{(7/2)}*e^{(2*(b*c - a*d)/d)}*\text{gamma}(-7/2, 2*(d*x + c)*b/d)/(d*x + c)^{(7/2)} + 14*\sqrt{2}*(-(d*x + c)*b/d)^{(7/2)}*e^{(-2*(b*c - a*d)/d)}*\text{gamma}(-7/2, -2*(d*x + c)*b/d)/(d*x + c)^{(7/2)} - 1/(d*x + c)^{(7/2)})/d$

**Fricas [B]** time = 3.57603, size = 3951, normalized size = 15.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="fricas")`

[Out]  $1/210*(64*\sqrt{2}*\sqrt{\pi}*((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*\cosh(b*x + a)^2*\cosh(-2*(b*c - a*d)/d) - (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*\cosh(b*x + a)^2*\sinh(-2*(b*c - a*d)/d) + ((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*\cosh(-2*(b*c - a*d)/d) - (b^3*$



$$\begin{aligned}
& d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) \operatorname{sinh}(-2(b c - a d) / d) * \operatorname{sinh}(b x + a)^2 + 2 * ((b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + \\
& 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) * \operatorname{cosh}(b x + a) * \operatorname{cosh}(-2(b c - a d) / d) - (b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x \\
& + b^3 c^4) * \operatorname{cosh}(b x + a) * \operatorname{sinh}(-2(b c - a d) / d)) * \operatorname{sinh}(b x + a) * \operatorname{sqrt}(b / d) \\
& * \operatorname{erf}(\operatorname{sqrt}(2) * \operatorname{sqrt}(d x + c) * \operatorname{sqrt}(b / d)) - 64 * \operatorname{sqrt}(2) * \operatorname{sqrt}(\pi) * ((b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) * \operatorname{cosh}(b x + \\
& a)^2 * \operatorname{cosh}(-2(b c - a d) / d) + (b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) * \operatorname{cosh}(b x + a)^2 * \operatorname{sinh}(-2(b c - a d) / d) + ( \\
& (b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) * \operatorname{cosh}(-2(b c - a d) / d) + (b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) * \operatorname{sinh}(-2(b c - a d) / d)) * \operatorname{sinh}(b x + a)^2 + 2 * \\
& ((b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) * \operatorname{cosh}(b x + a) * \operatorname{cosh}(-2(b c - a d) / d) + (b^3 d^4 x^4 + 4 b^3 c d^3 x^3 + 6 b^3 c^2 d^2 x^2 + 4 b^3 c^3 d x + b^3 c^4) * \operatorname{cosh}(b x + a) * \operatorname{sinh}(-2(b c - \\
& a d) / d)) * \operatorname{sinh}(b x + a) * \operatorname{sqrt}(-b / d) * \operatorname{erf}(\operatorname{sqrt}(2) * \operatorname{sqrt}(d x + c) * \operatorname{sqrt}(-b / d)) + \\
& (64 * b^3 d^3 x^3 + 64 * b^3 c^3 - 16 * b^2 c^2 d + 30 * d^3 * \operatorname{cosh}(b x + a)^2 - (64 * \\
& b^3 d^3 x^3 + 64 * b^3 c^3 + 16 * b^2 c^2 d + 12 * b * c * d^2 + 15 * d^3 + 16 * (12 * b^3 c \\
& * d^2 + b^2 * d^3) * x^2 + 4 * (48 * b^3 c^2 d + 8 * b^2 * c * d^2 + 3 * b * d^3) * x) * \operatorname{cosh}(b x \\
& + a)^4 - 4 * (64 * b^3 d^3 x^3 + 64 * b^3 c^3 + 16 * b^2 c^2 d + 12 * b * c * d^2 + 15 * d \\
& ^3 + 16 * (12 * b^3 c * d^2 + b^2 * d^3) * x^2 + 4 * (48 * b^3 c^2 d + 8 * b^2 * c * d^2 + 3 * b * \\
& d^3) * x) * \operatorname{cosh}(b x + a) * \operatorname{sinh}(b x + a)^3 - (64 * b^3 d^3 x^3 + 64 * b^3 c^3 + 16 * b \\
& ^2 c^2 d + 12 * b * c * d^2 + 15 * d^3 + 16 * (12 * b^3 c * d^2 + b^2 * d^3) * x^2 + 4 * (48 * b^3 \\
& c^2 d + 8 * b^2 * c * d^2 + 3 * b * d^3) * x) * \operatorname{sinh}(b x + a)^4 + 12 * b * c * d^2 - 15 * d^3 + \\
& 16 * (12 * b^3 c * d^2 - b^2 * d^3) * x^2 + 6 * (5 * d^3 - (64 * b^3 d^3 x^3 + 64 * b^3 c^3 \\
& + 16 * b^2 c^2 d + 12 * b * c * d^2 + 15 * d^3 + 16 * (12 * b^3 c * d^2 + b^2 * d^3) * x^2 + 4 * \\
& (48 * b^3 c^2 d + 8 * b^2 * c * d^2 + 3 * b * d^3) * x) * \operatorname{cosh}(b x + a)^2) * \operatorname{sinh}(b x + a)^2 \\
& + 4 * (48 * b^3 c^2 d - 8 * b^2 * c * d^2 + 3 * b * d^3) * x + 4 * (15 * d^3 * \operatorname{cosh}(b x + a) - (6 \\
& 4 * b^3 d^3 x^3 + 64 * b^3 c^3 + 16 * b^2 c^2 d + 12 * b * c * d^2 + 15 * d^3 + 16 * (12 * b^3 \\
& c * d^2 + b^2 * d^3) * x^2 + 4 * (48 * b^3 c^2 d + 8 * b^2 * c * d^2 + 3 * b * d^3) * x) * \operatorname{cosh}(b \\
& x + a)^3) * \operatorname{sinh}(b x + a) * \operatorname{sqrt}(d x + c)) / ((d^8 x^4 + 4 * c * d^7 x^3 + 6 * c^2 * d^6 x^2 + 4 * c^3 * d^5 x + c^4 * d^4) * \operatorname{cosh}(b x + a)^2 + 2 * (d^8 x^4 + 4 * c * d^7 x^3 + \\
& 6 * c^2 * d^6 x^2 + 4 * c^3 * d^5 x + c^4 * d^4) * \operatorname{cosh}(b x + a) * \operatorname{sinh}(b x + a) + (d^8 x^4 + 4 * c * d^7 x^3 + 6 * c^2 * d^6 x^2 + 4 * c^3 * d^5 x + c^4 * d^4) * \operatorname{sinh}(b x + a)^2)
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)\*\*2/(d\*x+c)\*\*(9/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx + a)^2}{(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^2/(d\*x+c)^(9/2),x, algorithm="giac")

[Out] integrate(sinh(b\*x + a)^2/(d\*x + c)^(9/2), x)

### 3.53 $\int (c + dx)^{5/2} \sinh^3(a + bx) dx$

**Optimal.** Leaf size=381

$$\frac{45\sqrt{\pi}d^{5/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}} - \frac{5\sqrt{\frac{\pi}{3}}d^{5/2}e^{\frac{3bc}{d}-3a}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} + \frac{45\sqrt{\pi}d^{5/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}} - \frac{5\sqrt{\frac{\pi}{3}}d^{5/2}e^{3a-\frac{3bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}$$

[Out]  $(-45*d^2*\operatorname{Sqrt}[c + d*x]*\operatorname{Cosh}[a + b*x])/(16*b^3) - (2*(c + d*x)^{(5/2)}*\operatorname{Cosh}[a + b*x])/(3*b) + (5*d^2*\operatorname{Sqrt}[c + d*x]*\operatorname{Cosh}[3*a + 3*b*x])/(144*b^3) + (45*d^{(5/2)}*E^{(-a + (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/\operatorname{Sqrt}[d]])/(64*b^{(7/2)}) - (5*d^{(5/2)}*E^{(-3*a + (3*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erf}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/\operatorname{Sqrt}[d]])/(576*b^{(7/2)}) + (45*d^{(5/2)}*E^{(a - (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/\operatorname{Sqrt}[d]])/(64*b^{(7/2)}) - (5*d^{(5/2)}*E^{(3*a - (3*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/\operatorname{Sqrt}[d]])/(576*b^{(7/2)}) + (5*d*(c + d*x)^{(3/2)}*\operatorname{Sinh}[a + b*x])/(3*b^2) + ((c + d*x)^{(5/2)}*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x]^2)/(3*b) - (5*d*(c + d*x)^{(3/2)}*\operatorname{Sinh}[a + b*x]^3)/(18*b^2)$

**Rubi [A]** time = 1.06608, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {3311, 3296, 3307, 2180, 2204, 2205, 3312}

$$\frac{45\sqrt{\pi}d^{5/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}} - \frac{5\sqrt{\frac{\pi}{3}}d^{5/2}e^{\frac{3bc}{d}-3a}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} + \frac{45\sqrt{\pi}d^{5/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}} - \frac{5\sqrt{\frac{\pi}{3}}d^{5/2}e^{3a-\frac{3bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^{(5/2)}*\operatorname{Sinh}[a + b*x]^3, x]$

[Out]  $(-45*d^2*\operatorname{Sqrt}[c + d*x]*\operatorname{Cosh}[a + b*x])/(16*b^3) - (2*(c + d*x)^{(5/2)}*\operatorname{Cosh}[a + b*x])/(3*b) + (5*d^2*\operatorname{Sqrt}[c + d*x]*\operatorname{Cosh}[3*a + 3*b*x])/(144*b^3) + (45*d^{(5/2)}*E^{(-a + (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/\operatorname{Sqrt}[d]])/(64*b^{(7/2)}) - (5*d^{(5/2)}*E^{(-3*a + (3*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erf}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/\operatorname{Sqrt}[d]])/(576*b^{(7/2)}) + (45*d^{(5/2)}*E^{(a - (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/\operatorname{Sqrt}[d]])/(64*b^{(7/2)}) - (5*d^{(5/2)}*E^{(3*a - (3*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/\operatorname{Sqrt}[d]])/(576*b^{(7/2)}) + (5*d*(c + d*x)^{(3/2)}*\operatorname{Sinh}[a + b*x])/(3*b^2) + ((c + d*x)^{(5/2)}*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x]^2)/(3*b) - (5*d*(c + d*x)^{(3/2)}*\operatorname{Sinh}[a + b*x]^3)/(18*b^2)$

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:> Simp[(d*m*(c + d*x)^(m - 1)*(b*SIN[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*SIN[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*SIN[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*cos[e + f*x]*(b*SIN[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
:> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)^(n_)], x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

### Rubi steps

$$\begin{aligned}
 \int (c + dx)^{5/2} \sinh^3(a + bx) dx &= \frac{(c + dx)^{5/2} \cosh(a + bx) \sinh^2(a + bx)}{3b} - \frac{5d(c + dx)^{3/2} \sinh^3(a + bx)}{18b^2} - \frac{2}{3} \int (c + dx)^{5/2} \\
 &= -\frac{2(c + dx)^{5/2} \cosh(a + bx)}{3b} + \frac{(c + dx)^{5/2} \cosh(a + bx) \sinh^2(a + bx)}{3b} - \frac{5d(c + dx)^{3/2} \sinh^3(a + bx)}{18b^2} \\
 &= -\frac{2(c + dx)^{5/2} \cosh(a + bx)}{3b} + \frac{5d(c + dx)^{3/2} \sinh(a + bx)}{3b^2} + \frac{(c + dx)^{5/2} \cosh(a + bx) \sinh^2(a + bx)}{3b} \\
 &= -\frac{45d^2 \sqrt{c + dx} \cosh(a + bx)}{16b^3} - \frac{2(c + dx)^{5/2} \cosh(a + bx)}{3b} + \frac{5d^2 \sqrt{c + dx} \cosh(3a + 3bx)}{144b^3} \\
 &= -\frac{45d^2 \sqrt{c + dx} \cosh(a + bx)}{16b^3} - \frac{2(c + dx)^{5/2} \cosh(a + bx)}{3b} + \frac{5d^2 \sqrt{c + dx} \cosh(3a + 3bx)}{144b^3} \\
 &= -\frac{45d^2 \sqrt{c + dx} \cosh(a + bx)}{16b^3} - \frac{2(c + dx)^{5/2} \cosh(a + bx)}{3b} + \frac{5d^2 \sqrt{c + dx} \cosh(3a + 3bx)}{144b^3} \\
 &= -\frac{45d^2 \sqrt{c + dx} \cosh(a + bx)}{16b^3} - \frac{2(c + dx)^{5/2} \cosh(a + bx)}{3b} + \frac{5d^2 \sqrt{c + dx} \cosh(3a + 3bx)}{144b^3}
 \end{aligned}$$

**Mathematica [A]** time = 9.44604, size = 243, normalized size = 0.64

$$\frac{d^3 \left( \sqrt{3} \sqrt{-\frac{b(c+dx)}{d}} \Gamma\left(\frac{7}{2}, -\frac{3b(c+dx)}{d}\right) \left( \sinh\left(3a - \frac{3bc}{d}\right) + \cosh\left(3a - \frac{3bc}{d}\right) \right) + \left( \sinh\left(a - \frac{bc}{d}\right) - \cosh\left(a - \frac{bc}{d}\right) \right) \left( \sqrt{\frac{b(c+dx)}{d}} \right) \right)}{d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^(5/2)\*Sinh[a + b\*x]^3,x]

[Out]  $-(d^3 \sqrt{3} \sqrt{-\frac{b(c+dx)}{d}} \Gamma\left[\frac{7}{2}, -\frac{3b(c+dx)}{d}\right] (\cosh[3a - \frac{3bc}{d}] + \sinh[3a - \frac{3bc}{d}]) + \sqrt{\frac{b(c+dx)}{d}} (-243 \Gamma\left[\frac{7}{2}, \frac{b(c+dx)}{d}\right] + \sqrt{3} \Gamma\left[\frac{7}{2}, \frac{3b(c+dx)}{d}\right] (\cosh[2a - \frac{2bc}{d}] - \sinh[2a - \frac{2bc}{d}])) + 243 \sqrt{-\frac{b(c+dx)}{d}} \Gamma\left[\frac{7}{2}, -\frac{b(c+dx)}{d}\right] (\cosh[2a - \frac{2bc}{d}] + \sinh[2a - \frac{2bc}{d}]) - \sqrt{3} \sqrt{-\frac{b(c+dx)}{d}} \Gamma\left[\frac{7}{2}, -\frac{3b(c+dx)}{d}\right] (\sinh[a - \frac{bc}{d}] - \cosh[a - \frac{bc}{d}]))}{648 b^4 \sqrt{c + dx}}$

---

**Maple [F]** time = 0.082, size = 0, normalized size = 0.

$$\int (dx + c)^{\frac{5}{2}} (\sinh(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^(5/2)*sinh(b*x+a)^3,x)`

[Out] `int((d*x+c)^(5/2)*sinh(b*x+a)^3,x)`

---

**Maxima [A]** time = 1.82584, size = 693, normalized size = 1.82

$$\frac{5\sqrt{3}\sqrt{\pi}d^3 \operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}} + \frac{5\sqrt{3}\sqrt{\pi}d^3 \operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}} - \frac{1215\sqrt{\pi}d^3 \operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}} - \frac{1215\sqrt{\pi}d^3 \operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)}{b^3\sqrt{\frac{b}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^(5/2)*sinh(b*x+a)^3,x, algorithm="maxima")`

[Out] `-1/1728*(5*sqrt(3)*sqrt(pi)*d^3*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a - 3*b*c/d)/(b^3*sqrt(-b/d)) + 5*sqrt(3)*sqrt(pi)*d^3*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d))*e^(-3*a + 3*b*c/d)/(b^3*sqrt(b/d)) - 1215*sqrt(pi)*d^3*erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/(b^3*sqrt(-b/d)) - 1215*sqrt(pi)*d^3*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^3*sqrt(b/d)) + 162*(4*(d*x + c)^(5/2)*b^2*d*e^(b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(b*c/d) + 15*sqrt(d*x + c)*d^3*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^3 - 6*(12*(d*x + c)^(5/2)*b^2*d*e^(3*b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(3*b*c/d) + 5*sqrt(d*x + c)*d^3*e^(3*b*c/d))*e^(-3*a - 3*(d*x + c)*b/d)/b^3 - 6*(12*(d*x + c)^(5/2)*b^2*d*e^(3*a) - 10*(d*x + c)^(3/2)*b*d^2*e^(3*a) + 5*sqrt(d*x + c)*d^3*e^(3*a))*e^(3*(d*x + c)*b/d - 3*b*c/d)/b^3 + 162*(4*(d*x + c)^(5/2)*b^2*d*e^a - 10*(d*x + c)^(3/2)*b*d^2*e^a + 15*sqrt(d*x + c)*d^3*e^a)*e^((d*x + c)*b/d - b*c/d)/b^3)/d`

---

**Fricas [B]** time = 3.42604, size = 4817, normalized size = 12.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(5/2)\*sinh(b\*x+a)^3,x, algorithm="fricas")

[Out] 
$$-1/1728*(5*\sqrt{3}*\sqrt{\pi}*(d^3*\cosh(b*x + a)^3*\cosh(-3*(b*c - a*d)/d) - d^3*\cosh(b*x + a)^3*\sinh(-3*(b*c - a*d)/d) + (d^3*\cosh(-3*(b*c - a*d)/d) - d^3*\sinh(-3*(b*c - a*d)/d))*\sinh(b*x + a)^3 + 3*(d^3*\cosh(b*x + a)*\cosh(-3*(b*c - a*d)/d) - d^3*\cosh(b*x + a)*\sinh(-3*(b*c - a*d)/d))*\sinh(b*x + a)^2 + 3*(d^3*\cosh(b*x + a)^2*\cosh(-3*(b*c - a*d)/d) - d^3*\cosh(b*x + a)^2*\sinh(-3*(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{b/d}*\operatorname{erf}(\sqrt{3}*\sqrt{d*x + c}*\sqrt{b/d}) - 5*\sqrt{3}*\sqrt{\pi}*(d^3*\cosh(b*x + a)^3*\cosh(-3*(b*c - a*d)/d) + d^3*\cosh(b*x + a)^3*\sinh(-3*(b*c - a*d)/d) + (d^3*\cosh(-3*(b*c - a*d)/d) + d^3*\sinh(-3*(b*c - a*d)/d))*\sinh(b*x + a)^3 + 3*(d^3*\cosh(b*x + a)*\cosh(-3*(b*c - a*d)/d) + d^3*\cosh(b*x + a)*\sinh(-3*(b*c - a*d)/d))*\sinh(b*x + a)^2 + 3*(d^3*\cosh(b*x + a)^2*\cosh(-3*(b*c - a*d)/d) + d^3*\cosh(b*x + a)^2*\sinh(-3*(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{-b/d}*\operatorname{erf}(\sqrt{3}*\sqrt{d*x + c}*\sqrt{-b/d}) - 1215*\sqrt{\pi}*(d^3*\cosh(b*x + a)^3*\cosh(-(b*c - a*d)/d) - d^3*\cosh(b*x + a)^3*\sinh(-(b*c - a*d)/d) + (d^3*\cosh(-(b*c - a*d)/d) - d^3*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a)^3 + 3*(d^3*\cosh(b*x + a)*\cosh(-(b*c - a*d)/d) - d^3*\cosh(b*x + a)*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a)^2 + 3*(d^3*\cosh(b*x + a)^2*\cosh(-(b*c - a*d)/d) - d^3*\cosh(b*x + a)^2*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{b/d}*\operatorname{erf}(\sqrt{d*x + c}*\sqrt{b/d}) + 1215*\sqrt{\pi}*(d^3*\cosh(b*x + a)^3*\cosh(-(b*c - a*d)/d) + d^3*\cosh(b*x + a)^3*\sinh(-(b*c - a*d)/d) + (d^3*\cosh(-(b*c - a*d)/d) + d^3*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a)^3 + 3*(d^3*\cosh(b*x + a)*\cosh(-(b*c - a*d)/d) + d^3*\cosh(b*x + a)*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a)^2 + 3*(d^3*\cosh(b*x + a)^2*\cosh(-(b*c - a*d)/d) + d^3*\cosh(b*x + a)^2*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{-b/d}*\operatorname{erf}(\sqrt{d*x + c}*\sqrt{-b/d}) - 6*(12*b^3*d^2*x^2 + (12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x))*\cosh(b*x + a)^6 + 6*(12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x))*\cosh(b*x + a)*\sinh(b*x + a)^5 + (12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x))*\sinh(b*x + a)^6 + 12*b^3*c^2 - 27*(4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x))*\cosh(b*x + a)^4 - 3*(36*b^3*d^2*x^2 + 36*b^3*c^2 - 90*b^2*c*d + 135*b*d^2 - 5*(12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x))*\cosh(b*x + a)^2 + 18*(4*b^3*c*d - 5*b^2*d^2)*x)*\sinh(b*x + a)^4 + 10*b^2*c*d + 4*(5*(12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x))*\cosh(b*x + a)^3 - 27*(4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x))*\cosh(b*x + a))*\sinh(b*x + a)^3 + 5*b*d^2 - 27*(4*b^3*d^2*x^2 + 4*b^3*c^2 + 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d + 5*b^2*d^2)*x))*\cosh(b*x + a)^2 - 3*(36*b^3*d^2*x^2 + 36*b^3*c^2 - 5*(12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x))*\cosh(b*x + a)^4 + 90*b^2*c*d + 135*b*d^2 + 54*(4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5$$

```
*b^2*d^2)*x)*cosh(b*x + a)^2 + 18*(4*b^3*c*d + 5*b^2*d^2)*x)*sinh(b*x + a)^
2 + 2*(12*b^3*c*d + 5*b^2*d^2)*x + 6*((12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2
*c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^5 - 18*(4*b^3*
d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*
cosh(b*x + a)^3 - 9*(4*b^3*d^2*x^2 + 4*b^3*c^2 + 10*b^2*c*d + 15*b*d^2 + 2*
(4*b^3*c*d + 5*b^2*d^2)*x)*cosh(b*x + a))*sinh(b*x + a))*sqrt(d*x + c))/(b^
4*cosh(b*x + a)^3 + 3*b^4*cosh(b*x + a)^2*sinh(b*x + a) + 3*b^4*cosh(b*x +
a)*sinh(b*x + a)^2 + b^4*sinh(b*x + a)^3)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*sinh(b*x+a)**3,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{\frac{5}{2}} \sinh(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/2)*sinh(b*x + a)^3, x)
```



### 3.54 $\int (c + dx)^{3/2} \sinh^3(a + bx) dx$

**Optimal.** Leaf size=325

$$\frac{9\sqrt{\pi}d^{3/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}} - \frac{\sqrt{\frac{\pi}{3}}d^{3/2}e^{\frac{3bc}{d}-3a}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} - \frac{9\sqrt{\pi}d^{3/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}} + \frac{\sqrt{\frac{\pi}{3}}d^{3/2}e^{3a-\frac{3bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}$$

[Out]  $(-2*(c + d*x)^{(3/2)}*\operatorname{Cosh}[a + b*x])/(3*b) + (9*d^{(3/2)}*E^{(-a + (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(32*b^{(5/2)}) - (d^{(3/2)}*E^{(-3*a + (3*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erf}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(96*b^{(5/2)}) - (9*d^{(3/2)}*E^{(a - (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(32*b^{(5/2)}) + (d^{(3/2)}*E^{(3*a - (3*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(96*b^{(5/2)}) + (d*\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[a + b*x])/b^2 + ((c + d*x)^{(3/2)}*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x]^2)/(3*b) - (d*\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[a + b*x]^3)/(6*b^2)$

**Rubi [A]** time = 0.799132, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {3311, 3296, 3308, 2180, 2204, 2205, 3312}

$$\frac{9\sqrt{\pi}d^{3/2}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}} - \frac{\sqrt{\frac{\pi}{3}}d^{3/2}e^{\frac{3bc}{d}-3a}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} - \frac{9\sqrt{\pi}d^{3/2}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}} + \frac{\sqrt{\frac{\pi}{3}}d^{3/2}e^{3a-\frac{3bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d*x)^{(3/2)}*\operatorname{Sinh}[a + b*x]^3, x]$

[Out]  $(-2*(c + d*x)^{(3/2)}*\operatorname{Cosh}[a + b*x])/(3*b) + (9*d^{(3/2)}*E^{(-a + (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(32*b^{(5/2)}) - (d^{(3/2)}*E^{(-3*a + (3*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erf}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(96*b^{(5/2)}) - (9*d^{(3/2)}*E^{(a - (b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(32*b^{(5/2)}) + (d^{(3/2)}*E^{(3*a - (3*b*c)/d)}*\operatorname{Sqrt}[\operatorname{Pi}/3]*\operatorname{Erfi}[(\operatorname{Sqrt}[3]*\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c + d*x])/ \operatorname{Sqrt}[d]])/(96*b^{(5/2)}) + (d*\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[a + b*x])/b^2 + ((c + d*x)^{(3/2)}*\operatorname{Cosh}[a + b*x]*\operatorname{Sinh}[a + b*x]^2)/(3*b) - (d*\operatorname{Sqrt}[c + d*x]*\operatorname{Sinh}[a + b*x]^3)/(6*b^2)$

#### Rule 3311

$\operatorname{Int}[(c + d*x)^m*(b*\operatorname{Sin}[e + f*x])^n, x] := \operatorname{Simp}[(d*m*(c + d*x)^{(m-1)}*(b*\operatorname{Sin}[e + f*x])^n)/(f^2*n^2), x] + (\operatorname{Dist}[(b^2*(n-1))/n, \operatorname{Int}[(c + d*x)^m*(b*\operatorname{Sin}[e + f*x])^{n-2}, x], x] - \operatorname{Dist}[($

```
d^2*m*(m - 1)/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*SIN[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*SIN[e + f*x])^(n - 1))/(f*n), x] /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

### Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

### Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

### Rubi steps

$$\begin{aligned}
\int (c + dx)^{3/2} \sinh^3(a + bx) dx &= \frac{(c + dx)^{3/2} \cosh(a + bx) \sinh^2(a + bx)}{3b} - \frac{d\sqrt{c + dx} \sinh^3(a + bx)}{6b^2} - \frac{2}{3} \int (c + dx)^{3/2} \sinh^2(a + bx) dx \\
&= -\frac{2(c + dx)^{3/2} \cosh(a + bx)}{3b} + \frac{(c + dx)^{3/2} \cosh(a + bx) \sinh^2(a + bx)}{3b} - \frac{d\sqrt{c + dx} \sinh^3(a + bx)}{6b^2} \\
&= -\frac{2(c + dx)^{3/2} \cosh(a + bx)}{3b} + \frac{d\sqrt{c + dx} \sinh(a + bx)}{b^2} + \frac{(c + dx)^{3/2} \cosh(a + bx) \sinh^2(a + bx)}{3b} \\
&= -\frac{2(c + dx)^{3/2} \cosh(a + bx)}{3b} + \frac{d\sqrt{c + dx} \sinh(a + bx)}{b^2} + \frac{(c + dx)^{3/2} \cosh(a + bx) \sinh^2(a + bx)}{3b} \\
&= -\frac{2(c + dx)^{3/2} \cosh(a + bx)}{3b} + \frac{d\sqrt{c + dx} \sinh(a + bx)}{b^2} + \frac{(c + dx)^{3/2} \cosh(a + bx) \sinh^2(a + bx)}{3b} \\
&= -\frac{2(c + dx)^{3/2} \cosh(a + bx)}{3b} + \frac{9d^{3/2} e^{-a + \frac{bc}{d}} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}} - \frac{d^{3/2} e^{-3a + \frac{3bc}{d}} \sqrt{\frac{\pi}{3}} \operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}
\end{aligned}$$

**Mathematica [A]** time = 3.89138, size = 243, normalized size = 0.75

$$d^2 \left( \sqrt{3} \sqrt{-\frac{b(c+dx)}{d}} \operatorname{Gamma}\left(\frac{5}{2}, -\frac{3b(c+dx)}{d}\right) \left( \sinh\left(3a - \frac{3bc}{d}\right) + \cosh\left(3a - \frac{3bc}{d}\right) \right) + \left( \sinh\left(a - \frac{bc}{d}\right) - \cosh\left(a - \frac{bc}{d}\right) \right) \left( 81 \sqrt{-\frac{b(c+dx)}{d}} \operatorname{Gamma}\left(\frac{5}{2}, -\frac{3b(c+dx)}{d}\right) + \sqrt{3} \sqrt{-\frac{b(c+dx)}{d}} \operatorname{Gamma}\left(\frac{5}{2}, -\frac{3b(c+dx)}{d}\right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^(3/2)\*Sinh[a + b\*x]^3,x]

[Out] (d^2\*(Sqrt[3]\*Sqrt[-((b\*(c + d\*x))/d)]\*Gamma[5/2, (-3\*b\*(c + d\*x))/d]\*(Cosh[3\*a - (3\*b\*c)/d] + Sinh[3\*a - (3\*b\*c)/d]) + (81\*Sqrt[-((b\*(c + d\*x))/d)]\*Gamma[5/2, -((b\*(c + d\*x))/d)]\*(Cosh[2\*a - (2\*b\*c)/d] + Sinh[2\*a - (2\*b\*c)/d]) + Sqrt[(b\*(c + d\*x))/d]\*(81\*Gamma[5/2, (b\*(c + d\*x))/d] + Sqrt[3]\*Gamma[5/2, (3\*b\*(c + d\*x))/d]\*(-Cosh[2\*a - (2\*b\*c)/d] + Sinh[2\*a - (2\*b\*c)/d]))\*(-Cosh[a - (b\*c)/d] + Sinh[a - (b\*c)/d]))/(216\*b^3\*Sqrt[c + d\*x])

**Maple [F]** time = 0.078, size = 0, normalized size = 0.

$$\int (dx + c)^{\frac{3}{2}} (\sinh(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^(3/2)*sinh(b*x+a)^3,x)`

[Out] `int((d*x+c)^(3/2)*sinh(b*x+a)^3,x)`

**Maxima [A]** time = 1.79347, size = 581, normalized size = 1.79

$$\frac{\sqrt{3}\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} - \frac{\sqrt{3}\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} - \frac{81\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} + \frac{81\sqrt{\pi}d^2 \operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^(3/2)*sinh(b*x+a)^3,x, algorithm="maxima")`

[Out] `1/288*(sqrt(3)*sqrt(pi)*d^2*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a - 3*b*c/d)/(b^2*sqrt(-b/d)) - sqrt(3)*sqrt(pi)*d^2*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d))*e^(-3*a + 3*b*c/d)/(b^2*sqrt(b/d)) - 81*sqrt(pi)*d^2*erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/(b^2*sqrt(-b/d)) + 81*sqrt(pi)*d^2*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^2*sqrt(b/d)) - 54*(2*(d*x + c)^(3/2)*b*d*e^(b*c/d) + 3*sqrt(d*x + c)*d^2*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^2 + 6*(2*(d*x + c)^(3/2)*b*d*e^(3*b*c/d) + sqrt(d*x + c)*d^2*e^(3*b*c/d))*e^(-3*a - 3*(d*x + c)*b/d)/b^2 + 6*(2*(d*x + c)^(3/2)*b*d*e^(3*a) - sqrt(d*x + c)*d^2*e^(3*a))*e^(3*(d*x + c)*b/d - 3*b*c/d)/b^2 - 54*(2*(d*x + c)^(3/2)*b*d*e^a - 3*sqrt(d*x + c)*d^2*e^a)*e^((d*x + c)*b/d - b*c/d)/b^2)/d`

**Fricas [B]** time = 3.30136, size = 3622, normalized size = 11.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^(3/2)*sinh(b*x+a)^3,x, algorithm="fricas")`

[Out] `-1/288*(sqrt(3)*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d^2*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^2*cosh(-3*(b*c - a*d)/d) - d^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-3*(b*c`

$$\begin{aligned}
& - a*d)/d) - d^2*\cosh(b*x + a)*\sinh(-3*(b*c - a*d)/d))*\sinh(b*x + a)^2 + 3* \\
& (d^2*\cosh(b*x + a)^2*\cosh(-3*(b*c - a*d)/d) - d^2*\cosh(b*x + a)^2*\sinh(-3*( \\
& b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{b/d)*\operatorname{erf}(\sqrt{3}*\sqrt{d*x + c})*\sqrt{b/d} \\
& ) + \sqrt{3}*\sqrt{\pi}*(d^2*\cosh(b*x + a)^3*\cosh(-3*(b*c - a*d)/d) + d^2*\cosh \\
& (b*x + a)^3*\sinh(-3*(b*c - a*d)/d) + (d^2*\cosh(-3*(b*c - a*d)/d) + d^2*\sinh \\
& (-3*(b*c - a*d)/d))*\sinh(b*x + a)^3 + 3*(d^2*\cosh(b*x + a)*\cosh(-3*(b*c - a \\
& *d)/d) + d^2*\cosh(b*x + a)*\sinh(-3*(b*c - a*d)/d))*\sinh(b*x + a)^2 + 3*(d^2 \\
& *\cosh(b*x + a)^2*\cosh(-3*(b*c - a*d)/d) + d^2*\cosh(b*x + a)^2*\sinh(-3*(b*c \\
& - a*d)/d))*\sinh(b*x + a))*\sqrt{-b/d)*\operatorname{erf}(\sqrt{3}*\sqrt{d*x + c})*\sqrt{-b/d)} \\
& - 81*\sqrt{\pi}*(d^2*\cosh(b*x + a)^3*\cosh(-(b*c - a*d)/d) - d^2*\cosh(b*x + a) \\
& ^3*\sinh(-(b*c - a*d)/d) + (d^2*\cosh(-(b*c - a*d)/d) - d^2*\sinh(-(b*c - a*d) \\
& /d))*\sinh(b*x + a)^3 + 3*(d^2*\cosh(b*x + a)*\cosh(-(b*c - a*d)/d) - d^2*\cosh \\
& (b*x + a)*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a)^2 + 3*(d^2*\cosh(b*x + a)^2*\co \\
& sh(-(b*c - a*d)/d) - d^2*\cosh(b*x + a)^2*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a \\
& ))*\sqrt{b/d)*\operatorname{erf}(\sqrt{d*x + c})*\sqrt{b/d)} - 81*\sqrt{\pi}*(d^2*\cosh(b*x + a)^ \\
& 3*\cosh(-(b*c - a*d)/d) + d^2*\cosh(b*x + a)^3*\sinh(-(b*c - a*d)/d) + (d^2*\co \\
& sh(-(b*c - a*d)/d) + d^2*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a)^3 + 3*(d^2*\cos \\
& h(b*x + a)*\cosh(-(b*c - a*d)/d) + d^2*\cosh(b*x + a)*\sinh(-(b*c - a*d)/d))*\s \\
& inh(b*x + a)^2 + 3*(d^2*\cosh(b*x + a)^2*\cosh(-(b*c - a*d)/d) + d^2*\cosh(b*x \\
& + a)^2*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a))*\sqrt{-b/d)*\operatorname{erf}(\sqrt{d*x + c})*\s \\
& qrt(-b/d)} - 6*((2*b^2*d*x + 2*b^2*c - b*d)*\cosh(b*x + a)^6 + 6*(2*b^2*d*x \\
& + 2*b^2*c - b*d)*\cosh(b*x + a)*\sinh(b*x + a)^5 + (2*b^2*d*x + 2*b^2*c - b*d \\
& )*\sinh(b*x + a)^6 - 9*(2*b^2*d*x + 2*b^2*c - 3*b*d)*\cosh(b*x + a)^4 - 3*(6* \\
& b^2*d*x + 6*b^2*c - 5*(2*b^2*d*x + 2*b^2*c - b*d)*\cosh(b*x + a)^2 - 9*b*d)* \\
& \sinh(b*x + a)^4 + 2*b^2*d*x + 4*(5*(2*b^2*d*x + 2*b^2*c - b*d)*\cosh(b*x + a \\
& )^3 - 9*(2*b^2*d*x + 2*b^2*c - 3*b*d)*\cosh(b*x + a))*\sinh(b*x + a)^3 + 2*b^ \\
& 2*c - 9*(2*b^2*d*x + 2*b^2*c + 3*b*d)*\cosh(b*x + a)^2 + 3*(5*(2*b^2*d*x + 2 \\
& *b^2*c - b*d)*\cosh(b*x + a)^4 - 6*b^2*d*x - 6*b^2*c - 18*(2*b^2*d*x + 2*b^2 \\
& *c - 3*b*d)*\cosh(b*x + a)^2 - 9*b*d))*\sinh(b*x + a)^2 + b*d + 6*((2*b^2*d*x \\
& + 2*b^2*c - b*d)*\cosh(b*x + a)^5 - 6*(2*b^2*d*x + 2*b^2*c - 3*b*d)*\cosh(b*x \\
& + a)^3 - 3*(2*b^2*d*x + 2*b^2*c + 3*b*d)*\cosh(b*x + a))*\sinh(b*x + a))*\sqr \\
& t(d*x + c))/(b^3*\cosh(b*x + a)^3 + 3*b^3*\cosh(b*x + a)^2*\sinh(b*x + a) + 3* \\
& b^3*\cosh(b*x + a)*\sinh(b*x + a)^2 + b^3*\sinh(b*x + a)^3)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*(3/2)\*sinh(b\*x+a)\*\*3,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^{\frac{3}{2}} \sinh(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^(3/2)\*sinh(b\*x+a)^3,x, algorithm="giac")

[Out] integrate((d\*x + c)^(3/2)\*sinh(b\*x + a)^3, x)

### 3.55 $\int \sqrt{c + dx} \sinh^3(a + bx) dx$

**Optimal.** Leaf size=275

$$\frac{3\sqrt{\pi}\sqrt{de}^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} - \frac{\sqrt{\frac{\pi}{3}}\sqrt{de}^{\frac{3bc}{d}-3a}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} + \frac{3\sqrt{\pi}\sqrt{de}^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} - \frac{\sqrt{\frac{\pi}{3}}\sqrt{de}^{3a-\frac{3bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}$$

```
[Out] (-3*Sqrt[c + d*x]*Cosh[a + b*x])/(4*b) + (Sqrt[c + d*x]*Cosh[3*a + 3*b*x])/
(12*b) + (3*Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/S
qrt[d]])/(16*b^(3/2)) - (Sqrt[d]*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[
3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (3*Sqrt[d]*E^(a - (b*c)/
d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[d]*
E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]
])/(48*b^(3/2))
```

**Rubi [A]** time = 0.521909, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3312, 3296, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi}\sqrt{de}^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} - \frac{\sqrt{\frac{\pi}{3}}\sqrt{de}^{\frac{3bc}{d}-3a}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} + \frac{3\sqrt{\pi}\sqrt{de}^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} - \frac{\sqrt{\frac{\pi}{3}}\sqrt{de}^{3a-\frac{3bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Sinh[a + b*x]^3, x]
```

```
[Out] (-3*Sqrt[c + d*x]*Cosh[a + b*x])/(4*b) + (Sqrt[c + d*x]*Cosh[3*a + 3*b*x])/
(12*b) + (3*Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/S
qrt[d]])/(16*b^(3/2)) - (Sqrt[d]*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[
3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (3*Sqrt[d]*E^(a - (b*c)/
d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[d]*
E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]
])/(48*b^(3/2))
```

#### Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps



$$\begin{aligned}
\int \sqrt{c+dx} \sinh^3(a+bx) dx &= i \int \left( \frac{3}{4} i \sqrt{c+dx} \sinh(a+bx) - \frac{1}{4} i \sqrt{c+dx} \sinh(3a+3bx) \right) dx \\
&= \frac{1}{4} \int \sqrt{c+dx} \sinh(3a+3bx) dx - \frac{3}{4} \int \sqrt{c+dx} \sinh(a+bx) dx \\
&= -\frac{3\sqrt{c+dx} \cosh(a+bx)}{4b} + \frac{\sqrt{c+dx} \cosh(3a+3bx)}{12b} - \frac{d \int \frac{\cosh(3a+3bx)}{\sqrt{c+dx}} dx}{24b} + \frac{(3d) \int \frac{\cosh(a+bx)}{\sqrt{c+dx}} dx}{8b} \\
&= -\frac{3\sqrt{c+dx} \cosh(a+bx)}{4b} + \frac{\sqrt{c+dx} \cosh(3a+3bx)}{12b} - \frac{d \int \frac{e^{-i(3ia+3ibx)}}{\sqrt{c+dx}} dx}{48b} - \frac{d \int \frac{e^{i(3ia+3ibx)}}{\sqrt{c+dx}} dx}{48b} \\
&= -\frac{3\sqrt{c+dx} \cosh(a+bx)}{4b} + \frac{\sqrt{c+dx} \cosh(3a+3bx)}{12b} - \frac{\text{Subst} \left( \int e^{i(3ia-\frac{3ibc}{d})-\frac{3bx^2}{d}} dx, x, \sqrt{c+dx} \right)}{24b} \\
&= -\frac{3\sqrt{c+dx} \cosh(a+bx)}{4b} + \frac{\sqrt{c+dx} \cosh(3a+3bx)}{12b} + \frac{3\sqrt{d} e^{-a+\frac{bc}{d}} \sqrt{\pi} \operatorname{erf} \left( \frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}} \right)}{16b^{3/2}} - \frac{\sqrt{d} e^{a+\frac{bc}{d}} \sqrt{\pi} \operatorname{erf} \left( \frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}} \right)}{16b^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.291349, size = 209, normalized size = 0.76

$$\frac{\sqrt{c+dx} e^{-3\left(a+\frac{bc}{d}\right)} \left( \sqrt{3} e^{6a} \sqrt{\frac{b(c+dx)}{d}} \operatorname{Gamma} \left( \frac{3}{2}, -\frac{3b(c+dx)}{d} \right) - 27 e^{4a+\frac{2bc}{d}} \sqrt{\frac{b(c+dx)}{d}} \operatorname{Gamma} \left( \frac{3}{2}, -\frac{b(c+dx)}{d} \right) + e^{\frac{4bc}{d}} \sqrt{-\frac{b(c+dx)}{d}} \left( \sqrt{3} \operatorname{erf} \left( \frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}} \right) - \sqrt{3} \operatorname{erf} \left( \frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}} \right) \right) \right)}{72b \sqrt{-\frac{b^2(c+dx)^2}{d^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[c + d\*x]\*Sinh[a + b\*x]^3,x]

[Out] (Sqrt[c + d\*x]\*(Sqrt[3]\*E^(6\*a)\*Sqrt[(b\*(c + d\*x))/d]\*Gamma[3/2, (-3\*b\*(c + d\*x))/d] - 27\*E^(4\*a + (2\*b\*c)/d)\*Sqrt[(b\*(c + d\*x))/d]\*Gamma[3/2, -(b\*(c + d\*x))/d] + E^((4\*b\*c)/d)\*Sqrt[-((b\*(c + d\*x))/d)]\*(-27\*E^(2\*a)\*Gamma[3/2, (b\*(c + d\*x))/d] + Sqrt[3]\*E^((2\*b\*c)/d)\*Gamma[3/2, (3\*b\*(c + d\*x))/d]))/(72\*b\*E^(3\*(a + (b\*c)/d))\*Sqrt[-((b^2\*(c + d\*x)^2)/d^2)])

**Maple [F]** time = 0.078, size = 0, normalized size = 0.

$$\int (\sinh(bx+a))^3 \sqrt{dx+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(b*x+a)^3*(d*x+c)^(1/2),x)`

[Out] `int(sinh(b*x+a)^3*(d*x+c)^(1/2),x)`

**Maxima [A]** time = 1.88603, size = 450, normalized size = 1.64

$$\frac{\sqrt{3}\sqrt{\pi d} \operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}} + \frac{\sqrt{3}\sqrt{\pi d} \operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b\sqrt{\frac{b}{d}}} - \frac{27\sqrt{\pi d} \operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}} - \frac{27\sqrt{\pi d} \operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b\sqrt{\frac{b}{d}}}$$

144d

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)^3*(d*x+c)^(1/2),x, algorithm="maxima")`

[Out]  $-1/144*(\sqrt{3}*\sqrt{\pi})*d*\operatorname{erf}(\sqrt{3}*\sqrt{d*x+c}*\sqrt{-b/d})*e^{(3*a-3*b*c/d)/(b*\sqrt{-b/d})} + \sqrt{3}*\sqrt{\pi})*d*\operatorname{erf}(\sqrt{3}*\sqrt{d*x+c}*\sqrt{b/d})*e^{(-3*a+3*b*c/d)/(b*\sqrt{b/d})} - 27*\sqrt{\pi})*d*\operatorname{erf}(\sqrt{d*x+c}*\sqrt{-b/d})*e^{(a-b*c/d)/(b*\sqrt{-b/d})} - 27*\sqrt{\pi})*d*\operatorname{erf}(\sqrt{d*x+c}*\sqrt{b/d})*e^{(-a+b*c/d)/(b*\sqrt{b/d})} - 6*\sqrt{d*x+c})*d*e^{(3*a+3*(d*x+c)*b/d-3*b*c/d)/b} + 54*\sqrt{d*x+c})*d*e^{(a+(d*x+c)*b/d-b*c/d)/b} + 54*\sqrt{d*x+c})*d*e^{(-a-(d*x+c)*b/d+b*c/d)/b} - 6*\sqrt{d*x+c})*d*e^{(-3*a-3*(d*x+c)*b/d+3*b*c/d)/b}/d$

**Fricas [B]** time = 2.99826, size = 2966, normalized size = 10.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)^3*(d*x+c)^(1/2),x, algorithm="fricas")`

[Out]  $-1/144*(\sqrt{3}*\sqrt{\pi})*(d*\cosh(b*x+a)^3*\cosh(-3*(b*c-a*d)/d) - d*\cosh(b*x+a)^3*\sinh(-3*(b*c-a*d)/d) + (d*\cosh(-3*(b*c-a*d)/d) - d*\sinh(-3*(b*c-a*d)/d))*\sinh(b*x+a)^3 + 3*(d*\cosh(b*x+a)*\cosh(-3*(b*c-a*d)/d) - d*\cosh(b*x+a)*\sinh(-3*(b*c-a*d)/d))*\sinh(b*x+a)^2 + 3*(d*\cosh(b*x+a)^2*\cosh(-3*(b*c-a*d)/d) - d*\cosh(b*x+a)^2*\sinh(-3*(b*c-a*d)/d))*\sinh(b*x+a)*\sqrt{b/d}*\operatorname{erf}(\sqrt{3}*\sqrt{d*x+c}*\sqrt{b/d}) - \sqrt{3}*\sqrt{\pi})*(d*\cosh(b*x+a)^3*\cosh(-3*(b*c-a*d)/d) + d*\cosh(b*x+a)^3*\sinh(-3*(b*c-a*d)/d) - 27*\sqrt{\pi})*d*\operatorname{erf}(\sqrt{d*x+c}*\sqrt{-b/d})*e^{(a-b*c/d)/(b*\sqrt{-b/d})} - 27*\sqrt{\pi})*d*\operatorname{erf}(\sqrt{d*x+c}*\sqrt{b/d})*e^{(-a+b*c/d)/(b*\sqrt{b/d})} - 6*\sqrt{d*x+c})*d*e^{(3*a+3*(d*x+c)*b/d-3*b*c/d)/b} + 54*\sqrt{d*x+c})*d*e^{(a+(d*x+c)*b/d-b*c/d)/b} + 54*\sqrt{d*x+c})*d*e^{(-a-(d*x+c)*b/d+b*c/d)/b} - 6*\sqrt{d*x+c})*d*e^{(-3*a-3*(d*x+c)*b/d+3*b*c/d)/b}/d$

```
(b*c - a*d)/d) + (d*cosh(-3*(b*c - a*d)/d) + d*sinh(-3*(b*c - a*d)/d))*sinh
(b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) + d*cosh(b*x + a)*s
inh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(b*x + a)^2*cosh(-3*(b*c
- a*d)/d) + d*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)*sqrt(-
b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d)) - 27*sqrt(pi)*(d*cosh(b*x + a)^3
*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + (d*cosh(-(
b*c - a*d)/d) - d*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a
)*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a
)^2 + 3*(d*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)^2*sinh(-(
b*c - a*d)/d))*sinh(b*x + a)*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 27*s
qrt(pi)*(d*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) + d*cosh(b*x + a)^3*sinh(-(
b*c - a*d)/d) + (d*cosh(-(b*c - a*d)/d) + d*sinh(-(b*c - a*d)/d))*sinh(b*x
+ a)^3 + 3*(d*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d*cosh(b*x + a)*sinh(-(b
*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) +
d*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a)*sqrt(-b/d)*erf(sqrt
(d*x + c)*sqrt(-b/d)) - 6*(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x +
a)^5 + b*sinh(b*x + a)^6 - 9*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 -
3*b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 - 9*b*cosh(b*x + a))*sinh(b*x
+ a)^3 - 9*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 - 18*b*cosh(b*x + a)
^2 - 3*b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 - 6*b*cosh(b*x + a)^3 - 3*
b*cosh(b*x + a))*sinh(b*x + a) + b)*sqrt(d*x + c))/(b^2*cosh(b*x + a)^3 + 3
*b^2*cosh(b*x + a)^2*sinh(b*x + a) + 3*b^2*cosh(b*x + a)*sinh(b*x + a)^2 +
b^2*sinh(b*x + a)^3)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c + dx} \sinh^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)\*\*3\*(d\*x+c)\*\*(1/2),x)

[Out] Integral(sqrt(c + d\*x)\*sinh(a + b\*x)\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx + c} \sinh(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(b*x+a)^3*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x + c)*sinh(b*x + a)^3, x)
```

$$3.56 \quad \int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}} dx$$

**Optimal.** Leaf size=228

$$\frac{3\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} - \frac{\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} - \frac{3\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} + \frac{\sqrt{\frac{\pi}{3}}e^{3a-\frac{3bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}$$

[Out] (3\*E^(-a + (b\*c)/d)\*Sqrt[Pi]\*Erf[(Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(8\*Sqrt[b]\*Sqrt[d]) - (E^(-3\*a + (3\*b\*c)/d)\*Sqrt[Pi/3]\*Erf[(Sqrt[3]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(8\*Sqrt[b]\*Sqrt[d]) - (3\*E^(a - (b\*c)/d)\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(8\*Sqrt[b]\*Sqrt[d]) + (E^(3\*a - (3\*b\*c)/d)\*Sqrt[Pi/3]\*Erfi[(Sqrt[3]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(8\*Sqrt[b]\*Sqrt[d])

**Rubi [A]** time = 0.401139, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3312, 3308, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} - \frac{\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} - \frac{3\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} + \frac{\sqrt{\frac{\pi}{3}}e^{3a-\frac{3bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[a + b\*x]^3/Sqrt[c + d\*x], x]

[Out] (3\*E^(-a + (b\*c)/d)\*Sqrt[Pi]\*Erf[(Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(8\*Sqrt[b]\*Sqrt[d]) - (E^(-3\*a + (3\*b\*c)/d)\*Sqrt[Pi/3]\*Erf[(Sqrt[3]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(8\*Sqrt[b]\*Sqrt[d]) - (3\*E^(a - (b\*c)/d)\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(8\*Sqrt[b]\*Sqrt[d]) + (E^(3\*a - (3\*b\*c)/d)\*Sqrt[Pi/3]\*Erfi[(Sqrt[3]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(8\*Sqrt[b]\*Sqrt[d])

### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

### Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

### Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{\sinh^3(a + bx)}{\sqrt{c + dx}} dx &= i \int \left( \frac{3i \sinh(a + bx)}{4\sqrt{c + dx}} - \frac{i \sinh(3a + 3bx)}{4\sqrt{c + dx}} \right) dx \\
 &= \frac{1}{4} \int \frac{\sinh(3a + 3bx)}{\sqrt{c + dx}} dx - \frac{3}{4} \int \frac{\sinh(a + bx)}{\sqrt{c + dx}} dx \\
 &= \frac{1}{8} \int \frac{e^{-i(3ia+3ibx)}}{\sqrt{c + dx}} dx - \frac{1}{8} \int \frac{e^{i(3ia+3ibx)}}{\sqrt{c + dx}} dx - \frac{3}{8} \int \frac{e^{-i(ia+ibx)}}{\sqrt{c + dx}} dx + \frac{3}{8} \int \frac{e^{i(ia+ibx)}}{\sqrt{c + dx}} dx \\
 &= \frac{\text{Subst} \left( \int e^{i \left( 3ia - \frac{3ibc}{d} \right) - \frac{3bx^2}{d}} dx, x, \sqrt{c + dx} \right)}{4d} + \frac{\text{Subst} \left( \int e^{-i \left( 3ia - \frac{3ibc}{d} \right) + \frac{3bx^2}{d}} dx, x, \sqrt{c + dx} \right)}{4d} + \frac{3 \text{Subst} \left( \int e^{-i(ia+ibx)} dx, x, \sqrt{c + dx} \right)}{8\sqrt{b}\sqrt{d}} - \frac{3 \text{Subst} \left( \int e^{i(ia+ibx)} dx, x, \sqrt{c + dx} \right)}{8\sqrt{b}\sqrt{d}} \\
 &= \frac{3e^{-a+\frac{bc}{d}} \sqrt{\pi} \text{erf} \left( \frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}} \right)}{8\sqrt{b}\sqrt{d}} - \frac{e^{-3a+\frac{3bc}{d}} \sqrt{\frac{\pi}{3}} \text{erf} \left( \frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}} \right)}{8\sqrt{b}\sqrt{d}} - \frac{3e^{a-\frac{bc}{d}} \sqrt{\pi} \text{erfi} \left( \frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}} \right)}{8\sqrt{b}\sqrt{d}} + \frac{e^{3a-\frac{3bc}{d}} \sqrt{\frac{\pi}{3}} \text{erfi} \left( \frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}} \right)}{8\sqrt{b}\sqrt{d}}
 \end{aligned}$$

**Mathematica [A]** time = 0.194076, size = 191, normalized size = 0.84

$$\frac{e^{-3\left(a+\frac{bc}{d}\right)} \left( \sqrt{3} e^{6a} \sqrt{-\frac{b(c+dx)}{d}} \text{Gamma} \left( \frac{1}{2}, -\frac{3b(c+dx)}{d} \right) - 9 e^{4a+\frac{2bc}{d}} \sqrt{-\frac{b(c+dx)}{d}} \text{Gamma} \left( \frac{1}{2}, -\frac{b(c+dx)}{d} \right) + e^{\frac{4bc}{d}} \sqrt{\frac{b(c+dx)}{d}} \left( \sqrt{3} e^{\frac{2bc}{d}} \text{Gamma} \left( \frac{1}{2}, -\frac{3b(c+dx)}{d} \right) - 9 e^{\frac{2bc}{d}} \text{Gamma} \left( \frac{1}{2}, -\frac{b(c+dx)}{d} \right) \right) \right)}{24b\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[a + b\*x]^3/Sqrt[c + d\*x],x]

[Out] (Sqrt[3]\*E^(6\*a)\*Sqrt[-((b\*(c + d\*x))/d)]\*Gamma[1/2, (-3\*b\*(c + d\*x))/d] - 9\*E^(4\*a + (2\*b\*c)/d)\*Sqrt[-((b\*(c + d\*x))/d)]\*Gamma[1/2, -((b\*(c + d\*x))/d)] + E^((4\*b\*c)/d)\*Sqrt[(b\*(c + d\*x))/d]\*(-9\*E^(2\*a)\*Gamma[1/2, (b\*(c + d\*x))/d] + Sqrt[3]\*E^((2\*b\*c)/d)\*Gamma[1/2, (3\*b\*(c + d\*x))/d]))/(24\*b\*E^(3\*(a + (b\*c)/d))\*Sqrt[c + d\*x])

**Maple [F]** time = 0.095, size = 0, normalized size = 0.

$$\int (\sinh(bx + a))^3 \frac{1}{\sqrt{dx + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(b\*x+a)^3/(d\*x+c)^(1/2),x)

[Out] int(sinh(b\*x+a)^3/(d\*x+c)^(1/2),x)

**Maxima [A]** time = 1.70079, size = 240, normalized size = 1.05

$$\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{-b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{\sqrt{\frac{-b}{d}}} - \frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{\sqrt{\frac{b}{d}}} - \frac{9\sqrt{\pi}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{-b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{\sqrt{\frac{-b}{d}}} + \frac{9\sqrt{\pi}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{\sqrt{\frac{b}{d}}}$$

$24d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^3/(d\*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/24\*(sqrt(3)\*sqrt(pi)\*erf(sqrt(3)\*sqrt(d\*x + c)\*sqrt(-b/d))\*e^(3\*a - 3\*b\*c/d)/sqrt(-b/d) - sqrt(3)\*sqrt(pi)\*erf(sqrt(3)\*sqrt(d\*x + c)\*sqrt(b/d))\*e^(-3\*a + 3\*b\*c/d)/sqrt(b/d) - 9\*sqrt(pi)\*erf(sqrt(d\*x + c)\*sqrt(-b/d))\*e^(a - b\*c/d)/sqrt(-b/d) + 9\*sqrt(pi)\*erf(sqrt(d\*x + c)\*sqrt(b/d))\*e^(-a + b\*c/d)/sqrt(b/d)/d

**Fricas [A]** time = 2.72496, size = 595, normalized size = 2.61

$$\frac{\sqrt{3}\sqrt{\pi}\sqrt{\frac{b}{d}}\left(\cosh\left(-\frac{3(bc-ad)}{d}\right) - \sinh\left(-\frac{3(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right) + \sqrt{3}\sqrt{\pi}\sqrt{-\frac{b}{d}}\left(\cosh\left(-\frac{3(bc-ad)}{d}\right) + \sinh\left(-\frac{3(bc-ad)}{d}\right)\right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^3/(d\*x+c)^(1/2),x, algorithm="fricas")

[Out]  $-1/24*(\sqrt{3}*\sqrt{\pi}*\sqrt{b/d}*(\cosh(-3*(b*c - a*d)/d) - \sinh(-3*(b*c - a*d)/d))*\operatorname{erf}(\sqrt{3}*\sqrt{d*x + c}*\sqrt{b/d}) + \sqrt{3}*\sqrt{\pi}*\sqrt{-b/d}*(\cosh(-3*(b*c - a*d)/d) + \sinh(-3*(b*c - a*d)/d))*\operatorname{erf}(\sqrt{3}*\sqrt{d*x + c})*\sqrt{-b/d}) - 9*\sqrt{\pi}*\sqrt{b/d}*(\cosh(-(b*c - a*d)/d) - \sinh(-(b*c - a*d)/d))*\operatorname{erf}(\sqrt{d*x + c}*\sqrt{b/d}) - 9*\sqrt{\pi}*\sqrt{-b/d}*(\cosh(-(b*c - a*d)/d) + \sinh(-(b*c - a*d)/d))*\operatorname{erf}(\sqrt{d*x + c}*\sqrt{-b/d}))/b$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(a + bx)}{\sqrt{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)\*\*3/(d\*x+c)\*\*(1/2),x)

[Out] Integral(sinh(a + b\*x)\*\*3/sqrt(c + d\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(bx + a)}{\sqrt{dx + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^3/(d\*x+c)^(1/2),x, algorithm="giac")

[Out] integrate(sinh(b\*x + a)^3/sqrt(d\*x + c), x)



$$3.57 \quad \int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}} dx$$

**Optimal.** Leaf size=246

$$-\frac{3\sqrt{\pi}\sqrt{b}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} + \frac{\sqrt{3\pi}\sqrt{b}e^{\frac{3bc}{d}-3a}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} - \frac{3\sqrt{\pi}\sqrt{b}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} + \frac{\sqrt{3\pi}\sqrt{b}e^{3a-\frac{3bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}$$

```
[Out] (-3*Sqrt[b]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])
/(4*d^(3/2)) + (Sqrt[b]*E^(-3*a + (3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b]
)*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/2)) - (3*Sqrt[b]*E^(a - (b*c)/d)*Sqrt[Pi
]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/2)) + (Sqrt[b]*E^(3*a - (3
*b*c)/d)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/
2)) - (2*Sinh[a + b*x]^3)/(d*Sqrt[c + d*x])
```

**Rubi [A]** time = 0.453834, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3313, 3307, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}\sqrt{b}e^{\frac{bc}{d}-a}\operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} + \frac{\sqrt{3\pi}\sqrt{b}e^{\frac{3bc}{d}-3a}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} - \frac{3\sqrt{\pi}\sqrt{b}e^{a-\frac{bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} + \frac{\sqrt{3\pi}\sqrt{b}e^{3a-\frac{3bc}{d}}\operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[a + b*x]^3/(c + d*x)^(3/2), x]
```

```
[Out] (-3*Sqrt[b]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])
/(4*d^(3/2)) + (Sqrt[b]*E^(-3*a + (3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b]
)*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/2)) - (3*Sqrt[b]*E^(a - (b*c)/d)*Sqrt[Pi
]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/2)) + (Sqrt[b]*E^(3*a - (3
*b*c)/d)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/
2)) - (2*Sinh[a + b*x]^3)/(d*Sqrt[c + d*x])
```

### Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Si
mp[(((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
LtQ[m, -1]
```

Rule 3307

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Dist[I/2, Int[(c + d\*x)^m/(E^(I\*k\*Pi)\*E^(I\*(e + f\*x))), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(I\*k\*Pi)\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_))^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rubi steps

$$\begin{aligned}
 \int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}} dx &= -\frac{2 \sinh^3(a+bx)}{d\sqrt{c+dx}} - \frac{(6b) \int \left( \frac{\cosh(a+bx)}{4\sqrt{c+dx}} - \frac{\cosh(3a+3bx)}{4\sqrt{c+dx}} \right) dx}{d} \\
 &= -\frac{2 \sinh^3(a+bx)}{d\sqrt{c+dx}} - \frac{(3b) \int \frac{\cosh(a+bx)}{\sqrt{c+dx}} dx}{2d} + \frac{(3b) \int \frac{\cosh(3a+3bx)}{\sqrt{c+dx}} dx}{2d} \\
 &= -\frac{2 \sinh^3(a+bx)}{d\sqrt{c+dx}} - \frac{(3b) \int \frac{e^{-i(i+ibx)}}{\sqrt{c+dx}} dx}{4d} - \frac{(3b) \int \frac{e^{i(i+ibx)}}{\sqrt{c+dx}} dx}{4d} + \frac{(3b) \int \frac{e^{-i(3ia+3ibx)}}{\sqrt{c+dx}} dx}{4d} + \frac{(3b) \int \frac{e^{i(3ia+3ibx)}}{\sqrt{c+dx}} dx}{4d} \\
 &= -\frac{2 \sinh^3(a+bx)}{d\sqrt{c+dx}} + \frac{(3b) \text{Subst} \left( \int e^{i \left( 3ia - \frac{3ibc}{d} \right) - \frac{3bx^2}{d}} dx, x, \sqrt{c+dx} \right)}{2d^2} - \frac{(3b) \text{Subst} \left( \int e^{i \left( ia - \frac{ibc}{d} \right) - \frac{bx^2}{d}} dx, x, \sqrt{c+dx} \right)}{2d^2} \\
 &= -\frac{3\sqrt{b}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} + \frac{\sqrt{b}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} - \frac{3\sqrt{b}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} + \dots
 \end{aligned}$$

**Mathematica [B]** time = 10.2652, size = 2058, normalized size = 8.37

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[Sinh[a + b\*x]^3/(c + d\*x)^(3/2), x]

[Out] 
$$\begin{aligned} & (-3 * (\text{Cosh}[a] * (-((1 + E^{(2*b*(c + d*x))/d})/E^{(b*(c + d*x))/d}) + \text{Sqrt} \\ & [-(b*(c + d*x))/d] * \text{Gamma}[1/2, -(b*(c + d*x))/d] + \text{Sqrt}[(b*(c + d*x))/d] \\ & * \text{Gamma}[1/2, (b*(c + d*x))/d] * \text{Sinh}[(b*c)/d]) / (d * \text{Sqrt}[c + d*x])) + (\text{Cosh}[(b*c)/d] * \\ & (\text{Sqrt}[-(b*(c + d*x))/d] * \text{Gamma}[1/2, -(b*(c + d*x))/d] - \text{Sqrt}[(b*(c + d*x))/d] * \\ & \text{Gamma}[1/2, (b*(c + d*x))/d] - 2 * \text{Sinh}[(b*(c + d*x))/d])) / (d * \text{Sqrt}[c + d*x]) \\ & + \text{Sinh}[a] * ((\text{Cosh}[(b*c)/d] * (-((1 + E^{(2*b*(c + d*x))/d})/E^{(b*(c + d*x))/d}) + \\ & \text{Sqrt}[-(b*(c + d*x))/d] * \text{Gamma}[1/2, -(b*(c + d*x))/d] + \text{Sqrt}[(b*(c + d*x))/d] * \\ & \text{Gamma}[1/2, (b*(c + d*x))/d])) / (d * \text{Sqrt}[c + d*x]) + (\text{Sinh}[(b*c)/d] * (-(\text{Sqrt}[-(b*(c + d*x))/d] * \\ & \text{Gamma}[1/2, -(b*(c + d*x))/d]) + \text{Sqrt}[(b*(c + d*x))/d] * \text{Gamma}[1/2, (b*(c + d*x))/d] + \\ & 2 * \text{Sinh}[(b*(c + d*x))/d])) / (d * \text{Sqrt}[c + d*x])) / 4 + (- (\text{Sinh}[3*a] * (-((1 + 2 * \text{Cosh}[(2*b*c)/d]) * \\ & (-((1 + E^{(6*b*(c + d*x))/d})/E^{(3*b*(c + d*x))/d}) + \text{Sqrt}[3] * \text{Sqrt}[-(b*(c + d*x))/d] * \\ & \text{Gamma}[1/2, (-3*b*(c + d*x))/d] + \text{Sqrt}[3] * \text{Sqrt}[(b*(c + d*x))/d] * \text{Gamma}[1/2, \\ & (3*b*(c + d*x))/d]) * \text{Sinh}[(b*c)/d]) / (d * \text{Sqrt}[c + d*x])) + (\text{Cosh}[(b*c)/d] * (-1 + \\ & 2 * \text{Cosh}[(2*b*c)/d]) * ((\text{Sqrt}[b] * \text{Sqrt}[6 * \text{Pi}] * (\text{Erf}[(\text{Sqrt}[3] * \text{Sqrt}[b] * \text{Sqrt}[c + d*x])/ \\ & \text{Sqrt}[d]] + \text{Erfi}[(\text{Sqrt}[3] * \text{Sqrt}[b] * \text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])) / \text{Sqrt}[d] - (2 * \text{Sqrt}[2] * \\ & \text{Sinh}[(3*b*(c + d*x))/d]) / \text{Sqrt}[c + d*x])) / (\text{Sqrt}[2] * d)) - \text{Cosh}[3*a] * ((\text{Cosh}[(b*c)/d] * \\ & (-1 + 2 * \text{Cosh}[(2*b*c)/d]) * (-((1 + E^{(6*b*(c + d*x))/d})/E^{(3*b*(c + d*x))/d}) + \\ & \text{Sqrt}[3] * \text{Sqrt}[-(b*(c + d*x))/d] * \text{Gamma}[1/2, (-3*b*(c + d*x))/d] + \text{Sqrt}[3] * \text{Sqrt}[(b*(c + d*x))/d] * \\ & \text{Gamma}[1/2, (3*b*(c + d*x))/d])) / (d * \text{Sqrt}[c + d*x]) - ((1 + 2 * \text{Cosh}[(2*b*c)/d]) * \text{Sinh}[(b*c)/d] * \\ & ((\text{Sqrt}[b] * \text{Sqrt}[6 * \text{Pi}] * (\text{Erf}[(\text{Sqrt}[3] * \text{Sqrt}[b] * \text{Sqrt}[c + d*x])/ \text{Sqrt}[d]] + \text{Erfi}[(\text{Sqrt}[3] * \\ & \text{Sqrt}[b] * \text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])) / \text{Sqrt}[d] - (2 * \text{Sqrt}[2] * \text{Sinh}[(3*b*(c + d*x))/d]) / \\ & \text{Sqrt}[c + d*x])) / (\text{Sqrt}[2] * d)) / 8 + (\text{Sinh}[3*a] * (-((1 + 2 * \text{Cosh}[(2*b*c)/d]) * (-((1 + \\ & E^{(6*b*(c + d*x))/d})/E^{(3*b*(c + d*x))/d}) + \text{Sqrt}[3] * \text{Sqrt}[-(b*(c + d*x))/d] * \\ & \text{Gamma}[1/2, (-3*b*(c + d*x))/d] + \text{Sqrt}[3] * \text{Sqrt}[(b*(c + d*x))/d] * \text{Gamma}[1/2, (3*b*(c + d*x))/d] * \\ & \text{Sinh}[(b*c)/d])) / (d * \text{Sqrt}[c + d*x])) + (\text{Cosh}[(b*c)/d] * (-1 + 2 * \text{Cosh}[(2*b*c)/d]) * \\ & ((\text{Sqrt}[b] * \text{Sqrt}[6 * \text{Pi}] * (\text{Erf}[(\text{Sqrt}[3] * \text{Sqrt}[b] * \text{Sqrt}[c + d*x])/ \text{Sqrt}[d]] + \text{Erfi}[(\text{Sqrt}[3] * \\ & \text{Sqrt}[b] * \text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])) / \text{Sqrt}[d] - (2 * \text{Sqrt}[2] * \text{Sinh}[(3*b*(c + d*x))/d]) / \\ & \text{Sqrt}[c + d*x])) / (\text{Sqrt}[2] * d)) + \text{Cosh}[3*a] * ((\text{Cosh}[(b*c)/d] * (-1 + 2 * \text{Cosh}[(2*b*c)/d]) * \\ & (-((1 + E^{(6*b*(c + d*x))/d})/E^{(3*b*(c + d*x))/d}) + \text{Sqrt}[3] * \text{Sqrt}[-(b*(c + d*x))/d] * \\ & \text{Gamma}[1/2, (-3*b*(c + d*x))/d] + \text{Sqrt}[3] * \text{Sqrt}[(b*(c + d*x))/d] * \text{Gamma}[1/2, (3*b*(c + d*x))/d] * \\ & \text{Sinh}[(b*c)/d])) / (d * \text{Sqrt}[c + d*x]) - ((1 + 2 * \text{Cosh}[(2*b*c)/d]) * \text{Sinh}[(b*c)/d] * \\ & ((\text{Sqrt}[b] * \text{Sqrt}[6 * \text{Pi}] * (\text{Erf}[(\text{Sqrt}[3] * \text{Sqrt}[b] * \text{Sqrt}[c + d*x])/ \text{Sqrt}[d]] + \text{Erfi}[(\text{Sqrt}[3] * \\ & \text{Sqrt}[b] * \text{Sqrt}[c + d*x])/ \text{Sqrt}[d]])) / \text{Sqrt}[d] - (2 * \text{Sqrt}[2] * \text{Sinh}[(3*b*(c + d*x))/d]) / \text{Sqrt}[c + d*x])) / (\text{Sqrt}[2] * d)) \end{aligned}$$

$$\begin{aligned} &+ d*x))/d))/\text{Sqrt}[c + d*x]))/(\text{Sqrt}[2]*d)))/8 + (\text{Cosh}[3*a]*(-((1 + 2*\text{Cosh}[(2 \\ &*b*c)/d]))*(-((1 + \text{E}^((6*b*(c + d*x))/d))/\text{E}^((3*b*(c + d*x))/d)) + \text{Sqrt}[3]*\text{S} \\ &\text{qrt}[-((b*(c + d*x))/d)]*\text{Gamma}[1/2, (-3*b*(c + d*x))/d] + \text{Sqrt}[3]*\text{Sqrt}[(b*(c \\ &+ d*x))/d]*\text{Gamma}[1/2, (3*b*(c + d*x))/d])* \text{Sinh}[(b*c)/d])/ (d*\text{Sqrt}[c + d*x]) \\ &)+ (\text{Cosh}[(b*c)/d]*(-1 + 2*\text{Cosh}[(2*b*c)/d]))*((\text{Sqrt}[b]*\text{Sqrt}[6*\text{Pi}]*(\text{Erf}[(\text{Sqrt} \\ &[3]*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]] + \text{Erfi}[(\text{Sqrt}[3]*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/ \\ &\text{Sqrt}[d]]))/\text{Sqrt}[d] - (2*\text{Sqrt}[2]*\text{Sinh}[(3*b*(c + d*x))/d])/ \text{Sqrt}[c + d*x]))/(\text{S} \\ &\text{qrt}[2]*d) + \text{Sinh}[3*a]*((\text{Cosh}[(b*c)/d]*(-1 + 2*\text{Cosh}[(2*b*c)/d]))*(-((1 + \text{E}^ \\ &((6*b*(c + d*x))/d))/\text{E}^((3*b*(c + d*x))/d)) + \text{Sqrt}[3]*\text{Sqrt}[-((b*(c + d*x))/d \\ &)]*\text{Gamma}[1/2, (-3*b*(c + d*x))/d] + \text{Sqrt}[3]*\text{Sqrt}[(b*(c + d*x))/d]*\text{Gamma}[1/2 \\ &, (3*b*(c + d*x))/d]))/(d*\text{Sqrt}[c + d*x]) - ((1 + 2*\text{Cosh}[(2*b*c)/d])* \text{Sinh}[(b \\ &*c)/d]*((\text{Sqrt}[b]*\text{Sqrt}[6*\text{Pi}]*(\text{Erf}[(\text{Sqrt}[3]*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]] + \\ &\text{Erfi}[(\text{Sqrt}[3]*\text{Sqrt}[b]*\text{Sqrt}[c + d*x])/ \text{Sqrt}[d]]))/\text{Sqrt}[d] - (2*\text{Sqrt}[2]*\text{Sinh} \\ &((3*b*(c + d*x))/d))/ \text{Sqrt}[c + d*x]))/(\text{Sqrt}[2]*d)))/4 \end{aligned}$$

**Maple [F]** time = 0.088, size = 0, normalized size = 0.

$$\int (\sinh(bx + a))^3 (dx + c)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(b\*x+a)^3/(d\*x+c)^(3/2),x)

[Out] int(sinh(b\*x+a)^3/(d\*x+c)^(3/2),x)

**Maxima [A]** time = 1.47258, size = 266, normalized size = 1.08

$$\frac{\sqrt{3}\sqrt{\frac{(dx+c)b}{d}}e^{\left(\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{3(dx+c)b}{d}\right)}{\sqrt{dx+c}} - \frac{\sqrt{3}\sqrt{-\frac{(dx+c)b}{d}}e^{\left(-\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{3(dx+c)b}{d}\right)}{\sqrt{dx+c}} - \frac{3\sqrt{\frac{(dx+c)b}{d}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}} + \frac{3\sqrt{-\frac{(dx+c)b}{d}}e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}$$

$8d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^3/(d\*x+c)^(3/2),x, algorithm="maxima")

[Out] 1/8\*(sqrt(3)\*sqrt((d\*x + c)\*b/d)\*e^(3\*(b\*c - a\*d)/d)\*gamma(-1/2, 3\*(d\*x + c)\*b/d)/sqrt(d\*x + c) - sqrt(3)\*sqrt(-(d\*x + c)\*b/d)\*e^(-3\*(b\*c - a\*d)/d)\*gamma(-1/2, -3\*(d\*x + c)\*b/d)/sqrt(d\*x + c) - 3\*sqrt((d\*x + c)\*b/d)\*e^(-a + b\*c/d)\*gamma(-1/2, (d\*x + c)\*b/d)/sqrt(d\*x + c) + 3\*sqrt(-(d\*x + c)\*b/d)\*e^(-a + b\*c/d)\*gamma(-1/2, -(d\*x + c)\*b/d)/sqrt(d\*x + c)

$a - b*c/d)*\text{gamma}(-1/2, -(d*x + c)*b/d)/\text{sqrt}(d*x + c))/d$

**Fricas [B]** time = 3.01363, size = 3312, normalized size = 13.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")`

[Out] 
$$\frac{1}{4} * (\text{sqrt}(3) * \text{sqrt}(\pi) * ((d*x + c) * \cosh(b*x + a)^3 * \cosh(-3*(b*c - a*d)/d) - (d*x + c) * \cosh(b*x + a)^3 * \sinh(-3*(b*c - a*d)/d) + ((d*x + c) * \cosh(-3*(b*c - a*d)/d) - (d*x + c) * \sinh(-3*(b*c - a*d)/d)) * \sinh(b*x + a)^3 + 3 * ((d*x + c) * \cosh(b*x + a) * \cosh(-3*(b*c - a*d)/d) - (d*x + c) * \sinh(-3*(b*c - a*d)/d)) * \sinh(b*x + a)^2 + 3 * ((d*x + c) * \cosh(b*x + a)^2 * \cosh(-3*(b*c - a*d)/d) - (d*x + c) * \sinh(-3*(b*c - a*d)/d)) * \sinh(b*x + a) * \text{sqrt}(b/d) * \text{erf}(\text{sqrt}(3) * \text{sqrt}(d*x + c) * \text{sqrt}(b/d)) - \text{sqrt}(3) * \text{sqrt}(\pi) * ((d*x + c) * \cosh(b*x + a)^3 * \cosh(-3*(b*c - a*d)/d) + (d*x + c) * \cosh(b*x + a)^3 * \sinh(-3*(b*c - a*d)/d) + ((d*x + c) * \cosh(-3*(b*c - a*d)/d) + (d*x + c) * \sinh(-3*(b*c - a*d)/d)) * \sinh(b*x + a)^3 + 3 * ((d*x + c) * \cosh(b*x + a) * \cosh(-3*(b*c - a*d)/d) + (d*x + c) * \sinh(-3*(b*c - a*d)/d)) * \sinh(b*x + a)^2 + 3 * ((d*x + c) * \cosh(b*x + a)^2 * \cosh(-3*(b*c - a*d)/d) + (d*x + c) * \cosh(b*x + a)^2 * \sinh(-3*(b*c - a*d)/d)) * \sinh(b*x + a) * \text{sqrt}(-b/d) * \text{erf}(\text{sqrt}(3) * \text{sqrt}(d*x + c) * \text{sqrt}(-b/d)) - 3 * \text{sqrt}(\pi) * ((d*x + c) * \cosh(b*x + a)^3 * \cosh(-(b*c - a*d)/d) - (d*x + c) * \cosh(b*x + a)^3 * \sinh(-(b*c - a*d)/d) + ((d*x + c) * \cosh(-(b*c - a*d)/d) - (d*x + c) * \sinh(-(b*c - a*d)/d)) * \sinh(b*x + a)^3 + 3 * ((d*x + c) * \cosh(b*x + a) * \cosh(-(b*c - a*d)/d) - (d*x + c) * \sinh(-(b*c - a*d)/d)) * \sinh(b*x + a)^2 + 3 * ((d*x + c) * \cosh(b*x + a)^2 * \cosh(-(b*c - a*d)/d) - (d*x + c) * \sinh(-(b*c - a*d)/d)) * \sinh(b*x + a) * \text{sqrt}(b/d) * \text{erf}(\text{sqrt}(d*x + c) * \text{sqrt}(b/d)) + 3 * \text{sqrt}(\pi) * ((d*x + c) * \cosh(b*x + a)^3 * \cosh(-(b*c - a*d)/d) + (d*x + c) * \cosh(b*x + a)^3 * \sinh(-(b*c - a*d)/d) + ((d*x + c) * \cosh(-(b*c - a*d)/d) + (d*x + c) * \sinh(-(b*c - a*d)/d)) * \sinh(b*x + a)^3 + 3 * ((d*x + c) * \cosh(b*x + a) * \cosh(-(b*c - a*d)/d) + (d*x + c) * \sinh(-(b*c - a*d)/d)) * \sinh(b*x + a)^2 + 3 * ((d*x + c) * \cosh(b*x + a)^2 * \cosh(-(b*c - a*d)/d) + (d*x + c) * \cosh(b*x + a)^2 * \sinh(-(b*c - a*d)/d)) * \sinh(b*x + a) * \text{sqrt}(-b/d) * \text{erf}(\text{sqrt}(d*x + c) * \text{sqrt}(-b/d)) - (\cosh(b*x + a)^6 + 6 * \cosh(b*x + a) * \sinh(b*x + a)^5 + \sinh(b*x + a)^6 + 3 * (5 * \cosh(b*x + a)^2 - 1) * \sinh(b*x + a)^4 - 3 * \cosh(b*x + a)^4 + 4 * (5 * \cosh(b*x + a)^3 - 3 * \cosh(b*x + a)) * \sinh(b*x + a)^3 + 3 * (5 * \cosh(b*x + a)^4 - 6 * \cosh(b*x + a)^2 + 1) * \sinh(b*x + a)^2 + 3 * \cosh(b*x + a)^2 + 6 * (\cosh(b*x + a)^5 - 2 * \cosh(b*x + a)^3 + \cosh(b*x + a)) * \sinh(b*x + a) - 1) * \text{sqrt}(d*x + c)) / ((d^2*x + c*d) * \cosh(b*x + a)^3 + 3 * (d^2*x + c*d) * \cosh(b*x + a)^2 * \sinh(b*x + a) + 3 * (d^2*x + c*d) * \cosh(b*x +$$

$a) \cdot \sinh(b \cdot x + a)^2 + (d^2 \cdot x + c \cdot d) \cdot \sinh(b \cdot x + a)^3$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(a + bx)}{(c + dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)\*\*3/(d\*x+c)\*\*(3/2),x)

[Out] Integral(sinh(a + b\*x)\*\*3/(c + d\*x)\*\*(3/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx + a)^3}{(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^3/(d\*x+c)^(3/2),x, algorithm="giac")

[Out] integrate(sinh(b\*x + a)^3/(d\*x + c)^(3/2), x)

$$3.58 \quad \int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}} dx$$

**Optimal.** Leaf size=277

$$\frac{\sqrt{\pi} b^{3/2} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} - \frac{\sqrt{3\pi} b^{3/2} e^{\frac{3bc}{d}-3a} \operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} - \frac{\sqrt{\pi} b^{3/2} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} + \frac{\sqrt{3\pi} b^{3/2} e^{3a-\frac{3bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}$$

[Out] (b^(3/2)\*E^(-a + (b\*c)/d)\*Sqrt[Pi]\*Erf[(Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(2\*d^(5/2)) - (b^(3/2)\*E^(-3\*a + (3\*b\*c)/d)\*Sqrt[3\*Pi]\*Erf[(Sqrt[3]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(2\*d^(5/2)) - (b^(3/2)\*E^(a - (b\*c)/d)\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(2\*d^(5/2)) + (b^(3/2)\*E^(3\*a - (3\*b\*c)/d)\*Sqrt[3\*Pi]\*Erfi[(Sqrt[3]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(2\*d^(5/2)) - (4\*b\*Cosh[a + b\*x]\*Sinh[a + b\*x]^2)/(d^2\*Sqrt[c + d\*x]) - (2\*Sinh[a + b\*x]^3)/(3\*d\*(c + d\*x)^(3/2))

**Rubi [A]** time = 0.672687, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3314, 3308, 2180, 2204, 2205, 3312}

$$\frac{\sqrt{\pi} b^{3/2} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} - \frac{\sqrt{3\pi} b^{3/2} e^{\frac{3bc}{d}-3a} \operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} - \frac{\sqrt{\pi} b^{3/2} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} + \frac{\sqrt{3\pi} b^{3/2} e^{3a-\frac{3bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[a + b\*x]^3/(c + d\*x)^(5/2), x]

[Out] (b^(3/2)\*E^(-a + (b\*c)/d)\*Sqrt[Pi]\*Erf[(Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(2\*d^(5/2)) - (b^(3/2)\*E^(-3\*a + (3\*b\*c)/d)\*Sqrt[3\*Pi]\*Erf[(Sqrt[3]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(2\*d^(5/2)) - (b^(3/2)\*E^(a - (b\*c)/d)\*Sqrt[Pi]\*Erfi[(Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(2\*d^(5/2)) + (b^(3/2)\*E^(3\*a - (3\*b\*c)/d)\*Sqrt[3\*Pi]\*Erfi[(Sqrt[3]\*Sqrt[b]\*Sqrt[c + d\*x])/Sqrt[d]])/(2\*d^(5/2)) - (4\*b\*Cosh[a + b\*x]\*Sinh[a + b\*x]^2)/(d^2\*Sqrt[c + d\*x]) - (2\*Sinh[a + b\*x]^3)/(3\*d\*(c + d\*x)^(3/2))

**Rule 3314**

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[((c + d\*x)^(m + 1)\*(b\*Sine[e + f\*x])^n)/(d\*(m + 1)), x] + (Dist[(b^2\*f^2\*n\*(n - 1))/(d^2\*(m + 1)\*(m + 2)), Int[(c + d\*x)^(m + 2)\*(b\*Sine[e +

```
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sine[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*(b*Sine[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

### Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

### Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]
```

### Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

### Rubi steps



$$\begin{aligned}
\int \frac{\sinh^3(a+bx)}{(c+dx)^{5/2}} dx &= -\frac{4b \cosh(a+bx) \sinh^2(a+bx)}{d^2 \sqrt{c+dx}} - \frac{2 \sinh^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{(8b^2) \int \frac{\sinh(a+bx)}{\sqrt{c+dx}} dx}{d^2} + \frac{(12b^2) \int \frac{\sinh^3(a+bx)}{\sqrt{c+dx}}}{d^2} \\
&= -\frac{4b \cosh(a+bx) \sinh^2(a+bx)}{d^2 \sqrt{c+dx}} - \frac{2 \sinh^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{(12ib^2) \int \left( \frac{3i \sinh(a+bx)}{4\sqrt{c+dx}} - \frac{i \sinh(3a+3bx)}{4\sqrt{c+dx}} \right) dx}{d^2} + \\
&= -\frac{4b \cosh(a+bx) \sinh^2(a+bx)}{d^2 \sqrt{c+dx}} - \frac{2 \sinh^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{(8b^2) \text{Subst} \left( \int e^{i \left( ia - \frac{ibc}{d} \right) - \frac{bx^2}{d}} dx, x, \sqrt{c+dx} \right)}{d^3} \\
&= -\frac{4b^{3/2} e^{-a+\frac{bc}{d}} \sqrt{\pi} \operatorname{erf} \left( \frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{d^{5/2}} + \frac{4b^{3/2} e^{a-\frac{bc}{d}} \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{d^{5/2}} - \frac{4b \cosh(a+bx) \sinh^2(a+bx)}{d^2 \sqrt{c+dx}} \\
&= -\frac{4b^{3/2} e^{-a+\frac{bc}{d}} \sqrt{\pi} \operatorname{erf} \left( \frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{d^{5/2}} + \frac{4b^{3/2} e^{a-\frac{bc}{d}} \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{d^{5/2}} - \frac{4b \cosh(a+bx) \sinh^2(a+bx)}{d^2 \sqrt{c+dx}} \\
&= \frac{b^{3/2} e^{-a+\frac{bc}{d}} \sqrt{\pi} \operatorname{erf} \left( \frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{2d^{5/2}} - \frac{b^{3/2} e^{-3a+\frac{3bc}{d}} \sqrt{3\pi} \operatorname{erf} \left( \frac{\sqrt{3} \sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{2d^{5/2}} - \frac{b^{3/2} e^{a-\frac{bc}{d}} \sqrt{\pi} \operatorname{erfi} \left( \frac{\sqrt{b} \sqrt{c+dx}}{\sqrt{d}} \right)}{2d^{5/2}} +
\end{aligned}$$

**Mathematica [A]** time = 3.0608, size = 253, normalized size = 0.91

$$e^{-3\left(a+\frac{bc}{d}\right)} \left( -3\sqrt{3}e^{6a}d \left( -\frac{b(c+dx)}{d} \right)^{3/2} \operatorname{Gamma} \left( \frac{1}{2}, -\frac{3b(c+dx)}{d} \right) + 3de^{4a+\frac{2bc}{d}} \left( -\frac{b(c+dx)}{d} \right)^{3/2} \operatorname{Gamma} \left( \frac{1}{2}, -\frac{b(c+dx)}{d} \right) - 3de^{2a+\frac{4bc}{d}} \left( \frac{b(c+dx)}{d} \right)^{3/2} \operatorname{Gamma} \left( \frac{1}{2}, \frac{b(c+dx)}{d} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[a + b\*x]^3/(c + d\*x)^(5/2), x]

[Out]  $(-3\sqrt{3}d^3 e^{6a} \left( -\frac{b(c+dx)}{d} \right)^{3/2} \operatorname{Gamma} \left( \frac{1}{2}, -\frac{3b(c+dx)}{d} \right) + 3de^{4a+\frac{2bc}{d}} \left( -\frac{b(c+dx)}{d} \right)^{3/2} \operatorname{Gamma} \left( \frac{1}{2}, -\frac{b(c+dx)}{d} \right) - 3de^{2a+\frac{4bc}{d}} \left( \frac{b(c+dx)}{d} \right)^{3/2} \operatorname{Gamma} \left( \frac{1}{2}, \frac{b(c+dx)}{d} \right) - 4e^{3(a+\frac{bc}{d})} \operatorname{Sinh}[a+b*x]^2 (6b(c+dx) \operatorname{Cosh}[a+b*x] + d \operatorname{Sinh}[a+b*x]) / (6d^2 e^{3(a+\frac{bc}{d})} (c+dx)^{3/2})$

**Maple [F]** time = 0.084, size = 0, normalized size = 0.

$$\int (\sinh(bx + a))^3 (dx + c)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(b\*x+a)^3/(d\*x+c)^(5/2),x)

[Out] int(sinh(b\*x+a)^3/(d\*x+c)^(5/2),x)

**Maxima [A]** time = 1.42798, size = 265, normalized size = 0.96

$$3 \left( \frac{\sqrt{3} \left( \frac{dx+c}{d} \right)^{\frac{3}{2}} e^{\left( \frac{3(bc-ad)}{d} \right)} \Gamma\left( -\frac{3}{2}, \frac{3(dx+c)b}{d} \right)}{(dx+c)^{\frac{3}{2}}} - \frac{\sqrt{3} \left( -\frac{dx+c}{d} \right)^{\frac{3}{2}} e^{\left( -\frac{3(bc-ad)}{d} \right)} \Gamma\left( -\frac{3}{2}, -\frac{3(dx+c)b}{d} \right)}{(dx+c)^{\frac{3}{2}}} - \frac{\left( \frac{dx+c}{d} \right)^{\frac{3}{2}} e^{\left( -a + \frac{bc}{d} \right)} \Gamma\left( -\frac{3}{2}, \frac{dx+c}{d} \right)}{(dx+c)^{\frac{3}{2}}} + \frac{\left( -\frac{dx+c}{d} \right)^{\frac{3}{2}} e^{\left( a - \frac{bc}{d} \right)} \Gamma\left( -\frac{3}{2}, -\frac{dx+c}{d} \right)}{(dx+c)^{\frac{3}{2}}} \right) \frac{1}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^3/(d\*x+c)^(5/2),x, algorithm="maxima")

[Out] 3/8\*(sqrt(3)\*((d\*x + c)\*b/d)^(3/2)\*e^(3\*(b\*c - a\*d)/d)\*gamma(-3/2, 3\*(d\*x + c)\*b/d)/(d\*x + c)^(3/2) - sqrt(3)\*(-(d\*x + c)\*b/d)^(3/2)\*e^(-3\*(b\*c - a\*d)/d)\*gamma(-3/2, -3\*(d\*x + c)\*b/d)/(d\*x + c)^(3/2) - ((d\*x + c)\*b/d)^(3/2)\*e^(-a + b\*c/d)\*gamma(-3/2, (d\*x + c)\*b/d)/(d\*x + c)^(3/2) + (-(d\*x + c)\*b/d)^(3/2)\*e^(a - b\*c/d)\*gamma(-3/2, -(d\*x + c)\*b/d)/(d\*x + c)^(3/2))/d

**Fricas [B]** time = 3.15274, size = 4826, normalized size = 17.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^3/(d\*x+c)^(5/2),x, algorithm="fricas")

[Out] -1/12\*(6\*sqrt(3)\*sqrt(pi)\*((b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*cosh(b\*x + a)^3\*cosh(-3\*(b\*c - a\*d)/d) - (b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*cosh(b\*x + a)^3\*sinh(-3\*(b\*c - a\*d)/d) + ((b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*cosh(-3\*(b\*c - a\*d)

$$\begin{aligned}
& /d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\sinh(-3*(b*c - a*d)/d))*\sinh(b*x + a) \\
& ^3 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)*\cosh(-3*(b*c - a*d)/d) \\
& ) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)*\sinh(-3*(b*c - a*d)/d))*\sinh(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)^2*\cosh(-3*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)^2*\sinh(-3*(b*c - a*d)/d))*\sinh(b*x + a)*\sqrt{b/d)*\operatorname{erf}(\sqrt{3})*\sqrt{d*x + c)*\sqrt{b/d)} \\
& ) + 6*\sqrt{3)*\sqrt{\pi))*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)^3*\cosh(-3*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)^3*\sinh(-3*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(-3*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\sinh(-3*(b*c - a*d)/d))*\sinh(b*x + a)^3 \\
& + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)*\cosh(-3*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)*\sinh(-3*(b*c - a*d)/d))*\sinh(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)^2*\cosh(-3*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)^2*\sinh(-3*(b*c - a*d)/d))*\sinh(b*x + a)*\sqrt{-b/d)*\operatorname{erf}(\sqrt{3})*\sqrt{d*x + c)*\sqrt{-b/d)} \\
& - 6*\sqrt{\pi))*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)^3*\cosh(-(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)^3*\sinh(-(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(-(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a)^3 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)*\cosh(-(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)^2*\cosh(-(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)^2*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a) \\
& )*\sqrt{b/d)*\operatorname{erf}(\sqrt{d*x + c)*\sqrt{b/d)} - 6*\sqrt{\pi))*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)^3*\cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)^3*\sinh(-(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a)^3 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)*\cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)^2*\cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*\cosh(b*x + a)^2*\sinh(-(b*c - a*d)/d))*\sinh(b*x + a) \\
& )*\sqrt{-b/d)*\operatorname{erf}(\sqrt{d*x + c)*\sqrt{-b/d)} + ((6*b*d*x + 6*b*c + d)*\cosh(b*x + a)^6 + 6*(6*b*d*x + 6*b*c + d)*\cosh(b*x + a)*\sinh(b*x + a)^5 + (6*b*d*x + 6*b*c + d)*\sinh(b*x + a)^6 - 3*(2*b*d*x + 2*b*c + d)*\cosh(b*x + a)^4 - 3*(2*b*d*x - 5*(6*b*d*x + 6*b*c + d)*\cosh(b*x + a)^2 + 2*b*c + d)*\sinh(b*x + a)^4 + 4*(5*(6*b*d*x + 6*b*c + d)*\cosh(b*x + a)^3 - 3*(2*b*d*x + 2*b*c + d)*\cosh(b*x + a))*\sinh(b*x + a)^3 + 6*b*d*x - 3*(2*b*d*x + 2*b*c - d)*\cosh(b*x + a)^2 + 3*(5*(6*b*d*x + 6*b*c + d)*\cosh(b*x + a)^4 - 2*b*d*x - 6*(2*b*d*x + 2*b*c + d)*\cosh(b*x + a)^2 - 2*b*c + d)*\sinh(b*x + a)^2 + 6*b*c + 6*((6*b*d*x + 6*b*c + d)*\cosh(b*x + a)^5 - 2*(2*b*d*x + 2*b*c + d)*\cosh(b*x + a)^3 - (2*b*d*x + 2*b*c - d)*\cosh(b*x + a))*\sinh(b*x + a) - d)*\sqrt{d*x + c))/((d^4*x^2 + 2*c*d^3*x + c^2*d^2)*\cosh(b*x + a)^3 + 3*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*\cosh(b*x + a)^2*\sinh(b*x + a) + 3*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*\cosh(b*x + a)*\sinh(b*x + a)^2 + (d^4*x^2 + 2*c*d^3*x + c^2*d^2)*\sinh(b*x + a)^3)
\end{aligned}$$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^3(a + bx)}{(c + dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)\*\*3/(d\*x+c)\*\*(5/2),x)

[Out] Integral(sinh(a + b\*x)\*\*3/(c + d\*x)\*\*(5/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx + a)^3}{(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^3/(d\*x+c)^(5/2),x, algorithm="giac")

[Out] integrate(sinh(b\*x + a)^3/(d\*x + c)^(5/2), x)

$$3.59 \quad \int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}} dx$$

**Optimal.** Leaf size=331

$$\frac{\sqrt{\pi} b^{5/2} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{3\sqrt{3}\pi b^{5/2} e^{\frac{3bc}{d}-3a} \operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{\sqrt{\pi} b^{5/2} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{3\sqrt{3}\pi b^{5/2} e^{3a-\frac{3bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}$$

[Out]  $-(b^{5/2} * E^{-a + (b*c)/d} * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erf}[(\operatorname{Sqrt}[b] * \operatorname{Sqrt}[c + d*x]) / \operatorname{Sqrt}[d]]) / (5*d^{7/2}) + (3*b^{5/2} * E^{-3*a + (3*b*c)/d} * \operatorname{Sqrt}[3*\operatorname{Pi}] * \operatorname{Erf}[(\operatorname{Sqrt}[3] * \operatorname{Sqrt}[b] * \operatorname{Sqrt}[c + d*x]) / \operatorname{Sqrt}[d]]) / (5*d^{7/2}) - (b^{5/2} * E^{a - (b*c)/d} * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(\operatorname{Sqrt}[b] * \operatorname{Sqrt}[c + d*x]) / \operatorname{Sqrt}[d]]) / (5*d^{7/2}) + (3*b^{5/2} * E^{3*a - (3*b*c)/d} * \operatorname{Sqrt}[3*\operatorname{Pi}] * \operatorname{Erfi}[(\operatorname{Sqrt}[3] * \operatorname{Sqrt}[b] * \operatorname{Sqrt}[c + d*x]) / \operatorname{Sqrt}[d]]) / (5*d^{7/2}) - (16*b^2 * \operatorname{Sinh}[a + b*x]) / (5*d^3 * \operatorname{Sqrt}[c + d*x]) - (4*b * \operatorname{Cosh}[a + b*x] * \operatorname{Sinh}[a + b*x]^2) / (5*d^2 * (c + d*x)^{3/2}) - (2 * \operatorname{Sinh}[a + b*x]^3) / (5*d * (c + d*x)^{5/2}) - (24*b^2 * \operatorname{Sinh}[a + b*x]^3) / (5*d^3 * \operatorname{Sqrt}[c + d*x])$

**Rubi [A]** time = 0.774647, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {3314, 3297, 3307, 2180, 2204, 2205, 3313}

$$\frac{\sqrt{\pi} b^{5/2} e^{\frac{bc}{d}-a} \operatorname{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{3\sqrt{3}\pi b^{5/2} e^{\frac{3bc}{d}-3a} \operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{\sqrt{\pi} b^{5/2} e^{a-\frac{bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{3\sqrt{3}\pi b^{5/2} e^{3a-\frac{3bc}{d}} \operatorname{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sinh}[a + b*x]^3 / (c + d*x)^{7/2}, x]$

[Out]  $-(b^{5/2} * E^{-a + (b*c)/d} * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erf}[(\operatorname{Sqrt}[b] * \operatorname{Sqrt}[c + d*x]) / \operatorname{Sqrt}[d]]) / (5*d^{7/2}) + (3*b^{5/2} * E^{-3*a + (3*b*c)/d} * \operatorname{Sqrt}[3*\operatorname{Pi}] * \operatorname{Erf}[(\operatorname{Sqrt}[3] * \operatorname{Sqrt}[b] * \operatorname{Sqrt}[c + d*x]) / \operatorname{Sqrt}[d]]) / (5*d^{7/2}) - (b^{5/2} * E^{a - (b*c)/d} * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(\operatorname{Sqrt}[b] * \operatorname{Sqrt}[c + d*x]) / \operatorname{Sqrt}[d]]) / (5*d^{7/2}) + (3*b^{5/2} * E^{3*a - (3*b*c)/d} * \operatorname{Sqrt}[3*\operatorname{Pi}] * \operatorname{Erfi}[(\operatorname{Sqrt}[3] * \operatorname{Sqrt}[b] * \operatorname{Sqrt}[c + d*x]) / \operatorname{Sqrt}[d]]) / (5*d^{7/2}) - (16*b^2 * \operatorname{Sinh}[a + b*x]) / (5*d^3 * \operatorname{Sqrt}[c + d*x]) - (4*b * \operatorname{Cosh}[a + b*x] * \operatorname{Sinh}[a + b*x]^2) / (5*d^2 * (c + d*x)^{3/2}) - (2 * \operatorname{Sinh}[a + b*x]^3) / (5*d * (c + d*x)^{5/2}) - (24*b^2 * \operatorname{Sinh}[a + b*x]^3) / (5*d^3 * \operatorname{Sqrt}[c + d*x])$

**Rule 3314**

$\operatorname{Int}[(c_.) + (d_.) * (x_.)^{(m_)} * ((b_.) * \sin[(e_.) + (f_.) * (x_.)])^{(n_)}, x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m + 1)} * (b * \sin[e + f*x])^n / (d * (m + 1)), x] + (\operatorname{Dist}[($

```
b^2*f^2*n*(n - 1)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x] /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

### Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma == True
```

### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

### Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

### Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^n -
1), x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
```

LtQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(a+bx)}{(c+dx)^{7/2}} dx &= -\frac{4b \cosh(a+bx) \sinh^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2 \sinh^3(a+bx)}{5d(c+dx)^{5/2}} + \frac{(8b^2) \int \frac{\sinh(a+bx)}{(c+dx)^{3/2}} dx}{5d^2} + \frac{(12b^2) \int \frac{\sinh^3(a+bx)}{(c+dx)^{3/2}} dx}{5d^2} \\
&= -\frac{16b^2 \sinh(a+bx)}{5d^3 \sqrt{c+dx}} - \frac{4b \cosh(a+bx) \sinh^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2 \sinh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2 \sinh^3(a+bx)}{5d^3 \sqrt{c+dx}} + \\
&= -\frac{16b^2 \sinh(a+bx)}{5d^3 \sqrt{c+dx}} - \frac{4b \cosh(a+bx) \sinh^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2 \sinh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2 \sinh^3(a+bx)}{5d^3 \sqrt{c+dx}} + \\
&= -\frac{16b^2 \sinh(a+bx)}{5d^3 \sqrt{c+dx}} - \frac{4b \cosh(a+bx) \sinh^2(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{2 \sinh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2 \sinh^3(a+bx)}{5d^3 \sqrt{c+dx}} + \\
&= \frac{8b^{5/2} e^{-a+\frac{bc}{d}} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{8b^{5/2} e^{-a-\frac{bc}{d}} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{16b^2 \sinh(a+bx)}{5d^3 \sqrt{c+dx}} - \frac{4b \cosh(a+bx)}{5d^2(c+dx)^{3/2}} \\
&= -\frac{b^{5/2} e^{-a+\frac{bc}{d}} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{3b^{5/2} e^{-3a+\frac{3bc}{d}} \sqrt{3\pi} \operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{b^{5/2} e^{-a-\frac{bc}{d}} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \dots
\end{aligned}$$

**Mathematica [B]** time = 18.1883, size = 3211, normalized size = 9.7

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[Sinh[a + b\*x]^3/(c + d\*x)^(7/2), x]

```

[Out] (-3*(Cosh[a]*((-2*E^((b*(c + d*x))/d)*(3*d^2 + 2*b*d*(c + d*x) + 4*b^2*(c + d*x)^2) + 8*d^2*(-((b*(c + d*x))/d))^(5/2)*Gamma[1/2, -((b*(c + d*x))/d)] + (-6*d^2 + 4*b*d*(c + d*x) - 8*b^2*(c + d*x)^2 + 8*b*d*E^((b*(c + d*x))/d)*(c + d*x)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (b*(c + d*x))/d])/E^((b*(c + d*x))/d))*Sinh[(b*c)/d])/(30*d^3*(c + d*x)^(5/2)) + (2*Cosh[(b*c)/d]*(-(b*(c + d*x)*(2*E^((b*(c + d*x))/d)*(d + 2*b*(c + d*x)) + 4*d*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, -((b*(c + d*x))/d)] + (2*(d - 2*b*(c + d*x) + 2*d*E^((b*(c + d*x))/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (b*(c + d*x))/d]))/E^((b*(c + d*x))/d)))/2 - 3*d^2*Sinh[(b*(c + d*x))/d])/(15*d^3*(c + d*x)^(5/2)

```

$$\begin{aligned}
& )) + \text{Sinh}[a] * ((\text{Cosh}[(b*c)/d] * (-2 * E^{((b*(c + d*x))/d)} * (3*d^2 + 2*b*d*(c + d*x) + 4*b^2*(c + d*x)^2) + 8*d^2 * (-((b*(c + d*x))/d))^{(5/2)} * \text{Gamma}[1/2, -((b*(c + d*x))/d)] + (-6*d^2 + 4*b*d*(c + d*x) - 8*b^2*(c + d*x)^2 + 8*b*d * E^{((b*(c + d*x))/d)} * (c + d*x) * ((b*(c + d*x))/d)^{(3/2)} * \text{Gamma}[1/2, (b*(c + d*x))/d]) / E^{((b*(c + d*x))/d)})) / (30*d^3*(c + d*x)^{(5/2)}) - (2 * \text{Sinh}[(b*c)/d] * (-b*(c + d*x) * (2 * E^{((b*(c + d*x))/d)} * (d + 2*b*(c + d*x)) + 4*d * (-((b*(c + d*x))/d))^{(3/2)} * \text{Gamma}[1/2, -((b*(c + d*x))/d)] + (2*(d - 2*b*(c + d*x) + 2*d * E^{((b*(c + d*x))/d)} * ((b*(c + d*x))/d)^{(3/2)} * \text{Gamma}[1/2, (b*(c + d*x))/d])) / E^{((b*(c + d*x))/d)})) / 2 - 3*d^2 * \text{Sinh}[(b*(c + d*x))/d]) / (15*d^3*(c + d*x)^{(5/2)})) / 4 + (- (\text{Sinh}[3*a] * (-((1 + 2 * \text{Cosh}[(2*b*c)/d]) * (-2 * E^{((3*b*(c + d*x))/d)} * (d^2 + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24 * \text{Sqrt}[3] * d^2 * (-((b*(c + d*x))/d))^{(5/2)} * \text{Gamma}[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b^2*(c + d*x)^2 + 24 * \text{Sqrt}[3] * d^2 * E^{((3*b*(c + d*x))/d)} * ((b*(c + d*x))/d)^{(5/2)} * \text{Gamma}[1/2, (3*b*(c + d*x))/d]) / E^{((3*b*(c + d*x))/d)} * \text{Sinh}[(b*c)/d]) / (10*d^3*(c + d*x)^{(5/2)}) - (2 * \text{Cosh}[(b*c)/d] * (-1 + 2 * \text{Cosh}[(2*b*c)/d]) * (-6*b^{(5/2)} * \text{Sqrt}[3 * \text{Pi}] * (c + d*x)^{(5/2)} * \text{Erf}[(\text{Sqrt}[3] * \text{Sqrt}[b] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]] - 6*b^{(5/2)} * \text{Sqrt}[3 * \text{Pi}] * (c + d*x)^{(5/2)} * \text{Erfi}[(\text{Sqrt}[3] * \text{Sqrt}[b] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]] + \text{Sqrt}[d] * (2*b*d*(c + d*x) * \text{Cosh}[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*x)^2) * \text{Sinh}[(3*b*(c + d*x))/d])) / (5*d^{(7/2)} * (c + d*x)^{(5/2)})) - \text{Cosh}[3*a] * ((\text{Cosh}[(b*c)/d] * (-1 + 2 * \text{Cosh}[(2*b*c)/d]) * (-2 * E^{((3*b*(c + d*x))/d)} * (d^2 + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24 * \text{Sqrt}[3] * d^2 * (-((b*(c + d*x))/d))^{(5/2)} * \text{Gamma}[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b^2*(c + d*x)^2 + 24 * \text{Sqrt}[3] * d^2 * E^{((3*b*(c + d*x))/d)} * ((b*(c + d*x))/d)^{(5/2)} * \text{Gamma}[1/2, (3*b*(c + d*x))/d]) / E^{((3*b*(c + d*x))/d)})) / (10*d^3*(c + d*x)^{(5/2)}) + (2 * (1 + 2 * \text{Cosh}[(2*b*c)/d]) * \text{Sinh}[(b*c)/d] * (-6*b^{(5/2)} * \text{Sqrt}[3 * \text{Pi}] * (c + d*x)^{(5/2)} * \text{Erf}[(\text{Sqrt}[3] * \text{Sqrt}[b] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]] - 6*b^{(5/2)} * \text{Sqrt}[3 * \text{Pi}] * (c + d*x)^{(5/2)} * \text{Erfi}[(\text{Sqrt}[3] * \text{Sqrt}[b] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]] + \text{Sqrt}[d] * (2*b*d*(c + d*x) * \text{Cosh}[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*x)^2) * \text{Sinh}[(3*b*(c + d*x))/d])) / (5*d^{(7/2)} * (c + d*x)^{(5/2)})) / 8 + (\text{Sinh}[3*a] * (-((1 + 2 * \text{Cosh}[(2*b*c)/d]) * (-2 * E^{((3*b*(c + d*x))/d)} * (d^2 + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24 * \text{Sqrt}[3] * d^2 * (-((b*(c + d*x))/d))^{(5/2)} * \text{Gamma}[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b^2*(c + d*x)^2 + 24 * \text{Sqrt}[3] * d^2 * E^{((3*b*(c + d*x))/d)} * ((b*(c + d*x))/d)^{(5/2)} * \text{Gamma}[1/2, (3*b*(c + d*x))/d]) / E^{((3*b*(c + d*x))/d)} * \text{Sinh}[(b*c)/d]) / (10*d^3*(c + d*x)^{(5/2)}) - (2 * \text{Cosh}[(b*c)/d] * (-1 + 2 * \text{Cosh}[(2*b*c)/d]) * (-6*b^{(5/2)} * \text{Sqrt}[3 * \text{Pi}] * (c + d*x)^{(5/2)} * \text{Erf}[(\text{Sqrt}[3] * \text{Sqrt}[b] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]] - 6*b^{(5/2)} * \text{Sqrt}[3 * \text{Pi}] * (c + d*x)^{(5/2)} * \text{Erfi}[(\text{Sqrt}[3] * \text{Sqrt}[b] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]] + \text{Sqrt}[d] * (2*b*d*(c + d*x) * \text{Cosh}[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*x)^2) * \text{Sinh}[(3*b*(c + d*x))/d])) / (5*d^{(7/2)} * (c + d*x)^{(5/2)})) + \text{Cosh}[3*a] * ((\text{Cosh}[(b*c)/d] * (-1 + 2 * \text{Cosh}[(2*b*c)/d]) * (-2 * E^{((3*b*(c + d*x))/d)} * (d^2 + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24 * \text{Sqrt}[3] * d^2 * (-((b*(c + d*x))/d))^{(5/2)} * \text{Gamma}[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b^2*(c + d*x)^2 + 24 * \text{Sqrt}[3] * d^2 * E^{((3*b*(c + d*x))/d)} * ((b*(c + d*x))/d)^{(5/2)} * \text{Gamma}[1/2, (3*b*(c + d*x))/d]) / E^{((3*b*(c + d*x))/d)})) / (10*d^3*(c + d*x)^{(5/2)}) + (2 * (1 + 2 * \text{Cosh}[(2*b*c)/d]) * \text{Sinh}[(b*c)/d] * (-6*b^{(5/2)} * \text{Sqrt}[3 * \text{Pi}] * (c + d*x)
\end{aligned}$$



$$\begin{aligned} &)^{(5/2)} * \text{Erf}[(\text{Sqrt}[3] * \text{Sqrt}[b] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]] - 6*b^{(5/2)} * \text{Sqrt}[3*Pi] \\ &* (c + d*x)^{(5/2)} * \text{Erfi}[(\text{Sqrt}[3] * \text{Sqrt}[b] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]] + \text{Sqrt}[d] * (2 \\ &* b*d*(c + d*x) * \text{Cosh}[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*x)^2) * \text{Sinh}[(3 \\ &* b*(c + d*x))/d])) / (5*d^{(7/2)}*(c + d*x)^{(5/2)})) / 8 + (\text{Cosh}[3*a] * (-((1 + 2* \\ &\text{Cosh}[(2*b*c)/d]) * (-2*E^{((3*b*(c + d*x))/d)} * (d^2 + 2*b*d*(c + d*x) + 12*b^2* \\ &(c + d*x)^2) + 24*\text{Sqrt}[3]*d^2 * (-((b*(c + d*x))/d))^{(5/2)} * \text{Gamma}[1/2, (-3*b*( \\ &c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b^2*(c + d*x)^2 + 24*\text{Sqrt}[3]* \\ &d^2 * E^{((3*b*(c + d*x))/d)} * ((b*(c + d*x))/d)^{(5/2)} * \text{Gamma}[1/2, (3*b*(c + d*x) \\ &)/d]) / E^{((3*b*(c + d*x))/d)} * \text{Sinh}[(b*c)/d]) / (10*d^3*(c + d*x)^{(5/2)}) - (2*C \\ &\text{osh}[(b*c)/d] * (-1 + 2*\text{Cosh}[(2*b*c)/d]) * (-6*b^{(5/2)} * \text{Sqrt}[3*Pi] * (c + d*x)^{(5/2)} \\ &)* \text{Erf}[(\text{Sqrt}[3] * \text{Sqrt}[b] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]] - 6*b^{(5/2)} * \text{Sqrt}[3*Pi] * (c + \\ &d*x)^{(5/2)} * \text{Erfi}[(\text{Sqrt}[3] * \text{Sqrt}[b] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]] + \text{Sqrt}[d] * (2*b*d*( \\ &c + d*x) * \text{Cosh}[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*x)^2) * \text{Sinh}[(3*b*(c \\ &+ d*x))/d])) / (5*d^{(7/2)}*(c + d*x)^{(5/2)})) + \text{Sinh}[3*a] * ((\text{Cosh}[(b*c)/d] * (-1 \\ &+ 2*\text{Cosh}[(2*b*c)/d]) * (-2*E^{((3*b*(c + d*x))/d)} * (d^2 + 2*b*d*(c + d*x) + 12* \\ &b^2*(c + d*x)^2) + 24*\text{Sqrt}[3]*d^2 * (-((b*(c + d*x))/d))^{(5/2)} * \text{Gamma}[1/2, (-3 \\ &* b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b^2*(c + d*x)^2 + 24*\text{Sqrt} \\ &[3]*d^2 * E^{((3*b*(c + d*x))/d)} * ((b*(c + d*x))/d)^{(5/2)} * \text{Gamma}[1/2, (3*b*(c + \\ &d*x))/d]) / E^{((3*b*(c + d*x))/d)})) / (10*d^3*(c + d*x)^{(5/2)}) + (2*(1 + 2*\text{Cosh} \\ &[(2*b*c)/d]) * \text{Sinh}[(b*c)/d] * (-6*b^{(5/2)} * \text{Sqrt}[3*Pi] * (c + d*x)^{(5/2)} * \text{Erf}[(\text{Sqrt} \\ &[3] * \text{Sqrt}[b] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]] - 6*b^{(5/2)} * \text{Sqrt}[3*Pi] * (c + d*x)^{(5/2)} * \\ &\text{Erfi}[(\text{Sqrt}[3] * \text{Sqrt}[b] * \text{Sqrt}[c + d*x]) / \text{Sqrt}[d]] + \text{Sqrt}[d] * (2*b*d*(c + d*x) * \text{Co} \\ &\text{sh}[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*x)^2) * \text{Sinh}[(3*b*(c + d*x))/d]) \\ &)) / (5*d^{(7/2)}*(c + d*x)^{(5/2)})) / 4 \end{aligned}$$

**Maple [F]** time = 0.089, size = 0, normalized size = 0.

$$\int (\sinh(bx + a))^3 (dx + c)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(b\*x+a)^3/(d\*x+c)^(7/2),x)

[Out] int(sinh(b\*x+a)^3/(d\*x+c)^(7/2),x)

**Maxima [A]** time = 1.42465, size = 266, normalized size = 0.8

$$3 \left( \frac{3 \sqrt{3} \left( \frac{(dx+c)b}{d} \right)^{\frac{5}{2}} e^{\left( \frac{3(bc-ad)}{d} \right)} \Gamma \left( -\frac{5}{2}, \frac{3(dx+c)b}{d} \right)}{(dx+c)^{\frac{5}{2}}} - \frac{3 \sqrt{3} \left( -\frac{(dx+c)b}{d} \right)^{\frac{5}{2}} e^{\left( -\frac{3(bc-ad)}{d} \right)} \Gamma \left( -\frac{5}{2}, -\frac{3(dx+c)b}{d} \right)}{(dx+c)^{\frac{5}{2}}} - \frac{\left( \frac{(dx+c)b}{d} \right)^{\frac{5}{2}} e^{\left( -a + \frac{bc}{d} \right)} \Gamma \left( -\frac{5}{2}, \frac{(dx+c)b}{d} \right)}{(dx+c)^{\frac{5}{2}}} + \frac{\left( -\frac{(dx+c)b}{d} \right)^{\frac{5}{2}} e^{\left( a - \frac{bc}{d} \right)} \Gamma \left( -\frac{5}{2}, -\frac{(dx+c)b}{d} \right)}{(dx+c)^{\frac{5}{2}}} \right) \frac{1}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^3/(d\*x+c)^(7/2),x, algorithm="maxima")

[Out] 3/8\*(3\*sqrt(3)\*((d\*x + c)\*b/d)^(5/2)\*e^(3\*(b\*c - a\*d)/d)\*gamma(-5/2, 3\*(d\*x + c)\*b/d)/(d\*x + c)^(5/2) - 3\*sqrt(3)\*(-(d\*x + c)\*b/d)^(5/2)\*e^(-3\*(b\*c - a\*d)/d)\*gamma(-5/2, -3\*(d\*x + c)\*b/d)/(d\*x + c)^(5/2) - ((d\*x + c)\*b/d)^(5/2)\*e^(-a + b\*c/d)\*gamma(-5/2, (d\*x + c)\*b/d)/(d\*x + c)^(5/2) + (-d\*x + c)\*b/d)^(5/2)\*e^(a - b\*c/d)\*gamma(-5/2, -(d\*x + c)\*b/d)/(d\*x + c)^(5/2))/d

**Fricas [B]** time = 3.81101, size = 7120, normalized size = 21.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^3/(d\*x+c)^(7/2),x, algorithm="fricas")

[Out] 1/20\*(12\*sqrt(3)\*sqrt(pi)\*((b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*cosh(b\*x + a)^3\*cosh(-3\*(b\*c - a\*d)/d) - (b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*cosh(b\*x + a)^3\*sinh(-3\*(b\*c - a\*d)/d) + ((b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*cosh(-3\*(b\*c - a\*d)/d) - (b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*sinh(-3\*(b\*c - a\*d)/d))\*sinh(b\*x + a)^3 + 3\*((b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*cosh(b\*x + a)\*cosh(-3\*(b\*c - a\*d)/d) - (b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*cosh(b\*x + a)\*sinh(-3\*(b\*c - a\*d)/d))\*sinh(b\*x + a)^2 + 3\*((b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*cosh(b\*x + a)^2\*cosh(-3\*(b\*c - a\*d)/d) - (b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*cosh(b\*x + a)^2\*sinh(-3\*(b\*c - a\*d)/d))\*sinh(b\*x + a)\*sqrt(b/d)\*erf(sqrt(3)\*sqrt(d\*x + c)\*sqrt(b/d)) - 12\*sqrt(3)\*sqrt(pi)\*((b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*cosh(b\*x + a)^3\*cosh(-3\*(b\*c - a\*d)/d) + (b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*cosh(b\*x + a)^3\*sinh(-3\*(b\*c - a\*d)/d) + ((b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*cosh(-3\*(b\*c - a\*d)/d) + (b^2\*d^3\*x^3 + 3\*b^2\*c\*d^2\*x^2 + 3\*b^2\*c^2\*d\*x + b^2\*c^3)\*sinh(-3\*(b\*c - a\*d)/d))\*sin

$$\begin{aligned}
& h(b*x + a)^3 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3) \\
& *cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3* \\
& b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^ \\
& 2 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + \\
& a)^2*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d \\
& *x + b^2*c^3)*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)*sqrt(- \\
& b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d)) - 4*sqrt(pi)*((b^2*d^3*x^3 + 3*b \\
& ^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d \\
& ) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a) \\
& ^3*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + \\
& b^2*c^3)*cosh(-(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2 \\
& *d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((b^2*d^3*x^3 + 3 \\
& *b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-(b*c - a*d)/d \\
& ) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a) \\
& *sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 \\
& + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - (b^2*d^3* \\
& x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-(b*c \\
& - a*d)/d))*sinh(b*x + a)*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 4*sqrt( \\
& pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a) \\
& )^3*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + \\
& b^2*c^3)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d^ \\
& 2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^ \\
& 2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^ \\
& 3 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + \\
& a)*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + \\
& b^2*c^3)*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((b^2*d^3 \\
& *x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*cosh(-(b* \\
& c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos \\
& h(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a)*sqrt(-b/d)*erf(sqrt(d*x + \\
& c)*sqrt(-b/d)) - ((12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2 \\
& *c*d + b*d^2)*x)*cosh(b*x + a)^6 + 6*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d \\
& + d^2 + 2*(12*b^2*c*d + b*d^2)*x)*cosh(b*x + a)*sinh(b*x + a)^5 + (12*b^2* \\
& d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2*c*d + b*d^2)*x)*sinh(b*x + \\
& a)^6 - 12*b^2*d^2*x^2 - (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*( \\
& 4*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^4 - (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c* \\
& d - 15*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2*c*d + b*d^2 \\
& )*x)*cosh(b*x + a)^2 + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*sinh(b*x + a)^4 - 1 \\
& 2*b^2*c^2 + 4*(5*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2*c \\
& *d + b*d^2)*x)*cosh(b*x + a)^3 - (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d \\
& ^2 + 2*(4*b^2*c*d + b*d^2)*x)*cosh(b*x + a))*sinh(b*x + a)^3 + 2*b*c*d + (4 \\
& *b^2*d^2*x^2 + 4*b^2*c^2 - 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d - b*d^2)*x)*cosh( \\
& b*x + a)^2 + (4*b^2*d^2*x^2 + 15*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d \\
& ^2 + 2*(12*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^4 + 4*b^2*c^2 - 2*b*c*d - 6*(4 \\
& *b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*cosh( \\
& b*x + a)^2 + 3*d^2 + 2*(4*b^2*c*d - b*d^2)*x)*sinh(b*x + a)^2 - d^2 - 2*(12
\end{aligned}$$

```
*b^2*c*d - b*d^2)*x + 2*(3*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2
*(12*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^5 - 2*(4*b^2*d^2*x^2 + 4*b^2*c^2 + 2
*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^3 + (4*b^2*d^2*x^2
+ 4*b^2*c^2 - 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d - b*d^2)*x)*cosh(b*x + a))*sin
h(b*x + a))*sqrt(d*x + c))/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)
*cosh(b*x + a)^3 + 3*(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*cosh(b
*x + a)^2*sinh(b*x + a) + 3*(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)
*cosh(b*x + a)*sinh(b*x + a)^2 + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3
*d^3)*sinh(b*x + a)^3)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)\*\*3/(d\*x+c)\*\*(7/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(bx+a)^3}{(dx+c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(b\*x+a)^3/(d\*x+c)^(7/2),x, algorithm="giac")

[Out] integrate(sinh(b\*x + a)^3/(d\*x + c)^(7/2), x)

### 3.60 $\int (dx)^{3/2} \sinh(fx) dx$

**Optimal.** Leaf size=111

$$-\frac{3\sqrt{\pi}d^{3/2}\operatorname{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} + \frac{3\sqrt{\pi}d^{3/2}\operatorname{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} - \frac{3d\sqrt{dx}\sinh(fx)}{2f^2} + \frac{(dx)^{3/2}\cosh(fx)}{f}$$

[Out]  $((d*x)^{(3/2)}*\operatorname{Cosh}[f*x])/f - (3*d^{(3/2)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[f]*\operatorname{Sqrt}[d*x])/(\operatorname{Sqrt}[d])])/(8*f^{(5/2)}) + (3*d^{(3/2)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[f]*\operatorname{Sqrt}[d*x])/(\operatorname{Sqrt}[d])])/(8*f^{(5/2)}) - (3*d*\operatorname{Sqrt}[d*x]*\operatorname{Sinh}[f*x])/(2*f^2)$

**Rubi [A]** time = 0.158884, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {3296, 3308, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}d^{3/2}\operatorname{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} + \frac{3\sqrt{\pi}d^{3/2}\operatorname{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} - \frac{3d\sqrt{dx}\sinh(fx)}{2f^2} + \frac{(dx)^{3/2}\cosh(fx)}{f}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d*x)^{(3/2)}*\operatorname{Sinh}[f*x], x]$

[Out]  $((d*x)^{(3/2)}*\operatorname{Cosh}[f*x])/f - (3*d^{(3/2)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[f]*\operatorname{Sqrt}[d*x])/(\operatorname{Sqrt}[d])])/(8*f^{(5/2)}) + (3*d^{(3/2)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[f]*\operatorname{Sqrt}[d*x])/(\operatorname{Sqrt}[d])])/(8*f^{(5/2)}) - (3*d*\operatorname{Sqrt}[d*x]*\operatorname{Sinh}[f*x])/(2*f^2)$

#### Rule 3296

$\operatorname{Int}[(c + d*x)^m \sin(e + f*x), x] \rightarrow -\operatorname{Simp}[(c + d*x)^m \cos(e + f*x)/f, x] + \operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{m-1} \cos(e + f*x), x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x \ \&\& \operatorname{GtQ}[m, 0]$

#### Rule 3308

$\operatorname{Int}[(c + d*x)^m \sin(e + f*x), x] \rightarrow \operatorname{Dist}[I/2, \operatorname{Int}[(c + d*x)^m / E^{I*(e + f*x)}, x], x] - \operatorname{Dist}[I/2, \operatorname{Int}[(c + d*x)^m * E^{I*(e + f*x)}, x], x] /; \operatorname{FreeQ}\{c, d, e, f, m\}, x$

#### Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

### Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]
```

### Rubi steps

$$\begin{aligned}
\int (dx)^{3/2} \sinh(fx) dx &= \frac{(dx)^{3/2} \cosh(fx)}{f} - \frac{(3d) \int \sqrt{dx} \cosh(fx) dx}{2f} \\
&= \frac{(dx)^{3/2} \cosh(fx)}{f} - \frac{3d\sqrt{dx} \sinh(fx)}{2f^2} + \frac{(3d^2) \int \frac{\sinh(fx)}{\sqrt{dx}} dx}{4f^2} \\
&= \frac{(dx)^{3/2} \cosh(fx)}{f} - \frac{3d\sqrt{dx} \sinh(fx)}{2f^2} - \frac{(3d^2) \int \frac{e^{-fx}}{\sqrt{dx}} dx}{8f^2} + \frac{(3d^2) \int \frac{e^{fx}}{\sqrt{dx}} dx}{8f^2} \\
&= \frac{(dx)^{3/2} \cosh(fx)}{f} - \frac{3d\sqrt{dx} \sinh(fx)}{2f^2} - \frac{(3d) \text{Subst}\left(\int e^{-\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{4f^2} + \frac{(3d) \text{Subst}\left(\int e^{\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{4f^2} \\
&= \frac{(dx)^{3/2} \cosh(fx)}{f} - \frac{3d^{3/2} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} + \frac{3d^{3/2} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} - \frac{3d\sqrt{dx} \sinh(fx)}{2f^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0128487, size = 50, normalized size = 0.45

$$\frac{d^2 \left( \sqrt{-fx} \operatorname{Gamma}\left(\frac{5}{2}, -fx\right) + \sqrt{fx} \operatorname{Gamma}\left(\frac{5}{2}, fx\right) \right)}{2f^3 \sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(d\*x)^(3/2)\*Sinh[f\*x],x]

[Out] (d^2\*(Sqrt[-(f\*x)]\*Gamma[5/2, -(f\*x)] + Sqrt[f\*x]\*Gamma[5/2, f\*x]))/(2\*f^3\*Sqrt[d\*x])

**Maple [C]** time = 0.052, size = 132, normalized size = 1.2

$$-2 \frac{(dx)^{3/2} \sqrt{2} \sqrt{\pi}}{x^{3/2} (if)^{3/2} f} \left( -\frac{\sqrt{x} \sqrt{2} (if)^{7/2} (-14 fx + 21) e^{fx}}{112 \sqrt{\pi} f^3} + \frac{\sqrt{x} \sqrt{2} (if)^{7/2} (14 fx + 21) e^{-fx}}{112 \sqrt{\pi} f^3} - \frac{3 (if)^{7/2} \sqrt{2} \operatorname{Erf}(\sqrt{x} \sqrt{f})}{32 f^{7/2}} + \frac{3}{32 f^{7/2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x)^(3/2)\*sinh(f\*x),x)

[Out] -2\*(d\*x)^(3/2)/x^(3/2)\*2^(1/2)/(I\*f)^(3/2)\*Pi^(1/2)/f\*(-1/112/Pi^(1/2)\*x^(1/2)\*2^(1/2)\*(I\*f)^(7/2)\*(-14\*f\*x+21)/f^3\*exp(f\*x)+1/112/Pi^(1/2)\*x^(1/2)\*2^(1/2)\*(I\*f)^(7/2)\*(14\*f\*x+21)/f^3\*exp(-f\*x)-3/32\*(I\*f)^(7/2)\*2^(1/2)/f^(7/2)\*erf(x^(1/2)\*f^(1/2))+3/32\*(I\*f)^(7/2)\*2^(1/2)/f^(7/2)\*erfi(x^(1/2)\*f^(1/2)))

**Maxima [B]** time = 1.15145, size = 236, normalized size = 2.13

$$16 (dx)^{\frac{5}{2}} \sinh(fx) - \frac{f \left( \frac{15 \sqrt{\pi} d^3 \operatorname{erf}\left(\sqrt{dx} \sqrt{\frac{f}{d}}\right)}{f^3 \sqrt{\frac{f}{d}}} - \frac{15 \sqrt{\pi} d^3 \operatorname{erf}\left(\sqrt{dx} \sqrt{-\frac{f}{d}}\right)}{f^3 \sqrt{-\frac{f}{d}}} + \frac{2 \left( 4 (dx)^{\frac{5}{2}} d f^2 - 10 (dx)^{\frac{3}{2}} d^2 f + 15 \sqrt{dx} d^3 \right) e^{(fx)}}{f^3} - \frac{2 \left( 4 (dx)^{\frac{5}{2}} d f^2 + 10 (dx)^{\frac{3}{2}} d^2 f + 15 \sqrt{dx} d^3 \right) e^{(-fx)}}{f^3} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*sinh(f\*x),x, algorithm="maxima")

[Out] 1/40\*(16\*(d\*x)^(5/2)\*sinh(f\*x) - f\*(15\*sqrt(pi)\*d^3\*erf(sqrt(d\*x)\*sqrt(f/d))/(f^3\*sqrt(f/d)) - 15\*sqrt(pi)\*d^3\*erf(sqrt(d\*x)\*sqrt(-f/d))/(f^3\*sqrt(-f/d)) + 2\*(4\*(d\*x)^(5/2)\*d\*f^2 - 10\*(d\*x)^(3/2)\*d^2\*f + 15\*sqrt(d\*x)\*d^3)\*e^(f\*x)/f^3 - 2\*(4\*(d\*x)^(5/2)\*d\*f^2 + 10\*(d\*x)^(3/2)\*d^2\*f + 15\*sqrt(d\*x)\*d^3)\*e^(-f\*x)/f^3)/d)

---

**Fricas [B]** time = 2.75208, size = 467, normalized size = 4.21

$$\frac{3\sqrt{\pi}(d^2 \cosh(fx) + d^2 \sinh(fx))\sqrt{\frac{f}{d}} \operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right) + 3\sqrt{\pi}(d^2 \cosh(fx) + d^2 \sinh(fx))\sqrt{-\frac{f}{d}} \operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right) - 2\left(2d^2 \cosh(fx) + 2d^2 \sinh(fx)\right)}{8(f^3 \cosh(fx) + f^3 \sinh(fx))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)^(3/2)\*sinh(f\*x),x, algorithm="fricas")

[Out] 
$$-1/8*(3*\sqrt{\pi}*(d^2*\cosh(f*x) + d^2*\sinh(f*x))*\sqrt{f/d}*\operatorname{erf}(\sqrt{d*x}*\sqrt{f/d}) + 3*\sqrt{\pi}*(d^2*\cosh(f*x) + d^2*\sinh(f*x))*\sqrt{-f/d}*\operatorname{erf}(\sqrt{d*x}*\sqrt{-f/d}) - 2*(2*d*f^2*x + (2*d*f^2*x - 3*d*f)*\cosh(f*x)^2 + 2*(2*d*f^2*x - 3*d*f)*\cosh(f*x)*\sinh(f*x) + (2*d*f^2*x - 3*d*f)*\sinh(f*x)^2 + 3*d*f)*\sqrt{d*x})/(f^3*\cosh(f*x) + f^3*\sinh(f*x))$$

---

**Sympy [C]** time = 152.835, size = 133, normalized size = 1.2

$$\frac{7d^{\frac{3}{2}}x^{\frac{3}{2}}\cosh(fx)\Gamma\left(\frac{7}{4}\right)}{4f\Gamma\left(\frac{11}{4}\right)} - \frac{21d^{\frac{3}{2}}\sqrt{x}\sinh(fx)\Gamma\left(\frac{7}{4}\right)}{8f^2\Gamma\left(\frac{11}{4}\right)} + \frac{21\sqrt{2}\sqrt{\pi}d^{\frac{3}{2}}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{xe^{\frac{i\pi}{4}}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{7}{4}\right)}{16f^{\frac{5}{2}}\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x)\*\*(3/2)\*sinh(f\*x),x)

[Out] 
$$7*d^{3/2}*x^{3/2}*\cosh(f*x)*\gamma(7/4)/(4*f*\gamma(11/4)) - 21*d^{3/2}*\sqrt{x}*\sinh(f*x)*\gamma(7/4)/(8*f^2*\gamma(11/4)) + 21*\sqrt{2}*\sqrt{\pi}*d^{3/2}*\exp(-3*I*\pi/4)*\operatorname{fresnels}(\sqrt{2}*\sqrt{f}*\sqrt{x})*\exp(I*\pi/4)/\sqrt{\pi}*\gamma(7/4)/(16*f^{5/2}*\gamma(11/4))$$

---

**Giac [A]** time = 1.36039, size = 194, normalized size = 1.75

$$\frac{3\sqrt{\pi}d^3 \operatorname{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}f^2} + \frac{2(2\sqrt{dxd^2fx+3\sqrt{dxd^2}})e^{-fx}}{f^2} - \frac{3\sqrt{\pi}d^3 \operatorname{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}f^2} - \frac{2(2\sqrt{dxd^2fx-3\sqrt{dxd^2}})e^{fx}}{f^2}$$

$8d$   $8d$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*sinh(f*x),x, algorithm="giac")
```

```
[Out] 1/8*(3*sqrt(pi)*d^3*erf(-sqrt(d*f)*sqrt(d*x)/d)/(sqrt(d*f)*f^2) + 2*(2*sqrt
(d*x)*d^2*f*x + 3*sqrt(d*x)*d^2)*e^(-f*x)/f^2)/d - 1/8*(3*sqrt(pi)*d^3*erf(
-sqrt(-d*f)*sqrt(d*x)/d)/(sqrt(-d*f)*f^2) - 2*(2*sqrt(d*x)*d^2*f*x - 3*sqrt
(d*x)*d^2)*e^(f*x)/f^2)/d
```

### 3.61 $\int \sqrt{dx} \sinh(fx) dx$

**Optimal.** Leaf size=92

$$-\frac{\sqrt{\pi}\sqrt{d}\operatorname{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} - \frac{\sqrt{\pi}\sqrt{d}\operatorname{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} + \frac{\sqrt{dx} \cosh(fx)}{f}$$

[Out] (Sqrt[d\*x]\*Cosh[f\*x])/f - (Sqrt[d]\*Sqrt[Pi]\*Erf[(Sqrt[f]\*Sqrt[d\*x])/Sqrt[d]])/(4\*f^(3/2)) - (Sqrt[d]\*Sqrt[Pi]\*Erfi[(Sqrt[f]\*Sqrt[d\*x])/Sqrt[d]])/(4\*f^(3/2))

**Rubi [A]** time = 0.110155, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {3296, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{d}\operatorname{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} - \frac{\sqrt{\pi}\sqrt{d}\operatorname{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} + \frac{\sqrt{dx} \cosh(fx)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d\*x]\*Sinh[f\*x], x]

[Out] (Sqrt[d\*x]\*Cosh[f\*x])/f - (Sqrt[d]\*Sqrt[Pi]\*Erf[(Sqrt[f]\*Sqrt[d\*x])/Sqrt[d]])/(4\*f^(3/2)) - (Sqrt[d]\*Sqrt[Pi]\*Erfi[(Sqrt[f]\*Sqrt[d\*x])/Sqrt[d]])/(4\*f^(3/2))

#### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

#### Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Dist[
I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2180

```
Int[(F_)^((g_.)*(e_.) + (f_.)*(x_))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{dx} \sinh(fx) dx &= \frac{\sqrt{dx} \cosh(fx)}{f} - \frac{d \int \frac{\cosh(fx)}{\sqrt{dx}} dx}{2f} \\
&= \frac{\sqrt{dx} \cosh(fx)}{f} - \frac{d \int \frac{e^{-fx}}{\sqrt{dx}} dx}{4f} - \frac{d \int \frac{e^{fx}}{\sqrt{dx}} dx}{4f} \\
&= \frac{\sqrt{dx} \cosh(fx)}{f} - \frac{\text{Subst}\left(\int e^{-\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{2f} - \frac{\text{Subst}\left(\int e^{\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{2f} \\
&= \frac{\sqrt{dx} \cosh(fx)}{f} - \frac{\sqrt{d}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} - \frac{\sqrt{d}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0164927, size = 49, normalized size = 0.53

$$\frac{d\left(\sqrt{fx}\Gamma\left(\frac{3}{2}, fx\right) - \sqrt{-fx}\Gamma\left(\frac{3}{2}, -fx\right)\right)}{2f^2\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d\*x]\*Sinh[f\*x], x]

[Out]  $(d * (-\text{Sqrt}[-(f*x)] * \text{Gamma}[3/2, -(f*x)]) + \text{Sqrt}[f*x] * \text{Gamma}[3/2, f*x]) / (2*f^2 * \text{Sqrt}[d*x])$

**Maple [C]** time = 0.024, size = 120, normalized size = 1.3

$$-\frac{\sqrt{\pi}\sqrt{2}}{f}\sqrt{dx}\left(\frac{\sqrt{2}e^{-fx}}{4\sqrt{\pi}f^2}\sqrt{x}(if)^{\frac{5}{2}} + \frac{\sqrt{2}e^{fx}}{4\sqrt{\pi}f^2}\sqrt{x}(if)^{\frac{5}{2}} - \frac{\sqrt{2}}{8}(if)^{\frac{5}{2}}\text{Erf}(\sqrt{x}\sqrt{f})f^{-\frac{5}{2}} - \frac{\sqrt{2}}{8}(if)^{\frac{5}{2}}\text{erfi}(\sqrt{x}\sqrt{f})f^{-\frac{5}{2}}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{if}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\sinh(f*x)*(d*x)^{(1/2)}, x)$

[Out]  $-\text{Pi}^{(1/2)}*(d*x)^{(1/2)}/x^{(1/2)}*2^{(1/2)}/(I*f)^{(1/2)}/f*(1/4/\text{Pi}^{(1/2)}*x^{(1/2)}*2^{(1/2)}*(I*f)^{(5/2)}/f^2*\exp(-f*x)+1/4/\text{Pi}^{(1/2)}*x^{(1/2)}*2^{(1/2)}*(I*f)^{(5/2)}/f^2*\exp(f*x)-1/8*(I*f)^{(5/2)}*2^{(1/2)}/f^{(5/2)}*\text{erf}(x^{(1/2)}*f^{(1/2)})-1/8*(I*f)^{(5/2)}*2^{(1/2)}/f^{(5/2)}*\text{erfi}(x^{(1/2)}*f^{(1/2)})$

**Maxima [B]** time = 1.22451, size = 201, normalized size = 2.18

$$8(dx)^{\frac{3}{2}}\sinh(fx) - \frac{f\left(\frac{3\sqrt{\pi}d^2\text{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{f^2\sqrt{\frac{f}{d}}} + \frac{3\sqrt{\pi}d^2\text{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{f^2\sqrt{-\frac{f}{d}}} + \frac{2\left(2(dx)^{\frac{3}{2}}df-3\sqrt{dx}d^2\right)e^{(fx)}}{f^2} - \frac{2\left(2(dx)^{\frac{3}{2}}df+3\sqrt{dx}d^2\right)e^{(-fx)}}{f^2}\right)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\sinh(f*x)*(d*x)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out]  $1/12*(8*(d*x)^{(3/2)}*\sinh(f*x) - f*(3*\text{sqrt}(\text{pi})*d^2*\text{erf}(\text{sqrt}(d*x)*\text{sqrt}(f/d)))/(f^2*\text{sqrt}(f/d)) + 3*\text{sqrt}(\text{pi})*d^2*\text{erf}(\text{sqrt}(d*x)*\text{sqrt}(-f/d))/(f^2*\text{sqrt}(-f/d)) + 2*(2*(d*x)^{(3/2)}*d*f - 3*\text{sqrt}(d*x)*d^2)*e^{(f*x)}/f^2 - 2*(2*(d*x)^{(3/2)}*d*f + 3*\text{sqrt}(d*x)*d^2)*e^{(-f*x)}/f^2)/d/d$

**Fricas [B]** time = 2.73524, size = 356, normalized size = 3.87

$$\frac{\sqrt{\pi}(d \cosh(fx) + d \sinh(fx))\sqrt{\frac{f}{d}}\text{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right) - \sqrt{\pi}(d \cosh(fx) + d \sinh(fx))\sqrt{-\frac{f}{d}}\text{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right) - 2(f \cosh(fx) + f \sinh(fx))}{4(f^2 \cosh(fx) + f^2 \sinh(fx))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f\*x)\*(d\*x)^(1/2),x, algorithm="fricas")

[Out] 
$$-1/4*(\sqrt{\pi}*(d*\cosh(f*x) + d*\sinh(f*x))*\sqrt{f/d}*\operatorname{erf}(\sqrt{d*x}*\sqrt{f/d})) - \sqrt{\pi}*(d*\cosh(f*x) + d*\sinh(f*x))*\sqrt{-f/d}*\operatorname{erf}(\sqrt{d*x}*\sqrt{-f/d}) - 2*(f*\cosh(f*x)^2 + 2*f*\cosh(f*x)*\sinh(f*x) + f*\sinh(f*x)^2 + f)*\sqrt{d*x})/(f^2*\cosh(f*x) + f^2*\sinh(f*x))$$

**Sympy [C]** time = 3.04992, size = 99, normalized size = 1.08

$$\frac{5\sqrt{d}\sqrt{x}\cosh(fx)\Gamma\left(\frac{5}{4}\right)}{4f\Gamma\left(\frac{9}{4}\right)} - \frac{5\sqrt{2}\sqrt{\pi}\sqrt{d}e^{-\frac{i\pi}{4}}C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{xe^{\frac{i\pi}{4}}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{8f^{\frac{3}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f\*x)\*(d\*x)\*\*(1/2),x)

[Out] 
$$5*\sqrt{d}*\sqrt{x}*\cosh(f*x)*\gamma(5/4)/(4*f*\gamma(9/4)) - 5*\sqrt{2}*\sqrt{\pi}*\sqrt{d}*\exp(-I*\pi/4)*\operatorname{fresnelc}(\sqrt{2}*\sqrt{f}*\sqrt{x}*\exp(I*\pi/4)/\sqrt{\pi})*\gamma(5/4)/(8*f**(3/2)*\gamma(9/4))$$

**Giac [A]** time = 1.40396, size = 136, normalized size = 1.48

$$\frac{\frac{\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}f} + \frac{\sqrt{\pi}d^2 \operatorname{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}f} + \frac{2\sqrt{dxd}e^{(fx)}}{f} + \frac{2\sqrt{dxd}e^{(-fx)}}{f}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f\*x)\*(d\*x)^(1/2),x, algorithm="giac")

[Out] 
$$1/4*(\sqrt{\pi}*d^2*\operatorname{erf}(-\sqrt{d*f}*\sqrt{d*x}/d)/(\sqrt{d*f}*f) + \sqrt{\pi}*d^2*\operatorname{erf}(-\sqrt{-d*f}*\sqrt{d*x}/d)/(\sqrt{-d*f}*f) + 2*\sqrt{d*x}*d*e^{(f*x)}/f + 2*\sqrt{d*x}*d*e^{(-f*x)}/f)/d$$

$$3.62 \quad \int \frac{\sinh(fx)}{\sqrt{dx}} dx$$

**Optimal.** Leaf size=77

$$\frac{\sqrt{\pi} \operatorname{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}} - \frac{\sqrt{\pi} \operatorname{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}$$

[Out]  $-(\operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erf}[(\operatorname{Sqrt}[f] * \operatorname{Sqrt}[d * x]) / \operatorname{Sqrt}[d]]) / (2 * \operatorname{Sqrt}[d] * \operatorname{Sqrt}[f]) + (\operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(\operatorname{Sqrt}[f] * \operatorname{Sqrt}[d * x]) / \operatorname{Sqrt}[d]]) / (2 * \operatorname{Sqrt}[d] * \operatorname{Sqrt}[f])$

**Rubi [A]** time = 0.0761857, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi} \operatorname{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}} - \frac{\sqrt{\pi} \operatorname{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sinh}[f * x] / \operatorname{Sqrt}[d * x], x]$

[Out]  $-(\operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erf}[(\operatorname{Sqrt}[f] * \operatorname{Sqrt}[d * x]) / \operatorname{Sqrt}[d]]) / (2 * \operatorname{Sqrt}[d] * \operatorname{Sqrt}[f]) + (\operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[(\operatorname{Sqrt}[f] * \operatorname{Sqrt}[d * x]) / \operatorname{Sqrt}[d]]) / (2 * \operatorname{Sqrt}[d] * \operatorname{Sqrt}[f])$

#### Rule 3308

$\operatorname{Int}[(c_. + (d_.)(x_.))^{(m_.)} \sin[(e_.) + (f_.)(x_.)], x\_Symbol] :> \operatorname{Dist}[I/2, \operatorname{Int}[(c + d * x)^m / E^{(I * (e + f * x))}, x], x] - \operatorname{Dist}[I/2, \operatorname{Int}[(c + d * x)^m * E^{(I * (e + f * x))}, x], x] /;$   $\operatorname{FreeQ}\{c, d, e, f, m\}, x]$

#### Rule 2180

$\operatorname{Int}[(F_)^{(g_.)} * ((e_.) + (f_.)(x_.)) / \operatorname{Sqrt}[(c_.) + (d_.)(x_.)], x\_Symbol] :> \operatorname{Dist}[2/d, \operatorname{Subst}[\operatorname{Int}[F^{(g * (e - (c * f) / d) + (f * g * x^2) / d)}, x], x, \operatorname{Sqrt}[c + d * x]], x] /;$   $\operatorname{FreeQ}\{F, c, d, e, f, g\}, x] \&\& \text{!} \$\operatorname{UseGamma} == \operatorname{True}$

#### Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

### Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

### Rubi steps

$$\begin{aligned} \int \frac{\sinh(fx)}{\sqrt{dx}} dx &= -\left(\frac{1}{2} \int \frac{e^{-fx}}{\sqrt{dx}} dx\right) + \frac{1}{2} \int \frac{e^{fx}}{\sqrt{dx}} dx \\ &= -\frac{\text{Subst}\left(\int e^{-\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{d} + \frac{\text{Subst}\left(\int e^{\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{d} \\ &= -\frac{\sqrt{\pi} \text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}} + \frac{\sqrt{\pi} \text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}} \end{aligned}$$

**Mathematica [A]** time = 0.0087227, size = 47, normalized size = 0.61

$$\frac{\sqrt{-fx} \text{Gamma}\left(\frac{1}{2}, -fx\right) + \sqrt{fx} \text{Gamma}\left(\frac{1}{2}, fx\right)}{2f\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[f*x]/Sqrt[d*x], x]
```

```
[Out] (Sqrt[-(f*x)]*Gamma[1/2, -(f*x)] + Sqrt[f*x]*Gamma[1/2, f*x])/(2*f*Sqrt[d*x
])
```

**Maple [C]** time = 0.022, size = 71, normalized size = 0.9

$$-\frac{\sqrt{\pi}\sqrt{2}}{2f}\sqrt{x}\sqrt{if}\left(-\frac{\sqrt{2}}{2}(if)^{\frac{3}{2}}\text{Erf}(\sqrt{x}\sqrt{f})f^{-\frac{3}{2}} + \frac{\sqrt{2}}{2}(if)^{\frac{3}{2}}\text{erfi}(\sqrt{x}\sqrt{f})f^{-\frac{3}{2}}\right)\frac{1}{\sqrt{dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(f*x)/(d*x)^(1/2),x)`

[Out]  $-1/2*\text{Pi}^{(1/2)}/(d*x)^{(1/2)}*x^{(1/2)}*2^{(1/2)}*(I*f)^{(1/2)}/f*(-1/2*(I*f)^{(3/2)}*2^{(1/2)}/f^{(3/2)}*\text{erf}(x^{(1/2)}*f^{(1/2)})+1/2*(I*f)^{(3/2)}*2^{(1/2)}/f^{(3/2)}*\text{erfi}(x^{(1/2)}*f^{(1/2)})$

**Maxima [B]** time = 1.21827, size = 157, normalized size = 2.04

$$\frac{4\sqrt{dx}\sinh(fx) - \frac{\left( \frac{2\sqrt{dx}e^{fx}}{f} - \frac{2\sqrt{dx}e^{-fx}}{f} + \frac{\sqrt{\pi d}\text{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{f\sqrt{\frac{f}{d}}} - \frac{\sqrt{\pi d}\text{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{f\sqrt{-\frac{f}{d}}}\right) f}{d}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x)/(d*x)^(1/2),x, algorithm="maxima")`

[Out]  $1/2*(4*\text{sqrt}(d*x)*\sinh(f*x) - (2*\text{sqrt}(d*x)*d*e^{(f*x)}/f - 2*\text{sqrt}(d*x)*d*e^{(-f*x)}/f + \text{sqrt}(\text{pi})*d*\text{erf}(\text{sqrt}(d*x)*\text{sqrt}(f/d))/f - \text{sqrt}(\text{pi})*d*\text{erf}(\text{sqrt}(d*x)*\text{sqrt}(-f/d))/f))*f/d/d$

**Fricas [A]** time = 2.63551, size = 138, normalized size = 1.79

$$\frac{\sqrt{\pi}\sqrt{\frac{f}{d}}\text{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right) + \sqrt{\pi}\sqrt{-\frac{f}{d}}\text{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x)/(d*x)^(1/2),x, algorithm="fricas")`

[Out]  $-1/2*(\text{sqrt}(\text{pi})*\text{sqrt}(f/d)*\text{erf}(\text{sqrt}(d*x)*\text{sqrt}(f/d)) + \text{sqrt}(\text{pi})*\text{sqrt}(-f/d)*\text{erf}(\text{sqrt}(d*x)*\text{sqrt}(-f/d)))/f$



**Sympy [C]** time = 1.47673, size = 70, normalized size = 0.91

$$\frac{3\sqrt{2}\sqrt{\pi}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{4\sqrt{d}\sqrt{f}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f\*x)/(d\*x)\*\*(1/2),x)

[Out] 3\*sqrt(2)\*sqrt(pi)\*exp(-3\*I\*pi/4)\*fresnels(sqrt(2)\*sqrt(f)\*sqrt(x)\*exp(I\*pi/4)/sqrt(pi))\*gamma(3/4)/(4\*sqrt(d)\*sqrt(f)\*gamma(7/4))

**Giac [A]** time = 1.314, size = 82, normalized size = 1.06

$$\frac{\frac{\sqrt{\pi d} \operatorname{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}} - \frac{\sqrt{\pi d} \operatorname{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f\*x)/(d\*x)^(1/2),x, algorithm="giac")

[Out] 1/2\*(sqrt(pi)\*d\*erf(-sqrt(d\*f)\*sqrt(d\*x)/d)/sqrt(d\*f) - sqrt(pi)\*d\*erf(-sqrt(-d\*f)\*sqrt(d\*x)/d)/sqrt(-d\*f))/d

### 3.63 $\int \frac{\sinh(fx)}{(dx)^{3/2}} dx$

**Optimal.** Leaf size=87

$$\frac{\sqrt{\pi}\sqrt{f}\operatorname{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{\pi}\sqrt{f}\operatorname{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sinh(fx)}{d\sqrt{dx}}$$

[Out] (Sqrt[f]\*Sqrt[Pi]\*Erf[(Sqrt[f]\*Sqrt[d\*x])/Sqrt[d]])/d^(3/2) + (Sqrt[f]\*Sqrt[Pi]\*Erfi[(Sqrt[f]\*Sqrt[d\*x])/Sqrt[d]])/d^(3/2) - (2\*Sinh[f\*x])/(d\*Sqrt[d\*x])

**Rubi [A]** time = 0.112284, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {3297, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\sqrt{f}\operatorname{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{\pi}\sqrt{f}\operatorname{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2\sinh(fx)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[f\*x]/(d\*x)^(3/2), x]

[Out] (Sqrt[f]\*Sqrt[Pi]\*Erf[(Sqrt[f]\*Sqrt[d\*x])/Sqrt[d]])/d^(3/2) + (Sqrt[f]\*Sqrt[Pi]\*Erfi[(Sqrt[f]\*Sqrt[d\*x])/Sqrt[d]])/d^(3/2) - (2\*Sinh[f\*x])/(d\*Sqrt[d\*x])

#### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

#### Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

f, m}, x] && IntegerQ[2\*k]

### Rule 2180

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))/Sqrt[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :  
 > Dist[2/d, Subst[Int[F^(g\*(e - (c\*f)/d) + (f\*g\*x^2)/d), x], x, Sqrt[c + d\*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma === True

### Rule 2204

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erfi[(c + d\*x)\*Rt[b\*Log[F], 2]])/(2\*d\*Rt[b\*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

### Rule 2205

Int[(F\_)^((a\_.) + (b\_.)\*((c\_.) + (d\_.)\*(x\_)^2), x\_Symbol] :> Simp[(F^a\*Sqrt[Pi]\*Erf[(c + d\*x)\*Rt[-(b\*Log[F]), 2]])/(2\*d\*Rt[-(b\*Log[F]), 2]), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

### Rubi steps

$$\begin{aligned}
 \int \frac{\sinh(fx)}{(dx)^{3/2}} dx &= -\frac{2 \sinh(fx)}{d\sqrt{dx}} + \frac{(2f) \int \frac{\cosh(fx)}{\sqrt{dx}} dx}{d} \\
 &= -\frac{2 \sinh(fx)}{d\sqrt{dx}} + \frac{f \int \frac{e^{-fx}}{\sqrt{dx}} dx}{d} + \frac{f \int \frac{e^{fx}}{\sqrt{dx}} dx}{d} \\
 &= -\frac{2 \sinh(fx)}{d\sqrt{dx}} + \frac{(2f) \text{Subst}\left(\int e^{-\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{d^2} + \frac{(2f) \text{Subst}\left(\int e^{\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{d^2} \\
 &= \frac{\sqrt{f}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{f}\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} - \frac{2 \sinh(fx)}{d\sqrt{dx}}
 \end{aligned}$$

**Mathematica [A]** time = 0.0198243, size = 49, normalized size = 0.56

$$\frac{x \left( \sqrt{-fx} \Gamma\left(\frac{1}{2}, -fx\right) - \sqrt{fx} \Gamma\left(\frac{1}{2}, fx\right) - 2 \sinh(fx) \right)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[f\*x]/(d\*x)^(3/2),x]

[Out] (x\*(Sqrt[-(f\*x)]\*Gamma[1/2, -(f\*x)] - Sqrt[f\*x]\*Gamma[1/2, f\*x] - 2\*Sinh[f\*x]))/(d\*x)^(3/2)

**Maple [C]** time = 0.022, size = 120, normalized size = 1.4

$$-\frac{\sqrt{\pi}\sqrt{2}}{4f}x^{\frac{3}{2}}(if)^{\frac{3}{2}}\left(2\frac{\sqrt{2}\sqrt{if}e^{-fx}}{\sqrt{\pi}\sqrt{xf}}-2\frac{\sqrt{2}\sqrt{if}e^{fx}}{\sqrt{\pi}\sqrt{xf}}+2\frac{\sqrt{2}\sqrt{if}\operatorname{Erf}(\sqrt{x}\sqrt{f})}{\sqrt{f}}+2\frac{\sqrt{2}\sqrt{if}\operatorname{erfi}(\sqrt{x}\sqrt{f})}{\sqrt{f}}\right)(dx)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(f\*x)/(d\*x)^(3/2),x)

[Out]  $-1/4*\pi^{(1/2)}/(d*x)^{(3/2)}*x^{(3/2)}*2^{(1/2)}*(I*f)^{(3/2)}/f*(2/\pi^{(1/2)}/x^{(1/2)}*2^{(1/2)}*(I*f)^{(1/2)}/f*\exp(-f*x)-2/\pi^{(1/2)}/x^{(1/2)}*2^{(1/2)}*(I*f)^{(1/2)}/f*\exp(f*x)+2*(I*f)^{(1/2)}*2^{(1/2)}/f^{(1/2)}*\operatorname{erf}(x^{(1/2)}*f^{(1/2)})+2*(I*f)^{(1/2)}*2^{(1/2)}/f^{(1/2)}*\operatorname{erfi}(x^{(1/2)}*f^{(1/2)})$

**Maxima [A]** time = 1.13607, size = 100, normalized size = 1.15

$$\frac{f\left(\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{\sqrt{\frac{f}{d}}}+\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{\sqrt{-\frac{f}{d}}}\right)}{d}-\frac{2\sinh(fx)}{\sqrt{dx}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f\*x)/(d\*x)^(3/2),x, algorithm="maxima")

[Out]  $(f*(\operatorname{sqrt}(\pi)*\operatorname{erf}(\operatorname{sqrt}(d*x)*\operatorname{sqrt}(f/d)))/\operatorname{sqrt}(f/d) + \operatorname{sqrt}(\pi)*\operatorname{erf}(\operatorname{sqrt}(d*x)*\operatorname{sqrt}(-f/d))/\operatorname{sqrt}(-f/d))/d - 2*\sinh(f*x)/\operatorname{sqrt}(d*x))/d$

**Fricas [B]** time = 2.69863, size = 355, normalized size = 4.08

$$\frac{\sqrt{\pi}(dx \cosh(fx) + dx \sinh(fx))\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right) - \sqrt{\pi}(dx \cosh(fx) + dx \sinh(fx))\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right) - \sqrt{dx}\cos(fx)}{d^2x \cosh(fx) + d^2x \sinh(fx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f\*x)/(d\*x)^(3/2),x, algorithm="fricas")

[Out] (sqrt(pi)\*(d\*x\*cosh(f\*x) + d\*x\*sinh(f\*x))\*sqrt(f/d)\*erf(sqrt(d\*x)\*sqrt(f/d)) - sqrt(pi)\*(d\*x\*cosh(f\*x) + d\*x\*sinh(f\*x))\*sqrt(-f/d)\*erf(sqrt(d\*x)\*sqrt(-f/d)) - sqrt(d\*x)\*(cosh(f\*x)^2 + 2\*cosh(f\*x)\*sinh(f\*x) + sinh(f\*x)^2 - 1)) / (d^2\*x\*cosh(f\*x) + d^2\*x\*sinh(f\*x))

**Sympy [C]** time = 7.42422, size = 94, normalized size = 1.08

$$\frac{\sqrt{2}\sqrt{\pi}\sqrt{f}e^{-\frac{i\pi}{4}}C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{2d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)} - \frac{\sinh(fx)\Gamma\left(\frac{1}{4}\right)}{2d^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f\*x)/(d\*x)\*\*(3/2),x)

[Out] sqrt(2)\*sqrt(pi)\*sqrt(f)\*exp(-I\*pi/4)\*fresnelc(sqrt(2)\*sqrt(f)\*sqrt(x)\*exp(I\*pi/4)/sqrt(pi))\*gamma(1/4)/(2\*d\*\*(3/2)\*gamma(5/4)) - sinh(f\*x)\*gamma(1/4)/(2\*d\*\*(3/2)\*sqrt(x)\*gamma(5/4))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx)}{(dx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f\*x)/(d\*x)^(3/2),x, algorithm="giac")

[Out] integrate(sinh(f\*x)/(d\*x)^(3/2), x)

### 3.64 $\int \frac{\sinh(fx)}{(dx)^{5/2}} dx$

**Optimal.** Leaf size=114

$$-\frac{2\sqrt{\pi}f^{3/2}\operatorname{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2\sqrt{\pi}f^{3/2}\operatorname{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{4f \cosh(fx)}{3d^2\sqrt{dx}} - \frac{2 \sinh(fx)}{3d(dx)^{3/2}}$$

[Out]  $(-4*f*\operatorname{Cosh}[f*x])/(3*d^2*\operatorname{Sqrt}[d*x]) - (2*f^{(3/2)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[f]*\operatorname{Sqrt}[d*x])/\operatorname{Sqrt}[d]])/(3*d^{(5/2)}) + (2*f^{(3/2)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[f]*\operatorname{Sqrt}[d*x])/\operatorname{Sqrt}[d]])/(3*d^{(5/2)}) - (2*\operatorname{Sinh}[f*x])/(3*d*(d*x)^{(3/2)})$

**Rubi [A]** time = 0.149009, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {3297, 3308, 2180, 2204, 2205}

$$-\frac{2\sqrt{\pi}f^{3/2}\operatorname{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2\sqrt{\pi}f^{3/2}\operatorname{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{4f \cosh(fx)}{3d^2\sqrt{dx}} - \frac{2 \sinh(fx)}{3d(dx)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Sinh}[f*x]/(d*x)^{(5/2)}, x]$

[Out]  $(-4*f*\operatorname{Cosh}[f*x])/(3*d^2*\operatorname{Sqrt}[d*x]) - (2*f^{(3/2)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[(\operatorname{Sqrt}[f]*\operatorname{Sqrt}[d*x])/\operatorname{Sqrt}[d]])/(3*d^{(5/2)}) + (2*f^{(3/2)}*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[(\operatorname{Sqrt}[f]*\operatorname{Sqrt}[d*x])/\operatorname{Sqrt}[d]])/(3*d^{(5/2)}) - (2*\operatorname{Sinh}[f*x])/(3*d*(d*x)^{(3/2)})$

#### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

#### Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2180

```
Int[(F_)^((g_.)*(e_.) + (f_.)*(x_))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(fx)}{(dx)^{5/2}} dx &= -\frac{2 \sinh(fx)}{3d(dx)^{3/2}} + \frac{(2f) \int \frac{\cosh(fx)}{(dx)^{3/2}} dx}{3d} \\
&= -\frac{4f \cosh(fx)}{3d^2 \sqrt{dx}} - \frac{2 \sinh(fx)}{3d(dx)^{3/2}} + \frac{(4f^2) \int \frac{\sinh(fx)}{\sqrt{dx}} dx}{3d^2} \\
&= -\frac{4f \cosh(fx)}{3d^2 \sqrt{dx}} - \frac{2 \sinh(fx)}{3d(dx)^{3/2}} - \frac{(2f^2) \int \frac{e^{-fx}}{\sqrt{dx}} dx}{3d^2} + \frac{(2f^2) \int \frac{e^{fx}}{\sqrt{dx}} dx}{3d^2} \\
&= -\frac{4f \cosh(fx)}{3d^2 \sqrt{dx}} - \frac{2 \sinh(fx)}{3d(dx)^{3/2}} - \frac{(4f^2) \text{Subst}\left(\int e^{-\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{3d^3} + \frac{(4f^2) \text{Subst}\left(\int e^{\frac{fx^2}{d}} dx, x, \sqrt{dx}\right)}{3d^3} \\
&= -\frac{4f \cosh(fx)}{3d^2 \sqrt{dx}} - \frac{2f^{3/2} \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2f^{3/2} \sqrt{\pi} \operatorname{erfi}\left(\frac{\sqrt{f} \sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{2 \sinh(fx)}{3d(dx)^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.0830307, size = 84, normalized size = 0.74

$$-\frac{xe^{-fx} \left( 2e^{fx} (-fx)^{3/2} \operatorname{Gamma}\left(\frac{1}{2}, -fx\right) - 2e^{fx} (fx)^{3/2} \operatorname{Gamma}\left(\frac{1}{2}, fx\right) + e^{2fx} + 2fxe^{2fx} + 2fx - 1 \right)}{3(dx)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[f\*x]/(d\*x)^(5/2),x]

[Out]  $-(x*(-1 + E^{(2*f*x)} + 2*f*x + 2*E^{(2*f*x)}*f*x + 2*E^{(f*x)}*(-(f*x))^{(3/2)}*Gamma[1/2, -(f*x)] - 2*E^{(f*x)}*(f*x)^{(3/2)}*Gamma[1/2, f*x]))/(3*E^{(f*x)}*(d*x)^{(5/2)})$

**Maple [C]** time = 0.024, size = 132, normalized size = 1.2

$$-\frac{\sqrt{\pi}\sqrt{2}}{8f}x^{\frac{5}{2}}(if)^{\frac{5}{2}}\left(-\frac{4\sqrt{2}(2fx+1)e^{fx}}{3\sqrt{\pi}f}x^{-\frac{3}{2}}\frac{1}{\sqrt{if}}+\frac{4\sqrt{2}(-2fx+1)e^{-fx}}{3\sqrt{\pi}f}x^{-\frac{3}{2}}\frac{1}{\sqrt{if}}-\frac{8\sqrt{2}}{3}\sqrt{f}\operatorname{Erf}(\sqrt{x}\sqrt{f})\frac{1}{\sqrt{if}}+\frac{8\sqrt{2}}{3}\sqrt{f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(f\*x)/(d\*x)^(5/2),x)

[Out]  $-1/8*\Pi^{(1/2)}/(d*x)^{(5/2)}*x^{(5/2)}*2^{(1/2)}*(I*f)^{(5/2)}/f*(-4/3*\Pi^{(1/2)}/x^{(3/2)}*2^{(1/2)}/(I*f)^{(1/2)}*(2*f*x+1)/f*\exp(f*x)+4/3*\Pi^{(1/2)}/x^{(3/2)}*2^{(1/2)}/(I*f)^{(1/2)}*(-2*f*x+1)/f*\exp(-f*x)-8/3/(I*f)^{(1/2)}*2^{(1/2)}*f^{(1/2)}*\operatorname{erf}(x^{(1/2)}*f^{(1/2)})+8/3/(I*f)^{(1/2)}*2^{(1/2)}*f^{(1/2)}*\operatorname{erfi}(x^{(1/2)}*f^{(1/2)})$

**Maxima [A]** time = 1.18749, size = 77, normalized size = 0.68

$$\frac{f\left(\frac{\sqrt{fx}\Gamma\left(-\frac{1}{2},fx\right)}{\sqrt{dx}}+\frac{\sqrt{-fx}\Gamma\left(-\frac{1}{2},-fx\right)}{\sqrt{dx}}\right)}{d}+\frac{2\sinh(fx)}{(dx)^{\frac{3}{2}}}$$


---


$$3d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f\*x)/(d\*x)^(5/2),x, algorithm="maxima")

[Out]  $-1/3*(f*(\operatorname{sqrt}(f*x)*\operatorname{gamma}(-1/2, f*x)/\operatorname{sqrt}(d*x) + \operatorname{sqrt}(-f*x)*\operatorname{gamma}(-1/2, -f*x)/\operatorname{sqrt}(d*x)))/d + 2*\sinh(f*x)/(d*x)^{(3/2)}/d$



**Fricas [B]** time = 2.64832, size = 454, normalized size = 3.98

$$\frac{2\sqrt{\pi}(dfx^2 \cosh(fx) + dfx^2 \sinh(fx))\sqrt{\frac{f}{d}} \operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right) + 2\sqrt{\pi}(dfx^2 \cosh(fx) + dfx^2 \sinh(fx))\sqrt{-\frac{f}{d}} \operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{3(d^3x^2 \cosh(fx) + d^3x^2 \sinh(fx))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f\*x)/(d\*x)^(5/2),x, algorithm="fricas")

[Out]  $-1/3*(2*\sqrt{\pi}*(d*f*x^2*\cosh(f*x) + d*f*x^2*\sinh(f*x))*\sqrt{f/d}*\operatorname{erf}(\sqrt{d*x}*\sqrt{f/d}) + 2*\sqrt{\pi}*(d*f*x^2*\cosh(f*x) + d*f*x^2*\sinh(f*x))*\sqrt{-f/d}*\operatorname{erf}(\sqrt{d*x}*\sqrt{-f/d}) + ((2*f*x + 1)*\cosh(f*x)^2 + 2*(2*f*x + 1)*\cosh(f*x)*\sinh(f*x) + (2*f*x + 1)*\sinh(f*x)^2 + 2*f*x - 1)*\sqrt{d*x})/(d^3*x^2*\cosh(f*x) + d^3*x^2*\sinh(f*x))$

**Sympy [C]** time = 175.807, size = 129, normalized size = 1.13

$$-\frac{\sqrt{2}\sqrt{\pi}f^{\frac{3}{2}}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{xe^{\frac{i\pi}{4}}}}{\sqrt{\pi}}\right)\Gamma\left(-\frac{1}{4}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{3}{4}\right)} + \frac{f \cosh(fx)\Gamma\left(-\frac{1}{4}\right)}{3d^{\frac{5}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)} + \frac{\sinh(fx)\Gamma\left(-\frac{1}{4}\right)}{6d^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f\*x)/(d\*x)\*\*(5/2),x)

[Out]  $-\sqrt{2}*\sqrt{\pi}*f**(3/2)*\exp(-3*I*\pi/4)*\operatorname{fresnels}(\sqrt{2}*\sqrt{f}*\sqrt{x}*\exp(I*\pi/4)/\sqrt{\pi})*\gamma(-1/4)/(3*d**(5/2)*\gamma(3/4)) + f*\cosh(f*x)*\gamma(-1/4)/(3*d**(5/2)*\sqrt{x}*\gamma(3/4)) + \sinh(f*x)*\gamma(-1/4)/(6*d**(5/2)*x**(3/2)*\gamma(3/4))$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx)}{(dx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x)/(d*x)^(5/2), x)
```

### 3.65 $\int \sqrt{c + dx} \operatorname{csch}(a + bx) dx$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\sqrt{c + dx} \operatorname{csch}(a + bx), x\right)$$

[Out] Unintegrable[Sqrt[c + d\*x]\*Csch[a + b\*x], x]

**Rubi [A]** time = 0.0281708, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \sqrt{c + dx} \operatorname{csch}(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[c + d\*x]\*Csch[a + b\*x], x]

[Out] Defer[Int][Sqrt[c + d\*x]\*Csch[a + b\*x], x]

Rubi steps

$$\int \sqrt{c + dx} \operatorname{csch}(a + bx) dx = \int \sqrt{c + dx} \operatorname{csch}(a + bx) dx$$

**Mathematica [A]** time = 23.4824, size = 0, normalized size = 0.

$$\int \sqrt{c + dx} \operatorname{csch}(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[c + d\*x]\*Csch[a + b\*x], x]

[Out] Integrate[Sqrt[c + d\*x]\*Csch[a + b\*x], x]

**Maple [A]** time = 0.042, size = 0, normalized size = 0.

$$\int \operatorname{csch}(bx + a) \sqrt{dx + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(b*x+a)*(d*x+c)^(1/2),x)`

[Out] `int(csch(b*x+a)*(d*x+c)^(1/2),x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx + c} \operatorname{csch}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(b*x+a)*(d*x+c)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(d*x + c)*csch(b*x + a), x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{dx + c} \operatorname{csch}(bx + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(b*x+a)*(d*x+c)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(d*x + c)*csch(b*x + a), x)`

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{c + dx} \operatorname{csch}(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*csch(a + b*x), x)
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{dx + c} \operatorname{csch}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x + c)*csch(b*x + a), x)
```

$$3.66 \quad \int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}} dx$$

**Optimal.** Leaf size=18

$$\text{Unintegrable}\left(\frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}}, x\right)$$

[Out] Unintegrable[Csch[a + b\*x]/Sqrt[c + d\*x], x]

**Rubi [A]** time = 0.0297394, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}} dx$$

Verification is Not applicable to the result.

[In] Int[Csch[a + b\*x]/Sqrt[c + d\*x], x]

[Out] Defer[Int][Csch[a + b\*x]/Sqrt[c + d\*x], x]

Rubi steps

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}} dx = \int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}} dx$$

**Mathematica [A]** time = 21.7864, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a+bx)}{\sqrt{c+dx}} dx$$

Verification is Not applicable to the result.

[In] Integrate[Csch[a + b\*x]/Sqrt[c + d\*x], x]

[Out] Integrate[Csch[a + b\*x]/Sqrt[c + d\*x], x]

---

**Maple [A]** time = 0.043, size = 0, normalized size = 0.

$$\int \operatorname{csch}(bx + a) \frac{1}{\sqrt{dx + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(b\*x+a)/(d\*x+c)^(1/2),x)

[Out] int(csch(b\*x+a)/(d\*x+c)^(1/2),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx + a)}{\sqrt{dx + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(b\*x+a)/(d\*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate(csch(b\*x + a)/sqrt(d\*x + c), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(bx + a)}{\sqrt{dx + c}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(b\*x+a)/(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] integral(csch(b\*x + a)/sqrt(d\*x + c), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(a + bx)}{\sqrt{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(csch(a + b*x)/sqrt(c + d*x), x)
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(bx + a)}{\sqrt{dx + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csch(b*x + a)/sqrt(d*x + c), x)
```



$$3.67 \quad \int \frac{\sinh^{\frac{3}{2}}(x)}{x^3} dx$$

**Optimal.** Leaf size=61

$$\frac{9}{8} \text{Unintegrable} \left( \frac{\sinh^{\frac{3}{2}}(x)}{x}, x \right) + \frac{3}{8} \text{Unintegrable} \left( \frac{1}{x\sqrt{\sinh(x)}}, x \right) - \frac{\sinh^{\frac{3}{2}}(x)}{2x^2} - \frac{3\sqrt{\sinh(x)} \cosh(x)}{4x}$$

[Out]  $(-3*\text{Cosh}[x]*\text{Sqrt}[\text{Sinh}[x]])/(4*x) - \text{Sinh}[x]^{(3/2)}/(2*x^2) + (3*\text{Unintegrable}[1/(x*\text{Sqrt}[\text{Sinh}[x]]), x])/8 + (9*\text{Unintegrable}[\text{Sinh}[x]^{(3/2)}/x, x])/8$

**Rubi [A]** time = 0.107143, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3} dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[\text{Sinh}[x]^{(3/2)}/x^3, x]$

[Out]  $(-3*\text{Cosh}[x]*\text{Sqrt}[\text{Sinh}[x]])/(4*x) - \text{Sinh}[x]^{(3/2)}/(2*x^2) + (3*\text{Defer}[\text{Int}][1/(x*\text{Sqrt}[\text{Sinh}[x]]), x])/8 + (9*\text{Defer}[\text{Int}][\text{Sinh}[x]^{(3/2)}/x, x])/8$

Rubi steps

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3} dx = -\frac{3 \cosh(x)\sqrt{\sinh(x)}}{4x} - \frac{\sinh^{\frac{3}{2}}(x)}{2x^2} + \frac{3}{8} \int \frac{1}{x\sqrt{\sinh(x)}} dx + \frac{9}{8} \int \frac{\sinh^{\frac{3}{2}}(x)}{x} dx$$

**Mathematica [A]** time = 6.01493, size = 0, normalized size = 0.

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sinh[x]^(3/2)/x^3,x]

[Out] Integrate[Sinh[x]^(3/2)/x^3, x]

---

**Maple [A]** time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} (\sinh(x))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(x)^(3/2)/x^3,x)

[Out] int(sinh(x)^(3/2)/x^3,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(x)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^(3/2)/x^3,x, algorithm="maxima")

[Out] integrate(sinh(x)^(3/2)/x^3, x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^(3/2)/x^3,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^{\frac{3}{2}}(x)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)\*\*(3/2)/x\*\*3,x)

[Out] Integral(sinh(x)\*\*(3/2)/x\*\*3, x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(x)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(x)^(3/2)/x^3,x, algorithm="giac")

[Out] integrate(sinh(x)^(3/2)/x^3, x)

$$3.68 \quad \int \left( \frac{x}{\sinh^{\frac{3}{2}}(x)} - x\sqrt{\sinh(x)} \right) dx$$

**Optimal.** Leaf size=20

$$4\sqrt{\sinh(x)} - \frac{2x \cosh(x)}{\sqrt{\sinh(x)}}$$

[Out] (-2\*x\*Cosh[x])/Sqrt[Sinh[x]] + 4\*Sqrt[Sinh[x]]

**Rubi [A]** time = 0.0583554, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {3315}

$$4\sqrt{\sinh(x)} - \frac{2x \cosh(x)}{\sqrt{\sinh(x)}}$$

Antiderivative was successfully verified.

[In] Int[x/Sinh[x]^(3/2) - x\*Sqrt[Sinh[x]],x]

[Out] (-2\*x\*Cosh[x])/Sqrt[Sinh[x]] + 4\*Sqrt[Sinh[x]]

### Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
  Simp[((c + d*x)*Cos[e + f*x]*(b*Sin[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
  (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sin[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

### Rubi steps

$$\begin{aligned} \int \left( \frac{x}{\sinh^{\frac{3}{2}}(x)} - x\sqrt{\sinh(x)} \right) dx &= \int \frac{x}{\sinh^{\frac{3}{2}}(x)} dx - \int x\sqrt{\sinh(x)} dx \\ &= -\frac{2x \cosh(x)}{\sqrt{\sinh(x)}} + 4\sqrt{\sinh(x)} \end{aligned}$$

**Mathematica [A]** time = 0.113241, size = 17, normalized size = 0.85

$$\frac{4 \sinh(x) - 2x \cosh(x)}{\sqrt{\sinh(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sinh[x]^(3/2) - x\*Sqrt[Sinh[x]],x]

[Out] (-2\*x\*Cosh[x] + 4\*Sinh[x])/Sqrt[Sinh[x]]

**Maple [F]** time = 0.075, size = 0, normalized size = 0.

$$\int x (\sinh(x))^{-\frac{3}{2}} - x \sqrt{\sinh(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/sinh(x)^(3/2)-x\*sinh(x)^(1/2),x)

[Out] int(x/sinh(x)^(3/2)-x\*sinh(x)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int -x \sqrt{\sinh(x)} + \frac{x}{\sinh(x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sinh(x)^(3/2)-x\*sinh(x)^(1/2),x, algorithm="maxima")

[Out] integrate(-x\*sqrt(sinh(x)) + x/sinh(x)^(3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int -\frac{x}{\sinh^{\frac{3}{2}}(x)} dx - \int x\sqrt{\sinh(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)**(3/2)-x*sinh(x)**(1/2),x)
```

```
[Out] -Integral(-x/sinh(x)**(3/2), x) - Integral(x*sqrt(sinh(x)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -x\sqrt{\sinh(x)} + \frac{x}{\sinh(x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(3/2)-x*sinh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x*sqrt(sinh(x)) + x/sinh(x)^(3/2), x)
```

$$3.69 \quad \int \left( \frac{x}{\sinh^{\frac{5}{2}}(x)} + \frac{x}{3\sqrt{\sinh(x)}} \right) dx$$

**Optimal.** Leaf size=24

$$-\frac{4}{3\sqrt{\sinh(x)}} - \frac{2x \cosh(x)}{3 \sinh^{\frac{3}{2}}(x)}$$

[Out]  $(-2*x*Cosh[x])/(3*Sinh[x]^{(3/2)}) - 4/(3*Sqrt[Sinh[x]])$

**Rubi [A]** time = 0.0602881, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$ , Rules used = {3315}

$$-\frac{4}{3\sqrt{\sinh(x)}} - \frac{2x \cosh(x)}{3 \sinh^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x/\text{Sinh}[x]^{(5/2)} + x/(3*\text{Sqrt}[\text{Sinh}[x]]), x]$

[Out]  $(-2*x*Cosh[x])/(3*Sinh[x]^{(3/2)}) - 4/(3*Sqrt[Sinh[x]])$

### Rule 3315

$\text{Int}[\left((c_.) + (d_.)*(x_.)\right)*\left((b_.)*\sin[(e_.) + (f_.)*(x_.)]\right)^{(n_.)}, x\_Symbol] \rightarrow$   
 $\text{Simp}[\left((c + d*x)*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(n + 1)}\right)/(b*f*(n + 1)), x] +$   
 $(\text{Dist}[(n + 2)/(b^2*(n + 1)), \text{Int}[(c + d*x)*(b*\text{Sin}[e + f*x])^{(n + 2)}, x], x$   
 $] - \text{Simp}[(d*(b*\text{Sin}[e + f*x])^{(n + 2)})/(b^2*f^2*(n + 1)*(n + 2)), x]) /;$  FreeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]

### Rubi steps

$$\begin{aligned} \int \left( \frac{x}{\sinh^{\frac{5}{2}}(x)} + \frac{x}{3\sqrt{\sinh(x)}} \right) dx &= \frac{1}{3} \int \frac{x}{\sqrt{\sinh(x)}} dx + \int \frac{x}{\sinh^{\frac{5}{2}}(x)} dx \\ &= -\frac{2x \cosh(x)}{3 \sinh^{\frac{3}{2}}(x)} - \frac{4}{3\sqrt{\sinh(x)}} \end{aligned}$$

**Mathematica [A]** time = 0.0705882, size = 22, normalized size = 0.92

$$\frac{1}{6}\sqrt{\sinh(x)}(-8\operatorname{csch}(x) - 4x \operatorname{coth}(x)\operatorname{csch}(x))$$

Antiderivative was successfully verified.

[In] Integrate[x/Sinh[x]^(5/2) + x/(3\*Sqrt[Sinh[x]]),x]

[Out] ((-8\*Csch[x] - 4\*x\*Coth[x]\*Csch[x])\*Sqrt[Sinh[x]])/6

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int x (\sinh(x))^{-\frac{5}{2}} + \frac{x}{3} \frac{1}{\sqrt{\sinh(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/sinh(x)^(5/2)+1/3\*x/sinh(x)^(1/2),x)

[Out] int(x/sinh(x)^(5/2)+1/3\*x/sinh(x)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{3\sqrt{\sinh(x)}} + \frac{x}{\sinh(x)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/sinh(x)^(5/2)+1/3\*x/sinh(x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/3\*x/sqrt(sinh(x)) + x/sinh(x)^(5/2), x)

**Fricas [B]** time = 2.621, size = 375, normalized size = 15.62

$$\frac{4\left((x+2)\cosh(x)^3 + 3(x+2)\cosh(x)\sinh(x)^2 + (x+2)\sinh(x)^3 + (x-2)\cosh(x) + (3(x+2)\cosh(x)^2 + x-2)\sinh(x)\right)}{3\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2(3\cosh(x)^2 - 1)\sinh(x)^2 - 2\cosh(x)^2 + 4(\cosh(x)^3 - \cosh(x))\right)}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] -4/3*((x + 2)*cosh(x)^3 + 3*(x + 2)*cosh(x)*sinh(x)^2 + (x + 2)*sinh(x)^3 +
(x - 2)*cosh(x) + (3*(x + 2)*cosh(x)^2 + x - 2)*sinh(x))*sqrt(sinh(x))/(co
sh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 -
2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{3x}{\sinh^2(x)} dx + \int \frac{x}{\sqrt{\sinh(x)}} dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)**(5/2)+1/3*x/sinh(x)**(1/2),x)
```

```
[Out] (Integral(3*x/sinh(x)**(5/2), x) + Integral(x/sqrt(sinh(x)), x))/3
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{3\sqrt{\sinh(x)}} + \frac{x}{\sinh(x)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(5/2)+1/3*x/sinh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/3*x/sqrt(sinh(x)) + x/sinh(x)^(5/2), x)
```

$$3.70 \quad \int \left( \frac{x}{\sinh^{\frac{7}{2}}(x)} + \frac{3}{5}x\sqrt{\sinh(x)} \right) dx$$

**Optimal.** Leaf size=47

$$-\frac{4}{15\sinh^{\frac{3}{2}}(x)} - \frac{12\sqrt{\sinh(x)}}{5} - \frac{2x \cosh(x)}{5\sinh^{\frac{5}{2}}(x)} + \frac{6x \cosh(x)}{5\sqrt{\sinh(x)}}$$

[Out]  $(-2*x*Cosh[x])/(5*Sinh[x]^(5/2)) - 4/(15*Sinh[x]^(3/2)) + (6*x*Cosh[x])/(5*\text{Sqrt}[Sinh[x]]) - (12*\text{Sqrt}[Sinh[x]])/5$

**Rubi [A]** time = 0.0807323, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$ , Rules used = {3315}

$$-\frac{4}{15\sinh^{\frac{3}{2}}(x)} - \frac{12\sqrt{\sinh(x)}}{5} - \frac{2x \cosh(x)}{5\sinh^{\frac{5}{2}}(x)} + \frac{6x \cosh(x)}{5\sqrt{\sinh(x)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x/\text{Sinh}[x]^{(7/2)} + (3*x*\text{Sqrt}[\text{Sinh}[x]])/5, x]$

[Out]  $(-2*x*Cosh[x])/(5*Sinh[x]^(5/2)) - 4/(15*Sinh[x]^(3/2)) + (6*x*Cosh[x])/(5*\text{Sqrt}[Sinh[x]]) - (12*\text{Sqrt}[Sinh[x]])/5$

### Rule 3315

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
  Simp[((c + d*x)*Cos[e + f*x]*(b*Sinh[e + f*x])^(n + 1))/(b*f*(n + 1)), x] +
  (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sinh[e + f*x])^(n + 2), x], x
] - Simp[(d*(b*Sinh[e + f*x])^(n + 2))/(b^2*f^2*(n + 1)*(n + 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

### Rubi steps

$$\begin{aligned}
\int \left( \frac{x}{\sinh^{\frac{7}{2}}(x)} + \frac{3}{5} x \sqrt{\sinh(x)} \right) dx &= \frac{3}{5} \int x \sqrt{\sinh(x)} dx + \int \frac{x}{\sinh^{\frac{7}{2}}(x)} dx \\
&= -\frac{2x \cosh(x)}{5 \sinh^{\frac{5}{2}}(x)} - \frac{4}{15 \sinh^{\frac{3}{2}}(x)} - \frac{3}{5} \int \frac{x}{\sinh^{\frac{3}{2}}(x)} dx + \frac{3}{5} \int x \sqrt{\sinh(x)} dx \\
&= -\frac{2x \cosh(x)}{5 \sinh^{\frac{5}{2}}(x)} - \frac{4}{15 \sinh^{\frac{3}{2}}(x)} + \frac{6x \cosh(x)}{5 \sqrt{\sinh(x)}} - \frac{12 \sqrt{\sinh(x)}}{5}
\end{aligned}$$

**Mathematica [A]** time = 0.116836, size = 33, normalized size = 0.7

$$\frac{46 \sinh(x) - 18 \sinh(3x) - 21x \cosh(x) + 9x \cosh(3x)}{30 \sinh^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sinh[x]^(7/2) + (3\*x\*Sqrt[Sinh[x]])/5,x]

[Out] (-21\*x\*Cosh[x] + 9\*x\*Cosh[3\*x] + 46\*Sinh[x] - 18\*Sinh[3\*x])/(30\*Sinh[x]^(5/2))

**Maple [F]** time = 0.092, size = 0, normalized size = 0.

$$\int x (\sinh(x))^{-\frac{7}{2}} + \frac{3x}{5} \sqrt{\sinh(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/sinh(x)^(7/2)+3/5\*x\*sinh(x)^(1/2),x)

[Out] int(x/sinh(x)^(7/2)+3/5\*x\*sinh(x)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{3}{5} x \sqrt{\sinh(x)} + \frac{x}{\sinh(x)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(3/5*x*sqrt(sinh(x)) + x/sinh(x)^(7/2), x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)**(7/2)+3/5*x*sinh(x)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{3}{5} x \sqrt{\sinh(x)} + \frac{x}{\sinh(x)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sinh(x)^(7/2)+3/5*x*sinh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(3/5*x*sqrt(sinh(x)) + x/sinh(x)^(7/2), x)
```

$$3.71 \quad \int \left( \frac{x^2}{\sinh^{\frac{3}{2}}(x)} - x^2 \sqrt{\sinh(x)} \right) dx$$

**Optimal.** Leaf size=58

$$-\frac{2x^2 \cosh(x)}{\sqrt{\sinh(x)}} + 8x\sqrt{\sinh(x)} - \frac{16i\sqrt{\sinh(x)}E\left(\frac{\pi}{4} - \frac{ix}{2} \middle| 2\right)}{\sqrt{i\sinh(x)}}$$

[Out]  $(-2*x^2*Cosh[x])/Sqrt[Sinh[x]] + 8*x*Sqrt[Sinh[x]] - ((16*I)*EllipticE[Pi/4 - (I/2)*x, 2]*Sqrt[Sinh[x]])/Sqrt[I*Sinh[x]]$

**Rubi [A]** time = 0.118467, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {3316, 2640, 2639}

$$-\frac{2x^2 \cosh(x)}{\sqrt{\sinh(x)}} + 8x\sqrt{\sinh(x)} - \frac{16i\sqrt{\sinh(x)}E\left(\frac{\pi}{4} - \frac{ix}{2} \middle| 2\right)}{\sqrt{i\sinh(x)}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/\text{Sinh}[x]^{(3/2)} - x^2*\text{Sqrt}[\text{Sinh}[x]], x]$

[Out]  $(-2*x^2*Cosh[x])/Sqrt[Sinh[x]] + 8*x*Sqrt[Sinh[x]] - ((16*I)*EllipticE[Pi/4 - (I/2)*x, 2]*Sqrt[Sinh[x]])/Sqrt[I*Sinh[x]]$

### Rule 3316

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*((b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(n+1)}]/(b*f*(n+1)), x] + (\text{Dist}[(n+2)/(b^2*(n+1)), \text{Int}[(c + d*x)^m*(b*\text{Sin}[e + f*x])^{(n+2)}, x], x] + \text{Dist}[(d^2*m*(m-1))/(b^2*f^2*(n+1)*(n+2)), \text{Int}[(c + d*x)^{(m-2)}*(b*\text{Sin}[e + f*x])^{(n+2)}, x], x] - \text{Simp}[(d*m*(c + d*x)^{(m-1)}*(b*\text{Sin}[e + f*x])^{(n+2)}]/(b^2*f^2*(n+1)*(n+2)), x]) /; \text{FreeQ}\{b, c, d, e, f\}, x] \&\& \text{LtQ}[n, -1] \&\& \text{NeQ}[n, -2] \&\& \text{GtQ}[m, 1]$

### Rule 2640

$\text{Int}[\text{Sqrt}[(b_.)*\sin[(c_.) + (d_.)*(x_.)]], x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[b*\text{Sin}[c + d*x]]/\text{Sqrt}[\text{Sin}[c + d*x]], \text{Int}[\text{Sqrt}[\text{Sin}[c + d*x]], x], x] /; \text{FreeQ}\{b, c, d\},$

x]

**Rule 2639**

Int[Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]]], x\_Symbol] := Simp[(2\*EllipticE[(1\*(c - P  
i/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

**Rubi steps**

$$\begin{aligned}
 \int \left( \frac{x^2}{\sinh^{\frac{3}{2}}(x)} - x^2 \sqrt{\sinh(x)} \right) dx &= \int \frac{x^2}{\sinh^{\frac{3}{2}}(x)} dx - \int x^2 \sqrt{\sinh(x)} dx \\
 &= -\frac{2x^2 \cosh(x)}{\sqrt{\sinh(x)}} + 8x \sqrt{\sinh(x)} - 8 \int \sqrt{\sinh(x)} dx \\
 &= -\frac{2x^2 \cosh(x)}{\sqrt{\sinh(x)}} + 8x \sqrt{\sinh(x)} - \frac{(8\sqrt{\sinh(x)}) \int \sqrt{i \sinh(x)} dx}{\sqrt{i \sinh(x)}} \\
 &= -\frac{2x^2 \cosh(x)}{\sqrt{\sinh(x)}} + 8x \sqrt{\sinh(x)} - \frac{16iE\left(\frac{\pi}{4} - \frac{ix}{2} \middle| 2\right) \sqrt{\sinh(x)}}{\sqrt{i \sinh(x)}}
 \end{aligned}$$

**Mathematica [C]** time = 1.2416, size = 68, normalized size = 1.17

$$\frac{2 \left( -8\sqrt{2}(\sinh(x) - \cosh(x))\sqrt{-\sinh(x)(\sinh(x) + \cosh(x))} {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; \cosh(2x) + \sinh(2x)\right) + x^2 \cosh(x) - 4(x - 2) \right)}{\sqrt{\sinh(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sinh[x]^(3/2) - x^2\*Sqrt[Sinh[x]],x]

[Out] (-2\*(x^2\*Cosh[x] - 4\*(-2 + x)\*Sinh[x] - 8\*Sqrt[2]\*Hypergeometric2F1[-1/4, 1/2, 3/4, Cosh[2\*x] + Sinh[2\*x]]\*(-Cosh[x] + Sinh[x])\*Sqrt[-(Sinh[x]\*(Cosh[x] + Sinh[x]))])/Sqrt[Sinh[x]]

**Maple [F]** time = 0.073, size = 0, normalized size = 0.

$$\int x^2 (\sinh(x))^{-\frac{3}{2}} - x^2 \sqrt{\sinh(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x)`

[Out] `int(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int -x^2\sqrt{\sinh(x)} + \frac{x^2}{\sinh(x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(-x^2*sqrt(sinh(x)) + x^2/sinh(x)^(3/2), x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int -\frac{x^2}{\sinh^{\frac{3}{2}}(x)} dx - \int x^2\sqrt{\sinh(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/sinh(x)**(3/2)-x**2*sinh(x)**(1/2),x)`

[Out] `-Integral(-x**2/sinh(x)**(3/2), x) - Integral(x**2*sqrt(sinh(x)), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -x^2\sqrt{\sinh(x)} + \frac{x^2}{\sinh(x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sinh(x)^(3/2)-x^2*sinh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-x^2*sqrt(sinh(x)) + x^2/sinh(x)^(3/2), x)
```



$$3.72 \quad \int (c + dx)^m (b \sinh(e + fx))^n dx$$

**Optimal.** Leaf size=20

$$\text{Unintegrable}((c + dx)^m (b \sinh(e + fx))^n, x)$$

[Out] Unintegrable[(c + d\*x)^m\*(b\*Sinh[e + f\*x])^n, x]

**Rubi [A]** time = 0.0464853, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (c + dx)^m (b \sinh(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m\*(b\*Sinh[e + f\*x])^n,x]

[Out] Defer[Int] [(c + d\*x)^m\*(b\*Sinh[e + f\*x])^n, x]

Rubi steps

$$\int (c + dx)^m (b \sinh(e + fx))^n dx = \int (c + dx)^m (b \sinh(e + fx))^n dx$$

**Mathematica [A]** time = 2.99417, size = 0, normalized size = 0.

$$\int (c + dx)^m (b \sinh(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m\*(b\*Sinh[e + f\*x])^n,x]

[Out] Integrate[(c + d\*x)^m\*(b\*Sinh[e + f\*x])^n, x]

**Maple [A]** time = 0.063, size = 0, normalized size = 0.

$$\int (dx + c)^m (b \sinh (fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*(b\*sinh(f\*x+e))^n,x)

[Out] int((d\*x+c)^m\*(b\*sinh(f\*x+e))^n,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m (b \sinh (fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(b\*sinh(f\*x+e))^n,x, algorithm="maxima")

[Out] integrate((d\*x + c)^m\*(b\*sinh(f\*x + e))^n, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx + c)^m (b \sinh (fx + e))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(b\*sinh(f\*x+e))^n,x, algorithm="fricas")

[Out] integral((d\*x + c)^m\*(b\*sinh(f\*x + e))^n, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int (b \sinh (e + fx))^n (c + dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(b*sinh(f*x+e))**n,x)
```

```
[Out] Integral((b*sinh(e + f*x))**n*(c + d*x)**m, x)
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m (b \sinh(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*sinh(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*(b*sinh(f*x + e))^n, x)
```

### 3.73 $\int (c + dx)^m \sinh^3(a + bx) dx$

**Optimal.** Leaf size=237

$$\frac{3^{-m-1} e^{3a - \frac{3bc}{d}} (c + dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{3b(c+dx)}{d}\right)}{8b} - \frac{3e^{a - \frac{bc}{d}} (c + dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{b(c+dx)}{d}\right)}{8b}$$

```
[Out] (3^(-1 - m)*E^(3*a - (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (-3*b*(c + d*x))/d
])/((8*b*(-((b*(c + d*x))/d))^m) - (3*E^(a - (b*c)/d)*(c + d*x)^m*Gamma[1 +
m, -((b*(c + d*x))/d)])/(8*b*(-((b*(c + d*x))/d))^m) - (3*E^(-a + (b*c)/d)*
(c + d*x)^m*Gamma[1 + m, (b*(c + d*x))/d])/(8*b*((b*(c + d*x))/d)^m) + (3^(-
-1 - m)*E^(-3*a + (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (3*b*(c + d*x))/d])/(
8*b*((b*(c + d*x))/d)^m)
```

**Rubi [A]** time = 0.317832, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {3312, 3308, 2181}

$$\frac{3^{-m-1} e^{3a - \frac{3bc}{d}} (c + dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{3b(c+dx)}{d}\right)}{8b} - \frac{3e^{a - \frac{bc}{d}} (c + dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{b(c+dx)}{d}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Sinh[a + b*x]^3,x]
```

```
[Out] (3^(-1 - m)*E^(3*a - (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (-3*b*(c + d*x))/d
])/((8*b*(-((b*(c + d*x))/d))^m) - (3*E^(a - (b*c)/d)*(c + d*x)^m*Gamma[1 +
m, -((b*(c + d*x))/d)])/(8*b*(-((b*(c + d*x))/d))^m) - (3*E^(-a + (b*c)/d)*
(c + d*x)^m*Gamma[1 + m, (b*(c + d*x))/d])/(8*b*((b*(c + d*x))/d)^m) + (3^(-
-1 - m)*E^(-3*a + (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (3*b*(c + d*x))/d])/(
8*b*((b*(c + d*x))/d)^m)
```

#### Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

#### Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

### Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

### Rubi steps

$$\begin{aligned} \int (c + dx)^m \sinh^3(a + bx) dx &= i \int \left( \frac{3}{4} i (c + dx)^m \sinh(a + bx) - \frac{1}{4} i (c + dx)^m \sinh(3a + 3bx) \right) dx \\ &= \frac{1}{4} \int (c + dx)^m \sinh(3a + 3bx) dx - \frac{3}{4} \int (c + dx)^m \sinh(a + bx) dx \\ &= \frac{1}{8} \int e^{-i(3ia+3ibx)} (c + dx)^m dx - \frac{1}{8} \int e^{i(3ia+3ibx)} (c + dx)^m dx - \frac{3}{8} \int e^{-i(ia+ibx)} (c + dx)^m dx \\ &= \frac{3^{-1-m} e^{3a - \frac{3bc}{d}} (c + dx)^m \left( -\frac{b(c+dx)}{d} \right)^{-m} \Gamma\left(1 + m, -\frac{3b(c+dx)}{d}\right) - 3e^{a - \frac{bc}{d}} (c + dx)^m \left( -\frac{b(c+dx)}{d} \right)^{-m}}{8b} \end{aligned}$$

**Mathematica [A]** time = 0.198479, size = 206, normalized size = 0.87

$$\frac{3^{-m-1} e^{-3\left(a + \frac{bc}{d}\right)} (c + dx)^m \left( -\frac{b^2(c+dx)^2}{d^2} \right)^{-m} \left( e^{6a} \left( b \left( \frac{c}{d} + x \right) \right)^m \text{Gamma}\left(m + 1, -\frac{3b(c+dx)}{d}\right) - 3^{m+2} e^{4a + \frac{2bc}{d}} \left( b \left( \frac{c}{d} + x \right) \right)^m \text{Gamma}\left(m + 1, -\frac{3b(c+dx)}{d}\right) \right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Sinh[a + b*x]^3,x]
```

```
[Out] (3^(-1 - m)*(c + d*x)^m*(E^(6*a)*(b*(c/d + x))^m*Gamma[1 + m, (-3*b*(c + d*x))/d] - 3^(2 + m)*E^(4*a + (2*b*c)/d)*(b*(c/d + x))^m*Gamma[1 + m, -(b*(c + d*x))/d]) + E^((4*b*c)/d)*(-(b*(c + d*x))/d))^m*(-(3^(2 + m)*E^(2*a)*Gamma[1 + m, (b*(c + d*x))/d]) + E^((2*b*c)/d)*Gamma[1 + m, (3*b*(c + d*x))/d]))/(8*b*E^(3*(a + (b*c)/d))*(-(b^2*(c + d*x)^2/d^2))^m)
```

**Maple [F]** time = 0.089, size = 0, normalized size = 0.

$$\int (dx + c)^m (\sinh(bx + a))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*sinh(b\*x+a)^3,x)

[Out] int((d\*x+c)^m\*sinh(b\*x+a)^3,x)

**Maxima [A]** time = 1.54627, size = 217, normalized size = 0.92

$$\frac{(dx + c)^{m+1} e^{\left(-3a + \frac{3bc}{d}\right)} E_{-m}\left(\frac{3(dx+c)b}{d}\right)}{8d} - \frac{3(dx + c)^{m+1} e^{\left(-a + \frac{bc}{d}\right)} E_{-m}\left(\frac{(dx+c)b}{d}\right)}{8d} + \frac{3(dx + c)^{m+1} e^{\left(a - \frac{bc}{d}\right)} E_{-m}\left(-\frac{(dx+c)b}{d}\right)}{8d} - \frac{(dx + c)^{m+1} e^{3a} E_{-m}\left(\frac{3(dx+c)b}{d}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*sinh(b\*x+a)^3,x, algorithm="maxima")

[Out] 1/8\*(d\*x + c)^(m + 1)\*e^(-3\*a + 3\*b\*c/d)\*exp\_integral\_e(-m, 3\*(d\*x + c)\*b/d)/d - 3/8\*(d\*x + c)^(m + 1)\*e^(-a + b\*c/d)\*exp\_integral\_e(-m, (d\*x + c)\*b/d)/d + 3/8\*(d\*x + c)^(m + 1)\*e^(a - b\*c/d)\*exp\_integral\_e(-m, -(d\*x + c)\*b/d)/d - 1/8\*(d\*x + c)^(m + 1)\*e^(3\*a - 3\*b\*c/d)\*exp\_integral\_e(-m, -3\*(d\*x + c)\*b/d)/d

**Fricas [A]** time = 2.92055, size = 799, normalized size = 3.37

$$\cosh\left(\frac{dm \log\left(\frac{3b}{d}\right) - 3bc + 3ad}{d}\right) \Gamma\left(m + 1, \frac{3(bdx+bc)}{d}\right) - 9 \cosh\left(\frac{dm \log\left(\frac{b}{d}\right) - bc + ad}{d}\right) \Gamma\left(m + 1, \frac{bdx+bc}{d}\right) - 9 \cosh\left(\frac{dm \log\left(-\frac{b}{d}\right) + bc - ad}{d}\right) \Gamma\left(m + 1, -\frac{bdx+bc}{d}\right) - 9 \cosh\left(\frac{dm \log\left(\frac{b}{d}\right) - bc + ad}{d}\right) \Gamma\left(m + 1, \frac{bdx+bc}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*sinh(b\*x+a)^3,x, algorithm="fricas")

[Out] 1/24\*(cosh((d\*m\*log(3\*b/d) - 3\*b\*c + 3\*a\*d)/d)\*gamma(m + 1, 3\*(b\*d\*x + b\*c)/d) - 9\*cosh((d\*m\*log(b/d) - b\*c + a\*d)/d)\*gamma(m + 1, (b\*d\*x + b\*c)/d) - 9\*cosh((d\*m\*log(-b/d) + b\*c - a\*d)/d)\*gamma(m + 1, -(b\*d\*x + b\*c)/d) - 9\*cosh((d\*m\*log(b/d) - b\*c + a\*d)/d)\*gamma(m + 1, (b\*d\*x + b\*c)/d))

```

9*cosh((d*m*log(-b/d) + b*c - a*d)/d)*gamma(m + 1, -(b*d*x + b*c)/d) + cosh
((d*m*log(-3*b/d) + 3*b*c - 3*a*d)/d)*gamma(m + 1, -3*(b*d*x + b*c)/d) - ga
mma(m + 1, 3*(b*d*x + b*c)/d)*sinh((d*m*log(3*b/d) - 3*b*c + 3*a*d)/d) + 9*
gamma(m + 1, (b*d*x + b*c)/d)*sinh((d*m*log(b/d) - b*c + a*d)/d) + 9*gamma(
m + 1, -(b*d*x + b*c)/d)*sinh((d*m*log(-b/d) + b*c - a*d)/d) - gamma(m + 1,
-3*(b*d*x + b*c)/d)*sinh((d*m*log(-3*b/d) + 3*b*c - 3*a*d)/d))/b

```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^m \sinh^3(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sinh(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**m*sinh(a + b*x)**3, x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \sinh(bx + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sinh(b*x + a)^3, x)
```

### 3.74 $\int (c + dx)^m \sinh^2(a + bx) dx$

**Optimal.** Leaf size=144

$$\frac{2^{-m-3} e^{2a - \frac{2bc}{d}} (c + dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{2b(c+dx)}{d}\right)}{b} - \frac{2^{-m-3} e^{\frac{2bc}{d} - 2a} (c + dx)^m \left(\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, \frac{2b(c+dx)}{d}\right)}{b}$$

[Out]  $-(c + d*x)^{(1 + m)}/(2*d*(1 + m)) + (2^{(-3 - m)}*E^{(2*a - (2*b*c)/d)}*(c + d*x)^m*\Gamma[1 + m, (-2*b*(c + d*x))/d])/(b*(-((b*(c + d*x))/d))^m) - (2^{(-3 - m)}*E^{(-2*a + (2*b*c)/d)}*(c + d*x)^m*\Gamma[1 + m, (2*b*(c + d*x))/d])/(b*((b*(c + d*x))/d)^m)$

**Rubi [A]** time = 0.197365, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {3312, 3307, 2181}

$$\frac{2^{-m-3} e^{2a - \frac{2bc}{d}} (c + dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{2b(c+dx)}{d}\right)}{b} - \frac{2^{-m-3} e^{\frac{2bc}{d} - 2a} (c + dx)^m \left(\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, \frac{2b(c+dx)}{d}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^m\*Sinh[a + b\*x]^2,x]

[Out]  $-(c + d*x)^{(1 + m)}/(2*d*(1 + m)) + (2^{(-3 - m)}*E^{(2*a - (2*b*c)/d)}*(c + d*x)^m*\Gamma[1 + m, (-2*b*(c + d*x))/d])/(b*(-((b*(c + d*x))/d))^m) - (2^{(-3 - m)}*E^{(-2*a + (2*b*c)/d)}*(c + d*x)^m*\Gamma[1 + m, (2*b*(c + d*x))/d])/(b*((b*(c + d*x))/d)^m)$

#### Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

#### Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]
```



Rule 2181

```
Int[(F_)^((g_)*(e_) + (f_)*(x_))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])/d))*(c + d*x))/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \int (c + dx)^m \sinh^2(a + bx) dx &= - \int \left( \frac{1}{2}(c + dx)^m - \frac{1}{2}(c + dx)^m \cosh(2a + 2bx) \right) dx \\ &= -\frac{(c + dx)^{1+m}}{2d(1+m)} + \frac{1}{2} \int (c + dx)^m \cosh(2a + 2bx) dx \\ &= -\frac{(c + dx)^{1+m}}{2d(1+m)} + \frac{1}{4} \int e^{-i(2ia+2ibx)}(c + dx)^m dx + \frac{1}{4} \int e^{i(2ia+2ibx)}(c + dx)^m dx \\ &= -\frac{(c + dx)^{1+m}}{2d(1+m)} + \frac{2^{-3-m} e^{2a - \frac{2bc}{d}} (c + dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, -\frac{2b(c+dx)}{d}\right)}{b} - \frac{2^{-3-m} e^{-2a + \frac{2bc}{d}} (c + dx)^m \left(\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, \frac{2b(c+dx)}{d}\right)}{b} \end{aligned}$$

**Mathematica [A]** time = 0.167818, size = 131, normalized size = 0.91

$$\frac{1}{8}(c + dx)^m \left( \frac{2^{-m} e^{2a - \frac{2bc}{d}} \left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{2b(c+dx)}{d}\right)}{b} - \frac{2^{-m} e^{\frac{2bc}{d} - 2a} \left(\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, \frac{2b(c+dx)}{d}\right)}{b} - \frac{1}{c} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^m\*Sinh[a + b\*x]^2,x]

[Out] ((c + d\*x)^m\*((-4\*(c + d\*x))/(d\*(1 + m)) + (E^(2\*a - (2\*b\*c)/d)\*Gamma[1 + m, (-2\*b\*(c + d\*x))/d])/(2^m\*b\*(-((b\*(c + d\*x))/d))^m) - (E^(-2\*a + (2\*b\*c)/d)\*Gamma[1 + m, (2\*b\*(c + d\*x))/d])/(2^m\*b\*((b\*(c + d\*x))/d)^m))/8

**Maple [F]** time = 0.074, size = 0, normalized size = 0.

$$\int (dx + c)^m (\sinh(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*sinh(b\*x+a)^2,x)

[Out] int((d\*x+c)^m\*sinh(b\*x+a)^2,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*sinh(b\*x+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.69236, size = 597, normalized size = 4.15

$$(dm + d) \cosh\left(\frac{dm \log\left(\frac{2b}{d}\right) - 2bc + 2ad}{d}\right) \Gamma\left(m + 1, \frac{2(bdx+bc)}{d}\right) - (dm + d) \cosh\left(\frac{dm \log\left(-\frac{2b}{d}\right) + 2bc - 2ad}{d}\right) \Gamma\left(m + 1, -\frac{2(bdx+bc)}{d}\right) - (dm + d) \cosh\left(\frac{dm \log\left(\frac{2b}{d}\right) - 2bc + 2ad}{d}\right) \Gamma\left(m + 1, \frac{2(bdx+bc)}{d}\right) - (dm + d) \cosh\left(\frac{dm \log\left(-\frac{2b}{d}\right) + 2bc - 2ad}{d}\right) \Gamma\left(m + 1, -\frac{2(bdx+bc)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*sinh(b\*x+a)^2,x, algorithm="fricas")

[Out] 
$$-1/8*((d*m + d)*\cosh((d*m*\log(2*b/d) - 2*b*c + 2*a*d)/d)*\gamma(m + 1, 2*(b*d*x + b*c)/d) - (d*m + d)*\cosh((d*m*\log(-2*b/d) + 2*b*c - 2*a*d)/d)*\gamma(m + 1, -2*(b*d*x + b*c)/d) - (d*m + d)*\gamma(m + 1, 2*(b*d*x + b*c)/d)*\sinh((d*m*\log(2*b/d) - 2*b*c + 2*a*d)/d) + (d*m + d)*\gamma(m + 1, -2*(b*d*x + b*c)/d)*\sinh((d*m*\log(-2*b/d) + 2*b*c - 2*a*d)/d) + 4*(b*d*x + b*c)*\cosh(m*\log(d*x + c)) + 4*(b*d*x + b*c)*\sinh(m*\log(d*x + c)))/(b*d*m + b*d)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^m \sinh^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m\*sinh(b\*x+a)\*\*2,x)

[Out] Integral((c + d\*x)\*\*m\*sinh(a + b\*x)\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \sinh(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*sinh(b\*x+a)^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^m\*sinh(b\*x + a)^2, x)

### 3.75 $\int (c + dx)^m \sinh(a + bx) dx$

**Optimal.** Leaf size=110

$$\frac{e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{b(c+dx)}{d}\right)}{2b} + \frac{e^{\frac{bc}{d}-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{b(c+dx)}{d}\right)}{2b}$$

[Out]  $(E^{(a - (b*c)/d)*(c + d*x)^m*\Gamma[1 + m, -((b*(c + d*x))/d)])/(2*b*(-((b*(c + d*x))/d))^m) + (E^{(-a + (b*c)/d)*(c + d*x)^m*\Gamma[1 + m, (b*(c + d*x))/d]})/(2*b*((b*(c + d*x))/d)^m)$

**Rubi [A]** time = 0.0930162, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3308, 2181}

$$\frac{e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{b(c+dx)}{d}\right)}{2b} + \frac{e^{\frac{bc}{d}-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{b(c+dx)}{d}\right)}{2b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^m*\text{Sinh}[a + b*x], x]$

[Out]  $(E^{(a - (b*c)/d)*(c + d*x)^m*\Gamma[1 + m, -((b*(c + d*x))/d)])/(2*b*(-((b*(c + d*x))/d))^m) + (E^{(-a + (b*c)/d)*(c + d*x)^m*\Gamma[1 + m, (b*(c + d*x))/d]})/(2*b*((b*(c + d*x))/d)^m)$

#### Rule 3308

$\text{Int}[(c + d*x)^m*\text{Sinh}[a + b*x], x] := \text{Dist}[I/2, \text{Int}[(c + d*x)^m/E^{(I*(e + f*x))}, x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m/E^{(I*(e + f*x))}, x], x] /;$   $\text{FreeQ}\{c, d, e, f, m\}, x]$

#### Rule 2181

$\text{Int}[(F + G*(e + f*x))^m*(c + d*x)^m, x] := -\text{Simp}[(F + G*(e + f*x))^m*(c + d*x)^m*\Gamma[m + 1, -((f*g*\text{Log}[F])/d)]/(d*(c + d*x))]/(d*(-((f*g*\text{Log}[F])/d))^{(\text{IntPart}[m] + 1)*(-((f*g*\text{Log}[F])/d))}*\Gamma[m]), x] /;$   $\text{FreeQ}\{F, c, d, e, f, g, m\}, x] \&\& \text{IntegerQ}[m]$

Rubi steps

$$\int (c + dx)^m \sinh(a + bx) dx = \frac{1}{2} \int e^{-i(ia+ibx)} (c + dx)^m dx - \frac{1}{2} \int e^{i(ia+ibx)} (c + dx)^m dx$$

$$= \frac{e^{a-\frac{bc}{d}} (c + dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, -\frac{b(c+dx)}{d}\right)}{2b} + \frac{e^{-a+\frac{bc}{d}} (c + dx)^m \left(\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, \frac{b(c+dx)}{d}\right)}{2b}$$

**Mathematica [A]** time = 0.0546071, size = 101, normalized size = 0.92

$$\frac{e^{-a-\frac{bc}{d}} (c + dx)^m \left(e^{2a} \left(-\frac{b(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m + 1, -\frac{b(c+dx)}{d}\right) + e^{\frac{2bc}{d}} \left(b\left(\frac{c}{d} + x\right)\right)^{-m} \text{Gamma}\left(m + 1, \frac{b(c+dx)}{d}\right)\right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^m\*Sinh[a + b\*x], x]

[Out] (E^(-a - (b\*c)/d)\*(c + d\*x)^m\*((E^(2\*a)\*Gamma[1 + m, -((b\*(c + d\*x))/d)])/(-((b\*(c + d\*x))/d))^m + (E^((2\*b\*c)/d)\*Gamma[1 + m, (b\*(c + d\*x))/d])/(b\*(c/d + x))^m))/(2\*b)

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int (dx + c)^m \sinh(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*sinh(b\*x+a), x)

[Out] int((d\*x+c)^m\*sinh(b\*x+a), x)

**Maxima [A]** time = 1.28157, size = 107, normalized size = 0.97

$$\frac{(dx + c)^{m+1} e^{\left(-a+\frac{bc}{d}\right)} E_{-m}\left(\frac{(dx+c)b}{d}\right)}{2d} - \frac{(dx + c)^{m+1} e^{\left(a-\frac{bc}{d}\right)} E_{-m}\left(-\frac{(dx+c)b}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*sinh(b\*x+a),x, algorithm="maxima")

[Out]  $\frac{1}{2}*(d*x + c)^{(m + 1)}*e^{(-a + b*c/d)*\exp\_integral\_e(-m, (d*x + c)*b/d)/d} - \frac{1}{2}*(d*x + c)^{(m + 1)}*e^{(a - b*c/d)*\exp\_integral\_e(-m, -(d*x + c)*b/d)/d}$

**Fricas [A]** time = 2.77917, size = 377, normalized size = 3.43

$$\frac{\cosh\left(\frac{dm \log\left(\frac{b}{d}\right) - bc + ad}{d}\right) \Gamma\left(m + 1, \frac{bdx + bc}{d}\right) + \cosh\left(\frac{dm \log\left(-\frac{b}{d}\right) + bc - ad}{d}\right) \Gamma\left(m + 1, -\frac{bdx + bc}{d}\right) - \Gamma\left(m + 1, \frac{bdx + bc}{d}\right) \sinh\left(\frac{dm \log\left(\frac{b}{d}\right) - bc}{d}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*sinh(b\*x+a),x, algorithm="fricas")

[Out]  $\frac{1}{2}*(\cosh((d*m*\log(b/d) - b*c + a*d)/d)*\gamma(m + 1, (b*d*x + b*c)/d) + \cosh((d*m*\log(-b/d) + b*c - a*d)/d)*\gamma(m + 1, -(b*d*x + b*c)/d) - \gamma(m + 1, (b*d*x + b*c)/d)*\sinh((d*m*\log(b/d) - b*c + a*d)/d) - \gamma(m + 1, -(b*d*x + b*c)/d)*\sinh((d*m*\log(-b/d) + b*c - a*d)/d))/b$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m\*sinh(b\*x+a),x)

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \sinh(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sinh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*sinh(b*x + a), x)
```

### 3.76 $\int (c + dx)^m \mathbf{csch}(a + bx) dx$

**Optimal.** Leaf size=16

Unintegrable(csch(a + bx)(c + dx)^m, x)

[Out] Unintegrable[(c + d\*x)^m\*Csch[a + b\*x], x]

**Rubi [A]** time = 0.020653, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (c + dx)^m \mathbf{csch}(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m\*Csch[a + b\*x], x]

[Out] Defer[Int] [(c + d\*x)^m\*Csch[a + b\*x], x]

Rubi steps

$$\int (c + dx)^m \mathbf{csch}(a + bx) dx = \int (c + dx)^m \mathbf{csch}(a + bx) dx$$

**Mathematica [A]** time = 6.42703, size = 0, normalized size = 0.

$$\int (c + dx)^m \mathbf{csch}(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m\*Csch[a + b\*x], x]

[Out] Integrate[(c + d\*x)^m\*Csch[a + b\*x], x]



**Maple [A]** time = 0.029, size = 0, normalized size = 0.

$$\int (dx + c)^m \operatorname{csch}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*csch(b\*x+a),x)

[Out] int((d\*x+c)^m\*csch(b\*x+a),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \operatorname{csch}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*csch(b\*x+a),x, algorithm="maxima")

[Out] integrate((d\*x + c)^m\*csch(b\*x + a), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}((dx + c)^m \operatorname{csch}(bx + a), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*csch(b\*x+a),x, algorithm="fricas")

[Out] integral((d\*x + c)^m\*csch(b\*x + a), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^m \operatorname{csch}(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csch(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*csch(a + b*x), x)
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \operatorname{csch}(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csch(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csch(b*x + a), x)
```

### 3.77 $\int (c + dx)^m \mathbf{csch}^2(a + bx) dx$

Optimal. Leaf size=18

Unintegrable( $\text{csch}^2(a + bx)(c + dx)^m, x$ )

[Out] Unintegrable[(c + d\*x)^m\*Csch[a + b\*x]^2, x]

**Rubi [A]** time = 0.0369156, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (c + dx)^m \text{csch}^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m\*Csch[a + b\*x]^2, x]

[Out] Defer[Int] [(c + d\*x)^m\*Csch[a + b\*x]^2, x]

Rubi steps

$$\int (c + dx)^m \text{csch}^2(a + bx) dx = \int (c + dx)^m \text{csch}^2(a + bx) dx$$

**Mathematica [A]** time = 3.77869, size = 0, normalized size = 0.

$$\int (c + dx)^m \text{csch}^2(a + bx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m\*Csch[a + b\*x]^2, x]

[Out] Integrate[(c + d\*x)^m\*Csch[a + b\*x]^2, x]

**Maple [A]** time = 0.033, size = 0, normalized size = 0.

$$\int (dx + c)^m (\operatorname{csch}(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*csch(b\*x+a)^2,x)

[Out] int((d\*x+c)^m\*csch(b\*x+a)^2,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \operatorname{csch}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*csch(b\*x+a)^2,x, algorithm="maxima")

[Out] integrate((d\*x + c)^m\*csch(b\*x + a)^2, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}((dx + c)^m \operatorname{csch}(bx + a)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*csch(b\*x+a)^2,x, algorithm="fricas")

[Out] integral((d\*x + c)^m\*csch(b\*x + a)^2, x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int (c + dx)^m \operatorname{csch}^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csch(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*csch(a + b*x)**2, x)
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m \operatorname{csch}(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csch(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*csch(b*x + a)^2, x)
```

### 3.78 $\int x^{3+m} \sinh(a + bx) dx$

**Optimal.** Leaf size=59

$$\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+4,bx)}{2b^4} - \frac{e^ax^m(-bx)^{-m}\Gamma(m+4,-bx)}{2b^4}$$

[Out]  $-(E^{-a}x^m\Gamma[4+m, -(b*x)])/(2*b^4*(-(b*x))^m) + (x^m\Gamma[4+m, b*x])/(2*b^4*E^a*(b*x)^m)$

**Rubi [A]** time = 0.0777343, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3308, 2181}

$$\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+4,bx)}{2b^4} - \frac{e^ax^m(-bx)^{-m}\Gamma(m+4,-bx)}{2b^4}$$

Antiderivative was successfully verified.

[In] Int[x^(3 + m)\*Sinh[a + b\*x],x]

[Out]  $-(E^{-a}x^m\Gamma[4+m, -(b*x)])/(2*b^4*(-(b*x))^m) + (x^m\Gamma[4+m, b*x])/(2*b^4*E^a*(b*x)^m)$

#### Rule 3308

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Dist[I/2, Int[(c + d\*x)^m/E^(I\*(e + f\*x)), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-(f\*g\*Log[F])/d)\*(c + d\*x])]/(d\*(-(f\*g\*Log[F])/d)^(IntPart[m] + 1)\*(-(f\*g\*Log[F])\*(c + d\*x)/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\int x^{3+m} \sinh(a + bx) dx = \frac{1}{2} \int e^{-i(i+ibx)} x^{3+m} dx - \frac{1}{2} \int e^{i(i+ibx)} x^{3+m} dx$$

$$= \frac{e^a x^m (-bx)^{-m} \Gamma(4 + m, -bx)}{2b^4} + \frac{e^{-a} x^m (bx)^{-m} \Gamma(4 + m, bx)}{2b^4}$$

**Mathematica [A]** time = 0.0258614, size = 54, normalized size = 0.92

$$\frac{e^{-a} x^m \left( (bx)^{-m} \text{Gamma}(m + 4, bx) - e^{2a} (-bx)^{-m} \text{Gamma}(m + 4, -bx) \right)}{2b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3 + m)\*Sinh[a + b\*x], x]

[Out] (x^m\*(-((E^(2\*a)\*Gamma[4 + m, -(b\*x)])/(-(b\*x))^m) + Gamma[4 + m, b\*x]/(b\*x)^m))/(2\*b^4\*E^a)

**Maple [C]** time = 0.042, size = 73, normalized size = 1.2

$$\frac{x^{4+m} \sinh(a)}{4+m} {}_1F_2\left(2 + \frac{m}{2}; \frac{1}{2}, 3 + \frac{m}{2}; \frac{x^2 b^2}{4}\right) + \frac{bx^{5+m} \cosh(a)}{5+m} {}_1F_2\left(\frac{5}{2} + \frac{m}{2}; \frac{3}{2}, \frac{7}{2} + \frac{m}{2}; \frac{x^2 b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3+m)\*sinh(b\*x+a), x)

[Out] 1/(4+m)\*x^(4+m)\*hypergeom([2+1/2\*m], [1/2, 3+1/2\*m], 1/4\*x^2\*b^2)\*sinh(a)+b/(5+m)\*x^(5+m)\*hypergeom([5/2+1/2\*m], [3/2, 7/2+1/2\*m], 1/4\*x^2\*b^2)\*cosh(a)

**Maxima [A]** time = 1.23068, size = 74, normalized size = 1.25

$$\frac{1}{2} (bx)^{-m-4} x^{m+4} e^{(-a)} \Gamma(m + 4, bx) - \frac{1}{2} (-bx)^{-m-4} x^{m+4} e^a \Gamma(m + 4, -bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3+m)\*sinh(b\*x+a), x, algorithm="maxima")

[Out]  $\frac{1}{2}(bx)^{-m-4}x^{m+4}e^{-a}\Gamma(m+4, bx) - \frac{1}{2}(-bx)^{-m-4}x^{m+4}e^a\Gamma(m+4, -bx)$

**Fricas [A]** time = 2.79044, size = 258, normalized size = 4.37

$$\frac{\cosh((m+3)\log(b)+a)\Gamma(m+4, bx) + \cosh((m+3)\log(-b)-a)\Gamma(m+4, -bx) - \Gamma(m+4, -bx)\sinh((m+3)\log(-b)-a) - \Gamma(m+4, bx)\sinh((m+3)\log(b)+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3+m)\*sinh(b\*x+a), x, algorithm="fricas")

[Out]  $\frac{1}{2}(\cosh((m+3)\log(b)+a)\Gamma(m+4, bx) + \cosh((m+3)\log(-b)-a)\Gamma(m+4, -bx) - \Gamma(m+4, -bx)\sinh((m+3)\log(-b)-a) - \Gamma(m+4, bx)\sinh((m+3)\log(b)+a))/b$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(3+m)\*sinh(b\*x+a), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{m+3} \sinh(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3+m)\*sinh(b\*x+a), x, algorithm="giac")

[Out] integrate(x^(m+3)\*sinh(b\*x+a), x)



### 3.79 $\int x^{2+m} \sinh(a + bx) dx$

**Optimal.** Leaf size=59

$$\frac{e^a x^m (-bx)^{-m} \Gamma(m+3, -bx)}{2b^3} + \frac{e^{-a} x^m (bx)^{-m} \Gamma(m+3, bx)}{2b^3}$$

[Out]  $(E^a x^m \Gamma[3 + m, -(b*x)]) / (2*b^3 * (-(b*x))^m) + (x^m \Gamma[3 + m, b*x]) / (2*b^3 * E^a * (b*x)^m)$

**Rubi [A]** time = 0.0730884, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3308, 2181}

$$\frac{e^a x^m (-bx)^{-m} \Gamma(m+3, -bx)}{2b^3} + \frac{e^{-a} x^m (bx)^{-m} \Gamma(m+3, bx)}{2b^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(2 + m)} \text{Sinh}[a + b*x], x]$

[Out]  $(E^a x^m \Gamma[3 + m, -(b*x)]) / (2*b^3 * (-(b*x))^m) + (x^m \Gamma[3 + m, b*x]) / (2*b^3 * E^a * (b*x)^m)$

#### Rule 3308

$\text{Int}[(c + d*x)^m \sin(e + f*x), x\_Symbol] \rightarrow \text{Dist}[I/2, \text{Int}[(c + d*x)^m / E^{I*(e + f*x)}, x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m * E^{I*(e + f*x)}, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x]$

#### Rule 2181

$\text{Int}[(F)^{(g*(e + f*x))} * (c + d*x)^m, x\_Symbol] \rightarrow -\text{Simp}[(F^{(g*(e - (c*f)/d))} * (c + d*x)^m \text{FracPart}[m] * \Gamma[m + 1, -(f*g*\text{Log}[F])/d]) / (d * (-(f*g*\text{Log}[F])/d))^{(\text{IntPart}[m] + 1)} * (-(f*g*\text{Log}[F]) * (c + d*x) / d)^{\text{FracPart}[m]}), x] /; \text{FreeQ}\{F, c, d, e, f, g, m\}, x \ \&\& \ !\text{IntegerQ}[m]$

#### Rubi steps

$$\int x^{2+m} \sinh(a+bx) dx = \frac{1}{2} \int e^{-i(i+ibx)} x^{2+m} dx - \frac{1}{2} \int e^{i(i+ibx)} x^{2+m} dx$$

$$= \frac{e^a x^m (-bx)^{-m} \Gamma(3+m, -bx)}{2b^3} + \frac{e^{-a} x^m (bx)^{-m} \Gamma(3+m, bx)}{2b^3}$$

**Mathematica [A]** time = 0.0192387, size = 53, normalized size = 0.9

$$\frac{e^{-a} x^m \left( e^{2a} (-bx)^{-m} \text{Gamma}(m+3, -bx) + (bx)^{-m} \text{Gamma}(m+3, bx) \right)}{2b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^(2+m)\*Sinh[a+b\*x],x]

[Out] (x^m\*((E^(2\*a)\*Gamma[3+m,-(b\*x)])/(-(b\*x))^m + Gamma[3+m,b\*x]/(b\*x)^m))/((2\*b^3\*E^a))

**Maple [C]** time = 0.048, size = 73, normalized size = 1.2

$$\frac{x^{3+m} \sinh(a)}{3+m} {}_1F_2\left(\frac{3}{2} + \frac{m}{2}; \frac{1}{2}, \frac{5}{2} + \frac{m}{2}; \frac{x^2 b^2}{4}\right) + \frac{bx^{4+m} \cosh(a)}{4+m} {}_1F_2\left(2 + \frac{m}{2}; \frac{3}{2}, 3 + \frac{m}{2}; \frac{x^2 b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(2+m)\*sinh(b\*x+a),x)

[Out] 1/(3+m)\*x^(3+m)\*hypergeom([3/2+1/2\*m],[1/2,5/2+1/2\*m],1/4\*x^2\*b^2)\*sinh(a)+b/(4+m)\*x^(4+m)\*hypergeom([2+1/2\*m],[3/2,3+1/2\*m],1/4\*x^2\*b^2)\*cosh(a)

**Maxima [A]** time = 1.3399, size = 74, normalized size = 1.25

$$\frac{1}{2} (bx)^{-m-3} x^{m+3} e^{(-a)} \Gamma(m+3, bx) - \frac{1}{2} (-bx)^{-m-3} x^{m+3} e^a \Gamma(m+3, -bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2+m)\*sinh(b\*x+a),x, algorithm="maxima")

[Out]  $\frac{1}{2}(bx)^{-m-3}x^{m+3}e^{-a}\Gamma(m+3, bx) - \frac{1}{2}(-bx)^{-m-3}x^{m+3}e^a\Gamma(m+3, -bx)$

**Fricas [A]** time = 2.73305, size = 258, normalized size = 4.37

$$\frac{\cosh((m+2)\log(b)+a)\Gamma(m+3, bx) + \cosh((m+2)\log(-b)-a)\Gamma(m+3, -bx) - \Gamma(m+3, -bx)\sinh((m+2)\log(b)+a) - \Gamma(m+3, bx)\sinh((m+2)\log(-b)-a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(2+m)*sinh(b*x+a), x, algorithm="fricas")`

[Out]  $\frac{1}{2}(\cosh((m+2)\log(b)+a)\Gamma(m+3, bx) + \cosh((m+2)\log(-b)-a)\Gamma(m+3, -bx) - \Gamma(m+3, -bx)\sinh((m+2)\log(-b)-a) - \Gamma(m+3, bx)\sinh((m+2)\log(b)+a))/b$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(2+m)*sinh(b*x+a), x)`

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{m+2} \sinh(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(2+m)*sinh(b*x+a), x, algorithm="giac")`

[Out] `integrate(x^(m+2)*sinh(b*x+a), x)`

### 3.80 $\int x^{1+m} \sinh(a + bx) dx$

**Optimal.** Leaf size=59

$$\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+2, bx)}{2b^2} - \frac{e^ax^m(-bx)^{-m}\Gamma(m+2, -bx)}{2b^2}$$

[Out]  $-(E^a x^m \Gamma[2 + m, -(b*x)]) / (2*b^2*(-(b*x))^m) + (x^m \Gamma[2 + m, b*x]) / (2*b^2 * E^a * (b*x)^m)$

**Rubi [A]** time = 0.0736857, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3308, 2181}

$$\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+2, bx)}{2b^2} - \frac{e^ax^m(-bx)^{-m}\Gamma(m+2, -bx)}{2b^2}$$

Antiderivative was successfully verified.

[In] Int[x^(1 + m)\*Sinh[a + b\*x], x]

[Out]  $-(E^a x^m \Gamma[2 + m, -(b*x)]) / (2*b^2*(-(b*x))^m) + (x^m \Gamma[2 + m, b*x]) / (2*b^2 * E^a * (b*x)^m)$

#### Rule 3308

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Dist[I/2, Int[(c + d\*x)^m/E^(I\*(e + f\*x)), x], x] - Dist[I/2, Int[(c + d\*x)^m \* E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-(f\*g\*Log[F])/d)\*(c + d\*x])]/(d\*(-(f\*g\*Log[F])/d)^(IntPart[m] + 1)\*(-(f\*g\*Log[F])\*(c + d\*x)/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\int x^{1+m} \sinh(a + bx) dx = \frac{1}{2} \int e^{-i(i+ibx)} x^{1+m} dx - \frac{1}{2} \int e^{i(i+ibx)} x^{1+m} dx$$

$$= -\frac{e^a x^m (-bx)^{-m} \Gamma(2 + m, -bx)}{2b^2} + \frac{e^{-a} x^m (bx)^{-m} \Gamma(2 + m, bx)}{2b^2}$$

**Mathematica [A]** time = 0.021923, size = 54, normalized size = 0.92

$$\frac{e^{-a} x^m \left( (bx)^{-m} \text{Gamma}(m + 2, bx) - e^{2a} (-bx)^{-m} \text{Gamma}(m + 2, -bx) \right)}{2b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^(1 + m)\*Sinh[a + b\*x], x]

[Out] (x^m\*(-((E^(2\*a)\*Gamma[2 + m, -(b\*x)])/(-(b\*x))^m) + Gamma[2 + m, b\*x]/(b\*x)^m))/(2\*b^2\*E^a)

**Maple [C]** time = 0.04, size = 73, normalized size = 1.2

$$\frac{x^{2+m} \sinh(a)}{2+m} {}_1F_2\left(1 + \frac{m}{2}; \frac{1}{2}, 2 + \frac{m}{2}; \frac{x^2 b^2}{4}\right) + \frac{bx^{3+m} \cosh(a)}{3+m} {}_1F_2\left(\frac{3}{2} + \frac{m}{2}; \frac{3}{2}, \frac{5}{2} + \frac{m}{2}; \frac{x^2 b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1+m)\*sinh(b\*x+a), x)

[Out] 1/(2+m)\*x^(2+m)\*hypergeom([1+1/2\*m], [1/2, 2+1/2\*m], 1/4\*x^2\*b^2)\*sinh(a)+b/(3+m)\*x^(3+m)\*hypergeom([3/2+1/2\*m], [3/2, 5/2+1/2\*m], 1/4\*x^2\*b^2)\*cosh(a)

**Maxima [A]** time = 1.29879, size = 74, normalized size = 1.25

$$\frac{1}{2} (bx)^{-m-2} x^{m+2} e^{(-a)} \Gamma(m + 2, bx) - \frac{1}{2} (-bx)^{-m-2} x^{m+2} e^a \Gamma(m + 2, -bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1+m)\*sinh(b\*x+a), x, algorithm="maxima")

[Out]  $\frac{1}{2}(bx)^{-m-2}x^{m+2}e^{-a}\Gamma(m+2, bx) - \frac{1}{2}(-bx)^{-m-2}x^{m+2}e^a\Gamma(m+2, -bx)$

**Fricas [A]** time = 2.74414, size = 258, normalized size = 4.37

$$\frac{\cosh((m+1)\log(b)+a)\Gamma(m+2, bx) + \cosh((m+1)\log(-b)-a)\Gamma(m+2, -bx) - \Gamma(m+2, -bx)\sinh((m+1)\log(-b)-a) - \Gamma(m+2, bx)\sinh((m+1)\log(b)+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1+m)*sinh(b*x+a), x, algorithm="fricas")`

[Out]  $\frac{1}{2}(\cosh((m+1)\log(b)+a)\Gamma(m+2, bx) + \cosh((m+1)\log(-b)-a)\Gamma(m+2, -bx) - \Gamma(m+2, -bx)\sinh((m+1)\log(-b)-a) - \Gamma(m+2, bx)\sinh((m+1)\log(b)+a))/b$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1+m)*sinh(b*x+a), x)`

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{m+1} \sinh(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1+m)*sinh(b*x+a), x, algorithm="giac")`

[Out] `integrate(x^(m+1)*sinh(b*x+a), x)`

### 3.81 $\int x^m \sinh(a + bx) dx$

**Optimal.** Leaf size=59

$$\frac{e^a x^m (-bx)^{-m} \Gamma(m+1, -bx)}{2b} + \frac{e^{-a} x^m (bx)^{-m} \Gamma(m+1, bx)}{2b}$$

[Out]  $(E^a x^m \Gamma[1 + m, -(b*x)]) / (2*b*(-(b*x))^m) + (x^m \Gamma[1 + m, b*x]) / (2*b * E^a * (b*x)^m)$

**Rubi [A]** time = 0.0701193, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {3308, 2181}

$$\frac{e^a x^m (-bx)^{-m} \Gamma(m+1, -bx)}{2b} + \frac{e^{-a} x^m (bx)^{-m} \Gamma(m+1, bx)}{2b}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Sinh[a + b\*x],x]

[Out]  $(E^a x^m \Gamma[1 + m, -(b*x)]) / (2*b*(-(b*x))^m) + (x^m \Gamma[1 + m, b*x]) / (2*b * E^a * (b*x)^m)$

#### Rule 3308

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Dist[I/2, Int[(c + d\*x)^m/E^(I\*(e + f\*x)), x], x] - Dist[I/2, Int[(c + d\*x)^m \* E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\int x^m \sinh(a + bx) dx = \frac{1}{2} \int e^{-i(i+ibx)} x^m dx - \frac{1}{2} \int e^{i(i+ibx)} x^m dx$$

$$= \frac{e^a x^m (-bx)^{-m} \Gamma(1+m, -bx)}{2b} + \frac{e^{-a} x^m (bx)^{-m} \Gamma(1+m, bx)}{2b}$$

**Mathematica [A]** time = 0.016536, size = 53, normalized size = 0.9

$$\frac{e^{-a} x^m \left( e^{2a} (-bx)^{-m} \text{Gamma}(m+1, -bx) + (bx)^{-m} \text{Gamma}(m+1, bx) \right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*Sinh[a + b\*x], x]

[Out] (x^m\*((E^(2\*a)\*Gamma[1 + m, -(b\*x)])/(-(b\*x))^m + Gamma[1 + m, b\*x]/(b\*x)^m))/ (2\*b\*E^a)

**Maple [C]** time = 0.034, size = 73, normalized size = 1.2

$$\frac{x^{1+m} \sinh(a)}{1+m} {}_1F_2\left(\frac{1}{2} + \frac{m}{2}; \frac{1}{2}, \frac{3}{2} + \frac{m}{2}; \frac{x^2 b^2}{4}\right) + \frac{bx^{2+m} \cosh(a)}{2+m} {}_1F_2\left(1 + \frac{m}{2}; \frac{3}{2}, 2 + \frac{m}{2}; \frac{x^2 b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*sinh(b\*x+a), x)

[Out] 1/(1+m)\*x^(1+m)\*hypergeom([1/2+1/2\*m], [1/2, 3/2+1/2\*m], 1/4\*x^2\*b^2)\*sinh(a)+ b/(2+m)\*x^(2+m)\*hypergeom([1+1/2\*m], [3/2, 2+1/2\*m], 1/4\*x^2\*b^2)\*cosh(a)

**Maxima [A]** time = 1.31893, size = 74, normalized size = 1.25

$$\frac{1}{2} (bx)^{-m-1} x^{m+1} e^{(-a)} \Gamma(m+1, bx) - \frac{1}{2} (-bx)^{-m-1} x^{m+1} e^a \Gamma(m+1, -bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*sinh(b\*x+a), x, algorithm="maxima")



[Out]  $\frac{1}{2}(bx)^{-m-1}x^{m+1}e^{-a}\Gamma(m+1, bx) - \frac{1}{2}(-bx)^{-m-1}x^{m+1}e^a\Gamma(m+1, -bx)$

**Fricas [A]** time = 2.70492, size = 225, normalized size = 3.81

$$\frac{\cosh(m \log(b) + a)\Gamma(m+1, bx) + \cosh(m \log(-b) - a)\Gamma(m+1, -bx) - \Gamma(m+1, -bx)\sinh(m \log(-b) - a) - \Gamma(m+1, bx)\sinh(m \log(b) + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*sinh(b*x+a),x, algorithm="fricas")`

[Out]  $\frac{1}{2}(\cosh(m \log(b) + a)\Gamma(m+1, bx) + \cosh(m \log(-b) - a)\Gamma(m+1, -bx) - \Gamma(m+1, -bx)\sinh(m \log(-b) - a) - \Gamma(m+1, bx)\sinh(m \log(b) + a))/b$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*sinh(b*x+a),x)`

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sinh(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*sinh(b*x+a),x, algorithm="giac")`

[Out] `integrate(x^m*sinh(b*x + a), x)`

### 3.82 $\int x^{-1+m} \sinh(a + bx) dx$

**Optimal.** Leaf size=49

$$\frac{1}{2}e^{-a}x^m(bx)^{-m}\Gamma(m, bx) - \frac{1}{2}e^ax^m(-bx)^{-m}\Gamma(m, -bx)$$

[Out]  $-(E^{-a}x^m\Gamma[m, -(b*x)])/(2*(-(b*x))^m) + (x^m\Gamma[m, b*x])/(2E^a*(b*x)^m)$

**Rubi [A]** time = 0.0694422, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3308, 2181}

$$\frac{1}{2}e^{-a}x^m(bx)^{-m}\Gamma(m, bx) - \frac{1}{2}e^ax^m(-bx)^{-m}\Gamma(m, -bx)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(-1 + m)}*\text{Sinh}[a + b*x], x]$

[Out]  $-(E^{-a}x^m\Gamma[m, -(b*x)])/(2*(-(b*x))^m) + (x^m\Gamma[m, b*x])/(2E^a*(b*x)^m)$

#### Rule 3308

$\text{Int}[\frac{((c_.) + (d_.)*(x_))^{(m_.)}*\sin[(e_.) + (f_.)*(x_)]}{x\_Symbol}] \rightarrow \text{Dist}[I/2, \text{Int}[(c + d*x)^m/E^{I*(e + f*x)}, x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m*E^{I*(e + f*x)}, x], x] /;$  FreeQ[{c, d, e, f, m}, x]

#### Rule 2181

$\text{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^{(m_)}, x\_Symbol] \rightarrow -\text{Simp}[(F^{(g*(e - (c*f)/d))}*(c + d*x)^{\text{FracPart}[m]}*\Gamma[m + 1, (-((f*g*\text{Log}[F])/d))*(c + d*x])]/(d*(-((f*g*\text{Log}[F])/d))^{\text{IntPart}[m] + 1}*(-((f*g*\text{Log}[F])*(c + d*x)/d))^{\text{FracPart}[m]}), x] /;$  FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned}\int x^{-1+m} \sinh(a+bx) dx &= \frac{1}{2} \int e^{-i(ia+ibx)} x^{-1+m} dx - \frac{1}{2} \int e^{i(ia+ibx)} x^{-1+m} dx \\ &= -\frac{1}{2} e^a x^m (-bx)^{-m} \Gamma(m, -bx) + \frac{1}{2} e^{-a} x^m (bx)^{-m} \Gamma(m, bx)\end{aligned}$$

**Mathematica [A]** time = 0.022222, size = 49, normalized size = 1.

$$\frac{1}{2} e^{-a} x^m (bx)^{-m} \text{Gamma}(m, bx) - \frac{1}{2} e^a x^m (-bx)^{-m} \text{Gamma}(m, -bx)$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + m)\*Sinh[a + b\*x], x]

[Out] -(E^a\*x^m\*Gamma[m, -(b\*x)])/(2\*(-(b\*x))^m) + (x^m\*Gamma[m, b\*x])/(2\*E^a\*(b\*x)^m)

**Maple [C]** time = 0.041, size = 67, normalized size = 1.4

$$\frac{x^m \sinh(a)}{m} {}_1F_2\left(\frac{m}{2}; \frac{1}{2}, 1 + \frac{m}{2}; \frac{x^2 b^2}{4}\right) + \frac{bx^{1+m} \cosh(a)}{1+m} {}_1F_2\left(\frac{1}{2} + \frac{m}{2}; \frac{3}{2}, \frac{3}{2} + \frac{m}{2}; \frac{x^2 b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-1+m)\*sinh(b\*x+a), x)

[Out] 1/m\*x^m\*hypergeom([1/2\*m], [1/2, 1+1/2\*m], 1/4\*x^2\*b^2)\*sinh(a)+b/(1+m)\*x^(1+m)\*hypergeom([1/2+1/2\*m], [3/2, 3/2+1/2\*m], 1/4\*x^2\*b^2)\*cosh(a)

**Maxima [A]** time = 1.21694, size = 58, normalized size = 1.18

$$\frac{x^m e^{(-a)} \Gamma(m, bx)}{2 (bx)^m} - \frac{x^m e^a \Gamma(m, -bx)}{2 (-bx)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+m)\*sinh(b\*x+a), x, algorithm="maxima")

[Out]  $\frac{1}{2}x^m e^{-a} \gamma(m, bx) / (bx)^m - \frac{1}{2}x^m e^a \gamma(m, -bx) / (-bx)^m$

**Fricas [A]** time = 2.69251, size = 236, normalized size = 4.82

$$\frac{\cosh((m-1)\log(b) + a)\Gamma(m, bx) + \cosh((m-1)\log(-b) - a)\Gamma(m, -bx) - \Gamma(m, -bx)\sinh((m-1)\log(-b) - a) - \Gamma(m, bx)\sinh((m-1)\log(b) + a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+m)*sinh(b*x+a), x, algorithm="fricas")`

[Out]  $\frac{1}{2}(\cosh((m-1)\log(b) + a)\gamma(m, bx) + \cosh((m-1)\log(-b) - a)\gamma(m, -bx) - \gamma(m, -bx)\sinh((m-1)\log(-b) - a) - \gamma(m, bx)\sinh((m-1)\log(b) + a))/b$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-1+m)*sinh(b*x+a), x)`

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{m-1} \sinh(bx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-1+m)*sinh(b*x+a), x, algorithm="giac")`

[Out] `integrate(x^(m - 1)*sinh(b*x + a), x)`

### 3.83 $\int x^{-2+m} \sinh(a + bx) dx$

**Optimal.** Leaf size=55

$$\frac{1}{2}e^a bx^m (-bx)^{-m} \Gamma(m-1, -bx) + \frac{1}{2}e^{-a} bx^m (bx)^{-m} \Gamma(m-1, bx)$$

[Out] (b\*E^a\*x^m\*Gamma[-1 + m, -(b\*x)])/(2\*(-(b\*x))^m) + (b\*x^m\*Gamma[-1 + m, b\*x])/ (2\*E^a\*(b\*x)^m)

**Rubi [A]** time = 0.069492, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3308, 2181}

$$\frac{1}{2}e^a bx^m (-bx)^{-m} \Gamma(m-1, -bx) + \frac{1}{2}e^{-a} bx^m (bx)^{-m} \Gamma(m-1, bx)$$

Antiderivative was successfully verified.

[In] Int[x^(-2 + m)\*Sinh[a + b\*x],x]

[Out] (b\*E^a\*x^m\*Gamma[-1 + m, -(b\*x)])/(2\*(-(b\*x))^m) + (b\*x^m\*Gamma[-1 + m, b\*x])/ (2\*E^a\*(b\*x)^m)

#### Rule 3308

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Dist[I/2, Int[(c + d\*x)^m/E^(I\*(e + f\*x)), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x])]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F])\*(c + d\*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]

#### Rubi steps

$$\begin{aligned}\int x^{-2+m} \sinh(a+bx) dx &= \frac{1}{2} \int e^{-i(ia+ibx)} x^{-2+m} dx - \frac{1}{2} \int e^{i(ia+ibx)} x^{-2+m} dx \\ &= \frac{1}{2} b e^a x^m (-bx)^{-m} \Gamma(-1+m, -bx) + \frac{1}{2} b e^{-a} x^m (bx)^{-m} \Gamma(-1+m, bx)\end{aligned}$$

**Mathematica [A]** time = 0.0177609, size = 51, normalized size = 0.93

$$\frac{1}{2} e^{-a} b x^m \left( e^{2a} (-bx)^{-m} \text{Gamma}(m-1, -bx) + (bx)^{-m} \text{Gamma}(m-1, bx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(-2 + m)\*Sinh[a + b\*x], x]

[Out] (b\*x^m\*((E^(2\*a)\*Gamma[-1 + m, -(b\*x)]))/(-(b\*x))^m + Gamma[-1 + m, b\*x]/(b\*x)^m))/(2\*E^a)

**Maple [C]** time = 0.046, size = 67, normalized size = 1.2

$$\frac{x^{-1+m} \sinh(a)}{-1+m} {}_1F_2\left(-\frac{1}{2} + \frac{m}{2}; \frac{1}{2}, \frac{1}{2} + \frac{m}{2}; \frac{x^2 b^2}{4}\right) + \frac{b x^m \cosh(a)}{m} {}_1F_2\left(\frac{m}{2}; \frac{3}{2}, 1 + \frac{m}{2}; \frac{x^2 b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-2+m)\*sinh(b\*x+a), x)

[Out] 1/(-1+m)\*x^(-1+m)\*hypergeom([-1/2+1/2\*m], [1/2, 1/2+1/2\*m], 1/4\*x^2\*b^2)\*sinh(a)+b/m\*x^m\*hypergeom([1/2\*m], [3/2, 1+1/2\*m], 1/4\*x^2\*b^2)\*cosh(a)

**Maxima [A]** time = 1.28518, size = 74, normalized size = 1.35

$$\frac{1}{2} (bx)^{-m+1} x^{m-1} e^{(-a)} \Gamma(m-1, bx) - \frac{1}{2} (-bx)^{-m+1} x^{m-1} e^a \Gamma(m-1, -bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-2+m)\*sinh(b\*x+a), x, algorithm="maxima")

[Out]  $\frac{1}{2}(bx)^{-m+1}x^{m-1}e^{-a}\Gamma(m-1, bx) - \frac{1}{2}(-bx)^{-m+1}x^{m-1}e^a\Gamma(m-1, -bx)$

**Fricas [A]** time = 2.68778, size = 258, normalized size = 4.69

$$\frac{\cosh((m-2)\log(b)+a)\Gamma(m-1, bx) + \cosh((m-2)\log(-b)-a)\Gamma(m-1, -bx) - \Gamma(m-1, -bx)\sinh((m-2)\log(-b)-a) - \Gamma(m-1, bx)\sinh((m-2)\log(b)+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-2+m)*sinh(b*x+a),x, algorithm="fricas")`

[Out]  $\frac{1}{2}(\cosh((m-2)\log(b)+a)\Gamma(m-1, bx) + \cosh((m-2)\log(-b)-a)\Gamma(m-1, -bx) - \Gamma(m-1, -bx)\sinh((m-2)\log(-b)-a) - \Gamma(m-1, bx)\sinh((m-2)\log(b)+a))/b$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-2+m)*sinh(b*x+a),x)`

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{m-2} \sinh(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-2+m)*sinh(b*x+a),x, algorithm="giac")`

[Out] `integrate(x^(m-2)*sinh(b*x+a), x)`

### 3.84 $\int x^{-3+m} \sinh(a + bx) dx$

**Optimal.** Leaf size=59

$$\frac{1}{2}e^{-a}b^2x^m(bx)^{-m}\Gamma(m-2, bx) - \frac{1}{2}e^ab^2x^m(-bx)^{-m}\Gamma(m-2, -bx)$$

[Out]  $-(b^2E^a x^m \Gamma[-2 + m, -(b*x)]) / (2*(-(b*x))^m) + (b^2 x^m \Gamma[-2 + m, b*x]) / (2E^a (b*x)^m)$

**Rubi [A]** time = 0.0708014, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3308, 2181}

$$\frac{1}{2}e^{-a}b^2x^m(bx)^{-m}\Gamma(m-2, bx) - \frac{1}{2}e^ab^2x^m(-bx)^{-m}\Gamma(m-2, -bx)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(-3 + m)} \text{Sinh}[a + b*x], x]$

[Out]  $-(b^2E^a x^m \Gamma[-2 + m, -(b*x)]) / (2*(-(b*x))^m) + (b^2 x^m \Gamma[-2 + m, b*x]) / (2E^a (b*x)^m)$

#### Rule 3308

$\text{Int}[(c_. + (d_.)(x_.))^{(m_.)} \sin[(e_.) + (f_.)(x_.)], x\_Symbol] \rightarrow \text{Dist}[I/2, \text{Int}[(c + d*x)^m / E^{I*(e + f*x)}, x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m * E^{I*(e + f*x)}, x], x] /;$   $\text{FreeQ}\{c, d, e, f, m\}, x]$

#### Rule 2181

$\text{Int}[(F_)^{(g_.)*((e_.) + (f_.)(x_.))} * ((c_.) + (d_.)(x_.))^{(m_.)}, x\_Symbol] \rightarrow -\text{Simp}[(F^{(g*(e - (c*f)/d)}) * (c + d*x)^{\text{FracPart}[m]} * \Gamma[m + 1, -(f*g*\text{Log}[F])/d]) * (c + d*x)] / (d * (-(f*g*\text{Log}[F])/d)^{(\text{IntPart}[m] + 1)} * (-(f*g*\text{Log}[F]) * (c + d*x) / d)^{\text{FracPart}[m]}), x] /;$   $\text{FreeQ}\{F, c, d, e, f, g, m\}, x] \&\& \text{!IntegerQ}[m]$

#### Rubi steps



$$\begin{aligned}\int x^{-3+m} \sinh(a + bx) dx &= \frac{1}{2} \int e^{-i(i+ibx)} x^{-3+m} dx - \frac{1}{2} \int e^{i(i+ibx)} x^{-3+m} dx \\ &= -\frac{1}{2} b^2 e^a x^m (-bx)^{-m} \Gamma(-2 + m, -bx) + \frac{1}{2} b^2 e^{-a} x^m (bx)^{-m} \Gamma(-2 + m, bx)\end{aligned}$$

**Mathematica [A]** time = 0.0217781, size = 54, normalized size = 0.92

$$\frac{1}{2} e^{-a} b^2 x^m \left( (bx)^{-m} \text{Gamma}(m - 2, bx) - e^{2a} (-bx)^{-m} \text{Gamma}(m - 2, -bx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(-3 + m)\*Sinh[a + b\*x], x]

[Out] (b^2\*x^m\*(-((E^(2\*a)\*Gamma[-2 + m, -(b\*x)])/(-(b\*x))^m) + Gamma[-2 + m, b\*x]/(b\*x)^m))/(2\*E^a)

**Maple [C]** time = 0.03, size = 71, normalized size = 1.2

$$\frac{x^{-2+m} \sinh(a)}{-2+m} {}_1F_2\left(-1 + \frac{m}{2}; \frac{1}{2}, \frac{m}{2}; \frac{x^2 b^2}{4}\right) + \frac{bx^{-1+m} \cosh(a)}{-1+m} {}_1F_2\left(-\frac{1}{2} + \frac{m}{2}; \frac{3}{2}, \frac{1}{2} + \frac{m}{2}; \frac{x^2 b^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-3+m)\*sinh(b\*x+a), x)

[Out] 1/(-2+m)\*x^(-2+m)\*hypergeom([-1+1/2\*m], [1/2, 1/2\*m], 1/4\*x^2\*b^2)\*sinh(a)+b/(-1+m)\*x^(-1+m)\*hypergeom([-1/2+1/2\*m], [3/2, 1/2+1/2\*m], 1/4\*x^2\*b^2)\*cosh(a)

**Maxima [A]** time = 1.32526, size = 74, normalized size = 1.25

$$\frac{1}{2} (bx)^{-m+2} x^{m-2} e^{(-a)} \Gamma(m - 2, bx) - \frac{1}{2} (-bx)^{-m+2} x^{m-2} e^a \Gamma(m - 2, -bx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-3+m)\*sinh(b\*x+a), x, algorithm="maxima")

[Out]  $\frac{1}{2}(bx)^{-m+2}x^{m-2}e^{-a}\Gamma(m-2, bx) - \frac{1}{2}(-bx)^{-m+2}x^{m-2}e^a\Gamma(m-2, -bx)$

**Fricas [A]** time = 2.69793, size = 258, normalized size = 4.37

$$\frac{\cosh((m-3)\log(b)+a)\Gamma(m-2, bx) + \cosh((m-3)\log(-b)-a)\Gamma(m-2, -bx) - \Gamma(m-2, -bx)\sinh((m-3)\log(-b)-a) - \Gamma(m-2, bx)\sinh((m-3)\log(b)+a)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-3+m)*sinh(b*x+a), x, algorithm="fricas")`

[Out]  $\frac{1}{2}(\cosh((m-3)\log(b)+a)\Gamma(m-2, bx) + \cosh((m-3)\log(-b)-a)\Gamma(m-2, -bx) - \Gamma(m-2, -bx)\sinh((m-3)\log(-b)-a) - \Gamma(m-2, bx)\sinh((m-3)\log(b)+a))/b$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(-3+m)*sinh(b*x+a), x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{m-3} \sinh(bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(-3+m)*sinh(b*x+a), x, algorithm="giac")`

[Out] `integrate(x^(m-3)*sinh(b*x+a), x)`

### 3.85 $\int x^{3+m} \sinh^2(a + bx) dx$

**Optimal.** Leaf size=86

$$\frac{e^{2a}2^{-m-6}x^m(-bx)^{-m}\Gamma(m+4,-2bx)}{b^4} - \frac{e^{-2a}2^{-m-6}x^m(bx)^{-m}\Gamma(m+4,2bx)}{b^4} - \frac{x^{m+4}}{2(m+4)}$$

[Out]  $-x^{4+m}/(2*(4+m)) - (2^{(-6-m)}*E^{(2*a)}*x^m*\Gamma[4+m,-2*b*x])/(b^4*(-(b*x))^m) - (2^{(-6-m)}*x^m*\Gamma[4+m,2*b*x])/(b^4*E^{(2*a)}*(b*x)^m)$

**Rubi [A]** time = 0.158529, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3312, 3307, 2181}

$$\frac{e^{2a}2^{-m-6}x^m(-bx)^{-m}\Gamma(m+4,-2bx)}{b^4} - \frac{e^{-2a}2^{-m-6}x^m(bx)^{-m}\Gamma(m+4,2bx)}{b^4} - \frac{x^{m+4}}{2(m+4)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(3+m)}*\text{Sinh}[a+bx]^2,x]$

[Out]  $-x^{4+m}/(2*(4+m)) - (2^{(-6-m)}*E^{(2*a)}*x^m*\Gamma[4+m,-2*b*x])/(b^4*(-(b*x))^m) - (2^{(-6-m)}*x^m*\Gamma[4+m,2*b*x])/(b^4*E^{(2*a)}*(b*x)^m)$

#### Rule 3312

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[e + f*x]^n, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x] \&\& \text{IGtQ}[n, 1] \&\& (!\text{RationalQ}[m] || (\text{GeQ}[m, -1] \&\& \text{LtQ}[m, 1]))$

#### Rule 3307

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + \text{Pi}*(k_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow \text{Dist}[I/2, \text{Int}[(c + d*x)^m/(E^{(I*k*Pi)}*E^{(I*(e + f*x)}), x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m*E^{(I*k*Pi)}*E^{(I*(e + f*x)}), x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x] \&\& \text{IntegerQ}[2*k]$

#### Rule 2181

$\text{Int}[(F_.)^{(g_.)*((e_.) + (f_.)*(x_.))}*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow -\text{Simp}[(F^{(g*(e - (c*f)/d)})*(c + d*x)^{\text{FracPart}[m]}*\Gamma[m + 1, -(f*g*\text{Log}[F])/d])*(c + d*x)]/(d*(-(f*g*\text{Log}[F])/d))^{(\text{IntPart}[m] + 1)*(-(f*g*\text{Log}[F])$

`](*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]`

### Rubi steps

$$\begin{aligned}
 \int x^{3+m} \sinh^2(a + bx) dx &= - \int \left( \frac{x^{3+m}}{2} - \frac{1}{2} x^{3+m} \cosh(2a + 2bx) \right) dx \\
 &= -\frac{x^{4+m}}{2(4+m)} + \frac{1}{2} \int x^{3+m} \cosh(2a + 2bx) dx \\
 &= -\frac{x^{4+m}}{2(4+m)} + \frac{1}{4} \int e^{-i(2ia+2ibx)} x^{3+m} dx + \frac{1}{4} \int e^{i(2ia+2ibx)} x^{3+m} dx \\
 &= -\frac{x^{4+m}}{2(4+m)} - \frac{2^{-6-m} e^{2a} x^m (-bx)^{-m} \Gamma(4+m, -2bx)}{b^4} - \frac{2^{-6-m} e^{-2a} x^m (bx)^{-m} \Gamma(4+m, 2bx)}{b^4}
 \end{aligned}$$

**Mathematica [A]** time = 0.126828, size = 79, normalized size = 0.92

$$\frac{1}{64} x^m \left( -\frac{e^{2a} 2^{-m} (-bx)^{-m} \text{Gamma}(m+4, -2bx)}{b^4} - \frac{e^{-2a} 2^{-m} (bx)^{-m} \text{Gamma}(m+4, 2bx)}{b^4} - \frac{32x^4}{m+4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(3+m)\*Sinh[a+b\*x]^2,x]

[Out] (x^m\*((-32\*x^4)/(4+m) - (E^(2\*a)\*Gamma[4+m, -2\*b\*x])/(2^m\*b^4\*(-(b\*x))^-m) - Gamma[4+m, 2\*b\*x]/(2^m\*b^4\*E^(2\*a)\*(b\*x)^m)))/64

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int x^{3+m} (\sinh(bx+a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3+m)\*sinh(b\*x+a)^2,x)

[Out] int(x^(3+m)\*sinh(b\*x+a)^2,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3+m)\*sinh(b\*x+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.74761, size = 427, normalized size = 4.97

$$\frac{4bx \cosh((m+3)\log(x)) + (m+4)\cosh((m+3)\log(2b) + 2a)\Gamma(m+4, 2bx) - (m+4)\cosh((m+3)\log(-2b) - 2a)\Gamma(m+4, -2bx)}{(b^2m^2 + 4b^2m + 4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3+m)\*sinh(b\*x+a)^2,x, algorithm="fricas")

[Out] 
$$\frac{-1/8*(4*b*x*\cosh((m+3)*\log(x)) + (m+4)*\cosh((m+3)*\log(2*b) + 2*a)*\gamma(m+4, 2*b*x) - (m+4)*\cosh((m+3)*\log(-2*b) - 2*a)*\gamma(m+4, -2*b*x) - (m+4)*\gamma(m+4, 2*b*x)*\sinh((m+3)*\log(2*b) + 2*a) + (m+4)*\gamma(m+4, -2*b*x)*\sinh((m+3)*\log(-2*b) - 2*a) + 4*b*x*\sinh((m+3)*\log(x)))/(b^2*m^2 + 4*b^2)}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(3+m)\*sinh(b\*x+a)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{m+3} \sinh(bx+a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 3)*sinh(b*x + a)^2, x)
```

### 3.86 $\int x^{2+m} \sinh^2(a + bx) dx$

**Optimal.** Leaf size=85

$$\frac{e^{2a}2^{-m-5}x^m(-bx)^{-m}\Gamma(m+3,-2bx)}{b^3} - \frac{e^{-2a}2^{-m-5}x^m(bx)^{-m}\Gamma(m+3,2bx)}{b^3} - \frac{x^{m+3}}{2(m+3)}$$

[Out]  $-x^{(3+m)}/(2*(3+m)) + (2^{(-5-m)}*E^{(2*a)}*x^m*\Gamma[3+m,-2*b*x])/(b^3*(-(b*x))^m) - (2^{(-5-m)}*x^m*\Gamma[3+m,2*b*x])/(b^3*E^{(2*a)}*(b*x)^m)$

**Rubi [A]** time = 0.135763, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3312, 3307, 2181}

$$\frac{e^{2a}2^{-m-5}x^m(-bx)^{-m}\Gamma(m+3,-2bx)}{b^3} - \frac{e^{-2a}2^{-m-5}x^m(bx)^{-m}\Gamma(m+3,2bx)}{b^3} - \frac{x^{m+3}}{2(m+3)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(2+m)}*\text{Sinh}[a + b*x]^2, x]$

[Out]  $-x^{(3+m)}/(2*(3+m)) + (2^{(-5-m)}*E^{(2*a)}*x^m*\Gamma[3+m,-2*b*x])/(b^3*(-(b*x))^m) - (2^{(-5-m)}*x^m*\Gamma[3+m,2*b*x])/(b^3*E^{(2*a)}*(b*x)^m)$

#### Rule 3312

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[e + f*x]^n, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x] \&\& \text{IGtQ}[n, 1] \&\& (!\text{RationalQ}[m] || (\text{GeQ}[m, -1] \&\& \text{LtQ}[m, 1]))$

#### Rule 3307

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + \text{Pi}*(k_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow \text{Dist}[I/2, \text{Int}[(c + d*x)^m/(E^{(I*k*Pi)}*E^{(I*(e + f*x)}), x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m*E^{(I*k*Pi)}*E^{(I*(e + f*x)}), x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x] \&\& \text{IntegerQ}[2*k]$

#### Rule 2181

$\text{Int}[(F_)^{((g_.)*(e_.) + (f_.)*(x_.))}*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow -\text{Simp}[(F^{(g*(e - (c*f)/d)})*(c + d*x)^{\text{FracPart}[m]}*\Gamma[m + 1, (-(f*g*\text{Log}[F])/d)]*(c + d*x)]/(d*(-(f*g*\text{Log}[F])/d))^{(\text{IntPart}[m] + 1)}*(-(f*g*\text{Log}[F]$

`](*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]`

### Rubi steps

$$\begin{aligned}
 \int x^{2+m} \sinh^2(a + bx) dx &= - \int \left( \frac{x^{2+m}}{2} - \frac{1}{2} x^{2+m} \cosh(2a + 2bx) \right) dx \\
 &= -\frac{x^{3+m}}{2(3+m)} + \frac{1}{2} \int x^{2+m} \cosh(2a + 2bx) dx \\
 &= -\frac{x^{3+m}}{2(3+m)} + \frac{1}{4} \int e^{-i(2ia+2ibx)} x^{2+m} dx + \frac{1}{4} \int e^{i(2ia+2ibx)} x^{2+m} dx \\
 &= -\frac{x^{3+m}}{2(3+m)} + \frac{2^{-5-m} e^{2a} x^m (-bx)^{-m} \Gamma(3+m, -2bx)}{b^3} - \frac{2^{-5-m} e^{-2a} x^m (bx)^{-m} \Gamma(3+m, 2bx)}{b^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.123141, size = 78, normalized size = 0.92

$$\frac{1}{32} x^m \left( \frac{e^{2a} 2^{-m} (-bx)^{-m} \text{Gamma}(m+3, -2bx)}{b^3} - \frac{e^{-2a} 2^{-m} (bx)^{-m} \text{Gamma}(m+3, 2bx)}{b^3} - \frac{16x^3}{m+3} \right)$$

Antiderivative was successfully verified.

[In] `Integrate[x^(2+m)*Sinh[a+b*x]^2,x]`

[Out] `(x^m*((-16*x^3)/(3+m) + (E^(2*a)*Gamma[3+m, -2*b*x])/(2^m*b^3*(-(b*x))^-m) - Gamma[3+m, 2*b*x]/(2^m*b^3*E^(2*a)*(b*x)^m)))/32`

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int x^{2+m} (\sinh(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(2+m)*sinh(b*x+a)^2,x)`

[Out] `int(x^(2+m)*sinh(b*x+a)^2,x)`



**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2+m)\*sinh(b\*x+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.75892, size = 427, normalized size = 5.02

$$\frac{4bx \cosh((m+2)\log(x)) + (m+3) \cosh((m+2)\log(2b) + 2a) \Gamma(m+3, 2bx) - (m+3) \cosh((m+2)\log(-2b) - 2a) \Gamma(m+3, -2bx)}{(b^2 m^2 + 6bm + 3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(2+m)\*sinh(b\*x+a)^2,x, algorithm="fricas")

[Out] 
$$\frac{-1/8*(4*b*x*\cosh((m+2)*\log(x)) + (m+3)*\cosh((m+2)*\log(2*b) + 2*a)*\gamma(m+3, 2*b*x) - (m+3)*\cosh((m+2)*\log(-2*b) - 2*a)*\gamma(m+3, -2*b*x) - (m+3)*\gamma(m+3, 2*b*x)*\sinh((m+2)*\log(2*b) + 2*a) + (m+3)*\gamma(m+3, -2*b*x)*\sinh((m+2)*\log(-2*b) - 2*a) + 4*b*x*\sinh((m+2)*\log(x)))/(b*m + 3*b)}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(2+m)\*sinh(b\*x+a)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{m+2} \sinh(bx+a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)*sinh(b*x + a)^2, x)
```

### 3.87 $\int x^{1+m} \sinh^2(a + bx) dx$

**Optimal.** Leaf size=86

$$\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\Gamma(m+2,-2bx)}{b^2} - \frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\Gamma(m+2,2bx)}{b^2} - \frac{x^{m+2}}{2(m+2)}$$

[Out]  $-x^{2+m}/(2*(2+m)) - (2^{(-4-m)}*E^{(2*a)}*x^m*\Gamma[2+m,-2*b*x])/(b^2*(-(b*x))^m) - (2^{(-4-m)}*x^m*\Gamma[2+m,2*b*x])/(b^2*E^{(2*a)}*(b*x)^m)$

**Rubi [A]** time = 0.139944, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3312, 3307, 2181}

$$\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\Gamma(m+2,-2bx)}{b^2} - \frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\Gamma(m+2,2bx)}{b^2} - \frac{x^{m+2}}{2(m+2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^{(1+m)}*\text{Sinh}[a+b*x]^2,x]$

[Out]  $-x^{2+m}/(2*(2+m)) - (2^{(-4-m)}*E^{(2*a)}*x^m*\Gamma[2+m,-2*b*x])/(b^2*(-(b*x))^m) - (2^{(-4-m)}*x^m*\Gamma[2+m,2*b*x])/(b^2*E^{(2*a)}*(b*x)^m)$

#### Rule 3312

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sin}[e + f*x]^n, x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x] \&\& \text{IGtQ}[n, 1] \&\& (!\text{RationalQ}[m] || (\text{GeQ}[m, -1] \&\& \text{LtQ}[m, 1]))$

#### Rule 3307

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + \text{Pi}*(k_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow \text{Dist}[I/2, \text{Int}[(c + d*x)^m/(E^{(I*k*Pi)}*E^{(I*(e + f*x)}), x], x] - \text{Dist}[I/2, \text{Int}[(c + d*x)^m*E^{(I*k*Pi)}*E^{(I*(e + f*x)}), x], x] /; \text{FreeQ}\{c, d, e, f, m\}, x] \&\& \text{IntegerQ}[2*k]$

#### Rule 2181

$\text{Int}[(F_.)^{(g_.)}*((e_.) + (f_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow -\text{Simp}[(F^{(g*(e - (c*f)/d)})*(c + d*x)^{\text{FracPart}[m]}*\Gamma[m + 1, (-(f*g*\text{Log}[F])/d)]*(c + d*x)]/(d*(-(f*g*\text{Log}[F])/d))^{(\text{IntPart}[m] + 1)}*(-(f*g*\text{Log}[F]$

`](*(c + d*x))/d))^FracPart[m]], x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]`

### Rubi steps

$$\begin{aligned}
 \int x^{1+m} \sinh^2(a + bx) dx &= - \int \left( \frac{x^{1+m}}{2} - \frac{1}{2} x^{1+m} \cosh(2a + 2bx) \right) dx \\
 &= -\frac{x^{2+m}}{2(2+m)} + \frac{1}{2} \int x^{1+m} \cosh(2a + 2bx) dx \\
 &= -\frac{x^{2+m}}{2(2+m)} + \frac{1}{4} \int e^{-i(2ia+2ibx)} x^{1+m} dx + \frac{1}{4} \int e^{i(2ia+2ibx)} x^{1+m} dx \\
 &= -\frac{x^{2+m}}{2(2+m)} - \frac{2^{-4-m} e^{2a} x^m (-bx)^{-m} \Gamma(2+m, -2bx)}{b^2} - \frac{2^{-4-m} e^{-2a} x^m (bx)^{-m} \Gamma(2+m, 2bx)}{b^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.130026, size = 79, normalized size = 0.92

$$\frac{1}{16} x^m \left( -\frac{e^{2a} 2^{-m} (-bx)^{-m} \text{Gamma}(m+2, -2bx)}{b^2} - \frac{e^{-2a} 2^{-m} (bx)^{-m} \text{Gamma}(m+2, 2bx)}{b^2} - \frac{8x^2}{m+2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(1+m)\*Sinh[a+b\*x]^2,x]

[Out] (x^m\*((-8\*x^2)/(2+m) - (E^(2\*a)\*Gamma[2+m, -2\*b\*x])/(2^m\*b^2\*(-(b\*x))^m) - Gamma[2+m, 2\*b\*x]/(2^m\*b^2\*E^(2\*a)\*(b\*x)^m)))/16

**Maple [F]** time = 0.076, size = 0, normalized size = 0.

$$\int x^{1+m} (\sinh(bx+a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1+m)\*sinh(b\*x+a)^2,x)

[Out] int(x^(1+m)\*sinh(b\*x+a)^2,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1+m)\*sinh(b\*x+a)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.7728, size = 427, normalized size = 4.97

$$\frac{4bx \cosh((m+1)\log(x)) + (m+2) \cosh((m+1)\log(2b) + 2a) \Gamma(m+2, 2bx) - (m+2) \cosh((m+1)\log(-2b) - 2a) \Gamma(m+2, -2bx)}{(b^2m^2 + 4bm + 2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1+m)\*sinh(b\*x+a)^2,x, algorithm="fricas")

[Out] 
$$\frac{-1/8*(4*b*x*cosh((m+1)*log(x)) + (m+2)*cosh((m+1)*log(2*b) + 2*a)*gamma(m+2, 2*b*x) - (m+2)*cosh((m+1)*log(-2*b) - 2*a)*gamma(m+2, -2*b*x) - (m+2)*gamma(m+2, 2*b*x)*sinh((m+1)*log(2*b) + 2*a) + (m+2)*gamma(m+2, -2*b*x)*sinh((m+1)*log(-2*b) - 2*a) + 4*b*x*sinh((m+1)*log(x))}{(b*m^2 + 4*b*m + 2*b^2)}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{m+1} \sinh^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(1+m)\*sinh(b\*x+a)\*\*2,x)

[Out] Integral(x\*\*(m + 1)\*sinh(a + b\*x)\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{m+1} \sinh(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)*sinh(b*x + a)^2, x)
```

### 3.88 $\int x^m \sinh^2(a + bx) dx$

**Optimal.** Leaf size=85

$$\frac{e^{2a} 2^{-m-3} x^m (-bx)^{-m} \Gamma(m+1, -2bx)}{b} - \frac{e^{-2a} 2^{-m-3} x^m (bx)^{-m} \Gamma(m+1, 2bx)}{b} - \frac{x^{m+1}}{2(m+1)}$$

[Out]  $-x^{(1+m)}/(2*(1+m)) + (2^{(-3-m)}*E^{(2*a)}*x^m*\Gamma[1+m, -2*b*x])/(b*(-(b*x))^m) - (2^{(-3-m)}*x^m*\Gamma[1+m, 2*b*x])/(b*E^{(2*a)}*(b*x)^m)$

**Rubi [A]** time = 0.12687, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3312, 3307, 2181}

$$\frac{e^{2a} 2^{-m-3} x^m (-bx)^{-m} \Gamma(m+1, -2bx)}{b} - \frac{e^{-2a} 2^{-m-3} x^m (bx)^{-m} \Gamma(m+1, 2bx)}{b} - \frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

[In] Int[x^m\*Sinh[a + b\*x]^2,x]

[Out]  $-x^{(1+m)}/(2*(1+m)) + (2^{(-3-m)}*E^{(2*a)}*x^m*\Gamma[1+m, -2*b*x])/(b*(-(b*x))^m) - (2^{(-3-m)}*x^m*\Gamma[1+m, 2*b*x])/(b*E^{(2*a)}*(b*x)^m)$

#### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)^(n\_)], x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

#### Rule 3307

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] := Dist[I/2, Int[(c + d\*x)^m/(E^(I\*k\*Pi)\*E^(I\*(e + f\*x))), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(I\*k\*Pi)\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] := -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-(f\*g\*Log[F])/d))\*(c + d\*x)]/(d\*(-(f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-(f\*g\*Log[F]

`](*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]`

### Rubi steps

$$\begin{aligned}
 \int x^m \sinh^2(a + bx) dx &= - \int \left( \frac{x^m}{2} - \frac{1}{2} x^m \cosh(2a + 2bx) \right) dx \\
 &= - \frac{x^{1+m}}{2(1+m)} + \frac{1}{2} \int x^m \cosh(2a + 2bx) dx \\
 &= - \frac{x^{1+m}}{2(1+m)} + \frac{1}{4} \int e^{-i(2ia+2ibx)} x^m dx + \frac{1}{4} \int e^{i(2ia+2ibx)} x^m dx \\
 &= - \frac{x^{1+m}}{2(1+m)} + \frac{2^{-3-m} e^{2a} x^m (-bx)^{-m} \Gamma(1+m, -2bx)}{b} - \frac{2^{-3-m} e^{-2a} x^m (bx)^{-m} \Gamma(1+m, 2bx)}{b}
 \end{aligned}$$

**Mathematica [A]** time = 0.104684, size = 76, normalized size = 0.89

$$\frac{1}{8} x^m \left( \frac{e^{2a} 2^{-m} (-bx)^{-m} \text{Gamma}(m+1, -2bx)}{b} - \frac{e^{-2a} 2^{-m} (bx)^{-m} \text{Gamma}(m+1, 2bx)}{b} - \frac{4x}{m+1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^m\*Sinh[a + b\*x]^2,x]

[Out] (x^m\*((-4\*x)/(1+m) + (E^(2\*a)\*Gamma[1+m, -2\*b\*x])/(2^m\*b\*(-(b\*x))^m) - Gamma[1+m, 2\*b\*x]/(2^m\*b\*E^(2\*a)\*(b\*x)^m)))/8

**Maple [F]** time = 0.053, size = 0, normalized size = 0.

$$\int x^m (\sinh(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*sinh(b\*x+a)^2,x)

[Out] int(x^m\*sinh(b\*x+a)^2,x)



**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>\*sinh(b\*x+a)<sup>2</sup>,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.70444, size = 375, normalized size = 4.41

$$\frac{4bx \cosh(m \log(x)) + (m+1) \cosh(m \log(2b) + 2a) \Gamma(m+1, 2bx) - (m+1) \cosh(m \log(-2b) - 2a) \Gamma(m+1, -2bx)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>m</sup>\*sinh(b\*x+a)<sup>2</sup>,x, algorithm="fricas")

[Out] 
$$-1/8*(4*b*x*cosh(m*log(x)) + (m+1)*cosh(m*log(2*b) + 2*a)*gamma(m+1, 2*b*x) - (m+1)*cosh(m*log(-2*b) - 2*a)*gamma(m+1, -2*b*x) - (m+1)*gamma(m+1, 2*b*x)*sinh(m*log(2*b) + 2*a) + (m+1)*gamma(m+1, -2*b*x)*sinh(m*log(-2*b) - 2*a) + 4*b*x*sinh(m*log(x)))/(b*m + b)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sinh^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*sinh(b\*x+a)\*\*2,x)

[Out] Integral(x\*\*m\*sinh(a + b\*x)\*\*2, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^m \sinh(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*sinh(b*x + a)^2, x)
```

### 3.89 $\int x^{-1+m} \sinh^2(a + bx) dx$

**Optimal.** Leaf size=72

$$e^{2a} (-2^{-m-2}) x^m (-bx)^{-m} \Gamma(m, -2bx) - e^{-2a} 2^{-m-2} x^m (bx)^{-m} \Gamma(m, 2bx) - \frac{x^m}{2m}$$

[Out]  $-x^m/(2*m) - (2^{(-2 - m)}*E^{(2*a)}*x^m*\Gamma[m, -2*b*x])/(-b*x)^m - (2^{(-2 - m)}*x^m*\Gamma[m, 2*b*x])/(E^{(2*a)}*(b*x)^m)$

**Rubi [A]** time = 0.126916, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3312, 3307, 2181}

$$e^{2a} (-2^{-m-2}) x^m (-bx)^{-m} \Gamma(m, -2bx) - e^{-2a} 2^{-m-2} x^m (bx)^{-m} \Gamma(m, 2bx) - \frac{x^m}{2m}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + m)\*Sinh[a + b\*x]^2,x]

[Out]  $-x^m/(2*m) - (2^{(-2 - m)}*E^{(2*a)}*x^m*\Gamma[m, -2*b*x])/(-b*x)^m - (2^{(-2 - m)}*x^m*\Gamma[m, 2*b*x])/(E^{(2*a)}*(b*x)^m)$

#### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_) ]^(n\_), x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

#### Rule 3307

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_) ], x\_Symbol] :> Dist[I/2, Int[(c + d\*x)^m/(E^(I\*k\*Pi)\*E^(I\*(e + f\*x))), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(I\*k\*Pi)\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x)])/ (d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F]

```
](c + d*x)/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

### Rubi steps

$$\begin{aligned}
 \int x^{-1+m} \sinh^2(a + bx) dx &= - \int \left( \frac{x^{-1+m}}{2} - \frac{1}{2} x^{-1+m} \cosh(2a + 2bx) \right) dx \\
 &= -\frac{x^m}{2m} + \frac{1}{2} \int x^{-1+m} \cosh(2a + 2bx) dx \\
 &= -\frac{x^m}{2m} + \frac{1}{4} \int e^{-i(2ia+2ibx)} x^{-1+m} dx + \frac{1}{4} \int e^{i(2ia+2ibx)} x^{-1+m} dx \\
 &= -\frac{x^m}{2m} - 2^{-2-m} e^{2a} x^m (-bx)^{-m} \Gamma(m, -2bx) - 2^{-2-m} e^{-2a} x^m (bx)^{-m} \Gamma(m, 2bx)
 \end{aligned}$$

**Mathematica [A]** time = 0.075868, size = 63, normalized size = 0.88

$$\frac{x^m \left( e^{2a} 2^{-m} m (-bx)^{-m} \text{Gamma}(m, -2bx) + e^{-2a} 2^{-m} m (bx)^{-m} \text{Gamma}(m, 2bx) + 2 \right)}{4m}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*Sinh[a + b*x]^2, x]
```

```
[Out] -(x^m*(2 + (E^(2*a)*m*Gamma[m, -2*b*x]))/(2^m*(-(b*x))^m) + (m*Gamma[m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)))/(4*m)
```

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int x^{-1+m} (\sinh(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*sinh(b*x+a)^2, x)
```

```
[Out] int(x^(-1+m)*sinh(b*x+a)^2, x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-1+m)</sup>\*sinh(b\*x+a)<sup>2</sup>,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.75443, size = 365, normalized size = 5.07

$$\frac{4bx \cosh((m-1)\log(x)) + m \cosh((m-1)\log(2b) + 2a)\Gamma(m, 2bx) - m \cosh((m-1)\log(-2b) - 2a)\Gamma(m, -2bx)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-1+m)</sup>\*sinh(b\*x+a)<sup>2</sup>,x, algorithm="fricas")

[Out] 
$$\frac{-1/8*(4*b*x*\cosh((m-1)*\log(x)) + m*\cosh((m-1)*\log(2*b) + 2*a)*\gamma(m, 2*b*x) - m*\cosh((m-1)*\log(-2*b) - 2*a)*\gamma(m, -2*b*x) - m*\gamma(m, 2*b*x)*\sinh((m-1)*\log(2*b) + 2*a) + m*\gamma(m, -2*b*x)*\sinh((m-1)*\log(-2*b) - 2*a) + 4*b*x*\sinh((m-1)*\log(x)))/(b*m)}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{m-1} \sinh^2(a + bx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-1+m)</sup>\*sinh(b\*x+a)<sup>2</sup>,x)

[Out] Integral(x<sup>(m-1)</sup>\*sinh(a + b\*x)<sup>2</sup>, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{m-1} \sinh(bx + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)*sinh(b*x + a)^2, x)
```

### 3.90 $\int x^{-2+m} \sinh^2(a + bx) dx$

**Optimal.** Leaf size=83

$$e^{2a}b2^{-m-1}x^m(-bx)^{-m}\Gamma(m-1, -2bx) - e^{-2a}b2^{-m-1}x^m(bx)^{-m}\Gamma(m-1, 2bx) + \frac{x^{m-1}}{2(1-m)}$$

[Out]  $x^{(-1+m)/(2*(1-m))} + (2^{(-1-m)*b}*E^{(2*a)}*x^m*\Gamma[-1+m, -2*b*x])/(-b*x)^m - (2^{(-1-m)*b*x^m*\Gamma[-1+m, 2*b*x]})/(E^{(2*a)}*(b*x)^m)$

**Rubi [A]** time = 0.136826, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3312, 3307, 2181}

$$e^{2a}b2^{-m-1}x^m(-bx)^{-m}\Gamma(m-1, -2bx) - e^{-2a}b2^{-m-1}x^m(bx)^{-m}\Gamma(m-1, 2bx) + \frac{x^{m-1}}{2(1-m)}$$

Antiderivative was successfully verified.

[In] Int[x^(-2 + m)\*Sinh[a + b\*x]^2, x]

[Out]  $x^{(-1+m)/(2*(1-m))} + (2^{(-1-m)*b}*E^{(2*a)}*x^m*\Gamma[-1+m, -2*b*x])/(-b*x)^m - (2^{(-1-m)*b*x^m*\Gamma[-1+m, 2*b*x]})/(E^{(2*a)}*(b*x)^m)$

#### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)^(n\_)], x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

#### Rule 3307

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Dist[I/2, Int[(c + d\*x)^m/(E^(I\*k\*Pi)\*E^(I\*(e + f\*x))), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(I\*k\*Pi)\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

#### Rule 2181

Int[(F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_)))\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] :> -Simp[(F^(g\*(e - (c\*f)/d)))\*(c + d\*x)^FracPart[m]\*Gamma[m + 1, (-((f\*g\*Log[F])/d))\*(c + d\*x)]/(d\*(-((f\*g\*Log[F])/d))^(IntPart[m] + 1)\*(-((f\*g\*Log[F]

`](*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]`

### Rubi steps

$$\begin{aligned}
 \int x^{-2+m} \sinh^2(a + bx) dx &= - \int \left( \frac{x^{-2+m}}{2} - \frac{1}{2} x^{-2+m} \cosh(2a + 2bx) \right) dx \\
 &= \frac{x^{-1+m}}{2(1-m)} + \frac{1}{2} \int x^{-2+m} \cosh(2a + 2bx) dx \\
 &= \frac{x^{-1+m}}{2(1-m)} + \frac{1}{4} \int e^{-i(2ia+2ibx)} x^{-2+m} dx + \frac{1}{4} \int e^{i(2ia+2ibx)} x^{-2+m} dx \\
 &= \frac{x^{-1+m}}{2(1-m)} + 2^{-1-m} b e^{2a} x^m (-bx)^{-m} \Gamma(-1+m, -2bx) - 2^{-1-m} b e^{-2a} x^m (bx)^{-m} \Gamma(-1+m, 2bx)
 \end{aligned}$$

**Mathematica [A]** time = 0.100091, size = 72, normalized size = 0.87

$$\frac{1}{2} x^m \left( e^{2a} b 2^{-m} (-bx)^{-m} \text{Gamma}(m-1, -2bx) - e^{-2a} b 2^{-m} (bx)^{-m} \text{Gamma}(m-1, 2bx) + \frac{1}{x - mx} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(-2 + m)\*Sinh[a + b\*x]^2,x]

[Out] (x^m\*((x - m\*x)^(-1) + (b\*E^(2\*a))\*Gamma[-1 + m, -2\*b\*x])/(2^m\*(-(b\*x))^m) - (b\*Gamma[-1 + m, 2\*b\*x])/(2^m\*E^(2\*a)\*(b\*x)^m))/2

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int x^{-2+m} (\sinh(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(-2+m)\*sinh(b\*x+a)^2,x)

[Out] int(x^(-2+m)\*sinh(b\*x+a)^2,x)



**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-2+m)</sup>\*sinh(b\*x+a)<sup>2</sup>,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.79577, size = 424, normalized size = 5.11

$$\frac{4bx \cosh((m-2)\log(x)) + (m-1) \cosh((m-2)\log(2b) + 2a) \Gamma(m-1, 2bx) - (m-1) \cosh((m-2)\log(-2b) - 2a) \Gamma(m-1, -2bx)}{(b^m - b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-2+m)</sup>\*sinh(b\*x+a)<sup>2</sup>,x, algorithm="fricas")

[Out] 
$$\frac{-1/8*(4*b*x*\cosh((m-2)*\log(x)) + (m-1)*\cosh((m-2)*\log(2*b) + 2*a)*\gamma(m-1, 2*b*x) - (m-1)*\cosh((m-2)*\log(-2*b) - 2*a)*\gamma(m-1, -2*b*x) - (m-1)*\gamma(m-1, 2*b*x)*\sinh((m-2)*\log(2*b) + 2*a) + (m-1)*\gamma(m-1, -2*b*x)*\sinh((m-2)*\log(-2*b) - 2*a) + 4*b*x*\sinh((m-2)*\log(x))}{(b^m - b)}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-2+m)</sup>\*sinh(b\*x+a)<sup>2</sup>,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{m-2} \sinh(bx+a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)*sinh(b*x + a)^2, x)
```

### 3.91 $\int x^{-3+m} \sinh^2(a + bx) dx$

**Optimal.** Leaf size=84

$$-e^{2a}b^22^{-m}x^m(-bx)^{-m}\Gamma(m-2,-2bx) - e^{-2a}b^22^{-m}x^m(bx)^{-m}\Gamma(m-2,2bx) + \frac{x^{m-2}}{2(2-m)}$$

[Out]  $x^{(-2+m)/(2*(2-m))} - (b^2E^{(2*a)}x^m\Gamma[-2+m, -2*b*x])/(2^m*(-(b*x))^m) - (b^2*x^m\Gamma[-2+m, 2*b*x])/(2^mE^{(2*a)}*(b*x)^m)$

**Rubi [A]** time = 0.142652, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$ , Rules used = {3312, 3307, 2181}

$$-e^{2a}b^22^{-m}x^m(-bx)^{-m}\Gamma(m-2,-2bx) - e^{-2a}b^22^{-m}x^m(bx)^{-m}\Gamma(m-2,2bx) + \frac{x^{m-2}}{2(2-m)}$$

Antiderivative was successfully verified.

[In] Int[x<sup>(-3 + m)</sup>\*Sinh[a + b\*x]<sup>2</sup>, x]

[Out]  $x^{(-2+m)/(2*(2-m))} - (b^2E^{(2*a)}x^m\Gamma[-2+m, -2*b*x])/(2^m*(-(b*x))^m) - (b^2*x^m\Gamma[-2+m, 2*b*x])/(2^mE^{(2*a)}*(b*x)^m)$

#### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_)</sup>\*sin[(e\_.) + (f\_.)\*(x\_)]<sup>(n\_)</sup>, x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)<sup>m</sup>, Sin[e + f\*x]<sup>n</sup>, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

#### Rule 3307

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_)</sup>\*sin[(e\_.) + Pi\*(k\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Dist[I/2, Int[(c + d\*x)<sup>m</sup>/(E<sup>(I\*k\*Pi)</sup>\*E<sup>(I\*(e + f\*x))</sup>), x], x] - Dist[I/2, Int[(c + d\*x)<sup>m</sup>\*E<sup>(I\*k\*Pi)</sup>\*E<sup>(I\*(e + f\*x))</sup>, x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2\*k]

#### Rule 2181

Int[(F\_)<sup>(g\_)</sup>\*((e\_.) + (f\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))<sup>(m\_)</sup>, x\_Symbol] :> -Simp[(F<sup>(g\*(e - (c\*f)/d))</sup>\*(c + d\*x)<sup>FracPart[m]</sup>\*Gamma[m + 1, (-(f\*g\*Log[F])/d)]\*(c + d\*x)]/(d\*(-(f\*g\*Log[F])/d))<sup>(IntPart[m] + 1)</sup>\*(-(f\*g\*Log[F]

```
]*(c + d*x)/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

### Rubi steps

$$\begin{aligned}
 \int x^{-3+m} \sinh^2(a + bx) dx &= - \int \left( \frac{x^{-3+m}}{2} - \frac{1}{2} x^{-3+m} \cosh(2a + 2bx) \right) dx \\
 &= \frac{x^{-2+m}}{2(2-m)} + \frac{1}{2} \int x^{-3+m} \cosh(2a + 2bx) dx \\
 &= \frac{x^{-2+m}}{2(2-m)} + \frac{1}{4} \int e^{-i(2ia+2ibx)} x^{-3+m} dx + \frac{1}{4} \int e^{i(2ia+2ibx)} x^{-3+m} dx \\
 &= \frac{x^{-2+m}}{2(2-m)} - 2^{-m} b^2 e^{2a} x^m (-bx)^{-m} \Gamma(-2+m, -2bx) - 2^{-m} b^2 e^{-2a} x^m (bx)^{-m} \Gamma(-2+m, 2bx)
 \end{aligned}$$

**Mathematica [A]** time = 0.111581, size = 77, normalized size = 0.92

$$x^m \left( e^{2a} b^2 (-2^{-m}) (-bx)^{-m} \text{Gamma}(m-2, -2bx) - e^{-2a} b^2 2^{-m} (bx)^{-m} \text{Gamma}(m-2, 2bx) + \frac{1}{(4-2m)x^2} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)*Sinh[a + b*x]^2, x]
```

```
[Out] x^m*(1/((4 - 2*m)*x^2) - (b^2*E^(2*a)*Gamma[-2 + m, -2*b*x])/(2^m*(-(b*x))^-m) - (b^2*Gamma[-2 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m))
```

**Maple [F]** time = 0.049, size = 0, normalized size = 0.

$$\int x^{-3+m} (\sinh(bx + a))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3+m)*sinh(b*x+a)^2, x)
```

```
[Out] int(x^(-3+m)*sinh(b*x+a)^2, x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-3+m)</sup>\*sinh(b\*x+a)<sup>2</sup>,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.78067, size = 427, normalized size = 5.08

$$\frac{4bx \cosh((m-3)\log(x)) + (m-2) \cosh((m-3)\log(2b) + 2a) \Gamma(m-2, 2bx) - (m-2) \cosh((m-3)\log(-2b) - 2a) \Gamma(m-2, -2bx)}{(b^m - 2^m)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-3+m)</sup>\*sinh(b\*x+a)<sup>2</sup>,x, algorithm="fricas")

[Out] 
$$\frac{-1/8*(4*b*x*cosh((m-3)*log(x)) + (m-2)*cosh((m-3)*log(2*b) + 2*a)*gamma(m-2, 2*b*x) - (m-2)*cosh((m-3)*log(-2*b) - 2*a)*gamma(m-2, -2*b*x) - (m-2)*gamma(m-2, 2*b*x)*sinh((m-3)*log(2*b) + 2*a) + (m-2)*gamma(m-2, -2*b*x)*sinh((m-3)*log(-2*b) - 2*a) + 4*b*x*sinh((m-3)*log(x)))/(b^m - 2^m)}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x<sup>(-3+m)</sup>\*sinh(b\*x+a)<sup>2</sup>,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int x^{m-3} \sinh(bx+a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*sinh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 3)*sinh(b*x + a)^2, x)
```

$$3.92 \quad \int \left( \frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)} + \frac{1}{3} x \sqrt{\operatorname{csch}(x)} \right) dx$$

**Optimal.** Leaf size=24

$$\frac{2x \cosh(x)}{3\sqrt{\operatorname{csch}(x)}} - \frac{4}{9\operatorname{csch}^{\frac{3}{2}}(x)}$$

[Out]  $-4/(9*\operatorname{Csch}[x]^{(3/2)}) + (2*x*\operatorname{Cosh}[x])/(3*\operatorname{Sqrt}[\operatorname{Csch}[x]])$

**Rubi [A]** time = 0.111688, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {4187, 4189}

$$\frac{2x \cosh(x)}{3\sqrt{\operatorname{csch}(x)}} - \frac{4}{9\operatorname{csch}^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x/\operatorname{Csch}[x]^{(3/2)} + (x*\operatorname{Sqrt}[\operatorname{Csch}[x]])/3, x]$

[Out]  $-4/(9*\operatorname{Csch}[x]^{(3/2)}) + (2*x*\operatorname{Cosh}[x])/(3*\operatorname{Sqrt}[\operatorname{Csch}[x]])$

#### Rule 4187

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_.)), x_Symbol] :>
Simp[(d*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[((c + d*x)*Cos[e + f*x]*(b*C
sc[e + f*x])^(n + 1))/(b*f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

#### Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
+ f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] && !IntegerQ[n]
```

#### Rubi steps

$$\begin{aligned}
\int \left( \frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)} + \frac{1}{3} x \sqrt{\operatorname{csch}(x)} \right) dx &= \frac{1}{3} \int x \sqrt{\operatorname{csch}(x)} dx + \int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)} dx \\
&= -\frac{4}{9 \operatorname{csch}^{\frac{3}{2}}(x)} + \frac{2x \cosh(x)}{3 \sqrt{\operatorname{csch}(x)}} - \frac{1}{3} \int x \sqrt{\operatorname{csch}(x)} dx + \frac{1}{3} \left( \sqrt{\operatorname{csch}(x)} \sqrt{-\sinh(x)} \right) \int \frac{1}{\sqrt{-\sinh(x)}} dx \\
&= -\frac{4}{9 \operatorname{csch}^{\frac{3}{2}}(x)} + \frac{2x \cosh(x)}{3 \sqrt{\operatorname{csch}(x)}}
\end{aligned}$$

**Mathematica [A]** time = 0.0940987, size = 17, normalized size = 0.71

$$\frac{2(3x \operatorname{coth}(x) - 2)}{9 \operatorname{csch}^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

[In] Integrate[x/Csch[x]^(3/2) + (x\*Sqrt[Csch[x]])/3,x]

[Out] (2\*(-2 + 3\*x\*Coth[x]))/(9\*Csch[x]^(3/2))

**Maple [F]** time = 0.077, size = 0, normalized size = 0.

$$\int x (\operatorname{csch}(x))^{-\frac{3}{2}} + \frac{x}{3} \sqrt{\operatorname{csch}(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/csch(x)^(3/2)+1/3\*x\*csch(x)^(1/2),x)

[Out] int(x/csch(x)^(3/2)+1/3\*x\*csch(x)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{3} x \sqrt{\operatorname{csch}(x)} + \frac{x}{\operatorname{csch}(x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(x/csch(x)^(3/2)+1/3\*x\*csch(x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/3\*x\*sqrt(csch(x)) + x/csch(x)^(3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/csch(x)^(3/2)+1/3\*x\*csch(x)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{3x}{\operatorname{csch}^2(x)} dx + \int x\sqrt{\operatorname{csch}(x)} dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/csch(x)\*\*(3/2)+1/3\*x\*csch(x)\*\*(1/2),x)

[Out] (Integral(3\*x/csch(x)\*\*(3/2), x) + Integral(x\*sqrt(csch(x)), x))/3

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{3} x\sqrt{\operatorname{csch}(x)} + \frac{x}{\operatorname{csch}(x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/csch(x)^(3/2)+1/3\*x\*csch(x)^(1/2),x, algorithm="giac")

[Out] integrate(1/3\*x\*sqrt(csch(x)) + x/csch(x)^(3/2), x)

$$3.93 \quad \int \left( \frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)} + \frac{3x}{5\sqrt{\operatorname{csch}(x)}} \right) dx$$

**Optimal.** Leaf size=24

$$\frac{2x \cosh(x)}{5\operatorname{csch}^{\frac{3}{2}}(x)} - \frac{4}{25\operatorname{csch}^{\frac{5}{2}}(x)}$$

[Out] -4/(25\*Csch[x]^(5/2)) + (2\*x\*Cosh[x])/(5\*Csch[x]^(3/2))

**Rubi [A]** time = 0.117555, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {4187, 4189}

$$\frac{2x \cosh(x)}{5\operatorname{csch}^{\frac{3}{2}}(x)} - \frac{4}{25\operatorname{csch}^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

[In] Int[x/Csch[x]^(5/2) + (3\*x)/(5\*Sqrt[Csch[x]]), x]

[Out] -4/(25\*Csch[x]^(5/2)) + (2\*x\*Cosh[x])/(5\*Csch[x]^(3/2))

#### Rule 4187

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_.)), x_Symbol] :>
  Simp[(d*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[((c + d*x)*Cos[e + f*x]*(b*C
sc[e + f*x])^(n + 1))/(b*f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

#### Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
+ f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] && !IntegerQ[n]
```

#### Rubi steps

$$\begin{aligned}
\int \left( \frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)} + \frac{3x}{5\sqrt{\operatorname{csch}(x)}} \right) dx &= \frac{3}{5} \int \frac{x}{\sqrt{\operatorname{csch}(x)}} dx + \int \frac{x}{\operatorname{csch}^{\frac{5}{2}}(x)} dx \\
&= -\frac{4}{25\operatorname{csch}^{\frac{5}{2}}(x)} + \frac{2x \cosh(x)}{5\operatorname{csch}^{\frac{3}{2}}(x)} - \frac{3}{5} \int \frac{x}{\sqrt{\operatorname{csch}(x)}} dx + \frac{3 \int x \sqrt{-\sinh(x)} dx}{5\sqrt{\operatorname{csch}(x)}\sqrt{-\sinh(x)}} \\
&= -\frac{4}{25\operatorname{csch}^{\frac{5}{2}}(x)} + \frac{2x \cosh(x)}{5\operatorname{csch}^{\frac{3}{2}}(x)}
\end{aligned}$$

**Mathematica [A]** time = 0.123822, size = 17, normalized size = 0.71

$$\frac{2(5x \coth(x) - 2)}{25\operatorname{csch}^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

[In] Integrate[x/Csch[x]^(5/2) + (3\*x)/(5\*Sqrt[Csch[x]]), x]

[Out] (2\*(-2 + 5\*x\*Coth[x]))/(25\*Csch[x]^(5/2))

**Maple [F]** time = 0.073, size = 0, normalized size = 0.

$$\int x (\operatorname{csch}(x))^{-\frac{5}{2}} + \frac{3x}{5} \frac{1}{\sqrt{\operatorname{csch}(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/csch(x)^(5/2)+3/5\*x/csch(x)^(1/2), x)

[Out] int(x/csch(x)^(5/2)+3/5\*x/csch(x)^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{3x}{5\sqrt{\operatorname{csch}(x)}} + \frac{x}{\operatorname{csch}(x)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(3/5*x/sqrt(csch(x)) + x/csch(x)^(5/2), x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{5x}{\operatorname{csch}^2(x)} dx + \int \frac{3x}{\sqrt{\operatorname{csch}(x)}} dx}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/csch(x)**(5/2)+3/5*x/csch(x)**(1/2),x)`

[Out] `(Integral(5*x/csch(x)**(5/2), x) + Integral(3*x/sqrt(csch(x)), x))/5`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{3x}{5\sqrt{\operatorname{csch}(x)}} + \frac{x}{\operatorname{csch}(x)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/csch(x)^(5/2)+3/5*x/csch(x)^(1/2),x, algorithm="giac")`

[Out] `integrate(3/5*x/sqrt(csch(x)) + x/csch(x)^(5/2), x)`

$$3.94 \quad \int \left( \frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)} - \frac{5}{21} x \sqrt{\operatorname{csch}(x)} \right) dx$$

**Optimal.** Leaf size=47

$$\frac{20}{63 \operatorname{csch}^{\frac{3}{2}}(x)} - \frac{4}{49 \operatorname{csch}^{\frac{7}{2}}(x)} + \frac{2x \cosh(x)}{7 \operatorname{csch}^{\frac{5}{2}}(x)} - \frac{10x \cosh(x)}{21 \sqrt{\operatorname{csch}(x)}}$$

[Out]  $-4/(49*\operatorname{Csch}[x]^{(7/2)}) + (2*x*\operatorname{Cosh}[x])/(7*\operatorname{Csch}[x]^{(5/2)}) + 20/(63*\operatorname{Csch}[x]^{(3/2)}) - (10*x*\operatorname{Cosh}[x])/(21*\operatorname{Sqrt}[\operatorname{Csch}[x]])$

**Rubi [A]** time = 0.129631, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {4187, 4189}

$$\frac{20}{63 \operatorname{csch}^{\frac{3}{2}}(x)} - \frac{4}{49 \operatorname{csch}^{\frac{7}{2}}(x)} + \frac{2x \cosh(x)}{7 \operatorname{csch}^{\frac{5}{2}}(x)} - \frac{10x \cosh(x)}{21 \sqrt{\operatorname{csch}(x)}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x/\operatorname{Csch}[x]^{(7/2)} - (5*x*\operatorname{Sqrt}[\operatorname{Csch}[x]])/21, x]$

[Out]  $-4/(49*\operatorname{Csch}[x]^{(7/2)}) + (2*x*\operatorname{Cosh}[x])/(7*\operatorname{Csch}[x]^{(5/2)}) + 20/(63*\operatorname{Csch}[x]^{(3/2)}) - (10*x*\operatorname{Cosh}[x])/(21*\operatorname{Sqrt}[\operatorname{Csch}[x]])$

#### Rule 4187

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :=
  Simp[(d*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[((c + d*x)*Cos[e + f*x]*(b*C
sc[e + f*x])^(n + 1))/(b*f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

#### Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] := Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
+ f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] && !IntegerQ[n]
```

#### Rubi steps

$$\begin{aligned}
\int \left( \frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)} - \frac{5}{21} x \sqrt{\operatorname{csch}(x)} \right) dx &= - \left( \frac{5}{21} \int x \sqrt{\operatorname{csch}(x)} dx \right) + \int \frac{x}{\operatorname{csch}^{\frac{7}{2}}(x)} dx \\
&= - \frac{4}{49 \operatorname{csch}^{\frac{7}{2}}(x)} + \frac{2x \cosh(x)}{7 \operatorname{csch}^{\frac{5}{2}}(x)} - \frac{5}{7} \int \frac{x}{\operatorname{csch}^{\frac{3}{2}}(x)} dx - \frac{1}{21} \left( 5 \sqrt{\operatorname{csch}(x)} \sqrt{-\sinh(x)} \right) \int \frac{1}{\sqrt{-\sinh(x)}} dx \\
&= - \frac{4}{49 \operatorname{csch}^{\frac{7}{2}}(x)} + \frac{2x \cosh(x)}{7 \operatorname{csch}^{\frac{5}{2}}(x)} + \frac{20}{63 \operatorname{csch}^{\frac{3}{2}}(x)} - \frac{10x \cosh(x)}{21 \sqrt{\operatorname{csch}(x)}} + \frac{5}{21} \int x \sqrt{\operatorname{csch}(x)} dx - \\
&= - \frac{4}{49 \operatorname{csch}^{\frac{7}{2}}(x)} + \frac{2x \cosh(x)}{7 \operatorname{csch}^{\frac{5}{2}}(x)} + \frac{20}{63 \operatorname{csch}^{\frac{3}{2}}(x)} - \frac{10x \cosh(x)}{21 \sqrt{\operatorname{csch}(x)}}
\end{aligned}$$

**Mathematica [A]** time = 0.109528, size = 45, normalized size = 0.96

$$\sqrt{\operatorname{csch}(x)} \left( -\frac{13}{42} x \sinh(2x) + \frac{1}{28} x \sinh(4x) + \frac{88}{441} \cosh(2x) - \frac{1}{98} \cosh(4x) - \frac{167}{882} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/Csch[x]^(7/2) - (5\*x\*Sqrt[Csch[x]])/21,x]

[Out] Sqrt[Csch[x]]\*(-167/882 + (88\*Cosh[2\*x])/441 - Cosh[4\*x]/98 - (13\*x\*Sinh[2\*x])/42 + (x\*Sinh[4\*x])/28)

**Maple [F]** time = 0.081, size = 0, normalized size = 0.

$$\int x (\operatorname{csch}(x))^{-\frac{7}{2}} - \frac{5x}{21} \sqrt{\operatorname{csch}(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/csch(x)^(7/2)-5/21\*x\*csch(x)^(1/2),x)

[Out] int(x/csch(x)^(7/2)-5/21\*x\*csch(x)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{5}{21} x \sqrt{\operatorname{csch}(x)} + \frac{x}{\operatorname{csch}(x)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x, algorithm="maxima")`

[Out] `integrate(-5/21*x*sqrt(csch(x)) + x/csch(x)^(7/2), x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int -\frac{21x}{\operatorname{csch}^2(x)} dx + \int 5x\sqrt{\operatorname{csch}(x)} dx}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/csch(x)**(7/2)-5/21*x*csch(x)**(1/2),x)`

[Out] `-(Integral(-21*x/csch(x)**(7/2), x) + Integral(5*x*sqrt(csch(x)), x))/21`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{5}{21} x\sqrt{\operatorname{csch}(x)} + \frac{x}{\operatorname{csch}(x)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/csch(x)^(7/2)-5/21*x*csch(x)^(1/2),x, algorithm="giac")`

[Out] `integrate(-5/21*x*sqrt(csch(x)) + x/csch(x)^(7/2), x)`

$$3.95 \quad \int \left( \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)} + \frac{1}{3}x^2\sqrt{\operatorname{csch}(x)} \right) dx$$

**Optimal.** Leaf size=76

$$-\frac{16}{27}i\sqrt{i\sinh(x)}\sqrt{\operatorname{csch}(x)}\operatorname{EllipticF}\left(\frac{\pi}{4} - \frac{ix}{2}, 2\right) + \frac{2x^2\cosh(x)}{3\sqrt{\operatorname{csch}(x)}} - \frac{8x}{9\operatorname{csch}^{\frac{3}{2}}(x)} + \frac{16\cosh(x)}{27\sqrt{\operatorname{csch}(x)}}$$

[Out] (-8\*x)/(9\*Csch[x]^(3/2)) + (16\*Cosh[x])/(27\*Sqrt[Csch[x]]) + (2\*x^2\*Cosh[x])/(3\*Sqrt[Csch[x]]) - ((16\*I)/27)\*Sqrt[Csch[x]]\*EllipticF[Pi/4 - (I/2)\*x, 2]\*Sqrt[I\*Sinh[x]]

**Rubi [A]** time = 0.207091, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {4188, 4189, 3769, 3771, 2641}

$$\frac{2x^2\cosh(x)}{3\sqrt{\operatorname{csch}(x)}} - \frac{8x}{9\operatorname{csch}^{\frac{3}{2}}(x)} + \frac{16\cosh(x)}{27\sqrt{\operatorname{csch}(x)}} - \frac{16}{27}i\sqrt{i\sinh(x)}\sqrt{\operatorname{csch}(x)}F\left(\frac{\pi}{4} - \frac{ix}{2} \middle| 2\right)$$

Antiderivative was successfully verified.

[In] Int[x^2/Csch[x]^(3/2) + (x^2\*Sqrt[Csch[x]])/3, x]

[Out] (-8\*x)/(9\*Csch[x]^(3/2)) + (16\*Cosh[x])/(27\*Sqrt[Csch[x]]) + (2\*x^2\*Cosh[x])/(3\*Sqrt[Csch[x]]) - ((16\*I)/27)\*Sqrt[Csch[x]]\*EllipticF[Pi/4 - (I/2)\*x, 2]\*Sqrt[I\*Sinh[x]]

### Rule 4188

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(n + 1)/(b^2*n), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n + 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^n, x], x]
+ Simp[((c + d*x)^m*Cos[e + f*x]*(b*Csc[e + f*x])^(n + 1))/(b*f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
```

### Rule 4189

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
```



+ f\*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] && !IntegerQ[n]

### Rule 3769

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^n\_], x\_Symbol] := Simp[(Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n + 1))/(b\*d\*n), x] + Dist[(n + 1)/(b^2\*n), Int[(b\*Csc[c + d\*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2\*n]

### Rule 3771

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^n\_], x\_Symbol] := Dist[(b\*Csc[c + d\*x])^n\*Sin[c + d\*x]^n, Int[1/Sin[c + d\*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

### Rule 2641

Int[1/Sqrt[sin[(c\_.) + (d\_.)\*(x\_)]], x\_Symbol] := Simp[(2\*EllipticF[(1\*(c - Pi/2 + d\*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned}
 \int \left( \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)} + \frac{1}{3} x^2 \sqrt{\operatorname{csch}(x)} \right) dx &= \frac{1}{3} \int x^2 \sqrt{\operatorname{csch}(x)} dx + \int \frac{x^2}{\operatorname{csch}^{\frac{3}{2}}(x)} dx \\
 &= -\frac{8x}{9\operatorname{csch}^{\frac{3}{2}}(x)} + \frac{2x^2 \cosh(x)}{3\sqrt{\operatorname{csch}(x)}} - \frac{1}{3} \int x^2 \sqrt{\operatorname{csch}(x)} dx + \frac{8}{9} \int \frac{1}{\operatorname{csch}^{\frac{3}{2}}(x)} dx + \frac{1}{3} \left( \sqrt{\operatorname{csch}(x)} \right) \\
 &= -\frac{8x}{9\operatorname{csch}^{\frac{3}{2}}(x)} + \frac{16 \cosh(x)}{27\sqrt{\operatorname{csch}(x)}} + \frac{2x^2 \cosh(x)}{3\sqrt{\operatorname{csch}(x)}} - \frac{8}{27} \int \sqrt{\operatorname{csch}(x)} dx \\
 &= -\frac{8x}{9\operatorname{csch}^{\frac{3}{2}}(x)} + \frac{16 \cosh(x)}{27\sqrt{\operatorname{csch}(x)}} + \frac{2x^2 \cosh(x)}{3\sqrt{\operatorname{csch}(x)}} - \frac{1}{27} \left( 8\sqrt{\operatorname{csch}(x)}\sqrt{i \sinh(x)} \right) \int \frac{1}{\sqrt{i \sinh(x)}} dx \\
 &= -\frac{8x}{9\operatorname{csch}^{\frac{3}{2}}(x)} + \frac{16 \cosh(x)}{27\sqrt{\operatorname{csch}(x)}} + \frac{2x^2 \cosh(x)}{3\sqrt{\operatorname{csch}(x)}} - \frac{16}{27} i \sqrt{\operatorname{csch}(x)} F\left(\frac{\pi}{4} - \frac{ix}{2} \middle| 2\right) \sqrt{i \sinh(x)}
 \end{aligned}$$

**Mathematica [A]** time = 0.143932, size = 63, normalized size = 0.83

$$\frac{1}{27} \sqrt{\operatorname{csch}(x)} \left( -16i \sqrt{i \sinh(x)} \operatorname{EllipticF}\left(\frac{1}{4}(\pi - 2ix), 2\right) + 9x^2 \sinh(2x) + 12x + 8 \sinh(2x) - 12x \cosh(2x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Csch[x]^(3/2) + (x^2\*Sqrt[Csch[x]])/3,x]

[Out] (Sqrt[Csch[x]]\*(12\*x - 12\*x\*Cosh[2\*x] - (16\*I)\*EllipticF[(Pi - (2\*I)\*x)/4, 2]\*Sqrt[I\*Sinh[x]] + 8\*Sinh[2\*x] + 9\*x^2\*Sinh[2\*x]))/27

**Maple [F]** time = 0.071, size = 0, normalized size = 0.

$$\int x^2 (\operatorname{csch}(x))^{-\frac{3}{2}} + \frac{x^2}{3} \sqrt{\operatorname{csch}(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/csch(x)^(3/2)+1/3\*x^2\*csch(x)^(1/2),x)

[Out] int(x^2/csch(x)^(3/2)+1/3\*x^2\*csch(x)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{3} x^2 \sqrt{\operatorname{csch}(x)} + \frac{x^2}{\operatorname{csch}(x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/csch(x)^(3/2)+1/3\*x^2\*csch(x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/3\*x^2\*sqrt(csch(x)) + x^2/csch(x)^(3/2), x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/csch(x)^(3/2)+1/3\*x^2\*csch(x)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{3x^2}{\operatorname{csch}^2(x)} dx + \int x^2 \sqrt{\operatorname{csch}(x)} dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/csch(x)\*\*(3/2)+1/3\*x\*\*2\*csch(x)\*\*(1/2), x)

[Out] (Integral(3\*x\*\*2/csch(x)\*\*(3/2), x) + Integral(x\*\*2\*sqrt(csch(x)), x))/3

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{3} x^2 \sqrt{\operatorname{csch}(x)} + \frac{x^2}{\operatorname{csch}(x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/csch(x)^(3/2)+1/3\*x^2\*csch(x)^(1/2), x, algorithm="giac")

[Out] integrate(1/3\*x^2\*sqrt(csch(x)) + x^2/csch(x)^(3/2), x)

### 3.96 $\int (c + dx)^3 (a + ia \sinh(e + fx)) dx$

**Optimal.** Leaf size=98

$$\frac{6iad^2(c + dx) \cosh(e + fx)}{f^3} - \frac{3iad(c + dx)^2 \sinh(e + fx)}{f^2} + \frac{ia(c + dx)^3 \cosh(e + fx)}{f} + \frac{a(c + dx)^4}{4d} - \frac{6iad^3 \sinh(e + fx)}{f^4}$$

[Out] (a\*(c + d\*x)^4)/(4\*d) + ((6\*I)\*a\*d^2\*(c + d\*x)\*Cosh[e + f\*x])/f^3 + (I\*a\*(c + d\*x)^3\*Cosh[e + f\*x])/f - ((6\*I)\*a\*d^3\*Sinh[e + f\*x])/f^4 - ((3\*I)\*a\*d\*(c + d\*x)^2\*Sinh[e + f\*x])/f^2

**Rubi [A]** time = 0.139952, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3317, 3296, 2637}

$$\frac{6iad^2(c + dx) \cosh(e + fx)}{f^3} - \frac{3iad(c + dx)^2 \sinh(e + fx)}{f^2} + \frac{ia(c + dx)^3 \cosh(e + fx)}{f} + \frac{a(c + dx)^4}{4d} - \frac{6iad^3 \sinh(e + fx)}{f^4}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^3\*(a + I\*a\*Sinh[e + f\*x]),x]

[Out] (a\*(c + d\*x)^4)/(4\*d) + ((6\*I)\*a\*d^2\*(c + d\*x)\*Cosh[e + f\*x])/f^3 + (I\*a\*(c + d\*x)^3\*Cosh[e + f\*x])/f - ((6\*I)\*a\*d^3\*Sinh[e + f\*x])/f^4 - ((3\*I)\*a\*d\*(c + d\*x)^2\*Sinh[e + f\*x])/f^2

#### Rule 3317

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*Sin[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int (c + dx)^3 (a + ia \sinh(e + fx)) dx &= \int (a(c + dx)^3 + ia(c + dx)^3 \sinh(e + fx)) dx \\
&= \frac{a(c + dx)^4}{4d} + (ia) \int (c + dx)^3 \sinh(e + fx) dx \\
&= \frac{a(c + dx)^4}{4d} + \frac{ia(c + dx)^3 \cosh(e + fx)}{f} - \frac{(3iad) \int (c + dx)^2 \cosh(e + fx) dx}{f} \\
&= \frac{a(c + dx)^4}{4d} + \frac{ia(c + dx)^3 \cosh(e + fx)}{f} - \frac{3iad(c + dx)^2 \sinh(e + fx)}{f^2} + \frac{(6iad^2) \int (c + dx) \cosh(e + fx) dx}{f^2} \\
&= \frac{a(c + dx)^4}{4d} + \frac{6iad^2(c + dx) \cosh(e + fx)}{f^3} + \frac{ia(c + dx)^3 \cosh(e + fx)}{f} - \frac{3iad(c + dx)^2 \sinh(e + fx)}{f^2} \\
&= \frac{a(c + dx)^4}{4d} + \frac{6iad^2(c + dx) \cosh(e + fx)}{f^3} + \frac{ia(c + dx)^3 \cosh(e + fx)}{f} - \frac{6iad^3 \sinh(e + fx)}{f^4}
\end{aligned}$$

**Mathematica [A]** time = 0.832429, size = 128, normalized size = 1.31

$$\frac{a(-12id(c^2 f^2 + 2cdf^2 x + d^2(f^2 x^2 + 2)) \sinh(e + fx) + 4if(c + dx)(c^2 f^2 + 2cdf^2 x + d^2(f^2 x^2 + 6)) \cosh(e + fx) + f^4)}{4f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + I*a*Sinh[e + f*x]),x]
```

```
[Out] (a*(f^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + (4*I)*f*(c + d*x)*(c^2*f^2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Cosh[e + f*x] - (12*I)*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x]))/(4*f^4)
```

**Maple [B]** time = 0.01, size = 494, normalized size = 5.

$$\frac{1}{f} \left( \frac{d^3 a (fx + e)^4}{4 f^3} - \frac{3 id e c^2 a \cosh(fx + e)}{f} - \frac{d^3 e a (fx + e)^3}{f^3} - \frac{3 id^3 e a ((fx + e)^2 \cosh(fx + e) - 2 (fx + e) \sinh(fx + e))}{f^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3\*(a+I\*a\*sinh(f\*x+e)),x)

[Out]  $\frac{1}{f} \left( \frac{1}{4} f^3 d^3 a (f*x+e)^4 - 3 I d e / f c^2 a \cosh(f*x+e) - \frac{1}{f^3 d^3 e a (f*x+e)^3} - 3 I / f^3 d^3 e a ((f*x+e)^2 \cosh(f*x+e) - 2 (f*x+e) \sinh(f*x+e) + 2 \cosh(f*x+e)) + \frac{1}{f^2 d^2 c a (f*x+e)^3} - I d^3 e^3 / f^3 a \cosh(f*x+e) + \frac{3}{2} / f^3 d^3 e^2 a (f*x+e)^2 - 6 I / f^2 d^2 e c a ((f*x+e) \cosh(f*x+e) - \sinh(f*x+e)) - \frac{3}{f^2 d^2 e c a (f*x+e)^2} + 3 I d^2 e^2 / f^2 c a \cosh(f*x+e) + \frac{3}{2} / f d c^2 a (f*x+e)^2 + 3 I / f^2 d^2 c a ((f*x+e)^2 \cosh(f*x+e) - 2 (f*x+e) \sinh(f*x+e) + 2 \cosh(f*x+e)) - d^3 e^3 / f^3 a (f*x+e) + I c^3 a \cosh(f*x+e) + 3 d^2 e^2 / f^2 c a (f*x+e) + 3 I / f^3 d^3 e^2 a ((f*x+e) \cosh(f*x+e) - \sinh(f*x+e)) - 3 d e / f c^2 a (f*x+e) + 3 I / f d c^2 a ((f*x+e) \cosh(f*x+e) - \sinh(f*x+e)) + c^3 a (f*x+e) + I / f^3 d^3 a ((f*x+e)^3 \cosh(f*x+e) - 3 (f*x+e)^2 \sinh(f*x+e) + 6 (f*x+e) \cosh(f*x+e) - 6 \sinh(f*x+e)) \right)$

**Maxima [B]** time = 1.0986, size = 317, normalized size = 3.23

$$\frac{1}{4} a d^3 x^4 + a c d^2 x^3 + \frac{3}{2} a c^2 d x^2 + a c^3 x + \frac{3}{2} i a c^2 d \left( \frac{(f x e^e - e^e) e^{(f x)}}{f^2} + \frac{(f x + 1) e^{(-f x - e)}}{f^2} \right) + \frac{3}{2} i a c d^2 \left( \frac{(f^2 x^2 e^e - 2 f x e^e + 2 e^e) e^{(f x)}}{f^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*(a+I\*a\*sinh(f\*x+e)),x, algorithm="maxima")

[Out]  $\frac{1}{4} a d^3 x^4 + a c d^2 x^3 + \frac{3}{2} a c^2 d x^2 + a c^3 x + \frac{3}{2} I a c^2 d \left( (f x e^e - e^e) e^{(f x)} / f^2 + (f x + 1) e^{(-f x - e)} / f^2 \right) + \frac{3}{2} I a c d^2 \left( (f^2 x^2 e^e - 2 f x e^e + 2 e^e) e^{(f x)} / f^3 + (f^2 x^2 + 2 f x + 2) e^{(-f x - e)} / f^3 \right) + \frac{1}{2} I a d^3 \left( (f^3 x^3 e^e - 3 f^2 x^2 e^e + 6 f x e^e - 6 e^e) e^{(f x)} / f^4 + (f^3 x^3 + 3 f^2 x^2 + 6 f x + 6) e^{(-f x - e)} / f^4 \right) + I a c^3 \cosh(f x + e) / f$

**Fricas [B]** time = 2.78733, size = 656, normalized size = 6.69

$$\left( 2i ad^3 f^3 x^3 + 2i ac^3 f^3 + 6i ac^2 d f^2 + 12i acd^2 f + 12i ad^3 + (6i acd^2 f^3 + 6i ad^3 f^2) x^2 + (6i ac^2 d f^3 + 12i acd^2 f^2 + 12i ad^3 f \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*(a+I\*a\*sinh(f\*x+e)),x, algorithm="fricas")

```
[Out] 1/4*(2*I*a*d^3*f^3*x^3 + 2*I*a*c^3*f^3 + 6*I*a*c^2*d*f^2 + 12*I*a*c*d^2*f +
12*I*a*d^3 + (6*I*a*c*d^2*f^3 + 6*I*a*d^3*f^2)*x^2 + (6*I*a*c^2*d*f^3 + 12
*I*a*c*d^2*f^2 + 12*I*a*d^3*f)*x + (2*I*a*d^3*f^3*x^3 + 2*I*a*c^3*f^3 - 6*I
*a*c^2*d*f^2 + 12*I*a*c*d^2*f - 12*I*a*d^3 + (6*I*a*c*d^2*f^3 - 6*I*a*d^3*f
^2)*x^2 + (6*I*a*c^2*d*f^3 - 12*I*a*c*d^2*f^2 + 12*I*a*d^3*f)*x)*e^(2*f*x +
2*e) + (a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^
4*x)*e^(f*x + e))*e^(-f*x - e)/f^4
```

---

**Sympy [A]** time = 2.22986, size = 573, normalized size = 5.85

$$ac^3x + \frac{3ac^2dx^2}{2} + acd^2x^3 + \frac{ad^3x^4}{4} + \left\{ \frac{((2iac^3f^{19}e^{3e} + 6iac^2df^{19}xe^{3e} + 6iac^2df^{18}e^{3e} + 6iacd^2f^{19}x^2e^{3e} + 12iacd^2f^{18}xe^{3e} + 12iacd^2f^{17}e^{3e} + 2iad^3f^{19}x^3e^{3e} + 2iad^3f^{19}x^3e^{3e}))}{x^4(iad^3e^{2e} - iad^3)e^{-e}} + \frac{x^3(iacd^2e^{2e} - iacd^2)e^{-e}}{2} + \frac{x^2(3iac^2de^{2e} - 3iac^2d)e^{-e}}{4} + \frac{x(iac^3e^{2e} - iac^3)e^{-e}}{2} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+I*a*sinh(f*x+e)),x)
```

```
[Out] a*c**3*x + 3*a*c**2*d*x**2/2 + a*c*d**2*x**3 + a*d**3*x**4/4 + Piecewise(((
(2*I*a*c**3*f**19*exp(3*e) + 6*I*a*c**2*d*f**19*x*exp(3*e) + 6*I*a*c**2*d*f
**18*exp(3*e) + 6*I*a*c*d**2*f**19*x**2*exp(3*e) + 12*I*a*c*d**2*f**18*x*ex
p(3*e) + 12*I*a*c*d**2*f**17*exp(3*e) + 2*I*a*d**3*f**19*x**3*exp(3*e) + 6*
I*a*d**3*f**18*x**2*exp(3*e) + 12*I*a*d**3*f**17*x*exp(3*e) + 12*I*a*d**3*f
**16*exp(3*e))*exp(-f*x) + (2*I*a*c**3*f**19*exp(5*e) + 6*I*a*c**2*d*f**19*
x*exp(5*e) - 6*I*a*c**2*d*f**18*exp(5*e) + 6*I*a*c*d**2*f**19*x**2*exp(5*e)
- 12*I*a*c*d**2*f**18*x*exp(5*e) + 12*I*a*c*d**2*f**17*exp(5*e) + 2*I*a*d*
**3*f**19*x**3*exp(5*e) - 6*I*a*d**3*f**18*x**2*exp(5*e) + 12*I*a*d**3*f**17
*x*exp(5*e) - 12*I*a*d**3*f**16*exp(5*e))*exp(f*x))*exp(-4*e)/(4*f**20), Ne
(4*f**20*exp(4*e), 0)), (x**4*(I*a*d**3*exp(2*e) - I*a*d**3)*exp(-e)/8 + x*
**3*(I*a*c*d**2*exp(2*e) - I*a*c*d**2)*exp(-e)/2 + x**2*(3*I*a*c**2*d*exp(2*
e) - 3*I*a*c**2*d)*exp(-e)/4 + x*(I*a*c**3*exp(2*e) - I*a*c**3)*exp(-e)/2,
True))
```

---

**Giac [B]** time = 1.29727, size = 356, normalized size = 3.63

$$\frac{1}{4} ad^3x^4 + acd^2x^3 + \frac{3}{2} ac^2dx^2 + ac^3x - \frac{(-iad^3f^3x^3 - 3iacd^2f^3x^2 - 3iac^2df^3x + 3iad^3f^2x^2 - iac^3f^3 + 6iacd^2f^2x + 3iad^3f^3x^3 - 3iacd^2f^3x^2 - 3iac^2df^3x + 3iad^3f^2x^2 - iac^3f^3 + 6iacd^2f^2x + 3iad^3f^3x^3)}{2f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x - 1/2*(-I*a*d^3*f^3*x^3 - 3*I*a*c*d^2*f^3*x^2 - 3*I*a*c^2*d*f^3*x + 3*I*a*d^3*f^2*x^2 - I*a*c^3*f^3 + 6*I*a*c*d^2*f^2*x + 3*I*a*c^2*d*f^2 - 6*I*a*d^3*f*x - 6*I*a*c*d^2*f + 6*I*a*d^3)*e^(f*x + e)/f^4 - 1/2*(-I*a*d^3*f^3*x^3 - 3*I*a*c*d^2*f^3*x^2 - 3*I*a*c^2*d*f^3*x - 3*I*a*d^3*f^2*x^2 - I*a*c^3*f^3 - 6*I*a*c*d^2*f^2*x - 3*I*a*c^2*d*f^2 - 6*I*a*d^3*f*x - 6*I*a*c*d^2*f - 6*I*a*d^3)*e^(-f*x - e)/f^4
```



### 3.97 $\int (c + dx)^2 (a + ia \sinh(e + fx)) dx$

**Optimal.** Leaf size=74

$$-\frac{2iad(c + dx) \sinh(e + fx)}{f^2} + \frac{ia(c + dx)^2 \cosh(e + fx)}{f} + \frac{a(c + dx)^3}{3d} + \frac{2iad^2 \cosh(e + fx)}{f^3}$$

[Out]  $(a*(c + d*x)^3)/(3*d) + ((2*I)*a*d^2*Cosh[e + f*x])/f^3 + (I*a*(c + d*x)^2*Cosh[e + f*x])/f - ((2*I)*a*d*(c + d*x)*Sinh[e + f*x])/f^2$

**Rubi [A]** time = 0.0970964, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3317, 3296, 2638}

$$-\frac{2iad(c + dx) \sinh(e + fx)}{f^2} + \frac{ia(c + dx)^2 \cosh(e + fx)}{f} + \frac{a(c + dx)^3}{3d} + \frac{2iad^2 \cosh(e + fx)}{f^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^2*(a + I*a*Sinh[e + f*x]),x]$

[Out]  $(a*(c + d*x)^3)/(3*d) + ((2*I)*a*d^2*Cosh[e + f*x])/f^3 + (I*a*(c + d*x)^2*Cosh[e + f*x])/f - ((2*I)*a*d*(c + d*x)*Sinh[e + f*x])/f^2$

#### Rule 3317

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(c + d*x)^m, (a + b*\sin[e + f*x])^n, x], x] /;$  FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

#### Rule 3296

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)*\sin[(e_.) + (f_.)*(x_.)]}, x\_Symbol] :> -\text{Simp}[(c + d*x)^m*\cos[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\cos[e + f*x], x], x] /;$  FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x\_Symbol] :> -\text{Simp}[\cos[c + d*x]/d, x] /;$  FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (c + dx)^2 (a + ia \sinh(e + fx)) dx &= \int (a(c + dx)^2 + ia(c + dx)^2 \sinh(e + fx)) dx \\
&= \frac{a(c + dx)^3}{3d} + (ia) \int (c + dx)^2 \sinh(e + fx) dx \\
&= \frac{a(c + dx)^3}{3d} + \frac{ia(c + dx)^2 \cosh(e + fx)}{f} - \frac{(2iad) \int (c + dx) \cosh(e + fx) dx}{f} \\
&= \frac{a(c + dx)^3}{3d} + \frac{ia(c + dx)^2 \cosh(e + fx)}{f} - \frac{2iad(c + dx) \sinh(e + fx)}{f^2} + \frac{(2iad^2) \int \sinh(e + fx) dx}{f} \\
&= \frac{a(c + dx)^3}{3d} + \frac{2iad^2 \cosh(e + fx)}{f^3} + \frac{ia(c + dx)^2 \cosh(e + fx)}{f} - \frac{2iad(c + dx) \sinh(e + fx)}{f^2}
\end{aligned}$$

**Mathematica [A]** time = 0.472278, size = 88, normalized size = 1.19

$$\frac{a(3i(c^2 f^2 + 2cdf^2 x + d^2(f^2 x^2 + 2)) \cosh(e + fx) + f^3 x(3c^2 + 3cdx + d^2 x^2) - 6idf(c + dx) \sinh(e + fx))}{3f^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2\*(a + I\*a\*Sinh[e + f\*x]),x]

[Out] (a\*(f^3\*x\*(3\*c^2 + 3\*c\*d\*x + d^2\*x^2) + (3\*I)\*(c^2\*f^2 + 2\*c\*d\*f^2\*x + d^2\*(2 + f^2\*x^2))\*Cosh[e + f\*x] - (6\*I)\*d\*f\*(c + d\*x)\*Sinh[e + f\*x]))/(3\*f^3)

**Maple [B]** time = 0.01, size = 249, normalized size = 3.4

$$\frac{1}{f} \left( \frac{ad^2 (fx + e)^3}{3f^2} + \frac{id^2 a \left( (fx + e)^2 \cosh(fx + e) - 2(fx + e) \sinh(fx + e) + 2 \cosh(fx + e) \right)}{f^2} - \frac{d^2 e a (fx + e)^2}{f^2} - \frac{2id^2 a \sinh(fx + e)}{f^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2\*(a+I\*a\*sinh(f\*x+e)),x)

[Out] 1/f\*(1/3/f^2\*d^2\*a\*(f\*x+e)^3+I/f^2\*d^2\*a\*((f\*x+e)^2\*cosh(f\*x+e)-2\*(f\*x+e)\*sinh(f\*x+e)+2\*cosh(f\*x+e))-1/f^2\*d^2\*e\*a\*(f\*x+e)^2-2\*I/f^2\*d^2\*e\*a\*((f\*x+e)\*cosh(f\*x+e)-sinh(f\*x+e))+1/f\*d\*c\*a\*(f\*x+e)^2+2\*I/f\*d\*c\*a\*((f\*x+e)\*cosh(f\*x+e)-sinh(f\*x+e)))

$e) - \sinh(f*x+e)) + d^2 * e^2 / f^2 * a * (f*x+e) + I * d^2 * e^2 / f^2 * a * \cosh(f*x+e) - 2 * d * e / f * c * a * (f*x+e) - 2 * I * d * e / f * c * a * \cosh(f*x+e) + c^2 * a * (f*x+e) + I * c^2 * a * \cosh(f*x+e)$

**Maxima [B]** time = 1.06462, size = 190, normalized size = 2.57

$$\frac{1}{3} ad^2 x^3 + acdx^2 + ac^2 x + iacd \left( \frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2} \right) + \frac{1}{2} iad^2 \left( \frac{(f^2 x^2 e^e - 2fxe^e + 2e^e)e^{(fx)}}{f^3} + \frac{(f^2 x^2 + 2fx + 2)e^{(-fx-e)}}{f^3} \right) + Iac^2 \cosh(fx+e)/f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+I\*a\*sinh(f\*x+e)),x, algorithm="maxima")

[Out]  $1/3 * a * d^2 * x^3 + a * c * d * x^2 + a * c^2 * x + I * a * c * d * ((f * x * e^e - e^e) * e^{(f * x)} / f^2 + (f * x + 1) * e^{(-f * x - e)} / f^2) + 1/2 * I * a * d^2 * ((f^2 * x^2 * e^e - 2 * f * x * e^e + 2 * e^e) * e^{(f * x)} / f^3 + (f^2 * x^2 + 2 * f * x + 2) * e^{(-f * x - e)} / f^3) + I * a * c^2 * \cosh(f * x + e) / f$

**Fricas [B]** time = 2.64756, size = 402, normalized size = 5.43

$$\frac{(3i ad^2 f^2 x^2 + 3i ac^2 f^2 + 6i acdf + 6i ad^2 + (6i acdf^2 + 6i ad^2 f)x + (3i ad^2 f^2 x^2 + 3i ac^2 f^2 - 6i acdf + 6i ad^2 + (6i acdf^2 + 6i ad^2 f)x + (3i ad^2 f^2 x^2 + 3i ac^2 f^2 - 6i acdf + 6i ad^2 + (6i acdf^2 + 6i ad^2 f)x) * e^{(2 * f * x + 2 * e)} + 2 * (a * d^2 * f^3 * x^3 + 3 * a * c * d * f^3 * x^2 + 3 * a * c^2 * f^3 * x) * e^{(f * x + e)}) * e^{(-f * x - e)} / f^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+I\*a\*sinh(f\*x+e)),x, algorithm="fricas")

[Out]  $1/6 * (3 * I * a * d^2 * f^2 * x^2 + 3 * I * a * c^2 * f^2 + 6 * I * a * c * d * f + 6 * I * a * d^2 + (6 * I * a * c * d * f^2 + 6 * I * a * d^2 * f) * x + (3 * I * a * d^2 * f^2 * x^2 + 3 * I * a * c^2 * f^2 - 6 * I * a * c * d * f + 6 * I * a * d^2 + (6 * I * a * c * d * f^2 - 6 * I * a * d^2 * f) * x) * e^{(2 * f * x + 2 * e)} + 2 * (a * d^2 * f^3 * x^3 + 3 * a * c * d * f^3 * x^2 + 3 * a * c^2 * f^3 * x) * e^{(f * x + e)}) * e^{(-f * x - e)} / f^3$

**Sympy [A]** time = 1.53159, size = 350, normalized size = 4.73

$$ac^2 x + acdx^2 + \frac{ad^2 x^3}{3} + \left\{ \frac{((2iac^2 f^{11} e^{2e} + 4iacdf^{11} x e^{2e} + 4iacdf^{10} e^{2e} + 2iad^2 f^{11} x^2 e^{2e} + 4iad^2 f^{10} x e^{2e} + 4iad^2 f^9 e^{2e}) e^{-fx} + (2iac^2 f^{11} e^{4e} + 4iacdf^{11} x e^{4e} - 4iacd^2 f^{11} e^{2e} - 4iacd^2 f^{10} x e^{2e} - 4iacd^2 f^9 e^{2e}) e^{-fx} + (2iac^2 f^{11} e^{2e} + 4iacdf^{11} x e^{2e} + 4iacdf^{10} e^{2e} + 2iad^2 f^{11} x^2 e^{2e} + 4iad^2 f^{10} x e^{2e} + 4iad^2 f^9 e^{2e}) e^{-fx}}{4f^{12}} + \frac{x^3 (iad^2 e^{2e} - iad^2) e^{-e}}{6} + \frac{x^2 (iacd e^{2e} - iacd) e^{-e}}{2} + \frac{x (iac^2 e^{2e} - iac^2) e^{-e}}{2} \right\}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2\*(a+I\*a\*sinh(f\*x+e)),x)

[Out] a\*c\*\*2\*x + a\*c\*d\*x\*\*2 + a\*d\*\*2\*x\*\*3/3 + Piecewise((((2\*I\*a\*c\*\*2\*f\*\*11\*exp(2\*e) + 4\*I\*a\*c\*d\*f\*\*11\*x\*exp(2\*e) + 4\*I\*a\*c\*d\*f\*\*10\*exp(2\*e) + 2\*I\*a\*d\*\*2\*f\*\*11\*x\*\*2\*exp(2\*e) + 4\*I\*a\*d\*\*2\*f\*\*10\*x\*exp(2\*e) + 4\*I\*a\*d\*\*2\*f\*\*9\*exp(2\*e))\*exp(-f\*x) + (2\*I\*a\*c\*\*2\*f\*\*11\*exp(4\*e) + 4\*I\*a\*c\*d\*f\*\*11\*x\*exp(4\*e) - 4\*I\*a\*c\*d\*f\*\*10\*exp(4\*e) + 2\*I\*a\*d\*\*2\*f\*\*11\*x\*\*2\*exp(4\*e) - 4\*I\*a\*d\*\*2\*f\*\*10\*x\*exp(4\*e) + 4\*I\*a\*d\*\*2\*f\*\*9\*exp(4\*e))\*exp(f\*x))\*exp(-3\*e)/(4\*f\*\*12), Ne(4\*f\*\*12\*exp(3\*e), 0)), (x\*\*3\*(I\*a\*d\*\*2\*exp(2\*e) - I\*a\*d\*\*2)\*exp(-e)/6 + x\*\*2\*(I\*a\*c\*d\*exp(2\*e) - I\*a\*c\*d)\*exp(-e)/2 + x\*(I\*a\*c\*\*2\*exp(2\*e) - I\*a\*c\*\*2)\*exp(-e)/2, True))

**Giac [B]** time = 1.26303, size = 205, normalized size = 2.77

$$\frac{1}{3} ad^2 x^3 + acdx^2 + ac^2 x - \frac{(-i ad^2 f^2 x^2 - 2i acd f^2 x - i ac^2 f^2 + 2i ad^2 f x + 2i acd f - 2i ad^2) e^{(fx+e)}}{2 f^3} + \frac{(i ad^2 f^2 x^2 + 2i acd f^2 x + i ac^2 f^2 - 2i ad^2 f x - 2i acd f + 2i ad^2) e^{-(fx+e)}}{2 f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+I\*a\*sinh(f\*x+e)),x, algorithm="giac")

[Out] 1/3\*a\*d^2\*x^3 + a\*c\*d\*x^2 + a\*c^2\*x - 1/2\*(-I\*a\*d^2\*f^2\*x^2 - 2\*I\*a\*c\*d\*f^2\*x - I\*a\*c^2\*f^2 + 2\*I\*a\*d^2\*f\*x + 2\*I\*a\*c\*d\*f - 2\*I\*a\*d^2)\*e^(f\*x + e)/f^3 + 1/2\*(I\*a\*d^2\*f^2\*x^2 + 2\*I\*a\*c\*d\*f^2\*x + I\*a\*c^2\*f^2 + 2\*I\*a\*d^2\*f\*x + 2\*I\*a\*c\*d\*f + 2\*I\*a\*d^2)\*e^(-f\*x - e)/f^3

### 3.98 $\int (c + dx)(a + ia \sinh(e + fx)) dx$

**Optimal.** Leaf size=50

$$\frac{ia(c + dx) \cosh(e + fx)}{f} + \frac{a(c + dx)^2}{2d} - \frac{iad \sinh(e + fx)}{f^2}$$

[Out]  $(a*(c + d*x)^2)/(2*d) + (I*a*(c + d*x)*Cosh[e + f*x])/f - (I*a*d*Sinh[e + f*x])/f^2$

**Rubi [A]** time = 0.0533599, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {3317, 3296, 2637}

$$\frac{ia(c + dx) \cosh(e + fx)}{f} + \frac{a(c + dx)^2}{2d} - \frac{iad \sinh(e + fx)}{f^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)*(a + I*a*Sinh[e + f*x]), x]$

[Out]  $(a*(c + d*x)^2)/(2*d) + (I*a*(c + d*x)*Cosh[e + f*x])/f - (I*a*d*Sinh[e + f*x])/f^2$

#### Rule 3317

$\text{Int}[(c + d*x)^m * (a + b*\sin[e + f*x])^n, x]$  /;  $\text{FreeQ}\{a, b, c, d, e, f, m\}, x$  &&  $\text{IGtQ}[n, 0]$  &&  $(\text{EqQ}[n, 1] \parallel \text{IGtQ}[m, 0] \parallel \text{NeQ}[a^2 - b^2, 0])$

#### Rule 3296

$\text{Int}[(c + d*x)^m * \sin[e + f*x], x]$  /;  $\text{FreeQ}\{c, d, e, f\}, x$  &&  $\text{GtQ}[m, 0]$

#### Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c + d*x)], x]$  /;  $\text{FreeQ}\{c, d\}, x$

Rubi steps

$$\begin{aligned}
\int (c + dx)(a + ia \sinh(e + fx)) dx &= \int (a(c + dx) + ia(c + dx) \sinh(e + fx)) dx \\
&= \frac{a(c + dx)^2}{2d} + (ia) \int (c + dx) \sinh(e + fx) dx \\
&= \frac{a(c + dx)^2}{2d} + \frac{ia(c + dx) \cosh(e + fx)}{f} - \frac{(iad) \int \cosh(e + fx) dx}{f} \\
&= \frac{a(c + dx)^2}{2d} + \frac{ia(c + dx) \cosh(e + fx)}{f} - \frac{iad \sinh(e + fx)}{f^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0842476, size = 48, normalized size = 0.96

$$\frac{a(2if(c + dx) \cosh(e + fx) + f^2x(2c + dx) - 2id \sinh(e + fx))}{2f^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)\*(a + I\*a\*Sinh[e + f\*x]),x]

[Out] (a\*(f^2\*x\*(2\*c + d\*x) + (2\*I)\*f\*(c + d\*x)\*Cosh[e + f\*x] - (2\*I)\*d\*Sinh[e + f\*x]))/(2\*f^2)

**Maple [B]** time = 0.009, size = 96, normalized size = 1.9

$$\frac{1}{f} \left( \frac{da (fx + e)^2}{2f} + \frac{ida ((fx + e) \cosh (fx + e) - \sinh (fx + e))}{f} - \frac{dea (fx + e)}{f} - \frac{idea \cosh (fx + e)}{f} + ca (fx + e) + ica \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)\*(a+I\*a\*sinh(f\*x+e)),x)

[Out] 1/f\*(1/2/f\*d\*a\*(f\*x+e)^2+I/f\*d\*a\*((f\*x+e)\*cosh(f\*x+e)-sinh(f\*x+e))-d\*e/f\*a\*(f\*x+e)-I\*d\*e/f\*a\*cosh(f\*x+e)+c\*a\*(f\*x+e)+I\*c\*a\*cosh(f\*x+e))

**Maxima [A]** time = 1.04286, size = 89, normalized size = 1.78

$$\frac{1}{2} adx^2 + acx + \frac{1}{2} i ad \left( \frac{(fxe^e - e^e)e^{fx}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2} \right) + \frac{i ac \cosh(fx + e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+I\*a\*sinh(f\*x+e)),x, algorithm="maxima")

[Out] 1/2\*a\*d\*x^2 + a\*c\*x + 1/2\*I\*a\*d\*((f\*x\*e^e - e^e)\*e^(f\*x)/f^2 + (f\*x + 1)\*e^(-f\*x - e)/f^2) + I\*a\*c\*cosh(f\*x + e)/f

**Fricas [A]** time = 2.63207, size = 192, normalized size = 3.84

$$\frac{\left( i adfx + i acf + i ad + (i adfx + i acf - i ad)e^{(2fx+2e)} + (adf^2x^2 + 2acf^2x)e^{(fx+e)} \right) e^{(-fx-e)}}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+I\*a\*sinh(f\*x+e)),x, algorithm="fricas")

[Out] 1/2\*(I\*a\*d\*f\*x + I\*a\*c\*f + I\*a\*d + (I\*a\*d\*f\*x + I\*a\*c\*f - I\*a\*d)\*e^(2\*f\*x + 2\*e) + (a\*d\*f^2\*x^2 + 2\*a\*c\*f^2\*x)\*e^(f\*x + e))\*e^(-f\*x - e)/f^2

**Sympy [A]** time = 0.943545, size = 177, normalized size = 3.54

$$acx + \frac{adx^2}{2} + \begin{cases} \frac{\left( (2iacf^5e^e + 2iadf^5xe^e + 2iadf^4e^e)e^{-fx} + (2iacf^5e^{3e} + 2iadf^5xe^{3e} - 2iadf^4e^{3e})e^{fx} \right) e^{-2e}}{4f^6} & \text{for } 4f^6e^{2e} \neq 0 \\ \frac{x^2(iade^{2e} - iad)e^{-e}}{4} + \frac{x(iace^{2e} - iac)e^{-e}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+I\*a\*sinh(f\*x+e)),x)

[Out] a\*c\*x + a\*d\*x\*\*2/2 + Piecewise((((2\*I\*a\*c\*f\*\*5\*exp(e) + 2\*I\*a\*d\*f\*\*5\*x\*exp(e) + 2\*I\*a\*d\*f\*\*4\*exp(e))\*exp(-f\*x) + (2\*I\*a\*c\*f\*\*5\*exp(3\*e) + 2\*I\*a\*d\*f\*\*5\*x\*exp(3\*e) - 2\*I\*a\*d\*f\*\*4\*exp(3\*e))\*exp(f\*x))\*exp(-2\*e)/(4\*f\*\*6), Ne(4\*f\*\*

```
6*exp(2*e), 0)), (x**2*(I*a*d*exp(2*e) - I*a*d)*exp(-e)/4 + x*(I*a*c*exp(2*
e) - I*a*c)*exp(-e)/2, True))
```

**Giac [A]** time = 1.29728, size = 96, normalized size = 1.92

$$\frac{1}{2}adx^2 + acx - \frac{(-i adfx - i acf + i ad)e^{(fx+e)}}{2f^2} - \frac{(-i adfx - i acf - i ad)e^{(-fx-e)}}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*x^2 + a*c*x - 1/2*(-I*a*d*f*x - I*a*c*f + I*a*d)*e^(f*x + e)/f^2 -
1/2*(-I*a*d*f*x - I*a*c*f - I*a*d)*e^(-f*x - e)/f^2
```



$$3.99 \quad \int \frac{a+ia \sinh(e+fx)}{c+dx} dx$$

**Optimal.** Leaf size=70

$$\frac{ia \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d} + \frac{ia \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d} + \frac{a \log(c+dx)}{d}$$

[Out] (a\*Log[c + d\*x])/d + (I\*a\*CoshIntegral[(c\*f)/d + f\*x]\*Sinh[e - (c\*f)/d])/d + (I\*a\*Cosh[e - (c\*f)/d]\*SinhIntegral[(c\*f)/d + f\*x])/d

**Rubi [A]** time = 0.161418, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {3317, 3303, 3298, 3301}

$$\frac{ia \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d} + \frac{ia \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d} + \frac{a \log(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + I\*a\*Sinh[e + f\*x])/(c + d\*x),x]

[Out] (a\*Log[c + d\*x])/d + (I\*a\*CoshIntegral[(c\*f)/d + f\*x]\*Sinh[e - (c\*f)/d])/d + (I\*a\*Cosh[e - (c\*f)/d]\*SinhIntegral[(c\*f)/d + f\*x])/d

### Rule 3317

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*Sinh[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{a + ia \sinh(e + fx)}{c + dx} dx &= \int \left( \frac{a}{c + dx} + \frac{ia \sinh(e + fx)}{c + dx} \right) dx \\ &= \frac{a \log(c + dx)}{d} + (ia) \int \frac{\sinh(e + fx)}{c + dx} dx \\ &= \frac{a \log(c + dx)}{d} + \left( ia \cosh \left( e - \frac{cf}{d} \right) \right) \int \frac{\sinh \left( \frac{cf}{d} + fx \right)}{c + dx} dx + \left( ia \sinh \left( e - \frac{cf}{d} \right) \right) \int \frac{\cosh \left( \frac{cf}{d} + fx \right)}{c + dx} dx \\ &= \frac{a \log(c + dx)}{d} + \frac{ia \operatorname{Chi} \left( \frac{cf}{d} + fx \right) \sinh \left( e - \frac{cf}{d} \right)}{d} + \frac{ia \cosh \left( e - \frac{cf}{d} \right) \operatorname{Shi} \left( \frac{cf}{d} + fx \right)}{d} \end{aligned}$$

**Mathematica [A]** time = 0.29515, size = 60, normalized size = 0.86

$$\frac{a \left( i \operatorname{Chi} \left( f \left( \frac{c}{d} + x \right) \right) \sinh \left( e - \frac{cf}{d} \right) + i \cosh \left( e - \frac{cf}{d} \right) \operatorname{Shi} \left( f \left( \frac{c}{d} + x \right) \right) + \log(c + dx) \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[e + f*x])/(c + d*x),x]
```

```
[Out] (a*(Log[c + d*x] + I*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] + I*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)]))/d
```

**Maple [A]** time = 0.045, size = 96, normalized size = 1.4

$$\frac{a \ln(dx + c)}{d} + \frac{i}{2} \frac{a}{d} e^{\frac{cf-de}{d}} \operatorname{Ei} \left( 1, fx + e + \frac{cf-de}{d} \right) - \frac{i}{2} \frac{a}{d} e^{-\frac{cf-de}{d}} \operatorname{Ei} \left( 1, -fx - e - \frac{cf-de}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*sinh(f*x+e))/(d*x+c),x)`

[Out] `a*ln(d*x+c)/d+1/2*I*a/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*I*a/d*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)`

**Maxima [A]** time = 1.20787, size = 96, normalized size = 1.37

$$\frac{1}{2} i a \left( \frac{e^{\left(-e+\frac{cf}{d}\right)} E_1\left(\frac{(dx+c)f}{d}\right)}{d} - \frac{e^{\left(e-\frac{cf}{d}\right)} E_1\left(-\frac{(dx+c)f}{d}\right)}{d} \right) + \frac{a \log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*sinh(f*x+e))/(d*x+c),x, algorithm="maxima")`

[Out] `1/2*I*a*(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d - e^(e - c*f/d)*exp_integral_e(1, -(d*x + c)*f/d)/d) + a*log(d*x + c)/d`

**Fricas [A]** time = 2.77263, size = 162, normalized size = 2.31

$$\frac{i a E_i\left(\frac{dfx+cf}{d}\right) e^{\left(\frac{de-cf}{d}\right)} - i a E_i\left(-\frac{dfx+cf}{d}\right) e^{\left(-\frac{de-cf}{d}\right)} + 2 a \log\left(\frac{dx+c}{d}\right)}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*sinh(f*x+e))/(d*x+c),x, algorithm="fricas")`

[Out] `1/2*(I*a*Ei((d*f*x + c*f)/d)*e^((d*e - c*f)/d) - I*a*Ei(-(d*f*x + c*f)/d)*e^(-(d*e - c*f)/d) + 2*a*log((d*x + c)/d))/d`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$a \left( \int \frac{i \sinh(e + fx)}{c + dx} dx + \int \frac{1}{c + dx} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))/(d\*x+c),x)

[Out] a\*(Integral(I\*sinh(e + f\*x)/(c + d\*x), x) + Integral(1/(c + d\*x), x))

**Giac [A]** time = 1.29067, size = 96, normalized size = 1.37

$$\frac{i a \operatorname{Ei}\left(-\frac{d f x+c f}{d}\right) e^{\left(\frac{c f}{d}-e\right)}-i a \operatorname{Ei}\left(\frac{d f x+c f}{d}\right) e^{\left(-\frac{c f}{d}+e\right)}-2 a \log (d x+c)}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))/(d\*x+c),x, algorithm="giac")

[Out] -1/2\*(I\*a\*Ei(-(d\*f\*x + c\*f)/d)\*e^(c\*f/d - e) - I\*a\*Ei((d\*f\*x + c\*f)/d)\*e^(-c\*f/d + e) - 2\*a\*log(d\*x + c))/d

$$3.100 \quad \int \frac{a+ia \sinh(e+fx)}{(c+dx)^2} dx$$

**Optimal.** Leaf size=95

$$\frac{iaf \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d^2} + \frac{iaf \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^2} - \frac{ia \sinh(e+fx)}{d(c+dx)} - \frac{a}{d(c+dx)}$$

[Out]  $-(a/(d*(c + d*x))) + (I*a*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d^2 - (I*a*Sinh[e + f*x])/(d*(c + d*x)) + (I*a*f*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^2$

**Rubi [A]** time = 0.189107, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {3317, 3297, 3303, 3298, 3301}

$$\frac{iaf \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d^2} + \frac{iaf \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^2} - \frac{ia \sinh(e+fx)}{d(c+dx)} - \frac{a}{d(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + I*a*Sinh[e + f*x])/(c + d*x)^2, x]$

[Out]  $-(a/(d*(c + d*x))) + (I*a*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d^2 - (I*a*Sinh[e + f*x])/(d*(c + d*x)) + (I*a*f*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^2$

### Rule 3317

$\operatorname{Int}[(c_.) + (d_.)*(x_)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)])^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(c + d*x)^m, (a + b*\sin[e + f*x])^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, m\}, x\} \&\& \operatorname{IGtQ}[n, 0] \&\& (\operatorname{EqQ}[n, 1] \parallel \operatorname{IGtQ}[m, 0] \parallel \operatorname{NeQ}[a^2 - b^2, 0])$

### Rule 3297

$\operatorname{Int}[(c_.) + (d_.)*(x_)]^{(m_.)}*\sin[(e_.) + (f_.)*(x_)], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m+1)}*\sin[e + f*x]/(d*(m+1)), x] - \operatorname{Dist}[f/(d*(m+1)), \operatorname{Int}[(c + d*x)^{(m+1)}*\cos[e + f*x], x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x\} \&\& \operatorname{LtQ}[m, -1]$

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + ia \sinh(e + fx)}{(c + dx)^2} dx &= \int \left( \frac{a}{(c + dx)^2} + \frac{ia \sinh(e + fx)}{(c + dx)^2} \right) dx \\
&= -\frac{a}{d(c + dx)} + (ia) \int \frac{\sinh(e + fx)}{(c + dx)^2} dx \\
&= -\frac{a}{d(c + dx)} - \frac{ia \sinh(e + fx)}{d(c + dx)} + \frac{(iaf) \int \frac{\cosh(e+fx)}{c+dx} dx}{d} \\
&= -\frac{a}{d(c + dx)} - \frac{ia \sinh(e + fx)}{d(c + dx)} + \frac{\left( ia f \cosh\left(e - \frac{cf}{d}\right) \right) \int \frac{\cosh\left(\frac{cf}{d} + fx\right)}{c+dx} dx}{d} + \frac{\left( ia f \sinh\left(e - \frac{cf}{d}\right) \right) \int}{d} \\
&= -\frac{a}{d(c + dx)} + \frac{ia f \cosh\left(e - \frac{cf}{d}\right) \text{Chi}\left(\frac{cf}{d} + fx\right)}{d^2} - \frac{ia \sinh(e + fx)}{d(c + dx)} + \frac{ia f \sinh\left(e - \frac{cf}{d}\right) \text{Shi}\left(\frac{cf}{d}\right)}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.4877, size = 83, normalized size = 0.87

$$\frac{ia \left( f(c + dx) \text{Chi}\left(f\left(\frac{c}{d} + x\right)\right) \cosh\left(e - \frac{cf}{d}\right) + f(c + dx) \sinh\left(e - \frac{cf}{d}\right) \text{Shi}\left(f\left(\frac{c}{d} + x\right)\right) - d(\sinh(e + fx) - i) \right)}{d^2(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Sinh[e + f\*x])/(c + d\*x)^2,x]

[Out] (I\*a\*(f\*(c + d\*x)\*Cosh[e - (c\*f)/d]\*CoshIntegral[f\*(c/d + x)] - d\*(-I + Sinh[e + f\*x]) + f\*(c + d\*x)\*Sinh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)])/(d^2\*(c + d\*x))

**Maple [A]** time = 0.053, size = 153, normalized size = 1.6

$$-\frac{a}{d(dx+c)} + \frac{\frac{i}{2}fae^{-fx-e}}{d(dfx+cf)} - \frac{\frac{i}{2}fa}{d^2} e^{\frac{cf-de}{d}} \operatorname{Ei}\left(1, fx+e + \frac{cf-de}{d}\right) - \frac{\frac{i}{2}fae^{fx+e}}{d^2} \left(\frac{cf}{d} + fx\right)^{-1} - \frac{\frac{i}{2}fa}{d^2} e^{-\frac{cf-de}{d}} \operatorname{Ei}\left(1, -fx-e - \frac{cf-de}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*sinh(f\*x+e))/(d\*x+c)^2,x)

[Out] -a/d/(d\*x+c)+1/2\*I\*a\*f\*exp(-f\*x-e)/d/(d\*f\*x+c\*f)-1/2\*I\*a\*f/d^2\*exp((c\*f-d\*e)/d)\*Ei(1,f\*x+e+(c\*f-d\*e)/d)-1/2\*I\*a\*f/d^2\*exp(f\*x+e)/(c\*f/d+f\*x)-1/2\*I\*a\*f/d^2\*exp(-(c\*f-d\*e)/d)\*Ei(1,-f\*x-e-(c\*f-d\*e)/d)

**Maxima [A]** time = 1.22058, size = 119, normalized size = 1.25

$$\frac{1}{2} i a \left( \frac{e^{\left(-e + \frac{cf}{d}\right)} E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d} - \frac{e^{\left(e - \frac{cf}{d}\right)} E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d} \right) - \frac{a}{d^2 x + cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))/(d\*x+c)^2,x, algorithm="maxima")

[Out] 1/2\*I\*a\*(e^(-e + c\*f/d)\*exp\_integral\_e(2, (d\*x + c)\*f/d)/((d\*x + c)\*d) - e^(e - c\*f/d)\*exp\_integral\_e(2, -(d\*x + c)\*f/d)/((d\*x + c)\*d)) - a/(d^2\*x + c\*d)

**Fricas [A]** time = 2.7878, size = 290, normalized size = 3.05

$$\frac{\left(-iade^{2fx+2e} + iad + \left(iadfx + iacf\right)Ei\left(\frac{dfx+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)} + \left(iadfx + iacf\right)Ei\left(-\frac{dfx+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)} - 2ad\right)e^{(fx+e)}e^{(-fx-e)}}{2(d^3x + cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))/(d\*x+c)^2,x, algorithm="fricas")

[Out] 1/2\*(-I\*a\*d\*e^(2\*f\*x + 2\*e) + I\*a\*d + ((I\*a\*d\*f\*x + I\*a\*c\*f)\*Ei((d\*f\*x + c\*f)/d)\*e^((d\*e - c\*f)/d) + (I\*a\*d\*f\*x + I\*a\*c\*f)\*Ei(-(d\*f\*x + c\*f)/d)\*e^(-(d\*e - c\*f)/d) - 2\*a\*d)\*e^(f\*x + e))\*e^(-f\*x - e)/(d^3\*x + c\*d^2)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))/(d\*x+c)\*\*2,x)

[Out] Timed out

**Giac [A]** time = 1.28447, size = 225, normalized size = 2.37

$$\frac{i\left(dfxEi\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)} + dfxEi\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)} + cfEi\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)} + cfEi\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)} - de^{(fx+e)} + de^{(-fx-e)}\right)}{2(d^3x + cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))/(d\*x+c)^2,x, algorithm="giac")

[Out] 1/2\*I\*(d\*f\*x\*Ei(-(d\*f\*x + c\*f)/d)\*e^(c\*f/d - e) + d\*f\*x\*Ei((d\*f\*x + c\*f)/d)\*e^(-c\*f/d + e) + c\*f\*Ei(-(d\*f\*x + c\*f)/d)\*e^(c\*f/d - e) + c\*f\*Ei((d\*f\*x + c\*f)/d)\*e^(-c\*f/d + e) - d\*e^(f\*x + e) + d\*e^(-f\*x - e))\*a/(d^3\*x + c\*d^2) - a/((d\*x + c)\*d)



$$3.101 \quad \int \frac{a+ia \sinh(e+fx)}{(c+dx)^3} dx$$

**Optimal.** Leaf size=131

$$\frac{iaf^2 \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{2d^3} + \frac{iaf^2 \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{2d^3} - \frac{iaf \cosh(e+fx)}{2d^2(c+dx)} - \frac{ia \sinh(e+fx)}{2d(c+dx)^2} - \frac{a}{2d(c+dx)}$$

[Out]  $-a/(2*d*(c + d*x)^2) - ((I/2)*a*f*Cosh[e + f*x])/(d^2*(c + d*x)) + ((I/2)*a*f^2*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^3 - ((I/2)*a*Sinh[e + f*x])/(d*(c + d*x)^2) + ((I/2)*a*f^2*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^3$

**Rubi [A]** time = 0.232228, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {3317, 3297, 3303, 3298, 3301}

$$\frac{iaf^2 \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{2d^3} + \frac{iaf^2 \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{2d^3} - \frac{iaf \cosh(e+fx)}{2d^2(c+dx)} - \frac{ia \sinh(e+fx)}{2d(c+dx)^2} - \frac{a}{2d(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + I*a*\operatorname{Sinh}[e + f*x])/(c + d*x)^3, x]$

[Out]  $-a/(2*d*(c + d*x)^2) - ((I/2)*a*f*Cosh[e + f*x])/(d^2*(c + d*x)) + ((I/2)*a*f^2*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^3 - ((I/2)*a*Sinh[e + f*x])/(d*(c + d*x)^2) + ((I/2)*a*f^2*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^3$

### Rule 3317

$\operatorname{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(c + d*x)^m, (a + b*\sin[e + f*x])^n, x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, m\}, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ (\text{EqQ}[n, 1] \ || \ \text{IGtQ}[m, 0] \ || \ \text{NeQ}[a^2 - b^2, 0])$

### Rule 3297

$\operatorname{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow \operatorname{Simp}[(c + d*x)^{(m+1)}*\sin[e + f*x]/(d*(m+1)), x] - \operatorname{Dist}[f/(d*(m+1)), \operatorname{Int}[(c + d*x)^{(m+1)}*\cos[e + f*x], x], x] /;$   $\text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{LtQ}[m, -1]$

]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + ia \sinh(e + fx)}{(c + dx)^3} dx &= \int \left( \frac{a}{(c + dx)^3} + \frac{ia \sinh(e + fx)}{(c + dx)^3} \right) dx \\
&= -\frac{a}{2d(c + dx)^2} + (ia) \int \frac{\sinh(e + fx)}{(c + dx)^3} dx \\
&= -\frac{a}{2d(c + dx)^2} - \frac{ia \sinh(e + fx)}{2d(c + dx)^2} + \frac{(iaf) \int \frac{\cosh(e+fx)}{(c+dx)^2} dx}{2d} \\
&= -\frac{a}{2d(c + dx)^2} - \frac{iaf \cosh(e + fx)}{2d^2(c + dx)} - \frac{ia \sinh(e + fx)}{2d(c + dx)^2} + \frac{(iaf^2) \int \frac{\sinh(e+fx)}{c+dx} dx}{2d^2} \\
&= -\frac{a}{2d(c + dx)^2} - \frac{iaf \cosh(e + fx)}{2d^2(c + dx)} - \frac{ia \sinh(e + fx)}{2d(c + dx)^2} + \frac{\left( ia f^2 \cosh \left( e - \frac{cf}{d} \right) \right) \int \frac{\sinh \left( \frac{cf}{d} + fx \right)}{c+dx} dx}{2d^2} \\
&= -\frac{a}{2d(c + dx)^2} - \frac{iaf \cosh(e + fx)}{2d^2(c + dx)} + \frac{ia f^2 \text{Chi} \left( \frac{cf}{d} + fx \right) \sinh \left( e - \frac{cf}{d} \right)}{2d^3} - \frac{ia \sinh(e + fx)}{2d(c + dx)^2} + \frac{ia}{2d}
\end{aligned}$$

**Mathematica [A]** time = 0.669285, size = 109, normalized size = 0.83

$$\frac{ia \left( f^2(c+dx)^2 \operatorname{Chi} \left( f \left( \frac{c}{d} + x \right) \right) \sinh \left( e - \frac{cf}{d} \right) + f^2(c+dx)^2 \operatorname{cosh} \left( e - \frac{cf}{d} \right) \operatorname{Shi} \left( f \left( \frac{c}{d} + x \right) \right) - d(f(c+dx) \operatorname{cosh}(e+fx) + d \right)}{2d^3(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Sinh[e + f\*x])/(c + d\*x)^3, x]

[Out] ((I/2)\*a\*(f^2\*(c + d\*x)^2\*CoshIntegral[f\*(c/d + x)]\*Sinh[e - (c\*f)/d] - d\*(f\*(c + d\*x)\*Cosh[e + f\*x] + d\*(-I + Sinh[e + f\*x]))) + f^2\*(c + d\*x)^2\*Cosh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)]/(d^3\*(c + d\*x)^2)

**Maple [B]** time = 0.06, size = 303, normalized size = 2.3

$$-\frac{a}{2d(dx+c)^2} - \frac{\frac{i}{4}af^3e^{-fx-e}x}{d(d^2f^2x^2 + 2cdf^2x + c^2f^2)} - \frac{\frac{i}{4}af^3e^{-fx-e}c}{d^2(d^2f^2x^2 + 2cdf^2x + c^2f^2)} + \frac{\frac{i}{4}af^2e^{-fx-e}}{d(d^2f^2x^2 + 2cdf^2x + c^2f^2)} + \frac{\frac{i}{4}af^2}{d^3}e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*sinh(f\*x+e))/(d\*x+c)^3, x)

[Out] -1/2\*a/d/(d\*x+c)^2-1/4\*I\*a\*f^3\*exp(-f\*x-e)/d/(d^2\*f^2\*x^2+2\*c\*d\*f^2\*x+c^2\*f^2)\*x-1/4\*I\*a\*f^3\*exp(-f\*x-e)/d^2/(d^2\*f^2\*x^2+2\*c\*d\*f^2\*x+c^2\*f^2)\*c+1/4\*I\*a\*f^2\*exp(-f\*x-e)/d/(d^2\*f^2\*x^2+2\*c\*d\*f^2\*x+c^2\*f^2)+1/4\*I\*a\*f^2/d^3\*exp((c\*f-d\*e)/d)\*Ei(1, f\*x+e+(c\*f-d\*e)/d)-1/4\*I\*a\*f^2/d^3\*exp(f\*x+e)/(c\*f/d+f\*x)^2-1/4\*I\*a\*f^2/d^3\*exp(f\*x+e)/(c\*f/d+f\*x)-1/4\*I\*a\*f^2/d^3\*exp(-(c\*f-d\*e)/d)\*Ei(1, -f\*x-e-(c\*f-d\*e)/d)

**Maxima [A]** time = 1.21992, size = 134, normalized size = 1.02

$$\frac{1}{2}ia \left( \frac{e^{\left(-e+\frac{cf}{d}\right)} E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d} - \frac{e^{\left(e-\frac{cf}{d}\right)} E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d} \right) - \frac{a}{2(d^3x^2 + 2cd^2x + c^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))/(d\*x+c)^3,x, algorithm="maxima")

[Out]  $\frac{1}{2}Ia*(e^{-e + cf/d}*\exp\_integral\_e(3, (d*x + c)*f/d)/((d*x + c)^{2*d}) - e^{(e - cf/d)}*\exp\_integral\_e(3, -(d*x + c)*f/d)/((d*x + c)^{2*d}) - 1/2*a/(d^3*x^2 + 2*c*d^2*x + c^2*d)$

**Fricas [A]** time = 2.72999, size = 471, normalized size = 3.6

$$\frac{\left(-i ad^2 fx - i acdf + i ad^2 + (-i ad^2 fx - i acdf - i ad^2)e^{(2fx+2e)} - \left(2 ad^2 - (i ad^2 f^2 x^2 + 2i acdf^2 x + i ac^2 f^2)\right)Ei\left(\frac{dfx+cf}{d}\right)\right)}{4(d^5 x^2 + 2 cd^4 x + c^2 d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))/(d\*x+c)^3,x, algorithm="fricas")

[Out]  $\frac{1}{4}*(-I*a*d^2*f*x - I*a*c*d*f + I*a*d^2 + (-I*a*d^2*f*x - I*a*c*d*f - I*a*d^2)*e^{(2*f*x + 2*e)} - (2*a*d^2 - (I*a*d^2*f^2*x^2 + 2*I*a*c*d*f^2*x + I*a*c^2*f^2)*Ei((d*f*x + c*f)/d)*e^{((d*e - c*f)/d)} - (-I*a*d^2*f^2*x^2 - 2*I*a*c*d*f^2*x - I*a*c^2*f^2)*Ei(-(d*f*x + c*f)/d)*e^{-((d*e - c*f)/d)}*e^{(f*x + e)})*e^{(-f*x - e)}/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))/(d\*x+c)\*\*3,x)

[Out] Timed out

**Giac [B]** time = 1.34846, size = 451, normalized size = 3.44

$$\frac{-i ad^2 f^2 x^2 Ei\left(-\frac{dfx+cf}{d}\right) e^{\left(\frac{cf}{d}-e\right)} + i ad^2 f^2 x^2 Ei\left(\frac{dfx+cf}{d}\right) e^{\left(-\frac{cf}{d}+e\right)} - 2i acdf^2 x Ei\left(-\frac{dfx+cf}{d}\right) e^{\left(\frac{cf}{d}-e\right)} + 2i acdf^2 x Ei\left(\frac{dfx+cf}{d}\right) e^{\left(-\frac{cf}{d}+e\right)}}{4(d^5 x^2 + 2 cd^4 x + c^2 d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(-I*a*d^2*f^2*x^2*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + I*a*d^2*f^2*x^2*
Ei((d*f*x + c*f)/d)*e^(-c*f/d + e) - 2*I*a*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e
^(c*f/d - e) + 2*I*a*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(-c*f/d + e) - I*a*c^2
*f^2*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + I*a*c^2*f^2*Ei((d*f*x + c*f)/d)*e
^(-c*f/d + e) - I*a*d^2*f*x*e^(f*x + e) - I*a*d^2*f*x*e^(-f*x - e) - I*a*c*
d*f*e^(f*x + e) - I*a*c*d*f*e^(-f*x - e) - I*a*d^2*e^(f*x + e) + I*a*d^2*e^
(-f*x - e) - 2*a*d^2)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

### 3.102 $\int (c + dx)^3 (a + ia \sinh(e + fx))^2 dx$

**Optimal.** Leaf size=245

$$\frac{12ia^2d^2(c+dx)\cosh(e+fx)}{f^3} - \frac{3a^2d^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{4f^3} + \frac{3a^2cd^2x}{4f^2} + \frac{3a^2d(c+dx)^2\sinh^2(e+fx)}{4f^2} - \frac{6ia^2d^2(c+dx)^2\sinh(e+fx)}{4f^3}$$

[Out] (3\*a^2\*c\*d^2\*x)/(4\*f^2) + (3\*a^2\*d^3\*x^2)/(8\*f^2) + (3\*a^2\*(c+d\*x)^4)/(8\*d) + ((12\*I)\*a^2\*d^2\*(c+d\*x)\*Cosh[e+f\*x])/f^3 + ((2\*I)\*a^2\*(c+d\*x)^3\*Cosh[e+f\*x])/f - ((12\*I)\*a^2\*d^3\*Sinh[e+f\*x])/f^4 - ((6\*I)\*a^2\*d\*(c+d\*x)^2\*Sinh[e+f\*x])/f^2 - (3\*a^2\*d^2\*(c+d\*x)\*Cosh[e+f\*x]\*Sinh[e+f\*x])/(4\*f^3) - (a^2\*(c+d\*x)^3\*Cosh[e+f\*x]\*Sinh[e+f\*x])/(2\*f) + (3\*a^2\*d^3\*Sinh[e+f\*x]^2)/(8\*f^4) + (3\*a^2\*d\*(c+d\*x)^2\*Sinh[e+f\*x]^2)/(4\*f^2)

**Rubi [A]** time = 0.287126, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$ , Rules used = {3317, 3296, 2637, 3311, 32, 3310}

$$\frac{12ia^2d^2(c+dx)\cosh(e+fx)}{f^3} - \frac{3a^2d^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{4f^3} + \frac{3a^2cd^2x}{4f^2} + \frac{3a^2d(c+dx)^2\sinh^2(e+fx)}{4f^2} - \frac{6ia^2d^2(c+dx)^2\sinh(e+fx)}{4f^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^3\*(a + I\*a\*Sinh[e + f\*x])^2,x]

[Out] (3\*a^2\*c\*d^2\*x)/(4\*f^2) + (3\*a^2\*d^3\*x^2)/(8\*f^2) + (3\*a^2\*(c+d\*x)^4)/(8\*d) + ((12\*I)\*a^2\*d^2\*(c+d\*x)\*Cosh[e+f\*x])/f^3 + ((2\*I)\*a^2\*(c+d\*x)^3\*Cosh[e+f\*x])/f - ((12\*I)\*a^2\*d^3\*Sinh[e+f\*x])/f^4 - ((6\*I)\*a^2\*d\*(c+d\*x)^2\*Sinh[e+f\*x])/f^2 - (3\*a^2\*d^2\*(c+d\*x)\*Cosh[e+f\*x]\*Sinh[e+f\*x])/(4\*f^3) - (a^2\*(c+d\*x)^3\*Cosh[e+f\*x]\*Sinh[e+f\*x])/(2\*f) + (3\*a^2\*d^3\*Sinh[e+f\*x]^2)/(8\*f^4) + (3\*a^2\*d\*(c+d\*x)^2\*Sinh[e+f\*x]^2)/(4\*f^2)

#### Rule 3317

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*Sinh[e + f\*x])^n, x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

#### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[
(d*m*(c + d*x)^(m - 1)*(b*SIN[e + f*x])^n)/(f^2*n^2), x] + (Dist[
(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*SIN[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*SIN[e + f*x])^n, x], x] -
Simp[(b*(c + d*x)^m*cos[e + f*x]*(b*SIN[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :=
Simp[(d*(b*SIN[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*SIN[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*SIN[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]
```

### Rubi steps

$$\begin{aligned}
\int (c + dx)^3 (a + ia \sinh(e + fx))^2 dx &= \int (a^2(c + dx)^3 + 2ia^2(c + dx)^3 \sinh(e + fx) - a^2(c + dx)^3 \sinh^2(e + fx)) dx \\
&= \frac{a^2(c + dx)^4}{4d} + (2ia^2) \int (c + dx)^3 \sinh(e + fx) dx - a^2 \int (c + dx)^3 \sinh^2(e + fx) dx \\
&= \frac{a^2(c + dx)^4}{4d} + \frac{2ia^2(c + dx)^3 \cosh(e + fx)}{f} - \frac{a^2(c + dx)^3 \cosh(e + fx) \sinh(e + fx)}{2f} \\
&= \frac{3a^2(c + dx)^4}{8d} + \frac{2ia^2(c + dx)^3 \cosh(e + fx)}{f} - \frac{6ia^2d(c + dx)^2 \sinh(e + fx)}{f^2} - \frac{3a^2d^2}{f^2} \\
&= \frac{3a^2cd^2x}{4f^2} + \frac{3a^2d^3x^2}{8f^2} + \frac{3a^2(c + dx)^4}{8d} + \frac{12ia^2d^2(c + dx) \cosh(e + fx)}{f^3} + \frac{2ia^2(c + dx)}{f^3} \\
&= \frac{3a^2cd^2x}{4f^2} + \frac{3a^2d^3x^2}{8f^2} + \frac{3a^2(c + dx)^4}{8d} + \frac{12ia^2d^2(c + dx) \cosh(e + fx)}{f^3} + \frac{2ia^2(c + dx)}{f^3}
\end{aligned}$$

**Mathematica [A]** time = 1.4905, size = 220, normalized size = 0.9

$$a^2 \left( -2f(c + dx) (2c^2f^2 + 4cdf^2x + d^2(2f^2x^2 + 3)) \sinh(2(e + fx)) - 96id(c^2f^2 + 2cdf^2x + d^2(f^2x^2 + 2)) \sinh(e + fx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3\*(a + I\*a\*Sinh[e + f\*x])^2,x]

[Out] (a^2\*(6\*f^4\*x\*(4\*c^3 + 6\*c^2\*d\*x + 4\*c\*d^2\*x^2 + d^3\*x^3) + (32\*I)\*f\*(c + d\*x)\*(c^2\*f^2 + 2\*c\*d\*f^2\*x + d^2\*(6 + f^2\*x^2))\*Cosh[e + f\*x] + 3\*d\*(2\*c^2\*f^2 + 4\*c\*d\*f^2\*x + d^2\*(1 + 2\*f^2\*x^2))\*Cosh[2\*(e + f\*x)] - (96\*I)\*d\*(c^2\*f^2 + 2\*c\*d\*f^2\*x + d^2\*(2 + f^2\*x^2))\*Sinh[e + f\*x] - 2\*f\*(c + d\*x)\*(2\*c^2\*f^2 + 4\*c\*d\*f^2\*x + d^2\*(3 + 2\*f^2\*x^2))\*Sinh[2\*(e + f\*x)])/(16\*f^4)

**Maple [B]** time = 0.018, size = 1082, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3\*(a+I\*a\*sinh(f\*x+e))^2,x)

[Out] 1/f\*(3/2/f^3\*d^3\*e^2\*a^2\*(f\*x+e)^2-1/f^3\*d^3\*e\*a^2\*(f\*x+e)^3+1/f^3\*d^3\*e^3\*a^2\*(1/2\*cosh(f\*x+e)\*sinh(f\*x+e)-1/2\*f\*x-1/2\*e)+3/2/f\*c^2\*d\*a^2\*(f\*x+e)^2+1



```

/f^2*c*d^2*a^2*(f*x+e)^3-3/f^2*c*d^2*a^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*
x+e)-1/6*(f*x+e)^3-1/2*(f*x+e)*cosh(f*x+e)^2+1/4*cosh(f*x+e)*sinh(f*x+e)+1/
4*f*x+1/4*e)-3/f*c^2*d*a^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)
^2-1/4*cosh(f*x+e)^2)+2*I/f^3*d^3*a^2*((f*x+e)^3*cosh(f*x+e)-3*(f*x+e)^2*si
nh(f*x+e)+6*(f*x+e)*cosh(f*x+e)-6*sinh(f*x+e))+3/f^3*d^3*e*a^2*(1/2*(f*x+e)
^2*cosh(f*x+e)*sinh(f*x+e)-1/6*(f*x+e)^3-1/2*(f*x+e)*cosh(f*x+e)^2+1/4*cosh
(f*x+e)*sinh(f*x+e)+1/4*f*x+1/4*e)-3/f^3*d^3*e^2*a^2*(1/2*(f*x+e)*cosh(f*x+
e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-1/f^3*d^3*e^3*a^2*(f*x+e)-1
/f^3*d^3*a^2*(1/2*(f*x+e)^3*cosh(f*x+e)*sinh(f*x+e)-1/8*(f*x+e)^4-3/4*(f*x+
e)^2*cosh(f*x+e)^2+3/4*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+3/8*(f*x+e)^2-3/8*co
sh(f*x+e)^2)+1/4/f^3*d^3*a^2*(f*x+e)^4+2*I*c^3*a^2*cosh(f*x+e)+6*I/f^2*c*d^
2*e^2*a^2*cosh(f*x+e)-12*I/f^2*c*d^2*e*a^2*((f*x+e)*cosh(f*x+e)-sinh(f*x+e)
)-6*I/f*c^2*d*e*a^2*cosh(f*x+e)+6*I/f*c^2*d*a^2*((f*x+e)*cosh(f*x+e)-sinh(f
*x+e))-2*I/f^3*d^3*e^3*a^2*cosh(f*x+e)+6*I/f^3*d^3*e^2*a^2*((f*x+e)*cosh(f*
x+e)-sinh(f*x+e))+3/f^2*c*d^2*e^2*a^2*(f*x+e)-3/f*c^2*d*e*a^2*(f*x+e)+3/f*c
^2*d*e*a^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)+6/f^2*c*d^2*e*a^2*(1
/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-6*I/f^3
*d^3*e*a^2*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)+2*cosh(f*x+e))-3/f^
2*c*d^2*e^2*a^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)-3/f^2*c*d^2*e*a
^2*(f*x+e)^2+6*I/f^2*c*d^2*a^2*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)
+2*cosh(f*x+e))-c^3*a^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)+c^3*a^2
*(f*x+e))

```

---

**Maxima [B]** time = 1.15161, size = 709, normalized size = 2.89

$$\frac{1}{4} a^2 d^3 x^4 + a^2 c d^2 x^3 + \frac{3}{2} a^2 c^2 d x^2 + \frac{3}{16} \left( 4x^2 - \frac{(2fxe^{2e} - e^{2e})e^{2fx}}{f^2} + \frac{(2fx+1)e^{(-2fx-2e)}}{f^2} \right) a^2 c^2 d + \frac{1}{16} \left( 8x^3 - \frac{3(2fx+1)e^{(-2fx-2e)}}{f^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="maxima")

[Out] 1/4\*a^2\*d^3\*x^4 + a^2\*c\*d^2\*x^3 + 3/2\*a^2\*c^2\*d\*x^2 + 3/16\*(4\*x^2 - (2\*f\*x\*e^(2\*e) - e^(2\*e))\*e^(2\*f\*x)/f^2 + (2\*f\*x + 1)\*e^(-2\*f\*x - 2\*e)/f^2)\*a^2\*c^2\*d + 1/16\*(8\*x^3 - 3\*(2\*f^2\*x^2\*e^(2\*e) - 2\*f\*x\*e^(2\*e) + e^(2\*e))\*e^(2\*f\*x)/f^3 + 3\*(2\*f^2\*x^2 + 2\*f\*x + 1)\*e^(-2\*f\*x - 2\*e)/f^3)\*a^2\*c\*d^2 + 1/32\*(4\*x^4 - (4\*f^3\*x^3\*e^(2\*e) - 6\*f^2\*x^2\*e^(2\*e) + 6\*f\*x\*e^(2\*e) - 3\*e^(2\*e))\*e^(2\*f\*x)/f^4 + (4\*f^3\*x^3 + 6\*f^2\*x^2 + 6\*f\*x + 3)\*e^(-2\*f\*x - 2\*e)/f^4)\*a^2\*d^3 + 1/8\*a^2\*c^3\*(4\*x - e^(2\*f\*x + 2\*e)/f + e^(-2\*f\*x - 2\*e)/f) + a^2\*c^3\*x + 3\*I\*a^2\*c^2\*d\*((f\*x\*e^e - e^e)\*e^(f\*x)/f^2 + (f\*x + 1)\*e^(-f\*x - e)/f^2) + 3\*I\*a^2\*c\*d^2\*((f^2\*x^2\*e^e - 2\*f\*x\*e^e + 2\*e^e)\*e^(f\*x)/f^3 + (f^2\*x^2 + 2\*f\*x + 2)\*e^(-f\*x - e)/f^3) + I\*a^2\*d^3\*((f^3\*x^3\*e^e - 3\*f^2\*x^2\*e

$$\frac{e^x + 6fx e^x - 6e^x}{f^4} + \frac{(f^3 x^3 + 3f^2 x^2 + 6fx + 6)e^{-fx - e}}{f^4} + 2I a^2 c^3 \frac{\cosh(fx + e)}{f}$$

**Fricas [B]** time = 3.27347, size = 1283, normalized size = 5.24

$$\left(4a^2 d^3 f^3 x^3 + 4a^2 c^3 f^3 + 6a^2 c^2 d f^2 + 6a^2 c d^2 f + 3a^2 d^3 + 6(2a^2 c d^2 f^3 + a^2 d^3 f^2)\right)x^2 + 6(2a^2 c^2 d f^3 + 2a^2 c d^2 f^2 + a^2 d^3 f)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="fricas")

[Out]  $\frac{1}{32}(4a^2 d^3 f^3 x^3 + 4a^2 c^3 f^3 + 6a^2 c^2 d f^2 + 6a^2 c d^2 f + 3a^2 d^3 + 6(2a^2 c d^2 f^3 + a^2 d^3 f^2))x^2 + 6(2a^2 c^2 d f^3 + 2a^2 c d^2 f^2 + a^2 d^3 f)x - (4a^2 d^3 f^3 x^3 + 4a^2 c^3 f^3 - 6a^2 c^2 d f^2 + 6a^2 c d^2 f - 3a^2 d^3 + 6(2a^2 c d^2 f^3 - a^2 d^3 f^2))x^2 + 6(2a^2 c^2 d f^3 - 2a^2 c d^2 f^2 + a^2 d^3 f)x)e^{(4fx + 4e)} + (32I a^2 d^3 f^3 x^3 + 32I a^2 c^3 f^3 - 96I a^2 c^2 d f^2 + 192I a^2 c d^2 f - 192I a^2 d^3 + (96I a^2 c d^2 f^3 - 96I a^2 d^3 f^2))x^2 + (96I a^2 c^2 d f^3 - 192I a^2 c d^2 f^2 + 192I a^2 d^3 f)x)e^{(3fx + 3e)} + 12(a^2 d^3 f^4 x^4 + 4a^2 c d^2 f^4 x^3 + 6a^2 c^2 d f^4 x^2 + 4a^2 c^3 f^4 x)e^{(2fx + 2e)} + (32I a^2 d^3 f^3 x^3 + 32I a^2 c^3 f^3 + 96I a^2 c^2 d f^2 + 192I a^2 c d^2 f + 192I a^2 d^3 + (96I a^2 c d^2 f^3 + 192I a^2 c d^2 f^2 + 192I a^2 d^3 f))x)e^{(fx + e)}e^{(-2fx - 2e)}/f^4$

**Sympy [A]** time = 4.62814, size = 1153, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*3\*(a+I\*a\*sinh(f\*x+e))\*\*2,x)

[Out]  $3a^{**2}c^{**3}x/2 + 9a^{**2}c^{**2}d^{**2}x^{**2}/4 + 3a^{**2}c^{**2}d^{**2}x^{**3}/2 + 3a^{**2}d^{**3}x^{**4}/8 + \text{Piecewise}(\left(\left(\left(128a^{**2}c^{**3}f^{**27}\exp(4e) + 384a^{**2}c^{**2}d^{**2}f^{**27}x\exp(4e) + 192a^{**2}c^{**2}d^{**2}f^{**26}\exp(4e) + 384a^{**2}c^{**2}d^{**2}f^{**27}x^{**2}\exp(4e) + 384a^{**2}c^{**2}d^{**2}f^{**26}x\exp(4e) + 192a^{**2}c^{**2}d^{**2}f^{**25}\exp(4e) + 128a^{**2}d^{**3}f^{**27}x^{**3}\exp(4e) + 192a^{**2}d^{**3}f^{**26}x^{**2}\exp(4e) + \right.\right.\right.$

```

192*a**2*d**3*f**25*x*exp(4*e) + 96*a**2*d**3*f**24*exp(4*e))*exp(-2*f*x) +
(-128*a**2*c**3*f**27*exp(8*e) - 384*a**2*c**2*d*f**27*x*exp(8*e) + 192*a*
*2*c**2*d*f**26*exp(8*e) - 384*a**2*c*d**2*f**27*x**2*exp(8*e) + 384*a**2*c
*d**2*f**26*x*exp(8*e) - 192*a**2*c*d**2*f**25*exp(8*e) - 128*a**2*d**3*f**
27*x**3*exp(8*e) + 192*a**2*d**3*f**26*x**2*exp(8*e) - 192*a**2*d**3*f**25*
x*exp(8*e) + 96*a**2*d**3*f**24*exp(8*e))*exp(2*f*x) + (1024*I*a**2*c**3*f*
*27*exp(5*e) + 3072*I*a**2*c**2*d*f**27*x*exp(5*e) + 3072*I*a**2*c**2*d*f**
26*exp(5*e) + 3072*I*a**2*c*d**2*f**27*x**2*exp(5*e) + 6144*I*a**2*c*d**2*f
**26*x*exp(5*e) + 6144*I*a**2*c*d**2*f**25*exp(5*e) + 1024*I*a**2*d**3*f**2
7*x**3*exp(5*e) + 3072*I*a**2*d**3*f**26*x**2*exp(5*e) + 6144*I*a**2*d**3*f
**25*x*exp(5*e) + 6144*I*a**2*d**3*f**24*exp(5*e))*exp(-f*x) + (1024*I*a**2
*c**3*f**27*exp(7*e) + 3072*I*a**2*c**2*d*f**27*x*exp(7*e) - 3072*I*a**2*c
**2*d*f**26*exp(7*e) + 3072*I*a**2*c*d**2*f**27*x**2*exp(7*e) - 6144*I*a**2*
c*d**2*f**26*x*exp(7*e) + 6144*I*a**2*c*d**2*f**25*exp(7*e) + 1024*I*a**2*d
**3*f**27*x**3*exp(7*e) - 3072*I*a**2*d**3*f**26*x**2*exp(7*e) + 6144*I*a**
2*d**3*f**25*x*exp(7*e) - 6144*I*a**2*d**3*f**24*exp(7*e))*exp(f*x))*exp(-6
*e)/(1024*f**28), Ne(1024*f**28*exp(6*e), 0)), (x**4*(-a**2*d**3*exp(4*e) +
4*I*a**2*d**3*exp(3*e) - 4*I*a**2*d**3*exp(e) - a**2*d**3)*exp(-2*e)/16 +
x**3*(-a**2*c*d**2*exp(4*e) + 4*I*a**2*c*d**2*exp(3*e) - 4*I*a**2*c*d**2*ex
p(e) - a**2*c*d**2)*exp(-2*e)/4 + x**2*(-3*a**2*c**2*d*exp(4*e) + 12*I*a**2
*c**2*d*exp(3*e) - 12*I*a**2*c**2*d*exp(e) - 3*a**2*c**2*d)*exp(-2*e)/8 + x
*(-a**2*c**3*exp(4*e) + 4*I*a**2*c**3*exp(3*e) - 4*I*a**2*c**3*exp(e) - a**
2*c**3)*exp(-2*e)/4, True))

```

---

**Giac [B]** time = 1.34564, size = 788, normalized size = 3.22

$$\frac{3}{8} a^2 d^3 x^4 + \frac{3}{2} a^2 c d^2 x^3 + \frac{9}{4} a^2 c^2 d x^2 + \frac{3}{2} a^2 c^3 x - \frac{(4 a^2 d^3 f^3 x^3 + 12 a^2 c d^2 f^3 x^2 + 12 a^2 c^2 d f^3 x - 6 a^2 d^3 f^2 x^2 + 4 a^2 c^3 f^3 - 12 a^2 c^3 f^4)}{32 f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="giac")

[Out]  $\frac{3}{8} a^2 d^3 x^4 + \frac{3}{2} a^2 c d^2 x^3 + \frac{9}{4} a^2 c^2 d x^2 + \frac{3}{2} a^2 c^3 x - \frac{1}{32} (4 a^2 d^3 f^3 x^3 + 12 a^2 c d^2 f^3 x^2 + 12 a^2 c^2 d f^3 x - 6 a^2 d^3 f^2 x^2 + 4 a^2 c^3 f^3 - 12 a^2 c^3 f^4) / f^4 + (I a^2 d^3 f^3 x^3 + 3 I a^2 c d^2 f^3 x^2 + 3 I a^2 c^2 d f^3 x - 3 I a^2 d^3 f^2 x^2 + I a^2 c^3 f^3 - 6 I a^2 c d^2 f^2 x - 3 I a^2 c^2 d f^2 + 6 I a^2 d^3 f x + 6 I a^2 c d^2 f - 6 I a^2 d^3) e^{(f x + e)} / f^4 + (I a^2 d^3 f^3 x^3 + 3 I a^2 c d^2 f^3 x^2 + 3 I a^2 c^2 d f^3 x + 3 I a^2 d^3 f^2 x^2 + I a^2 c^3 f^3 + 6 I a^2 c d^2 f^2 x + 3 I a^2 c^2 d f^2 + 6 I a^2 d^3 f x + 6 I a^2 c d^2 f - 6 I a^2 d^3) e^{(f x + e)} / f^4$

$$2*f + 6*I*a^2*d^3)*e^{(-f*x - e)/f^4} + 1/32*(4*a^2*d^3*f^3*x^3 + 12*a^2*c*d^2*f^3*x^2 + 12*a^2*c^2*d*f^3*x + 6*a^2*d^3*f^2*x^2 + 4*a^2*c^3*f^3 + 12*a^2*c*d^2*f^2*x + 6*a^2*c^2*d*f^2 + 6*a^2*d^3*f*x + 6*a^2*c*d^2*f + 3*a^2*d^3)*e^{(-2*f*x - 2*e)/f^4}$$

### 3.103 $\int (c + dx)^2 (a + ia \sinh(e + fx))^2 dx$

**Optimal.** Leaf size=174

$$\frac{a^2 d(c + dx) \sinh^2(e + fx)}{2f^2} - \frac{4ia^2 d(c + dx) \sinh(e + fx)}{f^2} + \frac{2ia^2 (c + dx)^2 \cosh(e + fx)}{f} - \frac{a^2 (c + dx)^2 \sinh(e + fx) \cosh(e + fx)}{2f}$$

[Out]  $(a^2 d^2 x)/(4f^2) + (a^2 (c + dx)^3)/(2d) + ((4I) a^2 d^2 \cosh[e + fx])/f^3 + ((2I) a^2 (c + dx)^2 \cosh[e + fx])/f - ((4I) a^2 d (c + dx) \sinh[e + fx])/f^2 - (a^2 d^2 \cosh[e + fx] \sinh[e + fx])/(4f^3) - (a^2 (c + dx)^2 \cosh[e + fx] \sinh[e + fx])/(2f) + (a^2 d (c + dx) \sinh[e + fx]^2)/(2f^2)$

**Rubi [A]** time = 0.197184, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$ , Rules used = {3317, 3296, 2638, 3311, 32, 2635, 8}

$$\frac{a^2 d(c + dx) \sinh^2(e + fx)}{2f^2} - \frac{4ia^2 d(c + dx) \sinh(e + fx)}{f^2} + \frac{2ia^2 (c + dx)^2 \cosh(e + fx)}{f} - \frac{a^2 (c + dx)^2 \sinh(e + fx) \cosh(e + fx)}{2f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + dx)^2 (a + I a \sinh[e + fx])^2, x]$

[Out]  $(a^2 d^2 x)/(4f^2) + (a^2 (c + dx)^3)/(2d) + ((4I) a^2 d^2 \cosh[e + fx])/f^3 + ((2I) a^2 (c + dx)^2 \cosh[e + fx])/f - ((4I) a^2 d (c + dx) \sinh[e + fx])/f^2 - (a^2 d^2 \cosh[e + fx] \sinh[e + fx])/(4f^3) - (a^2 (c + dx)^2 \cosh[e + fx] \sinh[e + fx])/(2f) + (a^2 d (c + dx) \sinh[e + fx]^2)/(2f^2)$

#### Rule 3317

$\text{Int}[(c + dx)^m (a + b \sin[e + fx])^n, x] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c + dx)^m (a + b \sin[e + fx])^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ (\text{EqQ}[n, 1] \ || \ \text{IGtQ}[m, 0] \ || \ \text{NeQ}[a^2 - b^2, 0])$

#### Rule 3296

$\text{Int}[(c + dx)^m \cos[e + fx], x] \rightarrow -\text{Simp}[(c + dx)^m \cos[e + fx]/f, x] + \text{Dist}[(d m)/f, \text{Int}[(c + dx)^{m-1} \cos[e + fx], x]]$

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

### Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

### Rubi steps

$$\begin{aligned}
\int (c + dx)^2 (a + ia \sinh(e + fx))^2 dx &= \int (a^2(c + dx)^2 + 2ia^2(c + dx)^2 \sinh(e + fx) - a^2(c + dx)^2 \sinh^2(e + fx)) dx \\
&= \frac{a^2(c + dx)^3}{3d} + (2ia^2) \int (c + dx)^2 \sinh(e + fx) dx - a^2 \int (c + dx)^2 \sinh^2(e + fx) dx \\
&= \frac{a^2(c + dx)^3}{3d} + \frac{2ia^2(c + dx)^2 \cosh(e + fx)}{f} - \frac{a^2(c + dx)^2 \cosh(e + fx) \sinh(e + fx)}{2f} \\
&= \frac{a^2(c + dx)^3}{2d} + \frac{2ia^2(c + dx)^2 \cosh(e + fx)}{f} - \frac{4ia^2 d(c + dx) \sinh(e + fx)}{f^2} - \frac{a^2 d^2 \cosh^2(e + fx)}{2f^2} \\
&= \frac{a^2 d^2 x}{4f^2} + \frac{a^2(c + dx)^3}{2d} + \frac{4ia^2 d^2 \cosh(e + fx)}{f^3} + \frac{2ia^2(c + dx)^2 \cosh(e + fx)}{f} - \frac{4ia^2 d(c + dx) \sinh(e + fx)}{f^2}
\end{aligned}$$

**Mathematica [A]** time = 0.713293, size = 189, normalized size = 1.09

$$a^2 (16i (c^2 f^2 + 2cdf^2 x + d^2 (f^2 x^2 + 2)) \cosh(e + fx) - 2c^2 f^2 \sinh(2(e + fx)) + 12c^2 f^3 x - 4cdf^2 x \sinh(2(e + fx)) - 32d^2 f^2 x^2 \cosh(2(e + fx)) - 32d^2 f^2 x^2 \sinh(2(e + fx))) / (8f^3)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2\*(a + I\*a\*Sinh[e + f\*x])^2,x]

[Out] (a^2\*(12\*c^2\*f^3\*x + 12\*c\*d\*f^3\*x^2 + 4\*d^2\*f^3\*x^3 + (16\*I)\*(c^2\*f^2 + 2\*c\*d\*f^2\*x + d^2\*(2 + f^2\*x^2))\*Cosh[e + f\*x] + 2\*d\*f\*(c + d\*x)\*Cosh[2\*(e + f\*x)] - (32\*I)\*c\*d\*f\*Sinh[e + f\*x] - (32\*I)\*d^2\*f\*x\*Sinh[e + f\*x] - d^2\*Sinh[2\*(e + f\*x)] - 2\*c^2\*f^2\*Sinh[2\*(e + f\*x)] - 4\*c\*d\*f^2\*x\*Sinh[2\*(e + f\*x)] - 2\*d^2\*f^2\*x^2\*Sinh[2\*(e + f\*x)]))/(8\*f^3)

**Maple [B]** time = 0.014, size = 550, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2\*(a+I\*a\*sinh(f\*x+e))^2,x)

[Out] 1/f\*(1/3/f^2\*d^2\*a^2\*(f\*x+e)^3-4\*I/f^2\*d^2\*e\*a^2\*((f\*x+e)\*cosh(f\*x+e)-sinh(f\*x+e))-1/f^2\*d^2\*a^2\*(1/2\*(f\*x+e)^2\*cosh(f\*x+e)\*sinh(f\*x+e)-1/6\*(f\*x+e)^3-1/2\*(f\*x+e)\*cosh(f\*x+e)^2+1/4\*cosh(f\*x+e)\*sinh(f\*x+e)+1/4\*f\*x+1/4\*e)-1/f^2\*

$$d^2 e^a (f x + e)^{-2} - 4 I / f c d e^a (f x + e) \cosh(f x + e) + 2 / f^2 d^2 e^a (1/2 (f x + e) \cosh(f x + e) \sinh(f x + e) - 1/4 (f x + e)^2 - 1/4 \cosh(f x + e)^2) + 1 / f^2 d^2 e^a (1/2 (f x + e) + 4 I / f c d a^2 ((f x + e) \cosh(f x + e) - \sinh(f x + e)) - 1 / f^2 d^2 e^a (1/2 \cosh(f x + e) \sinh(f x + e) - 1/2 f x - 1/2 e) + 1 / f c d a^2 (f x + e)^2 + 2 I / f^2 d^2 a^2 ((f x + e)^2 \cosh(f x + e) - 2 (f x + e) \sinh(f x + e) + 2 \cosh(f x + e)) - 2 / f c d a^2 (1/2 (f x + e) \cosh(f x + e) \sinh(f x + e) - 1/4 (f x + e)^2 - 1/4 \cosh(f x + e)^2) - 2 / f c d e^a (f x + e) + 2 I c^2 a^2 \cosh(f x + e) + 2 / f c d e^a (1/2 \cosh(f x + e) \sinh(f x + e) - 1/2 f x - 1/2 e) + c^2 a^2 (f x + e) + 2 I / f^2 d^2 e^a \cosh(f x + e) - c^2 a^2 (1/2 \cosh(f x + e) \sinh(f x + e) - 1/2 f x - 1/2 e))$$

**Maxima [B]** time = 1.26859, size = 440, normalized size = 2.53

$$\frac{1}{3} a^2 d^2 x^3 + a^2 c d x^2 + \frac{1}{8} \left( 4 x^2 - \frac{(2 f x e^{(2e)} - e^{(2e)}) e^{(2 f x)}}{f^2} + \frac{(2 f x + 1) e^{(-2 f x - 2e)}}{f^2} \right) a^2 c d + \frac{1}{48} \left( 8 x^3 - \frac{3(2 f^2 x^2 e^{(2e)} - 2 f x e^{(2e)})}{f^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="maxima")

[Out]  $\frac{1}{3} a^2 d^2 x^3 + a^2 c d x^2 + \frac{1}{8} (4 x^2 - \frac{(2 f x e^{(2e)} - e^{(2e)}) e^{(2 f x)}}{f^2} + \frac{(2 f x + 1) e^{(-2 f x - 2e)}}{f^2}) a^2 c d + \frac{1}{48} (8 x^3 - \frac{3(2 f^2 x^2 e^{(2e)} - 2 f x e^{(2e)})}{f^3})$

**Fricas [B]** time = 3.04527, size = 768, normalized size = 4.41

$$(2 a^2 d^2 f^2 x^2 + 2 a^2 c^2 f^2 + 2 a^2 c d f + a^2 d^2 + 2 (2 a^2 c d f^2 + a^2 d^2 f)) x - (2 a^2 d^2 f^2 x^2 + 2 a^2 c^2 f^2 - 2 a^2 c d f + a^2 d^2 + 2 (2 a^2 c d f^2 + a^2 d^2 f))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="fricas")

[Out]  $\frac{1}{16} (2 a^2 d^2 f^2 x^2 + 2 a^2 c^2 f^2 + 2 a^2 c d f + a^2 d^2 + 2 (2 a^2 c d f^2 + a^2 d^2 f)) x - (2 a^2 d^2 f^2 x^2 + 2 a^2 c^2 f^2 - 2 a^2 c d f + a^2 d^2 + 2 (2 a^2 c d f^2 + a^2 d^2 f))$



$$a^2d^2 + 2*(2*a^2*c*d*f^2 - a^2*d^2*f)*x)*e^{(4*f*x + 4*e)} + (16*I*a^2*d^2*f^2*x^2 + 16*I*a^2*c^2*f^2 - 32*I*a^2*c*d*f + 32*I*a^2*d^2 + (32*I*a^2*c*d*f^2 - 32*I*a^2*d^2*f)*x)*e^{(3*f*x + 3*e)} + 8*(a^2*d^2*f^3*x^3 + 3*a^2*c*d*f^3*x^2 + 3*a^2*c^2*f^3*x)*e^{(2*f*x + 2*e)} + (16*I*a^2*d^2*f^2*x^2 + 16*I*a^2*c^2*f^2 + 32*I*a^2*c*d*f + 32*I*a^2*d^2 + (32*I*a^2*c*d*f^2 + 32*I*a^2*d^2*f)*x)*e^{(f*x + e)})*e^{(-2*f*x - 2*e)}/f^3$$

**Sympy [A]** time = 3.20624, size = 706, normalized size = 4.06

$$\frac{3a^2c^2x}{2} + \frac{3a^2cdx^2}{2} + \frac{a^2d^2x^3}{2} + \left\{ \frac{\left( (32a^2c^2f^{17}e^{3e} + 64a^2cdf^{17}xe^{3e} + 32a^2cdf^{16}e^{3e} + 32a^2d^2f^{17}x^2e^{3e} + 32a^2d^2f^{16}xe^{3e} + 16a^2d^2f^{15}e^{3e}) \right) e^{-2fx} + (-32a^2c^2f^{17}x^3(-a^2d^2e^{4e} + 4ia^2d^2e^{3e} - 4ia^2d^2e^e - a^2d^2)e^{-2e}}{12} + \frac{x^2(-a^2cde^{4e} + 4ia^2cde^{3e} - 4ia^2cde^e - a^2cd)e^{-2e}}{4} + \frac{x(-a^2c^2e^{4e} + 4ia^2c^2e^{3e} - 4ia^2c^2e^e - a^2c^2d)e^{-2e}}{4} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2\*(a+I\*a\*sinh(f\*x+e))\*\*2,x)

[Out] 3\*a\*\*2\*c\*\*2\*x/2 + 3\*a\*\*2\*c\*d\*x\*\*2/2 + a\*\*2\*d\*\*2\*x\*\*3/2 + Piecewise((((32\*a\*\*2\*c\*\*2\*f\*\*17\*exp(3\*e) + 64\*a\*\*2\*c\*d\*f\*\*17\*x\*exp(3\*e) + 32\*a\*\*2\*c\*d\*f\*\*16\*e\*exp(3\*e) + 32\*a\*\*2\*d\*\*2\*f\*\*17\*x\*\*2\*exp(3\*e) + 32\*a\*\*2\*d\*\*2\*f\*\*16\*x\*exp(3\*e) + 16\*a\*\*2\*d\*\*2\*f\*\*15\*exp(3\*e))\*exp(-2\*f\*x) + (-32\*a\*\*2\*c\*\*2\*f\*\*17\*exp(7\*e) - 64\*a\*\*2\*c\*d\*f\*\*17\*x\*exp(7\*e) + 32\*a\*\*2\*c\*d\*f\*\*16\*exp(7\*e) - 32\*a\*\*2\*d\*\*2\*f\*\*17\*x\*\*2\*exp(7\*e) + 32\*a\*\*2\*d\*\*2\*f\*\*16\*x\*exp(7\*e) - 16\*a\*\*2\*d\*\*2\*f\*\*15\*exp(7\*e))\*exp(2\*f\*x) + (256\*I\*a\*\*2\*c\*\*2\*f\*\*17\*exp(4\*e) + 512\*I\*a\*\*2\*c\*d\*f\*\*17\*x\*exp(4\*e) + 512\*I\*a\*\*2\*c\*d\*f\*\*16\*exp(4\*e) + 256\*I\*a\*\*2\*d\*\*2\*f\*\*17\*x\*\*2\*exp(4\*e) + 512\*I\*a\*\*2\*d\*\*2\*f\*\*16\*x\*exp(4\*e) + 512\*I\*a\*\*2\*d\*\*2\*f\*\*15\*exp(4\*e))\*exp(-f\*x) + (256\*I\*a\*\*2\*c\*\*2\*f\*\*17\*exp(6\*e) + 512\*I\*a\*\*2\*c\*d\*f\*\*17\*x\*exp(6\*e) - 512\*I\*a\*\*2\*c\*d\*f\*\*16\*exp(6\*e) + 256\*I\*a\*\*2\*d\*\*2\*f\*\*17\*x\*\*2\*exp(6\*e) - 512\*I\*a\*\*2\*d\*\*2\*f\*\*16\*x\*exp(6\*e) + 512\*I\*a\*\*2\*d\*\*2\*f\*\*15\*exp(6\*e))\*exp(f\*x)))\*exp(-5\*e)/(256\*f\*\*18), Ne(256\*f\*\*18\*exp(5\*e), 0)), (x\*\*3\*(-a\*\*2\*d\*\*2\*exp(4\*e) + 4\*I\*a\*\*2\*d\*\*2\*exp(3\*e) - 4\*I\*a\*\*2\*d\*\*2\*exp(e) - a\*\*2\*d\*\*2)\*exp(-2\*e)/12 + x\*\*2\*(-a\*\*2\*c\*d\*exp(4\*e) + 4\*I\*a\*\*2\*c\*d\*exp(3\*e) - 4\*I\*a\*\*2\*c\*d\*exp(e) - a\*\*2\*c\*d)\*exp(-2\*e)/4 + x\*(-a\*\*2\*c\*\*2\*exp(4\*e) + 4\*I\*a\*\*2\*c\*\*2\*exp(3\*e) - 4\*I\*a\*\*2\*c\*\*2\*exp(e) - a\*\*2\*c\*\*2)\*exp(-2\*e)/4, True))

**Giac [B]** time = 1.26013, size = 455, normalized size = 2.61

$$\frac{1}{2} a^2 d^2 x^3 + \frac{3}{2} a^2 c d x^2 + \frac{3}{2} a^2 c^2 x - \frac{(2 a^2 d^2 f^2 x^2 + 4 a^2 c d f^2 x + 2 a^2 c^2 f^2 - 2 a^2 d^2 f x - 2 a^2 c d f + a^2 d^2) e^{(2 f x + 2 e)}}{16 f^3} + \frac{(i a^2 d^2 f^2 x^2 + 4 i a^2 c d f^2 x + 2 i a^2 c^2 f^2 - 2 i a^2 d^2 f x - 2 i a^2 c d f + i a^2 d^2) e^{(2 f x + 2 e)}}{16 f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="giac")

[Out]  $\frac{1}{2}a^2d^2x^3 + \frac{3}{2}a^2c*d*x^2 + \frac{3}{2}a^2c^2*x - \frac{1}{16}(2a^2d^2f^2x^2 + 4a^2c*d*f^2*x + 2a^2c^2f^2 - 2a^2d^2f*x - 2a^2c*d*f + a^2d^2) * e^{(2f*x + 2e)}/f^3 + (Ia^2d^2f^2x^2 + 2Ia^2c*d*f^2*x + Ia^2c^2f^2 - 2Ia^2d^2f*x - 2Ia^2c*d*f + 2Ia^2d^2) * e^{(f*x + e)}/f^3 - (-Ia^2d^2f^2x^2 - 2Ia^2c*d*f^2*x - Ia^2c^2f^2 - 2Ia^2d^2f*x - 2Ia^2c*d*f - 2Ia^2d^2) * e^{(-f*x - e)}/f^3 + \frac{1}{16}(2a^2d^2f^2x^2 + 4a^2c*d*f^2*x + 2a^2c^2f^2 + 2a^2d^2f*x + 2a^2c*d*f + a^2d^2) * e^{(-2f*x - 2e)}/f^3$

### 3.104 $\int (c + dx)(a + ia \sinh(e + fx))^2 dx$

**Optimal.** Leaf size=122

$$\frac{2ia^2(c + dx) \cosh(e + fx)}{f} - \frac{a^2(c + dx) \sinh(e + fx) \cosh(e + fx)}{2f} + \frac{a^2(c + dx)^2}{2d} + \frac{1}{2}a^2cx + \frac{a^2d \sinh^2(e + fx)}{4f^2} - \frac{2ia^2d}{4f^2}$$

[Out]  $(a^2c*x)/2 + (a^2d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) + ((2*I)*a^2*(c + d*x)*\text{Cosh}[e + f*x])/f - ((2*I)*a^2*d*\text{Sinh}[e + f*x])/f^2 - (a^2*(c + d*x)*\text{Cosh}[e + f*x]*\text{Sinh}[e + f*x])/(2*f) + (a^2*d*\text{Sinh}[e + f*x]^2)/(4*f^2)$

**Rubi [A]** time = 0.105015, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {3317, 3296, 2637, 3310}

$$\frac{2ia^2(c + dx) \cosh(e + fx)}{f} - \frac{a^2(c + dx) \sinh(e + fx) \cosh(e + fx)}{2f} + \frac{a^2(c + dx)^2}{2d} + \frac{1}{2}a^2cx + \frac{a^2d \sinh^2(e + fx)}{4f^2} - \frac{2ia^2d}{4f^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)*(a + I*a*\text{Sinh}[e + f*x])^2, x]$

[Out]  $(a^2c*x)/2 + (a^2d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) + ((2*I)*a^2*(c + d*x)*\text{Cosh}[e + f*x])/f - ((2*I)*a^2*d*\text{Sinh}[e + f*x])/f^2 - (a^2*(c + d*x)*\text{Cosh}[e + f*x]*\text{Sinh}[e + f*x])/(2*f) + (a^2*d*\text{Sinh}[e + f*x]^2)/(4*f^2)$

#### Rule 3317

$\text{Int}[(c + d*x)^m * (a + b*\text{Sin}[e + f*x])^n, x]$  /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

#### Rule 3296

$\text{Int}[(c + d*x)^m * \text{Cos}[e + f*x], x]$  /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
Simp[(d*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sine[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sine[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

### Rubi steps

$$\begin{aligned}
\int (c + dx)(a + ia \sinh(e + fx))^2 dx &= \int (a^2(c + dx) + 2ia^2(c + dx) \sinh(e + fx) - a^2(c + dx) \sinh^2(e + fx)) dx \\
&= \frac{a^2(c + dx)^2}{2d} + (2ia^2) \int (c + dx) \sinh(e + fx) dx - a^2 \int (c + dx) \sinh^2(e + fx) dx \\
&= \frac{a^2(c + dx)^2}{2d} + \frac{2ia^2(c + dx) \cosh(e + fx)}{f} - \frac{a^2(c + dx) \cosh(e + fx) \sinh(e + fx)}{2f} + \\
&= \frac{1}{2}a^2cx + \frac{1}{4}a^2dx^2 + \frac{a^2(c + dx)^2}{2d} + \frac{2ia^2(c + dx) \cosh(e + fx)}{f} - \frac{2ia^2d \sinh(e + fx)}{f^2}
\end{aligned}$$

**Mathematica [A]** time = 1.20878, size = 86, normalized size = 0.7

$$\frac{a^2(-2(3(e + fx)(-2cf + de - dfx) + f(c + dx) \sinh(2(e + fx)) + 8id \sinh(e + fx)) + 16if(c + dx) \cosh(e + fx) + d \cosh(e + fx))}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (a^2*((16*I)*f*(c + d*x)*Cosh[e + f*x] + d*Cosh[2*(e + f*x)] - 2*(3*(e + f*
x)*(d*e - 2*c*f - d*f*x) + (8*I)*d*Sinh[e + f*x] + f*(c + d*x)*Sinh[2*(e +
f*x)])))/(8*f^2)
```

**Maple [A]** time = 0.015, size = 215, normalized size = 1.8

$$\frac{1}{f} \left( \frac{da^2 (fx + e)^2}{2f} + \frac{2ida^2 ((fx + e) \cosh(fx + e) - \sinh(fx + e))}{f} - \frac{da^2 \left( \frac{(fx + e) \cosh(fx + e) \sinh(fx + e)}{2} - \frac{(fx + e)}{4} \right)}{f} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)*(a+I*a*sinh(f*x+e))^2,x)`

[Out]  $\frac{1}{f} \left( \frac{1}{2} \frac{d}{f} a^2 (f*x+e)^2 + 2 \frac{I}{f} d a^2 ((f*x+e) \cosh(f*x+e) - \sinh(f*x+e)) - \frac{1}{f} d a^2 \left( \frac{1}{2} (f*x+e) \cosh(f*x+e) \sinh(f*x+e) - \frac{1}{4} (f*x+e)^2 - \frac{1}{4} \cosh(f*x+e)^2 \right) - \frac{d e}{f a^2} (f*x+e) - 2 \frac{I d e}{f a^2} \cosh(f*x+e) + \frac{d e}{f a^2} \left( \frac{1}{2} \cosh(f*x+e) \sinh(f*x+e) - \frac{1}{2} f*x - \frac{1}{2} e \right) + c a^2 (f*x+e) + 2 I c a^2 \cosh(f*x+e) - c a^2 \left( \frac{1}{2} \cosh(f*x+e) \sinh(f*x+e) - \frac{1}{2} f*x - \frac{1}{2} e \right) \right)$

**Maxima [A]** time = 1.18573, size = 225, normalized size = 1.84

$$\frac{1}{2} a^2 dx^2 + \frac{1}{16} \left( 4x^2 - \frac{(2fxe^{2e} - e^{2e})e^{2fx}}{f^2} + \frac{(2fx+1)e^{(-2fx-2e)}}{f^2} \right) a^2 d + \frac{1}{8} a^2 c \left( 4x - \frac{e^{(2fx+2e)}}{f} + \frac{e^{(-2fx-2e)}}{f} \right) + a^2 cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*(a+I*a*sinh(f*x+e))^2,x, algorithm="maxima")`

[Out]  $\frac{1}{2} a^2 d x^2 + \frac{1}{16} (4x^2 - (2fxe^{2e} - e^{2e})e^{2fx}/f^2 + (2fx+1)e^{(-2fx-2e)}/f^2) a^2 d + \frac{1}{8} a^2 c (4x - e^{(2fx+2e)}/f + e^{(-2fx-2e)}/f) + a^2 c x + I a^2 d ((fxe^e - e^e)e^{fx}/f^2 + (fx+1)e^{(-fx-e)}/f^2) + 2 I a^2 c \cosh(fx+e)/f$

**Fricas [A]** time = 2.9555, size = 389, normalized size = 3.19

$$\frac{(2a^2dfx + 2a^2cf + a^2d - (2a^2dfx + 2a^2cf - a^2d)e^{4fx+4e}) + (16i a^2dfx + 16i a^2cf - 16i a^2d)e^{3fx+3e} + 12(a^2df^2x^2)}{16f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")`

[Out]  $\frac{1}{16} (2a^2d f x + 2a^2c f + a^2d - (2a^2d f x + 2a^2c f - a^2d) e^{(4fx+4e)}) + (16I a^2d f x + 16I a^2c f - 16I a^2d) e^{(3fx+3e)} + 12(a^2d f^2 x^2 + 2a^2c f^2 x) e^{(2fx+2e)} + (16I a^2d f x + 16I a^2c f + 16I a^2d) e^{(fx+e)} e^{(-2fx-2e)}/f^2$

---

**Sympy [A]** time = 2.06403, size = 364, normalized size = 2.98

$$\frac{3a^2cx}{2} + \frac{3a^2dx^2}{4} + \left\{ \frac{\left( (32a^2cf^9e^{2e} + 32a^2df^9xe^{2e} + 16a^2df^8e^{2e})e^{-2fx} + (-32a^2cf^9e^{6e} - 32a^2df^9xe^{6e} + 16a^2df^8e^{6e})e^{2fx} + (256ia^2cf^9e^{3e} + 256ia^2df^9xe^{3e} + 256ia^2c) \right)}{256f^{10}} \right. \\ \left. + \frac{x^2(-a^2de^{4e} + 4ia^2de^{3e} - 4ia^2de^e - a^2d)e^{-2e}}{8} + \frac{x(-a^2ce^{4e} + 4ia^2ce^{3e} - 4ia^2ce^e - a^2c)e^{-2e}}{4} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+I\*a\*sinh(f\*x+e))\*\*2,x)

[Out] 3\*a\*\*2\*c\*x/2 + 3\*a\*\*2\*d\*x\*\*2/4 + Piecewise((((32\*a\*\*2\*c\*f\*\*9\*exp(2\*e) + 32\*a\*\*2\*d\*f\*\*9\*x\*exp(2\*e) + 16\*a\*\*2\*d\*f\*\*8\*exp(2\*e))\*exp(-2\*f\*x) + (-32\*a\*\*2\*c\*f\*\*9\*exp(6\*e) - 32\*a\*\*2\*d\*f\*\*9\*x\*exp(6\*e) + 16\*a\*\*2\*d\*f\*\*8\*exp(6\*e))\*exp(2\*f\*x) + (256\*I\*a\*\*2\*c\*f\*\*9\*exp(3\*e) + 256\*I\*a\*\*2\*d\*f\*\*9\*x\*exp(3\*e) + 256\*I\*a\*\*2\*d\*f\*\*8\*exp(3\*e))\*exp(-f\*x) + (256\*I\*a\*\*2\*c\*f\*\*9\*exp(5\*e) + 256\*I\*a\*\*2\*d\*f\*\*9\*x\*exp(5\*e) - 256\*I\*a\*\*2\*d\*f\*\*8\*exp(5\*e))\*exp(f\*x))\*exp(-4\*e)/(256\*f\*\*10), Ne(256\*f\*\*10\*exp(4\*e), 0)), (x\*\*2\*(-a\*\*2\*d\*exp(4\*e) + 4\*I\*a\*\*2\*d\*exp(3\*e) - 4\*I\*a\*\*2\*d\*exp(e) - a\*\*2\*d)\*exp(-2\*e)/8 + x\*(-a\*\*2\*c\*exp(4\*e) + 4\*I\*a\*\*2\*c\*exp(3\*e) - 4\*I\*a\*\*2\*c\*exp(e) - a\*\*2\*c)\*exp(-2\*e)/4, True))

---

**Giac [A]** time = 1.24014, size = 215, normalized size = 1.76

$$\frac{3}{4}a^2dx^2 + \frac{3}{2}a^2cx - \frac{(2a^2dfx + 2a^2cf - a^2d)e^{(2fx+2e)}}{16f^2} + \frac{(ia^2dfx + ia^2cf - ia^2d)e^{(fx+e)}}{f^2} + \frac{(ia^2dfx + ia^2cf + ia^2d)e^{(-fx+e)}}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="giac")

[Out] 3/4\*a^2\*d\*x^2 + 3/2\*a^2\*c\*x - 1/16\*(2\*a^2\*d\*f\*x + 2\*a^2\*c\*f - a^2\*d)\*e^(2\*f\*x + 2\*e)/f^2 + (I\*a^2\*d\*f\*x + I\*a^2\*c\*f - I\*a^2\*d)\*e^(f\*x + e)/f^2 + (I\*a^2\*d\*f\*x + I\*a^2\*c\*f + I\*a^2\*d)\*e^(-f\*x - e)/f^2 + 1/16\*(2\*a^2\*d\*f\*x + 2\*a^2\*c\*f + a^2\*d)\*e^(-2\*f\*x - 2\*e)/f^2

$$3.105 \quad \int \frac{(a+ia \sinh(e+fx))^2}{c+dx} dx$$

**Optimal.** Leaf size=149

$$\frac{2ia^2 \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d} - \frac{a^2 \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{2d} - \frac{a^2 \sinh\left(2e - \frac{2cf}{d}\right) \operatorname{Shi}\left(2xf + \frac{2cf}{d}\right)}{2d} + \frac{2ia^2 \cos\left(\frac{cf}{d}\right)}{d}$$

[Out]  $-(a^2 \operatorname{Cosh}[2e - (2cf)/d] \operatorname{CoshIntegral}[(2cf)/d + 2fx])/(2d) + (3a^2 \operatorname{Log}[c + dx])/(2d) + ((2I)a^2 \operatorname{CoshIntegral}[(cf)/d + fx] \operatorname{Sinh}[e - (cf)/d])/d + ((2I)a^2 \operatorname{Cosh}[e - (cf)/d] \operatorname{SinhIntegral}[(cf)/d + fx])/d - (a^2 \operatorname{Sinh}[2e - (2cf)/d] \operatorname{SinhIntegral}[(2cf)/d + 2fx])/(2d)$

**Rubi [A]** time = 0.352419, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$ , Rules used = {3318, 3312, 3303, 3298, 3301}

$$\frac{2ia^2 \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d} - \frac{a^2 \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{2d} - \frac{a^2 \sinh\left(2e - \frac{2cf}{d}\right) \operatorname{Shi}\left(2xf + \frac{2cf}{d}\right)}{2d} + \frac{2ia^2 \cos\left(\frac{cf}{d}\right)}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + I a \operatorname{Sinh}[e + fx])^2/(c + dx), x]$

[Out]  $-(a^2 \operatorname{Cosh}[2e - (2cf)/d] \operatorname{CoshIntegral}[(2cf)/d + 2fx])/(2d) + (3a^2 \operatorname{Log}[c + dx])/(2d) + ((2I)a^2 \operatorname{CoshIntegral}[(cf)/d + fx] \operatorname{Sinh}[e - (cf)/d])/d + ((2I)a^2 \operatorname{Cosh}[e - (cf)/d] \operatorname{SinhIntegral}[(cf)/d + fx])/d - (a^2 \operatorname{Sinh}[2e - (2cf)/d] \operatorname{SinhIntegral}[(2cf)/d + 2fx])/(2d)$

### Rule 3318

$\operatorname{Int}[(c + d(x))^m (a + b \sin[e + f(x)])^n, x\_Symbol] \rightarrow \operatorname{Dist}[(2a)^n, \operatorname{Int}[(c + dx)^m \operatorname{Sin}[(1 + \frac{\pi a}{2b})/2 + (fx)/2]^{2n}], x] /;$  FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])

### Rule 3312

$\operatorname{Int}[(c + d(x))^m \sin[e + f(x)]^n, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrigReduce}[(c + dx)^m \operatorname{Sin}[e + fx]^n, x], x] /;$  FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \sinh(e + fx))^2}{c + dx} dx &= (4a^2) \int \frac{\sin^4\left(\frac{1}{2}\left(ie + \frac{\pi}{2}\right) + \frac{ifx}{2}\right)}{c + dx} dx \\
&= (4a^2) \int \left( \frac{3}{8(c + dx)} - \frac{\cosh(2e + 2fx)}{8(c + dx)} + \frac{i \sinh(e + fx)}{2(c + dx)} \right) dx \\
&= \frac{3a^2 \log(c + dx)}{2d} + (2ia^2) \int \frac{\sinh(e + fx)}{c + dx} dx - \frac{1}{2}a^2 \int \frac{\cosh(2e + 2fx)}{c + dx} dx \\
&= \frac{3a^2 \log(c + dx)}{2d} - \frac{1}{2} \left( a^2 \cosh\left(2e - \frac{2cf}{d}\right) \right) \int \frac{\cosh\left(\frac{2cf}{d} + 2fx\right)}{c + dx} dx + \left( 2ia^2 \cosh\left(e - \frac{cf}{d}\right) \right) \\
&= -\frac{a^2 \cosh\left(2e - \frac{2cf}{d}\right) \text{Chi}\left(\frac{2cf}{d} + 2fx\right)}{2d} + \frac{3a^2 \log(c + dx)}{2d} + \frac{2ia^2 \text{Chi}\left(\frac{cf}{d} + fx\right) \sinh\left(e - \frac{cf}{d}\right)}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.395477, size = 117, normalized size = 0.79

$$\frac{a^2 \left( -4i \text{Chi}\left(f\left(\frac{c}{d} + x\right)\right) \sinh\left(e - \frac{cf}{d}\right) + \text{Chi}\left(\frac{2f(c+dx)}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right) + \sinh\left(2e - \frac{2cf}{d}\right) \text{Shi}\left(\frac{2f(c+dx)}{d}\right) - 4i \cosh\left(e - \frac{cf}{d}\right) \right)}{2d}$$

Antiderivative was successfully verified.



[In] Integrate[(a + I\*a\*Sinh[e + f\*x])^2/(c + d\*x), x]

[Out]  $-(a^2 * (\text{Cosh}[2*e - (2*c*f)/d] * \text{CoshIntegral}[(2*f*(c + d*x))/d] - 3*\text{Log}[c + d*x] - (4*I)*\text{CoshIntegral}[f*(c/d + x)] * \text{Sinh}[e - (c*f)/d] - (4*I)*\text{Cosh}[e - (c*f)/d] * \text{SinhIntegral}[f*(c/d + x)] + \text{Sinh}[2*e - (2*c*f)/d] * \text{SinhIntegral}[(2*f*(c + d*x))/d])) / (2*d)$

**Maple [A]** time = 0.119, size = 193, normalized size = 1.3

$$\frac{-ia^2 e^{-\frac{cf-de}{d}} \text{Ei}\left(1, -fx - e - \frac{cf-de}{d}\right) + \frac{3a^2 \ln(dx+c)}{2d} + \frac{a^2}{4d} e^{2\frac{cf-de}{d}} \text{Ei}\left(1, 2fx + 2e + 2\frac{cf-de}{d}\right) + \frac{a^2}{4d} e^{-2\frac{cf-de}{d}} \text{Ei}\left(1, -\right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*sinh(f\*x+e))^2/(d\*x+c), x)

[Out]  $-I*a^2/d*\exp(-(c*f-d*e)/d)*\text{Ei}(1, -f*x-e-(c*f-d*e)/d)+3/2*a^2*\ln(d*x+c)/d+1/4*a^2/d*\exp(2*(c*f-d*e)/d)*\text{Ei}(1, 2*f*x+2*e+2*(c*f-d*e)/d)+1/4*a^2/d*\exp(-2*(c*f-d*e)/d)*\text{Ei}(1, -2*f*x-2*e-2*(c*f-d*e)/d)+I*a^2/d*\exp((c*f-d*e)/d)*\text{Ei}(1, f*x+e+(c*f-d*e)/d)$

**Maxima [A]** time = 1.38847, size = 203, normalized size = 1.36

$$\frac{1}{4} a^2 \left( \frac{e^{\left(-2e + \frac{2cf}{d}\right)} E_1\left(\frac{2(dx+c)f}{d}\right)}{d} + \frac{e^{\left(2e - \frac{2cf}{d}\right)} E_1\left(-\frac{2(dx+c)f}{d}\right)}{d} + \frac{2 \log(dx+c)}{d} \right) + i a^2 \left( \frac{e^{\left(-e + \frac{cf}{d}\right)} E_1\left(\frac{(dx+c)f}{d}\right)}{d} - \frac{e^{\left(e - \frac{cf}{d}\right)} E_1\left(-\frac{(dx+c)f}{d}\right)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))^2/(d\*x+c), x, algorithm="maxima")

[Out]  $1/4*a^2*(e^{(-2*e + 2*c*f/d)}*\text{exp\_integral\_e}(1, 2*(d*x + c)*f/d)/d + e^{(2*e - 2*c*f/d)}*\text{exp\_integral\_e}(1, -2*(d*x + c)*f/d)/d + 2*\log(d*x + c)/d) + I*a^2*(e^{(-e + c*f/d)}*\text{exp\_integral\_e}(1, (d*x + c)*f/d)/d - e^{(e - c*f/d)}*\text{exp\_integral\_e}(1, -(d*x + c)*f/d)/d) + a^2*\log(d*x + c)/d$

**Fricas [A]** time = 2.81089, size = 309, normalized size = 2.07

$$\frac{a^2 \operatorname{Ei}\left(\frac{2(dfxc+cf)}{d}\right) e^{\left(\frac{2(de-cf)}{d}\right)} - 4i a^2 \operatorname{Ei}\left(\frac{dfxc+cf}{d}\right) e^{\left(\frac{de-cf}{d}\right)} + 4i a^2 \operatorname{Ei}\left(-\frac{dfxc+cf}{d}\right) e^{\left(-\frac{de-cf}{d}\right)} + a^2 \operatorname{Ei}\left(-\frac{2(dfxc+cf)}{d}\right) e^{\left(-\frac{2(de-cf)}{d}\right)} - 6 a^2 \log\left(\frac{d*x+c}{d}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))^2/(d\*x+c),x, algorithm="fricas")

[Out]  $-1/4*(a^2 \operatorname{Ei}(2*(d*f*x + c*f)/d) * e^{(2*(d*e - c*f)/d)} - 4*I*a^2 \operatorname{Ei}((d*f*x + c*f)/d) * e^{((d*e - c*f)/d)} + 4*I*a^2 \operatorname{Ei}(-(d*f*x + c*f)/d) * e^{-((d*e - c*f)/d)} + a^2 \operatorname{Ei}(-2*(d*f*x + c*f)/d) * e^{-2*(d*e - c*f)/d} - 6*a^2 * \log((d*x + c)/d)) / d$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$a^2 \left( \int -\frac{\sinh^2(e+fx)}{c+dx} dx + \int \frac{2i \sinh(e+fx)}{c+dx} dx + \int \frac{1}{c+dx} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))^2/(d\*x+c),x)

[Out]  $a**2*(\operatorname{Integral}(-\sinh(e+fx)**2/(c+d*x), x) + \operatorname{Integral}(2*I*\sinh(e+fx)/(c+d*x), x) + \operatorname{Integral}(1/(c+d*x), x))$

**Giac [A]** time = 1.3336, size = 188, normalized size = 1.26

$$\frac{a^2 \operatorname{Ei}\left(-\frac{2(dfxc+cf)}{d}\right) e^{\left(\frac{2cf}{d}-2e\right)} + 4i a^2 \operatorname{Ei}\left(-\frac{dfxc+cf}{d}\right) e^{\left(\frac{cf}{d}-e\right)} - 4i a^2 \operatorname{Ei}\left(\frac{dfxc+cf}{d}\right) e^{\left(-\frac{cf}{d}+e\right)} + a^2 \operatorname{Ei}\left(\frac{2(dfxc+cf)}{d}\right) e^{\left(-\frac{2cf}{d}+2e\right)} - 6 a^2 \log\left(\frac{d*x+c}{d}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))^2/(d\*x+c),x, algorithm="giac")

[Out]  $-1/4*(a^2 \operatorname{Ei}(-2*(d*f*x + c*f)/d) * e^{(2*c*f/d - 2*e)} + 4*I*a^2 \operatorname{Ei}(-(d*f*x + c*f)/d) * e^{(c*f/d - e)} - 4*I*a^2 \operatorname{Ei}((d*f*x + c*f)/d) * e^{-c*f/d + e} + a^2 \operatorname{Ei}(2*(d*f*x + c*f)/d) * e^{-2*c*f/d + 2*e} - 6*a^2 * \log(d*x + c)) / d$

$$3.106 \quad \int \frac{(a+ia \sinh(e+fx))^2}{(c+dx)^2} dx$$

**Optimal.** Leaf size=170

$$-\frac{a^2 f \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right)}{d^2} + \frac{2ia^2 f \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d^2} + \frac{2ia^2 f \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^2} - \frac{a^2 f \cosh\left(e - \frac{cf}{d}\right) \operatorname{Chi}\left(xf + \frac{cf}{d}\right)}{d^2}$$

[Out]  $(-4*a^2*\operatorname{Cosh}[e/2 + (I/4)*\operatorname{Pi} + (f*x)/2]^4)/(d*(c + d*x)) + ((2*I)*a^2*f*\operatorname{Cosh}[e - (c*f)/d]*\operatorname{CoshIntegral}[(c*f)/d + f*x])/d^2 - (a^2*f*\operatorname{CoshIntegral}[(2*c*f)/d + 2*f*x]*\operatorname{Sinh}[2*e - (2*c*f)/d])/d^2 + ((2*I)*a^2*f*\operatorname{Sinh}[e - (c*f)/d]*\operatorname{ShiIntegral}[(c*f)/d + f*x])/d^2 - (a^2*f*\operatorname{Cosh}[2*e - (2*c*f)/d]*\operatorname{SinhIntegral}[(2*c*f)/d + 2*f*x])/d^2$

**Rubi [A]** time = 0.339995, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$ , Rules used = {3318, 3313, 3303, 3298, 3301}

$$-\frac{a^2 f \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \sinh\left(2e - \frac{2cf}{d}\right)}{d^2} + \frac{2ia^2 f \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d^2} + \frac{2ia^2 f \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^2} - \frac{a^2 f \cosh\left(e - \frac{cf}{d}\right) \operatorname{Chi}\left(xf + \frac{cf}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + I*a*\operatorname{Sinh}[e + f*x])^2/(c + d*x)^2, x]$

[Out]  $(-4*a^2*\operatorname{Cosh}[e/2 + (I/4)*\operatorname{Pi} + (f*x)/2]^4)/(d*(c + d*x)) + ((2*I)*a^2*f*\operatorname{Cosh}[e - (c*f)/d]*\operatorname{CoshIntegral}[(c*f)/d + f*x])/d^2 - (a^2*f*\operatorname{CoshIntegral}[(2*c*f)/d + 2*f*x]*\operatorname{Sinh}[2*e - (2*c*f)/d])/d^2 + ((2*I)*a^2*f*\operatorname{Sinh}[e - (c*f)/d]*\operatorname{ShiIntegral}[(c*f)/d + f*x])/d^2 - (a^2*f*\operatorname{Cosh}[2*e - (2*c*f)/d]*\operatorname{SinhIntegral}[(2*c*f)/d + 2*f*x])/d^2$

### Rule 3318

$\operatorname{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[(2*a)^n, \operatorname{Int}[(c + d*x)^m*\operatorname{Sin}[(1*(e + (\operatorname{Pi}*a)/(2*b)))/2 + (f*x)/2]^{(2*n)}, x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f, m\}, x$  &&  $\operatorname{EqQ}[a^2 - b^2, 0]$  &&  $\operatorname{IntegerQ}[n]$  &&  $(\operatorname{GtQ}[n, 0] \parallel \operatorname{IGtQ}[m, 0])$

### Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
LtQ[m, -1]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \sinh(e + fx))^2}{(c + dx)^2} dx &= (4a^2) \int \frac{\sin^4\left(\frac{1}{2}\left(ie + \frac{\pi}{2}\right) + \frac{ifx}{2}\right)}{(c + dx)^2} dx \\
&= -\frac{4a^2 \cosh^4\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{d(c + dx)} + \frac{(8ia^2 f) \int \left(\frac{\cosh(e+fx)}{4(c+dx)} + \frac{i \sinh(2e+2fx)}{8(c+dx)}\right) dx}{d} \\
&= -\frac{4a^2 \cosh^4\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{d(c + dx)} + \frac{(2ia^2 f) \int \frac{\cosh(e+fx)}{c+dx} dx}{d} - \frac{(a^2 f) \int \frac{\sinh(2e+2fx)}{c+dx} dx}{d} \\
&= -\frac{4a^2 \cosh^4\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{d(c + dx)} - \frac{\left(a^2 f \cosh\left(2e - \frac{2cf}{d}\right)\right) \int \frac{\sinh\left(\frac{2cf}{d} + 2fx\right)}{c+dx} dx}{d} + \frac{\left(2ia^2 f \cosh\left(e - \frac{cf}{d}\right)\right) \int \frac{\cosh\left(\frac{cf}{d} + fx\right)}{c+dx} dx}{d} \\
&= -\frac{4a^2 \cosh^4\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{d(c + dx)} + \frac{2ia^2 f \cosh\left(e - \frac{cf}{d}\right) \text{Chi}\left(\frac{cf}{d} + fx\right)}{d^2} - \frac{a^2 f \text{Chi}\left(\frac{2cf}{d} + 2fx\right) \sinh\left(e - \frac{cf}{d}\right)}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.661156, size = 214, normalized size = 1.26

$$a^2 \left( -2f(c + dx) \operatorname{Chi} \left( \frac{2f(c+dx)}{d} \right) \sinh \left( 2e - \frac{2cf}{d} \right) + 4if(c + dx) \operatorname{Chi} \left( f \left( \frac{c}{d} + x \right) \right) \cosh \left( e - \frac{cf}{d} \right) + 4idfx \sinh \left( e - \frac{cf}{d} \right) \operatorname{Shi} \left( f \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Sinh[e + f\*x])^2/(c + d\*x)^2,x]

[Out] (a^2\*(-3\*d + d\*Cosh[2\*(e + f\*x)] + (4\*I)\*f\*(c + d\*x)\*Cosh[e - (c\*f)/d]\*CoshIntegral[f\*(c/d + x)] - 2\*f\*(c + d\*x)\*CoshIntegral[(2\*f\*(c + d\*x))/d]\*Sinh[2\*e - (2\*c\*f)/d] - (4\*I)\*d\*Sinh[e + f\*x] + (4\*I)\*c\*f\*Sinh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)] + (4\*I)\*d\*f\*x\*Sinh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)] - 2\*c\*f\*Cosh[2\*e - (2\*c\*f)/d]\*SinhIntegral[(2\*f\*(c + d\*x))/d] - 2\*d\*f\*x\*Cosh[2\*e - (2\*c\*f)/d]\*SinhIntegral[(2\*f\*(c + d\*x))/d]))/(2\*d^2\*(c + d\*x))

**Maple [A]** time = 0.141, size = 313, normalized size = 1.8

$$\frac{-ia^2 f e^{fx+e}}{d^2} \left( \frac{cf}{d} + fx \right)^{-1} - \frac{ia^2 f}{d^2} e^{-\frac{cf-de}{d}} \operatorname{Ei} \left( 1, -fx - e - \frac{cf-de}{d} \right) - \frac{3a^2}{2d(dx+c)} + \frac{fa^2 e^{-2fx-2e}}{4d(dfx+cf)} - \frac{fa^2}{2d^2} e^{2\frac{cf-de}{d}} \operatorname{Ei} \left( 1, 2f \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*sinh(f\*x+e))^2/(d\*x+c)^2,x)

[Out] -I\*a^2\*f/d^2\*exp(f\*x+e)/(c\*f/d+f\*x)-I\*a^2\*f/d^2\*exp(-(c\*f-d\*e)/d)\*Ei(1,-f\*x-e-(c\*f-d\*e)/d)-3/2\*a^2/d/(d\*x+c)+1/4\*a^2\*f\*exp(-2\*f\*x-2\*e)/d/(d\*f\*x+c\*f)-1/2\*a^2\*f/d^2\*exp(2\*(c\*f-d\*e)/d)\*Ei(1,2\*f\*x+2\*e+2\*(c\*f-d\*e)/d)+1/4\*f\*a^2/d^2\*exp(2\*f\*x+2\*e)/(c\*f/d+f\*x)+1/2\*f\*a^2/d^2\*exp(-2\*(c\*f-d\*e)/d)\*Ei(1,-2\*f\*x-2\*e-2\*(c\*f-d\*e)/d)+I\*a^2\*f\*exp(-f\*x-e)/d/(d\*f\*x+c\*f)-I\*a^2\*f/d^2\*exp((c\*f-d\*e)/d)\*Ei(1,f\*x+e+(c\*f-d\*e)/d)

**Maxima [A]** time = 1.39495, size = 247, normalized size = 1.45

$$\frac{1}{4} a^2 \left( \frac{e^{\left(-2e+\frac{2cf}{d}\right)} E_2 \left( \frac{2(dx+c)f}{d} \right)}{(dx+c)d} + \frac{e^{\left(2e-\frac{2cf}{d}\right)} E_2 \left( -\frac{2(dx+c)f}{d} \right)}{(dx+c)d} - \frac{2}{d^2x+cd} \right) + ia^2 \left( \frac{e^{\left(-e+\frac{cf}{d}\right)} E_2 \left( \frac{(dx+c)f}{d} \right)}{(dx+c)d} - \frac{e^{\left(e-\frac{cf}{d}\right)} E_2 \left( -\frac{(dx+c)f}{d} \right)}{(dx+c)d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))^2/(d\*x+c)^2,x, algorithm="maxima")

[Out]  $\frac{1}{4}a^2(e^{-2e+2cf/d}\exp\_integral\_e(2, 2(d*x+c)*f/d)/((d*x+c)*d) + e^{(2e-2cf/d)}\exp\_integral\_e(2, -2(d*x+c)*f/d)/((d*x+c)*d) - 2/(d^2*x+c*d) + I*a^2(e^{-e+cf/d}\exp\_integral\_e(2, (d*x+c)*f/d)/((d*x+c)*d) - e^{(e-cf/d)}\exp\_integral\_e(2, -(d*x+c)*f/d)/((d*x+c)*d) - a^2/(d^2*x+c*d)$

**Fricas [A]** time = 2.7084, size = 581, normalized size = 3.42

$$\left( a^2 d e^{(4fx+4e)} - 4i a^2 d e^{(3fx+3e)} + 4i a^2 d e^{(fx+e)} + a^2 d - \left( 6a^2 d + 2(a^2 d f x + a^2 c f) \operatorname{Ei}\left(\frac{2(df x + cf)}{d}\right) e^{\left(\frac{2(de-cf)}{d}\right)} - (4i a^2 d f x + 4i a^2 c f) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))^2/(d\*x+c)^2,x, algorithm="fricas")

[Out]  $\frac{1}{4}(a^2 d e^{(4fx+4e)} - 4i a^2 d e^{(3fx+3e)} + 4i a^2 d e^{(fx+e)} + a^2 d - (6a^2 d + 2(a^2 d f x + a^2 c f) \operatorname{Ei}(2(dfx+cf)/d) e^{(2(de-cf)/d)} - (4i a^2 d f x + 4i a^2 c f) \operatorname{Ei}((dfx+cf)/d) e^{((dfe-cf)/d)} - (4i a^2 d f x + 4i a^2 c f) \operatorname{Ei}(-(dfx+cf)/d) e^{-(dfe-cf)/d} - 2(a^2 d f x + a^2 c f) \operatorname{Ei}(-2(dfx+cf)/d) e^{-2(dfe-cf)/d}) e^{(2fx+2e)}) e^{(-2fx-2e)}/(d^3 x + c d^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))\*\*2/(d\*x+c)\*\*2,x)

[Out] Timed out

**Giac [B]** time = 2.10966, size = 485, normalized size = 2.85

$$2a^2dfxEi\left(-\frac{2(dfxc+f)}{d}\right)e^{\left(\frac{2cf}{d}-2e\right)} + 4ia^2dfxEi\left(-\frac{dfxc+f}{d}\right)e^{\left(\frac{cf}{d}-e\right)} + 4ia^2dfxEi\left(\frac{dfxc+f}{d}\right)e^{\left(-\frac{cf}{d}+e\right)} - 2a^2dfxEi\left(\frac{2(dfxc+f)}{d}\right)$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))^2/(d\*x+c)^2,x, algorithm="giac")

[Out] 1/4\*(2\*a^2\*d\*f\*x\*Ei(-2\*(d\*f\*x + c\*f)/d)\*e^(2\*c\*f/d - 2\*e) + 4\*I\*a^2\*d\*f\*x\*Ei(-(d\*f\*x + c\*f)/d)\*e^(c\*f/d - e) + 4\*I\*a^2\*d\*f\*x\*Ei((d\*f\*x + c\*f)/d)\*e^(-c\*f/d + e) - 2\*a^2\*d\*f\*x\*Ei(2\*(d\*f\*x + c\*f)/d)\*e^(-2\*c\*f/d + 2\*e) + 2\*a^2\*c\*f\*Ei(-2\*(d\*f\*x + c\*f)/d)\*e^(2\*c\*f/d - 2\*e) + 4\*I\*a^2\*c\*f\*Ei(-(d\*f\*x + c\*f)/d)\*e^(c\*f/d - e) + 4\*I\*a^2\*c\*f\*Ei((d\*f\*x + c\*f)/d)\*e^(-c\*f/d + e) - 2\*a^2\*c\*f\*Ei(2\*(d\*f\*x + c\*f)/d)\*e^(-2\*c\*f/d + 2\*e) + a^2\*d\*e^(2\*f\*x + 2\*e) - 4\*I\*a^2\*d\*e^(f\*x + e) + 4\*I\*a^2\*d\*e^(-f\*x - e) + a^2\*d\*e^(-2\*f\*x - 2\*e))/(d^3\*x + c\*d^2) - 3/2\*a^2/((d\*x + c)\*d)

$$3.107 \quad \int \frac{(a+ia \sinh(e+fx))^2}{(c+dx)^3} dx$$

**Optimal.** Leaf size=236

$$\frac{ia^2 f^2 \text{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d^3} - \frac{a^2 f^2 \text{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{d^3} - \frac{a^2 f^2 \sinh\left(2e - \frac{2cf}{d}\right) \text{Shi}\left(2xf + \frac{2cf}{d}\right)}{d^3} + \dots$$

[Out]  $(-2*a^2*\text{Cosh}[e/2 + (I/4)*\text{Pi} + (f*x)/2]^4)/(d*(c + d*x)^2) - (a^2*f^2*\text{Cosh}[2*e - (2*c*f)/d]*\text{CoshIntegral}[(2*c*f)/d + 2*f*x])/d^3 + (I*a^2*f^2*\text{CoshIntegral}[(c*f)/d + f*x]*\text{Sinh}[e - (c*f)/d])/d^3 - (4*a^2*f*\text{Cosh}[e/2 + (I/4)*\text{Pi} + (f*x)/2]^3*\text{Sinh}[e/2 + (I/4)*\text{Pi} + (f*x)/2])/d^2*(c + d*x) + (I*a^2*f^2*\text{Cosh}[e - (c*f)/d]*\text{SinhIntegral}[(c*f)/d + f*x])/d^3 - (a^2*f^2*\text{Sinh}[2*e - (2*c*f)/d]*\text{SinhIntegral}[(2*c*f)/d + 2*f*x])/d^3$

**Rubi [A]** time = 0.530018, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$ , Rules used = {3318, 3314, 3312, 3303, 3298, 3301}

$$\frac{ia^2 f^2 \text{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d^3} - \frac{a^2 f^2 \text{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{d^3} - \frac{a^2 f^2 \sinh\left(2e - \frac{2cf}{d}\right) \text{Shi}\left(2xf + \frac{2cf}{d}\right)}{d^3} + \dots$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + I*a*\text{Sinh}[e + f*x])^2/(c + d*x)^3, x]$

[Out]  $(-2*a^2*\text{Cosh}[e/2 + (I/4)*\text{Pi} + (f*x)/2]^4)/(d*(c + d*x)^2) - (a^2*f^2*\text{Cosh}[2*e - (2*c*f)/d]*\text{CoshIntegral}[(2*c*f)/d + 2*f*x])/d^3 + (I*a^2*f^2*\text{CoshIntegral}[(c*f)/d + f*x]*\text{Sinh}[e - (c*f)/d])/d^3 - (4*a^2*f*\text{Cosh}[e/2 + (I/4)*\text{Pi} + (f*x)/2]^3*\text{Sinh}[e/2 + (I/4)*\text{Pi} + (f*x)/2])/d^2*(c + d*x) + (I*a^2*f^2*\text{Cosh}[e - (c*f)/d]*\text{SinhIntegral}[(c*f)/d + f*x])/d^3 - (a^2*f^2*\text{Sinh}[2*e - (2*c*f)/d]*\text{SinhIntegral}[(2*c*f)/d + 2*f*x])/d^3$

### Rule 3318

$\text{Int}[(c + d*x)^m*(a + b*\sin[(e + f*x)])^n, x\_Symbol] \rightarrow \text{Dist}[(2*a)^n, \text{Int}[(c + d*x)^m*\text{Sin}[(1*(e + (Pi*a)/(2*b)))/2 + (f*x)/2]^{2*n}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, x\} \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ (\text{GtQ}[n, 0] \ || \ \text{IGtQ}[m, 0])$

### Rule 3314



```

Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:= Simp[((c + d*x)^(m + 1)*(b*Ssin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Ssin[e + f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Ssin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*(b*Ssin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]

```

### Rule 3312

```

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

```

### Rule 3303

```

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

```

### Rule 3298

```

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

```

### Rule 3301

```

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \sinh(e + fx))^2}{(c + dx)^3} dx &= (4a^2) \int \frac{\sin^4\left(\frac{1}{2}\left(ie + \frac{\pi}{2}\right) + \frac{ifx}{2}\right)}{(c + dx)^3} dx \\
&= -\frac{2a^2 \cosh^4\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{d(c + dx)^2} - \frac{4a^2 f \cosh^3\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sinh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{d^2(c + dx)} + \frac{(6a^2 f^2) \int \frac{\sin^4\left(\frac{1}{2}\left(ie + \frac{\pi}{2}\right) + \frac{ifx}{2}\right)}{(c + dx)^3} dx}{d^2} \\
&= -\frac{2a^2 \cosh^4\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{d(c + dx)^2} - \frac{4a^2 f \cosh^3\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sinh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{d^2(c + dx)} - \frac{(6a^2 f^2) \int \left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{d^2} \\
&= -\frac{2a^2 \cosh^4\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{d(c + dx)^2} - \frac{4a^2 f \cosh^3\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sinh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{d^2(c + dx)} - \frac{(3ia^2 f^2) \int \frac{\sin^4\left(\frac{1}{2}\left(ie + \frac{\pi}{2}\right) + \frac{ifx}{2}\right)}{(c + dx)^3} dx}{d^2} \\
&= -\frac{2a^2 \cosh^4\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{d(c + dx)^2} - \frac{4a^2 f \cosh^3\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sinh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{d^2(c + dx)} - \frac{(a^2 f^2 \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right))}{d^2} \\
&= -\frac{2a^2 \cosh^4\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{d(c + dx)^2} - \frac{a^2 f^2 \cosh\left(2e - \frac{2cf}{d}\right) \text{Chi}\left(\frac{2cf}{d} + 2fx\right)}{d^3} + \frac{ia^2 f^2 \text{Chi}\left(\frac{cf}{d} + fx\right)}{d^3}
\end{aligned}$$

**Mathematica [A]** time = 2.35465, size = 198, normalized size = 0.84

$$\frac{a^2 \left(4if^2 \text{Chi}\left(f\left(\frac{c}{d} + x\right)\right) \sinh\left(e - \frac{cf}{d}\right) - 4f^2 \text{Chi}\left(\frac{2f(c+dx)}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right) - 4f^2 \sinh\left(2e - \frac{2cf}{d}\right) \text{Shi}\left(\frac{2f(c+dx)}{d}\right) + 4if^2 \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)\right)}{4d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Sinh[e + f\*x])^2/(c + d\*x)^3,x]

[Out] (a^2\*(-4\*f^2\*Cosh[2\*e - (2\*c\*f)/d]\*CoshIntegral[(2\*f\*(c + d\*x))/d] + (4\*I)\*f^2\*CoshIntegral[f\*(c/d + x)]\*Sinh[e - (c\*f)/d] + (d\*(-3\*d - (4\*I)\*f\*(c + d\*x))\*Cosh[e + f\*x] + d\*Cosh[2\*(e + f\*x)] - (4\*I)\*d\*Sinh[e + f\*x] + 2\*c\*f\*Sinh[2\*(e + f\*x)] + 2\*d\*f\*x\*Sinh[2\*(e + f\*x)]))/(c + d\*x)^2 + (4\*I)\*f^2\*Cosh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)] - 4\*f^2\*Sinh[2\*e - (2\*c\*f)/d]\*SinhIntegral[(2\*f\*(c + d\*x))/d))/(4\*d^3)

**Maple [B]** time = 0.154, size = 625, normalized size = 2.7

$$\frac{-\frac{i}{2}a^2 f^2 e^{fx+e}}{d^3} \left(\frac{cf}{d} + fx\right)^{-2} - \frac{\frac{i}{2}a^2 f^2 e^{fx+e}}{d^3} \left(\frac{cf}{d} + fx\right)^{-1} - \frac{\frac{i}{2}a^2 f^2}{d^3} e^{-\frac{cf-de}{d}} \text{Ei}\left(1, -fx - e - \frac{cf-de}{d}\right) - \frac{3a^2}{4d(dx+c)^2} - \frac{1}{4d(d^2f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x)`

[Out] 
$$-1/2*I*a^2*f^2/d^3*\exp(f*x+e)/(c*f/d+f*x)^2-1/2*I*a^2*f^2/d^3*\exp(f*x+e)/(c*f/d+f*x)-1/2*I*a^2*f^2/d^3*\exp(-(c*f-d*e)/d)*\text{Ei}(1,-f*x-e-(c*f-d*e)/d)-3/4*a^2/d/(d*x+c)^2-1/4*a^2*f^3*\exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x-1/4*a^2*f^3*\exp(-2*f*x-2*e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+1/8*a^2*f^2*\exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/2*a^2*f^2/d^3*\exp(2*(c*f-d*e)/d)*\text{Ei}(1,2*f*x+2*e+2*(c*f-d*e)/d)+1/8*f^2*a^2/d^3*\exp(2*f*x+2*e)/(c*f/d+f*x)^2+1/4*f^2*a^2/d^3*\exp(2*f*x+2*e)/(c*f/d+f*x)+1/2*f^2*a^2/d^3*\exp(-2*(c*f-d*e)/d)*\text{Ei}(1,-2*f*x-2*e-2*(c*f-d*e)/d)-1/2*I*a^2*f^3*\exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x-1/2*I*a^2*f^3*\exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+1/2*I*a^2*f^2*\exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/2*I*a^2*f^2/d^3*\exp((c*f-d*e)/d)*\text{Ei}(1,f*x+e+(c*f-d*e)/d)$$

**Maxima [A]** time = 1.39689, size = 277, normalized size = 1.17

$$-\frac{1}{4}a^2\left(\frac{1}{d^3x^2+2cd^2x+c^2d}-\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_3\left(\frac{2(dx+cf)}{d}\right)}{(dx+c)^2d}-\frac{e^{\left(2e-\frac{2cf}{d}\right)}E_3\left(-\frac{2(dx+cf)}{d}\right)}{(dx+c)^2d}\right)+ia^2\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+cf)}{d}\right)}{(dx+c)^2d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\frac{(dx+cf)}{d}\right)}{(dx+c)^2d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")`

[Out] 
$$-1/4*a^2*(1/(d^3*x^2+2*c*d^2*x+c^2*d)-e^{(-2*e+2*c*f/d)}*\exp\_integral\_e(3,2*(d*x+c)*f/d)/((d*x+c)^2*d)-e^{(2*e-2*c*f/d)}*\exp\_integral\_e(3,-2*(d*x+c)*f/d)/((d*x+c)^2*d))+I*a^2*(e^{(-e+c*f/d)}*\exp\_integral\_e(3,(d*x+c)*f/d)/((d*x+c)^2*d)-e^{(e-c*f/d)}*\exp\_integral\_e(3,-(d*x+c)*f/d)/((d*x+c)^2*d))-1/2*a^2/(d^3*x^2+2*c*d^2*x+c^2*d)$$

**Fricas [B]** time = 2.90521, size = 959, normalized size = 4.06

$$\left(2a^2d^2fx+2a^2cdf-a^2d^2-\left(2a^2d^2fx+2a^2cdf+a^2d^2\right)e^{(4fx+4e)}-\left(-4ia^2d^2fx-4ia^2cdf-4ia^2d^2\right)e^{(3fx+3e)}\right)+\left(6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*a^2*d^2*f*x + 2*a^2*c*d*f - a^2*d^2 - (2*a^2*d^2*f*x + 2*a^2*c*d*f
+ a^2*d^2)*e^(4*f*x + 4*e) - (-4*I*a^2*d^2*f*x - 4*I*a^2*c*d*f - 4*I*a^2*d^
2)*e^(3*f*x + 3*e) + (6*a^2*d^2 + 4*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^
2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d)*e^(2*(d*e - c*f)/d) - (4*I*a^2*d^2*f^2*x^2
+ 8*I*a^2*c*d*f^2*x + 4*I*a^2*c^2*f^2)*Ei((d*f*x + c*f)/d)*e^((d*e - c*f)/
d) - (-4*I*a^2*d^2*f^2*x^2 - 8*I*a^2*c*d*f^2*x - 4*I*a^2*c^2*f^2)*Ei(-(d*f*
x + c*f)/d)*e^(-(d*e - c*f)/d) + 4*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2
*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d)*e^(-2*(d*e - c*f)/d))*e^(2*f*x + 2*e) - (-
4*I*a^2*d^2*f*x - 4*I*a^2*c*d*f + 4*I*a^2*d^2)*e^(f*x + e))*e^(-2*f*x - 2*e
)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)**3,x)
```

```
[Out] Timed out
```

**Giac [B]** time = 1.40146, size = 953, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*a^2*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) + 4*I*a^2*
d^2*f^2*x^2*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) - 4*I*a^2*d^2*f^2*x^2*Ei((d*
f*x + c*f)/d)*e^(-c*f/d + e) + 4*a^2*d^2*f^2*x^2*Ei(2*(d*f*x + c*f)/d)*e^(-
2*c*f/d + 2*e) + 8*a^2*c*d*f^2*x*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) +
8*I*a^2*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) - 8*I*a^2*c*d*f^2*x*E
i((d*f*x + c*f)/d)*e^(-c*f/d + e) + 8*a^2*c*d*f^2*x*Ei(2*(d*f*x + c*f)/d)*e
```

$$\begin{aligned}
& ^{-2*c*f/d + 2*e} + 4*a^2*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^{(2*c*f/d - 2*e)} \\
& + 4*I*a^2*c^2*f^2*Ei(-(d*f*x + c*f)/d)*e^{(c*f/d - e)} - 4*I*a^2*c^2*f^2*Ei((d*f*x + c*f)/d)*e^{(-c*f/d + e)} + 4*a^2*c^2*f^2*Ei(2*(d*f*x + c*f)/d)*e^{(-2*c*f/d + 2*e)} \\
& - 2*a^2*d^2*f*x*e^{(2*f*x + 2*e)} + 4*I*a^2*d^2*f*x*e^{(f*x + e)} + 4*I*a^2*d^2*f*x*e^{(-f*x - e)} + 2*a^2*d^2*f*x*e^{(-2*f*x - 2*e)} - 2*a^2*c*d*f*e^{(2*f*x + 2*e)} \\
& + 4*I*a^2*c*d*f*e^{(f*x + e)} + 4*I*a^2*c*d*f*e^{(-f*x - e)} + 2*a^2*c*d*f*e^{(-2*f*x - 2*e)} - a^2*d^2*e^{(2*f*x + 2*e)} + 4*I*a^2*d^2*e^{(f*x + e)} \\
& - 4*I*a^2*d^2*e^{(-f*x - e)} - a^2*d^2*e^{(-2*f*x - 2*e)} + 6*a^2*d^2)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
\end{aligned}$$

$$3.108 \quad \int \frac{(c+dx)^3}{a+ia \sinh(e+fx)} dx$$

**Optimal.** Leaf size=132

$$-\frac{12d^2(c+dx)\text{PolyLog}\left(2, -ie^{e+fx}\right)}{af^3} + \frac{12d^3\text{PolyLog}\left(3, -ie^{e+fx}\right)}{af^4} - \frac{6d(c+dx)^2 \log\left(1 + ie^{e+fx}\right)}{af^2} + \frac{(c+dx)^3 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af}$$

[Out] (c + d\*x)^3/(a\*f) - (6\*d\*(c + d\*x)^2\*Log[1 + I\*E^(e + f\*x)])/(a\*f^2) - (12\*d^2\*(c + d\*x)\*PolyLog[2, (-I)\*E^(e + f\*x)])/(a\*f^3) + (12\*d^3\*PolyLog[3, (-I)\*E^(e + f\*x)])/(a\*f^4) + ((c + d\*x)^3\*Tanh[e/2 + (I/4)\*Pi + (f\*x)/2])/(a\*f)

**Rubi [A]** time = 0.302292, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$ , Rules used = {3318, 4184, 3716, 2190, 2531, 2282, 6589}

$$-\frac{12d^2(c+dx)\text{PolyLog}\left(2, -ie^{e+fx}\right)}{af^3} + \frac{12d^3\text{PolyLog}\left(3, -ie^{e+fx}\right)}{af^4} - \frac{6d(c+dx)^2 \log\left(1 + ie^{e+fx}\right)}{af^2} + \frac{(c+dx)^3 \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{af}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^3/(a + I\*a\*Sinh[e + f\*x]),x]

[Out] (c + d\*x)^3/(a\*f) - (6\*d\*(c + d\*x)^2\*Log[1 + I\*E^(e + f\*x)])/(a\*f^2) - (12\*d^2\*(c + d\*x)\*PolyLog[2, (-I)\*E^(e + f\*x)])/(a\*f^3) + (12\*d^3\*PolyLog[3, (-I)\*E^(e + f\*x)])/(a\*f^4) + ((c + d\*x)^3\*Tanh[e/2 + (I/4)\*Pi + (f\*x)/2])/(a\*f)

### Rule 3318

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] := Dist[(2\*a)^n, Int[(c + d\*x)^m\*Sin[(1\*(e + (Pi\*a)/(2\*b)))/2 + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])

### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_.)]^2\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] := -Simp[p((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m-1)\*Co

$t[e + f*x], x], x] /;$  FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 3716

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)], x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x)))/(E^(2\*I\*k\*Pi)\*(1 + E^(2\*(-I\*e) + f\*fz\*x))/E^(2\*I\*k\*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^3}{a+ia \sinh(e+fx)} dx &= \frac{\int (c+dx)^3 \csc^2\left(\frac{1}{2}\left(ie + \frac{\pi}{2}\right) + \frac{ifx}{2}\right) dx}{2a} \\
&= \frac{(c+dx)^3 \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af} - \frac{(3d) \int (c+dx)^2 \coth\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) dx}{af} \\
&= \frac{(c+dx)^3}{af} + \frac{(c+dx)^3 \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af} - \frac{(6id) \int \frac{e^{2\left(\frac{e}{2} + \frac{fx}{2}\right)} (c+dx)^2}{1+ie^{2\left(\frac{e}{2} + \frac{fx}{2}\right)}} dx}{af} \\
&= \frac{(c+dx)^3}{af} - \frac{6d(c+dx)^2 \log(1+ie^{e+fx})}{af^2} + \frac{(c+dx)^3 \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af} + \frac{(12d^2) \int (c+dx)}{af^3} \\
&= \frac{(c+dx)^3}{af} - \frac{6d(c+dx)^2 \log(1+ie^{e+fx})}{af^2} - \frac{12d^2(c+dx)\text{Li}_2(-ie^{e+fx})}{af^3} + \frac{(c+dx)^3 \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af} \\
&= \frac{(c+dx)^3}{af} - \frac{6d(c+dx)^2 \log(1+ie^{e+fx})}{af^2} - \frac{12d^2(c+dx)\text{Li}_2(-ie^{e+fx})}{af^3} + \frac{(c+dx)^3 \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af} \\
&= \frac{(c+dx)^3}{af} - \frac{6d(c+dx)^2 \log(1+ie^{e+fx})}{af^2} - \frac{12d^2(c+dx)\text{Li}_2(-ie^{e+fx})}{af^3} + \frac{12d^3\text{Li}_3(-ie^{e+fx})}{af^4} + \dots
\end{aligned}$$

**Mathematica [A]** time = 2.89412, size = 206, normalized size = 1.56

$$\frac{2 \left( \frac{3de^e \left( -\frac{2ide^{-e}(e^e-i)(f(c+dx)\text{PolyLog}(2,ie^{-e-fx})+d\text{PolyLog}(3,ie^{-e-fx}))}{f^3} + \frac{(e^{-e+i}(c+dx)^2 \log(1-ie^{-e-fx}))}{f} + \frac{e^{-e}(c+dx)^3}{3d} \right)}{-1-ie^e} \right) + \frac{(c+dx)^3 \sinh\left(\frac{fx}{2}\right)}{(\cosh\left(\frac{e}{2}\right)+i\sinh\left(\frac{e}{2}\right))(\cosh\left(\frac{1}{2}(e+fx)\right)+i\sinh\left(\frac{1}{2}(e+fx)\right))}}{af}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3/(a + I\*a\*Sinh[e + f\*x]),x]

[Out] (2\*((3\*d\*E^e\*((c + d\*x)^3/(3\*d\*E^e) + ((I + E^(-e))\*(c + d\*x)^2\*Log[1 - I\*E^(-e - f\*x)]))/f - ((2\*I)\*d\*(-I + E^e)\*(f\*(c + d\*x)\*PolyLog[2, I\*E^(-e - f\*x)] + d\*PolyLog[3, I\*E^(-e - f\*x)]))/(E^e\*f^3)))/(-1 - I\*E^e) + ((c + d\*x)^3\*Sinh[(f\*x)/2])/((Cosh[e/2] + I\*Sinh[e/2])\*(Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2])))/(a\*f)



---

**Maple [B]** time = 0.112, size = 435, normalized size = 3.3

$$\frac{2i(d^3x^3 + 3cd^2x^2 + 3c^2dx + c^3)}{fa(e^{fx+e} - i)} - 12 \frac{cd^2 \text{polylog}(2, -ie^{fx+e})}{af^3} - 4 \frac{d^3e^3}{f^4a} + 6 \frac{d \ln(e^{fx+e})c^2}{af^2} + 6 \frac{d^3e^2 \ln(1 + ie^{fx+e})}{f^4a} - 12$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3/(a+I\*a\*sinh(f\*x+e)),x)

[Out] 2\*I\*(d^3\*x^3+3\*c\*d^2\*x^2+3\*c^2\*d\*x+c^3)/f/a/(exp(f\*x+e)-I)-12\*d^2/f^3/a\*c\*polylog(2,-I\*exp(f\*x+e))-4\*d^3/f^4/a\*e^3+6\*d/f^2/a\*ln(exp(f\*x+e))\*c^2+6\*d^3/f^4/a\*e^2\*ln(1+I\*exp(f\*x+e))-12\*d^2/f^2/a\*c\*ln(1+I\*exp(f\*x+e))\*x-12\*d^2/f^3/a\*c\*ln(1+I\*exp(f\*x+e))\*e+12\*d^2/f^2/a\*c\*e\*x+2\*d^3/f/a\*x^3-6\*d^3/f^3/a\*e^2\*x-12\*d^2/f^3/a\*c\*e\*ln(exp(f\*x+e))+6\*d^2/f/a\*c\*x^2+6\*d^2/f^3/a\*c\*e^2-6\*d^3/f^4/a\*e^2\*ln(exp(f\*x+e)-I)+12\*d^2/f^3/a\*c\*e\*ln(exp(f\*x+e)-I)-12\*d^3/f^3/a\*polylog(2,-I\*exp(f\*x+e))\*x+12\*d^3\*polylog(3,-I\*exp(f\*x+e))/a/f^4-6\*d^3/f^2/a\*ln(1+I\*exp(f\*x+e))\*x^2-6\*d/f^2/a\*ln(exp(f\*x+e)-I)\*c^2+6\*d^3/f^4/a\*e^2\*ln(exp(f\*x+e))

---

**Maxima [B]** time = 1.68317, size = 320, normalized size = 2.42

$$6c^2d \left( \frac{xe^{(fx+e)}}{afe^{(fx+e)} - iaf} - \frac{\log\left(\left(e^{(fx+e)} - i\right)e^{(-e)}\right)}{af^2} \right) - \frac{2c^3}{\left(iae^{(-fx-e)} - a\right)f} + \frac{2id^3x^3 + 6icd^2x^2}{afe^{(fx+e)} - iaf} - \frac{12\left(fx \log\left(ie^{(fx+e)} + 1\right) + 1\right)}{af^3} + 12$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+I\*a\*sinh(f\*x+e)),x, algorithm="maxima")

[Out] 6\*c^2\*d\*(x\*e^(f\*x + e)/(a\*f\*e^(f\*x + e) - I\*a\*f) - log((e^(f\*x + e) - I)\*e^(-e))/(a\*f^2)) - 2\*c^3/((I\*a\*e^(-f\*x - e) - a)\*f) + (2\*I\*d^3\*x^3 + 6\*I\*c\*d^2\*x^2)/(a\*f\*e^(f\*x + e) - I\*a\*f) - 12\*(f\*x\*log(I\*e^(f\*x + e) + 1) + dilog(-I\*e^(f\*x + e)))\*c\*d^2/(a\*f^3) - 6\*(f^2\*x^2\*log(I\*e^(f\*x + e) + 1) + 2\*f\*x\*dilog(-I\*e^(f\*x + e)) - 2\*polylog(3, -I\*e^(f\*x + e)))\*d^3/(a\*f^4) + 2\*(d^3\*f^3\*x^3 + 3\*c\*d^2\*f^3\*x^2)/(a\*f^4)

---

**Fricas [C]** time = 2.72744, size = 863, normalized size = 6.54

$$\frac{-2id^3e^3 + 6icd^2e^2f - 6ic^2def^2 + 2ic^3f^3 + \left(12id^3fx + 12icd^2f - 12(d^3fx + cd^2f)e^{(fx+e)}\right)\text{Li}_2\left(-ie^{(fx+e)}\right) + 2(d^3f^3x^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+I\*a\*sinh(f\*x+e)),x, algorithm="fricas")

[Out]  $(-2I*d^3*e^3 + 6I*c*d^2*e^2*f - 6I*c^2*d*e*f^2 + 2I*c^3*f^3 + (12I*d^3*f*x + 12I*c*d^2*f - 12*(d^3*f*x + c*d^2*f)*e^{(f*x + e)})*dilog(-I*e^{(f*x + e)}) + 2*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2)*e^{(f*x + e)} + (6I*d^3*e^2 - 12I*c*d^2*e*f + 6I*c^2*d*f^2 - 6*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*e^{(f*x + e)})*log(e^{(f*x + e)} - I) + (6I*d^3*f^2*x^2 + 12I*c*d^2*f^2*x - 6I*d^3*e^2 + 12I*c*d^2*e*f - 6*(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*e^{(f*x + e)})*log(I*e^{(f*x + e)} + 1) + (12*d^3*e^{(f*x + e)} - 12I*d^3)*polylog(3, -I*e^{(f*x + e)})))/(a*f^4*e^{(f*x + e)} - I*a*f^4)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*3/(a+I\*a\*sinh(f\*x+e)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^3}{ia \sinh(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+I\*a\*sinh(f\*x+e)),x, algorithm="giac")

[Out] integrate((d\*x + c)^3/(I\*a\*sinh(f\*x + e) + a), x)

$$3.109 \quad \int \frac{(c+dx)^2}{a+ia \sinh(e+fx)} dx$$

**Optimal.** Leaf size=101

$$-\frac{4d^2 \text{PolyLog}\left(2, -ie^{e+fx}\right)}{af^3} - \frac{4d(c+dx) \log\left(1 + ie^{e+fx}\right)}{af^2} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{af} + \frac{(c+dx)^2}{af}$$

[Out] (c + d\*x)^2/(a\*f) - (4\*d\*(c + d\*x)\*Log[1 + I\*E^(e + f\*x)])/(a\*f^2) - (4\*d^2\*PolyLog[2, (-I)\*E^(e + f\*x)])/(a\*f^3) + ((c + d\*x)^2\*Tanh[e/2 + (I/4)\*Pi + (f\*x)/2])/(a\*f)

**Rubi [A]** time = 0.218708, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$ , Rules used = {3318, 4184, 3716, 2190, 2279, 2391}

$$-\frac{4d^2 \text{PolyLog}\left(2, -ie^{e+fx}\right)}{af^3} - \frac{4d(c+dx) \log\left(1 + ie^{e+fx}\right)}{af^2} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{af} + \frac{(c+dx)^2}{af}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^2/(a + I\*a\*Sinh[e + f\*x]),x]

[Out] (c + d\*x)^2/(a\*f) - (4\*d\*(c + d\*x)\*Log[1 + I\*E^(e + f\*x)])/(a\*f^2) - (4\*d^2\*PolyLog[2, (-I)\*E^(e + f\*x)])/(a\*f^3) + ((c + d\*x)^2\*Tanh[e/2 + (I/4)\*Pi + (f\*x)/2])/(a\*f)

### Rule 3318

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[(2\*a)^n, Int[(c + d\*x)^m\*Sin[(1\*(e + (Pi\*a)/(2\*b)))/2 + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])

### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m-1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)
*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^2}{a+ia \sinh(e+fx)} dx &= \frac{\int (c+dx)^2 \csc^2\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right) dx}{2a} \\
&= \frac{(c+dx)^2 \tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af} - \frac{(2d) \int (c+dx) \coth\left(\frac{e}{2}-\frac{i\pi}{4}+\frac{fx}{2}\right) dx}{af} \\
&= \frac{(c+dx)^2}{af} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af} - \frac{(4id) \int \frac{e^{\frac{2}{2}+\frac{fx}{2}}(c+dx)}{1+ie^{\frac{2}{2}+\frac{fx}{2}}} dx}{af} \\
&= \frac{(c+dx)^2}{af} - \frac{4d(c+dx) \log(1+ie^{e+fx})}{af^2} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{(4d^2) \int \log\left(1+\frac{e^{\frac{e}{2}+\frac{fx}{2}}}{1+ie^{\frac{e}{2}+\frac{fx}{2}}}\right) dx}{af^2} \\
&= \frac{(c+dx)^2}{af} - \frac{4d(c+dx) \log(1+ie^{e+fx})}{af^2} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af} + \frac{(4d^2) \text{Subst}\left(\int \frac{1}{1+ie^{\frac{e}{2}+\frac{fx}{2}}} dx\right)}{af^2} \\
&= \frac{(c+dx)^2}{af} - \frac{4d(c+dx) \log(1+ie^{e+fx})}{af^2} - \frac{4d^2 \text{Li}_2(-ie^{e+fx})}{af^3} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{af}
\end{aligned}$$

**Mathematica [A]** time = 2.21886, size = 150, normalized size = 1.49

$$\frac{2 \left( 2d^2 \text{PolyLog}\left(2, ie^{-e-fx}\right) + \frac{f^2(c+dx)^2 \sinh\left(\frac{fx}{2}\right)}{\left(\cosh\left(\frac{e}{2}\right)+i \sinh\left(\frac{e}{2}\right)\right)\left(\cosh\left(\frac{1}{2}(e+fx)\right)+i \sinh\left(\frac{1}{2}(e+fx)\right)\right)} + \frac{if(c+dx)(f(c+dx)+2d(1+ie^e) \log(1-ie^{-e-fx}))}{e^e-i} \right)}{af^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2/(a + I\*a\*Sinh[e + f\*x]), x]

[Out] (2\*((I\*f\*(c + d\*x)\*(f\*(c + d\*x) + 2\*d\*(1 + I\*E^e)\*Log[1 - I\*E^(-e - f\*x)])))/(-I + E^e) + 2\*d^2\*PolyLog[2, I\*E^(-e - f\*x)] + (f^2\*(c + d\*x)^2\*Sinh[(f\*x)/2])/((Cosh[e/2] + I\*Sinh[e/2])\*(Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2])))/(a\*f^3)

**Maple [B]** time = 0.056, size = 227, normalized size = 2.3

$$\frac{2i(d^2x^2 + 2cdx + c^2)}{fa(e^{fx+e} - i)} - 4 \frac{d \ln(e^{fx+e} - i)c}{af^2} + 4 \frac{d \ln(e^{fx+e})c}{af^2} + 2 \frac{d^2x^2}{fa} + 4 \frac{d^2ex}{af^2} + 2 \frac{d^2e^2}{f^3a} - 4 \frac{d^2 \ln(1 + ie^{fx+e})x}{af^2} - 4 \frac{d^2 \text{Li}_2(-ie^{e+fx})}{af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^2/(a+I*a*sinh(f*x+e)),x)`

[Out]  $2*I*(d^2*x^2+2*c*d*x+c^2)/f/a/(\exp(f*x+e)-I)-4*d/f^2/a*\ln(\exp(f*x+e)-I)*c+4*d/f^2/a*\ln(\exp(f*x+e))*c+2*d^2/f/a*x^2+4*d^2/f^2/a*e*x+2*d^2/f^3/a*e^2-4*d^2/f^2/a*\ln(1+I*\exp(f*x+e))*x-4*d^2/f^3/a*\ln(1+I*\exp(f*x+e))*e-4*d^2*\text{polylog}(2,-I*\exp(f*x+e))/a/f^3+4*d^2/f^3/a*e*\ln(\exp(f*x+e)-I)-4*d^2/f^3/a*e*\ln(\exp(f*x+e))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$d^2 \left( \frac{2ix^2}{afe^{(fx+e)} - iaf} - 4i \int \frac{x}{afe^{(fx+e)} - iaf} dx \right) + 4cd \left( \frac{xe^{(fx+e)}}{afe^{(fx+e)} - iaf} - \frac{\log\left(\left(e^{(fx+e)} - i\right)e^{(-e)}\right)}{af^2} \right) - \frac{2c^2}{(iae^{(-fx-e)} - a)f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")`

[Out]  $d^2*(2*I*x^2/(a*f*e^{(f*x + e)} - I*a*f) - 4*I*\text{integrate}(x/(a*f*e^{(f*x + e)} - I*a*f), x)) + 4*c*d*(x*e^{(f*x + e)}/(a*f*e^{(f*x + e)} - I*a*f) - \log((e^{(f*x + e)} - I)*e^{(-e)})/(a*f^2)) - 2*c^2/((I*a*e^{(-f*x - e)} - a)*f)$

**Fricas [B]** time = 2.69656, size = 483, normalized size = 4.78

$$\frac{2i d^2 e^2 - 4i c d e f + 2i c^2 f^2 - \left(4 d^2 e^{(fx+e)} - 4i d^2\right) \text{Li}_2\left(-i e^{(fx+e)}\right) + 2\left(d^2 f^2 x^2 + 2 c d f^2 x - d^2 e^2 + 2 c d e f\right) e^{(fx+e)} + \left(-4i d^2 e^{(fx+e)} - i a e^{(fx+e)}\right)}{af^3 e^{(fx+e)} - i a e^{(fx+e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^2/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")`

[Out]  $(2*I*d^2*e^2 - 4*I*c*d*e*f + 2*I*c^2*f^2 - (4*d^2*e^{(f*x + e)} - 4*I*d^2)*d*\ln(-I*e^{(f*x + e)}) + 2*(d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*e^{(f*x + e)} + (-4*I*d^2*e + 4*I*c*d*f + 4*(d^2*e - c*d*f)*e^{(f*x + e)})*\ln(e^{(f*x + e)} - I) + (4*I*d^2*f*x + 4*I*d^2*e - 4*(d^2*f*x + d^2*e)*e^{(f*x + e)})$

))\*log(I\*e^(f\*x + e) + 1))/(a\*f^3\*e^(f\*x + e) - I\*a\*f^3)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2/(a+I\*a\*sinh(f\*x+e)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^2}{i a \sinh(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+I\*a\*sinh(f\*x+e)),x, algorithm="giac")

[Out] integrate((d\*x + c)^2/(I\*a\*sinh(f\*x + e) + a), x)

$$3.110 \quad \int \frac{c+dx}{a+ia \sinh(e+fx)} dx$$

**Optimal.** Leaf size=63

$$\frac{(c+dx) \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{af} - \frac{2d \log\left(\cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)\right)}{af^2}$$

[Out]  $(-2*d*\text{Log}[\text{Cosh}[e/2 + (I/4)*\text{Pi} + (f*x)/2]])/(a*f^2) + ((c + d*x)*\text{Tanh}[e/2 + (I/4)*\text{Pi} + (f*x)/2])/(a*f)$

**Rubi [A]** time = 0.0739995, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3318, 4184, 3475}

$$\frac{(c+dx) \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{af} - \frac{2d \log\left(\cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)\right)}{af^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)/(a + I*a*\text{Sinh}[e + f*x]), x]$

[Out]  $(-2*d*\text{Log}[\text{Cosh}[e/2 + (I/4)*\text{Pi} + (f*x)/2]])/(a*f^2) + ((c + d*x)*\text{Tanh}[e/2 + (I/4)*\text{Pi} + (f*x)/2])/(a*f)$

### Rule 3318

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[(2*a)^n, \text{Int}[(c + d*x)^m*\text{Sin}[(1*(e + (\text{Pi}*a)/(2*b)))/2 + (f*x)/2]^{(2*n)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ (\text{GtQ}[n, 0] \ || \ \text{IGtQ}[m, 0])$

### Rule 4184

$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]^2*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cot}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cot}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

### Rule 3475



Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned} \int \frac{c + dx}{a + ia \sinh(e + fx)} dx &= \frac{\int (c + dx) \csc^2 \left( \frac{1}{2} \left( ie + \frac{\pi}{2} \right) + \frac{ifx}{2} \right) dx}{2a} \\ &= \frac{(c + dx) \tanh \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right)}{af} - \frac{d \int \coth \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) dx}{af} \\ &= -\frac{2d \log \left( \cosh \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right) \right)}{af^2} + \frac{(c + dx) \tanh \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right)}{af} \end{aligned}$$

**Mathematica [B]** time = 0.450552, size = 185, normalized size = 2.94

$$\frac{2cf \sinh \left( \frac{fx}{2} \right) + idfx \cosh \left( e + \frac{fx}{2} \right) - id \sinh \left( e + \frac{fx}{2} \right) \log(\cosh(e + fx)) + 2d \sinh \left( e + \frac{fx}{2} \right) \tan^{-1} \left( \sinh \left( \frac{fx}{2} \right) \operatorname{sech} \left( e + \frac{fx}{2} \right) \right)}{af^2 \left( \cosh \left( \frac{e}{2} \right) + i \sinh \left( \frac{e}{2} \right) \right) \left( \cosh \left( \frac{1}{2}(e + fx) \right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)/(a + I\*a\*Sinh[e + f\*x]),x]

[Out] (I\*d\*f\*x\*Cosh[e + (f\*x)/2] + Cosh[(f\*x)/2]\*((-2\*I)\*d\*ArcTan[Sech[e + (f\*x)/2]\*Sinh[(f\*x)/2]] - d\*Log[Cosh[e + f\*x]]) + 2\*c\*f\*Sinh[(f\*x)/2] + d\*f\*x\*Sinh[(f\*x)/2] + 2\*d\*ArcTan[Sech[e + (f\*x)/2]\*Sinh[(f\*x)/2]]\*Sinh[e + (f\*x)/2] - I\*d\*Log[Cosh[e + f\*x]]\*Sinh[e + (f\*x)/2])/(a\*f^2\*(Cosh[e/2] + I\*Sinh[e/2])\*(Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2]))

**Maple [A]** time = 0.056, size = 66, normalized size = 1.1

$$2 \frac{dx}{af} + 2 \frac{de}{af^2} + \frac{2i(dx+c)}{af(e^{fx+e}-i)} - 2 \frac{d \ln(e^{fx+e}-i)}{af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)/(a+I\*a\*sinh(f\*x+e)),x)

[Out]  $2*d/a/f*x+2*d/a/f^2*e+2*I*(d*x+c)/f/a/(exp(f*x+e)-I)-2*d/a/f^2*\ln(exp(f*x+e)-I)$

**Maxima [A]** time = 1.06809, size = 101, normalized size = 1.6

$$2d \left( \frac{x e^{(fx+e)}}{a f e^{(fx+e)} - i a f} - \frac{\log\left(\left(e^{(fx+e)} - i\right) e^{(-e)}\right)}{a f^2} \right) - \frac{2c}{\left(i a e^{(-fx-e)} - a\right) f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="maxima")`

[Out]  $2*d*(x*e^{(f*x + e)}/(a*f*e^{(f*x + e)} - I*a*f) - \log((e^{(f*x + e)} - I)*e^{(-e)})/(a*f^2)) - 2*c/((I*a*e^{(-f*x - e)} - a)*f)$

**Fricas [A]** time = 3.1936, size = 151, normalized size = 2.4

$$\frac{2 d f x e^{(f x+e)} + 2 i c f - \left(2 d e^{(f x+e)} - 2 i d\right) \log\left(e^{(f x+e)} - i\right)}{a f^2 e^{(f x+e)} - i a f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="fricas")`

[Out]  $(2*d*f*x*e^{(f*x + e)} + 2*I*c*f - (2*d*e^{(f*x + e)} - 2*I*d)*\log(e^{(f*x + e)} - I))/(a*f^2*e^{(f*x + e)} - I*a*f^2)$

**Sympy [A]** time = 0.689824, size = 56, normalized size = 0.89

$$\frac{2dx}{af} - \frac{2d \log(e^{fx} - ie^{-e})}{af^2} + \frac{(2ic + 2idx)e^{-e}}{af(e^{fx} - ie^{-e})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x)`

[Out]  $2*d*x/(a*f) - 2*d*\log(\exp(f*x) - I*\exp(-e))/(a*f**2) + (2*I*c + 2*I*d*x)*\exp(-e)/(a*f*(\exp(f*x) - I*\exp(-e)))$

**Giac [A]** time = 1.2167, size = 97, normalized size = 1.54

$$\frac{2dfxe^{(fx+e)} - 2de^{(fx+e)} \log\left(e^{(fx+e)} - i\right) + 2icf + 2id \log\left(e^{(fx+e)} - i\right)}{af^2e^{(fx+e)} - iaf^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="giac")`

[Out]  $(2*d*f*x*e^{(f*x + e)} - 2*d*e^{(f*x + e)}*\log(e^{(f*x + e)} - I) + 2*I*c*f + 2*I*d*\log(e^{(f*x + e)} - I))/(a*f^2*e^{(f*x + e)} - I*a*f^2)$

$$3.111 \quad \int \frac{1}{(c+dx)(a+ia \sinh(e+fx))} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a+ia \sinh(e+fx))}, x\right)$$

[Out] Unintegrable[1/((c + d\*x)\*(a + I\*a\*Sinh[e + f\*x])), x]

**Rubi [A]** time = 0.0641974, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(c+dx)(a+ia \sinh(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d\*x)\*(a + I\*a\*Sinh[e + f\*x])),x]

[Out] Defer[Int][1/((c + d\*x)\*(a + I\*a\*Sinh[e + f\*x])), x]

Rubi steps

$$\int \frac{1}{(c+dx)(a+ia \sinh(e+fx))} dx = \int \frac{1}{(c+dx)(a+ia \sinh(e+fx))} dx$$

**Mathematica [A]** time = 20.6237, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+ia \sinh(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d\*x)\*(a + I\*a\*Sinh[e + f\*x])),x]

[Out] Integrate[1/((c + d\*x)\*(a + I\*a\*Sinh[e + f\*x])), x]

---

**Maple [A]** time = 0.098, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)(a+ia \sinh(fx+e))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x+c)/(a+I\*a\*sinh(f\*x+e)),x)

[Out] int(1/(d\*x+c)/(a+I\*a\*sinh(f\*x+e)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$2id \int \frac{1}{-iad^2fx^2 - 2iacdfx - iac^2f + (ad^2fx^2e^e + 2acdfxe^e + ac^2fe^e)e^{(fx)}} dx + \frac{2i}{-iadfx - iacf + (adfxe^e + acfe^e)e^{(fx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+I\*a\*sinh(f\*x+e)),x, algorithm="maxima")

[Out] 2\*I\*d\*integrate(1/(-I\*a\*d^2\*f\*x^2 - 2\*I\*a\*c\*d\*f\*x - I\*a\*c^2\*f + (a\*d^2\*f\*x^2\*e^e + 2\*a\*c\*d\*f\*x\*e^e + a\*c^2\*f\*e^e)\*e^(f\*x)), x) + 2\*I/(-I\*a\*d\*f\*x - I\*a\*c\*f + (a\*d\*f\*x\*e^e + a\*c\*f\*e^e)\*e^(f\*x))

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\frac{\left(-iadfx - iacf + (adfx + acf)e^{(fx+e)}\right) \operatorname{integral}\left(\frac{2id}{-iad^2fx^2 - 2iacdfx - iac^2f + (ad^2fx^2 + 2acdfx + ac^2f)e^{(fx+e)}}, x\right) + 2i}{-iadfx - iacf + (adfx + acf)e^{(fx+e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+I\*a\*sinh(f\*x+e)),x, algorithm="fricas")

[Out] ((-I\*a\*d\*f\*x - I\*a\*c\*f + (a\*d\*f\*x + a\*c\*f)\*e^(f\*x + e))\*integral(2\*I\*d/(-I\*a\*d^2\*f\*x^2 - 2\*I\*a\*c\*d\*f\*x - I\*a\*c^2\*f + (a\*d^2\*f\*x^2 + 2\*a\*c\*d\*f\*x + a\*c^2\*f)\*e^(f\*x+e)), x) + 2\*I/(-I\*a\*d\*f\*x - I\*a\*c\*f + (a\*d\*f\*x + a\*c\*f)\*e^(f\*x + e))

```
2*f)*e^(f*x + e)), x) + 2*I)/(-I*a*d*f*x - I*a*c*f + (a*d*f*x + a*c*f)*e^(f
*x + e))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)(ia \sinh(fx + e) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(I*a*sinh(f*x + e) + a)), x)
```

$$3.112 \quad \int \frac{1}{(c+dx)^2(a+ia \sinh(e+fx))} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a+ia \sinh(e+fx))}, x\right)$$

[Out] Unintegrable[1/((c + d\*x)^2\*(a + I\*a\*Sinh[e + f\*x])), x]

**Rubi [A]** time = 0.0597691, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+ia \sinh(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d\*x)^2\*(a + I\*a\*Sinh[e + f\*x])), x]

[Out] Defer[Int][1/((c + d\*x)^2\*(a + I\*a\*Sinh[e + f\*x])), x]

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+ia \sinh(e+fx))} dx = \int \frac{1}{(c+dx)^2(a+ia \sinh(e+fx))} dx$$

**Mathematica [A]** time = 21.0866, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+ia \sinh(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d\*x)^2\*(a + I\*a\*Sinh[e + f\*x])), x]

[Out] Integrate[1/((c + d\*x)^2\*(a + I\*a\*Sinh[e + f\*x])), x]

---

**Maple [A]** time = 0.164, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2 (a+ia \sinh(fx+e))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x+c)^2/(a+I\*a\*sinh(f\*x+e)),x)

[Out] int(1/(d\*x+c)^2/(a+I\*a\*sinh(f\*x+e)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$4id \int \frac{1}{-iad^3fx^3 - 3iacd^2fx^2 - 3iac^2dfx - iac^3f + (ad^3fx^3e^e + 3acd^2fx^2e^e + 3ac^2dfxe^e + ac^3fe^e)e^{fx}} dx + \frac{1}{-iad^2fx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+I\*a\*sinh(f\*x+e)),x, algorithm="maxima")

[Out] 4\*I\*d\*integrate(1/(-I\*a\*d^3\*f\*x^3 - 3\*I\*a\*c\*d^2\*f\*x^2 - 3\*I\*a\*c^2\*d\*f\*x - I\*a\*c^3\*f + (a\*d^3\*f\*x^3\*e^e + 3\*a\*c\*d^2\*f\*x^2\*e^e + 3\*a\*c^2\*d\*f\*x\*e^e + a\*c^3\*f\*e^e)\*e^(f\*x)), x) + 2\*I/(-I\*a\*d^2\*f\*x^2 - 2\*I\*a\*c\*d\*f\*x - I\*a\*c^2\*f + (a\*d^2\*f\*x^2\*e^e + 2\*a\*c\*d\*f\*x\*e^e + a\*c^2\*f\*e^e)\*e^(f\*x))

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\frac{\left(-iad^2fx^2 - 2iacdfx - iac^2f + (ad^2fx^2 + 2acdfox + ac^2f)e^{fx+e}\right) \operatorname{integral}\left(\frac{4id}{-iad^3fx^3 - 3iacd^2fx^2 - 3iac^2dfx - iac^3f + (ad^3fx^3 + 3acd^2fx^2 + 3ac^2dfxe^e + ac^3fe^e)e^{fx}}\right)}{-iad^2fx^2 - 2iacdfx - iac^2f + (ad^2fx^2 + 2acdfox + ac^2f)e^{fx+e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+I\*a\*sinh(f\*x+e)),x, algorithm="fricas")

[Out] ((-I\*a\*d^2\*f\*x^2 - 2\*I\*a\*c\*d\*f\*x - I\*a\*c^2\*f + (a\*d^2\*f\*x^2 + 2\*a\*c\*d\*f\*x + a\*c^2\*f)\*e^(f\*x + e))\*integral(4\*I\*d/(-I\*a\*d^3\*f\*x^3 - 3\*I\*a\*c\*d^2\*f\*x^2 -



$$3*I*a*c^2*d*f*x - I*a*c^3*f + (a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f)*e^{(f*x + e)}, x) + 2*I)/(-I*a*d^2*f*x^2 - 2*I*a*c*d*f*x - I*a*c^2*f + (a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f)*e^{(f*x + e)})$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)\*\*2/(a+I\*a\*sinh(f\*x+e)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2(i a \sinh(fx+e)+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+I\*a\*sinh(f\*x+e)),x, algorithm="giac")

[Out] integrate(1/((d\*x + c)^2\*(I\*a\*sinh(f\*x + e) + a)), x)

$$3.113 \quad \int \frac{(c+dx)^3}{(a+ia \sinh(e+fx))^2} dx$$

**Optimal.** Leaf size=305

$$-\frac{4d^2(c+dx)\text{PolyLog}\left(2, -ie^{e+fx}\right)}{a^2f^3} + \frac{4d^3\text{PolyLog}\left(3, -ie^{e+fx}\right)}{a^2f^4} - \frac{2d^2(c+dx) \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{a^2f^3} - \frac{2d(c+dx)^2 \log\left(1 + \dots\right)}{a^2f^2}$$

```
[Out] (c + d*x)^3/(3*a^2*f) - (2*d*(c + d*x)^2*Log[1 + I*E^(e + f*x)])/(a^2*f^2)
+ (4*d^3*Log[Cosh[e/2 + (I/4)*Pi + (f*x)/2]])/(a^2*f^4) - (4*d^2*(c + d*x)*
PolyLog[2, (-I)*E^(e + f*x)])/(a^2*f^3) + (4*d^3*PolyLog[3, (-I)*E^(e + f*x
)])/(a^2*f^4) + (d*(c + d*x)^2*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2)/(2*a^2*f^2)
- (2*d^2*(c + d*x)*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(a^2*f^3) + ((c + d*x)
^3*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*a^2*f) + ((c + d*x)^3*Sech[e/2 + (I/4
)*Pi + (f*x)/2]^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(6*a^2*f)
```

**Rubi [A]** time = 0.397799, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$ , Rules used = {3318, 4186, 4184, 3475, 3716, 2190, 2531, 2282, 6589}

$$-\frac{4d^2(c+dx)\text{PolyLog}\left(2, -ie^{e+fx}\right)}{a^2f^3} + \frac{4d^3\text{PolyLog}\left(3, -ie^{e+fx}\right)}{a^2f^4} - \frac{2d^2(c+dx) \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{a^2f^3} - \frac{2d(c+dx)^2 \log\left(1 + \dots\right)}{a^2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (c + d*x)^3/(3*a^2*f) - (2*d*(c + d*x)^2*Log[1 + I*E^(e + f*x)])/(a^2*f^2)
+ (4*d^3*Log[Cosh[e/2 + (I/4)*Pi + (f*x)/2]])/(a^2*f^4) - (4*d^2*(c + d*x)*
PolyLog[2, (-I)*E^(e + f*x)])/(a^2*f^3) + (4*d^3*PolyLog[3, (-I)*E^(e + f*x
)])/(a^2*f^4) + (d*(c + d*x)^2*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2)/(2*a^2*f^2)
- (2*d^2*(c + d*x)*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(a^2*f^3) + ((c + d*x)
^3*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*a^2*f) + ((c + d*x)^3*Sech[e/2 + (I/4
)*Pi + (f*x)/2]^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(6*a^2*f)
```

### Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
(f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
```

, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])

### Rule 4186

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.))^(n\_)\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] := -Simp[(b^2\*(c + d\*x)^m\*Cot[e + f\*x]\*(b\*Csc[e + f\*x])^(n - 2))/(f\*(n - 1)), x] + (Dist[(b^2\*d^2\*m\*(m - 1))/(f^2\*(n - 1)\*(n - 2)), Int[(c + d\*x)^(m - 2)\*(b\*Csc[e + f\*x])^(n - 2), x], x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(c + d\*x)^m\*(b\*Csc[e + f\*x])^(n - 2), x], x] - Simp[(b^2\*d\*m\*(c + d\*x)^(m - 1)\*(b\*Csc[e + f\*x])^(n - 2))/(f^2\*(n - 1)\*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(c + d\*x)^m\*Cot[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

### Rule 3716

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)], x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x))/(E^(2\*I\*k\*Pi)\*(1 + E^(2\*(-I\*e) + f\*fz\*x))/E^(2\*I\*k\*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f}

, g, n}, x] && GtQ[m, 0]

### Rule 2282

```
Int[u_, x_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :=> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^3}{(a+ia\sinh(e+fx))^2} dx &= \frac{\int (c+dx)^3 \csc^4\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right) dx}{4a^2} \\
&= \frac{d(c+dx)^2 \operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2a^2 f^2} + \frac{(c+dx)^3 \operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right) \tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2 f} - \dots \\
&= \frac{d(c+dx)^2 \operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2a^2 f^2} - \frac{2d^2(c+dx) \tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{a^2 f^3} + \frac{(c+dx)^3 \tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2 f} \\
&= \frac{(c+dx)^3}{3a^2 f} + \frac{4d^3 \log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{a^2 f^4} + \frac{d(c+dx)^2 \operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2a^2 f^2} - \frac{2d^2(c+dx)}{a^2 f} \\
&= \frac{(c+dx)^3}{3a^2 f} - \frac{2d(c+dx)^2 \log(1+ie^{e+fx})}{a^2 f^2} + \frac{4d^3 \log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{a^2 f^4} + \frac{d(c+dx)^2 \operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{2a^2 f^2} \\
&= \frac{(c+dx)^3}{3a^2 f} - \frac{2d(c+dx)^2 \log(1+ie^{e+fx})}{a^2 f^2} + \frac{4d^3 \log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{a^2 f^4} - \frac{4d^2(c+dx)L}{a^2 f} \\
&= \frac{(c+dx)^3}{3a^2 f} - \frac{2d(c+dx)^2 \log(1+ie^{e+fx})}{a^2 f^2} + \frac{4d^3 \log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{a^2 f^4} - \frac{4d^2(c+dx)L}{a^2 f} \\
&= \frac{(c+dx)^3}{3a^2 f} - \frac{2d(c+dx)^2 \log(1+ie^{e+fx})}{a^2 f^2} + \frac{4d^3 \log\left(\cosh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)\right)}{a^2 f^4} - \frac{4d^2(c+dx)L}{a^2 f}
\end{aligned}$$

**Mathematica [A]** time = 6.24652, size = 443, normalized size = 1.45

$$\frac{2d\left(-6cd(1+ie^e)\operatorname{PolyLog}\left(2,ie^{-e-fx}\right)-6d^2(1+ie^e)\left(x\operatorname{PolyLog}\left(2,ie^{-e-fx}\right)+\frac{\operatorname{PolyLog}\left(3,ie^{-e-fx}\right)}{f}\right)\right)+\frac{3(1+ie^e)(2d^2-c^2f^2)(fx-\log(-e^{e+fx}+i))}{f}+3c^2f^2x+6cd(1+ie^e)fx\log(1-i)}{-1-ie^e}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3/(a + I\*a\*Sinh[e + f\*x])^2,x]

[Out] ((2\*d\*(-6\*d^2\*x + 3\*c^2\*f^2\*x + 3\*c\*d\*f^2\*x^2 + d^2\*f^2\*x^3 + 6\*c\*d\*(1 + I\*E^e)\*f\*x\*Log[1 - I\*E^(-e - f\*x)] + 3\*d^2\*(1 + I\*E^e)\*f\*x^2\*Log[1 - I\*E^(-e - f\*x)] + (3\*(1 + I\*E^e)\*(2\*d^2 - c^2\*f^2)\*(f\*x - Log[I - E^e + f\*x])))/f

$$- 6*c*d*(1 + I*E^e)*PolyLog[2, I*E^{-e - f*x}] - 6*d^2*(1 + I*E^e)*(x*PolyLog[2, I*E^{-e - f*x}] + PolyLog[3, I*E^{-e - f*x}]/f))/(-1 - I*E^e) + ((c + d*x)*(3*d*f*(c + d*x)*Cosh[(f*x)/2] + (6*I)*d^2*Cosh[e + (f*x)/2] + I*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-6 + f^2*x^2))*Cosh[e + (3*f*x)/2] + 3*(c^2*f^2 + 2*c*d*f^2*x + d^2*(-4 + f^2*x^2))*Sinh[(f*x)/2] + (3*I)*d*f*(c + d*x)*Sinh[e + (f*x)/2]))/((Cosh[e/2] + I*Sinh[e/2])*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))^3)/(3*a^2*f^3)$$

**Maple [B]** time = 0.141, size = 723, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3/(a+I\*a\*sinh(f\*x+e))^2,x)

[Out]  $\frac{2}{3}*(6*I*d^3*x-3*I*f*d^3*x^2*\exp(2*f*x+2*e)-I*f^2*d^3*x^3+3*f^2*d^3*x^3*\exp(f*x+e)-3*f*d^3*x^2*\exp(f*x+e)-3*f*c^2*d*\exp(f*x+e)-3*I*f^2*c*d^2*x^2+9*f^2*c*d^2*x^2*\exp(f*x+e)+9*f^2*c^2*d*x*\exp(f*x+e)-6*f*c*d^2*x*\exp(f*x+e)-3*I*f^2*c^2*d*x-6*I*c*d^2*\exp(2*f*x+2*e)-I*f^2*c^3-6*I*f*c*d^2*x*\exp(2*f*x+2*e)-6*I*d^3*x*\exp(2*f*x+2*e)+6*I*c*d^2-12*d^3*x*\exp(f*x+e)-12*c*d^2*\exp(f*x+e)+3*f^2*c^3*\exp(f*x+e)-3*I*f*c^2*d*\exp(2*f*x+2*e))/(\exp(f*x+e)-I)^3/f^3/a^2-4*d^2/f^2/a^2*\ln(1+I*\exp(f*x+e))*c*x-4*d^2/f^3/a^2*\ln(1+I*\exp(f*x+e))*c*e+4*d^2/f^2/a^2*c*e*x+4*d^2/f^3/a^2*\ln(\exp(f*x+e)-I)*c*e-4*d^2/f^3/a^2*\ln(\exp(f*x+e))*c*e+4*d^3/f^4/a^2*\ln(\exp(f*x+e)-I)+4*d^3*polylog(3,-I*\exp(f*x+e))/a^2/f^4-4*d^3/f^4/a^2*\ln(\exp(f*x+e))-4*d^2/f^3/a^2*c*polylog(2,-I*\exp(f*x+e))-2*d^3/f^3/a^2*e^2*x+2*d^2/f/a^2*c*x^2+2*d^2/f^3/a^2*c*e^2-2*d^3/f^2/a^2*\ln(1+I*\exp(f*x+e))*x^2-4*d^3/f^3/a^2*polylog(2,-I*\exp(f*x+e))*x-2*d^3/f^4/a^2*\ln(\exp(f*x+e)-I)*e^2+2*d/f^2/a^2*\ln(\exp(f*x+e))*c^2-2*d/f^2/a^2*\ln(\exp(f*x+e)-I)*c^2+2*d^3/f^4/a^2*\ln(\exp(f*x+e))*e^2+2/3*d^3/f/a^2*x^3+2*d^3/f^4/a^2*\ln(1+I*\exp(f*x+e))*e^2-4/3*d^3/f^4/a^2*e^3$

**Maxima [B]** time = 1.99941, size = 856, normalized size = 2.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="maxima")

```
[Out] c^2*d*(3*(2*f*x*e^(3*f*x + 3*e) + (-6*I*f*x*e^(2*e) - 2*I*e^(2*e))*e^(2*f*x)
) - 2*e^(f*x + e))/(3*a^2*f^2*e^(3*f*x + 3*e) - 9*I*a^2*f^2*e^(2*f*x + 2*e)
- 9*a^2*f^2*e^(f*x + e) + 3*I*a^2*f^2) - 2*log(-I*(I*e^(f*x + e) + 1)*e^(-
e))/(a^2*f^2)) + c^3*(6*e^(-f*x - e)/((9*a^2*e^(-f*x - e) - 9*I*a^2*e^(-2*f
*x - 2*e) - 3*a^2*e^(-3*f*x - 3*e) + 3*I*a^2)*f) + 2*I/((9*a^2*e^(-f*x - e)
- 9*I*a^2*e^(-2*f*x - 2*e) - 3*a^2*e^(-3*f*x - 3*e) + 3*I*a^2)*f)) + (-2*I
*d^3*f^2*x^3 - 6*I*c*d^2*f^2*x^2 + 12*I*d^3*x + 12*I*c*d^2 - (6*I*d^3*f*x^2
*e^(2*e) + 12*I*c*d^2*e^(2*e) + (12*I*c*d^2*f*e^(2*e) + 12*I*d^3*e^(2*e))*x
)*e^(2*f*x) + 6*(d^3*f^2*x^3*e^e - 4*c*d^2*e^e + (3*c*d^2*f^2*e^e - d^3*f*e
^e)*x^2 - 2*(c*d^2*f*e^e + 2*d^3*e^e)*x)*e^(f*x))/(3*a^2*f^3*e^(3*f*x + 3*e
) - 9*I*a^2*f^3*e^(2*f*x + 2*e) - 9*a^2*f^3*e^(f*x + e) + 3*I*a^2*f^3) - 4*
(f*x*log(I*e^(f*x + e) + 1) + dilog(-I*e^(f*x + e)))*c*d^2/(a^2*f^3) - 4*d^
3*x/(a^2*f^3) - 2*(f^2*x^2*log(I*e^(f*x + e) + 1) + 2*f*x*dilog(-I*e^(f*x +
e)) - 2*polylog(3, -I*e^(f*x + e)))*d^3/(a^2*f^4) + 4*d^3*log(e^(f*x + e)
- I)/(a^2*f^4) + 2/3*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2)/(a^2*f^4)
```

**Fricas [C]** time = 2.60388, size = 2147, normalized size = 7.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (2*I*d^3*e^3 + 6*I*c^2*d*e*f^2 - 2*I*c^3*f^3 - 12*I*d^3*e + (-6*I*c*d^2*e^2
+ 12*I*c*d^2)*f + (-12*I*d^3*f*x - 12*I*c*d^2*f - 12*(d^3*f*x + c*d^2*f)*e
^(3*f*x + 3*e) + (36*I*d^3*f*x + 36*I*c*d^2*f)*e^(2*f*x + 2*e) + 36*(d^3*f*x
+ c*d^2*f)*e^(f*x + e))*dilog(-I*e^(f*x + e)) + 2*(d^3*f^3*x^3 + 3*c*d^2*
f^3*x^2 + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - 6*d^3*e + 3*(c^2*d*f^3
- 2*d^3*f)*x)*e^(3*f*x + 3*e) + (-6*I*d^3*f^3*x^3 - 6*I*d^3*e^3 + 36*I*d^3*
e + (-18*I*c^2*d*e - 6*I*c^2*d)*f^2 + (-18*I*c*d^2*f^3 - 6*I*d^3*f^2)*x^2 +
(18*I*c*d^2*e^2 - 12*I*c*d^2)*f + (-18*I*c^2*d*f^3 - 12*I*c*d^2*f^2 + 24*I
*d^3*f)*x)*e^(2*f*x + 2*e) - 6*(d^3*f^2*x^2 + d^3*e^3 - c^3*f^3 - 6*d^3*e +
(3*c^2*d*e + c^2*d)*f^2 - (3*c*d^2*e^2 - 4*c*d^2)*f + 2*(c*d^2*f^2 - d^3*f
)*x)*e^(f*x + e) + (-6*I*d^3*e^2 + 12*I*c*d^2*e*f - 6*I*c^2*d*f^2 + 12*I*d^
3 - 6*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*d^3)*e^(3*f*x + 3*e) + (18*I*d
^3*e^2 - 36*I*c*d^2*e*f + 18*I*c^2*d*f^2 - 36*I*d^3)*e^(2*f*x + 2*e) + 18*(
d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 - 2*d^3)*e^(f*x + e))*log(e^(f*x + e) - I
) + (-6*I*d^3*f^2*x^2 - 12*I*c*d^2*f^2*x + 6*I*d^3*e^2 - 12*I*c*d^2*e*f - 6
*(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*e^(3*f*x + 3*e) + (1
8*I*d^3*f^2*x^2 + 36*I*c*d^2*f^2*x - 18*I*d^3*e^2 + 36*I*c*d^2*e*f)*e^(2*f*
x + 2*e) + 18*(d^3*f^2*x^2 + 2*c*d^2*f^2*x - d^3*e^2 + 2*c*d^2*e*f)*e^(f*x
```

+ e))\*log(I\*e^(f\*x + e) + 1) + (12\*d^3\*e^(3\*f\*x + 3\*e) - 36\*I\*d^3\*e^(2\*f\*x + 2\*e) - 36\*d^3\*e^(f\*x + e) + 12\*I\*d^3)\*polylog(3, -I\*e^(f\*x + e)))/(3\*a^2\*f^4\*e^(3\*f\*x + 3\*e) - 9\*I\*a^2\*f^4\*e^(2\*f\*x + 2\*e) - 9\*a^2\*f^4\*e^(f\*x + e) + 3\*I\*a^2\*f^4)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*3/(a+I\*a\*sinh(f\*x+e))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^3}{(i a \sinh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3/(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^3/(I\*a\*sinh(f\*x + e) + a)^2, x)



$$3.114 \quad \int \frac{(c+dx)^2}{(a+ia \sinh(e+fx))^2} dx$$

**Optimal.** Leaf size=241

$$-\frac{4d^2 \text{PolyLog}\left(2, -ie^{e+fx}\right)}{3a^2 f^3} - \frac{4d(c+dx) \log\left(1 + ie^{e+fx}\right)}{3a^2 f^2} + \frac{d(c+dx) \text{sech}^2\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{3a^2 f^2} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{3a^2 f}$$

```
[Out] (c + d*x)^2/(3*a^2*f) - (4*d*(c + d*x)*Log[1 + I*E^(e + f*x)])/(3*a^2*f^2)
- (4*d^2*PolyLog[2, (-I)*E^(e + f*x)])/(3*a^2*f^3) + (d*(c + d*x)*Sech[e/2
+ (I/4)*Pi + (f*x)/2]^2)/(3*a^2*f^2) - (2*d^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2
])/ (3*a^2*f^3) + ((c + d*x)^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/ (3*a^2*f) + (
(c + d*x)^2*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2
])/ (6*a^2*f)
```

**Rubi [A]** time = 0.277598, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$ , Rules used = {3318, 4186, 3767, 8, 4184, 3716, 2190, 2279, 2391}

$$-\frac{4d^2 \text{PolyLog}\left(2, -ie^{e+fx}\right)}{3a^2 f^3} - \frac{4d(c+dx) \log\left(1 + ie^{e+fx}\right)}{3a^2 f^2} + \frac{d(c+dx) \text{sech}^2\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{3a^2 f^2} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{3a^2 f}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + I*a*Sinh[e + f*x])^2,x]
```

```
[Out] (c + d*x)^2/(3*a^2*f) - (4*d*(c + d*x)*Log[1 + I*E^(e + f*x)])/(3*a^2*f^2)
- (4*d^2*PolyLog[2, (-I)*E^(e + f*x)])/(3*a^2*f^3) + (d*(c + d*x)*Sech[e/2
+ (I/4)*Pi + (f*x)/2]^2)/(3*a^2*f^2) - (2*d^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2
])/ (3*a^2*f^3) + ((c + d*x)^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/ (3*a^2*f) + (
(c + d*x)^2*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2
])/ (6*a^2*f)
```

### Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
(f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x]
+ (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x]
+ Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]
- Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x]
&& GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_.)]^(n_.), x_Symbol]
:> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol]
:> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_.)]^2*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x]
&& GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_.)], x_Symbol]
:> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*e) + f*fz*x)))/E^(2*I*k*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x]
&& IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_.))))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_.))))^(n_.), x_Symbol]
:> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x]
&& IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_.))))^(n_.)], x_Symbol]
```

```

:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

```

### Rule 2391

```

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^2}{(a+ia \sinh(e+fx))^2} dx &= \frac{\int (c+dx)^2 \csc^4\left(\frac{1}{2}\left(ie+\frac{\pi}{2}\right)+\frac{ifx}{2}\right) dx}{4a^2} \\
&= \frac{d(c+dx) \operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2 f^2} + \frac{(c+dx)^2 \operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right) \tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6a^2 f} - \int (c+dx) \operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right) dx \\
&= \frac{d(c+dx) \operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2 f^2} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2 f} + \frac{(c+dx)^2 \operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{6} \\
&= \frac{(c+dx)^2}{3a^2 f} + \frac{d(c+dx) \operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2 f^2} - \frac{2d^2 \tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2 f^3} + \frac{(c+dx)^2 \tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2 f} \\
&= \frac{(c+dx)^2}{3a^2 f} - \frac{4d(c+dx) \log(1+ie^{e+fx})}{3a^2 f^2} + \frac{d(c+dx) \operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2 f^2} - \frac{2d^2 \tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2 f} \\
&= \frac{(c+dx)^2}{3a^2 f} - \frac{4d(c+dx) \log(1+ie^{e+fx})}{3a^2 f^2} + \frac{d(c+dx) \operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2 f^2} - \frac{2d^2 \tanh\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2 f} \\
&= \frac{(c+dx)^2}{3a^2 f} - \frac{4d(c+dx) \log(1+ie^{e+fx})}{3a^2 f^2} - \frac{4d^2 \operatorname{Li}_2(-ie^{e+fx})}{3a^2 f^3} + \frac{d(c+dx) \operatorname{sech}^2\left(\frac{e}{2}+\frac{i\pi}{4}+\frac{fx}{2}\right)}{3a^2 f^2}
\end{aligned}$$

**Mathematica [A]** time = 3.55659, size = 269, normalized size = 1.12

$$4d^2 \operatorname{PolyLog}\left(2, ie^{-e-fx}\right) + \frac{i(c^2 f^2 + 2cdf^2 x + d^2(f^2 x^2 - 2)) \cosh\left(e + \frac{3fx}{2}\right) + \sinh\left(\frac{fx}{2}\right) (3c^2 f^2 + 6cdf^2 x + d^2(3f^2 x^2 - 4)) + 2idf(c+dx) \sinh\left(e + \frac{fx}{2}\right) + 2df(c+dx)}{\left(\cosh\left(\frac{e}{2}\right) + i \sinh\left(\frac{e}{2}\right)\right) \left(\cosh\left(\frac{1}{2}(e+fx)\right) + i \sinh\left(\frac{1}{2}(e+fx)\right)\right)^3}$$


---


$$3a^2 f^3$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2/(a + I\*a\*Sinh[e + f\*x])^2,x]

[Out] (((2\*I)\*f\*(c + d\*x)\*(f\*(c + d\*x) + 2\*d\*(1 + I\*E^e)\*Log[1 - I\*E^(-e - f\*x)]) / (-I + E^e) + 4\*d^2\*PolyLog[2, I\*E^(-e - f\*x)] + (2\*d\*f\*(c + d\*x)\*Cosh[(f\*x)/2] + (2\*I)\*d^2\*Cosh[e + (f\*x)/2] + I\*(c^2\*f^2 + 2\*c\*d\*f^2\*x + d^2\*(-2 + f^2\*x^2))\*Cosh[e + (3\*f\*x)/2] + (3\*c^2\*f^2 + 6\*c\*d\*f^2\*x + d^2\*(-4 + 3\*f^2\*x^2))\*Sinh[(f\*x)/2] + (2\*I)\*d\*f\*(c + d\*x)\*Sinh[e + (f\*x)/2]) / ((Cosh[e/2] + I\*Sinh[e/2])\*(Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2])^3) / (3\*a^2\*f^3)

**Maple [A]** time = 0.113, size = 374, normalized size = 1.6

$$\frac{-2if^2d^2x^2 - 4fd^2xe^{fx+e} - 4fcd e^{fx+e} - 8d^2e^{fx+e} + 4id^2 - 4id^2e^{2fx+2e} - 2if^2c^2 - 4if^2cdx - 4ifd^2xe^{2fx+2e} - 4ifcde^{2fx+2e}}{3(e^{fx+e} - i)^3 f^3 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2/(a+I\*a\*sinh(f\*x+e))^2,x)

[Out]  $\frac{2/3*(-I*f^2*d^2*x^2 - 2*f*d^2*x*exp(f*x+e) - 2*f*c*d*exp(f*x+e) - 4*d^2*exp(f*x+e) + 2*I*d^2 - 2*I*d^2*exp(2*f*x+2*e) - I*f^2*c^2 - 2*I*f^2*c*d*x - 2*I*f*d^2*x*exp(2*f*x+2*e) - 2*I*f*c*d*exp(2*f*x+2*e) + 3*f^2*d^2*x^2*exp(f*x+e) + 6*f^2*c*d*x*exp(f*x+e) + 3*f^2*c^2*exp(f*x+e))}{(exp(f*x+e) - I)^3/f^3/a^2 - 4/3*d/f^2/a^2*\ln(exp(f*x+e) - I)*c + 4/3*d/f^2/a^2*\ln(exp(f*x+e))*c + 2/3*d^2/f/a^2*x^2 + 4/3*d^2/f^2/a^2*e*x + 2/3*d^2/f^3/a^2*e^2 - 4/3*d^2/f^2/a^2*\ln(1 + I*exp(f*x+e))*x - 4/3*d^2/f^3/a^2*\ln(1 + I*exp(f*x+e))*e - 4/3*d^2*polylog(2, -I*exp(f*x+e))/a^2/f^3 + 4/3*d^2/f^3/a^2*e*\ln(exp(f*x+e) - I) - 4/3*d^2/f^3/a^2*e*\ln(exp(f*x+e))}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$d^2 \left( \frac{-2i f^2 x^2 - (4i f x e^{2e} + 4i e^{2e}) e^{2fx} + 2(3 f^2 x^2 e^e - 2 f x e^e - 4 e^e) e^{fx} + 4i}{3 a^2 f^3 e^{3fx+3e} - 9i a^2 f^3 e^{2fx+2e} - 9 a^2 f^3 e^{fx+e} + 3i a^2 f^3} - 4i \int \frac{x}{3(a^2 f e^{fx+e} - i a^2 f)} dx \right) + \frac{2}{3} cd$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="maxima")

[Out]  $d^2*((-2*I*f^2*x^2 - (4*I*f*x*e^{(2*e)} + 4*I*e^{(2*e)})e^{(2*f*x)} + 2*(3*f^2*x^2*e^e - 2*f*x*e^e - 4*e^e)e^{(f*x)} + 4*I)/(3*a^2*f^3*e^{(3*f*x + 3*e)} - 9*I$

```
*a^2*f^3*e^(2*f*x + 2*e) - 9*a^2*f^3*e^(f*x + e) + 3*I*a^2*f^3) - 4*I*integrate(1/3*x/(a^2*f*e^(f*x + e) - I*a^2*f), x)) + 2/3*c*d*(3*(2*f*x*e^(3*f*x + 3*e) + (-6*I*f*x*e^(2*e) - 2*I*e^(2*e))*e^(2*f*x) - 2*e^(f*x + e))/(3*a^2*f^2*e^(3*f*x + 3*e) - 9*I*a^2*f^2*e^(2*f*x + 2*e) - 9*a^2*f^2*e^(f*x + e) + 3*I*a^2*f^2) - 2*log(-I*(I*e^(f*x + e) + 1)*e^(-e))/(a^2*f^2)) + c^2*(6*e^(-f*x - e)/((9*a^2*e^(-f*x - e) - 9*I*a^2*e^(-2*f*x - 2*e) - 3*a^2*e^(-3*f*x - 3*e) + 3*I*a^2)*f) + 2*I/((9*a^2*e^(-f*x - e) - 9*I*a^2*e^(-2*f*x - 2*e) - 3*a^2*e^(-3*f*x - 3*e) + 3*I*a^2)*f))
```

**Fricas [B]** time = 2.50375, size = 1162, normalized size = 4.82

$$-2i d^2 e^2 + 4i c d e f - 2i c^2 f^2 + 4i d^2 - \left( 4 d^2 e^{(3fx+3e)} - 12i d^2 e^{(2fx+2e)} - 12 d^2 e^{(fx+e)} + 4i d^2 \right) \text{Li}_2 \left( -i e^{(fx+e)} \right) + 2 \left( d^2 f^2 x^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="fricas")

[Out] (-2\*I\*d^2\*e^2 + 4\*I\*c\*d\*e\*f - 2\*I\*c^2\*f^2 + 4\*I\*d^2 - (4\*d^2\*e^(3\*f\*x + 3\*e) - 12\*I\*d^2\*e^(2\*f\*x + 2\*e) - 12\*d^2\*e^(f\*x + e) + 4\*I\*d^2)\*dilog(-I\*e^(f\*x + e)) + 2\*(d^2\*f^2\*x^2 + 2\*c\*d\*f^2\*x - d^2\*e^2 + 2\*c\*d\*e\*f)\*e^(3\*f\*x + 3\*e) + (-6\*I\*d^2\*f^2\*x^2 + 6\*I\*d^2\*e^2 - 4\*I\*d^2 + (-12\*I\*c\*d\*e - 4\*I\*c\*d)\*f + (-12\*I\*c\*d\*f^2 - 4\*I\*d^2\*f)\*x)\*e^(2\*f\*x + 2\*e) + 2\*(3\*d^2\*e^2 + 3\*c^2\*f^2 - 2\*d^2\*f\*x - 4\*d^2 - 2\*(3\*c\*d\*e + c\*d)\*f)\*e^(f\*x + e) + (4\*I\*d^2\*e - 4\*I\*c\*d\*f + 4\*(d^2\*e - c\*d\*f)\*e^(3\*f\*x + 3\*e) + (-12\*I\*d^2\*e + 12\*I\*c\*d\*f)\*e^(2\*f\*x + 2\*e) - 12\*(d^2\*e - c\*d\*f)\*e^(f\*x + e))\*log(e^(f\*x + e) - I) + (-4\*I\*d^2\*f\*x - 4\*I\*d^2\*e - 4\*(d^2\*f\*x + d^2\*e)\*e^(3\*f\*x + 3\*e) + (12\*I\*d^2\*f\*x + 12\*I\*d^2\*e)\*e^(2\*f\*x + 2\*e) + 12\*(d^2\*f\*x + d^2\*e)\*e^(f\*x + e))\*log(I\*e^(f\*x + e) + 1))/(3\*a^2\*f^3\*e^(3\*f\*x + 3\*e) - 9\*I\*a^2\*f^3\*e^(2\*f\*x + 2\*e) - 9\*a^2\*f^3\*e^(f\*x + e) + 3\*I\*a^2\*f^3)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2/(a+I\*a\*sinh(f\*x+e))\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^2}{(i a \sinh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^2/(I\*a\*sinh(f\*x + e) + a)^2, x)

$$3.115 \quad \int \frac{c+dx}{(a+ia \sinh(e+fx))^2} dx$$

**Optimal.** Leaf size=158

$$\frac{(c+dx) \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{3a^2f} + \frac{(c+dx) \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{6a^2f} + \frac{d \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{6a^2f^2} - \frac{2d \log\left(\cos\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)\right)}{6a^2f^2}$$

```
[Out] (-2*d*Log[Cosh[e/2 + (I/4)*Pi + (f*x)/2]])/(3*a^2*f^2) + (d*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2)/(6*a^2*f^2) + ((c + d*x)*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*a^2*f) + ((c + d*x)*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(6*a^2*f)
```

**Rubi [A]** time = 0.109139, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {3318, 4185, 4184, 3475}

$$\frac{(c+dx) \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{3a^2f} + \frac{(c+dx) \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{6a^2f} + \frac{d \operatorname{sech}^2\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{6a^2f^2} - \frac{2d \log\left(\cos\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)\right)}{6a^2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + I*a*Sinh[e + f*x])^2, x]
```

```
[Out] (-2*d*Log[Cosh[e/2 + (I/4)*Pi + (f*x)/2]])/(3*a^2*f^2) + (d*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2)/(6*a^2*f^2) + ((c + d*x)*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*a^2*f) + ((c + d*x)*Sech[e/2 + (I/4)*Pi + (f*x)/2]^2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(6*a^2*f)
```

### Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_)), x_Symbol] :> -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
```

, x] - Simp[(b^2\*d\*(b\*Csc[e + f\*x])^(n - 2))/(f^2\*(n - 1)\*(n - 2)), x] /;  
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]

### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_.)]^2\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned} \int \frac{c + dx}{(a + ia \sinh(e + fx))^2} dx &= \frac{\int (c + dx) \csc^4\left(\frac{1}{2}\left(ie + \frac{\pi}{2}\right) + \frac{ifx}{2}\right) dx}{4a^2} \\ &= \frac{d \operatorname{sech}^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{6a^2 f^2} + \frac{(c + dx) \operatorname{sech}^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{6a^2 f} - \frac{\int (c + dx) \operatorname{csc}^2\left(\frac{1}{2}\left(ie + \frac{\pi}{2}\right) + \frac{ifx}{2}\right) dx}{6a^2 f} \\ &= \frac{d \operatorname{sech}^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{6a^2 f^2} + \frac{(c + dx) \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{3a^2 f} + \frac{(c + dx) \operatorname{sech}^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{6a^2 f} \\ &= -\frac{2d \log\left(\cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)\right)}{3a^2 f^2} + \frac{d \operatorname{sech}^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{6a^2 f^2} + \frac{(c + dx) \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{3a^2 f} + \frac{(c + dx) \operatorname{sech}^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{6a^2 f} \end{aligned}$$

**Mathematica [A]** time = 1.08094, size = 241, normalized size = 1.53

$$\frac{\left(\sinh\left(\frac{1}{2}(e + fx)\right) - i \cosh\left(\frac{1}{2}(e + fx)\right)\right) \left(\cosh\left(\frac{3}{2}(e + fx)\right) \left(2cf + 2d \tan^{-1}\left(\tanh\left(\frac{1}{2}(e + fx)\right)\right)\right) - id \log(\cosh(e + fx))\right) - \dots}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)/(a + I\*a\*Sinh[e + f\*x])^2,x]

[Out] (((-1)\*Cosh[(e + f\*x)/2] + Sinh[(e + f\*x)/2])\*(d\*Cosh[(e + f\*x)/2]\*(-2\*I + 3\*e + 3\*f\*x - 6\*ArcTan[Tanh[(e + f\*x)/2]] + (3\*I)\*Log[Cosh[e + f\*x]]) + Cos h[(3\*(e + f\*x))/2]\*(-(d\*e) + 2\*c\*f + d\*f\*x + 2\*d\*ArcTan[Tanh[(e + f\*x)/2]]



$- I*d*\text{Log}[\text{Cosh}[e + f*x]] + (2*I)*((-I)*d + 2*d*e - 3*c*f - d*f*x - 4*d*\text{ArcTan}[\text{Tanh}[(e + f*x)/2]] + d*\text{Cosh}[e + f*x]*(e + f*x - 2*\text{ArcTan}[\text{Tanh}[(e + f*x)/2]]) + I*\text{Log}[\text{Cosh}[e + f*x]] + (2*I)*d*\text{Log}[\text{Cosh}[e + f*x]]*\text{Sinh}[(e + f*x)/2]))/(6*a^2*f^2*(-I + \text{Sinh}[e + f*x])^2)$

**Maple [A]** time = 0.103, size = 113, normalized size = 0.7

$$\frac{2dx}{3a^2f} + \frac{2de}{3f^2a^2} - \frac{\frac{2i}{3}(3ifdx e^{fx+e} + 3ifce^{fx+e} - ide^{fx+e} + dfx + de^2fx+2e + cf)}{(e^{fx+e} - i)^3 f^2 a^2} - \frac{2d \ln(e^{fx+e} - i)}{3f^2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)/(a+I\*a\*sinh(f\*x+e))^2,x)

[Out]  $\frac{2}{3}d/f/a^2*x + \frac{2}{3}d/f^2/a^2*e - \frac{2}{3}I*(3*I*f*d*x*\exp(f*x+e) + 3*I*f*c*\exp(f*x+e) - I*d*\exp(f*x+e) + d*f*x + d*\exp(2*f*x+2*e) + c*f)/( \exp(f*x+e) - I)^3/f^2/a^2 - \frac{2}{3}d/f^2/a^2*\ln(\exp(f*x+e) - I)$

**Maxima [B]** time = 1.19648, size = 346, normalized size = 2.19

$$\frac{1}{3}d \left( \frac{3 \left( 2fxe^{(3fx+3e)} + (-6ifxe^{(2e)} - 2ie^{(2e)})e^{(2fx)} - 2e^{(fx+e)} \right)}{3a^2f^2e^{(3fx+3e)} - 9ia^2f^2e^{(2fx+2e)} - 9a^2f^2e^{(fx+e)} + 3ia^2f^2} - \frac{2 \log \left( -i \left( ie^{(fx+e)} + 1 \right) e^{(-e)} \right)}{a^2f^2} \right) + c \left( \frac{1}{9a^2e^{(-fx-e)} - 9ia^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="maxima")

[Out]  $\frac{1}{3}d*(3*(2*f*x*e^{(3*f*x + 3*e)} + (-6*I*f*x*e^{(2*e)} - 2*I*e^{(2*e)})*e^{(2*f*x)} - 2*e^{(f*x + e)})/(3*a^2*f^2*e^{(3*f*x + 3*e)} - 9*I*a^2*f^2*e^{(2*f*x + 2*e)} - 9*a^2*f^2*e^{(f*x + e)} + 3*I*a^2*f^2) - 2*\log(-I*(I*e^{(f*x + e)} + 1)*e^{(-e)})/(a^2*f^2)) + c*(6*e^{(-f*x - e)})/((9*a^2*e^{(-f*x - e)} - 9*I*a^2*e^{(-2*f*x - 2*e)} - 3*a^2*e^{(-3*f*x - 3*e)} + 3*I*a^2)*f) + 2*I/((9*a^2*e^{(-f*x - e)} - 9*I*a^2*e^{(-2*f*x - 2*e)} - 3*a^2*e^{(-3*f*x - 3*e)} + 3*I*a^2)*f))$

**Fricas [A]** time = 2.53945, size = 398, normalized size = 2.52

$$\frac{2dfxe^{(3fx+3e)} - 2icf + (-6idfx - 2id)e^{(2fx+2e)} + 2(3cf - d)e^{(fx+e)} - (2de^{(3fx+3e)} - 6ide^{(2fx+2e)} - 6de^{(fx+e)} + 2id)}{3a^2f^2e^{(3fx+3e)} - 9ia^2f^2e^{(2fx+2e)} - 9a^2f^2e^{(fx+e)} + 3ia^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="fricas")

[Out] (2\*d\*f\*x\*e^(3\*f\*x + 3\*e) - 2\*I\*c\*f + (-6\*I\*d\*f\*x - 2\*I\*d)\*e^(2\*f\*x + 2\*e) + 2\*(3\*c\*f - d)\*e^(f\*x + e) - (2\*d\*e^(3\*f\*x + 3\*e) - 6\*I\*d\*e^(2\*f\*x + 2\*e) - 6\*d\*e^(f\*x + e) + 2\*I\*d)\*log(e^(f\*x + e) - I))/(3\*a^2\*f^2\*e^(3\*f\*x + 3\*e) - 9\*I\*a^2\*f^2\*e^(2\*f\*x + 2\*e) - 9\*a^2\*f^2\*e^(f\*x + e) + 3\*I\*a^2\*f^2)

**Sympy [A]** time = 8.02081, size = 173, normalized size = 1.09

$$\frac{\frac{2ide^{-2fx}}{3a^2f^2} - \frac{2ice^{3e}+2idxe^{3e}}{3a^2f} - \frac{(6cfe^{2e}+6dfxe^{2e}+2de^{2e})e^{-fx}}{3a^2f^2}}{-ie^{3e} - 3e^{2e}e^{-fx} + 3ie^e e^{-2fx} + e^{-3fx}} - \frac{2dx}{3a^2f} - \frac{2d \log(i e^e + e^{-fx})}{3a^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+I\*a\*sinh(f\*x+e))\*\*2,x)

[Out] (2\*I\*d\*exp(e)\*exp(-2\*f\*x)/(3\*a\*\*2\*f\*\*2) - (2\*I\*c\*exp(3\*e) + 2\*I\*d\*x\*exp(3\*e)))/(3\*a\*\*2\*f) - (6\*c\*f\*exp(2\*e) + 6\*d\*f\*x\*exp(2\*e) + 2\*d\*exp(2\*e))\*exp(-f\*x)/(3\*a\*\*2\*f\*\*2))/(-I\*exp(3\*e) - 3\*exp(2\*e)\*exp(-f\*x) + 3\*I\*exp(e)\*exp(-2\*f\*x) + exp(-3\*f\*x)) - 2\*d\*x/(3\*a\*\*2\*f) - 2\*d\*log(I\*exp(e) + exp(-f\*x))/(3\*a\*\*2\*f\*\*2)

**Giac [A]** time = 1.32547, size = 285, normalized size = 1.8

$$\frac{2dfxe^{(3fx+3e)} - 6idfxe^{(2fx+2e)} + 6cfe^{(fx+e)} - 2de^{(3fx+3e)} \log(e^{(fx+e)} - i) + 6ide^{(2fx+2e)} \log(e^{(fx+e)} - i) + 6de^{(fx+e)} \log(e^{(fx+e)} - i)}{3a^2f^2e^{(3fx+3e)} - 9ia^2f^2e^{(2fx+2e)} - 9a^2f^2e^{(fx+e)} + 3ia^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="giac")

```
[Out] (2*d*f*x*e^(3*f*x + 3*e) - 6*I*d*f*x*e^(2*f*x + 2*e) + 6*c*f*e^(f*x + e) -
2*d*e^(3*f*x + 3*e)*log(e^(f*x + e) - I) + 6*I*d*e^(2*f*x + 2*e)*log(e^(f*x
+ e) - I) + 6*d*e^(f*x + e)*log(e^(f*x + e) - I) - 2*I*c*f - 2*I*d*e^(2*f*
x + 2*e) - 2*d*e^(f*x + e) - 2*I*d*log(e^(f*x + e) - I))/(3*a^2*f^2*e^(3*f*
x + 3*e) - 9*I*a^2*f^2*e^(2*f*x + 2*e) - 9*a^2*f^2*e^(f*x + e) + 3*I*a^2*f^
2)
```

$$3.116 \quad \int \frac{1}{(c+dx)(a+ia \sinh(e+fx))^2} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a+ia \sinh(e+fx))^2}, x\right)$$

[Out] Unintegrable[1/((c + d\*x)\*(a + I\*a\*Sinh[e + f\*x])^2), x]

**Rubi [A]** time = 0.0604981, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(c+dx)(a+ia \sinh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d\*x)\*(a + I\*a\*Sinh[e + f\*x])^2), x]

[Out] Defer[Int][1/((c + d\*x)\*(a + I\*a\*Sinh[e + f\*x])^2), x]

Rubi steps

$$\int \frac{1}{(c+dx)(a+ia \sinh(e+fx))^2} dx = \int \frac{1}{(c+dx)(a+ia \sinh(e+fx))^2} dx$$

**Mathematica [A]** time = 36.1759, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+ia \sinh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d\*x)\*(a + I\*a\*Sinh[e + f\*x])^2), x]

[Out] Integrate[1/((c + d\*x)\*(a + I\*a\*Sinh[e + f\*x])^2), x]

---

**Maple [A]** time = 0.823, size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)(a + ia \sinh(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x+c)/(a+I\*a\*sinh(f\*x+e))^2,x)

[Out] int(1/(d\*x+c)/(a+I\*a\*sinh(f\*x+e))^2,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{-2i d^2 f^2 x^2 - 4i c d f^2 x - 2i c^2 f^2 + 4i d^2 + (2i d^2 f x e^{2e} + 2i d^2 f^2 x^2 e^{2e} + 2i c d f^2 x e^{2e} - 4i d^2 e^{2e}) e^{2f x} + 2*(3*d^2*f^2*x^2*e^e + 3*c^2*f^2*e^e + c*d*f*e^e - 4*d^2*e^e + (6*c*d*f^2*e^e + d^2*f*e^e)*x) * e^{f*x}}{3i a^2 d^3 f^3 x^3 + 9i a^2 c d^2 f^3 x^2 + 9i a^2 c^2 d f^3 x + 3i a^2 c^3 f^3 + 3(a^2 d^3 f^3 x^3 e^{3e} + 3 a^2 c d^2 f^3 x^2 e^{3e} + 3 a^2 c^2 d f^3 x e^{3e} + a^2 c^3 f^3 e^{3e}) e^{3f x} + (-9 I a^2 d^3 f^3 x^3 e^{2e} - 27 I a^2 c d^2 f^3 x^2 e^{2e} - 27 I a^2 c^2 d f^3 x e^{2e} - 9 I a^2 c^3 f^3 e^{2e}) e^{2f x} - 9(a^2 d^3 f^3 x^3 e^e + 3 a^2 c d^2 f^3 x^2 e^e + 3 a^2 c^2 d f^3 x e^e + a^2 c^3 f^3 e^e) e^{f x}} - \text{integrate}(2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 6*d^3)/(3*a^2*d^4*f^3*x^4 + 12*a^2*c*d^3*f^3*x^3 + 18*a^2*c^2*d^2*f^3*x^2 + 12*a^2*c^3*d*f^3*x + 3*a^2*c^4*f^3 + (3*I*a^2*d^4*f^3*x^4*e^e + 12*I*a^2*c*d^3*f^3*x^3*e^e + 18*I*a^2*c^2*d^2*f^3*x^2*e^e + 12*I*a^2*c^3*d*f^3*x*e^e + 3*I*a^2*c^4*f^3*e^e)*e^{f*x}), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="maxima")

[Out] (-2\*I\*d^2\*f^2\*x^2 - 4\*I\*c\*d\*f^2\*x - 2\*I\*c^2\*f^2 + 4\*I\*d^2 + (2\*I\*d^2\*f\*x\*e^e + 2\*I\*c\*d\*f\*e^e - 4\*I\*d^2\*e^e))\*e^(2\*f\*x) + 2\*(3\*d^2\*f^2\*x^2\*e^e + 3\*c^2\*f^2\*e^e + c\*d\*f\*e^e - 4\*d^2\*e^e + (6\*c\*d\*f^2\*e^e + d^2\*f\*e^e)\*x) \* e^{f\*x})/(3\*I\*a^2\*d^3\*f^3\*x^3 + 9\*I\*a^2\*c\*d^2\*f^3\*x^2 + 9\*I\*a^2\*c^2\*d\*f^3\*x + 3\*I\*a^2\*c^3\*f^3 + 3\*(a^2\*d^3\*f^3\*x^3\*e^{3e} + 3\*a^2\*c\*d^2\*f^3\*x^2\*e^{3e} + 3\*a^2\*c^2\*d\*f^3\*x\*e^{3e} + a^2\*c^3\*f^3\*e^{3e}))\*e^{3\*f\*x} + (-9\*I\*a^2\*d^3\*f^3\*x^3\*e^{2e} - 27\*I\*a^2\*c\*d^2\*f^3\*x^2\*e^{2e} - 27\*I\*a^2\*c^2\*d\*f^3\*x\*e^{2e} - 9\*I\*a^2\*c^3\*f^3\*e^{2e}))\*e^{2\*f\*x} - 9\*(a^2\*d^3\*f^3\*x^3\*e^e + 3\*a^2\*c\*d^2\*f^3\*x^2\*e^e + 3\*a^2\*c^2\*d\*f^3\*x\*e^e + a^2\*c^3\*f^3\*e^e)\*e^{f\*x}) - \text{integrate}(2\*(d^3\*f^2\*x^2 + 2\*c\*d^2\*f^2\*x + c^2\*d\*f^2 - 6\*d^3)/(3\*a^2\*d^4\*f^3\*x^4 + 12\*a^2\*c\*d^3\*f^3\*x^3 + 18\*a^2\*c^2\*d^2\*f^3\*x^2 + 12\*a^2\*c^3\*d\*f^3\*x + 3\*a^2\*c^4\*f^3 + (3\*I\*a^2\*d^4\*f^3\*x^4\*e^e + 12\*I\*a^2\*c\*d^3\*f^3\*x^3\*e^e + 18\*I\*a^2\*c^2\*d^2\*f^3\*x^2\*e^e + 12\*I\*a^2\*c^3\*d\*f^3\*x\*e^e + 3\*I\*a^2\*c^4\*f^3\*e^e)\*e^{f\*x}), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (-2*I*d^2*f^2*x^2 - 4*I*c*d*f^2*x - 2*I*c^2*f^2 + 4*I*d^2 + (2*I*d^2*f*x +
2*I*c*d*f - 4*I*d^2)*e^(2*f*x + 2*e) + 2*(3*d^2*f^2*x^2 + 3*c^2*f^2 + c*d*f
- 4*d^2 + (6*c*d*f^2 + d^2*f)*x)*e^(f*x + e) + (3*I*a^2*d^3*f^3*x^3 + 9*I*
a^2*c*d^2*f^3*x^2 + 9*I*a^2*c^2*d*f^3*x + 3*I*a^2*c^3*f^3 + 3*(a^2*d^3*f^3*
x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(3*f*x + 3*e
) + (-9*I*a^2*d^3*f^3*x^3 - 27*I*a^2*c*d^2*f^3*x^2 - 27*I*a^2*c^2*d*f^3*x -
9*I*a^2*c^3*f^3)*e^(2*f*x + 2*e) - 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^
2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(f*x + e))*integral((2*I*d^3*f^2*x^2
+ 4*I*c*d^2*f^2*x + 2*I*c^2*d*f^2 - 12*I*d^3)/(-3*I*a^2*d^4*f^3*x^4 - 12*I
*a^2*c*d^3*f^3*x^3 - 18*I*a^2*c^2*d^2*f^3*x^2 - 12*I*a^2*c^3*d*f^3*x - 3*I*
a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*
x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^(f*x + e)), x)/(3*I*a^2*d^3*f^3*x
^3 + 9*I*a^2*c*d^2*f^3*x^2 + 9*I*a^2*c^2*d*f^3*x + 3*I*a^2*c^3*f^3 + 3*(a^2
*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(3*
f*x + 3*e) + (-9*I*a^2*d^3*f^3*x^3 - 27*I*a^2*c*d^2*f^3*x^2 - 27*I*a^2*c^2*
d*f^3*x - 9*I*a^2*c^3*f^3)*e^(2*f*x + 2*e) - 9*(a^2*d^3*f^3*x^3 + 3*a^2*c*d
^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3)*e^(f*x + e))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)(ia \sinh(fx+e)+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(I*a*sinh(f*x + e) + a)^2), x)
```

$$3.117 \quad \int \frac{1}{(c+dx)^2(a+ia \sinh(e+fx))^2} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a+ia \sinh(e+fx))^2}, x\right)$$

[Out] Unintegrable[1/((c + d\*x)^2\*(a + I\*a\*Sinh[e + f\*x])^2), x]

**Rubi [A]** time = 0.0574974, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+ia \sinh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d\*x)^2\*(a + I\*a\*Sinh[e + f\*x])^2), x]

[Out] Defer[Int][1/((c + d\*x)^2\*(a + I\*a\*Sinh[e + f\*x])^2), x]

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+ia \sinh(e+fx))^2} dx = \int \frac{1}{(c+dx)^2(a+ia \sinh(e+fx))^2} dx$$

**Mathematica [A]** time = 37.9065, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+ia \sinh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d\*x)^2\*(a + I\*a\*Sinh[e + f\*x])^2), x]

[Out] Integrate[1/((c + d\*x)^2\*(a + I\*a\*Sinh[e + f\*x])^2), x]



---

**Maple [A]** time = 1.103, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2 (a+ia \sinh(fx+e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x+c)^2/(a+I\*a\*sinh(f\*x+e))^2,x)

[Out] int(1/(d\*x+c)^2/(a+I\*a\*sinh(f\*x+e))^2,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="maxima")

[Out]  $(-2*I*d^2*f^2*x^2 - 4*I*c*d*f^2*x - 2*I*c^2*f^2 + 12*I*d^2 + (4*I*d^2*f*x*e^{(2*e)} + 4*I*c*d*f*e^{(2*e)} - 12*I*d^2*e^{(2*e)})e^{(2*f*x)} + 2*(3*d^2*f^2*x^2 * e^e + 3*c^2*f^2*e^e + 2*c*d*f*e^e - 12*d^2*e^e + 2*(3*c*d*f^2*e^e + d^2*f * e^e)*x)e^{(f*x)}) / (3*I*a^2*d^4*f^3*x^4 + 12*I*a^2*c*d^3*f^3*x^3 + 18*I*a^2*c^2*d^2*f^3*x^2 + 12*I*a^2*c^3*d*f^3*x + 3*I*a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 * e^{(3*e)} + 4*a^2*c*d^3*f^3*x^3 * e^{(3*e)} + 6*a^2*c^2*d^2*f^3*x^2 * e^{(3*e)} + 4 * a^2*c^3*d*f^3*x * e^{(3*e)} + a^2*c^4*f^3 * e^{(3*e)})e^{(3*f*x)} + (-9*I*a^2*d^4*f^3 * x^4 * e^{(2*e)} - 36*I*a^2*c*d^3*f^3*x^3 * e^{(2*e)} - 54*I*a^2*c^2*d^2*f^3*x^2 * e^{(2*e)} - 36*I*a^2*c^3*d*f^3*x * e^{(2*e)} - 9*I*a^2*c^4*f^3 * e^{(2*e)})e^{(2*f*x)} - 9*(a^2*d^4*f^3*x^4 * e^e + 4*a^2*c*d^3*f^3*x^3 * e^e + 6*a^2*c^2*d^2*f^3*x^2 * e^e + 4*a^2*c^3*d*f^3*x * e^e + a^2*c^4*f^3 * e^e)e^{(f*x)}) - integrate(4*(d^3 * f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 12*d^3) / (3*a^2*d^5*f^3*x^5 + 15*a^2 * c*d^4*f^3*x^4 + 30*a^2*c^2*d^3*f^3*x^3 + 30*a^2*c^3*d^2*f^3*x^2 + 15*a^2*c^4 * d*f^3*x + 3*a^2*c^5*f^3 + (3*I*a^2*d^5*f^3*x^5 * e^e + 15*I*a^2*c*d^4*f^3 * x^4 * e^e + 30*I*a^2*c^2*d^3*f^3*x^3 * e^e + 30*I*a^2*c^3*d^2*f^3*x^2 * e^e + 15*I * a^2*c^4*d*f^3*x * e^e + 3*I*a^2*c^5*f^3 * e^e)e^{(f*x)}), x)$

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="fricas")

[Out]  $(-2*I*d^2*f^2*x^2 - 4*I*c*d*f^2*x - 2*I*c^2*f^2 + 12*I*d^2 + (4*I*d^2*f*x + 4*I*c*d*f - 12*I*d^2)*e^{(2*f*x + 2*e)} + 2*(3*d^2*f^2*x^2 + 3*c^2*f^2 + 2*c*d*f - 12*d^2 + 2*(3*c*d*f^2 + d^2*f)*x)*e^{(f*x + e)} + (3*I*a^2*d^4*f^3*x^4 + 12*I*a^2*c*d^3*f^3*x^3 + 18*I*a^2*c^2*d^2*f^3*x^2 + 12*I*a^2*c^3*d*f^3*x + 3*I*a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^{(3*f*x + 3*e)} + (-9*I*a^2*d^4*f^3*x^4 - 36*I*a^2*c*d^3*f^3*x^3 - 54*I*a^2*c^2*d^2*f^3*x^2 - 36*I*a^2*c^3*d*f^3*x - 9*I*a^2*c^4*f^3)*e^{(2*f*x + 2*e)} - 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^{(f*x + e)})*integral((4*I*d^3*f^2*x^2 + 8*I*c*d^2*f^2*x + 4*I*c^2*d*f^2 - 48*I*d^3)/(-3*I*a^2*d^5*f^3*x^5 - 15*I*a^2*c*d^4*f^3*x^4 - 30*I*a^2*c^2*d^3*f^3*x^3 - 30*I*a^2*c^3*d^2*f^3*x^2 - 15*I*a^2*c^4*d*f^3*x - 3*I*a^2*c^5*f^3 + 3*(a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3)*e^{(f*x + e)}), x)/(3*I*a^2*d^4*f^3*x^4 + 12*I*a^2*c*d^3*f^3*x^3 + 18*I*a^2*c^2*d^2*f^3*x^2 + 12*I*a^2*c^3*d*f^3*x + 3*I*a^2*c^4*f^3 + 3*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^{(3*f*x + 3*e)} + (-9*I*a^2*d^4*f^3*x^4 - 36*I*a^2*c*d^3*f^3*x^3 - 54*I*a^2*c^2*d^2*f^3*x^2 - 36*I*a^2*c^3*d*f^3*x - 9*I*a^2*c^4*f^3)*e^{(2*f*x + 2*e)} - 9*(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3)*e^{(f*x + e)})$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)\*\*2/(a+I\*a\*sinh(f\*x+e))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2 (ia \sinh(fx+e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(I*a*sinh(f*x + e) + a)^2), x)
```

### 3.118 $\int x^4 \sqrt{a + ia \sinh(e + fx)} dx$

**Optimal.** Leaf size=181

$$-\frac{16x^3 \sqrt{a + ia \sinh(e + fx)}}{f^2} + \frac{96x^2 \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)}}{f^3} - \frac{384x \sqrt{a + ia \sinh(e + fx)}}{f^4} + \frac{768 \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)}}{f^5}$$

[Out] (-384\*x\*Sqrt[a + I\*a\*Sinh[e + f\*x]])/f^4 - (16\*x^3\*Sqrt[a + I\*a\*Sinh[e + f\*x]])/f^2 + (768\*Sqrt[a + I\*a\*Sinh[e + f\*x]]\*Tanh[e/2 + (I/4)\*Pi + (f\*x)/2])/f^5 + (96\*x^2\*Sqrt[a + I\*a\*Sinh[e + f\*x]]\*Tanh[e/2 + (I/4)\*Pi + (f\*x)/2])/f^3 + (2\*x^4\*Sqrt[a + I\*a\*Sinh[e + f\*x]]\*Tanh[e/2 + (I/4)\*Pi + (f\*x)/2])/f

**Rubi [A]** time = 0.212391, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3319, 3296, 2638}

$$-\frac{16x^3 \sqrt{a + ia \sinh(e + fx)}}{f^2} + \frac{96x^2 \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)}}{f^3} - \frac{384x \sqrt{a + ia \sinh(e + fx)}}{f^4} + \frac{768 \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)}}{f^5}$$

Antiderivative was successfully verified.

[In] Int[x^4\*Sqrt[a + I\*a\*Sinh[e + f\*x]],x]

[Out] (-384\*x\*Sqrt[a + I\*a\*Sinh[e + f\*x]])/f^4 - (16\*x^3\*Sqrt[a + I\*a\*Sinh[e + f\*x]])/f^2 + (768\*Sqrt[a + I\*a\*Sinh[e + f\*x]]\*Tanh[e/2 + (I/4)\*Pi + (f\*x)/2])/f^5 + (96\*x^2\*Sqrt[a + I\*a\*Sinh[e + f\*x]]\*Tanh[e/2 + (I/4)\*Pi + (f\*x)/2])/f^3 + (2\*x^4\*Sqrt[a + I\*a\*Sinh[e + f\*x]]\*Tanh[e/2 + (I/4)\*Pi + (f\*x)/2])/f

#### Rule 3319

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Dist[((2\*a)^IntPart[n]\*(a + b\*Sin[e + f\*x])^FracPart[n])/Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*FracPart[n]), Int[(c + d\*x)^m\*Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m-1)\*Cos[e + f\*x], x], x]

$e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

### Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x\_Symbol] \text{ :> } -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

### Rubi steps

$$\begin{aligned} \int x^4 \sqrt{a + ia \sinh(e + fx)} dx &= \left( \text{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right) \int x^4 \sinh \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) dx \\ &= \frac{2x^4 \sqrt{a + ia \sinh(e + fx)} \tanh \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right)}{f} - \frac{\left( 8 \text{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right)}{f} \\ &= -\frac{16x^3 \sqrt{a + ia \sinh(e + fx)}}{f^2} + \frac{2x^4 \sqrt{a + ia \sinh(e + fx)} \tanh \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right)}{f} - \frac{\left( 48 \text{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right)}{f^2} \\ &= -\frac{16x^3 \sqrt{a + ia \sinh(e + fx)}}{f^2} + \frac{96x^2 \sqrt{a + ia \sinh(e + fx)} \tanh \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right)}{f^3} + \frac{2x^4 \sqrt{a + ia \sinh(e + fx)}}{f^3} \\ &= -\frac{384x \sqrt{a + ia \sinh(e + fx)}}{f^4} - \frac{16x^3 \sqrt{a + ia \sinh(e + fx)}}{f^2} + \frac{96x^2 \sqrt{a + ia \sinh(e + fx)}}{f^3} \\ &= -\frac{384x \sqrt{a + ia \sinh(e + fx)}}{f^4} - \frac{16x^3 \sqrt{a + ia \sinh(e + fx)}}{f^2} + \frac{768 \sqrt{a + ia \sinh(e + fx)}}{f^5} \end{aligned}$$

**Mathematica [A]** time = 0.23611, size = 141, normalized size = 0.78

$$\frac{2\sqrt{a + ia \sinh(e + fx)} \left( (f^4 x^4 - 8if^3 x^3 + 48f^2 x^2 - 192ifx + 384) \sinh \left( \frac{1}{2}(e + fx) \right) + i(f^4 x^4 + 8if^3 x^3 + 48f^2 x^2 + 192ifx + 384) \cosh \left( \frac{1}{2}(e + fx) \right) \right)}{f^5 \left( \cosh \left( \frac{1}{2}(e + fx) \right) + i \sinh \left( \frac{1}{2}(e + fx) \right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*Sqrt[a + I\*a\*Sinh[e + f\*x]],x]

[Out] (2\*(I\*(384 + (192\*I)\*f\*x + 48\*f^2\*x^2 + (8\*I)\*f^3\*x^3 + f^4\*x^4)\*Cosh[(e + f\*x)/2] + (384 - (192\*I)\*f\*x + 48\*f^2\*x^2 - (8\*I)\*f^3\*x^3 + f^4\*x^4)\*Sinh[(e + f\*x)/2])\*Sqrt[a + I\*a\*Sinh[e + f\*x]]/(f^5\*(Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2]))

---

**Maple [A]** time = 0.087, size = 174, normalized size = 1.

$$\frac{i\sqrt{2}\left(ix^4f^4 + f^4x^4e^{fx+e} + 8ix^3f^3 - 8f^3x^3e^{fx+e} + 48ix^2f^2 + 48f^2x^2e^{fx+e} + 192ixf - 192fxe^{fx+e} + 384i + 384e^{fx+e}\right)}{\left(i e^{2fx+2e} - i + 2e^{fx+e}\right) f^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(a+I*a*sinh(f*x+e))^(1/2),x)`

[Out]  $I*2^{(1/2)}*(a*(I*\exp(2*f*x+2*e)-I+2*\exp(f*x+e))*\exp(-f*x-e))^{(1/2)}/(I*\exp(2*f*x+2*e)-I+2*\exp(f*x+e))*(I*x^4*f^4+f^4*x^4*\exp(f*x+e)+8*I*x^3*f^3-8*f^3*x^3*\exp(f*x+e)+48*I*x^2*f^2+48*f^2*x^2*\exp(f*x+e)+192*I*x*f-192*f*x*\exp(f*x+e)+384*I+384*\exp(f*x+e))*(\exp(f*x+e)-I)/f^5$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ia \sinh(fx + e) + ax^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(I*a*sinh(f*x + e) + a)*x^4, x)`

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^4 \sqrt{a(i \sinh(e + fx) + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(a+I\*a\*sinh(f\*x+e))\*\*(1/2),x)

[Out] Integral(x\*\*4\*sqrt(a\*(I\*sinh(e + f\*x) + 1)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ia \sinh(fx + e) + ax^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(a+I\*a\*sinh(f\*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(I\*a\*sinh(f\*x + e) + a)\*x^4, x)

### 3.119 $\int x^3 \sqrt{a + ia \sinh(e + fx)} dx$

**Optimal.** Leaf size=136

$$-\frac{12x^2 \sqrt{a + ia \sinh(e + fx)}}{f^2} - \frac{96 \sqrt{a + ia \sinh(e + fx)}}{f^4} + \frac{48x \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)}}{f^3} + \frac{2x^3 \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)}}{f^3}$$

```
[Out] (-96*Sqrt[a + I*a*Sinh[e + f*x]])/f^4 - (12*x^2*Sqrt[a + I*a*Sinh[e + f*x]])/f^2 + (48*x*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/f^3 + (2*x^3*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/f^3
```

**Rubi [A]** time = 0.172217, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3319, 3296, 2638}

$$-\frac{12x^2 \sqrt{a + ia \sinh(e + fx)}}{f^2} - \frac{96 \sqrt{a + ia \sinh(e + fx)}}{f^4} + \frac{48x \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)}}{f^3} + \frac{2x^3 \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)}}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] (-96*Sqrt[a + I*a*Sinh[e + f*x]])/f^4 - (12*x^2*Sqrt[a + I*a*Sinh[e + f*x]])/f^2 + (48*x*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/f^3 + (2*x^3*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/f^3
```

#### Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Dist[((2*a)^(IntPart[n])*(a + b*Sinh[e + f*x])^(FracPart[n]))/Sin[e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sinh[e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

#### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cosh[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cosh[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```



**Rule 2638**

`Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

**Rubi steps**

$$\begin{aligned}
 \int x^3 \sqrt{a + ia \sinh(e + fx)} dx &= \left( \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right) \int x^3 \sinh \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) dx \\
 &= \frac{2x^3 \sqrt{a + ia \sinh(e + fx)} \tanh \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right)}{f} - \frac{\left( 6 \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right)}{f} \\
 &= -\frac{12x^2 \sqrt{a + ia \sinh(e + fx)}}{f^2} + \frac{2x^3 \sqrt{a + ia \sinh(e + fx)} \tanh \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right)}{f} - \frac{\left( 24i \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right)}{f} \\
 &= -\frac{12x^2 \sqrt{a + ia \sinh(e + fx)}}{f^2} + \frac{48x \sqrt{a + ia \sinh(e + fx)} \tanh \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right)}{f^3} + \frac{2x^3 \sqrt{a + ia \sinh(e + fx)}}{f^3} \\
 &= -\frac{96 \sqrt{a + ia \sinh(e + fx)}}{f^4} - \frac{12x^2 \sqrt{a + ia \sinh(e + fx)}}{f^2} + \frac{48x \sqrt{a + ia \sinh(e + fx)} \tanh \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right)}{f^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.300401, size = 125, normalized size = 0.92

$$\frac{2\sqrt{a + ia \sinh(e + fx)} \left( (f^3 x^3 - 6if^2 x^2 + 24fx - 48i) \sinh \left( \frac{1}{2}(e + fx) \right) + i(f^3 x^3 + 6if^2 x^2 + 24fx + 48i) \cosh \left( \frac{1}{2}(e + fx) \right) \right)}{f^4 \left( \cosh \left( \frac{1}{2}(e + fx) \right) + i \sinh \left( \frac{1}{2}(e + fx) \right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Sqrt[a + I\*a\*Sinh[e + f\*x]],x]

[Out] (2\*(I\*(48\*I + 24\*f\*x + (6\*I)\*f^2\*x^2 + f^3\*x^3)\*Cosh[(e + f\*x)/2] + (-48\*I + 24\*f\*x - (6\*I)\*f^2\*x^2 + f^3\*x^3)\*Sinh[(e + f\*x)/2])\*Sqrt[a + I\*a\*Sinh[e + f\*x]])/(f^4\*(Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2]))

**Maple [A]** time = 0.062, size = 151, normalized size = 1.1

$$\frac{i\sqrt{2} \left( ix^3 f^3 + f^3 x^3 e^{fx+e} + 6ix^2 f^2 - 6f^2 x^2 e^{fx+e} + 24ixf + 24fx e^{fx+e} + 48i - 48e^{fx+e} \right) (e^{fx+e} - i)}{(ie^{2fx+2e} - i + 2e^{fx+e}) f^4} \sqrt{a (ie^{2fx+2e} - i + 2e^{fx+e})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+I*a*sinh(f*x+e))^(1/2),x)
```

```
[Out] I*2^(1/2)*(a*(I*exp(2*f*x+2*e)-I+2*exp(f*x+e))*exp(-f*x-e))^(1/2)/(I*exp(2*f*x+2*e)-I+2*exp(f*x+e))*(I*x^3*f^3+f^3*x^3*exp(f*x+e)+6*I*x^2*f^2-6*f^2*x^2*exp(f*x+e)+24*I*x*f+24*f*x*exp(f*x+e)+48*I-48*exp(f*x+e))*(exp(f*x+e)-I)/f^4
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ia \sinh(fx + e) + ax^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)*x^3, x)
```

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \sqrt{a(i \sinh(e + fx) + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+I*a*sinh(f*x+e))**(1/2),x)
```

[Out] `Integral(x**3*sqrt(a*(I*sinh(e + f*x) + 1)), x)`

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ia \sinh(fx + e) + ax^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(I*a*sinh(f*x + e) + a)*x^3, x)`

### 3.120 $\int x^2 \sqrt{a + ia \sinh(e + fx)} dx$

**Optimal.** Leaf size=111

$$-\frac{8x\sqrt{a + ia \sinh(e + fx)}}{f^2} + \frac{16 \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)\sqrt{a + ia \sinh(e + fx)}}{f^3} + \frac{2x^2 \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)\sqrt{a + ia \sinh(e + fx)}}{f}$$

[Out]  $(-8*x*\text{Sqrt}[a + I*a*\text{Sinh}[e + f*x]])/f^2 + (16*\text{Sqrt}[a + I*a*\text{Sinh}[e + f*x]]*\text{Tanh}[e/2 + (I/4)*\text{Pi} + (f*x)/2])/f^3 + (2*x^2*\text{Sqrt}[a + I*a*\text{Sinh}[e + f*x]]*\text{Tanh}[e/2 + (I/4)*\text{Pi} + (f*x)/2])/f$

**Rubi [A]** time = 0.141345, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3319, 3296, 2638}

$$-\frac{8x\sqrt{a + ia \sinh(e + fx)}}{f^2} + \frac{16 \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)\sqrt{a + ia \sinh(e + fx)}}{f^3} + \frac{2x^2 \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)\sqrt{a + ia \sinh(e + fx)}}{f}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*\text{Sqrt}[a + I*a*\text{Sinh}[e + f*x]],x]$

[Out]  $(-8*x*\text{Sqrt}[a + I*a*\text{Sinh}[e + f*x]])/f^2 + (16*\text{Sqrt}[a + I*a*\text{Sinh}[e + f*x]]*\text{Tanh}[e/2 + (I/4)*\text{Pi} + (f*x)/2])/f^3 + (2*x^2*\text{Sqrt}[a + I*a*\text{Sinh}[e + f*x]]*\text{Tanh}[e/2 + (I/4)*\text{Pi} + (f*x)/2])/f$

#### Rule 3319

$\text{Int}[\left((c_.) + (d_.)*(x_.)\right)^{(m_.)}*\left((a_.) + (b_.)*\sin\left[(e_.) + (f_.)*(x_.)\right]\right)^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}\left[\left((2*a)^{\text{IntPart}[n]}*(a + b*\sin[e + f*x])^{\text{FracPart}[n]}\right)/\sin\left[e/2 + (a*\text{Pi})/(4*b) + (f*x)/2\right]^{(2*\text{FracPart}[n])}, \text{Int}[(c + d*x)^m*\sin[e/2 + (a*\text{Pi})/(4*b) + (f*x)/2]^{(2*n)}, x], x\right] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[n + 1/2] \ \&\& \ (\text{GtQ}[n, 0] \ || \ \text{IGtQ}[m, 0])$

#### Rule 3296

$\text{Int}[\left((c_.) + (d_.)*(x_.)\right)^{(m_.)}*\sin\left[(e_.) + (f_.)*(x_.)\right], x\_Symbol] \rightarrow -\text{Simp}\left[\left((c + d*x)^m*\cos[e + f*x]\right)/f, x\right] + \text{Dist}\left[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\cos[e + f*x], x], x\right] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \ \text{GtQ}[m, 0]$

Rule 2638

`Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned} \int x^2 \sqrt{a + ia \sinh(e + fx)} dx &= \left( \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right) \int x^2 \sinh \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) dx \\ &= \frac{2x^2 \sqrt{a + ia \sinh(e + fx)} \tanh \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right)}{f} - \frac{\left( 4 \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right)}{f} \\ &= -\frac{8x \sqrt{a + ia \sinh(e + fx)}}{f^2} + \frac{2x^2 \sqrt{a + ia \sinh(e + fx)} \tanh \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right)}{f} - \frac{\left( 8 \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right)}{f} \\ &= -\frac{8x \sqrt{a + ia \sinh(e + fx)}}{f^2} + \frac{16 \sqrt{a + ia \sinh(e + fx)} \tanh \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right)}{f^3} + \frac{2x^2 \sqrt{a + ia \sinh(e + fx)}}{f} \end{aligned}$$

**Mathematica [A]** time = 0.223754, size = 105, normalized size = 0.95

$$\frac{2\sqrt{a + ia \sinh(e + fx)} \left( (f^2 x^2 - 4ifx + 8) \sinh \left( \frac{1}{2}(e + fx) \right) + i(f^2 x^2 + 4ifx + 8) \cosh \left( \frac{1}{2}(e + fx) \right) \right)}{f^3 \left( \cosh \left( \frac{1}{2}(e + fx) \right) + i \sinh \left( \frac{1}{2}(e + fx) \right) \right)}$$

Antiderivative was successfully verified.

[In] `Integrate[x^2*Sqrt[a + I*a*Sinh[e + f*x]],x]`

[Out] `(2*(I*(8 + (4*I)*f*x + f^2*x^2)*Cosh[(e + f*x)/2] + (8 - (4*I)*f*x + f^2*x^2)*Sinh[(e + f*x)/2])*Sqrt[a + I*a*Sinh[e + f*x]]/(f^3*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))`

**Maple [A]** time = 0.06, size = 128, normalized size = 1.2

$$\frac{i\sqrt{2} \left( ix^2 f^2 + f^2 x^2 e^{fx+e} + 4ixf - 4fxe^{fx+e} + 8i + 8e^{fx+e} \right) \left( e^{fx+e} - i \right)}{\left( ie^{2fx+2e} - i + 2e^{fx+e} \right) f^3} \sqrt{a \left( ie^{2fx+2e} - i + 2e^{fx+e} \right) e^{-fx-e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+I*a*sinh(f*x+e))^(1/2),x)
```

```
[Out] I*2^(1/2)*(a*(I*exp(2*f*x+2*e)-I+2*exp(f*x+e))*exp(-f*x-e))^(1/2)/(I*exp(2*f*x+2*e)-I+2*exp(f*x+e))*(I*x^2*f^2+f^2*x^2*exp(f*x+e)+4*I*x*f-4*f*x*exp(f*x+e)+8*I+8*exp(f*x+e))*(exp(f*x+e)-I)/f^3
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ia \sinh(fx + e) + ax^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)*x^2, x)
```

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{a(i \sinh(e + fx) + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*(I*sinh(e + f*x) + 1)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ia \sinh(fx + e) + ax^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(a+I\*a\*sinh(f\*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(I\*a\*sinh(f\*x + e) + a)\*x^2, x)

### 3.121 $\int x \sqrt{a + ia \sinh(e + fx)} dx$

**Optimal.** Leaf size=66

$$\frac{2x \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)}}{f} - \frac{4\sqrt{a + ia \sinh(e + fx)}}{f^2}$$

[Out]  $(-4*\text{Sqrt}[a + I*a*\text{Sinh}[e + f*x]])/f^2 + (2*x*\text{Sqrt}[a + I*a*\text{Sinh}[e + f*x]]*\text{Tanh}[e/2 + (I/4)*\text{Pi} + (f*x)/2])/f$

**Rubi [A]** time = 0.0753615, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {3319, 3296, 2638}

$$\frac{2x \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)}}{f} - \frac{4\sqrt{a + ia \sinh(e + fx)}}{f^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*\text{Sqrt}[a + I*a*\text{Sinh}[e + f*x]], x]$

[Out]  $(-4*\text{Sqrt}[a + I*a*\text{Sinh}[e + f*x]])/f^2 + (2*x*\text{Sqrt}[a + I*a*\text{Sinh}[e + f*x]]*\text{Tanh}[e/2 + (I/4)*\text{Pi} + (f*x)/2])/f$

#### Rule 3319

$\text{Int}[\text{((c_.) + (d_.)*(x_.))}^{(m_.)} * \text{((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])}^{(n_.)}, x\_Symbol] \text{ :> Dist}[\text{((2*a)}^{IntPart[n]} * (a + b*\text{Sin}[e + f*x])^{FracPart[n]}) / \text{Sin}[e/2 + (a*\text{Pi})/(4*b) + (f*x)/2]^{(2*FracPart[n])}, \text{Int}[(c + d*x)^m * \text{Sin}[e/2 + (a*\text{Pi})/(4*b) + (f*x)/2]^{(2*n)}, x], x] \text{ /; FreeQ}\{a, b, c, d, e, f, m\}, x \text{ \&\& E} \text{qQ}[a^2 - b^2, 0] \text{ \&\& IntegerQ}[n + 1/2] \text{ \&\& (GtQ}[n, 0] \text{ || IGtQ}[m, 0]$

#### Rule 3296

$\text{Int}[\text{((c_.) + (d_.)*(x_.))}^{(m_.)} * \sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \text{ :> -Simp}[(c + d*x)^m * \text{Cos}[e + f*x] / f, x] + \text{Dist}[(d*m) / f, \text{Int}[(c + d*x)^{(m-1)} * \text{Cos}[e + f*x], x], x] \text{ /; FreeQ}\{c, d, e, f\}, x \text{ \&\& GtQ}[m, 0]$

#### Rule 2638



Int[sin[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned} \int x\sqrt{a + ia \sinh(e + fx)} dx &= \left( \operatorname{csch}\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) \sqrt{a + ia \sinh(e + fx)} \right) \int x \sinh\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) dx \\ &= \frac{2x\sqrt{a + ia \sinh(e + fx)} \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{f} - \frac{\left(2\operatorname{csch}\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) \sqrt{a + ia \sinh(e + fx)}\right)}{f} \\ &= -\frac{4\sqrt{a + ia \sinh(e + fx)}}{f^2} + \frac{2x\sqrt{a + ia \sinh(e + fx)} \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{f} \end{aligned}$$

**Mathematica [A]** time = 0.16385, size = 87, normalized size = 1.32

$$\frac{2\sqrt{a + ia \sinh(e + fx)} \left( (fx - 2i) \sinh\left(\frac{1}{2}(e + fx)\right) + (-2 + ifx) \cosh\left(\frac{1}{2}(e + fx)\right) \right)}{f^2 \left( \cosh\left(\frac{1}{2}(e + fx)\right) + i \sinh\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Sqrt[a + I\*a\*Sinh[e + f\*x]], x]

[Out] (2\*((-2 + I\*f\*x)\*Cosh[(e + f\*x)/2] + (-2\*I + f\*x)\*Sinh[(e + f\*x)/2])\*Sqrt[a + I\*a\*Sinh[e + f\*x]]/(f^2\*(Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2]))

**Maple [A]** time = 0.057, size = 105, normalized size = 1.6

$$\frac{i\sqrt{2} \left( ixf + fx e^{fx+e} + 2i - 2 e^{fx+e} \right) \left( e^{fx+e} - i \right)}{\left( ie^{2fx+2e} - i + 2 e^{fx+e} \right) f^2} \sqrt{a \left( ie^{2fx+2e} - i + 2 e^{fx+e} \right) e^{-fx-e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+I\*a\*sinh(f\*x+e))^(1/2), x)

[Out] I\*2^(1/2)\*(a\*(I\*exp(2\*f\*x+2\*e)-I+2\*exp(f\*x+e))\*exp(-f\*x-e))^(1/2)/(I\*exp(2\*f\*x+2\*e)-I+2\*exp(f\*x+e))\*(I\*x\*f+f\*x\*exp(f\*x+e)+2\*I-2\*exp(f\*x+e))\*(exp(f\*x+e

) - I) / f^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ia \sinh(fx + e) + ax} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+I\*a\*sinh(f\*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(I\*a\*sinh(f\*x + e) + a)\*x, x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+I\*a\*sinh(f\*x+e))^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sqrt{a(i \sinh(e + fx) + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+I\*a\*sinh(f\*x+e))\*\*(1/2),x)

[Out] Integral(x\*sqrt(a\*(I\*sinh(e + f\*x) + 1)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ia \sinh(fx + e) + ax} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)*x, x)
```

$$3.122 \quad \int \frac{\sqrt{a+ia \sinh(e+fx)}}{x} dx$$

**Optimal.** Leaf size=125

$$i \sinh\left(\frac{1}{4}(2e - i\pi)\right) \text{Chi}\left(\frac{fx}{2}\right) \text{sech}\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)} + i \cosh\left(\frac{1}{4}(2e - i\pi)\right) \text{Shi}\left(\frac{fx}{2}\right) \text{sech}\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)$$

[Out] I\*CoshIntegral[(f\*x)/2]\*Sech[e/2 + (I/4)\*Pi + (f\*x)/2]\*Sinh[(2\*e - I\*Pi)/4]  
\*Sqrt[a + I\*a\*Sinh[e + f\*x]] + I\*Cosh[(2\*e - I\*Pi)/4]\*Sech[e/2 + (I/4)\*Pi +  
(f\*x)/2]\*Sqrt[a + I\*a\*Sinh[e + f\*x]]\*SinhIntegral[(f\*x)/2]

**Rubi [A]** time = 0.145539, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$ , Rules used = {3319, 3303, 3298, 3301}

$$i \sinh\left(\frac{1}{4}(2e - i\pi)\right) \text{Chi}\left(\frac{fx}{2}\right) \text{sech}\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)} + i \cosh\left(\frac{1}{4}(2e - i\pi)\right) \text{Shi}\left(\frac{fx}{2}\right) \text{sech}\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + I\*a\*Sinh[e + f\*x]]/x,x]

[Out] I\*CoshIntegral[(f\*x)/2]\*Sech[e/2 + (I/4)\*Pi + (f\*x)/2]\*Sinh[(2\*e - I\*Pi)/4]  
\*Sqrt[a + I\*a\*Sinh[e + f\*x]] + I\*Cosh[(2\*e - I\*Pi)/4]\*Sech[e/2 + (I/4)\*Pi +  
(f\*x)/2]\*Sqrt[a + I\*a\*Sinh[e + f\*x]]\*SinhIntegral[(f\*x)/2]

### Rule 3319

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_),  
x\_Symbol] :> Dist[((2\*a)^(IntPart[n])\*(a + b\*Sinh[e + f\*x])^(FracPart[n]))/Sin[  
e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*FracPart[n]), Int[(c + d\*x)^m\*Sinh[e/2 + (a  
\*Pi)/(4\*b) + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E  
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*  
e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f  
) /d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] &&  
NeQ[d\*e - c\*f, 0]

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + ia \sinh(e + fx)}}{x} dx &= \left( \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right) \int \frac{\sinh \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right)}{x} dx \\ &= \left( \cosh \left( \frac{1}{4}(2e - i\pi) \right) \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right) \int \frac{\sinh \left( \frac{fx}{2} \right)}{x} dx + \left( \operatorname{csch} \left( \frac{1}{4}(2e - i\pi) \right) \sqrt{a + ia \sinh(e + fx)} \right) \int \frac{\cosh \left( \frac{fx}{2} \right)}{x} dx \\ &= i \operatorname{Chi} \left( \frac{fx}{2} \right) \operatorname{sech} \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right) \sinh \left( \frac{1}{4}(2e - i\pi) \right) \sqrt{a + ia \sinh(e + fx)} + i \cosh \left( \frac{1}{4}(2e - i\pi) \right) \sqrt{a + ia \sinh(e + fx)} \operatorname{Shi} \left( \frac{fx}{2} \right) \end{aligned}$$

**Mathematica [A]** time = 0.158604, size = 96, normalized size = 0.77

$$\frac{\sqrt{a + ia \sinh(e + fx)} \left( \operatorname{Chi} \left( \frac{fx}{2} \right) \left( \cosh \left( \frac{e}{2} \right) + i \sinh \left( \frac{e}{2} \right) \right) + \left( \sinh \left( \frac{e}{2} \right) + i \cosh \left( \frac{e}{2} \right) \right) \operatorname{Shi} \left( \frac{fx}{2} \right) \right)}{\cosh \left( \frac{1}{2}(e + fx) \right) + i \sinh \left( \frac{1}{2}(e + fx) \right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Sinh[e + f*x]]/x,x]
```

```
[Out] (Sqrt[a + I*a*Sinh[e + f*x]]*(CoshIntegral[(f*x)/2]*(Cosh[e/2] + I*Sinh[e/2]) + (I*Cosh[e/2] + Sinh[e/2])*SinhIntegral[(f*x)/2]))/(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2])
```

**Maple [F]** time = 0.305, size = 0, normalized size = 0.

$$\int \frac{1}{x} \sqrt{a + ia \sinh(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*sinh(f*x+e))^(1/2)/x,x)`

[Out] `int((a+I*a*sinh(f*x+e))^(1/2)/x,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ia \sinh(fx + e) + a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*sinh(f*x+e))^(1/2)/x,x, algorithm="maxima")`

[Out] `integrate(sqrt(I*a*sinh(f*x + e) + a)/x, x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*sinh(f*x+e))^(1/2)/x,x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a(i \sinh(e + fx) + 1)}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*sinh(f*x+e))**(1/2)/x,x)`

[Out] Integral(sqrt(a\*(I\*sinh(e + f\*x) + 1))/x, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ia \sinh(fx + e) + a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))^(1/2)/x,x, algorithm="giac")

[Out] integrate(sqrt(I\*a\*sinh(f\*x + e) + a)/x, x)

### 3.123 $\int \frac{\sqrt{a+ia \sinh(e+fx)}}{x^2} dx$

**Optimal.** Leaf size=149

$$\frac{1}{2}f \sinh\left(\frac{1}{4}(2e + i\pi)\right) \text{Chi}\left(\frac{fx}{2}\right) \text{sech}\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)} + \frac{1}{2}f \cosh\left(\frac{1}{4}(2e + i\pi)\right) \text{Shi}\left(\frac{fx}{2}\right) \text{sech}\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)}$$

[Out] -(Sqrt[a + I\*a\*Sinh[e + f\*x]]/x) + (f\*CoshIntegral[(f\*x)/2]\*Sech[e/2 + (I/4)\*Pi + (f\*x)/2]\*Sinh[(2\*e + I\*Pi)/4]\*Sqrt[a + I\*a\*Sinh[e + f\*x]])/2 + (f\*CoshIntegral[(2\*e + I\*Pi)/4]\*Sech[e/2 + (I/4)\*Pi + (f\*x)/2]\*Sqrt[a + I\*a\*Sinh[e + f\*x]])\*SinhIntegral[(f\*x)/2])/2

**Rubi [A]** time = 0.175202, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {3319, 3297, 3303, 3298, 3301}

$$\frac{1}{2}f \sinh\left(\frac{1}{4}(2e + i\pi)\right) \text{Chi}\left(\frac{fx}{2}\right) \text{sech}\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)} + \frac{1}{2}f \cosh\left(\frac{1}{4}(2e + i\pi)\right) \text{Shi}\left(\frac{fx}{2}\right) \text{sech}\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + I\*a\*Sinh[e + f\*x]]/x^2,x]

[Out] -(Sqrt[a + I\*a\*Sinh[e + f\*x]]/x) + (f\*CoshIntegral[(f\*x)/2]\*Sech[e/2 + (I/4)\*Pi + (f\*x)/2]\*Sinh[(2\*e + I\*Pi)/4]\*Sqrt[a + I\*a\*Sinh[e + f\*x]])/2 + (f\*CoshIntegral[(2\*e + I\*Pi)/4]\*Sech[e/2 + (I/4)\*Pi + (f\*x)/2]\*Sqrt[a + I\*a\*Sinh[e + f\*x]])\*SinhIntegral[(f\*x)/2])/2

#### Rule 3319

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Dist[((2\*a)^IntPart[n]\*(a + b\*Sinh[e + f\*x])^FracPart[n])/Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*FracPart[n]), Int[(c + d\*x)^m\*Sinh[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])

#### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Simp[(c + d\*x)^(m + 1)\*Sin[e + f\*x]/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]



]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + ia \sinh(e + fx)}}{x^2} dx &= \left( \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right) \int \frac{\sinh \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right)}{x^2} dx \\
&= -\frac{\sqrt{a + ia \sinh(e + fx)}}{x} + \frac{1}{2} \left( f \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right) \int \frac{\cosh \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right)}{x} dx \\
&= -\frac{\sqrt{a + ia \sinh(e + fx)}}{x} - \frac{1}{2} \left( if \cosh \left( \frac{1}{4}(2e + i\pi) \right) \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right) \int \frac{1}{x} dx \\
&= -\frac{\sqrt{a + ia \sinh(e + fx)}}{x} + \frac{1}{2} f \operatorname{Chi} \left( \frac{fx}{2} \right) \operatorname{sech} \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right) \sinh \left( \frac{1}{4}(2e + i\pi) \right) \sqrt{a + ia \sinh(e + fx)}
\end{aligned}$$

**Mathematica [A]** time = 0.25353, size = 133, normalized size = 0.89

$$\frac{\sqrt{a + ia \sinh(e + fx)} \left( fx \operatorname{Chi} \left( \frac{fx}{2} \right) \left( \sinh \left( \frac{e}{2} \right) + i \cosh \left( \frac{e}{2} \right) \right) + fx \left( \cosh \left( \frac{e}{2} \right) + i \sinh \left( \frac{e}{2} \right) \right) \operatorname{Shi} \left( \frac{fx}{2} \right) - 2 \left( \cosh \left( \frac{1}{2}(e + fx) \right) \right)}{2x \left( \cosh \left( \frac{1}{2}(e + fx) \right) + i \sinh \left( \frac{1}{2}(e + fx) \right) \right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + I*a*Sinh[e + f*x]]/x^2,x]
```

```
[Out] (Sqrt[a + I*a*Sinh[e + f*x]]*(f*x*CoshIntegral[(f*x)/2]*(I*Cosh[e/2] + Sinh[e/2]) - 2*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]) + f*x*(Cosh[e/2] + I*Sinh[e/2])*SinhIntegral[(f*x)/2]))/(2*x*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))
```

**Maple [F]** time = 0.062, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \sqrt{a + ia \sinh(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(f*x+e))^(1/2)/x^2,x)
```

```
[Out] int((a+I*a*sinh(f*x+e))^(1/2)/x^2,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ia \sinh(fx + e) + a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(I*a*sinh(f*x + e) + a)/x^2, x)
```

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(1/2)/x^2,x, algorithm="fricas")
```

[Out] Exception raised: UnboundLocalError

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a(i \sinh(e + fx) + 1)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))\*\*(1/2)/x\*\*2,x)

[Out] Integral(sqrt(a\*(I\*sinh(e + f\*x) + 1))/x\*\*2, x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ia \sinh(fx + e) + a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))^(1/2)/x^2,x, algorithm="giac")

[Out] integrate(sqrt(I\*a\*sinh(f\*x + e) + a)/x^2, x)

$$3.124 \quad \int \frac{\sqrt{a+ia \sinh(e+fx)}}{x^3} dx$$

**Optimal.** Leaf size=204

$$\frac{1}{8}if^2 \sinh\left(\frac{1}{4}(2e - i\pi)\right) \text{Chi}\left(\frac{fx}{2}\right) \text{sech}\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)} + \frac{1}{8}if^2 \cosh\left(\frac{1}{4}(2e - i\pi)\right) \text{Shi}\left(\frac{fx}{2}\right) \text{sech}\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)$$

```
[Out] -Sqrt[a + I*a*Sinh[e + f*x]]/(2*x^2) + (I/8)*f^2*CoshIntegral[(f*x)/2]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[(2*e - I*Pi)/4]*Sqrt[a + I*a*Sinh[e + f*x]] + (I/8)*f^2*Cosh[(2*e - I*Pi)/4]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*SinhIntegral[(f*x)/2] - (f*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(4*x)
```

---

**Rubi [A]** time = 0.198161, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {3319, 3297, 3303, 3298, 3301}

$$\frac{1}{8}if^2 \sinh\left(\frac{1}{4}(2e - i\pi)\right) \text{Chi}\left(\frac{fx}{2}\right) \text{sech}\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)} + \frac{1}{8}if^2 \cosh\left(\frac{1}{4}(2e - i\pi)\right) \text{Shi}\left(\frac{fx}{2}\right) \text{sech}\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + I*a*Sinh[e + f*x]]/x^3,x]
```

```
[Out] -Sqrt[a + I*a*Sinh[e + f*x]]/(2*x^2) + (I/8)*f^2*CoshIntegral[(f*x)/2]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[(2*e - I*Pi)/4]*Sqrt[a + I*a*Sinh[e + f*x]] + (I/8)*f^2*Cosh[(2*e - I*Pi)/4]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*SinhIntegral[(f*x)/2] - (f*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(4*x)
```

### Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sinh[e + f*x])^FracPart[n])/Sin[e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sinh[e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + ia \sinh(e + fx)}}{x^3} dx &= \left( \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right) \int \frac{\sinh \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right)}{x^3} dx \\
&= -\frac{\sqrt{a + ia \sinh(e + fx)}}{2x^2} + \frac{1}{4} \left( f \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right) \int \frac{\cosh \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right)}{x^2} dx \\
&= -\frac{\sqrt{a + ia \sinh(e + fx)}}{2x^2} - \frac{f \sqrt{a + ia \sinh(e + fx)} \tanh \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right)}{4x} - \frac{1}{8} \left( if^2 \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right) \int \frac{1}{x} dx \\
&= -\frac{\sqrt{a + ia \sinh(e + fx)}}{2x^2} - \frac{f \sqrt{a + ia \sinh(e + fx)} \tanh \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right)}{4x} + \frac{1}{8} \left( f^2 \cosh \left( \frac{1}{4}(2e - i\pi) \right) \sqrt{a + ia \sinh(e + fx)} \right) \ln|x| \\
&= -\frac{\sqrt{a + ia \sinh(e + fx)}}{2x^2} + \frac{1}{8} if^2 \operatorname{Chi} \left( \frac{fx}{2} \right) \operatorname{sech} \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right) \sinh \left( \frac{1}{4}(2e - i\pi) \right) \sqrt{a + ia \sinh(e + fx)} + \frac{1}{8} \left( f^2 \cosh \left( \frac{1}{4}(2e - i\pi) \right) \sqrt{a + ia \sinh(e + fx)} \right) \ln|x|
\end{aligned}$$

**Mathematica [A]** time = 0.339556, size = 170, normalized size = 0.83

$$\frac{\sqrt{a + ia \sinh(e + fx)} \left( f^2 x^2 \operatorname{Chi} \left( \frac{fx}{2} \right) \left( \cosh \left( \frac{e}{2} \right) + i \sinh \left( \frac{e}{2} \right) \right) + f^2 x^2 \left( \sinh \left( \frac{e}{2} \right) + i \cosh \left( \frac{e}{2} \right) \right) \operatorname{Shi} \left( \frac{fx}{2} \right) - 2fx \sinh \left( \frac{1}{2}(e + fx) \right) \right)}{8x^2 \left( \cosh \left( \frac{1}{2}(e + fx) \right) + i \sinh \left( \frac{1}{2}(e + fx) \right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + I\*a\*Sinh[e + f\*x]]/x^3,x]

[Out] (Sqrt[a + I\*a\*Sinh[e + f\*x]]\*(-4\*Cosh[(e + f\*x)/2] - (2\*I)\*f\*x\*Cosh[(e + f\*x)/2] + f^2\*x^2\*CoshIntegral[(f\*x)/2]\*(Cosh[e/2] + I\*Sinh[e/2]) - (4\*I)\*Sinh[(e + f\*x)/2] - 2\*f\*x\*Sinh[(e + f\*x)/2] + f^2\*x^2\*(I\*Cosh[e/2] + Sinh[e/2])\*SinhIntegral[(f\*x)/2]))/(8\*x^2\*(Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2]))

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} \sqrt{a + ia \sinh(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*sinh(f\*x+e))^(1/2)/x^3,x)

[Out] int((a+I\*a\*sinh(f\*x+e))^(1/2)/x^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ia \sinh(fx + e) + a}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate(sqrt(I\*a\*sinh(f\*x + e) + a)/x^3, x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))^(1/2)/x^3,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a(i \sinh(e + fx) + 1)}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))\*\*(1/2)/x\*\*3,x)

[Out] Integral(sqrt(a\*(I\*sinh(e + f\*x) + 1))/x\*\*3, x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ia \sinh(fx + e) + a}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))^(1/2)/x^3,x, algorithm="giac")

[Out] integrate(sqrt(I\*a\*sinh(f\*x + e) + a)/x^3, x)

### 3.125 $\int x^3(a + ia \sinh(e + fx))^{3/2} dx$

**Optimal.** Leaf size=377

$$\frac{16ax^2\sqrt{a + ia \sinh(e + fx)}}{f^2} - \frac{8ax^2 \cosh^2\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)\sqrt{a + ia \sinh(e + fx)}}{3f^2} - \frac{1280a\sqrt{a + ia \sinh(e + fx)}}{9f^4} + \frac{640ax \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{3f}$$

```
[Out] (-1280*a*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^4) - (16*a*x^2*Sqrt[a + I*a*Sinh[e + f*x]])/f^2 - (64*a*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]])/(27*f^4) - (8*a*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f^2) + (32*a*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^3) + (4*a*x^3*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f) + (640*a*x*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(9*f^3) + (8*a*x^3*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*f)
```

---

**Rubi [A]** time = 0.351429, antiderivative size = 377, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {3319, 3311, 3296, 2638, 3310}

$$\frac{16ax^2\sqrt{a + ia \sinh(e + fx)}}{f^2} - \frac{8ax^2 \cosh^2\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)\sqrt{a + ia \sinh(e + fx)}}{3f^2} - \frac{1280a\sqrt{a + ia \sinh(e + fx)}}{9f^4} + \frac{640ax \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] (-1280*a*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^4) - (16*a*x^2*Sqrt[a + I*a*Sinh[e + f*x]])/f^2 - (64*a*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]])/(27*f^4) - (8*a*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f^2) + (32*a*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^3) + (4*a*x^3*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f) + (640*a*x*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(9*f^3) + (8*a*x^3*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*f)
```

Rule 3319



```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] := Dist[((2*a)^IntPart[n]*(a + b*SIN[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*SIN[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
 := Simp[(d*m*(c + d*x)^(m - 1)*(b*SIN[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*SIN[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*SIN[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*cos[e + f*x]*(b*SIN[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
Simp[(d*(b*SIN[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*SIN[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*cos[e + f*x]*(b
*SIN[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

### Rubi steps

$$\begin{aligned}
\int x^3(a + ia \sinh(e + fx))^{3/2} dx &= -\left(\left(2a \operatorname{csch}\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) \sqrt{a + ia \sinh(e + fx)}\right) \int x^3 \sinh^3\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) dx\right) \\
&= -\frac{8ax^2 \cosh^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sqrt{a + ia \sinh(e + fx)}}{3f^2} + \frac{4ax^3 \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sinh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{3f} \\
&= -\frac{64a \cosh^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sqrt{a + ia \sinh(e + fx)}}{27f^4} - \frac{8ax^2 \cosh^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sqrt{a + ia \sinh(e + fx)}}{3f^2} \\
&= -\frac{16ax^2 \sqrt{a + ia \sinh(e + fx)}}{f^2} - \frac{64a \cosh^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sqrt{a + ia \sinh(e + fx)}}{27f^4} - \frac{8ax^2 \cosh^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sqrt{a + ia \sinh(e + fx)}}{3f^2} \\
&= -\frac{128a \sqrt{a + ia \sinh(e + fx)}}{9f^4} - \frac{16ax^2 \sqrt{a + ia \sinh(e + fx)}}{f^2} - \frac{64a \cosh^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sqrt{a + ia \sinh(e + fx)}}{27f^4} \\
&= -\frac{1280a \sqrt{a + ia \sinh(e + fx)}}{9f^4} - \frac{16ax^2 \sqrt{a + ia \sinh(e + fx)}}{f^2} - \frac{64a \cosh^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sqrt{a + ia \sinh(e + fx)}}{27f^4}
\end{aligned}$$

**Mathematica [A]** time = 1.40348, size = 269, normalized size = 0.71

$$\frac{a(\sinh(e + fx) - i)\sqrt{a + ia \sinh(e + fx)}\left(-81if^3x^3 \sinh\left(\frac{1}{2}(e + fx)\right) + 9if^3x^3 \sinh\left(\frac{3}{2}(e + fx)\right) - 486f^2x^2 \sinh\left(\frac{1}{2}(e + fx)\right)\right)}{27f^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + I\*a\*Sinh[e + f\*x])^(3/2),x]

[Out] -(a\*(-I + Sinh[e + f\*x])\*Sqrt[a + I\*a\*Sinh[e + f\*x]]\*(81\*(48\*I + 24\*f\*x + (6\*I)\*f^2\*x^2 + f^3\*x^3)\*Cosh[(e + f\*x)/2] + (-16\*I + 24\*f\*x - (18\*I)\*f^2\*x^2 + 9\*f^3\*x^3)\*Cosh[(3\*(e + f\*x))/2] - 3888\*Sinh[(e + f\*x)/2] - (1944\*I)\*f\*x\*Sinh[(e + f\*x)/2] - 486\*f^2\*x^2\*Sinh[(e + f\*x)/2] - (81\*I)\*f^3\*x^3\*Sinh[(e + f\*x)/2] - 16\*Sinh[(3\*(e + f\*x))/2] + (24\*I)\*f\*x\*Sinh[(3\*(e + f\*x))/2] - 18\*f^2\*x^2\*Sinh[(3\*(e + f\*x))/2] + (9\*I)\*f^3\*x^3\*Sinh[(3\*(e + f\*x))/2]))/(27\*f^4\*(Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2])^3)

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int x^3 (a + ia \sinh(fx + e))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a+I*a*sinh(f*x+e))^(3/2),x)`

[Out] `int(x^3*(a+I*a*sinh(f*x+e))^(3/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (i a \sinh(fx + e) + a)^{\frac{3}{2}} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")`

[Out] `integrate((I*a*sinh(f*x + e) + a)^(3/2)*x^3, x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+I*a*sinh(f*x+e))**(3/2),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (i a \sinh(fx + e) + a)^{\frac{3}{2}} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)*x^3, x)
```

### 3.126 $\int x^2(a + ia \sinh(e + fx))^{3/2} dx$

**Optimal.** Leaf size=303

$$-\frac{32ax\sqrt{a + ia \sinh(e + fx)}}{3f^2} + \frac{32a \sinh^2\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)}}{27f^3} + \frac{224a \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{27f^3}$$

```
[Out] (-32*a*x*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f^2) - (16*a*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^2) + (4*a*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f) + (224*a*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(9*f^3) + (8*a*x^2*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*f) + (32*a*Sinh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(27*f^3)
```

**Rubi [A]** time = 0.25098, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {3319, 3311, 3296, 2638, 2633}

$$-\frac{32ax\sqrt{a + ia \sinh(e + fx)}}{3f^2} + \frac{32a \sinh^2\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)}}{27f^3} + \frac{224a \tanh\left(\frac{e}{2} + \frac{fx}{2}\right)}{27f^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] (-32*a*x*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f^2) - (16*a*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^2) + (4*a*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f) + (224*a*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(9*f^3) + (8*a*x^2*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*f) + (32*a*Sinh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(27*f^3)
```

#### Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[((2*a)^(IntPart[n])*(a + b*Sinh[e + f*x])^(FracPart[n])/Sin[e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sinh[e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
```

$qQ[a^2 - b^2, 0] \&\& \text{IntegerQ}[n + 1/2] \&\& (\text{GtQ}[n, 0] \parallel \text{IGtQ}[m, 0])$

### Rule 3311

$\text{Int}[(c + d x)^m (b \sin(e + f x))^n, x_{\text{Symbol}}] \rightarrow \text{Simp}[(d^m (c + d x)^{m-1} (b \sin(e + f x))^n / (f^2 n^2), x] + (\text{Dist}[(b^2 (n-1))/n, \text{Int}[(c + d x)^m (b \sin(e + f x))^{n-2}, x], x] - \text{Dist}[(d^2 m (m-1)) / (f^2 n^2), \text{Int}[(c + d x)^{m-2} (b \sin(e + f x))^n, x], x] - \text{Simp}[(b (c + d x)^m \cos(e + f x) (b \sin(e + f x))^{n-1} / (f n), x]) /;$   
 $\text{FreeQ}\{b, c, d, e, f\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{GtQ}[m, 1]$

### Rule 3296

$\text{Int}[(c + d x)^m \sin(e + f x), x_{\text{Symbol}}] \rightarrow -\text{Simp}[(c + d x)^m \cos(e + f x) / f, x] + \text{Dist}[(d m) / f, \text{Int}[(c + d x)^{m-1} \cos(e + f x), x], x] /;$   
 $\text{FreeQ}\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$

### Rule 2638

$\text{Int}[\sin(c + d x), x_{\text{Symbol}}] \rightarrow -\text{Simp}[\cos(c + d x) / d, x] /;$   
 $\text{FreeQ}\{c, d\}, x]$

### Rule 2633

$\text{Int}[\sin(c + d x)^n, x_{\text{Symbol}}] \rightarrow -\text{Dist}[d^{-1}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{(n-1)/2}, x], x], x, \cos(c + d x)], x] /;$   
 $\text{FreeQ}\{c, d\}, x] \&\& \text{IGtQ}[(n-1)/2, 0]$

### Rubi steps

$$\begin{aligned}
\int x^2(a + ia \sinh(e + fx))^{3/2} dx &= -\left(2a \operatorname{csch}\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) \sqrt{a + ia \sinh(e + fx)}\right) \int x^2 \sinh^3\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) dx \\
&= -\frac{16ax \cosh^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sqrt{a + ia \sinh(e + fx)}}{9f^2} + \frac{4ax^2 \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sinh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{3f} \\
&= -\frac{16ax \cosh^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sqrt{a + ia \sinh(e + fx)}}{9f^2} + \frac{4ax^2 \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sinh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{3f} \\
&= -\frac{32ax \sqrt{a + ia \sinh(e + fx)}}{3f^2} - \frac{16ax \cosh^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sqrt{a + ia \sinh(e + fx)}}{9f^2} + \frac{4ax^2 \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sinh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{3f} \\
&= -\frac{32ax \sqrt{a + ia \sinh(e + fx)}}{3f^2} - \frac{16ax \cosh^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sqrt{a + ia \sinh(e + fx)}}{9f^2} + \frac{4ax^2 \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sinh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{3f}
\end{aligned}$$

**Mathematica [A]** time = 1.1252, size = 173, normalized size = 0.57

$$\frac{a(\sinh(e + fx) - i)\sqrt{a + ia \sinh(e + fx)}\left(81(f^2x^2 + 4ifx + 8)\cosh\left(\frac{1}{2}(e + fx)\right) + (9f^2x^2 - 12ifx + 8)\cosh\left(\frac{3}{2}(e + fx)\right)\right) - 27f^3\left(\cosh\left(\frac{1}{2}(e + fx)\right) + i\sinh\left(\frac{1}{2}(e + fx)\right)\right)}{27f^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + I\*a\*Sinh[e + f\*x])^(3/2),x]

[Out]  $-(a(81(8 + (4I)f*x + f^2*x^2)*\operatorname{Cosh}[(e + f*x)/2] + (8 - (12I)f*x + 9f^2*x^2)*\operatorname{Cosh}[(3*(e + f*x))/2] + (2I)*(-4*(80 - (42I)f*x + 9f^2*x^2) + (8 + (12I)f*x + 9f^2*x^2)*\operatorname{Cosh}[e + f*x])*\operatorname{Sinh}[(e + f*x)/2])*(-I + \operatorname{Sinh}[e + f*x])*\operatorname{Sqrt}[a + I*a*\operatorname{Sinh}[e + f*x]])/(27*f^3*(\operatorname{Cosh}[(e + f*x)/2] + I*\operatorname{Sinh}[(e + f*x)/2]))^3$

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int x^2 (a + ia \sinh(fx + e))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a+I\*a\*sinh(f\*x+e))^(3/2),x)

[Out] `int(x^2*(a+I*a*sinh(f*x+e))^(3/2),x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (i a \sinh(fx + e) + a)^{\frac{3}{2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")`

[Out] `integrate((I*a*sinh(f*x + e) + a)^(3/2)*x^2, x)`

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+I*a*sinh(f*x+e))**(3/2),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (i a \sinh(fx + e) + a)^{\frac{3}{2}} x^2 dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)*x^2, x)
```

### 3.127 $\int x(a + ia \sinh(e + fx))^{3/2} dx$

**Optimal.** Leaf size=185

$$-\frac{16a\sqrt{a + ia \sinh(e + fx)}}{3f^2} - \frac{8a \cosh^2\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)\sqrt{a + ia \sinh(e + fx)}}{9f^2} + \frac{8ax \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)\sqrt{a + ia \sinh(e + fx)}}{3f}$$

```
[Out] (-16*a*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f^2) - (8*a*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^2) + (4*a*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f) + (8*a*x*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*f)
```

**Rubi [A]** time = 0.133372, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {3319, 3310, 3296, 2638}

$$-\frac{16a\sqrt{a + ia \sinh(e + fx)}}{3f^2} - \frac{8a \cosh^2\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)\sqrt{a + ia \sinh(e + fx)}}{9f^2} + \frac{8ax \tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)\sqrt{a + ia \sinh(e + fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] (-16*a*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f^2) - (8*a*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]])/(9*f^2) + (4*a*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]])/(3*f) + (8*a*x*Sqrt[a + I*a*Sinh[e + f*x]]*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(3*f)
```

#### Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[((2*a)^(IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

#### Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
  Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int x(a + ia \sinh(e + fx))^{3/2} dx &= -\left(\left(2a \operatorname{csch}\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) \sqrt{a + ia \sinh(e + fx)}\right) \int x \sinh^3\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) dx\right) \\
&= -\frac{8a \cosh^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sqrt{a + ia \sinh(e + fx)}}{9f^2} + \frac{4ax \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sinh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{3f} \\
&= -\frac{8a \cosh^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sqrt{a + ia \sinh(e + fx)}}{9f^2} + \frac{4ax \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sinh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{3f} \\
&= -\frac{16a \sqrt{a + ia \sinh(e + fx)}}{3f^2} - \frac{8a \cosh^2\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sqrt{a + ia \sinh(e + fx)}}{9f^2} + \frac{4ax \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \sinh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{3f}
\end{aligned}$$

**Mathematica [A]** time = 0.776093, size = 138, normalized size = 0.75

$$\frac{a(\sinh(e + fx) - i)\sqrt{a + ia \sinh(e + fx)}\left(27(fx + 2i) \cosh\left(\frac{1}{2}(e + fx)\right) + (3fx - 2i) \cosh\left(\frac{3}{2}(e + fx)\right) + 2i \sinh\left(\frac{1}{2}(e + fx)\right)\right)}{9f^2\left(\cosh\left(\frac{1}{2}(e + fx)\right) + i \sinh\left(\frac{1}{2}(e + fx)\right)\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + I*a*Sinh[e + f*x])^(3/2), x]
```

```
[Out] -(a*(27*(2*I + f*x)*Cosh[(e + f*x)/2] + (-2*I + 3*f*x)*Cosh[(3*(e + f*x))/2]
) + (2*I)*(28*I - 12*f*x + (2*I + 3*f*x)*Cosh[e + f*x])*Sinh[(e + f*x)/2])*
(-I + Sinh[e + f*x])*Sqrt[a + I*a*Sinh[e + f*x]]/(9*f^2*(Cosh[(e + f*x)/2]
+ I*Sinh[(e + f*x)/2])^3)
```

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int x (a + ia \sinh(fx + e))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+I*a*sinh(f*x+e))^(3/2),x)
```

```
[Out] int(x*(a+I*a*sinh(f*x+e))^(3/2),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ia \sinh(fx + e) + a)^{\frac{3}{2}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)*x, x)
```

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+I\*a\*sinh(f\*x+e))\*\*(3/2),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (i a \sinh(fx + e) + a)^{\frac{3}{2}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+I\*a\*sinh(f\*x+e))^(3/2),x, algorithm="giac")

[Out] integrate((I\*a\*sinh(f\*x + e) + a)^(3/2)\*x, x)

$$3.128 \quad \int \frac{(a+ia \sinh(e+fx))^{3/2}}{x} dx$$

**Optimal.** Leaf size=261

$$\frac{3}{2}ia \sinh\left(\frac{1}{4}(2e - i\pi)\right) \text{Chi}\left(\frac{fx}{2}\right) \text{sech}\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)} + \frac{1}{2}ia \sinh\left(\frac{1}{4}(6e + i\pi)\right) \text{Chi}\left(\frac{3fx}{2}\right) \text{sech}\left(\frac{e}{2}\right)$$

[Out] ((3\*I)/2)\*a\*CoshIntegral[(f\*x)/2]\*Sech[e/2 + (I/4)\*Pi + (f\*x)/2]\*Sinh[(2\*e - I\*Pi)/4]\*Sqrt[a + I\*a\*Sinh[e + f\*x]] + (I/2)\*a\*CoshIntegral[(3\*f\*x)/2]\*Sech[e/2 + (I/4)\*Pi + (f\*x)/2]\*Sinh[(6\*e + I\*Pi)/4]\*Sqrt[a + I\*a\*Sinh[e + f\*x]] + ((3\*I)/2)\*a\*Cosh[(2\*e - I\*Pi)/4]\*Sech[e/2 + (I/4)\*Pi + (f\*x)/2]\*Sqrt[a + I\*a\*Sinh[e + f\*x]]\*SinhIntegral[(f\*x)/2] + (I/2)\*a\*Cosh[(6\*e + I\*Pi)/4]\*Sech[e/2 + (I/4)\*Pi + (f\*x)/2]\*Sqrt[a + I\*a\*Sinh[e + f\*x]]\*SinhIntegral[(3\*f\*x)/2]

**Rubi [A]** time = 0.273379, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {3319, 3312, 3303, 3298, 3301}

$$\frac{3}{2}ia \sinh\left(\frac{1}{4}(2e - i\pi)\right) \text{Chi}\left(\frac{fx}{2}\right) \text{sech}\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)} + \frac{1}{2}ia \sinh\left(\frac{1}{4}(6e + i\pi)\right) \text{Chi}\left(\frac{3fx}{2}\right) \text{sech}\left(\frac{e}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + I\*a\*Sinh[e + f\*x])^(3/2)/x,x]

[Out] ((3\*I)/2)\*a\*CoshIntegral[(f\*x)/2]\*Sech[e/2 + (I/4)\*Pi + (f\*x)/2]\*Sinh[(2\*e - I\*Pi)/4]\*Sqrt[a + I\*a\*Sinh[e + f\*x]] + (I/2)\*a\*CoshIntegral[(3\*f\*x)/2]\*Sech[e/2 + (I/4)\*Pi + (f\*x)/2]\*Sinh[(6\*e + I\*Pi)/4]\*Sqrt[a + I\*a\*Sinh[e + f\*x]] + ((3\*I)/2)\*a\*Cosh[(2\*e - I\*Pi)/4]\*Sech[e/2 + (I/4)\*Pi + (f\*x)/2]\*Sqrt[a + I\*a\*Sinh[e + f\*x]]\*SinhIntegral[(f\*x)/2] + (I/2)\*a\*Cosh[(6\*e + I\*Pi)/4]\*Sech[e/2 + (I/4)\*Pi + (f\*x)/2]\*Sqrt[a + I\*a\*Sinh[e + f\*x]]\*SinhIntegral[(3\*f\*x)/2]

### Rule 3319

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Dist[((2\*a)^IntPart[n]\*(a + b\*Sinh[e + f\*x])^FracPart[n])/Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*FracPart[n]), Int[(c + d\*x)^m\*Sinh[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])

Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + ia \sinh(e + fx))^{3/2}}{x} dx &= - \left( \left( 2a \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right) \int \frac{\sinh^3 \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right)}{x} dx \right) \\
 &= - \left( \left( 2ia \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right) \int \left( \frac{3i \sinh \left( \frac{1}{4}(2e - i\pi) + \frac{fx}{2} \right)}{4x} + \dots \right) \right) \\
 &= \frac{1}{2} \left( a \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right) \int \frac{\sinh \left( \frac{1}{4}(6e + i\pi) + \frac{3fx}{2} \right)}{x} dx + \frac{1}{2} \left( 3a \dots \right) \\
 &= \frac{1}{2} \left( 3a \cosh \left( \frac{1}{4}(2e - i\pi) \right) \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right) \int \frac{\sinh \left( \frac{fx}{2} \right)}{x} dx + \dots \\
 &= \frac{3}{2} ia \operatorname{Chi} \left( \frac{fx}{2} \right) \operatorname{sech} \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right) \sinh \left( \frac{1}{4}(2e - i\pi) \right) \sqrt{a + ia \sinh(e + fx)} + \frac{1}{2} ia \operatorname{Chi} \left( \dots \right)
 \end{aligned}$$

**Mathematica [A]** time = 0.784664, size = 146, normalized size = 0.56

$$\frac{a\sqrt{a + ia \sinh(e + fx)} \left( 3\text{Chi}\left(\frac{fx}{2}\right) \left( \cosh\left(\frac{e}{2}\right) + i \sinh\left(\frac{e}{2}\right) \right) - \text{Chi}\left(\frac{3fx}{2}\right) \left( \cosh\left(\frac{3e}{2}\right) - i \sinh\left(\frac{3e}{2}\right) \right) + \left( \sinh\left(\frac{e}{2}\right) + i \cosh\left(\frac{e}{2}\right) \right) \right)}{2 \left( \cosh\left(\frac{1}{2}(e + fx)\right) + i \sinh\left(\frac{1}{2}(e + fx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Sinh[e + f\*x])^(3/2)/x,x]

[Out] (a\*Sqrt[a + I\*a\*Sinh[e + f\*x]]\*(3\*CoshIntegral[(f\*x)/2]\*(Cosh[e/2] + I\*Sinh[e/2]) - CoshIntegral[(3\*f\*x)/2]\*(Cosh[(3\*e)/2] - I\*Sinh[(3\*e)/2]) + (I\*Cosh[e/2] + Sinh[e/2])\*(3\*SinhIntegral[(f\*x)/2] + (1 + (2\*I)\*Sinh[e])\*SinhIntegral[(3\*f\*x)/2]))/(2\*(Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2]))

**Maple [F]** time = 0.045, size = 0, normalized size = 0.

$$\int \frac{1}{x} (a + ia \sinh(fx + e))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*sinh(f\*x+e))^(3/2)/x,x)

[Out] int((a+I\*a\*sinh(f\*x+e))^(3/2)/x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia \sinh(fx + e) + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))^(3/2)/x,x, algorithm="maxima")

[Out] integrate((I\*a\*sinh(f\*x + e) + a)^(3/2)/x, x)



**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))^(3/2)/x,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))\*\*(3/2)/x,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \sinh(fx + e) + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))^(3/2)/x,x, algorithm="giac")

[Out] integrate((I\*a\*sinh(f\*x + e) + a)^(3/2)/x, x)

$$3.129 \quad \int \frac{(a+ia \sinh(e+fx))^{3/2}}{x^2} dx$$

**Optimal.** Leaf size=302

$$-\frac{3}{4}af \sinh\left(\frac{1}{4}(6e - i\pi)\right) \text{Chi}\left(\frac{3fx}{2}\right) \text{sech}\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)} + \frac{3}{4}af \sinh\left(\frac{1}{4}(2e + i\pi)\right) \text{Chi}\left(\frac{fx}{2}\right) \text{sech}\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)}$$

```
[Out] (-2*a*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]])/x - (3*
a*f*CoshIntegral[(3*f*x)/2]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[(6*e - I*Pi
)/4]*Sqrt[a + I*a*Sinh[e + f*x]])/4 + (3*a*f*CoshIntegral[(f*x)/2]*Sech[e/2
+ (I/4)*Pi + (f*x)/2]*Sinh[(2*e + I*Pi)/4]*Sqrt[a + I*a*Sinh[e + f*x]])/4
+ (3*a*f*Cosh[(2*e + I*Pi)/4]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*S
inh[e + f*x]]*SinhIntegral[(f*x)/2])/4 - (3*a*f*Cosh[(6*e - I*Pi)/4]*Sech[e
/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*SinhIntegral[(3*f*x)/2
])/4
```

**Rubi [A]** time = 0.289703, antiderivative size = 302, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {3319, 3313, 3303, 3298, 3301}

$$-\frac{3}{4}af \sinh\left(\frac{1}{4}(6e - i\pi)\right) \text{Chi}\left(\frac{3fx}{2}\right) \text{sech}\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)} + \frac{3}{4}af \sinh\left(\frac{1}{4}(2e + i\pi)\right) \text{Chi}\left(\frac{fx}{2}\right) \text{sech}\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(e + fx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[e + f*x])^(3/2)/x^2,x]
```

```
[Out] (-2*a*Cosh[e/2 + (I/4)*Pi + (f*x)/2]^2*Sqrt[a + I*a*Sinh[e + f*x]])/x - (3*
a*f*CoshIntegral[(3*f*x)/2]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sinh[(6*e - I*Pi
)/4]*Sqrt[a + I*a*Sinh[e + f*x]])/4 + (3*a*f*CoshIntegral[(f*x)/2]*Sech[e/2
+ (I/4)*Pi + (f*x)/2]*Sinh[(2*e + I*Pi)/4]*Sqrt[a + I*a*Sinh[e + f*x]])/4
+ (3*a*f*Cosh[(2*e + I*Pi)/4]*Sech[e/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*S
inh[e + f*x]]*SinhIntegral[(f*x)/2])/4 - (3*a*f*Cosh[(6*e - I*Pi)/4]*Sech[e
/2 + (I/4)*Pi + (f*x)/2]*Sqrt[a + I*a*Sinh[e + f*x]]*SinhIntegral[(3*f*x)/2
])/4
```

### Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sinh[e + f*x])^FracPart[n])/Sin[
```

$e/2 + (a\pi)/(4b) + (f*x)/2]^{(2*\text{FracPart}[n])}$ , Int[(c + d\*x)^m\*Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])

### Rule 3313

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] :> Simp[((c + d\*x)^(m + 1)\*Sin[e + f\*x]^n)/(d\*(m + 1)), x] - Dist[(f\*n)/(d\*(m + 1)), Int[ExpandTrigReduce[(c + d\*x)^(m + 1), Cos[e + f\*x]\*Sin[e + f\*x]^(n - 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \sinh(e + fx))^{3/2}}{x^2} dx &= - \left( \left( 2a \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right) \int \frac{\sinh^3 \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right)}{x^2} dx \right) \\
&= - \frac{2a \cosh^2 \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)}}{x} + \left( 3af \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right) \\
&= - \frac{2a \cosh^2 \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)}}{x} + \frac{1}{4} \left( 3af \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)} \right) \\
&= - \frac{2a \cosh^2 \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)}}{x} + \frac{1}{4} \left( 3iaf \cosh \left( \frac{1}{4}(6e - i\pi) \right) \operatorname{csch} \left( \frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2} \right) \right) \\
&= - \frac{2a \cosh^2 \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right) \sqrt{a + ia \sinh(e + fx)}}{x} - \frac{3}{4} af \operatorname{Chi} \left( \frac{3fx}{2} \right) \operatorname{sech} \left( \frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2} \right) \operatorname{si}
\end{aligned}$$

**Mathematica [A]** time = 0.973868, size = 243, normalized size = 0.8

$$a(\sinh(e + fx) - i)\sqrt{a + ia \sinh(e + fx)} \left( -3fx \operatorname{Chi} \left( \frac{fx}{2} \right) \left( \cosh \left( \frac{e}{2} \right) - i \sinh \left( \frac{e}{2} \right) \right) - 3fx \operatorname{Chi} \left( \frac{3fx}{2} \right) \left( \cosh \left( \frac{3e}{2} \right) + i \sinh \left( \frac{3e}{2} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Sinh[e + f\*x])^(3/2)/x^2,x]

[Out] (a\*(-I + Sinh[e + f\*x])\*Sqrt[a + I\*a\*Sinh[e + f\*x]]\*((-6\*I)\*Cosh[(e + f\*x)/2] + (2\*I)\*Cosh[(3\*(e + f\*x))/2] - 3\*f\*x\*CoshIntegral[(f\*x)/2]\*(Cosh[e/2] - I\*Sinh[e/2]) - 3\*f\*x\*CoshIntegral[(3\*f\*x)/2]\*(Cosh[(3\*e)/2] + I\*Sinh[(3\*e)/2]) + 6\*Sinh[(e + f\*x)/2] + 2\*Sinh[(3\*(e + f\*x))/2] + (3\*I)\*f\*x\*Cosh[e/2]\*SinhIntegral[(f\*x)/2] - 3\*f\*x\*Sinh[e/2]\*SinhIntegral[(f\*x)/2] - (3\*I)\*f\*x\*Cosh[(3\*e)/2]\*SinhIntegral[(3\*f\*x)/2] - 3\*f\*x\*Sinh[(3\*e)/2]\*SinhIntegral[(3\*f\*x)/2]))/(4\*x\*(Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2])^3)

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (a + ia \sinh(fx + e))^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*sinh(f*x+e))^(3/2)/x^2,x)`

[Out] `int((a+I*a*sinh(f*x+e))^(3/2)/x^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \sinh(f x + e) + a)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*sinh(f*x+e))^(3/2)/x^2,x, algorithm="maxima")`

[Out] `integrate((I*a*sinh(f*x + e) + a)^(3/2)/x^2, x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*sinh(f*x+e))^(3/2)/x^2,x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*sinh(f*x+e))**(3/2)/x**2,x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \sinh(fx + e) + a)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(f*x+e))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)^(3/2)/x^2, x)
```

### 3.130 $\int x^3(a + ia \sinh(c + dx))^{5/2} dx$

**Optimal.** Leaf size=638

$$\frac{128a^2x^2\sqrt{a + ia \sinh(c + dx)}}{5d^2} - \frac{48a^2x^2 \cosh^4\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\sqrt{a + ia \sinh(c + dx)}}{25d^2} - \frac{64a^2x^2 \cosh^2\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\sqrt{a + ia \sinh(c + dx)}}{15d^2}$$

```
[Out] (-265216*a^2*Sqrt[a + I*a*Sinh[c + d*x]]/(1125*d^4) - (128*a^2*x^2*Sqrt[a + I*a*Sinh[c + d*x]]/(5*d^2) - (17408*a^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^2*Sqrt[a + I*a*Sinh[c + d*x]]/(3375*d^4) - (64*a^2*x^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^2*Sqrt[a + I*a*Sinh[c + d*x]]/(15*d^2) - (384*a^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]]/(625*d^4) - (48*a^2*x^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]]/(25*d^2) + (8704*a^2*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]/(1125*d^3) + (32*a^2*x^3*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]/(15*d) + (192*a^2*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]/(125*d^3) + (8*a^2*x^3*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]/(5*d) + (132608*a^2*x*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(1125*d^3) + (64*a^2*x^3*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(15*d)
```

**Rubi [A]** time = 0.639757, antiderivative size = 638, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {3319, 3311, 3296, 2638, 3310}

$$\frac{128a^2x^2\sqrt{a + ia \sinh(c + dx)}}{5d^2} - \frac{48a^2x^2 \cosh^4\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\sqrt{a + ia \sinh(c + dx)}}{25d^2} - \frac{64a^2x^2 \cosh^2\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\sqrt{a + ia \sinh(c + dx)}}{15d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + I*a*Sinh[c + d*x])^(5/2),x]
```

```
[Out] (-265216*a^2*Sqrt[a + I*a*Sinh[c + d*x]]/(1125*d^4) - (128*a^2*x^2*Sqrt[a + I*a*Sinh[c + d*x]]/(5*d^2) - (17408*a^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^2*Sqrt[a + I*a*Sinh[c + d*x]]/(3375*d^4) - (64*a^2*x^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^2*Sqrt[a + I*a*Sinh[c + d*x]]/(15*d^2) - (384*a^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]]/(625*d^4) - (48*a^2*x^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]]/(25*d^2) + (8704*a^2*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]/(1125*d^3) + (32*a^2*x^3*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]/(15*d) + (192*a^2*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]/(125*d^3) + (8*a^2*x^3*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3*Sinh[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]/(5*d) + (132608*a^2*x*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(1125*d^3) + (64*a^2*x^3*Sqrt[a + I*a*Sinh[c + d*x]]*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(15*d)
```

$$\begin{aligned} & a + I*a*\text{Sinh}[c + d*x]]/(1125*d^3) + (32*a^2*x^3*\text{Cosh}[c/2 + (I/4)*\text{Pi} + (d*x) \\ & )/2]*\text{Sinh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]]/(15*d) + ( \\ & 192*a^2*x*\text{Cosh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]^3*\text{Sinh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]*\text{S} \\ & \text{qrt}[a + I*a*\text{Sinh}[c + d*x]]/(125*d^3) + (8*a^2*x^3*\text{Cosh}[c/2 + (I/4)*\text{Pi} + (d \\ & *x)/2]^3*\text{Sinh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]]/(5*d) \\ & + (132608*a^2*x*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]]*\text{Tanh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]) \\ & /((1125*d^3) + (64*a^2*x^3*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]]*\text{Tanh}[c/2 + (I/4)*\text{Pi} + \\ & (d*x)/2]))/(15*d) \end{aligned}$$

### Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
  x_Symbol] := Dist[((2*a)^IntPart[n]*(a + b*Sine[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sine[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sine[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sine[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sine[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
Simp[(d*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sine[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sine[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```



Rubi steps

$$\begin{aligned}
\int x^3(a + ia \sinh(c + dx))^{5/2} dx &= \left(4a^2 \operatorname{csch}\left(\frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}\right) \int x^3 \sinh^5\left(\frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2}\right) dx \\
&= -\frac{48a^2 x^2 \cosh^4\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{25d^2} + \frac{8a^2 x^3 \cosh^3\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sinh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{625d^4} \\
&= -\frac{64a^2 x^2 \cosh^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{15d^2} - \frac{384a^2 \cosh^4\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{625d^4} \\
&= -\frac{17408a^2 \cosh^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{3375d^4} - \frac{64a^2 x^2 \cosh^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{15d^2} \\
&= -\frac{128a^2 x^2 \sqrt{a + ia \sinh(c + dx)}}{5d^2} - \frac{17408a^2 \cosh^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{3375d^4} \\
&= -\frac{34816a^2 \sqrt{a + ia \sinh(c + dx)}}{1125d^4} - \frac{128a^2 x^2 \sqrt{a + ia \sinh(c + dx)}}{5d^2} - \frac{17408a^2 \cosh^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{3375d^4} \\
&= -\frac{265216a^2 \sqrt{a + ia \sinh(c + dx)}}{1125d^4} - \frac{128a^2 x^2 \sqrt{a + ia \sinh(c + dx)}}{5d^2} - \frac{17408a^2 \cosh^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{3375d^4}
\end{aligned}$$

**Mathematica [B]** time = 7.43947, size = 2918, normalized size = 4.57

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[x^3\*(a + I\*a\*Sinh[c + d\*x])^(5/2),x]

[Out] (2\*(((−1/135000 − I/135000)\*Cosh[5\*(c/2 + (d\*x)/2)])/d^3 + ((1/135000 + I/135000)\*Sinh[5\*(c/2 + (d\*x)/2)])/d^3)\*(1296\*I − (3240\*I)\*c + (4050\*I)\*c^2 − (3375\*I)\*c^3 + (6480\*I)\*(c/2 + (d\*x)/2) − (16200\*I)\*c\*(c/2 + (d\*x)/2) + (20250\*I)\*c^2\*(c/2 + (d\*x)/2) + (16200\*I)\*(c/2 + (d\*x)/2)^2 − (40500\*I)\*c\*(c/2 + (d\*x)/2)^2 + (27000\*I)\*(c/2 + (d\*x)/2)^3 − 50000\*Cosh[2\*(c/2 + (d\*x)/2)] + 75000\*c\*Cosh[2\*(c/2 + (d\*x)/2)] − 56250\*c^2\*Cosh[2\*(c/2 + (d\*x)/2)] + 28125\*c^3\*Cosh[2\*(c/2 + (d\*x)/2)] − 150000\*(c/2 + (d\*x)/2)\*Cosh[2\*(c/2 + (d\*x)/2)] + 225000\*c\*(c/2 + (d\*x)/2)\*Cosh[2\*(c/2 + (d\*x)/2)] − 168750\*c^2\*(c/2 + (d\*x)/2)\*Cosh[2\*(c/2 + (d\*x)/2)] − 225000\*(c/2 + (d\*x)/2)^2\*Cosh[2\*(c/2 + (d\*x)/2)] + 337500\*c\*(c/2 + (d\*x)/2)^2\*Cosh[2\*(c/2 + (d\*x)/2)] − 225000\*(c/2 + (d\*x)/2)^3\*Cosh[2\*(c/2 + (d\*x)/2)] − (8100000\*I)\*Cosh[4\*(c/2 + (d\*x)/2)] + (4050000\*I)\*c\*Cosh[4\*(c/2 + (d\*x)/2)] − (1012500\*I)\*c^2\*Cosh[4\*(c/2 + (d\*x)/2)]

$$\begin{aligned}
& (d*x)/2] + (168750*I)*c^3*\text{Cosh}[4*(c/2 + (d*x)/2)] - (8100000*I)*(c/2 + (d*x)/2)*\text{Cosh}[4*(c/2 + (d*x)/2)] + (4050000*I)*c*(c/2 + (d*x)/2)*\text{Cosh}[4*(c/2 + (d*x)/2)] - (1012500*I)*c^2*(c/2 + (d*x)/2)*\text{Cosh}[4*(c/2 + (d*x)/2)] - (4050000*I)*(c/2 + (d*x)/2)^2*\text{Cosh}[4*(c/2 + (d*x)/2)] + (2025000*I)*c*(c/2 + (d*x)/2)^2*\text{Cosh}[4*(c/2 + (d*x)/2)] - (1350000*I)*(c/2 + (d*x)/2)^3*\text{Cosh}[4*(c/2 + (d*x)/2)] + 8100000*\text{Cosh}[6*(c/2 + (d*x)/2)] + 4050000*c*\text{Cosh}[6*(c/2 + (d*x)/2)] + 1012500*c^2*\text{Cosh}[6*(c/2 + (d*x)/2)] + 168750*c^3*\text{Cosh}[6*(c/2 + (d*x)/2)] - 8100000*(c/2 + (d*x)/2)*\text{Cosh}[6*(c/2 + (d*x)/2)] - 4050000*c*(c/2 + (d*x)/2)*\text{Cosh}[6*(c/2 + (d*x)/2)] - 1012500*c^2*(c/2 + (d*x)/2)*\text{Cosh}[6*(c/2 + (d*x)/2)] + 4050000*(c/2 + (d*x)/2)^2*\text{Cosh}[6*(c/2 + (d*x)/2)] + 2025000*c*(c/2 + (d*x)/2)^2*\text{Cosh}[6*(c/2 + (d*x)/2)] - 1350000*(c/2 + (d*x)/2)^3*\text{Cosh}[6*(c/2 + (d*x)/2)] + (50000*I)*\text{Cosh}[8*(c/2 + (d*x)/2)] + (75000*I)*c*\text{Cosh}[8*(c/2 + (d*x)/2)] + (56250*I)*c^2*\text{Cosh}[8*(c/2 + (d*x)/2)] + (28125*I)*c^3*\text{Cosh}[8*(c/2 + (d*x)/2)] - (150000*I)*(c/2 + (d*x)/2)*\text{Cosh}[8*(c/2 + (d*x)/2)] - (225000*I)*c*(c/2 + (d*x)/2)*\text{Cosh}[8*(c/2 + (d*x)/2)] - (168750*I)*c^2*(c/2 + (d*x)/2)*\text{Cosh}[8*(c/2 + (d*x)/2)] + (225000*I)*(c/2 + (d*x)/2)^2*\text{Cosh}[8*(c/2 + (d*x)/2)] + (337500*I)*c*(c/2 + (d*x)/2)^2*\text{Cosh}[8*(c/2 + (d*x)/2)] - (225000*I)*(c/2 + (d*x)/2)^3*\text{Cosh}[8*(c/2 + (d*x)/2)] - 1296*\text{Cosh}[10*(c/2 + (d*x)/2)] - 3240*c*\text{Cosh}[10*(c/2 + (d*x)/2)] - 4050*c^2*\text{Cosh}[10*(c/2 + (d*x)/2)] - 3375*c^3*\text{Cosh}[10*(c/2 + (d*x)/2)] + 6480*(c/2 + (d*x)/2)*\text{Cosh}[10*(c/2 + (d*x)/2)] + 16200*c*(c/2 + (d*x)/2)*\text{Cosh}[10*(c/2 + (d*x)/2)] + 20250*c^2*(c/2 + (d*x)/2)*\text{Cosh}[10*(c/2 + (d*x)/2)] - 16200*(c/2 + (d*x)/2)^2*\text{Cosh}[10*(c/2 + (d*x)/2)] - 40500*c*(c/2 + (d*x)/2)^2*\text{Cosh}[10*(c/2 + (d*x)/2)] + 27000*(c/2 + (d*x)/2)^3*\text{Cosh}[10*(c/2 + (d*x)/2)] - 50000*\text{Sinh}[2*(c/2 + (d*x)/2)] + 75000*c*\text{Sinh}[2*(c/2 + (d*x)/2)] - 56250*c^2*\text{Sinh}[2*(c/2 + (d*x)/2)] + 28125*c^3*\text{Sinh}[2*(c/2 + (d*x)/2)] - 150000*(c/2 + (d*x)/2)*\text{Sinh}[2*(c/2 + (d*x)/2)] + 225000*c*(c/2 + (d*x)/2)*\text{Sinh}[2*(c/2 + (d*x)/2)] - 168750*c^2*(c/2 + (d*x)/2)*\text{Sinh}[2*(c/2 + (d*x)/2)] - 225000*(c/2 + (d*x)/2)^2*\text{Sinh}[2*(c/2 + (d*x)/2)] + 337500*c*(c/2 + (d*x)/2)^2*\text{Sinh}[2*(c/2 + (d*x)/2)] - 225000*(c/2 + (d*x)/2)^3*\text{Sinh}[2*(c/2 + (d*x)/2)] - (8100000*I)*\text{Sinh}[4*(c/2 + (d*x)/2)] + (4050000*I)*c*\text{Sinh}[4*(c/2 + (d*x)/2)] - (1012500*I)*c^2*\text{Sinh}[4*(c/2 + (d*x)/2)] + (168750*I)*c^3*\text{Sinh}[4*(c/2 + (d*x)/2)] - (8100000*I)*(c/2 + (d*x)/2)*\text{Sinh}[4*(c/2 + (d*x)/2)] + (4050000*I)*c*(c/2 + (d*x)/2)*\text{Sinh}[4*(c/2 + (d*x)/2)] - (1012500*I)*c^2*(c/2 + (d*x)/2)*\text{Sinh}[4*(c/2 + (d*x)/2)] - (4050000*I)*(c/2 + (d*x)/2)^2*\text{Sinh}[4*(c/2 + (d*x)/2)] + (2025000*I)*c*(c/2 + (d*x)/2)^2*\text{Sinh}[4*(c/2 + (d*x)/2)] - (1350000*I)*(c/2 + (d*x)/2)^3*\text{Sinh}[4*(c/2 + (d*x)/2)] + 8100000*\text{Sinh}[6*(c/2 + (d*x)/2)] + 4050000*c*\text{Sinh}[6*(c/2 + (d*x)/2)] + 1012500*c^2*\text{Sinh}[6*(c/2 + (d*x)/2)] + 168750*c^3*\text{Sinh}[6*(c/2 + (d*x)/2)] - 8100000*(c/2 + (d*x)/2)*\text{Sinh}[6*(c/2 + (d*x)/2)] - 4050000*c*(c/2 + (d*x)/2)*\text{Sinh}[6*(c/2 + (d*x)/2)] - 1012500*c^2*(c/2 + (d*x)/2)*\text{Sinh}[6*(c/2 + (d*x)/2)] + 4050000*(c/2 + (d*x)/2)^2*\text{Sinh}[6*(c/2 + (d*x)/2)] + 2025000*c*(c/2 + (d*x)/2)^2*\text{Sinh}[6*(c/2 + (d*x)/2)] - 1350000*(c/2 + (d*x)/2)^3*\text{Sinh}[6*(c/2 + (d*x)/2)] + (50000*I)*\text{Sinh}[8*(c/2 + (d*x)/2)] + (75000*I)*c*\text{Sinh}[8*(c/2 + (d*x)/2)] + (56250*I)*c^2*\text{Sinh}[8*(c/2 + (d*x)/2)] + (28125*I)*c^3*\text{Sinh}[8*(c/2 + (d*x)/2)] - (150000*I)*(c/2 + (d*x)/2)*\text{Sinh}[8*(c/2 + (d*x)/2)]
\end{aligned}$$

$$\begin{aligned}
 & /2 + (d*x)/2] - (225000*I)*c*(c/2 + (d*x)/2)*\text{Sinh}[8*(c/2 + (d*x)/2)] - (16 \\
 & 8750*I)*c^2*(c/2 + (d*x)/2)*\text{Sinh}[8*(c/2 + (d*x)/2)] + (225000*I)*(c/2 + (d* \\
 & x)/2)^2*\text{Sinh}[8*(c/2 + (d*x)/2)] + (337500*I)*c*(c/2 + (d*x)/2)^2*\text{Sinh}[8*(c/ \\
 & 2 + (d*x)/2)] - (225000*I)*(c/2 + (d*x)/2)^3*\text{Sinh}[8*(c/2 + (d*x)/2)] - 1296 \\
 & *\text{Sinh}[10*(c/2 + (d*x)/2)] - 3240*c*\text{Sinh}[10*(c/2 + (d*x)/2)] - 4050*c^2*\text{Sinh} \\
 & [10*(c/2 + (d*x)/2)] - 3375*c^3*\text{Sinh}[10*(c/2 + (d*x)/2)] + 6480*(c/2 + (d*x \\
 & )/2)*\text{Sinh}[10*(c/2 + (d*x)/2)] + 16200*c*(c/2 + (d*x)/2)*\text{Sinh}[10*(c/2 + (d*x \\
 & )/2)] + 20250*c^2*(c/2 + (d*x)/2)*\text{Sinh}[10*(c/2 + (d*x)/2)] - 16200*(c/2 + ( \\
 & d*x)/2)^2*\text{Sinh}[10*(c/2 + (d*x)/2)] - 40500*c*(c/2 + (d*x)/2)^2*\text{Sinh}[10*(c/2 \\
 & + (d*x)/2)] + 27000*(c/2 + (d*x)/2)^3*\text{Sinh}[10*(c/2 + (d*x)/2)]*(a + I*a*S \\
 & \text{inh}[c + d*x])^(5/2))/(d*(\text{Cosh}[c/2 + (d*x)/2] + I*\text{Sinh}[c/2 + (d*x)/2])^5)
 \end{aligned}$$

**Maple [F]** time = 0.063, size = 0, normalized size = 0.

$$\int x^3 (a + ia \sinh(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(a+I\*a\*sinh(d\*x+c))^(5/2),x)

[Out] int(x^3\*(a+I\*a\*sinh(d\*x+c))^(5/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ia \sinh(dx + c) + a)^{\frac{5}{2}} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(a+I\*a\*sinh(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((I\*a\*sinh(d\*x + c) + a)^(5/2)\*x^3, x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (i a \sinh(dx + c) + a)^{\frac{5}{2}} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)*x^3, x)
```

### 3.131 $\int x^2(a + ia \sinh(c + dx))^{5/2} dx$

**Optimal.** Leaf size=506

$$-\frac{256a^2x\sqrt{a+ia\sinh(c+dx)}}{15d^2} + \frac{64a^2\sinh^4\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\tanh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{125d^3} + \frac{2432a^2\sinh^2\left(\frac{c}{2}\right)}{125d^3}$$

[Out]  $(-256*a^2*x*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]])/(15*d^2) - (128*a^2*x*\text{Cosh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]^2*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]])/(45*d^2) - (32*a^2*x*\text{Cosh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]^4*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]])/(25*d^2) + (32*a^2*x^2*\text{Cosh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]*\text{Sinh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]])/(15*d) + (8*a^2*x^2*\text{Cosh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]^3*\text{Sinh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]])/(5*d) + (9536*a^2*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]]*\text{Tanh}[c/2 + (I/4)*\text{Pi} + (d*x)/2])/(225*d^3) + (64*a^2*x^2*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]]*\text{Tanh}[c/2 + (I/4)*\text{Pi} + (d*x)/2])/(15*d) + (2432*a^2*\text{Sinh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]^2*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]]*\text{Tanh}[c/2 + (I/4)*\text{Pi} + (d*x)/2])/(675*d^3) + (64*a^2*\text{Sinh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]^4*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]]*\text{Tanh}[c/2 + (I/4)*\text{Pi} + (d*x)/2])/(125*d^3)$

**Rubi [A]** time = 0.389703, antiderivative size = 506, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {3319, 3311, 3296, 2638, 2633}

$$-\frac{256a^2x\sqrt{a+ia\sinh(c+dx)}}{15d^2} + \frac{64a^2\sinh^4\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\tanh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\sqrt{a+ia\sinh(c+dx)}}{125d^3} + \frac{2432a^2\sinh^2\left(\frac{c}{2}\right)}{125d^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*(a + I*a*\text{Sinh}[c + d*x])^{5/2}, x]$

[Out]  $(-256*a^2*x*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]])/(15*d^2) - (128*a^2*x*\text{Cosh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]^2*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]])/(45*d^2) - (32*a^2*x*\text{Cosh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]^4*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]])/(25*d^2) + (32*a^2*x^2*\text{Cosh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]*\text{Sinh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]])/(15*d) + (8*a^2*x^2*\text{Cosh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]^3*\text{Sinh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]])/(5*d) + (9536*a^2*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]]*\text{Tanh}[c/2 + (I/4)*\text{Pi} + (d*x)/2])/(225*d^3) + (64*a^2*x^2*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]]*\text{Tanh}[c/2 + (I/4)*\text{Pi} + (d*x)/2])/(15*d) + (2432*a^2*\text{Sinh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]^2*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]]*\text{Tanh}[c/2 + (I/4)*\text{Pi} + (d*x)/2])/(675*d^3) + (64*a^2*\text{Sinh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]^4*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]]*\text{Tanh}[c/2 + (I/4)*\text{Pi} + (d*x)/2])/(125*d^3)$

$(d*x)/2]^4*\text{Sqrt}[a + I*a*\text{Sinh}[c + d*x]]*\text{Tanh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]/(125*d^3)$

### Rule 3319

$\text{Int}[(c_.) + (d_.)*(x_)]^{(m_)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^{(n_)}, x\_Symbol] \rightarrow \text{Dist}[(2*a)^{\text{IntPart}[n]}*(a + b*\sin[e + f*x])^{\text{FracPart}[n]}/\sin[e/2 + (a*\text{Pi})/(4*b) + (f*x)/2]^{2*\text{FracPart}[n]}], \text{Int}[(c + d*x)^m*\sin[e/2 + (a*\text{Pi})/(4*b) + (f*x)/2]^{2*n}], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{E}qQ[a^2 - b^2, 0] \&\& \text{IntegerQ}[n + 1/2] \&\& (\text{GtQ}[n, 0] \parallel \text{IGtQ}[m, 0])$

### Rule 3311

$\text{Int}[(c_.) + (d_.)*(x_)]^{(m_)}*((b_.)*\sin[(e_.) + (f_.)*(x_)]^{(n_)}, x\_Symbol] \rightarrow \text{Simp}[(d*m*(c + d*x)^{(m-1)}*(b*\sin[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n-1))/n, \text{Int}[(c + d*x)^m*(b*\sin[e + f*x])^{(n-2)}], x], x] - \text{Dist}[(d^2*m*(m-1))/(f^2*n^2), \text{Int}[(c + d*x)^{(m-2)}*(b*\sin[e + f*x])^n], x] - \text{Simp}[(b*(c + d*x)^m*\cos[e + f*x]*(b*\sin[e + f*x])^{(n-1)}]/(f*n), x]) /; \text{FreeQ}\{b, c, d, e, f\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{GtQ}[m, 1]$

### Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_)]^{(m_)}*\sin[(e_.) + (f_.)*(x_)], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\cos[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\cos[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$

### Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_)]], x\_Symbol] \rightarrow -\text{Simp}[\cos[c + d*x]/d, x] /; \text{FreeQ}\{c, d\}, x]$

### Rule 2633

$\text{Int}[\sin[(c_.) + (d_.)*(x_)]^{(n_)}, x\_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{(n-1)/2}], x], x], x, \cos[c + d*x]], x] /; \text{FreeQ}\{c, d\}, x] \&\& \text{IGtQ}[(n-1)/2, 0]$

### Rubi steps

$$\begin{aligned}
\int x^2(a + ia \sinh(c + dx))^{5/2} dx &= \left(4a^2 \operatorname{csch}\left(\frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}\right) \int x^2 \sinh^5\left(\frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2}\right) dx \\
&= -\frac{32a^2 x \cosh^4\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{25d^2} + \frac{8a^2 x^2 \cosh^3\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sinh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{25d^2} \\
&= -\frac{128a^2 x \cosh^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{45d^2} - \frac{32a^2 x \cosh^4\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{25d^2} \\
&= -\frac{128a^2 x \cosh^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{45d^2} - \frac{32a^2 x \cosh^4\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{25d^2} \\
&= -\frac{256a^2 x \sqrt{a + ia \sinh(c + dx)}}{15d^2} - \frac{128a^2 x \cosh^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{45d^2} \\
&= -\frac{256a^2 x \sqrt{a + ia \sinh(c + dx)}}{15d^2} - \frac{128a^2 x \cosh^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{45d^2}
\end{aligned}$$

**Mathematica [A]** time = 1.80694, size = 300, normalized size = 0.59

$$a^2 \sqrt{a + ia \sinh(c + dx)} \left( 33750d^2 x^2 \sinh\left(\frac{1}{2}(c + dx)\right) - 5625d^2 x^2 \sinh\left(\frac{3}{2}(c + dx)\right) - 675d^2 x^2 \sinh\left(\frac{5}{2}(c + dx)\right) - 675id^2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + I\*a\*Sinh[c + d\*x])^(5/2),x]

[Out] (a^2\*Sqrt[a + I\*a\*Sinh[c + d\*x]]\*((33750\*I)\*(8 + (4\*I)\*d\*x + d^2\*x^2)\*Cosh[(c + d\*x)/2] + 625\*(8\*I + 12\*d\*x + (9\*I)\*d^2\*x^2)\*Cosh[(3\*(c + d\*x))/2] - (216\*I)\*Cosh[(5\*(c + d\*x))/2] + 540\*d\*x\*Cosh[(5\*(c + d\*x))/2] - (675\*I)\*d^2\*x^2\*Cosh[(5\*(c + d\*x))/2] + 270000\*Sinh[(c + d\*x)/2] - (135000\*I)\*d\*x\*Sinh[(c + d\*x)/2] + 33750\*d^2\*x^2\*Sinh[(c + d\*x)/2] - 5000\*Sinh[(3\*(c + d\*x))/2] - (7500\*I)\*d\*x\*Sinh[(3\*(c + d\*x))/2] - 5625\*d^2\*x^2\*Sinh[(3\*(c + d\*x))/2] - 216\*Sinh[(5\*(c + d\*x))/2] + (540\*I)\*d\*x\*Sinh[(5\*(c + d\*x))/2] - 675\*d^2\*x^2\*Sinh[(5\*(c + d\*x))/2]))/(6750\*d^3\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]))

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int x^2 (a + ia \sinh(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+I*a*sinh(d*x+c))^(5/2),x)
```

```
[Out] int(x^2*(a+I*a*sinh(d*x+c))^(5/2),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (i a \sinh(dx + c) + a)^{\frac{5}{2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)*x^2, x)
```

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```



---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (i a \sinh(dx + c) + a)^{\frac{5}{2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")`

[Out] `integrate((I*a*sinh(d*x + c) + a)^(5/2)*x^2, x)`

### 3.132 $\int x(a + ia \sinh(c + dx))^{5/2} dx$

**Optimal.** Leaf size=312

$$\frac{128a^2\sqrt{a + ia \sinh(c + dx)}}{15d^2} - \frac{16a^2 \cosh^4\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\sqrt{a + ia \sinh(c + dx)}}{25d^2} - \frac{64a^2 \cosh^2\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\sqrt{a + ia \sinh(c + dx)}}{45d^2}$$

[Out]  $(-128a^2\sqrt{a + I*a*\text{Sinh}[c + d*x]})/(15*d^2) - (64*a^2*\text{Cosh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]^2*\sqrt{a + I*a*\text{Sinh}[c + d*x]})/(45*d^2) - (16*a^2*\text{Cosh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]^4*\sqrt{a + I*a*\text{Sinh}[c + d*x]})/(25*d^2) + (32*a^2*x*\text{Cosh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]*\text{Sinh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]*\sqrt{a + I*a*\text{Sinh}[c + d*x]})/(15*d) + (8*a^2*x*\text{Cosh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]^3*\text{Sinh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]*\sqrt{a + I*a*\text{Sinh}[c + d*x]})/(5*d) + (64*a^2*x*\sqrt{a + I*a*\text{Sinh}[c + d*x]}*\text{Tanh}[c/2 + (I/4)*\text{Pi} + (d*x)/2])/(15*d)$

**Rubi [A]** time = 0.208157, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {3319, 3310, 3296, 2638}

$$\frac{128a^2\sqrt{a + ia \sinh(c + dx)}}{15d^2} - \frac{16a^2 \cosh^4\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\sqrt{a + ia \sinh(c + dx)}}{25d^2} - \frac{64a^2 \cosh^2\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\sqrt{a + ia \sinh(c + dx)}}{45d^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(a + I*a*\text{Sinh}[c + d*x])^{5/2}, x]$

[Out]  $(-128*a^2*\sqrt{a + I*a*\text{Sinh}[c + d*x]})/(15*d^2) - (64*a^2*\text{Cosh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]^2*\sqrt{a + I*a*\text{Sinh}[c + d*x]})/(45*d^2) - (16*a^2*\text{Cosh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]^4*\sqrt{a + I*a*\text{Sinh}[c + d*x]})/(25*d^2) + (32*a^2*x*\text{Cosh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]*\text{Sinh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]*\sqrt{a + I*a*\text{Sinh}[c + d*x]})/(15*d) + (8*a^2*x*\text{Cosh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]^3*\text{Sinh}[c/2 + (I/4)*\text{Pi} + (d*x)/2]*\sqrt{a + I*a*\text{Sinh}[c + d*x]})/(5*d) + (64*a^2*x*\sqrt{a + I*a*\text{Sinh}[c + d*x]}*\text{Tanh}[c/2 + (I/4)*\text{Pi} + (d*x)/2])/(15*d)$

#### Rule 3319

$\text{Int}[\frac{((c_.) + (d_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}}{x\_Symbol}, x] \rightarrow \text{Dist}[\frac{((2*a)^{\text{IntPart}[n]}*(a + b*\sin[e + f*x])^{\text{FracPart}[n]})}{\sin[e/2 + (a*\text{Pi})/(4*b) + (f*x)/2]^{(2*\text{FracPart}[n])}}, \text{Int}[(c + d*x)^m*\sin[e/2 + (a*\text{Pi})/(4*b) + (f*x)/2]^{(2*n)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\& \text{E}$

qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])

### Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
  Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
 \int x(a + ia \sinh(c + dx))^{5/2} dx &= \left(4a^2 \operatorname{csch}\left(\frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}\right) \int x \sinh^5\left(\frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2}\right) dx \\
 &= -\frac{16a^2 \cosh^4\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{25d^2} + \frac{8a^2 x \cosh^3\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sinh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{5d} \\
 &= -\frac{64a^2 \cosh^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{45d^2} - \frac{16a^2 \cosh^4\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{25d^2} \\
 &= -\frac{64a^2 \cosh^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{45d^2} - \frac{16a^2 \cosh^4\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{25d^2} \\
 &= -\frac{128a^2 \sqrt{a + ia \sinh(c + dx)}}{15d^2} - \frac{64a^2 \cosh^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{45d^2} - \frac{16a^2 \cosh^4\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}}{25d^2}
 \end{aligned}$$

**Mathematica [A]** time = 1.38736, size = 218, normalized size = 0.7

$$a^2(\sinh(c + dx) - i)^2 \sqrt{a + ia \sinh(c + dx)} \left(-2250dx \sinh\left(\frac{1}{2}(c + dx)\right) + 4500i \sinh\left(\frac{1}{2}(c + dx)\right) + 375dx \sinh\left(\frac{3}{2}(c + dx)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + I\*a\*Sinh[c + d\*x])^(5/2),x]

[Out] (a^2\*(-I + Sinh[c + d\*x])^2\*Sqrt[a + I\*a\*Sinh[c + d\*x]]\*(2250\*(2 - I\*d\*x)\*Cosh[(c + d\*x)/2] + (-250 - (375\*I)\*d\*x)\*Cosh[(3\*(c + d\*x))/2] - 18\*Cosh[(5\*(c + d\*x))/2] + (45\*I)\*d\*x\*Cosh[(5\*(c + d\*x))/2] + (4500\*I)\*Sinh[(c + d\*x)/2] - 2250\*d\*x\*Sinh[(c + d\*x)/2] + (250\*I)\*Sinh[(3\*(c + d\*x))/2] + 375\*d\*x\*Sinh[(3\*(c + d\*x))/2] - (18\*I)\*Sinh[(5\*(c + d\*x))/2] + 45\*d\*x\*Sinh[(5\*(c + d\*x))/2]))/(450\*d^2\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^5)

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int x (a + ia \sinh(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a+I\*a\*sinh(d\*x+c))^(5/2),x)

[Out] int(x\*(a+I\*a\*sinh(d\*x+c))^(5/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ia \sinh(dx + c) + a)^{\frac{5}{2}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a+I\*a\*sinh(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((I\*a\*sinh(d\*x + c) + a)^(5/2)\*x, x)

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (i a \sinh(dx + c) + a)^{\frac{5}{2}} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)*x, x)
```

$$3.133 \quad \int \frac{(a+ia \sinh(c+dx))^{5/2}}{x} dx$$

**Optimal.** Leaf size=403

$$-\frac{1}{4}ia^2 \sinh\left(\frac{5c}{2} - \frac{i\pi}{4}\right) \text{Chi}\left(\frac{5dx}{2}\right) \text{sech}\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(c + dx)} + \frac{5}{2}ia^2 \sinh\left(\frac{1}{4}(2c - i\pi)\right) \text{Chi}\left(\frac{dx}{2}\right) \text{sech}\left(\frac{c}{2}\right)$$

[Out] (-I/4)\*a^2\*CoshIntegral[(5\*d\*x)/2]\*Sech[c/2 + (I/4)\*Pi + (d\*x)/2]\*Sinh[(5\*c)/2 - (I/4)\*Pi]\*Sqrt[a + I\*a\*Sinh[c + d\*x]] + ((5\*I)/2)\*a^2\*CoshIntegral[(d\*x)/2]\*Sech[c/2 + (I/4)\*Pi + (d\*x)/2]\*Sinh[(2\*c - I\*Pi)/4]\*Sqrt[a + I\*a\*Sinh[c + d\*x]] + ((5\*I)/4)\*a^2\*CoshIntegral[(3\*d\*x)/2]\*Sech[c/2 + (I/4)\*Pi + (d\*x)/2]\*Sinh[(6\*c + I\*Pi)/4]\*Sqrt[a + I\*a\*Sinh[c + d\*x]] + ((5\*I)/2)\*a^2\*Cosh[(2\*c - I\*Pi)/4]\*Sech[c/2 + (I/4)\*Pi + (d\*x)/2]\*Sqrt[a + I\*a\*Sinh[c + d\*x]]\*SinhIntegral[(d\*x)/2] + ((5\*I)/4)\*a^2\*Cosh[(6\*c + I\*Pi)/4]\*Sech[c/2 + (I/4)\*Pi + (d\*x)/2]\*Sqrt[a + I\*a\*Sinh[c + d\*x]]\*SinhIntegral[(3\*d\*x)/2] - (I/4)\*a^2\*Cosh[(5\*c)/2 - (I/4)\*Pi]\*Sech[c/2 + (I/4)\*Pi + (d\*x)/2]\*Sqrt[a + I\*a\*Sinh[c + d\*x]]\*SinhIntegral[(5\*d\*x)/2]

**Rubi [A]** time = 0.417715, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {3319, 3312, 3303, 3298, 3301}

$$-\frac{1}{4}ia^2 \sinh\left(\frac{5c}{2} - \frac{i\pi}{4}\right) \text{Chi}\left(\frac{5dx}{2}\right) \text{sech}\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(c + dx)} + \frac{5}{2}ia^2 \sinh\left(\frac{1}{4}(2c - i\pi)\right) \text{Chi}\left(\frac{dx}{2}\right) \text{sech}\left(\frac{c}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + I\*a\*Sinh[c + d\*x])^(5/2)/x,x]

[Out] (-I/4)\*a^2\*CoshIntegral[(5\*d\*x)/2]\*Sech[c/2 + (I/4)\*Pi + (d\*x)/2]\*Sinh[(5\*c)/2 - (I/4)\*Pi]\*Sqrt[a + I\*a\*Sinh[c + d\*x]] + ((5\*I)/2)\*a^2\*CoshIntegral[(d\*x)/2]\*Sech[c/2 + (I/4)\*Pi + (d\*x)/2]\*Sinh[(2\*c - I\*Pi)/4]\*Sqrt[a + I\*a\*Sinh[c + d\*x]] + ((5\*I)/4)\*a^2\*CoshIntegral[(3\*d\*x)/2]\*Sech[c/2 + (I/4)\*Pi + (d\*x)/2]\*Sinh[(6\*c + I\*Pi)/4]\*Sqrt[a + I\*a\*Sinh[c + d\*x]] + ((5\*I)/2)\*a^2\*Cosh[(2\*c - I\*Pi)/4]\*Sech[c/2 + (I/4)\*Pi + (d\*x)/2]\*Sqrt[a + I\*a\*Sinh[c + d\*x]]\*SinhIntegral[(d\*x)/2] + ((5\*I)/4)\*a^2\*Cosh[(6\*c + I\*Pi)/4]\*Sech[c/2 + (I/4)\*Pi + (d\*x)/2]\*Sqrt[a + I\*a\*Sinh[c + d\*x]]\*SinhIntegral[(3\*d\*x)/2] - (I/4)\*a^2\*Cosh[(5\*c)/2 - (I/4)\*Pi]\*Sech[c/2 + (I/4)\*Pi + (d\*x)/2]\*Sqrt[a + I\*a\*Sinh[c + d\*x]]\*SinhIntegral[(5\*d\*x)/2]

**Rule 3319**

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
  x_Symbol] := Dist[((2*a)^(IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \sinh(c + dx))^{5/2}}{x} dx &= \left(4a^2 \operatorname{csch}\left(\frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}\right) \int \frac{\sinh^5\left(\frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2}\right)}{x} dx \\
&= -\left(\left(4ia^2 \operatorname{csch}\left(\frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}\right) \int \left(\frac{5i \sinh\left(\frac{1}{4}(2c - i\pi) + \frac{dx}{2}\right)}{8x} + \frac{5i \sinh\left(\frac{1}{4}(2c - i\pi) + \frac{dx}{2}\right)}{8x}\right) dx\right) \\
&= -\left(\frac{1}{4} \left(a^2 \operatorname{csch}\left(\frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}\right) \int \frac{\sinh\left(\frac{1}{4}(10c - i\pi) + \frac{5dx}{2}\right)}{x} dx\right) + \frac{1}{4} \\
&= -\left(\frac{1}{4} \left(a^2 \cosh\left(\frac{5c}{2} - \frac{i\pi}{4}\right) \operatorname{csch}\left(\frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2}\right) \sqrt{a + ia \sinh(c + dx)}\right) \int \frac{\sinh\left(\frac{5dx}{2}\right)}{x} dx\right) + \\
&= -\frac{1}{4} ia^2 \operatorname{Chi}\left(\frac{5dx}{2}\right) \operatorname{sech}\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \sinh\left(\frac{5c}{2} - \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(c + dx)} + \frac{5}{2} ia^2 \operatorname{Chi}\left(\frac{5dx}{2}\right)
\end{aligned}$$

**Mathematica [A]** time = 1.49706, size = 242, normalized size = 0.6

$$a^2(\sinh(c + dx) - i)^2 \sqrt{a + ia \sinh(c + dx)} \left( i \sinh\left(\frac{5c}{2}\right) \operatorname{Chi}\left(\frac{5dx}{2}\right) + \cosh\left(\frac{5c}{2}\right) \operatorname{Chi}\left(\frac{5dx}{2}\right) - 10 \left( \cosh\left(\frac{c}{2}\right) + i \sinh\left(\frac{c}{2}\right) \right) \operatorname{Chi}\left(\frac{5dx}{2}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Sinh[c + d\*x])^(5/2)/x,x]

[Out] (a^2\*(-I + Sinh[c + d\*x])^2\*Sqrt[a + I\*a\*Sinh[c + d\*x]]\*(Cosh[(5\*c)/2]\*CoshIntegral[(5\*d\*x)/2] - 10\*CoshIntegral[(d\*x)/2]\*(Cosh[c/2] + I\*Sinh[c/2]) + 5\*CoshIntegral[(3\*d\*x)/2]\*(Cosh[(3\*c)/2] - I\*Sinh[(3\*c)/2]) + I\*CoshIntegral[(5\*d\*x)/2]\*Sinh[(5\*c)/2] - (10\*I)\*Cosh[c/2]\*SinhIntegral[(d\*x)/2] - 10\*Sinh[c/2]\*SinhIntegral[(d\*x)/2] - (5\*I)\*Cosh[(3\*c)/2]\*SinhIntegral[(3\*d\*x)/2] + 5\*Sinh[(3\*c)/2]\*SinhIntegral[(3\*d\*x)/2] + I\*Cosh[(5\*c)/2]\*SinhIntegral[(5\*d\*x)/2] + Sinh[(5\*c)/2]\*SinhIntegral[(5\*d\*x)/2]))/(4\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^5)

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x} (a + ia \sinh(dx + c))^{\frac{5}{2}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+I*a*sinh(d*x+c))^(5/2)/x,x)`

[Out] `int((a+I*a*sinh(d*x+c))^(5/2)/x,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \sinh(dx + c) + a)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*sinh(d*x+c))^(5/2)/x,x, algorithm="maxima")`

[Out] `integrate((I*a*sinh(d*x + c) + a)^(5/2)/x, x)`

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*sinh(d*x+c))^(5/2)/x,x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+I*a*sinh(d*x+c))**(5/2)/x,x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \sinh(dx + c) + a)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(d\*x+c))^(5/2)/x,x, algorithm="giac")

[Out] integrate((I\*a\*sinh(d\*x + c) + a)^(5/2)/x, x)

$$3.134 \quad \int \frac{(a+ia \sinh(c+dx))^{5/2}}{x^2} dx$$

**Optimal.** Leaf size=444

$$-\frac{5}{8}a^2d \sinh\left(\frac{5c}{2} + \frac{i\pi}{4}\right) \text{Chi}\left(\frac{5dx}{2}\right) \text{sech}\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(c + dx)} - \frac{15}{8}a^2d \sinh\left(\frac{1}{4}(6c - i\pi)\right) \text{Chi}\left(\frac{3dx}{2}\right) \text{sech}\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(c + dx)}$$

```
[Out] (-4*a^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/x - (5*a^2*d*CoshIntegral[(5*d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(5*c)/2 + (I/4)*Pi]*Sqrt[a + I*a*Sinh[c + d*x]])/8 - (15*a^2*d*CoshIntegral[(3*d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(6*c - I*Pi)/4]*Sqrt[a + I*a*Sinh[c + d*x]])/8 + (5*a^2*d*CoshIntegral[(d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(2*c + I*Pi)/4]*Sqrt[a + I*a*Sinh[c + d*x]])/4 + (5*a^2*d*Cosh[(2*c + I*Pi)/4]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(d*x)/2])/4 - (15*a^2*d*Cosh[(6*c - I*Pi)/4]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(3*d*x)/2])/8 - (5*a^2*d*Cosh[(5*c)/2 + (I/4)*Pi]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(5*d*x)/2])/8
```

---

**Rubi [A]** time = 0.436698, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {3319, 3313, 3303, 3298, 3301}

$$-\frac{5}{8}a^2d \sinh\left(\frac{5c}{2} + \frac{i\pi}{4}\right) \text{Chi}\left(\frac{5dx}{2}\right) \text{sech}\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(c + dx)} - \frac{15}{8}a^2d \sinh\left(\frac{1}{4}(6c - i\pi)\right) \text{Chi}\left(\frac{3dx}{2}\right) \text{sech}\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) \sqrt{a + ia \sinh(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(5/2)/x^2,x]
```

```
[Out] (-4*a^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/x - (5*a^2*d*CoshIntegral[(5*d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(5*c)/2 + (I/4)*Pi]*Sqrt[a + I*a*Sinh[c + d*x]])/8 - (15*a^2*d*CoshIntegral[(3*d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(6*c - I*Pi)/4]*Sqrt[a + I*a*Sinh[c + d*x]])/8 + (5*a^2*d*CoshIntegral[(d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(2*c + I*Pi)/4]*Sqrt[a + I*a*Sinh[c + d*x]])/4 + (5*a^2*d*Cosh[(2*c + I*Pi)/4]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(d*x)/2])/4 - (15*a^2*d*Cosh[(6*c - I*Pi)/4]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(3*d*x)/2])/8 - (5*a^2*d*Cosh[(5*c)/2 + (I/4)*Pi]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(5*d*x)/2])/8
```

+ d\*x]]\*SinhIntegral[(5\*d\*x)/2])/8

### Rule 3319

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_),  
x\_Symbol] :> Dist[((2\*a)^IntPart[n]\*(a + b\*SIN[e + f\*x])^FracPart[n])/Sin[  
e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*FracPart[n]), Int[(c + d\*x)^m\*SIN[e/2 + (a  
\*Pi)/(4\*b) + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E  
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])

### Rule 3313

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] :> Si  
mp[((c + d\*x)^(m + 1)\*Sin[e + f\*x]^n)/(d\*(m + 1)), x] - Dist[(f\*n)/(d\*(m +  
1)), Int[ExpandTrigReduce[(c + d\*x)^(m + 1), Cos[e + f\*x]\*Sin[e + f\*x]^(n -  
1), x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&  
LtQ[m, -1]

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*  
e - c\*f)/d], Int[SIN[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[SIN[(d\*e - c\*f  
)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] &&  
NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbo  
l] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f  
, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbo  
l] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz  
}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{(a + ia \sinh(c + dx))^{5/2}}{x^2} dx &= \left( 4a^2 \operatorname{csch} \left( \frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2} \right) \sqrt{a + ia \sinh(c + dx)} \right) \int \frac{\sinh^5 \left( \frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2} \right)}{x^2} dx \\
&= -\frac{4a^2 \cosh^4 \left( \frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2} \right) \sqrt{a + ia \sinh(c + dx)}}{x} + \left( 10a^2 d \operatorname{csch} \left( \frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2} \right) \sqrt{a + ia \sinh(c + dx)} \right) \int \frac{1}{x} dx \\
&= -\frac{4a^2 \cosh^4 \left( \frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2} \right) \sqrt{a + ia \sinh(c + dx)}}{x} - \frac{1}{8} \left( 5a^2 d \operatorname{csch} \left( \frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2} \right) \sqrt{a + ia \sinh(c + dx)} \right) \int \frac{1}{x} dx \\
&= -\frac{4a^2 \cosh^4 \left( \frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2} \right) \sqrt{a + ia \sinh(c + dx)}}{x} + \frac{1}{8} \left( 5ia^2 d \cosh \left( \frac{5c}{2} + \frac{i\pi}{4} \right) \operatorname{csch} \left( \frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2} \right) \right) \int \frac{1}{x} dx \\
&= -\frac{4a^2 \cosh^4 \left( \frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2} \right) \sqrt{a + ia \sinh(c + dx)}}{x} - \frac{5}{8} a^2 d \operatorname{Chi} \left( \frac{5dx}{2} \right) \operatorname{sech} \left( \frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2} \right) \int \frac{1}{x} dx
\end{aligned}$$

**Mathematica [A]** time = 2.43392, size = 347, normalized size = 0.78

$$a^2(\sinh(c + dx) - i)^2 \sqrt{a + ia \sinh(c + dx)} \left( 5dx \sinh \left( \frac{5c}{2} \right) \operatorname{Chi} \left( \frac{5dx}{2} \right) + 5idx \cosh \left( \frac{5c}{2} \right) \operatorname{Chi} \left( \frac{5dx}{2} \right) - 10idx \left( \cosh \left( \frac{c}{2} \right) - i \sinh \left( \frac{c}{2} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + I\*a\*Sinh[c + d\*x])^(5/2)/x^2,x]

[Out] (a^2\*(-I + Sinh[c + d\*x])^2\*Sqrt[a + I\*a\*Sinh[c + d\*x]]\*(20\*Cosh[(c + d\*x)/2] - 10\*Cosh[(3\*(c + d\*x))/2] - 2\*Cosh[(5\*(c + d\*x))/2] + (5\*I)\*d\*x\*Cosh[(5\*c)/2]\*CoshIntegral[(5\*d\*x)/2] - (10\*I)\*d\*x\*CoshIntegral[(d\*x)/2]\*(Cosh[c/2] - I\*Sinh[c/2]) + 15\*d\*x\*CoshIntegral[(3\*d\*x)/2]\*((-I)\*Cosh[(3\*c)/2] + Sinh[(3\*c)/2]) + 5\*d\*x\*CoshIntegral[(5\*d\*x)/2]\*Sinh[(5\*c)/2] + (20\*I)\*Sinh[(c + d\*x)/2] + (10\*I)\*Sinh[(3\*(c + d\*x))/2] - (2\*I)\*Sinh[(5\*(c + d\*x))/2] - 10\*d\*x\*Cosh[c/2]\*SinhIntegral[(d\*x)/2] - (10\*I)\*d\*x\*Sinh[c/2]\*SinhIntegral[(d\*x)/2] + 15\*d\*x\*Cosh[(3\*c)/2]\*SinhIntegral[(3\*d\*x)/2] - (15\*I)\*d\*x\*Sinh[(3\*c)/2]\*SinhIntegral[(3\*d\*x)/2] + 5\*d\*x\*Cosh[(5\*c)/2]\*SinhIntegral[(5\*d\*x)/2] + (5\*I)\*d\*x\*Sinh[(5\*c)/2]\*SinhIntegral[(5\*d\*x)/2]))/(8\*x\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^5)

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (a + ia \sinh(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*sinh(d\*x+c))^(5/2)/x^2,x)

[Out] int((a+I\*a\*sinh(d\*x+c))^(5/2)/x^2,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia \sinh(dx + c) + a)^{\frac{5}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(d\*x+c))^(5/2)/x^2,x, algorithm="maxima")

[Out] integrate((I\*a\*sinh(d\*x + c) + a)^(5/2)/x^2, x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(d\*x+c))^(5/2)/x^2,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))**(5/2)/x**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \sinh(dx + c) + a)^{\frac{5}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)/x^2, x)
```

$$3.135 \quad \int \frac{(a+ia \sinh(c+dx))^{5/2}}{x^3} dx$$

**Optimal.** Leaf size=536

$$-\frac{25}{32}ia^2d^2 \sinh\left(\frac{5c}{2} - \frac{i\pi}{4}\right) \text{Chi}\left(\frac{5dx}{2}\right) \text{sech}\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) \sqrt{a+ia \sinh(c+dx)} + \frac{5}{16}ia^2d^2 \sinh\left(\frac{1}{4}(2c-i\pi)\right) \text{Chi}\left(\frac{dx}{2}\right) \text{sech}\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) \sqrt{a+ia \sinh(c+dx)}$$

```
[Out] (-2*a^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/x^2 -
  ((25*I)/32)*a^2*d^2*CoshIntegral[(5*d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]
  *Sinh[(5*c)/2 - (I/4)*Pi]*Sqrt[a + I*a*Sinh[c + d*x]] + ((5*I)/16)*a^2*d^2*
  CoshIntegral[(d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(2*c - I*Pi)/4]*S
  qrt[a + I*a*Sinh[c + d*x]] + ((45*I)/32)*a^2*d^2*CoshIntegral[(3*d*x)/2]*Se
  ch[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(6*c + I*Pi)/4]*Sqrt[a + I*a*Sinh[c + d*x
  ]] - (5*a^2*d*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3*Sinh[c/2 + (I/4)*Pi + (d*x)/
  2]*Sqrt[a + I*a*Sinh[c + d*x]])/x + ((5*I)/16)*a^2*d^2*Cosh[(2*c - I*Pi)/4]
  *Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(d
  *x)/2] + ((45*I)/32)*a^2*d^2*Cosh[(6*c + I*Pi)/4]*Sech[c/2 + (I/4)*Pi + (d*
  x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(3*d*x)/2] - ((25*I)/32)*a^2
  *d^2*Cosh[(5*c)/2 - (I/4)*Pi]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*S
  inh[c + d*x]]*SinhIntegral[(5*d*x)/2]
```

---

**Rubi [A]** time = 0.638273, antiderivative size = 536, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3319, 3314, 3312, 3303, 3298, 3301}

$$-\frac{25}{32}ia^2d^2 \sinh\left(\frac{5c}{2} - \frac{i\pi}{4}\right) \text{Chi}\left(\frac{5dx}{2}\right) \text{sech}\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) \sqrt{a+ia \sinh(c+dx)} + \frac{5}{16}ia^2d^2 \sinh\left(\frac{1}{4}(2c-i\pi)\right) \text{Chi}\left(\frac{dx}{2}\right) \text{sech}\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) \sqrt{a+ia \sinh(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + I*a*Sinh[c + d*x])^(5/2)/x^3,x]
```

```
[Out] (-2*a^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^4*Sqrt[a + I*a*Sinh[c + d*x]])/x^2 -
  ((25*I)/32)*a^2*d^2*CoshIntegral[(5*d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]
  *Sinh[(5*c)/2 - (I/4)*Pi]*Sqrt[a + I*a*Sinh[c + d*x]] + ((5*I)/16)*a^2*d^2*
  CoshIntegral[(d*x)/2]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(2*c - I*Pi)/4]*S
  qrt[a + I*a*Sinh[c + d*x]] + ((45*I)/32)*a^2*d^2*CoshIntegral[(3*d*x)/2]*Se
  ch[c/2 + (I/4)*Pi + (d*x)/2]*Sinh[(6*c + I*Pi)/4]*Sqrt[a + I*a*Sinh[c + d*x
  ]] - (5*a^2*d*Cosh[c/2 + (I/4)*Pi + (d*x)/2]^3*Sinh[c/2 + (I/4)*Pi + (d*x)/
  2]*Sqrt[a + I*a*Sinh[c + d*x]])/x + ((5*I)/16)*a^2*d^2*Cosh[(2*c - I*Pi)/4]
```



```
*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(d
*x)/2] + ((45*I)/32)*a^2*d^2*Cosh[(6*c + I*Pi)/4]*Sech[c/2 + (I/4)*Pi + (d*
x)/2]*Sqrt[a + I*a*Sinh[c + d*x]]*SinhIntegral[(3*d*x)/2] - ((25*I)/32)*a^2
*d^2*Cosh[(5*c)/2 - (I/4)*Pi]*Sech[c/2 + (I/4)*Pi + (d*x)/2]*Sqrt[a + I*a*S
inh[c + d*x]]*SinhIntegral[(5*d*x)/2]
```

### Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
  x_Symbol] := Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
 := Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

### Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
 := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{(a + ia \sinh(c + dx))^{5/2}}{x^3} dx &= \left( 4a^2 \operatorname{csch} \left( \frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2} \right) \sqrt{a + ia \sinh(c + dx)} \right) \int \frac{\sinh^5 \left( \frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2} \right)}{x^3} dx \\ &= -\frac{2a^2 \cosh^4 \left( \frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2} \right) \sqrt{a + ia \sinh(c + dx)}}{x^2} - \frac{5a^2 d \cosh^3 \left( \frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2} \right) \sinh \left( \frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2} \right)}{x} \\ &= -\frac{2a^2 \cosh^4 \left( \frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2} \right) \sqrt{a + ia \sinh(c + dx)}}{x^2} - \frac{5a^2 d \cosh^3 \left( \frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2} \right) \sinh \left( \frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2} \right)}{x} \\ &= -\frac{2a^2 \cosh^4 \left( \frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2} \right) \sqrt{a + ia \sinh(c + dx)}}{x^2} - \frac{5a^2 d \cosh^3 \left( \frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2} \right) \sinh \left( \frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2} \right)}{x} \\ &= -\frac{2a^2 \cosh^4 \left( \frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2} \right) \sqrt{a + ia \sinh(c + dx)}}{x^2} - \frac{5a^2 d \cosh^3 \left( \frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2} \right) \sinh \left( \frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2} \right)}{x} \\ &= -\frac{2a^2 \cosh^4 \left( \frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2} \right) \sqrt{a + ia \sinh(c + dx)}}{x^2} - \frac{25}{32} ia^2 d^2 \operatorname{Chi} \left( \frac{5dx}{2} \right) \operatorname{sech} \left( \frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2} \right) \end{aligned}$$

**Mathematica [B]** time = 7.68558, size = 4751, normalized size = 8.86

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(a + I*a*Sinh[c + d*x])^(5/2)/x^3,x]
```

```
[Out] (2*((1/128 + I/128)*Cosh[5*(c/2 + (d*x)/2)] - (1/128 + I/128)*Sinh[5*(c/2 + (d*x)/2)])*(a + I*a*Sinh[c + d*x])^(5/2)*((-4*I)*d^3 - (10*I)*c*d^3 + (20*I)*d^3*(c/2 + (d*x)/2) + 20*d^3*Cosh[2*(c/2 + (d*x)/2)] + 30*c*d^3*Cosh[2*(c/2 + (d*x)/2)] - 60*d^3*(c/2 + (d*x)/2)*Cosh[2*(c/2 + (d*x)/2)] + (40*I)*d^3*Cosh[4*(c/2 + (d*x)/2)] + (20*I)*c*d^3*Cosh[4*(c/2 + (d*x)/2)] - (40*I)*d^3*(c/2 + (d*x)/2)*Cosh[4*(c/2 + (d*x)/2)] - 40*d^3*Cosh[6*(c/2 + (d*x)/2)] + 20*c*d^3*Cosh[6*(c/2 + (d*x)/2)] - 40*d^3*(c/2 + (d*x)/2)*Cosh[6*(c/2 + (d*x)/2)] - (20*I)*d^3*Cosh[8*(c/2 + (d*x)/2)] + (30*I)*c*d^3*Cosh[8*(c/2
```

$$\begin{aligned}
& + (d*x)/2] - (60*I)*d^3*(c/2 + (d*x)/2)*Cosh[8*(c/2 + (d*x)/2)] + 4*d^3*Co \\
& sh[10*(c/2 + (d*x)/2)] - 10*c*d^3*Cosh[10*(c/2 + (d*x)/2)] + 20*d^3*(c/2 + \\
& (d*x)/2)*Cosh[10*(c/2 + (d*x)/2)] - (10*I)*c^2*d^3*Cosh[c/2 - 5*(c/2 + (d*x \\
& )/2)]*CoshIntegral[(d*x)/2] + (40*I)*c*d^3*(c/2 + (d*x)/2)*Cosh[c/2 - 5*(c/ \\
& 2 + (d*x)/2)]*CoshIntegral[(d*x)/2] - (40*I)*d^3*(c/2 + (d*x)/2)^2*Cosh[c/2 \\
& - 5*(c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] + 10*c^2*d^3*Cosh[c/2 + 5*(c/2 \\
& + (d*x)/2)]*CoshIntegral[(d*x)/2] - 40*c*d^3*(c/2 + (d*x)/2)*Cosh[c/2 + 5*( \\
& c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] + 40*d^3*(c/2 + (d*x)/2)^2*Cosh[c/2 + \\
& 5*(c/2 + (d*x)/2)]*CoshIntegral[(d*x)/2] - 45*c^2*d^3*Cosh[(3*c)/2 - 5*(c/ \\
& 2 + (d*x)/2)]*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)] + 180*c*d^3*(c/2 + \\
& (d*x)/2)*Cosh[(3*c)/2 - 5*(c/2 + (d*x)/2)]*CoshIntegral[(-3*c)/2 + 3*(c/2 \\
& + (d*x)/2)] - 180*d^3*(c/2 + (d*x)/2)^2*Cosh[(3*c)/2 - 5*(c/2 + (d*x)/2)]*C \\
& oshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)] + (45*I)*c^2*d^3*Cosh[(3*c)/2 + 5 \\
& *(c/2 + (d*x)/2)]*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)] - (180*I)*c*d^ \\
& 3*(c/2 + (d*x)/2)*Cosh[(3*c)/2 + 5*(c/2 + (d*x)/2)]*CoshIntegral[(-3*c)/2 + \\
& 3*(c/2 + (d*x)/2)] + (180*I)*d^3*(c/2 + (d*x)/2)^2*Cosh[(3*c)/2 + 5*(c/2 + \\
& (d*x)/2)]*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)] + (25*I)*c^2*d^3*Cosh \\
& [(5*c)/2 - 5*(c/2 + (d*x)/2)]*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)] - \\
& (100*I)*c*d^3*(c/2 + (d*x)/2)*Cosh[(5*c)/2 - 5*(c/2 + (d*x)/2)]*CoshIntegra \\
& l[(-5*c)/2 + 5*(c/2 + (d*x)/2)] + (100*I)*d^3*(c/2 + (d*x)/2)^2*Cosh[(5*c)/ \\
& 2 - 5*(c/2 + (d*x)/2)]*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)] - 25*c^2* \\
& d^3*Cosh[(5*c)/2 + 5*(c/2 + (d*x)/2)]*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x \\
& )/2)] + 100*c*d^3*(c/2 + (d*x)/2)*Cosh[(5*c)/2 + 5*(c/2 + (d*x)/2)]*CoshInt \\
& egral[(-5*c)/2 + 5*(c/2 + (d*x)/2)] - 100*d^3*(c/2 + (d*x)/2)^2*Cosh[(5*c)/ \\
& 2 + 5*(c/2 + (d*x)/2)]*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)] + 20*d^3* \\
& Sinh[2*(c/2 + (d*x)/2)] + 30*c*d^3*Sinh[2*(c/2 + (d*x)/2)] - 60*d^3*(c/2 + \\
& (d*x)/2)*Sinh[2*(c/2 + (d*x)/2)] + (40*I)*d^3*Sinh[4*(c/2 + (d*x)/2)] + (20 \\
& *I)*c*d^3*Sinh[4*(c/2 + (d*x)/2)] - (40*I)*d^3*(c/2 + (d*x)/2)*Sinh[4*(c/2 \\
& + (d*x)/2)] - 40*d^3*Sinh[6*(c/2 + (d*x)/2)] + 20*c*d^3*Sinh[6*(c/2 + (d*x) \\
& /2)] - 40*d^3*(c/2 + (d*x)/2)*Sinh[6*(c/2 + (d*x)/2)] - (20*I)*d^3*Sinh[8*( \\
& c/2 + (d*x)/2)] + (30*I)*c*d^3*Sinh[8*(c/2 + (d*x)/2)] - (60*I)*d^3*(c/2 + \\
& (d*x)/2)*Sinh[8*(c/2 + (d*x)/2)] + 4*d^3*Sinh[10*(c/2 + (d*x)/2)] - 10*c*d^ \\
& 3*Sinh[10*(c/2 + (d*x)/2)] + 20*d^3*(c/2 + (d*x)/2)*Sinh[10*(c/2 + (d*x)/2) \\
& ] + (10*I)*c^2*d^3*CoshIntegral[(d*x)/2]*Sinh[c/2 - 5*(c/2 + (d*x)/2)] - (4 \\
& 0*I)*c*d^3*(c/2 + (d*x)/2)*CoshIntegral[(d*x)/2]*Sinh[c/2 - 5*(c/2 + (d*x)/ \\
& 2)] + (40*I)*d^3*(c/2 + (d*x)/2)^2*CoshIntegral[(d*x)/2]*Sinh[c/2 - 5*(c/2 \\
& + (d*x)/2)] + 45*c^2*d^3*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)]*Sinh[(3 \\
& *c)/2 - 5*(c/2 + (d*x)/2)] - 180*c*d^3*(c/2 + (d*x)/2)*CoshIntegral[(-3*c)/ \\
& 2 + 3*(c/2 + (d*x)/2)]*Sinh[(3*c)/2 - 5*(c/2 + (d*x)/2)] + 180*d^3*(c/2 + ( \\
& d*x)/2)^2*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)]*Sinh[(3*c)/2 - 5*(c/2 \\
& + (d*x)/2)] - (25*I)*c^2*d^3*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)]*Sin \\
& h[(5*c)/2 - 5*(c/2 + (d*x)/2)] + (100*I)*c*d^3*(c/2 + (d*x)/2)*CoshIntegral \\
& [(-5*c)/2 + 5*(c/2 + (d*x)/2)]*Sinh[(5*c)/2 - 5*(c/2 + (d*x)/2)] - (100*I)* \\
& d^3*(c/2 + (d*x)/2)^2*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)]*Sinh[(5*c) \\
& /2 - 5*(c/2 + (d*x)/2)] + 10*c^2*d^3*CoshIntegral[(d*x)/2]*Sinh[c/2 + 5*(c/
\end{aligned}$$

$$\begin{aligned}
& 2 + (d*x)/2] - 40*c*d^3*(c/2 + (d*x)/2)*CoshIntegral[(d*x)/2]*Sinh[c/2 + 5 \\
& *(c/2 + (d*x)/2)] + 40*d^3*(c/2 + (d*x)/2)^2*CoshIntegral[(d*x)/2]*Sinh[c/2 \\
& + 5*(c/2 + (d*x)/2)] + (45*I)*c^2*d^3*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d* \\
& x)/2)]*Sinh[(3*c)/2 + 5*(c/2 + (d*x)/2)] - (180*I)*c*d^3*(c/2 + (d*x)/2)*Co \\
& shIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)]*Sinh[(3*c)/2 + 5*(c/2 + (d*x)/2)] \\
& + (180*I)*d^3*(c/2 + (d*x)/2)^2*CoshIntegral[(-3*c)/2 + 3*(c/2 + (d*x)/2)]* \\
& Sinh[(3*c)/2 + 5*(c/2 + (d*x)/2)] - 25*c^2*d^3*CoshIntegral[(-5*c)/2 + 5*(c \\
& /2 + (d*x)/2)]*Sinh[(5*c)/2 + 5*(c/2 + (d*x)/2)] + 100*c*d^3*(c/2 + (d*x)/2 \\
& )*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)]*Sinh[(5*c)/2 + 5*(c/2 + (d*x)/ \\
& 2)] - 100*d^3*(c/2 + (d*x)/2)^2*CoshIntegral[(-5*c)/2 + 5*(c/2 + (d*x)/2)]* \\
& Sinh[(5*c)/2 + 5*(c/2 + (d*x)/2)] + (10*I)*c^2*d^3*Cosh[c/2 - 5*(c/2 + (d*x \\
& )/2)]*SinhIntegral[(d*x)/2] - (40*I)*c*d^3*(c/2 + (d*x)/2)*Cosh[c/2 - 5*(c/ \\
& 2 + (d*x)/2)]*SinhIntegral[(d*x)/2] + (40*I)*d^3*(c/2 + (d*x)/2)^2*Cosh[c/2 \\
& - 5*(c/2 + (d*x)/2)]*SinhIntegral[(d*x)/2] + 10*c^2*d^3*Cosh[c/2 + 5*(c/2 \\
& + (d*x)/2)]*SinhIntegral[(d*x)/2] - 40*c*d^3*(c/2 + (d*x)/2)*Cosh[c/2 + 5*( \\
& c/2 + (d*x)/2)]*SinhIntegral[(d*x)/2] + 40*d^3*(c/2 + (d*x)/2)^2*Cosh[c/2 + \\
& 5*(c/2 + (d*x)/2)]*SinhIntegral[(d*x)/2] - (10*I)*c^2*d^3*Sinh[c/2 - 5*(c/ \\
& 2 + (d*x)/2)]*SinhIntegral[(d*x)/2] + (40*I)*c*d^3*(c/2 + (d*x)/2)*Sinh[c/2 \\
& - 5*(c/2 + (d*x)/2)]*SinhIntegral[(d*x)/2] - (40*I)*d^3*(c/2 + (d*x)/2)^2* \\
& Sinh[c/2 - 5*(c/2 + (d*x)/2)]*SinhIntegral[(d*x)/2] + 10*c^2*d^3*Sinh[c/2 + \\
& 5*(c/2 + (d*x)/2)]*SinhIntegral[(d*x)/2] - 40*c*d^3*(c/2 + (d*x)/2)*Sinh[c \\
& /2 + 5*(c/2 + (d*x)/2)]*SinhIntegral[(d*x)/2] + 40*d^3*(c/2 + (d*x)/2)^2*Si \\
& nh[c/2 + 5*(c/2 + (d*x)/2)]*SinhIntegral[(d*x)/2] + (25*I)*c^2*d^3*Cosh[(5* \\
& c)/2 - 5*(c/2 + (d*x)/2)]*SinhIntegral[(5*c)/2 - 5*(c/2 + (d*x)/2)] - (100* \\
& I)*c*d^3*(c/2 + (d*x)/2)*Cosh[(5*c)/2 - 5*(c/2 + (d*x)/2)]*SinhIntegral[(5* \\
& c)/2 - 5*(c/2 + (d*x)/2)] + (100*I)*d^3*(c/2 + (d*x)/2)^2*Cosh[(5*c)/2 - 5* \\
& (c/2 + (d*x)/2)]*SinhIntegral[(5*c)/2 - 5*(c/2 + (d*x)/2)] + 25*c^2*d^3*Cos \\
& h[(5*c)/2 + 5*(c/2 + (d*x)/2)]*SinhIntegral[(5*c)/2 - 5*(c/2 + (d*x)/2)] - \\
& 100*c*d^3*(c/2 + (d*x)/2)*Cosh[(5*c)/2 + 5*(c/2 + (d*x)/2)]*SinhIntegral[(5 \\
& *c)/2 - 5*(c/2 + (d*x)/2)] + 100*d^3*(c/2 + (d*x)/2)^2*Cosh[(5*c)/2 + 5*(c/ \\
& 2 + (d*x)/2)]*SinhIntegral[(5*c)/2 - 5*(c/2 + (d*x)/2)] - (25*I)*c^2*d^3*Si \\
& nh[(5*c)/2 - 5*(c/2 + (d*x)/2)]*SinhIntegral[(5*c)/2 - 5*(c/2 + (d*x)/2)] + \\
& (100*I)*c*d^3*(c/2 + (d*x)/2)*Sinh[(5*c)/2 - 5*(c/2 + (d*x)/2)]*SinhIntegr \\
& al[(5*c)/2 - 5*(c/2 + (d*x)/2)] - (100*I)*d^3*(c/2 + (d*x)/2)^2*Sinh[(5*c)/ \\
& 2 - 5*(c/2 + (d*x)/2)]*SinhIntegral[(5*c)/2 - 5*(c/2 + (d*x)/2)] + 25*c^2*d \\
& ^3*Sinh[(5*c)/2 + 5*(c/2 + (d*x)/2)]*SinhIntegral[(5*c)/2 - 5*(c/2 + (d*x)/ \\
& 2)] - 100*c*d^3*(c/2 + (d*x)/2)*Sinh[(5*c)/2 + 5*(c/2 + (d*x)/2)]*SinhIntegr \\
& al[(5*c)/2 - 5*(c/2 + (d*x)/2)] + 100*d^3*(c/2 + (d*x)/2)^2*Sinh[(5*c)/2 + \\
& 5*(c/2 + (d*x)/2)]*SinhIntegral[(5*c)/2 - 5*(c/2 + (d*x)/2)] - 45*c^2*d^3* \\
& Cosh[(3*c)/2 - 5*(c/2 + (d*x)/2)]*SinhIntegral[(3*c)/2 - 3*(c/2 + (d*x)/2)] \\
& + 180*c*d^3*(c/2 + (d*x)/2)*Cosh[(3*c)/2 - 5*(c/2 + (d*x)/2)]*SinhIntegral \\
& [(3*c)/2 - 3*(c/2 + (d*x)/2)] - 180*d^3*(c/2 + (d*x)/2)^2*Cosh[(3*c)/2 - 5* \\
& (c/2 + (d*x)/2)]*SinhIntegral[(3*c)/2 - 3*(c/2 + (d*x)/2)] - (45*I)*c^2*d^3 \\
& *Cosh[(3*c)/2 + 5*(c/2 + (d*x)/2)]*SinhIntegral[(3*c)/2 - 3*(c/2 + (d*x)/2) \\
& ] + (180*I)*c*d^3*(c/2 + (d*x)/2)*Cosh[(3*c)/2 + 5*(c/2 + (d*x)/2)]*SinhInt
\end{aligned}$$

```

egral[(3*c)/2 - 3*(c/2 + (d*x)/2)] - (180*I)*d^3*(c/2 + (d*x)/2)^2*Cosh[(3*
c)/2 + 5*(c/2 + (d*x)/2)]*SinhIntegral[(3*c)/2 - 3*(c/2 + (d*x)/2)] + 45*c^
2*d^3*Sinh[(3*c)/2 - 5*(c/2 + (d*x)/2)]*SinhIntegral[(3*c)/2 - 3*(c/2 + (d*
x)/2)] - 180*c*d^3*(c/2 + (d*x)/2)*Sinh[(3*c)/2 - 5*(c/2 + (d*x)/2)]*SinhIn
tegral[(3*c)/2 - 3*(c/2 + (d*x)/2)] + 180*d^3*(c/2 + (d*x)/2)^2*Sinh[(3*c)/
2 - 5*(c/2 + (d*x)/2)]*SinhIntegral[(3*c)/2 - 3*(c/2 + (d*x)/2)] - (45*I)*c
^2*d^3*Sinh[(3*c)/2 + 5*(c/2 + (d*x)/2)]*SinhIntegral[(3*c)/2 - 3*(c/2 + (d
*x)/2)] + (180*I)*c*d^3*(c/2 + (d*x)/2)*Sinh[(3*c)/2 + 5*(c/2 + (d*x)/2)]*S
inhIntegral[(3*c)/2 - 3*(c/2 + (d*x)/2)] - (180*I)*d^3*(c/2 + (d*x)/2)^2*Si
nh[(3*c)/2 + 5*(c/2 + (d*x)/2)]*SinhIntegral[(3*c)/2 - 3*(c/2 + (d*x)/2)])
/(d*(-c + 2*(c/2 + (d*x)/2))^2*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2]
)^5)

```

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{x^3} (a + ia \sinh(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+I*a*sinh(d*x+c))^(5/2)/x^3,x)
```

```
[Out] int((a+I*a*sinh(d*x+c))^(5/2)/x^3,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia \sinh(dx + c) + a)^{\frac{5}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)/x^3, x)
```

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))**(5/2)/x**3,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \sinh(dx + c) + a)^{\frac{5}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+I*a*sinh(d*x+c))^(5/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(d*x + c) + a)^(5/2)/x^3, x)
```

$$3.136 \quad \int \frac{x^3}{\sqrt{a+ia \sinh(e+fx)}} dx$$

**Optimal.** Leaf size=493

$$\frac{12ix^2 \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, -e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{f^2 \sqrt{a+ia \sinh(e+fx)}} - \frac{12ix^2 \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{f^2 \sqrt{a+ia \sinh(e+fx)}} - \frac{48ix \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{f^2 \sqrt{a+ia \sinh(e+fx)}}$$

```
[Out] ((4*I)*x^3*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/(f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((12*I)*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((12*I)*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((48*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^3*Sqrt[a + I*a*Sinh[e + f*x]]) + ((48*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^3*Sqrt[a + I*a*Sinh[e + f*x]]) + ((96*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[4, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^4*Sqrt[a + I*a*Sinh[e + f*x]]) - ((96*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[4, E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^4*Sqrt[a + I*a*Sinh[e + f*x]])]
```

**Rubi [A]** time = 0.260732, antiderivative size = 493, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {3319, 4182, 2531, 6609, 2282, 6589}

$$\frac{12ix^2 \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, -e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{f^2 \sqrt{a+ia \sinh(e+fx)}} - \frac{12ix^2 \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{f^2 \sqrt{a+ia \sinh(e+fx)}} - \frac{48ix \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{f^2 \sqrt{a+ia \sinh(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] ((4*I)*x^3*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/(f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((12*I)*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((12*I)*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((48*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^3*Sqrt[a + I*a*Sinh[e + f*x]]) + ((48*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLo
```

```
g[3, E^((2*e - I*Pi)/4 + (f*x)/2)]/(f^3*Sqrt[a + I*a*Sinh[e + f*x]]) + ((9
6*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[4, -E^((2*e - I*Pi)/4 + (f*x)/2
)]/(f^4*Sqrt[a + I*a*Sinh[e + f*x]]) - ((96*I)*Cosh[e/2 + (I/4)*Pi + (f*x)
/2]*PolyLog[4, E^((2*e - I*Pi)/4 + (f*x)/2)]/(f^4*Sqrt[a + I*a*Sinh[e + f
x]]))
```

### Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sinh[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sinh[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x))))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x))))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_.))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```



(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int [PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp [PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int \frac{x^3}{\sqrt{a + ia \sinh(e + fx)}} dx &= \frac{\sinh\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) \int x^3 \operatorname{csch}\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) dx}{\sqrt{a + ia \sinh(e + fx)}} \\
 &= \frac{4ix^3 \tanh^{-1}\left(e^{\frac{1}{4}(2e - i\pi) + \frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{f\sqrt{a + ia \sinh(e + fx)}} - \frac{\left(6 \sinh\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right)\right) \int x^2 \log\left(1 - e^{-i\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right)}\right) dx}{f\sqrt{a + ia \sinh(e + fx)}} \\
 &= \frac{4ix^3 \tanh^{-1}\left(e^{\frac{1}{4}(2e - i\pi) + \frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{f\sqrt{a + ia \sinh(e + fx)}} + \frac{12ix^2 \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \operatorname{Li}_2\left(-e^{\frac{1}{4}(2e - i\pi)}\right)}{f^2\sqrt{a + ia \sinh(e + fx)}} \\
 &= \frac{4ix^3 \tanh^{-1}\left(e^{\frac{1}{4}(2e - i\pi) + \frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{f\sqrt{a + ia \sinh(e + fx)}} + \frac{12ix^2 \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \operatorname{Li}_2\left(-e^{\frac{1}{4}(2e - i\pi)}\right)}{f^2\sqrt{a + ia \sinh(e + fx)}} \\
 &= \frac{4ix^3 \tanh^{-1}\left(e^{\frac{1}{4}(2e - i\pi) + \frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{f\sqrt{a + ia \sinh(e + fx)}} + \frac{12ix^2 \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \operatorname{Li}_2\left(-e^{\frac{1}{4}(2e - i\pi)}\right)}{f^2\sqrt{a + ia \sinh(e + fx)}} \\
 &= \frac{4ix^3 \tanh^{-1}\left(e^{\frac{1}{4}(2e - i\pi) + \frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{f\sqrt{a + ia \sinh(e + fx)}} + \frac{12ix^2 \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \operatorname{Li}_2\left(-e^{\frac{1}{4}(2e - i\pi)}\right)}{f^2\sqrt{a + ia \sinh(e + fx)}}
 \end{aligned}$$

**Mathematica [A]** time = 1.27041, size = 331, normalized size = 0.67

$$\frac{(1 - i)(-1)^{3/4} \left( \cosh\left(\frac{1}{2}(e + fx)\right) + i \sinh\left(\frac{1}{2}(e + fx)\right) \right) \left( -6f^2x^2 \operatorname{PolyLog}\left(2, -(-1)^{3/4} e^{\frac{1}{2}(e + fx)}\right) + 6f^2x^2 \operatorname{PolyLog}\left(2, (-1)^3\right) \right)}{f^2\sqrt{a + ia \sinh(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[a + I\*a\*Sinh[e + f\*x]], x]

```
[Out] ((1 - I)*(-1)^(3/4)*((2*I)*e^3*ArcTan[(-1)^(1/4)*E^((e + f*x)/2)] + e^3*Log
[1 - (-1)^(3/4)*E^((e + f*x)/2)] + f^3*x^3*Log[1 - (-1)^(3/4)*E^((e + f*x)/
2)] - e^3*Log[1 + (-1)^(3/4)*E^((e + f*x)/2)] - f^3*x^3*Log[1 + (-1)^(3/4)*
E^((e + f*x)/2)] - 6*f^2*x^2*PolyLog[2, -((-1)^(3/4)*E^((e + f*x)/2))] + 6*
f^2*x^2*PolyLog[2, (-1)^(3/4)*E^((e + f*x)/2)] + 24*f*x*PolyLog[3, -((-1)^(
3/4)*E^((e + f*x)/2))] - 24*f*x*PolyLog[3, (-1)^(3/4)*E^((e + f*x)/2)] - 48
*PolyLog[4, -((-1)^(3/4)*E^((e + f*x)/2))] + 48*PolyLog[4, (-1)^(3/4)*E^((e
+ f*x)/2)]*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))/(f^4*Sqrt[a + I*a*S
inh[e + f*x]])
```

**Maple [F]** time = 0.201, size = 0, normalized size = 0.

$$\int x^3 \frac{1}{\sqrt{a + ia \sinh(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+I*a*sinh(f*x+e))^(1/2),x)
```

```
[Out] int(x^3/(a+I*a*sinh(f*x+e))^(1/2),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{ia \sinh(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/sqrt(I*a*sinh(f*x + e) + a), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{2i \sqrt{\frac{1}{2}} \sqrt{ia e^{(2fx+2e)} + 2ae^{(fx+e)} - ia x^3 e^{\left(\frac{1}{2}fx + \frac{1}{2}e\right)}}}{ae^{(2fx+2e)} - 2iae^{(fx+e)} - a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")`

[Out] `integral(-2*I*sqrt(1/2)*sqrt(I*a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*x^3*e^(1/2*f*x + 1/2*e)/(a*e^(2*f*x + 2*e) - 2*I*a*e^(f*x + e) - a), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a(i \sinh(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(a+I*a*sinh(f*x+e))**(1/2),x)`

[Out] `Integral(x**3/sqrt(a*(I*sinh(e + f*x) + 1)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{ia \sinh(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")`

[Out] `integrate(x^3/sqrt(I*a*sinh(f*x + e) + a), x)`

$$3.137 \quad \int \frac{x^2}{\sqrt{a+ia \sinh(e+fx)}} dx$$

**Optimal.** Leaf size=349

$$\frac{8ix \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, -e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{f^2 \sqrt{a+ia \sinh(e+fx)}} - \frac{8ix \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{f^2 \sqrt{a+ia \sinh(e+fx)}} - \frac{16i \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(3, -e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{f^3 \sqrt{a+ia \sinh(e+fx)}} + \frac{16i \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(3, e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{f^3 \sqrt{a+ia \sinh(e+fx)}}$$

```
[Out] ((4*I)*x^2*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/(f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((8*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((8*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((16*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^3*Sqrt[a + I*a*Sinh[e + f*x]]) + ((16*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^3*Sqrt[a + I*a*Sinh[e + f*x]])
```

**Rubi [A]** time = 0.2019, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {3319, 4182, 2531, 2282, 6589}

$$\frac{8ix \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, -e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{f^2 \sqrt{a+ia \sinh(e+fx)}} - \frac{8ix \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{f^2 \sqrt{a+ia \sinh(e+fx)}} - \frac{16i \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(3, -e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{f^3 \sqrt{a+ia \sinh(e+fx)}} + \frac{16i \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(3, e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{f^3 \sqrt{a+ia \sinh(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] ((4*I)*x^2*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/(f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((8*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((8*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((16*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^3*Sqrt[a + I*a*Sinh[e + f*x]]) + ((16*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, E^((2*e - I*Pi)/4 + (f*x)/2)])/(f^3*Sqrt[a + I*a*Sinh[e + f*x]])
```

**Rule 3319**

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
  x_Symbol] := Dist[((2*a)^(IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{a + ia \sinh(e + fx)}} dx &= \frac{\sinh\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) \int x^2 \operatorname{csch}\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) dx}{\sqrt{a + ia \sinh(e + fx)}} \\
&= \frac{4ix^2 \tanh^{-1}\left(e^{\frac{1}{4}(2e - i\pi) + \frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{f\sqrt{a + ia \sinh(e + fx)}} - \frac{\left(4 \sinh\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right)\right) \int x \log\left(1 - e^{-i\left(\frac{ie}{2} + \frac{fx}{2}\right)}\right) dx}{f\sqrt{a + ia \sinh(e + fx)}} \\
&= \frac{4ix^2 \tanh^{-1}\left(e^{\frac{1}{4}(2e - i\pi) + \frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{f\sqrt{a + ia \sinh(e + fx)}} + \frac{8ix \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \operatorname{Li}_2\left(-e^{\frac{1}{4}(2e - i\pi) + \frac{fx}{2}}\right)}{f^2\sqrt{a + ia \sinh(e + fx)}} \\
&= \frac{4ix^2 \tanh^{-1}\left(e^{\frac{1}{4}(2e - i\pi) + \frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{f\sqrt{a + ia \sinh(e + fx)}} + \frac{8ix \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \operatorname{Li}_2\left(-e^{\frac{1}{4}(2e - i\pi) + \frac{fx}{2}}\right)}{f^2\sqrt{a + ia \sinh(e + fx)}} \\
&= \frac{4ix^2 \tanh^{-1}\left(e^{\frac{1}{4}(2e - i\pi) + \frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{f\sqrt{a + ia \sinh(e + fx)}} + \frac{8ix \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \operatorname{Li}_2\left(-e^{\frac{1}{4}(2e - i\pi) + \frac{fx}{2}}\right)}{f^2\sqrt{a + ia \sinh(e + fx)}}
\end{aligned}$$

**Mathematica [A]** time = 0.973604, size = 276, normalized size = 0.79

$$(1 + i)(-1)^{3/4} \left( \sinh\left(\frac{1}{2}(e + fx)\right) - i \cosh\left(\frac{1}{2}(e + fx)\right) \right) \left( -4fx \operatorname{PolyLog}\left(2, (-1)^{3/4} e^{\frac{1}{2}(e+fx)}\right) + 4fx \operatorname{PolyLog}\left(2, (-1)^{3/4} e^{\frac{1}{2}(e+fx)}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[a + I\*a\*Sinh[e + f\*x]],x]

[Out]  $((1 + I)(-1)^{3/4}((-2I)e^2 \operatorname{ArcTan}((-1)^{1/4}E^{(e + f*x)/2}) - e^2 \operatorname{Log}[1 - (-1)^{3/4}E^{(e + f*x)/2}] + f^2 x^2 \operatorname{Log}[1 - (-1)^{3/4}E^{(e + f*x)/2}] + e^2 \operatorname{Log}[1 + (-1)^{3/4}E^{(e + f*x)/2}] - f^2 x^2 \operatorname{Log}[1 + (-1)^{3/4}E^{(e + f*x)/2}] - 4fx \operatorname{PolyLog}[2, -((-1)^{3/4}E^{(e + f*x)/2}]) + 4fx \operatorname{PolyLog}[2, (-1)^{3/4}E^{(e + f*x)/2}] + 8 \operatorname{PolyLog}[3, -((-1)^{3/4}E^{(e + f*x)/2}]) - 8 \operatorname{PolyLog}[3, (-1)^{3/4}E^{(e + f*x)/2}]) * ((-I) \operatorname{Cosh}[(e + f*x)/2] + \operatorname{Sinh}[(e + f*x)/2])) / (f^3 \operatorname{Sqrt}[a + I*a*Sinh[e + f*x]])$

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int x^2 \frac{1}{\sqrt{a + ia \sinh(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a+I*a*sinh(f*x+e))^(1/2),x)`

[Out] `int(x^2/(a+I*a*sinh(f*x+e))^(1/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{ia \sinh(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^2/sqrt(I*a*sinh(f*x + e) + a), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{2i \sqrt{\frac{1}{2}} \sqrt{iae^{(2fx+2e)} + 2ae^{(fx+e)} - iax^2e^{\left(\frac{1}{2}fx + \frac{1}{2}e\right)}}}{ae^{(2fx+2e)} - 2iae^{(fx+e)} - a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")`

[Out] `integral(-2*I*sqrt(1/2)*sqrt(I*a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*x^2*e^(1/2*f*x + 1/2*e)/(a*e^(2*f*x + 2*e) - 2*I*a*e^(f*x + e) - a), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a(i \sinh(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a*(I*sinh(e + f*x) + 1)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{ia \sinh(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(I*a*sinh(f*x + e) + a), x)
```



$$3.138 \quad \int \frac{x}{\sqrt{a+ia \sinh(e+fx)}} dx$$

**Optimal.** Leaf size=207

$$\frac{4i \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, -e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{f^2 \sqrt{a+ia \sinh(e+fx)}} - \frac{4i \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{f^2 \sqrt{a+ia \sinh(e+fx)}} + \frac{4ix \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{f \sqrt{a+ia \sinh(e+fx)}}$$

```
[Out] ((4*I)*x*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/(f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((4*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)]/(f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((4*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)]/(f^2*Sqrt[a + I*a*Sinh[e + f*x]]])
```

**Rubi [A]** time = 0.104719, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {3319, 4182, 2279, 2391}

$$\frac{4i \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, -e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{f^2 \sqrt{a+ia \sinh(e+fx)}} - \frac{4i \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{f^2 \sqrt{a+ia \sinh(e+fx)}} + \frac{4ix \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{f \sqrt{a+ia \sinh(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] ((4*I)*x*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/(f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((4*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)]/(f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((4*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)]/(f^2*Sqrt[a + I*a*Sinh[e + f*x]]])
```

### Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\int \frac{x}{\sqrt{a + ia \sinh(e + fx)}} dx = \frac{\sinh\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) \int x \operatorname{csch}\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) dx}{\sqrt{a + ia \sinh(e + fx)}}$$

$$= \frac{4ix \tanh^{-1}\left(e^{\frac{1}{4}(2e - i\pi) + \frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{f\sqrt{a + ia \sinh(e + fx)}} - \frac{\left(2 \sinh\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right)\right) \int \log\left(1 - e^{-i\left(\frac{ie}{2} + \frac{\pi}{4}\right)}\right) dx}{f\sqrt{a + ia \sinh(e + fx)}}$$

$$= \frac{4ix \tanh^{-1}\left(e^{\frac{1}{4}(2e - i\pi) + \frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{f\sqrt{a + ia \sinh(e + fx)}} - \frac{\left(4 \sinh\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right)\right) \operatorname{Subst}\left(\int \frac{\log(1-x)}{x} dx\right)}{f^2\sqrt{a + ia \sinh(e + fx)}}$$

$$= \frac{4ix \tanh^{-1}\left(e^{\frac{1}{4}(2e - i\pi) + \frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{f\sqrt{a + ia \sinh(e + fx)}} + \frac{4i \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right) \operatorname{Li}_2\left(-e^{\frac{1}{4}(2e - i\pi) + \frac{fx}{2}}\right)}{f^2\sqrt{a + ia \sinh(e + fx)}}$$

**Mathematica [A]** time = 0.505013, size = 221, normalized size = 1.07

$$\frac{\sqrt{2} \left( \cosh\left(\frac{1}{2}(e + fx)\right) + i \sinh\left(\frac{1}{2}(e + fx)\right) \right) \left( -2i \left( \operatorname{PolyLog}\left(2, -\sqrt[4]{-1} e^{-\frac{e}{2} - \frac{fx}{2}}\right) - \operatorname{PolyLog}\left(2, \sqrt[4]{-1} e^{-\frac{e}{2} - \frac{fx}{2}}\right) \right) - \frac{1}{2}(2ie + 2ifx) \right)}{f^2\sqrt{a + ia \sinh(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + I*a*Sinh[e + f*x]],x]
```

```
[Out] (Sqrt[2]*(-2*e*ArcTan[(I + Tanh[(e + f*x)/4])/Sqrt[2]] + I*Pi*ArcTan[(I + Tanh[(e + f*x)/4])/Sqrt[2]] - (((2*I)*e + Pi + (2*I)*f*x)*(Log[1 - (-1)^(1/4)*E^(-e/2 - (f*x)/2)] - Log[1 + (-1)^(1/4)*E^(-e/2 - (f*x)/2)]))/2 - (2*I)*(PolyLog[2, -((-1)^(1/4)*E^(-e/2 - (f*x)/2))] - PolyLog[2, (-1)^(1/4)*E^(-e/2 - (f*x)/2)]))*(Cosh[(e + f*x)/2] + I*Sinh[(e + f*x)/2]))/(f^2*Sqrt[a + I*a*Sinh[e + f*x]])
```

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int x \frac{1}{\sqrt{a + ia \sinh(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

```
[Out] int(x/(a+I*a*sinh(f*x+e))^(1/2),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{ia \sinh(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(I*a*sinh(f*x + e) + a), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{2i \sqrt{\frac{1}{2}} \sqrt{ia e^{(2fx+2e)} + 2ae^{(fx+e)} - iaxe^{\left(\frac{1}{2}fx + \frac{1}{2}e\right)}}}{ae^{(2fx+2e)} - 2iae^{(fx+e)} - a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-2*I*sqrt(1/2)*sqrt(I*a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*x
*e^(1/2*f*x + 1/2*e)/(a*e^(2*f*x + 2*e) - 2*I*a*e^(f*x + e) - a), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a(i \sinh(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a*(I*sinh(e + f*x) + 1)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{ia \sinh(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+I*a*sinh(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(I*a*sinh(f*x + e) + a), x)
```

$$3.139 \quad \int \frac{1}{x\sqrt{a+ia \sinh(e+fx)}} dx$$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{x\sqrt{a+ia \sinh(e+fx)}}, x\right)$$

[Out] Unintegrable[1/(x\*Sqrt[a + I\*a\*Sinh[e + f\*x]]), x]

**Rubi [A]** time = 0.0776774, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{x\sqrt{a+ia \sinh(e+fx)}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*Sqrt[a + I\*a\*Sinh[e + f\*x]]), x]

[Out] Defer[Int][1/(x\*Sqrt[a + I\*a\*Sinh[e + f\*x]]), x]

Rubi steps

$$\int \frac{1}{x\sqrt{a+ia \sinh(e+fx)}} dx = \int \frac{1}{x\sqrt{a+ia \sinh(e+fx)}} dx$$

**Mathematica [A]** time = 3.84339, size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+ia \sinh(e+fx)}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*Sqrt[a + I\*a\*Sinh[e + f\*x]]), x]

[Out] Integrate[1/(x\*Sqrt[a + I\*a\*Sinh[e + f\*x]]), x]

---

**Maple [A]** time = 0.052, size = 0, normalized size = 0.

$$\int \frac{1}{x \sqrt{a + ia \sinh(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+I\*a\*sinh(f\*x+e))^(1/2),x)

[Out] int(1/x/(a+I\*a\*sinh(f\*x+e))^(1/2),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ia \sinh(fx + e) + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+I\*a\*sinh(f\*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(I\*a\*sinh(f\*x + e) + a)\*x), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{2i \sqrt{\frac{1}{2}} \sqrt{iae^{(2fx+2e)} + 2ae^{(fx+e)} - iae^{\left(\frac{1}{2}fx + \frac{1}{2}e\right)}}}{axe^{(2fx+2e)} - 2i axe^{(fx+e)} - ax}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+I\*a\*sinh(f\*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(-2\*I\*sqrt(1/2)\*sqrt(I\*a\*e^(2\*f\*x + 2\*e) + 2\*a\*e^(f\*x + e) - I\*a)\*e^(1/2\*f\*x + 1/2\*e)/(a\*x\*e^(2\*f\*x + 2\*e) - 2\*I\*a\*x\*e^(f\*x + e) - a\*x), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a(i\sinh(e+fx)+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+I\*a\*sinh(f\*x+e))\*\*(1/2),x)

[Out] Integral(1/(x\*sqrt(a\*(I\*sinh(e + f\*x) + 1))), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ia\sinh(fx+e)+ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+I\*a\*sinh(f\*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(I\*a\*sinh(f\*x + e) + a)\*x), x)

$$3.140 \quad \int \frac{1}{x^2 \sqrt{a+ia \sinh(e+fx)}} dx$$

**Optimal.** Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{x^2 \sqrt{a+ia \sinh(e+fx)}}, x\right)$$

[Out] Unintegrable[1/(x^2\*Sqrt[a + I\*a\*Sinh[e + f\*x]]), x]

**Rubi [A]** time = 0.0757779, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{x^2 \sqrt{a+ia \sinh(e+fx)}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2\*Sqrt[a + I\*a\*Sinh[e + f\*x]]),x]

[Out] Defer[Int][1/(x^2\*Sqrt[a + I\*a\*Sinh[e + f\*x]]), x]

Rubi steps

$$\int \frac{1}{x^2 \sqrt{a+ia \sinh(e+fx)}} dx = \int \frac{1}{x^2 \sqrt{a+ia \sinh(e+fx)}} dx$$

**Mathematica [A]** time = 3.95241, size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{a+ia \sinh(e+fx)}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2\*Sqrt[a + I\*a\*Sinh[e + f\*x]]),x]

[Out] Integrate[1/(x^2\*Sqrt[a + I\*a\*Sinh[e + f\*x]]), x]



---

**Maple [A]** time = 0.053, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} \frac{1}{\sqrt{a + ia \sinh(fx + e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+I\*a\*sinh(f\*x+e))^(1/2),x)

[Out] int(1/x^2/(a+I\*a\*sinh(f\*x+e))^(1/2),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ia \sinh(fx + e) + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+I\*a\*sinh(f\*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(I\*a\*sinh(f\*x + e) + a)\*x^2), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( -\frac{2i \sqrt{\frac{1}{2}} \sqrt{ia e^{(2fx+2e)} + 2ae^{(fx+e)} - ia e^{\left(\frac{1}{2}fx + \frac{1}{2}e\right)}}}{ax^2 e^{(2fx+2e)} - 2iax^2 e^{(fx+e)} - ax^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+I\*a\*sinh(f\*x+e))^(1/2),x, algorithm="fricas")

[Out] integral(-2\*I\*sqrt(1/2)\*sqrt(I\*a\*e^(2\*f\*x + 2\*e) + 2\*a\*e^(f\*x + e) - I\*a)\*e^(1/2\*f\*x + 1/2\*e)/(a\*x^2\*e^(2\*f\*x + 2\*e) - 2\*I\*a\*x^2\*e^(f\*x + e) - a\*x^2), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{a(i \sinh(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+I\*a\*sinh(f\*x+e))\*\*(1/2),x)

[Out] Integral(1/(x\*\*2\*sqrt(a\*(I\*sinh(e + f\*x) + 1))), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ia \sinh(fx + e) + ax^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+I\*a\*sinh(f\*x+e))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(I\*a\*sinh(f\*x + e) + a)\*x^2), x)

$$3.141 \quad \int \frac{x^3}{(a+ia \sinh(e+fx))^{3/2}} dx$$

**Optimal.** Leaf size=807

$$\frac{\tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)x^3}{2af\sqrt{i \sinh(e+fx)a+a}} + \frac{i \tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)x^3}{af\sqrt{i \sinh(e+fx)a+a}} + \frac{3i \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, -e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^2\sqrt{i \sinh(e+fx)a+a}}$$

[Out] (3\*x^2)/(a\*f^2\*Sqrt[a + I\*a\*Sinh[e + f\*x]]) - ((24\*I)\*x\*ArcTanh[E^((2\*e - I\*Pi)/4 + (f\*x)/2)]\*Cosh[e/2 + (I/4)\*Pi + (f\*x)/2])/(a\*f^3\*Sqrt[a + I\*a\*Sinh[e + f\*x]]) + (I\*x^3\*ArcTanh[E^((2\*e - I\*Pi)/4 + (f\*x)/2)]\*Cosh[e/2 + (I/4)\*Pi + (f\*x)/2])/(a\*f\*Sqrt[a + I\*a\*Sinh[e + f\*x]]) - ((24\*I)\*Cosh[e/2 + (I/4)\*Pi + (f\*x)/2]\*PolyLog[2, -E^((2\*e - I\*Pi)/4 + (f\*x)/2)])/(a\*f^4\*Sqrt[a + I\*a\*Sinh[e + f\*x]]) + ((3\*I)\*x^2\*Cosh[e/2 + (I/4)\*Pi + (f\*x)/2]\*PolyLog[2, -E^((2\*e - I\*Pi)/4 + (f\*x)/2)])/(a\*f^2\*Sqrt[a + I\*a\*Sinh[e + f\*x]]) + ((24\*I)\*Cosh[e/2 + (I/4)\*Pi + (f\*x)/2]\*PolyLog[2, E^((2\*e - I\*Pi)/4 + (f\*x)/2)])/(a\*f^4\*Sqrt[a + I\*a\*Sinh[e + f\*x]]) - ((3\*I)\*x^2\*Cosh[e/2 + (I/4)\*Pi + (f\*x)/2]\*PolyLog[2, E^((2\*e - I\*Pi)/4 + (f\*x)/2)])/(a\*f^2\*Sqrt[a + I\*a\*Sinh[e + f\*x]]) - ((12\*I)\*x\*Cosh[e/2 + (I/4)\*Pi + (f\*x)/2]\*PolyLog[3, -E^((2\*e - I\*Pi)/4 + (f\*x)/2)])/(a\*f^3\*Sqrt[a + I\*a\*Sinh[e + f\*x]]) + ((12\*I)\*x\*Cosh[e/2 + (I/4)\*Pi + (f\*x)/2]\*PolyLog[3, E^((2\*e - I\*Pi)/4 + (f\*x)/2)])/(a\*f^3\*Sqrt[a + I\*a\*Sinh[e + f\*x]]) + ((24\*I)\*Cosh[e/2 + (I/4)\*Pi + (f\*x)/2]\*PolyLog[4, -E^((2\*e - I\*Pi)/4 + (f\*x)/2)])/(a\*f^4\*Sqrt[a + I\*a\*Sinh[e + f\*x]]) - ((24\*I)\*Cosh[e/2 + (I/4)\*Pi + (f\*x)/2]\*PolyLog[4, E^((2\*e - I\*Pi)/4 + (f\*x)/2)])/(a\*f^4\*Sqrt[a + I\*a\*Sinh[e + f\*x]]) + (x^3\*Tanh[e/2 + (I/4)\*Pi + (f\*x)/2])/(2\*a\*f\*Sqrt[a + I\*a\*Sinh[e + f\*x]])

**Rubi [A]** time = 0.437724, antiderivative size = 807, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {3319, 4186, 4182, 2279, 2391, 2531, 6609, 2282, 6589}

$$\frac{\tanh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)x^3}{2af\sqrt{i \sinh(e+fx)a+a}} + \frac{i \tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)x^3}{af\sqrt{i \sinh(e+fx)a+a}} + \frac{3i \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, -e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^2\sqrt{i \sinh(e+fx)a+a}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + I\*a\*Sinh[e + f\*x])^(3/2),x]

[Out] (3\*x^2)/(a\*f^2\*Sqrt[a + I\*a\*Sinh[e + f\*x]]) - ((24\*I)\*x\*ArcTanh[E^((2\*e - I\*Pi)/4 + (f\*x)/2)]\*Cosh[e/2 + (I/4)\*Pi + (f\*x)/2])/(a\*f^3\*Sqrt[a + I\*a\*Sinh

$$\begin{aligned}
& [e + f*x]]) + (I*x^3*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4) \\
& *Pi + (f*x)/2])/(a*f*Sqrt[a + I*a*Sinh[e + f*x]]) - ((24*I)*Cosh[e/2 + (I/4) \\
& )*Pi + (f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^4*Sqrt[a + \\
& I*a*Sinh[e + f*x]]) + ((3*I)*x^2*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, \\
& -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) + ((24*I) \\
& )*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)]) \\
& /((a*f^4*Sqrt[a + I*a*Sinh[e + f*x]]) - ((3*I)*x^2*Cosh[e/2 + (I/4)*Pi + (f* \\
& x)/2]*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e \\
& + f*x]]) - ((12*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, -E^((2*e - I \\
& *Pi)/4 + (f*x)/2)])/(a*f^3*Sqrt[a + I*a*Sinh[e + f*x]]) + ((12*I)*x*Cosh[e/ \\
& 2 + (I/4)*Pi + (f*x)/2]*PolyLog[3, E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^3*Sq \\
& rt[a + I*a*Sinh[e + f*x]]) + ((24*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog \\
& [4, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^4*Sqrt[a + I*a*Sinh[e + f*x]]) - ( \\
& (24*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[4, E^((2*e - I*Pi)/4 + (f*x)/ \\
& 2)])/(a*f^4*Sqrt[a + I*a*Sinh[e + f*x]]) + (x^3*Tanh[e/2 + (I/4)*Pi + (f*x) \\
& /2])/(2*a*f*Sqrt[a + I*a*Sinh[e + f*x]])
\end{aligned}$$

### Rule 3319

```

Int[((c_.) + (d_.)*(x_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.),
  x_Symbol] := Dist[((2*a)^(IntPart[n])*a + b*Sinh[e + f*x])^(FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sinh[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])

```

### Rule 4186

```

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.), x_Symbo
l] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

```

### Rule 4182

```

Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x
_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_))))^(n_)]*((f_) + (g_)
*(x_)^(m_)), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_) + (f_)*(x_))^(m_)*PolyLog[n_, (d_)*((F_)^((c_)*((a_) + (b_
)*(x_))))^(p_)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(a + ia \sinh(e + fx))^{3/2}} dx &= -\frac{\sinh\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) \int x^3 \operatorname{csch}^3\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) dx}{2a\sqrt{a + ia \sinh(e + fx)}} \\
&= \frac{3x^2}{af^2\sqrt{a + ia \sinh(e + fx)}} + \frac{x^3 \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{2af\sqrt{a + ia \sinh(e + fx)}} + \frac{\sinh\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) \int x^3 \operatorname{csch}\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) dx}{4a\sqrt{a + ia \sinh(e + fx)}} \\
&= \frac{3x^2}{af^2\sqrt{a + ia \sinh(e + fx)}} - \frac{24ix \tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af^3\sqrt{a + ia \sinh(e + fx)}} + \frac{ix^3 \tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^3\sqrt{a + ia \sinh(e + fx)}} \\
&= \frac{3x^2}{af^2\sqrt{a + ia \sinh(e + fx)}} - \frac{24ix \tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af^3\sqrt{a + ia \sinh(e + fx)}} + \frac{ix^3 \tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^3\sqrt{a + ia \sinh(e + fx)}} \\
&= \frac{3x^2}{af^2\sqrt{a + ia \sinh(e + fx)}} - \frac{24ix \tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af^3\sqrt{a + ia \sinh(e + fx)}} + \frac{ix^3 \tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^3\sqrt{a + ia \sinh(e + fx)}} \\
&= \frac{3x^2}{af^2\sqrt{a + ia \sinh(e + fx)}} - \frac{24ix \tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af^3\sqrt{a + ia \sinh(e + fx)}} + \frac{ix^3 \tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^3\sqrt{a + ia \sinh(e + fx)}} \\
&= \frac{3x^2}{af^2\sqrt{a + ia \sinh(e + fx)}} - \frac{24ix \tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af^3\sqrt{a + ia \sinh(e + fx)}} + \frac{ix^3 \tanh^{-1}\left(e^{\frac{1}{4}(2e-i\pi)+\frac{fx}{2}}\right)}{af^3\sqrt{a + ia \sinh(e + fx)}}
\end{aligned}$$

**Mathematica [A]** time = 3.23932, size = 546, normalized size = 0.68

$$\frac{\left(\cosh\left(\frac{1}{2}(e + fx)\right) + i \sinh\left(\frac{1}{2}(e + fx)\right)\right) \left(\left(\frac{1}{2} - \frac{i}{2}\right) (-1)^{3/4} \left(\cosh\left(\frac{1}{2}(e + fx)\right) + i \sinh\left(\frac{1}{2}(e + fx)\right)\right)\right)^2 \left(-6(f^2x^2 - 8) \operatorname{PolyLog}\right)}{1}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + I\*a\*Sinh[e + f\*x])^(3/2), x]

[Out] ((Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2])\*(f^2\*x^2\*(6 + I\*f\*x)\*(Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2]) + (1/2 - I/2)\*(-1)^(3/4)\*(-48\*e\*ArcTanh[(-1)^(3/4)\*E^((e + f\*x)/2)] + 2\*e^3\*ArcTanh[(-1)^(3/4)\*E^((e + f\*x)/2)] - 24\*e\*Log[1 - (-1)^(3/4)\*E^((e + f\*x)/2)] + e^3\*Log[1 - (-1)^(3/4)\*E^((e + f\*x)/2)] - 24\*f\*x\*Log[1 - (-1)^(3/4)\*E^((e + f\*x)/2)] + f^3\*x^3\*Log[1 - (-1)^(3/4)\*E^((e + f\*x)/2)] + 24\*e\*Log[1 + (-1)^(3/4)\*E^((e + f\*x)/2)] - e^3\*Log[1 +

$$(-1)^{3/4}E^{(e+fx)/2}] + 24fx \text{Log}[1 + (-1)^{3/4}E^{(e+fx)/2}] - f^3x^3 \text{Log}[1 + (-1)^{3/4}E^{(e+fx)/2}] - 6(-8 + f^2x^2) \text{PolyLog}[2, -((-1)^{3/4}E^{(e+fx)/2})] + 6(-8 + f^2x^2) \text{PolyLog}[2, (-1)^{3/4}E^{(e+fx)/2}] + 24fx \text{PolyLog}[3, -((-1)^{3/4}E^{(e+fx)/2})] - 24fx \text{PolyLog}[3, (-1)^{3/4}E^{(e+fx)/2}] - 48 \text{PolyLog}[4, -((-1)^{3/4}E^{(e+fx)/2})] + 48 \text{PolyLog}[4, (-1)^{3/4}E^{(e+fx)/2}] * (\text{Cosh}[(e+fx)/2] + I \text{Sinh}[(e+fx)/2])^2 + 2f^3x^3 \text{Sinh}[(e+fx)/2]) / (2f^4(a + I a \text{Sinh}[(e+fx)/2])^{3/2})$$

**Maple [F]** time = 0.047, size = 0, normalized size = 0.

$$\int x^3 (a + ia \sinh(fx + e))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+I\*a\*sinh(f\*x+e))^(3/2),x)

[Out] int(x^3/(a+I\*a\*sinh(f\*x+e))^(3/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(ia \sinh(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+I\*a\*sinh(f\*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate(x^3/(I\*a\*sinh(f\*x + e) + a)^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\sqrt{\frac{1}{2}} \left( (-ifx^3 - 6ix^2)e^{(2fx+2e)} + (fx^3 - 6x^2)e^{(fx+e)} \right) \sqrt{iae^{(2fx+2e)} + 2ae^{(fx+e)} - iae^{(-\frac{1}{2}fx - \frac{1}{2}e)}} + \left( a^2f^2e^{(3fx+3e)} - 3ia^2f^2e^{(2fx+2e)} - 3a^2f^2e^{(3fx+3e)} - 3ia^2f^2e^{(2fx+2e)} - 3a^2f^2e^{(3fx+3e)} - 3ia^2f^2e^{(2fx+2e)} - 3a^2f^2e^{(3fx+3e)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(1/2)*((-I*f*x^3 - 6*I*x^2)*e^(2*f*x + 2*e) + (f*x^3 - 6*x^2)*e^(f*x + e))*sqrt(I*a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*e^(-1/2*f*x - 1/2*e) + (a^2*f^2*e^(3*f*x + 3*e) - 3*I*a^2*f^2*e^(2*f*x + 2*e) - 3*a^2*f^2*e^(f*x + e) + I*a^2*f^2)*integral(sqrt(1/2)*(-I*f^2*x^3 + 24*I*x)*sqrt(I*a*e^(2*f*x + 2*e) + 2*a*e^(f*x + e) - I*a)*e^(1/2*f*x + 1/2*e)/(2*a^2*f^2*e^(2*f*x + 2*e) - 4*I*a^2*f^2*e^(f*x + e) - 2*a^2*f^2), x))/(a^2*f^2*e^(3*f*x + 3*e) - 3*I*a^2*f^2*e^(2*f*x + 2*e) - 3*a^2*f^2*e^(f*x + e) + I*a^2*f^2)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a(i \sinh(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+I*a*sinh(f*x+e))**(3/2),x)
```

```
[Out] Integral(x**3/(a*(I*sinh(e + f*x) + 1))**(3/2), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(i a \sinh(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+I*a*sinh(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/(I*a*sinh(f*x + e) + a)^(3/2), x)
```



$$3.142 \quad \int \frac{x^2}{(a+ia \sinh(e+fx))^{3/2}} dx$$

**Optimal.** Leaf size=506

$$\frac{2ix \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, -e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{af^2 \sqrt{a + ia \sinh(e + fx)}} - \frac{2ix \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{af^2 \sqrt{a + ia \sinh(e + fx)}} - \frac{4i \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{af^2 \sqrt{a + ia \sinh(e + fx)}}$$

```
[Out] (2*x)/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - (4*ArcTan[Sinh[e/2 + (I/4)*Pi +
(f*x)/2]]*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/(a*f^3*Sqrt[a + I*a*Sinh[e + f*x
]]) + (I*x^2*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f
*x)/2])/(a*f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((2*I)*x*Cosh[e/2 + (I/4)*Pi +
(f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sin
h[e + f*x]]) - ((2*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e -
I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((4*I)*Cosh[e/2
+ (I/4)*Pi + (f*x)/2]*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^3*Sq
rt[a + I*a*Sinh[e + f*x]]) + ((4*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[
3, E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^3*Sqrt[a + I*a*Sinh[e + f*x]]) + (x^
2*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(2*a*f*Sqrt[a + I*a*Sinh[e + f*x]])
```

**Rubi [A]** time = 0.311229, antiderivative size = 506, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {3319, 4186, 3770, 4182, 2531, 2282, 6589}

$$\frac{2ix \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, -e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{af^2 \sqrt{a + ia \sinh(e + fx)}} - \frac{2ix \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{af^2 \sqrt{a + ia \sinh(e + fx)}} - \frac{4i \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right)}{af^2 \sqrt{a + ia \sinh(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + I*a*Sinh[e + f*x])^(3/2),x]
```

```
[Out] (2*x)/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - (4*ArcTan[Sinh[e/2 + (I/4)*Pi +
(f*x)/2]]*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/(a*f^3*Sqrt[a + I*a*Sinh[e + f*x
]]) + (I*x^2*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f
*x)/2])/(a*f*Sqrt[a + I*a*Sinh[e + f*x]]) + ((2*I)*x*Cosh[e/2 + (I/4)*Pi +
(f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sin
h[e + f*x]]) - ((2*I)*x*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e -
I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - ((4*I)*Cosh[e/2
+ (I/4)*Pi + (f*x)/2]*PolyLog[3, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^3*Sq
rt[a + I*a*Sinh[e + f*x]]) + ((4*I)*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[
```

3,  $E^{((2e - I\pi)/4 + (f*x)/2)}/(a*f^3*\sqrt{a + I*a*\sinh[e + f*x]}) + (x^2*\tanh[e/2 + (I/4)*\pi + (f*x)/2])/(2*a*f*\sqrt{a + I*a*\sinh[e + f*x]})$

### Rule 3319

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[((2\*a)^IntPart[n]\*(a + b\*Sin[e + f\*x])^FracPart[n])/Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*FracPart[n]), Int[(c + d\*x)^m\*Sin[e/2 + (a\*Pi)/(4\*b) + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])

### Rule 4186

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.))^(n\_)\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] := -Simp[(b^2\*(c + d\*x)^m\*Cot[e + f\*x]\*(b\*Csc[e + f\*x])^(n - 2))/(f\*(n - 1)), x] + (Dist[(b^2\*d^2\*m\*(m - 1))/(f^2\*(n - 1)\*(n - 2)), Int[(c + d\*x)^(m - 2)\*(b\*Csc[e + f\*x])^(n - 2), x], x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(c + d\*x)^m\*(b\*Csc[e + f\*x])^(n - 2), x], x] - Simp[(b^2\*d\*m\*(c + d\*x)^(m - 1)\*(b\*Csc[e + f\*x])^(n - 2))/(f^2\*(n - 1)\*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

### Rule 4182

Int[csc[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)]/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\int \frac{x^2}{(a + ia \sinh(e + fx))^{3/2}} dx = -\frac{\sinh\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) \int x^2 \operatorname{csch}^3\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) dx}{2a\sqrt{a + ia \sinh(e + fx)}}$$

$$= \frac{2x}{af^2\sqrt{a + ia \sinh(e + fx)}} + \frac{x^2 \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{2af\sqrt{a + ia \sinh(e + fx)}} + \frac{\sinh\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) \int x^2 \operatorname{csch}\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) dx}{4a\sqrt{a + ia \sinh(e + fx)}}$$

$$= \frac{2x}{af^2\sqrt{a + ia \sinh(e + fx)}} - \frac{4 \tan^{-1}\left(\sinh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af^3\sqrt{a + ia \sinh(e + fx)}} + \frac{ix^2 \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af^2\sqrt{a + ia \sinh(e + fx)}}$$

$$= \frac{2x}{af^2\sqrt{a + ia \sinh(e + fx)}} - \frac{4 \tan^{-1}\left(\sinh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af^3\sqrt{a + ia \sinh(e + fx)}} + \frac{ix^2 \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af^2\sqrt{a + ia \sinh(e + fx)}}$$

$$= \frac{2x}{af^2\sqrt{a + ia \sinh(e + fx)}} - \frac{4 \tan^{-1}\left(\sinh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af^3\sqrt{a + ia \sinh(e + fx)}} + \frac{ix^2 \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af^2\sqrt{a + ia \sinh(e + fx)}}$$

$$= \frac{2x}{af^2\sqrt{a + ia \sinh(e + fx)}} - \frac{4 \tan^{-1}\left(\sinh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af^3\sqrt{a + ia \sinh(e + fx)}} + \frac{ix^2 \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af^2\sqrt{a + ia \sinh(e + fx)}}$$

**Mathematica [A]** time = 1.76413, size = 384, normalized size = 0.76

$$\frac{\left(\cosh\left(\frac{1}{2}(e + fx)\right) + i \sinh\left(\frac{1}{2}(e + fx)\right)\right) \left(-\left(\frac{1}{2} - \frac{i}{2}\right) (-1)^{3/4} \left(\cosh\left(\frac{1}{2}(e + fx)\right) + i \sinh\left(\frac{1}{2}(e + fx)\right)\right)\right)^2 \left(4fx \operatorname{PolyLog}\left(2, -\frac{1}{2}(e + fx)\right)\right)}{af^2\sqrt{a + ia \sinh(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + I\*a\*Sinh[e + f\*x])^(3/2),x]

[Out] ((Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2])\*(f\*x\*(4 + I\*f\*x)\*(Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2]) - (1/2 - I/2)\*(-1)^(3/4)\*(-16\*ArcTanh[(-1)^(3/4)\*E^((e + f\*x)/2)] + 2\*e^2\*ArcTanh[(-1)^(3/4)\*E^((e + f\*x)/2)] + e^2\*Log[1 - (-1)^(3/4)\*E^((e + f\*x)/2)] - f^2\*x^2\*Log[1 - (-1)^(3/4)\*E^((e + f\*x)/2)] - e^2\*Log[1 + (-1)^(3/4)\*E^((e + f\*x)/2)] + f^2\*x^2\*Log[1 + (-1)^(3/4)\*E^((e + f\*x)/2)] + 4\*f\*x\*PolyLog[2, -((-1)^(3/4)\*E^((e + f\*x)/2))] - 4\*f\*x\*PolyLog[2, (-1)^(3/4)\*E^((e + f\*x)/2)] - 8\*PolyLog[3, -((-1)^(3/4)\*E^((e + f\*x)/2))] + 8\*PolyLog[3, (-1)^(3/4)\*E^((e + f\*x)/2)]\*(Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2])^2 + 2\*f^2\*x^2\*Sinh[(e + f\*x)/2]))/(2\*f^3\*(a + I\*a\*Sinh[e + f\*x])^(3/2))

**Maple [F]** time = 0.047, size = 0, normalized size = 0.

$$\int x^2 (a + ia \sinh(fx + e))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+I\*a\*sinh(f\*x+e))^(3/2),x)

[Out] int(x^2/(a+I\*a\*sinh(f\*x+e))^(3/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(ia \sinh(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+I\*a\*sinh(f\*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate(x^2/(I\*a\*sinh(f\*x + e) + a)^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\frac{1}{2}}\left((-i f x^2 - 4 i x)e^{(2 f x+2 e)} + (f x^2 - 4 x)e^{(f x+e)}\right)\sqrt{i a e^{(2 f x+2 e)} + 2 a e^{(f x+e)} - i a e^{\left(-\frac{1}{2} f x-\frac{1}{2} e\right)}} + \left(a^2 f^2 e^{(3 f x+3 e)} - 3 i a^2 f^2 e^{(2 f x+2 e)}\right)}{a^2 f^2 e^{(3 f x+3 e)} - 3 i a^2 f^2 e^{(2 f x+2 e)} - 3 a^2 f^2 e^{(f x+e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+I\*a\*sinh(f\*x+e))^(3/2),x, algorithm="fricas")

[Out] (sqrt(1/2)\*((-I\*f\*x^2 - 4\*I\*x)\*e^(2\*f\*x + 2\*e) + (f\*x^2 - 4\*x)\*e^(f\*x + e)) \*sqrt(I\*a\*e^(2\*f\*x + 2\*e) + 2\*a\*e^(f\*x + e) - I\*a)\*e^(-1/2\*f\*x - 1/2\*e) + (a^2\*f^2\*e^(3\*f\*x + 3\*e) - 3\*I\*a^2\*f^2\*e^(2\*f\*x + 2\*e) - 3\*a^2\*f^2\*e^(f\*x + e) + I\*a^2\*f^2)\*integral(sqrt(1/2)\*(-I\*f^2\*x^2 + 8\*I)\*sqrt(I\*a\*e^(2\*f\*x + 2\*e) + 2\*a\*e^(f\*x + e) - I\*a)\*e^(1/2\*f\*x + 1/2\*e)/(2\*a^2\*f^2\*e^(2\*f\*x + 2\*e) - 4\*I\*a^2\*f^2\*e^(f\*x + e) - 2\*a^2\*f^2), x))/(a^2\*f^2\*e^(3\*f\*x + 3\*e) - 3\*I\*a^2\*f^2\*e^(2\*f\*x + 2\*e) - 3\*a^2\*f^2\*e^(f\*x + e) + I\*a^2\*f^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(a(i \sinh(e + fx) + 1)\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(a+I\*a\*sinh(f\*x+e))\*\*(3/2),x)

[Out] Integral(x\*\*2/(a\*(I\*sinh(e + f\*x) + 1))\*\*(3/2), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(i a \sinh(f x+e)+a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+I\*a\*sinh(f\*x+e))^(3/2),x, algorithm="giac")

```
[Out] integrate(x^2/(I*a*sinh(f*x + e) + a)^(3/2), x)
```

$$3.143 \quad \int \frac{x}{(a+ia \sinh(e+fx))^{3/2}} dx$$

**Optimal.** Leaf size=288

$$\frac{i \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, -e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{af^2 \sqrt{a + ia \sinh(e + fx)}} - \frac{i \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{af^2 \sqrt{a + ia \sinh(e + fx)}} + \frac{1}{af^2 \sqrt{a + ia \sinh(e + fx)}}$$

```
[Out] 1/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) + (I*x*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/(a*f*Sqrt[a + I*a*Sinh[e + f*x]]) + (I*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - (I*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) + (x*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(2*a*f*Sqrt[a + I*a*Sinh[e + f*x]])
```

**Rubi [A]** time = 0.169333, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {3319, 4185, 4182, 2279, 2391}

$$\frac{i \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, -e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{af^2 \sqrt{a + ia \sinh(e + fx)}} - \frac{i \cosh\left(\frac{e}{2} + \frac{fx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, e^{\frac{fx}{2} + \frac{1}{4}(2e-i\pi)}\right)}{af^2 \sqrt{a + ia \sinh(e + fx)}} + \frac{1}{af^2 \sqrt{a + ia \sinh(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + I*a*Sinh[e + f*x])^(3/2), x]
```

```
[Out] 1/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) + (I*x*ArcTanh[E^((2*e - I*Pi)/4 + (f*x)/2)]*Cosh[e/2 + (I/4)*Pi + (f*x)/2])/(a*f*Sqrt[a + I*a*Sinh[e + f*x]]) + (I*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, -E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) - (I*Cosh[e/2 + (I/4)*Pi + (f*x)/2]*PolyLog[2, E^((2*e - I*Pi)/4 + (f*x)/2)])/(a*f^2*Sqrt[a + I*a*Sinh[e + f*x]]) + (x*Tanh[e/2 + (I/4)*Pi + (f*x)/2])/(2*a*f*Sqrt[a + I*a*Sinh[e + f*x]])
```

### Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Dist[((2*a)^IntPart[n]*(a + b*Sin[e + f*x])^FracPart[n])/Sin[e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Sin[e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
```

$qQ[a^2 - b^2, 0] \&\& \text{IntegerQ}[n + 1/2] \&\& (\text{GtQ}[n, 0] \parallel \text{IGtQ}[m, 0])$

### Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
-Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x)) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps



$$\begin{aligned}
\int \frac{x}{(a + ia \sinh(e + fx))^{3/2}} dx &= -\frac{\sinh\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) \int x \operatorname{csch}^3\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) dx}{2a\sqrt{a + ia \sinh(e + fx)}} \\
&= \frac{1}{af^2\sqrt{a + ia \sinh(e + fx)}} + \frac{x \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{2af\sqrt{a + ia \sinh(e + fx)}} + \frac{\sinh\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) \int x \operatorname{csch}\left(\frac{e}{2} - \frac{i\pi}{4} + \frac{fx}{2}\right) dx}{4a\sqrt{a + ia \sinh(e + fx)}} \\
&= \frac{1}{af^2\sqrt{a + ia \sinh(e + fx)}} + \frac{ix \tanh^{-1}\left(e^{\frac{1}{4}(2e - i\pi) + \frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af\sqrt{a + ia \sinh(e + fx)}} + \frac{x \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{2af\sqrt{a + ia \sinh(e + fx)}} \\
&= \frac{1}{af^2\sqrt{a + ia \sinh(e + fx)}} + \frac{ix \tanh^{-1}\left(e^{\frac{1}{4}(2e - i\pi) + \frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af\sqrt{a + ia \sinh(e + fx)}} + \frac{x \tanh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{2af\sqrt{a + ia \sinh(e + fx)}} \\
&= \frac{1}{af^2\sqrt{a + ia \sinh(e + fx)}} + \frac{ix \tanh^{-1}\left(e^{\frac{1}{4}(2e - i\pi) + \frac{fx}{2}}\right) \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af\sqrt{a + ia \sinh(e + fx)}} + \frac{i \cosh\left(\frac{e}{2} + \frac{i\pi}{4} + \frac{fx}{2}\right)}{af^2\sqrt{a + ia \sinh(e + fx)}}
\end{aligned}$$

**Mathematica [A]** time = 0.852845, size = 332, normalized size = 1.15

$$\left(\cosh\left(\frac{1}{2}(e + fx)\right) + i \sinh\left(\frac{1}{2}(e + fx)\right)\right) \left( \frac{i \left(\cosh\left(\frac{1}{2}(e + fx)\right) + i \sinh\left(\frac{1}{2}(e + fx)\right)\right)^2 \left(-2 \operatorname{PolyLog}\left(2, -\sqrt[4]{-1} e^{-\frac{e}{2} - \frac{fx}{2}}\right) + 2 \operatorname{PolyLog}\left(2, \sqrt[4]{-1} e^{-\frac{e}{2} - \frac{fx}{2}}\right)\right) + \frac{1}{2} i (2e + \pi + 2fx) \left(\log\left[1 - (-1)^{1/4} e^{-\frac{e}{2} - \frac{fx}{2}}\right] - \log\left[1 + (-1)^{1/4} e^{-\frac{e}{2} - \frac{fx}{2}}\right]\right) - 2 \operatorname{PolyLog}\left[2, -(-1)^{1/4} e^{-\frac{e}{2} - \frac{fx}{2}}\right] + 2 \operatorname{PolyLog}\left[2, (-1)^{1/4} e^{-\frac{e}{2} - \frac{fx}{2}}\right]}{\sqrt{2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + I\*a\*Sinh[e + f\*x])^(3/2),x]

[Out] ((Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2])\*((2 + I\*f\*x)\*(Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2]) - Sqrt[2]\*e\*ArcTan[(I + Tanh[(e + f\*x)/4])/Sqrt[2]]\*(Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2])^2 + (I\*(Pi\*ArcTan[(I + Tanh[(e + f\*x)/4])/Sqrt[2]] + (I/2)\*((2\*I)\*e + Pi + (2\*I)\*f\*x)\*(Log[1 - (-1)^(1/4)\*E^(-e/2 - (f\*x)/2)] - Log[1 + (-1)^(1/4)\*E^(-e/2 - (f\*x)/2)]) - 2\*PolyLog[2, -((-1)^(1/4)\*E^(-e/2 - (f\*x)/2)]) + 2\*PolyLog[2, (-1)^(1/4)\*E^(-e/2 - (f\*x)/2)])\*(Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2])^2)/Sqrt[2] + 2\*f\*x\*Sinh[(e + f\*x)/2]))/(2\*f^2\*(a + I\*a\*Sinh[e + f\*x])^(3/2))

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int x (a + ia \sinh(fx + e))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+I\*a\*sinh(f\*x+e))^(3/2),x)

[Out] int(x/(a+I\*a\*sinh(f\*x+e))^(3/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(ia \sinh(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+I\*a\*sinh(f\*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate(x/(I\*a\*sinh(f\*x + e) + a)^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\sqrt{\frac{1}{2}} \left( (-ifx - 2i)e^{2fx+2e} + (fx - 2)e^{fx+e} \right) \sqrt{iae^{2fx+2e} + 2ae^{fx+e} - iae^{-\frac{1}{2}fx - \frac{1}{2}e}} + \left( a^2 f^2 e^{3fx+3e} - 3i a^2 f^2 e^{2fx+2e} \right)$$


---


$$a^2 f^2 e^{3fx+3e} - 3i a^2 f^2 e^{2fx+2e} - 3 a^2 f^2 e^{fx+e} + i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+I\*a\*sinh(f\*x+e))^(3/2),x, algorithm="fricas")

[Out] (sqrt(1/2)\*((-I\*f\*x - 2\*I)\*e^(2\*f\*x + 2\*e) + (f\*x - 2)\*e^(f\*x + e))\*sqrt(I\*a\*e^(2\*f\*x + 2\*e) + 2\*a\*e^(f\*x + e) - I\*a)\*e^(-1/2\*f\*x - 1/2\*e) + (a^2\*f^2\*e^(3\*f\*x + 3\*e) - 3\*I\*a^2\*f^2\*e^(2\*f\*x + 2\*e) - 3\*a^2\*f^2\*e^(f\*x + e) + I\*a^2\*f^2)\*integral(-I\*sqrt(1/2)\*sqrt(I\*a\*e^(2\*f\*x + 2\*e) + 2\*a\*e^(f\*x + e) - I\*a)\*x\*e^(1/2\*f\*x + 1/2\*e)/(2\*a^2\*e^(2\*f\*x + 2\*e) - 4\*I\*a^2\*e^(f\*x + e) - 2

$*a^2), x))/(a^2*f^2*e^{(3*f*x + 3*e)} - 3*I*a^2*f^2*e^{(2*f*x + 2*e)} - 3*a^2*f^2*e^{(f*x + e)} + I*a^2*f^2)$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a(i \sinh(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+I\*a\*sinh(f\*x+e))\*\*(3/2),x)

[Out] Integral(x/(a\*(I\*sinh(e + f\*x) + 1))\*\*(3/2), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(i a \sinh(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+I\*a\*sinh(f\*x+e))^(3/2),x, algorithm="giac")

[Out] integrate(x/(I\*a\*sinh(f\*x + e) + a)^(3/2), x)

$$3.144 \quad \int \frac{1}{x(a+ia \sinh(e+fx))^{3/2}} dx$$

**Optimal.** Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{x(a+ia \sinh(e+fx))^{3/2}}, x\right)$$

[Out] Unintegrable[1/(x\*(a + I\*a\*Sinh[e + f\*x])^(3/2)), x]

**Rubi [A]** time = 0.0876549, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{x(a+ia \sinh(e+fx))^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + I\*a\*Sinh[e + f\*x])^(3/2)),x]

[Out] Defer[Int][1/(x\*(a + I\*a\*Sinh[e + f\*x])^(3/2)), x]

Rubi steps

$$\int \frac{1}{x(a+ia \sinh(e+fx))^{3/2}} dx = \int \frac{1}{x(a+ia \sinh(e+fx))^{3/2}} dx$$

**Mathematica [A]** time = 21.6249, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+ia \sinh(e+fx))^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + I\*a\*Sinh[e + f\*x])^(3/2)),x]

[Out] Integrate[1/(x\*(a + I\*a\*Sinh[e + f\*x])^(3/2)), x]

---

**Maple [A]** time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x} (a + ia \sinh(fx + e))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+I\*a\*sinh(f\*x+e))^(3/2),x)

[Out] int(1/x/(a+I\*a\*sinh(f\*x+e))^(3/2),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ia \sinh(fx + e) + a)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+I\*a\*sinh(f\*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate(1/((I\*a\*sinh(f\*x + e) + a)^(3/2)\*x), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\sqrt{\frac{1}{2}} \left( (-ifx + 2i)e^{(2fx+2e)} + (fx + 2)e^{(fx+e)} \right) \sqrt{iae^{(2fx+2e)} + 2ae^{(fx+e)} - iae^{\left(\frac{1}{2}fx - \frac{1}{2}e\right)}} + \left( a^2 f^2 x^2 e^{(3fx+3e)} - 3i a^2 f^2 x^2 e^{(2fx+2e)} - 3 a^2 f^2 \right)$$


---


$$a^2 f^2 x^2 e^{(3fx+3e)} - 3i a^2 f^2 x^2 e^{(2fx+2e)} - 3 a^2 f^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+I\*a\*sinh(f\*x+e))^(3/2),x, algorithm="fricas")

[Out] (sqrt(1/2)\*((-I\*f\*x + 2\*I)\*e^(2\*f\*x + 2\*e) + (f\*x + 2)\*e^(f\*x + e))\*sqrt(I\*a\*e^(2\*f\*x + 2\*e) + 2\*a\*e^(f\*x + e) - I\*a)\*e^(-1/2\*f\*x - 1/2\*e) + (a^2\*f^2\*x^2\*e^(3\*f\*x + 3\*e) - 3\*I\*a^2\*f^2\*x^2\*e^(2\*f\*x + 2\*e) - 3\*a^2\*f^2\*x^2\*e^(f\*x + e) + I\*a^2\*f^2\*x^2)\*integral(sqrt(1/2)\*(-I\*f^2\*x^2 + 8\*I)\*sqrt(I\*a\*e^(2

$*f*x + 2*e) + 2*a*e^{(f*x + e) - I*a}*e^{(1/2*f*x + 1/2*e)/(2*a^2*f^2*x^3*e^{(2*f*x + 2*e) - 4*I*a^2*f^2*x^3*e^{(f*x + e) - 2*a^2*f^2*x^3})}$ , x)) / (a^2\*f^2\*x^2\*e^{(3\*f\*x + 3\*e) - 3\*I\*a^2\*f^2\*x^2\*e^{(2\*f\*x + 2\*e) - 3\*a^2\*f^2\*x^2\*e^{(f\*x + e) + I\*a^2\*f^2\*x^2}})

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x \left( a \left( i \sinh(e + fx) + 1 \right) \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+I\*a\*sinh(f\*x+e))\*\*(3/2),x)

[Out] Integral(1/(x\*(a\*(I\*sinh(e + f\*x) + 1))\*\*(3/2)), x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left( i a \sinh(fx + e) + a \right)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+I\*a\*sinh(f\*x+e))^(3/2),x, algorithm="giac")

[Out] integrate(1/((I\*a\*sinh(f\*x + e) + a)^(3/2)\*x), x)

$$3.145 \quad \int \frac{1}{x^2(a+ia \sinh(e+fx))^{3/2}} dx$$

**Optimal.** Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{x^2(a+ia \sinh(e+fx))^{3/2}}, x\right)$$

[Out] Unintegrable[1/(x^2\*(a + I\*a\*Sinh[e + f\*x])^(3/2)), x]

**Rubi [A]** time = 0.085883, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{x^2(a+ia \sinh(e+fx))^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x^2\*(a + I\*a\*Sinh[e + f\*x])^(3/2)), x]

[Out] Defer[Int][1/(x^2\*(a + I\*a\*Sinh[e + f\*x])^(3/2)), x]

Rubi steps

$$\int \frac{1}{x^2(a+ia \sinh(e+fx))^{3/2}} dx = \int \frac{1}{x^2(a+ia \sinh(e+fx))^{3/2}} dx$$

**Mathematica [A]** time = 24.0865, size = 0, normalized size = 0.

$$\int \frac{1}{x^2(a+ia \sinh(e+fx))^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x^2\*(a + I\*a\*Sinh[e + f\*x])^(3/2)), x]

[Out] Integrate[1/(x^2\*(a + I\*a\*Sinh[e + f\*x])^(3/2)), x]

---

**Maple [A]** time = 0.044, size = 0, normalized size = 0.

$$\int \frac{1}{x^2} (a + ia \sinh(fx + e))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+I\*a\*sinh(f\*x+e))^(3/2),x)

[Out] int(1/x^2/(a+I\*a\*sinh(f\*x+e))^(3/2),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ia \sinh(fx + e) + a)^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+I\*a\*sinh(f\*x+e))^(3/2),x, algorithm="maxima")

[Out] integrate(1/((I\*a\*sinh(f\*x + e) + a)^(3/2)\*x^2), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{\frac{1}{2}} \left( (-i f x + 4i) e^{(2 f x + 2 e)} + (f x + 4) e^{(f x + e)} \right) \sqrt{i a e^{(2 f x + 2 e)} + 2 a e^{(f x + e)} - i a e^{(-\frac{1}{2} f x - \frac{1}{2} e)}} + \left( a^2 f^2 x^3 e^{(3 f x + 3 e)} - 3 i a^2 f^2 x^3 e^{(2 f x + 2 e)} \right)}{a^2 f^2 x^3 e^{(3 f x + 3 e)} - 3 i a^2 f^2 x^3 e^{(2 f x + 2 e)} - 3 a^2 f^2 x^3 e^{(f x + e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+I\*a\*sinh(f\*x+e))^(3/2),x, algorithm="fricas")

[Out] (sqrt(1/2)\*((-I\*f\*x + 4\*I)\*e^(2\*f\*x + 2\*e) + (f\*x + 4)\*e^(f\*x + e))\*sqrt(I\*a\*e^(2\*f\*x + 2\*e) + 2\*a\*e^(f\*x + e) - I\*a)\*e^(-1/2\*f\*x - 1/2\*e) + (a^2\*f^2\*x^3\*e^(3\*f\*x + 3\*e) - 3\*I\*a^2\*f^2\*x^3\*e^(2\*f\*x + 2\*e) - 3\*a^2\*f^2\*x^3\*e^(f\*x + e) + I\*a^2\*f^2\*x^3)\*integral(sqrt(1/2)\*(-I\*f^2\*x^2 + 24\*I)\*sqrt(I\*a\*e^(



$2fx + 2e) + 2ae^{(fx + e)} - I a e^{(1/2fx + 1/2e)} / (2a^2 f^2 x^4 e^{(2fx + 2e)} - 4I a^2 f^2 x^4 e^{(fx + e)} - 2a^2 f^2 x^4), x) / (a^2 f^2 x^3 e^{(3fx + 3e)} - 3I a^2 f^2 x^3 e^{(2fx + 2e)} - 3a^2 f^2 x^3 e^{(fx + e)} + I a^2 f^2 x^3)$

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a (i \sinh(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(a+I\*a\*sinh(f\*x+e))\*\*(3/2),x)

[Out] Integral(1/(x\*\*2\*(a\*(I\*sinh(e + f\*x) + 1))\*\*(3/2)), x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(i a \sinh(fx + e) + a)^{\frac{3}{2}} x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+I\*a\*sinh(f\*x+e))^(3/2),x, algorithm="giac")

[Out] integrate(1/((I\*a\*sinh(f\*x + e) + a)^(3/2)\*x^2), x)

$$3.146 \quad \int \frac{x^3}{(a+ia \sinh(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=1016

result too large to display

```
[Out] -(1/(a^2*d^4*Sqrt[a + I*a*Sinh[c + d*x]])) + (9*x^2)/(8*a^2*d^2*Sqrt[a + I*
a*Sinh[c + d*x]]) - ((10*I)*x*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)]*Cosh[c/
2 + (I/4)*Pi + (d*x)/2])/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (((3*I)/8)
*x^3*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)]*Cosh[c/2 + (I/4)*Pi + (d*x)/2])/
(a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) - ((10*I)*Cosh[c/2 + (I/4)*Pi + (d*x)/2
]*PolyLog[2, -E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^4*Sqrt[a + I*a*Sinh[c +
d*x]]) + (((9*I)/8)*x^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, -E^((2*c
- I*Pi)/4 + (d*x)/2)))/(a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) + ((10*I)*Cos
h[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*
d^4*Sqrt[a + I*a*Sinh[c + d*x]]) - (((9*I)/8)*x^2*Cosh[c/2 + (I/4)*Pi + (d*
x)/2]*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^2*Sqrt[a + I*a*Sinh[
c + d*x]]) - (((9*I)/2)*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, -E^((2*
c - I*Pi)/4 + (d*x)/2)))/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (((9*I)/2)
*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, E^((2*c - I*Pi)/4 + (d*x)/2)])
/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + ((9*I)*Cosh[c/2 + (I/4)*Pi + (d*x)
/2]*PolyLog[4, -E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^4*Sqrt[a + I*a*Sinh[c
+ d*x]]) - ((9*I)*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[4, E^((2*c - I*Pi
)/4 + (d*x)/2)))/(a^2*d^4*Sqrt[a + I*a*Sinh[c + d*x]]) + (x^2*Sech[c/2 + (I
/4)*Pi + (d*x)/2]^2)/(4*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - (x*Tanh[c/2
+ (I/4)*Pi + (d*x)/2])/(2*a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (3*x^3*Tan
h[c/2 + (I/4)*Pi + (d*x)/2])/(16*a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) + (x^3*
Sech[c/2 + (I/4)*Pi + (d*x)/2]^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(8*a^2*d*S
qrt[a + I*a*Sinh[c + d*x]])
```

**Rubi [A]** time = 0.676576, antiderivative size = 1016, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {3319, 4186, 4185, 4182, 2279, 2391, 2531, 6609, 2282, 6589}

$$\frac{\operatorname{sech}^2\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) \tanh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) x^3}{8a^2 d \sqrt{i \sinh(c+dx) a + a}} + \frac{3 \tanh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) x^3}{16a^2 d \sqrt{i \sinh(c+dx) a + a}} + \frac{3i \tanh^{-1}\left(e^{\frac{1}{4}(2c-i\pi) + \frac{dx}{2}}\right) \cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) x^3}{8a^2 d \sqrt{i \sinh(c+dx) a + a}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + I\*a\*Sinh[c + d\*x])^(5/2), x]

```
[Out] -(1/(a^2*d^4*Sqrt[a + I*a*Sinh[c + d*x]])) + (9*x^2)/(8*a^2*d^2*Sqrt[a + I*
a*Sinh[c + d*x]]) - ((10*I)*x*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)]*Cosh[c/
2 + (I/4)*Pi + (d*x)/2])/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (((3*I)/8)
*x^3*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)]*Cosh[c/2 + (I/4)*Pi + (d*x)/2])/
(a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) - ((10*I)*Cosh[c/2 + (I/4)*Pi + (d*x)/2
]*PolyLog[2, -E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^4*Sqrt[a + I*a*Sinh[c +
d*x]]) + (((9*I)/8)*x^2*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, -E^((2*c
- I*Pi)/4 + (d*x)/2)])/(a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) + ((10*I)*Cos
h[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*
d^4*Sqrt[a + I*a*Sinh[c + d*x]]) - (((9*I)/8)*x^2*Cosh[c/2 + (I/4)*Pi + (d*
x)/2]*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^2*Sqrt[a + I*a*Sinh[
c + d*x]]) - (((9*I)/2)*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, -E^((2*
c - I*Pi)/4 + (d*x)/2)])/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (((9*I)/2)
*x*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, E^((2*c - I*Pi)/4 + (d*x)/2)])
/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + ((9*I)*Cosh[c/2 + (I/4)*Pi + (d*x)
/2]*PolyLog[4, -E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^4*Sqrt[a + I*a*Sinh[c
+ d*x]]) - ((9*I)*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[4, E^((2*c - I*Pi
)/4 + (d*x)/2)])/(a^2*d^4*Sqrt[a + I*a*Sinh[c + d*x]]) + (x^2*Sech[c/2 + (I
/4)*Pi + (d*x)/2]^2)/(4*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - (x*Tanh[c/2
+ (I/4)*Pi + (d*x)/2])/(2*a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (3*x^3*Tan
h[c/2 + (I/4)*Pi + (d*x)/2])/(16*a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) + (x^3*
Sech[c/2 + (I/4)*Pi + (d*x)/2]^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(8*a^2*d*S
qrt[a + I*a*Sinh[c + d*x]])
```

### Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
  x_Symbol] := Dist[((2*a)^IntPart[n]*(a + b*Ssin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Ssin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

### Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_)), x_Symbol] :=
```

```
-Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
)*(x_)))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_.)*((a_.)*(v_)^(n_.))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\int \frac{x^3}{(a + ia \sinh(c + dx))^{5/2}} dx = \frac{\sinh\left(\frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2}\right) \int x^3 \operatorname{csch}^5\left(\frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2}\right) dx}{4a^2 \sqrt{a + ia \sinh(c + dx)}}$$

$$= \frac{x^2 \operatorname{sech}^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{4a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} + \frac{x^3 \operatorname{sech}^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \tanh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{8a^2 d \sqrt{a + ia \sinh(c + dx)}} - \frac{\left(3 \sinh\left(\frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2}\right)\right)}{4a^2 d^2 \sqrt{a + ia \sinh(c + dx)}}$$

$$= -\frac{1}{a^2 d^4 \sqrt{a + ia \sinh(c + dx)}} + \frac{9x^2}{8a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} + \frac{x^2 \operatorname{sech}^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{4a^2 d^2 \sqrt{a + ia \sinh(c + dx)}}$$

$$= -\frac{1}{a^2 d^4 \sqrt{a + ia \sinh(c + dx)}} + \frac{9x^2}{8a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} - \frac{10ix \tanh^{-1}\left(e^{\frac{1}{4}(2c - i\pi) + \frac{dx}{2}}\right)}{a^2 d^3 \sqrt{a + ia \sinh(c + dx)}}$$

$$= -\frac{1}{a^2 d^4 \sqrt{a + ia \sinh(c + dx)}} + \frac{9x^2}{8a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} - \frac{10ix \tanh^{-1}\left(e^{\frac{1}{4}(2c - i\pi) + \frac{dx}{2}}\right)}{a^2 d^3 \sqrt{a + ia \sinh(c + dx)}}$$

$$= -\frac{1}{a^2 d^4 \sqrt{a + ia \sinh(c + dx)}} + \frac{9x^2}{8a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} - \frac{10ix \tanh^{-1}\left(e^{\frac{1}{4}(2c - i\pi) + \frac{dx}{2}}\right)}{a^2 d^3 \sqrt{a + ia \sinh(c + dx)}}$$

$$= -\frac{1}{a^2 d^4 \sqrt{a + ia \sinh(c + dx)}} + \frac{9x^2}{8a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} - \frac{10ix \tanh^{-1}\left(e^{\frac{1}{4}(2c - i\pi) + \frac{dx}{2}}\right)}{a^2 d^3 \sqrt{a + ia \sinh(c + dx)}}$$

$$= -\frac{1}{a^2 d^4 \sqrt{a + ia \sinh(c + dx)}} + \frac{9x^2}{8a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} - \frac{10ix \tanh^{-1}\left(e^{\frac{1}{4}(2c - i\pi) + \frac{dx}{2}}\right)}{a^2 d^3 \sqrt{a + ia \sinh(c + dx)}}$$

**Mathematica [A]** time = 4.53243, size = 1200, normalized size = 1.18

result too large to display

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + I\*a\*Sinh[c + d\*x])^(5/2),x]

[Out] ((Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])\*(-48\*Cosh[(c + d\*x)/2] + (8\*I)\*c\*Cosh[(c + d\*x)/2] + 70\*c^2\*Cosh[(c + d\*x)/2] - (11\*I)\*c^3\*Cosh[(c + d\*x)/2] - (8\*I)\*(c + d\*x)\*Cosh[(c + d\*x)/2] - 140\*c\*(c + d\*x)\*Cosh[(c + d\*x)/2] + (33\*I)\*c^2\*(c + d\*x)\*Cosh[(c + d\*x)/2] + 70\*(c + d\*x)^2\*Cosh[(c + d\*x)/2] - (33\*I)\*c\*(c + d\*x)^2\*Cosh[(c + d\*x)/2] + (11\*I)\*(c + d\*x)^3\*Cosh[(c + d\*x)/2] + 16\*Cosh[(3\*(c + d\*x))/2] + (8\*I)\*c\*Cosh[(3\*(c + d\*x))/2] - 18\*c^2\*Cosh[(3\*(c + d\*x))/2] - (3\*I)\*c^3\*Cosh[(3\*(c + d\*x))/2] - (8\*I)\*(c + d\*x)\*Cosh[(3\*(c + d\*x))/2] + 36\*c\*(c + d\*x)\*Cosh[(3\*(c + d\*x))/2] + (9\*I)\*c^2\*(c + d\*x)\*Cosh[(3\*(c + d\*x))/2] - 18\*(c + d\*x)^2\*Cosh[(3\*(c + d\*x))/2] - (9\*I)\*c\*(c + d\*x)^2\*Cosh[(3\*(c + d\*x))/2] + (3\*I)\*(c + d\*x)^3\*Cosh[(3\*(c + d\*x))/2] + (1 - I)\*(-1)^(3/4)\*(-160\*c\*ArcTanh[(-1)^(3/4)\*E^((c + d\*x)/2)] + 6\*c^3\*ArcTanh[(-1)^(3/4)\*E^((c + d\*x)/2)] - 80\*c\*Log[1 - (-1)^(3/4)\*E^((c + d\*x)/2)] + 3\*c^3\*Log[1 - (-1)^(3/4)\*E^((c + d\*x)/2)] - 80\*d\*x\*Log[1 - (-1)^(3/4)\*E^((c + d\*x)/2)] + 3\*d^3\*x^3\*Log[1 - (-1)^(3/4)\*E^((c + d\*x)/2)] + 80\*c\*Log[1 + (-1)^(3/4)\*E^((c + d\*x)/2)] - 3\*c^3\*Log[1 + (-1)^(3/4)\*E^((c + d\*x)/2)] + 80\*d\*x\*Log[1 + (-1)^(3/4)\*E^((c + d\*x)/2)] - 3\*d^3\*x^3\*Log[1 + (-1)^(3/4)\*E^((c + d\*x)/2)] - 2\*(-80 + 9\*d^2\*x^2)\*PolyLog[2, -((-1)^(3/4)\*E^((c + d\*x)/2))] + 2\*(-80 + 9\*d^2\*x^2)\*PolyLog[2, (-1)^(3/4)\*E^((c + d\*x)/2)] + 72\*d\*x\*PolyLog[3, -((-1)^(3/4)\*E^((c + d\*x)/2))] - 72\*d\*x\*PolyLog[3, (-1)^(3/4)\*E^((c + d\*x)/2)] - 144\*PolyLog[4, -((-1)^(3/4)\*E^((c + d\*x)/2))] + 144\*PolyLog[4, (-1)^(3/4)\*E^((c + d\*x)/2)]\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^4 - (48\*I)\*Sinh[(c + d\*x)/2] + 8\*c\*Sinh[(c + d\*x)/2] + (70\*I)\*c^2\*Sinh[(c + d\*x)/2] - 11\*c^3\*Sinh[(c + d\*x)/2] - 8\*(c + d\*x)\*Sinh[(c + d\*x)/2] - (140\*I)\*c\*(c + d\*x)\*Sinh[(c + d\*x)/2] + 33\*c^2\*(c + d\*x)\*Sinh[(c + d\*x)/2] + (70\*I)\*(c + d\*x)^2\*Sinh[(c + d\*x)/2] - 33\*c\*(c + d\*x)^2\*Sinh[(c + d\*x)/2] + 11\*(c + d\*x)^3\*Sinh[(c + d\*x)/2] - (16\*I)\*Sinh[(3\*(c + d\*x))/2] - 8\*c\*Sinh[(3\*(c + d\*x))/2] + (18\*I)\*c^2\*Sinh[(3\*(c + d\*x))/2] + 3\*c^3\*Sinh[(3\*(c + d\*x))/2] + 8\*(c + d\*x)\*Sinh[(3\*(c + d\*x))/2] - (36\*I)\*c\*(c + d\*x)\*Sinh[(3\*(c + d\*x))/2] - 9\*c^2\*(c + d\*x)\*Sinh[(3\*(c + d\*x))/2] + (18\*I)\*(c + d\*x)^2\*Sinh[(3\*(c + d\*x))/2] + 9\*c\*(c + d\*x)^2\*Sinh[(3\*(c + d\*x))/2] - 3\*(c + d\*x)^3\*Sinh[(3\*(c + d\*x))/2]))/(32\*d^4\*(a + I\*a\*Sinh[c + d\*x])^(5/2))

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int x^3 (a + ia \sinh(dx + c))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+I\*a\*sinh(d\*x+c))^(5/2),x)

[Out]  $\int (x^3/(a+I*a*\sinh(d*x+c))^{5/2}, x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(i a \sinh(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^3/(a+I*a*\sinh(d*x+c))^{5/2}, x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}(x^3/(I*a*\sinh(d*x + c) + a)^{5/2}, x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\sqrt{\frac{1}{2}}((-3i d^3 x^3 - 18i d^2 x^2 + 8i dx + 16i)e^{(4dx+4c)} - (11 d^3 x^3 + 70 d^2 x^2 - 8 dx - 48)e^{(3dx+3c)} + (-11i d^3 x^3 + 70i d^2 x^2 + 8i$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^3/(a+I*a*\sinh(d*x+c))^{5/2}, x, \text{algorithm}="fricas")$

[Out]  $(\text{sqrt}(1/2)*((-3*I*d^3*x^3 - 18*I*d^2*x^2 + 8*I*d*x + 16*I)*e^{(4*d*x + 4*c)} - (11*d^3*x^3 + 70*d^2*x^2 - 8*d*x - 48)*e^{(3*d*x + 3*c)} + (-11*I*d^3*x^3 + 70*I*d^2*x^2 + 8*I*d*x - 48*I)*e^{(2*d*x + 2*c)} - (3*d^3*x^3 - 18*d^2*x^2 - 8*d*x + 16)*e^{(d*x + c)})*\text{sqrt}(I*a*e^{(2*d*x + 2*c)} + 2*a*e^{(d*x + c)} - I*a)*e^{(-1/2*d*x - 1/2*c)} + (8*a^3*d^4*e^{(5*d*x + 5*c)} - 40*I*a^3*d^4*e^{(4*d*x + 4*c)} - 80*a^3*d^4*e^{(3*d*x + 3*c)} + 80*I*a^3*d^4*e^{(2*d*x + 2*c)} + 40*a^3*d^4*e^{(d*x + c)} - 8*I*a^3*d^4)*\text{integral}(\text{sqrt}(1/2)*(-3*I*d^2*x^3 + 80*I*x)*\text{sqrt}(I*a*e^{(2*d*x + 2*c)} + 2*a*e^{(d*x + c)} - I*a)*e^{(1/2*d*x + 1/2*c)})/(16*a^3*d^2*e^{(2*d*x + 2*c)} - 32*I*a^3*d^2*e^{(d*x + c)} - 16*a^3*d^2), x)/(8*a^3*d^4*e^{(5*d*x + 5*c)} - 40*I*a^3*d^4*e^{(4*d*x + 4*c)} - 80*a^3*d^4*e^{(3*d*x + 3*c)} + 80*I*a^3*d^4*e^{(2*d*x + 2*c)} + 40*a^3*d^4*e^{(d*x + c)} - 8*I*a^3*d^4)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(a+I\*a\*sinh(d\*x+c))\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(i a \sinh(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+I\*a\*sinh(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(x^3/(I\*a\*sinh(d\*x + c) + a)^(5/2), x)



$$3.147 \quad \int \frac{x^2}{(a+ia \sinh(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=689

$$\frac{3ix \cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, -e^{\frac{dx}{2} + \frac{1}{4}(2c-i\pi)}\right)}{4a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} - \frac{3ix \cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, e^{\frac{dx}{2} + \frac{1}{4}(2c-i\pi)}\right)}{4a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} - \frac{3i \cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)}{2a^2}$$

```
[Out] (3*x)/(4*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - (5*ArcTan[Sinh[c/2 + (I/4)*
Pi + (d*x)/2]]*Cosh[c/2 + (I/4)*Pi + (d*x)/2])/(3*a^2*d^3*Sqrt[a + I*a*Sinh
[c + d*x]]) + (((3*I)/8)*x^2*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)]*Cosh[c/2
+ (I/4)*Pi + (d*x)/2])/(a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) + (((3*I)/4)*x*
Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, -E^((2*c - I*Pi)/4 + (d*x)/2)])/(
a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - (((3*I)/4)*x*Cosh[c/2 + (I/4)*Pi + (
d*x)/2]*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)])/(a^2*d^2*Sqrt[a + I*a*Sin
h[c + d*x]]) - (((3*I)/2)*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, -E^((2*
c - I*Pi)/4 + (d*x)/2)])/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (((3*I)/2)
*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, E^((2*c - I*Pi)/4 + (d*x)/2)])/(
a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (x*Sech[c/2 + (I/4)*Pi + (d*x)/2]^2)
/(6*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - Tanh[c/2 + (I/4)*Pi + (d*x)/2]/(
6*a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (3*x^2*Tanh[c/2 + (I/4)*Pi + (d*x)
/2])/(16*a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) + (x^2*Sech[c/2 + (I/4)*Pi + (d
*x)/2]^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(8*a^2*d*Sqrt[a + I*a*Sinh[c + d*x
]])
```

**Rubi [A]** time = 0.460092, antiderivative size = 689, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$ , Rules used = {3319, 4186, 3768, 3770, 4182, 2531, 2282, 6589}

$$\frac{3ix \cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, -e^{\frac{dx}{2} + \frac{1}{4}(2c-i\pi)}\right)}{4a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} - \frac{3ix \cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, e^{\frac{dx}{2} + \frac{1}{4}(2c-i\pi)}\right)}{4a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} - \frac{3i \cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)}{2a^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + I\*a\*Sinh[c + d\*x])^(5/2), x]

```
[Out] (3*x)/(4*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - (5*ArcTan[Sinh[c/2 + (I/4)*
Pi + (d*x)/2]]*Cosh[c/2 + (I/4)*Pi + (d*x)/2])/(3*a^2*d^3*Sqrt[a + I*a*Sinh
[c + d*x]]) + (((3*I)/8)*x^2*ArcTanh[E^((2*c - I*Pi)/4 + (d*x)/2)]*Cosh[c/2
+ (I/4)*Pi + (d*x)/2])/(a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) + (((3*I)/4)*x*
```

```

Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[2, -E^((2*c - I*Pi)/4 + (d*x)/2)]/(
a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - (((3*I)/4)*x*Cosh[c/2 + (I/4)*Pi + (
d*x)/2]*PolyLog[2, E^((2*c - I*Pi)/4 + (d*x)/2)]/(a^2*d^2*Sqrt[a + I*a*Sin
h[c + d*x]]) - (((3*I)/2)*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, -E^((2*
c - I*Pi)/4 + (d*x)/2)]/(a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (((3*I)/2)
*Cosh[c/2 + (I/4)*Pi + (d*x)/2]*PolyLog[3, E^((2*c - I*Pi)/4 + (d*x)/2)]/(
a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (x*Sech[c/2 + (I/4)*Pi + (d*x)/2]^2)
/(6*a^2*d^2*Sqrt[a + I*a*Sinh[c + d*x]]) - Tanh[c/2 + (I/4)*Pi + (d*x)/2]/(
6*a^2*d^3*Sqrt[a + I*a*Sinh[c + d*x]]) + (3*x^2*Tanh[c/2 + (I/4)*Pi + (d*x)
/2])/(16*a^2*d*Sqrt[a + I*a*Sinh[c + d*x]]) + (x^2*Sech[c/2 + (I/4)*Pi + (d
*x)/2]^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(8*a^2*d*Sqrt[a + I*a*Sinh[c + d*x
]])

```

### Rule 3319

```

Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] := Dist[((2*a)^IntPart[n]*(a + b*Ssin[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*Ssin[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])

```

### Rule 4186

```

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

```

### Rule 3768

```

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]

```

### Rule 3770

```

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]

```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x]
+ Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)], x], x])
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol]
:> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)])/(b*c*n*Log[F]), x]
+ Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x]
/; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol]
:> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]
/; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)]
/; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_]
/; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x]
/; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps



$$(-1)^{3/4} * E^{((c + d*x)/2)} + 72 * \text{PolyLog}[3, (-1)^{3/4} * E^{((c + d*x)/2)}] * (\text{Cosh}[(c + d*x)/2] + I * \text{Sinh}[(c + d*x)/2])^4 + 24 * d^2 * x^2 * \text{Sinh}[(c + d*x)/2] + 2 * (-8 + 9 * d^2 * x^2) * (\text{Cosh}[(c + d*x)/2] + I * \text{Sinh}[(c + d*x)/2])^2 * \text{Sinh}[(c + d*x)/2]) / (48 * d^3 * (a + I * a * \text{Sinh}[c + d*x])^{5/2})$$

**Maple [F]** time = 0.048, size = 0, normalized size = 0.

$$\int x^2 (a + ia \sinh(dx + c))^{-5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+I\*a\*sinh(d\*x+c))^(5/2),x)

[Out] int(x^2/(a+I\*a\*sinh(d\*x+c))^(5/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(ia \sinh(dx + c) + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+I\*a\*sinh(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(x^2/(I\*a\*sinh(d\*x + c) + a)^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\sqrt{\frac{1}{2}} \left( (-9i d^2 x^2 - 36i dx + 8i) e^{(4dx+4c)} - (33 d^2 x^2 + 140 dx - 8) e^{(3dx+3c)} + (-33i d^2 x^2 + 140i dx + 8i) e^{(2dx+2c)} - (9 d^2 x^2 - \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+I\*a\*sinh(d\*x+c))^(5/2),x, algorithm="fricas")

```
[Out] (sqrt(1/2)*((-9*I*d^2*x^2 - 36*I*d*x + 8*I)*e^(4*d*x + 4*c) - (33*d^2*x^2 +
140*d*x - 8)*e^(3*d*x + 3*c) + (-33*I*d^2*x^2 + 140*I*d*x + 8*I)*e^(2*d*x
+ 2*c) - (9*d^2*x^2 - 36*d*x - 8)*e^(d*x + c))*sqrt(I*a*e^(2*d*x + 2*c) + 2
*a*e^(d*x + c) - I*a)*e^(-1/2*d*x - 1/2*c) + (24*a^3*d^3*e^(5*d*x + 5*c) -
120*I*a^3*d^3*e^(4*d*x + 4*c) - 240*a^3*d^3*e^(3*d*x + 3*c) + 240*I*a^3*d^3
*e^(2*d*x + 2*c) + 120*a^3*d^3*e^(d*x + c) - 24*I*a^3*d^3)*integral(sqrt(1/
2)*(-9*I*d^2*x^2 + 80*I)*sqrt(I*a*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - I*a)*
e^(1/2*d*x + 1/2*c)/(48*a^3*d^2*e^(2*d*x + 2*c) - 96*I*a^3*d^2*e^(d*x + c)
- 48*a^3*d^2), x))/(24*a^3*d^3*e^(5*d*x + 5*c) - 120*I*a^3*d^3*e^(4*d*x + 4
*c) - 240*a^3*d^3*e^(3*d*x + 3*c) + 240*I*a^3*d^3*e^(2*d*x + 2*c) + 120*a^3
*d^3*e^(d*x + c) - 24*I*a^3*d^3)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+I*a*sinh(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(i a \sinh(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+I*a*sinh(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(I*a*sinh(d*x + c) + a)^(5/2), x)
```

$$3.148 \quad \int \frac{x}{(a+ia \sinh(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=416

$$\frac{3i \cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, -e^{\frac{dx}{2} + \frac{1}{4}(2c-i\pi)}\right)}{8a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} - \frac{3i \cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, e^{\frac{dx}{2} + \frac{1}{4}(2c-i\pi)}\right)}{8a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} + \frac{3}{8a^2 d^2 \sqrt{a + ia \sinh(c + dx)}}$$

[Out] 3/(8\*a^2\*d^2\*Sqrt[a + I\*a\*Sinh[c + d\*x]]) + (((3\*I)/8)\*x\*ArcTanh[E^((2\*c - I\*Pi)/4 + (d\*x)/2)]\*Cosh[c/2 + (I/4)\*Pi + (d\*x)/2])/(a^2\*d\*Sqrt[a + I\*a\*Sinh[c + d\*x]]) + (((3\*I)/8)\*Cosh[c/2 + (I/4)\*Pi + (d\*x)/2]\*PolyLog[2, -E^((2\*c - I\*Pi)/4 + (d\*x)/2)])/(a^2\*d^2\*Sqrt[a + I\*a\*Sinh[c + d\*x]]) - (((3\*I)/8)\*Cosh[c/2 + (I/4)\*Pi + (d\*x)/2]\*PolyLog[2, E^((2\*c - I\*Pi)/4 + (d\*x)/2)])/(a^2\*d^2\*Sqrt[a + I\*a\*Sinh[c + d\*x]]) + Sech[c/2 + (I/4)\*Pi + (d\*x)/2]^2/(12\*a^2\*d^2\*Sqrt[a + I\*a\*Sinh[c + d\*x]]) + (3\*x\*Tanh[c/2 + (I/4)\*Pi + (d\*x)/2])/(16\*a^2\*d\*Sqrt[a + I\*a\*Sinh[c + d\*x]]) + (x\*Sech[c/2 + (I/4)\*Pi + (d\*x)/2]^2\*Tanh[c/2 + (I/4)\*Pi + (d\*x)/2])/(8\*a^2\*d\*Sqrt[a + I\*a\*Sinh[c + d\*x]])

**Rubi [A]** time = 0.242906, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {3319, 4185, 4182, 2279, 2391}

$$\frac{3i \cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, -e^{\frac{dx}{2} + \frac{1}{4}(2c-i\pi)}\right)}{8a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} - \frac{3i \cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right) \text{PolyLog}\left(2, e^{\frac{dx}{2} + \frac{1}{4}(2c-i\pi)}\right)}{8a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} + \frac{3}{8a^2 d^2 \sqrt{a + ia \sinh(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + I\*a\*Sinh[c + d\*x])^(5/2), x]

[Out] 3/(8\*a^2\*d^2\*Sqrt[a + I\*a\*Sinh[c + d\*x]]) + (((3\*I)/8)\*x\*ArcTanh[E^((2\*c - I\*Pi)/4 + (d\*x)/2)]\*Cosh[c/2 + (I/4)\*Pi + (d\*x)/2])/(a^2\*d\*Sqrt[a + I\*a\*Sinh[c + d\*x]]) + (((3\*I)/8)\*Cosh[c/2 + (I/4)\*Pi + (d\*x)/2]\*PolyLog[2, -E^((2\*c - I\*Pi)/4 + (d\*x)/2)])/(a^2\*d^2\*Sqrt[a + I\*a\*Sinh[c + d\*x]]) - (((3\*I)/8)\*Cosh[c/2 + (I/4)\*Pi + (d\*x)/2]\*PolyLog[2, E^((2\*c - I\*Pi)/4 + (d\*x)/2)])/(a^2\*d^2\*Sqrt[a + I\*a\*Sinh[c + d\*x]]) + Sech[c/2 + (I/4)\*Pi + (d\*x)/2]^2/(12\*a^2\*d^2\*Sqrt[a + I\*a\*Sinh[c + d\*x]]) + (3\*x\*Tanh[c/2 + (I/4)\*Pi + (d\*x)/2])/(16\*a^2\*d\*Sqrt[a + I\*a\*Sinh[c + d\*x]]) + (x\*Sech[c/2 + (I/4)\*Pi + (d\*x)/2]^2\*Tanh[c/2 + (I/4)\*Pi + (d\*x)/2])/(8\*a^2\*d\*Sqrt[a + I\*a\*Sinh[c + d\*x]])

Rule 3319

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
  x_Symbol] := Dist[((2*a)^IntPart[n]*(a + b*SIN[e + f*x])^FracPart[n])/Sin[
e/2 + (a*Pi)/(4*b) + (f*x)/2]^(2*FracPart[n]), Int[(c + d*x)^m*SIN[e/2 + (a
*Pi)/(4*b) + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_)), x_Symbol] :=
-Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps



$$\begin{aligned}
\int \frac{x}{(a + ia \sinh(c + dx))^{5/2}} dx &= \frac{\sinh\left(\frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2}\right) \int x \operatorname{csch}^5\left(\frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2}\right) dx}{4a^2 \sqrt{a + ia \sinh(c + dx)}} \\
&= \frac{\operatorname{sech}^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{12a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} + \frac{x \operatorname{sech}^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right) \tanh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{8a^2 d \sqrt{a + ia \sinh(c + dx)}} - \frac{\left(3 \sinh\left(\frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2}\right)\right)}{16a^2 d \sqrt{a + ia \sinh(c + dx)}} \\
&= \frac{3}{8a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} + \frac{\operatorname{sech}^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{12a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} + \frac{3x \tanh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{16a^2 d \sqrt{a + ia \sinh(c + dx)}} \\
&= \frac{3}{8a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} + \frac{3ix \tanh^{-1}\left(e^{\frac{1}{4}(2c - i\pi) + \frac{dx}{2}}\right) \cosh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{8a^2 d \sqrt{a + ia \sinh(c + dx)}} + \frac{\operatorname{sech}^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{12a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} \\
&= \frac{3}{8a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} + \frac{3ix \tanh^{-1}\left(e^{\frac{1}{4}(2c - i\pi) + \frac{dx}{2}}\right) \cosh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{8a^2 d \sqrt{a + ia \sinh(c + dx)}} + \frac{\operatorname{sech}^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{12a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} \\
&= \frac{3}{8a^2 d^2 \sqrt{a + ia \sinh(c + dx)}} + \frac{3ix \tanh^{-1}\left(e^{\frac{1}{4}(2c - i\pi) + \frac{dx}{2}}\right) \cosh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{8a^2 d \sqrt{a + ia \sinh(c + dx)}} + \frac{3i \cosh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{8a^2 d \sqrt{a + ia \sinh(c + dx)}} + \frac{\operatorname{sech}^2\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{12a^2 d^2 \sqrt{a + ia \sinh(c + dx)}}
\end{aligned}$$

**Mathematica [A]** time = 1.57683, size = 411, normalized size = 0.99

$$\left(\cosh\left(\frac{1}{2}(c + dx)\right) + i \sinh\left(\frac{1}{2}(c + dx)\right)\right) \left( \frac{9i \left(\cosh\left(\frac{1}{2}(c + dx)\right) + i \sinh\left(\frac{1}{2}(c + dx)\right)\right)^4 \left( -2 \operatorname{PolyLog}\left(2, -\sqrt[4]{-1} e^{-\frac{c}{2} - \frac{dx}{2}}\right) + 2 \operatorname{PolyLog}\left(2, \sqrt[4]{-1} e^{-\frac{c}{2} - \frac{dx}{2}}\right) + \frac{1}{2} i \right)}{\sqrt{2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + I\*a\*Sinh[c + d\*x])^(5/2), x]

[Out] ((Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])\*(4\*(2 + (3\*I)\*d\*x)\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]) + 9\*(2 + I\*d\*x)\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^3 - 9\*sqrt[2]\*c\*ArcTan[(I + Tanh[(c + d\*x)/4])/sqrt[2]]\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^4 + ((9\*I)\*(Pi\*ArcTan[(I + Tanh[(c + d\*x)/4])/sqrt[2]] + (I/2)\*((2\*I)\*c + Pi + (2\*I)\*d\*x)\*(Log[1 - (-1)^(1/4)\*E^(-c/2 - (d\*x)/2)] - Log[1 + (-1)^(1/4)\*E^(-c/2 - (d\*x)/2)]) - 2\*PolyLog[2, -((-1)^(1/4)\*E^(-c/2 - (d\*x)/2)]) + 2\*PolyLog[2, (-1)^(1/4)\*E^(-c/2 - (d\*x)/2)])\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^4)/sqrt[2] + 24\*d\*x\*Sinh[(c + d\*x)/2] + 18\*d\*x\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^2\*Sinh[(c + d\*x)

/2]))/(48\*d^2\*(a + I\*a\*Sinh[c + d\*x])^(5/2))

**Maple [F]** time = 0.045, size = 0, normalized size = 0.

$$\int x (a + ia \sinh(dx + c))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+I\*a\*sinh(d\*x+c))^(5/2),x)

[Out] int(x/(a+I\*a\*sinh(d\*x+c))^(5/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(ia \sinh(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+I\*a\*sinh(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(x/(I\*a\*sinh(d\*x + c) + a)^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\sqrt{\frac{1}{2}} \left( (-9i dx - 18i) e^{(4dx+4c)} - (33 dx + 70) e^{(3dx+3c)} + (-33i dx + 70i) e^{(2dx+2c)} - 9(dx-2) e^{(dx+c)} \right) \sqrt{ia e^{(2dx+2c)} + 2ae^{(dx+c)}}$$

$$24 a^3 d^2 e^{(5dx+5c)} - 120 i a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+I\*a\*sinh(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] (sqrt(1/2)\*((-9\*I\*d\*x - 18\*I)\*e^(4\*d\*x + 4\*c) - (33\*d\*x + 70)\*e^(3\*d\*x + 3\*c) + (-33\*I\*d\*x + 70\*I)\*e^(2\*d\*x + 2\*c) - 9\*(d\*x - 2)\*e^(d\*x + c))\*sqrt(I\*a

```
*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - I*a)*e^(-1/2*d*x - 1/2*c) + (24*a^3*d^
2*e^(5*d*x + 5*c) - 120*I*a^3*d^2*e^(4*d*x + 4*c) - 240*a^3*d^2*e^(3*d*x +
3*c) + 240*I*a^3*d^2*e^(2*d*x + 2*c) + 120*a^3*d^2*e^(d*x + c) - 24*I*a^3*d
^2)*integral(-3*I*sqrt(1/2)*sqrt(I*a*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - I*
a)*x*e^(1/2*d*x + 1/2*c)/(16*a^3*e^(2*d*x + 2*c) - 32*I*a^3*e^(d*x + c) - 1
6*a^3), x)/(24*a^3*d^2*e^(5*d*x + 5*c) - 120*I*a^3*d^2*e^(4*d*x + 4*c) - 2
40*a^3*d^2*e^(3*d*x + 3*c) + 240*I*a^3*d^2*e^(2*d*x + 2*c) + 120*a^3*d^2*e^
(d*x + c) - 24*I*a^3*d^2)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+I\*a\*sinh(d\*x+c))\*\*(5/2),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(i a \sinh(dx + c) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+I\*a\*sinh(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(x/(I\*a\*sinh(d\*x + c) + a)^(5/2), x)

$$3.149 \quad \int \frac{1}{x(a+ia \sinh(c+dx))^{5/2}} dx$$

**Optimal.** Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{x(a+ia \sinh(c+dx))^{5/2}}, x\right)$$

[Out] Unintegrable[1/(x\*(a + I\*a\*Sinh[c + d\*x])^(5/2)), x]

**Rubi [A]** time = 0.0900751, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{x(a+ia \sinh(c+dx))^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x\*(a + I\*a\*Sinh[c + d\*x])^(5/2)),x]

[Out] Defer[Int][1/(x\*(a + I\*a\*Sinh[c + d\*x])^(5/2)), x]

Rubi steps

$$\int \frac{1}{x(a+ia \sinh(c+dx))^{5/2}} dx = \int \frac{1}{x(a+ia \sinh(c+dx))^{5/2}} dx$$

**Mathematica [A]** time = 35.33, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+ia \sinh(c+dx))^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x\*(a + I\*a\*Sinh[c + d\*x])^(5/2)),x]

[Out] Integrate[1/(x\*(a + I\*a\*Sinh[c + d\*x])^(5/2)), x]

---

**Maple [A]** time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{x} (a + ia \sinh(dx + c))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+I\*a\*sinh(d\*x+c))^(5/2),x)

[Out] int(1/x/(a+I\*a\*sinh(d\*x+c))^(5/2),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ia \sinh(dx + c) + a)^{\frac{5}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+I\*a\*sinh(d\*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate(1/((I\*a\*sinh(d\*x + c) + a)^(5/2)\*x), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\sqrt{\frac{1}{2}} \left( (-9i d^3 x^3 + 18i d^2 x^2 + 8i dx - 48i) e^{4dx+4c} - (33 d^3 x^3 - 70 d^2 x^2 - 8 dx + 144) e^{3dx+3c} + (-33i d^3 x^3 - 70i d^2 x^2 + 8 dx - 48i) e^{2dx+2c} \right) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+I\*a\*sinh(d\*x+c))^(5/2),x, algorithm="fricas")

[Out] (sqrt(1/2))\*((-9\*I\*d^3\*x^3 + 18\*I\*d^2\*x^2 + 8\*I\*d\*x - 48\*I)\*e^(4\*d\*x + 4\*c) - (33\*d^3\*x^3 - 70\*d^2\*x^2 - 8\*d\*x + 144)\*e^(3\*d\*x + 3\*c) + (-33\*I\*d^3\*x^3 - 70\*I\*d^2\*x^2 + 8\*I\*d\*x + 144\*I)\*e^(2\*d\*x + 2\*c) - (9\*d^3\*x^3 + 18\*d^2\*x^2 - 8\*d\*x - 48)\*e^(d\*x + c))\*sqrt(I\*a\*e^(2\*d\*x + 2\*c) + 2\*a\*e^(d\*x + c) - I\*a)\*e^(-1/2\*d\*x - 1/2\*c) + (24\*a^3\*d^4\*x^4\*e^(5\*d\*x + 5\*c) - 120\*I\*a^3\*d^4\*x

$$\begin{aligned}
& ^4e^{(4dx + 4c)} - 240a^3d^4x^4e^{(3dx + 3c)} + 240Ia^3d^4x^4e^{(2dx + 2c)} + 120a^3d^4x^4e^{(dx + c)} - 24Ia^3d^4x^4 \int \sqrt{\frac{1}{2}} \cdot (-9Id^4x^4 + 80Id^2x^2 - 384I) \cdot \sqrt{Iae^{(2dx + 2c)} + 2ae^{(dx + c)} - Ia} \cdot e^{(1/2dx + 1/2c)} / (48a^3d^4x^5e^{(2dx + 2c)} - 96Ia^3d^4x^5e^{(dx + c)} - 48a^3d^4x^5) , x) / (24a^3d^4x^4e^{(5dx + 5c)} - 120Ia^3d^4x^4e^{(4dx + 4c)} - 240a^3d^4x^4e^{(3dx + 3c)} + 240Ia^3d^4x^4e^{(2dx + 2c)} + 120a^3d^4x^4e^{(dx + c)} - 24Ia^3d^4x^4)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+I\*a\*sinh(d\*x+c))\*\*(5/2),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(ia \sinh(dx + c) + a)^{\frac{5}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+I\*a\*sinh(d\*x+c))^(5/2),x, algorithm="giac")

[Out] integrate(1/((I\*a\*sinh(d\*x + c) + a)^(5/2)\*x), x)

$$3.150 \quad \int \frac{\sqrt[3]{a+ia \sinh(e+fx)}}{x} dx$$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\frac{\sqrt[3]{a+ia \sinh(e+fx)}}{x}, x\right)$$

[Out] Unintegrable[(a + I\*a\*Sinh[e + f\*x])^(1/3)/x, x]

**Rubi [A]** time = 0.0733574, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sqrt[3]{a+ia \sinh(e+fx)}}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(a + I\*a\*Sinh[e + f\*x])^(1/3)/x,x]

[Out] Defer[Int] [(a + I\*a\*Sinh[e + f\*x])^(1/3)/x, x]

Rubi steps

$$\int \frac{\sqrt[3]{a+ia \sinh(e+fx)}}{x} dx = \int \frac{\sqrt[3]{a+ia \sinh(e+fx)}}{x} dx$$

**Mathematica [A]** time = 3.97186, size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a+ia \sinh(e+fx)}}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + I\*a\*Sinh[e + f\*x])^(1/3)/x,x]

[Out] Integrate[(a + I\*a\*Sinh[e + f\*x])^(1/3)/x, x]

---

**Maple [A]** time = 0.073, size = 0, normalized size = 0.

$$\int \frac{1}{x} \sqrt[3]{a + ia \sinh(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+I\*a\*sinh(f\*x+e))^(1/3)/x,x)

[Out] int((a+I\*a\*sinh(f\*x+e))^(1/3)/x,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ia \sinh(fx + e) + a)^{\frac{1}{3}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))^(1/3)/x,x, algorithm="maxima")

[Out] integrate((I\*a\*sinh(f\*x + e) + a)^(1/3)/x, x)

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))^(1/3)/x,x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{a(i \sinh(e + fx) + 1)}}{x} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))\*\*(1/3)/x,x)

[Out] Integral((a\*(I\*sinh(e + f\*x) + 1))\*\*(1/3)/x, x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(i a \sinh(fx + e) + a)^{\frac{1}{3}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+I\*a\*sinh(f\*x+e))^(1/3)/x,x, algorithm="giac")

[Out] integrate((I\*a\*sinh(f\*x + e) + a)^(1/3)/x, x)

$$\mathbf{3.151} \quad \int (c + dx)^m (a + ia \sinh(e + fx))^n dx$$

Optimal. Leaf size=25

$$\text{Unintegrable}((c + dx)^m (a + ia \sinh(e + fx))^n, x)$$

[Out] Unintegrable[(c + d\*x)^m\*(a + I\*a\*Sinh[e + f\*x])^n, x]

**Rubi [A]** time = 0.0514336, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (c + dx)^m (a + ia \sinh(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m\*(a + I\*a\*Sinh[e + f\*x])^n,x]

[Out] Defer[Int] [(c + d\*x)^m\*(a + I\*a\*Sinh[e + f\*x])^n, x]

Rubi steps

$$\int (c + dx)^m (a + ia \sinh(e + fx))^n dx = \int (c + dx)^m (a + ia \sinh(e + fx))^n dx$$

**Mathematica [A]** time = 3.91554, size = 0, normalized size = 0.

$$\int (c + dx)^m (a + ia \sinh(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m\*(a + I\*a\*Sinh[e + f\*x])^n,x]

[Out] Integrate[(c + d\*x)^m\*(a + I\*a\*Sinh[e + f\*x])^n, x]

**Maple [A]** time = 0.048, size = 0, normalized size = 0.

$$\int (dx + c)^m (a + ia \sinh (fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*(a+I\*a\*sinh(f\*x+e))^n,x)

[Out] int((d\*x+c)^m\*(a+I\*a\*sinh(f\*x+e))^n,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m (ia \sinh (fx + e) + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+I\*a\*sinh(f\*x+e))^n,x, algorithm="maxima")

[Out] integrate((d\*x + c)^m\*(I\*a\*sinh(f\*x + e) + a)^n, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx + c)^m \left(\frac{1}{2} \left(ia e^{2fx+2e} + 2ae^{fx+e} - ia\right)e^{-fx-e}\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+I\*a\*sinh(f\*x+e))^n,x, algorithm="fricas")

[Out] integral((d\*x + c)^m\*(1/2\*(I\*a\*e^(2\*f\*x + 2\*e) + 2\*a\*e^(f\*x + e) - I\*a)\*e^(-f\*x - e))^n, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m\*(a+I\*a\*sinh(f\*x+e))\*\*n,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m (i a \sinh(fx + e) + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+I\*a\*sinh(f\*x+e))^n,x, algorithm="giac")

[Out] integrate((d\*x + c)^m\*(I\*a\*sinh(f\*x + e) + a)^n, x)

### 3.152 $\int (c + dx)^m (a + ia \sinh(e + fx))^3 dx$

**Optimal.** Leaf size=410

$$\frac{ia^3 3^{-m-1} e^{3e - \frac{3cf}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, -\frac{3f(c+dx)}{d}\right)}{8f} - \frac{3a^3 2^{-m-3} e^{2e - \frac{2cf}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, -\frac{2f(c+dx)}{d}\right)}{f}$$

[Out]  $(5a^3(c + dx)^{(1+m)})/(2d(1+m)) - ((I/8)3^{(-1-m)}a^3E^{(3e - (3cf)/d)}(c + dx)^m \text{Gamma}[1+m, (-3f(c + dx))/d])/(f((f(c + dx))/d))^m - (3 \cdot 2^{(-3-m)}a^3E^{(2e - (2cf)/d)}(c + dx)^m \text{Gamma}[1+m, (-2f(c + dx))/d])/(f((f(c + dx))/d))^m + (((15I)/8)a^3E^{(e - (cf)/d)}(c + dx)^m \text{Gamma}[1+m, -(f(c + dx))/d])/(f((f(c + dx))/d))^m + (((15I)/8)a^3E^{(-e + (cf)/d)}(c + dx)^m \text{Gamma}[1+m, (f(c + dx))/d])/(f((f(c + dx))/d))^m + (3 \cdot 2^{(-3-m)}a^3E^{(-2e + (2cf)/d)}(c + dx)^m \text{Gamma}[1+m, (2f(c + dx))/d])/(f((f(c + dx))/d))^m - ((I/8)3^{(-1-m)}a^3E^{(-3e + (3cf)/d)}(c + dx)^m \text{Gamma}[1+m, (3f(c + dx))/d])/(f((f(c + dx))/d))^m$

**Rubi [A]** time = 0.602034, antiderivative size = 410, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$ , Rules used = {3318, 3312, 3307, 2181, 3308}

$$\frac{ia^3 3^{-m-1} e^{3e - \frac{3cf}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, -\frac{3f(c+dx)}{d}\right)}{8f} - \frac{3a^3 2^{-m-3} e^{2e - \frac{2cf}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, -\frac{2f(c+dx)}{d}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^m\*(a + I\*a\*Sinh[e + f\*x])^3,x]

[Out]  $(5a^3(c + dx)^{(1+m)})/(2d(1+m)) - ((I/8)3^{(-1-m)}a^3E^{(3e - (3cf)/d)}(c + dx)^m \text{Gamma}[1+m, (-3f(c + dx))/d])/(f((f(c + dx))/d))^m - (3 \cdot 2^{(-3-m)}a^3E^{(2e - (2cf)/d)}(c + dx)^m \text{Gamma}[1+m, (-2f(c + dx))/d])/(f((f(c + dx))/d))^m + (((15I)/8)a^3E^{(e - (cf)/d)}(c + dx)^m \text{Gamma}[1+m, -(f(c + dx))/d])/(f((f(c + dx))/d))^m + (((15I)/8)a^3E^{(-e + (cf)/d)}(c + dx)^m \text{Gamma}[1+m, (f(c + dx))/d])/(f((f(c + dx))/d))^m + (3 \cdot 2^{(-3-m)}a^3E^{(-2e + (2cf)/d)}(c + dx)^m \text{Gamma}[1+m, (2f(c + dx))/d])/(f((f(c + dx))/d))^m - ((I/8)3^{(-1-m)}a^3E^{(-3e + (3cf)/d)}(c + dx)^m \text{Gamma}[1+m, (3f(c + dx))/d])/(f((f(c + dx))/d))^m$

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
(f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:= -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x]])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^m (a + ia \sinh(e + fx))^3 dx &= (8a^3) \int (c + dx)^m \sin^6\left(\frac{1}{2}\left(ie + \frac{\pi}{2}\right) + \frac{ifx}{2}\right) dx \\
&= (8a^3) \int \left(\frac{5}{16}(c + dx)^m - \frac{3}{16}(c + dx)^m \cosh(2e + 2fx) + \frac{15}{32}i(c + dx)^m \sinh(e + fx)\right) dx \\
&= \frac{5a^3(c + dx)^{1+m}}{2d(1+m)} - \frac{1}{4}(ia^3) \int (c + dx)^m \sinh(3e + 3fx) dx + \frac{1}{4}(15ia^3) \int (c + dx)^m \sinh(e + fx) dx \\
&= \frac{5a^3(c + dx)^{1+m}}{2d(1+m)} - \frac{1}{8}(ia^3) \int e^{-i(3ie+3ifx)}(c + dx)^m dx + \frac{1}{8}(ia^3) \int e^{i(3ie+3ifx)}(c + dx)^m dx \\
&= \frac{5a^3(c + dx)^{1+m}}{2d(1+m)} - \frac{i3^{-1-m}a^3e^{3e-\frac{3cf}{d}}(c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{3f(c+dx)}{d}\right)}{8f}
\end{aligned}$$

**Mathematica [A]** time = 1.58512, size = 339, normalized size = 0.83

$$\frac{1}{24}a^3(c + dx)^m \left( -\frac{i3^{-m}e^{3e-\frac{3cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{3f(c+dx)}{d}\right)}{f} - \frac{9 \cdot 2^{-m}e^{2e-\frac{2cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2f(c+dx)}{d}\right)}{f} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^m\*(a + I\*a\*Sinh[e + f\*x])^3,x]

[Out] (a^3\*(c + d\*x)^m\*((60\*(c + d\*x))/(d\*(1 + m)) - (I\*E^(3\*e - (3\*c\*f)/d)\*Gamma[1 + m, (-3\*f\*(c + d\*x))/d])/(3^m\*f\*(-((f\*(c + d\*x))/d))^m) - (9\*E^(2\*e - (2\*c\*f)/d)\*Gamma[1 + m, (-2\*f\*(c + d\*x))/d])/(2^m\*f\*(-((f\*(c + d\*x))/d))^m) + ((45\*I)\*E^(e - (c\*f)/d)\*Gamma[1 + m, -((f\*(c + d\*x))/d)])/(f\*(-((f\*(c + d\*x))/d))^m) + ((45\*I)\*E^(-e + (c\*f)/d)\*Gamma[1 + m, (f\*(c + d\*x))/d])/(f\*((f\*(c + d\*x))/d)^m) + (9\*E^(-2\*e + (2\*c\*f)/d)\*Gamma[1 + m, (2\*f\*(c + d\*x))/d])/(2^m\*f\*((f\*(c + d\*x))/d)^m) - (I\*E^(-3\*e + (3\*c\*f)/d)\*Gamma[1 + m, (3\*f\*(c + d\*x))/d])/(3^m\*f\*((f\*(c + d\*x))/d)^m))/24

**Maple [F]** time = 0.12, size = 0, normalized size = 0.

$$\int (dx + c)^m (a + ia \sinh(fx + e))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x)`

[Out] `int((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.8322, size = 857, normalized size = 2.09

$$\left(-i a^3 d m - i a^3 d\right) e^{\left(-\frac{d m \log\left(\frac{3 f}{d}\right)+3 d e-3 c f}{d}\right)} \Gamma\left(m+1, \frac{3(d f x+c f)}{d}\right) + 9\left(a^3 d m + a^3 d\right) e^{\left(-\frac{d m \log\left(\frac{2 f}{d}\right)+2 d e-2 c f}{d}\right)} \Gamma\left(m+1, \frac{2(d f x+c f)}{d}\right) + (45i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*(a+I*a*sinh(f*x+e))^3,x, algorithm="fricas")`

[Out]  $1/24*((-I*a^3*d*m - I*a^3*d)*e^{-(d*m*\log(3*f/d) + 3*d*e - 3*c*f)/d}*\text{gamma}(m + 1, 3*(d*f*x + c*f)/d) + 9*(a^3*d*m + a^3*d)*e^{-(d*m*\log(2*f/d) + 2*d*e - 2*c*f)/d}*\text{gamma}(m + 1, 2*(d*f*x + c*f)/d) + (45*I*a^3*d*m + 45*I*a^3*d)*e^{-(d*m*\log(f/d) + d*e - c*f)/d}*\text{gamma}(m + 1, (d*f*x + c*f)/d) + (45*I*a^3*d*m + 45*I*a^3*d)*e^{-(d*m*\log(-f/d) - d*e + c*f)/d}*\text{gamma}(m + 1, -(d*f*x + c*f)/d) - 9*(a^3*d*m + a^3*d)*e^{-(d*m*\log(-2*f/d) - 2*d*e + 2*c*f)/d}*\text{gamma}(m + 1, -2*(d*f*x + c*f)/d) + (-I*a^3*d*m - I*a^3*d)*e^{-(d*m*\log(-3*f/d) - 3*d*e + 3*c*f)/d}*\text{gamma}(m + 1, -3*(d*f*x + c*f)/d) + 60*(a^3*d*f*x + a^3*c*f)*(d*x + c)^m/(d*f*m + d*f)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m\*(a+I\*a\*sinh(f\*x+e))\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (i a \sinh(fx + e) + a)^3 (dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+I\*a\*sinh(f\*x+e))^3,x, algorithm="giac")

[Out] integrate((I\*a\*sinh(f\*x + e) + a)^3\*(d\*x + c)^m, x)

### 3.153 $\int (c + dx)^m (a + ia \sinh(e + fx))^2 dx$

**Optimal.** Leaf size=268

$$\frac{a^2 2^{-m-3} e^{2e - \frac{2cf}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2f(c+dx)}{d}\right)}{f} + \frac{ia^2 e^{e - \frac{cf}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{f(c+dx)}{d}\right)}{f}$$

[Out]  $(3a^2(c + dx)^{(1+m)})/(2d(1+m)) - (2^{-(3-m)}a^2E^{(2e - (2cf)/d)}(c + dx)^m \Gamma[1+m, (-2f(c + dx))/d])/(f((f(c + dx))/d)^m) + (Ia^2E^{(e - (cf)/d)}(c + dx)^m \Gamma[1+m, -(f(c + dx))/d])/(f((f(c + dx))/d)^m) + (Ia^2E^{(-e + (cf)/d)}(c + dx)^m \Gamma[1+m, (f(c + dx))/d])/(f((f(c + dx))/d)^m) + (2^{-(3-m)}a^2E^{(-2e + (2cf)/d)}(c + dx)^m \Gamma[1+m, (2f(c + dx))/d])/(f((f(c + dx))/d)^m)$

**Rubi [A]** time = 0.361244, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$ , Rules used = {3318, 3312, 3307, 2181, 3308}

$$\frac{a^2 2^{-m-3} e^{2e - \frac{2cf}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2f(c+dx)}{d}\right)}{f} + \frac{ia^2 e^{e - \frac{cf}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{f(c+dx)}{d}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[(c + dx)^m\*(a + I\*a\*Sinh[e + f\*x])^2,x]

[Out]  $(3a^2(c + dx)^{(1+m)})/(2d(1+m)) - (2^{-(3-m)}a^2E^{(2e - (2cf)/d)}(c + dx)^m \Gamma[1+m, (-2f(c + dx))/d])/(f((f(c + dx))/d)^m) + (Ia^2E^{(e - (cf)/d)}(c + dx)^m \Gamma[1+m, -(f(c + dx))/d])/(f((f(c + dx))/d)^m) + (Ia^2E^{(-e + (cf)/d)}(c + dx)^m \Gamma[1+m, (f(c + dx))/d])/(f((f(c + dx))/d)^m) + (2^{-(3-m)}a^2E^{(-2e + (2cf)/d)}(c + dx)^m \Gamma[1+m, (2f(c + dx))/d])/(f((f(c + dx))/d)^m)$

#### Rule 3318

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Dist[(2\*a)^n, Int[(c + dx)^m\*Sin[(1\*(e + (Pi\*a)/(2\*b)))/2 + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])

#### Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int
[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

### Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

### Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:= -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]* (c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
negerQ[m]
```

### Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

### Rubi steps

$$\begin{aligned}
\int (c + dx)^m (a + ia \sinh(e + fx))^2 dx &= (4a^2) \int (c + dx)^m \sin^4 \left( \frac{1}{2} \left( ie + \frac{\pi}{2} \right) + \frac{ifx}{2} \right) dx \\
&= (4a^2) \int \left( \frac{3}{8} (c + dx)^m - \frac{1}{8} (c + dx)^m \cosh(2e + 2fx) + \frac{1}{2} i (c + dx)^m \sinh(e + fx) \right) dx \\
&= \frac{3a^2 (c + dx)^{1+m}}{2d(1+m)} + (2ia^2) \int (c + dx)^m \sinh(e + fx) dx - \frac{1}{2} a^2 \int (c + dx)^m \cosh(2e + 2fx) dx \\
&= \frac{3a^2 (c + dx)^{1+m}}{2d(1+m)} + (ia^2) \int e^{-i(ie+ifx)} (c + dx)^m dx - (ia^2) \int e^{i(ie+ifx)} (c + dx)^m dx \\
&= \frac{3a^2 (c + dx)^{1+m}}{2d(1+m)} - \frac{2^{-3-m} a^2 e^{2e - \frac{2cf}{d}} (c + dx)^m \left( -\frac{f(c+dx)}{d} \right)^{-m} \Gamma \left( 1 + m, -\frac{2f(c+dx)}{d} \right)}{f} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.799054, size = 229, normalized size = 0.85

$$\frac{1}{8}a^2(c+dx)^m \left( -\frac{2^{-m}e^{2e-\frac{2cf}{d}} \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{2f(c+dx)}{d}\right)}{f} + \frac{8ie^{e-\frac{cf}{d}} \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{f(c+dx)}{d}\right)}{f} \right) +$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^m\*(a + I\*a\*Sinh[e + f\*x])^2,x]

[Out] (a^2\*(c + d\*x)^m\*((12\*(c + d\*x))/(d\*(1 + m)) - (E^(2\*e - (2\*c\*f)/d)\*Gamma[1 + m, (-2\*f\*(c + d\*x))/d])/(2^m\*f\*(-((f\*(c + d\*x))/d))^m) + ((8\*I)\*E^(e - (c\*f)/d)\*Gamma[1 + m, -((f\*(c + d\*x))/d)])/(f\*(-((f\*(c + d\*x))/d))^m) + ((8\*I)\*E^(-e + (c\*f)/d)\*Gamma[1 + m, (f\*(c + d\*x))/d])/(f\*((f\*(c + d\*x))/d)^m) + (E^(-2\*e + (2\*c\*f)/d)\*Gamma[1 + m, (2\*f\*(c + d\*x))/d])/(2^m\*f\*((f\*(c + d\*x))/d)^m))/8

**Maple [F]** time = 0.101, size = 0, normalized size = 0.

$$\int (dx + c)^m (a + ia \sinh(fx + e))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*(a+I\*a\*sinh(f\*x+e))^2,x)

[Out] int((d\*x+c)^m\*(a+I\*a\*sinh(f\*x+e))^2,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.6165, size = 581, normalized size = 2.17

$$\frac{(a^2 dm + a^2 d) e^{\left(-\frac{dm \log\left(\frac{2f}{d}\right) + 2de - 2cf}{d}\right)} \Gamma\left(m + 1, \frac{2(dfx + cf)}{d}\right) + (8i a^2 dm + 8i a^2 d) e^{\left(-\frac{dm \log\left(\frac{f}{d}\right) + de - cf}{d}\right)} \Gamma\left(m + 1, \frac{dfx + cf}{d}\right) + (8i a^2 dm + 8i a^2 d) e^{\left(-\frac{dm \log\left(\frac{f}{d}\right) + de - cf}{d}\right)} \Gamma\left(m + 1, \frac{dfx + cf}{d}\right) + (8i a^2 dm + 8i a^2 d) e^{\left(-\frac{dm \log\left(\frac{f}{d}\right) + de - cf}{d}\right)} \Gamma\left(m + 1, \frac{dfx + cf}{d}\right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="fricas")

[Out] 1/8\*((a^2\*d\*m + a^2\*d)\*e^(-(d\*m\*log(2\*f/d) + 2\*d\*e - 2\*c\*f)/d)\*gamma(m + 1, 2\*(d\*f\*x + c\*f)/d) + (8\*I\*a^2\*d\*m + 8\*I\*a^2\*d)\*e^(-(d\*m\*log(f/d) + d\*e - c\*f)/d)\*gamma(m + 1, (d\*f\*x + c\*f)/d) + (8\*I\*a^2\*d\*m + 8\*I\*a^2\*d)\*e^(-(d\*m\*log(-f/d) - d\*e + c\*f)/d)\*gamma(m + 1, -(d\*f\*x + c\*f)/d) - (a^2\*d\*m + a^2\*d)\*e^(-(d\*m\*log(-2\*f/d) - 2\*d\*e + 2\*c\*f)/d)\*gamma(m + 1, -2\*(d\*f\*x + c\*f)/d) + 12\*(a^2\*d\*f\*x + a^2\*c\*f)\*(d\*x + c)^m/(d\*f\*m + d\*f)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m\*(a+I\*a\*sinh(f\*x+e))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (i a \sinh(fx + e) + a)^2 (dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate((I\*a\*sinh(f\*x + e) + a)^2\*(d\*x + c)^m, x)

### 3.154 $\int (c + dx)^m (a + ia \sinh(e + fx)) dx$

**Optimal.** Leaf size=135

$$\frac{iae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f} + \frac{iae^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{2f} + \frac{a(c+dx)^{m+1}}{d}$$

[Out] (a\*(c + d\*x)^(1 + m))/(d\*(1 + m)) + ((I/2)\*a\*E^(e - (c\*f)/d)\*(c + d\*x)^m\*Gamma[1 + m, -((f\*(c + d\*x))/d)]/(f\*(-((f\*(c + d\*x))/d))^m) + ((I/2)\*a\*E^(-e + (c\*f)/d)\*(c + d\*x)^m\*Gamma[1 + m, (f\*(c + d\*x))/d])/(f\*((f\*(c + d\*x))/d)^m)

**Rubi [A]** time = 0.153541, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {3317, 3308, 2181}

$$\frac{iae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f} + \frac{iae^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{2f} + \frac{a(c+dx)^{m+1}}{d}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^m\*(a + I\*a\*Sinh[e + f\*x]),x]

[Out] (a\*(c + d\*x)^(1 + m))/(d\*(1 + m)) + ((I/2)\*a\*E^(e - (c\*f)/d)\*(c + d\*x)^m\*Gamma[1 + m, -((f\*(c + d\*x))/d)]/(f\*(-((f\*(c + d\*x))/d))^m) + ((I/2)\*a\*E^(-e + (c\*f)/d)\*(c + d\*x)^m\*Gamma[1 + m, (f\*(c + d\*x))/d])/(f\*((f\*(c + d\*x))/d)^m)

#### Rule 3317

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*Sinh[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

#### Rule 3308

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Dist[I/2, Int[(c + d\*x)^m/E^(I\*(e + f\*x)), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

Rule 2181

```
Int[(F_)^((g_)*(e_) + (f_)*(x_))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])/d))*(c + d*x)/d)^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^m (a + ia \sinh(e + fx)) dx &= \int (a(c + dx)^m + ia(c + dx)^m \sinh(e + fx)) dx \\
&= \frac{a(c + dx)^{1+m}}{d(1+m)} + (ia) \int (c + dx)^m \sinh(e + fx) dx \\
&= \frac{a(c + dx)^{1+m}}{d(1+m)} + \frac{1}{2}(ia) \int e^{-i(e+ifx)} (c + dx)^m dx - \frac{1}{2}(ia) \int e^{i(e+ifx)} (c + dx)^m dx \\
&= \frac{a(c + dx)^{1+m}}{d(1+m)} + \frac{iae^{-\frac{cf}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{f(c+dx)}{d}\right)}{2f} + \frac{iae^{-e+\frac{cf}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{f(c+dx)}{d}\right)}{2f}
\end{aligned}$$

**Mathematica [A]** time = 0.55988, size = 207, normalized size = 1.53

$$\frac{ae^{-\frac{cf}{d}} e^{-e} (c + dx)^m (\sinh(e + fx) - i) \left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m} \left(de^{2e}(m+1) \left(f\left(\frac{c}{d} + x\right)\right)^m \Gamma\left(m+1, -\frac{f(c+dx)}{d}\right) + d(m+1)e^{\frac{2cf}{d}}\right)}{2df(m+1) \left(\cosh\left(\frac{1}{2}(e + fx)\right) + i \sinh\left(\frac{1}{2}(e + fx)\right)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^m\*(a + I\*a\*Sinh[e + f\*x]), x]

[Out] -(a\*E^(-e - (c\*f)/d)\*(c + d\*x)^m\*((-2\*I)\*E^(e + (c\*f)/d)\*f\*(c + d\*x)\*(-((f^2\*(c + d\*x)^2)/d^2))^m + d\*E^(2\*e)\*(1 + m)\*(f\*(c/d + x))^m\*Gamma[1 + m, -((f\*(c + d\*x))/d)] + d\*E^((2\*c\*f)/d)\*(1 + m)\*(-((f\*(c + d\*x))/d))^m\*Gamma[1 + m, (f\*(c + d\*x))/d])\*(-I + Sinh[e + f\*x]))/(2\*d\*f\*(1 + m)\*(-((f^2\*(c + d\*x)^2)/d^2))^m\*(Cosh[(e + f\*x)/2] + I\*Sinh[(e + f\*x)/2])^2)

**Maple [F]** time = 0.056, size = 0, normalized size = 0.

$$\int (dx + c)^m (a + ia \sinh(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^m*(a+I*a*sinh(f*x+e)),x)`

[Out] `int((d*x+c)^m*(a+I*a*sinh(f*x+e)),x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*(a+I*a*sinh(f*x+e)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.54164, size = 301, normalized size = 2.23

$$\frac{(i adm + i ad)e^{\left(-\frac{dm \log\left(\frac{f}{d}\right) + de - cf}{d}\right)} \Gamma\left(m + 1, \frac{dfx + cf}{d}\right) + (i adm + i ad)e^{\left(-\frac{dm \log\left(-\frac{f}{d}\right) - de + cf}{d}\right)} \Gamma\left(m + 1, -\frac{dfx + cf}{d}\right) + 2(adfx + acf)(dx)}{2(dfm + df)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*(a+I*a*sinh(f*x+e)),x, algorithm="fricas")`

[Out] `1/2*((I*a*d*m + I*a*d)*e^(-(d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*x + c*f)/d) + (I*a*d*m + I*a*d)*e^(-(d*m*log(-f/d) - d*e + c*f)/d)*gamma(m + 1, -(d*f*x + c*f)/d) + 2*(a*d*f*x + a*c*f)*(d*x + c)^m/(d*f*m + d*f)`

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((d*x+c)**m*(a+I*a*sinh(f*x+e)),x)
```

```
[Out] Exception raised: TypeError
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (i a \sinh(fx + e) + a)(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+I*a*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((I*a*sinh(f*x + e) + a)*(d*x + c)^m, x)
```

$$3.155 \quad \int \frac{(c+dx)^m}{a+ia \sinh(e+fx)} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable}\left(\frac{(c+dx)^m}{a+ia \sinh(e+fx)}, x\right)$$

[Out] Unintegrable[(c + d\*x)^m/(a + I\*a\*Sinh[e + f\*x]), x]

**Rubi [A]** time = 0.0561594, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(c+dx)^m}{a+ia \sinh(e+fx)} dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m/(a + I\*a\*Sinh[e + f\*x]),x]

[Out] Defer[Int] [(c + d\*x)^m/(a + I\*a\*Sinh[e + f\*x]), x]

Rubi steps

$$\int \frac{(c+dx)^m}{a+ia \sinh(e+fx)} dx = \int \frac{(c+dx)^m}{a+ia \sinh(e+fx)} dx$$

**Mathematica [A]** time = 4.27846, size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{a+ia \sinh(e+fx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m/(a + I\*a\*Sinh[e + f\*x]),x]

[Out] Integrate[(c + d\*x)^m/(a + I\*a\*Sinh[e + f\*x]), x]

---

**Maple [A]** time = 0.034, size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{a + ia \sinh(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m/(a+I\*a\*sinh(f\*x+e)),x)

[Out] int((d\*x+c)^m/(a+I\*a\*sinh(f\*x+e)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{ia \sinh(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+I\*a\*sinh(f\*x+e)),x, algorithm="maxima")

[Out] integrate((d\*x + c)^m/(I\*a\*sinh(f\*x + e) + a), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\frac{(afe^{(fx+e)} - iaf) \operatorname{integral}\left(-\frac{2i(dx+c)^m dm}{-i adfx - i acf + (adfx+acf)e^{(fx+e)}}, x\right) + 2i(dx+c)^m}{afe^{(fx+e)} - iaf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+I\*a\*sinh(f\*x+e)),x, algorithm="fricas")

[Out] ((a\*f\*e^(f\*x + e) - I\*a\*f)\*integral(-2\*I\*(d\*x + c)^m\*d\*m/(-I\*a\*d\*f\*x - I\*a\*c\*f + (a\*d\*f\*x + a\*c\*f)\*e^(f\*x + e)), x) + 2\*I\*(d\*x + c)^m)/(a\*f\*e^(f\*x + e) - I\*a\*f)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m/(a+I\*a\*sinh(f\*x+e)),x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{i a \sinh(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+I\*a\*sinh(f\*x+e)),x, algorithm="giac")

[Out] integrate((d\*x + c)^m/(I\*a\*sinh(f\*x + e) + a), x)

$$3.156 \quad \int \frac{(c+dx)^m}{(a+ia \sinh(e+fx))^2} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable}\left(\frac{(c+dx)^m}{(a+ia \sinh(e+fx))^2}, x\right)$$

[Out] Unintegrable[(c + d\*x)^m/(a + I\*a\*Sinh[e + f\*x])^2, x]

**Rubi [A]** time = 0.0572106, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(c+dx)^m}{(a+ia \sinh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m/(a + I\*a\*Sinh[e + f\*x])^2,x]

[Out] Defer[Int] [(c + d\*x)^m/(a + I\*a\*Sinh[e + f\*x])^2, x]

Rubi steps

$$\int \frac{(c+dx)^m}{(a+ia \sinh(e+fx))^2} dx = \int \frac{(c+dx)^m}{(a+ia \sinh(e+fx))^2} dx$$

**Mathematica [A]** time = 15.7454, size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{(a+ia \sinh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m/(a + I\*a\*Sinh[e + f\*x])^2,x]

[Out] Integrate[(c + d\*x)^m/(a + I\*a\*Sinh[e + f\*x])^2, x]

---

**Maple [A]** time = 0.058, size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{(a + ia \sinh(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m/(a+I\*a\*sinh(f\*x+e))^2,x)

[Out] int((d\*x+c)^m/(a+I\*a\*sinh(f\*x+e))^2,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{(ia \sinh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="maxima")

[Out] integrate((d\*x + c)^m/(I\*a\*sinh(f\*x + e) + a)^2, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\left(-2i d^2 f^2 x^2 - 4i c d f^2 x - 2i c^2 f^2 + 2i d^2 m^2 - 2i d^2 m + (-2i d^2 f m x - 2i d^2 m^2 + (-2i c d f + 2i d^2) m)\right) e^{(2fx+2e)} + 2(3 d^2 f^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+I\*a\*sinh(f\*x+e))^2,x, algorithm="fricas")

[Out] ((-2\*I\*d^2\*f^2\*x^2 - 4\*I\*c\*d\*f^2\*x - 2\*I\*c^2\*f^2 + 2\*I\*d^2\*m^2 - 2\*I\*d^2\*m + (-2\*I\*d^2\*f\*m\*x - 2\*I\*d^2\*m^2 + (-2\*I\*c\*d\*f + 2\*I\*d^2)\*m)\*e^(2\*f\*x + 2\*e) + 2\*(3\*d^2\*f^2\*x^2 + 3\*c^2\*f^2 - 2\*d^2\*m^2 - (c\*d\*f - 2\*d^2)\*m + (6\*c\*d\*f^2 - d^2\*f\*m)\*x)\*e^(f\*x + e))\*(d\*x + c)^m + (3\*I\*a^2\*d^2\*f^3\*x^2 + 6\*I\*a^2\*c

```

*d*f^3*x + 3*I*a^2*c^2*f^3 + 3*(a^2*d^2*f^3*x^2 + 2*a^2*c*d*f^3*x + a^2*c^2
*f^3)*e^(3*f*x + 3*e) + (-9*I*a^2*d^2*f^3*x^2 - 18*I*a^2*c*d*f^3*x - 9*I*a^
2*c^2*f^3)*e^(2*f*x + 2*e) - 9*(a^2*d^2*f^3*x^2 + 2*a^2*c*d*f^3*x + a^2*c^2
*f^3)*e^(f*x + e))*integral((-2*I*d^3*f^2*m*x^2 - 4*I*c*d^2*f^2*m*x + 2*I*d
^3*m^3 - 6*I*d^3*m^2 + (-2*I*c^2*d*f^2 + 4*I*d^3)*m)*(d*x + c)^m/(-3*I*a^2*
d^3*f^3*x^3 - 9*I*a^2*c*d^2*f^3*x^2 - 9*I*a^2*c^2*d*f^3*x - 3*I*a^2*c^3*f^3
+ 3*(a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f
^3)*e^(f*x + e)), x)/(3*I*a^2*d^2*f^3*x^2 + 6*I*a^2*c*d*f^3*x + 3*I*a^2*c^
2*f^3 + 3*(a^2*d^2*f^3*x^2 + 2*a^2*c*d*f^3*x + a^2*c^2*f^3)*e^(3*f*x + 3*e)
+ (-9*I*a^2*d^2*f^3*x^2 - 18*I*a^2*c*d*f^3*x - 9*I*a^2*c^2*f^3)*e^(2*f*x +
2*e) - 9*(a^2*d^2*f^3*x^2 + 2*a^2*c*d*f^3*x + a^2*c^2*f^3)*e^(f*x + e))

```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+I*a*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{(ia \sinh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+I*a*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(I*a*sinh(f*x + e) + a)^2, x)
```

### 3.157 $\int (c + dx)^3 (a + b \sinh(e + fx)) dx$

**Optimal.** Leaf size=89

$$\frac{a(c + dx)^4}{4d} + \frac{6bd^2(c + dx) \cosh(e + fx)}{f^3} - \frac{3bd(c + dx)^2 \sinh(e + fx)}{f^2} + \frac{b(c + dx)^3 \cosh(e + fx)}{f} - \frac{6bd^3 \sinh(e + fx)}{f^4}$$

[Out] (a\*(c + d\*x)^4)/(4\*d) + (6\*b\*d^2\*(c + d\*x)\*Cosh[e + f\*x])/f^3 + (b\*(c + d\*x)^3\*Cosh[e + f\*x])/f - (6\*b\*d^3\*Sinh[e + f\*x])/f^4 - (3\*b\*d\*(c + d\*x)^2\*Sinh[e + f\*x])/f^2

**Rubi [A]** time = 0.142754, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3317, 3296, 2637}

$$\frac{a(c + dx)^4}{4d} + \frac{6bd^2(c + dx) \cosh(e + fx)}{f^3} - \frac{3bd(c + dx)^2 \sinh(e + fx)}{f^2} + \frac{b(c + dx)^3 \cosh(e + fx)}{f} - \frac{6bd^3 \sinh(e + fx)}{f^4}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^3\*(a + b\*Sinh[e + f\*x]),x]

[Out] (a\*(c + d\*x)^4)/(4\*d) + (6\*b\*d^2\*(c + d\*x)\*Cosh[e + f\*x])/f^3 + (b\*(c + d\*x)^3\*Cosh[e + f\*x])/f - (6\*b\*d^3\*Sinh[e + f\*x])/f^4 - (3\*b\*d\*(c + d\*x)^2\*Sinh[e + f\*x])/f^2

#### Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

#### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

#### Rule 2637



```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
 \int (c + dx)^3 (a + b \sinh(e + fx)) dx &= \int (a(c + dx)^3 + b(c + dx)^3 \sinh(e + fx)) dx \\
 &= \frac{a(c + dx)^4}{4d} + b \int (c + dx)^3 \sinh(e + fx) dx \\
 &= \frac{a(c + dx)^4}{4d} + \frac{b(c + dx)^3 \cosh(e + fx)}{f} - \frac{(3bd) \int (c + dx)^2 \cosh(e + fx) dx}{f} \\
 &= \frac{a(c + dx)^4}{4d} + \frac{b(c + dx)^3 \cosh(e + fx)}{f} - \frac{3bd(c + dx)^2 \sinh(e + fx)}{f^2} + \frac{(6bd^2) \int (c + dx) \sinh(e + fx) dx}{f^2} \\
 &= \frac{a(c + dx)^4}{4d} + \frac{6bd^2(c + dx) \cosh(e + fx)}{f^3} + \frac{b(c + dx)^3 \cosh(e + fx)}{f} - \frac{3bd(c + dx)^2 \sinh(e + fx)}{f^2} \\
 &= \frac{a(c + dx)^4}{4d} + \frac{6bd^2(c + dx) \cosh(e + fx)}{f^3} + \frac{b(c + dx)^3 \cosh(e + fx)}{f} - \frac{6bd^3 \sinh(e + fx)}{f^4}
 \end{aligned}$$

**Mathematica [A]** time = 0.45088, size = 123, normalized size = 1.38

$$\frac{1}{4}ax(6c^2dx + 4c^3 + 4cd^2x^2 + d^3x^3) - \frac{3bd(c^2f^2 + 2cdf^2x + d^2(f^2x^2 + 2))\sinh(e + fx)}{f^4} + \frac{b(c + dx)(c^2f^2 + 2cdf^2x + d^3f^2x^2 + d^4f^2x^3)}{f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*Sinh[e + f*x]),x]
```

```
[Out] (a*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 + (b*(c + d*x)*(c^2*f^2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Cosh[e + f*x])/f^3 - (3*b*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x])/f^4
```

**Maple [B]** time = 0.01, size = 482, normalized size = 5.4

$$\frac{1}{f} \left( \frac{d^3 a (fx + e)^4}{4 f^3} + \frac{d^3 b \left( (fx + e)^3 \cosh(fx + e) - 3 (fx + e)^2 \sinh(fx + e) + 6 (fx + e) \cosh(fx + e) - 6 \sinh(fx + e) \right)}{f^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^3*(a+b*sinh(f*x+e)),x)`

[Out]  $\frac{1}{f} \left( \frac{1}{4} f^3 d^3 a (f*x+e)^4 + \frac{1}{f^3} d^3 b \left( (f*x+e)^3 \cosh(f*x+e) - 3(f*x+e)^2 \sinh(f*x+e) + 6(f*x+e) \cosh(f*x+e) - 6 \sinh(f*x+e) \right) - \frac{1}{f^3} d^3 e a (f*x+e)^3 - \frac{3}{f^3} d^3 e b \left( (f*x+e)^2 \cosh(f*x+e) - 2(f*x+e) \sinh(f*x+e) + 2 \cosh(f*x+e) \right) + \frac{3}{2} f^3 d^3 e^2 a (f*x+e)^2 + \frac{3}{f^3} d^3 e^2 b \left( (f*x+e) \cosh(f*x+e) - \sinh(f*x+e) \right) - d^3 e^3 / f^3 a (f*x+e) - \frac{1}{f^3} d^3 e^3 b \cosh(f*x+e) + \frac{1}{f^2} d^2 c a (f*x+e)^3 + \frac{3}{f^2} c d^2 b \left( (f*x+e)^2 \cosh(f*x+e) - 2(f*x+e) \sinh(f*x+e) + 2 \cosh(f*x+e) \right) - \frac{3}{f^2} d^2 e c a (f*x+e)^2 - \frac{6}{f^2} c d^2 e b \left( (f*x+e) \cosh(f*x+e) - \sinh(f*x+e) \right) + 3 d^2 e^2 / f^2 c a (f*x+e) + \frac{3}{f^2} c d^2 e^2 b \cosh(f*x+e) + \frac{3}{2} f d c^2 a (f*x+e)^2 + \frac{3}{f} c^2 d b \left( (f*x+e) \cosh(f*x+e) - \sinh(f*x+e) \right) - 3 d e / f c^2 a (f*x+e) - \frac{3}{f} c^2 d e b \cosh(f*x+e) + c^3 a (f*x+e) + b c^3 \cosh(f*x+e) \right)$

**Maxima [B]** time = 1.31172, size = 316, normalized size = 3.55

$$\frac{1}{4} a d^3 x^4 + a c d^2 x^3 + \frac{3}{2} a c^2 d x^2 + a c^3 x + \frac{3}{2} b c^2 d \left( \frac{(f x e^e - e^e) e^{(f x)}}{f^2} + \frac{(f x + 1) e^{(-f x - e)}}{f^2} \right) + \frac{3}{2} b c d^2 \left( \frac{(f^2 x^2 e^e - 2 f x e^e + 2 e^e) e^{(f x)}}{f^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^3*(a+b*sinh(f*x+e)),x, algorithm="maxima")`

[Out]  $\frac{1}{4} a d^3 x^4 + a c d^2 x^3 + \frac{3}{2} a c^2 d x^2 + a c^3 x + \frac{3}{2} b c^2 d \left( \frac{(f^2 x^2 e^e - 2 f x e^e + 2 e^e) e^{(f x)}}{f^3} + \frac{(f x + 1) e^{(-f x - e)}}{f^2} \right) + \frac{3}{2} b c d^2 \left( \frac{(f^2 x^2 e^e - 2 f x e^e + 2 e^e) e^{(f x)}}{f^3} + \frac{(f^2 x^2 + 2 f x + 2) e^{(-f x - e)}}{f^3} \right) + \frac{1}{2} b d^3 \left( \frac{(f^3 x^3 e^e - 3 f^2 x^2 e^e + 6 f x e^e - 6 e^e) e^{(f x)}}{f^4} + \frac{(f^3 x^3 + 3 f^2 x^2 + 6 f x + 6) e^{(-f x - e)}}{f^4} \right) + b c^3 \cosh(f x + e) / f$

**Fricas [A]** time = 2.42436, size = 365, normalized size = 4.1

$$\frac{a d^3 f^4 x^4 + 4 a c d^2 f^4 x^3 + 6 a c^2 d f^4 x^2 + 4 a c^3 f^4 x + 4 (b d^3 f^3 x^3 + 3 b c d^2 f^3 x^2 + b c^3 f^3 + 6 b c d^2 f + 3 (b c^2 d f^3 + 2 b d^3 f) x) \cosh(f x + e)}{4 f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^3*(a+b*sinh(f*x+e)),x, algorithm="fricas")`

[Out]  $\frac{1}{4}(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x + 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + b*c^3*f^3 + 6*b*c*d^2*f + 3*(b*c^2*d*f^3 + 2*b*d^3*f)*x)*\cosh(f*x + e) - 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2 + 2*b*d^3)*\sinh(f*x + e))/f^4$

**Sympy [A]** time = 1.93265, size = 264, normalized size = 2.97

$$\left\{ \begin{array}{l} ac^3x + \frac{3ac^2dx^2}{2} + acd^2x^3 + \frac{ad^3x^4}{4} + \frac{bc^3 \cosh(e+fx)}{f} + \frac{3bc^2dx \cosh(e+fx)}{f} - \frac{3bc^2d \sinh(e+fx)}{f^2} + \frac{3bcd^2x^2 \cosh(e+fx)}{f} - \frac{6bcd^2x \sinh(e+fx)}{f^2} \\ (a + b \sinh(e)) \left( c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*3\*(a+b\*sinh(f\*x+e)),x)

[Out] Piecewise((a\*c\*\*3\*x + 3\*a\*c\*\*2\*d\*x\*\*2/2 + a\*c\*d\*\*2\*x\*\*3 + a\*d\*\*3\*x\*\*4/4 + b\*c\*\*3\*cosh(e + f\*x)/f + 3\*b\*c\*\*2\*d\*x\*cosh(e + f\*x)/f - 3\*b\*c\*\*2\*d\*sinh(e + f\*x)/f\*\*2 + 3\*b\*c\*d\*\*2\*x\*\*2\*cosh(e + f\*x)/f - 6\*b\*c\*d\*\*2\*x\*sinh(e + f\*x)/f\*\*2 + 6\*b\*c\*d\*\*2\*cosh(e + f\*x)/f\*\*3 + b\*d\*\*3\*x\*\*3\*cosh(e + f\*x)/f - 3\*b\*d\*\*3\*x\*\*2\*sinh(e + f\*x)/f\*\*2 + 6\*b\*d\*\*3\*x\*cosh(e + f\*x)/f\*\*3 - 6\*b\*d\*\*3\*sinh(e + f\*x)/f\*\*4, Ne(f, 0)), ((a + b\*sinh(e))\*(c\*\*3\*x + 3\*c\*\*2\*d\*x\*\*2/2 + c\*d\*\*2\*x\*\*3 + d\*\*3\*x\*\*4/4), True))

**Giac [B]** time = 1.26699, size = 351, normalized size = 3.94

$$\frac{1}{4} ad^3x^4 + acd^2x^3 + \frac{3}{2} ac^2dx^2 + ac^3x + \frac{(bd^3f^3x^3 + 3bcd^2f^3x^2 + 3bc^2df^3x - 3bd^3f^2x^2 + bc^3f^3 - 6bcd^2f^2x - 3bc^2df^2)}{2f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*(a+b\*sinh(f\*x+e)),x, algorithm="giac")

[Out]  $\frac{1}{4}a*d^3*x^4 + a*c*d^2*x^3 + \frac{3}{2}a*c^2*d*x^2 + a*c^3*x + \frac{1}{2}*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x - 3*b*d^3*f^2*x^2 + b*c^3*f^3 - 6*b*c*d^2*f^2*x - 3*b*c^2*d*f^2 + 6*b*d^3*f*x + 6*b*c*d^2*f - 6*b*d^3)*e^{(f*x + e)}/f^4 + \frac{1}{2}*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + 3*b*d^3*f^2*x^2 + b*c^3*f^3 + 6*b*c*d^2*f^2*x + 3*b*c^2*d*f^2 + 6*b*d^3*f*x + 6*b*c*d^2*f + 6*b*d^3)*e^{(-f*x - e)}/f^4$

### 3.158 $\int (c + dx)^2 (a + b \sinh(e + fx)) dx$

**Optimal.** Leaf size=67

$$\frac{a(c + dx)^3}{3d} - \frac{2bd(c + dx) \sinh(e + fx)}{f^2} + \frac{b(c + dx)^2 \cosh(e + fx)}{f} + \frac{2bd^2 \cosh(e + fx)}{f^3}$$

[Out] (a\*(c + d\*x)^3)/(3\*d) + (2\*b\*d^2\*Cosh[e + f\*x])/f^3 + (b\*(c + d\*x)^2\*Cosh[e + f\*x])/f - (2\*b\*d\*(c + d\*x)\*Sinh[e + f\*x])/f^2

**Rubi [A]** time = 0.0957637, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3317, 3296, 2638}

$$\frac{a(c + dx)^3}{3d} - \frac{2bd(c + dx) \sinh(e + fx)}{f^2} + \frac{b(c + dx)^2 \cosh(e + fx)}{f} + \frac{2bd^2 \cosh(e + fx)}{f^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^2\*(a + b\*Sinh[e + f\*x]),x]

[Out] (a\*(c + d\*x)^3)/(3\*d) + (2\*b\*d^2\*Cosh[e + f\*x])/f^3 + (b\*(c + d\*x)^2\*Cosh[e + f\*x])/f - (2\*b\*d\*(c + d\*x)\*Sinh[e + f\*x])/f^2

#### Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sinh[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

#### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

#### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^2(a + b \sinh(e + fx)) dx &= \int (a(c + dx)^2 + b(c + dx)^2 \sinh(e + fx)) dx \\
&= \frac{a(c + dx)^3}{3d} + b \int (c + dx)^2 \sinh(e + fx) dx \\
&= \frac{a(c + dx)^3}{3d} + \frac{b(c + dx)^2 \cosh(e + fx)}{f} - \frac{(2bd) \int (c + dx) \cosh(e + fx) dx}{f} \\
&= \frac{a(c + dx)^3}{3d} + \frac{b(c + dx)^2 \cosh(e + fx)}{f} - \frac{2bd(c + dx) \sinh(e + fx)}{f^2} + \frac{(2bd^2) \int \sinh(e + fx) dx}{f^2} \\
&= \frac{a(c + dx)^3}{3d} + \frac{2bd^2 \cosh(e + fx)}{f^3} + \frac{b(c + dx)^2 \cosh(e + fx)}{f} - \frac{2bd(c + dx) \sinh(e + fx)}{f^2}
\end{aligned}$$

**Mathematica [A]** time = 0.308752, size = 83, normalized size = 1.24

$$\frac{1}{3}ax(3c^2 + 3cdx + d^2x^2) + \frac{b(c^2f^2 + 2cdf^2x + d^2(f^2x^2 + 2)) \cosh(e + fx)}{f^3} - \frac{2bd(c + dx) \sinh(e + fx)}{f^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2\*(a + b\*Sinh[e + f\*x]),x]

[Out] (a\*x\*(3\*c^2 + 3\*c\*d\*x + d^2\*x^2))/3 + (b\*(c^2\*f^2 + 2\*c\*d\*f^2\*x + d^2\*(2 + f^2\*x^2))\*Cosh[e + f\*x])/f^3 - (2\*b\*d\*(c + d\*x)\*Sinh[e + f\*x])/f^2

**Maple [B]** time = 0.007, size = 240, normalized size = 3.6

$$\frac{1}{f} \left( \frac{ad^2 (fx + e)^3}{3f^2} + \frac{d^2b \left( (fx + e)^2 \cosh(fx + e) - 2 (fx + e) \sinh(fx + e) + 2 \cosh(fx + e) \right)}{f^2} - \frac{d^2ea (fx + e)^2}{f^2} - 2 \frac{d^2b \sinh(fx + e)}{f^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2\*(a+b\*sinh(f\*x+e)),x)

[Out] 1/f\*(1/3/f^2\*d^2\*a\*(f\*x+e)^3+1/f^2\*d^2\*b\*((f\*x+e)^2\*cosh(f\*x+e)-2\*(f\*x+e)\*sinh(f\*x+e)+2\*cosh(f\*x+e))-1/f^2\*d^2\*e\*a\*(f\*x+e)^2-2/f^2\*d^2\*e\*b\*((f\*x+e)\*cosh(f\*x+e)-sinh(f\*x+e))+d^2\*e^2/f^2\*a\*(f\*x+e)+1/f^2\*d^2\*e^2\*b\*cosh(f\*x+e)+1/

$$f*d*c*a*(f*x+e)^2+2/f*c*d*b*((f*x+e)*\cosh(f*x+e)-\sinh(f*x+e))-2*d*e/f*c*a*(f*x+e)-2/f*c*d*e*b*\cosh(f*x+e)+c^2*a*(f*x+e)+b*c^2*\cosh(f*x+e)$$

**Maxima [B]** time = 1.1858, size = 188, normalized size = 2.81

$$\frac{1}{3}ad^2x^3 + acdx^2 + ac^2x + bcd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} + \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{1}{2}bd^2\left(\frac{(f^2x^2e^e - 2fxe^e + 2e^e)e^{(fx)}}{f^3} + \frac{(f^2x^2 + 2fx)}{f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+b\*sinh(f\*x+e)),x, algorithm="maxima")

[Out] 1/3\*a\*d^2\*x^3 + a\*c\*d\*x^2 + a\*c^2\*x + b\*c\*d\*((f\*x\*e^e - e^e)\*e^(f\*x)/f^2 + (f\*x + 1)\*e^(-f\*x - e)/f^2) + 1/2\*b\*d^2\*((f^2\*x^2\*e^e - 2\*f\*x\*e^e + 2\*e^e)\*e^(f\*x)/f^3 + (f^2\*x^2 + 2\*f\*x + 2)\*e^(-f\*x - e)/f^3) + b\*c^2\*cosh(f\*x + e)/f

**Fricas [A]** time = 2.4565, size = 231, normalized size = 3.45

$$\frac{ad^2f^3x^3 + 3acdf^3x^2 + 3ac^2f^3x + 3(bd^2f^2x^2 + 2bcd f^2x + bc^2f^2 + 2bd^2)\cosh(fx + e) - 6(bd^2fx + bcd f)\sinh(fx + e)}{3f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+b\*sinh(f\*x+e)),x, algorithm="fricas")

[Out] 1/3\*(a\*d^2\*f^3\*x^3 + 3\*a\*c\*d\*f^3\*x^2 + 3\*a\*c^2\*f^3\*x + 3\*(b\*d^2\*f^2\*x^2 + 2\*b\*c\*d\*f^2\*x + b\*c^2\*f^2 + 2\*b\*d^2)\*cosh(f\*x + e) - 6\*(b\*d^2\*f\*x + b\*c\*d\*f)\*sinh(f\*x + e))/f^3

**Sympy [A]** time = 0.879971, size = 151, normalized size = 2.25

$$\left\{ \begin{array}{l} ac^2x + acdx^2 + \frac{ad^2x^3}{3} + \frac{bc^2 \cosh(e+fx)}{f} + \frac{2bcdx \cosh(e+fx)}{f} - \frac{2bcd \sinh(e+fx)}{f^2} + \frac{bd^2x^2 \cosh(e+fx)}{f} - \frac{2bd^2x \sinh(e+fx)}{f^2} + \frac{2bd^2 \cosh(e+fx)}{f^3} \\ (a + b \sinh(e)) \left( c^2x + cdx^2 + \frac{d^2x^3}{3} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2\*(a+b\*sinh(f\*x+e)),x)

[Out] Piecewise((a\*c\*\*2\*x + a\*c\*d\*x\*\*2 + a\*d\*\*2\*x\*\*3/3 + b\*c\*\*2\*cosh(e + f\*x)/f + 2\*b\*c\*d\*x\*cosh(e + f\*x)/f - 2\*b\*c\*d\*sinh(e + f\*x)/f\*\*2 + b\*d\*\*2\*x\*\*2\*cosh(e + f\*x)/f - 2\*b\*d\*\*2\*x\*sinh(e + f\*x)/f\*\*2 + 2\*b\*d\*\*2\*cosh(e + f\*x)/f\*\*3, N e(f, 0)), ((a + b\*sinh(e))\*(c\*\*2\*x + c\*d\*x\*\*2 + d\*\*2\*x\*\*3/3), True))

**Giac [B]** time = 1.32828, size = 200, normalized size = 2.99

$$\frac{1}{3} ad^2x^3 + acdx^2 + ac^2x + \frac{(bd^2f^2x^2 + 2bcd f^2x + bc^2f^2 - 2bd^2fx - 2bcdf + 2bd^2)e^{(fx+e)}}{2f^3} + \frac{(bd^2f^2x^2 + 2bcd f^2x + bc^2f^2 - 2bd^2fx - 2bcdf + 2bd^2)e^{-(fx+e)}}{2f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+b\*sinh(f\*x+e)),x, algorithm="giac")

[Out] 1/3\*a\*d^2\*x^3 + a\*c\*d\*x^2 + a\*c^2\*x + 1/2\*(b\*d^2\*f^2\*x^2 + 2\*b\*c\*d\*f^2\*x + b\*c^2\*f^2 - 2\*b\*d^2\*f\*x - 2\*b\*c\*d\*f + 2\*b\*d^2)\*e^(f\*x + e)/f^3 + 1/2\*(b\*d^2\*f^2\*x^2 + 2\*b\*c\*d\*f^2\*x + b\*c^2\*f^2 + 2\*b\*d^2\*f\*x + 2\*b\*c\*d\*f + 2\*b\*d^2)\*e^(-f\*x - e)/f^3

### 3.159 $\int (c + dx)(a + b \sinh(e + fx)) dx$

**Optimal.** Leaf size=45

$$\frac{a(c + dx)^2}{2d} + \frac{b(c + dx) \cosh(e + fx)}{f} - \frac{bd \sinh(e + fx)}{f^2}$$

[Out]  $(a*(c + d*x)^2)/(2*d) + (b*(c + d*x)*Cosh[e + f*x])/f - (b*d*Sinh[e + f*x])/f^2$

**Rubi [A]** time = 0.047893, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$ , Rules used = {3317, 3296, 2637}

$$\frac{a(c + dx)^2}{2d} + \frac{b(c + dx) \cosh(e + fx)}{f} - \frac{bd \sinh(e + fx)}{f^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)\*(a + b\*Sinh[e + f\*x]),x]

[Out]  $(a*(c + d*x)^2)/(2*d) + (b*(c + d*x)*Cosh[e + f*x])/f - (b*d*Sinh[e + f*x])/f^2$

#### Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sinh[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

#### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

#### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```



Rubi steps

$$\begin{aligned}
\int (c + dx)(a + b \sinh(e + fx)) dx &= \int (a(c + dx) + b(c + dx) \sinh(e + fx)) dx \\
&= \frac{a(c + dx)^2}{2d} + b \int (c + dx) \sinh(e + fx) dx \\
&= \frac{a(c + dx)^2}{2d} + \frac{b(c + dx) \cosh(e + fx)}{f} - \frac{(bd) \int \cosh(e + fx) dx}{f} \\
&= \frac{a(c + dx)^2}{2d} + \frac{b(c + dx) \cosh(e + fx)}{f} - \frac{bd \sinh(e + fx)}{f^2}
\end{aligned}$$

**Mathematica [A]** time = 0.111022, size = 43, normalized size = 0.96

$$\frac{1}{2}ax(2c + dx) + \frac{b(c + dx) \cosh(e + fx)}{f} - \frac{bd \sinh(e + fx)}{f^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)\*(a + b\*Sinh[e + f\*x]),x]

[Out] (a\*x\*(2\*c + d\*x))/2 + (b\*(c + d\*x)\*Cosh[e + f\*x])/f - (b\*d\*Sinh[e + f\*x])/f^2

**Maple [B]** time = 0.007, size = 91, normalized size = 2.

$$\frac{1}{f} \left( \frac{da (fx + e)^2}{2f} + \frac{bd ((fx + e) \cosh (fx + e) - \sinh (fx + e))}{f} - \frac{dea (fx + e)}{f} - \frac{deb \cosh (fx + e)}{f} + ca (fx + e) + cb \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)\*(a+b\*sinh(f\*x+e)),x)

[Out] 1/f\*(1/2/f\*d\*a\*(f\*x+e)^2+1/f\*d\*b\*((f\*x+e)\*cosh(f\*x+e)-sinh(f\*x+e))-d\*e/f\*a\*(f\*x+e)-d\*e/f\*b\*cosh(f\*x+e)+c\*a\*(f\*x+e)+c\*b\*cosh(f\*x+e))

**Maxima [A]** time = 1.30236, size = 88, normalized size = 1.96

$$\frac{1}{2} adx^2 + acx + \frac{1}{2} bd \left( \frac{(fxe^e - e^e)e^{fx}}{f^2} + \frac{(fx+1)e^{-fx-e}}{f^2} \right) + \frac{bc \cosh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+b\*sinh(f\*x+e)),x, algorithm="maxima")

[Out] 1/2\*a\*d\*x^2 + a\*c\*x + 1/2\*b\*d\*((f\*x\*e^e - e^e)\*e^(f\*x)/f^2 + (f\*x + 1)\*e^(-f\*x - e)/f^2) + b\*c\*cosh(f\*x + e)/f

**Fricas [A]** time = 2.46015, size = 128, normalized size = 2.84

$$\frac{adf^2x^2 + 2acf^2x - 2bd \sinh(fx+e) + 2(bdfx + bcf) \cosh(fx+e)}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+b\*sinh(f\*x+e)),x, algorithm="fricas")

[Out] 1/2\*(a\*d\*f^2\*x^2 + 2\*a\*c\*f^2\*x - 2\*b\*d\*sinh(f\*x + e) + 2\*(b\*d\*f\*x + b\*c\*f)\*cosh(f\*x + e))/f^2

**Sympy [A]** time = 0.36472, size = 68, normalized size = 1.51

$$\begin{cases} acx + \frac{adx^2}{2} + \frac{bc \cosh(e+fx)}{f} + \frac{bdx \cosh(e+fx)}{f} - \frac{bd \sinh(e+fx)}{f^2} & \text{for } f \neq 0 \\ (a + b \sinh(e)) \left( cx + \frac{dx^2}{2} \right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+b\*sinh(f\*x+e)),x)

[Out] Piecewise((a\*c\*x + a\*d\*x\*\*2/2 + b\*c\*cosh(e + f\*x)/f + b\*d\*x\*cosh(e + f\*x)/f - b\*d\*sinh(e + f\*x)/f\*\*2, Ne(f, 0)), ((a + b\*sinh(e))\*(c\*x + d\*x\*\*2/2), True))

---

**Giac [A]** time = 1.28085, size = 89, normalized size = 1.98

$$\frac{1}{2} adx^2 + acx + \frac{(bdfx + bcf - bd)e^{(fx+e)}}{2f^2} + \frac{(bdfx + bcf + bd)e^{(-fx-e)}}{2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+b\*sinh(f\*x+e)),x, algorithm="giac")

[Out] 1/2\*a\*d\*x^2 + a\*c\*x + 1/2\*(b\*d\*f\*x + b\*c\*f - b\*d)\*e^(f\*x + e)/f^2 + 1/2\*(b\*d\*f\*x + b\*c\*f + b\*d)\*e^(-f\*x - e)/f^2

$$3.160 \quad \int \frac{a+b \sinh(e+fx)}{c+dx} dx$$

**Optimal.** Leaf size=64

$$\frac{a \log(c+dx)}{d} + \frac{b \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d} + \frac{b \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d}$$

[Out] (a\*Log[c + d\*x])/d + (b\*CoshIntegral[(c\*f)/d + f\*x]\*Sinh[e - (c\*f)/d])/d + (b\*Cosh[e - (c\*f)/d]\*SinhIntegral[(c\*f)/d + f\*x])/d

**Rubi [A]** time = 0.124337, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3317, 3303, 3298, 3301}

$$\frac{a \log(c+dx)}{d} + \frac{b \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d} + \frac{b \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Sinh[e + f\*x])/(c + d\*x), x]

[Out] (a\*Log[c + d\*x])/d + (b\*CoshIntegral[(c\*f)/d + f\*x]\*Sinh[e - (c\*f)/d])/d + (b\*Cosh[e - (c\*f)/d]\*SinhIntegral[(c\*f)/d + f\*x])/d

#### Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

#### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

#### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{a + b \sinh(e + fx)}{c + dx} dx &= \int \left( \frac{a}{c + dx} + \frac{b \sinh(e + fx)}{c + dx} \right) dx \\ &= \frac{a \log(c + dx)}{d} + b \int \frac{\sinh(e + fx)}{c + dx} dx \\ &= \frac{a \log(c + dx)}{d} + \left( b \cosh \left( e - \frac{cf}{d} \right) \right) \int \frac{\sinh \left( \frac{cf}{d} + fx \right)}{c + dx} dx + \left( b \sinh \left( e - \frac{cf}{d} \right) \right) \int \frac{\cosh \left( \frac{cf}{d} + fx \right)}{c + dx} dx \\ &= \frac{a \log(c + dx)}{d} + \frac{b \operatorname{Chi} \left( \frac{cf}{d} + fx \right) \sinh \left( e - \frac{cf}{d} \right)}{d} + \frac{b \cosh \left( e - \frac{cf}{d} \right) \operatorname{Shi} \left( \frac{cf}{d} + fx \right)}{d} \end{aligned}$$

**Mathematica [A]** time = 0.144335, size = 57, normalized size = 0.89

$$\frac{a \log(c + dx) + b \operatorname{Chi} \left( f \left( \frac{c}{d} + x \right) \right) \sinh \left( e - \frac{cf}{d} \right) + b \cosh \left( e - \frac{cf}{d} \right) \operatorname{Shi} \left( f \left( \frac{c}{d} + x \right) \right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x])/(c + d*x), x]
```

```
[Out] (a*Log[c + d*x] + b*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] + b*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)])/d
```

**Maple [A]** time = 0.023, size = 94, normalized size = 1.5

$$\frac{a \ln(dx + c)}{d} + \frac{b}{2d} e^{\frac{cf-de}{d}} \operatorname{Ei} \left( 1, fx + e + \frac{cf - de}{d} \right) - \frac{b}{2d} e^{-\frac{cf-de}{d}} \operatorname{Ei} \left( 1, -fx - e - \frac{cf - de}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sinh(f*x+e))/(d*x+c),x)`

[Out]  $a \ln(d*x+c)/d + 1/2*b/d*\exp((c*f-d*e)/d)*Ei(1, f*x+e+(c*f-d*e)/d) - 1/2*b/d*\exp(-(c*f-d*e)/d)*Ei(1, -f*x-e-(c*f-d*e)/d)$

**Maxima [A]** time = 1.42502, size = 96, normalized size = 1.5

$$\frac{1}{2} b \left( \frac{e^{\left(-e + \frac{cf}{d}\right)} E_1\left(\frac{(dx+c)f}{d}\right) - e^{\left(e - \frac{cf}{d}\right)} E_1\left(-\frac{(dx+c)f}{d}\right)}{d} \right) + \frac{a \log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e))/(d*x+c),x, algorithm="maxima")`

[Out]  $1/2*b*(e^{(-e + c*f/d)}*\exp\_integral\_e(1, (d*x + c)*f/d)/d - e^{(e - c*f/d)}*\exp\_integral\_e(1, -(d*x + c)*f/d)/d) + a*\log(d*x + c)/d$

**Fricas [A]** time = 2.4936, size = 230, normalized size = 3.59

$$\frac{\left(bEi\left(\frac{dfx+cf}{d}\right) - bEi\left(-\frac{dfx+cf}{d}\right)\right) \cosh\left(-\frac{de-cf}{d}\right) + 2a \log(dx+c) - \left(bEi\left(\frac{dfx+cf}{d}\right) + bEi\left(-\frac{dfx+cf}{d}\right)\right) \sinh\left(-\frac{de-cf}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e))/(d*x+c),x, algorithm="fricas")`

[Out]  $1/2*((b*Ei((d*f*x + c*f)/d) - b*Ei(-(d*f*x + c*f)/d))*\cosh(-(d*e - c*f)/d) + 2*a*\log(d*x + c) - (b*Ei((d*f*x + c*f)/d) + b*Ei(-(d*f*x + c*f)/d))*\sinh(-(d*e - c*f)/d))/d$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \sinh(e + fx)}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sinh(f\*x+e))/(d\*x+c),x)

[Out] Integral((a + b\*sinh(e + f\*x))/(c + d\*x), x)

**Giac [A]** time = 1.24907, size = 95, normalized size = 1.48

$$\frac{b\operatorname{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)} - b\operatorname{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)} - 2a\log(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sinh(f\*x+e))/(d\*x+c),x, algorithm="giac")

[Out]  $-1/2*(b*\operatorname{Ei}(-(d*f*x + c*f)/d)*e^{(c*f/d - e)} - b*\operatorname{Ei}((d*f*x + c*f)/d)*e^{(-c*f/d + e)} - 2*a*\log(d*x + c))/d$

$$3.161 \quad \int \frac{a+b \sinh(e+fx)}{(c+dx)^2} dx$$

**Optimal.** Leaf size=87

$$-\frac{a}{d(c+dx)} + \frac{bf \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d^2} + \frac{bf \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^2} - \frac{b \sinh(e+fx)}{d(c+dx)}$$

[Out]  $-(a/(d*(c + d*x))) + (b*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d^2 - (b*Sinh[e + f*x])/(d*(c + d*x)) + (b*f*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^2$

**Rubi [A]** time = 0.169289, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3317, 3297, 3303, 3298, 3301}

$$-\frac{a}{d(c+dx)} + \frac{bf \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d^2} + \frac{bf \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^2} - \frac{b \sinh(e+fx)}{d(c+dx)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{Sinh}[e + f*x])/(c + d*x)^2, x]$

[Out]  $-(a/(d*(c + d*x))) + (b*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d^2 - (b*Sinh[e + f*x])/(d*(c + d*x)) + (b*f*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^2$

### Rule 3317

$\operatorname{Int}[(c + d*x)^m * (a + b*\sin[e + f*x])^n, x]$   
 $\operatorname{Int}[(c + d*x)^m * (a + b*\sin[e + f*x])^n, x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f, m\}, x$  &&  $\operatorname{IGtQ}[n, 0]$  &&  $(\operatorname{EqQ}[n, 1] \mid \mid \operatorname{IGtQ}[m, 0] \mid \mid \operatorname{NeQ}[a^2 - b^2, 0])$

### Rule 3297

$\operatorname{Int}[(c + d*x)^m * \sin[e + f*x], x]$   
 $\operatorname{Int}[(c + d*x)^m * \sin[e + f*x], x] /;$   $\operatorname{FreeQ}\{c, d, e, f\}, x$  &&  $\operatorname{LtQ}[m, -1]$



Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f,
fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz},
x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \sinh(e + fx)}{(c + dx)^2} dx &= \int \left( \frac{a}{(c + dx)^2} + \frac{b \sinh(e + fx)}{(c + dx)^2} \right) dx \\
&= -\frac{a}{d(c + dx)} + b \int \frac{\sinh(e + fx)}{(c + dx)^2} dx \\
&= -\frac{a}{d(c + dx)} - \frac{b \sinh(e + fx)}{d(c + dx)} + \frac{(bf) \int \frac{\cosh(e + fx)}{c + dx} dx}{d} \\
&= -\frac{a}{d(c + dx)} - \frac{b \sinh(e + fx)}{d(c + dx)} + \frac{\left( bf \cosh\left(e - \frac{cf}{d}\right) \right) \int \frac{\cosh\left(\frac{cf}{d} + fx\right)}{c + dx} dx}{d} + \frac{\left( bf \sinh\left(e - \frac{cf}{d}\right) \right) \int \frac{\sinh\left(\frac{cf}{d} + fx\right)}{c + dx} dx}{d} \\
&= -\frac{a}{d(c + dx)} + \frac{bf \cosh\left(e - \frac{cf}{d}\right) \text{Chi}\left(\frac{cf}{d} + fx\right)}{d^2} - \frac{b \sinh(e + fx)}{d(c + dx)} + \frac{bf \sinh\left(e - \frac{cf}{d}\right) \text{Shi}\left(\frac{cf}{d} + fx\right)}{d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.366188, size = 71, normalized size = 0.82

$$\frac{-\frac{d(a+b \sinh(e+fx))}{c+dx} + bf \text{Chi}\left(f\left(\frac{c}{d} + x\right)\right) \cosh\left(e - \frac{cf}{d}\right) + bf \sinh\left(e - \frac{cf}{d}\right) \text{Shi}\left(f\left(\frac{c}{d} + x\right)\right)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Sinh[e + f\*x])/(c + d\*x)^2,x]

[Out] (b\*f\*Cosh[e - (c\*f)/d]\*CoshIntegral[f\*(c/d + x)] - (d\*(a + b\*Sinh[e + f\*x])/(c + d\*x) + b\*f\*Sinh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)])/d^2

**Maple [A]** time = 0.022, size = 149, normalized size = 1.7

$$-\frac{a}{d(dx+c)} + \frac{f b e^{-fx-e}}{2d(dfx+cf)} - \frac{fb}{2d^2} e^{\frac{cf-de}{d}} \operatorname{Ei}\left(1, fx+e + \frac{cf-de}{d}\right) - \frac{fb e^{fx+e}}{2d^2} \left(\frac{cf}{d} + fx\right)^{-1} - \frac{fb}{2d^2} e^{-\frac{cf-de}{d}} \operatorname{Ei}\left(1, -fx-e - \frac{cf-de}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sinh(f\*x+e))/(d\*x+c)^2,x)

[Out] -a/d/(d\*x+c)+1/2\*b\*f\*exp(-f\*x-e)/d/(d\*f\*x+c\*f)-1/2\*b\*f/d^2\*exp((c\*f-d\*e)/d)\*Ei(1,f\*x+e+(c\*f-d\*e)/d)-1/2\*f\*b/d^2\*exp(f\*x+e)/(c\*f/d+f\*x)-1/2\*f\*b/d^2\*exp(-(c\*f-d\*e)/d)\*Ei(1,-f\*x-e-(c\*f-d\*e)/d)

**Maxima [A]** time = 1.45451, size = 119, normalized size = 1.37

$$\frac{1}{2} b \left( \frac{e^{\left(-e+\frac{cf}{d}\right)} E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d} - \frac{e^{\left(e-\frac{cf}{d}\right)} E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d} \right) - \frac{a}{d^2 x + cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sinh(f\*x+e))/(d\*x+c)^2,x, algorithm="maxima")

[Out] 1/2\*b\*(e^(-e + c\*f/d)\*exp\_integral\_e(2, (d\*x + c)\*f/d)/((d\*x + c)\*d) - e^(e - c\*f/d)\*exp\_integral\_e(2, -(d\*x + c)\*f/d)/((d\*x + c)\*d)) - a/(d^2\*x + c\*d)

**Fricas [A]** time = 2.47105, size = 351, normalized size = 4.03

$$\frac{2bd \sinh(fx+e) + 2ad - \left( (bdfx+bcf) \operatorname{Ei}\left(\frac{dfx+cf}{d}\right) + (bdfx+bcf) \operatorname{Ei}\left(-\frac{dfx+cf}{d}\right) \right) \cosh\left(-\frac{de-cf}{d}\right) + \left( (bdfx+bcf) \operatorname{Ei}\left(\frac{dfx+cf}{d}\right) - (bdfx+bcf) \operatorname{Ei}\left(-\frac{dfx+cf}{d}\right) \right) \sinh\left(-\frac{de-cf}{d}\right)}{2(d^3x + cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sinh(f\*x+e))/(d\*x+c)^2,x, algorithm="fricas")

[Out] 
$$-1/2*(2*b*d*sinh(f*x + e) + 2*a*d - ((b*d*f*x + b*c*f)*Ei((d*f*x + c*f)/d) + (b*d*f*x + b*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) + ((b*d*f*x + b*c*f)*Ei((d*f*x + c*f)/d) - (b*d*f*x + b*c*f)*Ei(-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d)/(d^3*x + c*d^2)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sinh(f\*x+e))/(d\*x+c)\*\*2,x)

[Out] Timed out

**Giac [A]** time = 1.3074, size = 225, normalized size = 2.59

$$\frac{\left(d f x E i\left(-\frac{d f x+c f}{d}\right) e^{\left(\frac{c f}{d}-e\right)}+d f x E i\left(\frac{d f x+c f}{d}\right) e^{\left(-\frac{c f}{d}+e\right)}+c f E i\left(-\frac{d f x+c f}{d}\right) e^{\left(\frac{c f}{d}-e\right)}+c f E i\left(\frac{d f x+c f}{d}\right) e^{\left(-\frac{c f}{d}+e\right)}-d e^{(f x+e)}+d e^{(-f x-e)}\right)}{2\left(d^3 x+c d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sinh(f\*x+e))/(d\*x+c)^2,x, algorithm="giac")

[Out] 
$$1/2*(d*f*x*Ei(-(d*f*x + c*f)/d)*e^{(c*f/d - e)} + d*f*x*Ei((d*f*x + c*f)/d)*e^{(-c*f/d + e)} + c*f*Ei(-(d*f*x + c*f)/d)*e^{(c*f/d - e)} + c*f*Ei((d*f*x + c*f)/d)*e^{(-c*f/d + e)} - d*e^{(f*x + e)} + d*e^{(-f*x - e)})*b/(d^3*x + c*d^2) - a/((d*x + c)*d)$$

$$3.162 \quad \int \frac{a+b \sinh(e+fx)}{(c+dx)^3} dx$$

**Optimal.** Leaf size=123

$$-\frac{a}{2d(c+dx)^2} + \frac{bf^2 \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{2d^3} + \frac{bf^2 \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{2d^3} - \frac{bf \cosh(e+fx)}{2d^2(c+dx)} - \frac{b \sinh(e+fx)}{2d(c+dx)^2}$$

[Out] -a/(2\*d\*(c + d\*x)^2) - (b\*f\*Cosh[e + f\*x])/(2\*d^2\*(c + d\*x)) + (b\*f^2\*CoshIntegral[(c\*f)/d + f\*x]\*Sinh[e - (c\*f)/d])/(2\*d^3) - (b\*Sinh[e + f\*x])/(2\*d\*(c + d\*x)^2) + (b\*f^2\*Cosh[e - (c\*f)/d]\*SinhIntegral[(c\*f)/d + f\*x])/(2\*d^3)

**Rubi [A]** time = 0.205068, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3317, 3297, 3303, 3298, 3301}

$$-\frac{a}{2d(c+dx)^2} + \frac{bf^2 \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{2d^3} + \frac{bf^2 \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{2d^3} - \frac{bf \cosh(e+fx)}{2d^2(c+dx)} - \frac{b \sinh(e+fx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Sinh[e + f\*x])/(c + d\*x)^3,x]

[Out] -a/(2\*d\*(c + d\*x)^2) - (b\*f\*Cosh[e + f\*x])/(2\*d^2\*(c + d\*x)) + (b\*f^2\*CoshIntegral[(c\*f)/d + f\*x]\*Sinh[e - (c\*f)/d])/(2\*d^3) - (b\*Sinh[e + f\*x])/(2\*d\*(c + d\*x)^2) + (b\*f^2\*Cosh[e - (c\*f)/d]\*SinhIntegral[(c\*f)/d + f\*x])/(2\*d^3)

### Rule 3317

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*Sin[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

### Rule 3297

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := Simp[(c + d\*x)^(m + 1)\*Sin[e + f\*x]/(d\*(m + 1)), x] - Dist[f/(d\*(m + 1)), Int[(c + d\*x)^(m + 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1

]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \sinh(e + fx)}{(c + dx)^3} dx &= \int \left( \frac{a}{(c + dx)^3} + \frac{b \sinh(e + fx)}{(c + dx)^3} \right) dx \\
&= -\frac{a}{2d(c + dx)^2} + b \int \frac{\sinh(e + fx)}{(c + dx)^3} dx \\
&= -\frac{a}{2d(c + dx)^2} - \frac{b \sinh(e + fx)}{2d(c + dx)^2} + \frac{(bf) \int \frac{\cosh(e + fx)}{(c + dx)^2} dx}{2d} \\
&= -\frac{a}{2d(c + dx)^2} - \frac{bf \cosh(e + fx)}{2d^2(c + dx)} - \frac{b \sinh(e + fx)}{2d(c + dx)^2} + \frac{(bf^2) \int \frac{\sinh(e + fx)}{c + dx} dx}{2d^2} \\
&= -\frac{a}{2d(c + dx)^2} - \frac{bf \cosh(e + fx)}{2d^2(c + dx)} - \frac{b \sinh(e + fx)}{2d(c + dx)^2} + \frac{\left( bf^2 \cosh\left(e - \frac{cf}{d}\right) \right) \int \frac{\sinh\left(\frac{cf}{d} + fx\right)}{c + dx} dx}{2d^2} + \\
&= -\frac{a}{2d(c + dx)^2} - \frac{bf \cosh(e + fx)}{2d^2(c + dx)} + \frac{bf^2 \operatorname{Chi}\left(\frac{cf}{d} + fx\right) \sinh\left(e - \frac{cf}{d}\right)}{2d^3} - \frac{b \sinh(e + fx)}{2d(c + dx)^2} + \frac{bf^2}{2d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.639493, size = 95, normalized size = 0.77

$$\frac{-\frac{d(d(a+b \sinh(e+fx))+bf(c+dx) \cosh(e+fx))}{(c+dx)^2} + bf^2 \text{Chi}\left(f\left(\frac{c}{d} + x\right)\right) \sinh\left(e - \frac{cf}{d}\right) + bf^2 \cosh\left(e - \frac{cf}{d}\right) \text{Shi}\left(f\left(\frac{c}{d} + x\right)\right)}{2d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Sinh[e + f\*x])/(c + d\*x)^3,x]

[Out] (b\*f^2\*CoshIntegral[f\*(c/d + x)]\*Sinh[e - (c\*f)/d] - (d\*(b\*f\*(c + d\*x)\*Cosh[e + f\*x] + d\*(a + b\*Sinh[e + f\*x])))/(c + d\*x)^2 + b\*f^2\*Cosh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)]/(2\*d^3)

**Maple [B]** time = 0.024, size = 296, normalized size = 2.4

$$\frac{a}{2d(dx+c)^2} - \frac{bf^3e^{-fx-ex}}{4d(d^2f^2x^2 + 2dcf^2x + c^2f^2)} - \frac{bf^3e^{-fx-ec}}{4d^2(d^2f^2x^2 + 2dcf^2x + c^2f^2)} + \frac{f^2be^{-fx-e}}{4d(d^2f^2x^2 + 2dcf^2x + c^2f^2)} + \frac{f^2e^{-fx-e}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sinh(f\*x+e))/(d\*x+c)^3,x)

[Out]  $-1/2*a/d/(d*x+c)^2 - 1/4*b*f^3*\exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x - 1/4*b*f^3*\exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c + 1/4*b*f^2*\exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2) + 1/4*b*f^2/d^3*\exp((c*f-d*e)/d)*\text{Ei}(1, f*x+e+(c*f-d*e)/d) - 1/4*f^2*b/d^3*\exp(f*x+e)/(c*f/d+f*x)^2 - 1/4*f^2*b/d^3*\exp(f*x+e)/(c*f/d+f*x) - 1/4*f^2*b/d^3*\exp(-(c*f-d*e)/d)*\text{Ei}(1, -f*x-e-(c*f-d*e)/d)$

**Maxima [A]** time = 1.46596, size = 134, normalized size = 1.09

$$\frac{1}{2}b \left( \frac{e^{\left(-e+\frac{cf}{d}\right)} E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d} - \frac{e^{\left(e-\frac{cf}{d}\right)} E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d} \right) - \frac{a}{2(d^3x^2 + 2cd^2x + c^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sinh(f\*x+e))/(d\*x+c)^3,x, algorithm="maxima")

[Out]  $\frac{1}{2}b(e^{-e + cf/d})\exp\_integral\_e(3, (dx + c)f/d)/((dx + c)^{2d}) - e^{-(e - cf/d)}\exp\_integral\_e(3, -(dx + c)f/d)/((dx + c)^{2d}) - \frac{1}{2}a/(d^3x^2 + 2cd^2x + c^2d)$

**Fricas [B]** time = 2.45782, size = 572, normalized size = 4.65

$$2bd^2 \sinh(fx + e) + 2ad^2 + 2(bd^2fx + bcdf) \cosh(fx + e) - \left( (bd^2f^2x^2 + 2bcdf^2x + bc^2f^2) \operatorname{Ei}\left(\frac{dfx+cf}{d}\right) - (bd^2f^2x^2 \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e))/(d*x+c)^3,x, algorithm="fricas")`

[Out]  $-\frac{1}{4}(2bd^2\sinh(fx + e) + 2ad^2 + 2(bd^2fx + bcdf)\cosh(fx + e) - ((bd^2f^2x^2 + 2bcdf^2x + bc^2f^2)\operatorname{Ei}((dfx + cf)/d) - (bd^2f^2x^2 + 2bcdf^2x + bc^2f^2)\operatorname{Ei}(-(dfx + cf)/d))\cosh(-(de - cf)/d) + ((bd^2f^2x^2 + 2bcdf^2x + bc^2f^2)\operatorname{Ei}((dfx + cf)/d) + (bd^2f^2x^2 + 2bcdf^2x + bc^2f^2)\operatorname{Ei}(-(dfx + cf)/d))\sinh(-(de - cf)/d))/(d^5x^2 + 2cd^4x + c^2d^3)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e))/(d*x+c)**3,x)`

[Out] Timed out

**Giac [B]** time = 1.24726, size = 441, normalized size = 3.59

$$bd^2f^2x^2\operatorname{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)} - bd^2f^2x^2\operatorname{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}+e\right)} + 2bcdf^2x\operatorname{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(\frac{cf}{d}-e\right)} - 2bcdf^2x\operatorname{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(-\frac{cf}{d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sinh(f\*x+e))/(d\*x+c)^3,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/4*(b*d^2*f^2*x^2*Ei(-(d*f*x + c*f)/d)*e^{(c*f/d - e)} - b*d^2*f^2*x^2*Ei((d*f*x + c*f)/d)*e^{(-c*f/d + e)} + 2*b*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^{(c*f/d - e)} - 2*b*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^{(-c*f/d + e)} + b*c^2*f^2*Ei(-(d*f*x + c*f)/d)*e^{(c*f/d - e)} - b*c^2*f^2*Ei((d*f*x + c*f)/d)*e^{(-c*f/d + e)} \\ & + b*d^2*f*x*e^{(f*x + e)} + b*d^2*f*x*e^{(-f*x - e)} + b*c*d*f*e^{(f*x + e)} + b*c*d*f*e^{(-f*x - e)} + b*d^2*e^{(f*x + e)} - b*d^2*e^{(-f*x - e)} + 2*a*d^2)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3) \end{aligned}$$



### 3.163 $\int (c + dx)^3 (a + b \sinh(e + fx))^2 dx$

**Optimal.** Leaf size=250

$$\frac{a^2(c + dx)^4}{4d} + \frac{12abd^2(c + dx) \cosh(e + fx)}{f^3} - \frac{6abd(c + dx)^2 \sinh(e + fx)}{f^2} + \frac{2ab(c + dx)^3 \cosh(e + fx)}{f} - \frac{12abd^3 \sinh(e + fx)}{f^4}$$

[Out]  $(-3*b^2*c*d^2*x)/(4*f^2) - (3*b^2*d^3*x^2)/(8*f^2) + (a^2*(c + d*x)^4)/(4*d) - (b^2*(c + d*x)^4)/(8*d) + (12*a*b*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + (2*a*b*(c + d*x)^3*Cosh[e + f*x])/f - (12*a*b*d^3*Sinh[e + f*x])/f^4 - (6*a*b*d*(c + d*x)^2*Sinh[e + f*x])/f^2 + (3*b^2*d^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(4*f^3) + (b^2*(c + d*x)^3*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) - (3*b^2*d^3*Sinh[e + f*x]^2)/(8*f^4) - (3*b^2*d*(c + d*x)^2*Sinh[e + f*x]^2)/(4*f^2)$

**Rubi [A]** time = 0.289963, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {3317, 3296, 2637, 3311, 32, 3310}

$$\frac{a^2(c + dx)^4}{4d} + \frac{12abd^2(c + dx) \cosh(e + fx)}{f^3} - \frac{6abd(c + dx)^2 \sinh(e + fx)}{f^2} + \frac{2ab(c + dx)^3 \cosh(e + fx)}{f} - \frac{12abd^3 \sinh(e + fx)}{f^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)^3*(a + b*\text{Sinh}[e + f*x])^2, x]$

[Out]  $(-3*b^2*c*d^2*x)/(4*f^2) - (3*b^2*d^3*x^2)/(8*f^2) + (a^2*(c + d*x)^4)/(4*d) - (b^2*(c + d*x)^4)/(8*d) + (12*a*b*d^2*(c + d*x)*Cosh[e + f*x])/f^3 + (2*a*b*(c + d*x)^3*Cosh[e + f*x])/f - (12*a*b*d^3*Sinh[e + f*x])/f^4 - (6*a*b*d*(c + d*x)^2*Sinh[e + f*x])/f^2 + (3*b^2*d^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(4*f^3) + (b^2*(c + d*x)^3*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) - (3*b^2*d^3*Sinh[e + f*x]^2)/(8*f^4) - (3*b^2*d*(c + d*x)^2*Sinh[e + f*x]^2)/(4*f^2)$

#### Rule 3317

$\text{Int}[(c + d*x)^m*(a + b*\text{Sin}[e + f*x])^n, x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ (\text{EqQ}[n, 1] \ || \ \text{IGtQ}[m, 0] \ || \ \text{NeQ}[a^2 - b^2, 0])$

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbo
l] := Simp[(d*m*(c + d*x)^(m - 1)*(b*SIn[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*SIn[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*SIn[e + f*x])^n, x]
- Simp[(b*(c + d*x)^m*cos[e + f*x]*(b*SIn[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
Simp[(d*(b*SIn[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*SIn[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*cos[e + f*x]*(b
*SIn[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^3 (a + b \sinh(e + fx))^2 dx &= \int (a^2(c + dx)^3 + 2ab(c + dx)^3 \sinh(e + fx) + b^2(c + dx)^3 \sinh^2(e + fx)) dx \\
&= \frac{a^2(c + dx)^4}{4d} + (2ab) \int (c + dx)^3 \sinh(e + fx) dx + b^2 \int (c + dx)^3 \sinh^2(e + fx) dx \\
&= \frac{a^2(c + dx)^4}{4d} + \frac{2ab(c + dx)^3 \cosh(e + fx)}{f} + \frac{b^2(c + dx)^3 \cosh(e + fx) \sinh(e + fx)}{2f} \\
&= \frac{a^2(c + dx)^4}{4d} - \frac{b^2(c + dx)^4}{8d} + \frac{2ab(c + dx)^3 \cosh(e + fx)}{f} - \frac{6abd(c + dx)^2 \sinh(e + fx)}{f^2} \\
&= -\frac{3b^2cd^2x}{4f^2} - \frac{3b^2d^3x^2}{8f^2} + \frac{a^2(c + dx)^4}{4d} - \frac{b^2(c + dx)^4}{8d} + \frac{12abd^2(c + dx) \cosh(e + fx)}{f^3} \\
&= -\frac{3b^2cd^2x}{4f^2} - \frac{3b^2d^3x^2}{8f^2} + \frac{a^2(c + dx)^4}{4d} - \frac{b^2(c + dx)^4}{8d} + \frac{12abd^2(c + dx) \cosh(e + fx)}{f^3}
\end{aligned}$$

**Mathematica [A]** time = 1.38262, size = 235, normalized size = 0.94

$$\frac{2(f^4x(2a^2 - b^2)(6c^2dx + 4c^3 + 4cd^2x^2 + d^3x^3) - 48abd(c^2f^2 + 2cdf^2x + d^2(f^2x^2 + 2)) \sinh(e + fx) + b^2f(c + dx)(2f^2x^2 + 2))}{16f^4}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^3\*(a + b\*Sinh[e + f\*x])^2,x]

[Out] (32\*a\*b\*f\*(c + d\*x)\*(c^2\*f^2 + 2\*c\*d\*f^2\*x + d^2\*(6 + f^2\*x^2))\*Cosh[e + f\*x] - 3\*b^2\*d\*(2\*c^2\*f^2 + 4\*c\*d\*f^2\*x + d^2\*(1 + 2\*f^2\*x^2))\*Cosh[2\*(e + f\*x)] + 2\*((2\*a^2 - b^2)\*f^4\*x\*(4\*c^3 + 6\*c^2\*d\*x + 4\*c\*d^2\*x^2 + d^3\*x^3) - 48\*a\*b\*d\*(c^2\*f^2 + 2\*c\*d\*f^2\*x + d^2\*(2 + f^2\*x^2))\*Sinh[e + f\*x] + b^2\*f\*(c + d\*x)\*(2\*c^2\*f^2 + 4\*c\*d\*f^2\*x + d^2\*(3 + 2\*f^2\*x^2))\*Sinh[2\*(e + f\*x)])/(16\*f^4)

**Maple [B]** time = 0.014, size = 1061, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^3\*(a+b\*sinh(f\*x+e))^2,x)

```
[Out] 1/f*(c^3*b^2*(1/2*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)+3/2/f^3*d^3*e^2*a^
2*(f*x+e)^2-1/f^3*d^3*e*a^2*(f*x+e)^3+3/2/f*c^2*d*a^2*(f*x+e)^2+1/f^2*c*d^2
*a^2*(f*x+e)^3-1/f^3*d^3*e^3*a^2*(f*x+e)+2/f^3*d^3*a*b*((f*x+e)^3*cosh(f*x+
e)-3*(f*x+e)^2*sinh(f*x+e)+6*(f*x+e)*cosh(f*x+e)-6*sinh(f*x+e))-3/f^3*d^3*e
*b^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)-1/6*(f*x+e)^3-1/2*(f*x+e)*cosh(
f*x+e)^2+1/4*cosh(f*x+e)*sinh(f*x+e)+1/4*f*x+1/4*e)+3/f^3*d^3*e^2*b^2*(1/2*
(f*x+e)*cosh(f*x+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)+3/f^2*c*d^
2*b^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)-1/6*(f*x+e)^3-1/2*(f*x+e)*cosh
(f*x+e)^2+1/4*cosh(f*x+e)*sinh(f*x+e)+1/4*f*x+1/4*e)-1/f^3*d^3*e^3*b^2*(1/2
*cosh(f*x+e)*sinh(f*x+e)-1/2*f*x-1/2*e)+3/f*c^2*d*b^2*(1/2*(f*x+e)*cosh(f*x
+e)*sinh(f*x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)+1/f^3*d^3*b^2*(1/2*(f*x+e)
^3*cosh(f*x+e)*sinh(f*x+e)-1/8*(f*x+e)^4-3/4*(f*x+e)^2*cosh(f*x+e)^2+3/4*(f
*x+e)*cosh(f*x+e)*sinh(f*x+e)+3/8*(f*x+e)^2-3/8*cosh(f*x+e)^2)+2*c^3*a*b*co
sh(f*x+e)+1/4/f^3*d^3*a^2*(f*x+e)^4+3/f^2*c*d^2*e^2*b^2*(1/2*cosh(f*x+e)*si
nh(f*x+e)-1/2*f*x-1/2*e)-6/f^2*c*d^2*e*b^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*
x+e)-1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)+6/f*c^2*d*a*b*((f*x+e)*cosh(f*x+e)-si
nh(f*x+e))-2/f^3*d^3*e^3*a*b*cosh(f*x+e)-3/f*c^2*d*e*b^2*(1/2*cosh(f*x+e)*s
inh(f*x+e)-1/2*f*x-1/2*e)+6/f^3*d^3*e^2*a*b*((f*x+e)*cosh(f*x+e)-sinh(f*x+e
))+6/f^2*c*d^2*a*b*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)+2*cosh(f*x+
e))-6/f^3*d^3*e*a*b*((f*x+e)^2*cosh(f*x+e)-2*(f*x+e)*sinh(f*x+e)+2*cosh(f*x
+e))+6/f^2*c*d^2*e^2*a*b*cosh(f*x+e)-12/f^2*c*d^2*e*a*b*((f*x+e)*cosh(f*x+e
)-sinh(f*x+e))-6/f*c^2*d*e*a*b*cosh(f*x+e)+3/f^2*c*d^2*e^2*a^2*(f*x+e)-3/f*
c^2*d*e*a^2*(f*x+e)-3/f^2*c*d^2*e*a^2*(f*x+e)^2+c^3*a^2*(f*x+e))
```

---

**Maxima [B]** time = 1.3345, size = 702, normalized size = 2.81

$$\frac{1}{4}a^2d^3x^4 + a^2cd^2x^3 + \frac{3}{2}a^2c^2dx^2 - \frac{3}{16}\left(4x^2 - \frac{(2fxe^{(2e)} - e^{(2e)})e^{(2fx)}}{f^2} + \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}\right)b^2c^2d - \frac{1}{16}\left(8x^3 - \frac{3(2f^2x^2 + 2fx + 1)e^{(-2fx-2e)}}{f^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sinh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 - 3/16*(4*x^2 - (2*f*x*
e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 + (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*c^
2*d - 1/16*(8*x^3 - 3*(2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*
x)/f^3 + 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*b^2*c*d^2 - 1/32*(
4*x^4 - (4*f^3*x^3*e^(2*e) - 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) - 3*e^(2*e))
*e^(2*f*x)/f^4 + (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x - 2*e)/f^4)*
b^2*d^3 - 1/8*b^2*c^3*(4*x - e^(2*f*x + 2*e)/f + e^(-2*f*x - 2*e)/f) + a^2*
c^3*x + 3*a*b*c^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 + (f*x + 1)*e^(-f*x - e)/f
^2) + 3*a*b*c*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 + (f^2*x^2
```

$$+ 2*f*x + 2)*e^{(-f*x - e)/f^3} + a*b*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6*f*x*e^e - 6*e^e)*e^{(f*x)/f^4} + (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^{(-f*x - e)/f^4}) + 2*a*b*c^3*cosh(f*x + e)/f$$

**Fricas [A]** time = 2.4951, size = 880, normalized size = 3.52

$$2(2a^2 - b^2)d^3f^4x^4 + 8(2a^2 - b^2)cd^2f^4x^3 + 12(2a^2 - b^2)c^2df^4x^2 + 8(2a^2 - b^2)c^3f^4x - 3(2b^2d^3f^2x^2 + 4b^2cd^2f^2x +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^3\*(a+b\*sinh(f\*x+e))^2,x, algorithm="fricas")

[Out]  $\frac{1}{16}*(2*(2*a^2 - b^2)*d^3*f^4*x^4 + 8*(2*a^2 - b^2)*c*d^2*f^4*x^3 + 12*(2*a^2 - b^2)*c^2*d*f^4*x^2 + 8*(2*a^2 - b^2)*c^3*f^4*x - 3*(2*b^2*d^3*f^2*x^2 + 4*b^2*c*d^2*f^2*x + 2*b^2*c^2*d*f^2 + b^2*d^3)*cosh(f*x + e)^2 - 3*(2*b^2*d^3*f^2*x^2 + 4*b^2*c*d^2*f^2*x + 2*b^2*c^2*d*f^2 + b^2*d^3)*sinh(f*x + e)^2 + 32*(a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + a*b*c^3*f^3 + 6*a*b*c*d^2*f + 3*(a*b*c^2*d*f^3 + 2*a*b*d^3*f)*x)*cosh(f*x + e) - 4*(24*a*b*d^3*f^2*x^2 + 48*a*b*c*d^2*f^2*x + 24*a*b*c^2*d*f^2 + 48*a*b*d^3 - (2*b^2*d^3*f^3*x^3 + 6*b^2*c*d^2*f^3*x^2 + 2*b^2*c^3*f^3 + 3*b^2*c*d^2*f + 3*(2*b^2*c^2*d*f^3 + b^2*d^3*f)*x)*cosh(f*x + e))*sinh(f*x + e))/f^4$

**Sympy [A]** time = 5.31346, size = 779, normalized size = 3.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*3\*(a+b\*sinh(f\*x+e))\*\*2,x)

[Out] Piecewise((a\*\*2\*c\*\*3\*x + 3\*a\*\*2\*c\*\*2\*d\*x\*\*2/2 + a\*\*2\*c\*d\*\*2\*x\*\*3 + a\*\*2\*d\*\*3\*x\*\*4/4 + 2\*a\*b\*c\*\*3\*cosh(e + f\*x)/f + 6\*a\*b\*c\*\*2\*d\*x\*cosh(e + f\*x)/f - 6\*a\*b\*c\*\*2\*d\*sinh(e + f\*x)/f\*\*2 + 6\*a\*b\*c\*d\*\*2\*x\*\*2\*cosh(e + f\*x)/f - 12\*a\*b\*c\*d\*\*2\*x\*sinh(e + f\*x)/f\*\*2 + 12\*a\*b\*c\*d\*\*2\*cosh(e + f\*x)/f\*\*3 + 2\*a\*b\*d\*\*3\*x\*\*3\*cosh(e + f\*x)/f - 6\*a\*b\*d\*\*3\*x\*\*2\*sinh(e + f\*x)/f\*\*2 + 12\*a\*b\*d\*\*3\*x\*cosh(e + f\*x)/f\*\*3 - 12\*a\*b\*d\*\*3\*sinh(e + f\*x)/f\*\*4 + b\*\*2\*c\*\*3\*x\*sinh(e + f\*x)\*\*2/2 - b\*\*2\*c\*\*3\*x\*cosh(e + f\*x)\*\*2/2 + b\*\*2\*c\*\*3\*sinh(e + f\*x)\*cosh(e + f\*x)/(2\*f) + 3\*b\*\*2\*c\*\*2\*d\*x\*\*2\*sinh(e + f\*x)\*\*2/4 - 3\*b\*\*2\*c\*\*2\*d\*x\*\*2\*

```

cosh(e + f*x)**2/4 + 3*b**2*c**2*d*x*sinh(e + f*x)*cosh(e + f*x)/(2*f) - 3*
b**2*c**2*d*sinh(e + f*x)**2/(4*f**2) + b**2*c*d**2*x**3*sinh(e + f*x)**2/2
- b**2*c*d**2*x**3*cosh(e + f*x)**2/2 + 3*b**2*c*d**2*x**2*sinh(e + f*x)*c
osh(e + f*x)/(2*f) - 3*b**2*c*d**2*x**2*sinh(e + f*x)**2/(4*f**2) - 3*b**2*c*d
**2*x*cosh(e + f*x)**2/(4*f**2) + 3*b**2*c*d**2*sinh(e + f*x)*cosh(e + f*x)
/(4*f**3) + b**2*d**3*x**4*sinh(e + f*x)**2/8 - b**2*d**3*x**4*cosh(e + f*x
)**2/8 + b**2*d**3*x**3*sinh(e + f*x)*cosh(e + f*x)/(2*f) - 3*b**2*d**3*x**
2*sinh(e + f*x)**2/(8*f**2) - 3*b**2*d**3*x**2*cosh(e + f*x)**2/(8*f**2) +
3*b**2*d**3*x*sinh(e + f*x)*cosh(e + f*x)/(4*f**3) - 3*b**2*d**3*sinh(e +
f*x)**2/(8*f**4), Ne(f, 0)), ((a + b*sinh(e))**2*(c**3*x + 3*c**2*d*x**2/2 +
c*d**2*x**3 + d**3*x**4/4), True))

```

**Giac [B]** time = 1.26267, size = 813, normalized size = 3.25

$$\frac{1}{4}a^2d^3x^4 - \frac{1}{8}b^2d^3x^4 + a^2cd^2x^3 - \frac{1}{2}b^2cd^2x^3 + \frac{3}{2}a^2c^2dx^2 - \frac{3}{4}b^2c^2dx^2 + a^2c^3x - \frac{1}{2}b^2c^3x + \frac{(4b^2d^3f^3x^3 + 12b^2cd^2f^3x^2 + 12b^2c^2d^3f^3x + 6b^2d^3f^2x^2 + 4b^2c^3f^3 - 12b^2c*d^2*f^2*x - 6b^2*c^2*d*f^2 + 6b^2*d^3*f*x + 6b^2*c*d^2*f - 3b^2*d^3)*e^{(2*f*x + 2*e)}}{f^4} + \frac{(a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3*x - 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 - 6*a*b*c*d^2*f^2*x - 3*a*b*c^2*d*f^2 + 6*a*b*d^3*f*x + 6*a*b*c*d^2*f - 6*a*b*d^3)*e^{(f*x + e)}}{f^4} + \frac{(a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3*x + 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 + 6*a*b*c*d^2*f^2*x + 3*a*b*c^2*d*f^2 + 6*a*b*d^3*f*x + 6*a*b*c*d^2*f + 6*a*b*d^3)*e^{(-f*x - e)}}{f^4} - \frac{1}{32}(4*b^2*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x + 6*b^2*d^3*f^2*x^2 + 4*b^2*c^3*f^3 + 12*b^2*c*d^2*f^2*x + 6*b^2*c^2*d*f^2 + 6*b^2*d^3*f*x + 6*b^2*c*d^2*f + 3*b^2*d^3)*e^{(-2*f*x - 2*e)}/f^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

```

[Out] 1/4*a^2*d^3*x^4 - 1/8*b^2*d^3*x^4 + a^2*c*d^2*x^3 - 1/2*b^2*c*d^2*x^3 + 3/2
*a^2*c^2*d*x^2 - 3/4*b^2*c^2*d*x^2 + a^2*c^3*x - 1/2*b^2*c^3*x + 1/32*(4*b^
2*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x - 6*b^2*d^3*f^2*x
^2 + 4*b^2*c^3*f^3 - 12*b^2*c*d^2*f^2*x - 6*b^2*c^2*d*f^2 + 6*b^2*d^3*f*x +
6*b^2*c*d^2*f - 3*b^2*d^3)*e^(2*f*x + 2*e)/f^4 + (a*b*d^3*f^3*x^3 + 3*a*b*
c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3*x - 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 - 6*a*b
*c*d^2*f^2*x - 3*a*b*c^2*d*f^2 + 6*a*b*d^3*f*x + 6*a*b*c*d^2*f - 6*a*b*d^3)
*e^(f*x + e)/f^4 + (a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3
*x + 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 + 6*a*b*c*d^2*f^2*x + 3*a*b*c^2*d*f^2
+ 6*a*b*d^3*f*x + 6*a*b*c*d^2*f + 6*a*b*d^3)*e^(-f*x - e)/f^4 - 1/32*(4*b^2
*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x + 6*b^2*d^3*f^2*x^
2 + 4*b^2*c^3*f^3 + 12*b^2*c*d^2*f^2*x + 6*b^2*c^2*d*f^2 + 6*b^2*d^3*f*x +
6*b^2*c*d^2*f + 3*b^2*d^3)*e^(-2*f*x - 2*e)/f^4

```

### 3.164 $\int (c + dx)^2 (a + b \sinh(e + fx))^2 dx$

**Optimal.** Leaf size=182

$$\frac{a^2(c + dx)^3}{3d} - \frac{4abd(c + dx) \sinh(e + fx)}{f^2} + \frac{2ab(c + dx)^2 \cosh(e + fx)}{f} + \frac{4abd^2 \cosh(e + fx)}{f^3} - \frac{b^2d(c + dx) \sinh^2(e + fx)}{2f^2}$$

```
[Out] -(b^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(3*d) - (b^2*(c + d*x)^3)/(6*d) +
(4*a*b*d^2*Cosh[e + f*x])/f^3 + (2*a*b*(c + d*x)^2*Cosh[e + f*x])/f - (4*a*
b*d*(c + d*x)*Sinh[e + f*x])/f^2 + (b^2*d^2*Cosh[e + f*x]*Sinh[e + f*x])/(4
*f^3) + (b^2*(c + d*x)^2*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) - (b^2*d*(c + d
*x)*Sinh[e + f*x]^2)/(2*f^2)
```

**Rubi [A]** time = 0.202018, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$ , Rules used = {3317, 3296, 2638, 3311, 32, 2635, 8}

$$\frac{a^2(c + dx)^3}{3d} - \frac{4abd(c + dx) \sinh(e + fx)}{f^2} + \frac{2ab(c + dx)^2 \cosh(e + fx)}{f} + \frac{4abd^2 \cosh(e + fx)}{f^3} - \frac{b^2d(c + dx) \sinh^2(e + fx)}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] -(b^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(3*d) - (b^2*(c + d*x)^3)/(6*d) +
(4*a*b*d^2*Cosh[e + f*x])/f^3 + (2*a*b*(c + d*x)^2*Cosh[e + f*x])/f - (4*a*
b*d*(c + d*x)*Sinh[e + f*x])/f^2 + (b^2*d^2*Cosh[e + f*x]*Sinh[e + f*x])/(4
*f^3) + (b^2*(c + d*x)^2*Cosh[e + f*x]*Sinh[e + f*x])/(2*f) - (b^2*d*(c + d
*x)*Sinh[e + f*x]^2)/(2*f^2)
```

#### Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sinh[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

#### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:= Simp[(d*m*(c + d*x)^(m - 1)*(b*Ssin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Ssin[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Ssin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Ssin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
]
```

### Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

### Rubi steps



$$\begin{aligned}
\int (c + dx)^2 (a + b \sinh(e + fx))^2 dx &= \int (a^2(c + dx)^2 + 2ab(c + dx)^2 \sinh(e + fx) + b^2(c + dx)^2 \sinh^2(e + fx)) dx \\
&= \frac{a^2(c + dx)^3}{3d} + (2ab) \int (c + dx)^2 \sinh(e + fx) dx + b^2 \int (c + dx)^2 \sinh^2(e + fx) dx \\
&= \frac{a^2(c + dx)^3}{3d} + \frac{2ab(c + dx)^2 \cosh(e + fx)}{f} + \frac{b^2(c + dx)^2 \cosh(e + fx) \sinh(e + fx)}{2f} \\
&= \frac{a^2(c + dx)^3}{3d} - \frac{b^2(c + dx)^3}{6d} + \frac{2ab(c + dx)^2 \cosh(e + fx)}{f} - \frac{4abd(c + dx) \sinh(e + fx)}{f^2} \\
&= -\frac{b^2 d^2 x}{4f^2} + \frac{a^2(c + dx)^3}{3d} - \frac{b^2(c + dx)^3}{6d} + \frac{4abd^2 \cosh(e + fx)}{f^3} + \frac{2ab(c + dx)^2 \cosh(e + fx)}{f}
\end{aligned}$$

**Mathematica [A]** time = 0.752984, size = 249, normalized size = 1.37

$$\frac{24a^2c^2f^3x + 24a^2cdf^3x^2 + 8a^2d^2f^3x^3 + 48ab(c^2f^2 + 2cdf^2x + d^2(f^2x^2 + 2)) \cosh(e + fx) - 96abcdf \sinh(e + fx) - 6b^2d^2x}{24f^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2\*(a + b\*Sinh[e + f\*x])^2,x]

[Out] (24\*a^2\*c^2\*f^3\*x - 12\*b^2\*c^2\*f^3\*x + 24\*a^2\*c\*d\*f^3\*x^2 - 12\*b^2\*c\*d\*f^3\*x^2 + 8\*a^2\*d^2\*f^3\*x^3 - 4\*b^2\*d^2\*f^3\*x^3 + 48\*a\*b\*(c^2\*f^2 + 2\*c\*d\*f^2\*x + d^2\*(2 + f^2\*x^2))\*Cosh[e + f\*x] - 6\*b^2\*d\*f\*(c + d\*x)\*Cosh[2\*(e + f\*x)] - 96\*a\*b\*c\*d\*f\*Sinh[e + f\*x] - 96\*a\*b\*d^2\*f\*x\*Sinh[e + f\*x] + 3\*b^2\*d^2\*Sinh[2\*(e + f\*x)] + 6\*b^2\*c^2\*f^2\*Sinh[2\*(e + f\*x)] + 12\*b^2\*c\*d\*f^2\*x\*Sinh[2\*(e + f\*x)] + 6\*b^2\*d^2\*f^2\*x^2\*Sinh[2\*(e + f\*x)])/(24\*f^3)

**Maple [B]** time = 0.016, size = 535, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2\*(a+b\*sinh(f\*x+e))^2,x)

[Out] 1/f\*(1/3/f^2\*d^2\*a^2\*(f\*x+e)^3+2/f^2\*d^2\*a\*b\*((f\*x+e)^2\*cosh(f\*x+e)-2\*(f\*x+e)\*sinh(f\*x+e)+2\*cosh(f\*x+e))+1/f^2\*d^2\*b^2\*(1/2\*(f\*x+e)^2\*cosh(f\*x+e)\*sinh

$$(f*x+e)-1/6*(f*x+e)^3-1/2*(f*x+e)*\cosh(f*x+e)^2+1/4*\cosh(f*x+e)*\sinh(f*x+e)+1/4*f*x+1/4*e)-1/f^2*d^2*e*a^2*(f*x+e)^2-4/f^2*d^2*e*a*b*((f*x+e)*\cosh(f*x+e)-\sinh(f*x+e))-2/f^2*d^2*e*b^2*(1/2*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e)-1/4*(f*x+e)^2-1/4*\cosh(f*x+e)^2)+1/f^2*d^2*e^2*a^2*(f*x+e)+2/f^2*d^2*e^2*a*b*\cosh(f*x+e)+1/f^2*d^2*e^2*b^2*(1/2*\cosh(f*x+e)*\sinh(f*x+e)-1/2*f*x-1/2*e)+1/f*c*d*a^2*(f*x+e)^2+4/f*c*d*a*b*((f*x+e)*\cosh(f*x+e)-\sinh(f*x+e))+2/f*c*d*b^2*(1/2*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e)-1/4*(f*x+e)^2-1/4*\cosh(f*x+e)^2)-2/f*c*d*e*a^2*(f*x+e)-4/f*c*d*e*a*b*\cosh(f*x+e)-2/f*c*d*e*b^2*(1/2*\cosh(f*x+e)*\sinh(f*x+e)-1/2*f*x-1/2*e)+c^2*a^2*(f*x+e)+2*c^2*a*b*\cosh(f*x+e)+c^2*b^2*(1/2*\cosh(f*x+e)*\sinh(f*x+e)-1/2*f*x-1/2*e))$$

**Maxima [A]** time = 1.27115, size = 435, normalized size = 2.39

$$\frac{1}{3}a^2d^2x^3 + a^2cdx^2 - \frac{1}{8}\left(4x^2 - \frac{(2fxe^{(2e)} - e^{(2e)})e^{(2fx)}}{f^2} + \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}\right)b^2cd - \frac{1}{48}\left(8x^3 - \frac{3(2f^2x^2e^{(2e)} - 2fxe^{(2e)})}{f^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+b\*sinh(f\*x+e))^2,x, algorithm="maxima")

[Out]  $\frac{1}{3}a^2d^2x^3 + a^2cdx^2 - \frac{1}{8}(4x^2 - \frac{(2fxe^{(2e)} - e^{(2e)})e^{(2fx)}}{f^2} + \frac{(2fx+1)e^{(-2fx-2e)}}{f^2})b^2cd - \frac{1}{48}(8x^3 - \frac{3(2f^2x^2e^{(2e)} - 2fxe^{(2e)})}{f^3})$

**Fricas [A]** time = 2.47877, size = 541, normalized size = 2.97

$$2(2a^2 - b^2)d^2f^3x^3 + 6(2a^2 - b^2)cdf^3x^2 + 6(2a^2 - b^2)c^2f^3x - 3(b^2d^2fx + b^2cdf)\cosh(fx + e)^2 - 3(b^2d^2fx + b^2cdf)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2\*(a+b\*sinh(f\*x+e))^2,x, algorithm="fricas")

```
[Out] 1/12*(2*(2*a^2 - b^2)*d^2*f^3*x^3 + 6*(2*a^2 - b^2)*c*d*f^3*x^2 + 6*(2*a^2 - b^2)*c^2*f^3*x - 3*(b^2*d^2*f*x + b^2*c*d*f)*cosh(f*x + e)^2 - 3*(b^2*d^2*f*x + b^2*c*d*f)*sinh(f*x + e)^2 + 24*(a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2 + 2*a*b*d^2)*cosh(f*x + e) - 3*(16*a*b*d^2*f*x + 16*a*b*c*d*f - (2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*f^2 + b^2*d^2)*cosh(f*x + e))*sinh(f*x + e))/f^3
```

**Sympy [A]** time = 2.35746, size = 456, normalized size = 2.51

$$\left\{ \begin{array}{l} a^2 c^2 x + a^2 c d x^2 + \frac{a^2 d^2 x^3}{3} + \frac{2 a b c^2 \cosh(e+f x)}{3} + \frac{4 a b c d x \cosh(e+f x)}{f} - \frac{4 a b c d \sinh(e+f x)}{f^2} + \frac{2 a b d^2 x^2 \cosh(e+f x)}{f} - \frac{4 a b d^2 x \sinh(e+f x)}{f^2} + \frac{4 a b d^2 \cosh(e+f x)}{f^2} \\ (a + b \sinh(e))^2 \left( c^2 x + c d x^2 + \frac{d^2 x^3}{3} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*sinh(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**2*x + a**2*c*d*x**2 + a**2*d**2*x**3/3 + 2*a*b*c**2*cosh(e + f*x)/f + 4*a*b*c*d*x*cosh(e + f*x)/f - 4*a*b*c*d*sinh(e + f*x)/f**2 + 2*a*b*d**2*x**2*cosh(e + f*x)/f - 4*a*b*d**2*x*sinh(e + f*x)/f**2 + 4*a*b*d**2*cosh(e + f*x)/f**3 + b**2*c**2*x*sinh(e + f*x)**2/2 - b**2*c**2*x*cosh(e + f*x)**2/2 + b**2*c**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) + b**2*c*d*x**2*sinh(e + f*x)**2/2 - b**2*c*d*x**2*cosh(e + f*x)**2/2 + b**2*c*d*x*sinh(e + f*x)*cosh(e + f*x)/f - b**2*c*d*sinh(e + f*x)**2/(2*f**2) + b**2*d**2*x**3*sinh(e + f*x)**2/6 - b**2*d**2*x**3*cosh(e + f*x)**2/6 + b**2*d**2*x**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*d**2*x*sinh(e + f*x)**2/(4*f**2) - b**2*d**2*x*cosh(e + f*x)**2/(4*f**2) + b**2*d**2*sinh(e + f*x)*cosh(e + f*x)/(4*f**3), Ne(f, 0)), ((a + b*sinh(e))**2*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

**Giac [B]** time = 1.2962, size = 470, normalized size = 2.58

$$\frac{1}{3} a^2 d^2 x^3 - \frac{1}{6} b^2 d^2 x^3 + a^2 c d x^2 - \frac{1}{2} b^2 c d x^2 + a^2 c^2 x - \frac{1}{2} b^2 c^2 x + \frac{(2 b^2 d^2 f^2 x^2 + 4 b^2 c d f^2 x + 2 b^2 c^2 f^2 - 2 b^2 d^2 f x - 2 b^2 c d f)}{16 f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*d^2*x^3 - 1/6*b^2*d^2*x^3 + a^2*c*d*x^2 - 1/2*b^2*c*d*x^2 + a^2*c^2
*x - 1/2*b^2*c^2*x + 1/16*(2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*
f^2 - 2*b^2*d^2*f*x - 2*b^2*c*d*f + b^2*d^2)*e^(2*f*x + 2*e)/f^3 + (a*b*d^2
*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2 - 2*a*b*d^2*f*x - 2*a*b*c*d*f + 2*
a*b*d^2)*e^(f*x + e)/f^3 + (a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2
+ 2*a*b*d^2*f*x + 2*a*b*c*d*f + 2*a*b*d^2)*e^(-f*x - e)/f^3 - 1/16*(2*b^2*
d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*f^2 + 2*b^2*d^2*f*x + 2*b^2*c*d*f
+ b^2*d^2)*e^(-2*f*x - 2*e)/f^3
```

### 3.165 $\int (c + dx)(a + b \sinh(e + fx))^2 dx$

**Optimal.** Leaf size=116

$$\frac{a^2(c + dx)^2}{2d} + \frac{2ab(c + dx) \cosh(e + fx)}{f} - \frac{2abd \sinh(e + fx)}{f^2} + \frac{b^2(c + dx) \sinh(e + fx) \cosh(e + fx)}{2f} - \frac{1}{2}b^2cx - \frac{b^2d \sinh(e + fx)^2}{4f^2}$$

[Out]  $-(b^2*c*x)/2 - (b^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) + (2*a*b*(c + d*x)*\text{Cosh}[e + f*x])/f - (2*a*b*d*\text{Sinh}[e + f*x])/f^2 + (b^2*(c + d*x)*\text{Cosh}[e + f*x]*\text{Sinh}[e + f*x])/(2*f) - (b^2*d*\text{Sinh}[e + f*x]^2)/(4*f^2)$

**Rubi [A]** time = 0.105548, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {3317, 3296, 2637, 3310}

$$\frac{a^2(c + dx)^2}{2d} + \frac{2ab(c + dx) \cosh(e + fx)}{f} - \frac{2abd \sinh(e + fx)}{f^2} + \frac{b^2(c + dx) \sinh(e + fx) \cosh(e + fx)}{2f} - \frac{1}{2}b^2cx - \frac{b^2d \sinh(e + fx)^2}{4f^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x)*(a + b*\text{Sinh}[e + f*x])^2, x]$

[Out]  $-(b^2*c*x)/2 - (b^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) + (2*a*b*(c + d*x)*\text{Cosh}[e + f*x])/f - (2*a*b*d*\text{Sinh}[e + f*x])/f^2 + (b^2*(c + d*x)*\text{Cosh}[e + f*x]*\text{Sinh}[e + f*x])/(2*f) - (b^2*d*\text{Sinh}[e + f*x]^2)/(4*f^2)$

#### Rule 3317

$\text{Int}[(c + d*x)^m * (a + b*\text{Sin}[e + f*x])^n, x]$  /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

#### Rule 3296

$\text{Int}[(c + d*x)^m * \text{Cos}[e + f*x], x]$  /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
Simp[(d*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sine[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sine[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

### Rubi steps

$$\begin{aligned} \int (c + dx)(a + b \sinh(e + fx))^2 dx &= \int (a^2(c + dx) + 2ab(c + dx) \sinh(e + fx) + b^2(c + dx) \sinh^2(e + fx)) dx \\ &= \frac{a^2(c + dx)^2}{2d} + (2ab) \int (c + dx) \sinh(e + fx) dx + b^2 \int (c + dx) \sinh^2(e + fx) dx \\ &= \frac{a^2(c + dx)^2}{2d} + \frac{2ab(c + dx) \cosh(e + fx)}{f} + \frac{b^2(c + dx) \cosh(e + fx) \sinh(e + fx)}{2f} - \frac{b^2(c + dx) \sinh^2(e + fx)}{2f} \\ &= -\frac{1}{2}b^2cx - \frac{1}{4}b^2dx^2 + \frac{a^2(c + dx)^2}{2d} + \frac{2ab(c + dx) \cosh(e + fx)}{f} - \frac{2abd \sinh(e + fx)}{f^2} + \frac{b^2(c + dx) \sinh^2(e + fx)}{2f} \end{aligned}$$

**Mathematica [A]** time = 0.745308, size = 98, normalized size = 0.84

$$\frac{2(a^2 - b^2)(e + fx)(d(e - fx) - 2cf) - 16abf(c + dx) \cosh(e + fx) + 16abd \sinh(e + fx) - 2b^2f(c + dx) \sinh(2(e + fx))}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] -(2*(2*a^2 - b^2)*(e + f*x)*(-2*c*f + d*(e - f*x)) - 16*a*b*f*(c + d*x)*Cos
h[e + f*x] + b^2*d*Cosh[2*(e + f*x)] + 16*a*b*d*Sinh[e + f*x] - 2*b^2*f*(c
+ d*x)*Sinh[2*(e + f*x)])/(8*f^2)
```

**Maple [A]** time = 0.014, size = 208, normalized size = 1.8

$$\frac{1}{f} \left( \frac{da^2 (fx + e)^2}{2f} + 2 \frac{dab((fx + e) \cosh(fx + e) - \sinh(fx + e))}{f} + \frac{db^2}{f} \left( \frac{(fx + e) \cosh(fx + e) \sinh(fx + e)}{2} - \frac{(fx + e) \sinh^2(fx + e)}{4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)*(a+b*sinh(f*x+e))^2,x)`

[Out]  $1/f*(1/2/f*d*a^2*(f*x+e)^2+2/f*d*a*b*((f*x+e)*\cosh(f*x+e)-\sinh(f*x+e))+1/f*d*b^2*(1/2*(f*x+e)*\cosh(f*x+e)*\sinh(f*x+e)-1/4*(f*x+e)^2-1/4*\cosh(f*x+e)^2)-d*e/f*a^2*(f*x+e)-2*d*e/f*a*b*\cosh(f*x+e)-d*e/f*b^2*(1/2*\cosh(f*x+e)*\sinh(f*x+e)-1/2*f*x-1/2*e)+c*a^2*(f*x+e)+2*c*a*b*\cosh(f*x+e)+c*b^2*(1/2*\cosh(f*x+e)*\sinh(f*x+e)-1/2*f*x-1/2*e))$

**Maxima [A]** time = 1.25498, size = 221, normalized size = 1.91

$$\frac{1}{2} a^2 dx^2 - \frac{1}{16} \left( 4x^2 - \frac{(2fxe^{2e} - e^{2e})e^{2fx}}{f^2} + \frac{(2fx+1)e^{(-2fx-2e)}}{f^2} \right) b^2 d - \frac{1}{8} b^2 c \left( 4x - \frac{e^{(2fx+2e)}}{f} + \frac{e^{(-2fx-2e)}}{f} \right) + a^2 cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*(a+b*sinh(f*x+e))^2,x, algorithm="maxima")`

[Out]  $1/2*a^2*d*x^2 - 1/16*(4*x^2 - (2*f*x*e^{(2*e)} - e^{(2*e)})*e^{(2*f*x)}/f^2 + (2*f*x + 1)*e^{(-2*f*x - 2*e)}/f^2)*b^2*d - 1/8*b^2*c*(4*x - e^{(2*f*x + 2*e)}/f + e^{(-2*f*x - 2*e)}/f) + a^2*c*x + a*b*d*((f*x*e^e - e^e)*e^{(f*x)}/f^2 + (f*x + 1)*e^{(-f*x - e)}/f^2) + 2*a*b*c*\cosh(f*x + e)/f$

**Fricas [A]** time = 2.29991, size = 294, normalized size = 2.53

$$\frac{2(2a^2 - b^2)df^2x^2 + 4(2a^2 - b^2)cf^2x - b^2d \cosh(fx + e)^2 - b^2d \sinh(fx + e)^2 + 16(abdfx + abcf) \cosh(fx + e) - 4(4ab*d - (b^2*d*f*x + b^2*c*f)*\cosh(f*x + e))*\sinh(f*x + e))/f^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)*(a+b*sinh(f*x+e))^2,x, algorithm="fricas")`

[Out]  $1/8*(2*(2*a^2 - b^2)*d*f^2*x^2 + 4*(2*a^2 - b^2)*c*f^2*x - b^2*d*\cosh(f*x + e)^2 - b^2*d*\sinh(f*x + e)^2 + 16*(a*b*d*f*x + a*b*c*f)*\cosh(f*x + e) - 4*(4*a*b*d - (b^2*d*f*x + b^2*c*f)*\cosh(f*x + e))*\sinh(f*x + e))/f^2$

**Sympy [A]** time = 0.935067, size = 219, normalized size = 1.89

$$\left\{ \begin{array}{l} a^2 cx + \frac{a^2 dx^2}{2} + \frac{2abc \cosh(e+fx)}{f} + \frac{2abdx \cosh(e+fx)}{f} - \frac{2abd \sinh(e+fx)}{f^2} + \frac{b^2 cx \sinh^2(e+fx)}{2} - \frac{b^2 cx \cosh^2(e+fx)}{2} + \frac{b^2 c \sinh(e+fx) \cosh(e+fx)}{2f} \\ (a + b \sinh(e))^2 \left( cx + \frac{dx^2}{2} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+b\*sinh(f\*x+e))\*\*2,x)

[Out] Piecewise((a\*\*2\*c\*x + a\*\*2\*d\*x\*\*2/2 + 2\*a\*b\*c\*cosh(e + f\*x)/f + 2\*a\*b\*d\*x\*cosh(e + f\*x)/f - 2\*a\*b\*d\*sinh(e + f\*x)/f\*\*2 + b\*\*2\*c\*x\*sinh(e + f\*x)\*\*2/2 - b\*\*2\*c\*x\*cosh(e + f\*x)\*\*2/2 + b\*\*2\*c\*sinh(e + f\*x)\*cosh(e + f\*x)/(2\*f) + b\*\*2\*d\*x\*\*2\*sinh(e + f\*x)\*\*2/4 - b\*\*2\*d\*x\*\*2\*cosh(e + f\*x)\*\*2/4 + b\*\*2\*d\*x\*sinh(e + f\*x)\*cosh(e + f\*x)/(2\*f) - b\*\*2\*d\*sinh(e + f\*x)\*\*2/(4\*f\*\*2), Ne(f, 0)), ((a + b\*sinh(e))\*\*2\*(c\*x + d\*x\*\*2/2), True))

**Giac [A]** time = 1.23683, size = 220, normalized size = 1.9

$$\frac{1}{2} a^2 dx^2 - \frac{1}{4} b^2 dx^2 + a^2 cx - \frac{1}{2} b^2 cx + \frac{(2b^2 d f x + 2b^2 c f - b^2 d) e^{(2fx+2e)}}{16 f^2} + \frac{(abdfx + abcf - abd) e^{(fx+e)}}{f^2} + \frac{(abdfx + abcf - abd) e^{(fx+e)}}{f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*(a+b\*sinh(f\*x+e))^2,x, algorithm="giac")

[Out] 1/2\*a^2\*d\*x^2 - 1/4\*b^2\*d\*x^2 + a^2\*c\*x - 1/2\*b^2\*c\*x + 1/16\*(2\*b^2\*d\*f\*x + 2\*b^2\*c\*f - b^2\*d)\*e^(2\*f\*x + 2\*e)/f^2 + (a\*b\*d\*f\*x + a\*b\*c\*f - a\*b\*d)\*e^(f\*x + e)/f^2 + (a\*b\*d\*f\*x + a\*b\*c\*f + a\*b\*d)\*e^(-f\*x - e)/f^2 - 1/16\*(2\*b^2\*d\*f\*x + 2\*b^2\*c\*f + b^2\*d)\*e^(-2\*f\*x - 2\*e)/f^2



$$3.166 \quad \int \frac{(a+b \sinh(e+fx))^2}{c+dx} dx$$

**Optimal.** Leaf size=156

$$\frac{a^2 \log(c+dx)}{d} + \frac{2ab \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d} + \frac{2ab \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d} + \frac{b^2 \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{2d}$$

[Out] (b^2\*Cosh[2\*e - (2\*c\*f)/d]\*CoshIntegral[(2\*c\*f)/d + 2\*f\*x])/(2\*d) + (a^2\*Log[c + d\*x])/d - (b^2\*Log[c + d\*x])/(2\*d) + (2\*a\*b\*CoshIntegral[(c\*f)/d + f\*x]\*Sinh[e - (c\*f)/d])/d + (2\*a\*b\*Cosh[e - (c\*f)/d]\*SinhIntegral[(c\*f)/d + f\*x])/d + (b^2\*Sinh[2\*e - (2\*c\*f)/d]\*SinhIntegral[(2\*c\*f)/d + 2\*f\*x])/(2\*d)

**Rubi [A]** time = 0.325149, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3317, 3303, 3298, 3301, 3312}

$$\frac{a^2 \log(c+dx)}{d} + \frac{2ab \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d} + \frac{2ab \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d} + \frac{b^2 \operatorname{Chi}\left(2xf + \frac{2cf}{d}\right) \cosh\left(2e - \frac{2cf}{d}\right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Sinh[e + f\*x])^2/(c + d\*x),x]

[Out] (b^2\*Cosh[2\*e - (2\*c\*f)/d]\*CoshIntegral[(2\*c\*f)/d + 2\*f\*x])/(2\*d) + (a^2\*Log[c + d\*x])/d - (b^2\*Log[c + d\*x])/(2\*d) + (2\*a\*b\*CoshIntegral[(c\*f)/d + f\*x]\*Sinh[e - (c\*f)/d])/d + (2\*a\*b\*Cosh[e - (c\*f)/d]\*SinhIntegral[(c\*f)/d + f\*x])/d + (b^2\*Sinh[2\*e - (2\*c\*f)/d]\*SinhIntegral[(2\*c\*f)/d + 2\*f\*x])/(2\*d)

### Rule 3317

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*Sinh[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] &&

NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rule 3312

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)^m, Sin[e + f\*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + b \sinh(e + fx))^2}{c + dx} dx &= \int \left( \frac{a^2}{c + dx} + \frac{2ab \sinh(e + fx)}{c + dx} + \frac{b^2 \sinh^2(e + fx)}{c + dx} \right) dx \\
 &= \frac{a^2 \log(c + dx)}{d} + (2ab) \int \frac{\sinh(e + fx)}{c + dx} dx + b^2 \int \frac{\sinh^2(e + fx)}{c + dx} dx \\
 &= \frac{a^2 \log(c + dx)}{d} - b^2 \int \left( \frac{1}{2(c + dx)} - \frac{\cosh(2e + 2fx)}{2(c + dx)} \right) dx + \left( 2ab \cosh \left( e - \frac{cf}{d} \right) \right) \int \frac{\sinh \left( \frac{c}{d} + fx \right)}{c + dx} dx \\
 &= \frac{a^2 \log(c + dx)}{d} - \frac{b^2 \log(c + dx)}{2d} + \frac{2ab \operatorname{Chi} \left( \frac{cf}{d} + fx \right) \sinh \left( e - \frac{cf}{d} \right)}{d} + \frac{2ab \cosh \left( e - \frac{cf}{d} \right) \operatorname{Shi} \left( \frac{c}{d} + fx \right)}{d} \\
 &= \frac{a^2 \log(c + dx)}{d} - \frac{b^2 \log(c + dx)}{2d} + \frac{2ab \operatorname{Chi} \left( \frac{cf}{d} + fx \right) \sinh \left( e - \frac{cf}{d} \right)}{d} + \frac{2ab \cosh \left( e - \frac{cf}{d} \right) \operatorname{Shi} \left( \frac{c}{d} + fx \right)}{d} \\
 &= \frac{b^2 \cosh \left( 2e - \frac{2cf}{d} \right) \operatorname{Chi} \left( \frac{2cf}{d} + 2fx \right)}{2d} + \frac{a^2 \log(c + dx)}{d} - \frac{b^2 \log(c + dx)}{2d} + \frac{2ab \operatorname{Chi} \left( \frac{cf}{d} + fx \right) \sinh \left( e - \frac{cf}{d} \right)}{d}
 \end{aligned}$$

**Mathematica [A]** time = 0.297739, size = 134, normalized size = 0.86

$$\frac{2a^2 \log(c + dx) + 4ab \operatorname{Chi} \left( f \left( \frac{c}{d} + x \right) \right) \sinh \left( e - \frac{cf}{d} \right) + 4ab \cosh \left( e - \frac{cf}{d} \right) \operatorname{Shi} \left( f \left( \frac{c}{d} + x \right) \right) + b^2 \operatorname{Chi} \left( \frac{2f(c+dx)}{d} \right) \cosh \left( 2e - \frac{2cf}{d} \right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Sinh[e + f\*x])^2/(c + d\*x),x]

[Out] (b^2\*Cosh[2\*e - (2\*c\*f)/d]\*CoshIntegral[(2\*f\*(c + d\*x))/d] + 2\*a^2\*Log[c + d\*x] - b^2\*Log[c + d\*x] + 4\*a\*b\*CoshIntegral[f\*(c/d + x)]\*Sinh[e - (c\*f)/d] + 4\*a\*b\*Cosh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)] + b^2\*Sinh[2\*e - (2\*c\*f)/d]\*SinhIntegral[(2\*f\*(c + d\*x))/d])/(2\*d)

**Maple [A]** time = 0.108, size = 201, normalized size = 1.3

$$-\frac{ab}{d}e^{-\frac{cf-de}{d}}\text{Ei}\left(1,-fx-e-\frac{cf-de}{d}\right)+\frac{a^2\ln(dx+c)}{d}-\frac{b^2\ln(dx+c)}{2d}-\frac{b^2}{4d}e^{2\frac{cf-de}{d}}\text{Ei}\left(1,2fx+2e+2\frac{cf-de}{d}\right)-\frac{b^2}{4d}e^{-\frac{cf-de}{d}}\text{Ei}\left(1,2fx+2e-2\frac{cf-de}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sinh(f\*x+e))^2/(d\*x+c),x)

[Out] -a\*b/d\*exp(-(c\*f-d\*e)/d)\*Ei(1,-f\*x-e-(c\*f-d\*e)/d)+a^2\*ln(d\*x+c)/d-1/2\*b^2\*ln(d\*x+c)/d-1/4\*b^2/d\*exp(2\*(c\*f-d\*e)/d)\*Ei(1,2\*f\*x+2\*e+2\*(c\*f-d\*e)/d)-1/4\*b^2/d\*exp(-2\*(c\*f-d\*e)/d)\*Ei(1,-2\*f\*x-2\*e-2\*(c\*f-d\*e)/d)+a\*b/d\*exp((c\*f-d\*e)/d)\*Ei(1,f\*x+e+(c\*f-d\*e)/d)

**Maxima [A]** time = 1.59282, size = 200, normalized size = 1.28

$$-\frac{1}{4}b^2\left(\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_1\left(\frac{2(dx+c)f}{d}\right)}{d}+\frac{e^{\left(2e-\frac{2cf}{d}\right)}E_1\left(-\frac{2(dx+c)f}{d}\right)}{d}+\frac{2\log(dx+c)}{d}\right)+ab\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sinh(f\*x+e))^2/(d\*x+c),x, algorithm="maxima")

[Out] -1/4\*b^2\*(e^(-2\*e + 2\*c\*f/d)\*exp\_integral\_e(1, 2\*(d\*x + c)\*f/d)/d + e^(2\*e - 2\*c\*f/d)\*exp\_integral\_e(1, -2\*(d\*x + c)\*f/d)/d + 2\*log(d\*x + c)/d) + a\*b\*(e^(-e + c\*f/d)\*exp\_integral\_e(1, (d\*x + c)\*f/d)/d - e^(e - c\*f/d)\*exp\_integral\_e(1, -(d\*x + c)\*f/d)/d) + a^2\*log(d\*x + c)/d

---

**Fricas [A]** time = 2.50693, size = 483, normalized size = 3.1

$$4 \left( ab \operatorname{Ei} \left( \frac{dfx+cf}{d} \right) - ab \operatorname{Ei} \left( -\frac{dfx+cf}{d} \right) \right) \cosh \left( -\frac{de-cf}{d} \right) + \left( b^2 \operatorname{Ei} \left( \frac{2(dfx+cf)}{d} \right) + b^2 \operatorname{Ei} \left( -\frac{2(dfx+cf)}{d} \right) \right) \cosh \left( -\frac{2(de-cf)}{d} \right) + 2 \left( 2a^2 - \right.$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sinh(f\*x+e))^2/(d\*x+c),x, algorithm="fricas")

[Out]  $\frac{1}{4} * (4 * (a * b * \operatorname{Ei}((d * f * x + c * f) / d) - a * b * \operatorname{Ei}(- (d * f * x + c * f) / d)) * \cosh(- (d * e - c * f) / d) + (b^2 * \operatorname{Ei}(2 * (d * f * x + c * f) / d) + b^2 * \operatorname{Ei}(- 2 * (d * f * x + c * f) / d)) * \cosh(- 2 * (d * e - c * f) / d) + 2 * (2 * a^2 - b^2) * \log(d * x + c) - 4 * (a * b * \operatorname{Ei}((d * f * x + c * f) / d) + a * b * \operatorname{Ei}(- (d * f * x + c * f) / d)) * \sinh(- (d * e - c * f) / d) - (b^2 * \operatorname{Ei}(2 * (d * f * x + c * f) / d) - b^2 * \operatorname{Ei}(- 2 * (d * f * x + c * f) / d)) * \sinh(- 2 * (d * e - c * f) / d)) / d$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \sinh(e + fx))^2}{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sinh(f\*x+e))^2/(d\*x+c),x)

[Out] Integral((a + b\*sinh(e + f\*x))^2/(c + d\*x), x)

---

**Giac [A]** time = 1.29421, size = 200, normalized size = 1.28

$$\frac{b^2 \operatorname{Ei} \left( -\frac{2(dfx+cf)}{d} \right) e^{\left( \frac{2cf}{d} - 2e \right)} - 4ab \operatorname{Ei} \left( -\frac{dfx+cf}{d} \right) e^{\left( \frac{cf}{d} - e \right)} + 4ab \operatorname{Ei} \left( \frac{dfx+cf}{d} \right) e^{\left( -\frac{cf}{d} + e \right)} + b^2 \operatorname{Ei} \left( \frac{2(dfx+cf)}{d} \right) e^{\left( -\frac{2cf}{d} + 2e \right)} + 4a^2 \log \left( \right)}{4d}$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sinh(f\*x+e))^2/(d\*x+c),x, algorithm="giac")

```
[Out] 1/4*(b^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d - 2*e) - 4*a*b*Ei(-(d*f*x + c*f)/d)*e^(c*f/d - e) + 4*a*b*Ei((d*f*x + c*f)/d)*e^(-c*f/d + e) + b^2*Ei(2*(d*f*x + c*f)/d)*e^(-2*c*f/d + 2*e) + 4*a^2*log(d*x + c) - 2*b^2*log(d*x + c))  
/d
```

$$3.167 \quad \int \frac{(a+b \sinh(e+fx))^2}{(c+dx)^2} dx$$

**Optimal.** Leaf size=183

$$-\frac{a^2}{d(c+dx)} + \frac{2abf \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d^2} + \frac{2abf \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^2} - \frac{2ab \sinh(e+fx)}{d(c+dx)} + \frac{b^2 f \operatorname{Chi}\left(2xf - \frac{2cf}{d}\right)}{d^2}$$

[Out]  $-(a^2/(d*(c + d*x))) + (2*a*b*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d^2 + (b^2*f*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d^2 - (2*a*b*Sinh[e + f*x]/(d*(c + d*x)) - (b^2*Sinh[e + f*x]^2)/(d*(c + d*x))) + (2*a*b*f*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^2 + (b^2*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d^2$

**Rubi [A]** time = 0.350092, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$ , Rules used = {3317, 3297, 3303, 3298, 3301, 3313, 12}

$$-\frac{a^2}{d(c+dx)} + \frac{2abf \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \cosh\left(e - \frac{cf}{d}\right)}{d^2} + \frac{2abf \sinh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^2} - \frac{2ab \sinh(e+fx)}{d(c+dx)} + \frac{b^2 f \operatorname{Chi}\left(2xf - \frac{2cf}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*\operatorname{Sinh}[e + f*x])^2/(c + d*x)^2, x]$

[Out]  $-(a^2/(d*(c + d*x))) + (2*a*b*f*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d^2 + (b^2*f*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d^2 - (2*a*b*Sinh[e + f*x]/(d*(c + d*x)) - (b^2*Sinh[e + f*x]^2)/(d*(c + d*x))) + (2*a*b*f*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^2 + (b^2*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d^2$

### Rule 3317

$\operatorname{Int}[(c + d*x)^m * (a + b*\sin[e + f*x])^n, x]$   
 $\operatorname{Int}[\operatorname{ExpandIntegrand}[(c + d*x)^m, (a + b*\sin[e + f*x])^n, x], x]$  /;  $\operatorname{FreeQ}\{a, b, c, d, e, f, m\}, x$  &&  $\operatorname{IGtQ}[n, 0]$  &&  $(\operatorname{EqQ}[n, 1] \mid \mid \operatorname{IGtQ}[m, 0] \mid \mid \operatorname{NeQ}[a^2 - b^2, 0])$

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m + 1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n - 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] && LtQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sinh(e + fx))^2}{(c + dx)^2} dx &= \int \left( \frac{a^2}{(c + dx)^2} + \frac{2ab \sinh(e + fx)}{(c + dx)^2} + \frac{b^2 \sinh^2(e + fx)}{(c + dx)^2} \right) dx \\
&= -\frac{a^2}{d(c + dx)} + (2ab) \int \frac{\sinh(e + fx)}{(c + dx)^2} dx + b^2 \int \frac{\sinh^2(e + fx)}{(c + dx)^2} dx \\
&= -\frac{a^2}{d(c + dx)} - \frac{2ab \sinh(e + fx)}{d(c + dx)} - \frac{b^2 \sinh^2(e + fx)}{d(c + dx)} + \frac{(2abf) \int \frac{\cosh(e+fx)}{c+dx} dx}{d} - \frac{(2ib^2f) \int \frac{\sinh(e+fx)}{c+dx} dx}{d} \\
&= -\frac{a^2}{d(c + dx)} - \frac{2ab \sinh(e + fx)}{d(c + dx)} - \frac{b^2 \sinh^2(e + fx)}{d(c + dx)} + \frac{(b^2f) \int \frac{\sinh(2e+2fx)}{c+dx} dx}{d} + \frac{(2abf \cosh(e+fx))}{d} \\
&= -\frac{a^2}{d(c + dx)} + \frac{2abf \cosh\left(e - \frac{cf}{d}\right) \text{Chi}\left(\frac{cf}{d} + fx\right)}{d^2} - \frac{2ab \sinh(e + fx)}{d(c + dx)} - \frac{b^2 \sinh^2(e + fx)}{d(c + dx)} + \frac{b^2f \text{Chi}\left(\frac{2cf}{d} + 2fx\right) \sinh\left(2e - \frac{2cf}{d}\right)}{d^2} \\
&= -\frac{a^2}{d(c + dx)} + \frac{2abf \cosh\left(e - \frac{cf}{d}\right) \text{Chi}\left(\frac{cf}{d} + fx\right)}{d^2} + \frac{b^2f \text{Chi}\left(\frac{2cf}{d} + 2fx\right) \sinh\left(2e - \frac{2cf}{d}\right)}{d^2} - \frac{2ab \sinh(e + fx)}{d(c + dx)} - \frac{b^2 \sinh^2(e + fx)}{d(c + dx)}
\end{aligned}$$

**Mathematica [A]** time = 0.641991, size = 232, normalized size = 1.27

$$-\frac{2a^2d + 4abf(c + dx)\text{Chi}\left(f\left(\frac{c}{d} + x\right)\right) \cosh\left(e - \frac{cf}{d}\right) + 4abcf \sinh\left(e - \frac{cf}{d}\right) \text{Shi}\left(f\left(\frac{c}{d} + x\right)\right) + 4abdfx \sinh\left(e - \frac{cf}{d}\right) \text{Shi}\left(f\left(\frac{c}{d} + x\right)\right)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Sinh[e + f\*x])^2/(c + d\*x)^2,x]

[Out] 
$$\begin{aligned}
&(-2a^2d + b^2d - b^2d \text{Cosh}[2*(e + f*x)] + 4a*b*f*(c + d*x)*\text{Cosh}[e - (c*f)/d] * \text{CoshIntegral}[f*(c/d + x)] + 2*b^2*f*(c + d*x)*\text{CoshIntegral}[(2*f*(c + d*x))/d] * \text{Sinh}[2*e - (2*c*f)/d] - 4a*b*d*\text{Sinh}[e + f*x] + 4a*b*c*f*\text{Sinh}[e - (c*f)/d] * \text{SinhIntegral}[f*(c/d + x)] + 4a*b*d*f*x*\text{Sinh}[e - (c*f)/d] * \text{SinhIntegral}[f*(c/d + x)] + 2*b^2*c*f*\text{Cosh}[2*e - (2*c*f)/d] * \text{SinhIntegral}[(2*f*(c + d*x))/d] + 2*b^2*d*f*x*\text{Cosh}[2*e - (2*c*f)/d] * \text{SinhIntegral}[(2*f*(c + d*x))/d]) / (2*d^2*(c + d*x))
\end{aligned}$$

**Maple [A]** time = 0.128, size = 319, normalized size = 1.7

$$-\frac{abfe^{fx+e}}{d^2} \left(\frac{cf}{d} + fx\right)^{-1} - \frac{abf}{d^2} e^{-\frac{cf-de}{d}} \text{Ei}\left(1, -fx - e - \frac{cf-de}{d}\right) - \frac{a^2}{d(dx+c)} + \frac{b^2}{2d(dx+c)} - \frac{fb^2e^{-2fx-2e}}{4d(dfxc+cf)} + \frac{fb^2}{2d^2} e^{\frac{cf-d}{d}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sinh(f*x+e))^2/(d*x+c)^2,x)`

[Out]  $-a*b*f/d^2*\exp(f*x+e)/(c*f/d+f*x)-a*b*f/d^2*\exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)-a^2/d/(d*x+c)+1/2*b^2/d/(d*x+c)-1/4*b^2*f*\exp(-2*f*x-2*e)/d/(d*f*x+c*f)+1/2*b^2*f/d^2*\exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1/4*f*b^2/d^2*\exp(2*f*x+2*e)/(c*f/d+f*x)-1/2*f*b^2/d^2*\exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)+a*b*f*\exp(-f*x-e)/d/(d*f*x+c*f)-a*b*f/d^2*\exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)$

**Maxima [A]** time = 1.50236, size = 244, normalized size = 1.33

$$-\frac{1}{4}b^2\left(\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_2\left(\frac{2(dx+c)f}{d}\right)}{(dx+c)d} + \frac{e^{\left(2e-\frac{2cf}{d}\right)}E_2\left(-\frac{2(dx+c)f}{d}\right)}{(dx+c)d} - \frac{2}{d^2x+cd}\right) + ab\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d} - \frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")`

[Out]  $-1/4*b^2*(e^{(-2*e + 2*c*f/d)}*\exp\_integral\_e(2, 2*(d*x + c)*f/d)/((d*x + c)*d) + e^{(2*e - 2*c*f/d)}*\exp\_integral\_e(2, -2*(d*x + c)*f/d)/((d*x + c)*d) - 2/(d^2*x + c*d) + a*b*(e^{(-e + c*f/d)}*\exp\_integral\_e(2, (d*x + c)*f/d)/((d*x + c)*d) - e^{(e - c*f/d)}*\exp\_integral\_e(2, -(d*x + c)*f/d)/((d*x + c)*d)) - a^2/(d^2*x + c*d)$

**Fricas [A]** time = 2.56421, size = 778, normalized size = 4.25

$$b^2d \cosh(fx + e)^2 + b^2d \sinh(fx + e)^2 + 4abd \sinh(fx + e) + (2a^2 - b^2)d - 2\left((abdfx + abcf)Ei\left(\frac{dfx+cf}{d}\right) + (abdf$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")`

[Out]  $-1/2*(b^2*d*\cosh(f*x + e)^2 + b^2*d*\sinh(f*x + e)^2 + 4*a*b*d*\sinh(f*x + e) + (2*a^2 - b^2)*d - 2*((a*b*d*f*x + a*b*c*f)*Ei((d*f*x + c*f)/d) + (a*b*d*$

$$f*x + a*b*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) - ((b^2*d*f*x + b^2*c*f)*Ei(2*(d*f*x + c*f)/d) - (b^2*d*f*x + b^2*c*f)*Ei(-2*(d*f*x + c*f)/d))*cosh(-2*(d*e - c*f)/d) + 2*((a*b*d*f*x + a*b*c*f)*Ei((d*f*x + c*f)/d) - (a*b*d*f*x + a*b*c*f)*Ei(-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) + ((b^2*d*f*x + b^2*c*f)*Ei(2*(d*f*x + c*f)/d) + (b^2*d*f*x + b^2*c*f)*Ei(-2*(d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/(d^3*x + c*d^2)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \sinh(e + fx))^2}{(c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sinh(f\*x+e))\*\*2/(d\*x+c)\*\*2,x)

[Out] Integral((a + b\*sinh(e + f\*x))\*\*2/(c + d\*x)\*\*2, x)

**Giac [A]** time = 2.0866, size = 487, normalized size = 2.66

$$2b^2dfxEi\left(-\frac{2(dfxc+f)}{d}\right)e^{\left(\frac{2cf}{d}-2e\right)} - 4abdfxEi\left(-\frac{dfxc+f}{d}\right)e^{\left(\frac{cf}{d}-e\right)} - 4abdfxEi\left(\frac{dfxc+f}{d}\right)e^{\left(-\frac{cf}{d}+e\right)} - 2b^2dfxEi\left(\frac{2(dfxc+f)}{d}\right)e^{\left(\frac{2(dfxc+f)}{d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sinh(f\*x+e))^2/(d\*x+c)^2,x, algorithm="giac")

[Out]  $-1/4*(2*b^2*d*f*x*Ei(-2*(d*f*x + c*f)/d))*e^{(2*c*f/d - 2*e)} - 4*a*b*d*f*x*Ei(-(d*f*x + c*f)/d)*e^{(c*f/d - e)} - 4*a*b*d*f*x*Ei((d*f*x + c*f)/d)*e^{(-c*f/d + e)} - 2*b^2*d*f*x*Ei(2*(d*f*x + c*f)/d)*e^{(-2*c*f/d + 2*e)} + 2*b^2*c*f*Ei(-2*(d*f*x + c*f)/d)*e^{(2*c*f/d - 2*e)} - 4*a*b*c*f*Ei(-(d*f*x + c*f)/d)*e^{(c*f/d - e)} - 4*a*b*c*f*Ei((d*f*x + c*f)/d)*e^{(-c*f/d + e)} - 2*b^2*c*f*Ei(2*(d*f*x + c*f)/d)*e^{(-2*c*f/d + 2*e)} + b^2*d*e^{(2*f*x + 2*e)} + 4*a*b*d*e^{(f*x + e)} - 4*a*b*d*e^{(-f*x - e)} + b^2*d*e^{(-2*f*x - 2*e))/(d^3*x + c*d^2) - 1/2*(2*a^2 - b^2)/((d*x + c)*d)$

$$3.168 \quad \int \frac{(a+b \sinh(e+fx))^2}{(c+dx)^3} dx$$

**Optimal.** Leaf size=242

$$-\frac{a^2}{2d(c+dx)^2} + \frac{abf^2 \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d^3} + \frac{abf^2 \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^3} - \frac{abf \cosh(e+fx)}{d^2(c+dx)} - \frac{ab \sinh(e+fx)}{d(c+dx)}$$

[Out]  $-a^2/(2*d*(c + d*x)^2) - (a*b*f*Cosh[e + f*x])/(d^2*(c + d*x)) + (b^2*f^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d^3 + (a*b*f^2*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^3 - (a*b*Sinh[e + f*x])/(d*(c + d*x)^2) - (b^2*f*Cosh[e + f*x]*Sinh[e + f*x])/(d^2*(c + d*x)) - (b^2*Sinh[e + f*x]^2)/(2*d*(c + d*x)^2) + (a*b*f^2*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^3 + (b^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d^3$

**Rubi [A]** time = 0.446764, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {3317, 3297, 3303, 3298, 3301, 3314, 31, 3312}

$$-\frac{a^2}{2d(c+dx)^2} + \frac{abf^2 \operatorname{Chi}\left(xf + \frac{cf}{d}\right) \sinh\left(e - \frac{cf}{d}\right)}{d^3} + \frac{abf^2 \cosh\left(e - \frac{cf}{d}\right) \operatorname{Shi}\left(xf + \frac{cf}{d}\right)}{d^3} - \frac{abf \cosh(e+fx)}{d^2(c+dx)} - \frac{ab \sinh(e+fx)}{d(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Sinh[e + f\*x])^2/(c + d\*x)^3, x]

[Out]  $-a^2/(2*d*(c + d*x)^2) - (a*b*f*Cosh[e + f*x])/(d^2*(c + d*x)) + (b^2*f^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d^3 + (a*b*f^2*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^3 - (a*b*Sinh[e + f*x])/(d*(c + d*x)^2) - (b^2*f*Cosh[e + f*x]*Sinh[e + f*x])/(d^2*(c + d*x)) - (b^2*Sinh[e + f*x]^2)/(2*d*(c + d*x)^2) + (a*b*f^2*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^3 + (b^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d^3$

**Rule 3317**

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*Sinh[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[

$m, 0] \parallel \text{NeQ}[a^2 - b^2, 0]$ )

### Rule 3297

$\text{Int}[(c + d*x)^m * \sin[e + f*x], x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{m+1} * \text{Sin}[e + f*x] / (d*(m+1)), x] - \text{Dist}[f / (d*(m+1)), \text{Int}[(c + d*x)^{m+1} * \text{Cos}[e + f*x], x], x] /;$   $\text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{LtQ}[m, -1]$

### Rule 3303

$\text{Int}[\sin[e + f*x] / (c + d*x), x\_Symbol] \rightarrow \text{Dist}[\text{Cos}[(d*e - c*f)/d], \text{Int}[\text{Sin}[c*f/d + f*x] / (c + d*x), x], x] + \text{Dist}[\text{Sin}[(d*e - c*f)/d], \text{Int}[\text{Cos}[c*f/d + f*x] / (c + d*x), x], x] /;$   $\text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{NeQ}[d*e - c*f, 0]$

### Rule 3298

$\text{Int}[\sin[e + \text{Complex}[0, fz]] * f * x / (c + d*x), x\_Symbol] \rightarrow \text{Simp}[(I * \text{SinhIntegral}[c*f*fz/d + f*fz*x] / d, x] /;$   $\text{FreeQ}\{c, d, e, f, fz\}, x\} \&\& \text{EqQ}[d*e - c*f*fz*I, 0]$

### Rule 3301

$\text{Int}[\sin[e + \text{Complex}[0, fz]] * f * x / (c + d*x), x\_Symbol] \rightarrow \text{Simp}[\text{CoshIntegral}[c*f*fz/d + f*fz*x] / d, x] /;$   $\text{FreeQ}\{c, d, e, f, fz\}, x\} \&\& \text{EqQ}[d*(e - \text{Pi}/2) - c*f*fz*I, 0]$

### Rule 3314

$\text{Int}[(c + d*x)^m * (b * \sin[e + f*x])^n, x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^{m+1} * (b * \text{Sin}[e + f*x])^n / (d*(m+1)), x] + (\text{Dist}[(b^2 * f^2 * n * (n-1)) / (d^2 * (m+1) * (m+2)), \text{Int}[(c + d*x)^{m+2} * (b * \text{Sin}[e + f*x])^{n-2}, x], x] - \text{Dist}[(f^2 * n^2) / (d^2 * (m+1) * (m+2)), \text{Int}[(c + d*x)^{m+2} * (b * \text{Sin}[e + f*x])^n, x], x] - \text{Simp}[(b * f * n * (c + d*x)^{m+2} * \text{Cos}[e + f*x] * (b * \text{Sin}[e + f*x])^{n-1}) / (d^2 * (m+1) * (m+2)), x]) /;$   $\text{FreeQ}\{b, c, d, e, f\}, x\} \&\& \text{GtQ}[n, 1] \&\& \text{LtQ}[m, -2]$

### Rule 31

$\text{Int}[(a + b*x)^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]] / b, x] /;$   $\text{FreeQ}\{a, b\}, x\}$

### Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + b \sinh(e + fx))^2}{(c + dx)^3} dx &= \int \left( \frac{a^2}{(c + dx)^3} + \frac{2ab \sinh(e + fx)}{(c + dx)^3} + \frac{b^2 \sinh^2(e + fx)}{(c + dx)^3} \right) dx \\
&= -\frac{a^2}{2d(c + dx)^2} + (2ab) \int \frac{\sinh(e + fx)}{(c + dx)^3} dx + b^2 \int \frac{\sinh^2(e + fx)}{(c + dx)^3} dx \\
&= -\frac{a^2}{2d(c + dx)^2} - \frac{ab \sinh(e + fx)}{d(c + dx)^2} - \frac{b^2 f \cosh(e + fx) \sinh(e + fx)}{d^2(c + dx)} - \frac{b^2 \sinh^2(e + fx)}{2d(c + dx)^2} + \dots \\
&= -\frac{a^2}{2d(c + dx)^2} - \frac{abf \cosh(e + fx)}{d^2(c + dx)} + \frac{b^2 f^2 \log(c + dx)}{d^3} - \frac{ab \sinh(e + fx)}{d(c + dx)^2} - \frac{b^2 f \cosh(e + fx)}{d^2(c + dx)} \\
&= -\frac{a^2}{2d(c + dx)^2} - \frac{abf \cosh(e + fx)}{d^2(c + dx)} - \frac{ab \sinh(e + fx)}{d(c + dx)^2} - \frac{b^2 f \cosh(e + fx) \sinh(e + fx)}{d^2(c + dx)} - \dots \\
&= -\frac{a^2}{2d(c + dx)^2} - \frac{abf \cosh(e + fx)}{d^2(c + dx)} + \frac{abf^2 \text{Chi}\left(\frac{cf}{d} + fx\right) \sinh\left(e - \frac{cf}{d}\right)}{d^3} - \frac{ab \sinh(e + fx)}{d(c + dx)^2} \\
&= -\frac{a^2}{2d(c + dx)^2} - \frac{abf \cosh(e + fx)}{d^2(c + dx)} + \frac{b^2 f^2 \cosh\left(2e - \frac{2cf}{d}\right) \text{Chi}\left(\frac{2cf}{d} + 2fx\right)}{d^3} + \frac{abf^2 \text{Chi}\left(\frac{cf}{d}\right)}{d^3}
\end{aligned}$$

**Mathematica [A]** time = 0.960048, size = 395, normalized size = 1.63

$$-2a^2d^2 + 4abc^2f^2 \cosh\left(e - \frac{cf}{d}\right) \text{Shi}\left(f\left(\frac{c}{d} + x\right)\right) + 4abf^2(c + dx)^2 \text{Chi}\left(f\left(\frac{c}{d} + x\right)\right) \sinh\left(e - \frac{cf}{d}\right) + 4abd^2f^2x^2 \cosh\left(e - \frac{cf}{d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] (-2*a^2*d^2 + b^2*d^2 - 4*a*b*c*d*f*Cosh[e + f*x] - 4*a*b*d^2*f*x*Cosh[e +
f*x] - b^2*d^2*Cosh[2*(e + f*x)] + 4*b^2*f^2*(c + d*x)^2*Cosh[2*e - (2*c*f)
/d]*CoshIntegral[(2*f*(c + d*x))/d] + 4*a*b*f^2*(c + d*x)^2*CoshIntegral[f*(
c/d + x)]*Sinh[e - (c*f)/d] - 4*a*b*d^2*Sinh[e + f*x] - 2*b^2*c*d*f*Sinh[2
*(e + f*x)] - 2*b^2*d^2*f*x*Sinh[2*(e + f*x)] + 4*a*b*c^2*f^2*Cosh[e - (c*f
```

)/d]\*SinhIntegral[f\*(c/d + x)] + 8\*a\*b\*c\*d\*f^2\*x\*Cosh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)] + 4\*a\*b\*d^2\*f^2\*x^2\*Cosh[e - (c\*f)/d]\*SinhIntegral[f\*(c/d + x)] + 4\*b^2\*c^2\*f^2\*Sinh[2\*e - (2\*c\*f)/d]\*SinhIntegral[(2\*f\*(c + d\*x))/d] + 8\*b^2\*c\*d\*f^2\*x\*Sinh[2\*e - (2\*c\*f)/d]\*SinhIntegral[(2\*f\*(c + d\*x))/d] + 4\*b^2\*d^2\*f^2\*x^2\*Sinh[2\*e - (2\*c\*f)/d]\*SinhIntegral[(2\*f\*(c + d\*x))/d]/(4\*d^3\*(c + d\*x)^2)

**Maple [B]** time = 0.135, size = 626, normalized size = 2.6

$$-\frac{abf^2e^{fx+e}}{2d^3}\left(\frac{cf}{d}+fx\right)^{-2}-\frac{abf^2e^{fx+e}}{2d^3}\left(\frac{cf}{d}+fx\right)^{-1}-\frac{abf^2}{2d^3}e^{-\frac{cf-de}{d}}\text{Ei}\left(1,-fx-e-\frac{cf-de}{d}\right)-\frac{a^2}{2d(dx+c)^2}+\frac{b^2}{4d(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*sinh(f\*x+e))^2/(d\*x+c)^3,x)

[Out] 
$$-1/2*a*b*f^2/d^3*\exp(f*x+e)/(c*f/d+f*x)^2-1/2*a*b*f^2/d^3*\exp(f*x+e)/(c*f/d+f*x)-1/2*a*b*f^2/d^3*\exp(-(c*f-d*e)/d)*\text{Ei}(1,-f*x-e-(c*f-d*e)/d)-1/2*a^2/d/(d*x+c)^2+1/4*b^2/d/(d*x+c)^2+1/4*b^2*f^3*\exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x+1/4*b^2*f^3*\exp(-2*f*x-2*e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c-1/8*b^2*f^2*\exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-1/2*b^2*f^2/d^3*\exp(2*(c*f-d*e)/d)*\text{Ei}(1,2*f*x+2*e+2*(c*f-d*e)/d)-1/8*f^2*b^2/d^3*\exp(2*f*x+2*e)/(c*f/d+f*x)^2-1/4*f^2*b^2/d^3*\exp(2*f*x+2*e)/(c*f/d+f*x)-1/2*f^2*b^2/d^3*\exp(-2*(c*f-d*e)/d)*\text{Ei}(1,-2*f*x-2*e-2*(c*f-d*e)/d)-1/2*a*b*f^3*\exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x-1/2*a*b*f^3*\exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c+1/2*a*b*f^2*\exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)+1/2*a*b*f^2/d^3*\exp((c*f-d*e)/d)*\text{Ei}(1,f*x+e+(c*f-d*e)/d)$$

**Maxima [A]** time = 1.58655, size = 274, normalized size = 1.13

$$\frac{1}{4}b^2\left(\frac{1}{d^3x^2+2cd^2x+c^2d}-\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_3\left(\frac{2(dx+c)f}{d}\right)}{(dx+c)^2d}-\frac{e^{\left(2e-\frac{2cf}{d}\right)}E_3\left(-\frac{2(dx+c)f}{d}\right)}{(dx+c)^2d}\right)+ab\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}-\frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sinh(f\*x+e))^2/(d\*x+c)^3,x, algorithm="maxima")

```
[Out] 1/4*b^2*(1/(d^3*x^2 + 2*c*d^2*x + c^2*d) - e^(-2*e + 2*c*f/d)*exp_integral_
e(3, 2*(d*x + c)*f/d)/((d*x + c)^2*d) - e^(2*e - 2*c*f/d)*exp_integral_e(3,
-2*(d*x + c)*f/d)/((d*x + c)^2*d)) + a*b*(e^(-e + c*f/d)*exp_integral_e(3,
(d*x + c)*f/d)/((d*x + c)^2*d) - e^(e - c*f/d)*exp_integral_e(3, -(d*x + c
)*f/d)/((d*x + c)^2*d)) - 1/2*a^2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

---

**Fricas [B]** time = 2.47013, size = 1237, normalized size = 5.11

$$b^2 d^2 \cosh^2(fx + e) + b^2 d^2 \sinh^2(fx + e) + (2a^2 - b^2)d^2 + 4(abd^2 fx + abcd f) \cosh(fx + e) - 2\left((abd^2 f^2 x^2 + 2abcd f x + abc^2 d^2) \cosh(fx + e) - (abd^2 f^2 x^2 + 2abcd f x + abc^2 d^2) \sinh(fx + e)\right)$$


---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*d^2*cosh(f*x + e)^2 + b^2*d^2*sinh(f*x + e)^2 + (2*a^2 - b^2)*d^2
+ 4*(a*b*d^2*f*x + a*b*c*d*f)*cosh(f*x + e) - 2*((a*b*d^2*f^2*x^2 + 2*a*b*
c*d*f^2*x + a*b*c^2*f^2)*Ei((d*f*x + c*f)/d) - (a*b*d^2*f^2*x^2 + 2*a*b*c*d
*f^2*x + a*b*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) - 2*((b^2*
d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d) + (b^2*d
^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*cosh(-2
*(d*e - c*f)/d) + 4*(a*b*d^2 + (b^2*d^2*f*x + b^2*c*d*f)*cosh(f*x + e))*sin
h(f*x + e) + 2*((a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*Ei((d*f*x
+ c*f)/d) + (a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*Ei(-(d*f*x +
c*f)/d))*sinh(-(d*e - c*f)/d) + 2*((b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^
2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d) - (b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2
*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/(d^5*x^2 + 2*c*d^
4*x + c^2*d^3)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \sinh(e + fx))^2}{(c + dx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e))^2/(d*x+c)^3,x)
```

[Out] Integral((a + b\*sinh(e + f\*x))\*\*2/(c + d\*x)\*\*3, x)

**Giac [B]** time = 1.30969, size = 948, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*sinh(f\*x+e))^2/(d\*x+c)^3,x, algorithm="giac")

[Out] 
$$\begin{aligned} & 1/8*(4*b^2*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^{(2*c*f/d - 2*e)} - 4*a*b*d^2 \\ & *f^2*x^2*Ei(-(d*f*x + c*f)/d)*e^{(c*f/d - e)} + 4*a*b*d^2*f^2*x^2*Ei((d*f*x + \\ & c*f)/d)*e^{(-c*f/d + e)} + 4*b^2*d^2*f^2*x^2*Ei(2*(d*f*x + c*f)/d)*e^{(-2*c*f \\ & /d + 2*e)} + 8*b^2*c*d*f^2*x*Ei(-2*(d*f*x + c*f)/d)*e^{(2*c*f/d - 2*e)} - 8*a* \\ & b*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^{(c*f/d - e)} + 8*a*b*c*d*f^2*x*Ei((d*f*x \\ & + c*f)/d)*e^{(-c*f/d + e)} + 8*b^2*c*d*f^2*x*Ei(2*(d*f*x + c*f)/d)*e^{(-2*c*f/d \\ & + 2*e)} + 4*b^2*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^{(2*c*f/d - 2*e)} - 4*a*b*c \\ & ^2*f^2*Ei(-(d*f*x + c*f)/d)*e^{(c*f/d - e)} + 4*a*b*c^2*f^2*Ei((d*f*x + c*f)/ \\ & d)*e^{(-c*f/d + e)} + 4*b^2*c^2*f^2*Ei(2*(d*f*x + c*f)/d)*e^{(-2*c*f/d + 2*e)} \\ & - 2*b^2*d^2*f*x*e^{(2*f*x + 2*e)} - 4*a*b*d^2*f*x*e^{(f*x + e)} - 4*a*b*d^2*f*x \\ & *e^{(-f*x - e)} + 2*b^2*d^2*f*x*e^{(-2*f*x - 2*e)} - 2*b^2*c*d*f*e^{(2*f*x + 2*e)} \\ & ) - 4*a*b*c*d*f*e^{(f*x + e)} - 4*a*b*c*d*f*e^{(-f*x - e)} + 2*b^2*c*d*f*e^{(-2* \\ & f*x - 2*e)} - b^2*d^2*e^{(2*f*x + 2*e)} - 4*a*b*d^2*e^{(f*x + e)} + 4*a*b*d^2*e^{ \\ & (-f*x - e)} - b^2*d^2*e^{(-2*f*x - 2*e)} - 4*a^2*d^2 + 2*b^2*d^2)/(d^5*x^2 + 2 \\ & *c*d^4*x + c^2*d^3) \end{aligned}$$



$$3.169 \quad \int \frac{(c+dx)^3}{a+b \sinh(e+fx)} dx$$

**Optimal.** Leaf size=404

$$-\frac{6d^2(c+dx)\text{PolyLog}\left(3, -\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^3\sqrt{a^2+b^2}} + \frac{6d^2(c+dx)\text{PolyLog}\left(3, -\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^3\sqrt{a^2+b^2}} + \frac{3d(c+dx)^2\text{PolyLog}\left(2, -\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\sqrt{a^2+b^2}} -$$

```
[Out] ((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]*f) - ((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]*f) + (3*d*(c + d*x)^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^2) - (3*d*(c + d*x)^2*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^2) - (6*d^2*(c + d*x)*PolyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^3) + (6*d^2*(c + d*x)*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^3) + (6*d^3*PolyLog[4, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^4) - (6*d^3*PolyLog[4, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^4)
```

**Rubi [A]** time = 0.825822, antiderivative size = 404, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$ , Rules used = {3322, 2264, 2190, 2531, 6609, 2282, 6589}

$$-\frac{6d^2(c+dx)\text{PolyLog}\left(3, -\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^3\sqrt{a^2+b^2}} + \frac{6d^2(c+dx)\text{PolyLog}\left(3, -\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^3\sqrt{a^2+b^2}} + \frac{3d(c+dx)^2\text{PolyLog}\left(2, -\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\sqrt{a^2+b^2}} -$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^3/(a + b\*Sinh[e + f\*x]), x]

```
[Out] ((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]*f) - ((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]*f) + (3*d*(c + d*x)^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^2) - (3*d*(c + d*x)^2*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^2) - (6*d^2*(c + d*x)*PolyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^3) + (6*d^2*(c + d*x)*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^3) + (6*d^3*PolyLog[4, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^4) - (6*d^3*PolyLog[4, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^4)
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
 \int \frac{(c + dx)^3}{a + b \sinh(e + fx)} dx &= 2 \int \frac{e^{e+fx}(c + dx)^3}{-b + 2ae^{e+fx} + be^{2(e+fx)}} dx \\
 &= \frac{(2b) \int \frac{e^{e+fx}(c+dx)^3}{2a-2\sqrt{a^2+b^2}+2be^{e+fx}} dx}{\sqrt{a^2+b^2}} - \frac{(2b) \int \frac{e^{e+fx}(c+dx)^3}{2a+2\sqrt{a^2+b^2}+2be^{e+fx}} dx}{\sqrt{a^2+b^2}} \\
 &= \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(3d) \int (c + dx)^2 \log\left(1 + \frac{2be^{e+fx}}{2a-2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2} \\
 &= \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} + \frac{3d(c + dx)^2 \text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2} \\
 &= \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} + \frac{3d(c + dx)^2 \text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2} \\
 &= \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} + \frac{3d(c + dx)^2 \text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2} \\
 &= \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} + \frac{3d(c + dx)^2 \text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2} \\
 &= \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c + dx)^3 \log\left(1 + \frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} + \frac{3d(c + dx)^2 \text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.260614, size = 318, normalized size = 0.79

$$\frac{3d\left(f^2(c+dx)^2 \text{PolyLog}\left(2, \frac{be^{e+fx}}{\sqrt{a^2+b^2}-a}\right) - 2df(c+dx) \text{PolyLog}\left(3, \frac{be^{e+fx}}{\sqrt{a^2+b^2}-a}\right) + 2d^2 \text{PolyLog}\left(4, \frac{be^{e+fx}}{\sqrt{a^2+b^2}-a}\right)\right)}{f^3} - \frac{3d\left(f^2(c+dx)^2 \text{PolyLog}\left(2, -\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right) - 2df(c+dx) \text{PolyLog}\left(3, -\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right) + 2d^2 \text{PolyLog}\left(4, -\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)\right)}{f^3}$$


---


$$f\sqrt{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*Sinh[e + f*x]),x]
```

```
[Out] ((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])] - (c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])] + (3*d*(f^2*(c + d*x)^2*PolyLog[2, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*f*(c + d*x)*PolyLog[3, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])] + 2*d^2*PolyLog[4, (b*E^(e + f*x))/(-a + Sqrt[a^2 + b^2])])/f^3 - (3*d*(f^2*(c + d*x)^2*PolyLog[2, -(b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*(c + d*x)*PolyLog[3, -(b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])] + 2*d^2*PolyLog[4, -(b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/f^3)/(Sqrt[a^2 + b^2]*f)
```

**Maple [F]** time = 0.168, size = 0, normalized size = 0.

$$\int \frac{(dx + c)^3}{a + b \sinh(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*sinh(f*x+e)),x)
```

```
[Out] int((d*x+c)^3/(a+b*sinh(f*x+e)),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [C]** time = 2.72142, size = 2412, normalized size = 5.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] (6*b*d^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(f*x + e) + a*sinh(f*x + e)
) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))/b - 6*b*d^3
*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(f*x + e) + a*sinh(f*x + e) - (b*c
osh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))/b + 3*(b*d^3*f^2*x^
2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(f*x
+ e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^
2)/b^2) - b)/b + 1) - 3*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*sqr
t((a^2 + b^2)/b^2)*dilog((a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x +
e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b*d^3*e^3 - 3*b
*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*log(2*b*c
osh(f*x + e) + 2*b*sinh(f*x + e) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b*d^
3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt((a^2 + b^2)/b^2
)*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) - 2*b*sqrt((a^2 + b^2)/b^2) + 2
*a) + (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 - 3*
b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(f*x + e
) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/
b^2) - b)/b) - (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3
*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cos
h(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^
2 + b^2)/b^2) - b)/b) - 6*(b*d^3*f*x + b*c*d^2*f)*sqrt((a^2 + b^2)/b^2)*pol
ylog(3, (a*cosh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x
+ e))*sqrt((a^2 + b^2)/b^2))/b) + 6*(b*d^3*f*x + b*c*d^2*f)*sqrt((a^2 + b^2
)/b^2)*polylog(3, (a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b
*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))/b))/((a^2 + b^2)*f^4)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*sinh(f*x+e)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^3}{b \sinh(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*sinh(f*x + e) + a), x)
```

$$3.170 \quad \int \frac{(c+dx)^2}{a+b \sinh(e+fx)} dx$$

**Optimal.** Leaf size=296

$$\frac{2d(c+dx)\text{PolyLog}\left(2, -\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\sqrt{a^2+b^2}} - \frac{2d(c+dx)\text{PolyLog}\left(2, -\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^2\sqrt{a^2+b^2}} - \frac{2d^2\text{PolyLog}\left(3, -\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^3\sqrt{a^2+b^2}} + \frac{2d^2\text{PolyLog}\left(3, -\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^3\sqrt{a^2+b^2}}$$

[Out]  $((c+d*x)^2*\text{Log}[1+(b*E^{(e+f*x)})/(a-\text{Sqrt}[a^2+b^2])])/( \text{Sqrt}[a^2+b^2]*f) - ((c+d*x)^2*\text{Log}[1+(b*E^{(e+f*x)})/(a+\text{Sqrt}[a^2+b^2])])/( \text{Sqrt}[a^2+b^2]*f) + (2*d*(c+d*x)*\text{PolyLog}[2, -((b*E^{(e+f*x)})/(a-\text{Sqrt}[a^2+b^2]))])/( \text{Sqrt}[a^2+b^2]*f^2) - (2*d*(c+d*x)*\text{PolyLog}[2, -((b*E^{(e+f*x)})/(a+\text{Sqrt}[a^2+b^2]))])/( \text{Sqrt}[a^2+b^2]*f^2) - (2*d^2*\text{PolyLog}[3, -((b*E^{(e+f*x)})/(a-\text{Sqrt}[a^2+b^2]))])/( \text{Sqrt}[a^2+b^2]*f^3) + (2*d^2*\text{PolyLog}[3, -((b*E^{(e+f*x)})/(a+\text{Sqrt}[a^2+b^2]))])/( \text{Sqrt}[a^2+b^2]*f^3)$

**Rubi [A]** time = 0.668979, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {3322, 2264, 2190, 2531, 2282, 6589}

$$\frac{2d(c+dx)\text{PolyLog}\left(2, -\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\sqrt{a^2+b^2}} - \frac{2d(c+dx)\text{PolyLog}\left(2, -\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^2\sqrt{a^2+b^2}} - \frac{2d^2\text{PolyLog}\left(3, -\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^3\sqrt{a^2+b^2}} + \frac{2d^2\text{PolyLog}\left(3, -\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^3\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c+d*x)^2/(a+b*\text{Sinh}[e+f*x]),x]$

[Out]  $((c+d*x)^2*\text{Log}[1+(b*E^{(e+f*x)})/(a-\text{Sqrt}[a^2+b^2])])/( \text{Sqrt}[a^2+b^2]*f) - ((c+d*x)^2*\text{Log}[1+(b*E^{(e+f*x)})/(a+\text{Sqrt}[a^2+b^2])])/( \text{Sqrt}[a^2+b^2]*f) + (2*d*(c+d*x)*\text{PolyLog}[2, -((b*E^{(e+f*x)})/(a-\text{Sqrt}[a^2+b^2]))])/( \text{Sqrt}[a^2+b^2]*f^2) - (2*d*(c+d*x)*\text{PolyLog}[2, -((b*E^{(e+f*x)})/(a+\text{Sqrt}[a^2+b^2]))])/( \text{Sqrt}[a^2+b^2]*f^2) - (2*d^2*\text{PolyLog}[3, -((b*E^{(e+f*x)})/(a-\text{Sqrt}[a^2+b^2]))])/( \text{Sqrt}[a^2+b^2]*f^3) + (2*d^2*\text{PolyLog}[3, -((b*E^{(e+f*x)})/(a+\text{Sqrt}[a^2+b^2]))])/( \text{Sqrt}[a^2+b^2]*f^3)$

### Rule 3322

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}/((a_.) + (b_.)*\sin[(e_.) + (\text{Complex}[0, fz\_])*(f_.)*(x_.)]), x\_Symbol] := \text{Dist}[2, \text{Int}[(c + d*x)^m*E^{-(I*e + f*fz*x)}]/(-I*b + 2*a*E^{-(I*e + f*fz*x)} + I*b*E^{(2*(-I*e + f*fz*x))}), x], x] /; F$

reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

#### Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.)\*(F\_)^(v\_)), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^(c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n])]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

#### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rubi steps



$$\begin{aligned}
\int \frac{(c+dx)^2}{a+b\sinh(e+fx)} dx &= 2 \int \frac{e^{e+fx}(c+dx)^2}{-b+2ae^{e+fx}+be^{2(e+fx)}} dx \\
&= \frac{(2b) \int \frac{e^{e+fx}(c+dx)^2}{2a-2\sqrt{a^2+b^2}+2be^{e+fx}} dx}{\sqrt{a^2+b^2}} - \frac{(2b) \int \frac{e^{e+fx}(c+dx)^2}{2a+2\sqrt{a^2+b^2}+2be^{e+fx}} dx}{\sqrt{a^2+b^2}} \\
&= \frac{(c+dx)^2 \log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c+dx)^2 \log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(2d) \int (c+dx) \log\left(1+\frac{2be^{e+fx}}{2a-2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} \\
&= \frac{(c+dx)^2 \log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c+dx)^2 \log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} + \frac{2d(c+dx)\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2} \\
&= \frac{(c+dx)^2 \log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c+dx)^2 \log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} + \frac{2d(c+dx)\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2} \\
&= \frac{(c+dx)^2 \log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c+dx)^2 \log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} + \frac{2d(c+dx)\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2}
\end{aligned}$$

**Mathematica [A]** time = 0.155571, size = 233, normalized size = 0.79

$$\frac{2d\left(f(c+dx)\text{PolyLog}\left(2,\frac{be^{e+fx}}{\sqrt{a^2+b^2}-a}\right)-d\text{PolyLog}\left(3,\frac{be^{e+fx}}{\sqrt{a^2+b^2}-a}\right)\right)}{f^2} - \frac{2d\left(f(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)-d\text{PolyLog}\left(3,-\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)\right)}{f^2} + (c+dx)^2 \log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2/(a + b\*Sinh[e + f\*x]),x]

[Out] ((c + d\*x)^2\*Log[1 + (b\*E^(e + f\*x))/(a - Sqrt[a^2 + b^2]]) - (c + d\*x)^2\*Log[1 + (b\*E^(e + f\*x))/(a + Sqrt[a^2 + b^2]]) + (2\*d\*(f\*(c + d\*x)\*PolyLog[2, (b\*E^(e + f\*x))/(-a + Sqrt[a^2 + b^2]]) - d\*PolyLog[3, (b\*E^(e + f\*x))/(-a + Sqrt[a^2 + b^2])]))/f^2 - (2\*d\*(f\*(c + d\*x)\*PolyLog[2, -((b\*E^(e + f\*x))/(a + Sqrt[a^2 + b^2]))] - d\*PolyLog[3, -((b\*E^(e + f\*x))/(a + Sqrt[a^2 + b^2])])))/f^2)/(Sqrt[a^2 + b^2]\*f)

**Maple [F]** time = 0.12, size = 0, normalized size = 0.

$$\int \frac{(dx + c)^2}{a + b \sinh(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2/(a+b\*sinh(f\*x+e)),x)

[Out] int((d\*x+c)^2/(a+b\*sinh(f\*x+e)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+b\*sinh(f\*x+e)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 2.5205, size = 1733, normalized size = 5.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+b\*sinh(f\*x+e)),x, algorithm="fricas")

[Out] 
$$-(2*b*d^2*\sqrt{(a^2 + b^2)/b^2}*\text{polylog}(3, (a*\cosh(f*x + e) + a*\sinh(f*x + e) + (b*\cosh(f*x + e) + b*\sinh(f*x + e))*\sqrt{(a^2 + b^2)/b^2})/b) - 2*b*d^2*\sqrt{(a^2 + b^2)/b^2}*\text{polylog}(3, (a*\cosh(f*x + e) + a*\sinh(f*x + e) - (b*\cosh(f*x + e) + b*\sinh(f*x + e))*\sqrt{(a^2 + b^2)/b^2})/b) - 2*(b*d^2*f*x + b*c*d*f)*\sqrt{(a^2 + b^2)/b^2}*\text{dilog}((a*\cosh(f*x + e) + a*\sinh(f*x + e) + (b*\cosh(f*x + e) + b*\sinh(f*x + e))*\sqrt{(a^2 + b^2)/b^2}) - b)/b + 1) + 2*(b*d^2*f*x + b*c*d*f)*\sqrt{(a^2 + b^2)/b^2}*\text{dilog}((a*\cosh(f*x + e) + a*\sinh(f*x + e) - (b*\cosh(f*x + e) + b*\sinh(f*x + e))*\sqrt{(a^2 + b^2)/b^2}) - b)/b + 1) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*\sqrt{(a^2 + b^2)/b^2}*\log(2*b*\cosh(f*x + e) + 2*b*\sinh(f*x + e) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - (b*$$

$$d^2e^2 - 2bcde^2f + b^2c^2f^2) \sqrt{(a^2 + b^2)/b^2} \log(2b \cosh(fx + e) + 2b \sinh(fx + e) - 2b \sqrt{(a^2 + b^2)/b^2} + 2a) - (bd^2f^2x^2 + 2bcdf^2x - bd^2e^2 + 2bcde^2f) \sqrt{(a^2 + b^2)/b^2} \log(-a \cosh(fx + e) + a \sinh(fx + e) + (b \cosh(fx + e) + b \sinh(fx + e)) \sqrt{(a^2 + b^2)/b^2} - b)/b) + (bd^2f^2x^2 + 2bcdf^2x - bd^2e^2 + 2bcde^2f) \sqrt{(a^2 + b^2)/b^2} \log(-a \cosh(fx + e) + a \sinh(fx + e) - (b \cosh(fx + e) + b \sinh(fx + e)) \sqrt{(a^2 + b^2)/b^2} - b)/b) / ((a^2 + b^2) f^3)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2/(a+b\*sinh(f\*x+e)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^2}{b \sinh(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+b\*sinh(f\*x+e)),x, algorithm="giac")

[Out] integrate((d\*x + c)^2/(b\*sinh(f\*x + e) + a), x)

$$3.171 \quad \int \frac{c+dx}{a+b \sinh(e+fx)} dx$$

**Optimal.** Leaf size=187

$$\frac{d\text{PolyLog}\left(2, -\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\sqrt{a^2+b^2}} - \frac{d\text{PolyLog}\left(2, -\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^2\sqrt{a^2+b^2}} + \frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{f\sqrt{a^2+b^2}} - \frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{f\sqrt{a^2+b^2}}$$

```
[Out] ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]*f) - ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]*f) + (d*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^2) - (d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^2)
```

**Rubi [A]** time = 0.369, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$ , Rules used = {3322, 2264, 2190, 2279, 2391}

$$\frac{d\text{PolyLog}\left(2, -\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2\sqrt{a^2+b^2}} - \frac{d\text{PolyLog}\left(2, -\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^2\sqrt{a^2+b^2}} + \frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right)}{f\sqrt{a^2+b^2}} - \frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right)}{f\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Sinh[e + f*x]),x]
```

```
[Out] ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]*f) - ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2])])/(Sqrt[a^2 + b^2]*f) + (d*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^2) - (d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))])/(Sqrt[a^2 + b^2]*f^2)
```

### Rule 3322

```
Int[((c._) + (d._)*(x._))^(m._)/((a._) + (b._)*sin[(e._) + (Complex[0, fz_])* (f._)*(x._)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(- (I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[(c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{c + dx}{a + b \sinh(e + fx)} dx &= 2 \int \frac{e^{e+fx}(c + dx)}{-b + 2ae^{e+fx} + be^{2(e+fx)}} dx \\
&= \frac{(2b) \int \frac{e^{e+fx}(c+dx)}{2a-2\sqrt{a^2+b^2}+2be^{e+fx}} dx}{\sqrt{a^2+b^2}} - \frac{(2b) \int \frac{e^{e+fx}(c+dx)}{2a+2\sqrt{a^2+b^2}+2be^{e+fx}} dx}{\sqrt{a^2+b^2}} \\
&= \frac{(c + dx) \log\left(1 + \frac{be^{e+fx}}{a - \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c + dx) \log\left(1 + \frac{be^{e+fx}}{a + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{d \int \log\left(1 + \frac{2be^{e+fx}}{2a-2\sqrt{a^2+b^2}}\right) dx}{\sqrt{a^2+b^2}f} + \dots \\
&= \frac{(c + dx) \log\left(1 + \frac{be^{e+fx}}{a - \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c + dx) \log\left(1 + \frac{be^{e+fx}}{a + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{d \operatorname{Subst}\left(\int \frac{\log\left(1 + \frac{2bx}{2a-2\sqrt{a^2+b^2}}\right)}{x} dx, \dots\right)}{\sqrt{a^2+b^2}f^2} \\
&= \frac{(c + dx) \log\left(1 + \frac{be^{e+fx}}{a - \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} - \frac{(c + dx) \log\left(1 + \frac{be^{e+fx}}{a + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f} + \frac{d \operatorname{Li}_2\left(-\frac{be^{e+fx}}{a - \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2} - \frac{d \operatorname{Li}_2\left(-\frac{be^{e+fx}}{a + \sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}f^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0365069, size = 142, normalized size = 0.76

$$\frac{d\text{PolyLog}\left(2, \frac{be^{e+fx}}{\sqrt{a^2+b^2-a}}\right) - d\text{PolyLog}\left(2, -\frac{be^{e+fx}}{\sqrt{a^2+b^2+a}}\right) + f(c+dx)\left(\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right) - \log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2+a}}+1\right)\right)}{f^2\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)/(a + b\*Sinh[e + f\*x]), x]

[Out] (f\*(c + d\*x)\*(Log[1 + (b\*E^(e + f\*x))/(a - Sqrt[a^2 + b^2]] - Log[1 + (b\*E^(e + f\*x))/(a + Sqrt[a^2 + b^2]])] + d\*PolyLog[2, (b\*E^(e + f\*x))/(a - Sqrt[a^2 + b^2]]) - d\*PolyLog[2, -(b\*E^(e + f\*x))/(a + Sqrt[a^2 + b^2]])]/(Sqrt[a^2 + b^2]\*f^2)

**Maple [B]** time = 0.054, size = 393, normalized size = 2.1

$$-2 \frac{c}{f\sqrt{a^2+b^2}} \text{Arctanh}\left(\frac{1}{2} \frac{2be^{fx+e} + 2a}{\sqrt{a^2+b^2}}\right) + \frac{dx}{f} \ln\left(\left(-be^{fx+e} + \sqrt{a^2+b^2} - a\right)\left(-a + \sqrt{a^2+b^2}\right)^{-1}\right) \frac{1}{\sqrt{a^2+b^2}} + \frac{de}{f^2} \ln\left(\left(-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)/(a+b\*sinh(f\*x+e)), x)

[Out] -2/f\*c/(a^2+b^2)^(1/2)\*arctanh(1/2\*(2\*b\*exp(f\*x+e)+2\*a)/(a^2+b^2)^(1/2))+1/f\*d/(a^2+b^2)^(1/2)\*ln((-b\*exp(f\*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))) \*x+1/f^2\*d/(a^2+b^2)^(1/2)\*ln((-b\*exp(f\*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))) \*e-1/f\*d/(a^2+b^2)^(1/2)\*ln((b\*exp(f\*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))) \*x-1/f^2\*d/(a^2+b^2)^(1/2)\*ln((b\*exp(f\*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))) \*e+1/f^2\*d/(a^2+b^2)^(1/2)\*dilog((-b\*exp(f\*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/f^2\*d/(a^2+b^2)^(1/2)\*dilog((b\*exp(f\*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/f^2\*d\*e/(a^2+b^2)^(1/2)\*arctanh(1/2\*(2\*b\*exp(f\*x+e)+2\*a)/(a^2+b^2)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*sinh(f\*x+e)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.54519, size = 1127, normalized size = 6.03

$$bd\sqrt{\frac{a^2+b^2}{b^2}}\operatorname{Li}_2\left(\frac{a\cosh(fx+e)+a\sinh(fx+e)+(b\cosh(fx+e)+b\sinh(fx+e))\sqrt{\frac{a^2+b^2}{b^2}-b}}{b}+1\right)-bd\sqrt{\frac{a^2+b^2}{b^2}}\operatorname{Li}_2\left(\frac{a\cosh(fx+e)+a\sinh(fx+e)-(b\cosh(fx+e)+b\sinh(fx+e))\sqrt{\frac{a^2+b^2}{b^2}-b}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*sinh(f\*x+e)),x, algorithm="fricas")

[Out] (b\*d\*sqrt((a^2 + b^2)/b^2)\*dilog((a\*cosh(f\*x + e) + a\*sinh(f\*x + e) + (b\*cosh(f\*x + e) + b\*sinh(f\*x + e))\*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - b\*d\*sqrt((a^2 + b^2)/b^2)\*dilog((a\*cosh(f\*x + e) + a\*sinh(f\*x + e) - (b\*cosh(f\*x + e) + b\*sinh(f\*x + e))\*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b\*d\*e - b\*c\*f)\*sqrt((a^2 + b^2)/b^2)\*log(2\*b\*cosh(f\*x + e) + 2\*b\*sinh(f\*x + e) + 2\*b\*sqrt((a^2 + b^2)/b^2) + 2\*a) - (b\*d\*e - b\*c\*f)\*sqrt((a^2 + b^2)/b^2)\*log(2\*b\*cosh(f\*x + e) + 2\*b\*sinh(f\*x + e) - 2\*b\*sqrt((a^2 + b^2)/b^2) + 2\*a) + (b\*d\*f\*x + b\*d\*e)\*sqrt((a^2 + b^2)/b^2)\*log(-(a\*cosh(f\*x + e) + a\*sinh(f\*x + e) + (b\*cosh(f\*x + e) + b\*sinh(f\*x + e))\*sqrt((a^2 + b^2)/b^2) - b)/b) - (b\*d\*f\*x + b\*d\*e)\*sqrt((a^2 + b^2)/b^2)\*log(-(a\*cosh(f\*x + e) + a\*sinh(f\*x + e) - (b\*cosh(f\*x + e) + b\*sinh(f\*x + e))\*sqrt((a^2 + b^2)/b^2) - b)/b))/((a^2 + b^2)\*f^2)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*sinh(f\*x+e)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{b \sinh(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*sinh(f\*x+e)),x, algorithm="giac")

[Out] integrate((d\*x + c)/(b\*sinh(f\*x + e) + a), x)



$$3.172 \quad \int \frac{1}{(c+dx)(a+b \sinh(e+fx))} dx$$

**Optimal.** Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a+b \sinh(e+fx))}, x\right)$$

[Out] Unintegrable[1/((c + d\*x)\*(a + b\*Sinh[e + f\*x])), x]

**Rubi [A]** time = 0.0635657, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(c+dx)(a+b \sinh(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d\*x)\*(a + b\*Sinh[e + f\*x])), x]

[Out] Defer[Int][1/((c + d\*x)\*(a + b\*Sinh[e + f\*x])), x]

Rubi steps

$$\int \frac{1}{(c+dx)(a+b \sinh(e+fx))} dx = \int \frac{1}{(c+dx)(a+b \sinh(e+fx))} dx$$

**Mathematica [A]** time = 0.944279, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+b \sinh(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d\*x)\*(a + b\*Sinh[e + f\*x])), x]

[Out] Integrate[1/((c + d\*x)\*(a + b\*Sinh[e + f\*x])), x]

---

**Maple [A]** time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)(a + b \sinh(fx + e))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x+c)/(a+b\*sinh(f\*x+e)),x)

[Out] int(1/(d\*x+c)/(a+b\*sinh(f\*x+e)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)(b \sinh(fx + e) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+b\*sinh(f\*x+e)),x, algorithm="maxima")

[Out] integrate(1/((d\*x + c)\*(b\*sinh(f\*x + e) + a)), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{adx + ac + (bdx + bc) \sinh(fx + e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+b\*sinh(f\*x+e)),x, algorithm="fricas")

[Out] integral(1/(a\*d\*x + a\*c + (b\*d\*x + b\*c)\*sinh(f\*x + e)), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e)),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)(b \sinh(fx+e)+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*sinh(f*x + e) + a)), x)
```

$$3.173 \quad \int \frac{1}{(c+dx)^2(a+b \sinh(e+fx))} dx$$

**Optimal.** Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a+b \sinh(e+fx))}, x\right)$$

[Out] Unintegrable[1/((c + d\*x)^2\*(a + b\*Sinh[e + f\*x])), x]

**Rubi [A]** time = 0.0595487, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b \sinh(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d\*x)^2\*(a + b\*Sinh[e + f\*x])),x]

[Out] Defer[Int][1/((c + d\*x)^2\*(a + b\*Sinh[e + f\*x])), x]

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b \sinh(e+fx))} dx = \int \frac{1}{(c+dx)^2(a+b \sinh(e+fx))} dx$$

**Mathematica [A]** time = 0.938603, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+b \sinh(e+fx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d\*x)^2\*(a + b\*Sinh[e + f\*x])),x]

[Out] Integrate[1/((c + d\*x)^2\*(a + b\*Sinh[e + f\*x])), x]

---

**Maple [A]** time = 0.036, size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)^2 (a + b \sinh(fx + e))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x+c)^2/(a+b\*sinh(f\*x+e)),x)

[Out] int(1/(d\*x+c)^2/(a+b\*sinh(f\*x+e)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)^2 (b \sinh(fx + e) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+b\*sinh(f\*x+e)),x, algorithm="maxima")

[Out] integrate(1/((d\*x + c)^2\*(b\*sinh(f\*x + e) + a)), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{ad^2x^2 + 2acdx + ac^2 + (bd^2x^2 + 2bcdx + bc^2) \sinh(fx + e)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+b\*sinh(f\*x+e)),x, algorithm="fricas")

[Out] integral(1/(a\*d^2\*x^2 + 2\*a\*c\*d\*x + a\*c^2 + (b\*d^2\*x^2 + 2\*b\*c\*d\*x + b\*c^2)\*sinh(f\*x + e)), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)\*\*2/(a+b\*sinh(f\*x+e)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)^2 (b \sinh(fx + e) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+b\*sinh(f\*x+e)),x, algorithm="giac")

[Out] integrate(1/((d\*x + c)^2\*(b\*sinh(f\*x + e) + a)), x)

$$3.174 \quad \int \frac{(c+dx)^2}{(a+b \sinh(e+fx))^2} dx$$

**Optimal.** Leaf size=549

$$\frac{2ad(c+dx)\text{PolyLog}\left(2, -\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2(a^2+b^2)^{3/2}} - \frac{2ad(c+dx)\text{PolyLog}\left(2, -\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^2(a^2+b^2)^{3/2}} + \frac{2d^2\text{PolyLog}\left(2, -\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^3(a^2+b^2)} + \frac{2d^2\text{PolyLog}\left(2, -\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^3(a^2+b^2)}$$

```
[Out] -((c + d*x)^2/((a^2 + b^2)*f)) + (2*d*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]])/((a^2 + b^2)*f^2) + (a*(c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*f) + (2*d*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]])/((a^2 + b^2)*f^2) - (a*(c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*f) + (2*d^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)*f^3) + (2*a*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)^(3/2)*f^2) + (2*d^2*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 + b^2)*f^3) - (2*a*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 + b^2)^(3/2)*f^2) - (2*a*d^2*PolyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)^(3/2)*f^3) + (2*a*d^2*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 + b^2)^(3/2)*f^3) - (b*(c + d*x)^2*Cosh[e + f*x])/((a^2 + b^2)*f*(a + b*Sinh[e + f*x]))
```

**Rubi [A]** time = 1.03665, antiderivative size = 549, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 10, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {3324, 3322, 2264, 2190, 2531, 2282, 6589, 5561, 2279, 2391}

$$\frac{2ad(c+dx)\text{PolyLog}\left(2, -\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2(a^2+b^2)^{3/2}} - \frac{2ad(c+dx)\text{PolyLog}\left(2, -\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^2(a^2+b^2)^{3/2}} + \frac{2d^2\text{PolyLog}\left(2, -\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^3(a^2+b^2)} + \frac{2d^2\text{PolyLog}\left(2, -\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)}{f^3(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Sinh[e + f*x])^2,x]
```

```
[Out] -((c + d*x)^2/((a^2 + b^2)*f)) + (2*d*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]])/((a^2 + b^2)*f^2) + (a*(c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*f) + (2*d*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]])/((a^2 + b^2)*f^2) - (a*(c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*f) + (2*d^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)*f^3) + (2*a*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)^(3/2)*f^2) - (2*d^2*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 + b^2)*f^3) - (2*a*d^2*PolyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)^(3/2)*f^3) + (2*a*d^2*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 + b^2)^(3/2)*f^3) - (b*(c + d*x)^2*Cosh[e + f*x])/((a^2 + b^2)*f*(a + b*Sinh[e + f*x]))
```

$$\frac{((a^2 + b^2)^{3/2} f^2 + (2*d^2*PolyLog[2, -((b*E^{(e + f*x)))/(a + Sqrt[a^2 + b^2])])]/((a^2 + b^2)*f^3) - (2*a*d*(c + d*x)*PolyLog[2, -((b*E^{(e + f*x)))/(a + Sqrt[a^2 + b^2])])]/((a^2 + b^2)^{3/2} f^2) - (2*a*d^2*PolyLog[3, -((b*E^{(e + f*x)))/(a - Sqrt[a^2 + b^2])])]/((a^2 + b^2)^{3/2} f^3) + (2*a*d^2*PolyLog[3, -((b*E^{(e + f*x)))/(a + Sqrt[a^2 + b^2])])]/((a^2 + b^2)^{3/2} f^3) - (b*(c + d*x)^2*Cosh[e + f*x])/((a^2 + b^2)*f*(a + b*Sinh[e + f*x]))$$

### Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] := Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sin[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
+ b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

### Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_]
*(f_.)*(x_))]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.)/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```



, g, n}, x] && GtQ[m, 0]

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)])*((e_.) + (f_.)*(x_))^(m_.)]/((a_.) + (b_.)*Sin h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))]^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^2}{(a+b\sinh(e+fx))^2} dx &= -\frac{b(c+dx)^2 \cosh(e+fx)}{(a^2+b^2)f(a+b\sinh(e+fx))} + \frac{a \int \frac{(c+dx)^2}{a+b\sinh(e+fx)} dx}{a^2+b^2} + \frac{(2bd) \int \frac{(c+dx) \cosh(e+fx)}{a+b\sinh(e+fx)} dx}{(a^2+b^2)f} \\
&= -\frac{(c+dx)^2}{(a^2+b^2)f} - \frac{b(c+dx)^2 \cosh(e+fx)}{(a^2+b^2)f(a+b\sinh(e+fx))} + \frac{(2a) \int \frac{e^{e+fx}(c+dx)^2}{-b+2ae^{e+fx}+be^{2(e+fx)}} dx}{a^2+b^2} + \frac{(2bd) \int \frac{dx}{a-}}{(a^2+b^2)f} \\
&= -\frac{(c+dx)^2}{(a^2+b^2)f} + \frac{2d(c+dx) \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)f^2} + \frac{2d(c+dx) \log\left(1 + \frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)f^2} - \frac{b(c+dx)}{(a^2+b^2)} \\
&= -\frac{(c+dx)^2}{(a^2+b^2)f} + \frac{2d(c+dx) \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)f^2} + \frac{a(c+dx)^2 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}f} + \frac{2d(c+dx)}{(a^2+b^2)} \\
&= -\frac{(c+dx)^2}{(a^2+b^2)f} + \frac{2d(c+dx) \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)f^2} + \frac{a(c+dx)^2 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}f} + \frac{2d(c+dx)}{(a^2+b^2)} \\
&= -\frac{(c+dx)^2}{(a^2+b^2)f} + \frac{2d(c+dx) \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)f^2} + \frac{a(c+dx)^2 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}f} + \frac{2d(c+dx)}{(a^2+b^2)} \\
&= -\frac{(c+dx)^2}{(a^2+b^2)f} + \frac{2d(c+dx) \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)f^2} + \frac{a(c+dx)^2 \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}f} + \frac{2d(c+dx)}{(a^2+b^2)}
\end{aligned}$$

**Mathematica [A]** time = 1.93532, size = 428, normalized size = 0.78

$$\frac{a\left(-2df(c+dx)\text{PolyLog}\left(2, \frac{be^{e+fx}}{\sqrt{a^2+b^2}-a}\right)+2df(c+dx)\text{PolyLog}\left(2, -\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)+2d^2\text{PolyLog}\left(3, \frac{be^{e+fx}}{\sqrt{a^2+b^2}-a}\right)-2d^2\text{PolyLog}\left(3, -\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right)-f^2(c+dx)^2 \log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)+\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^2/(a + b\*Sinh[e + f\*x])^2,x]

[Out]  $(-f^2(c+dx)^2 + 2d*f*(c+dx)*\text{Log}[1 + (b*E^{(e+f*x)})/(a - \text{Sqrt}[a^2 + b^2])]) + 2*d*f*(c+dx)*\text{Log}[1 + (b*E^{(e+f*x)})/(a + \text{Sqrt}[a^2 + b^2])]$   
 $+ 2*d^2*\text{PolyLog}[2, (b*E^{(e+f*x)})/(-a + \text{Sqrt}[a^2 + b^2])] + 2*d^2*\text{PolyLog}[2, -((b*E^{(e+f*x)})/(a + \text{Sqrt}[a^2 + b^2]))] - (a*(-f^2*(c+dx)^2*\text{Log}[1$

$$+ (bE^{(e+fx)})/(a - \text{Sqrt}[a^2 + b^2])) + f^2(c + dx)^2 \text{Log}[1 + (bE^{(e+fx)})/(a + \text{Sqrt}[a^2 + b^2])] - 2df(c + dx) \text{PolyLog}[2, (bE^{(e+fx)})/(-a + \text{Sqrt}[a^2 + b^2])] + 2df(c + dx) \text{PolyLog}[2, -((bE^{(e+fx)})/(a + \text{Sqrt}[a^2 + b^2]))] + 2d^2 \text{PolyLog}[3, (bE^{(e+fx)})/(-a + \text{Sqrt}[a^2 + b^2])] - 2d^2 \text{PolyLog}[3, -((bE^{(e+fx)})/(a + \text{Sqrt}[a^2 + b^2])))]/\text{Sqrt}[a^2 + b^2] - (bf^2(c + dx)^2 \text{Cosh}[e + fx])/(a + b \text{Sinh}[e + fx])/((a^2 + b^2)f^3)$$

**Maple [F]** time = 0.29, size = 0, normalized size = 0.

$$\int \frac{(dx + c)^2}{(a + b \sinh(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^2/(a+b\*sinh(f\*x+e))^2,x)

[Out] int((d\*x+c)^2/(a+b\*sinh(f\*x+e))^2,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+b\*sinh(f\*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 3.52775, size = 8910, normalized size = 16.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+b\*sinh(f\*x+e))^2,x, algorithm="fricas")

```

[Out] -(2*(a^2*b + b^3)*d^2*e^2 - 4*(a^2*b + b^3)*c*d*e*f + 2*(a^2*b + b^3)*c^2*f
^2 + 2*((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*c*d*f^2*x - (a^2*b + b^
3)*d^2*e^2 + 2*(a^2*b + b^3)*c*d*e*f)*cosh(f*x + e)^2 + 2*((a^2*b + b^3)*d^
2*f^2*x^2 + 2*(a^2*b + b^3)*c*d*f^2*x - (a^2*b + b^3)*d^2*e^2 + 2*(a^2*b +
b^3)*c*d*e*f)*sinh(f*x + e)^2 + 2*(a*b^2*d^2*cosh(f*x + e)^2 + a*b^2*d^2*si
nh(f*x + e)^2 + 2*a^2*b*d^2*cosh(f*x + e) - a*b^2*d^2 + 2*(a*b^2*d^2*cosh(f
*x + e) + a^2*b*d^2)*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cos
h(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^
2 + b^2)/b^2))/b) - 2*(a*b^2*d^2*cosh(f*x + e)^2 + a*b^2*d^2*sinh(f*x + e)^
2 + 2*a^2*b*d^2*cosh(f*x + e) - a*b^2*d^2 + 2*(a*b^2*d^2*cosh(f*x + e) + a^
2*b*d^2)*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(f*x + e) +
a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2
))/b) + 2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*c*d*f^2*x - 2*(a^3 +
a*b^2)*d^2*e^2 + 4*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*cosh(f*x
+ e) - 2*((a^2*b + b^3)*d^2*cosh(f*x + e)^2 + (a^2*b + b^3)*d^2*sinh(f*x +
e)^2 + 2*(a^3 + a*b^2)*d^2*cosh(f*x + e) - (a^2*b + b^3)*d^2 + 2*((a^2*b +
b^3)*d^2*cosh(f*x + e) + (a^3 + a*b^2)*d^2)*sinh(f*x + e) - (a*b^2*d^2*f*x
+ a*b^2*c*d*f - (a*b^2*d^2*f*x + a*b^2*c*d*f)*cosh(f*x + e)^2 - (a*b^2*d^2
*f*x + a*b^2*c*d*f)*sinh(f*x + e)^2 - 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*cosh(
f*x + e) - 2*(a^2*b*d^2*f*x + a^2*b*c*d*f + (a*b^2*d^2*f*x + a*b^2*c*d*f)*c
osh(f*x + e))*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))*dilog((a*cosh(f*x + e)
+ a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 + b^2)/b^
2) - b)/b + 1) - 2*((a^2*b + b^3)*d^2*cosh(f*x + e)^2 + (a^2*b + b^3)*d^2*si
nh(f*x + e)^2 + 2*(a^3 + a*b^2)*d^2*cosh(f*x + e) - (a^2*b + b^3)*d^2 + 2*
((a^2*b + b^3)*d^2*cosh(f*x + e) + (a^3 + a*b^2)*d^2)*sinh(f*x + e) + (a*b^
2*d^2*f*x + a*b^2*c*d*f - (a*b^2*d^2*f*x + a*b^2*c*d*f)*cosh(f*x + e)^2 - (
a*b^2*d^2*f*x + a*b^2*c*d*f)*sinh(f*x + e)^2 - 2*(a^2*b*d^2*f*x + a^2*b*c*d
*f)*cosh(f*x + e) - 2*(a^2*b*d^2*f*x + a^2*b*c*d*f + (a*b^2*d^2*f*x + a*b^2
*c*d*f)*cosh(f*x + e))*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))*dilog((a*cosh(
f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2
+ b^2)/b^2) - b)/b + 1) - (2*(a^2*b + b^3)*d^2*e - 2*(a^2*b + b^3)*c*d*f -
2*((a^2*b + b^3)*d^2*e - (a^2*b + b^3)*c*d*f)*cosh(f*x + e)^2 - 2*((a^2*b +
b^3)*d^2*e - (a^2*b + b^3)*c*d*f)*sinh(f*x + e)^2 - 4*((a^3 + a*b^2)*d^2*e
- (a^3 + a*b^2)*c*d*f)*cosh(f*x + e) - 4*((a^3 + a*b^2)*d^2*e - (a^3 + a*b
^2)*c*d*f + ((a^2*b + b^3)*d^2*e - (a^2*b + b^3)*c*d*f)*cosh(f*x + e))*sinh
(f*x + e) + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2 - (a*b^2*d^2*e
^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*sinh(f*x + e)^2 - 2*(a^2*b*d^2*e^2 - 2*a^2*b
*c*d*e*f + a^2*b*c^2*f^2)*cosh(f*x + e) - 2*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*
f + a^2*b*c^2*f^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(
f*x + e))*sinh(f*x + e))*sqrt((a^2 + b^2)/b^2))*log(2*b*cosh(f*x + e) + 2*b
*sinh(f*x + e) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (2*(a^2*b + b^3)*d^2*e
- 2*(a^2*b + b^3)*c*d*f - 2*((a^2*b + b^3)*d^2*e - (a^2*b + b^3)*c*d*f)*cos
h(f*x + e)^2 - 2*((a^2*b + b^3)*d^2*e - (a^2*b + b^3)*c*d*f)*sinh(f*x + e)^
2 - 4*((a^3 + a*b^2)*d^2*e - (a^3 + a*b^2)*c*d*f)*cosh(f*x + e) - 4*((a^3 +

```

$$\begin{aligned}
& a*b^2*d^2*e - (a^3 + a*b^2)*c*d*f + ((a^2*b + b^3)*d^2*e - (a^2*b + b^3)* \\
& c*d*f)*\cosh(f*x + e))*\sinh(f*x + e) - (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a* \\
& b^2*c^2*f^2 - (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*\cosh(f*x + \\
& e)^2 - (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*\sinh(f*x + e)^2 - \\
& 2*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*\cosh(f*x + e) - 2*(a^2* \\
& b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d* \\
& e*f + a*b^2*c^2*f^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{(a^2 + b^2)/b^2})*\log(2*b*\cosh(f*x + e) + 2*b*\sinh(f*x + e) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) \\
& + (2*(a^2*b + b^3)*d^2*f*x + 2*(a^2*b + b^3)*d^2*e - 2*((a^2*b + b^3)*d^2*f*x + (a^ \\
& 2*b + b^3)*d^2*e)*\cosh(f*x + e)^2 - 2*((a^2*b + b^3)*d^2*f*x + (a^ \\
& 2*b + b^3)*d^2*e)*\sinh(f*x + e)^2 - 4*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2) \\
& )*d^2*e)*\cosh(f*x + e) - 4*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d^2*e + ( \\
& (a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*d^2*e)*\cosh(f*x + e))*\sinh(f*x + e) + \\
& (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f - \\
& (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f)* \\
& \cosh(f*x + e)^2 - (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + \\
& 2*a*b^2*c*d*e*f)*\sinh(f*x + e)^2 - 2*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*c*d*f^2*x \\
& - a^2*b*d^2*e^2 + 2*a^2*b*c*d*e*f)*\cosh(f*x + e) - 2*(a^2*b*d^2*f^2*x^2 + \\
& 2*a^2*b*c*d*f^2*x - a^2*b*d^2*e^2 + 2*a^2*b*c*d*e*f + (a*b^2*d^2*f^2*x^2 + \\
& 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f)*\cosh(f*x + e))*\sinh(f* \\
& x + e))*\sqrt{(a^2 + b^2)/b^2})*\log(-(a*\cosh(f*x + e) + a*\sinh(f*x + e) + (b \\
& *\cosh(f*x + e) + b*\sinh(f*x + e))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + (2*(a^2*b \\
& + b^3)*d^2*f*x + 2*(a^2*b + b^3)*d^2*e - 2*((a^2*b + b^3)*d^2*f*x + (a^2*b \\
& + b^3)*d^2*e)*\cosh(f*x + e)^2 - 2*((a^2*b + b^3)*d^2*f*x + (a^2*b + b^3)*d \\
& ^2*e)*\sinh(f*x + e)^2 - 4*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d^2*e)*\cos \\
& h(f*x + e) - 4*((a^3 + a*b^2)*d^2*f*x + (a^3 + a*b^2)*d^2*e + ((a^2*b + b^3) \\
& )*d^2*f*x + (a^2*b + b^3)*d^2*e)*\cosh(f*x + e))*\sinh(f*x + e) - (a*b^2*d^2* \\
& f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f - (a*b^2*d^2* \\
& f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f)*\cosh(f*x + e) \\
& )^2 - (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d*f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d* \\
& e*f)*\sinh(f*x + e)^2 - 2*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*c*d*f^2*x - a^2*b*d^2 \\
& *e^2 + 2*a^2*b*c*d*e*f)*\cosh(f*x + e) - 2*(a^2*b*d^2*f^2*x^2 + 2*a^2*b*c*d* \\
& f^2*x - a^2*b*d^2*e^2 + 2*a^2*b*c*d*e*f + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*c*d* \\
& f^2*x - a*b^2*d^2*e^2 + 2*a*b^2*c*d*e*f)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{ \\
& ((a^2 + b^2)/b^2)}*\log(-(a*\cosh(f*x + e) + a*\sinh(f*x + e) - (b*\cosh(f*x + \\
& e) + b*\sinh(f*x + e))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + 2*((a^3 + a*b^2)*d^2* \\
& f^2*x^2 + 2*(a^3 + a*b^2)*c*d*f^2*x - 2*(a^3 + a*b^2)*d^2*e^2 + 4*(a^3 + a* \\
& b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2 + 2*((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^ \\
& 2*b + b^3)*c*d*f^2*x - (a^2*b + b^3)*d^2*e^2 + 2*(a^2*b + b^3)*c*d*e*f)*\cos \\
& h(f*x + e))*\sinh(f*x + e))/((a^4*b + 2*a^2*b^3 + b^5)*f^3*\cosh(f*x + e)^2 + \\
& (a^4*b + 2*a^2*b^3 + b^5)*f^3*\sinh(f*x + e)^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4) \\
& )*f^3*\cosh(f*x + e) - (a^4*b + 2*a^2*b^3 + b^5)*f^3 + 2*((a^4*b + 2*a^2*b^3 \\
& + b^5)*f^3*\cosh(f*x + e) + (a^5 + 2*a^3*b^2 + a*b^4)*f^3)*\sinh(f*x + e))
\end{aligned}$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*2/(a+b\*sinh(f\*x+e))\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^2}{(b \sinh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^2/(a+b\*sinh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^2/(b\*sinh(f\*x + e) + a)^2, x)

$$3.175 \quad \int \frac{c+dx}{(a+b \sinh(e+fx))^2} dx$$

**Optimal.** Leaf size=254

$$\frac{adPolyLog\left(2, -\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2(a^2+b^2)^{3/2}} - \frac{adPolyLog\left(2, -\frac{be^{e+fx}}{\sqrt{a^2+b^2+a}}\right)}{f^2(a^2+b^2)^{3/2}} + \frac{a(c+dx) \log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}} + 1\right)}{f(a^2+b^2)^{3/2}} - \frac{a(c+dx) \log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2+a}} + 1\right)}{f(a^2+b^2)^{3/2}}$$

[Out] (a\*(c + d\*x)\*Log[1 + (b\*E^(e + f\*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*f) - (a\*(c + d\*x)\*Log[1 + (b\*E^(e + f\*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*f) + (d\*Log[a + b\*Sinh[e + f\*x]])/((a^2 + b^2)\*f^2) + (a\*d\*PolyLog[2, -((b\*E^(e + f\*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)\*f^2) - (a\*d\*PolyLog[2, -((b\*E^(e + f\*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)\*f^2) - (b\*(c + d\*x)\*Cosh[e + f\*x])/((a^2 + b^2)\*f\*(a + b\*Sinh[e + f\*x]))

**Rubi [A]** time = 0.442247, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {3324, 3322, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{adPolyLog\left(2, -\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{f^2(a^2+b^2)^{3/2}} - \frac{adPolyLog\left(2, -\frac{be^{e+fx}}{\sqrt{a^2+b^2+a}}\right)}{f^2(a^2+b^2)^{3/2}} + \frac{a(c+dx) \log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}} + 1\right)}{f(a^2+b^2)^{3/2}} - \frac{a(c+dx) \log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2+a}} + 1\right)}{f(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)/(a + b\*Sinh[e + f\*x])^2, x]

[Out] (a\*(c + d\*x)\*Log[1 + (b\*E^(e + f\*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*f) - (a\*(c + d\*x)\*Log[1 + (b\*E^(e + f\*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*f) + (d\*Log[a + b\*Sinh[e + f\*x]])/((a^2 + b^2)\*f^2) + (a\*d\*PolyLog[2, -((b\*E^(e + f\*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)\*f^2) - (a\*d\*PolyLog[2, -((b\*E^(e + f\*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)\*f^2) - (b\*(c + d\*x)\*Cosh[e + f\*x])/((a^2 + b^2)\*f\*(a + b\*Sinh[e + f\*x]))

**Rule 3324**

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)/((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]^2, x\_ Symbol] :> Simp[(b\*(c + d\*x)^m\*Cos[e + f\*x])/(f\*(a^2 - b^2)\*(a + b\*Sin[e + f\*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d\*x)^m/(a + b\*Sin[e + f\*x]), x],

$x] - \text{Dist}[(b*d*m)/(f*(a^2 - b^2)), \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x]]/(a + b*\text{Sin}[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[m, 0]$

### Rule 3322

$\text{Int}[(c + d*x)^m/(a + b*\text{sin}[e + f*x] + \text{Complex}[0, fz]*f*x)], x\_Symbol] :> \text{Dist}[2, \text{Int}[(c + d*x)^m*\text{E}^{-(I*e + f*fz*x)} / (-I*b + 2*a*\text{E}^{-(I*e + f*fz*x)} + I*b*\text{E}^{2*(-I*e + f*fz*x)})], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, fz\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[m, 0]$

### Rule 2264

$\text{Int}[(F + d*x)^m/(a + b*(F + d*x)^u + c*(F + d*x)^v)], x\_Symbol] :> \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c)/q, \text{Int}[(f + g*x)^m*(F + d*x)^u/(b - q + 2*c*(F + d*x)^u), x], x] - \text{Dist}[(2*c)/q, \text{Int}[(f + g*x)^m*(F + d*x)^u/(b + q + 2*c*(F + d*x)^u), x], x] /; \text{FreeQ}\{F, a, b, c, f, g\}, x] \&\& \text{EqQ}[v, 2*u] \&\& \text{LinearQ}[u, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[m, 0]$

### Rule 2190

$\text{Int}[(F + d*x)^n*(c + d*x)^m/(a + b*(F + d*x)^n)], x\_Symbol] :> \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F + d*x)^n)/a]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + (b*(F + d*x)^n)/a]], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

### Rule 2279

$\text{Int}[\text{Log}[a + b*(F + d*x)^n], x\_Symbol] :> \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F + d*x)^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \&\& \text{GtQ}[a, 0]$

### Rule 2391

$\text{Int}[\text{Log}[c + d*(F + d*x)^n]/(F + d*x), x\_Symbol] :> -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$

### Rule 2668

$\text{Int}[\text{cos}[e + f*x]^p*(a + b*\text{sin}[e + f*x])^m], x\_Symbol] :> \text{Dist}[1/(b^p*f), \text{Subst}[\text{Int}[(a + x)^m*(b^2 - x^2)^{(p-1)/2}], x], x, b*\text{Sin}[e + f*x], x] /; \text{FreeQ}\{a, b, e, f, m\}, x] \&\& \text{IntegerQ}[(p-1)/2] \&\& \text{NeQ}[a^2 - b^2, 0]$



Rule 31

Int[((a\_) + (b\_.)\*(x\_))^-1, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned}
\int \frac{c + dx}{(a + b \sinh(e + fx))^2} dx &= -\frac{b(c + dx) \cosh(e + fx)}{(a^2 + b^2) f(a + b \sinh(e + fx))} + \frac{a \int \frac{c+dx}{a+b \sinh(e+fx)} dx}{a^2 + b^2} + \frac{(bd) \int \frac{\cosh(e+fx)}{a+b \sinh(e+fx)} dx}{(a^2 + b^2) f} \\
&= -\frac{b(c + dx) \cosh(e + fx)}{(a^2 + b^2) f(a + b \sinh(e + fx))} + \frac{(2a) \int \frac{e^{e+fx}(c+dx)}{-b+2ae^{e+fx}+be^{2(e+fx)}} dx}{a^2 + b^2} + \frac{d \operatorname{Subst}\left(\int \frac{1}{a+x} dx, x, b \sinh(e + fx)\right)}{(a^2 + b^2) f} \\
&= \frac{d \log(a + b \sinh(e + fx))}{(a^2 + b^2) f^2} - \frac{b(c + dx) \cosh(e + fx)}{(a^2 + b^2) f(a + b \sinh(e + fx))} + \frac{(2ab) \int \frac{e^{e+fx}(c+dx)}{2a-2\sqrt{a^2+b^2}+2be^{e+fx}} dx}{(a^2 + b^2)^{3/2}} \\
&= \frac{a(c + dx) \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2 + b^2)^{3/2} f} - \frac{a(c + dx) \log\left(1 + \frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2 + b^2)^{3/2} f} + \frac{d \log(a + b \sinh(e + fx))}{(a^2 + b^2) f^2} \\
&= \frac{a(c + dx) \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2 + b^2)^{3/2} f} - \frac{a(c + dx) \log\left(1 + \frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2 + b^2)^{3/2} f} + \frac{d \log(a + b \sinh(e + fx))}{(a^2 + b^2) f^2} \\
&= \frac{a(c + dx) \log\left(1 + \frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2 + b^2)^{3/2} f} - \frac{a(c + dx) \log\left(1 + \frac{be^{e+fx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2 + b^2)^{3/2} f} + \frac{d \log(a + b \sinh(e + fx))}{(a^2 + b^2) f^2}
\end{aligned}$$

**Mathematica [A]** time = 1.07478, size = 194, normalized size = 0.76

$$\frac{a \left( d \operatorname{PolyLog}\left(2, \frac{be^{e+fx}}{\sqrt{a^2+b^2}-a}\right) - d \operatorname{PolyLog}\left(2, -\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}\right) + f(c+dx) \left( \log\left(\frac{be^{e+fx}}{a-\sqrt{a^2+b^2}}+1\right) - \log\left(\frac{be^{e+fx}}{\sqrt{a^2+b^2}+a}+1\right) \right) \right)}{\sqrt{a^2+b^2}} - \frac{bf(c+dx) \cosh(e+fx)}{a+b \sinh(e+fx)} + d \log(a + b \sinh(e + fx))$$


---


$$f^2 (a^2 + b^2)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)/(a + b\*Sinh[e + f\*x])^2, x]

[Out] (d\*Log[a + b\*Sinh[e + f\*x]] + (a\*(f\*(c + d\*x)\*(Log[1 + (b\*E^(e + f\*x))]/(a - Sqrt[a^2 + b^2])) - Log[1 + (b\*E^(e + f\*x))/(a + Sqrt[a^2 + b^2]])) + d\*Po

lyLog[2, (b\*E^(e + f\*x))/(-a + Sqrt[a^2 + b^2])] - d\*PolyLog[2, -((b\*E^(e + f\*x))/(a + Sqrt[a^2 + b^2]))]/Sqrt[a^2 + b^2] - (b\*f\*(c + d\*x)\*Cosh[e + f\*x])/(a + b\*Sinh[e + f\*x])/((a^2 + b^2)\*f^2)

**Maple [B]** time = 0.103, size = 519, normalized size = 2.

$$2 \frac{(dx + c)(ae^{fx+e} - b)}{f(a^2 + b^2)(be^{2fx+2e} + 2ae^{fx+e} - b)} - 2 \frac{d \ln(e^{fx+e})}{(a^2 + b^2)f^2} + \frac{d \ln(be^{2fx+2e} + 2ae^{fx+e} - b)}{(a^2 + b^2)f^2} - 2 \frac{ac}{(a^2 + b^2)^{3/2}f} \operatorname{Arctanh}\left(\frac{1}{2} \frac{2}{f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)/(a+b\*sinh(f\*x+e))^2,x)

[Out] 2\*(d\*x+c)\*(a\*exp(f\*x+e)-b)/f/(a^2+b^2)/(b\*exp(2\*f\*x+2\*e)+2\*a\*exp(f\*x+e)-b)-2/(a^2+b^2)/f^2\*d\*ln(exp(f\*x+e))+1/(a^2+b^2)/f^2\*d\*ln(b\*exp(2\*f\*x+2\*e)+2\*a\*exp(f\*x+e)-b)-2/(a^2+b^2)^(3/2)/f\*a\*c\*arctanh(1/2\*(2\*b\*exp(f\*x+e)+2\*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(3/2)/f\*d\*a\*ln((-b\*exp(f\*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))\*x+1/(a^2+b^2)^(3/2)/f^2\*d\*a\*ln((-b\*exp(f\*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))\*e-1/(a^2+b^2)^(3/2)/f\*d\*a\*ln((b\*exp(f\*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))\*x-1/(a^2+b^2)^(3/2)/f^2\*d\*a\*ln((b\*exp(f\*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))\*e+1/(a^2+b^2)^(3/2)/f^2\*d\*a\*dilog((-b\*exp(f\*x+e)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/(a^2+b^2)^(3/2)/f^2\*d\*a\*dilog((b\*exp(f\*x+e)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/(a^2+b^2)^(3/2)/f^2\*a\*d\*e\*arctanh(1/2\*(2\*b\*exp(f\*x+e)+2\*a)/(a^2+b^2)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*sinh(f\*x+e))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 3.115, size = 4073, normalized size = 16.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)/(a+b\*sinh(f\*x+e))^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & (2*(a^2*b + b^3)*d*e - 2*(a^2*b + b^3)*c*f - 2*((a^2*b + b^3)*d*f*x + (a^2* \\ & b + b^3)*d*e)*\cosh(f*x + e)^2 - 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e) \\ & * \sinh(f*x + e)^2 + (a*b^2*d*\cosh(f*x + e)^2 + a*b^2*d*\sinh(f*x + e)^2 + 2*a \\ & ^2*b*d*\cosh(f*x + e) - a*b^2*d + 2*(a*b^2*d*\cosh(f*x + e) + a^2*b*d)*\sinh(f \\ & *x + e))*\sqrt{(a^2 + b^2)/b^2}* \operatorname{dilog}((a*\cosh(f*x + e) + a*\sinh(f*x + e) + ( \\ & b*\cosh(f*x + e) + b*\sinh(f*x + e))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - (a*b \\ & ^2*d*\cosh(f*x + e)^2 + a*b^2*d*\sinh(f*x + e)^2 + 2*a^2*b*d*\cosh(f*x + e) - \\ & a*b^2*d + 2*(a*b^2*d*\cosh(f*x + e) + a^2*b*d)*\sinh(f*x + e))*\sqrt{(a^2 + b^ \\ & 2)/b^2}* \operatorname{dilog}((a*\cosh(f*x + e) + a*\sinh(f*x + e) - (b*\cosh(f*x + e) + b*\sin \\ & h(f*x + e))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - (a*b^2*d*f*x + a*b^2*d*e - \\ & (a*b^2*d*f*x + a*b^2*d*e)*\cosh(f*x + e)^2 - (a*b^2*d*f*x + a*b^2*d*e)*\sinh( \\ & f*x + e)^2 - 2*(a^2*b*d*f*x + a^2*b*d*e)*\cosh(f*x + e) - 2*(a^2*b*d*f*x + a \\ & ^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{(a^ \\ & 2 + b^2)/b^2}* \log(-(a*\cosh(f*x + e) + a*\sinh(f*x + e) + (b*\cosh(f*x + e) + \\ & b*\sinh(f*x + e))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + (a*b^2*d*f*x + a*b^2*d*e - \\ & (a*b^2*d*f*x + a*b^2*d*e)*\cosh(f*x + e)^2 - (a*b^2*d*f*x + a*b^2*d*e)*\sinh \\ & (f*x + e)^2 - 2*(a^2*b*d*f*x + a^2*b*d*e)*\cosh(f*x + e) - 2*(a^2*b*d*f*x + \\ & a^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{(a \\ & ^2 + b^2)/b^2}* \log(-(a*\cosh(f*x + e) + a*\sinh(f*x + e) - (b*\cosh(f*x + e) + \\ & b*\sinh(f*x + e))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 2*((a^3 + a*b^2)*d*f*x + \\ & 2*(a^3 + a*b^2)*d*e - (a^3 + a*b^2)*c*f)*\cosh(f*x + e) + ((a^2*b + b^3)*d*c \\ & osh(f*x + e)^2 + (a^2*b + b^3)*d*\sinh(f*x + e)^2 + 2*(a^3 + a*b^2)*d*\cosh(f \\ & *x + e) - (a^2*b + b^3)*d + 2*((a^2*b + b^3)*d*\cosh(f*x + e) + (a^3 + a*b^2) \\ & )*d)*\sinh(f*x + e) - (a*b^2*d*e - a*b^2*c*f - (a*b^2*d*e - a*b^2*c*f)*\cosh( \\ & f*x + e)^2 - (a*b^2*d*e - a*b^2*c*f)*\sinh(f*x + e)^2 - 2*(a^2*b*d*e - a^2*b \\ & *c*f)*\cosh(f*x + e) - 2*(a^2*b*d*e - a^2*b*c*f + (a*b^2*d*e - a*b^2*c*f)*co \\ & sh(f*x + e))*\sinh(f*x + e))*\sqrt{(a^2 + b^2)/b^2})*\log(2*b*\cosh(f*x + e) + \\ & 2*b*\sinh(f*x + e) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + ((a^2*b + b^3)*d*cos \\ & h(f*x + e)^2 + (a^2*b + b^3)*d*\sinh(f*x + e)^2 + 2*(a^3 + a*b^2)*d*\cosh(f*x \\ & + e) - (a^2*b + b^3)*d + 2*((a^2*b + b^3)*d*\cosh(f*x + e) + (a^3 + a*b^2)* \\ & d)*\sinh(f*x + e) + (a*b^2*d*e - a*b^2*c*f - (a*b^2*d*e - a*b^2*c*f)*\cosh(f \\ & x + e)^2 - (a*b^2*d*e - a*b^2*c*f)*\sinh(f*x + e)^2 - 2*(a^2*b*d*e - a^2*b*c \\ & *f)*\cosh(f*x + e) - 2*(a^2*b*d*e - a^2*b*c*f + (a*b^2*d*e - a*b^2*c*f)*cosh \\ & (f*x + e))*\sinh(f*x + e))*\sqrt{(a^2 + b^2)/b^2})*\log(2*b*\cosh(f*x + e) + 2* \\ & b*\sinh(f*x + e) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - 2*((a^3 + a*b^2)*d*f*x \\ & + 2*(a^3 + a*b^2)*d*e - (a^3 + a*b^2)*c*f + 2*((a^2*b + b^3)*d*f*x + (a^2* \end{aligned}$$

```

b + b^3)*d*e)*cosh(f*x + e))*sinh(f*x + e))/((a^4*b + 2*a^2*b^3 + b^5)*f^2*
cosh(f*x + e)^2 + (a^4*b + 2*a^2*b^3 + b^5)*f^2*sinh(f*x + e)^2 + 2*(a^5 +
2*a^3*b^2 + a*b^4)*f^2*cosh(f*x + e) - (a^4*b + 2*a^2*b^3 + b^5)*f^2 + 2*((
a^4*b + 2*a^2*b^3 + b^5)*f^2*cosh(f*x + e) + (a^5 + 2*a^3*b^2 + a*b^4)*f^2)
*sinh(f*x + e))

```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{dx + c}{(b \sinh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*sinh(f*x + e) + a)^2, x)
```

$$3.176 \quad \int \frac{1}{(c+dx)(a+b \sinh(e+fx))^2} dx$$

**Optimal.** Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)(a+b \sinh(e+fx))^2}, x\right)$$

[Out] Unintegrable[1/((c + d\*x)\*(a + b\*Sinh[e + f\*x])^2), x]

**Rubi [A]** time = 0.0615198, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(c+dx)(a+b \sinh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d\*x)\*(a + b\*Sinh[e + f\*x])^2), x]

[Out] Defer[Int][1/((c + d\*x)\*(a + b\*Sinh[e + f\*x])^2), x]

Rubi steps

$$\int \frac{1}{(c+dx)(a+b \sinh(e+fx))^2} dx = \int \frac{1}{(c+dx)(a+b \sinh(e+fx))^2} dx$$

**Mathematica [A]** time = 53.3547, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)(a+b \sinh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d\*x)\*(a + b\*Sinh[e + f\*x])^2), x]

[Out] Integrate[1/((c + d\*x)\*(a + b\*Sinh[e + f\*x])^2), x]

---

**Maple [A]** time = 0.283, size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)(a + b \sinh(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x+c)/(a+b\*sinh(f\*x+e))^2,x)

[Out] int(1/(d\*x+c)/(a+b\*sinh(f\*x+e))^2,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$2 \left( a e^{(fx+e)} - b \right)$$

---


$$a^2bcf + b^3cf + (a^2bdf + b^3df)x - (a^2bcfe^{(2e)} + b^3cfe^{(2e)} + (a^2bdfe^{(2e)} + b^3dfe^{(2e)})x)e^{(2fx)} - 2(a^3cfe^e + ab^2cfe^e + ($$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)/(a+b\*sinh(f\*x+e))^2,x, algorithm="maxima")

[Out]  $-2*(a*e^{(f*x + e)} - b)/(a^2*b*c*f + b^3*c*f + (a^2*b*d*f + b^3*d*f)*x - (a^2*b*c*f*e^{(2*e)} + b^3*c*f*e^{(2*e)} + (a^2*b*d*f*e^{(2*e)} + b^3*d*f*e^{(2*e)})*x)*e^{(2*f*x)} - 2*(a^3*c*f*e^e + a*b^2*c*f*e^e + (a^3*d*f*e^e + a*b^2*d*f*e^e)*x)*e^{(f*x)}) + \text{integrate}(2*(b*d - (a*d*f*x*e^e + (c*f*e^e + d*e^e)*a)*e^{(f*x)})/(a^2*b*c^2*f + b^3*c^2*f + (a^2*b*d^2*f + b^3*d^2*f)*x^2 + 2*(a^2*b*c*d*f + b^3*c*d*f)*x - (a^2*b*c^2*f*e^{(2*e)} + b^3*c^2*f*e^{(2*e)} + (a^2*b*d^2*f*e^{(2*e)} + b^3*d^2*f*e^{(2*e)})*x^2 + 2*(a^2*b*c*d*f*e^{(2*e)} + b^3*c*d*f*e^{(2*e)})*x)*e^{(2*f*x)} - 2*(a^3*c^2*f*e^e + a*b^2*c^2*f*e^e + (a^3*d^2*f*e^e + a*b^2*d^2*f*e^e)*x^2 + 2*(a^3*c*d*f*e^e + a*b^2*c*d*f*e^e)*x)*e^{(f*x)}), x)$

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{a^2dx + a^2c + (b^2dx + b^2c) \sinh(fx + e)^2 + 2(abdx + abc) \sinh(fx + e)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*sinh(f*x + e)^2 + 2*(a*b*d*x + a*b*c)*sinh(f*x + e)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e))**2,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)(b \sinh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*sinh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*sinh(f*x + e) + a)^2), x)
```

$$3.177 \quad \int \frac{1}{(c+dx)^2(a+b \sinh(e+fx))^2} dx$$

**Optimal.** Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(c+dx)^2(a+b \sinh(e+fx))^2}, x\right)$$

[Out] Unintegrable[1/((c + d\*x)^2\*(a + b\*Sinh[e + f\*x])^2), x]

**Rubi [A]** time = 0.0579459, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b \sinh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c + d\*x)^2\*(a + b\*Sinh[e + f\*x])^2),x]

[Out] Defer[Int][1/((c + d\*x)^2\*(a + b\*Sinh[e + f\*x])^2), x]

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b \sinh(e+fx))^2} dx = \int \frac{1}{(c+dx)^2(a+b \sinh(e+fx))^2} dx$$

**Mathematica [A]** time = 55.645, size = 0, normalized size = 0.

$$\int \frac{1}{(c+dx)^2(a+b \sinh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c + d\*x)^2\*(a + b\*Sinh[e + f\*x])^2),x]

[Out] Integrate[1/((c + d\*x)^2\*(a + b\*Sinh[e + f\*x])^2), x]



---

**Maple [A]** time = 0.47, size = 0, normalized size = 0.

$$\int \frac{1}{(dx+c)^2 (a+b \sinh (fx+e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*x+c)^2/(a+b\*sinh(f\*x+e))^2,x)

[Out] int(1/(d\*x+c)^2/(a+b\*sinh(f\*x+e))^2,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$a^2bc^2f + b^3c^2f + (a^2bd^2f + b^3d^2f)x^2 + 2(a^2bcd f + b^3cdf)x - (a^2bc^2fe^{(2e)} + b^3c^2fe^{(2e)} + (a^2bd^2fe^{(2e)} + b^3d^2fe^{(2e)}))e^{(2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+b\*sinh(f\*x+e))^2,x, algorithm="maxima")

[Out]  $-2*(a*e^{(f*x+e)} - b)/(a^2*b*c^2*f + b^3*c^2*f + (a^2*b*d^2*f + b^3*d^2*f)*x^2 + 2*(a^2*b*c*d*f + b^3*c*d*f)*x - (a^2*b*c^2*f*e^{(2e)} + b^3*c^2*f*e^{(2e)} + (a^2*b*d^2*f*e^{(2e)} + b^3*d^2*f*e^{(2e)}))*x^2 + 2*(a^2*b*c*d*f*e^{(2e)} + b^3*c*d*f*e^{(2e)})*x)*e^{(2*f*x)} - 2*(a^3*c^2*f*e^e + a*b^2*c^2*f*e^e + (a^3*d^2*f*e^e + a*b^2*d^2*f*e^e))*x^2 + 2*(a^3*c*d*f*e^e + a*b^2*c*d*f*e^e)*x)*e^{(f*x)} + integrate(2*(2*b*d - (a*d*f*x*e^e + (c*f*e^e + 2*d*e^e)*a)*e^{(f*x)})/(a^2*b*c^3*f + b^3*c^3*f + (a^2*b*d^3*f + b^3*d^3*f)*x^3 + 3*(a^2*b*c*d^2*f + b^3*c*d^2*f)*x^2 + 3*(a^2*b*c^2*d*f + b^3*c^2*d*f)*x - (a^2*b*c^3*f*e^{(2e)} + b^3*c^3*f*e^{(2e)} + (a^2*b*d^3*f*e^{(2e)} + b^3*d^3*f*e^{(2e)}))*x^3 + 3*(a^2*b*c*d^2*f*e^{(2e)} + b^3*c*d^2*f*e^{(2e)})*x^2 + 3*(a^2*b*c^2*d*f*e^{(2e)} + b^3*c^2*d*f*e^{(2e)})*x)*e^{(2*f*x)} - 2*(a^3*c^3*f*e^e + a*b^2*c^3*f*e^e + (a^3*d^3*f*e^e + a*b^2*d^3*f*e^e))*x^3 + 3*(a^3*c*d^2*f*e^e + a*b^2*c*d^2*f*e^e)*x^2 + 3*(a^3*c^2*d*f*e^e + a*b^2*c^2*d*f*e^e)*x)*e^{(f*x)}, x)$

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{a^2 d^2 x^2 + 2 a^2 c d x + a^2 c^2 + (b^2 d^2 x^2 + 2 b^2 c d x + b^2 c^2) \sinh(fx + e)^2 + 2 (abd^2 x^2 + 2 abcdx + abc^2) \sinh(fx + e)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+b\*sinh(f\*x+e))^2,x, algorithm="fricas")

[Out] integral(1/(a^2\*d^2\*x^2 + 2\*a^2\*c\*d\*x + a^2\*c^2 + (b^2\*d^2\*x^2 + 2\*b^2\*c\*d\*x + b^2\*c^2)\*sinh(f\*x + e)^2 + 2\*(a\*b\*d^2\*x^2 + 2\*a\*b\*c\*d\*x + a\*b\*c^2)\*sinh(f\*x + e)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)\*\*2/(a+b\*sinh(f\*x+e))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dx + c)^2 (b \sinh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*x+c)^2/(a+b\*sinh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate(1/((d\*x + c)^2\*(b\*sinh(f\*x + e) + a)^2), x)

$$3.178 \quad \int \frac{e+fx}{(a+b \sinh(c+dx))^3} dx$$

**Optimal.** Leaf size=544

$$\frac{3a^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2d^2 (a^2+b^2)^{5/2}} - \frac{3a^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{2d^2 (a^2+b^2)^{5/2}} - \frac{f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2d^2 (a^2+b^2)^{3/2}} + \frac{f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{2d^2 (a^2+b^2)^{3/2}}$$

```
[Out] (3*a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(2*(a^2 +
b^2)^(5/2)*d) - ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
(2*(a^2 + b^2)^(3/2)*d) - (3*a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])])/(2*(a^2 + b^2)^(5/2)*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2])])/(2*(a^2 + b^2)^(3/2)*d) + (3*a*f*Log[a + b*Sinh[c
+ d*x]])/(2*(a^2 + b^2)^2*d^2) + (3*a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/(2*(a^2 + b^2)^(5/2)*d^2) - (f*PolyLog[2, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/(2*(a^2 + b^2)^(3/2)*d^2) - (3*a^2*f*PolyLo
g[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(2*(a^2 + b^2)^(5/2)*d^2) +
(f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(2*(a^2 + b^2)^(3
/2)*d^2) - (b*(e + f*x)*Cosh[c + d*x])/(2*(a^2 + b^2)*d*(a + b*Sinh[c + d*x
])^2) - f/(2*(a^2 + b^2)*d^2*(a + b*Sinh[c + d*x])) - (3*a*b*(e + f*x)*Cosh
[c + d*x])/(2*(a^2 + b^2)^2*d*(a + b*Sinh[c + d*x]))
```

---

**Rubi [A]** time = 2.08106, antiderivative size = 544, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 11, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$ , Rules used = {3325, 3324, 3322, 2264, 2190, 2279, 2391, 2668, 31, 6742, 32}

$$\frac{3a^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2d^2 (a^2+b^2)^{5/2}} - \frac{3a^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{2d^2 (a^2+b^2)^{5/2}} - \frac{f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2d^2 (a^2+b^2)^{3/2}} + \frac{f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{2d^2 (a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(e + f*x)/(a + b*Sinh[c + d*x])^3, x]
```

```
[Out] (3*a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(2*(a^2 +
b^2)^(5/2)*d) - ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
(2*(a^2 + b^2)^(3/2)*d) - (3*a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2])])/(2*(a^2 + b^2)^(5/2)*d) + ((e + f*x)*Log[1 + (b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2])])/(2*(a^2 + b^2)^(3/2)*d) + (3*a*f*Log[a + b*Sinh[c
+ d*x]])/(2*(a^2 + b^2)^2*d^2) + (3*a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/(2*(a^2 + b^2)^(5/2)*d^2) - (f*PolyLog[2, -((b*E^(c +
```

$$\frac{d*x)}{(a - \sqrt{a^2 + b^2})})]/(2*(a^2 + b^2)^{(3/2)*d^2)} - (3*a^2*f*PolyLog[2, -((b*E^c + d*x)/(a + \sqrt{a^2 + b^2}))])/(2*(a^2 + b^2)^{(5/2)*d^2)} + (f*PolyLog[2, -((b*E^c + d*x)/(a + \sqrt{a^2 + b^2}))])/(2*(a^2 + b^2)^{(3/2)*d^2)} - (b*(e + f*x)*Cosh[c + d*x])/(2*(a^2 + b^2)*d*(a + b*\sinh[c + d*x]))^2 - f/(2*(a^2 + b^2)*d^2*(a + b*\sinh[c + d*x])) - (3*a*b*(e + f*x)*Cosh[c + d*x])/(2*(a^2 + b^2)^2*d*(a + b*\sinh[c + d*x]))$$

### Rule 3325

$$\text{Int}[\frac{((c_.) + (d_.)*(x_))^m}{((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)])^n}, x\_Symbol] \rightarrow -\text{Simp}[\frac{b*(c + d*x)^m*\cos[e + f*x]*(a + b*\sin[e + f*x])^{n+1}}{f*(n+1)*(a^2 - b^2)}, x] + (\text{Dist}[a/(a^2 - b^2), \text{Int}[(c + d*x)^m*(a + b*\sin[e + f*x])^{n+1}, x], x] - \text{Dist}[(b*(n+2))/((n+1)*(a^2 - b^2)), \text{Int}[(c + d*x)^m*\sin[e + f*x]*(a + b*\sin[e + f*x])^{n+1}, x], x] + \text{Dist}[(b*d*m)/(f*(n+1)*(a^2 - b^2)), \text{Int}[(c + d*x)^{m-1}*\cos[e + f*x]*(a + b*\sin[e + f*x])^{n+1}, x], x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{ILtQ}[n, -2] \&\& \text{IGtQ}[m, 0]$$

### Rule 3324

$$\text{Int}[\frac{((c_.) + (d_.)*(x_))^m}{((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^2}, x\_Symbol] \rightarrow \text{Simp}[\frac{b*(c + d*x)^m*\cos[e + f*x]}{f*(a^2 - b^2)*(a + b*\sin[e + f*x])}, x] + (\text{Dist}[a/(a^2 - b^2), \text{Int}[(c + d*x)^m/(a + b*\sin[e + f*x]), x], x] - \text{Dist}[(b*d*m)/(f*(a^2 - b^2)), \text{Int}[(c + d*x)^{m-1}*\cos[e + f*x]/(a + b*\sin[e + f*x]), x], x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[m, 0]$$

### Rule 3322

$$\text{Int}[\frac{((c_.) + (d_.)*(x_))^m}{((a_.) + (b_.)*\sin[(e_.) + (\text{Complex}[0, fz])*(f_.)*(x_)]}), x\_Symbol] \rightarrow \text{Dist}[2, \text{Int}[\frac{(c + d*x)^m*E^{-(I*e) + f*fz*x}}{-(I*b) + 2*a*E^{-(I*e) + f*fz*x} + I*b*E^{2*(-(I*e) + f*fz*x)}}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, fz\}, x \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[m, 0]$$

### Rule 2264

$$\text{Int}[\frac{(F_.)^{u_}*((f_.) + (g_.)*(x_))^m}{((a_.) + (b_.)*(F_.)^{u_} + (c_.)*(F_.)^{v_})}, x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c)/q, \text{Int}[\frac{(f + g*x)^m*F^u}{(b - q + 2*c*F^u)}, x], x] - \text{Dist}[(2*c)/q, \text{Int}[\frac{(f + g*x)^m*F^u}{(b + q + 2*c*F^u)}, x], x]) /; \text{FreeQ}\{F, a, b, c, f, g\}, x \&\& \text{EqQ}[v, 2*u] \&\& \text{LinearQ}[u, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[m, 0]$$

### Rule 2190

$$\text{Int}[\frac{((F_.)^{((g_.)*((e_.) + (f_.)*(x_)))^n}{((c_.) + (d_.)*(x_))^m}}{((c_.) + (d_.)*(x_))^m}$$

```
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2668

```
Int[cos[(e_) + (f_)*(x_)]^(p_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^(m
_)), x_Symbol] := Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

### Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 32

```
Int[((a_) + (b_)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{e+fx}{(a+b\sinh(c+dx))^3} dx &= -\frac{b(e+fx)\cosh(c+dx)}{2(a^2+b^2)d(a+b\sinh(c+dx))^2} + \frac{a\int\frac{e+fx}{(a+b\sinh(c+dx))^2}dx}{a^2+b^2} - \frac{b\int\frac{(e+fx)\sinh(c+dx)}{(a+b\sinh(c+dx))^2}dx}{2(a^2+b^2)} + \frac{bf}{2(a^2+b^2)} \\
&= -\frac{b(e+fx)\cosh(c+dx)}{2(a^2+b^2)d(a+b\sinh(c+dx))^2} - \frac{ab(e+fx)\cosh(c+dx)}{(a^2+b^2)^2d(a+b\sinh(c+dx))} + \frac{a^2\int\frac{e+fx}{a+b\sinh(c+dx)}dx}{(a^2+b^2)^2} \\
&= -\frac{b(e+fx)\cosh(c+dx)}{2(a^2+b^2)d(a+b\sinh(c+dx))^2} - \frac{f}{2(a^2+b^2)d^2(a+b\sinh(c+dx))} - \frac{ab(e+fx)\cosh(c+dx)}{(a^2+b^2)^2d(a+b\sinh(c+dx))} \\
&= \frac{af\log(a+b\sinh(c+dx))}{(a^2+b^2)^2d^2} - \frac{b(e+fx)\cosh(c+dx)}{2(a^2+b^2)d(a+b\sinh(c+dx))^2} - \frac{f}{2(a^2+b^2)d^2(a+b\sinh(c+dx))} \\
&= \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}d} - \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}d} + \frac{af\log(a+b\sinh(c+dx))}{(a^2+b^2)^2d^2} \\
&= \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}d} - \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}d} - \frac{a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}d} \\
&= \frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{5/2}d} - \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}d} - \frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{5/2}d} \\
&= \frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{5/2}d} - \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}d} - \frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{5/2}d} \\
&= \frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{5/2}d} - \frac{(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}d} - \frac{3a^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{5/2}d}
\end{aligned}$$

**Mathematica [A]** time = 9.85403, size = 836, normalized size = 1.54

$$\frac{-bde\cosh(c+dx)+bcf\cosh(c+dx)-bf(c+dx)\cosh(c+dx)}{2(a^2+b^2)d^2(a+b\sinh(c+dx))^2} - \frac{6\sqrt{a^2+b^2}f\tan^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{-a^2-b^2}}\right)a^2-4\sqrt{-a^2-b^2}de\tanh\left(\frac{c+dx}{2}\right)}{2(a^2+b^2)d^2(a+b\sinh(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)/(a + b\*Sinh[c + d\*x])^3,x]

[Out] 
$$\begin{aligned} & -(-3*a*\sqrt{-(a^2 + b^2)^2}*f*(c + d*x) + 6*a^2*\sqrt{a^2 + b^2}*f*\text{ArcTan}[(a \\ & + b*E^{(c + d*x)})/\sqrt{-a^2 - b^2}] - 4*a^2*\sqrt{-a^2 - b^2}*d*e*\text{ArcTanh}[(a \\ & + b*E^{(c + d*x)})/\sqrt{a^2 + b^2}] + 2*b^2*\sqrt{-a^2 - b^2}*d*e*\text{ArcTanh}[(a \\ & + b*E^{(c + d*x)})/\sqrt{a^2 + b^2}] + 6*a^2*\sqrt{-a^2 - b^2}*f*\text{ArcTanh}[(a + b \\ & *E^{(c + d*x)})/\sqrt{a^2 + b^2}] + 4*a^2*\sqrt{-a^2 - b^2}*c*f*\text{ArcTanh}[(a + b*E \\ & ^{(c + d*x)})/\sqrt{a^2 + b^2}] - 2*b^2*\sqrt{-a^2 - b^2}*c*f*\text{ArcTanh}[(a + b*E \\ & ^{(c + d*x)})/\sqrt{a^2 + b^2}] + 2*a^2*\sqrt{-a^2 - b^2}*f*(c + d*x)*\text{Log}[1 + ( \\ & b*E^{(c + d*x)})/(a - \sqrt{a^2 + b^2})] - b^2*\sqrt{-a^2 - b^2}*f*(c + d*x)*\text{Lo} \\ & \text{g}[1 + (b*E^{(c + d*x)})/(a - \sqrt{a^2 + b^2})] - 2*a^2*\sqrt{-a^2 - b^2}*f*(c \\ & + d*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \sqrt{a^2 + b^2})] + 3*a*\sqrt{-(a \\ & ^2 + b^2)^2}*f*\text{Log}[2*a*E^{(c + d*x)} + b*(-1 + E^{(2*(c + d*x))})] + \sqrt{-a^2 \\ & - b^2}*(2*a^2 - b^2)*f*\text{PolyLog}[2, (b*E^{(c + d*x)})/(-a + \sqrt{a^2 + b^2})] + \\ & \sqrt{-a^2 - b^2}*(-2*a^2 + b^2)*f*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \sqrt{a \\ & ^2 + b^2}))]/(2*(-(a^2 + b^2)^2)^{(3/2)}*d^2) + (-(b*d*e*\text{Cosh}[c + d*x]) + b* \\ & c*f*\text{Cosh}[c + d*x] - b*f*(c + d*x)*\text{Cosh}[c + d*x])/(2*(a^2 + b^2)*d^2*(a + b* \\ & \text{Sinh}[c + d*x])^2) + (-(a^2*f) - b^2*f - 3*a*b*d*e*\text{Cosh}[c + d*x] + 3*a*b*c*f \\ & *\text{Cosh}[c + d*x] - 3*a*b*f*(c + d*x)*\text{Cosh}[c + d*x])/(2*(a^2 + b^2)^2*d^2*(a + \\ & b*\text{Sinh}[c + d*x])) \end{aligned}$$

---

**Maple [B]** time = 0.192, size = 1232, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)/(a+b\*sinh(d\*x+c))^3,x)

[Out] 
$$\begin{aligned} & (2*a^2*b*d*f*x*\exp(3*d*x+3*c) - b^3*d*f*x*\exp(3*d*x+3*c) + 6*a^3*d*f*x*\exp(2*d* \\ & x+2*c) + 2*a^2*b*d*e*\exp(3*d*x+3*c) - 3*a*b^2*d*f*x*\exp(2*d*x+2*c) - b^3*d*e*\exp( \\ & 3*d*x+3*c) + 6*a^3*d*e*\exp(2*d*x+2*c) - 10*a^2*b*d*f*x*\exp(d*x+c) - a^2*b*f*\exp(3 \\ & *d*x+3*c) - 3*a*b^2*d*e*\exp(2*d*x+2*c) - b^3*d*f*x*\exp(d*x+c) - b^3*f*\exp(3*d*x+3 \\ & *c) - 2*a^3*f*\exp(2*d*x+2*c) - 10*a^2*b*d*e*\exp(d*x+c) + 3*a*b^2*d*f*x - 2*a*b^2*f* \\ & \exp(2*d*x+2*c) - b^3*d*e*\exp(d*x+c) + a^2*b*f*\exp(d*x+c) + 3*a*b^2*d*e + b^3*f*\exp( \\ & d*x+c))/d^2/(a^2+b^2)^2/(b*\exp(2*d*x+2*c) + 2*a*\exp(d*x+c) - b)^2 - 1/d^2/(a^2+b^ \\ & 2)^{(5/2)}*b^2*f*c*\text{arctanh}(1/2*(2*b*\exp(d*x+c) + 2*a)/(a^2+b^2)^{(1/2})) + 3/2/d^2/ \\ & (a^2+b^2)^2*a*f*\ln(b*\exp(2*d*x+2*c) + 2*a*\exp(d*x+c) - b) - 3/d^2/(a^2+b^2)^2*a*f \\ & *\ln(\exp(d*x+c)) - 2/d/(a^2+b^2)^{(5/2)}*a^2*e*\text{arctanh}(1/2*(2*b*\exp(d*x+c) + 2*a)/ \\ & (a^2+b^2)^{(1/2})) + 2/d^2/(a^2+b^2)^{(5/2)}*a^2*f*c*\text{arctanh}(1/2*(2*b*\exp(d*x+c) + \\ & 2*a)/(a^2+b^2)^{(1/2})) + 1/d/(a^2+b^2)^{(5/2)}*a^2*f*\ln((-b*\exp(d*x+c) + (a^2+b^2) \end{aligned}$$

$$\begin{aligned} & \frac{1}{d} \frac{(-a + (a^2 + b^2)^{1/2})}{(-a + (a^2 + b^2)^{1/2})} * x + \frac{1}{d^2} \frac{1}{(a^2 + b^2)^{5/2}} * a^2 * f * \ln\left(\frac{-b * \exp(dx + c) + (a^2 + b^2)^{1/2} - a}{(-a + (a^2 + b^2)^{1/2})}\right) \\ & - \frac{1}{d} \frac{1}{(a^2 + b^2)^{5/2}} * a^2 * f * \ln\left(\frac{(b * \exp(dx + c) + (a^2 + b^2)^{1/2} + a)}{(a + (a^2 + b^2)^{1/2})}\right) * x - \frac{1}{d^2} \frac{1}{(a^2 + b^2)^{5/2}} * a^2 * f * \ln\left(\frac{(b * \exp(dx + c) + (a^2 + b^2)^{1/2} + a)}{(a + (a^2 + b^2)^{1/2})}\right) \\ & + \frac{1}{d^2} \frac{1}{(a^2 + b^2)^{5/2}} * a^2 * f * \operatorname{dilog}\left(\frac{-b * \exp(dx + c) + (a^2 + b^2)^{1/2} - a}{(-a + (a^2 + b^2)^{1/2})}\right) \\ & - \frac{1}{d^2} \frac{1}{(a^2 + b^2)^{5/2}} * a^2 * f * \operatorname{dilog}\left(\frac{(b * \exp(dx + c) + (a^2 + b^2)^{1/2} + a)}{(a + (a^2 + b^2)^{1/2})}\right) \\ & + \frac{1}{d} \frac{1}{(a^2 + b^2)^{5/2}} * b^2 * e * \operatorname{arctanh}\left(\frac{1/2 * (2 * b * \exp(dx + c) + 2 * a)}{(a^2 + b^2)^{1/2}}\right) \\ & - \frac{1}{2} \frac{1}{d} \frac{1}{(a^2 + b^2)^{5/2}} * b^2 * f * \ln\left(\frac{-b * \exp(dx + c) + (a^2 + b^2)^{1/2} - a}{(-a + (a^2 + b^2)^{1/2})}\right) * x \\ & - \frac{1}{2} \frac{1}{d^2} \frac{1}{(a^2 + b^2)^{5/2}} * b^2 * f * \ln\left(\frac{-b * \exp(dx + c) + (a^2 + b^2)^{1/2} - a}{(-a + (a^2 + b^2)^{1/2})}\right) * c \\ & + \frac{1}{2} \frac{1}{d} \frac{1}{(a^2 + b^2)^{5/2}} * b^2 * f * \ln\left(\frac{(b * \exp(dx + c) + (a^2 + b^2)^{1/2} + a)}{(a + (a^2 + b^2)^{1/2})}\right) * x \\ & + \frac{1}{2} \frac{1}{d^2} \frac{1}{(a^2 + b^2)^{5/2}} * b^2 * f * \ln\left(\frac{(b * \exp(dx + c) + (a^2 + b^2)^{1/2} + a)}{(a + (a^2 + b^2)^{1/2})}\right) * c \\ & - \frac{1}{2} \frac{1}{d^2} \frac{1}{(a^2 + b^2)^{5/2}} * b^2 * f * \operatorname{dilog}\left(\frac{-b * \exp(dx + c) + (a^2 + b^2)^{1/2} - a}{(-a + (a^2 + b^2)^{1/2})}\right) \\ & + \frac{1}{2} \frac{1}{d^2} \frac{1}{(a^2 + b^2)^{5/2}} * b^2 * f * \operatorname{dilog}\left(\frac{(b * \exp(dx + c) + (a^2 + b^2)^{1/2} + a)}{(a + (a^2 + b^2)^{1/2})}\right) \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)/(a+b\*sinh(d\*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 3.71103, size = 13728, normalized size = 25.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)/(a+b\*sinh(d\*x+c))^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -\frac{1}{2} * (6 * ((a^3 * b^2 + a * b^4) * d * f * x + (a^3 * b^2 + a * b^4) * c * f) * \cosh(dx + c)^4 + \\ & 6 * ((a^3 * b^2 + a * b^4) * d * f * x + (a^3 * b^2 + a * b^4) * c * f) * \sinh(dx + c)^4 + 2 * (( \\ & 10 * a^4 * b + 11 * a^2 * b^3 + b^5) * d * f * x - (2 * a^4 * b + a^2 * b^3 - b^5) * d * e + (a^4 * b \\ & + 2 * a^2 * b^3 + b^5 + 12 * (a^4 * b + a^2 * b^3) * c) * f) * \cosh(dx + c)^3 + 2 * ((10 * a^4 * b \\ & + 11 * a^2 * b^3 + b^5) * d * f * x - (2 * a^4 * b + a^2 * b^3 - b^5) * d * e + (a^4 * b + 2 * \\ & a^2 * b^3 + b^5 + 12 * (a^4 * b + a^2 * b^3) * c) * f + 12 * ((a^3 * b^2 + a * b^4) * d * f * x + ( \end{aligned}$$



$$\begin{aligned}
& a^3 b^2 + a b^4) * c * f) * \cosh(d * x + c)) * \sinh(d * x + c)^3 - 6 * (a^3 b^2 + a b^4) * \\
& d * e + 6 * (a^3 b^2 + a b^4) * c * f + 2 * (3 * (2 * a^5 + a^3 b^2 - a b^4) * d * f * x - 3 * (2 \\
& * a^5 + a^3 b^2 - a b^4) * d * e + 2 * (a^5 + 2 * a^3 b^2 + a b^4 + 3 * (2 * a^5 + a^3 b \\
& ^2 - a b^4) * c) * f) * \cosh(d * x + c)^2 + 2 * (3 * (2 * a^5 + a^3 b^2 - a b^4) * d * f * x - \\
& 3 * (2 * a^5 + a^3 b^2 - a b^4) * d * e + 18 * ((a^3 b^2 + a b^4) * d * f * x + (a^3 b^2 + \\
& a b^4) * c * f) * \cosh(d * x + c)^2 + 2 * (a^5 + 2 * a^3 b^2 + a b^4 + 3 * (2 * a^5 + a^3 b \\
& ^2 - a b^4) * c) * f + 3 * ((10 * a^4 * b + 11 * a^2 * b^3 + b^5) * d * f * x - (2 * a^4 * b + a^2 * \\
& b^3 - b^5) * d * e + (a^4 * b + 2 * a^2 * b^3 + b^5 + 12 * (a^4 * b + a^2 * b^3) * c) * f) * \cosh \\
& (d * x + c)) * \sinh(d * x + c)^2 - ((2 * a^2 * b^3 - b^5) * f * \cosh(d * x + c)^4 + (2 * a^2 * \\
& b^3 - b^5) * f * \sinh(d * x + c)^4 + 4 * (2 * a^3 * b^2 - a b^4) * f * \cosh(d * x + c)^3 + 2 * \\
& (4 * a^4 * b - 4 * a^2 * b^3 + b^5) * f * \cosh(d * x + c)^2 + 4 * ((2 * a^2 * b^3 - b^5) * f * \cosh \\
& (d * x + c) + (2 * a^3 * b^2 - a b^4) * f) * \sinh(d * x + c)^3 - 4 * (2 * a^3 * b^2 - a b^4) * \\
& f * \cosh(d * x + c) + 2 * (3 * (2 * a^2 * b^3 - b^5) * f * \cosh(d * x + c)^2 + 6 * (2 * a^3 * b^2 - \\
& a b^4) * f * \cosh(d * x + c) + (4 * a^4 * b - 4 * a^2 * b^3 + b^5) * f) * \sinh(d * x + c)^2 + \\
& (2 * a^2 * b^3 - b^5) * f + 4 * ((2 * a^2 * b^3 - b^5) * f * \cosh(d * x + c)^3 + 3 * (2 * a^3 * b^2 \\
& - a b^4) * f * \cosh(d * x + c)^2 + (4 * a^4 * b - 4 * a^2 * b^3 + b^5) * f * \cosh(d * x + c) - \\
& (2 * a^3 * b^2 - a b^4) * f) * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} * \operatorname{dilog}((a * \cosh( \\
& d * x + c) + a * \sinh(d * x + c) + (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \sqrt{(a^2 \\
& + b^2) / b^2} - b) / b + 1) + ((2 * a^2 * b^3 - b^5) * f * \cosh(d * x + c)^4 + (2 * a^2 * b^3 \\
& - b^5) * f * \sinh(d * x + c)^4 + 4 * (2 * a^3 * b^2 - a b^4) * f * \cosh(d * x + c)^3 + 2 * (4 * \\
& a^4 * b - 4 * a^2 * b^3 + b^5) * f * \cosh(d * x + c)^2 + 4 * ((2 * a^2 * b^3 - b^5) * f * \cosh(d * x \\
& + c) + (2 * a^3 * b^2 - a b^4) * f) * \sinh(d * x + c)^3 - 4 * (2 * a^3 * b^2 - a b^4) * f * c \\
& osh(d * x + c) + 2 * (3 * (2 * a^2 * b^3 - b^5) * f * \cosh(d * x + c)^2 + 6 * (2 * a^3 * b^2 - a \\
& b^4) * f * \cosh(d * x + c) + (4 * a^4 * b - 4 * a^2 * b^3 + b^5) * f) * \sinh(d * x + c)^2 + (2 * \\
& a^2 * b^3 - b^5) * f + 4 * ((2 * a^2 * b^3 - b^5) * f * \cosh(d * x + c)^3 + 3 * (2 * a^3 * b^2 - \\
& a b^4) * f * \cosh(d * x + c)^2 + (4 * a^4 * b - 4 * a^2 * b^3 + b^5) * f * \cosh(d * x + c) - (2 \\
& * a^3 * b^2 - a b^4) * f) * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} * \operatorname{dilog}((a * \cosh(d * x \\
& + c) + a * \sinh(d * x + c) - (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \sqrt{(a^2 + b \\
& ^2) / b^2} - b) / b + 1) - (((2 * a^2 * b^3 - b^5) * d * f * x + (2 * a^2 * b^3 - b^5) * c * f) * c \\
& osh(d * x + c)^4 + ((2 * a^2 * b^3 - b^5) * d * f * x + (2 * a^2 * b^3 - b^5) * c * f) * \sinh(d * x \\
& + c)^4 + (2 * a^2 * b^3 - b^5) * d * f * x + 4 * ((2 * a^3 * b^2 - a b^4) * d * f * x + (2 * a^3 * b \\
& ^2 - a b^4) * c * f) * \cosh(d * x + c)^3 + 4 * ((2 * a^3 * b^2 - a b^4) * d * f * x + (2 * a^3 * b \\
& ^2 - a b^4) * c * f + ((2 * a^2 * b^3 - b^5) * d * f * x + (2 * a^2 * b^3 - b^5) * c * f) * \cosh(d * x \\
& + c)) * \sinh(d * x + c)^3 + (2 * a^2 * b^3 - b^5) * c * f + 2 * ((4 * a^4 * b - 4 * a^2 * b^3 + \\
& b^5) * d * f * x + (4 * a^4 * b - 4 * a^2 * b^3 + b^5) * c * f) * \cosh(d * x + c)^2 + 2 * ((4 * a^4 * b \\
& - 4 * a^2 * b^3 + b^5) * d * f * x + (4 * a^4 * b - 4 * a^2 * b^3 + b^5) * c * f + 3 * ((2 * a^2 * b^3 \\
& - b^5) * d * f * x + (2 * a^2 * b^3 - b^5) * c * f) * \cosh(d * x + c)^2 + 6 * ((2 * a^3 * b^2 - a \\
& b^4) * d * f * x + (2 * a^3 * b^2 - a b^4) * c * f) * \cosh(d * x + c)) * \sinh(d * x + c)^2 - 4 * (( \\
& 2 * a^3 * b^2 - a b^4) * d * f * x + (2 * a^3 * b^2 - a b^4) * c * f) * \cosh(d * x + c) - 4 * ((2 * a \\
& ^3 * b^2 - a b^4) * d * f * x - ((2 * a^2 * b^3 - b^5) * d * f * x + (2 * a^2 * b^3 - b^5) * c * f) * c \\
& osh(d * x + c)^3 + (2 * a^3 * b^2 - a b^4) * c * f - 3 * ((2 * a^3 * b^2 - a b^4) * d * f * x + ( \\
& 2 * a^3 * b^2 - a b^4) * c * f) * \cosh(d * x + c)^2 - ((4 * a^4 * b - 4 * a^2 * b^3 + b^5) * d * f * \\
& x + (4 * a^4 * b - 4 * a^2 * b^3 + b^5) * c * f) * \cosh(d * x + c)) * \sinh(d * x + c)) * \sqrt{(a^ \\
& 2 + b^2) / b^2} * \log(-(a * \cosh(d * x + c) + a * \sinh(d * x + c) + (b * \cosh(d * x + c) + \\
& b * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b) + (((2 * a^2 * b^3 - b^5) * d * f * x
\end{aligned}$$

$$\begin{aligned}
& + (2a^2b^3 - b^5)cf \cosh(dx + c)^4 + ((2a^2b^3 - b^5)dfx + (2a^2b^3 - b^5)cf) \sinh(dx + c)^4 + (2a^2b^3 - b^5)dfx + 4((2a^3b^2 - ab^4)dfx + (2a^3b^2 - ab^4)cf) \cosh(dx + c)^3 + 4((2a^3b^2 - ab^4)dfx + (2a^3b^2 - ab^4)cf) \cosh(dx + c) \sinh(dx + c)^3 + (2a^2b^3 - b^5)cf + 2((4a^4b - 4a^2b^3 + b^5)dfx + (4a^4b - 4a^2b^3 + b^5)cf) \cosh(dx + c)^2 + 2((4a^4b - 4a^2b^3 + b^5)dfx + (4a^4b - 4a^2b^3 + b^5)cf) \cosh(dx + c) \sinh(dx + c)^2 - 4((2a^3b^2 - ab^4)dfx + (2a^3b^2 - ab^4)cf) \cosh(dx + c) - 4((2a^3b^2 - ab^4)dfx - ((2a^2b^3 - b^5)dfx + (2a^2b^3 - b^5)cf) \cosh(dx + c)^3 + (2a^3b^2 - ab^4)cf - 3((2a^3b^2 - ab^4)dfx + (2a^3b^2 - ab^4)cf) \cosh(dx + c)^2 - ((4a^4b - 4a^2b^3 + b^5)dfx + (4a^4b - 4a^2b^3 + b^5)cf) \cosh(dx + c) \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} \log(-a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b) - 2((2a^4b + a^2b^3 - b^5)dfx - (10a^4b + 11a^2b^3 + b^5)dfe + (a^4b + 2a^2b^3 + b^5 + 12(a^4b + a^2b^3)c)f) \cosh(dx + c) - (3(a^3b^2 + ab^4)f \cosh(dx + c)^4 + 3(a^3b^2 + ab^4)f \sinh(dx + c)^4 + 12(a^4b + a^2b^3)f \cosh(dx + c)^3 + 6(2a^5 + a^3b^2 - ab^4)f \cosh(dx + c)^2 + 12((a^3b^2 + ab^4)f \cosh(dx + c) + (a^4b + a^2b^3)f) \sinh(dx + c)^3 - 12(a^4b + a^2b^3)f \cosh(dx + c) + 6(3(a^3b^2 + ab^4)f \cosh(dx + c)^2 + 6(a^4b + a^2b^3)f \cosh(dx + c) + (2a^5 + a^3b^2 - ab^4)f) \sinh(dx + c)^2 + 3(a^3b^2 + ab^4)f + 12((a^3b^2 + ab^4)f \cosh(dx + c)^3 + 3(a^4b + a^2b^3)f \cosh(dx + c)^2 + (2a^5 + a^3b^2 - ab^4)f \cosh(dx + c) - (a^4b + a^2b^3)f) \sinh(dx + c) - (((2a^2b^3 - b^5)dfe - (2a^2b^3 - b^5)cf) \cosh(dx + c)^4 + ((2a^2b^3 - b^5)dfe - (2a^2b^3 - b^5)cf) \sinh(dx + c)^4 + 4((2a^3b^2 - ab^4)dfe - (2a^3b^2 - ab^4)cf) \cosh(dx + c)^3 + 4((2a^3b^2 - ab^4)dfe - (2a^3b^2 - ab^4)cf) \cosh(dx + c) \sinh(dx + c)^3 + (2a^2b^3 - b^5)dfe - (2a^2b^3 - b^5)cf + 2((4a^4b - 4a^2b^3 + b^5)dfe - (4a^4b - 4a^2b^3 + b^5)cf) \cosh(dx + c)^2 + 2((4a^4b - 4a^2b^3 + b^5)dfe - (4a^4b - 4a^2b^3 + b^5)cf) \cosh(dx + c) \sinh(dx + c)^2 + 6((2a^3b^2 - ab^4)dfe - (2a^3b^2 - ab^4)cf) \cosh(dx + c) \sinh(dx + c)^2 - 4((2a^3b^2 - ab^4)dfe - (2a^3b^2 - ab^4)cf) \cosh(dx + c) + 4(((2a^2b^3 - b^5)dfe - (2a^2b^3 - b^5)cf) \cosh(dx + c)^3 - (2a^3b^2 - ab^4)dfe + (2a^3b^2 - ab^4)cf + 3((2a^3b^2 - ab^4)dfe - (2a^3b^2 - ab^4)cf) \cosh(dx + c)^2 + ((4a^4b - 4a^2b^3 + b^5)dfe - (4a^4b - 4a^2b^3 + b^5)cf) \cosh(dx + c) \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} \log(2b \cosh(dx + c) + 2b \sinh(dx + c) + 2b \sqrt{(a^2 + b^2)/b^2} + 2a) - (3(a^3b^2 + ab^4)f \cosh(dx + c)^4 + 3(a^3b^2 + ab^4)f \sinh(dx + c)^4 + 12(a^4b + a^2b^3)f \cosh(dx + c)^3 + 6(2a^5 + a^3b^2 - ab^4)f \cosh(dx + c)^2 + 12((a^3b^2 + ab^4)f \cosh(dx + c) + (a^4b + a^2b^3)f) \sinh(dx + c)^3 - 12(a^4b + a^
\end{aligned}$$

$$\begin{aligned}
& 2*b^3)*f*\cosh(d*x + c) + 6*(3*(a^3*b^2 + a*b^4)*f*\cosh(d*x + c)^2 + 6*(a^4*b + a^2*b^3)*f*\cosh(d*x + c) + (2*a^5 + a^3*b^2 - a*b^4)*f)*\sinh(d*x + c)^2 \\
& + 3*(a^3*b^2 + a*b^4)*f + 12*((a^3*b^2 + a*b^4)*f*\cosh(d*x + c)^3 + 3*(a^4*b + a^2*b^3)*f*\cosh(d*x + c)^2 + (2*a^5 + a^3*b^2 - a*b^4)*f*\cosh(d*x + c) \\
& - (a^4*b + a^2*b^3)*f)*\sinh(d*x + c) + (((2*a^2*b^3 - b^5)*d*e - (2*a^2*b^3 - b^5)*c*f)*\cosh(d*x + c)^4 + ((2*a^2*b^3 - b^5)*d*e - (2*a^2*b^3 - b^5)*c*f)*\sinh(d*x + c)^4 + 4*((2*a^3*b^2 - a*b^4)*d*e - (2*a^3*b^2 - a*b^4)*c*f)*\cosh(d*x + c)^3 + 4*((2*a^3*b^2 - a*b^4)*d*e - (2*a^3*b^2 - a*b^4)*c*f + ((2*a^2*b^3 - b^5)*d*e - (2*a^2*b^3 - b^5)*c*f)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (2*a^2*b^3 - b^5)*d*e - (2*a^2*b^3 - b^5)*c*f + 2*((4*a^4*b - 4*a^2*b^3 + b^5)*d*e - (4*a^4*b - 4*a^2*b^3 + b^5)*c*f)*\cosh(d*x + c)^2 + 2*((4*a^4*b - 4*a^2*b^3 + b^5)*d*e - (4*a^4*b - 4*a^2*b^3 + b^5)*c*f + 3*((2*a^2*b^3 - b^5)*d*e - (2*a^2*b^3 - b^5)*c*f)*\cosh(d*x + c)^2 + 6*((2*a^3*b^2 - a*b^4)*d*e - (2*a^3*b^2 - a*b^4)*c*f)*\cosh(d*x + c))*\sinh(d*x + c)^2 - 4*((2*a^3*b^2 - a*b^4)*d*e - (2*a^3*b^2 - a*b^4)*c*f)*\cosh(d*x + c) + 4*((2*a^2*b^3 - b^5)*d*e - (2*a^2*b^3 - b^5)*c*f)*\cosh(d*x + c)^3 - (2*a^3*b^2 - a*b^4)*d*e + (2*a^3*b^2 - a*b^4)*c*f + 3*((2*a^3*b^2 - a*b^4)*d*e - (2*a^3*b^2 - a*b^4)*c*f)*\cosh(d*x + c)^2 + ((4*a^4*b - 4*a^2*b^3 + b^5)*d*e - (4*a^4*b - 4*a^2*b^3 + b^5)*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}) * \log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2}) + 2*a) - 2*((2*a^4*b + a^2*b^3 - b^5)*d*f*x - 12*((a^3*b^2 + a*b^4)*d*f*x + (a^3*b^2 + a*b^4)*c*f)*\cosh(d*x + c)^3 - (10*a^4*b + 11*a^2*b^3 + b^5)*d*e - 3*((10*a^4*b + 11*a^2*b^3 + b^5)*d*f*x - (2*a^4*b + a^2*b^3 - b^5)*d*e + (a^4*b + 2*a^2*b^3 + b^5 + 12*(a^4*b + a^2*b^3)*c)*f)*\cosh(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5 + 12*(a^4*b + a^2*b^3)*c)*f - 2*(3*(2*a^5 + a^3*b^2 - a*b^4)*d*f*x - 3*(2*a^5 + a^3*b^2 - a*b^4)*c)*f)*\cosh(d*x + c))*\sinh(d*x + c))/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d^2*\cosh(d*x + c)^4 + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d^2*\sinh(d*x + c)^4 + 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d^2*\cosh(d*x + c)^3 + 2*(2*a^8 + 5*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 - b^8)*d^2*\cosh(d*x + c)^2 - 4*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d^2*\cosh(d*x + c) + 4*((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d^2*\cosh(d*x + c) + (a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d^2)*\sinh(d*x + c)^3 + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d^2 + 2*(3*(a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d^2*\cosh(d*x + c)^2 + 6*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d^2*\cosh(d*x + c) + (2*a^8 + 5*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 - b^8)*d^2)*\sinh(d*x + c)^2 + 4*((a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d^2*\cosh(d*x + c)^3 + 3*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d^2*\cosh(d*x + c)^2 + (2*a^8 + 5*a^6*b^2 + 3*a^4*b^4 - a^2*b^6 - b^8)*d^2*\cosh(d*x + c) - (a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d^2)*\sinh(d*x + c))
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)/(a+b\*sinh(d\*x+c))\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{fx + e}{(b \sinh(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)/(a+b\*sinh(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((f\*x + e)/(b\*sinh(d\*x + c) + a)^3, x)

$$3.179 \quad \int \frac{1}{(e+fx)(a+b \sinh(c+dx))^3} dx$$

**Optimal.** Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(e+fx)(a+b \sinh(c+dx))^3}, x\right)$$

[Out] Unintegrable[1/((e + f\*x)\*(a + b\*Sinh[c + d\*x])^3), x]

**Rubi [A]** time = 0.0622738, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(e+fx)(a+b \sinh(c+dx))^3} dx$$

Verification is Not applicable to the result.

[In] Int[1/((e + f\*x)\*(a + b\*Sinh[c + d\*x])^3), x]

[Out] Defer[Int][1/((e + f\*x)\*(a + b\*Sinh[c + d\*x])^3), x]

Rubi steps

$$\int \frac{1}{(e+fx)(a+b \sinh(c+dx))^3} dx = \int \frac{1}{(e+fx)(a+b \sinh(c+dx))^3} dx$$

**Mathematica [A]** time = 104.981, size = 0, normalized size = 0.

$$\int \frac{1}{(e+fx)(a+b \sinh(c+dx))^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((e + f\*x)\*(a + b\*Sinh[c + d\*x])^3), x]

[Out] Integrate[1/((e + f\*x)\*(a + b\*Sinh[c + d\*x])^3), x]

---

**Maple [A]** time = 0.61, size = 0, normalized size = 0.

$$\int \frac{1}{(fx + e)(a + b \sinh(dx + c))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(f\*x+e)/(a+b\*sinh(d\*x+c))^3,x)

[Out] int(1/(f\*x+e)/(a+b\*sinh(d\*x+c))^3,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(f\*x+e)/(a+b\*sinh(d\*x+c))^3,x, algorithm="maxima")

[Out] (3\*a\*b^2\*d\*f\*x + 3\*a\*b^2\*d\*e + ((2\*d\*e + f)\*a^2\*b\*e^(3\*c) - (d\*e - f)\*b^3\*e^(3\*c) + (2\*a^2\*b\*d\*f\*e^(3\*c) - b^3\*d\*f\*e^(3\*c))\*x)\*e^(3\*d\*x) + (2\*(3\*d\*e + f)\*a^3\*e^(2\*c) - (3\*d\*e - 2\*f)\*a\*b^2\*e^(2\*c) + 3\*(2\*a^3\*d\*f\*e^(2\*c) - a\*b^2\*d\*f\*e^(2\*c))\*x)\*e^(2\*d\*x) - ((10\*d\*e + f)\*a^2\*b\*e^c + (d\*e + f)\*b^3\*e^c + (10\*a^2\*b\*d\*f\*e^c + b^3\*d\*f\*e^c)\*x)\*e^(d\*x))/(a^4\*b^2\*d^2\*e^2 + 2\*a^2\*b^4\*d^2\*e^2 + b^6\*d^2\*e^2 + (a^4\*b^2\*d^2\*f^2 + 2\*a^2\*b^4\*d^2\*f^2 + b^6\*d^2\*f^2)\*x^2 + 2\*(a^4\*b^2\*d^2\*e\*f + 2\*a^2\*b^4\*d^2\*e\*f + b^6\*d^2\*e\*f)\*x + (a^4\*b^2\*d^2\*e^2\*e^(4\*c) + 2\*a^2\*b^4\*d^2\*e^2\*e^(4\*c) + b^6\*d^2\*e^2\*e^(4\*c) + (a^4\*b^2\*d^2\*f^2\*e^(4\*c) + 2\*a^2\*b^4\*d^2\*f^2\*e^(4\*c) + b^6\*d^2\*f^2\*e^(4\*c))\*x^2 + 2\*(a^4\*b^2\*d^2\*e\*f\*e^(4\*c) + 2\*a^2\*b^4\*d^2\*e\*f\*e^(4\*c) + b^6\*d^2\*e\*f\*e^(4\*c))\*x)\*e^(4\*d\*x) + 4\*(a^5\*b\*d^2\*e^2\*e^(3\*c) + 2\*a^3\*b^3\*d^2\*e^2\*e^(3\*c) + a\*b^5\*d^2\*e^2\*e^(3\*c) + (a^5\*b\*d^2\*f^2\*e^(3\*c) + 2\*a^3\*b^3\*d^2\*f^2\*e^(3\*c) + a\*b^5\*d^2\*f^2\*e^(3\*c))\*x^2 + 2\*(a^5\*b\*d^2\*e\*f\*e^(3\*c) + 2\*a^3\*b^3\*d^2\*e\*f\*e^(3\*c) + a\*b^5\*d^2\*e\*f\*e^(3\*c))\*x)\*e^(3\*d\*x) + 2\*(2\*a^6\*d^2\*e^2\*e^(2\*c) + 3\*a^4\*b^2\*d^2\*e^2\*e^(2\*c) - b^6\*d^2\*e^2\*e^(2\*c) + (2\*a^6\*d^2\*f^2\*e^(2\*c) + 3\*a^4\*b^2\*d^2\*f^2\*e^(2\*c) - b^6\*d^2\*f^2\*e^(2\*c))\*x^2 + 2\*(2\*a^6\*d^2\*e\*f\*e^(2\*c) + 3\*a^4\*b^2\*d^2\*e\*f\*e^(2\*c) - b^6\*d^2\*e\*f\*e^(2\*c))\*x)\*e^(2\*d\*x) - 4\*(a^5\*b\*d^2\*e^2\*e^c + 2\*a^3\*b^3\*d^2\*e^2\*e^c + a\*b^5\*d^2\*e^2\*e^c + (a^5\*b\*d^2\*f^2\*e^c + 2\*a^3\*b^3\*d^2\*f^2\*e^c + a\*b^5\*d^2\*f^2\*e^c)\*x^2 + 2\*(a^5\*b\*d^2\*e\*f\*e^c + 2\*a^3\*b^3\*d^2\*e\*f\*e^c + a\*b^5\*d^2\*e\*f\*e^c)\*x)\*e^(d\*x)) + integrate((3\*a\*b\*d\*f^2\*x + 3\*a\*b\*d\*e\*f - ((2\*d^2\*e^2 + 3\*d\*e\*f + 2\*f^2)\*a^2\*e^c - (d^2\*e^

$$2 - 2*f^2)*b^2*e^c + (2*a^2*d^2*f^2*e^c - b^2*d^2*f^2*e^c)*x^2 - (2*b^2*d^2*e*f*e^c - (4*d^2*e*f + 3*d*f^2)*a^2*e^c)*x)*e^{(d*x)}/(a^4*b*d^2*e^3 + 2*a^2*b^3*d^2*e^3 + b^5*d^2*e^3 + (a^4*b*d^2*f^3 + 2*a^2*b^3*d^2*f^3 + b^5*d^2*f^3)*x^3 + 3*(a^4*b*d^2*e*f^2 + 2*a^2*b^3*d^2*e*f^2 + b^5*d^2*e*f^2)*x^2 + 3*(a^4*b*d^2*e^2*f + 2*a^2*b^3*d^2*e^2*f + b^5*d^2*e^2*f)*x - (a^4*b*d^2*e^3*e^{(2*c)} + 2*a^2*b^3*d^2*e^3*e^{(2*c)} + b^5*d^2*e^3*e^{(2*c)} + (a^4*b*d^2*f^3*e^{(2*c)} + 2*a^2*b^3*d^2*f^3*e^{(2*c)} + b^5*d^2*f^3*e^{(2*c)}))*x^3 + 3*(a^4*b*d^2*e*f^2*e^{(2*c)} + 2*a^2*b^3*d^2*e*f^2*e^{(2*c)} + b^5*d^2*e*f^2*e^{(2*c)})*x^2 + 3*(a^4*b*d^2*e^2*f*e^{(2*c)} + 2*a^2*b^3*d^2*e^2*f*e^{(2*c)} + b^5*d^2*e^2*f*e^{(2*c)})*x)*e^{(2*d*x)} - 2*(a^5*d^2*e^3*e^c + 2*a^3*b^2*d^2*e^3*e^c + a*b^4*d^2*e^3*e^c + (a^5*d^2*f^3*e^c + 2*a^3*b^2*d^2*f^3*e^c + a*b^4*d^2*f^3*e^c)*x^3 + 3*(a^5*d^2*e*f^2*e^c + 2*a^3*b^2*d^2*e*f^2*e^c + a*b^4*d^2*e*f^2*e^c)*x^2 + 3*(a^5*d^2*e^2*f*e^c + 2*a^3*b^2*d^2*e^2*f*e^c + a*b^4*d^2*e^2*f*e^c)*x)*e^{(d*x)}, x$$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^3fx + a^3e + (b^3fx + b^3e)\sinh(dx + c)^3 + 3(ab^2fx + ab^2e)\sinh(dx + c)^2 + 3(a^2bfx + a^2be)\sinh(dx + c)}\right), x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(f\*x+e)/(a+b\*sinh(d\*x+c))^3,x, algorithm="fricas")

[Out] integral(1/(a^3\*f\*x + a^3\*e + (b^3\*f\*x + b^3\*e)\*sinh(d\*x + c)^3 + 3\*(a\*b^2\*f\*x + a\*b^2\*e)\*sinh(d\*x + c)^2 + 3\*(a^2\*b\*f\*x + a^2\*b\*e)\*sinh(d\*x + c)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(f\*x+e)/(a+b\*sinh(d\*x+c))\*\*3,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(fx + e)(b \sinh(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(f*x+e)/(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((f*x + e)*(b*sinh(d*x + c) + a)^3), x)
```



$$3.180 \quad \int \frac{1}{(e+fx)^2(a+b \sinh(c+dx))^3} dx$$

**Optimal.** Leaf size=22

$$\text{Unintegrable}\left(\frac{1}{(e+fx)^2(a+b \sinh(c+dx))^3}, x\right)$$

[Out] Unintegrable[1/((e + f\*x)^2\*(a + b\*Sinh[c + d\*x])^3), x]

**Rubi [A]** time = 0.0611184, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(e+fx)^2(a+b \sinh(c+dx))^3} dx$$

Verification is Not applicable to the result.

[In] Int[1/((e + f\*x)^2\*(a + b\*Sinh[c + d\*x])^3), x]

[Out] Defer[Int][1/((e + f\*x)^2\*(a + b\*Sinh[c + d\*x])^3), x]

Rubi steps

$$\int \frac{1}{(e+fx)^2(a+b \sinh(c+dx))^3} dx = \int \frac{1}{(e+fx)^2(a+b \sinh(c+dx))^3} dx$$

**Mathematica [A]** time = 98.0677, size = 0, normalized size = 0.

$$\int \frac{1}{(e+fx)^2(a+b \sinh(c+dx))^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((e + f\*x)^2\*(a + b\*Sinh[c + d\*x])^3), x]

[Out] Integrate[1/((e + f\*x)^2\*(a + b\*Sinh[c + d\*x])^3), x]

---

**Maple [A]** time = 0.79, size = 0, normalized size = 0.

$$\int \frac{1}{(fx + e)^2 (a + b \sinh(dx + c))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(f\*x+e)^2/(a+b\*sinh(d\*x+c))^3,x)

[Out] int(1/(f\*x+e)^2/(a+b\*sinh(d\*x+c))^3,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(f\*x+e)^2/(a+b\*sinh(d\*x+c))^3,x, algorithm="maxima")

[Out] (3\*a\*b^2\*d\*f\*x + 3\*a\*b^2\*d\*e + (2\*(d\*e + f)\*a^2\*b\*e^(3\*c) - (d\*e - 2\*f)\*b^3\*e^(3\*c) + (2\*a^2\*b\*d\*f\*e^(3\*c) - b^3\*d\*f\*e^(3\*c))\*x)\*e^(3\*d\*x) + (2\*(3\*d\*e + 2\*f)\*a^3\*e^(2\*c) - (3\*d\*e - 4\*f)\*a\*b^2\*e^(2\*c) + 3\*(2\*a^3\*d\*f\*e^(2\*c) - a\*b^2\*d\*f\*e^(2\*c))\*x)\*e^(2\*d\*x) - (2\*(5\*d\*e + f)\*a^2\*b\*e^c + (d\*e + 2\*f)\*b^3\*e^c + (10\*a^2\*b\*d\*f\*e^c + b^3\*d\*f\*e^c)\*x)\*e^(d\*x))/(a^4\*b^2\*d^2\*e^3 + 2\*a^2\*b^4\*d^2\*e^3 + b^6\*d^2\*e^3 + (a^4\*b^2\*d^2\*f^3 + 2\*a^2\*b^4\*d^2\*f^3 + b^6\*d^2\*f^3)\*x^3 + 3\*(a^4\*b^2\*d^2\*e\*f^2 + 2\*a^2\*b^4\*d^2\*e\*f^2 + b^6\*d^2\*e\*f^2)\*x^2 + 3\*(a^4\*b^2\*d^2\*e^2\*f + 2\*a^2\*b^4\*d^2\*e^2\*f + b^6\*d^2\*e^2\*f)\*x + (a^4\*b^2\*d^2\*e^3\*e^(4\*c) + 2\*a^2\*b^4\*d^2\*e^3\*e^(4\*c) + b^6\*d^2\*e^3\*e^(4\*c) + (a^4\*b^2\*d^2\*f^3\*e^(4\*c) + 2\*a^2\*b^4\*d^2\*f^3\*e^(4\*c) + b^6\*d^2\*f^3\*e^(4\*c))\*x^3 + 3\*(a^4\*b^2\*d^2\*e\*f^2\*e^(4\*c) + 2\*a^2\*b^4\*d^2\*e\*f^2\*e^(4\*c) + b^6\*d^2\*e\*f^2\*e^(4\*c))\*x^2 + 3\*(a^4\*b^2\*d^2\*e^2\*f\*e^(4\*c) + 2\*a^2\*b^4\*d^2\*e^2\*f\*e^(4\*c) + b^6\*d^2\*e^2\*f\*e^(4\*c))\*x) + 4\*(a^5\*b\*d^2\*e^3\*e^(3\*c) + 2\*a^3\*b^3\*d^2\*e^3\*e^(3\*c) + a\*b^5\*d^2\*e^3\*e^(3\*c) + (a^5\*b\*d^2\*f^3\*e^(3\*c) + 2\*a^3\*b^3\*d^2\*f^3\*e^(3\*c) + a\*b^5\*d^2\*f^3\*e^(3\*c))\*x^3 + 3\*(a^5\*b\*d^2\*e\*f^2\*e^(3\*c) + 2\*a^3\*b^3\*d^2\*e\*f^2\*e^(3\*c) + a\*b^5\*d^2\*e\*f^2\*e^(3\*c))\*x^2 + 3\*(a^5\*b\*d^2\*e^2\*f\*e^(3\*c) + 2\*a^3\*b^3\*d^2\*e^2\*f\*e^(3\*c) + a\*b^5\*d^2\*e^2\*f\*e^(3\*c))\*x) + 2\*(2\*a^6\*d^2\*e^3\*e^(2\*c) + 3\*a^4\*b^2\*d^2\*e^3\*e^(2\*c) - b^6\*d^2\*e^3\*e^(2\*c) + (2\*a^6\*d^2\*f^3\*e^(2\*c) + 3\*a^4\*b^2\*d^2\*f^3\*e^(2\*c) - b^6\*d^2\*f^3\*e^(2\*c))\*x^3 + 3\*(2\*a^6\*d^2\*e\*f^2\*e^(2\*c) + 3\*a^4\*b^2\*d^2\*e\*f^2\*e^(2\*c) - b^6\*d^2\*e\*f^2\*e^(2\*c))\*x^2 + 3\*(2\*a^6\*d^2\*e^2\*f\*e^(2\*c) + 3\*a^4\*b^2

```

*d^2*e^2*f*e^(2*c) - b^6*d^2*e^2*f*e^(2*c))*x)*e^(2*d*x) - 4*(a^5*b*d^2*e^3
*e^c + 2*a^3*b^3*d^2*e^3*e^c + a*b^5*d^2*e^3*e^c + (a^5*b*d^2*f^3*e^c + 2*a
^3*b^3*d^2*f^3*e^c + a*b^5*d^2*f^3*e^c)*x^3 + 3*(a^5*b*d^2*e*f^2*e^c + 2*a
^3*b^3*d^2*e*f^2*e^c + a*b^5*d^2*e*f^2*e^c)*x^2 + 3*(a^5*b*d^2*e^2*f*e^c + 2
*a^3*b^3*d^2*e^2*f*e^c + a*b^5*d^2*e^2*f*e^c)*x)*e^(d*x)) + integrate((6*a*
b*d*f^2*x + 6*a*b*d*e*f - (2*(d^2*e^2 + 3*d*e*f + 3*f^2)*a^2*e^c - (d^2*e^2
- 6*f^2)*b^2*e^c + (2*a^2*d^2*f^2*e^c - b^2*d^2*f^2*e^c)*x^2 - 2*(b^2*d^2*
e*f*e^c - (2*d^2*e*f + 3*d*f^2)*a^2*e^c)*x)*e^(d*x))/(a^4*b*d^2*e^4 + 2*a^2
*b^3*d^2*e^4 + b^5*d^2*e^4 + (a^4*b*d^2*f^4 + 2*a^2*b^3*d^2*f^4 + b^5*d^2*f
^4)*x^4 + 4*(a^4*b*d^2*e*f^3 + 2*a^2*b^3*d^2*e*f^3 + b^5*d^2*e*f^3)*x^3 + 6
*(a^4*b*d^2*e^2*f^2 + 2*a^2*b^3*d^2*e^2*f^2 + b^5*d^2*e^2*f^2)*x^2 + 4*(a^4
*b*d^2*e^3*f + 2*a^2*b^3*d^2*e^3*f + b^5*d^2*e^3*f)*x - (a^4*b*d^2*e^4*e^(2
*c) + 2*a^2*b^3*d^2*e^4*e^(2*c) + b^5*d^2*e^4*e^(2*c) + (a^4*b*d^2*f^4*e^(2
*c) + 2*a^2*b^3*d^2*f^4*e^(2*c) + b^5*d^2*f^4*e^(2*c))*x^4 + 4*(a^4*b*d^2*e
*f^3*e^(2*c) + 2*a^2*b^3*d^2*e*f^3*e^(2*c) + b^5*d^2*e*f^3*e^(2*c))*x^3 + 6
*(a^4*b*d^2*e^2*f^2*e^(2*c) + 2*a^2*b^3*d^2*e^2*f^2*e^(2*c) + b^5*d^2*e^2*f
^2*e^(2*c))*x^2 + 4*(a^4*b*d^2*e^3*f*e^(2*c) + 2*a^2*b^3*d^2*e^3*f*e^(2*c)
+ b^5*d^2*e^3*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^5*d^2*e^4*e^c + 2*a^3*b^2*d^2*
e^4*e^c + a*b^4*d^2*e^4*e^c + (a^5*d^2*f^4*e^c + 2*a^3*b^2*d^2*f^4*e^c + a*
b^4*d^2*f^4*e^c)*x^4 + 4*(a^5*d^2*e*f^3*e^c + 2*a^3*b^2*d^2*e*f^3*e^c + a*b
^4*d^2*e*f^3*e^c)*x^3 + 6*(a^5*d^2*e^2*f^2*e^c + 2*a^3*b^2*d^2*e^2*f^2*e^c
+ a*b^4*d^2*e^2*f^2*e^c)*x^2 + 4*(a^5*d^2*e^3*f*e^c + 2*a^3*b^2*d^2*e^3*f*e
^c + a*b^4*d^2*e^3*f*e^c)*x)*e^(d*x)), x)

```

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{a^3 f^2 x^2 + 2 a^3 e f x + a^3 e^2 + (b^3 f^2 x^2 + 2 b^3 e f x + b^3 e^2) \sinh(dx + c)^3 + 3(ab^2 f^2 x^2 + 2 ab^2 e f x + ab^2 e^2) \sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(f\*x+e)^2/(a+b\*sinh(d\*x+c))^3,x, algorithm="fricas")

[Out] integral(1/(a^3\*f^2\*x^2 + 2\*a^3\*e\*f\*x + a^3\*e^2 + (b^3\*f^2\*x^2 + 2\*b^3\*e\*f\*x + b^3\*e^2)\*sinh(d\*x + c)^3 + 3\*(a\*b^2\*f^2\*x^2 + 2\*a\*b^2\*e\*f\*x + a\*b^2\*e^2)\*sinh(d\*x + c)^2 + 3\*(a^2\*b\*f^2\*x^2 + 2\*a^2\*b\*e\*f\*x + a^2\*b\*e^2)\*sinh(d\*x + c)), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(f\*x+e)\*\*2/(a+b\*sinh(d\*x+c))\*\*3,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(fx + e)^2 (b \sinh(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(f\*x+e)^2/(a+b\*sinh(d\*x+c))^3,x, algorithm="giac")

[Out] integrate(1/((f\*x + e)^2\*(b\*sinh(d\*x + c) + a)^3), x)

$$\mathbf{3.181} \quad \int (c + dx)^m (a + b \sinh(e + fx))^n dx$$

**Optimal.** Leaf size=22

$$\text{Unintegrable}((c + dx)^m (a + b \sinh(e + fx))^n, x)$$

[Out] Unintegrable[(c + d\*x)^m\*(a + b\*Sinh[e + f\*x])^n, x]

**Rubi [A]** time = 0.0546259, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int (c + dx)^m (a + b \sinh(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m\*(a + b\*Sinh[e + f\*x])^n,x]

[Out] Defer[Int] [(c + d\*x)^m\*(a + b\*Sinh[e + f\*x])^n, x]

Rubi steps

$$\int (c + dx)^m (a + b \sinh(e + fx))^n dx = \int (c + dx)^m (a + b \sinh(e + fx))^n dx$$

**Mathematica [A]** time = 4.44785, size = 0, normalized size = 0.

$$\int (c + dx)^m (a + b \sinh(e + fx))^n dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m\*(a + b\*Sinh[e + f\*x])^n,x]

[Out] Integrate[(c + d\*x)^m\*(a + b\*Sinh[e + f\*x])^n, x]

**Maple [A]** time = 0.042, size = 0, normalized size = 0.

$$\int (dx + c)^m (a + b \sinh(fx + e))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*(a+b\*sinh(f\*x+e))^n,x)

[Out] int((d\*x+c)^m\*(a+b\*sinh(f\*x+e))^n,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m (b \sinh(fx + e) + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+b\*sinh(f\*x+e))^n,x, algorithm="maxima")

[Out] integrate((d\*x + c)^m\*(b\*sinh(f\*x + e) + a)^n, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((dx + c)^m (b \sinh(fx + e) + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+b\*sinh(f\*x+e))^n,x, algorithm="fricas")

[Out] integral((d\*x + c)^m\*(b\*sinh(f\*x + e) + a)^n, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*sinh(f*x+e))**n,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int (dx + c)^m (b \sinh(fx + e) + a)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e))^n,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*(b*sinh(f*x + e) + a)^n, x)
```

### 3.182 $\int (c + dx)^m (a + b \sinh(e + fx))^3 dx$

**Optimal.** Leaf size=543

$$\frac{3a^2 b e^{-\frac{cf}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{f(c+dx)}{d}\right)}{2f} + \frac{3a^2 b e^{\frac{cf}{d}-e} (c + dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{f(c+dx)}{d}\right)}{2f} +$$

[Out]  $(a^3(c + dx)^{(1+m)})/(d(1+m)) - (3ab^2(c + dx)^{(1+m)})/(2d(1+m)) + (3^{(-1-m)}b^3E^{(3e - (3cf)/d)}(c + dx)^m \Gamma[1+m, (-3f(c + dx)/d)]/(8f^{(-((f(c + dx))/d))^m}) + (3^{2(-3-m)}ab^2E^{(2e - (2cf)/d)}(c + dx)^m \Gamma[1+m, (-2f(c + dx)/d)]/(f^{(-((f(c + dx))/d))^m}) + (3a^2bE^{(e - (cf)/d)}(c + dx)^m \Gamma[1+m, -((f(c + dx))/d)])/(2f^{(-((f(c + dx))/d))^m}) - (3b^3E^{(e - (cf)/d)}(c + dx)^m \Gamma[1+m, -((f(c + dx))/d)])/(8f^{(-((f(c + dx))/d))^m}) + (3a^2bE^{(-e + (cf)/d)}(c + dx)^m \Gamma[1+m, (f(c + dx)/d)]/(2f^{((f(c + dx))/d))^m}) - (3b^3E^{(-e + (cf)/d)}(c + dx)^m \Gamma[1+m, (f(c + dx)/d)]/(8f^{((f(c + dx))/d))^m}) - (3^{2(-3-m)}ab^2E^{(-2e + (2cf)/d)}(c + dx)^m \Gamma[1+m, (2f(c + dx)/d)]/(f^{((f(c + dx))/d))^m}) + (3^{(-1-m)}b^3E^{(-3e + (3cf)/d)}(c + dx)^m \Gamma[1+m, (3f(c + dx)/d)]/(8f^{((f(c + dx))/d))^m})$

**Rubi [A]** time = 0.807379, antiderivative size = 543, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3317, 3308, 2181, 3312, 3307}

$$\frac{3a^2 b e^{-\frac{cf}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, -\frac{f(c+dx)}{d}\right)}{2f} + \frac{3a^2 b e^{\frac{cf}{d}-e} (c + dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1, \frac{f(c+dx)}{d}\right)}{2f} +$$

Antiderivative was successfully verified.

[In] Int[(c + dx)^m\*(a + b\*Sinh[e + f\*x])^3,x]

[Out]  $(a^3(c + dx)^{(1+m)})/(d(1+m)) - (3ab^2(c + dx)^{(1+m)})/(2d(1+m)) + (3^{(-1-m)}b^3E^{(3e - (3cf)/d)}(c + dx)^m \Gamma[1+m, (-3f(c + dx)/d)]/(8f^{(-((f(c + dx))/d))^m}) + (3^{2(-3-m)}ab^2E^{(2e - (2cf)/d)}(c + dx)^m \Gamma[1+m, (-2f(c + dx)/d)]/(f^{(-((f(c + dx))/d))^m}) + (3a^2bE^{(e - (cf)/d)}(c + dx)^m \Gamma[1+m, -((f(c + dx))/d)])/(2f^{(-((f(c + dx))/d))^m}) - (3b^3E^{(e - (cf)/d)}(c + dx)^m \Gamma[1+m, -((f(c + dx))/d)])/(8f^{(-((f(c + dx))/d))^m}) + (3a^2bE^{(-e + (cf)/d)}(c + dx)^m \Gamma[1+m, (f(c + dx)/d)]/(2f^{((f(c + dx))/d))^m}) - (3b^3E^{(-e + (cf)/d)}(c + dx)^m \Gamma[1+m, (f(c + dx)/d)]/(8f^{((f(c + dx))/d))^m}) - (3^{2(-3-m)}ab^2E^{(-2e + (2cf)/d)}(c + dx)^m \Gamma[1+m, (2f(c + dx)/d)]/(f^{((f(c + dx))/d))^m}) + (3^{(-1-m)}b^3E^{(-3e + (3cf)/d)}(c + dx)^m \Gamma[1+m, (3f(c + dx)/d)]/(8f^{((f(c + dx))/d))^m})$



$$\begin{aligned} & /d)^m) - (3*b^3*E^{(-e + (c*f)/d)*(c + d*x)^m}*Gamma[1 + m, (f*(c + d*x))/d]) \\ & / (8*f*((f*(c + d*x))/d)^m) - (3*2^{(-3 - m)*a*b^2*E^{(-2*e + (2*c*f)/d)*(c + \\ & d*x)^m}*Gamma[1 + m, (2*f*(c + d*x))/d]) / (f*((f*(c + d*x))/d)^m) + (3^{(-1 - \\ & m)*b^3*E^{(-3*e + (3*c*f)/d)*(c + d*x)^m}*Gamma[1 + m, (3*f*(c + d*x))/d]) / (8 \\ & *f*((f*(c + d*x))/d)^m) \end{aligned}$$

### Rule 3317

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Int[ExpandIntegrand[(c + d*x)^m, (a + b*SIN[e + f*x])^n, x],
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

### Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

### Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:= -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

### Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

### Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

### Rubi steps

$$\begin{aligned}
\int (c + dx)^m (a + b \sinh(e + fx))^3 dx &= \int (a^3(c + dx)^m + 3a^2b(c + dx)^m \sinh(e + fx) + 3ab^2(c + dx)^m \sinh^2(e + fx) + b^3(c + dx)^m \sinh^3(e + fx)) dx \\
&= \frac{a^3(c + dx)^{1+m}}{d(1+m)} + (3a^2b) \int (c + dx)^m \sinh(e + fx) dx + (3ab^2) \int (c + dx)^m \sinh^2(e + fx) dx + b^3 \int (c + dx)^m \sinh^3(e + fx) dx \\
&= \frac{a^3(c + dx)^{1+m}}{d(1+m)} + \frac{1}{2} (3a^2b) \int e^{-i(i e + i f x)} (c + dx)^m dx - \frac{1}{2} (3a^2b) \int e^{i(i e + i f x)} (c + dx)^m dx \\
&= \frac{a^3(c + dx)^{1+m}}{d(1+m)} - \frac{3ab^2(c + dx)^{1+m}}{2d(1+m)} + \frac{3a^2be^{-\frac{cf}{d}}(c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{f(c+dx)}{d}\right)}{2f} \\
&= \frac{a^3(c + dx)^{1+m}}{d(1+m)} - \frac{3ab^2(c + dx)^{1+m}}{2d(1+m)} + \frac{3a^2be^{-\frac{cf}{d}}(c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{f(c+dx)}{d}\right)}{2f} \\
&= \frac{a^3(c + dx)^{1+m}}{d(1+m)} - \frac{3ab^2(c + dx)^{1+m}}{2d(1+m)} + \frac{3^{-1-m}b^3e^{3e-\frac{3cf}{d}}(c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{f(c+dx)}{d}\right)}{8f}
\end{aligned}$$

**Mathematica [A]** time = 1.74719, size = 448, normalized size = 0.83

$$2^{-m-3}3^{-m-1}e^{-3\left(\frac{cf}{d}+e\right)}(c+dx)^m\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(2^me^{\frac{3cf}{d}}\left(b^3d(m+1)e^{\frac{3cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^m\Gamma\left(m+1,\frac{3f(c+dx)}{d}\right)+4ae^{3e}f3^{m+1}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^m\*(a + b\*Sinh[e + f\*x])^3,x]

[Out] (2^(-3 - m)\*3^(-1 - m)\*(c + d\*x)^m\*(2^m\*b^3\*d\*E^(6\*e)\*(1 + m)\*((f\*(c + d\*x))/d)^m\*Gamma[1 + m, (-3\*f\*(c + d\*x))/d] + 3^(2 + m)\*a\*b^2\*d\*E^(5\*e + (c\*f)/d)\*(1 + m)\*(f\*(c/d + x))^m\*Gamma[1 + m, (-2\*f\*(c + d\*x))/d] - 2^m\*3^(2 + m)\*b\*(-4\*a^2 + b^2)\*d\*E^(4\*e + (2\*c\*f)/d)\*(1 + m)\*((f\*(c + d\*x))/d)^m\*Gamma[1 + m, -((f\*(c + d\*x))/d)] - 2^m\*3^(2 + m)\*b\*(-4\*a^2 + b^2)\*d\*E^(2\*e + (4\*c\*f)/d)\*(1 + m)\*(-((f\*(c + d\*x))/d))^m\*Gamma[1 + m, (f\*(c + d\*x))/d] - 3^(2 + m)\*a\*b^2\*d\*E^(e + (5\*c\*f)/d)\*(1 + m)\*(-((f\*(c + d\*x))/d))^m\*Gamma[1 + m, (2\*f\*(c + d\*x))/d] + 2^m\*E^((3\*c\*f)/d)\*(4\*3^(1 + m)\*a\*(2\*a^2 - 3\*b^2)\*E^(3\*e)\*f\*(c + d\*x)\*(-((f^2\*(c + d\*x)^2)/d^2))^m + b^3\*d\*E^((3\*c\*f)/d)\*(1 + m)\*(-((f\*(c + d\*x))/d))^m\*Gamma[1 + m, (3\*f\*(c + d\*x))/d]))/(d\*E^(3\*(e + (c\*f)/d))\*f\*(1 + m)\*(-((f^2\*(c + d\*x)^2)/d^2))^m)

**Maple [F]** time = 0.108, size = 0, normalized size = 0.

$$\int (dx + c)^m (a + b \sinh(fx + e))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m\*(a+b\*sinh(f\*x+e))^3,x)

[Out] int((d\*x+c)^m\*(a+b\*sinh(f\*x+e))^3,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+b\*sinh(f\*x+e))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.96735, size = 1885, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+b\*sinh(f\*x+e))^3,x, algorithm="fricas")

[Out] 
$$\frac{1}{24} \left( (b^3 d^m + b^3 d) \cosh\left(\frac{d m \log(3f/d) + 3d e - 3c f}{d}\right) \Gamma(m+1, \frac{3(d f x + c f)}{d}) - 9(a b^2 d^m + a b^2 d) \cosh\left(\frac{d m \log(2f/d) + 2d e - 2c f}{d}\right) \Gamma(m+1, \frac{2(d f x + c f)}{d}) + 9((4 a^2 b - b^3) d^m + (4 a^2 b - b^3) d) \cosh\left(\frac{d m \log(f/d) + d e - c f}{d}\right) \Gamma(m+1, \frac{d f x + c f}{d}) + 9((4 a^2 b - b^3) d^m + (4 a^2 b - b^3) d) \cosh\left(\frac{d m \log(-f/d) - d e + c f}{d}\right) \Gamma(m+1, \frac{-(d f x + c f)}{d}) + 9(a b^2 d^m + a b^2 d) \cosh\left(\frac{d m \log(-2f/d) - 2d e + 2c f}{d}\right) \Gamma(m+1, \frac{-2(d f x + c f)}{d}) + (b^3 d^m + b^3 d) \cosh\left(\frac{d m \log(-3f/d) - 3d e + 3c f}{d}\right) \Gamma(m+1, \frac{-3(d f x + c f)}{d}) - (b^3 d^m + b^3 d) \Gamma(m+1, \frac{3(d f x + c f)}{d}) \sinh\left(\frac{d m \log(3f/d) + 3d e - 3c f}{d}\right) + 9(a b^2 d^m + a b^2 d) \Gamma(m+1, \frac{2(d f x + c f)}{d}) \right)$$

$$\begin{aligned} & (d*f*x + c*f)/d)*\sinh((d*m*\log(2*f/d) + 2*d*e - 2*c*f)/d) - 9*((4*a^2*b - b^3)*d*m + (4*a^2*b - b^3)*d)*\gamma(m + 1, (d*f*x + c*f)/d)*\sinh((d*m*\log(f/d) + d*e - c*f)/d) - 9*((4*a^2*b - b^3)*d*m + (4*a^2*b - b^3)*d)*\gamma(m + 1, -(d*f*x + c*f)/d)*\sinh((d*m*\log(-f/d) - d*e + c*f)/d) - 9*(a*b^2*d*m + a*b^2*d)*\gamma(m + 1, -2*(d*f*x + c*f)/d)*\sinh((d*m*\log(-2*f/d) - 2*d*e + 2*c*f)/d) - (b^3*d*m + b^3*d)*\gamma(m + 1, -3*(d*f*x + c*f)/d)*\sinh((d*m*\log(-3*f/d) - 3*d*e + 3*c*f)/d) + 12*((2*a^3 - 3*a*b^2)*d*f*x + (2*a^3 - 3*a*b^2)*c*f)*\cosh(m*\log(d*x + c)) + 12*((2*a^3 - 3*a*b^2)*d*f*x + (2*a^3 - 3*a*b^2)*c*f)*\sinh(m*\log(d*x + c)))/(d*f*m + d*f) \end{aligned}$$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m\*(a+b\*sinh(f\*x+e))\*\*3,x)

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(fx + e) + a)^3 (dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+b\*sinh(f\*x+e))^3,x, algorithm="giac")

[Out] integrate((b\*sinh(f\*x + e) + a)^3\*(d\*x + c)^m, x)

### 3.183 $\int (c + dx)^m (a + b \sinh(e + fx))^2 dx$

**Optimal.** Leaf size=281

$$\frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{f} + \frac{abe^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{f} + \dots$$

[Out]  $(a^2(c+dx)^{(1+m)})/(d(1+m)) - (b^2(c+dx)^{(1+m)})/(2d(1+m)) + (2^{(-3-m)}b^2E^{(2e-(2cf)/d)}(c+dx)^m\Gamma[1+m,(-2f(c+dx))/d])/(f(-((f(c+dx))/d))^m) + (abE^e(e-(cf)/d)(c+dx)^m\Gamma[1+m,(-(f(c+dx))/d)])/(f(-((f(c+dx))/d))^m) + (abE^{-e+cf/d})(c+dx)^m\Gamma[1+m,(f(c+dx))/d])/(f((f(c+dx))/d)^m) - (2^{(-3-m)}b^2E^{(-2e+(2cf)/d)}(c+dx)^m\Gamma[1+m,(2f(c+dx))/d])/(f((f(c+dx))/d)^m)$

**Rubi [A]** time = 0.374911, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {3317, 3308, 2181, 3312, 3307}

$$\frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{f} + \frac{abe^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{f} + \dots$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^m\*(a + b\*Sinh[e + f\*x])^2,x]

[Out]  $(a^2(c+dx)^{(1+m)})/(d(1+m)) - (b^2(c+dx)^{(1+m)})/(2d(1+m)) + (2^{(-3-m)}b^2E^{(2e-(2cf)/d)}(c+dx)^m\Gamma[1+m,(-2f(c+dx))/d])/(f(-((f(c+dx))/d))^m) + (abE^e(e-(cf)/d)(c+dx)^m\Gamma[1+m,(-(f(c+dx))/d)])/(f(-((f(c+dx))/d))^m) + (abE^{-e+cf/d})(c+dx)^m\Gamma[1+m,(f(c+dx))/d])/(f((f(c+dx))/d)^m) - (2^{(-3-m)}b^2E^{(-2e+(2cf)/d)}(c+dx)^m\Gamma[1+m,(2f(c+dx))/d])/(f((f(c+dx))/d)^m)$

#### Rule 3317

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*Sin[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && (!RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}
\int (c + dx)^m (a + b \sinh(e + fx))^2 dx &= \int (a^2(c + dx)^m + 2ab(c + dx)^m \sinh(e + fx) + b^2(c + dx)^m \sinh^2(e + fx)) dx \\
&= \frac{a^2(c + dx)^{1+m}}{d(1+m)} + (2ab) \int (c + dx)^m \sinh(e + fx) dx + b^2 \int (c + dx)^m \sinh^2(e + fx) dx \\
&= \frac{a^2(c + dx)^{1+m}}{d(1+m)} + (ab) \int e^{-i(i e + i f x)} (c + dx)^m dx - (ab) \int e^{i(i e + i f x)} (c + dx)^m dx - b^2 \int (c + dx)^m \cosh^2(e + fx) dx \\
&= \frac{a^2(c + dx)^{1+m}}{d(1+m)} - \frac{b^2(c + dx)^{1+m}}{2d(1+m)} + \frac{abe^{e-\frac{cf}{d}}(c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{f(c+dx)}{d}\right)}{f} \\
&= \frac{a^2(c + dx)^{1+m}}{d(1+m)} - \frac{b^2(c + dx)^{1+m}}{2d(1+m)} + \frac{abe^{e-\frac{cf}{d}}(c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{f(c+dx)}{d}\right)}{f} \\
&= \frac{a^2(c + dx)^{1+m}}{d(1+m)} - \frac{b^2(c + dx)^{1+m}}{2d(1+m)} + \frac{2^{-3-m} b^2 e^{2e-\frac{2cf}{d}} (c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m, -\frac{f(c+dx)}{d}\right)}{f}
\end{aligned}$$

**Mathematica [A]** time = 0.764251, size = 254, normalized size = 0.9

$$(c + dx)^m \left( 8abd(m+1)e^{e-\frac{cf}{d}} \left(-\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, -\frac{f(c+dx)}{d}\right) + 8abd(m+1)e^{\frac{cf}{d}-e} \left(\frac{f(c+dx)}{d}\right)^{-m} \text{Gamma}\left(m+1, \frac{f(c+dx)}{d}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^m\*(a + b\*Sinh[e + f\*x])^2,x]

[Out] ((c + d\*x)^m\*(8\*a^2\*f\*(c + d\*x) - 4\*b^2\*f\*(c + d\*x) + (b^2\*d\*E^(2\*e - (2\*c\*f)/d))\*(1 + m)\*Gamma[1 + m, (-2\*f\*(c + d\*x))/d])/(2^m\*(-((f\*(c + d\*x))/d))^m) + (8\*a\*b\*d\*E^(e - (c\*f)/d)\*(1 + m)\*Gamma[1 + m, -((f\*(c + d\*x))/d)])/( -((f\*(c + d\*x))/d))^m + (8\*a\*b\*d\*E^(-e + (c\*f)/d)\*(1 + m)\*Gamma[1 + m, (f\*(c + d\*x))/d])/((f\*(c + d\*x))/d)^m - (b^2\*d\*E^(-2\*e + (2\*c\*f)/d)\*(1 + m)\*Gamma[1 + m, (2\*f\*(c + d\*x))/d])/(2^m\*((f\*(c + d\*x))/d)^m)/(8\*d\*f\*(1 + m))

**Maple [F]** time = 0.089, size = 0, normalized size = 0.

$$\int (dx + c)^m (a + b \sinh(fx + e))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x+c)^m*(a+b*sinh(f*x+e))^2,x)`

[Out] `int((d*x+c)^m*(a+b*sinh(f*x+e))^2,x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*(a+b*sinh(f*x+e))^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.49271, size = 1191, normalized size = 4.24

$$(b^2dm + b^2d) \cosh\left(\frac{dm \log\left(\frac{2f}{d}\right) + 2de - 2cf}{d}\right) \Gamma\left(m + 1, \frac{2(dfx+cf)}{d}\right) - 8(abdm + abd) \cosh\left(\frac{dm \log\left(\frac{f}{d}\right) + de - cf}{d}\right) \Gamma\left(m + 1, \frac{dfx+cf}{d}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x+c)^m*(a+b*sinh(f*x+e))^2,x, algorithm="fricas")`

[Out] `-1/8*((b^2*d*m + b^2*d)*cosh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) - 8*(a*b*d*m + a*b*d)*cosh((d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*x + c*f)/d) - 8*(a*b*d*m + a*b*d)*cosh((d*m*log(-f/d) - d*e + c*f)/d)*gamma(m + 1, -(d*f*x + c*f)/d) - (b^2*d*m + b^2*d)*cosh((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*gamma(m + 1, -2*(d*f*x + c*f)/d) - (b^2*d*m + b^2*d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d) + 8*(a*b*d*m + a*b*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh((d*m*log(f/d) + d*e - c*f)/d) + 8*(a*b*d*m + a*b*d)*gamma(m + 1, -(d*f*x + c*f)/d)*sinh((d*m*log(-f/d) - d*e + c*f)/d) + (b^2*d*m + b^2*d)*gamma(m + 1, -2*(d*f*x + c*f)/d)*sinh((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d) - 4*((2*a^2 - b^2)*d*f*x + (2*a^2 - b^2)*c*f)*cosh(m*log(d*x + c)) - 4*((2*a^2 - b^2)*d*f*x + (2*a^2 - b^2)*c*f)*sinh(m*log(d*x + c))/(d*f*m + d*f)`



**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m\*(a+b\*sinh(f\*x+e))\*\*2,x)

[Out] Exception raised: TypeError

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(fx + e) + a)^2 (dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m\*(a+b\*sinh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate((b\*sinh(f\*x + e) + a)^2\*(d\*x + c)^m, x)

### 3.184 $\int (c + dx)^m (a + b \sinh(e + fx)) dx$

**Optimal.** Leaf size=131

$$\frac{be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f} + \frac{be^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{2f} + \frac{a(c+dx)^{m+1}}{d(m+1)}$$

[Out] (a\*(c + d\*x)^(1 + m))/(d\*(1 + m)) + (b\*E^(e - (c\*f)/d)\*(c + d\*x)^m\*Gamma[1 + m, -(f\*(c + d\*x))/d])/(2\*f\*(-(f\*(c + d\*x))/d)^m) + (b\*E^(-e + (c\*f)/d)\*(c + d\*x)^m\*Gamma[1 + m, (f\*(c + d\*x))/d])/(2\*f\*((f\*(c + d\*x))/d)^m)

**Rubi [A]** time = 0.148223, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {3317, 3308, 2181}

$$\frac{be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f} + \frac{be^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{2f} + \frac{a(c+dx)^{m+1}}{d(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x)^m\*(a + b\*Sinh[e + f\*x]),x]

[Out] (a\*(c + d\*x)^(1 + m))/(d\*(1 + m)) + (b\*E^(e - (c\*f)/d)\*(c + d\*x)^m\*Gamma[1 + m, -(f\*(c + d\*x))/d])/(2\*f\*(-(f\*(c + d\*x))/d)^m) + (b\*E^(-e + (c\*f)/d)\*(c + d\*x)^m\*Gamma[1 + m, (f\*(c + d\*x))/d])/(2\*f\*((f\*(c + d\*x))/d)^m)

#### Rule 3317

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(c + d\*x)^m, (a + b\*sin[e + f\*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[m, 0] || NeQ[a^2 - b^2, 0])

#### Rule 3308

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> Dist[I/2, Int[(c + d\*x)^m/E^(I\*(e + f\*x)), x], x] - Dist[I/2, Int[(c + d\*x)^m\*E^(I\*(e + f\*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]

#### Rule 2181

```
Int[(F_)^((g_)*(e_) + (f_)*(x_))*((c_) + (d_)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Log[F])/d))*(c + d*x])]/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F])*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !IntegerQ[m]
```

### Rubi steps

$$\begin{aligned} \int (c + dx)^m (a + b \sinh(e + fx)) dx &= \int (a(c + dx)^m + b(c + dx)^m \sinh(e + fx)) dx \\ &= \frac{a(c + dx)^{1+m}}{d(1+m)} + b \int (c + dx)^m \sinh(e + fx) dx \\ &= \frac{a(c + dx)^{1+m}}{d(1+m)} + \frac{1}{2}b \int e^{-i(e+ifx)}(c + dx)^m dx - \frac{1}{2}b \int e^{i(e+ifx)}(c + dx)^m dx \\ &= \frac{a(c + dx)^{1+m}}{d(1+m)} + \frac{be^{-\frac{cf}{d}}(c + dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, -\frac{f(c+dx)}{d}\right)}{2f} + \frac{be^{-e+\frac{cf}{d}}(c + dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1 + m, \frac{f(c+dx)}{d}\right)}{2f} \end{aligned}$$

**Mathematica [A]** time = 0.191807, size = 118, normalized size = 0.9

$$\frac{1}{2}(c + dx)^m \left( \frac{be^{\frac{cf}{d}-e} \left(f\left(\frac{c}{d} + x\right)\right)^{-m} \Gamma\left(m + 1, \frac{f(c+dx)}{d}\right)}{f} + \frac{be^{-\frac{cf}{d}} \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m + 1, -\frac{f(c+dx)}{d}\right)}{f} + \frac{2a(c + dx)^m}{d(m + 1)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x)^m\*(a + b\*Sinh[e + f\*x]),x]

[Out] ((c + d\*x)^m\*((2\*a\*(c + d\*x))/(d\*(1 + m)) + (b\*E^(e - (c\*f)/d)\*Gamma[1 + m, -((f\*(c + d\*x))/d)])/(f\*(-((f\*(c + d\*x))/d))^m) + (b\*E^(-e + (c\*f)/d)\*Gamma[1 + m, (f\*(c + d\*x))/d])/(f\*(f\*(c/d + x))^m))/2

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int (dx + c)^m (a + b \sinh(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*sinh(f*x+e)),x)
```

```
[Out] int((d*x+c)^m*(a+b*sinh(f*x+e)),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 2.73051, size = 585, normalized size = 4.47

$$(b d m + b d) \cosh\left(\frac{d m \log\left(\frac{f}{d}\right) + d e - c f}{d}\right) \Gamma\left(m + 1, \frac{d f x + c f}{d}\right) + (b d m + b d) \cosh\left(\frac{d m \log\left(-\frac{f}{d}\right) - d e + c f}{d}\right) \Gamma\left(m + 1, -\frac{d f x + c f}{d}\right) - (b d m + b d)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*((b*d*m + b*d)*cosh((d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*x +
c*f)/d) + (b*d*m + b*d)*cosh((d*m*log(-f/d) - d*e + c*f)/d)*gamma(m + 1, -
(d*f*x + c*f)/d) - (b*d*m + b*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh((d*m*lo
g(f/d) + d*e - c*f)/d) - (b*d*m + b*d)*gamma(m + 1, -(d*f*x + c*f)/d)*sinh(
(d*m*log(-f/d) - d*e + c*f)/d) + 2*(a*d*f*x + a*c*f)*cosh(m*log(d*x + c)) +
2*(a*d*f*x + a*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*sinh(f*x+e)),x)
```

```
[Out] Exception raised: TypeError
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(fx + e) + a)(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*sinh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e) + a)*(d*x + c)^m, x)
```

$$3.185 \quad \int \frac{(c+dx)^m}{a+b \sinh(e+fx)} dx$$

**Optimal.** Leaf size=22

$$\text{Unintegrable}\left(\frac{(c+dx)^m}{a+b \sinh(e+fx)}, x\right)$$

[Out] Unintegrable[(c + d\*x)^m/(a + b\*Sinh[e + f\*x]), x]

**Rubi [A]** time = 0.0567216, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(c+dx)^m}{a+b \sinh(e+fx)} dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m/(a + b\*Sinh[e + f\*x]),x]

[Out] Defer[Int] [(c + d\*x)^m/(a + b\*Sinh[e + f\*x]), x]

Rubi steps

$$\int \frac{(c+dx)^m}{a+b \sinh(e+fx)} dx = \int \frac{(c+dx)^m}{a+b \sinh(e+fx)} dx$$

**Mathematica [A]** time = 1.19354, size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{a+b \sinh(e+fx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m/(a + b\*Sinh[e + f\*x]),x]

[Out] Integrate[(c + d\*x)^m/(a + b\*Sinh[e + f\*x]), x]

---

**Maple [A]** time = 0.03, size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{a + b \sinh(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m/(a+b\*sinh(f\*x+e)),x)

[Out] int((d\*x+c)^m/(a+b\*sinh(f\*x+e)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{b \sinh(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+b\*sinh(f\*x+e)),x, algorithm="maxima")

[Out] integrate((d\*x + c)^m/(b\*sinh(f\*x + e) + a), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx + c)^m}{b \sinh(fx + e) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+b\*sinh(f\*x+e)),x, algorithm="fricas")

[Out] integral((d\*x + c)^m/(b\*sinh(f\*x + e) + a), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(c + dx)^m}{a + b \sinh(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m/(a+b\*sinh(f\*x+e)),x)

[Out] Integral((c + d\*x)\*\*m/(a + b\*sinh(e + f\*x)), x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{b \sinh(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+b\*sinh(f\*x+e)),x, algorithm="giac")

[Out] integrate((d\*x + c)^m/(b\*sinh(f\*x + e) + a), x)



$$3.186 \quad \int \frac{(c+dx)^m}{(a+b \sinh(e+fx))^2} dx$$

**Optimal.** Leaf size=22

$$\text{Unintegrable} \left( \frac{(c+dx)^m}{(a+b \sinh(e+fx))^2}, x \right)$$

[Out] Unintegrable[(c + d\*x)^m/(a + b\*Sinh[e + f\*x])^2, x]

**Rubi [A]** time = 0.055234, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(c+dx)^m}{(a+b \sinh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[(c + d\*x)^m/(a + b\*Sinh[e + f\*x])^2,x]

[Out] Defer[Int] [(c + d\*x)^m/(a + b\*Sinh[e + f\*x])^2, x]

Rubi steps

$$\int \frac{(c+dx)^m}{(a+b \sinh(e+fx))^2} dx = \int \frac{(c+dx)^m}{(a+b \sinh(e+fx))^2} dx$$

**Mathematica [A]** time = 5.30235, size = 0, normalized size = 0.

$$\int \frac{(c+dx)^m}{(a+b \sinh(e+fx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x)^m/(a + b\*Sinh[e + f\*x])^2,x]

[Out] Integrate[(c + d\*x)^m/(a + b\*Sinh[e + f\*x])^2, x]

---

**Maple [A]** time = 0.068, size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{(a + b \sinh(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x+c)^m/(a+b\*sinh(f\*x+e))^2,x)

[Out] int((d\*x+c)^m/(a+b\*sinh(f\*x+e))^2,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{(b \sinh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+b\*sinh(f\*x+e))^2,x, algorithm="maxima")

[Out] integrate((d\*x + c)^m/(b\*sinh(f\*x + e) + a)^2, x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx + c)^m}{b^2 \sinh(fx + e)^2 + 2ab \sinh(fx + e) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+b\*sinh(f\*x+e))^2,x, algorithm="fricas")

[Out] integral((d\*x + c)^m/(b^2\*sinh(f\*x + e)^2 + 2\*a\*b\*sinh(f\*x + e) + a^2), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(c + dx)^m}{(a + b \sinh(e + fx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)\*\*m/(a+b\*sinh(f\*x+e))\*\*2,x)

[Out] Integral((c + d\*x)\*\*m/(a + b\*sinh(e + f\*x))\*\*2, x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx + c)^m}{(b \sinh(fx + e) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x+c)^m/(a+b\*sinh(f\*x+e))^2,x, algorithm="giac")

[Out] integrate((d\*x + c)^m/(b\*sinh(f\*x + e) + a)^2, x)

$$3.187 \quad \int \frac{(e+fx)^3 \sinh(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=163

$$-\frac{12if^2(e+fx)\text{PolyLog}(2, -ie^{c+dx})}{ad^3} + \frac{12if^3\text{PolyLog}(3, -ie^{c+dx})}{ad^4} - \frac{6if(e+fx)^2 \log(1+ie^{c+dx})}{ad^2} + \frac{i(e+fx)^3 \tanh\left(\frac{c}{2} + \dots\right)}{ad}$$

[Out] (I\*(e + f\*x)^3)/(a\*d) - ((I/4)\*(e + f\*x)^4)/(a\*f) - ((6\*I)\*f\*(e + f\*x)^2\*Log[1 + I\*E^(c + d\*x)])/(a\*d^2) - ((12\*I)\*f^2\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*d^3) + ((12\*I)\*f^3\*PolyLog[3, (-I)\*E^(c + d\*x)])/(a\*d^4) + (I\*(e + f\*x)^3\*Tanh[c/2 + (I/4)\*Pi + (d\*x)/2])/(a\*d)

**Rubi [A]** time = 0.356154, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.31$ , Rules used = {5557, 32, 3318, 4184, 3716, 2190, 2531, 2282, 6589}

$$-\frac{12if^2(e+fx)\text{PolyLog}(2, -ie^{c+dx})}{ad^3} + \frac{12if^3\text{PolyLog}(3, -ie^{c+dx})}{ad^4} - \frac{6if(e+fx)^2 \log(1+ie^{c+dx})}{ad^2} + \frac{i(e+fx)^3 \tanh\left(\frac{c}{2} + \dots\right)}{ad}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Sinh[c + d\*x])/(a + I\*a\*Sinh[c + d\*x]), x]

[Out] (I\*(e + f\*x)^3)/(a\*d) - ((I/4)\*(e + f\*x)^4)/(a\*f) - ((6\*I)\*f\*(e + f\*x)^2\*Log[1 + I\*E^(c + d\*x)])/(a\*d^2) - ((12\*I)\*f^2\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*d^3) + ((12\*I)\*f^3\*PolyLog[3, (-I)\*E^(c + d\*x)])/(a\*d^4) + (I\*(e + f\*x)^3\*Tanh[c/2 + (I/4)\*Pi + (d\*x)/2])/(a\*d)

### Rule 5557

Int[(((e\_.) + (f\_.)\*(x\_.))^(m\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Sinh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sinh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 32

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
(f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_.)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \sinh(c+dx)}{a+ia \sinh(c+dx)} dx &= i \int \frac{(e+fx)^3}{a+ia \sinh(c+dx)} dx - \frac{i \int (e+fx)^3 dx}{a} \\
&= -\frac{i(e+fx)^4}{4af} + \frac{i \int (e+fx)^3 \operatorname{csc}^2\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{id x}{2}\right) dx}{2a} \\
&= -\frac{i(e+fx)^4}{4af} + \frac{i(e+fx)^3 \tanh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{ad} - \frac{(3if) \int (e+fx)^2 \operatorname{coth}\left(\frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2}\right) dx}{ad} \\
&= \frac{i(e+fx)^3}{ad} - \frac{i(e+fx)^4}{4af} + \frac{i(e+fx)^3 \tanh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{ad} + \frac{(6f) \int \frac{e^{2\left(\frac{c}{2} + \frac{dx}{2}\right)} (e+fx)^2}{1+ie^{2\left(\frac{c}{2} + \frac{dx}{2}\right)}} dx}{ad} \\
&= \frac{i(e+fx)^3}{ad} - \frac{i(e+fx)^4}{4af} - \frac{6if(e+fx)^2 \log(1+ie^{c+dx})}{ad^2} + \frac{i(e+fx)^3 \tanh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{ad} + \dots \\
&= \frac{i(e+fx)^3}{ad} - \frac{i(e+fx)^4}{4af} - \frac{6if(e+fx)^2 \log(1+ie^{c+dx})}{ad^2} - \frac{12if^2(e+fx) \operatorname{Li}_2(-ie^{c+dx})}{ad^3} + \dots \\
&= \frac{i(e+fx)^3}{ad} - \frac{i(e+fx)^4}{4af} - \frac{6if(e+fx)^2 \log(1+ie^{c+dx})}{ad^2} - \frac{12if^2(e+fx) \operatorname{Li}_2(-ie^{c+dx})}{ad^3} + \dots \\
&= \frac{i(e+fx)^3}{ad} - \frac{i(e+fx)^4}{4af} - \frac{6if(e+fx)^2 \log(1+ie^{c+dx})}{ad^2} - \frac{12if^2(e+fx) \operatorname{Li}_2(-ie^{c+dx})}{ad^3} + \dots
\end{aligned}$$

**Mathematica [A]** time = 3.27641, size = 232, normalized size = 1.42

$$\frac{8(6i(-e^c+i)f^2(d(e+fx)\operatorname{PolyLog}(2,ie^{-c-dx})+f\operatorname{PolyLog}(3,ie^{-c-dx}))+3(1+ie^c)d^2f(e+fx)^2 \log(1-ie^{-c-dx})+d^3(e+fx)^3)}{(e^c-i)d^4} + \frac{8i \sinh\left(\frac{dx}{2}\right)(e+fx)}{d(\cosh\left(\frac{c}{2}\right)+i \sinh\left(\frac{c}{2}\right))\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}$$

4a

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^3\*Sinh[c + d\*x])/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] 
$$\frac{((-I)*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) - (8*(d^3*(e + f*x)^3 + 3*d^2*(1 + I*E^c)*f*(e + f*x)^2*\text{Log}[1 - I*E^{(-c - d*x)}] + (6*I)*(I - E^c)*f^2*(d*(e + f*x)*\text{PolyLog}[2, I*E^{(-c - d*x)}] + f*\text{PolyLog}[3, I*E^{(-c - d*x)}]))/(d^4*(-I + E^c)) + ((8*I)*(e + f*x)^3*\text{Sinh}[(d*x)/2])/(d*(\text{Cosh}[c/2] + I*\text{Sinh}[c/2]))*(\text{Cosh}[(c + d*x)/2] + I*\text{Sinh}[(c + d*x)/2]))/(4*a)}$$

**Maple [B]** time = 0.103, size = 501, normalized size = 3.1

$$\frac{12if^3\text{polylog}(3, -ie^{dx+c})}{ad^4} - \frac{6if^3\ln(1 + ie^{dx+c})x^2}{ad^2} - \frac{12if^3\text{polylog}(2, -ie^{dx+c})x}{ad^3} + \frac{6ief^2c^2}{ad^3} - 2\frac{x^3f^3 + 3ef^2x^2 + 3e^2}{da(e^{dx+c} - i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*sinh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] 
$$12*I*f^3*\text{polylog}(3, -I*\exp(d*x+c))/a/d^4 - 6*I/d^2/a*f^3*\ln(1+I*\exp(d*x+c))*x^2 - 12*I/d^3/a*f^3*\text{polylog}(2, -I*\exp(d*x+c))*x + 6*I/d^3/a*e*f^2*c^2 - 2*(f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x + e^3)/d/a/(\exp(d*x+c) - I) - 6*I/d^2/a*\ln(\exp(d*x+c) - I)*e^2*f + 12*I/d^3/a*e*f^2*c*\ln(\exp(d*x+c) - I) - 4*I/d^4/a*f^3*c^3 - 12*I/d^3/a*e*f^2*\text{polylog}(2, -I*\exp(d*x+c)) + 6*I/d^4/a*f^3*c^2*\ln(1+I*\exp(d*x+c)) + 12*I/d^2/a*e*f^2*c*x - 6*I/d^3/a*f^3*c^2*x - 1/4*I/a*x^4*f^3 - 12*I/d^3/a*e*f^2*c*\ln(\exp(d*x+c)) - I/a*e^3*x - 6*I/d^4/a*f^3*c^2*\ln(\exp(d*x+c) - I) + 2*I/d/a*f^3*x^3 - 12*I/d^3/a*e*f^2*\ln(1+I*\exp(d*x+c))*c - I/a*e*f^2*x^3 - 12*I/d^2/a*e*f^2*\ln(1+I*\exp(d*x+c))*x + 6*I/d^4/a*f^3*c^2*\ln(\exp(d*x+c)) + 6*I/d/a*e*f^2*x^2 - 3/2*I/a*e^2*f*x^2 + 6*I/d^2/a*\ln(\exp(d*x+c))*e^2*f$$

**Maxima [B]** time = 1.7308, size = 428, normalized size = 2.63

$$\frac{3}{2}e^2f\left(\frac{-idx^2 + (dx^2e^c - 4xe^c)e^{(dx)}}{iade^{(dx+c)} + ad} - \frac{4i\log\left(\left(e^{(dx+c)} - i\right)e^{(-c)}\right)}{ad^2}\right) - e^3\left(\frac{i(dx+c)}{ad} + \frac{2}{(ae^{(-dx-c)} + ia)d}\right) - \frac{df^3x^4 + 24ef^2x^2}{da(e^{dx+c} - i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*sinh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$3/2*e^2*f*((-I*d*x^2 + (d*x^2*e^c - 4*x*e^c)*e^{(d*x)})/(I*a*d*e^{(d*x + c)} + a*d) - 4*I*\log((e^{(d*x + c)} - I)*e^{(-c)})/(a*d^2)) - e^3*(I*(d*x + c)/(a*d)$$

$$+ 2/((a*e^{(-d*x - c)} + I*a*d)) - 1/4*(d*f^3*x^4 + 24*e*f^2*x^2 + 4*(d*e*f^2 + 2*f^3)*x^3 + (I*d*f^3*x^4*e^c + 4*I*d*e*f^2*x^3*e^c)*e^{(d*x)})/(a*d*e^{(d*x + c)} - I*a*d) - 12*I*(d*x*\log(I*e^{(d*x + c)} + 1) + \operatorname{dilog}(-I*e^{(d*x + c)})) * e*f^2/(a*d^3) - 6*I*(d^2*x^2*\log(I*e^{(d*x + c)} + 1) + 2*d*x*\operatorname{dilog}(-I*e^{(d*x + c)})) - 2*\operatorname{polylog}(3, -I*e^{(d*x + c)}) * f^3/(a*d^4) + (2*I*d^3*f^3*x^3 + 6*I*d^3*e*f^2*x^2)/(a*d^4)$$

**Fricas [C]** time = 2.48288, size = 1096, normalized size = 6.72

$$d^4 f^3 x^4 + 4 d^4 e f^2 x^3 + 6 d^4 e^2 f x^2 + 4 d^4 e^3 x + 8 d^3 e^3 - 24 c d^2 e^2 f + 24 c^2 d e f^2 - 8 c^3 f^3 + (48 d f^3 x + 48 d e f^2 - (-48 i d f^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2 + 4*d^4*e^3*x + 8*d^3*e^3 - 24*c*d^2*e^2*f + 24*c^2*d*e*f^2 - 8*c^3*f^3 + (48*d*f^3*x + 48*d*e*f^2 - (-48*I*d*f^3*x - 48*I*d*e*f^2)*e^{(d*x + c)})*\operatorname{dilog}(-I*e^{(d*x + c)}) - (-I*d^4*f^3*x^4 + 24*I*c*d^2*e^2*f - 24*I*c^2*d*e*f^2 + 8*I*c^3*f^3 + (-4*I*d^4*e*f^2 + 8*I*d^3*f^3)*x^3 + (-6*I*d^4*e^2*f + 24*I*d^3*e*f^2)*x^2 + (-4*I*d^4*e^3 + 24*I*d^3*e^2*f)*x)*e^{(d*x + c)} + (24*d^2*e^2*f - 48*c*d*e*f^2 + 24*c^2*f^3 - (-24*I*d^2*e^2*f + 48*I*c*d*e*f^2 - 24*I*c^2*f^3)*e^{(d*x + c)})*\log(e^{(d*x + c)} - I) + (24*d^2*f^3*x^2 + 48*d^2*e*f^2*x + 48*c*d*e*f^2 - 24*c^2*f^3 - (-24*I*d^2*f^3*x^2 - 48*I*d^2*e*f^2*x - 48*I*c*d*e*f^2 + 24*I*c^2*f^3)*e^{(d*x + c)})*\log(I*e^{(d*x + c)} + 1) - (48*I*f^3*e^{(d*x + c)} + 48*f^3)*\operatorname{polylog}(3, -I*e^{(d*x + c)})/(4*a*d^4*e^{(d*x + c)} - 4*I*a*d^4)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```



**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \sinh(dx + c)}{ia \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sinh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

$$3.188 \quad \int \frac{(e+fx)^2 \sinh(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=130

$$-\frac{4if^2 \text{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^3} - \frac{4if(e+fx) \log\left(1 + ie^{c+dx}\right)}{ad^2} + \frac{i(e+fx)^2 \tanh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)}{ad} + \frac{i(e+fx)^2}{ad} - \frac{i(e+fx)^3}{3af}$$

[Out] (I\*(e + f\*x)^2)/(a\*d) - ((I/3)\*(e + f\*x)^3)/(a\*f) - ((4\*I)\*f\*(e + f\*x)\*Log[1 + I\*E^(c + d\*x)])/(a\*d^2) - ((4\*I)\*f^2\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*d^3) + (I\*(e + f\*x)^2\*Tanh[c/2 + (I/4)\*Pi + (d\*x)/2])/(a\*d)

**Rubi [A]** time = 0.272064, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$ , Rules used = {5557, 32, 3318, 4184, 3716, 2190, 2279, 2391}

$$-\frac{4if^2 \text{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^3} - \frac{4if(e+fx) \log\left(1 + ie^{c+dx}\right)}{ad^2} + \frac{i(e+fx)^2 \tanh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)}{ad} + \frac{i(e+fx)^2}{ad} - \frac{i(e+fx)^3}{3af}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Sinh[c + d\*x])/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (I\*(e + f\*x)^2)/(a\*d) - ((I/3)\*(e + f\*x)^3)/(a\*f) - ((4\*I)\*f\*(e + f\*x)\*Log[1 + I\*E^(c + d\*x)])/(a\*d^2) - ((4\*I)\*f^2\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*d^3) + (I\*(e + f\*x)^2\*Tanh[c/2 + (I/4)\*Pi + (d\*x)/2])/(a\*d)

### Rule 5557

Int[(((e\_.) + (f\_.)\*(x\_.))^(m\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Sinh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sinh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 32

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
(f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.)/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \sinh(c+dx)}{a+ia \sinh(c+dx)} dx &= i \int \frac{(e+fx)^2}{a+ia \sinh(c+dx)} dx - \frac{i \int (e+fx)^2 dx}{a} \\
&= -\frac{i(e+fx)^3}{3af} + \frac{i \int (e+fx)^2 \csc^2\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{idx}{2}\right) dx}{2a} \\
&= -\frac{i(e+fx)^3}{3af} + \frac{i(e+fx)^2 \tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad} - \frac{(2if) \int (e+fx) \coth\left(\frac{c}{2}-\frac{i\pi}{4}+\frac{dx}{2}\right) dx}{ad} \\
&= \frac{i(e+fx)^2}{ad} - \frac{i(e+fx)^3}{3af} + \frac{i(e+fx)^2 \tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad} + \frac{(4f) \int \frac{e^{2\left(\frac{c}{2}+\frac{dx}{2}\right)}(e+fx)}{1+ie^{2\left(\frac{c}{2}+\frac{dx}{2}\right)}} dx}{ad} \\
&= \frac{i(e+fx)^2}{ad} - \frac{i(e+fx)^3}{3af} - \frac{4if(e+fx) \log(1+ie^{c+dx})}{ad^2} + \frac{i(e+fx)^2 \tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad} + \\
&= \frac{i(e+fx)^2}{ad} - \frac{i(e+fx)^3}{3af} - \frac{4if(e+fx) \log(1+ie^{c+dx})}{ad^2} + \frac{i(e+fx)^2 \tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad} + \\
&= \frac{i(e+fx)^2}{ad} - \frac{i(e+fx)^3}{3af} - \frac{4if(e+fx) \log(1+ie^{c+dx})}{ad^2} - \frac{4if^2 \text{Li}_2(-ie^{c+dx})}{ad^3} + \frac{i(e+fx)^2 \tanh\left(\frac{c}{2}+\frac{i\pi}{4}+\frac{dx}{2}\right)}{ad}
\end{aligned}$$

**Mathematica [A]** time = 2.45513, size = 188, normalized size = 1.45

$$\frac{3(4(1+ie^c)f^2 \text{PolyLog}(2, ie^{-c-dx}) - 2d(e+fx)(d(e+fx)+2(1+ie^c)f \log(1-ie^{-c-dx})))}{(e^c-i)d^3} + \frac{6i \sinh\left(\frac{dx}{2}\right)(e+fx)^2}{d(\cosh\left(\frac{c}{2}\right)+i \sinh\left(\frac{c}{2}\right))(\cosh\left(\frac{1}{2}(c+dx)\right)+i \sinh\left(\frac{1}{2}(c+dx)\right))} - ix(3e^2 + \dots)$$


---

3a

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Sinh[c + d\*x])/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] ((-I)\*x\*(3\*e^2 + 3\*e\*f\*x + f^2\*x^2) + (3\*(-2\*d\*(e + f\*x)\*(d\*(e + f\*x) + 2\*(1 + I\*E^c)\*f\*Log[1 - I\*E^(-c - d\*x)]) + 4\*(1 + I\*E^c)\*f^2\*PolyLog[2, I\*E^(-c - d\*x)])))/(d^3\*(-I + E^c)) + ((6\*I)\*(e + f\*x)^2\*Sinh[(d\*x)/2])/(d\*(Cosh[c/2] + I\*Sinh[c/2])\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]))/(3\*a)

---

**Maple [B]** time = 0.069, size = 269, normalized size = 2.1

$$\frac{-\frac{i}{3}x^3f^2}{a} - \frac{iefx^2}{a} - \frac{ie^2x}{a} - 2\frac{x^2f^2 + 2efx + e^2}{da(e^{dx+c} - i)} - \frac{4i \ln(e^{dx+c} - i)ef}{ad^2} + \frac{4i \ln(e^{dx+c})ef}{ad^2} + \frac{2if^2x^2}{da} + \frac{4if^2cx}{ad^2} + \frac{2if^2c^2}{ad^3} -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)`

[Out] 
$$-1/3*I/a*x^3*f^2 - I/a*e*f*x^2 - I/a*e^2*x - 2*(f^2*x^2 + 2*e*f*x + e^2)/d/a/(exp(d*x+c) - I) - 4*I/a/d^2*\ln(exp(d*x+c) - I)*e*f + 4*I/a/d^2*\ln(exp(d*x+c))*e*f + 2*I/a/d*f^2*x^2 + 4*I/a/d^2*f^2*c*x + 2*I/a/d^3*f^2*c^2 - 4*I/a/d^2*f^2*\ln(1 + I*exp(d*x+c))*x - 4*I/a/d^3*f^2*\ln(1 + I*exp(d*x+c))*c - 4*I*f^2*polylog(2, -I*exp(d*x+c))/a/d^3 + 4*I/a/d^3*f^2*c*\ln(exp(d*x+c) - I) - 4*I/a/d^3*f^2*c*\ln(exp(d*x+c))$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}f^2\left(\frac{2idx^3e^{(dx+c)} + 2dx^3 + 12x^2}{ade^{(dx+c)} - iad} - 24 \int \frac{x}{ade^{(dx+c)} - iad} dx\right) + ef\left(\frac{-idx^2 + (dx^2e^c - 4xe^c)e^{(dx)}}{iade^{(dx+c)} + ad} - \frac{4i \log((e^{(dx+c)} - i))}{ad^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out] 
$$-1/6*f^2*((2*I*d*x^3*e^{(d*x + c)} + 2*d*x^3 + 12*x^2)/(a*d*e^{(d*x + c)} - I*a*d) - 24*\integrate(x/(a*d*e^{(d*x + c)} - I*a*d), x)) + e*f*((-I*d*x^2 + (d*x^2*e^c - 4*x*e^c)*e^{(d*x)})/(I*a*d*e^{(d*x + c)} + a*d) - 4*I*\log((e^{(d*x + c)} - I)*e^{(-c)})/(a*d^2)) - e^2*(I*(d*x + c)/(a*d) + 2/((a*e^{(-d*x - c)} + I*a)*d))$$

**Fricas [B]** time = 2.46449, size = 649, normalized size = 4.99

$$\frac{d^3f^2x^3 + 3d^3efx^2 + 3d^3e^2x + 6d^2e^2 - 12cdef + 6c^2f^2 - (-12if^2e^{(dx+c)} - 12f^2)\text{Li}_2(-ie^{(dx+c)}) - (-id^3f^2x^3 + 12icd^2f^2x^2 + 12icd^2efx^2 + 12icd^2e^2x + 6d^2e^2 - 12cdef + 6c^2f^2)}{ad^3(e^{(dx+c)} - i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")`

```
[Out] -(d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*e^2*x + 6*d^2*e^2 - 12*c*d*e*f + 6*c^
2*f^2 - (-12*I*f^2*e^(d*x + c) - 12*f^2)*dilog(-I*e^(d*x + c)) - (-I*d^3*f^
2*x^3 + 12*I*c*d*e*f - 6*I*c^2*f^2 + (-3*I*d^3*e*f + 6*I*d^2*f^2)*x^2 + (-3
*I*d^3*e^2 + 12*I*d^2*e*f)*x)*e^(d*x + c) + (12*d*e*f - 12*c*f^2 - (-12*I*d
*e*f + 12*I*c*f^2)*e^(d*x + c))*log(e^(d*x + c) - I) + (12*d*f^2*x + 12*c*f
^2 - (-12*I*d*f^2*x - 12*I*c*f^2)*e^(d*x + c))*log(I*e^(d*x + c) + 1))/(3*a
*d^3*e^(d*x + c) - 3*I*a*d^3)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \sinh(dx + c)}{ia \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sinh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

$$3.189 \quad \int \frac{(e+fx) \sinh(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=90

$$-\frac{2if \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\right)}{ad^2} + \frac{i(e+fx) \tanh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)}{ad} - \frac{icx}{a} - \frac{ifx^2}{2a}$$

[Out]  $((-I)*e*x)/a - ((I/2)*f*x^2)/a - ((2*I)*f*Log[Cosh[c/2 + (I/4)*Pi + (d*x)/2]])/(a*d^2) + (I*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)$

**Rubi [A]** time = 0.112382, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {5557, 3318, 4184, 3475}

$$-\frac{2if \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\right)}{ad^2} + \frac{i(e+fx) \tanh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)}{ad} - \frac{icx}{a} - \frac{ifx^2}{2a}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(e + f*x)*\text{Sinh}[c + d*x]]/(a + I*a*\text{Sinh}[c + d*x]), x]$

[Out]  $((-I)*e*x)/a - ((I/2)*f*x^2)/a - ((2*I)*f*Log[Cosh[c/2 + (I/4)*Pi + (d*x)/2]])/(a*d^2) + (I*(e + f*x)*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)$

### Rule 5557

$\text{Int}[\frac{((e_.) + (f_.)*(x_))^{(m_.)}*\text{Sinh}[(c_.) + (d_.)*(x_)]^{(n_.)}}{(a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_)]}, x\_Symbol] \rightarrow \text{Dist}[1/b, \text{Int}[(e + f*x)^m*\text{Sinh}[c + d*x]^{(n-1)}, x], x] - \text{Dist}[a/b, \text{Int}[(e + f*x)^m*\text{Sinh}[c + d*x]^{(n-1)}]/(a + b*\text{Sinh}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[n, 0]$

### Rule 3318

$\text{Int}[\frac{((c_.) + (d_.)*(x_))^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^{(n_.)}}{x\_Symbol] \rightarrow \text{Dist}[(2*a)^n, \text{Int}[(c + d*x)^m*\text{Sin}[(1*(e + (Pi*a)/(2*b)))/2 + (f*x)/2]^{(2*n)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ (\text{GtQ}[n, 0] \ || \ \text{IGtQ}[m, 0])]$

### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{(e + fx) \sinh(c + dx)}{a + ia \sinh(c + dx)} dx &= i \int \frac{e + fx}{a + ia \sinh(c + dx)} dx - \frac{i \int (e + fx) dx}{a} \\ &= -\frac{iox}{a} - \frac{ifx^2}{2a} + \frac{i \int (e + fx) \csc^2\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{idx}{2}\right) dx}{2a} \\ &= -\frac{iox}{a} - \frac{ifx^2}{2a} + \frac{i(e + fx) \tanh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{ad} - \frac{(if) \int \coth\left(\frac{c}{2} - \frac{i\pi}{4} + \frac{dx}{2}\right) dx}{ad} \\ &= -\frac{iox}{a} - \frac{ifx^2}{2a} - \frac{2if \log\left(\cosh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)\right)}{ad^2} + \frac{i(e + fx) \tanh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{ad} \end{aligned}$$

**Mathematica [B]** time = 0.637151, size = 239, normalized size = 2.66

$$\frac{-i \cosh\left(\frac{dx}{2}\right) \left(2f \log(\cosh(c + dx)) + 4if \tan^{-1}\left(\sinh\left(\frac{dx}{2}\right) \operatorname{sech}\left(c + \frac{dx}{2}\right)\right) + d^2 x(2e + fx)\right) + 2d^2 ex \sinh\left(c + \frac{dx}{2}\right) + d^2 fx^2}{2ad^2 \left(\cosh\left(\frac{c}{2}\right) + I \sinh\left(\frac{c}{2}\right)\right) \left(\cosh\left(\frac{c + dx}{2}\right) + I \sinh\left(\frac{c + dx}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (-2*d*f*x*Cosh[c + (d*x)/2] - I*Cosh[(d*x)/2]*(d^2*x*(2*e + f*x) + (4*I)*f*
ArcTan[Sech[c + (d*x)/2]*Sinh[(d*x)/2]] + 2*f*Log[Cosh[c + d*x]]) + (4*I)*d
*e*Sinh[(d*x)/2] + (2*I)*d*f*x*Sinh[(d*x)/2] + 2*d^2*e*x*Sinh[c + (d*x)/2]
+ d^2*f*x^2*Sinh[c + (d*x)/2] + (4*I)*f*ArcTan[Sech[c + (d*x)/2]*Sinh[(d*x)
/2]]*Sinh[c + (d*x)/2] + 2*f*Log[Cosh[c + d*x]]*Sinh[c + (d*x)/2])/(2*a*d^2
*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2]))
```



**Maple [A]** time = 0.079, size = 86, normalized size = 1.

$$\frac{-\frac{i}{2}fx^2}{a} - \frac{ie^x}{a} + \frac{2ifx}{da} + \frac{2ifc}{ad^2} - 2\frac{fx+e}{da(e^{dx+c}-i)} - \frac{2if\ln(e^{dx+c}-i)}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)\*sinh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)

[Out]  $-1/2*I*f*x^2/a - I*e*x/a + 2*I*f/a/d*x + 2*I*f/a/d^2*c - 2*(f*x+e)/d/a/(exp(d*x+c)-I) - 2*I*f/a/d^2*\ln(exp(d*x+c)-I)$

**Maxima [A]** time = 1.26702, size = 146, normalized size = 1.62

$$\frac{1}{2}f\left(\frac{-i dx^2 + (dx^2 e^c - 4 x e^c)e^{(dx)}}{i a d e^{(dx+c)} + a d} - \frac{4i \log\left(\left(e^{(dx+c)} - i\right)e^{(-c)}\right)}{ad^2}\right) - e\left(\frac{i(dx+c)}{ad} + \frac{2}{(ae^{(-dx-c)} + ia)d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $1/2*f*((-I*d*x^2 + (d*x^2*e^c - 4*x*e^c)*e^{(d*x)})/(I*a*d*e^{(d*x+c)} + a*d) - 4*I*\log((e^{(d*x+c)} - I)*e^{(-c)})/(a*d^2)) - e*(I*(d*x+c)/(a*d) + 2/((a*e^{(-d*x-c)} + I*a)*d))$

**Fricas [A]** time = 2.53768, size = 235, normalized size = 2.61

$$\frac{d^2fx^2 + 2d^2ex + 4de - (-id^2fx^2 + (-2id^2e + 4idf)x)e^{(dx+c)} - (-4ife^{(dx+c)} - 4f)\log(e^{(dx+c)} - i)}{2ad^2e^{(dx+c)} - 2iad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $-(d^2*f*x^2 + 2*d^2*e*x + 4*d*e - (-I*d^2*f*x^2 + (-2*I*d^2*e + 4*I*d*f)*x)*e^{(d*x+c)} - (-4*I*f*e^{(d*x+c)} - 4*f)*\log(e^{(d*x+c)} - I))/(2*a*d^2*e^{(d*x+c)} - 2*I*a*d^2)$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [B]** time = 1.21546, size = 180, normalized size = 2.

$$\frac{i d^2 f x^2 e^{(dx+2c)} + d^2 f x^2 e^c + 2i d^2 x e^{(dx+2c+1)} - 4i d f x e^{(dx+2c)} + 2 d^2 x e^{(c+1)} + 4i f e^{(dx+2c)} \log(e^{(dx+c)} - i) + 4 f e^c \log(e^{(dx+c)} - I)}{2(ad^2 e^{(dx+2c)} - i ad^2 e^c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] 
$$\frac{-1/2*(I*d^2*f*x^2*e^{(d*x + 2*c)} + d^2*f*x^2*e^c + 2*I*d^2*x*e^{(d*x + 2*c + 1)} - 4*I*d*f*x*e^{(d*x + 2*c)} + 2*d^2*x*e^{(c + 1)} + 4*I*f*e^{(d*x + 2*c)}*\log(e^{(d*x + c)} - I) + 4*f*e^c*\log(e^{(d*x + c)} - I) + 4*d*e^{(c + 1)})}{(a*d^2*e^{(d*x + 2*c)} - I*a*d^2*e^c)}$$

$$3.190 \quad \int \frac{\sinh(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=35

$$-\frac{\cosh(c+dx)}{d(a+ia \sinh(c+dx))} - \frac{ix}{a}$$

[Out]  $((-I)*x)/a - \text{Cosh}[c + d*x]/(d*(a + I*a*\text{Sinh}[c + d*x]))$

**Rubi [A]** time = 0.0438023, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2735, 2648}

$$-\frac{\cosh(c+dx)}{d(a+ia \sinh(c+dx))} - \frac{ix}{a}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sinh}[c + d*x]/(a + I*a*\text{Sinh}[c + d*x]), x]$

[Out]  $((-I)*x)/a - \text{Cosh}[c + d*x]/(d*(a + I*a*\text{Sinh}[c + d*x]))$

#### Rule 2735

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]), x\_Symbol] \rightarrow \text{Simp}[(b*x)/d, x] - \text{Dist}[(b*c - a*d)/d, \text{Int}[1/(c + d*\sin[e + f*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

#### Rule 2648

$\text{Int}[(a_.) + (b_.)*\sin[(c_.) + (d_.)*(x_)]^{-1}, x\_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/(d*(b + a*\sin[c + d*x])), x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

#### Rubi steps

$$\begin{aligned} \int \frac{\sinh(c+dx)}{a+ia \sinh(c+dx)} dx &= -\frac{ix}{a} + i \int \frac{1}{a+ia \sinh(c+dx)} dx \\ &= -\frac{ix}{a} - \frac{\cosh(c+dx)}{d(a+ia \sinh(c+dx))} \end{aligned}$$

**Mathematica [A]** time = 0.206753, size = 61, normalized size = 1.74

$$\frac{i \cosh(c + dx) \left( 1 - \frac{\sinh^{-1}(\sinh(c+dx))(\sinh(c+dx)-i)}{\sqrt{\cosh^2(c+dx)}} \right)}{ad(\sinh(c + dx) - i)}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d\*x]/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (I\*Cosh[c + d\*x]\*(1 - (ArcSinh[Sinh[c + d\*x]]\*(-I + Sinh[c + d\*x])))/Sqrt[Cosh[c + d\*x]^2]))/(a\*d\*(-I + Sinh[c + d\*x]))

**Maple [A]** time = 0.026, size = 67, normalized size = 1.9

$$\frac{-i}{da} \ln \left( \tanh \left( \frac{dx}{2} + \frac{c}{2} \right) + 1 \right) + \frac{2i}{da} \left( -i + \tanh \left( \frac{dx}{2} + \frac{c}{2} \right) \right)^{-1} + \frac{i}{da} \ln \left( \tanh \left( \frac{dx}{2} + \frac{c}{2} \right) - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] -I/d/a\*ln(tanh(1/2\*d\*x+1/2\*c)+1)+2\*I/d/a/(-I+tanh(1/2\*d\*x+1/2\*c))+I/d/a\*ln(tanh(1/2\*d\*x+1/2\*c)-1)

**Maxima [A]** time = 1.1521, size = 49, normalized size = 1.4

$$-\frac{i(dx+c)}{ad} - \frac{2}{(ae^{(-dx-c)} + ia)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -I\*(d\*x + c)/(a\*d) - 2/((a\*e^(-d\*x - c) + I\*a)\*d)

**Fricas [A]** time = 2.32153, size = 78, normalized size = 2.23

$$\frac{-i dx e^{(dx+c)} - dx - 2}{ad e^{(dx+c)} - i ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] (-I\*d\*x\*e^(d\*x + c) - d\*x - 2)/(a\*d\*e^(d\*x + c) - I\*a\*d)

**Sympy [A]** time = 0.352585, size = 24, normalized size = 0.69

$$-\frac{ix}{a} - \frac{2e^{-c}}{ad(e^{dx} - ie^{-c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] -I\*x/a - 2\*exp(-c)/(a\*d\*(exp(d\*x) - I\*exp(-c)))

**Giac [A]** time = 1.23488, size = 46, normalized size = 1.31

$$-\frac{i(dx+c)}{ad} - \frac{2i}{ad(i e^{(dx+c)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] -I\*(d\*x + c)/(a\*d) - 2\*I/(a\*d\*(I\*e^(d\*x + c) + 1))

$$3.191 \quad \int \frac{\sinh(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

**Optimal.** Leaf size=31

$$\text{Unintegrable} \left( \frac{\sinh(c+dx)}{(e+fx)(a+ia \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[Sinh[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0504119, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Sinh[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])),x]

[Out] Defer[Int][Sinh[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx = \int \frac{\sinh(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

**Mathematica [A]** time = 48.143, size = 0, normalized size = 0.

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sinh[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])),x]

[Out] Integrate[Sinh[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

---

**Maple [A]** time = 0.091, size = 0, normalized size = 0.

$$\int \frac{\sinh(dx + c)}{(fx + e)(a + ia \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d\*x+c)/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] int(sinh(d\*x+c)/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-2f \int \frac{1}{-i adf^2x^2 - 2i adefx - i ade^2 + (adf^2x^2e^c + 2 adefxe^c + ade^2e^c)e^{(dx)}} dx - \frac{2}{-i adfx - i ade + (adfxe^c + adee^c)e^{(d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -2\*f\*integrate(1/(-I\*a\*d\*f^2\*x^2 - 2\*I\*a\*d\*e\*f\*x - I\*a\*d\*e^2 + (a\*d\*f^2\*x^2 \* e^c + 2\*a\*d\*e\*f\*x\*e^c + a\*d\*e^2\*e^c)\*e^(d\*x)), x) - 2/(-I\*a\*d\*f\*x - I\*a\*d\*e + (a\*d\*f\*x\*e^c + a\*d\*e\*e^c)\*e^(d\*x)) - I\*log(f\*x + e)/(a\*f)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\frac{(-i adfx - i ade + (adfx + ade)e^{(dx+c)}) \operatorname{integral}\left(-\frac{dfx+de-(idf x-i de)e^{(dx+c)}+2f}{-i adf^2x^2-2i adefx-i ade^2+(adf^2x^2+2 adefx+ade^2)e^{(dx+c)}}, x\right) - 2}{-i adfx - i ade + (adfx + ade)e^{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] ((-I\*a\*d\*f\*x - I\*a\*d\*e + (a\*d\*f\*x + a\*d\*e)\*e^(d\*x + c))\*integral(-(d\*f\*x + d\*e - (-I\*d\*f\*x - I\*d\*e)\*e^(d\*x + c) + 2\*f)/(-I\*a\*d\*f^2\*x^2 - 2\*I\*a\*d\*e\*f\*x

$$\frac{-I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^{(d*x + c)}, x) - 2)}{(-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^{(d*x + c)})}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx + c)}{(fx + e)(ia \sinh(dx + c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate(sinh(d\*x + c)/((f\*x + e)\*(I\*a\*sinh(d\*x + c) + a)), x)



$$3.192 \quad \int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

**Optimal.** Leaf size=31

$$\text{Unintegrable}\left(\frac{\sinh(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Sinh[c + d\*x]/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0511999, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Sinh[c + d\*x]/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])),x]

[Out] Defer[Int][Sinh[c + d\*x]/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx = \int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

**Mathematica [A]** time = 41.715, size = 0, normalized size = 0.

$$\int \frac{\sinh(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sinh[c + d\*x]/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])),x]

[Out] Integrate[Sinh[c + d\*x]/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

---

**Maple [A]** time = 0.102, size = 0, normalized size = 0.

$$\int \frac{\sinh(dx + c)}{(fx + e)^2 (a + ia \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d\*x+c)/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] int(sinh(d\*x+c)/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-4f \int \frac{1}{-i adf^3 x^3 - 3i adef^2 x^2 - 3i ade^2 fx - i ade^3 + (adf^3 x^3 e^c + 3 adef^2 x^2 e^c + 3 ade^2 f x e^c + ade^3 e^c) e^{(dx)}} dx + \frac{1}{2(-i adf^3 x^3 - 3i adef^2 x^2 - 3i ade^2 fx - i ade^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -4\*f\*integrate(1/(-I\*a\*d\*f^3\*x^3 - 3\*I\*a\*d\*e\*f^2\*x^2 - 3\*I\*a\*d\*e^2\*f\*x - I\*a\*d\*e^3 + (a\*d\*f^3\*x^3\*e^c + 3\*a\*d\*e\*f^2\*x^2\*e^c + 3\*a\*d\*e^2\*f\*x\*e^c + a\*d\*e^3\*e^c)\*e^(d\*x)), x) + 1/2\*(2\*d\*f\*x + 2\*d\*e + (2\*I\*d\*f\*x\*e^c + 2\*I\*d\*e\*e^c)\*e^(d\*x) - 4\*f)/(-I\*a\*d\*f^3\*x^2 - 2\*I\*a\*d\*e\*f^2\*x - I\*a\*d\*e^2\*f + (a\*d\*f^3\*x^2\*e^c + 2\*a\*d\*e\*f^2\*x\*e^c + a\*d\*e^2\*f\*e^c)\*e^(d\*x))

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\frac{(-i adf^2 x^2 - 2i adefx - i ade^2 + (adf^2 x^2 + 2 adefx + ade^2) e^{(dx+c)}) \operatorname{integral} \left( \frac{dfx+de - (-i dfx - i de) e^{(dx+c)}}{-i adf^3 x^3 - 3i adef^2 x^2 - 3i ade^2 fx - i ade^3 + (adf^3 x^3 + 3 adef^2 x^2 + 3 ade^2 fx + ade^3) e^{(dx+c)}}}{-i adf^2 x^2 - 2i adefx - i ade^2 + (adf^2 x^2 + 2 adefx + ade^2) e^{(dx+c)}} \right)}{(-i adf^2 x^2 - 2i adefx - i ade^2 + (adf^2 x^2 + 2 adefx + ade^2) e^{(dx+c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

```
[Out] ((-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x +
a*d*e^2)*e^(d*x + c))*integral(-(d*f*x + d*e - (-I*d*f*x - I*d*e)*e^(d*x +
c) + 4*f)/(-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x - I*a*d*e^
3 + (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*e^(d*x + c)),
x) - 2)/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d
*e*f*x + a*d*e^2)*e^(d*x + c))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)}{(fx+e)^2 (ia \sinh(dx+c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

$$3.193 \quad \int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=241

$$\frac{12f^2(e+fx)\text{PolyLog}(2, -ie^{c+dx})}{ad^3} - \frac{12f^3\text{PolyLog}(3, -ie^{c+dx})}{ad^4} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} + \frac{6f(e+fx)^2 \log(1+ie^{c+dx})}{ad^2}$$

[Out]  $-\left(\frac{(e+fx)^3}{ad}\right) + \frac{(e+fx)^4}{4af} - \left(\frac{6I f^2 (e+fx) \cosh[c+d*x]}{ad^3}\right) - \left(\frac{I (e+fx)^3 \cosh[c+d*x]}{ad}\right) + \frac{6f (e+fx)^2 \log[1+I E^{c+d*x}]}{ad^2} + \frac{12f^2 (e+fx) \text{PolyLog}[2, (-I) E^{c+d*x}]}{ad^3} - \frac{12f^3 \text{PolyLog}[3, (-I) E^{c+d*x}]}{ad^4} + \frac{6I f^3 \sinh[c+d*x]}{ad^4} + \frac{3I f (e+fx)^2 \sinh[c+d*x]}{ad^2} - \frac{(e+fx)^3 \tanh[c/2 + (I/4)\pi + (d*x)/2]}{ad}$

**Rubi [A]** time = 0.526142, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$ , Rules used = {5557, 3296, 2637, 32, 3318, 4184, 3716, 2190, 2531, 2282, 6589}

$$\frac{12f^2(e+fx)\text{PolyLog}(2, -ie^{c+dx})}{ad^3} - \frac{12f^3\text{PolyLog}(3, -ie^{c+dx})}{ad^4} - \frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} + \frac{6f(e+fx)^2 \log(1+ie^{c+dx})}{ad^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Sinh[c + d\*x]^2)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out]  $-\left(\frac{(e+fx)^3}{ad}\right) + \frac{(e+fx)^4}{4af} - \left(\frac{6I f^2 (e+fx) \cosh[c+d*x]}{ad^3}\right) - \left(\frac{I (e+fx)^3 \cosh[c+d*x]}{ad}\right) + \frac{6f (e+fx)^2 \log[1+I E^{c+d*x}]}{ad^2} + \frac{12f^2 (e+fx) \text{PolyLog}[2, (-I) E^{c+d*x}]}{ad^3} - \frac{12f^3 \text{PolyLog}[3, (-I) E^{c+d*x}]}{ad^4} + \frac{6I f^3 \sinh[c+d*x]}{ad^4} + \frac{3I f (e+fx)^2 \sinh[c+d*x]}{ad^2} - \frac{(e+fx)^3 \tanh[c/2 + (I/4)\pi + (d*x)/2]}{ad}$

**Rule 5557**

Int[(((e\_.) + (f\_.)\*(x\_.))^(m\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] :> Dist[1/b, Int[(e + f\*x)^m\*Sinh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sinh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Dist[(2*a)^n, Int[(c + d*x)^m*sin[(1*(e + (Pi*a)/(2*b)))/2 +
(f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp
p[((c + d*x)^m*cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps



$$d^4 x (4e^3 + 6e^2 f x + 4e f^2 x^2 + f^3 x^3) \operatorname{Cosh}[(d x)/2] - (2I) d (e + f x) (6f^2 + d^2 (e + f x)^2) \operatorname{Cosh}[c + (d x)/2] - (2I) d (e + f x) (6f^2 + d^2 (e + f x)^2) \operatorname{Cosh}[c + (3d x)/2] - 6d^2 e^2 f \operatorname{Cosh}[2c + (3d x)/2] - 12f^3 \operatorname{Cosh}[2c + (3d x)/2] - 12d^2 e f^2 x \operatorname{Cosh}[2c + (3d x)/2] - 6d^2 f^3 x^2 \operatorname{Cosh}[2c + (3d x)/2] - 10d^3 e^3 \operatorname{Sinh}[(d x)/2] - 12d e f^2 \operatorname{Sinh}[(d x)/2] - 30d^3 e^2 f x \operatorname{Sinh}[(d x)/2] - 12d f^3 x \operatorname{Sinh}[(d x)/2] - 30d^3 e f^2 x^2 \operatorname{Sinh}[(d x)/2] - 10d^3 f^3 x^3 \operatorname{Sinh}[(d x)/2] + (6I) d^2 e^2 f \operatorname{Sinh}[c + (d x)/2] + (12I) f^3 \operatorname{Sinh}[c + (d x)/2] + (4I) d^4 e^3 x \operatorname{Sinh}[c + (d x)/2] + (12I) d^2 e f^2 x \operatorname{Sinh}[c + (d x)/2] + (6I) d^4 e^2 f x^2 \operatorname{Sinh}[c + (d x)/2] + (6I) d^2 f^3 x^2 \operatorname{Sinh}[c + (d x)/2] + (4I) d^4 e f^2 x^3 \operatorname{Sinh}[c + (d x)/2] + I d^4 f^3 x^4 \operatorname{Sinh}[c + (d x)/2] + (6I) d^2 e^2 f \operatorname{Sinh}[c + (3d x)/2] + (12I) f^3 \operatorname{Sinh}[c + (3d x)/2] + (12I) d^2 e f^2 x \operatorname{Sinh}[c + (3d x)/2] + (6I) d^2 f^3 x^2 \operatorname{Sinh}[c + (3d x)/2] + 2d^3 e^3 \operatorname{Sinh}[2c + (3d x)/2] + 12d e f^2 \operatorname{Sinh}[2c + (3d x)/2] + 6d^3 e^2 f x \operatorname{Sinh}[2c + (3d x)/2] + 12d f^3 x \operatorname{Sinh}[2c + (3d x)/2] + 6d^3 e f^2 x^2 \operatorname{Sinh}[2c + (3d x)/2] + 2d^3 f^3 x^3 \operatorname{Sinh}[2c + (3d x)/2]) / ((\operatorname{Cosh}[c/2] + I \operatorname{Sinh}[c/2]) * (\operatorname{Cosh}[(c + d x)/2] + I \operatorname{Sinh}[(c + d x)/2])) / (4a d^4)$$

**Maple [B]** time = 0.183, size = 688, normalized size = 2.9

$$12 \frac{ef^2 \ln(1 + ie^{dx+c}) x}{ad^2} + 12 \frac{ef^2 \ln(1 + ie^{dx+c}) c}{ad^3} + 12 \frac{ef^2 c \ln(e^{dx+c})}{ad^3} - 12 \frac{ef^2 c \ln(e^{dx+c} - i)}{ad^3} - 12 \frac{ef^2 cx}{ad^2} + 4 \frac{c^3 f^3}{ad^4} - 2 \frac{x^3}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)`

[Out]  $12f^2/d^2/a * \ln(1 + I \exp(dx+c)) * x + 12f^2/d^3/a * \ln(1 + I \exp(dx+c)) * c + 12f^2/d^3/a * \ln(\exp(dx+c)) - 12f^2/d^3/a * \ln(\exp(dx+c) - I) - 12f^2/d^2/a * c * x + 4f^3/d^4/a * c^3 - 2f^3/d/a * x^3 - 1/2 * I * (d^3 f^3 x^3 + 3d^3 e f^2 x^2 + 3d^3 e^2 f x - 3d^2 f^3 x^2 + d^3 e^3 - 6d^2 e f^2 x - 3d^2 e^2 f + 6d f^3 x + 6d e f^2 - 6f^3) / a / d^4 * \exp(dx+c) - 2 * I * (f^3 x^3 + 3e f^2 x^2 + 3e^2 f x + e^3) / d / (\exp(dx+c) - I) - 12f^3 \operatorname{polylog}(3, -I \exp(dx+c)) / a / d^4 + 6f^3/d^3/a * c^2 * x - 6f/d^2/a * \ln(\exp(dx+c)) * e^2 + 6f^3/d^2/a * \ln(1 + I \exp(dx+c)) * x^2 - 6f^3/d^4/a * \ln(1 + I \exp(dx+c)) * c^2 - 6f^2/d^3/a * e * c^2 + 6f/d^2/a * \ln(\exp(dx+c) - I) * e^2 + 6f^3/d^4/a * c^2 * \ln(\exp(dx+c) - I) - 6f^2/d/a * e * x^2 - 6f^3/d^4/a * c^2 * \ln(\exp(dx+c)) + 12f^3/d^3/a * \operatorname{polylog}(2, -I \exp(dx+c)) * x + 12f^2/d^3/a * \operatorname{polylog}(2, -I \exp(dx+c)) + 1/a * e f^2 x^3 + 3/2/a * e^2 f x^2 - 1/2 * I * (d^3 f^3 x^3 + 3d^3 e f^2 x^2 + 3d^3 e^2 f x + 3d^2 f^3 x^2 + d^3 e^3 + 6d^2 e f^2 x + 3d^2 e^2 f + 6d f^3 x + 6d e f^2 + 6f^3) / a / d^4 * \exp(-d*x-c) + 1/4/a * x^4 * f^3 + 1/a * e^3 * x$



---

**Maxima [B]** time = 1.98274, size = 906, normalized size = 3.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -3/4*e^2*f*(4*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) - (-2*I*d^2*x^2*e^c - 2*I*d*x*e^c - (2*I*d*x*e^(3*c) - 2*I*e^(3*c))*e^(2*d*x) + 2*(d^2*x^2*e^(2*c) - 3*d*x*e^(2*c) + e^(2*c))*e^(d*x) - 2*(d*x + 1)*e^(-d*x) - 2*I*e^c)/(a*d^2*e^(d*x + 2*c) - I*a*d^2*e^c) - 8*log((e^(d*x + c) - I)*e^(-c))/(a*d^2) + 1/4*e^3*(4*(d*x + c)/(a*d) + 2*(-5*I*e^(-d*x - c) + 1)/((I*a*e^(-d*x - c) + a*e^(-2*d*x - 2*c))*d) - 2*I*e^(-d*x - c)/(a*d)) + 1/4*(-I*d^4*f^3*x^4 - 12*I*d*e*f^2 - (4*I*d^4*e*f^2 + 10*I*d^3*f^3)*x^3 - 12*I*f^3 - (30*I*d^3*e*f^2 + 6*I*d^2*f^3)*x^2 - (12*I*d^2*e*f^2 + 12*I*d*f^3)*x - (2*I*d^3*f^3*x^3*e^(2*c) + (6*I*d^3*e*f^2 - 6*I*d^2*f^3)*x^2*e^(2*c) + (-12*I*d^2*e*f^2 + 12*I*d*f^3)*x*e^(2*c) + (12*I*d*e*f^2 - 12*I*f^3)*e^(2*c))*e^(2*d*x) + (d^4*f^3*x^4*e^c + 2*(2*d^4*e*f^2 - d^3*f^3)*x^3*e^c - 6*(d^3*e*f^2 - d^2*f^3)*x^2*e^c + 12*(d^2*e*f^2 - d*f^3)*x*e^c - 12*(d*e*f^2 - f^3)*e^c)*e^(d*x))/(a*d^4*e^(d*x + c) - I*a*d^4) + 12*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*e*f^2/(a*d^3) + 6*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4) - 2*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2)/(a*d^4)
```

---

**Fricas [C]** time = 2.6279, size = 1904, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*d^3*f^3*x^3 + 2*d^3*e^3 + 6*d^2*e^2*f + 12*d*e*f^2 + 12*f^3 + 6*(d^3*e*f^2 + d^2*f^3)*x^2 + 6*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*x - (48*(d*f^3*x + d*e*f^2)*e^(2*d*x + 2*c) + (-48*I*d*f^3*x - 48*I*d*e*f^2)*e^(d*x + c))*dilog(-I*e^(d*x + c)) - (-2*I*d^3*f^3*x^3 - 2*I*d^3*e^3 + 6*I*d^2*e^2*f - 12*I*d*e*f^2 + 12*I*f^3 + (-6*I*d^3*e*f^2 + 6*I*d^2*f^3)*x^2 + (-6*I*d^3*e^2*
```

$$\begin{aligned}
& f + 12*I*d^2*e*f^2 - 12*I*d*f^3)*x)*e^{(3*d*x + 3*c)} - (d^4*f^3*x^4 - 2*d^3* \\
& e^3 - 6*(4*c - 1)*d^2*e^2*f + 12*(2*c^2 - 1)*d*e*f^2 - 4*(2*c^3 - 3)*f^3 + \\
& 2*(2*d^4*e*f^2 - 5*d^3*f^3)*x^3 + 6*(d^4*e^2*f - 5*d^3*e*f^2 + d^2*f^3)*x^2 \\
& + 2*(2*d^4*e^3 - 15*d^3*e^2*f + 6*d^2*e*f^2 - 6*d*f^3)*x)*e^{(2*d*x + 2*c)} \\
& - (-I*d^4*f^3*x^4 - 10*I*d^3*e^3 + (24*I*c - 6*I)*d^2*e^2*f + (-24*I*c^2 - \\
& 12*I)*d*e*f^2 + (8*I*c^3 - 12*I)*f^3 + (-4*I*d^4*e*f^2 - 2*I*d^3*f^3)*x^3 + \\
& (-6*I*d^4*e^2*f - 6*I*d^3*e*f^2 - 6*I*d^2*f^3)*x^2 + (-4*I*d^4*e^3 - 6*I*d^ \\
& ^3*e^2*f - 12*I*d^2*e*f^2 - 12*I*d*f^3)*x)*e^{(d*x + c)} - (24*(d^2*e^2*f - 2 \\
& *c*d*e*f^2 + c^2*f^3)*e^{(2*d*x + 2*c)} + (-24*I*d^2*e^2*f + 48*I*c*d*e*f^2 - \\
& 24*I*c^2*f^3)*e^{(d*x + c)})*log(e^{(d*x + c)} - I) - (24*(d^2*f^3*x^2 + 2*d^2 \\
& *e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*e^{(2*d*x + 2*c)} + (-24*I*d^2*f^3*x^2 - 48 \\
& *I*d^2*e*f^2*x - 48*I*c*d*e*f^2 + 24*I*c^2*f^3)*e^{(d*x + c)})*log(I*e^{(d*x + \\
& c)} + 1) + 48*(f^3*e^{(2*d*x + 2*c)} - I*f^3*e^{(d*x + c)})*polylog(3, -I*e^{(d* \\
& x + c)))/(4*a*d^4*e^{(2*d*x + 2*c)} - 4*I*a*d^4*e^{(d*x + c)})
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*sinh(d\*x+c)\*\*2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \sinh(dx + c)^2}{ia \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*sinh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*sinh(d\*x + c)^2/(I\*a\*sinh(d\*x + c) + a), x)

$$3.194 \quad \int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=184

$$\frac{4f^2 \text{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^3} + \frac{4f(e+fx) \log\left(1 + ie^{c+dx}\right)}{ad^2} + \frac{2if(e+fx) \sinh(c+dx)}{ad^2} - \frac{2if^2 \cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2 \coth(c+dx)}{ad}$$

[Out]  $-\left(\frac{(e+fx)^2}{ad}\right) + \frac{(e+fx)^3}{3af} - \frac{((2I)f^2 \cosh[c+dx])}{(ad^3)} - \frac{(I(e+fx)^2 \cosh[c+dx])}{(ad)} + \frac{(4f(e+fx) \log[1 + Ie^{c+dx}])}{(ad^2)} + \frac{(4f^2 \text{PolyLog}[2, (-I)E^{c+dx}])}{(ad^3)} + \frac{((2I)f(e+fx) \sinh[c+dx])}{(ad^2)} - \frac{((e+fx)^2 \tanh[c/2 + (I/4)\pi + (dx)/2])}{(ad)}$

**Rubi [A]** time = 0.390758, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$ , Rules used = {5557, 3296, 2638, 32, 3318, 4184, 3716, 2190, 2279, 2391}

$$\frac{4f^2 \text{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^3} + \frac{4f(e+fx) \log\left(1 + ie^{c+dx}\right)}{ad^2} + \frac{2if(e+fx) \sinh(c+dx)}{ad^2} - \frac{2if^2 \cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2 \coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Sinh[c + d\*x]^2)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out]  $-\left(\frac{(e+fx)^2}{ad}\right) + \frac{(e+fx)^3}{3af} - \frac{((2I)f^2 \cosh[c+dx])}{(ad^3)} - \frac{(I(e+fx)^2 \cosh[c+dx])}{(ad)} + \frac{(4f(e+fx) \log[1 + Ie^{c+dx}])}{(ad^2)} + \frac{(4f^2 \text{PolyLog}[2, (-I)E^{c+dx}])}{(ad^3)} + \frac{((2I)f(e+fx) \sinh[c+dx])}{(ad^2)} - \frac{((e+fx)^2 \tanh[c/2 + (I/4)\pi + (dx)/2])}{(ad)}$

### Rule 5557

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Sinh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sinh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
(f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Sim
p[((c + d*x)^m*cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[(c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+ia \sinh(c+dx)} dx &= i \int \frac{(e+fx)^2 \sinh(c+dx)}{a+ia \sinh(c+dx)} dx - \frac{i \int (e+fx)^2 \sinh(c+dx) dx}{a} \\
&= -\frac{i(e+fx)^2 \cosh(c+dx)}{ad} + \frac{\int (e+fx)^2 dx}{a} + \frac{(2if) \int (e+fx) \cosh(c+dx) dx}{ad} - \int \frac{1}{a+ia \sinh(c+dx)} dx \\
&= \frac{(e+fx)^3}{3af} - \frac{i(e+fx)^2 \cosh(c+dx)}{ad} + \frac{2if(e+fx) \sinh(c+dx)}{ad^2} - \frac{\int (e+fx)^2 \csc^2\left(\frac{1}{2}(c+dx)\right) dx}{2a} \\
&= \frac{(e+fx)^3}{3af} - \frac{2if^2 \cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2 \cosh(c+dx)}{ad} + \frac{2if(e+fx) \sinh(c+dx)}{ad^2} \\
&= -\frac{(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} - \frac{2if^2 \cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2 \cosh(c+dx)}{ad} + \frac{2if(e+fx) \sinh(c+dx)}{ad^2} \\
&= -\frac{(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} - \frac{2if^2 \cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2 \cosh(c+dx)}{ad} + \frac{4f(e+fx) \log\left(\frac{1-i \tanh\left(\frac{1}{2}(c+dx)\right)}{1+i \tanh\left(\frac{1}{2}(c+dx)\right)}\right)}{ad} \\
&= -\frac{(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} - \frac{2if^2 \cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2 \cosh(c+dx)}{ad} + \frac{4f(e+fx) \log\left(\frac{1-i \tanh\left(\frac{1}{2}(c+dx)\right)}{1+i \tanh\left(\frac{1}{2}(c+dx)\right)}\right)}{ad} \\
&= -\frac{(e+fx)^2}{ad} + \frac{(e+fx)^3}{3af} - \frac{2if^2 \cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2 \cosh(c+dx)}{ad} + \frac{4f(e+fx) \log\left(\frac{1-i \tanh\left(\frac{1}{2}(c+dx)\right)}{1+i \tanh\left(\frac{1}{2}(c+dx)\right)}\right)}{ad}
\end{aligned}$$

**Mathematica [A]** time = 3.4604, size = 260, normalized size = 1.41

$$\frac{6 \left( \frac{d(e+fx) \left( 2(e^c - i) f \log(1 - i e^{-c-dx}) - i d(e+fx) \right)}{e^c - i} - 2f^2 \text{PolyLog}(2, i e^{-c-dx}) \right)}{d^3} - \frac{3i \cosh(dx) \left( \cosh(c) (d^2 (e+fx)^2 + 2f^2) - 2df \sinh(c) (e+fx) \right)}{d^3} - \frac{3i \sinh(dx) \left( \sinh(c) (d^2 (e+fx)^2 + 2f^2) - 2df \cosh(c) (e+fx) \right)}{d^3}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Sinh[c + d\*x]^2)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (x\*(3\*e^2 + 3\*e\*f\*x + f^2\*x^2) + (6\*((d\*(e + f\*x)\*((-I)\*d\*(e + f\*x) + 2\*(-I + E^c)\*f\*Log[1 - I\*E^(-c - d\*x)])))/(-I + E^c) - 2\*f^2\*PolyLog[2, I\*E^(-c - d\*x)]))/d^3 - ((3\*I)\*Cosh[d\*x]\*((2\*f^2 + d^2\*(e + f\*x)^2)\*Cosh[c] - 2\*d\*f\*(e + f\*x)\*Sinh[c]))/d^3 - ((3\*I)\*(-2\*d\*f\*(e + f\*x)\*Cosh[c] + (2\*f^2 + d^2\*(e + f\*x)^2)\*Sinh[c])\*Sinh[d\*x])/d^3 - (6\*(e + f\*x)^2\*Sinh[(d\*x)/2])/(d\*(Cosh[c/2] + I\*Sinh[c/2])\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]))/(3\*a)

**Maple [B]** time = 0.13, size = 374, normalized size = 2.

$$\frac{x^3 f^2}{3a} + \frac{efx^2}{a} + \frac{e^2 x}{a} - \frac{2i(x^2 f^2 + 2efx + e^2)}{da(e^{dx+c} - i)} - \frac{\frac{i}{2}(f^2 x^2 d^2 + 2d^2 efx + d^2 e^2 - 2df^2 x - 2efd + 2f^2)e^{dx+c}}{ad^3} - \frac{\frac{i}{2}(f^2 x^2 d^2 + 2d^2 efx + d^2 e^2 - 2df^2 x - 2efd + 2f^2)e^{dx+c}}{ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*sinh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] 1/3/a\*x^3\*f^2+1/a\*e\*f\*x^2+1/a\*e^2\*x-2\*I\*(f^2\*x^2+2\*e\*f\*x+e^2)/d/a/(exp(d\*x+c)-I)-1/2\*I\*(d^2\*f^2\*x^2+2\*d^2\*e\*f\*x+d^2\*e^2-2\*d\*f^2\*x-2\*d\*e\*f+2\*f^2)/a/d^3\*exp(d\*x+c)-1/2\*I\*(d^2\*f^2\*x^2+2\*d^2\*e\*f\*x+d^2\*e^2+2\*d\*f^2\*x+2\*d\*e\*f+2\*f^2)/a/d^3\*exp(-d\*x-c)+4\*f/d^2/a\*ln(exp(d\*x+c)-I)\*e-4\*f/d^2/a\*ln(exp(d\*x+c))\*e-2\*f^2\*x^2/a/d-4\*f^2/d^2/a\*c\*x-2\*f^2/d^3/a\*c^2+4\*f^2/d^2/a\*ln(1+I\*exp(d\*x+c))\*x+4\*f^2/d^3/a\*ln(1+I\*exp(d\*x+c))\*c+4\*f^2\*polylog(2,-I\*exp(d\*x+c))/a/d^3-4\*f^2/d^3/a\*c\*ln(exp(d\*x+c)-I)+4\*f^2/d^3/a\*c\*ln(exp(d\*x+c))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}ef\left(\frac{4xe^{(dx+c)}}{ade^{(dx+c)} - iad} - \frac{-2id^2x^2e^c - 2idxe^c - (2idxe^{(3c)} - 2ie^{(3c)})e^{(2dx)} + 2(d^2x^2e^{(2c)} - 3dxe^{(2c)} + e^{(2c)})e^{(dx)} - 2(dx + 1)e^{(2c)}}{ad^2e^{(dx+2c)} - iad^2e^c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sinh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

```
[Out] -1/2*e*f*(4*x*e^(d*x + c)/(a*d*e^(d*x + c) - I*a*d) - (-2*I*d^2*x^2*e^c - 2
*I*d*x*e^c - (2*I*d*x*e^(3*c) - 2*I*e^(3*c))*e^(2*d*x) + 2*(d^2*x^2*e^(2*c)
- 3*d*x*e^(2*c) + e^(2*c))*e^(d*x) - 2*(d*x + 1)*e^(-d*x) - 2*I*e^c)/(a*d^
2*e^(d*x + 2*c) - I*a*d^2*e^c) - 8*log((e^(d*x + c) - I)*e^(-c))/(a*d^2)) +
1/12*f^2*((-4*I*d^3*x^3 - 30*I*d^2*x^2 - 12*I*d*x - (6*I*d^2*x^2*e^(2*c) -
12*I*d*x*e^(2*c) + 12*I*e^(2*c))*e^(2*d*x) + 2*(2*d^3*x^3*e^c - 3*d^2*x^2*
e^c + 6*d*x*e^c - 6*e^c)*e^(d*x) - 12*I)/(a*d^3*e^(d*x + c) - I*a*d^3) + 48
*I*integrate(x/(a*d*e^(d*x + c) - I*a*d), x)) + 1/4*e^2*(4*(d*x + c)/(a*d)
+ 2*(-5*I*e^(-d*x - c) + 1)/((I*a*e^(-d*x - c) + a*e^(-2*d*x - 2*c))*d) - 2
*I*e^(-d*x - c)/(a*d))
```

**Fricas [B]** time = 2.72295, size = 1123, normalized size = 6.1

$$3d^2f^2x^2 + 3d^2e^2 + 6def + 6f^2 + 6(d^2ef + df^2)x - 24(f^2e^{2dx+2c} - if^2e^{dx+c})\text{Li}_2(-ie^{dx+c}) - (-3id^2f^2x^2 - 3id^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(3*d^2*f^2*x^2 + 3*d^2*e^2 + 6*d*e*f + 6*f^2 + 6*(d^2*e*f + d*f^2)*x - 24*
(f^2*e^(2*d*x + 2*c) - I*f^2*e^(d*x + c))*dilog(-I*e^(d*x + c)) - (-3*I*d^2
*f^2*x^2 - 3*I*d^2*e^2 + 6*I*d*e*f - 6*I*f^2 + (-6*I*d^2*e*f + 6*I*d*f^2)*x
)*e^(3*d*x + 3*c) - (2*d^3*f^2*x^3 - 3*d^2*e^2 - 6*(4*c - 1)*d*e*f + 6*(2*c
^2 - 1)*f^2 + 3*(2*d^3*e*f - 5*d^2*f^2)*x^2 + 6*(d^3*e^2 - 5*d^2*e*f + d*f^
2)*x)*e^(2*d*x + 2*c) - (-2*I*d^3*f^2*x^3 - 15*I*d^2*e^2 + (24*I*c - 6*I)*d
*e*f + (-12*I*c^2 - 6*I)*f^2 + (-6*I*d^3*e*f - 3*I*d^2*f^2)*x^2 + (-6*I*d^3
*e^2 - 6*I*d^2*e*f - 6*I*d*f^2)*x)*e^(d*x + c) - (24*(d*e*f - c*f^2)*e^(2*d
*x + 2*c) + (-24*I*d*e*f + 24*I*c*f^2)*e^(d*x + c))*log(e^(d*x + c) - I) -
(24*(d*f^2*x + c*f^2)*e^(2*d*x + 2*c) + (-24*I*d*f^2*x - 24*I*c*f^2)*e^(d*x
+ c))*log(I*e^(d*x + c) + 1))/(6*a*d^3*e^(2*d*x + 2*c) - 6*I*a*d^3*e^(d*x
+ c))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \sinh(dx + c)^2}{ia \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sinh(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```



$$3.195 \quad \int \frac{(e+fx) \sinh^2(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=119

$$\frac{if \sinh(c+dx)}{ad^2} + \frac{2f \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\right)}{ad^2} - \frac{i(e+fx) \cosh(c+dx)}{ad} - \frac{(e+fx) \tanh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)}{ad} + \frac{ex}{a} + \frac{fx^2}{2a}$$

[Out] (e\*x)/a + (f\*x^2)/(2\*a) - (I\*(e + f\*x)\*Cosh[c + d\*x])/(a\*d) + (2\*f\*Log[Cosh[c/2 + (I/4)\*Pi + (d\*x)/2]])/(a\*d^2) + (I\*f\*Sinh[c + d\*x])/(a\*d^2) - ((e + f\*x)\*Tanh[c/2 + (I/4)\*Pi + (d\*x)/2])/(a\*d)

**Rubi [A]** time = 0.177117, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {5557, 3296, 2637, 3318, 4184, 3475}

$$\frac{if \sinh(c+dx)}{ad^2} + \frac{2f \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\right)}{ad^2} - \frac{i(e+fx) \cosh(c+dx)}{ad} - \frac{(e+fx) \tanh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)}{ad} + \frac{ex}{a} + \frac{fx^2}{2a}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Sinh[c + d\*x]^2)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (e\*x)/a + (f\*x^2)/(2\*a) - (I\*(e + f\*x)\*Cosh[c + d\*x])/(a\*d) + (2\*f\*Log[Cosh[c/2 + (I/4)\*Pi + (d\*x)/2]])/(a\*d^2) + (I\*f\*Sinh[c + d\*x])/(a\*d^2) - ((e + f\*x)\*Tanh[c/2 + (I/4)\*Pi + (d\*x)/2])/(a\*d)

### Rule 5557

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)])^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] :> Dist[1/b, Int[(e + f\*x)^m\*Sinh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sinh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 3296

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /;  
FreeQ[{c, d}, x]

Rule 3318

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Dist[(2\*a)^n, Int[(c + d\*x)^m\*Sin[(1\*(e + (Pi\*a)/(2\*b)))/2 + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])

Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[p((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m-1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{(e+fx) \sinh^2(c+dx)}{a+ia \sinh(c+dx)} dx &= i \int \frac{(e+fx) \sinh(c+dx)}{a+ia \sinh(c+dx)} dx - \frac{i \int (e+fx) \sinh(c+dx) dx}{a} \\ &= -\frac{i(e+fx) \cosh(c+dx)}{ad} + \frac{\int (e+fx) dx}{a} + \frac{(if) \int \cosh(c+dx) dx}{ad} - \int \frac{e+fx}{a+ia \sinh(c+dx)} dx \\ &= \frac{ex}{a} + \frac{fx^2}{2a} - \frac{i(e+fx) \cosh(c+dx)}{ad} + \frac{if \sinh(c+dx)}{ad^2} - \frac{\int (e+fx) \csc^2\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{idx}{2}\right)}{2a} \\ &= \frac{ex}{a} + \frac{fx^2}{2a} - \frac{i(e+fx) \cosh(c+dx)}{ad} + \frac{if \sinh(c+dx)}{ad^2} - \frac{(e+fx) \tanh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{ad} + \frac{f}{a} \\ &= \frac{ex}{a} + \frac{fx^2}{2a} - \frac{i(e+fx) \cosh(c+dx)}{ad} + \frac{2f \log\left(\cosh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)\right)}{ad^2} + \frac{if \sinh(c+dx)}{ad^2} - \frac{(e+fx) \tanh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{ad} + \frac{f}{a} \end{aligned}$$

**Mathematica [A]** time = 1.08925, size = 238, normalized size = 2.

$$\left(\sinh\left(\frac{1}{2}(c+dx)\right) - i \cosh\left(\frac{1}{2}(c+dx)\right)\right) \left(\cosh\left(\frac{1}{2}(c+dx)\right)\right) \left(c^2(-f) - 2id(e+fx) \cosh(c+dx) + 2cde + 2if \sinh(c+dx) - \dots\right)$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Sinh[c + d\*x]^2)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (((-I)\*Cosh[(c + d\*x)/2] + Sinh[(c + d\*x)/2])\*(Sinh[(c + d\*x)/2]\*(I\*(2\*I + c + d\*x)\*(2\*d\*e - c\*f + d\*f\*x) - 4\*f\*ArcTan[Tanh[(c + d\*x)/2]] + 2\*d\*(e + f\*x)\*Cosh[c + d\*x] + (2\*I)\*f\*Log[Cosh[c + d\*x]] - 2\*f\*Sinh[c + d\*x]) + Cosh[(c + d\*x)/2]\*(2\*c\*d\*e - (2\*I)\*c\*f - c^2\*f + 2\*d^2\*e\*x - (2\*I)\*d\*f\*x + d^2\*f\*x^2 + (4\*I)\*f\*ArcTan[Tanh[(c + d\*x)/2]] - (2\*I)\*d\*(e + f\*x)\*Cosh[c + d\*x] + 2\*f\*Log[Cosh[c + d\*x]] + (2\*I)\*f\*Sinh[c + d\*x])))/(2\*a\*d^2\*(-I + Sinh[c + d\*x]))

**Maple [A]** time = 0.106, size = 134, normalized size = 1.1

$$\frac{fx^2}{2a} + \frac{ex}{a} - \frac{i}{2} \frac{(dfx + de - f)e^{dx+c}}{ad^2} - \frac{i}{2} \frac{(dfx + de + f)e^{-dx-c}}{ad^2} - 2 \frac{fx}{da} - 2 \frac{cf}{ad^2} - \frac{2i(fx + e)}{da(e^{dx+c} - i)} + 2 \frac{f \ln(e^{dx+c} - i)}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)\*sinh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] 1/2\*f\*x^2/a+e\*x/a-1/2\*I\*(d\*f\*x+d\*e-f)/a/d^2\*exp(d\*x+c)-1/2\*I\*(d\*f\*x+d\*e+f)/a/d^2\*exp(-d\*x-c)-2\*f\*x/a/d-2\*f/a/d^2\*c-2\*I\*(f\*x+e)/d/a/(exp(d\*x+c)-I)+2\*f/a/d^2\*ln(exp(d\*x+c)-I)

**Maxima [B]** time = 1.3415, size = 325, normalized size = 2.73

$$-\frac{1}{4} f \left( \frac{4xe^{(dx+c)}}{ade^{(dx+c)} - iad} - \frac{-2id^2x^2e^c - 2idxe^c - (2idxe^{(3c)} - 2ie^{(3c)})e^{(2dx)} + 2(d^2x^2e^{(2c)} - 3dxe^{(2c)} + e^{(2c)})e^{(dx)} - 2(dx + c)e^{(2c)}}{ad^2e^{(dx+2c)} - iad^2e^c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -1/4\*f\*(4\*x\*e^(d\*x + c)/(a\*d\*e^(d\*x + c) - I\*a\*d) - (-2\*I\*d^2\*x^2\*e^c - 2\*I\*d\*x\*e^c - (2\*I\*d\*x\*e^(3\*c) - 2\*I\*e^(3\*c))\*e^(2\*d\*x) + 2\*(d^2\*x^2\*e^(2\*c) - 3\*d\*x\*e^(2\*c) + e^(2\*c))\*e^(d\*x) - 2\*(d\*x + 1)\*e^(-d\*x) - 2\*I\*e^c)/(a\*d^2\*e^(d\*x + 2\*c) - I\*a\*d^2\*e^c) - 8\*log((e^(d\*x + c) - I)\*e^(-c))/(a\*d^2) + 1

$$\frac{4e^{4(dx+c)}(ad) + 2(-5Ie^{-dx-c} + 1)/((Iae^{-dx-c} + ae^{-2dx-2c}))d - 2Ie^{-dx-c}/(ad)}{2ad^2e^{2dx+2c} - 2iad^2e^{(dx+c)}}$$

**Fricas [A]** time = 2.66552, size = 416, normalized size = 3.5

$$\frac{dfx + de - (-idfx - ide + if)e^{(3dx+3c)} - (d^2fx^2 - de + (2d^2e - 5df)x + f)e^{(2dx+2c)} - (-id^2fx^2 - 5ide + (-2id^2e - i}}{2ad^2e^{2dx+2c} - 2iad^2e^{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $-(d*f*x + d*e - (-I*d*f*x - I*d*e + I*f))*e^{(3*d*x + 3*c)} - (d^2*f*x^2 - d*e + (2*d^2*e - 5*d*f)*x + f)*e^{(2*d*x + 2*c)} - (-I*d^2*f*x^2 - 5*I*d*e + (-2*I*d^2*e - I*d*f)*x - I*f)*e^{(d*x + c)} - 4*(f*e^{(2*d*x + 2*c)} - I*f*e^{(d*x + c)})*\log(e^{(d*x + c)} - I) + f)/(2*a*d^2*e^{(2*d*x + 2*c)} - 2*I*a*d^2*e^{(d*x + c)})$

**Sympy [A]** time = 2.36023, size = 449, normalized size = 3.77

$$\left\{ \begin{array}{ll} \frac{((-2ia^3d^5ee^c - 2ia^3d^5fxe^c - 2ia^3d^4fe^c)e^{-dx} + (-2ia^3d^5ee^{3c} - 2ia^3d^5fxe^{3c} + 2ia^3d^4fe^{3c})e^{dx})e^{-2c}}{x^2(ife^{6c} + 4fe^{5c} - 7ife^{4c} - 8fe^{3c} + 7ife^{2c} + 4fe^c - if)} + \frac{4a^4d^6}{-4ae^{5c} + 16iae^{4c} + 24ae^{3c} - 16iae^{2c} - 4ae^c} + \frac{x(iee^{8c} + 6ee^{7c} - 16iee^{6c} - 26ee^{5c} + 30iee^{4c} + 26ee^{3c} - 16iee^{2c} - 6ee^c + ie)}{-2ae^{7c} + 12iae^{6c} + 30ae^{5c} - 40iae^{4c} - 30ae^{3c} + 12iae^{2c} + 2ae^c} & \text{for } 4a^4d^6e^{2c} \neq 0 \\ & \text{otherwise} \end{array} \right. + \frac{fx^2}{2a} + \frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)\*\*2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Piecewise(((((-2Ia\*\*3d\*\*5e\*exp(c) - 2Ia\*\*3d\*\*5f\*x\*exp(c) - 2Ia\*\*3d\*\*4f\*exp(c))\*exp(-d\*x) + (-2Ia\*\*3d\*\*5e\*exp(3\*c) - 2Ia\*\*3d\*\*5f\*x\*exp(3\*c) + 2Ia\*\*3d\*\*4f\*exp(3\*c))\*exp(d\*x))\*exp(-2\*c)/(4a\*\*4d\*\*6), Ne(4a\*\*4d\*\*6\*exp(2\*c), 0)), (x\*\*2\*(I\*f\*exp(6\*c) + 4\*f\*exp(5\*c) - 7\*I\*f\*exp(4\*c) - 8\*f\*exp(3\*c) + 7\*I\*f\*exp(2\*c) + 4\*f\*exp(c) - I\*f)/(-4a\*exp(5\*c) + 16Ia\*exp(4\*c) + 24a\*exp(3\*c) - 16Ia\*exp(2\*c) - 4a\*exp(c)) + x\*(I\*e\*exp(8\*c) + 6\*e\*exp(7\*c) - 16I\*e\*exp(6\*c) - 26\*e\*exp(5\*c) + 30I\*e\*exp(4\*c) + 26\*e\*exp(3\*c) - 16I\*e\*exp(2\*c) - 6\*e\*exp(c) + I\*e)/(-2a\*exp(7\*c) + 12Ia\*exp(6\*c) + 30a\*exp(5\*c) - 40Ia\*exp(4\*c) - 30a\*exp(3\*c) + 12Ia\*exp(2\*c) + 2a\*exp(c)), True)) + f\*x\*\*2/(2\*a) + x\*(d\*e - 2\*f)/(a\*d) - (2I\*e + 2I\*f\*x)\*exp(-c)/(a\*d\*(exp(d\*x) - I\*exp(-c))) + 2\*f\*log(exp(d\*x) - I\*exp(-c))/(

a\*d\*\*2)

---

**Giac [B]** time = 1.52809, size = 367, normalized size = 3.08

$$d^2 f x^2 e^{(2 dx+3c)} - i d^2 f x^2 e^{(dx+2c)} - i d f x e^{(3 dx+4c)} + 2 d^2 x e^{(2 dx+3c+1)} - 5 d f x e^{(2 dx+3c)} - 2 i d^2 x e^{(dx+2c+1)} - i d f x e^{(dx+2c)} - d$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out]  $\frac{1}{2}*(d^2*f*x^2*e^{(2*d*x + 3*c)} - I*d^2*f*x^2*e^{(d*x + 2*c)} - I*d*f*x*e^{(3*d*x + 4*c)} + 2*d^2*x*e^{(2*d*x + 3*c + 1)} - 5*d*f*x*e^{(2*d*x + 3*c)} - 2*I*d^2*x*e^{(d*x + 2*c + 1)} - I*d*f*x*e^{(d*x + 2*c)} - d*f*x*e^c + 4*f*e^{(2*d*x + 3*c)}*\log(e^{(d*x + c)} - I) - 4*I*f*e^{(d*x + 2*c)}*\log(e^{(d*x + c)} - I) - I*d*e^{(3*d*x + 4*c + 1)} + I*f*e^{(3*d*x + 4*c)} - d*e^{(2*d*x + 3*c + 1)} + f*e^{(2*d*x + 3*c)} - 5*I*d*e^{(d*x + 2*c + 1)} - I*f*e^{(d*x + 2*c)} - d*e^{(c + 1)} - f*e^c)/(a*d^2*e^{(2*d*x + 3*c)} - I*a*d^2*e^{(d*x + 2*c)})$

$$3.196 \quad \int \frac{\sinh^2(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=52

$$-\frac{i \cosh(c+dx)}{ad} - \frac{i \cosh(c+dx)}{ad(1+i \sinh(c+dx))} + \frac{x}{a}$$

[Out] x/a - (I\*Cosh[c + d\*x])/(a\*d) - (I\*Cosh[c + d\*x])/(a\*d\*(1 + I\*Sinh[c + d\*x]))

**Rubi [A]** time = 0.08879, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2746, 12, 2735, 2648}

$$-\frac{i \cosh(c+dx)}{ad} - \frac{i \cosh(c+dx)}{ad(1+i \sinh(c+dx))} + \frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d\*x]^2/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] x/a - (I\*Cosh[c + d\*x])/(a\*d) - (I\*Cosh[c + d\*x])/(a\*d\*(1 + I\*Sinh[c + d\*x]))

#### Rule 2746

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(b^2*Cos[e + f*x])/(d*f), x] + Dist[1/d, Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/(c_. + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

Int[((a\_) + (b\_)\*sin[(c\_) + (d\_)\*(x\_)])^(-1), x\_Symbol] := -Simp[Cos[c + d\*x]/(d\*(b + a\*Sin[c + d\*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sinh^2(c + dx)}{a + ia \sinh(c + dx)} dx &= -\frac{i \cosh(c + dx)}{ad} + \frac{i \int \frac{a \sinh(c+dx)}{a+ia \sinh(c+dx)} dx}{a} \\ &= -\frac{i \cosh(c + dx)}{ad} + i \int \frac{\sinh(c + dx)}{a + ia \sinh(c + dx)} dx \\ &= \frac{x}{a} - \frac{i \cosh(c + dx)}{ad} - \int \frac{1}{a + ia \sinh(c + dx)} dx \\ &= \frac{x}{a} - \frac{i \cosh(c + dx)}{ad} - \frac{i \cosh(c + dx)}{d(a + ia \sinh(c + dx))} \end{aligned}$$

**Mathematica [A]** time = 0.220904, size = 59, normalized size = 1.13

$$\frac{\cosh(c + dx) \left( \frac{\sinh^{-1}(\sinh(c+dx))}{\sqrt{\cosh^2(c+dx)}} + \frac{-2-i \sinh(c+dx)}{\sinh(c+dx)-i} \right)}{ad}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d\*x]^2/(a + I\*a\*Sinh[c + d\*x]), x]

[Out] (Cosh[c + d\*x]\*(ArcSinh[Sinh[c + d\*x]]/Sqrt[Cosh[c + d\*x]^2] + (-2 - I\*Sinh[c + d\*x])/(-I + Sinh[c + d\*x]))/(a\*d)

**Maple [B]** time = 0.036, size = 107, normalized size = 2.1

$$-2 \frac{1}{da(-i + \tanh(1/2 dx + c/2))} - \frac{i}{da} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} + \frac{1}{da} \ln \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right) + \frac{i}{da} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)), x)

[Out]  $-2/d/a/(-I+\tanh(1/2*d*x+1/2*c))-I/d/a/(\tanh(1/2*d*x+1/2*c)+1)+1/d/a*\ln(\tanh(1/2*d*x+1/2*c)+1)+I/d/a/(\tanh(1/2*d*x+1/2*c)-1)-1/d/a*\ln(\tanh(1/2*d*x+1/2*c)-1)$

**Maxima [A]** time = 1.00892, size = 100, normalized size = 1.92

$$\frac{dx + c}{ad} + \frac{-5ie^{(-dx-c)} + 1}{2(iae^{(-dx-c)} + ae^{(-2dx-2c)})d} - \frac{ie^{(-dx-c)}}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out]  $(d*x + c)/(a*d) + 1/2*(-5*I*e^{(-d*x - c)} + 1)/((I*a*e^{(-d*x - c)} + a*e^{(-2*d*x - 2*c)})*d) - 1/2*I*e^{(-d*x - c)}/(a*d)$

**Fricas [A]** time = 2.46326, size = 178, normalized size = 3.42

$$\frac{(2dx - 1)e^{(2dx+2c)} + (-2idx - 5i)e^{(dx+c)} - ie^{(3dx+3c)} - 1}{2ade^{(2dx+2c)} - 2iade^{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")`

[Out]  $((2*d*x - 1)*e^{(2*d*x + 2*c)} + (-2*I*d*x - 5*I)*e^{(d*x + c)} - I*e^{(3*d*x + 3*c)} - 1)/(2*a*d*e^{(2*d*x + 2*c)} - 2*I*a*d*e^{(d*x + c)})$

**Sympy [A]** time = 0.747901, size = 102, normalized size = 1.96

$$\begin{cases} \frac{(-2iade^{2c}e^{dx}-2iade^{-dx})e^{-c}}{4a^2d^2} & \text{for } 4a^2d^2e^c \neq 0 \\ x\left(-\frac{(ie^{2c}-2e^c-i)e^{-c}}{2a}-\frac{1}{a}\right) & \text{otherwise} \end{cases} + \frac{x}{a} - \frac{2ie^{-c}}{ad(e^{dx}-ie^{-c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)`



```
[Out] Piecewise((( -2*I*a*d*exp(2*c)*exp(d*x) - 2*I*a*d*exp(-d*x))*exp(-c)/(4*a**2
*d**2), Ne(4*a**2*d**2*exp(c), 0)), (x*(-(I*exp(2*c) - 2*exp(c) - I)*exp(-c
)/(2*a) - 1/a), True)) + x/a - 2*I*exp(-c)/(a*d*(exp(d*x) - I*exp(-c)))
```

**Giac [A]** time = 1.41128, size = 89, normalized size = 1.71

$$\frac{dx + c}{ad} - \frac{i e^{(dx+c)}}{2ad} + \frac{(5 e^{(dx+c)} - i) e^{(-dx-c)}}{2ad(i e^{(dx+c)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (d*x + c)/(a*d) - 1/2*I*e^(d*x + c)/(a*d) + 1/2*(5*e^(d*x + c) - I)*e^(-d*x
- c)/(a*d*(I*e^(d*x + c) + 1))
```

$$3.197 \quad \int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

**Optimal.** Leaf size=33

$$\text{Unintegrable} \left( \frac{\sinh^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[Sinh[c + d\*x]^2/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0778347, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Sinh[c + d\*x]^2/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Sinh[c + d\*x]^2/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx = \int \frac{\sinh^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.001, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[Sinh[c + d\*x]^2/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] \$Aborted

---

**Maple [A]** time = 0.128, size = 0, normalized size = 0.

$$\int \frac{(\sinh(dx + c))^2}{(fx + e)(a + ia \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)`

[Out] `int(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-2if \int \frac{1}{-iadf^2x^2 - 2iade fx - iade^2 + (adf^2x^2e^c + 2adefxe^c + ade^2e^c)e^{(dx)}} dx - \frac{ie^{(-c+\frac{de}{f})}E_1\left(\frac{(fx+e)d}{f}\right)}{2af} + \frac{ie^{(c-\frac{de}{f})}E_1\left(-\frac{(fx+e)d}{f}\right)}{2af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `-2*I*f*integrate(1/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) - 1/2*I*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(a*f) + 1/2*I*e^(c - d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(a*f) - 2*I/(-I*a*d*f*x - I*a*d*e + (a*d*f*x*e^c + a*d*e*e^c)*e^(d*x)) + log(f*x + e)/(a*f)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\frac{(-iadfx - iade + (adfx + ade)e^{(dx+c)}) \operatorname{integral} \left( \frac{dfx+de+(-idf x-i de)e^{(3dx+3c)}+(dfx+de)e^{(2dx+2c)}+(-idf x-i de-4i f)e^{(dx+c)}}{2(adf^2x^2+2adefx+ade^2)e^{(2dx+2c)}+(-2iadf^2x^2-4i adefx-2i ade^2)e^{(dx+c)}} \right), x}{-iadfx - iade + (adfx + ade)e^{(dx+c)}} - 2i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")`

```
[Out] ((-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))*integral((d*f*x + d
*e + (-I*d*f*x - I*d*e)*e^(3*d*x + 3*c) + (d*f*x + d*e)*e^(2*d*x + 2*c) + (
-I*d*f*x - I*d*e - 4*I*f)*e^(d*x + c))/(2*(a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*
e^2)*e^(2*d*x + 2*c) + (-2*I*a*d*f^2*x^2 - 4*I*a*d*e*f*x - 2*I*a*d*e^2)*e^(
d*x + c)), x) - 2*I)/(-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)^2}{(fx+e)(ia \sinh(dx+c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^2/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

$$3.198 \quad \int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Sinh[c + d\*x]^2/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0751888, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Sinh[c + d\*x]^2/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])),x]

[Out] Defer[Int][Sinh[c + d\*x]^2/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx = \int \frac{\sinh^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.002, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[Sinh[c + d\*x]^2/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])),x]

[Out] \$Aborted

---

**Maple [A]** time = 0.218, size = 0, normalized size = 0.

$$\int \frac{(\sinh(dx + c))^2}{(fx + e)^2 (a + ia \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d\*x+c)^2/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] int(sinh(d\*x+c)^2/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-4if \int \frac{1}{-iadf^3x^3 - 3iade^2fx - iade^3 + (adf^3x^3e^c + 3adf^2x^2e^c + 3ade^2fxe^c + ade^3e^c)e^{(dx)}} dx + \frac{1}{4(-iadf^3x^3 - 3iade^2fx - iade^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)^2/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -4\*I\*f\*integrate(1/(-I\*a\*d\*f^3\*x^3 - 3\*I\*a\*d\*e\*f^2\*x^2 - 3\*I\*a\*d\*e^2\*f\*x - I\*a\*d\*e^3 + (a\*d\*f^3\*x^3\*e^c + 3\*a\*d\*e\*f^2\*x^2\*e^c + 3\*a\*d\*e^2\*f\*x\*e^c + a\*d\*e^3\*e^c)\*e^(d\*x)), x) + 1/4\*(4\*I\*d\*f\*x + 4\*I\*d\*e - 4\*(d\*f\*x\*e^c + d\*e\*e^c)\*e^(d\*x) - 8\*I\*f)/(-I\*a\*d\*f^3\*x^2 - 2\*I\*a\*d\*e\*f^2\*x - I\*a\*d\*e^2\*f + (a\*d\*f^3\*x^2\*e^c + 2\*a\*d\*e\*f^2\*x\*e^c + a\*d\*e^2\*f\*e^c)\*e^(d\*x)) - 1/2\*I\*e^(-c + d\*e/f)\*exp\_integral\_e(2, (f\*x + e)\*d/f)/((f\*x + e)\*a\*f) + 1/2\*I\*e^(c - d\*e/f)\*exp\_integral\_e(2, -(f\*x + e)\*d/f)/((f\*x + e)\*a\*f)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\frac{(-iadf^2x^2 - 2iade^2fx - iade^2 + (adf^2x^2 + 2adefx + ade^2)e^{(dx+c)}) \operatorname{integral} \left( \frac{dfx+de+(-idf x-i de)e^{(3dx+3c)}+(dfx+de)e^{(2dx+2c)}}{2(adf^3x^3+3adef^2x^2+3ade^2fx+ade^3)e^{(2dx+2c)}} + (-iadf^3x^3 - 3iade^2fx - iade^3) \right)}{-iadf^2x^2 - 2iade^2fx - iade^2 + (adf^2x^2 + 2adefx + ade^2)e^{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))*integral((d*f*x + d*e + (-I*d*f*x - I*d*e)*e^(3*d*x + 3*c) + (d*f*x + d*e)*e^(2*d*x + 2*c) + (-I*d*f*x - I*d*e - 8*I*f)*e^(d*x + c)))/(2*(a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*e^(2*d*x + 2*c) + (-2*I*a*d*f^3*x^3 - 6*I*a*d*e*f^2*x^2 - 6*I*a*d*e^2*f*x - 2*I*a*d*e^3)*e^(d*x + c)), x) - 2*I)/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)^2}{(fx+e)^2 (ia \sinh(dx+c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^2/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

$$3.199 \quad \int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=393

$$\frac{12if^2(e+fx)\text{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^3} - \frac{12if^3\text{PolyLog}\left(3, -ie^{c+dx}\right)}{ad^4} + \frac{6f^2(e+fx) \cosh(c+dx)}{ad^3} - \frac{3if^2(e+fx) \sinh(c+dx)}{4ad^3}$$

[Out] (((3\*I)/4)\*e\*f^2\*x)/(a\*d^2) + (((3\*I)/8)\*f^3\*x^2)/(a\*d^2) - (I\*(e + f\*x)^3)/(a\*d) + (((3\*I)/8)\*(e + f\*x)^4)/(a\*f) + (6\*f^2\*(e + f\*x)\*Cosh[c + d\*x])/(a\*d^3) + ((e + f\*x)^3\*Cosh[c + d\*x])/(a\*d) + ((6\*I)\*f\*(e + f\*x)^2\*Log[1 + I\*E^(c + d\*x)])/(a\*d^2) + ((12\*I)\*f^2\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*d^3) - ((12\*I)\*f^3\*PolyLog[3, (-I)\*E^(c + d\*x)])/(a\*d^4) - (6\*f^3\*Sinh[c + d\*x])/(a\*d^4) - (3\*f\*(e + f\*x)^2\*Sinh[c + d\*x])/(a\*d^2) - (((3\*I)/4)\*f^2\*(e + f\*x)\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(a\*d^3) - ((I/2)\*(e + f\*x)^3\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(a\*d) + (((3\*I)/8)\*f^3\*Sinh[c + d\*x]^2)/(a\*d^4) + (((3\*I)/4)\*f\*(e + f\*x)^2\*Sinh[c + d\*x]^2)/(a\*d^2) - (I\*(e + f\*x)^3\*Tanh[c/2 + (I/4)\*Pi + (d\*x)/2])/(a\*d)

**Rubi [A]** time = 0.702415, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$ , Rules used = {5557, 3311, 32, 3310, 3296, 2637, 3318, 4184, 3716, 2190, 2531, 2282, 6589}

$$\frac{12if^2(e+fx)\text{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^3} - \frac{12if^3\text{PolyLog}\left(3, -ie^{c+dx}\right)}{ad^4} + \frac{6f^2(e+fx) \cosh(c+dx)}{ad^3} - \frac{3if^2(e+fx) \sinh(c+dx)}{4ad^3}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Sinh[c + d\*x]^3)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (((3\*I)/4)\*e\*f^2\*x)/(a\*d^2) + (((3\*I)/8)\*f^3\*x^2)/(a\*d^2) - (I\*(e + f\*x)^3)/(a\*d) + (((3\*I)/8)\*(e + f\*x)^4)/(a\*f) + (6\*f^2\*(e + f\*x)\*Cosh[c + d\*x])/(a\*d^3) + ((e + f\*x)^3\*Cosh[c + d\*x])/(a\*d) + ((6\*I)\*f\*(e + f\*x)^2\*Log[1 + I\*E^(c + d\*x)])/(a\*d^2) + ((12\*I)\*f^2\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*d^3) - ((12\*I)\*f^3\*PolyLog[3, (-I)\*E^(c + d\*x)])/(a\*d^4) - (6\*f^3\*Sinh[c + d\*x])/(a\*d^4) - (3\*f\*(e + f\*x)^2\*Sinh[c + d\*x])/(a\*d^2) - (((3\*I)/4)\*f^2\*(e + f\*x)\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(a\*d^3) - ((I/2)\*(e + f\*x)^3\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(a\*d) + (((3\*I)/8)\*f^3\*Sinh[c + d\*x]^2)/(a\*d^4) + (((3\*I)/4)\*f\*(e + f\*x)^2\*Sinh[c + d\*x]^2)/(a\*d^2) - (I\*(e + f\*x)^3\*Tanh[c/2



+ (I/4)\*Pi + (d\*x)/2]/(a\*d)

### Rule 5557

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Sinh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sinh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_.), x\_Symbol] := Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*Sinh[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*Sinh[e + f\*x])^(n - 2), x], x] - Dist[(d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*Sinh[e + f\*x])^n, x], x] - Simp[(b\*(c + d\*x)^m\*Cosh[e + f\*x]\*(b\*Sinh[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

### Rule 3310

Int[((c\_.) + (d\_.)\*(x\_))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_.), x\_Symbol] := Simp[(d\*(b\*Sinh[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)\*(b\*Sinh[e + f\*x])^(n - 2), x], x] - Simp[(b\*(c + d\*x)\*Cosh[e + f\*x]\*(b\*Sinh[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[((c + d\*x)^m\*Cosh[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cosh[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Dist[(2*a)^(n), Int[(c + d*x)^(m)*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
(f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

#### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp
p[((c + d*x)^(m)*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

#### Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^(m)*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

#### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^(m)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^(m)*PolyLog[2, -(e*(F^(c*(a + b*x
))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+ia \sinh(c+dx)} dx &= i \int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+ia \sinh(c+dx)} dx - \frac{i \int (e+fx)^3 \sinh^2(c+dx) dx}{a} \\
&= -\frac{i(e+fx)^3 \cosh(c+dx) \sinh(c+dx)}{2ad} + \frac{3if(e+fx)^2 \sinh^2(c+dx)}{4ad^2} + \frac{i \int (e+fx)^3 dx}{2a} \\
&= \frac{i(e+fx)^4}{8af} + \frac{(e+fx)^3 \cosh(c+dx)}{ad} - \frac{3if^2(e+fx) \cosh(c+dx) \sinh(c+dx)}{4ad^3} - \frac{i(e+fx)^3}{2a} \\
&= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} + \frac{3i(e+fx)^4}{8af} + \frac{(e+fx)^3 \cosh(c+dx)}{ad} - \frac{3f(e+fx)^2 \sinh(c+dx)}{ad^2} \\
&= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} + \frac{3i(e+fx)^4}{8af} + \frac{6f^2(e+fx) \cosh(c+dx)}{ad^3} + \frac{(e+fx)^3 \cosh(c+dx)}{ad} \\
&= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} - \frac{i(e+fx)^3}{ad} + \frac{3i(e+fx)^4}{8af} + \frac{6f^2(e+fx) \cosh(c+dx)}{ad^3} + \frac{(e+fx)^3 \cosh(c+dx)}{ad} \\
&= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} - \frac{i(e+fx)^3}{ad} + \frac{3i(e+fx)^4}{8af} + \frac{6f^2(e+fx) \cosh(c+dx)}{ad^3} + \frac{(e+fx)^3 \cosh(c+dx)}{ad} \\
&= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} - \frac{i(e+fx)^3}{ad} + \frac{3i(e+fx)^4}{8af} + \frac{6f^2(e+fx) \cosh(c+dx)}{ad^3} + \frac{(e+fx)^3 \cosh(c+dx)}{ad} \\
&= \frac{3ief^2x}{4ad^2} + \frac{3if^3x^2}{8ad^2} - \frac{i(e+fx)^3}{ad} + \frac{3i(e+fx)^4}{8af} + \frac{6f^2(e+fx) \cosh(c+dx)}{ad^3} + \frac{(e+fx)^3 \cosh(c+dx)}{ad}
\end{aligned}$$

**Mathematica [A]** time = 7.31088, size = 376, normalized size = 0.96

$$-\frac{192if^2(d(e+fx)\text{PolyLog}(2,ie^{-c-dx})+f\text{PolyLog}(3,ie^{-c-dx}))}{d^4} - \frac{6if^2(e+fx) \sinh(2(c+dx))}{d^3} + \frac{96f^2(e+fx) \cosh(c+dx)}{d^3} + \frac{96if(e+fx)^2 \log(1-ie^{-c-dx})}{d^2} - \frac{48}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sinh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((24*I)*e^3*x + (36*I)*e^2*f*x^2 + (24*I)*e*f^2*x^3 + (6*I)*f^3*x^4 + (32*(e + f*x)^3)/(d*(-I + E^c)) + (96*f^2*(e + f*x)*Cosh[c + d*x])/d^3 + (16*(e + f*x)^3*Cosh[c + d*x])/d + ((3*I)*f^3*Cosh[2*(c + d*x)])/d^4 + ((6*I)*f*(e + f*x)^2*Cosh[2*(c + d*x)])/d^2 + ((96*I)*f*(e + f*x)^2*Log[1 - I*E^(-c - d*x)])/d^2 - ((192*I)*f^2*(d*(e + f*x)*PolyLog[2, I*E^(-c - d*x)] + f*PolyLog[3, I*E^(-c - d*x)]))/d^4 - ((32*I)*(e + f*x)^3*Sinh[(d*x)/2])/(d*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[(c + d*x)/2] + I*Sinh[(c + d*x)/2])) - (96*f^3*Sinh[c + d*x])/d^4 - (48*f*(e + f*x)^2*Sinh[c + d*x])/d^2 - ((6*I)*f^2*(e + f*x)*Sinh[2*(c + d*x)])/d^3 - ((4*I)*(e + f*x)^3*Sinh[2*(c + d*x)])/d/(16*a)
```

**Maple [B]** time = 0.155, size = 928, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 2*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(exp(d*x+c)-I)+6*I/d^2/a*f^3*ln(1+I*exp(d*x+c))*x^2-6*I/d^2/a*ln(exp(d*x+c))*e^2*f-6*I/d/a*e*f^2*x^2+6*I/d^2/a*ln(exp(d*x+c)-I)*e^2*f+6*I/d^3/a*f^3*c^2*x+6*I/d^4/a*f^3*c^2*ln(exp(d*x+c)-I)+12*I/d^3/a*e*f^2*polylog(2,-I*exp(d*x+c))-6*I/d^4/a*f^3*c^2*ln(exp(d*x+c))-6*I/d^3/a*e*f^2*c^2+12*I/d^3/a*f^3*polylog(2,-I*exp(d*x+c))*x-6*I/d^4/a*f^3*c^2*ln(1+I*exp(d*x+c))-12*I/d^2/a*e*f^2*c*x+12*I/d^3/a*e*f^2*ln(1+I*exp(d*x+c))*c+12*I/d^2/a*e*f^2*ln(1+I*exp(d*x+c))*x+12*I/d^3/a*e*f^2*c*ln(exp(d*x+c))-12*I*f^3*polylog(3,-I*exp(d*x+c))/a/d^4-2*I/d/a*f^3*x^3+4*I/d^4/a*f^3*c^3-1/32*I*(4*d^3*f^3*x^3+12*d^3*e*f^2*x^2+12*d^3*e^2*f*x-6*d^2*f^3*x^2+4*d^3*e^3-12*d^2*e*f^2*x-6*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2-3*f^3)/a/d^4*exp(2*d*x+2*c)-12*I/d^3/a*e*f^2*c*ln(exp(d*x+c)-I)+3/8*I/a*x^4*f^3+3/2*I/a*e^3*x+1/32*I*(4*d^3*f^3*x^3+12*d^3*e*f^2*x^2+12*d^3*e^2*f*x+6*d^2*f^3*x^2+4*d^3*e^3+12*d^2*e*f^2*x+6*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2+3*f^3)/a/d^4*exp(-2*d*x-2*c)+1/2*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x+3*d^2*f^3*x^2+d^3*e^3+6*d^2*e*f^2*x+3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2+6*f^3)/a/d^4*exp(-d*x-c)+1/2*(d^3*f^3*x^3+3*d^3*e*f^2*x^2+3*d^3*e^2*f*x-3*d^2*f^3*x^2+d^3*e^3-6*d^2*e*f^2*x-3*d^2*e^2*f+6*d*f^3*x+6*d*e*f^2-6*f^3)/a/d^4*exp(d*x+c)+3/2*I/a*e*f^2*x^3+9/4*I/a*e^2*f*x^2
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

---

**Fricas [C]** time = 2.78172, size = 2449, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (4*d^3*f^3*x^3 + 4*d^3*e^3 + 6*d^2*e^2*f + 6*d*e*f^2 + 3*f^3 + 6*(2*d^3*e*f^2 + d^2*f^3)*x^2 + 6*(2*d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*x + ((384*I*d*f^3*x + 384*I*d*e*f^2)*e^(3*d*x + 3*c) + 384*(d*f^3*x + d*e*f^2)*e^(2*d*x + 2*c))*dilog(-I*e^(d*x + c)) + (-4*I*d^3*f^3*x^3 - 4*I*d^3*e^3 + 6*I*d^2*e^2*f - 6*I*d*e*f^2 + 3*I*f^3 + (-12*I*d^3*e*f^2 + 6*I*d^2*f^3)*x^2 + (-12*I*d^3*e^2*f + 12*I*d^2*e*f^2 - 6*I*d*f^3)*x)*e^(5*d*x + 5*c) + 3*(4*d^3*f^3*x^3 + 4*d^3*e^3 - 14*d^2*e^2*f + 30*d*e*f^2 - 31*f^3 + 2*(6*d^3*e*f^2 - 7*d^2*f^3)*x^2 + 2*(6*d^3*e^2*f - 14*d^2*e*f^2 + 15*d*f^3)*x)*e^(4*d*x + 4*c) + (12*I*d^4*f^3*x^4 - 16*I*d^3*e^3 + (-192*I*c + 48*I)*d^2*e^2*f + (192*I*c^2 - 96*I)*d*e*f^2 + (-64*I*c^3 + 96*I)*f^3 + (48*I*d^4*e*f^2 - 80*I*d^3*f^3)*x^3 + (72*I*d^4*e^2*f - 240*I*d^3*e*f^2 + 48*I*d^2*f^3)*x^2 + (48*I*d^4*e^3 - 240*I*d^3*e^2*f + 96*I*d^2*e*f^2 - 96*I*d*f^3)*x)*e^(3*d*x + 3*c) + 4*(3*d^4*f^3*x^4 + 20*d^3*e^3 - 12*(4*c - 1)*d^2*e^2*f + 24*(2*c^2 + 1)*d*e*f^2 - 8*(2*c^3 - 3)*f^3 + 4*(3*d^4*e*f^2 + d^3*f^3)*x^3 + 6*(3*d^4*e^2*f + 2*d^3*e*f^2 + 2*d^2*f^3)*x^2 + 12*(d^4*e^3 + d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*x)*e^(2*d*x + 2*c) + (-12*I*d^3*f^3*x^3 - 12*I*d^3*e^3 - 42*I*d^2*e^2*f - 90*I*d*e*f^2 - 93*I*f^3 + (-36*I*d^3*e*f^2 - 42*I*d^2*f^3)*x^2 + (-36*I*d^3*e^2*f - 84*I*d^2*e*f^2 - 90*I*d*f^3)*x)*e^(d*x + c) + ((192*I*d^2*e^2*f - 384*I*c*d*e*f^2 + 192*I*c^2*f^3)*e^(3*d*x + 3*c) + 192*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*e^(2*d*x + 2*c))*log(e^(d*x + c) - I) + ((192*I*d^2*f^3*x^2
```

```
+ 384*I*d^2*e*f^2*x + 384*I*c*d*e*f^2 - 192*I*c^2*f^3)*e^(3*d*x + 3*c) + 1
92*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*e^(2*d*x + 2*c))*1
og(I*e^(d*x + c) + 1) + (-384*I*f^3*e^(3*d*x + 3*c) - 384*f^3*e^(2*d*x + 2*
c))*polylog(3, -I*e^(d*x + c)))/(32*a*d^4*e^(3*d*x + 3*c) - 32*I*a*d^4*e^(2
*d*x + 2*c))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \sinh(dx + c)^3}{ia \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sinh(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

$$3.200 \quad \int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=287

$$\frac{4if^2 \text{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^3} + \frac{4if(e+fx) \log\left(1 + ie^{c+dx}\right)}{ad^2} + \frac{if(e+fx) \sinh^2(c+dx)}{2ad^2} - \frac{2f(e+fx) \sinh(c+dx)}{ad^2} + \frac{2f^2 c}{ad^2}$$

[Out]  $((I/4)*f^2*x)/(a*d^2) - (I*(e + f*x)^2)/(a*d) + ((I/2)*(e + f*x)^3)/(a*f) + (2*f^2*Cosh[c + d*x])/(a*d^3) + ((e + f*x)^2*Cosh[c + d*x])/(a*d) + ((4*I)*f*(e + f*x)*Log[1 + I*E^(c + d*x)])/(a*d^2) + ((4*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3) - (2*f*(e + f*x)*Sinh[c + d*x])/(a*d^2) - ((I/4)*f^2*Cosh[c + d*x]*Sinh[c + d*x])/(a*d^3) - ((I/2)*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(a*d) + ((I/2)*f*(e + f*x)*Sinh[c + d*x]^2)/(a*d^2) - (I*(e + f*x)^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)$

**Rubi [A]** time = 0.546854, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 13, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$ , Rules used = {5557, 3311, 32, 2635, 8, 3296, 2638, 3318, 4184, 3716, 2190, 2279, 2391}

$$\frac{4if^2 \text{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^3} + \frac{4if(e+fx) \log\left(1 + ie^{c+dx}\right)}{ad^2} + \frac{if(e+fx) \sinh^2(c+dx)}{2ad^2} - \frac{2f(e+fx) \sinh(c+dx)}{ad^2} + \frac{2f^2 c}{ad^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Sinh[c + d\*x]^3)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out]  $((I/4)*f^2*x)/(a*d^2) - (I*(e + f*x)^2)/(a*d) + ((I/2)*(e + f*x)^3)/(a*f) + (2*f^2*Cosh[c + d*x])/(a*d^3) + ((e + f*x)^2*Cosh[c + d*x])/(a*d) + ((4*I)*f*(e + f*x)*Log[1 + I*E^(c + d*x)])/(a*d^2) + ((4*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3) - (2*f*(e + f*x)*Sinh[c + d*x])/(a*d^2) - ((I/4)*f^2*Cosh[c + d*x]*Sinh[c + d*x])/(a*d^3) - ((I/2)*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(a*d) + ((I/2)*f*(e + f*x)*Sinh[c + d*x]^2)/(a*d^2) - (I*(e + f*x)^2*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)$

**Rule 5557**

Int[(((e\_.) + (f\_.)\*(x\_.))^(m\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Sinh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sinh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]

] && IGtQ[n, 0]

### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*Sin[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*Sin[e + f\*x])^(n - 2), x], x] - Dist[(d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*Sin[e + f\*x])^n, x], x] - Simp[(b\*(c + d\*x)^m\*Cos[e + f\*x]\*(b\*Sin[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x]\*(b\*Sin[c + d\*x])^(n - 1))/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Sin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rule 3318

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Dist[(2\*a)^n, Int[(c + d\*x)^m\*Sin[(1\*(e + (Pi\*a)/(2\*b)))/2 + (f\*x)/2]^(2\*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])



Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+ia \sinh(c+dx)} dx &= i \int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+ia \sinh(c+dx)} dx - \frac{i \int (e+fx)^2 \sinh^2(c+dx) dx}{a} \\
&= -\frac{i(e+fx)^2 \cosh(c+dx) \sinh(c+dx)}{2ad} + \frac{if(e+fx) \sinh^2(c+dx)}{2ad^2} + \frac{i \int (e+fx)^2 dx}{2a} + \dots \\
&= \frac{i(e+fx)^3}{6af} + \frac{(e+fx)^2 \cosh(c+dx)}{ad} - \frac{if^2 \cosh(c+dx) \sinh(c+dx)}{4ad^3} - \frac{i(e+fx)^2 \cosh(c+dx)}{2ad} + \dots \\
&= \frac{if^2x}{4ad^2} + \frac{i(e+fx)^3}{2af} + \frac{(e+fx)^2 \cosh(c+dx)}{ad} - \frac{2f(e+fx) \sinh(c+dx)}{ad^2} - \frac{if^2 \cosh(c+dx)}{4ad^2} + \dots \\
&= \frac{if^2x}{4ad^2} + \frac{i(e+fx)^3}{2af} + \frac{2f^2 \cosh(c+dx)}{ad^3} + \frac{(e+fx)^2 \cosh(c+dx)}{ad} - \frac{2f(e+fx) \sinh(c+dx)}{ad^2} + \dots \\
&= \frac{if^2x}{4ad^2} - \frac{i(e+fx)^2}{ad} + \frac{i(e+fx)^3}{2af} + \frac{2f^2 \cosh(c+dx)}{ad^3} + \frac{(e+fx)^2 \cosh(c+dx)}{ad} - \frac{2f(e+fx) \sinh(c+dx)}{ad^2} + \dots \\
&= \frac{if^2x}{4ad^2} - \frac{i(e+fx)^2}{ad} + \frac{i(e+fx)^3}{2af} + \frac{2f^2 \cosh(c+dx)}{ad^3} + \frac{(e+fx)^2 \cosh(c+dx)}{ad} + \frac{4if(e+fx) \sinh(c+dx)}{ad^2} + \dots \\
&= \frac{if^2x}{4ad^2} - \frac{i(e+fx)^2}{ad} + \frac{i(e+fx)^3}{2af} + \frac{2f^2 \cosh(c+dx)}{ad^3} + \frac{(e+fx)^2 \cosh(c+dx)}{ad} + \frac{4if(e+fx) \sinh(c+dx)}{ad^2} + \dots \\
&= \frac{if^2x}{4ad^2} - \frac{i(e+fx)^2}{ad} + \frac{i(e+fx)^3}{2af} + \frac{2f^2 \cosh(c+dx)}{ad^3} + \frac{(e+fx)^2 \cosh(c+dx)}{ad} + \frac{4if(e+fx) \sinh(c+dx)}{ad^2} + \dots
\end{aligned}$$

**Mathematica [B]** time = 5.13595, size = 1661, normalized size = 5.79

result too large to display

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Sinh[c + d\*x]^3)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] ((-6\*I)\*d^2\*e^2\*E^c\*Cosh[(3\*d\*x)/2] + 6\*d^2\*e^2\*E^(4\*c)\*Cosh[(3\*d\*x)/2] - (14\*I)\*d\*e\*E^c\*f\*Cosh[(3\*d\*x)/2] - 14\*d\*e\*E^(4\*c)\*f\*Cosh[(3\*d\*x)/2] - (15\*I)\*E^c\*f^2\*Cosh[(3\*d\*x)/2] + 15\*E^(4\*c)\*f^2\*Cosh[(3\*d\*x)/2] - (12\*I)\*d^2\*e\*E^c\*f\*x\*Cosh[(3\*d\*x)/2] + 12\*d^2\*e\*E^(4\*c)\*f\*x\*Cosh[(3\*d\*x)/2] - (14\*I)\*d\*e^c\*f^2\*x\*Cosh[(3\*d\*x)/2] - 14\*d\*E^(4\*c)\*f^2\*x\*Cosh[(3\*d\*x)/2] - (6\*I)\*d^2\*E^c\*f^2\*x^2\*Cosh[(3\*d\*x)/2] + 6\*d^2\*E^(4\*c)\*f^2\*x^2\*Cosh[(3\*d\*x)/2] + 2\*d^2\*e^c

$$\begin{aligned}
& 2*\text{Cosh}[(5*d*x)/2] - (2*I)*d^2*e^2*E^(5*c)*\text{Cosh}[(5*d*x)/2] + 2*d*e*f*\text{Cosh}[(5*d*x)/2] \\
& + (2*I)*d*e*E^(5*c)*f*\text{Cosh}[(5*d*x)/2] + f^2*\text{Cosh}[(5*d*x)/2] - I*E^(5*c)*f^2*\text{Cosh}[(5*d*x)/2] \\
& + 4*d^2*e*f*x*\text{Cosh}[(5*d*x)/2] - (4*I)*d^2*e*E^(5*c)*f*x*\text{Cosh}[(5*d*x)/2] + 2*d*f^2*x*\text{Cosh}[(5*d*x)/2] \\
& + (2*I)*d*E^(5*c)*f^2*x*\text{Cosh}[(5*d*x)/2] + 2*d^2*f^2*x^2*\text{Cosh}[(5*d*x)/2] - (2*I)*d^2*E^(5*c)*f^2*x^2*\text{Cosh}[(5*d*x)/2] \\
& + 8*E^(2*c)*\text{Cosh}[(d*x)/2]*(2*(1 - I*E^c)*f^2 + 2*d*(1 + I*E^c)*f*(e + f*x) + d^2*(5 - I*E^c)*(e + f*x)^2 + d^3*(1 + I*E^c)*x*(3*e^2 + 3*e*f*x + f^2*x^2) \\
& + 8*d*(1 + I*E^c)*f*(e + f*x)*\text{Log}[1 - I*E^(-c - d*x)]) - 40*d^2*e^2*E^(2*c)*\text{Sinh}[(d*x)/2] - (8*I)*d^2*e^2*E^(3*c)*\text{Sinh}[(d*x)/2] - 16*d*e*E^(2*c)*f*\text{Sinh}[(d*x)/2] \\
& + (16*I)*d*e*E^(3*c)*f*\text{Sinh}[(d*x)/2] - 16*E^(2*c)*f^2*\text{Sinh}[(d*x)/2] - (16*I)*E^(3*c)*f^2*\text{Sinh}[(d*x)/2] - 24*d^3*e^2*E^(2*c)*x*\text{Sinh}[(d*x)/2] \\
& + (24*I)*d^3*e^2*E^(3*c)*x*\text{Sinh}[(d*x)/2] - 80*d^2*e*E^(2*c)*f*x*\text{Sinh}[(d*x)/2] - (16*I)*d^2*e*E^(3*c)*f*x*\text{Sinh}[(d*x)/2] - 16*d*E^(2*c)*f^2*x*\text{Sinh}[(d*x)/2] \\
& + (16*I)*d*E^(3*c)*f^2*x*\text{Sinh}[(d*x)/2] - 24*d^3*e*E^(2*c)*f*x^2*\text{Sinh}[(d*x)/2] + (24*I)*d^3*e*E^(3*c)*f*x^2*\text{Sinh}[(d*x)/2] - 40*d^2*E^(2*c)*f^2*x^2*\text{Sinh}[(d*x)/2] \\
& - (8*I)*d^2*E^(3*c)*f^2*x^2*\text{Sinh}[(d*x)/2] - 8*d^3*E^(2*c)*f^2*x^3*\text{Sinh}[(d*x)/2] + (8*I)*d^3*E^(3*c)*f^2*x^3*\text{Sinh}[(d*x)/2] - 64*d*e*E^(2*c)*f*\text{Log}[1 - I*E^(-c - d*x)]*\text{Sinh}[(d*x)/2] \\
& + (64*I)*d*e*E^(3*c)*f*\text{Log}[1 - I*E^(-c - d*x)]*\text{Sinh}[(d*x)/2] - 64*d*E^(2*c)*f^2*x*\text{Log}[1 - I*E^(-c - d*x)]*\text{Sinh}[(d*x)/2] \\
& + (64*I)*d*E^(3*c)*f^2*x*\text{Log}[1 - I*E^(-c - d*x)]*\text{Sinh}[(d*x)/2] + (1 - I*E^c)*\text{Sinh}[(d*x)/2]) + (6*I)*d^2*e^2*E^c*\text{Sinh}[(3*d*x)/2] \\
& + 6*d^2*e^2*E^(4*c)*\text{Sinh}[(3*d*x)/2] + (14*I)*d*e*E^c*f*\text{Sinh}[(3*d*x)/2] - 14*d*e*E^(4*c)*f*\text{Sinh}[(3*d*x)/2] \\
& + (15*I)*E^c*f^2*\text{Sinh}[(3*d*x)/2] + 15*E^(4*c)*f^2*\text{Sinh}[(3*d*x)/2] + (12*I)*d^2*e*E^c*f*x*\text{Sinh}[(3*d*x)/2] + 12*d^2*e*E^(4*c)*f*x*\text{Sinh}[(3*d*x)/2] \\
& + (14*I)*d*E^c*f^2*x*\text{Sinh}[(3*d*x)/2] - 14*d*E^(4*c)*f^2*x*\text{Sinh}[(3*d*x)/2] + (6*I)*d^2*E^c*f^2*x^2*\text{Sinh}[(3*d*x)/2] + 6*d^2*E^(4*c)*f^2*x^2*\text{Sinh}[(3*d*x)/2] \\
& - 2*d^2*e^2*\text{Sinh}[(5*d*x)/2] - (2*I)*d^2*e^2*E^(5*c)*\text{Sinh}[(5*d*x)/2] - 2*d*e*f*\text{Sinh}[(5*d*x)/2] + (2*I)*d*e*E^(5*c)*f*\text{Sinh}[(5*d*x)/2] \\
& - f^2*\text{Sinh}[(5*d*x)/2] - I*E^(5*c)*f^2*\text{Sinh}[(5*d*x)/2] - 4*d^2*e*f*x*\text{Sinh}[(5*d*x)/2] - (4*I)*d^2*e*E^(5*c)*f*x*\text{Sinh}[(5*d*x)/2] - 2*d*f^2*x*\text{Sinh}[(5*d*x)/2] \\
& + (2*I)*d*E^(5*c)*f^2*x*\text{Sinh}[(5*d*x)/2] - 2*d^2*f^2*x^2*\text{Sinh}[(5*d*x)/2] - (2*I)*d^2*E^(5*c)*f^2*x^2*\text{Sinh}[(5*d*x)/2]) / (16*a*d^3*E^(2*c)*((-I + E^c)*\text{Cosh}[(d*x)/2] + (I + E^c)*\text{Sinh}[(d*x)/2]))
\end{aligned}$$

**Maple [A]** time = 0.128, size = 508, normalized size = 1.8

$$\frac{-2if^2c^2}{ad^3} + \frac{i}{16} \frac{(2f^2x^2d^2 + 4d^2efx + 2d^2e^2 + 2df^2x + 2efd + f^2)e^{-2dx-2c}}{ad^3} - \frac{2if^2x^2}{da} - \frac{4i \ln(e^{dx+c})ef}{ad^2} + \frac{(f^2x^2d^2 + 2f^2x^2d + f^2)}{ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -2*I/a/d^3*f^2*c^2+1/16*I*(2*d^2*f^2*x^2+4*d^2*e*f*x+2*d^2*e^2+2*d*f^2*x+2*d*e*f+f^2)/a/d^3*exp(-2*d*x-2*c)-2*I/a/d*f^2*x^2-4*I/a/d^2*ln(exp(d*x+c))*f+1/2*(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2-2*d*f^2*x-2*d*e*f+2*f^2)/a/d^3*exp(d*x+c)+1/2*(d^2*f^2*x^2+2*d^2*e*f*x+d^2*e^2+2*d*f^2*x+2*d*e*f+2*f^2)/a/d^3*exp(-d*x-c)+4*I/a/d^3*f^2*ln(1+I*exp(d*x+c))*c+2*(f^2*x^2+2*e*f*x+e^2)/d/a/(exp(d*x+c)-I)-4*I/a/d^3*f^2*c*ln(exp(d*x+c)-I)-1/16*I*(2*d^2*f^2*x^2+4*d^2*e*f*x+2*d^2*e^2-2*d*f^2*x-2*d*e*f+f^2)/a/d^3*exp(2*d*x+2*c)+4*I/a/d^3*f^2*c*ln(exp(d*x+c))+3/2*I/a*e^2*x-4*I/a/d^2*f^2*c*x+1/2*I/a*x^3*f^2+4*I*f^2*polylog(2,-I*exp(d*x+c))/a/d^3+3/2*I/a*e*f*x^2+4*I/a/d^2*ln(exp(d*x+c)-I)*e*f+4*I/a/d^2*f^2*ln(1+I*exp(d*x+c))*x
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

**Fricas [B]** time = 2.67821, size = 1419, normalized size = 4.94

$$2d^2f^2x^2 + 2d^2e^2 + 2def + f^2 + 2(2d^2ef + df^2)x + (64if^2e^{(3dx+3c)} + 64f^2e^{(2dx+2c)})\text{Li}_2(-ie^{(dx+c)}) + (-2id^2f^2x^2 - 2id^2e^2 - 2idf + f^2)x + (64I^2f^2e^{(3dx+3c)} + 64I^2f^2e^{(2dx+2c)})\text{dilog}(-Ie^{(dx+c)}) + (-2I^2d^2f^2x^2 - 2I^2d^2e^2 + 2I^2d^2ef - I^2f^2 + (-4I^2d^2ef + 2I^2d^2f^2)x)e^{(5dx+5c)} + (6d^2f^2x^2 + 6d^2e^2 - 14d^2ef + 15f^2 + 2(6d^2ef - 7d^2f^2)x)e^{(4dx+4c)} + (8I^2d^3f^2x^3 - 8I^2d^2e^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*d^2*f^2*x^2 + 2*d^2*e^2 + 2*d*e*f + f^2 + 2*(2*d^2*e*f + d*f^2)*x + (64*I*f^2*e^(3*d*x + 3*c) + 64*f^2*e^(2*d*x + 2*c))*dilog(-I*e^(d*x + c)) + (-2*I*d^2*f^2*x^2 - 2*I*d^2*e^2 + 2*I*d^2*e*f - I*f^2 + (-4*I*d^2*e*f + 2*I*d^2*f^2)*x)*e^(5*d*x + 5*c) + (6*d^2*f^2*x^2 + 6*d^2*e^2 - 14*d^2*e*f + 15*f^2 + 2*(6*d^2*e*f - 7*d^2*f^2)*x)*e^(4*d*x + 4*c) + (8*I*d^3*f^2*x^3 - 8*I*d^2*e^2 +
```

$$\begin{aligned}
& (-64*I*c + 16*I)*d*e*f + (32*I*c^2 - 16*I)*f^2 + (24*I*d^3*e*f - 40*I*d^2* \\
& f^2)*x^2 + (24*I*d^3*e^2 - 80*I*d^2*e*f + 16*I*d*f^2)*x)*e^{(3*d*x + 3*c)} + \\
& 8*(d^3*f^2*x^3 + 5*d^2*e^2 - 2*(4*c - 1)*d*e*f + 2*(2*c^2 + 1)*f^2 + (3*d^3 \\
& *e*f + d^2*f^2)*x^2 + (3*d^3*e^2 + 2*d^2*e*f + 2*d*f^2)*x)*e^{(2*d*x + 2*c)} \\
& + (-6*I*d^2*f^2*x^2 - 6*I*d^2*e^2 - 14*I*d*e*f - 15*I*f^2 + (-12*I*d^2*e*f \\
& - 14*I*d*f^2)*x)*e^{(d*x + c)} + ((64*I*d*e*f - 64*I*c*f^2)*e^{(3*d*x + 3*c)} + \\
& 64*(d*e*f - c*f^2)*e^{(2*d*x + 2*c)})*\log(e^{(d*x + c)} - I) + ((64*I*d*f^2*x \\
& + 64*I*c*f^2)*e^{(3*d*x + 3*c)} + 64*(d*f^2*x + c*f^2)*e^{(2*d*x + 2*c)})*\log(I \\
& *e^{(d*x + c)} + 1))/(16*a*d^3*e^{(3*d*x + 3*c)} - 16*I*a*d^3*e^{(2*d*x + 2*c)})
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*sinh(d\*x+c)\*\*3/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \sinh(dx + c)^3}{i a \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sinh(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*sinh(d\*x + c)^3/(I\*a\*sinh(d\*x + c) + a), x)

$$3.201 \quad \int \frac{(e+fx) \sinh^3(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=175

$$\frac{if \sinh^2(c+dx)}{4ad^2} - \frac{f \sinh(c+dx)}{ad^2} + \frac{2if \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\right)}{ad^2} + \frac{(e+fx) \cosh(c+dx)}{ad} - \frac{i(e+fx) \tanh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)}{ad}$$

[Out] (((3\*I)/2)\*e\*x)/a + (((3\*I)/4)\*f\*x^2)/a + ((e + f\*x)\*Cosh[c + d\*x])/(a\*d) + ((2\*I)\*f\*Log[Cosh[c/2 + (I/4)\*Pi + (d\*x)/2]])/(a\*d^2) - (f\*Sinh[c + d\*x])/(a\*d^2) - ((I/2)\*(e + f\*x)\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(a\*d) + ((I/4)\*f\*Sinh[c + d\*x]^2)/(a\*d^2) - (I\*(e + f\*x)\*Tanh[c/2 + (I/4)\*Pi + (d\*x)/2])/(a\*d)

**Rubi [A]** time = 0.262787, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$ , Rules used = {5557, 3310, 3296, 2637, 3318, 4184, 3475}

$$\frac{if \sinh^2(c+dx)}{4ad^2} - \frac{f \sinh(c+dx)}{ad^2} + \frac{2if \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\right)}{ad^2} + \frac{(e+fx) \cosh(c+dx)}{ad} - \frac{i(e+fx) \tanh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)}{ad}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Sinh[c + d\*x]^3)/(a + I\*a\*Sinh[c + d\*x]), x]

[Out] (((3\*I)/2)\*e\*x)/a + (((3\*I)/4)\*f\*x^2)/a + ((e + f\*x)\*Cosh[c + d\*x])/(a\*d) + ((2\*I)\*f\*Log[Cosh[c/2 + (I/4)\*Pi + (d\*x)/2]])/(a\*d^2) - (f\*Sinh[c + d\*x])/(a\*d^2) - ((I/2)\*(e + f\*x)\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(a\*d) + ((I/4)\*f\*Sinh[c + d\*x]^2)/(a\*d^2) - (I\*(e + f\*x)\*Tanh[c/2 + (I/4)\*Pi + (d\*x)/2])/(a\*d)

### Rule 5557

Int[(((e\_.) + (f\_.)\*(x\_.))^(m\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Sinh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sinh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 3310

Int[((c\_.) + (d\_.)\*(x\_.))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]^(n\_.), x\_Symbol] := Simp[(d\*(b\*Sinh[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c

```
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
(f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx) \sinh^3(c+dx)}{a+ia \sinh(c+dx)} dx &= i \int \frac{(e+fx) \sinh^2(c+dx)}{a+ia \sinh(c+dx)} dx - \frac{i \int (e+fx) \sinh^2(c+dx) dx}{a} \\
&= -\frac{i(e+fx) \cosh(c+dx) \sinh(c+dx)}{2ad} + \frac{if \sinh^2(c+dx)}{4ad^2} + \frac{i \int (e+fx) dx}{2a} + \frac{\int (e+fx) \sinh(c+dx) dx}{a} \\
&= \frac{3iex}{2a} + \frac{ifx^2}{4a} + \frac{(e+fx) \cosh(c+dx)}{ad} - \frac{i(e+fx) \cosh(c+dx) \sinh(c+dx)}{2ad} + \frac{if \sinh^2(c+dx)}{4ad^2} \\
&= \frac{3iex}{2a} + \frac{3ifx^2}{4a} + \frac{(e+fx) \cosh(c+dx)}{ad} - \frac{f \sinh(c+dx)}{ad^2} - \frac{i(e+fx) \cosh(c+dx) \sinh(c+dx)}{2ad} \\
&= \frac{3iex}{2a} + \frac{3ifx^2}{4a} + \frac{(e+fx) \cosh(c+dx)}{ad} - \frac{f \sinh(c+dx)}{ad^2} - \frac{i(e+fx) \cosh(c+dx) \sinh(c+dx)}{2ad} \\
&= \frac{3iex}{2a} + \frac{3ifx^2}{4a} + \frac{(e+fx) \cosh(c+dx)}{ad} + \frac{2if \log\left(\cosh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)\right)}{ad^2} - \frac{f \sinh(c+dx)}{ad^2}
\end{aligned}$$

**Mathematica [A]** time = 1.80156, size = 325, normalized size = 1.86

$$\left(\cosh\left(\frac{1}{2}(c+dx)\right) + i \sinh\left(\frac{1}{2}(c+dx)\right)\right) \left(\cosh\left(\frac{1}{2}(c+dx)\right)\right) \left(2\left(-3c^2f - d(e+fx) \sinh(2(c+dx)) + 6cde + 4if \sinh(c+dx)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Sinh[c + d\*x]^3)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] ((Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])\*(Cosh[(c + d\*x)/2]\*((-8\*I)\*d\*(e + f\*x)\*Cosh[c + d\*x] + f\*Cosh[2\*(c + d\*x)] + 2\*(6\*c\*d\*e - (4\*I)\*c\*f - 3\*c^2\*f + 6\*d^2\*e\*x - (4\*I)\*d\*f\*x + 3\*d^2\*f\*x^2 + (8\*I)\*f\*ArcTan[Tanh[(c + d\*x)/2]] + 4\*f\*Log[Cosh[c + d\*x]] + (4\*I)\*f\*Sinh[c + d\*x] - d\*(e + f\*x)\*Sinh[2\*(c + d\*x)])) + Sinh[(c + d\*x)/2]\*(8\*d\*(e + f\*x)\*Cosh[c + d\*x] + I\*(f\*Cosh[2\*(c + d\*x)] + 2\*((8\*I)\*d\*e + 6\*c\*d\*e - (4\*I)\*c\*f - 3\*c^2\*f + 6\*d^2\*e\*x + (4\*I)\*d\*f\*x + 3\*d^2\*f\*x^2 + (8\*I)\*f\*ArcTan[Tanh[(c + d\*x)/2]] + 4\*f\*Log[Cosh[c + d\*x]] + (4\*I)\*f\*Sinh[c + d\*x] - d\*(e + f\*x)\*Sinh[2\*(c + d\*x)])))))/(8\*a\*d^2\*(-I + Sinh[c + d\*x]))

**Maple [A]** time = 0.109, size = 197, normalized size = 1.1

$$\frac{3i}{4} \frac{fx^2}{a} + \frac{3i}{2} \frac{ex}{a} - \frac{i}{16} \frac{(2dfx + 2de - f)e^{2dx+2c}}{ad^2} + \frac{(dfx + de - f)e^{dx+c}}{2ad^2} + \frac{(dfx + de + f)e^{-dx-c}}{2ad^2} + \frac{i}{16} \frac{(2dfx + 2de + f)e^{2dx+2c}}{ad^2}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 3/4*I*f*x^2/a+3/2*I*e*x/a-1/16*I*(2*d*f*x+2*d*e-f)/a/d^2*exp(2*d*x+2*c)+1/2
*(d*f*x+d*e-f)/a/d^2*exp(d*x+c)+1/2*(d*f*x+d*e+f)/a/d^2*exp(-d*x-c)+1/16*I*
(2*d*f*x+2*d*e+f)/a/d^2*exp(-2*d*x-2*c)-2*I*f/a/d*x-2*I*f/a/d^2*c+2*(f*x+e)
/d/a/(exp(d*x+c)-I)+2*I*f/a/d^2*ln(exp(d*x+c)-I)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

**Fricas [A]** time = 2.55633, size = 586, normalized size = 3.35

$$2dfx + 2de + (-2idfx - 2ide + if)e^{(5dx+5c)} + (6dfx + 6de - 7f)e^{(4dx+4c)} + (12id^2fx^2 - 8ide + (24id^2e - 40idf)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*d*f*x + 2*d*e + (-2*I*d*f*x - 2*I*d*e + I*f)*e^(5*d*x + 5*c) + (6*d*f*x
+ 6*d*e - 7*f)*e^(4*d*x + 4*c) + (12*I*d^2*f*x^2 - 8*I*d*e + (24*I*d^2*e -
40*I*d*f)*x + 8*I*f)*e^(3*d*x + 3*c) + 4*(3*d^2*f*x^2 + 10*d*e + 2*(3*d^2*e
+ d*f)*x + 2*f)*e^(2*d*x + 2*c) + (-6*I*d*f*x - 6*I*d*e - 7*I*f)*e^(d*x +
c) - 32*(-I*f*e^(3*d*x + 3*c) - f*e^(2*d*x + 2*c))*log(e^(d*x + c) - I) + f
)/(16*a*d^2*e^(3*d*x + 3*c) - 16*I*a*d^2*e^(2*d*x + 2*c))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)\*\*3/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [B]** time = 1.43709, size = 479, normalized size = 2.74

$$12i d^2 f x^2 e^{(3dx+4c)} + 12 d^2 f x^2 e^{(2dx+3c)} - 2i d f x e^{(5dx+6c)} + 6 d f x e^{(4dx+5c)} + 24i d^2 x e^{(3dx+4c+1)} - 40i d f x e^{(3dx+4c)} + 24 d^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] (12\*I\*d^2\*f\*x^2\*e^(3\*d\*x + 4\*c) + 12\*d^2\*f\*x^2\*e^(2\*d\*x + 3\*c) - 2\*I\*d\*f\*x\*e^(5\*d\*x + 6\*c) + 6\*d\*f\*x\*e^(4\*d\*x + 5\*c) + 24\*I\*d^2\*x\*e^(3\*d\*x + 4\*c + 1) - 40\*I\*d\*f\*x\*e^(3\*d\*x + 4\*c) + 24\*d^2\*x\*e^(2\*d\*x + 3\*c + 1) + 8\*d\*f\*x\*e^(2\*d\*x + 3\*c) - 6\*I\*d\*f\*x\*e^(d\*x + 2\*c) + 2\*d\*f\*x\*e^c + 32\*I\*f\*e^(3\*d\*x + 4\*c)\*log(e^(d\*x + c) - I) + 32\*f\*e^(2\*d\*x + 3\*c)\*log(e^(d\*x + c) - I) - 2\*I\*d\*e^(5\*d\*x + 6\*c + 1) + I\*f\*e^(5\*d\*x + 6\*c) + 6\*d\*e^(4\*d\*x + 5\*c + 1) - 7\*f\*e^(4\*d\*x + 5\*c) - 8\*I\*d\*e^(3\*d\*x + 4\*c + 1) + 8\*I\*f\*e^(3\*d\*x + 4\*c) + 40\*d\*e^(2\*d\*x + 3\*c + 1) + 8\*f\*e^(2\*d\*x + 3\*c) - 6\*I\*d\*e^(d\*x + 2\*c + 1) - 7\*I\*f\*e^(d\*x + 2\*c) + 2\*d\*e^(c + 1) + f\*e^c)/(16\*a\*d^2\*e^(3\*d\*x + 4\*c) - 16\*I\*a\*d^2\*e^(2\*d\*x + 3\*c))

$$3.202 \quad \int \frac{\sinh^3(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=83

$$\frac{2 \cosh(c+dx)}{ad} - \frac{\sinh^2(c+dx) \cosh(c+dx)}{d(a+ia \sinh(c+dx))} - \frac{3i \sinh(c+dx) \cosh(c+dx)}{2ad} + \frac{3ix}{2a}$$

[Out] (((3\*I)/2)\*x)/a + (2\*Cosh[c + d\*x])/(a\*d) - (((3\*I)/2)\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(a\*d) - (Cosh[c + d\*x]\*Sinh[c + d\*x]^2)/(d\*(a + I\*a\*Sinh[c + d\*x]))

**Rubi [A]** time = 0.0795302, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {2767, 2734}

$$\frac{2 \cosh(c+dx)}{ad} - \frac{\sinh^2(c+dx) \cosh(c+dx)}{d(a+ia \sinh(c+dx))} - \frac{3i \sinh(c+dx) \cosh(c+dx)}{2ad} + \frac{3ix}{2a}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d\*x]^3/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (((3\*I)/2)\*x)/a + (2\*Cosh[c + d\*x])/(a\*d) - (((3\*I)/2)\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(a\*d) - (Cosh[c + d\*x]\*Sinh[c + d\*x]^2)/(d\*(a + I\*a\*Sinh[c + d\*x]))

#### Rule 2767

Int[((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_)/((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> -Simp[((b\*c - a\*d)\*Cos[e + f\*x]\*(c + d\*Sin[e + f\*x])^(n - 1))/(a\*f\*(a + b\*Sin[e + f\*x])), x] - Dist[d/(a\*b), Int[(c + d\*Sin[e + f\*x])^(n - 2)\*Simp[b\*d\*(n - 1) - a\*c\*n + (b\*c\*(n - 1) - a\*d\*n)\*Sin[e + f\*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2\*n] || EqQ[c, 0])

#### Rule 2734

Int[((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[((2\*a\*c + b\*d)\*x)/2, x] + (-Simp[((b\*c + a\*d)\*Cos[e + f\*x])/f, x] - Simp[(b\*d\*Cos[e + f\*x]\*Sin[e + f\*x])/(2\*f), x]) /; Free

$Q[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

### Rubi steps

$$\int \frac{\sinh^3(c + dx)}{a + ia \sinh(c + dx)} dx = -\frac{\cosh(c + dx) \sinh^2(c + dx)}{d(a + ia \sinh(c + dx))} + \frac{\int \sinh(c + dx)(2a - 3ia \sinh(c + dx)) dx}{a^2}$$

$$= \frac{3ix}{2a} + \frac{2 \cosh(c + dx)}{ad} - \frac{3i \cosh(c + dx) \sinh(c + dx)}{2ad} - \frac{\cosh(c + dx) \sinh^2(c + dx)}{d(a + ia \sinh(c + dx))}$$

**Mathematica [A]** time = 0.163607, size = 109, normalized size = 1.31

$$\frac{(3\sqrt{1 + i \sinh(c + dx)} \sinh^{-1}(\sinh(c + dx)) + \sqrt{1 - i \sinh(c + dx)} (-i \sinh^2(c + dx) + \sinh(c + dx) - 4i)) \cosh(c + dx)}{2ad\sqrt{1 - i \sinh(c + dx)}(\sinh(c + dx) - i)}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d\*x]^3/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (Cosh[c + d\*x]\*(3\*ArcSinh[Sinh[c + d\*x]]\*Sqrt[1 + I\*Sinh[c + d\*x]] + Sqrt[1 - I\*Sinh[c + d\*x]]\*(-4\*I + Sinh[c + d\*x] - I\*Sinh[c + d\*x]^2)))/(2\*a\*d\*Sqrt[1 - I\*Sinh[c + d\*x]]\*(-I + Sinh[c + d\*x]))

**Maple [B]** time = 0.041, size = 196, normalized size = 2.4

$$\frac{-2i}{da} \left(-i + \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-1} + \frac{i}{2da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^{-2} + \frac{3i}{2da} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) + \frac{1}{da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x)

[Out] -2\*I/d/a/(-I+tanh(1/2\*d\*x+1/2\*c))+1/2\*I/d/a/(tanh(1/2\*d\*x+1/2\*c)+1)^2+3/2\*I/d/a\*ln(tanh(1/2\*d\*x+1/2\*c)+1)+1/d/a/(tanh(1/2\*d\*x+1/2\*c)+1)-1/2\*I/d/a/(tanh(1/2\*d\*x+1/2\*c)+1)-3/2\*I/d/a\*ln(tanh(1/2\*d\*x+1/2\*c)-1)-1/2\*I/d/a/(tanh(1/2\*d\*x+1/2\*c)-1)^2-1/d/a/(tanh(1/2\*d\*x+1/2\*c)-1)-1/2\*I/d/a/(tanh(1/2\*d\*x+1/2\*c)-1)

---

**Maxima [A]** time = 1.02886, size = 132, normalized size = 1.59

$$\frac{3i(dx+c)}{2ad} + \frac{3ie^{(-dx-c)} + 20e^{(-2dx-2c)} + 1}{8(iae^{(-2dx-2c)} + ae^{(-3dx-3c)})d} + \frac{i(-4ie^{(-dx-c)} + e^{(-2dx-2c)})}{8ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 3/2\*I\*(d\*x + c)/(a\*d) + 1/8\*(3\*I\*e^(-d\*x - c) + 20\*e^(-2\*d\*x - 2\*c) + 1)/((I\*a\*e^(-2\*d\*x - 2\*c) + a\*e^(-3\*d\*x - 3\*c))\*d) + 1/8\*I\*(-4\*I\*e^(-d\*x - c) + e^(-2\*d\*x - 2\*c))/(a\*d)

---

**Fricas [A]** time = 2.59988, size = 243, normalized size = 2.93

$$\frac{(12i dx - 4i)e^{(3dx+3c)} + 4(3dx + 5)e^{(2dx+2c)} - ie^{(5dx+5c)} + 3e^{(4dx+4c)} - 3ie^{(dx+c)} + 1}{8ade^{(3dx+3c)} - 8iade^{(2dx+2c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] ((12\*I\*d\*x - 4\*I)\*e^(3\*d\*x + 3\*c) + 4\*(3\*d\*x + 5)\*e^(2\*d\*x + 2\*c) - I\*e^(5\*d\*x + 5\*c) + 3\*e^(4\*d\*x + 4\*c) - 3\*I\*e^(d\*x + c) + 1)/(8\*a\*d\*e^(3\*d\*x + 3\*c) - 8\*I\*a\*d\*e^(2\*d\*x + 2\*c))

---

**Sympy [A]** time = 1.39073, size = 180, normalized size = 2.17

$$\begin{cases} \frac{(-32ia^3d^3e^{5c}e^{2dx} + 128a^3d^3e^{4c}e^{dx} + 128a^3d^3e^{2c}e^{-dx} + 32ia^3d^3e^c e^{-2dx})e^{-3c}}{256a^4d^4} & \text{for } 256a^4d^4e^{3c} \neq 0 \\ x \left( -\frac{(ie^{4c} - 2e^{3c} - 6ie^{2c} + 2e^c + i)e^{-2c}}{4a} - \frac{3i}{2a} \right) & \text{otherwise} \end{cases} + \frac{3ix}{2a} + \frac{2e^c}{ad(ie^c + e^{-dx})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)\*\*3/(a+I\*a\*sinh(d\*x+c)),x)

```
[Out] Piecewise((( -32*I*a**3*d**3*exp(5*c)*exp(2*d*x) + 128*a**3*d**3*exp(4*c)*exp(d*x) + 128*a**3*d**3*exp(2*c)*exp(-d*x) + 32*I*a**3*d**3*exp(c)*exp(-2*d*x))*exp(-3*c)/(256*a**4*d**4), Ne(256*a**4*d**4*exp(3*c), 0)), (x*(-(I*exp(4*c) - 2*exp(3*c) - 6*I*exp(2*c) + 2*exp(c) + I)*exp(-2*c)/(4*a) - 3*I/(2*a)), True)) + 3*I*x/(2*a) + 2*exp(c)/(a*d*(I*exp(c) + exp(-d*x)))
```

**Giac [A]** time = 1.47046, size = 127, normalized size = 1.53

$$\frac{3i(dx+c)}{2ad} + \frac{(20e^{(2dx+2c)} - 3ie^{(dx+c)} + 1)e^{(-2dx-2c)}}{8ad(e^{(dx+c)} - i)} - \frac{iade^{(2dx+2c)} - 4ade^{(dx+c)}}{8a^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/2*I*(d*x + c)/(a*d) + 1/8*(20*e^(2*d*x + 2*c) - 3*I*e^(d*x + c) + 1)*e^(-2*d*x - 2*c)/(a*d*(e^(d*x + c) - I)) - 1/8*(I*a*d*e^(2*d*x + 2*c) - 4*a*d*e^(d*x + c))/(a^2*d^2)
```

$$3.203 \quad \int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Optimal. Leaf size=33

$$\text{Unintegrable} \left( \frac{\sinh^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[Sinh[c + d\*x]^3/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0780933, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Sinh[c + d\*x]^3/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])),x]

[Out] Defer[Int][Sinh[c + d\*x]^3/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx = \int \frac{\sinh^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.003, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[Sinh[c + d\*x]^3/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])),x]

[Out] \$Aborted

---

**Maple [A]** time = 0.137, size = 0, normalized size = 0.

$$\int \frac{(\sinh(dx+c))^3}{(fx+e)(a+ia\sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d\*x+c)^3/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] int(sinh(d\*x+c)^3/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)^3/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\frac{(-i adfx - i ade + (adfx + ade)e^{(dx+c)}) \operatorname{integral} \left( -\frac{dfx+de - (-idfx - ide)e^{(5dx+5c)} - (dfx+de)e^{(4dx+4c)} - (4idfx+4ide)e^{(3dx+3c)} - 4(dfx+de+2f)}{4(adf^2x^2+2adefx+ade^2)e^{(3dx+3c)} + (-4iadf^2x^2-8iade fx-4iade^2)e^{(2dx+2c)}} \right)}{-i adfx - i ade + (adfx + ade)e^{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)^3/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] ((-I\*a\*d\*f\*x - I\*a\*d\*e + (a\*d\*f\*x + a\*d\*e)\*e^(d\*x + c))\*integral(-(d\*f\*x + d\*e - (-I\*d\*f\*x - I\*d\*e)\*e^(5\*d\*x + 5\*c) - (d\*f\*x + d\*e)\*e^(4\*d\*x + 4\*c) - (4\*I\*d\*f\*x + 4\*I\*d\*e)\*e^(3\*d\*x + 3\*c) - 4\*(d\*f\*x + d\*e + 2\*f)\*e^(2\*d\*x + 2\*c) - (I\*d\*f\*x + I\*d\*e)\*e^(d\*x + c))/(4\*(a\*d\*f^2\*x^2 + 2\*a\*d\*e\*f\*x + a\*d\*e^2)\*e^(3\*d\*x + 3\*c) + (-4\*I\*a\*d\*f^2\*x^2 - 8\*I\*a\*d\*e\*f\*x - 4\*I\*a\*d\*e^2)\*e^(2\*d\*x + 2\*c))



```
*x + 2*c)), x) + 2)/(-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)^3}{(fx+e)(ia \sinh(dx+c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^3/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

$$3.204 \quad \int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

**Optimal.** Leaf size=33

$$\text{Unintegrable}\left(\frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Sinh[c + d\*x]^3/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0786479, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Sinh[c + d\*x]^3/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Sinh[c + d\*x]^3/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx = \int \frac{\sinh^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.002, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[Sinh[c + d\*x]^3/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] \$Aborted

---

**Maple [A]** time = 0.157, size = 0, normalized size = 0.

$$\int \frac{(\sinh(dx + c))^3}{(fx + e)^2 (a + ia \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d\*x+c)^3/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] int(sinh(d\*x+c)^3/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)^3/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\frac{(-i adf^2x^2 - 2i adefx - i ade^2 + (adf^2x^2 + 2 adefx + ade^2)e^{(dx+c)}) \operatorname{integral} \left( -\frac{dfx+de - (-idf x - ide)e^{(5dx+5c)} - (dfx+de)e^{(4dx+4c)}}{4(adf^3x^3 + 3 adef^2x^2 + 3 ade^2fx + ade^3)e^{(3dx+c)}} \right)}{-i adf^2x^2 - 2i adefx - i ade^2 + (adf^2x^2 + 2 adefx + ade^2)e^{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)^3/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] ((-I\*a\*d\*f^2\*x^2 - 2\*I\*a\*d\*e\*f\*x - I\*a\*d\*e^2 + (a\*d\*f^2\*x^2 + 2\*a\*d\*e\*f\*x + a\*d\*e^2)\*e^(d\*x + c))\*integral(-(d\*f\*x + d\*e - (-I\*d\*f\*x - I\*d\*e)\*e^(5\*d\*x + 5\*c) - (d\*f\*x + d\*e)\*e^(4\*d\*x + 4\*c) - (4\*I\*d\*f\*x + 4\*I\*d\*e)\*e^(3\*d\*x +

$$\begin{aligned} & 3*c) - 4*(d*f*x + d*e + 4*f)*e^{(2*d*x + 2*c)} - (I*d*f*x + I*d*e)*e^{(d*x + c)} \\ & ))/(4*(a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*e^{(3*d*x + 3*c)} \\ & + (-4*I*a*d*f^3*x^3 - 12*I*a*d*e*f^2*x^2 - 12*I*a*d*e^2*f*x - 4*I*a*d*e^3)*e^{(2*d*x + 2*c)}), x) + 2)/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 \\ & + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^{(d*x + c)}) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)\*\*3/(f\*x+e)\*\*2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)^3}{(fx+e)^2 (ia \sinh(dx+c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)^3/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate(sinh(d\*x + c)^3/((f\*x + e)^2\*(I\*a\*sinh(d\*x + c) + a)), x)

$$3.205 \quad \int \frac{(e+fx)^3 \operatorname{csch}(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=313

$$\frac{12if^2(e+fx)\operatorname{PolyLog}(2, -ie^{c+dx})}{ad^3} + \frac{6f^2(e+fx)\operatorname{PolyLog}(3, -e^{c+dx})}{ad^3} - \frac{6f^2(e+fx)\operatorname{PolyLog}(3, e^{c+dx})}{ad^3} - \frac{3f(e+fx)^2}{ad^3}$$

```
[Out] ((-I)*(e + f*x)^3)/(a*d) - (2*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) + ((6
*I)*f*(e + f*x)^2*Log[1 + I*E^(c + d*x)])/(a*d^2) - (3*f*(e + f*x)^2*PolyLo
g[2, -E^(c + d*x)])/(a*d^2) + ((12*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c +
d*x)])/(a*d^3) + (3*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)])/(a*d^2) + (6*f^2
*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) - ((12*I)*f^3*PolyLog[3, (-I)*
E^(c + d*x)])/(a*d^4) - (6*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a*d^3) -
(6*f^3*PolyLog[4, -E^(c + d*x)])/(a*d^4) + (6*f^3*PolyLog[4, E^(c + d*x)])
/(a*d^4) - (I*(e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

**Rubi [A]** time = 0.482013, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$ , Rules used = {5575, 4182, 2531, 6609, 2282, 6589, 3318, 4184, 3716, 2190}

$$\frac{12if^2(e+fx)\operatorname{PolyLog}(2, -ie^{c+dx})}{ad^3} + \frac{6f^2(e+fx)\operatorname{PolyLog}(3, -e^{c+dx})}{ad^3} - \frac{6f^2(e+fx)\operatorname{PolyLog}(3, e^{c+dx})}{ad^3} - \frac{3f(e+fx)^2}{ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((-I)*(e + f*x)^3)/(a*d) - (2*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) + ((6
*I)*f*(e + f*x)^2*Log[1 + I*E^(c + d*x)])/(a*d^2) - (3*f*(e + f*x)^2*PolyLo
g[2, -E^(c + d*x)])/(a*d^2) + ((12*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c +
d*x)])/(a*d^3) + (3*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)])/(a*d^2) + (6*f^2
*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) - ((12*I)*f^3*PolyLog[3, (-I)*
E^(c + d*x)])/(a*d^4) - (6*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a*d^3) -
(6*f^3*PolyLog[4, -E^(c + d*x)])/(a*d^4) + (6*f^3*PolyLog[4, E^(c + d*x)])
/(a*d^4) - (I*(e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

**Rule 5575**

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Csch[c
```

+ d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Csch[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 4182

Int[csc[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)]/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(p\_.)], x\_Symbol] :> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p]]/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\* (F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^(n), Int[(c + d*x)^(m)*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
(f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

#### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Simp
p[((c + d*x)^(m)*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

#### Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^(m)*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

#### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^(m)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \operatorname{csch}(c+dx)}{a+ia \sinh(c+dx)} dx &= -\left(i \int \frac{(e+fx)^3}{a+ia \sinh(c+dx)} dx\right) + \frac{\int (e+fx)^3 \operatorname{csch}(c+dx) dx}{a} \\
&= -\frac{2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{i \int (e+fx)^3 \operatorname{csc}^2\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{idx}{2}\right) dx}{2a} - \frac{(3f) \int (e+fx)^2 \operatorname{lo}}{ad} \\
&= -\frac{2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{3f(e+fx)^2 \operatorname{Li}_2(-e^{c+dx})}{ad^2} + \frac{3f(e+fx)^2 \operatorname{Li}_2(e^{c+dx})}{ad^2} - \frac{i(e+f}{ad^2} \\
&= -\frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{3f(e+fx)^2 \operatorname{Li}_2(-e^{c+dx})}{ad^2} + \frac{3f(e+fx)^2 \operatorname{Li}_2(e^{c+dx})}{ad^2} \\
&= -\frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} + \frac{6if(e+fx)^2 \log(1+ie^{c+dx})}{ad^2} - \frac{3f(e+fx)^2 \operatorname{Li}_2}{ad^2} \\
&= -\frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} + \frac{6if(e+fx)^2 \log(1+ie^{c+dx})}{ad^2} - \frac{3f(e+fx)^2 \operatorname{Li}_2}{ad^2} \\
&= -\frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} + \frac{6if(e+fx)^2 \log(1+ie^{c+dx})}{ad^2} - \frac{3f(e+fx)^2 \operatorname{Li}_2}{ad^2} \\
&= -\frac{i(e+fx)^3}{ad} - \frac{2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} + \frac{6if(e+fx)^2 \log(1+ie^{c+dx})}{ad^2} - \frac{3f(e+fx)^2 \operatorname{Li}_2}{ad^2}
\end{aligned}$$

**Mathematica [A]** time = 5.59883, size = 363, normalized size = 1.16

$$-3f(d^2(e+fx)^2 \operatorname{PolyLog}(2, -\sinh(c+dx) - \cosh(c+dx))) - 2df(e+fx) \operatorname{PolyLog}(3, -\sinh(c+dx) - \cosh(c+dx)) +$$

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^3\*Csch[c + d\*x])/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] ((2\*d^3\*(e + f\*x)^3)/(-I + E^c) - 2\*d^3\*(e + f\*x)^3\*ArcTanh[Cosh[c + d\*x] + Sinh[c + d\*x]] + (6\*I)\*d^2\*f\*(e + f\*x)^2\*Log[1 - I\*E^(-c - d\*x)] - (12\*I)\*f^2\*(d\*(e + f\*x)\*PolyLog[2, I\*E^(-c - d\*x)] + f\*PolyLog[3, I\*E^(-c - d\*x)]) - 3\*f\*(d^2\*(e + f\*x)^2\*PolyLog[2, -Cosh[c + d\*x] - Sinh[c + d\*x]] - 2\*d\*f\*(e + f\*x)\*PolyLog[3, -Cosh[c + d\*x] - Sinh[c + d\*x]] + 2\*f^2\*PolyLog[4, -Cosh[c + d\*x] - Sinh[c + d\*x]]) + 3\*f\*(d^2\*(e + f\*x)^2\*PolyLog[2, Cosh[c + d\*x] + Sinh[c + d\*x]])



$$x] + \text{Sinh}[c + d*x]] - 2*d*f*(e + f*x)*\text{PolyLog}[3, \text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]] + 2*f^2*\text{PolyLog}[4, \text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]] - ((2*I)*d^3*(e + f*x)^3*\text{Sinh}[(d*x)/2])/((\text{Cosh}[c/2] + I*\text{Sinh}[c/2])*(\text{Cosh}[(c + d*x)/2] + I*\text{Sinh}[(c + d*x)/2]))/(a*d^4)$$

**Maple [B]** time = 0.268, size = 1034, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)^3*\text{csch}(d*x+c)/(a+I*a*\text{sinh}(d*x+c)), x)$

[Out]  $2*(f^3*x^3+3*e*f^2*x^2+3*e^2*f*x+e^3)/d/a/(\exp(d*x+c)-I)+6*I/d^2/a*f^3*\ln(1+I*\exp(d*x+c))*x^2-6*I/d^2/a*\ln(\exp(d*x+c))*e^2*f-6*I/d/a*e*f^2*x^2+6*I/d^2/a*\ln(\exp(d*x+c)-I)*e^2*f+6*I/d^3/a*f^3*c^2*x+6*I/d^4/a*f^3*c^2*\ln(\exp(d*x+c)-I)+12*I/d^3/a*e*f^2*\text{polylog}(2,-I*\exp(d*x+c))-6*I/d^4/a*f^3*c^2*\ln(\exp(d*x+c))-6*I/d^3/a*e*f^2*c^2+12*I/d^3/a*f^3*\text{polylog}(2,-I*\exp(d*x+c))*x-6*I/d^4/a*f^3*c^2*\ln(1+I*\exp(d*x+c))-12*I/d^2/a*e*f^2*c*x+12*I/d^3/a*e*f^2*\ln(1+I*\exp(d*x+c))*c+12*I/d^2/a*e*f^2*\ln(1+I*\exp(d*x+c))*x+12*I/d^3/a*e*f^2*c*\ln(\exp(d*x+c))-12*I*f^3*\text{polylog}(3,-I*\exp(d*x+c))/a/d^4-6*f^3*\text{polylog}(4,-\exp(d*x+c))/a/d^4+6*f^3*\text{polylog}(4,\exp(d*x+c))/a/d^4-2*I/d/a*f^3*x^3+4*I/d^4/a*f^3*c^3+3/d^3/a*e*f^2*c^2*\ln(\exp(d*x+c)-1)-12*I/d^3/a*e*f^2*c*\ln(\exp(d*x+c)-I)-3/d^2/a*e^2*f*c*\ln(\exp(d*x+c)-1)+3/d^2/a*\ln(1-\exp(d*x+c))*c*e^2*f+3/d/a*\ln(1-\exp(d*x+c))*e^2*f*x-3/d/a*\ln(\exp(d*x+c)+1)*e^2*f*x-3/d^3/a*e*f^2*c^2*\ln(1-\exp(d*x+c))+3/d/a*e*f^2*\ln(1-\exp(d*x+c))*x^2+6/d^2/a*e*f^2*\text{polylog}(2,\exp(d*x+c))*x-3/d/a*e*f^2*\ln(\exp(d*x+c)+1))*x^2-6/d^2/a*e*f^2*\text{polylog}(2,-\exp(d*x+c))*x+1/d/a*e^3*\ln(\exp(d*x+c)-1)-1/d/a*e^3*\ln(\exp(d*x+c)+1)+3/d^2/a*f^3*\text{polylog}(2,\exp(d*x+c))*x^2-6/d^3/a*f^3*\text{polylog}(3,\exp(d*x+c))*x+3/d^2/a*e^2*f*\text{polylog}(2,\exp(d*x+c))-3/d^2/a*e^2*f*\text{polylog}(2,-\exp(d*x+c))-6/d^3/a*e*f^2*\text{polylog}(3,\exp(d*x+c))+6/d^3/a*e*f^2*\text{polylog}(3,-\exp(d*x+c))-1/d^4/a*f^3*c^3*\ln(\exp(d*x+c)-1)-1/d/a*f^3*\ln(\exp(d*x+c)+1))*x^3-3/d^2/a*f^3*\text{polylog}(2,-\exp(d*x+c))*x^2+6/d^3/a*f^3*\text{polylog}(3,-\exp(d*x+c))*x+1/d/a*f^3*\ln(1-\exp(d*x+c))*x^3+1/d^4/a*f^3*\ln(1-\exp(d*x+c))*c^3$

**Maxima [B]** time = 1.96947, size = 783, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*csch(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-e^3(\log(e^{-dx-c})+1)/(a*d) - \log(e^{-dx-c})-1)/(a*d) - 2/((a*e^{-dx-c})+I*a)*d) - 6*I*e^{2*f*x}/(a*d) - 3*(d*x*\log(e^{d*x+c})+1) + \operatorname{dilog}(-e^{d*x+c})) * e^{2*f}/(a*d^2) + 3*(d*x*\log(-e^{d*x+c})+1) + \operatorname{dilog}(e^{d*x+c})) * e^{2*f}/(a*d^2) + 6*I*e^{2*f}*\log(I*e^{d*x+c})+1)/(a*d^2) + 2*(f^3*x^3 + 3*e*f^2*x^2 + 3*e^2*f*x)/(a*d*e^{d*x+c}) - I*a*d) - 3*(d^2*x^2*\log(e^{d*x+c})+1) + 2*d*x*\operatorname{dilog}(-e^{d*x+c})) - 2*\operatorname{polylog}(3, -e^{d*x+c})) * e*f^2/(a*d^3) + 3*(d^2*x^2*\log(-e^{d*x+c})+1) + 2*d*x*\operatorname{dilog}(e^{d*x+c})) - 2*\operatorname{polylog}(3, e^{d*x+c})) * e*f^2/(a*d^3) + 12*I*(d*x*\log(I*e^{d*x+c})+1) + \operatorname{dilog}(-I*e^{d*x+c})) * e*f^2/(a*d^3) - (d^3*x^3*\log(e^{d*x+c})+1) + 3*d^2*x^2*\operatorname{dilog}(-e^{d*x+c})) - 6*d*x*\operatorname{polylog}(3, -e^{d*x+c})) + 6*\operatorname{polylog}(4, -e^{d*x+c})) * f^3/(a*d^4) + (d^3*x^3*\log(-e^{d*x+c})+1) + 3*d^2*x^2*\operatorname{dilog}(e^{d*x+c})) - 6*d*x*\operatorname{polylog}(3, e^{d*x+c})) + 6*\operatorname{polylog}(4, e^{d*x+c})) * f^3/(a*d^4) + 6*I*(d^2*x^2*\log(I*e^{d*x+c})+1) + 2*d*x*\operatorname{dilog}(-I*e^{d*x+c})) - 2*\operatorname{polylog}(3, -I*e^{d*x+c})) * f^3/(a*d^4) - (2*I*d^3*f^3*x^3 + 6*I*d^3*e*f^2*x^2)/(a*d^4)$

**Fricas [C]** time = 2.82403, size = 2367, normalized size = 7.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*csch(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $(2*d^3*e^3 - 6*c*d^2*e^2*f + 6*c^2*d*e*f^2 - 2*c^3*f^3 + (12*d*f^3*x + 12*d*e*f^2 + (12*I*d*f^3*x + 12*I*d*e*f^2)*e^{d*x+c})*\operatorname{dilog}(-I*e^{d*x+c})) + (3*I*d^2*f^3*x^2 + 6*I*d^2*e*f^2*x + 3*I*d^2*e^2*f - 3*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*e^{d*x+c})*\operatorname{dilog}(-e^{d*x+c})) + (-3*I*d^2*f^3*x^2 - 6*I*d^2*e*f^2*x - 3*I*d^2*e^2*f + 3*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*e^{d*x+c})*\operatorname{dilog}(e^{d*x+c})) + (-2*I*d^3*f^3*x^3 - 6*I*d^3*e*f^2*x^2 - 6*I*d^3*e^2*f*x - 6*I*c*d^2*e^2*f + 6*I*c^2*d*e*f^2 - 2*I*c^3*f^3)*e^{d*x+c} + (I*d^3*f^3*x^3 + 3*I*d^3*e*f^2*x^2 + 3*I*d^3*e^2*f*x + I*d^3*e^3 - (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + d^3*e^3)*e^{d*x+c})*\log(e^{d*x+c}+1) + (6*d^2*e^2*f - 12*c*d*e*f^2 + 6*c^2*f^3 + (6*I*d^2*e^2*f - 12*I*c*d*e*f^2 + 6*I*c^2*f^3)*e^{d*x+c})*\log(e^{d*x+c}) - I) + (-I*d^3*e^3 + 3*I*c*d^2*e^2*f - 3*I*c^2*d*e*f^2 + I*c^3*f^3 + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*e^{d*x+c})*\log(e^{d*x+c}) - 1) + (6*d^2*f^3*x^2 + 12*d^2*e*f^2*x + 12*c*d*e*f^2 - 6*c^2*f^3 + (6*I*d^2*f^3*x^2 + 12*I*d^2*e*f^2*x + 12*I*c*d*e*f^2 - 6*I*c^2*f^3)*e^{d*x+c})*\log(I*e^{d*x+c}+1) + (-I*d^3*f^3*x^3 - 3*I*d^3*e*f^2*x^2 - 3*I*d^3*e^2*f*x - 3*I*c$

```
*d^2*e^2*f + 3*I*c^2*d*e*f^2 - I*c^3*f^3 + (d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 +
  3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*e^(d*x + c))*log(
-e^(d*x + c) + 1) - 6*(f^3*e^(d*x + c) - I*f^3)*polylog(4, -e^(d*x + c)) +
6*(f^3*e^(d*x + c) - I*f^3)*polylog(4, e^(d*x + c)) + (-12*I*f^3*e^(d*x + c)
) - 12*f^3)*polylog(3, -I*e^(d*x + c)) + (-6*I*d*f^3*x - 6*I*d*e*f^2 + 6*(d
*f^3*x + d*e*f^2)*e^(d*x + c))*polylog(3, -e^(d*x + c)) + (6*I*d*f^3*x + 6*
I*d*e*f^2 - 6*(d*f^3*x + d*e*f^2)*e^(d*x + c))*polylog(3, e^(d*x + c)))/(a*
d^4*e^(d*x + c) - I*a*d^4)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*csch(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \operatorname{csch}(dx + c)}{ia \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*csch(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*csch(d\*x + c)/(I\*a\*sinh(d\*x + c) + a), x)

$$3.206 \quad \int \frac{(e+fx)^2 \operatorname{csch}(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=224

$$-\frac{2f(e+fx)\operatorname{PolyLog}(2, -e^{c+dx})}{ad^2} + \frac{2f(e+fx)\operatorname{PolyLog}(2, e^{c+dx})}{ad^2} + \frac{4if^2\operatorname{PolyLog}(2, -ie^{c+dx})}{ad^3} + \frac{2f^2\operatorname{PolyLog}(3, -e^{c+dx})}{ad^3}$$

[Out]  $((-I)*(e + f*x)^2)/(a*d) - (2*(e + f*x)^2*\operatorname{ArcTanh}[E^{(c + d*x)}])/(a*d) + ((4*I)*f*(e + f*x)*\operatorname{Log}[1 + I*E^{(c + d*x)}])/(a*d^2) - (2*f*(e + f*x)*\operatorname{PolyLog}[2, -E^{(c + d*x)}])/(a*d^2) + ((4*I)*f^2*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x)}])/(a*d^3) + (2*f*(e + f*x)*\operatorname{PolyLog}[2, E^{(c + d*x)}])/(a*d^2) + (2*f^2*\operatorname{PolyLog}[3, -E^{(c + d*x)}])/(a*d^3) - (2*f^2*\operatorname{PolyLog}[3, E^{(c + d*x)}])/(a*d^3) - (I*(e + f*x)^2*\operatorname{Tanh}[c/2 + (I/4)*\operatorname{Pi} + (d*x)/2])/(a*d)$

**Rubi [A]** time = 0.345685, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 11, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$ , Rules used = {5575, 4182, 2531, 2282, 6589, 3318, 4184, 3716, 2190, 2279, 2391}

$$-\frac{2f(e+fx)\operatorname{PolyLog}(2, -e^{c+dx})}{ad^2} + \frac{2f(e+fx)\operatorname{PolyLog}(2, e^{c+dx})}{ad^2} + \frac{4if^2\operatorname{PolyLog}(2, -ie^{c+dx})}{ad^3} + \frac{2f^2\operatorname{PolyLog}(3, -e^{c+dx})}{ad^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e + f*x)^2*\operatorname{Csch}[c + d*x]/(a + I*a*\operatorname{Sinh}[c + d*x]), x]$

[Out]  $((-I)*(e + f*x)^2)/(a*d) - (2*(e + f*x)^2*\operatorname{ArcTanh}[E^{(c + d*x)}])/(a*d) + ((4*I)*f*(e + f*x)*\operatorname{Log}[1 + I*E^{(c + d*x)}])/(a*d^2) - (2*f*(e + f*x)*\operatorname{PolyLog}[2, -E^{(c + d*x)}])/(a*d^2) + ((4*I)*f^2*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x)}])/(a*d^3) + (2*f*(e + f*x)*\operatorname{PolyLog}[2, E^{(c + d*x)}])/(a*d^2) + (2*f^2*\operatorname{PolyLog}[3, -E^{(c + d*x)}])/(a*d^3) - (2*f^2*\operatorname{PolyLog}[3, E^{(c + d*x)}])/(a*d^3) - (I*(e + f*x)^2*\operatorname{Tanh}[c/2 + (I/4)*\operatorname{Pi} + (d*x)/2])/(a*d)$

### Rule 5575

$\operatorname{Int}[(\operatorname{Csch}[(c_.) + (d_.)*(x_.)]^{(n_.)}*((e_.) + (f_.)*(x_.))^{(m_.)})/((a_.) + (b_.)*\operatorname{Sinh}[(c_.) + (d_.)*(x_.)]), x\_Symbol] \rightarrow \operatorname{Dist}[1/a, \operatorname{Int}[(e + f*x)^m*\operatorname{Csch}[c + d*x]^n, x], x] - \operatorname{Dist}[b/a, \operatorname{Int}[(e + f*x)^m*\operatorname{Csch}[c + d*x]^{(n-1)}/(a + b*\operatorname{Sinh}[c + d*x]), x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \operatorname{IGtQ}[m, 0] \ \&\& \ \operatorname{IGtQ}[n, 0]$

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 + (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \operatorname{csch}(c+dx)}{a+ia \sinh(c+dx)} dx &= -\left(i \int \frac{(e+fx)^2}{a+ia \sinh(c+dx)} dx\right) + \frac{\int (e+fx)^2 \operatorname{csch}(c+dx) dx}{a} \\
&= -\frac{2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{i \int (e+fx)^2 \operatorname{csc}^2\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{idx}{2}\right) dx}{2a} - \frac{(2f) \int (e+fx) \log(e+fx) dx}{a} \\
&= -\frac{2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2f(e+fx) \operatorname{Li}_2(-e^{c+dx})}{ad^2} + \frac{2f(e+fx) \operatorname{Li}_2(e^{c+dx})}{ad^2} - \frac{i(e+fx)^2}{ad} \\
&= -\frac{i(e+fx)^2}{ad} - \frac{2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2f(e+fx) \operatorname{Li}_2(-e^{c+dx})}{ad^2} + \frac{2f(e+fx) \operatorname{Li}_2(e^{c+dx})}{ad^2} \\
&= -\frac{i(e+fx)^2}{ad} - \frac{2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} + \frac{4if(e+fx) \log(1+ie^{c+dx})}{ad^2} - \frac{2f(e+fx) \operatorname{Li}_2(e^{c+dx})}{ad^2} \\
&= -\frac{i(e+fx)^2}{ad} - \frac{2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} + \frac{4if(e+fx) \log(1+ie^{c+dx})}{ad^2} - \frac{2f(e+fx) \operatorname{Li}_2(e^{c+dx})}{ad^2} \\
&= -\frac{i(e+fx)^2}{ad} - \frac{2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} + \frac{4if(e+fx) \log(1+ie^{c+dx})}{ad^2} - \frac{2f(e+fx) \operatorname{Li}_2(e^{c+dx})}{ad^2}
\end{aligned}$$

**Mathematica [A]** time = 4.15301, size = 275, normalized size = 1.23

$$\frac{2d(e+fx)(2(e^c-i)f \log(1-ie^{-c-dx})-id(e+fx))-4(e^c-i)f^2 \operatorname{PolyLog}(2,ie^{-c-dx})}{-1-ie^c} - 2df(e+fx) \operatorname{PolyLog}(2,-e^{c+dx}) + 2df(e+fx) \operatorname{PolyLog}(2,$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Csch[c + d\*x])/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (d^2\*(e + f\*x)^2\*Log[1 - E^(c + d\*x)] - d^2\*(e + f\*x)^2\*Log[1 + E^(c + d\*x)] + (2\*d\*(e + f\*x)\*((-I)\*d\*(e + f\*x) + 2\*(-I + E^c)\*f\*Log[1 - I\*E^(-c - d\*x)]) - 4\*(-I + E^c)\*f^2\*PolyLog[2, I\*E^(-c - d\*x)])/(-1 - I\*E^c) - 2\*d\*f\*(e + f\*x)\*PolyLog[2, -E^(c + d\*x)] + 2\*d\*f\*(e + f\*x)\*PolyLog[2, E^(c + d\*x)] + 2\*f^2\*PolyLog[3, -E^(c + d\*x)] - 2\*f^2\*PolyLog[3, E^(c + d\*x)] - ((2\*I)\*d^2\*(e + f\*x)^2\*Sinh[(d\*x)/2])/((Cosh[c/2] + I\*Sinh[c/2])\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]))/(a\*d^3)

**Maple [B]** time = 0.131, size = 573, normalized size = 2.6

$$2 \frac{x^2 f^2 + 2 e f x + e^2}{d a (e^{d x+c} - i)} + \frac{4 i \ln(e^{d x+c} - i) e f}{a d^2} - \frac{4 i \ln(e^{d x+c}) e f}{a d^2} - \frac{4 i f^2 c x}{a d^2} + \frac{4 i f^2 \ln(1 + i e^{d x+c}) x}{a d^2} + \frac{4 i f^2 \ln(1 + i e^{d x+c}) c}{a d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*csh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)

[Out]  $2*(f^2*x^2+2*e*f*x+e^2)/d/a/(exp(d*x+c)-I)+4*I/a/d^2*\ln(exp(d*x+c)-I)*e*f-4*I/a/d^2*\ln(exp(d*x+c))*e*f-4*I/a/d^2*f^2*c*x+4*I/a/d^2*f^2*\ln(1+I*exp(d*x+c))*x+4*I/a/d^3*f^2*\ln(1+I*exp(d*x+c))*c-4*I/a/d^3*f^2*c*\ln(exp(d*x+c)-I)+4*I/a/d^3*f^2*c*\ln(exp(d*x+c))+2/a/d^2*\ln(1-exp(d*x+c))*c*e*f-2/a/d*\ln(exp(d*x+c)+1)*e*f*x+2/a/d*\ln(1-exp(d*x+c))*e*f*x-2/a/d^2*e*f*c*\ln(exp(d*x+c)-1)-2*f^2*polylog(3,exp(d*x+c))/a/d^3+2*f^2*polylog(3,-exp(d*x+c))/a/d^3+4*I*f^2*polylog(2,-I*exp(d*x+c))/a/d^3+1/a/d*e^2*\ln(exp(d*x+c)-1)-1/a/d*e^2*\ln(exp(d*x+c)+1)-2/a/d^2*f^2*polylog(2,-exp(d*x+c))*x-1/a/d^3*f^2*c^2*\ln(1-exp(d*x+c))+1/a/d*f^2*\ln(1-exp(d*x+c))*x^2+1/a/d^3*f^2*c^2*\ln(exp(d*x+c)-1)-2/a/d^2*e*f*polylog(2,-exp(d*x+c))+2/a/d^2*e*f*polylog(2,exp(d*x+c))-2*I/a/d*f^2*x^2-2*I/a/d^3*f^2*c^2+2/a/d^2*f^2*polylog(2,exp(d*x+c))*x-1/a/d*f^2*\ln(exp(d*x+c)+1)*x^2$

**Maxima [A]** time = 1.85616, size = 468, normalized size = 2.09

$$-e^2 \left( \frac{\log(e^{-d x-c} + 1)}{a d} - \frac{\log(e^{-d x-c} - 1)}{a d} - \frac{2}{(a e^{-d x-c} + i a) d} \right) - \frac{2 i f^2 x^2}{a d} - \frac{4 i e f x}{a d} + \frac{2(f^2 x^2 + 2 e f x)}{a d e^{d x+c} - i a d} - \frac{2(dx \log(e^{d x+c}))}{a d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*csh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-e^2*(\log(e^{-d*x-c} + 1)/(a*d) - \log(e^{-d*x-c} - 1)/(a*d) - 2/((a*e^{-d*x-c} + I*a)*d)) - 2*I*f^2*x^2/(a*d) - 4*I*e*f*x/(a*d) + 2*(f^2*x^2 + 2*e*f*x)/(a*d*e^{d*x+c} - I*a*d) - 2*(d*x*\log(e^{d*x+c} + 1) + \text{dilog}(-e^{d*x+c})) * e*f/(a*d^2) + 2*(d*x*\log(-e^{d*x+c} + 1) + \text{dilog}(e^{d*x+c})) * e*f/(a*d^2) + 4*I*e*f*\log(I*e^{d*x+c} + 1)/(a*d^2) - (d^2*x^2*\log(e^{d*x+c} + 1) + 2*d*x*\text{dilog}(-e^{d*x+c})) - 2*polylog(3, -e^{d*x+c})) * f^2/(a*d^3) + (d^2*x^2*\log(-e^{d*x+c} + 1) + 2*d*x*\text{dilog}(e^{d*x+c})) - 2*polylog(3, e^{d*x+c})) * f^2/(a*d^3) + 4*I*(d*x*\log(I*e^{d*x+c} + 1) + \text{dilog}(-I*e^{d*x+c})) * f^2/(a*d^3)$



---

**Fricas [C]** time = 2.71007, size = 1368, normalized size = 6.11

$$\frac{2d^2e^2 - 4cdef + 2c^2f^2 + (4if^2e^{dx+c} + 4f^2)\text{Li}_2(-ie^{dx+c}) + (2idf^2x + 2idef - 2(df^2x + def)e^{dx+c})\text{Li}_2(-e^{dx+c})}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cscsch(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] (2\*d^2\*e^2 - 4\*c\*d\*e\*f + 2\*c^2\*f^2 + (4\*I\*f^2\*e^(d\*x + c) + 4\*f^2)\*dilog(-I\*e^(d\*x + c)) + (2\*I\*d\*f^2\*x + 2\*I\*d\*e\*f - 2\*(d\*f^2\*x + d\*e\*f)\*e^(d\*x + c))\*dilog(-e^(d\*x + c)) + (-2\*I\*d\*f^2\*x - 2\*I\*d\*e\*f + 2\*(d\*f^2\*x + d\*e\*f)\*e^(d\*x + c))\*dilog(e^(d\*x + c)) + (-2\*I\*d^2\*f^2\*x^2 - 4\*I\*d^2\*e\*f\*x - 4\*I\*c\*d\*e\*f + 2\*I\*c^2\*f^2)\*e^(d\*x + c) + (I\*d^2\*f^2\*x^2 + 2\*I\*d^2\*e\*f\*x + I\*d^2\*e^2 - (d^2\*f^2\*x^2 + 2\*d^2\*e\*f\*x + d^2\*e^2)\*e^(d\*x + c))\*log(e^(d\*x + c) + 1) + (4\*d\*e\*f - 4\*c\*f^2 + (4\*I\*d\*e\*f - 4\*I\*c\*f^2)\*e^(d\*x + c))\*log(e^(d\*x + c) - I) + (-I\*d^2\*e^2 + 2\*I\*c\*d\*e\*f - I\*c^2\*f^2 + (d^2\*e^2 - 2\*c\*d\*e\*f + c^2\*f^2)\*e^(d\*x + c))\*log(e^(d\*x + c) - 1) + (4\*d\*f^2\*x + 4\*c\*f^2 + (4\*I\*d\*f^2\*x + 4\*I\*c\*f^2)\*e^(d\*x + c))\*log(I\*e^(d\*x + c) + 1) + (-I\*d^2\*f^2\*x^2 - 2\*I\*d^2\*e\*f\*x - 2\*I\*c\*d\*e\*f + I\*c^2\*f^2 + (d^2\*f^2\*x^2 + 2\*d^2\*e\*f\*x + 2\*c\*d\*e\*f - c^2\*f^2)\*e^(d\*x + c))\*log(-e^(d\*x + c) + 1) + 2\*(f^2\*e^(d\*x + c) - I\*f^2)\*polylog(3, -e^(d\*x + c)) - 2\*(f^2\*e^(d\*x + c) - I\*f^2)\*polylog(3, e^(d\*x + c)))/(a\*d^3\*e^(d\*x + c) - I\*a\*d^3)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cscsch(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \operatorname{csch}(dx + c)}{ia \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*csch(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

$$3.207 \quad \int \frac{(e+fx)\operatorname{csch}(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=126

$$-\frac{f \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{ad^2} + \frac{f \operatorname{PolyLog}\left(2, e^{c+dx}\right)}{ad^2} + \frac{2if \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\right)}{ad^2} - \frac{2(e+fx) \tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{i(e+fx)}{ad}$$

[Out]  $(-2*(e + f*x)*\operatorname{ArcTanh}[E^{(c + d*x)}])/(a*d) + ((2*I)*f*\operatorname{Log}[\operatorname{Cosh}[c/2 + (I/4)*Pi + (d*x)/2]])/(a*d^2) - (f*\operatorname{PolyLog}[2, -E^{(c + d*x)}])/(a*d^2) + (f*\operatorname{PolyLog}[2, E^{(c + d*x)}])/(a*d^2) - (I*(e + f*x)*\operatorname{Tanh}[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)$

**Rubi [A]** time = 0.147479, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$ , Rules used = {5575, 4182, 2279, 2391, 3318, 4184, 3475}

$$-\frac{f \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{ad^2} + \frac{f \operatorname{PolyLog}\left(2, e^{c+dx}\right)}{ad^2} + \frac{2if \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\right)}{ad^2} - \frac{2(e+fx) \tanh^{-1}\left(e^{c+dx}\right)}{ad} - \frac{i(e+fx)}{ad}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\frac{(e + f*x)*\operatorname{Csch}[c + d*x]}{(a + I*a*\operatorname{Sinh}[c + d*x])}, x]$

[Out]  $(-2*(e + f*x)*\operatorname{ArcTanh}[E^{(c + d*x)}])/(a*d) + ((2*I)*f*\operatorname{Log}[\operatorname{Cosh}[c/2 + (I/4)*Pi + (d*x)/2]])/(a*d^2) - (f*\operatorname{PolyLog}[2, -E^{(c + d*x)}])/(a*d^2) + (f*\operatorname{PolyLog}[2, E^{(c + d*x)}])/(a*d^2) - (I*(e + f*x)*\operatorname{Tanh}[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)$

### Rule 5575

$\operatorname{Int}[\frac{\operatorname{Csch}[c + d*x]^n * ((e + f*x)^m)}{(a + b*\operatorname{Sinh}[c + d*x])}, x, \text{Symbol}] \rightarrow \operatorname{Dist}[1/a, \operatorname{Int}[(e + f*x)^m * \operatorname{Csch}[c + d*x]^n, x], x] - \operatorname{Dist}[b/a, \operatorname{Int}[(e + f*x)^m * \operatorname{Csch}[c + d*x]^{n-1} / (a + b*\operatorname{Sinh}[c + d*x]), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I GtQ[n, 0]

### Rule 4182

$\operatorname{Int}[\operatorname{csc}[c + d*x] * (\operatorname{Complex}[0, fz]) * (e + f*fz*x)^m, x, \text{Symbol}] \rightarrow \operatorname{Simp}[(-2*(c + d*x)^m * \operatorname{ArcTanh}[E^{-(I*e + f*fz*x)}]) / (f*fz*I), x]$

```

+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

```

### Rule 2279

```

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

```

### Rule 2391

```

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

```

### Rule 3318

```

Int[((c_.) + (d_.)*(x_)^(m_.))*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
(f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])

```

### Rule 4184

```

Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_)^(m_.)), x_Symbol] := -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

```

### Rule 3475

```

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)} dx &= -\left(i \int \frac{e+fx}{a+ia\sinh(c+dx)} dx\right) + \frac{\int (e+fx)\operatorname{csch}(c+dx) dx}{a} \\
&= -\frac{2(e+fx)\tanh^{-1}(e^{c+dx})}{ad} - \frac{i \int (e+fx) \operatorname{csc}^2\left(\frac{1}{2}\left(ic + \frac{\pi}{2}\right) + \frac{idx}{2}\right) dx}{2a} - \frac{f \int \log(1-e^{c+dx})}{ad} \\
&= -\frac{2(e+fx)\tanh^{-1}(e^{c+dx})}{ad} - \frac{i(e+fx)\tanh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)}{ad} - \frac{f \operatorname{Subst}\left(\int \frac{\log(1-x)}{x} dx, x, e^{c+dx}\right)}{ad^2} \\
&= -\frac{2(e+fx)\tanh^{-1}(e^{c+dx})}{ad} + \frac{2if \log\left(\cosh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)\right)}{ad^2} - \frac{f \operatorname{Li}_2(-e^{c+dx})}{ad^2} + \frac{f \operatorname{Li}_2(e^{c+dx})}{ad^2}
\end{aligned}$$

**Mathematica [B]** time = 1.25237, size = 345, normalized size = 2.74

$$\left(\cosh\left(\frac{1}{2}(c+dx)\right) + i \sinh\left(\frac{1}{2}(c+dx)\right)\right) \left(f\left(\cosh\left(\frac{1}{2}(c+dx)\right) + i \sinh\left(\frac{1}{2}(c+dx)\right)\right)\right) \left(\operatorname{PolyLog}\left(2, -e^{-c-dx}\right) - \operatorname{PolyLog}\left(2, e^{c+dx}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Csch[c + d\*x])/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] ((Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])\*(f\*(c + d\*x)\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]) - 2\*f\*ArcTan[Tanh[(c + d\*x)/2]]\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]) + I\*f\*Log[Cosh[c + d\*x]]\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]) + d\*e\*Log[Tanh[(c + d\*x)/2]]\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]) - c\*f\*Log[Tanh[(c + d\*x)/2]]\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]) + f\*((c + d\*x)\*(Log[1 - E^(-c - d\*x)] - Log[1 + E^(-c - d\*x)]) + PolyLog[2, -E^(-c - d\*x)] - PolyLog[2, E^(-c - d\*x)])\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]) - (2\*I)\*d\*(e + f\*x)\*Sinh[(c + d\*x)/2]))/(d^2\*(a + I\*a\*Sinh[c + d\*x]))

**Maple [A]** time = 0.132, size = 211, normalized size = 1.7

$$2 \frac{fx+e}{da(e^{dx+c}-i)} + \frac{f \operatorname{polylog}\left(2, e^{dx+c}\right)}{ad^2} - \frac{f \operatorname{polylog}\left(2, -e^{dx+c}\right)}{ad^2} - \frac{2if \ln\left(e^{dx+c}\right)}{ad^2} + \frac{e \ln\left(e^{dx+c}-1\right)}{da} - \frac{e \ln\left(e^{dx+c}+1\right)}{da} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)\*csch(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)

[Out]  $2*(f*x+e)/d/a/(exp(d*x+c)-1)+f*polylog(2,exp(d*x+c))/a/d^2-f*polylog(2,-exp(d*x+c))/a/d^2-2*I/d^2/a*f*ln(exp(d*x+c))+1/d/a*e*ln(exp(d*x+c)-1)-1/d/a*e*ln(exp(d*x+c)+1)+1/d/a*ln(1-exp(d*x+c))*f*x+1/d^2/a*ln(1-exp(d*x+c))*c*f-1/d/a*ln(exp(d*x+c)+1)*f*x-1/d^2/a*f*c*ln(exp(d*x+c)-1)+2*I*f/a/d^2*ln(exp(d*x+c)-1)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2f\left(\frac{xe^{(dx+c)}}{iade^{(dx+c)}+ad} + \frac{i\log((e^{(dx+c)}-i)e^{(-c)})}{ad^2} + \int \frac{x}{2(ae^{(dx+c)}+a)} dx + \int \frac{x}{2(ae^{(dx+c)}-a)} dx\right) - e\left(\frac{\log(e^{(-dx-c)}+1)}{ad} - \log\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $2*f*(x*e^{(d*x+c)}/(I*a*d*e^{(d*x+c)}+a*d)+I*\log((e^{(d*x+c)}-I)*e^{(-c)})/(a*d^2)+\text{integrate}(1/2*x/(a*e^{(d*x+c)}+a),x)+\text{integrate}(1/2*x/(a*e^{(d*x+c)}-a),x)-e*(\log(e^{(-d*x-c)}+1)/(a*d)-\log(e^{(-d*x-c)}-1)/(a*d)-2/((a*e^{(-d*x-c)}+I*a)*d))$

**Fricas [B]** time = 2.62454, size = 548, normalized size = 4.35

$$-2i d f x e^{(dx+c)} + 2 d e - (f e^{(dx+c)} - i f) \text{Li}_2(-e^{(dx+c)}) + (f e^{(dx+c)} - i f) \text{Li}_2(e^{(dx+c)}) + (i d f x + i d e - (d f x + d e) e^{(dx+c)}) \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $(-2*I*d*f*x*e^{(d*x+c)}+2*d*e-(f*e^{(d*x+c)}-I*f)*\text{dilog}(-e^{(d*x+c)})+(f*e^{(d*x+c)}-I*f)*\text{dilog}(e^{(d*x+c)})+(I*d*f*x+I*d*e-(d*f*x+d*e)*e^{(d*x+c)})*\log(e^{(d*x+c)}+1)+(2*I*f*e^{(d*x+c)}+2*f)*\log(e^{(d*x+c)}-1)+(-I*d*e+I*c*f+(d*e-c*f)*e^{(d*x+c)})*\log(e^{(d*x+c)}-1)+(-I*d*f*x-I*c*f+(d*f*x+c*f)*e^{(d*x+c)})*\log(-e^{(d*x+c)}+1))/(a*d^2*e^{(d*x+c)}-I*a*d^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \operatorname{csch}(dx + c)}{ia \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)\*csch(d\*x + c)/(I\*a\*sinh(d\*x + c) + a), x)

$$3.208 \quad \int \frac{\operatorname{csch}(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=41

$$-\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{\cosh(c+dx)}{d(a+ia \sinh(c+dx))}$$

[Out] -(ArcTanh[Cosh[c + d\*x]]/(a\*d)) + Cosh[c + d\*x]/(d\*(a + I\*a\*Sinh[c + d\*x]))

**Rubi [A]** time = 0.0599134, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {2747, 3770, 2648}

$$-\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{\cosh(c+dx)}{d(a+ia \sinh(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d\*x]/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] -(ArcTanh[Cosh[c + d\*x]]/(a\*d)) + Cosh[c + d\*x]/(d\*(a + I\*a\*Sinh[c + d\*x]))

#### Rule 2747

Int[1/(((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])), x\_Symbol] :> Dist[b/(b\*c - a\*d), Int[1/(a + b\*Sin[e + f\*x]), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*Sin[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

#### Rule 2648

Int[((a\_) + (b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] :> -Simp[Cos[c + d\*x]/(d\*(b + a\*Sin[c + d\*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

#### Rubi steps



$$\int \frac{\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)} dx = -\left(i \int \frac{1}{a+ia\sinh(c+dx)} dx\right) + \frac{\int \operatorname{csch}(c+dx) dx}{a}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{\cosh(c+dx)}{d(a+ia\sinh(c+dx))}$$

**Mathematica [A]** time = 0.0642838, size = 52, normalized size = 1.27

$$\frac{\operatorname{sech}(c+dx) \left( i \sinh(c+dx) + \sqrt{\cosh^2(c+dx)} \tanh^{-1} \left( \sqrt{\cosh^2(c+dx)} \right) - 1 \right)}{ad}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d\*x]/(a + I\*a\*Sinh[c + d\*x]), x]

[Out] -((Sech[c + d\*x]\*(-1 + ArcTanh[Sqrt[Cosh[c + d\*x]^2]]\*Sqrt[Cosh[c + d\*x]^2] + I\*Sinh[c + d\*x]))/(a\*d))

**Maple [A]** time = 0.037, size = 42, normalized size = 1.

$$\frac{-2i}{da} \left( -i + \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^{-1} + \frac{1}{da} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d\*x+c)/(a+I\*a\*sinh(d\*x+c)), x)

[Out] -2\*I/d/a/(-I+tanh(1/2\*d\*x+1/2\*c))+1/d/a\*ln(tanh(1/2\*d\*x+1/2\*c))

**Maxima [A]** time = 1.08649, size = 84, normalized size = 2.05

$$-\frac{\log(e^{-dx-c} + 1)}{ad} + \frac{\log(e^{-dx-c} - 1)}{ad} + \frac{2}{(ae^{-dx-c} + ia)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-\log(e^{-d*x - c} + 1)/(a*d) + \log(e^{-d*x - c} - 1)/(a*d) + 2/((a*e^{-d*x - c} + I*a)*d)$

**Fricas [A]** time = 2.49623, size = 154, normalized size = 3.76

$$-\frac{(e^{(dx+c)} - i) \log(e^{(dx+c)} + 1) - (e^{(dx+c)} - i) \log(e^{(dx+c)} - 1) - 2}{ade^{(dx+c)} - i ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $-((e^{(d*x + c)} - I)*\log(e^{(d*x + c)} + 1) - (e^{(d*x + c)} - I)*\log(e^{(d*x + c)} - 1) - 2)/(a*d*e^{(d*x + c)} - I*a*d)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{csch}(c+dx)}{i \sinh(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Integral(csch(c + d\*x)/(I\*sinh(c + d\*x) + 1), x)/a

**Giac [A]** time = 1.24345, size = 72, normalized size = 1.76

$$-\frac{\log(e^{(dx+c)} + 1)}{ad} + \frac{\log(|e^{(dx+c)} - 1|)}{ad} + \frac{2}{ad(e^{(dx+c)} - i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

```
[Out] -log(e^(d*x + c) + 1)/(a*d) + log(abs(e^(d*x + c) - 1))/(a*d) + 2/(a*d*(e^(d*x + c) - I))
```

$$3.209 \quad \int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

**Optimal.** Leaf size=31

$$\text{Unintegrable}\left(\frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Csch[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0501509, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Csch[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Csch[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx = \int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

**Mathematica [A]** time = 46.1088, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Csch[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Integrate[Csch[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

---

**Maple [A]** time = 0.742, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx + c)}{(fx + e)(a + ia \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)`

[Out] `int(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$2f \int \frac{1}{-iadf^2x^2 - 2iade fx - iade^2 + (adf^2x^2e^c + 2adefxe^c + ade^2e^c)e^{(dx)}} dx + \frac{2}{-iadf x - iade + (adfxe^c + adee^c)e^{(dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `2*f*integrate(1/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)), x) + 2/(-I*a*d*f*x - I*a*d*e + (a*d*f*x*e^c + a*d*e*e^c)*e^(d*x)) + 2*integrate(1/2/(a*f*x + a*e + (a*f*x*e^c + a*e*e^c)*e^(d*x)), x) + 2*integrate(-1/2/(a*f*x + a*e - (a*f*x*e^c + a*e*e^c)*e^(d*x)), x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\frac{(-iadf x - iade + (adfx + ade)e^{(dx+c)}) \operatorname{integral} \left( \frac{2(dx+de+f)e^{(2dx+2c)} + (-2idfx-2ide)e^{(dx+c)} - 2f}{iadf^2x^2+2iade fx+iade^2+(adf^2x^2+2adefx+ade^2)e^{(3dx+3c)}+(-iadf^2x^2-2iade fx-iade^2)e^{(2dx+2c)}} \right)}{-iadf x - iade + (adfx + ade)e^{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")`

[Out] `((-I*a*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))*integral((2*(d*f*x + d*e + f)*e^(2*d*x + 2*c) + (-2*I*d*f*x - 2*I*d*e)*e^(d*x + c) - 2*f)/(I*a`

```
*d*f^2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e
^2)*e^(3*d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2)*e^(2*d*x
+ 2*c) - (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c)), x) + 2)/(-I*a
*d*f*x - I*a*d*e + (a*d*f*x + a*d*e)*e^(d*x + c))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx + c)}{(fx + e)(ia \sinh(dx + c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(csch(d*x + c)/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

$$3.210 \quad \int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

**Optimal.** Leaf size=31

$$\text{Unintegrable}\left(\frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Csch[c + d\*x]/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.050252, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Csch[c + d\*x]/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Csch[c + d\*x]/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx = \int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

**Mathematica [A]** time = 57.7485, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Csch[c + d\*x]/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Integrate[Csch[c + d\*x]/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

---

**Maple [A]** time = 1.383, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx + c)}{(fx + e)^2 (a + ia \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)`

[Out] `int(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$4f \int \frac{1}{-i adf^3 x^3 - 3i adef^2 x^2 - 3i ade^2 fx - i ade^3 + (adf^3 x^3 e^c + 3 adef^2 x^2 e^c + 3 ade^2 fx e^c + ade^3 e^c) e^{(dx)}} dx + \frac{1}{-i adf^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `4*f*integrate(1/(-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x - I*a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c + a*d*e^3*e^c)*e^(d*x)), x) + 2/(-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)) + 2*integrate(1/2/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 + (a*f^2*x^2*e^c + 2*a*e*f*x*e^c + a*e^2*e^c)*e^(d*x)), x) + 2*integrate(-1/2/(a*f^2*x^2 + 2*a*e*f*x + a*e^2 - (a*f^2*x^2*e^c + 2*a*e*f*x*e^c + a*e^2*e^c)*e^(d*x)), x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\frac{(-i adf^2 x^2 - 2i adefx - i ade^2 + (adf^2 x^2 + 2 adefx + ade^2) e^{(dx+c)}) \operatorname{integral} \left( \frac{1}{i adf^3 x^3 + 3i adef^2 x^2 + 3i ade^2 fx + i ade^3 + (adf^3 x^3 + 3 adef^2 x^2 + 3 ade^2 fx + ade^3) e^{(dx+c)}} \right)}{-i adf^2 x^2 - 2i adefx - i ade^2 + (adf^2 x^2 + 2 adefx + ade^2) e^{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")`



```
[Out] ((-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x +
a*d*e^2)*e^(d*x + c))*integral((2*(d*f*x + d*e + 2*f)*e^(2*d*x + 2*c) + (-
2*I*d*f*x - 2*I*d*e)*e^(d*x + c) - 4*f)/(I*a*d*f^3*x^3 + 3*I*a*d*e*f^2*x^2
+ 3*I*a*d*e^2*f*x + I*a*d*e^3 + (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*
f*x + a*d*e^3)*e^(3*d*x + 3*c) + (-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*
a*d*e^2*f*x - I*a*d*e^3)*e^(2*d*x + 2*c) - (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 +
3*a*d*e^2*f*x + a*d*e^3)*e^(d*x + c)), x) + 2)/(-I*a*d*f^2*x^2 - 2*I*a*d*e*
f*x - I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.211 \quad \int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=419

$$\frac{12f^2(e+fx)\operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^3} + \frac{3f^2(e+fx)\operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{ad^3} - \frac{6if^2(e+fx)\operatorname{PolyLog}\left(3, -e^{c+dx}\right)}{ad^3} + \frac{6if^2(e+fx)}{ad^3}$$

[Out]  $(-2*(e + f*x)^3)/(a*d) + ((2*I)*(e + f*x)^3*\operatorname{ArcTanh}[E^{(c + d*x)}])/(a*d) - ((e + f*x)^3*\operatorname{Coth}[c + d*x])/(a*d) + (6*f*(e + f*x)^2*\operatorname{Log}[1 + I*E^{(c + d*x)}])/(a*d^2) + (3*f*(e + f*x)^2*\operatorname{Log}[1 - E^{(2*(c + d*x))}])/(a*d^2) + ((3*I)*f*(e + f*x)^2*\operatorname{PolyLog}[2, -E^{(c + d*x)}])/(a*d^2) + (12*f^2*(e + f*x)*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x)}])/(a*d^3) - ((3*I)*f*(e + f*x)^2*\operatorname{PolyLog}[2, E^{(c + d*x)}])/(a*d^2) + (3*f^2*(e + f*x)*\operatorname{PolyLog}[2, E^{(2*(c + d*x))}])/(a*d^3) - ((6*I)*f^2*(e + f*x)*\operatorname{PolyLog}[3, -E^{(c + d*x)}])/(a*d^3) - (12*f^3*\operatorname{PolyLog}[3, (-I)*E^{(c + d*x)}])/(a*d^4) + ((6*I)*f^2*(e + f*x)*\operatorname{PolyLog}[3, E^{(c + d*x)}])/(a*d^3) - (3*f^3*\operatorname{PolyLog}[3, E^{(2*(c + d*x))}])/(2*a*d^4) + ((6*I)*f^3*\operatorname{PolyLog}[4, -E^{(c + d*x)}])/(a*d^4) - ((6*I)*f^3*\operatorname{PolyLog}[4, E^{(c + d*x)}])/(a*d^4) - ((e + f*x)^3*\operatorname{Tanh}[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)$

**Rubi [A]** time = 0.805277, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$ , Rules used = {5575, 4184, 3716, 2190, 2531, 2282, 6589, 4182, 6609, 3318}

$$\frac{12f^2(e+fx)\operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^3} + \frac{3f^2(e+fx)\operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{ad^3} - \frac{6if^2(e+fx)\operatorname{PolyLog}\left(3, -e^{c+dx}\right)}{ad^3} + \frac{6if^2(e+fx)}{ad^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\frac{(e + f*x)^3*\operatorname{Csch}[c + d*x]^2}{(a + I*a*\operatorname{Sinh}[c + d*x])}, x]$

[Out]  $(-2*(e + f*x)^3)/(a*d) + ((2*I)*(e + f*x)^3*\operatorname{ArcTanh}[E^{(c + d*x)}])/(a*d) - ((e + f*x)^3*\operatorname{Coth}[c + d*x])/(a*d) + (6*f*(e + f*x)^2*\operatorname{Log}[1 + I*E^{(c + d*x)}])/(a*d^2) + (3*f*(e + f*x)^2*\operatorname{Log}[1 - E^{(2*(c + d*x))}])/(a*d^2) + ((3*I)*f*(e + f*x)^2*\operatorname{PolyLog}[2, -E^{(c + d*x)}])/(a*d^2) + (12*f^2*(e + f*x)*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x)}])/(a*d^3) - ((3*I)*f*(e + f*x)^2*\operatorname{PolyLog}[2, E^{(c + d*x)}])/(a*d^2) + (3*f^2*(e + f*x)*\operatorname{PolyLog}[2, E^{(2*(c + d*x))}])/(a*d^3) - ((6*I)*f^2*(e + f*x)*\operatorname{PolyLog}[3, -E^{(c + d*x)}])/(a*d^3) - (12*f^3*\operatorname{PolyLog}[3, (-I)*E^{(c + d*x)}])/(a*d^4) + ((6*I)*f^2*(e + f*x)*\operatorname{PolyLog}[3, E^{(c + d*x)}])/(a*d^3) - (3*f^3*\operatorname{PolyLog}[3, E^{(2*(c + d*x))}])/(2*a*d^4) + ((6*I)*f^3*\operatorname{PolyLog}[4, -E^{(c + d*x)}])/(a*d^4) - ((6*I)*f^3*\operatorname{PolyLog}[4, E^{(c + d*x)}])/(a*d^4) - ((e + f$

$x^3 \tanh[c/2 + (I/4)\pi + (d*x)/2] / (a*d)$

### Rule 5575

Int[(Csch[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Csch[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Csch[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 3716

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)], x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x))/(E^(2\*I\*k\*Pi)\*(1 + E^(2\*(-I\*e) + f\*fz\*x))/E^(2\*I\*k\*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[(((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a))/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ

```
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :=> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :=> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x)] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :=> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :=> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
(f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)}{a+ia \sinh(c+dx)} dx &= -\left(i \int \frac{(e+fx)^3 \operatorname{csch}(c+dx)}{a+ia \sinh(c+dx)} dx\right) + \frac{\int (e+fx)^3 \operatorname{csch}^2(c+dx) dx}{a} \\
&= -\frac{(e+fx)^3 \operatorname{coth}(c+dx)}{ad} - \frac{i \int (e+fx)^3 \operatorname{csch}(c+dx) dx}{a} + \frac{(3f) \int (e+fx)^2 \operatorname{coth}(c+dx) dx}{ad} \\
&= -\frac{(e+fx)^3}{ad} + \frac{2i(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{(e+fx)^3 \operatorname{coth}(c+dx)}{ad} - \frac{\int (e+fx)^3 \operatorname{csc}^2\left(\frac{1}{2}\right)}{2} \\
&= -\frac{(e+fx)^3}{ad} + \frac{2i(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{(e+fx)^3 \operatorname{coth}(c+dx)}{ad} + \frac{3f(e+fx)^2 \log(1)}{ad^2} \\
&= -\frac{2(e+fx)^3}{ad} + \frac{2i(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{(e+fx)^3 \operatorname{coth}(c+dx)}{ad} + \frac{3f(e+fx)^2 \log(1)}{ad^2} \\
&= -\frac{2(e+fx)^3}{ad} + \frac{2i(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{(e+fx)^3 \operatorname{coth}(c+dx)}{ad} + \frac{6f(e+fx)^2 \log(1)}{ad^2} \\
&= -\frac{2(e+fx)^3}{ad} + \frac{2i(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{(e+fx)^3 \operatorname{coth}(c+dx)}{ad} + \frac{6f(e+fx)^2 \log(1)}{ad^2} \\
&= -\frac{2(e+fx)^3}{ad} + \frac{2i(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{(e+fx)^3 \operatorname{coth}(c+dx)}{ad} + \frac{6f(e+fx)^2 \log(1)}{ad^2} \\
&= -\frac{2(e+fx)^3}{ad} + \frac{2i(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{(e+fx)^3 \operatorname{coth}(c+dx)}{ad} + \frac{6f(e+fx)^2 \log(1)}{ad^2}
\end{aligned}$$

**Mathematica [B]** time = 17.2794, size = 1042, normalized size = 2.49

$$\frac{2i(d^3(e+fx)^3 + 3d^2(1+ie^c)f \log(1-ie^{-c-dx}))(e+fx)^2 + 6i(i-e^c)f^2(d(e+fx)\operatorname{PolyLog}(2, ie^{-c-dx}) + f\operatorname{PolyLog}(3, ie^{-c-dx}))}{ad^4(-i+e^c)}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^3\*Csch[c + d\*x]^2)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] ((-2\*I)\*(d^3\*(e + f\*x)^3 + 3\*d^2\*(1 + I\*E^c)\*f\*(e + f\*x)^2\*Log[1 - I\*E^(-c - d\*x)] + (6\*I)\*(I - E^c)\*f^2\*(d\*(e + f\*x)\*PolyLog[2, I\*E^(-c - d\*x)] + f\*PolyLog[3, I\*E^(-c - d\*x)])))/(a\*d^4\*(-I + E^c)) + (-d^3\*(e + f\*x)^3\*(-1 + Coth[c])) + I\*d^2\*e^2\*(d\*e + (3\*I)\*f)\*(d\*x - Log[1 - Cosh[c + d\*x] - Sinh[c

$$\begin{aligned}
& + d*x]]) + 3*d^2*e*f*(I*d*e + 2*f)*x*\text{Log}[1 + \text{Cosh}[c + d*x] - \text{Sinh}[c + d*x] \\
& ] + 3*d^2*f^2*(I*d*e + f)*x^2*\text{Log}[1 + \text{Cosh}[c + d*x] - \text{Sinh}[c + d*x]] + I*d^ \\
& 3*f^3*x^3*\text{Log}[1 + \text{Cosh}[c + d*x] - \text{Sinh}[c + d*x]] + 3*d^2*e*f*((-I)*d*e + 2* \\
& f)*x*\text{Log}[1 - \text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]] + 3*d^2*f^2*((-I)*d*e + f)*x^2* \\
& \text{Log}[1 - \text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]] - I*d^3*f^3*x^3*\text{Log}[1 - \text{Cosh}[c + d*x] \\
& ] + \text{Sinh}[c + d*x]] - I*d^2*e^2*(d*e - (3*I)*f)*(d*x - \text{Log}[1 + \text{Cosh}[c + d*x] \\
& + \text{Sinh}[c + d*x]]) + (3*I)*d*e*(d*e + (2*I)*f)*f*\text{PolyLog}[2, \text{Cosh}[c + d*x] - \\
& \text{Sinh}[c + d*x]] - (3*I)*d*e*(d*e - (2*I)*f)*f*\text{PolyLog}[2, -\text{Cosh}[c + d*x] + \text{S} \\
& \text{inh}[c + d*x]] + (6*I)*(d*e + I*f)*f^2*(d*x*\text{PolyLog}[2, \text{Cosh}[c + d*x] - \text{Sinh}[ \\
& c + d*x]] + \text{PolyLog}[3, \text{Cosh}[c + d*x] - \text{Sinh}[c + d*x]]) - 6*f^2*(I*d*e + f)* \\
& (d*x*\text{PolyLog}[2, -\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]] + \text{PolyLog}[3, -\text{Cosh}[c + d*x] \\
& + \text{Sinh}[c + d*x]]) + (3*I)*f^3*(d^2*x^2*\text{PolyLog}[2, \text{Cosh}[c + d*x] - \text{Sinh}[c + \\
& d*x]] + 2*(d*x*\text{PolyLog}[3, \text{Cosh}[c + d*x] - \text{Sinh}[c + d*x]] + \text{PolyLog}[4, \text{Cosh} \\
& [c + d*x] - \text{Sinh}[c + d*x]])) - (3*I)*f^3*(d^2*x^2*\text{PolyLog}[2, -\text{Cosh}[c + d*x] \\
& + \text{Sinh}[c + d*x]] + 2*(d*x*\text{PolyLog}[3, -\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]] + \text{Pol} \\
& \text{yLog}[4, -\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]])))/(a*d^4) + (\text{Sech}[c/2]*\text{Sech}[c/2 + \\
& (d*x)/2]*(-e^3*\text{Sinh}[(d*x)/2]) - 3*e^2*f*x*\text{Sinh}[(d*x)/2] - 3*e*f^2*x^2*\text{Sinh} \\
& [(d*x)/2] - f^3*x^3*\text{Sinh}[(d*x)/2]))/(2*a*d) + (\text{Csch}[c/2]*\text{Csch}[c/2 + (d*x)/2] \\
& )*(e^3*\text{Sinh}[(d*x)/2] + 3*e^2*f*x*\text{Sinh}[(d*x)/2] + 3*e*f^2*x^2*\text{Sinh}[(d*x)/2] \\
& + f^3*x^3*\text{Sinh}[(d*x)/2]))/(2*a*d) - (2*(e^3*\text{Sinh}[(d*x)/2] + 3*e^2*f*x*\text{Sinh} \\
& (d*x)/2] + 3*e*f^2*x^2*\text{Sinh}[(d*x)/2] + f^3*x^3*\text{Sinh}[(d*x)/2]))/(a*d*(\text{Cosh}[c \\
& /2] + I*\text{Sinh}[c/2])*(\text{Cosh}[c/2 + (d*x)/2] + I*\text{Sinh}[c/2 + (d*x)/2]))
\end{aligned}$$

**Maple [B]** time = 0.266, size = 1535, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)^3*\text{csch}(d*x+c)^2/(a+I*a*\text{sinh}(d*x+c)),x)$

[Out]  $12*f^2/d^2/a*e*\ln(1+I*\exp(d*x+c))*x+12*f^2/d^3/a*e*\ln(1+I*\exp(d*x+c))*c+24*f^2/d^3/a*e*c*\ln(\exp(d*x+c))-12*f^2/d^3/a*e*c*\ln(\exp(d*x+c)-I)-24*f^2/d^2/a*e*c*x+8*f^3/d^4/a*c^3-4*f^3/d/a*x^3-6*f^3*\text{polylog}(3,-\exp(d*x+c))/a/d^4-6*f^3*\text{polylog}(3,\exp(d*x+c))/a/d^4+6*I*f^3*\text{polylog}(4,-\exp(d*x+c))/a/d^4-6*I*f^3*\text{polylog}(4,\exp(d*x+c))/a/d^4-12*f^3*\text{polylog}(3,-I*\exp(d*x+c))/a/d^4-2*I*(f^3*x^3*\exp(2*d*x+2*c)+3*e*f^2*x^2*\exp(2*d*x+2*c)+3*e^2*f*x*\exp(2*d*x+2*c)-2*f^3*x^3-I*\exp(d*x+c)*f^3*x^3+e^3*\exp(2*d*x+2*c)-6*e*f^2*x^2-3*I*\exp(d*x+c)*e*f^2*x^2-6*e^2*f*x-3*I*\exp(d*x+c)*e^2*f*x-2*e^3-I*\exp(d*x+c)*e^3)/(\exp(2*d*x+2*c)-1)/(\exp(d*x+c)-I)/d/a+12*f^3/d^3/a*c^2*x-12*f/d^2/a*\ln(\exp(d*x+c))*e^2+6*f^3/d^2/a*\ln(1+I*\exp(d*x+c))*x^2-6*f^3/d^4/a*\ln(1+I*\exp(d*x+c))*c^2-12*f^2/d^3/a*e*c^2+6*f/d^2/a*\ln(\exp(d*x+c)-I)*e^2+6*f^3/d^4/a*c^2*\ln(\exp(d*x+$

```

c)-I)-12*f^2/d/a*e*x^2-12*f^3/d^4/a*c^2*ln(exp(d*x+c))+12*f^3/d^3/a*polylog
(2,-I*exp(d*x+c))*x+12*f^2/d^3/a*e*polylog(2,-I*exp(d*x+c))+I/d/a*e^3*ln(ex
p(d*x+c)+1)+6/d^3/a*f^3*polylog(2,-exp(d*x+c))*x+6/d^3/a*f^3*polylog(2,exp(
d*x+c))*x+3/d^2/a*f^3*ln(exp(d*x+c)+1)*x^2+3/d^2/a*f^3*ln(1-exp(d*x+c))*x^2
-3/d^4/a*f^3*c^2*ln(1-exp(d*x+c))+3/d^2/a*e^2*f*ln(exp(d*x+c)-1)+3/d^2/a*e^
2*f*ln(exp(d*x+c)+1)+6/d^3/a*e*f^2*polylog(2,exp(d*x+c))+6/d^3/a*e*f^2*poly
log(2,-exp(d*x+c))+3/d^4/a*f^3*c^2*ln(exp(d*x+c)-1)-I/d/a*e^3*ln(exp(d*x+c)
-1)+3*I/d^2/a*e^2*f*c*ln(exp(d*x+c)-1)-3*I/d^3/a*e*f^2*c^2*ln(exp(d*x+c)-1)
-3*I/d^2/a*ln(1-exp(d*x+c))*c*e^2*f+6*I/d^2/a*e*f^2*polylog(2,-exp(d*x+c))*
x-3*I/d/a*ln(1-exp(d*x+c))*e^2*f*x+3*I/d/a*ln(exp(d*x+c)+1)*e^2*f*x+3*I/d^3
/a*e*f^2*c^2*ln(1-exp(d*x+c))-3*I/d/a*e*f^2*ln(1-exp(d*x+c))*x^2-6*I/d^2/a*
e*f^2*polylog(2,exp(d*x+c))*x+3*I/d/a*e*f^2*ln(exp(d*x+c)+1)*x^2+6/d^2/a*e*
f^2*ln(1-exp(d*x+c))*x-6/d^3/a*e*f^2*c*ln(exp(d*x+c)-1)+6/d^3/a*e*f^2*ln(1-
exp(d*x+c))*c+6/d^2/a*e*f^2*ln(exp(d*x+c)+1)*x+I/d^4/a*f^3*c^3*ln(exp(d*x+c)
-1)+I/d/a*f^3*ln(exp(d*x+c)+1)*x^3-3*I/d^2/a*e^2*f*polylog(2,exp(d*x+c))+3
*I/d^2/a*e^2*f*polylog(2,-exp(d*x+c))+6*I/d^3/a*e*f^2*polylog(3,exp(d*x+c))
-6*I/d^3/a*e*f^2*polylog(3,-exp(d*x+c))+6*I/d^3/a*f^3*polylog(3,exp(d*x+c))
*x+3*I/d^2/a*f^3*polylog(2,-exp(d*x+c))*x^2-6*I/d^3/a*f^3*polylog(3,-exp(d*
x+c))*x-I/d/a*f^3*ln(1-exp(d*x+c))*x^3-I/d^4/a*f^3*ln(1-exp(d*x+c))*c^3-3*I
/d^2/a*f^3*polylog(2,exp(d*x+c))*x^2

```

---

**Maxima [B]** time = 2.22416, size = 1268, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((f*x+e)^3*csh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)

```

```

[Out] -e^3*(4*(e^(-d*x - c) - I*e^(-2*d*x - 2*c) + 2*I)/((2*a*e^(-d*x - c) - 2*I*
a*e^(-2*d*x - 2*c) - 2*a*e^(-3*d*x - 3*c) + 2*I*a)*d) - I*log(e^(-d*x - c)
+ 1)/(a*d) + I*log(e^(-d*x - c) - 1)/(a*d)) - 12*e^2*f*x/(a*d) + 3*e^2*f*lo
g(e^(d*x + c) + 1)/(a*d^2) + 6*e^2*f*log(e^(d*x + c) - I)/(a*d^2) + 3*e^2*f
*log(e^(d*x + c) - 1)/(a*d^2) + (4*I*f^3*x^3 + 12*I*e*f^2*x^2 + 12*I*e^2*f*
x - (2*I*f^3*x^3*e^(2*c) + 6*I*e*f^2*x^2*e^(2*c) + 6*I*e^2*f*x*e^(2*c)))*e^(
2*d*x) - 2*(f^3*x^3*e^c + 3*e*f^2*x^2*e^c + 3*e^2*f*x*e^c)*e^(d*x))/(a*d*e^
(3*d*x + 3*c) - I*a*d*e^(2*d*x + 2*c) - a*d*e^(d*x + c) + I*a*d) + 12*(d*x*
log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*e*f^2/(a*d^3) + I*(d^3*x^3*
log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^
(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*f^3/(a*d^4) - I*(d^3*x^3*log(-e^(d
*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c))

```

$$\begin{aligned}
& + 6*\text{polylog}(4, e^{(d*x + c)}) * f^3 / (a*d^4) + 6*(d^2*x^2*\log(I*e^{(d*x + c)} + \\
& 1) + 2*d*x*\text{dilog}(-I*e^{(d*x + c)}) - 2*\text{polylog}(3, -I*e^{(d*x + c)}) * f^3 / (a*d^4 \\
& ) - 3*(-I*d*e^{2*f} - 2*e*f^2)*(d*x*\log(e^{(d*x + c)} + 1) + \text{dilog}(-e^{(d*x + c)} \\
& )) / (a*d^3) + 3*(-I*d*e^{2*f} + 2*e*f^2)*(d*x*\log(-e^{(d*x + c)} + 1) + \text{dilog}(e^{(d*x + c)} \\
& )) / (a*d^3) + (d^2*x^2*\log(e^{(d*x + c)} + 1) + 2*d*x*\text{dilog}(-e^{(d*x + c)} \\
& c)) - 2*\text{polylog}(3, -e^{(d*x + c)}) * (3*I*d*e*f^2 + 3*f^3) / (a*d^4) - (d^2*x^2 \\
& * \log(-e^{(d*x + c)} + 1) + 2*d*x*\text{dilog}(e^{(d*x + c)}) - 2*\text{polylog}(3, e^{(d*x + c)} \\
& )) * (3*I*d*e*f^2 - 3*f^3) / (a*d^4) - 1/4*(I*d^4*f^3*x^4 + (4*I*d*e*f^2 + 4*f^3) \\
& * d^3*x^3 + (6*I*d^2*e^{2*f} + 12*d*e*f^2)*d^2*x^2) / (a*d^4) + 1/4*(I*d^4*f^3 \\
& * x^4 + (4*I*d*e*f^2 - 4*f^3)*d^3*x^3 + (6*I*d^2*e^{2*f} - 12*d*e*f^2)*d^2*x^2 \\
& ) / (a*d^4) - 2*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2) / (a*d^4)
\end{aligned}$$

**Fricas [C]** time = 3.03668, size = 5860, normalized size = 13.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (4*I*d^3*e^3 - 12*I*c*d^2*e^2*f + 12*I*c^2*d*e*f^2 - 4*I*c^3*f^3 + (12*I*d*f^3*x + 12*I*d*e*f^2 + 12*(d*f^3*x + d*e*f^2)*e^(3*d*x + 3*c) + (-12*I*d*f^3*x - 12*I*d*e*f^2)*e^(2*d*x + 2*c) - 12*(d*f^3*x + d*e*f^2)*e^(d*x + c))*dilog(-I*e^(d*x + c)) - (3*d^2*f^3*x^2 + 3*d^2*e^2*f - 6*I*d*e*f^2 + (6*d^2*e*f^2 - 6*I*d*f^3)*x - (3*I*d^2*f^3*x^2 + 3*I*d^2*e^2*f + 6*d*e*f^2 - 6*(-I*d^2*e*f^2 - d*f^3)*x)*e^(3*d*x + 3*c) - (3*d^2*f^3*x^2 + 3*d^2*e^2*f - 6*I*d*e*f^2 + (6*d^2*e*f^2 - 6*I*d*f^3)*x)*e^(2*d*x + 2*c) - (-3*I*d^2*f^3*x^2 - 3*I*d^2*e^2*f - 6*d*e*f^2 - 6*(I*d^2*e*f^2 + d*f^3)*x)*e^(d*x + c))*dilog(-e^(d*x + c)) + (3*d^2*f^3*x^2 + 3*d^2*e^2*f + 6*I*d*e*f^2 + (6*d^2*e*f^2 + 6*I*d*f^3)*x + (-3*I*d^2*f^3*x^2 - 3*I*d^2*e^2*f + 6*d*e*f^2 - 6*(I*d^2*e*f^2 - d*f^3)*x)*e^(3*d*x + 3*c) - (3*d^2*f^3*x^2 + 3*d^2*e^2*f + 6*I*d*e*f^2 + (6*d^2*e*f^2 + 6*I*d*f^3)*x)*e^(2*d*x + 2*c) + (3*I*d^2*f^3*x^2 + 3*I*d^2*e^2*f - 6*d*e*f^2 - 6*(-I*d^2*e*f^2 + d*f^3)*x)*e^(d*x + c))*dilog(e^(d*x + c)) - 4*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*e^(3*d*x + 3*c) + (2*I*d^3*f^3*x^3 + 6*I*d^3*e*f^2*x^2 + 6*I*d^3*e^2*f*x - 2*I*d^3*e^3 + 12*I*c*d^2*e^2*f - 12*I*c^2*d*e*f^2 + 4*I*c^3*f^3)*e^(2*d*x + 2*c) + 2*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x - d^3*e^3 + 6*c*d^2*e^2*f - 6*c^2*d*e*f^2 + 2*c^3*f^3)*e^(d*x + c) - (d^3*f^3*x^3 + d^3*e^3 - 3*I*d^2*e^2*f + (3*d^3*e*f^2 - 3*I*d^2*f^3)*x^2 + 3*(d^3*e^2*f - 2*I*d^2*e*f^2)*x - (I*d^3*f^3*x^3 + I*d^3*e^3 + 3*d^2*e^2*f - 3*(-I*d^3*e*f^2 - d^2*f^3)*x^2 + (3*I*d^3*e^2*f + 6*d^2*e*f^2)*x)*e^
```



$$\begin{aligned}
& (3dx + 3c) - (d^3f^3x^3 + d^3e^3 - 3I*d^2*e^2*f + (3d^3*e*f^2 - 3I*d^2*f^3)*x^2 + 3*(d^3*e^2*f - 2I*d^2*e*f^2)*x)*e^{(2dx + 2c)} - (-I*d^3*f^3*x^3 - I*d^3*e^3 - 3d^2*e^2*f - 3*(I*d^3*e*f^2 + d^2*f^3)*x^2 + (-3I*d^3*e^2*f - 6d^2*e*f^2)*x)*e^{(dx + c)}*\log(e^{(dx + c)} + 1) + (6I*d^2*e^2*f - 12I*c*d*e*f^2 + 6I*c^2*f^3 + 6*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3))*e^{(3dx + 3c)} + (-6I*d^2*e^2*f + 12I*c*d*e*f^2 - 6I*c^2*f^3)*e^{(2dx + 2c)} - 6*(d^2*e^2*f - 2*c*d*e*f^2 + c^2*f^3)*e^{(dx + c)}*\log(e^{(dx + c)} - 1) + (d^3*e^3 - (3c - 3I)*d^2*e^2*f + 3*(c^2 - 2I*c)*d*e*f^2 - (c^3 - 3I*c^2)*f^3 + (-I*d^3*e^3 - 3*(-I*c - 1)*d^2*e^2*f + (-3I*c^2 - 6*c)*d*e*f^2 + (I*c^3 + 3*c^2)*f^3)*e^{(3dx + 3c)} - (d^3*e^3 - (3c - 3I)*d^2*e^2*f + 3*(c^2 - 2I*c)*d*e*f^2 - (c^3 - 3I*c^2)*f^3)*e^{(2dx + 2c)} + (I*d^3*e^3 - 3*(I*c + 1)*d^2*e^2*f + (3I*c^2 + 6*c)*d*e*f^2 + (-I*c^3 - 3*c^2)*f^3)*e^{(dx + c)}*\log(e^{(dx + c)} - 1) + (6I*d^2*f^3*x^2 + 12I*d^2*e*f^2*x + 12I*c*d*e*f^2 - 6I*c^2*f^3 + 6*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3))*e^{(3dx + 3c)} + (-6I*d^2*f^3*x^2 - 12I*d^2*e*f^2*x - 12I*c*d*e*f^2 + 6I*c^2*f^3)*e^{(2dx + 2c)} - 6*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*e^{(dx + c)}*\log(I*e^{(dx + c)} + 1) + (d^3*f^3*x^3 + 3*c*d^2*e^2*f - 3*(c^2 - 2I*c)*d*e*f^2 + (c^3 - 3I*c^2)*f^3 + (3d^3*e*f^2 + 3I*d^2*f^3)*x^2 + 3*(d^3*e^2*f + 2I*d^2*e*f^2)*x + (-I*d^3*f^3*x^3 - 3I*c*d^2*e^2*f + (3I*c^2 + 6*c)*d*e*f^2 + (-I*c^3 - 3*c^2)*f^3 - 3*(I*d^3*e*f^2 - d^2*f^3)*x^2 + (-3I*d^3*e^2*f + 6*d^2*e*f^2)*x)*e^{(3dx + 3c)} - (d^3*f^3*x^3 + 3*c*d^2*e^2*f - 3*(c^2 - 2I*c)*d*e*f^2 + (c^3 - 3I*c^2)*f^3 + (3d^3*e*f^2 + 3I*d^2*f^3)*x^2 + 3*(d^3*e^2*f + 2I*d^2*e*f^2)*x)*e^{(2dx + 2c)} + (I*d^3*f^3*x^3 + 3I*c*d^2*e^2*f + (-3I*c^2 - 6*c)*d*e*f^2 + (I*c^3 + 3*c^2)*f^3 - 3*(-I*d^3*e*f^2 + d^2*f^3)*x^2 + (3I*d^3*e^2*f - 6*d^2*e*f^2)*x)*e^{(dx + c)}*\log(-e^{(dx + c)} + 1) + (6I*f^3*e^{(3dx + 3c)} + 6*f^3*e^{(2dx + 2c)} - 6I*f^3*e^{(dx + c)} - 6*f^3)*polylog(4, -e^{(dx + c)}) + (-6I*f^3*e^{(3dx + 3c)} - 6*f^3*e^{(2dx + 2c)} + 6I*f^3*e^{(dx + c)} + 6*f^3)*polylog(4, e^{(dx + c)}) - (12*f^3*e^{(3dx + 3c)} - 12I*f^3*e^{(2dx + 2c)} - 12*f^3*e^{(dx + c)} + 12I*f^3)*polylog(3, -I*e^{(dx + c)}) + (6*d*f^3*x + 6*d*e*f^2 - 6I*f^3 + (-6I*d*f^3*x - 6I*d*e*f^2 - 6*f^3)*e^{(3dx + 3c)} - 6*(d*f^3*x + d*e*f^2 - I*f^3)*e^{(2dx + 2c)} + (6I*d*f^3*x + 6I*d*e*f^2 + 6*f^3)*e^{(dx + c)})*polylog(3, -e^{(dx + c)}) - (6*d*f^3*x + 6*d*e*f^2 + 6I*f^3 - (6I*d*f^3*x + 6I*d*e*f^2 - 6*f^3)*e^{(3dx + 3c)} - 6*(d*f^3*x + d*e*f^2 + I*f^3)*e^{(2dx + 2c)} - (-6I*d*f^3*x - 6I*d*e*f^2 + 6*f^3)*e^{(dx + c)})*polylog(3, e^{(dx + c)}))/(a*d^4*e^{(3dx + 3c)} - I*a*d^4*e^{(2dx + 2c)} - a*d^4*e^{(dx + c)} + I*a*d^4)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.212 \quad \int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=296

$$\frac{2if(e+fx)\operatorname{PolyLog}(2, -e^{c+dx})}{ad^2} - \frac{2if(e+fx)\operatorname{PolyLog}(2, e^{c+dx})}{ad^2} + \frac{4f^2\operatorname{PolyLog}(2, -ie^{c+dx})}{ad^3} + \frac{f^2\operatorname{PolyLog}(2, e^{2(c+dx)})}{ad^3}$$

[Out]  $(-2*(e + f*x)^2)/(a*d) + ((2*I)*(e + f*x)^2*\operatorname{ArcTanh}[E^{(c + d*x)}])/(a*d) - ((e + f*x)^2*\operatorname{Coth}[c + d*x])/(a*d) + (4*f*(e + f*x)*\operatorname{Log}[1 + I*E^{(c + d*x)}])/(a*d^2) + (2*f*(e + f*x)*\operatorname{Log}[1 - E^{(2*(c + d*x))}])/(a*d^2) + ((2*I)*f*(e + f*x)*\operatorname{PolyLog}[2, -E^{(c + d*x)}])/(a*d^2) + (4*f^2*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x)}])/(a*d^3) - ((2*I)*f*(e + f*x)*\operatorname{PolyLog}[2, E^{(c + d*x)}])/(a*d^2) + (f^2*\operatorname{PolyLog}[2, E^{(2*(c + d*x))}])/(a*d^3) - ((2*I)*f^2*\operatorname{PolyLog}[3, -E^{(c + d*x)}])/(a*d^3) + ((2*I)*f^2*\operatorname{PolyLog}[3, E^{(c + d*x)}])/(a*d^3) - ((e + f*x)^2*\operatorname{Tanh}[c/2 + (I/4)*\operatorname{Pi} + (d*x)/2])/(a*d)$

**Rubi [A]** time = 0.583369, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 11, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$ , Rules used = {5575, 4184, 3716, 2190, 2279, 2391, 4182, 2531, 2282, 6589, 3318}

$$\frac{2if(e+fx)\operatorname{PolyLog}(2, -e^{c+dx})}{ad^2} - \frac{2if(e+fx)\operatorname{PolyLog}(2, e^{c+dx})}{ad^2} + \frac{4f^2\operatorname{PolyLog}(2, -ie^{c+dx})}{ad^3} + \frac{f^2\operatorname{PolyLog}(2, e^{2(c+dx)})}{ad^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e + f*x)^2*\operatorname{Csch}[c + d*x]^2/(a + I*a*\operatorname{Sinh}[c + d*x]), x]$

[Out]  $(-2*(e + f*x)^2)/(a*d) + ((2*I)*(e + f*x)^2*\operatorname{ArcTanh}[E^{(c + d*x)}])/(a*d) - ((e + f*x)^2*\operatorname{Coth}[c + d*x])/(a*d) + (4*f*(e + f*x)*\operatorname{Log}[1 + I*E^{(c + d*x)}])/(a*d^2) + (2*f*(e + f*x)*\operatorname{Log}[1 - E^{(2*(c + d*x))}])/(a*d^2) + ((2*I)*f*(e + f*x)*\operatorname{PolyLog}[2, -E^{(c + d*x)}])/(a*d^2) + (4*f^2*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x)}])/(a*d^3) - ((2*I)*f*(e + f*x)*\operatorname{PolyLog}[2, E^{(c + d*x)}])/(a*d^2) + (f^2*\operatorname{PolyLog}[2, E^{(2*(c + d*x))}])/(a*d^3) - ((2*I)*f^2*\operatorname{PolyLog}[3, -E^{(c + d*x)}])/(a*d^3) + ((2*I)*f^2*\operatorname{PolyLog}[3, E^{(c + d*x)}])/(a*d^3) - ((e + f*x)^2*\operatorname{Tanh}[c/2 + (I/4)*\operatorname{Pi} + (d*x)/2])/(a*d)$

### Rule 5575

$\operatorname{Int}[(\operatorname{Csch}[c_.] + (d_.)*(x_)]^{(n_.)}*((e_.) + (f_.)*(x_))^{(m_.)}]/((a_.) + (b_.)*\operatorname{Sinh}[c_.] + (d_.)*(x_)), x\_Symbol] \rightarrow \operatorname{Dist}[1/a, \operatorname{Int}[(e + f*x)^m*\operatorname{Csch}[c$

+ d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Csch[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

#### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3716

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)], x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x)))/(E^(2\*I\*k\*Pi)\*(1 + E^(2\*(-I\*e) + f\*fz\*x)))/E^(2\*I\*k\*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/((b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/((b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (d\_.)\*(x\_)))^(n\_.)], x\_Symbol] := Dist[1/((d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 4182

Int[csc[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)]/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] := Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
(f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx)}{a+ia \sinh(c+dx)} dx &= -\left(i \int \frac{(e+fx)^2 \operatorname{csch}(c+dx)}{a+ia \sinh(c+dx)} dx\right) + \frac{\int (e+fx)^2 \operatorname{csch}^2(c+dx) dx}{a} \\
&= -\frac{(e+fx)^2 \operatorname{coth}(c+dx)}{ad} - \frac{i \int (e+fx)^2 \operatorname{csch}(c+dx) dx}{a} + \frac{(2f) \int (e+fx) \operatorname{coth}(c+dx) dx}{ad} \\
&= -\frac{(e+fx)^2}{ad} + \frac{2i(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{(e+fx)^2 \operatorname{coth}(c+dx)}{ad} - \frac{\int (e+fx)^2 \operatorname{csc}^2\left(\frac{1}{2}\right)}{2a} \\
&= -\frac{(e+fx)^2}{ad} + \frac{2i(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{(e+fx)^2 \operatorname{coth}(c+dx)}{ad} + \frac{2f(e+fx) \log(1-)}{ad^2} \\
&= -\frac{2(e+fx)^2}{ad} + \frac{2i(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{(e+fx)^2 \operatorname{coth}(c+dx)}{ad} + \frac{2f(e+fx) \log(1-)}{ad^2} \\
&= -\frac{2(e+fx)^2}{ad} + \frac{2i(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{(e+fx)^2 \operatorname{coth}(c+dx)}{ad} + \frac{4f(e+fx) \log(1-)}{ad^2} \\
&= -\frac{2(e+fx)^2}{ad} + \frac{2i(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{(e+fx)^2 \operatorname{coth}(c+dx)}{ad} + \frac{4f(e+fx) \log(1-)}{ad^2} \\
&= -\frac{2(e+fx)^2}{ad} + \frac{2i(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{(e+fx)^2 \operatorname{coth}(c+dx)}{ad} + \frac{4f(e+fx) \log(1-)}{ad^2}
\end{aligned}$$

**Mathematica [B]** time = 13.0869, size = 715, normalized size = 2.42

$$\frac{-2(e^{2c}-1)f(f+ide)\operatorname{PolyLog}(2,-e^{-c-dx})+2i(e^{2c}-1)f(de+if)\operatorname{PolyLog}(2,e^{-c-dx})-2i(e^{2c}-1)f^2(dx\operatorname{PolyLog}(2,$$

Antiderivative was successfully verified.

[In] Integrate[((e+f\*x)^2\*Csch[c+d\*x]^2)/(a+I\*a\*Sinh[c+d\*x]),x]

[Out] (2\*((d\*(e+f\*x)\*((-I)\*d\*(e+f\*x)+2\*(-I+E^c)\*f\*Log[1-I\*E^(-c-d\*x)]))/(-I+E^c)-2\*f^2\*PolyLog[2,I\*E^(-c-d\*x)]))/(a\*d^3)+(-2\*d^2\*(e+f\*x)^2+2\*d\*(-1+E^(2\*c))\*f\*((-I)\*d\*e+f)\*x\*Log[1-E^(-c-d\*x)]-I\*d^2\*(-1+E^(2\*c))\*f^2\*x^2\*Log[1-E^(-c-d\*x)]+2\*d\*(-1+E^(2\*c))\*f\*(I\*d\*e+f)\*x\*Log[1+E^(-c-d\*x)]+I\*d^2\*(-1+E^(2\*c))\*f^2\*x^2\*Log[1+E^(-c-d\*x)]+I\*d\*e\*(-1+E^(2\*c))\*(d\*e+(2\*I)\*f)\*(d\*x-Log[1-E^(c+d\*x)])+d\*e\*(1-E^(2\*c))\*(I\*d\*e+2\*f)\*(d\*x-Log[1+E^(c+d\*x)])-2\*(-1+E

$$\begin{aligned} &^{(2*c)})*f*(I*d*e + f)*\text{PolyLog}[2, -E^{(-c - d*x)}] + (2*I)*(-1 + E^{(2*c)})*(d*e \\ &+ I*f)*f*\text{PolyLog}[2, E^{(-c - d*x)}] - (2*I)*(-1 + E^{(2*c)})*f^2*(d*x*\text{PolyLog}[ \\ &2, -E^{(-c - d*x)}] + \text{PolyLog}[3, -E^{(-c - d*x)}]) + (2*I)*(-1 + E^{(2*c)})*f^2*( \\ &d*x*\text{PolyLog}[2, E^{(-c - d*x)}] + \text{PolyLog}[3, E^{(-c - d*x)}])/(a*d^3*(-1 + E^{(2 \\ &*c)})) + (\text{Sech}[c/2]*\text{Sech}[c/2 + (d*x)/2]*(-e^{2*\text{Sinh}}[(d*x)/2]) - 2*e*f*x*\text{Sinh} \\ &[(d*x)/2] - f^2*x^2*\text{Sinh}[(d*x)/2]))/(2*a*d) + (\text{Csch}[c/2]*\text{Csch}[c/2 + (d*x)/2 \\ &]*(e^{2*\text{Sinh}}[(d*x)/2] + 2*e*f*x*\text{Sinh}[(d*x)/2] + f^2*x^2*\text{Sinh}[(d*x)/2]))/(2*a \\ &*d) - (2*(e^{2*\text{Sinh}}[(d*x)/2] + 2*e*f*x*\text{Sinh}[(d*x)/2] + f^2*x^2*\text{Sinh}[(d*x)/2] \\ &))/(a*d*(\text{Cosh}[c/2] + I*\text{Sinh}[c/2])*(\text{Cosh}[c/2 + (d*x)/2] + I*\text{Sinh}[c/2 + (d*x) \\ &/2])) \end{aligned}$$

**Maple [B]** time = 0.151, size = 847, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)^{2*c}\text{csch}(d*x+c)^2/(a+I*a*\sinh(d*x+c)), x)$

[Out]  $-2*I/d^2/a*f^2*\text{polylog}(2, \exp(d*x+c))*x + 2*I/d^2/a*f^2*\text{polylog}(2, -\exp(d*x+c))$   
 $*x - I/d^3/a*f^2*c^2*\ln(\exp(d*x+c)-1) + 2*I/d^2/a*e*f*\text{polylog}(2, -\exp(d*x+c)) - 2*$   
 $I/d^2/a*e*f*\text{polylog}(2, \exp(d*x+c)) + I/d^3/a*f^2*c^2*\ln(1-\exp(d*x+c)) + I/d/a*f^2$   
 $*\ln(\exp(d*x+c)+1)*x^2 - I/d/a*f^2*\ln(1-\exp(d*x+c))*x^2 - 2*I*(f^2*x^2*\exp(2*d*$   
 $x+2*c) + 2*e*f*x*\exp(2*d*x+2*c) + e^{2*\exp(2*d*x+2*c)} - 2*x^2*f^2 - I*\exp(d*x+c)*f^2$   
 $*x^2 - 4*e*f*x - 2*I*\exp(d*x+c)*e*f*x - 2*e^2 - I*\exp(d*x+c)*e^2)/(\exp(2*d*x+2*c) - 1$   
 $)/(\exp(d*x+c) - I)/d/a + 2*f^2*\text{polylog}(2, -\exp(d*x+c))/a/d^3 + 2*f^2*\text{polylog}(2, \exp$   
 $(d*x+c))/a/d^3 - 4*f^2*x^2/a/d + 2*I*f^2*\text{polylog}(3, \exp(d*x+c))/a/d^3 - 2*I*f^2*\text{po}$   
 $\text{lylog}(3, -\exp(d*x+c))/a/d^3 + 4*f^2*\text{polylog}(2, -I*\exp(d*x+c))/a/d^3 - 2*I/d/a*\ln($   
 $1-\exp(d*x+c))*e*f*x + 2*I/d/a*\ln(\exp(d*x+c)+1)*e*f*x - 2*I/d^2/a*\ln(1-\exp(d*x+c$   
 $))*c*e*f + 2*I/d^2/a*e*f*c*\ln(\exp(d*x+c)-1) - 4*f^2/d^3/a*c^2 + 4*f/d^2/a*\ln(\exp($   
 $d*x+c) - I)*e - 8*f/d^2/a*\ln(\exp(d*x+c))*e - 8*f^2/d^2/a*c*x + 4*f^2/d^2/a*\ln(1+I*e$   
 $\exp(d*x+c))*x + 4*f^2/d^3/a*\ln(1+I*\exp(d*x+c))*c - 4*f^2/d^3/a*c*\ln(\exp(d*x+c) - I$   
 $) + 8*f^2/d^3/a*c*\ln(\exp(d*x+c)) + 2/d^2/a*f^2*\ln(\exp(d*x+c)+1)*x + 2/d^2/a*e*f*\ln$   
 $(\exp(d*x+c)-1) + 2/d^2/a*e*f*\ln(\exp(d*x+c)+1) - 2/d^3/a*f^2*c*\ln(\exp(d*x+c)-1)$   
 $+ 2/d^2/a*f^2*\ln(1-\exp(d*x+c))*x + 2/d^3/a*f^2*\ln(1-\exp(d*x+c))*c + I/d/a*e^2*\ln$   
 $(\exp(d*x+c)+1) - I/d/a*e^2*\ln(\exp(d*x+c)-1)$

**Maxima [B]** time = 2.0813, size = 814, normalized size = 2.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -e^2*(4*(e^(-d*x - c) - I*e^(-2*d*x - 2*c) + 2*I)/((2*a*e^(-d*x - c) - 2*I*a*e^(-2*d*x - 2*c) - 2*a*e^(-3*d*x - 3*c) + 2*I*a)*d) - I*log(e^(-d*x - c) + 1)/(a*d) + I*log(e^(-d*x - c) - 1)/(a*d)) - 2*f^2*x^2/(a*d) - 8*e*f*x/(a*d) + (4*I*f^2*x^2 + 8*I*e*f*x - (2*I*f^2*x^2*e^(2*c) + 4*I*e*f*x*e^(2*c))*e^(2*d*x) - 2*(f^2*x^2*e^c + 2*e*f*x*e^c)*e^(d*x))/(a*d*e^(3*d*x + 3*c) - I*a*d*e^(2*d*x + 2*c) - a*d*e^(d*x + c) + I*a*d) + 2*e*f*log(e^(d*x + c) + 1)/(a*d^2) + 4*e*f*log(e^(d*x + c) - 1)/(a*d^2) + 2*e*f*log(e^(d*x + c) - 1)/(a*d^2) + I*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c))) - 2*polylog(3, -e^(d*x + c))*f^2/(a*d^3) - I*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c))*f^2/(a*d^3) + 4*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)))*f^2/(a*d^3) + (2*I*d*e*f + 2*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a*d^3) - (2*I*d*e*f - 2*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a*d^3) - 1/3*(I*d^3*f^2*x^3 + (3*I*d*e*f + 3*f^2)*d^2*x^2)/(a*d^3) + 1/3*(I*d^3*f^2*x^3 + (3*I*d*e*f - 3*f^2)*d^2*x^2)/(a*d^3)
```

**Fricas [C]** time = 2.76264, size = 3237, normalized size = 10.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (4*I*d^2*e^2 - 8*I*c*d*e*f + 4*I*c^2*f^2 + (4*f^2*e^(3*d*x + 3*c) - 4*I*f^2*e^(2*d*x + 2*c) - 4*f^2*e^(d*x + c) + 4*I*f^2)*dilog(-I*e^(d*x + c)) - (2*d*f^2*x + 2*d*e*f - 2*I*f^2 - (2*I*d*f^2*x + 2*I*d*e*f + 2*f^2)*e^(3*d*x + 3*c) - 2*(d*f^2*x + d*e*f - I*f^2)*e^(2*d*x + 2*c) - (-2*I*d*f^2*x - 2*I*d*e*f - 2*f^2)*e^(d*x + c))*dilog(-e^(d*x + c)) + (2*d*f^2*x + 2*d*e*f + 2*I*f^2 + (-2*I*d*f^2*x - 2*I*d*e*f + 2*f^2)*e^(3*d*x + 3*c) - 2*(d*f^2*x + d*e*f + I*f^2)*e^(2*d*x + 2*c) + (2*I*d*f^2*x + 2*I*d*e*f - 2*f^2)*e^(d*x + c))*dilog(e^(d*x + c)) - 4*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(3*d*x + 3*c) + (2*I*d^2*f^2*x^2 + 4*I*d^2*e*f*x - 2*I*d^2*e^2 + 8*I*c*d*e*f - 4*I*c^2*f^2)*e^(2*d*x + 2*c) + 2*(d^2*f^2*x^2 + 2*d^2*e*f*x - d^2*e^2 + 4*c*d*e*f - 2*c^2*f^2)*e^(d*x + c) - (d^2*f^2*x^2 + d^2*e^2 - 2*I*d*e*f + (2*d^2*e*f - 2*I*d*f^2)*x - (I*d^2*f^2*x^2 + I*d^2*e^2 + 2*d*e*f - 2*(-I*
```



$$\begin{aligned}
& d^2 * e * f - d * f^2 * x) * e^{(3 * d * x + 3 * c)} - (d^2 * f^2 * x^2 + d^2 * e^2 - 2 * I * d * e * f + \\
& (2 * d^2 * e * f - 2 * I * d * f^2) * x) * e^{(2 * d * x + 2 * c)} - (-I * d^2 * f^2 * x^2 - I * d^2 * e^2 - \\
& 2 * d * e * f - 2 * (I * d^2 * e * f + d * f^2) * x) * e^{(d * x + c)} * \log(e^{(d * x + c)} + 1) + (4 * I \\
& * d * e * f - 4 * I * c * f^2 + 4 * (d * e * f - c * f^2) * e^{(3 * d * x + 3 * c)} + (-4 * I * d * e * f + 4 * I * \\
& c * f^2) * e^{(2 * d * x + 2 * c)} - 4 * (d * e * f - c * f^2) * e^{(d * x + c)} * \log(e^{(d * x + c)} - I \\
& ) + (d^2 * e^2 - (2 * c - 2 * I) * d * e * f + (c^2 - 2 * I * c) * f^2 + (-I * d^2 * e^2 - 2 * (-I * \\
& c - 1) * d * e * f + (-I * c^2 - 2 * c) * f^2) * e^{(3 * d * x + 3 * c)} - (d^2 * e^2 - (2 * c - 2 * I) \\
& * d * e * f + (c^2 - 2 * I * c) * f^2) * e^{(2 * d * x + 2 * c)} + (I * d^2 * e^2 - 2 * (I * c + 1) * d * e * \\
& f + (I * c^2 + 2 * c) * f^2) * e^{(d * x + c)} * \log(e^{(d * x + c)} - 1) + (4 * I * d * f^2 * x + 4 \\
& * I * c * f^2 + 4 * (d * f^2 * x + c * f^2) * e^{(3 * d * x + 3 * c)} + (-4 * I * d * f^2 * x - 4 * I * c * f^2) \\
& * e^{(2 * d * x + 2 * c)} - 4 * (d * f^2 * x + c * f^2) * e^{(d * x + c)} * \log(I * e^{(d * x + c)} + 1) \\
& + (d^2 * f^2 * x^2 + 2 * c * d * e * f - (c^2 - 2 * I * c) * f^2 + (2 * d^2 * e * f + 2 * I * d * f^2) * x \\
& + (-I * d^2 * f^2 * x^2 - 2 * I * c * d * e * f + (I * c^2 + 2 * c) * f^2 - 2 * (I * d^2 * e * f - d * f^2) \\
& * x) * e^{(3 * d * x + 3 * c)} - (d^2 * f^2 * x^2 + 2 * c * d * e * f - (c^2 - 2 * I * c) * f^2 + (2 * d^2 \\
& * e * f + 2 * I * d * f^2) * x) * e^{(2 * d * x + 2 * c)} + (I * d^2 * f^2 * x^2 + 2 * I * c * d * e * f + (-I * c \\
& ^2 - 2 * c) * f^2 - 2 * (-I * d^2 * e * f + d * f^2) * x) * e^{(d * x + c)} * \log(-e^{(d * x + c)} + 1 \\
& ) + (-2 * I * f^2 * e^{(3 * d * x + 3 * c)} - 2 * f^2 * e^{(2 * d * x + 2 * c)} + 2 * I * f^2 * e^{(d * x + c)} \\
& + 2 * f^2) * \text{polylog}(3, -e^{(d * x + c)}) + (2 * I * f^2 * e^{(3 * d * x + 3 * c)} + 2 * f^2 * e^{(2 * \\
& d * x + 2 * c)} - 2 * I * f^2 * e^{(d * x + c)} - 2 * f^2) * \text{polylog}(3, e^{(d * x + c)})) / (a * d^3 * e \\
& ^{(3 * d * x + 3 * c)} - I * a * d^3 * e^{(2 * d * x + 2 * c)} - a * d^3 * e^{(d * x + c)} + I * a * d^3)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cscsch(d\*x+c)\*\*2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cscsch(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.213 \quad \int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=163

$$\frac{if \operatorname{PolyLog}(2, -e^{c+dx})}{ad^2} - \frac{if \operatorname{PolyLog}(2, e^{c+dx})}{ad^2} + \frac{f \log(\sinh(c+dx))}{ad^2} + \frac{2f \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\right)}{ad^2} + \frac{2i(e+fx) \tanh^{-1}\left(\frac{e^{c+dx} - 1}{e^{c+dx} + 1}\right)}{ad}$$

[Out]  $((2*I)*(e + f*x)*\operatorname{ArcTanh}[E^{(c + d*x)}])/(a*d) - ((e + f*x)*\operatorname{Coth}[c + d*x])/(a*d) + (2*f*\operatorname{Log}[\operatorname{Cosh}[c/2 + (I/4)*\operatorname{Pi} + (d*x)/2]])/(a*d^2) + (f*\operatorname{Log}[\operatorname{Sinh}[c + d*x]])/(a*d^2) + (I*f*\operatorname{PolyLog}[2, -E^{(c + d*x)}])/(a*d^2) - (I*f*\operatorname{PolyLog}[2, E^{(c + d*x)}])/(a*d^2) - ((e + f*x)*\operatorname{Tanh}[c/2 + (I/4)*\operatorname{Pi} + (d*x)/2])/(a*d)$

**Rubi [A]** time = 0.228043, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$ , Rules used = {5575, 4184, 3475, 4182, 2279, 2391, 3318}

$$\frac{if \operatorname{PolyLog}(2, -e^{c+dx})}{ad^2} - \frac{if \operatorname{PolyLog}(2, e^{c+dx})}{ad^2} + \frac{f \log(\sinh(c+dx))}{ad^2} + \frac{2f \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\right)}{ad^2} + \frac{2i(e+fx) \tanh^{-1}\left(\frac{e^{c+dx} - 1}{e^{c+dx} + 1}\right)}{ad}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\frac{(e + f*x)*\operatorname{Csch}[c + d*x]^2}{(a + I*a*\operatorname{Sinh}[c + d*x])}, x]$

[Out]  $((2*I)*(e + f*x)*\operatorname{ArcTanh}[E^{(c + d*x)}])/(a*d) - ((e + f*x)*\operatorname{Coth}[c + d*x])/(a*d) + (2*f*\operatorname{Log}[\operatorname{Cosh}[c/2 + (I/4)*\operatorname{Pi} + (d*x)/2]])/(a*d^2) + (f*\operatorname{Log}[\operatorname{Sinh}[c + d*x]])/(a*d^2) + (I*f*\operatorname{PolyLog}[2, -E^{(c + d*x)}])/(a*d^2) - (I*f*\operatorname{PolyLog}[2, E^{(c + d*x)}])/(a*d^2) - ((e + f*x)*\operatorname{Tanh}[c/2 + (I/4)*\operatorname{Pi} + (d*x)/2])/(a*d)$

### Rule 5575

$\operatorname{Int}[(\operatorname{Csch}[(c_.) + (d_.)*(x_)]^{(n_.)}*((e_.) + (f_.)*(x_))^{(m_.)})/((a_.) + (b_.)*\operatorname{Sinh}[(c_.) + (d_.)*(x_)]), x\_Symbol] \rightarrow \operatorname{Dist}[1/a, \operatorname{Int}[(e + f*x)^m*\operatorname{Csch}[c + d*x]^n, x], x] - \operatorname{Dist}[b/a, \operatorname{Int}[(e + f*x)^m*\operatorname{Csch}[c + d*x]^{(n-1)}/(a + b*\operatorname{Sinh}[c + d*x]), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 4184

$\operatorname{Int}[\operatorname{csc}[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow -\operatorname{Sim}p[\frac{(c + d*x)^m*\operatorname{Cot}[e + f*x]}{f}, x] + \operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Cot}[e + f*x], x], x]$

$t[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{GtQ}[m, 0]$

### Rule 3475

$\text{Int}[\tan[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

### Rule 4182

$\text{Int}[\text{csc}[(e_.) + (\text{Complex}[0, fz\_])*(f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(-2*(c + d*x)^m*\text{ArcTanh}[E^{-(I*e) + f*fz*x}])/(f*fz*I), x] + (-\text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 - E^{-(I*e) + f*fz*x}], x], x] + \text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + E^{-(I*e) + f*fz*x}], x], x]) /; \text{FreeQ}\{c, d, e, f, fz\}, x\} \ \&\& \ \text{IGtQ}[m, 0]$

### Rule 2279

$\text{Int}[\text{Log}[(a_) + (b_.)*((F_)^{((e_.)*((c_.) + (d_.)*(x_.)))})^{(n_.)}], x\_Symbol] \rightarrow \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \ \&\& \ \text{GtQ}[a, 0]$

### Rule 2391

$\text{Int}[\text{Log}[(c_.)*((d_) + (e_.)*(x_)^{(n_.)})]/(x_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x\} \ \&\& \ \text{EqQ}[c*d, 1]$

### Rule 3318

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)}*((a_) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[(2*a)^n, \text{Int}[(c + d*x)^m*\text{Sin}[(1*(e + (\text{Pi}*a)/(2*b)))/2 + (f*x)/2]^{(2*n)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x\} \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ (\text{GtQ}[n, 0] \ || \ \text{IGtQ}[m, 0])$

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)} dx &= -\left(i \int \frac{(e+fx)\operatorname{csch}(c+dx)}{a+ia\sinh(c+dx)} dx\right) + \frac{\int (e+fx)\operatorname{csch}^2(c+dx) dx}{a} \\
&= -\frac{(e+fx)\operatorname{coth}(c+dx)}{ad} - \frac{i \int (e+fx)\operatorname{csch}(c+dx) dx}{a} + \frac{f \int \operatorname{coth}(c+dx) dx}{ad} - \int \frac{e}{a+ia\sinh(c+dx)} dx \\
&= \frac{2i(e+fx)\tanh^{-1}(e^{c+dx})}{ad} - \frac{(e+fx)\operatorname{coth}(c+dx)}{ad} + \frac{f \log(\sinh(c+dx))}{ad^2} - \frac{\int (e+fx)\operatorname{csch}(c+dx) dx}{a} \\
&= \frac{2i(e+fx)\tanh^{-1}(e^{c+dx})}{ad} - \frac{(e+fx)\operatorname{coth}(c+dx)}{ad} + \frac{f \log(\sinh(c+dx))}{ad^2} - \frac{(e+fx)\tanh^{-1}(e^{c+dx})}{a} \\
&= \frac{2i(e+fx)\tanh^{-1}(e^{c+dx})}{ad} - \frac{(e+fx)\operatorname{coth}(c+dx)}{ad} + \frac{2f \log\left(\cosh\left(\frac{c}{2} + \frac{i\pi}{4} + \frac{dx}{2}\right)\right)}{ad^2} + \frac{f \log(e^{c+dx})}{ad}
\end{aligned}$$

**Mathematica [B]** time = 5.43604, size = 454, normalized size = 2.79

$$\left(\cosh\left(\frac{1}{2}(c+dx)\right) + i \sinh\left(\frac{1}{2}(c+dx)\right)\right) \left(-2if\left(\cosh\left(\frac{1}{2}(c+dx)\right) + i \sinh\left(\frac{1}{2}(c+dx)\right)\right)\right) \left(\operatorname{PolyLog}\left(2, -e^{-c-dx}\right) - \operatorname{PolyLog}\left(2, e^{-c-dx}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Csch[c + d\*x]^2)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] ((Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])\*(-(d\*(e + f\*x)\*Cosh[(c + d\*x)/2] + (I + Coth[(c + d\*x)/2])) + (4\*I)\*f\*ArcTan[Tanh[(c + d\*x)/2]]\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]) + 2\*f\*Log[Cosh[c + d\*x]]\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]) + 2\*f\*Log[Sinh[c + d\*x]]\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]) + (2\*I)\*c\*f\*Log[Tanh[(c + d\*x)/2]]\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]) - (2\*I)\*f\*((c + d\*x)\*(Log[1 - E^(-c - d\*x)] - Log[1 + E^(-c - d\*x)]) + PolyLog[2, -E^(-c - d\*x)] - PolyLog[2, E^(-c - d\*x)])\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]) - 4\*d\*(e + f\*x)\*Sinh[(c + d\*x)/2] + 2\*f\*(c + d\*x)\*((-I)\*Cosh[(c + d\*x)/2] + Sinh[(c + d\*x)/2]) + 2\*d\*e\*Log[Tanh[(c + d\*x)/2]]\*((-I)\*Cosh[(c + d\*x)/2] + Sinh[(c + d\*x)/2]) - I\*d\*(e + f\*x)\*Sinh[(c + d\*x)/2]\*(-I + Tanh[(c + d\*x)/2]))/(2\*d^2\*(a + I\*a\*Sinh[c + d\*x]))

**Maple [B]** time = 0.151, size = 316, normalized size = 1.9

$$\frac{-2i(fxe^{2dx+2c} + ee^{2dx+2c} - 2fx - ie^{dx+c}fx - 2e - ie^{dx+c}e)}{(e^{2dx+2c} - 1)(e^{dx+c} - i)da} - \frac{ifpolylog(2, e^{dx+c})}{ad^2} + \frac{ifpolylog(2, -e^{dx+c})}{ad^2} - 4 \frac{f \ln(e^{dx+c})}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)`

[Out]  $-2*I*(f*x*\exp(2*d*x+2*c)+e*\exp(2*d*x+2*c)-2*f*x-I*\exp(d*x+c)*f*x-2*e-I*\exp(d*x+c)*e)/(\exp(2*d*x+2*c)-1)/(\exp(d*x+c)-I)/d/a-I*f*\operatorname{polylog}(2,\exp(d*x+c))/a/d^2+I*f*\operatorname{polylog}(2,-\exp(d*x+c))/a/d^2-4/d^2/a*f*\ln(\exp(d*x+c))+1/d^2/a*f*\ln(\exp(d*x+c)-1)+1/d^2/a*f*\ln(\exp(d*x+c)+1)-I/d/a*e*\ln(\exp(d*x+c)-1)+I/d/a*e*\ln(\exp(d*x+c)+1)-I/d/a*\ln(1-\exp(d*x+c))*f*x-I/d^2/a*\ln(1-\exp(d*x+c))*c*f+I/d/a*\ln(\exp(d*x+c)+1)*f*x+2*f/a/d^2*\ln(\exp(d*x+c)-I)+I/d^2/a*f*c*\ln(\exp(d*x+c)-1)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-4id \int \frac{x}{4(ade^{dx+c} + ad)} dx + 4id \int \frac{x}{4(ade^{dx+c} - ad)} dx + \frac{4(xe^{3dx+3c} - ix)}{2ade^{3dx+3c} - 2iade^{2dx+2c} - 2ade^{dx+c} + 2iad} + \frac{2(d}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out]  $-(4*I*d*\operatorname{integrate}(1/4*x/(a*d*e^{d*x+c} + a*d), x) + 4*I*d*\operatorname{integrate}(1/4*x/(a*d*e^{d*x+c} - a*d), x) + 4*(x*e^{3*d*x+3*c} - I*x)/(2*a*d*e^{3*d*x+3*c} - 2*I*a*d*e^{2*d*x+2*c} - 2*a*d*e^{d*x+c} + 2*I*a*d) + 2*(d*x+c)/(a*d^2) - 2*\log((e^{d*x+c} - I)*e^{-c}))/a*d^2 - \log(e^{d*x+c} + 1)/a*d^2 - \log(e^{d*x+c} - 1)/a*d^2)*f - e*(4*(e^{-d*x-c} - I*e^{-2*d*x-2*c} + 2*I))/((2*a*e^{-d*x-c} - 2*I*a*e^{-2*d*x-2*c} - 2*a*e^{-3*d*x-3*c} + 2*I*a)*d) - I*\log(e^{-d*x-c} + 1)/a*d + I*\log(e^{-d*x-c} - 1)/a*d)$

**Fricas [B]** time = 2.59178, size = 1272, normalized size = 7.8

$$\frac{4ide - 2icf + (ife^{3dx+3c} + fe^{2dx+2c} - ife^{dx+c} - f)\operatorname{Li}_2(-e^{dx+c}) + (-ife^{3dx+3c} - fe^{2dx+2c} + ife^{dx+c} + f)\operatorname{Li}_2(e^{dx+c})}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")`

```
[Out] (4*I*d*e - 2*I*c*f + (I*f*e^(3*d*x + 3*c) + f*e^(2*d*x + 2*c) - I*f*e^(d*x + c) - f)*dilog(-e^(d*x + c)) + (-I*f*e^(3*d*x + 3*c) - f*e^(2*d*x + 2*c) + I*f*e^(d*x + c) + f)*dilog(e^(d*x + c)) - 2*(2*d*f*x + c*f)*e^(3*d*x + 3*c) + (2*I*d*f*x - 2*I*d*e + 2*I*c*f)*e^(2*d*x + 2*c) + 2*(d*f*x - d*e + c*f)*e^(d*x + c) - (d*f*x + d*e - (I*d*f*x + I*d*e + f))*e^(3*d*x + 3*c) - (d*f*x + d*e - I*f)*e^(2*d*x + 2*c) - (-I*d*f*x - I*d*e - f)*e^(d*x + c) - I*f*log(e^(d*x + c) + 1) + (2*f*e^(3*d*x + 3*c) - 2*I*f*e^(2*d*x + 2*c) - 2*f*e^(d*x + c) + 2*I*f)*log(e^(d*x + c) - I) + (d*e - (c - I)*f + (-I*d*e + (I*c + 1)*f))*e^(3*d*x + 3*c) - (d*e - (c - I)*f)*e^(2*d*x + 2*c) + (I*d*e + (-I*c - 1)*f)*e^(d*x + c))*log(e^(d*x + c) - 1) + (d*f*x + c*f + (-I*d*f*x - I*c*f))*e^(3*d*x + 3*c) - (d*f*x + c*f)*e^(2*d*x + 2*c) + (I*d*f*x + I*c*f)*e^(d*x + c))*log(-e^(d*x + c) + 1))/(a*d^2*e^(3*d*x + 3*c) - I*a*d^2*e^(2*d*x + 2*c) - a*d^2*e^(d*x + c) + I*a*d^2)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \operatorname{csch}(dx + c)^2}{i a \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*csch(d*x + c)^2/(I*a*sinh(d*x + c) + a), x)
```

$$3.214 \quad \int \frac{\operatorname{csch}^2(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=57

$$-\frac{2 \operatorname{coth}(c+dx)}{ad} + \frac{i \tanh^{-1}(\operatorname{cosh}(c+dx))}{ad} + \frac{\operatorname{coth}(c+dx)}{d(a+ia \sinh(c+dx))}$$

[Out] (I\*ArcTanh[Cosh[c + d\*x]])/(a\*d) - (2\*Coth[c + d\*x])/(a\*d) + Coth[c + d\*x]/(d\*(a + I\*a\*Sinh[c + d\*x]))

**Rubi [A]** time = 0.0856202, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$ , Rules used = {2768, 2748, 3767, 8, 3770}

$$-\frac{2 \operatorname{coth}(c+dx)}{ad} + \frac{i \tanh^{-1}(\operatorname{cosh}(c+dx))}{ad} + \frac{\operatorname{coth}(c+dx)}{d(a+ia \sinh(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d\*x]^2/(a + I\*a\*Sinh[c + d\*x]), x]

[Out] (I\*ArcTanh[Cosh[c + d\*x]])/(a\*d) - (2\*Coth[c + d\*x])/(a\*d) + Coth[c + d\*x]/(d\*(a + I\*a\*Sinh[c + d\*x]))

#### Rule 2768

Int[((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_)/((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> -Simp[(b^2\*Cos[e + f\*x]\*(c + d\*Sin[e + f\*x])^(n + 1))/(a\*f\*(b\*c - a\*d)\*(a + b\*Sin[e + f\*x])), x] + Dist[d/(a\*(b\*c - a\*d)), Int[(c + d\*Sin[e + f\*x])^n\*(a^n - b\*(n + 1)\*Sin[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2\*n] || EqQ[c, 0])

#### Rule 2748

Int[((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[c, Int[(b\*Sin[e + f\*x])^m, x], x] + Dist[d/b, Int[(b\*Sin[e + f\*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

#### Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

### Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{\operatorname{csch}^2(c + dx)}{a + ia \sinh(c + dx)} dx &= \frac{\operatorname{coth}(c + dx)}{d(a + ia \sinh(c + dx))} - \frac{\int \operatorname{csch}^2(c + dx)(-2a + ia \sinh(c + dx)) dx}{a^2} \\
 &= \frac{\operatorname{coth}(c + dx)}{d(a + ia \sinh(c + dx))} - \frac{i \int \operatorname{csch}(c + dx) dx}{a} + \frac{2 \int \operatorname{csch}^2(c + dx) dx}{a} \\
 &= \frac{i \tanh^{-1}(\cosh(c + dx))}{ad} + \frac{\operatorname{coth}(c + dx)}{d(a + ia \sinh(c + dx))} - \frac{(2i) \operatorname{Subst}(\int 1 dx, x, -i \operatorname{coth}(c + dx))}{ad} \\
 &= \frac{i \tanh^{-1}(\cosh(c + dx))}{ad} - \frac{2 \operatorname{coth}(c + dx)}{ad} + \frac{\operatorname{coth}(c + dx)}{d(a + ia \sinh(c + dx))}
 \end{aligned}$$

**Mathematica [A]** time = 0.205343, size = 61, normalized size = 1.07

$$\frac{\operatorname{sech}(c + dx) \left( 2 \sinh(c + dx) + \operatorname{csch}(c + dx) - i \sqrt{\cosh^2(c + dx)} \tanh^{-1} \left( \sqrt{\cosh^2(c + dx)} \right) + i \right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + I*a*Sinh[c + d*x]), x]
```

```
[Out] -((Sech[c + d*x]*(I - I*ArcTanh[Sqrt[Cosh[c + d*x]^2]]*Sqrt[Cosh[c + d*x]^2] + Csch[c + d*x] + 2*Sinh[c + d*x]))/(a*d))
```



**Maple [A]** time = 0.04, size = 79, normalized size = 1.4

$$-\frac{1}{2da} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{2da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-1} - \frac{i}{da} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 2 \frac{1}{da(-i + \tanh(1/2 dx + c/2))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] -1/2/d/a\*tanh(1/2\*d\*x+1/2\*c)-1/2/d/a/tanh(1/2\*d\*x+1/2\*c)-I/d/a\*ln(tanh(1/2\*d\*x+1/2\*c))-2/d/a/(-I+tanh(1/2\*d\*x+1/2\*c))

**Maxima [B]** time = 1.07214, size = 149, normalized size = 2.61

$$\frac{4(e^{-dx-c} - i e^{-2dx-2c} + 2i)}{(2ae^{-dx-c} - 2i ae^{-2dx-2c} - 2ae^{-3dx-3c} + 2ia)d} + \frac{i \log(e^{-dx-c} + 1)}{ad} - \frac{i \log(e^{-dx-c} - 1)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -4\*(e^(-d\*x - c) - I\*e^(-2\*d\*x - 2\*c) + 2\*I)/((2\*a\*e^(-d\*x - c) - 2\*I\*a\*e^(-2\*d\*x - 2\*c) - 2\*a\*e^(-3\*d\*x - 3\*c) + 2\*I\*a)\*d) + I\*log(e^(-d\*x - c) + 1)/(a\*d) - I\*log(e^(-d\*x - c) - 1)/(a\*d)

**Fricas [B]** time = 2.56622, size = 378, normalized size = 6.63

$$\frac{(i e^{(3dx+3c)} + e^{(2dx+2c)} - i e^{(dx+c)} - 1) \log(e^{(dx+c)} + 1) + (-i e^{(3dx+3c)} - e^{(2dx+2c)} + i e^{(dx+c)} + 1) \log(e^{(dx+c)} - 1) - 2i e^{(2dx+2c)}}{ade^{(3dx+3c)} - i ade^{(2dx+2c)} - ade^{(dx+c)} + i ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] ((I\*e^(3\*d\*x + 3\*c) + e^(2\*d\*x + 2\*c) - I\*e^(d\*x + c) - 1)\*log(e^(d\*x + c) + 1) + (-I\*e^(3\*d\*x + 3\*c) - e^(2\*d\*x + 2\*c) + I\*e^(d\*x + c) + 1)\*log(e^(d\*x + c) - 1) - 2\*I\*e^(2\*d\*x + 2\*c) - 2\*e^(d\*x + c) + 4\*I)/(a\*d\*e^(3\*d\*x + 3\*c) - I\*a\*d\*e^(2\*d\*x + 2\*c) - a\*d\*e^(d\*x + c) + I\*a\*d)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)\*\*2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [A]** time = 1.2282, size = 128, normalized size = 2.25

$$\frac{i \log(e^{(dx+c)} + 1)}{ad} - \frac{i \log(|e^{(dx+c)} - 1|)}{ad} + \frac{2(e^{(2dx+2c)} - i e^{(dx+c)} - 2)}{ad(i e^{(3dx+3c)} + e^{(2dx+2c)} - i e^{(dx+c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] I\*log(e^(d\*x + c) + 1)/(a\*d) - I\*log(abs(e^(d\*x + c) - 1))/(a\*d) + 2\*(e^(2\*d\*x + 2\*c) - I\*e^(d\*x + c) - 2)/(a\*d\*(I\*e^(3\*d\*x + 3\*c) + e^(2\*d\*x + 2\*c) - I\*e^(d\*x + c) - 1))

$$3.215 \quad \int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Csch[c + d\*x]^2/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.075928, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Csch[c + d\*x]^2/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Csch[c + d\*x]^2/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx = \int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

**Mathematica [A]** time = 131.951, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Csch[c + d\*x]^2/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Integrate[Csch[c + d\*x]^2/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

**Maple [A]** time = 0.571, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx + c))^2}{(fx + e)(a + ia \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d\*x+c)^2/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] int(csch(d\*x+c)^2/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-4if \int \frac{1}{-2i adf^2x^2 - 4i adefx - 2i ade^2 + 2(adf^2x^2e^c + 2 adefxe^c + ade^2e^c)e^{(dx)}} dx - \frac{1}{2i adfx + 2i ade + 2(adfxe^{(3c)} + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^2/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-4I*f*\operatorname{integrate}(1/(-2I*a*d*f^2*x^2 - 4I*a*d*e*f*x - 2I*a*d*e^2 + 2*(a*d*f^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c))*e^{(d*x)}, x) - 4*(I*e^{(2*d*x + 2*c)} + e^{(d*x + c)} - 2*I)/(2*I*a*d*f*x + 2*I*a*d*e + 2*(a*d*f*x*e^{(3*c)} + a*d*e*e^{(3*c)}))*e^{(3*d*x)} + (-2*I*a*d*f*x*e^{(2*c)} - 2*I*a*d*e*e^{(2*c)})*e^{(2*d*x)} - 2*(a*d*f*x*e^c + a*d*e*e^c)*e^{(d*x)} - 4*\operatorname{integrate}(-1/4*(I*d*f*x + I*d*e + f)/(a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2 - (a*d*f^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c))*e^{(d*x)}, x) - 4*\operatorname{integrate}(1/4*(I*d*f*x + I*d*e - f)/(a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2 + (a*d*f^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c))*e^{(d*x)}, x)$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\frac{(i adfx + i ade + (adfx + ade)e^{(3dx+3c)} + (-i adfx - i ade)e^{(2dx+2c)} - (adfx + ade)e^{(dx+c)}) \operatorname{integral}\left(\frac{1}{i adf^2x^2+2i adefx+i ade^2+ \dots}\right)}{i adfx + i ade + (adfx + ade)e^{(3dx+3c)} + (-i adfx - i ade)e^{(2dx+2c)} - (adfx + ade)e^{(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((I*a*d*f*x + I*a*d*e + (a*d*f*x + a*d*e)*e^(3*d*x + 3*c) + (-I*a*d*f*x - I
*a*d*e)*e^(2*d*x + 2*c) - (a*d*f*x + a*d*e)*e^(d*x + c))*integral((( -2*I*d*
f*x - 2*I*d*e - 2*I*f)*e^(2*d*x + 2*c) - 2*(d*f*x + d*e + f)*e^(d*x + c) +
4*I*f)/(I*a*d*f^2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*
f*x + a*d*e^2)*e^(3*d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^
2)*e^(2*d*x + 2*c) - (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c)), x)
- 2*I*e^(2*d*x + 2*c) - 2*e^(d*x + c) + 4*I)/(I*a*d*f*x + I*a*d*e + (a*d*f
*x + a*d*e)*e^(3*d*x + 3*c) + (-I*a*d*f*x - I*a*d*e)*e^(2*d*x + 2*c) - (a*d
*f*x + a*d*e)*e^(d*x + c))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.216 \quad \int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

**Optimal.** Leaf size=33

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Csch[c + d\*x]^2/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0744917, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Csch[c + d\*x]^2/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Csch[c + d\*x]^2/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx = \int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.001, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[Csch[c + d\*x]^2/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] \$Aborted

---

**Maple [A]** time = 1.329, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx + c))^2}{(fx + e)^2 (a + ia \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)`

[Out] `int(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-4if \int \frac{1}{-iadf^3x^3 - 3iade^2fx - iade^3 + (adf^3x^3e^c + 3ade^2fx^2e^c + 3ade^2fxe^c + ade^3e^c)e^{(dx)}} dx - \frac{1}{2iadf^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `-4*I*f*integrate(1/(-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x - I*a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c + a*d*e^3*e^c)*e^(d*x)), x) - 4*(I*e^(2*d*x + 2*c) + e^(d*x + c) - 2*I)/(2*I*a*d*f^2*x^2 + 4*I*a*d*e*f*x + 2*I*a*d*e^2 + 2*(a*d*f^2*x^2*e^(3*c) + 2*a*d*e*f*x*e^(3*c) + a*d*e^2*e^(3*c))*e^(3*d*x) + (-2*I*a*d*f^2*x^2*e^(2*c) - 4*I*a*d*e*f*x*e^(2*c) - 2*I*a*d*e^2*e^(2*c))*e^(2*d*x) - 2*(a*d*f^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c)*e^(d*x)) - 4*integrate(-1/4*(I*d*f*x + I*d*e + 2*f)/(a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3 - (a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c + a*d*e^3*e^c)*e^(d*x)), x) - 4*integrate(1/4*(I*d*f*x + I*d*e - 2*f)/(a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3 + (a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c + a*d*e^3*e^c)*e^(d*x)), x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$(i adf^2x^2 + 2i adefx + i ade^2 + (adf^2x^2 + 2 adefx + ade^2)e^{(3dx+3c)} + (-i adf^2x^2 - 2i adefx - i ade^2)e^{(2dx+2c)} - (adf^2x^2$$

$$i adf^2x^2 + 2i adefx + i ade^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((I*a*d*f^2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(3*d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2)*e^(2*d*x + 2*c) - (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))*integral((-2*I*d*f*x - 2*I*d*e - 4*I*f)*e^(2*d*x + 2*c) - 2*(d*f*x + d*e + 2*f)*e^(d*x + c) + 8*I*f)/(I*a*d*f^3*x^3 + 3*I*a*d*e*f^2*x^2 + 3*I*a*d*e^2*f*x + I*a*d*e^3 + (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*e^(3*d*x + 3*c) + (-I*a*d*f^3*x^3 - 3*I*a*d*e*f^2*x^2 - 3*I*a*d*e^2*f*x - I*a*d*e^3)*e^(2*d*x + 2*c) - (a*d*f^3*x^3 + 3*a*d*e*f^2*x^2 + 3*a*d*e^2*f*x + a*d*e^3)*e^(d*x + c)), x) - 2*I*e^(2*d*x + 2*c) - 2*e^(d*x + c) + 4*I)/(I*a*d*f^2*x^2 + 2*I*a*d*e*f*x + I*a*d*e^2 + (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(3*d*x + 3*c) + (-I*a*d*f^2*x^2 - 2*I*a*d*e*f*x - I*a*d*e^2)*e^(2*d*x + 2*c) - (a*d*f^2*x^2 + 2*a*d*e*f*x + a*d*e^2)*e^(d*x + c))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```



$$3.217 \quad \int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=546

$$\frac{12if^2(e+fx)\operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^3} - \frac{3if^2(e+fx)\operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{ad^3} - \frac{9f^2(e+fx)\operatorname{PolyLog}\left(3, -e^{c+dx}\right)}{ad^3} + \frac{9f^2(e+fx)\operatorname{PolyLog}\left(3, e^{c+dx}\right)}{ad^3}$$

```
[Out] ((2*I)*(e + f*x)^3)/(a*d) - (6*f^2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^3)
+ (3*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) + (I*(e + f*x)^3*Coth[c + d*x]
)/(a*d) - (3*f*(e + f*x)^2*Csch[c + d*x])/(2*a*d^2) - ((e + f*x)^3*Coth[c +
d*x]*Csch[c + d*x])/(2*a*d) - ((6*I)*f*(e + f*x)^2*Log[1 + I*E^(c + d*x)])
/(a*d^2) - ((3*I)*f*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a*d^2) - (3*f^3*
PolyLog[2, -E^(c + d*x)])/(a*d^4) + (9*f*(e + f*x)^2*PolyLog[2, -E^(c + d*x
)])/((2*a*d^2) - ((12*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3)
+ (3*f^3*PolyLog[2, E^(c + d*x)])/(a*d^4) - (9*f*(e + f*x)^2*PolyLog[2, E^
(c + d*x)])/(2*a*d^2) - ((3*I)*f^2*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(
a*d^3) - (9*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) + ((12*I)*f^3*P
olyLog[3, (-I)*E^(c + d*x)])/(a*d^4) + (9*f^2*(e + f*x)*PolyLog[3, E^(c + d
*x)])/(a*d^3) + (((3*I)/2)*f^3*PolyLog[3, E^(2*(c + d*x))])/(a*d^4) + (9*f^
3*PolyLog[4, -E^(c + d*x)])/(a*d^4) - (9*f^3*PolyLog[4, E^(c + d*x)])/(a*d^
4) + (I*(e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

**Rubi [A]** time = 1.21183, antiderivative size = 546, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 13, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$ , Rules used = {5575, 4186, 4182, 2279, 2391, 2531, 6609, 2282, 6589, 4184, 3716, 2190, 3318}

$$\frac{12if^2(e+fx)\operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^3} - \frac{3if^2(e+fx)\operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{ad^3} - \frac{9f^2(e+fx)\operatorname{PolyLog}\left(3, -e^{c+dx}\right)}{ad^3} + \frac{9f^2(e+fx)\operatorname{PolyLog}\left(3, e^{c+dx}\right)}{ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Csch[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((2*I)*(e + f*x)^3)/(a*d) - (6*f^2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^3)
+ (3*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) + (I*(e + f*x)^3*Coth[c + d*x]
)/(a*d) - (3*f*(e + f*x)^2*Csch[c + d*x])/(2*a*d^2) - ((e + f*x)^3*Coth[c +
d*x]*Csch[c + d*x])/(2*a*d) - ((6*I)*f*(e + f*x)^2*Log[1 + I*E^(c + d*x)])
/(a*d^2) - ((3*I)*f*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a*d^2) - (3*f^3*
PolyLog[2, -E^(c + d*x)])/(a*d^4) + (9*f*(e + f*x)^2*PolyLog[2, -E^(c + d*x
)])/((2*a*d^2) - ((12*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3)
+ (3*f^3*PolyLog[2, E^(c + d*x)])/(a*d^4) - (9*f*(e + f*x)^2*PolyLog[2, E^
(c + d*x)])/(2*a*d^2) - ((3*I)*f^2*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(
a*d^3) - (9*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) + ((12*I)*f^3*P
olyLog[3, (-I)*E^(c + d*x)])/(a*d^4) + (9*f^2*(e + f*x)*PolyLog[3, E^(c + d
*x)])/(a*d^3) + (((3*I)/2)*f^3*PolyLog[3, E^(2*(c + d*x))])/(a*d^4) + (9*f^
3*PolyLog[4, -E^(c + d*x)])/(a*d^4) - (9*f^3*PolyLog[4, E^(c + d*x)])/(a*d^
4) + (I*(e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)
```

```

]])/(2*a*d^2) - ((12*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)]/(a*d^3)
+ (3*f^3*PolyLog[2, E^(c + d*x)]/(a*d^4) - (9*f*(e + f*x)^2*PolyLog[2, E^
(c + d*x)]/(2*a*d^2) - ((3*I)*f^2*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(
a*d^3) - (9*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)]/(a*d^3) + ((12*I)*f^3*P
olyLog[3, (-I)*E^(c + d*x)]/(a*d^4) + (9*f^2*(e + f*x)*PolyLog[3, E^(c + d
*x)]/(a*d^3) + (((3*I)/2)*f^3*PolyLog[3, E^(2*(c + d*x))])/(a*d^4) + (9*f^
3*PolyLog[4, -E^(c + d*x)]/(a*d^4) - (9*f^3*PolyLog[4, E^(c + d*x)]/(a*d^
4) + (I*(e + f*x)^3*Tanh[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)

```

### Rule 5575

```

Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Csch[c
+ d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a +
b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]

```

### Rule 4186

```

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:= -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

```

### Rule 4182

```

Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

```

### Rule 2279

```

Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

```

### Rule 2391

```

Int[Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*(a_.) + (b_.
)*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x)))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 3318

```
Int[((c_) + (d_)*(x_))^(m_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(n_)
, x_Symbol] := Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
(f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx)}{a+ia \sinh(c+dx)} dx &= -\left( i \int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)}{a+ia \sinh(c+dx)} dx \right) + \frac{\int (e+fx)^3 \operatorname{csch}^3(c+dx) dx}{a} \\
&= -\frac{3f(e+fx)^2 \operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+fx)^3 \operatorname{coth}(c+dx) \operatorname{csch}(c+dx)}{2ad} - \frac{i \int (e+fx)^3 \operatorname{csch}^2(c+dx) dx}{a} \\
&= -\frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} + \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} + \frac{i(e+fx)^3 \operatorname{coth}(c+dx)}{ad} - \frac{3 \int (e+fx)^3 \operatorname{csch}(c+dx) dx}{a} \\
&= \frac{i(e+fx)^3}{ad} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} + \frac{3(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} + \frac{i(e+fx)^3 \operatorname{coth}(c+dx)}{ad} \\
&= \frac{i(e+fx)^3}{ad} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} + \frac{3(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} + \frac{i(e+fx)^3 \operatorname{coth}(c+dx)}{ad} \\
&= \frac{2i(e+fx)^3}{ad} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} + \frac{3(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} + \frac{i(e+fx)^3 \operatorname{coth}(c+dx)}{ad} \\
&= \frac{2i(e+fx)^3}{ad} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} + \frac{3(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} + \frac{i(e+fx)^3 \operatorname{coth}(c+dx)}{ad} \\
&= \frac{2i(e+fx)^3}{ad} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} + \frac{3(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} + \frac{i(e+fx)^3 \operatorname{coth}(c+dx)}{ad} \\
&= \frac{2i(e+fx)^3}{ad} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} + \frac{3(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} + \frac{i(e+fx)^3 \operatorname{coth}(c+dx)}{ad} \\
&= \frac{2i(e+fx)^3}{ad} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} + \frac{3(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} + \frac{i(e+fx)^3 \operatorname{coth}(c+dx)}{ad} \\
&= \frac{2i(e+fx)^3}{ad} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} + \frac{3(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} + \frac{i(e+fx)^3 \operatorname{coth}(c+dx)}{ad}
\end{aligned}$$

**Mathematica [B]** time = 68.6696, size = 2479, normalized size = 4.54

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^3\*Csch[c + d\*x]^3)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (-3\*e^3\*Log[Tanh[(c + d\*x)/2]])/(2\*a\*d) + (3\*e\*f^2\*Log[Tanh[(c + d\*x)/2]])/(a\*d^3) - (9\*e^2\*f\*(-(c\*Log[Tanh[(c + d\*x)/2]]) - I\*((I\*c + I\*d\*x)\*(Log[1 - E^(I\*(I\*c + I\*d\*x))]) - Log[1 + E^(I\*(I\*c + I\*d\*x))]) + I\*(PolyLog[2, -E^(I\*(I\*c + I\*d\*x))]) - PolyLog[2, E^(I\*(I\*c + I\*d\*x))])))/(2\*a\*d^2) + (3\*f^3\*(

$$\begin{aligned}
& -(c \operatorname{Log}[\operatorname{Tanh}[(c + d*x)/2]]) - I*((I*c + I*d*x)*(\operatorname{Log}[1 - E^{(I*(I*c + I*d*x))}] \\
& - \operatorname{Log}[1 + E^{(I*(I*c + I*d*x))}]) + I*(\operatorname{PolyLog}[2, -E^{(I*(I*c + I*d*x))}] - \operatorname{Poly} \\
& \operatorname{Log}[2, E^{(I*(I*c + I*d*x))}]))/(a*d^4) - (2*(d^3*(e + f*x)^3 + 3*d^2*(1 \\
& + I*E^c)*f*(e + f*x)^2*\operatorname{Log}[1 - I*E^{(-c - d*x)}] + (6*I)*(I - E^c)*f^2*(d*(e \\
& + f*x)*\operatorname{PolyLog}[2, I*E^{(-c - d*x)}] + f*\operatorname{PolyLog}[3, I*E^{(-c - d*x)}]))/(a*d^4 \\
& *(-I + E^c)) + ((I/2)*E^c*f^3*\operatorname{Csch}[c]*((2*d^3*x^3)/E^{(2*c)} - 3*d^2*(1 - E^{(-2*c)}) \\
& *x^2*\operatorname{Log}[1 - E^{(-c - d*x)}] - 3*d^2*(1 - E^{(-2*c)})*x^2*\operatorname{Log}[1 + E^{(-c - d*x)}] \\
& + 6*(1 - E^{(-2*c)})*(d*x*\operatorname{PolyLog}[2, -E^{(-c - d*x)}] + \operatorname{PolyLog}[3, -E^{(-c - d*x)}]) \\
& + 6*(1 - E^{(-2*c)})*(d*x*\operatorname{PolyLog}[2, E^{(-c - d*x)}] + \operatorname{PolyLog}[3, E^{(-c - d*x)}])) \\
& )/(a*d^4) + (9*e*f^2*(d^2*x^2*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x] + \operatorname{Sinh}[c + d*x]] + d*x*\operatorname{PolyLog}[2, \\
& -\operatorname{Cosh}[c + d*x] - \operatorname{Sinh}[c + d*x]] - d*x*\operatorname{PolyLog}[2, \operatorname{Cosh}[c + d*x] + \operatorname{Sinh}[c + d*x]] \\
& - \operatorname{PolyLog}[3, -\operatorname{Cosh}[c + d*x] - \operatorname{Sinh}[c + d*x]] + \operatorname{PolyLog}[3, \operatorname{Cosh}[c + d*x] + \operatorname{Sinh}[c + d*x]])) \\
& )/(a*d^3) - (3*f^3*(-2*d^3*x^3*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x] + \operatorname{Sinh}[c + d*x]] - 3*d^2*x^2*\operatorname{PolyLog}[2, \\
& -\operatorname{Cosh}[c + d*x] - \operatorname{Sinh}[c + d*x]] + 3*d^2*x^2*\operatorname{PolyLog}[2, \operatorname{Cosh}[c + d*x] + \operatorname{Sinh}[c + d*x]] \\
& + 6*d*x*\operatorname{PolyLog}[3, -\operatorname{Cosh}[c + d*x] - \operatorname{Sinh}[c + d*x]] - 6*d*x*\operatorname{PolyLog}[3, \operatorname{Cosh}[c + d*x] \\
& + \operatorname{Sinh}[c + d*x]] - 6*\operatorname{PolyLog}[4, -\operatorname{Cosh}[c + d*x] - \operatorname{Sinh}[c + d*x]] + 6*\operatorname{Poly} \\
& \operatorname{Log}[4, \operatorname{Cosh}[c + d*x] + \operatorname{Sinh}[c + d*x]]))/(2*a*d^4) + ((3*I)*e^2*f*\operatorname{Csch}[c] \\
& *(-d*x*\operatorname{Cosh}[c]) + \operatorname{Log}[\operatorname{Cosh}[d*x]*\operatorname{Sinh}[c] + \operatorname{Cosh}[c]*\operatorname{Sinh}[d*x]]*\operatorname{Sinh}[c]))/(a \\
& d^2*(-\operatorname{Cosh}[c]^2 + \operatorname{Sinh}[c]^2)) + (\operatorname{Csch}[c]*\operatorname{Csch}[c + d*x]^2*(3*e^2*f*\operatorname{Cosh}[(d*x) \\
& ]/2 + 6*e*f^2*x*\operatorname{Cosh}[(d*x)/2] + 3*f^3*x^2*\operatorname{Cosh}[(d*x)/2] + 3*e^2*f*\operatorname{Cosh}[(3* \\
& d*x)/2] + 6*e*f^2*x*\operatorname{Cosh}[(3*d*x)/2] + 3*f^3*x^2*\operatorname{Cosh}[(3*d*x)/2] + (5*I)*d*e \\
& ^3*\operatorname{Cosh}[c - (d*x)/2] + (15*I)*d*e^2*f*x*\operatorname{Cosh}[c - (d*x)/2] + (15*I)*d*e*f^2*x \\
& ^2*\operatorname{Cosh}[c - (d*x)/2] + (5*I)*d*f^3*x^3*\operatorname{Cosh}[c - (d*x)/2] - I*d*e^3*\operatorname{Cosh}[c \\
& + (d*x)/2] - (3*I)*d*e^2*f*x*\operatorname{Cosh}[c + (d*x)/2] - (3*I)*d*e*f^2*x^2*\operatorname{Cosh}[c + \\
& (d*x)/2] - I*d*f^3*x^3*\operatorname{Cosh}[c + (d*x)/2] - 3*e^2*f*\operatorname{Cosh}[2*c + (d*x)/2] - 6 \\
& *e*f^2*x*\operatorname{Cosh}[2*c + (d*x)/2] - 3*f^3*x^2*\operatorname{Cosh}[2*c + (d*x)/2] + I*d*e^3*\operatorname{Cosh} \\
& [c + (3*d*x)/2] + (3*I)*d*e^2*f*x*\operatorname{Cosh}[c + (3*d*x)/2] + (3*I)*d*e*f^2*x^2*\operatorname{C} \\
& \operatorname{osh}[c + (3*d*x)/2] + I*d*f^3*x^3*\operatorname{Cosh}[c + (3*d*x)/2] - 3*e^2*f*\operatorname{Cosh}[2*c + ( \\
& 3*d*x)/2] - 6*e*f^2*x*\operatorname{Cosh}[2*c + (3*d*x)/2] - 3*f^3*x^2*\operatorname{Cosh}[2*c + (3*d*x)/ \\
& 2] - (3*I)*d*e^3*\operatorname{Cosh}[3*c + (3*d*x)/2] - (9*I)*d*e^2*f*x*\operatorname{Cosh}[3*c + (3*d*x) \\
& /2] - (9*I)*d*e*f^2*x^2*\operatorname{Cosh}[3*c + (3*d*x)/2] - (3*I)*d*f^3*x^3*\operatorname{Cosh}[3*c + \\
& (3*d*x)/2] - (4*I)*d*e^3*\operatorname{Cosh}[c + (5*d*x)/2] - (12*I)*d*e^2*f*x*\operatorname{Cosh}[c + (5 \\
& *d*x)/2] - (12*I)*d*e*f^2*x^2*\operatorname{Cosh}[c + (5*d*x)/2] - (4*I)*d*f^3*x^3*\operatorname{Cosh}[c \\
& + (5*d*x)/2] + (2*I)*d*e^3*\operatorname{Cosh}[3*c + (5*d*x)/2] + (6*I)*d*e^2*f*x*\operatorname{Cosh}[3*c \\
& + (5*d*x)/2] + (6*I)*d*e*f^2*x^2*\operatorname{Cosh}[3*c + (5*d*x)/2] + (2*I)*d*f^3*x^3*\operatorname{C} \\
& \operatorname{osh}[3*c + (5*d*x)/2] - d*e^3*\operatorname{Sinh}[(d*x)/2] - 3*d*e^2*f*x*\operatorname{Sinh}[(d*x)/2] - 3* \\
& d*e*f^2*x^2*\operatorname{Sinh}[(d*x)/2] - d*f^3*x^3*\operatorname{Sinh}[(d*x)/2] - d*e^3*\operatorname{Sinh}[(3*d*x)/2] \\
& - 3*d*e^2*f*x*\operatorname{Sinh}[(3*d*x)/2] - 3*d*e*f^2*x^2*\operatorname{Sinh}[(3*d*x)/2] - d*f^3*x^3* \\
& \operatorname{Sinh}[(3*d*x)/2] + (3*I)*e^2*f*\operatorname{Sinh}[c - (d*x)/2] + (6*I)*e*f^2*x*\operatorname{Sinh}[c - (d \\
& *x)/2] + (3*I)*f^3*x^2*\operatorname{Sinh}[c - (d*x)/2] + (3*I)*e^2*f*\operatorname{Sinh}[c + (d*x)/2] + \\
& (6*I)*e*f^2*x*\operatorname{Sinh}[c + (d*x)/2] + (3*I)*f^3*x^2*\operatorname{Sinh}[c + (d*x)/2] - 3*d*e^3 \\
& *\operatorname{Sinh}[2*c + (d*x)/2] - 9*d*e^2*f*x*\operatorname{Sinh}[2*c + (d*x)/2] - 9*d*e*f^2*x^2*\operatorname{Sinh} \\
& [2*c + (d*x)/2] - 3*d*f^3*x^3*\operatorname{Sinh}[2*c + (d*x)/2] + (3*I)*e^2*f*\operatorname{Sinh}[c + (3 \\
& *d*x)/2] + (6*I)*e*f^2*x*\operatorname{Sinh}[c + (3*d*x)/2] + (3*I)*f^3*x^2*\operatorname{Sinh}[c + (3*d*
\end{aligned}$$

$$\begin{aligned} & x)/2] - d*e^3*\text{Sinh}[2*c + (3*d*x)/2] - 3*d*e^2*f*x*\text{Sinh}[2*c + (3*d*x)/2] - 3 \\ & *d*e*f^2*x^2*\text{Sinh}[2*c + (3*d*x)/2] - d*f^3*x^3*\text{Sinh}[2*c + (3*d*x)/2] - (3*I \\ & )*e^2*f*\text{Sinh}[3*c + (3*d*x)/2] - (6*I)*e*f^2*x*\text{Sinh}[3*c + (3*d*x)/2] - (3*I) \\ & *f^3*x^2*\text{Sinh}[3*c + (3*d*x)/2] + 2*d*e^3*\text{Sinh}[2*c + (5*d*x)/2] + 6*d*e^2*f* \\ & x*\text{Sinh}[2*c + (5*d*x)/2] + 6*d*e*f^2*x^2*\text{Sinh}[2*c + (5*d*x)/2] + 2*d*f^3*x^3 \\ & *\text{Sinh}[2*c + (5*d*x)/2]))/(8*a*d^2*(\text{Cosh}[c/2] + I*\text{Sinh}[c/2])*(\text{Cosh}[c/2 + (d* \\ & x)/2] + I*\text{Sinh}[c/2 + (d*x)/2])) - ((3*I)*e*f^2*\text{Csch}[c]*\text{Sech}[c]*(-((d^2*x^2) \\ & /E^{\text{ArcTanh}[\text{Tanh}[c]}) + (I*(-(d*x*(-\text{Pi} + (2*I)*\text{ArcTanh}[\text{Tanh}[c])))) - \text{Pi}*\text{Log}[1 \\ & + E^{(2*d*x)}] - 2*(I*d*x + I*\text{ArcTanh}[\text{Tanh}[c]])*\text{Log}[1 - E^{(2*I)*(I*d*x + I* \\ & \text{ArcTanh}[\text{Tanh}[c]])}] + \text{Pi}*\text{Log}[\text{Cosh}[d*x]] + (2*I)*\text{ArcTanh}[\text{Tanh}[c]]*\text{Log}[I*\text{Sinh} \\ & [d*x + \text{ArcTanh}[\text{Tanh}[c]]]]) + I*\text{PolyLog}[2, E^{(2*I)*(I*d*x + I*\text{ArcTanh}[\text{Tanh}[c] \\ & ])}])*\text{Tanh}[c])/ \text{Sqrt}[1 - \text{Tanh}[c]^2]))/(a*d^3*\text{Sqrt}[\text{Sech}[c]^2*(\text{Cosh}[c]^2 - \text{Si} \\ & \text{nh}[c]^2)]) \end{aligned}$$

**Maple [B]** time = 0.256, size = 2058, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)^3*\text{csch}(d*x+c)^3/(a+I*a*\text{sinh}(d*x+c)), x)$

[Out] 
$$\begin{aligned} & -6*I/a/d^2*\ln(1-\exp(d*x+c))*e*f^2*x-6*I/a/d^2*\ln(\exp(d*x+c)+1)*e*f^2*x+24*I \\ & /a/d^2*c*e*f^2*x-6*I/a/d^3*\ln(1-\exp(d*x+c))*c*e*f^2+6*I/a/d^3*e*f^2*c*\ln(\exp \\ & (d*x+c)-1)-24*I/a/d^3*e*f^2*c*\ln(\exp(d*x+c))+6*I/d^4/a*f^3*c^2*\ln(1+I*\exp( \\ & d*x+c))-6*I/d^2/a*\ln(\exp(d*x+c)-I)*e^2*f-12*I/d^3/a*f^3*\text{polylog}(2, -I*\exp(d* \\ & x+c))*x-6*I/d^2/a*f^3*\ln(1+I*\exp(d*x+c))*x^2-12*I/d^3/a*e*f^2*\text{polylog}(2, -I* \\ & \exp(d*x+c))-6*I/d^4/a*f^3*c^2*\ln(\exp(d*x+c)-I)-12*I/d^2/a*e*f^2*\ln(1+I*\exp( \\ & d*x+c))*x-12*I/d^3/a*e*f^2*\ln(1+I*\exp(d*x+c))*c+12*I/d^3/a*e*f^2*c*\ln(\exp(d \\ & *x+c)-I)-3*f^3*\text{polylog}(2, -\exp(d*x+c))/a/d^4+3*f^3*\text{polylog}(2, \exp(d*x+c))/a/d \\ & ^4+9*f^3*\text{polylog}(4, -\exp(d*x+c))/a/d^4-9*f^3*\text{polylog}(4, \exp(d*x+c))/a/d^4+12* \\ & I*f^3*\text{polylog}(3, -I*\exp(d*x+c))/a/d^4-(-3*I*f^3*x^2*\exp(3*d*x+3*c)-3*I*d*e^3 \\ & *\exp(3*d*x+3*c)-3*I*e^2*f*\exp(3*d*x+3*c)+4*d*e^3+3*I*d*e*f^2*x^2*\exp(d*x+c) \\ & -5*d*f^3*x^3*\exp(2*d*x+2*c)-6*e*f^2*x*\exp(2*d*x+2*c)+3*d*f^3*x^3*\exp(4*d*x+ \\ & 4*c)+6*e*f^2*x*\exp(4*d*x+4*c)+I*d*f^3*x^3*\exp(d*x+c)+9*d*e*f^2*x^2*\exp(4*d* \\ & x+4*c)+9*d*e^2*f*x*\exp(4*d*x+4*c)-6*I*e*f^2*x*\exp(3*d*x+3*c)-3*I*d*f^3*x^3* \\ & \exp(3*d*x+3*c)+4*d*f^3*x^3+3*I*d*e^2*f*x*\exp(d*x+c)-15*d*e*f^2*x^2*\exp(2*d* \\ & x+2*c)-15*d*e^2*f*x*\exp(2*d*x+2*c)+12*d*e*f^2*x^2+12*d*e^2*f*x+3*I*\exp(d*x+ \\ & c)*e^2*f+3*I*f^3*x^2*\exp(d*x+c)+6*I*e*f^2*x*\exp(d*x+c)+I*d*e^3*\exp(d*x+c)-9 \\ & *I*d*e*f^2*x^2*\exp(3*d*x+3*c)-9*I*d*e^2*f*x*\exp(3*d*x+3*c)-3*f^3*x^2*\exp(2* \\ & d*x+2*c)-3*e^2*f*\exp(2*d*x+2*c)+3*f^3*x^2*\exp(4*d*x+4*c)+3*d*e^3*\exp(4*d*x+ \\ & 4*c)+3*e^2*f*\exp(4*d*x+4*c)-5*d*e^3*\exp(2*d*x+2*c))/(\exp(2*d*x+2*c)-1)^2/d^ \end{aligned}$$

$$\begin{aligned}
& 2/(\exp(d*x+c)-1)/a-9/2/d^3/a*e*f^2*c^2*\ln(\exp(d*x+c)-1)+9/2/d^2/a*e^2*f*c*\ln(\exp(d*x+c)-1)-9/2/d^2/a*\ln(1-\exp(d*x+c))*c*e^2*f-9/2/d/a*\ln(1-\exp(d*x+c))*e^2*f*x+9/2/d/a*\ln(\exp(d*x+c)+1)*e^2*f*x+9/2/d^3/a*e*f^2*c^2*\ln(1-\exp(d*x+c))-9/2/d/a*e*f^2*\ln(1-\exp(d*x+c))*x^2-9/d^2/a*e*f^2*polylog(2,\exp(d*x+c))*x+9/2/d/a*e*f^2*\ln(\exp(d*x+c)+1)*x^2+9/d^2/a*e*f^2*polylog(2,-\exp(d*x+c))*x-3*I/a/d^4*f^3*c^2*\ln(\exp(d*x+c)-1)-12*I/a/d^3*f^3*c^2*x+12*I/a/d*e*f^2*x^2+3*I/a/d^4*f^3*c^2*\ln(1-\exp(d*x+c))-3*I/a/d^2*f^3*\ln(1-\exp(d*x+c))*x^2-3*I/a/d^2*f^3*\ln(\exp(d*x+c)+1)*x^2-6*I/a/d^3*f^3*polylog(2,-\exp(d*x+c))*x-6*I/a/d^3*f^3*polylog(2,\exp(d*x+c))*x+12*I/a/d^4*f^3*c^2*\ln(\exp(d*x+c))-3*I/a/d^2*e^2*f*\ln(\exp(d*x+c)-1)-3*I/a/d^2*e^2*f*\ln(\exp(d*x+c)+1)+12*I/a/d^2*e^2*f*\ln(\exp(d*x+c))+12*I/a/d^3*c^2*e*f^2-6*I/a/d^3*e*f^2*polylog(2,\exp(d*x+c))-6*I/a/d^3*e*f^2*polylog(2,-\exp(d*x+c))+3/a/d^3*e*f^2*\ln(\exp(d*x+c)-1)-3/a/d^3*e*f^2*\ln(\exp(d*x+c)+1)+3/a/d^3*f^3*\ln(1-\exp(d*x+c))*x-3/2/d/a*e^3*\ln(\exp(d*x+c)-1)+3/2/d/a*e^3*\ln(\exp(d*x+c)+1)-3/a/d^3*f^3*\ln(\exp(d*x+c)+1)*x+3/a/d^4*f^3*c*\ln(1-\exp(d*x+c))-3/a/d^4*f^3*c*\ln(\exp(d*x+c)-1)+4*I/a/d*f^3*x^3-8*I/a/d^4*f^3*c^3+6*I/a/d^4*f^3*polylog(3,\exp(d*x+c))+6*I/a/d^4*f^3*polylog(3,-\exp(d*x+c))-9/2/d^2/a*f^3*polylog(2,\exp(d*x+c))*x^2+9/d^3/a*f^3*polylog(3,\exp(d*x+c))*x-9/2/d^2/a*e^2*f*polylog(2,\exp(d*x+c))+9/2/d^2/a*e^2*f*polylog(2,-\exp(d*x+c))+9/d^3/a*e*f^2*polylog(3,\exp(d*x+c))-9/d^3/a*e*f^2*polylog(3,-\exp(d*x+c))+3/2/d^4/a*f^3*c^3*\ln(\exp(d*x+c)-1)+3/2/d/a*f^3*\ln(\exp(d*x+c)+1)*x^3+9/2/d^2/a*f^3*polylog(2,-\exp(d*x+c))*x^2-9/d^3/a*f^3*polylog(3,-\exp(d*x+c))*x-3/2/d/a*f^3*\ln(1-\exp(d*x+c))*x^3-3/2/d^4/a*f^3*\ln(1-\exp(d*x+c))*c^3
\end{aligned}$$


---

**Maxima [B]** time = 2.74902, size = 1777, normalized size = 3.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*c\*sch(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned}
& -1/2*e^3*(16*(-I*e^{(-d*x - c)} - 5*e^{(-2*d*x - 2*c)} + 3*I*e^{(-3*d*x - 3*c)} + 3*e^{(-4*d*x - 4*c)} + 4)/((8*a*e^{(-d*x - c)} - 16*I*a*e^{(-2*d*x - 2*c)} - 16*a*e^{(-3*d*x - 3*c)} + 8*I*a*e^{(-4*d*x - 4*c)} + 8*a*e^{(-5*d*x - 5*c)} + 8*I*a*d) - 3*\log(e^{(-d*x - c)} + 1)/(a*d) + 3*\log(e^{(-d*x - c)} - 1)/(a*d)) + 6*I*e^2*f*x/(a*d) - 6*I*e^2*f*\log(I*e^{(d*x + c)} + 1)/(a*d^2) - (4*d*f^3*x^3 + 12*d*e*f^2*x^2 + 12*d*e^2*f*x + 3*(d*f^3*x^3*e^{(4*c)} + e^2*f*e^{(4*c)} + (3*d*e*f^2 + f^3)*x^2*e^{(4*c)} + (3*d*e^2*f + 2*e*f^2)*x*e^{(4*c)})*e^{(4*d*x)} + (-3*I*d*f^3*x^3*e^{(3*c)} - 3*I*e^2*f*e^{(3*c)} + (-9*I*d*e*f^2 - 3*I*f^3)*x^2*e^{(3*c)} + (-9*I*d*e^2*f - 6*I*e*f^2)*x*e^{(3*c)})*e^{(3*d*x)} - (5*d*f^3*x^3*e^{(2*
\end{aligned}$$



$$\begin{aligned}
& c) + 3e^{2f}e^{(2c)} + 3(5d*ef^2 + f^3)*x^2e^{(2c)} + 3(5d*e^{2f} + 2e \\
& *f^2)*x*e^{(2c)}*e^{(2dx)} + (I*d*f^3*x^3*e^c + 3I*e^{2f}*e^c + (3I*d*ef^ \\
& 2 + 3I*f^3)*x^2*e^c + (3I*d*e^{2f} + 6I*e*ef^2)*x*e^c)*e^{(dx)})/(a*d^2*e^{( \\
& 5*d*x + 5*c)} - I*a*d^2*e^{(4*d*x + 4*c)} - 2*a*d^2*e^{(3*d*x + 3*c)} + 2*I*a*d^ \\
& 2*e^{(2*d*x + 2*c)} + a*d^2*e^{(d*x + c)} - I*a*d^2) - 12*I*(d*x*log(I*e^{(d*x + \\
& c)} + 1) + dilog(-I*e^{(d*x + c)}))*ef^2/(a*d^3) + 3/2*(d^3*x^3*log(e^{(d*x + \\
& c)} + 1) + 3*d^2*x^2*dilog(-e^{(d*x + c)}) - 6*d*x*polylog(3, -e^{(d*x + c)}) + \\
& 6*polylog(4, -e^{(d*x + c)}))*f^3/(a*d^4) - 3/2*(d^3*x^3*log(-e^{(d*x + c)} + \\
& 1) + 3*d^2*x^2*dilog(e^{(d*x + c)}) - 6*d*x*polylog(3, e^{(d*x + c)}) + 6*polyl \\
& og(4, e^{(d*x + c)}))*f^3/(a*d^4) - 6*I*(d^2*x^2*log(I*e^{(d*x + c)} + 1) + 2*d \\
& *x*dilog(-I*e^{(d*x + c)}) - 2*polylog(3, -I*e^{(d*x + c)}))*f^3/(a*d^4) - 3*(- \\
& I*d*e^{2f} + ef^2)*x/(a*d^2) - 3*(-I*d*e^{2f} - ef^2)*x/(a*d^2) + 3*(-I*d*e \\
& ^2f - ef^2)*log(e^{(d*x + c)} + 1)/(a*d^3) + 3*(-I*d*e^{2f} + ef^2)*log(e^{( \\
& d*x + c)} - 1)/(a*d^3) - 3/2*(d^2*x^2*log(-e^{(d*x + c)} + 1) + 2*d*x*dilog(e^{ \\
& (d*x + c)}) - 2*polylog(3, e^{(d*x + c)}))* (3*d*ef^2 + 2*I*f^3)/(a*d^4) + 3/2 \\
& *(d^2*x^2*log(e^{(d*x + c)} + 1) + 2*d*x*dilog(-e^{(d*x + c)}) - 2*polylog(3, - \\
& e^{(d*x + c)}))* (3*d*ef^2 - 2*I*f^3)/(a*d^4) + 1/2*(9*d^2*e^{2f} - 12*I*d*ef \\
& ^2 - 6*f^3)*(d*x*log(e^{(d*x + c)} + 1) + dilog(-e^{(d*x + c)}))/(a*d^4) - 1/2* \\
& (9*d^2*e^{2f} + 12*I*d*ef^2 - 6*f^3)*(d*x*log(-e^{(d*x + c)} + 1) + dilog(e^{( \\
& d*x + c)}))/(a*d^4) + 1/8*(3*d^4*f^3*x^4 + 4*(3*d*ef^2 + 2*I*f^3)*d^3*x^3 + \\
& (18*d^2*e^{2f} + 24*I*d*ef^2 - 12*f^3)*d^2*x^2)/(a*d^4) - 1/8*(3*d^4*f^3*x \\
& ^4 + 4*(3*d*ef^2 - 2*I*f^3)*d^3*x^3 + (18*d^2*e^{2f} - 24*I*d*ef^2 - 12*f^ \\
& 3)*d^2*x^2)/(a*d^4) + (2*I*d^3*f^3*x^3 + 6*I*d^3*ef^2*x^2)/(a*d^4)
\end{aligned}$$

**Fricas [C]** time = 3.72538, size = 10116, normalized size = 18.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csc(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(8*d^3*e^3 - 24*c*d^2*e^2*f + 24*c^2*d*ef^2 - 8*c^3*f^3 + (24*d*f^3*x + 2
4*d*ef^2 - (-24*I*d*f^3*x - 24*I*d*ef^2))*e^(5*d*x + 5*c) + 24*(d*f^3*x +
d*ef^2)*e^(4*d*x + 4*c) - (48*I*d*f^3*x + 48*I*d*ef^2)*e^(3*d*x + 3*c) -
48*(d*f^3*x + d*ef^2)*e^(2*d*x + 2*c) - (-24*I*d*f^3*x - 24*I*d*ef^2)*e^(
d*x + c))*dilog(-I*e^(d*x + c)) - (-9*I*d^2*f^3*x^2 - 9*I*d^2*e^2*f - 12*d*
ef^2 + 6*I*f^3 - 6*(3*I*d^2*ef^2 + 2*d*f^3)*x + (9*d^2*f^3*x^2 + 9*d^2*e^
2*f - 12*I*d*ef^2 - 6*f^3 + (18*d^2*ef^2 - 12*I*d*f^3)*x)*e^(5*d*x + 5*c)
+ (-9*I*d^2*f^3*x^2 - 9*I*d^2*e^2*f - 12*d*ef^2 + 6*I*f^3 - 6*(3*I*d^2*ef
^2 + 2*d*f^3)*x)*e^(4*d*x + 4*c) - (18*d^2*f^3*x^2 + 18*d^2*e^2*f - 24*I*d
```

$$\begin{aligned}
& *e^f^2 - 12f^3 + (36d^2 * e^f^2 - 24I * d * f^3) * x) * e^{(3d * x + 3c)} + (18I * d^2 * f^3 * x^2 + 18I * d^2 * e^2 * f + 24d * e * f^2 - 12I * f^3 - 12 * (-3I * d^2 * e^f^2 - 2 * d * f^3) * x) * e^{(2d * x + 2c)} + (9d^2 * f^3 * x^2 + 9d^2 * e^2 * f - 12I * d * e * f^2 - 6f^3 + (18d^2 * e^f^2 - 12I * d * f^3) * x) * e^{(d * x + c)}) * \operatorname{dilog}(-e^{(d * x + c)}) - (9I * d^2 * f^3 * x^2 + 9I * d^2 * e^2 * f - 12d * e * f^2 - 6I * f^3 - 6 * (-3I * d^2 * e^f^2 + 2 * d * f^3) * x - (9d^2 * f^3 * x^2 + 9d^2 * e^2 * f + 12I * d * e * f^2 - 6f^3 + (18d^2 * e^f^2 + 12I * d * f^3) * x) * e^{(5d * x + 5c)} + (9I * d^2 * f^3 * x^2 + 9I * d^2 * e^2 * f - 12d * e * f^2 - 6I * f^3 - 6 * (-3I * d^2 * e^f^2 + 2 * d * f^3) * x) * e^{(4d * x + 4c)} + (18d^2 * f^3 * x^2 + 18d^2 * e^2 * f + 24I * d * e * f^2 - 12f^3 + (36d^2 * e^f^2 + 24I * d * f^3) * x) * e^{(3d * x + 3c)} + (-18I * d^2 * f^3 * x^2 - 18I * d^2 * e^2 * f + 24d * e * f^2 + 12I * f^3 - 12 * (3I * d^2 * e^f^2 - 2 * d * f^3) * x) * e^{(2d * x + 2c)} - (9d^2 * f^3 * x^2 + 9d^2 * e^2 * f + 12I * d * e * f^2 - 6f^3 + (18d^2 * e^f^2 + 12I * d * f^3) * x) * e^{(d * x + c)}) * \operatorname{dilog}(e^{(d * x + c)}) - (8I * d^3 * f^3 * x^3 + 24I * d^3 * e^f^2 * x^2 + 24I * d^3 * e^2 * f * x + 24I * c * d^2 * e^2 * f - 24I * c^2 * d * e * f^2 + 8I * c^3 * f^3) * e^{(5d * x + 5c)} - 2 * (d^3 * f^3 * x^3 - 3d^3 * e^3 + 3 * (4c - 1) * d^2 * e^2 * f - 12c^2 * d * e * f^2 + 4c^3 * f^3 + 3 * (d^3 * e^f^2 - d^2 * f^3) * x^2 + 3 * (d^3 * e^2 * f - 2d^2 * e * f^2) * x) * e^{(4d * x + 4c)} - (-10I * d^3 * f^3 * x^3 + 6I * d^3 * e^3 + (-48I * c + 6I) * d^2 * e^2 * f + 48I * c^2 * d * e * f^2 - 16I * c^3 * f^3 + (-30I * d^3 * e^f^2 + 6I * d^2 * f^3) * x^2 + (-30I * d^3 * e^2 * f + 12I * d^2 * e * f^2) * x) * e^{(3d * x + 3c)} + 2 * (3d^3 * f^3 * x^3 - 5d^3 * e^3 + 3 * (8c - 1) * d^2 * e^2 * f - 24c^2 * d * e * f^2 + 8c^3 * f^3 + 3 * (3d^3 * e^f^2 - d^2 * f^3) * x^2 + 3 * (3d^3 * e^2 * f - 2d^2 * e * f^2) * x) * e^{(2d * x + 2c)} - (6I * d^3 * f^3 * x^3 - 2I * d^3 * e^3 + (24I * c - 6I) * d^2 * e^2 * f - 24I * c^2 * d * e * f^2 + 8I * c^3 * f^3 + (18I * d^3 * e^f^2 - 6I * d^2 * f^3) * x^2 + (18I * d^3 * e^2 * f - 12I * d^2 * e * f^2) * x) * e^{(d * x + c)} - (-3I * d^3 * f^3 * x^3 - 3I * d^3 * e^3 - 6d^2 * e^2 * f + 6I * d * e * f^2 - 3 * (3I * d^3 * e^f^2 + 2d^2 * f^3) * x^2 + (-9I * d^3 * e^2 * f - 12d^2 * e * f^2 + 6I * d * f^3) * x + (3d^3 * f^3 * x^3 + 3d^3 * e^3 - 6I * d^2 * e^2 * f - 6d * e * f^2 + (9d^3 * e^f^2 - 6I * d^2 * f^3) * x^2 + 3 * (3d^3 * e^2 * f - 4I * d^2 * e * f^2 - 2d * f^3) * x) * e^{(5d * x + 5c)} + (-3I * d^3 * f^3 * x^3 - 3I * d^3 * e^3 - 6d^2 * e^2 * f + 6I * d * e * f^2 - 3 * (3I * d^3 * e^f^2 + 2d^2 * f^3) * x^2 + (-9I * d^3 * e^2 * f - 12d^2 * e * f^2 + 6I * d * f^3) * x) * e^{(4d * x + 4c)} - (6d^3 * f^3 * x^3 + 6d^3 * e^3 - 12I * d^2 * e^2 * f - 12d * e * f^2 + (18d^3 * e^f^2 - 12I * d^2 * f^3) * x^2 + 6 * (3d^3 * e^2 * f - 4I * d^2 * e * f^2 - 2d * f^3) * x) * e^{(3d * x + 3c)} + (6I * d^3 * f^3 * x^3 + 6I * d^3 * e^3 + 12d^2 * e^2 * f - 12I * d * e * f^2 - 6 * (-3I * d^3 * e^f^2 - 2d^2 * f^3) * x^2 + (18I * d^3 * e^2 * f + 24d^2 * e * f^2 - 12I * d * f^3) * x) * e^{(2d * x + 2c)} + (3d^3 * f^3 * x^3 + 3d^3 * e^3 - 6I * d^2 * e^2 * f - 6d * e * f^2 + (9d^3 * e^f^2 - 6I * d^2 * f^3) * x^2 + 3 * (3d^3 * e^2 * f - 4I * d^2 * e * f^2 - 2d * f^3) * x) * e^{(d * x + c)}) * \log(e^{(d * x + c)} + 1) + (12d^2 * e^2 * f - 24c * d * e * f^2 + 12c^2 * f^3 - (-12I * d^2 * e^2 * f + 24I * c * d * e * f^2 - 12I * c^2 * f^3) * e^{(5d * x + 5c)} + 12 * (d^2 * e^2 * f - 2c * d * e * f^2 + c^2 * f^3) * e^{(4d * x + 4c)} - (24I * d^2 * e^2 * f - 48I * c * d * e * f^2 + 24I * c^2 * f^3) * e^{(3d * x + 3c)} - 24 * (d^2 * e^2 * f - 2c * d * e * f^2 + c^2 * f^3) * e^{(2d * x + 2c)} - (-12I * d^2 * e^2 * f + 24I * c * d * e * f^2 - 12I * c^2 * f^3) * e^{(d * x + c)}) * \log(e^{(d * x + c)} - 1) - (3I * d^3 * e^3 - 3 * (3I * c + 2) * d^2 * e^2 * f + (9I * c^2 + 12c - 6I) * d * e * f^2 + (-3I * c^3 - 6c^2 + 6I * c) * f^3 - (3d^3 * e^3 - (9c - 6I) * d^2 * e^2 * f + 3 * (3c^2 - 4I * c - 2) * d * e * f^2 - (3c^3 - 6I * c^2 - 6c) * f^3) * e^{(5d * x + 5c)} + (3I * d^3 * e^3 - 3 * (3I * c + 2) * d^2 * e^2 * f + (9I * c^2
\end{aligned}$$

$$\begin{aligned}
& 2 + 12*c - 6*I)*d*e*f^2 + (-3*I*c^3 - 6*c^2 + 6*I*c)*f^3)*e^{(4*d*x + 4*c)} + \\
& (6*d^3*e^3 - (18*c - 12*I)*d^2*e^2*f + 6*(3*c^2 - 4*I*c - 2)*d*e*f^2 - (6* \\
& c^3 - 12*I*c^2 - 12*c)*f^3)*e^{(3*d*x + 3*c)} + (-6*I*d^3*e^3 - 6*(-3*I*c - 2) \\
& )*d^2*e^2*f + (-18*I*c^2 - 24*c + 12*I)*d*e*f^2 + (6*I*c^3 + 12*c^2 - 12*I* \\
& c)*f^3)*e^{(2*d*x + 2*c)} - (3*d^3*e^3 - (9*c - 6*I)*d^2*e^2*f + 3*(3*c^2 - 4 \\
& *I*c - 2)*d*e*f^2 - (3*c^3 - 6*I*c^2 - 6*c)*f^3)*e^{(d*x + c)})*\log(e^{(d*x + \\
& c)} - 1) + (12*d^2*f^3*x^2 + 24*d^2*e*f^2*x + 24*c*d*e*f^2 - 12*c^2*f^3 - (- \\
& 12*I*d^2*f^3*x^2 - 24*I*d^2*e*f^2*x - 24*I*c*d*e*f^2 + 12*I*c^2*f^3)*e^{(5*d \\
& *x + 5*c)} + 12*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*e^{(4*d \\
& *x + 4*c)} - (24*I*d^2*f^3*x^2 + 48*I*d^2*e*f^2*x + 48*I*c*d*e*f^2 - 24*I*c^ \\
& 2*f^3)*e^{(3*d*x + 3*c)} - 24*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^ \\
& 2*f^3)*e^{(2*d*x + 2*c)} - (-12*I*d^2*f^3*x^2 - 24*I*d^2*e*f^2*x - 24*I*c*d*e \\
& *f^2 + 12*I*c^2*f^3)*e^{(d*x + c)})*\log(I*e^{(d*x + c)} + 1) - (3*I*d^3*f^3*x^3 \\
& + 9*I*c*d^2*e^2*f + (-9*I*c^2 - 12*c)*d*e*f^2 + (3*I*c^3 + 6*c^2 - 6*I*c)* \\
& f^3 - 3*(-3*I*d^3*e*f^2 + 2*d^2*f^3)*x^2 + (9*I*d^3*e^2*f - 12*d^2*e*f^2 - \\
& 6*I*d*f^3)*x - (3*d^3*f^3*x^3 + 9*c*d^2*e^2*f - 3*(3*c^2 - 4*I*c)*d*e*f^2 + \\
& (3*c^3 - 6*I*c^2 - 6*c)*f^3 + (9*d^3*e*f^2 + 6*I*d^2*f^3)*x^2 + 3*(3*d^3*e \\
& ^2*f + 4*I*d^2*e*f^2 - 2*d*f^3)*x)*e^{(5*d*x + 5*c)} + (3*I*d^3*f^3*x^3 + 9*I \\
& *c*d^2*e^2*f + (-9*I*c^2 - 12*c)*d*e*f^2 + (3*I*c^3 + 6*c^2 - 6*I*c)*f^3 - \\
& 3*(-3*I*d^3*e*f^2 + 2*d^2*f^3)*x^2 + (9*I*d^3*e^2*f - 12*d^2*e*f^2 - 6*I*d* \\
& f^3)*x)*e^{(4*d*x + 4*c)} + (6*d^3*f^3*x^3 + 18*c*d^2*e^2*f - 6*(3*c^2 - 4*I* \\
& c)*d*e*f^2 + (6*c^3 - 12*I*c^2 - 12*c)*f^3 + (18*d^3*e*f^2 + 12*I*d^2*f^3)* \\
& x^2 + 6*(3*d^3*e^2*f + 4*I*d^2*e*f^2 - 2*d*f^3)*x)*e^{(3*d*x + 3*c)} + (-6*I* \\
& d^3*f^3*x^3 - 18*I*c*d^2*e^2*f + (18*I*c^2 + 24*c)*d*e*f^2 + (-6*I*c^3 - 12 \\
& *c^2 + 12*I*c)*f^3 - 6*(3*I*d^3*e*f^2 - 2*d^2*f^3)*x^2 + (-18*I*d^3*e^2*f + \\
& 24*d^2*e*f^2 + 12*I*d*f^3)*x)*e^{(2*d*x + 2*c)} - (3*d^3*f^3*x^3 + 9*c*d^2*e \\
& ^2*f - 3*(3*c^2 - 4*I*c)*d*e*f^2 + (3*c^3 - 6*I*c^2 - 6*c)*f^3 + (9*d^3*e*f \\
& ^2 + 6*I*d^2*f^3)*x^2 + 3*(3*d^3*e^2*f + 4*I*d^2*e*f^2 - 2*d*f^3)*x)*e^{(d*x \\
& + c)})*\log(-e^{(d*x + c)} + 1) - (18*f^3*e^{(5*d*x + 5*c)} - 18*I*f^3*e^{(4*d*x \\
& + 4*c)} - 36*f^3*e^{(3*d*x + 3*c)} + 36*I*f^3*e^{(2*d*x + 2*c)} + 18*f^3*e^{(d*x \\
& + c)} - 18*I*f^3)*\text{polylog}(4, -e^{(d*x + c)}) + (18*f^3*e^{(5*d*x + 5*c)} - 18*I* \\
& f^3*e^{(4*d*x + 4*c)} - 36*f^3*e^{(3*d*x + 3*c)} + 36*I*f^3*e^{(2*d*x + 2*c)} + 1 \\
& 8*f^3*e^{(d*x + c)} - 18*I*f^3)*\text{polylog}(4, e^{(d*x + c)}) - (24*I*f^3*e^{(5*d*x \\
& + 5*c)} + 24*f^3*e^{(4*d*x + 4*c)} - 48*I*f^3*e^{(3*d*x + 3*c)} - 48*f^3*e^{(2*d* \\
& x + 2*c)} + 24*I*f^3*e^{(d*x + c)} + 24*f^3)*\text{polylog}(3, -I*e^{(d*x + c)}) - (18* \\
& I*d*f^3*x + 18*I*d*e*f^2 + 12*f^3 - 6*(3*d*f^3*x + 3*d*e*f^2 - 2*I*f^3)*e^{( \\
& 5*d*x + 5*c)} + (18*I*d*f^3*x + 18*I*d*e*f^2 + 12*f^3)*e^{(4*d*x + 4*c)} + 12* \\
& (3*d*f^3*x + 3*d*e*f^2 - 2*I*f^3)*e^{(3*d*x + 3*c)} + (-36*I*d*f^3*x - 36*I*d \\
& *e*f^2 - 24*f^3)*e^{(2*d*x + 2*c)} - 6*(3*d*f^3*x + 3*d*e*f^2 - 2*I*f^3)*e^{(d \\
& *x + c)})*\text{polylog}(3, -e^{(d*x + c)}) - (-18*I*d*f^3*x - 18*I*d*e*f^2 + 12*f^3 \\
& + 6*(3*d*f^3*x + 3*d*e*f^2 + 2*I*f^3)*e^{(5*d*x + 5*c)} + (-18*I*d*f^3*x - 18 \\
& *I*d*e*f^2 + 12*f^3)*e^{(4*d*x + 4*c)} - 12*(3*d*f^3*x + 3*d*e*f^2 + 2*I*f^3) \\
& *e^{(3*d*x + 3*c)} + (36*I*d*f^3*x + 36*I*d*e*f^2 - 24*f^3)*e^{(2*d*x + 2*c)} + \\
& 6*(3*d*f^3*x + 3*d*e*f^2 + 2*I*f^3)*e^{(d*x + c)})*\text{polylog}(3, e^{(d*x + c)})/ \\
& (2*a*d^4*e^{(5*d*x + 5*c)} - 2*I*a*d^4*e^{(4*d*x + 4*c)} - 4*a*d^4*e^{(3*d*x + 3
\end{aligned}$$

$*c) + 4*I*a*d^4*e^{(2*d*x + 2*c)} + 2*a*d^4*e^{(d*x + c)} - 2*I*a*d^4)$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*csch(d\*x+c)\*\*3/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*csch(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.218 \quad \int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=368

$$\frac{3f(e+fx)\operatorname{PolyLog}(2, -e^{c+dx})}{ad^2} - \frac{3f(e+fx)\operatorname{PolyLog}(2, e^{c+dx})}{ad^2} - \frac{4if^2\operatorname{PolyLog}(2, -ie^{c+dx})}{ad^3} - \frac{if^2\operatorname{PolyLog}(2, e^{2(c+dx)})}{ad^3}$$

[Out]  $((2*I)*(e + f*x)^2)/(a*d) + (3*(e + f*x)^2*\operatorname{ArcTanh}[E^(c + d*x)])/(a*d) - (f^2*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/(a*d^3) + (I*(e + f*x)^2*\operatorname{Coth}[c + d*x])/(a*d) - (f*(e + f*x)*\operatorname{Csch}[c + d*x])/(a*d^2) - ((e + f*x)^2*\operatorname{Coth}[c + d*x]*\operatorname{Csch}[c + d*x])/(2*a*d) - ((4*I)*f*(e + f*x)*\operatorname{Log}[1 + I*E^(c + d*x)])/(a*d^2) - ((2*I)*f*(e + f*x)*\operatorname{Log}[1 - E^(2*(c + d*x))])/(a*d^2) + (3*f*(e + f*x)*\operatorname{PolyLog}[2, -E^(c + d*x)])/(a*d^2) - ((4*I)*f^2*\operatorname{PolyLog}[2, (-I)*E^(c + d*x)])/(a*d^3) - (3*f*(e + f*x)*\operatorname{PolyLog}[2, E^(c + d*x)])/(a*d^2) - (I*f^2*\operatorname{PolyLog}[2, E^(2*(c + d*x))])/(a*d^3) - (3*f^2*\operatorname{PolyLog}[3, -E^(c + d*x)])/(a*d^3) + (3*f^2*\operatorname{PolyLog}[3, E^(c + d*x)])/(a*d^3) + (I*(e + f*x)^2*\operatorname{Tanh}[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)$

**Rubi [A]** time = 0.855921, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 13, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$ , Rules used = {5575, 4186, 3770, 4182, 2531, 2282, 6589, 4184, 3716, 2190, 2279, 2391, 3318}

$$\frac{3f(e+fx)\operatorname{PolyLog}(2, -e^{c+dx})}{ad^2} - \frac{3f(e+fx)\operatorname{PolyLog}(2, e^{c+dx})}{ad^2} - \frac{4if^2\operatorname{PolyLog}(2, -ie^{c+dx})}{ad^3} - \frac{if^2\operatorname{PolyLog}(2, e^{2(c+dx)})}{ad^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\frac{(e + f*x)^2*\operatorname{Csch}[c + d*x]^3}{(a + I*a*\operatorname{Sinh}[c + d*x])}, x]$

[Out]  $((2*I)*(e + f*x)^2)/(a*d) + (3*(e + f*x)^2*\operatorname{ArcTanh}[E^(c + d*x)])/(a*d) - (f^2*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/(a*d^3) + (I*(e + f*x)^2*\operatorname{Coth}[c + d*x])/(a*d) - (f*(e + f*x)*\operatorname{Csch}[c + d*x])/(a*d^2) - ((e + f*x)^2*\operatorname{Coth}[c + d*x]*\operatorname{Csch}[c + d*x])/(2*a*d) - ((4*I)*f*(e + f*x)*\operatorname{Log}[1 + I*E^(c + d*x)])/(a*d^2) - ((2*I)*f*(e + f*x)*\operatorname{Log}[1 - E^(2*(c + d*x))])/(a*d^2) + (3*f*(e + f*x)*\operatorname{PolyLog}[2, -E^(c + d*x)])/(a*d^2) - ((4*I)*f^2*\operatorname{PolyLog}[2, (-I)*E^(c + d*x)])/(a*d^3) - (3*f*(e + f*x)*\operatorname{PolyLog}[2, E^(c + d*x)])/(a*d^2) - (I*f^2*\operatorname{PolyLog}[2, E^(2*(c + d*x))])/(a*d^3) - (3*f^2*\operatorname{PolyLog}[3, -E^(c + d*x)])/(a*d^3) + (3*f^2*\operatorname{PolyLog}[3, E^(c + d*x)])/(a*d^3) + (I*(e + f*x)^2*\operatorname{Tanh}[c/2 + (I/4)*Pi + (d*x)/2])/(a*d)$

Rule 5575

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_]*(f_.)*(x_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x))]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

$(F\_)[v\_]$  /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 3716

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)], x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x))/(E^(2\*I\*k\*Pi)\*(1 + E^(2\*(-I\*e) + f\*fz\*x))/E^(2\*I\*k\*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 3318

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_.)), x\_Symbol] := Dist[(2\*a)^n, Int[(c + d\*x)^m\*Sin[(1\*(e + (Pi\*a)/(2\*b)))]/2 +

$(f*x)/2]^{(2*n)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ (\text{GtQ}[n, 0] \ || \ \text{IGtQ}[m, 0])$

### Rubi steps

$$\begin{aligned}
 \int \frac{(e+fx)^2 \text{csch}^3(c+dx)}{a+ia \sinh(c+dx)} dx &= -\left( i \int \frac{(e+fx)^2 \text{csch}^2(c+dx)}{a+ia \sinh(c+dx)} dx \right) + \frac{\int (e+fx)^2 \text{csch}^3(c+dx) dx}{a} \\
 &= -\frac{f(e+fx) \text{csch}(c+dx)}{ad^2} - \frac{(e+fx)^2 \coth(c+dx) \text{csch}(c+dx)}{2ad} - \frac{i \int (e+fx)^2 \text{csch}^2(c+dx) dx}{a} \\
 &= \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e+fx)^2 \coth(c+dx)}{ad} - \frac{f(e+fx)}{ad} \\
 &= \frac{i(e+fx)^2}{ad} + \frac{3(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e+fx)^2 \coth(c+dx)}{ad} \\
 &= \frac{i(e+fx)^2}{ad} + \frac{3(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e+fx)^2 \coth(c+dx)}{ad} \\
 &= \frac{2i(e+fx)^2}{ad} + \frac{3(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e+fx)^2 \coth(c+dx)}{ad} \\
 &= \frac{2i(e+fx)^2}{ad} + \frac{3(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e+fx)^2 \coth(c+dx)}{ad} \\
 &= \frac{2i(e+fx)^2}{ad} + \frac{3(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e+fx)^2 \coth(c+dx)}{ad} \\
 &= \frac{2i(e+fx)^2}{ad} + \frac{3(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{i(e+fx)^2 \coth(c+dx)}{ad}
 \end{aligned}$$

**Mathematica [B]** time = 17.0436, size = 1378, normalized size = 3.74

result too large to display

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Csch[c + d\*x]^3)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out]  $(-2*d*(e + f*x)*(d*(e + f*x) + 2*(1 + I*E^c)*f*\text{Log}[1 - I*E^{-(c - d*x)}]) + 4*(1 + I*E^c)*f^2*\text{PolyLog}[2, I*E^{-(c - d*x)}])/(a*d^3*(-I + E^c)) + ((2*I)*d^*$



$$\begin{aligned}
& 2*(e + f*x)^2*(-1 + \text{Coth}[c]) + (3*d^2*e^2 + (4*I)*d*e*f - 2*f^2)*(d*x - \text{Log}[1 - \text{Cosh}[c + d*x] - \text{Sinh}[c + d*x]]) + 2*d*(3*d*e - (2*I)*f)*f*x*\text{Log}[1 + \text{Cosh}[c + d*x] - \text{Sinh}[c + d*x]] + 3*d^2*f^2*x^2*\text{Log}[1 + \text{Cosh}[c + d*x] - \text{Sinh}[c + d*x]] - 2*d*(3*d*e + (2*I)*f)*f*x*\text{Log}[1 - \text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]] - 3*d^2*f^2*x^2*\text{Log}[1 - \text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]] - (3*d^2*e^2 - (4*I)*d*e*f - 2*f^2)*(d*x - \text{Log}[1 + \text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]]) + 2*(3*d*e + (2*I)*f)*f*\text{PolyLog}[2, \text{Cosh}[c + d*x] - \text{Sinh}[c + d*x]] - 2*(3*d*e - (2*I)*f)*f*\text{PolyLog}[2, -\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]] + 6*f^2*(d*x*\text{PolyLog}[2, \text{Cosh}[c + d*x] - \text{Sinh}[c + d*x]] + \text{PolyLog}[3, \text{Cosh}[c + d*x] - \text{Sinh}[c + d*x]]) - 6*f^2*(d*x*\text{PolyLog}[2, -\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]] + \text{PolyLog}[3, -\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]])/(2*a*d^3) + (\text{Csch}[c]*\text{Csch}[c + d*x]^2*(2*e*f*\text{Cosh}[(d*x)/2] + 2*f^2*x*\text{Cosh}[(d*x)/2] + 2*e*f*\text{Cosh}[(3*d*x)/2] + 2*f^2*x*\text{Cosh}[(3*d*x)/2] + (5*I)*d*e^2*\text{Cosh}[c - (d*x)/2] + (10*I)*d*e*f*x*\text{Cosh}[c - (d*x)/2] + (5*I)*d*f^2*x^2*\text{Cosh}[c - (d*x)/2] - I*d*e^2*\text{Cosh}[c + (d*x)/2] - (2*I)*d*e*f*x*\text{Cosh}[c + (d*x)/2] - I*d*f^2*x^2*\text{Cosh}[c + (d*x)/2] - 2*e*f*\text{Cosh}[2*c + (d*x)/2] - 2*f^2*x*\text{Cosh}[2*c + (d*x)/2] + I*d*e^2*\text{Cosh}[c + (3*d*x)/2] + (2*I)*d*e*f*x*\text{Cosh}[c + (3*d*x)/2] + I*d*f^2*x^2*\text{Cosh}[c + (3*d*x)/2] - 2*e*f*\text{Cosh}[2*c + (3*d*x)/2] - 2*f^2*x*\text{Cosh}[2*c + (3*d*x)/2] - (3*I)*d*e^2*\text{Cosh}[3*c + (3*d*x)/2] - (6*I)*d*e*f*x*\text{Cosh}[3*c + (3*d*x)/2] - (3*I)*d*f^2*x^2*\text{Cosh}[3*c + (3*d*x)/2] - (4*I)*d*e^2*\text{Cosh}[c + (5*d*x)/2] - (8*I)*d*e*f*x*\text{Cosh}[c + (5*d*x)/2] - (4*I)*d*f^2*x^2*\text{Cosh}[c + (5*d*x)/2] + (2*I)*d*e^2*\text{Cosh}[3*c + (5*d*x)/2] + (4*I)*d*e*f*x*\text{Cosh}[3*c + (5*d*x)/2] + (2*I)*d*f^2*x^2*\text{Cosh}[3*c + (5*d*x)/2] - d*e^2*\text{Sinh}[(d*x)/2] - 2*d*e*f*x*\text{Sinh}[(d*x)/2] - d*f^2*x^2*\text{Sinh}[(d*x)/2] - d*e^2*\text{Sinh}[(3*d*x)/2] - 2*d*e*f*x*\text{Sinh}[(3*d*x)/2] - d*f^2*x^2*\text{Sinh}[(3*d*x)/2] + (2*I)*e*f*\text{Sinh}[c - (d*x)/2] + (2*I)*f^2*x*\text{Sinh}[c - (d*x)/2] + (2*I)*e*f*\text{Sinh}[c + (d*x)/2] + (2*I)*f^2*x*\text{Sinh}[c + (d*x)/2] - 3*d*e^2*\text{Sinh}[2*c + (d*x)/2] - 6*d*e*f*x*\text{Sinh}[2*c + (d*x)/2] - 3*d*f^2*x^2*\text{Sinh}[2*c + (d*x)/2] + (2*I)*e*f*\text{Sinh}[c + (3*d*x)/2] + (2*I)*f^2*x*\text{Sinh}[c + (3*d*x)/2] - d*e^2*\text{Sinh}[2*c + (3*d*x)/2] - 2*d*e*f*x*\text{Sinh}[2*c + (3*d*x)/2] - d*f^2*x^2*\text{Sinh}[2*c + (3*d*x)/2] - (2*I)*e*f*\text{Sinh}[3*c + (3*d*x)/2] - (2*I)*f^2*x*\text{Sinh}[3*c + (3*d*x)/2] + 2*d*e^2*\text{Sinh}[2*c + (5*d*x)/2] + 4*d*e*f*x*\text{Sinh}[2*c + (5*d*x)/2] + 2*d*f^2*x^2*\text{Sinh}[2*c + (5*d*x)/2]))/(8*a*d^2*(\text{Cosh}[c/2] + I*\text{Sinh}[c/2])*(\text{Cosh}[c/2 + (d*x)/2] + I*\text{Sinh}[c/2 + (d*x)/2]))
\end{aligned}$$


---

**Maple [B]** time = 0.192, size = 1107, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*csch(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x)

```
[Out] -3/a/d^2*ln(1-exp(d*x+c))*c*e*f+3/a/d*ln(exp(d*x+c)+1)*e*f*x-3/a/d*ln(1-exp
(d*x+c))*e*f*x+3/a/d^2*e*f*c*ln(exp(d*x+c)-1)-4*I/a/d^2*ln(exp(d*x+c)-I)*e*
f-4*I/a/d^2*f^2*ln(1+I*exp(d*x+c))*x-4*I/a/d^3*f^2*ln(1+I*exp(d*x+c))*c+4*I
/a/d^3*f^2*c*ln(exp(d*x+c)-I)-4*I*f^2*polylog(2,-I*exp(d*x+c))/a/d^3+3*f^2*
polylog(3,exp(d*x+c))/a/d^3-3*f^2*polylog(3,-exp(d*x+c))/a/d^3-1/a/d^3*f^2*
ln(exp(d*x+c)+1)+1/a/d^3*f^2*ln(exp(d*x+c)-1)+4*I/a/d^3*c^2*f^2-2*I/a/d^3*f
^2*polylog(2,-exp(d*x+c))-2*I/a/d^3*f^2*polylog(2,exp(d*x+c))+4*I/a/d*f^2*x
^2-3/2/a/d*e^2*ln(exp(d*x+c)-1)+3/2/a/d*e^2*ln(exp(d*x+c)+1)-2*I/a/d^2*f*e*
ln(exp(d*x+c)+1)+2*I/a/d^3*f^2*c*ln(exp(d*x+c)-1)-8*I/a/d^3*f^2*c*ln(exp(d*
x+c))-2*I/a/d^3*ln(1-exp(d*x+c))*c*f^2+8*I/a/d^2*c*f^2*x-2*I/a/d^2*ln(exp(d
*x+c)+1)*f^2*x-2*I/a/d^2*ln(1-exp(d*x+c))*f^2*x+8*I/a/d^2*e*f*ln(exp(d*x+c)
)-2*I/a/d^2*f*e*ln(exp(d*x+c)-1)+3/a/d^2*f^2*polylog(2,-exp(d*x+c))*x+3/2/a
/d^3*f^2*c^2*ln(1-exp(d*x+c))-3/2/a/d*f^2*ln(1-exp(d*x+c))*x^2-3/2/a/d^3*f^
2*c^2*ln(exp(d*x+c)-1)+3/a/d^2*e*f*polylog(2,-exp(d*x+c))-3/a/d^2*e*f*polyl
og(2,exp(d*x+c))-3/a/d^2*f^2*polylog(2,exp(d*x+c))*x+3/2/a/d*f^2*ln(exp(d*x
+c)+1)*x^2-(6*d*e*f*x*exp(4*d*x+4*c)+4*d*e^2-5*d*e^2*exp(2*d*x+2*c)+2*I*d*e
*f*x*exp(d*x+c)+2*I*exp(d*x+c)*e*f-10*d*e*f*x*exp(2*d*x+2*c)-3*I*d*e^2*exp(
3*d*x+3*c)+I*d*f^2*x^2*exp(d*x+c)+8*d*e*f*x+4*d*f^2*x^2+3*d*e^2*exp(4*d*x+4
*c)+2*f^2*x*exp(4*d*x+4*c)+2*e*f*exp(4*d*x+4*c)-2*f^2*x*exp(2*d*x+2*c)-2*e*
f*exp(2*d*x+2*c)+3*d*f^2*x^2*exp(4*d*x+4*c)+I*d*e^2*exp(d*x+c)-3*I*d*f^2*x^
2*exp(3*d*x+3*c)-2*I*e*f*exp(3*d*x+3*c)-6*I*d*e*f*x*exp(3*d*x+3*c)+2*I*f^2*
x*exp(d*x+c)-2*I*f^2*x*exp(3*d*x+3*c)-5*d*f^2*x^2*exp(2*d*x+2*c))/(exp(2*d*
x+2*c)-1)^2/d^2/(exp(d*x+c)-I)/a
```

---

**Maxima [B]** time = 2.44375, size = 1165, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] -1/2*e^2*(16*(-I*e^(-d*x - c) - 5*e^(-2*d*x - 2*c) + 3*I*e^(-3*d*x - 3*c) +
3*e^(-4*d*x - 4*c) + 4)/((8*a*e^(-d*x - c) - 16*I*a*e^(-2*d*x - 2*c) - 16*
a*e^(-3*d*x - 3*c) + 8*I*a*e^(-4*d*x - 4*c) + 8*a*e^(-5*d*x - 5*c) + 8*I*a)
*d) - 3*log(e^(-d*x - c) + 1)/(a*d) + 3*log(e^(-d*x - c) - 1)/(a*d)) + 2*I*
f^2*x^2/(a*d) + 4*I*e*f*x/(a*d) - (4*d*f^2*x^2 + 8*d*e*f*x + (3*d*f^2*x^2*e
^(4*c) + 2*e*f*e^(4*c) + 2*(3*d*e*f + f^2)*x*e^(4*c)))*e^(4*d*x) + (-3*I*d*f
^2*x^2*e^(3*c) - 2*I*e*f*e^(3*c) + (-6*I*d*e*f - 2*I*f^2)*x*e^(3*c))*e^(3*d
*x) - (5*d*f^2*x^2*e^(2*c) + 2*e*f*e^(2*c) + 2*(5*d*e*f + f^2)*x*e^(2*c))*e
^(2*d*x) + (I*d*f^2*x^2*e^c + 2*I*e*f*e^c + (2*I*d*e*f + 2*I*f^2)*x*e^c)*e^
```

$$\begin{aligned} & (d*x))/(a*d^2*e^(5*d*x + 5*c) - I*a*d^2*e^(4*d*x + 4*c) - 2*a*d^2*e^(3*d*x \\ & + 3*c) + 2*I*a*d^2*e^(2*d*x + 2*c) + a*d^2*e^(d*x + c) - I*a*d^2) - 4*I*e*f \\ & *log(I*e^(d*x + c) + 1)/(a*d^2) + 3/2*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x \\ & *dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*f^2/(a*d^3) - 3/2*(d^2*x \\ & ^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + \\ & c)))*f^2/(a*d^3) - 4*I*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d*x + c)) \\ & )*f^2/(a*d^3) + (2*I*d*e*f + f^2)*x/(a*d^2) + (2*I*d*e*f - f^2)*x/(a*d^2) + \\ & (3*d*e*f - 2*I*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a*d^ \\ & 3) - (3*d*e*f + 2*I*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/( \\ & a*d^3) - (2*I*d*e*f + f^2)*log(e^(d*x + c) + 1)/(a*d^3) - (2*I*d*e*f - f^2) \\ & *log(e^(d*x + c) - 1)/(a*d^3) + 1/2*(d^3*f^2*x^3 + (3*d*e*f + 2*I*f^2)*d^2* \\ & x^2)/(a*d^3) - 1/2*(d^3*f^2*x^3 + (3*d*e*f - 2*I*f^2)*d^2*x^2)/(a*d^3) \end{aligned}$$

**Fricas [C]** time = 2.95854, size = 5378, normalized size = 14.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csc(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(8*d^2*e^2 - 16*c*d*e*f + 8*c^2*f^2 - (-8*I*f^2*e^(5*d*x + 5*c) - 8*f^2*e^(4*d*x + 4*c) + 16*I*f^2*e^(3*d*x + 3*c) + 16*f^2*e^(2*d*x + 2*c) - 8*I*f^2*e^(d*x + c) - 8*f^2)*dilog(-I*e^(d*x + c)) - (-6*I*d*f^2*x - 6*I*d*e*f - 4*f^2 + 2*(3*d*f^2*x + 3*d*e*f - 2*I*f^2)*e^(5*d*x + 5*c) + (-6*I*d*f^2*x - 6*I*d*e*f - 4*f^2)*e^(4*d*x + 4*c) - 4*(3*d*f^2*x + 3*d*e*f - 2*I*f^2)*e^(3*d*x + 3*c) + (12*I*d*f^2*x + 12*I*d*e*f + 8*f^2)*e^(2*d*x + 2*c) + 2*(3*d*f^2*x + 3*d*e*f - 2*I*f^2)*e^(d*x + c))*dilog(-e^(d*x + c)) - (6*I*d*f^2*x + 6*I*d*e*f - 4*f^2 - 2*(3*d*f^2*x + 3*d*e*f + 2*I*f^2)*e^(5*d*x + 5*c) + (6*I*d*f^2*x + 6*I*d*e*f - 4*f^2)*e^(4*d*x + 4*c) + 4*(3*d*f^2*x + 3*d*e*f + 2*I*f^2)*e^(3*d*x + 3*c) + (-12*I*d*f^2*x - 12*I*d*e*f + 8*f^2)*e^(2*d*x + 2*c) - 2*(3*d*f^2*x + 3*d*e*f + 2*I*f^2)*e^(d*x + c))*dilog(e^(d*x + c)) - (8*I*d^2*f^2*x^2 + 16*I*d^2*e*f*x + 16*I*c*d*e*f - 8*I*c^2*f^2)*e^(5*d*x + 5*c) - 2*(d^2*f^2*x^2 - 3*d^2*e^2 + 2*(4*c - 1)*d*e*f - 4*c^2*f^2 + 2*(d^2*e*f - d*f^2)*x)*e^(4*d*x + 4*c) - (-10*I*d^2*f^2*x^2 + 6*I*d^2*e^2 + (-32*I*c + 4*I)*d*e*f + 16*I*c^2*f^2 + (-20*I*d^2*e*f + 4*I*d*f^2)*x)*e^(3*d*x + 3*c) + 2*(3*d^2*f^2*x^2 - 5*d^2*e^2 + 2*(8*c - 1)*d*e*f - 8*c^2*f^2 + 2*(3*d^2*e*f - d*f^2)*x)*e^(2*d*x + 2*c) - (6*I*d^2*f^2*x^2 - 2*I*d^2*e^2 + (16*I*c - 4*I)*d*e*f - 8*I*c^2*f^2 + (12*I*d^2*e*f - 4*I*d*f^2)*x)*e^(d*x + c) - (-3*I*d^2*f^2*x^2 - 3*I*d^2*e^2 - 4*d*e*f + 2*I*f^2 - 2*(3*I*d^2*e*f + 2*d*f^2)*x + (3*d^2*f^2*x^2 + 3*d^2*e^2 - 4*I*d*e*f - 2*f^2 + (6*d^2*e*f - 4
```

```

*I*d*f^2)*x)*e^(5*d*x + 5*c) + (-3*I*d^2*f^2*x^2 - 3*I*d^2*e^2 - 4*d*e*f +
2*I*f^2 - 2*(3*I*d^2*e*f + 2*d*f^2)*x)*e^(4*d*x + 4*c) - (6*d^2*f^2*x^2 + 6
*d^2*e^2 - 8*I*d*e*f - 4*f^2 + (12*d^2*e*f - 8*I*d*f^2)*x)*e^(3*d*x + 3*c)
+ (6*I*d^2*f^2*x^2 + 6*I*d^2*e^2 + 8*d*e*f - 4*I*f^2 - 4*(-3*I*d^2*e*f - 2*
d*f^2)*x)*e^(2*d*x + 2*c) + (3*d^2*f^2*x^2 + 3*d^2*e^2 - 4*I*d*e*f - 2*f^2
+ (6*d^2*e*f - 4*I*d*f^2)*x)*e^(d*x + c))*log(e^(d*x + c) + 1) + (8*d*e*f -
8*c*f^2 - (-8*I*d*e*f + 8*I*c*f^2)*e^(5*d*x + 5*c) + 8*(d*e*f - c*f^2)*e^(
4*d*x + 4*c) - (16*I*d*e*f - 16*I*c*f^2)*e^(3*d*x + 3*c) - 16*(d*e*f - c*f^
2)*e^(2*d*x + 2*c) - (-8*I*d*e*f + 8*I*c*f^2)*e^(d*x + c))*log(e^(d*x + c)
- I) - (3*I*d^2*e^2 - 2*(3*I*c + 2)*d*e*f + (3*I*c^2 + 4*c - 2*I)*f^2 - (3*
d^2*e^2 - (6*c - 4*I)*d*e*f + (3*c^2 - 4*I*c - 2)*f^2)*e^(5*d*x + 5*c) + (3
*I*d^2*e^2 - 2*(3*I*c + 2)*d*e*f + (3*I*c^2 + 4*c - 2*I)*f^2)*e^(4*d*x + 4*
c) + (6*d^2*e^2 - (12*c - 8*I)*d*e*f + 2*(3*c^2 - 4*I*c - 2)*f^2)*e^(3*d*x
+ 3*c) + (-6*I*d^2*e^2 - 4*(-3*I*c - 2)*d*e*f + (-6*I*c^2 - 8*c + 4*I)*f^2)
*e^(2*d*x + 2*c) - (3*d^2*e^2 - (6*c - 4*I)*d*e*f + (3*c^2 - 4*I*c - 2)*f^2
)*e^(d*x + c))*log(e^(d*x + c) - 1) + (8*d*f^2*x + 8*c*f^2 - (-8*I*d*f^2*x
- 8*I*c*f^2)*e^(5*d*x + 5*c) + 8*(d*f^2*x + c*f^2)*e^(4*d*x + 4*c) - (16*I*
d*f^2*x + 16*I*c*f^2)*e^(3*d*x + 3*c) - 16*(d*f^2*x + c*f^2)*e^(2*d*x + 2*c
) - (-8*I*d*f^2*x - 8*I*c*f^2)*e^(d*x + c))*log(I*e^(d*x + c) + 1) - (3*I*d
^2*f^2*x^2 + 6*I*c*d*e*f + (-3*I*c^2 - 4*c)*f^2 - 2*(-3*I*d^2*e*f + 2*d*f^2
)*x - (3*d^2*f^2*x^2 + 6*c*d*e*f - (3*c^2 - 4*I*c)*f^2 + (6*d^2*e*f + 4*I*d
*f^2)*x)*e^(5*d*x + 5*c) + (3*I*d^2*f^2*x^2 + 6*I*c*d*e*f + (-3*I*c^2 - 4*c
)*f^2 - 2*(-3*I*d^2*e*f + 2*d*f^2)*x)*e^(4*d*x + 4*c) + (6*d^2*f^2*x^2 + 12
*c*d*e*f - 2*(3*c^2 - 4*I*c)*f^2 + (12*d^2*e*f + 8*I*d*f^2)*x)*e^(3*d*x + 3
*c) + (-6*I*d^2*f^2*x^2 - 12*I*c*d*e*f + (6*I*c^2 + 8*c)*f^2 - 4*(3*I*d^2*e
*f - 2*d*f^2)*x)*e^(2*d*x + 2*c) - (3*d^2*f^2*x^2 + 6*c*d*e*f - (3*c^2 - 4*
I*c)*f^2 + (6*d^2*e*f + 4*I*d*f^2)*x)*e^(d*x + c))*log(-e^(d*x + c) + 1) +
(6*f^2*e^(5*d*x + 5*c) - 6*I*f^2*e^(4*d*x + 4*c) - 12*f^2*e^(3*d*x + 3*c) +
12*I*f^2*e^(2*d*x + 2*c) + 6*f^2*e^(d*x + c) - 6*I*f^2)*polylog(3, -e^(d*x
+ c)) - (6*f^2*e^(5*d*x + 5*c) - 6*I*f^2*e^(4*d*x + 4*c) - 12*f^2*e^(3*d*x
+ 3*c) + 12*I*f^2*e^(2*d*x + 2*c) + 6*f^2*e^(d*x + c) - 6*I*f^2)*polylog(3
, e^(d*x + c)))/(2*a*d^3*e^(5*d*x + 5*c) - 2*I*a*d^3*e^(4*d*x + 4*c) - 4*a*
d^3*e^(3*d*x + 3*c) + 4*I*a*d^3*e^(2*d*x + 2*c) + 2*a*d^3*e^(d*x + c) - 2*I
*a*d^3)

```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*csch(d\*x+c)\*\*3/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \operatorname{csch}(dx + c)^3}{ia \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^2*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")`

[Out] `integrate((f*x + e)^2*csch(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)`

$$3.219 \quad \int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=214

$$\frac{3f\operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{2ad^2} - \frac{3f\operatorname{PolyLog}\left(2, e^{c+dx}\right)}{2ad^2} - \frac{f\operatorname{csch}(c+dx)}{2ad^2} - \frac{if \log(\sinh(c+dx))}{ad^2} - \frac{2if \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\right)}{ad^2}$$

[Out] (3\*(e + f\*x)\*ArcTanh[E^(c + d\*x)]/(a\*d) + (I\*(e + f\*x)\*Coth[c + d\*x])/(a\*d) - (f\*Csch[c + d\*x])/(2\*a\*d^2) - ((e + f\*x)\*Coth[c + d\*x]\*Csch[c + d\*x])/(2\*a\*d) - ((2\*I)\*f\*Log[Cosh[c/2 + (I/4)\*Pi + (d\*x)/2]])/(a\*d^2) - (I\*f\*Log[Sinh[c + d\*x]])/(a\*d^2) + (3\*f\*PolyLog[2, -E^(c + d\*x)])/(2\*a\*d^2) - (3\*f\*PolyLog[2, E^(c + d\*x)])/(2\*a\*d^2) + (I\*(e + f\*x)\*Tanh[c/2 + (I/4)\*Pi + (d\*x)/2])/(a\*d)

**Rubi [A]** time = 0.367409, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 8, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$ , Rules used = {5575, 4185, 4182, 2279, 2391, 4184, 3475, 3318}

$$\frac{3f\operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{2ad^2} - \frac{3f\operatorname{PolyLog}\left(2, e^{c+dx}\right)}{2ad^2} - \frac{f\operatorname{csch}(c+dx)}{2ad^2} - \frac{if \log(\sinh(c+dx))}{ad^2} - \frac{2if \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2} + \frac{i\pi}{4}\right)\right)}{ad^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Csch[c + d\*x]^3)/(a + I\*a\*Sinh[c + d\*x]), x]

[Out] (3\*(e + f\*x)\*ArcTanh[E^(c + d\*x)]/(a\*d) + (I\*(e + f\*x)\*Coth[c + d\*x])/(a\*d) - (f\*Csch[c + d\*x])/(2\*a\*d^2) - ((e + f\*x)\*Coth[c + d\*x]\*Csch[c + d\*x])/(2\*a\*d) - ((2\*I)\*f\*Log[Cosh[c/2 + (I/4)\*Pi + (d\*x)/2]])/(a\*d^2) - (I\*f\*Log[Sinh[c + d\*x]])/(a\*d^2) + (3\*f\*PolyLog[2, -E^(c + d\*x)])/(2\*a\*d^2) - (3\*f\*PolyLog[2, E^(c + d\*x)])/(2\*a\*d^2) + (I\*(e + f\*x)\*Tanh[c/2 + (I/4)\*Pi + (d\*x)/2])/(a\*d)

### Rule 5575

Int[(Csch[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] :> Dist[1/a, Int[(e + f\*x)^m\*Csch[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Csch[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :=
  -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x]
  + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
  , x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :=
  Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]]/(f*fz*I), x]
  + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
  ], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
  f*fz*x)]]], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] :=
  Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
  )^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
  , -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp
  [((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
  t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
  *x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
  , x_Symbol] := Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
  (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
  , 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)} dx &= -\left(i \int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+ia\sinh(c+dx)} dx\right) + \frac{\int (e+fx)\operatorname{csch}^3(c+dx) dx}{a} \\
&= -\frac{f\operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+fx)\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{i \int (e+fx)\operatorname{csch}^2(c+dx) dx}{a} - \frac{\int (e+fx)\operatorname{csch}^3(c+dx) dx}{a} \\
&= \frac{(e+fx)\tanh^{-1}(e^{c+dx})}{ad} + \frac{i(e+fx)\operatorname{coth}(c+dx)}{ad} - \frac{f\operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+fx)\operatorname{coth}(c+dx)}{2ad} \\
&= \frac{3(e+fx)\tanh^{-1}(e^{c+dx})}{ad} + \frac{i(e+fx)\operatorname{coth}(c+dx)}{ad} - \frac{f\operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+fx)\operatorname{coth}(c+dx)}{2ad} \\
&= \frac{3(e+fx)\tanh^{-1}(e^{c+dx})}{ad} + \frac{i(e+fx)\operatorname{coth}(c+dx)}{ad} - \frac{f\operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+fx)\operatorname{coth}(c+dx)}{2ad} \\
&= \frac{3(e+fx)\tanh^{-1}(e^{c+dx})}{ad} + \frac{i(e+fx)\operatorname{coth}(c+dx)}{ad} - \frac{f\operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+fx)\operatorname{coth}(c+dx)}{2ad}
\end{aligned}$$

**Mathematica [B]** time = 2.88705, size = 541, normalized size = 2.53

$$\frac{\left(\cosh\left(\frac{1}{2}(c+dx)\right) + i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(-12f\left(\cosh\left(\frac{1}{2}(c+dx)\right) + i\sinh\left(\frac{1}{2}(c+dx)\right)\right)\left(\operatorname{PolyLog}\left(2, -e^{-c-dx}\right) - \operatorname{PolyLog}\left(2, e^{-c-dx}\right)\right)\right)}{(a + I a \sinh(c + dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Csch[c + d\*x]^3)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] ((Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])\*((2\*I)\*(I\*f + 2\*d\*(e + f\*x))\*Cosh[(c + d\*x)/2]\*(I + Coth[(c + d\*x)/2]) - d\*(e + f\*x)\*(I + Coth[(c + d\*x)/2])\*Csch[(c + d\*x)/2] - 8\*f\*(c + d\*x)\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]) + 16\*f\*ArcTan[Tanh[(c + d\*x)/2]]\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]) - 12\*d\*e\*Log[Tanh[(c + d\*x)/2]]\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]) + 12\*c\*f\*Log[Tanh[(c + d\*x)/2]]\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]) - 12\*f\*((c + d\*x)\*(Log[1 - E^(-c - d\*x)] - Log[1 + E^(-c - d\*x)]) + PolyLog[2, -E^(-c - d\*x)] - PolyLog[2, E^(-c - d\*x)]))\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]) + (16\*I)\*d\*(e + f\*x)\*Sinh[(c + d\*x)/2] + 8\*f\*Log[Cosh[c + d\*x]]\*((-I)\*Cosh[(c + d\*x)/2] + Sinh[(c + d\*x)/2]) + 8\*f\*Log[Sinh[c + d\*x]]\*((-I)\*Cosh[(c + d\*x)/2] + Sinh[(c + d\*x)/2]) + 2\*(f + (2\*I)\*d\*(e + f\*x))\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])\*Tanh[(c + d\*x)/2] - I\*d\*(e + f\*x)\*Ssch[(c + d\*x)/2]\*(-I + Tanh[(c + d\*x)/2]))/(8\*d^2\*(a + I\*a\*Sinh[c + d\*x]))



**Maple [B]** time = 0.184, size = 423, normalized size = 2.

$$\frac{-3 idfxe^{3dx+3c} - 5 dfxe^{2dx+2c} + 3 dfxe^{4dx+4c} + idfxe^{dx+c} - 5 dee^{2dx+2c} + 3 dee^{4dx+4c} - 3 idee^{3dx+3c} + 4 dfx + fe^{4a}}{(e^{2dx+2c} - 1)^2 d^2 (e^{dx+c} - i) a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)`

[Out] 
$$-(-3I*d*f*x*\exp(3*d*x+3*c)-5*d*f*x*\exp(2*d*x+2*c)+3*d*f*x*\exp(4*d*x+4*c)+I*d*f*x*\exp(d*x+c)-5*d*e*\exp(2*d*x+2*c)+3*d*e*\exp(4*d*x+4*c)-3*I*d*e*\exp(3*d*x+3*c)+4*d*f*x+f*\exp(4*d*x+4*c)+I*\exp(d*x+c)*f-I*\exp(3*d*x+3*c)*f+4*d*e-f*\exp(2*d*x+2*c)+I*d*e*\exp(d*x+c))/(\exp(2*d*x+2*c)-1)^2/d^2/(\exp(d*x+c)-I)/a-3/2*f*\operatorname{polylog}(2,\exp(d*x+c))/a/d^2+3/2*f*\operatorname{polylog}(2,-\exp(d*x+c))/a/d^2-2*I*f/a/d^2*\ln(\exp(d*x+c)-I)+4*I/d^2/a*f*\ln(\exp(d*x+c))-I/d^2/a*f*\ln(\exp(d*x+c)-1)-I/d^2/a*f*\ln(\exp(d*x+c)+1)-3/2/d/a*e*\ln(\exp(d*x+c)-1)+3/2/d/a*e*\ln(\exp(d*x+c)+1)-3/2/d/a*\ln(1-\exp(d*x+c))*f*x-3/2/d^2/a*\ln(1-\exp(d*x+c))*c*f+3/2/d/a*\ln(\exp(d*x+c)+1)*f*x+3/2/d^2/a*f*c*\ln(\exp(d*x+c)-1)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\left(24d \int \frac{x}{16(ade^{(dx+c)} + ad)} dx + 24d \int \frac{x}{16(ade^{(dx+c)} - ad)} dx + \frac{8(2dxe^{(5dx+5c)} + 2idx + (idxe^{(4c)} + ie^{(4c)})e^{(4dx)} - (2dxe^{(5dx+5c)} + 8ad^2e^{(4dx+4c)} - 16idxe^{(4c)} + 2ie^{(4c)})e^{(4dx)}}{8iad^2e^{(5dx+5c)} + 8ad^2e^{(4dx+4c)} - 16idxe^{(4c)} + 2ie^{(4c)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out] 
$$-(24*d*\operatorname{integrate}(1/16*x/(a*d*e^{(d*x+c)} + a*d), x) + 24*d*\operatorname{integrate}(1/16*x/(a*d*e^{(d*x+c)} - a*d), x) + 8*(2*d*x*e^{(5*d*x+5*c)} + 2*I*d*x + (I*d*x*e^{(4*c)} + I*e^{(4*c)})*e^{(4*d*x)} - (d*x*e^{(3*c)} - e^{(3*c)})*e^{(3*d*x)} + (-I*d*x*e^{(2*c)} - I*e^{(2*c)})*e^{(2*d*x)} + (d*x*e^c - e^c)*e^{(d*x)})/(8*I*a*d^2*e^{(5*d*x+5*c)} + 8*a*d^2*e^{(4*d*x+4*c)} - 16*I*a*d^2*e^{(3*d*x+3*c)} - 16*a*d^2*e^{(2*d*x+2*c)} + 8*I*a*d^2*e^{(d*x+c)} + 8*a*d^2) - 2*I*(d*x+c)/(a*d^2) + 2*I*\log((e^{(d*x+c)} - I)*e^{(-c)})/(a*d^2) + I*\log(e^{(d*x+c)} + 1)/(a*d^2) + I*\log(e^{(d*x+c)} - 1)/(a*d^2))*f - 1/2*e*(16*(-I*e^{(-d*x-c)} - 5*e^{(-2*d*x-2*c)} + 3*I*e^{(-3*d*x-3*c)} + 3*e^{(-4*d*x-4*c)} + 4)/((8*a*e^{(-d*x-c)} - 16*I*a*e^{(-2*d*x-2*c)} - 16*a*e^{(-3*d*x-3*c)} + 8*I*a*e^{(-4*d*x-4*c)} + 8*a*e^{(-5*d*x-5*c)} + 8*I*a)*d) - 3*\log(e^{(-d*x-c)} + 1)/(a*d)$$

+ 3\*log(e<sup>-(d\*x - c) - 1</sup>/(a\*d))

**Fricas [B]** time = 2.77223, size = 2163, normalized size = 10.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -(8*d*e - 4*c*f - (3*f*e^{(5*d*x + 5*c)} - 3*I*f*e^{(4*d*x + 4*c)} - 6*f*e^{(3*d*x + 3*c)} + 6*I*f*e^{(2*d*x + 2*c)} + 3*f*e^{(d*x + c)} - 3*I*f)*\text{dilog}(-e^{(d*x + c)})) + (3*f*e^{(5*d*x + 5*c)} - 3*I*f*e^{(4*d*x + 4*c)} - 6*f*e^{(3*d*x + 3*c)} + 6*I*f*e^{(2*d*x + 2*c)} + 3*f*e^{(d*x + c)} - 3*I*f)*\text{dilog}(e^{(d*x + c)}) - (8*I*d*f*x + 4*I*c*f)*e^{(5*d*x + 5*c)} - 2*(d*f*x - 3*d*e + (2*c - 1)*f)*e^{(4*d*x + 4*c)} - (-10*I*d*f*x + 6*I*d*e + (-8*I*c + 2*I)*f)*e^{(3*d*x + 3*c)} + 2*(3*d*f*x - 5*d*e + (4*c - 1)*f)*e^{(2*d*x + 2*c)} - (6*I*d*f*x - 2*I*d*e + (4*I*c - 2*I)*f)*e^{(d*x + c)} - (-3*I*d*f*x - 3*I*d*e + (3*d*f*x + 3*d*e - 2*I*f)*e^{(5*d*x + 5*c)} + (-3*I*d*f*x - 3*I*d*e - 2*f)*e^{(4*d*x + 4*c)} - 2*(3*d*f*x + 3*d*e - 2*I*f)*e^{(3*d*x + 3*c)} + (6*I*d*f*x + 6*I*d*e + 4*f)*e^{(2*d*x + 2*c)} + (3*d*f*x + 3*d*e - 2*I*f)*e^{(d*x + c)} - 2*f)*\log(e^{(d*x + c)} + 1) - (-4*I*f*e^{(5*d*x + 5*c)} - 4*f*e^{(4*d*x + 4*c)} + 8*I*f*e^{(3*d*x + 3*c)} + 8*f*e^{(2*d*x + 2*c)} - 4*I*f*e^{(d*x + c)} - 4*f)*\log(e^{(d*x + c)} - 1) - (3*I*d*e + (-3*I*c - 2)*f - (3*d*e - (3*c - 2*I)*f)*e^{(5*d*x + 5*c)} + (3*I*d*e + (-3*I*c - 2)*f)*e^{(4*d*x + 4*c)} + (6*d*e - (6*c - 4*I)*f)*e^{(3*d*x + 3*c)} - 2*(3*I*d*e + (-3*I*c - 2)*f)*e^{(2*d*x + 2*c)} - (3*d*e - (3*c - 2*I)*f)*e^{(d*x + c)})*\log(e^{(d*x + c)} - 1) - (3*I*d*f*x + 3*I*c*f - 3*(d*f*x + c*f)*e^{(5*d*x + 5*c)} + (3*I*d*f*x + 3*I*c*f)*e^{(4*d*x + 4*c)} + 6*(d*f*x + c*f)*e^{(3*d*x + 3*c)} + (-6*I*d*f*x - 6*I*c*f)*e^{(2*d*x + 2*c)} - 3*(d*f*x + c*f)*e^{(d*x + c)})*\log(-e^{(d*x + c)} + 1))/(2*a*d^2*e^{(5*d*x + 5*c)} - 2*I*a*d^2*e^{(4*d*x + 4*c)} - 4*a*d^2*e^{(3*d*x + 3*c)} + 4*I*a*d^2*e^{(2*d*x + 2*c)} + 2*a*d^2*e^{(d*x + c)} - 2*I*a*d^2) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)\*\*3/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \operatorname{csch}(dx + c)^3}{ia \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)\*csch(d\*x + c)^3/(I\*a\*sinh(d\*x + c) + a), x)

$$3.220 \quad \int \frac{\operatorname{csch}^3(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=87

$$\frac{2i \coth(c+dx)}{ad} + \frac{3 \tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{3 \coth(c+dx) \operatorname{csch}(c+dx)}{2ad} + \frac{\coth(c+dx) \operatorname{csch}(c+dx)}{d(a+ia \sinh(c+dx))}$$

[Out] (3\*ArcTanh[Cosh[c + d\*x]])/(2\*a\*d) + ((2\*I)\*Coth[c + d\*x])/(a\*d) - (3\*Coth[c + d\*x]\*Csch[c + d\*x])/(2\*a\*d) + (Coth[c + d\*x]\*Csch[c + d\*x])/(d\*(a + I\*a\*Sinh[c + d\*x]))

**Rubi [A]** time = 0.12336, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2768, 2748, 3768, 3770, 3767, 8}

$$\frac{2i \coth(c+dx)}{ad} + \frac{3 \tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{3 \coth(c+dx) \operatorname{csch}(c+dx)}{2ad} + \frac{\coth(c+dx) \operatorname{csch}(c+dx)}{d(a+ia \sinh(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d\*x]^3/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (3\*ArcTanh[Cosh[c + d\*x]])/(2\*a\*d) + ((2\*I)\*Coth[c + d\*x])/(a\*d) - (3\*Coth[c + d\*x]\*Csch[c + d\*x])/(2\*a\*d) + (Coth[c + d\*x]\*Csch[c + d\*x])/(d\*(a + I\*a\*Sinh[c + d\*x]))

### Rule 2768

Int[((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_)/((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> -Simp[(b^2\*Cos[e + f\*x]\*(c + d\*Sin[e + f\*x])^(n + 1))/(a\*f\*(b\*c - a\*d)\*(a + b\*Sin[e + f\*x])), x] + Dist[d/(a\*(b\*c - a\*d)), Int[(c + d\*Sin[e + f\*x])^n\*(a\*n - b\*(n + 1)\*Sin[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2\*n] || EqQ[c, 0])

### Rule 2748

Int[((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[c, Int[(b\*Sin[e + f\*x])^m, x], x] + Dist[d/b, Int[(b\*Sin[e + f\*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}^3(c+dx)}{a+ia\sinh(c+dx)} dx &= \frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{d(a+ia\sinh(c+dx))} - \frac{\int \operatorname{csch}^3(c+dx)(-3a+2ia\sinh(c+dx)) dx}{a^2} \\ &= \frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{d(a+ia\sinh(c+dx))} - \frac{(2i) \int \operatorname{csch}^2(c+dx) dx}{a} + \frac{3 \int \operatorname{csch}^3(c+dx) dx}{a} \\ &= -\frac{3 \operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad} + \frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{d(a+ia\sinh(c+dx))} - \frac{3 \int \operatorname{csch}(c+dx) dx}{2a} - \frac{2 \operatorname{Subst}[\int \operatorname{csch}^3(x) dx, x, \operatorname{coth}(c+dx)]}{2a} \\ &= \frac{3 \tanh^{-1}(\cosh(c+dx))}{2ad} + \frac{2i \operatorname{coth}(c+dx)}{ad} - \frac{3 \operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad} + \frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{d(a+ia\sinh(c+dx))} \end{aligned}$$

**Mathematica [A]** time = 0.41949, size = 90, normalized size = 1.03

$$\frac{4i \tanh(c+dx) + 4i \operatorname{csch}(2(c+dx)) - 3 \operatorname{sech}(c+dx) + \operatorname{csch}^2(c+dx)(-\operatorname{sech}(c+dx)) + 3\sqrt{\cosh^2(c+dx)} \operatorname{sech}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d\*x]^3/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] ((4\*I)\*Csch[2\*(c + d\*x)] - 3\*Sech[c + d\*x] + 3\*ArcTanh[Sqrt[Cosh[c + d\*x]^2]]\*Sqrt[Cosh[c + d\*x]^2]\*Sech[c + d\*x] - Csch[c + d\*x]^2\*Sech[c + d\*x] + (4\*I)\*Tanh[c + d\*x])/(2\*a\*d)

**Maple [A]** time = 0.049, size = 119, normalized size = 1.4

$$\frac{2i}{da} \left( -i + \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^{-1} + \frac{i}{da} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{1}{8da} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2 - \frac{1}{8da} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^{-2} + \frac{i}{da} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x)

[Out] 2\*I/d/a/(-I+tanh(1/2\*d\*x+1/2\*c))+1/2\*I/d/a\*tanh(1/2\*d\*x+1/2\*c)+1/8/d/a\*tanh(1/2\*d\*x+1/2\*c)^2-1/8/d/a/tanh(1/2\*d\*x+1/2\*c)^2+1/2\*I/d/a/tanh(1/2\*d\*x+1/2\*c)-3/2/d/a\*ln(tanh(1/2\*d\*x+1/2\*c))

**Maxima [A]** time = 1.15247, size = 213, normalized size = 2.45

$$\frac{8 \left( -i e^{(-dx-c)} - 5 e^{(-2dx-2c)} + 3i e^{(-3dx-3c)} + 3 e^{(-4dx-4c)} + 4 \right)}{\left( 8 a e^{(-dx-c)} - 16i a e^{(-2dx-2c)} - 16 a e^{(-3dx-3c)} + 8i a e^{(-4dx-4c)} + 8 a e^{(-5dx-5c)} + 8i a \right) d} + \frac{3 \log \left( e^{(-dx-c)} + 1 \right)}{2 a d} - \frac{3 \log \left( e^{(-dx-c)} - 1 \right)}{2 a d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -8\*(-I\*e^(-d\*x - c) - 5\*e^(-2\*d\*x - 2\*c) + 3\*I\*e^(-3\*d\*x - 3\*c) + 3\*e^(-4\*d\*x - 4\*c) + 4)/((8\*a\*e^(-d\*x - c) - 16\*I\*a\*e^(-2\*d\*x - 2\*c) - 16\*a\*e^(-3\*d\*x - 3\*c) + 8\*I\*a\*e^(-4\*d\*x - 4\*c) + 8\*a\*e^(-5\*d\*x - 5\*c) + 8\*I\*a)\*d) + 3/2\*log(e^(-d\*x - c) + 1)/(a\*d) - 3/2\*log(e^(-d\*x - c) - 1)/(a\*d)

**Fricas [B]** time = 2.6035, size = 640, normalized size = 7.36

$$\frac{\left( 3 e^{(5dx+5c)} - 3i e^{(4dx+4c)} - 6 e^{(3dx+3c)} + 6i e^{(2dx+2c)} + 3 e^{(dx+c)} - 3i \right) \log \left( e^{(dx+c)} + 1 \right) - \left( 3 e^{(5dx+5c)} - 3i e^{(4dx+4c)} - 6 e^{(3dx+3c)} + 6i e^{(2dx+2c)} + 3 e^{(dx+c)} - 3i \right)}{2 a d e^{(5dx+5c)} - 2i a d e^{(4dx+4c)} - 4 a d e^{(3dx+3c)} + 4i a d e^{(2dx+2c)} + 2 a d e^{(dx+c)} - 2i a d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")`

[Out]  $((3e^{(5dx+5c)} - 3Ie^{(4dx+4c)} - 6e^{(3dx+3c)} + 6Ie^{(2dx+x+2c)} + 3e^{(dx+c)} - 3I)\log(e^{(dx+c)} + 1) - (3e^{(5dx+5c)} - 3Ie^{(4dx+4c)} - 6e^{(3dx+3c)} + 6Ie^{(2dx+x+2c)} + 3e^{(dx+c)} - 3I)\log(e^{(dx+c)} - 1) - 6e^{(4dx+4c)} + 6Ie^{(3dx+3c)} + 10e^{(2dx+x+2c)} - 2Ie^{(dx+c)} - 8)/(2ad e^{(5dx+5c)} - 2Iad e^{(4dx+4c)} - 4ad e^{(3dx+3c)} + 4Iad e^{(2dx+x+2c)} + 2ad e^{(dx+c)} - 2Iad)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)`

[Out] Timed out

**Giac [A]** time = 1.21453, size = 142, normalized size = 1.63

$$\frac{3 \log(e^{(dx+c)} + 1)}{2ad} - \frac{3 \log(|e^{(dx+c)} - 1|)}{2ad} - \frac{e^{(3dx+3c)} - 2ie^{(2dx+2c)} + e^{(dx+c)} + 2i}{ad(e^{(2dx+2c)} - 1)^2} - \frac{2i}{ad(i e^{(dx+c)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")`

[Out]  $3/2 \log(e^{(dx+c)} + 1)/(a*d) - 3/2 \log(\text{abs}(e^{(dx+c)} - 1))/(a*d) - (e^{(3dx+3c)} - 2Ie^{(2dx+2c)} + e^{(dx+c)} + 2I)/(a*d*(e^{(2dx+2c)} - 1)^2) - 2I/(a*d*(Ie^{(dx+c)} + 1))$

$$3.221 \quad \int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Csch[c + d\*x]^3/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0749999, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Csch[c + d\*x]^3/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Csch[c + d\*x]^3/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx = \int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.023, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[Csch[c + d\*x]^3/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] \$Aborted



---

**Maple [A]** time = 1.332, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx + c))^3}{(fx + e)(a + ia \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)`

[Out] `int(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `-8*f*integrate(1/(-4*I*a*d*f^2*x^2 - 8*I*a*d*e*f*x - 4*I*a*d*e^2 + 4*(a*d*f^2*x^2*e^c + 2*a*d*e*f*x*e^c + a*d*e^2*e^c))*e^(d*x)), x) - 8*(4*d*f*x + 4*d*e + (3*d*f*x*e^(4*c) + (3*d*e - f)*e^(4*c))*e^(4*d*x) + (-3*I*d*f*x*e^(3*c) + (-3*I*d*e + I*f)*e^(3*c))*e^(3*d*x) - (5*d*f*x*e^(2*c) + (5*d*e - f)*e^(2*c))*e^(2*d*x) + (I*d*f*x*e^c + (I*d*e - I*f)*e^c)*e^(d*x))/(-8*I*a*d^2*f^2*x^2 - 16*I*a*d^2*e*f*x - 8*I*a*d^2*e^2 + 8*(a*d^2*f^2*x^2*e^(5*c) + 2*a*d^2*e*f*x*e^(5*c) + a*d^2*e^2*e^(5*c))*e^(5*d*x) + (-8*I*a*d^2*f^2*x^2*e^(4*c) - 16*I*a*d^2*e*f*x*e^(4*c) - 8*I*a*d^2*e^2*e^(4*c))*e^(4*d*x) - 16*(a*d^2*f^2*x^2*e^(3*c) + 2*a*d^2*e*f*x*e^(3*c) + a*d^2*e^2*e^(3*c))*e^(3*d*x) + (16*I*a*d^2*f^2*x^2*e^(2*c) + 32*I*a*d^2*e*f*x*e^(2*c) + 16*I*a*d^2*e^2*e^(2*c))*e^(2*d*x) + 8*(a*d^2*f^2*x^2*e^c + 2*a*d^2*e*f*x*e^c + a*d^2*e^2*e^c)*e^(d*x)) - 8*integrate(1/16*(3*d^2*f^2*x^2 + 3*d^2*e^2 + 2*I*d*e*f - 2*f^2 + (6*d^2*e*f + 2*I*d*f^2)*x)/(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^2*f^3*x^3*e^c + 3*a*d^2*e*f^2*x^2*e^c + 3*a*d^2*e^2*f*x*e^c + a*d^2*e^3*e^c))*e^(d*x)), x) - 8*integrate(-1/16*(3*d^2*f^2*x^2 + 3*d^2*e^2 - 2*I*d*e*f - 2*f^2 + (6*d^2*e*f - 2*I*d*f^2)*x)/(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 - (a*d^2*f^3*x^3*e^c + 3*a*d^2*e*f^2*x^2*e^c + 3*a*d^2*e^2*f*x*e^c + a*d^2*e^3*e^c))*e^(d*x)), x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^3/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-(4*d*f*x + 4*d*e + (3*d*f*x + 3*d*e - f)*e^{(4*d*x + 4*c)} - (3*I*d*f*x + 3*I*d*e - I*f)*e^{(3*d*x + 3*c)} - (5*d*f*x + 5*d*e - f)*e^{(2*d*x + 2*c)} - (-I*d*f*x - I*d*e + I*f)*e^{(d*x + c)} - (-I*a*d^2*f^2*x^2 - 2*I*a*d^2*e*f*x - I*a*d^2*e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*e^{(5*d*x + 5*c)} + (-I*a*d^2*f^2*x^2 - 2*I*a*d^2*e*f*x - I*a*d^2*e^2)*e^{(4*d*x + 4*c)} - 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*e^{(3*d*x + 3*c)} + (2*I*a*d^2*f^2*x^2 + 4*I*a*d^2*e*f*x + 2*I*a*d^2*e^2)*e^{(2*d*x + 2*c)} + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*e^{(d*x + c)})*integral((4*d*f^2*x + 4*d*e*f - (3*d^2*f^2*x^2 + 3*d^2*e^2 + 2*d*e*f - 2*f^2 + 2*(3*d^2*e*f + d*f^2)*x)*e^{(2*d*x + 2*c)} + (3*I*d^2*f^2*x^2 + 3*I*d^2*e^2 + 2*I*d*e*f - 2*I*f^2 + (6*I*d^2*e*f + 2*I*d*f^2)*x)*e^{(d*x + c)})/(I*a*d^2*f^3*x^3 + 3*I*a*d^2*e*f^2*x^2 + 3*I*a*d^2*e^2*f*x + I*a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^{(3*d*x + 3*c)} + (-I*a*d^2*f^3*x^3 - 3*I*a*d^2*e*f^2*x^2 - 3*I*a*d^2*e^2*f*x - I*a*d^2*e^3)*e^{(2*d*x + 2*c)} - (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^{(d*x + c)}), x)/(-I*a*d^2*f^2*x^2 - 2*I*a*d^2*e*f*x - I*a*d^2*e^2 + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*e^{(5*d*x + 5*c)} + (-I*a*d^2*f^2*x^2 - 2*I*a*d^2*e*f*x - I*a*d^2*e^2)*e^{(4*d*x + 4*c)} - 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*e^{(3*d*x + 3*c)} + (2*I*a*d^2*f^2*x^2 + 4*I*a*d^2*e*f*x + 2*I*a*d^2*e^2)*e^{(2*d*x + 2*c)} + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*e^{(d*x + c)})$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)\*\*3/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.222 \quad \int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

**Optimal.** Leaf size=33

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Csch[c + d\*x]^3/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0739772, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Csch[c + d\*x]^3/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Csch[c + d\*x]^3/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx = \int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.047, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[Csch[c + d\*x]^3/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] \$Aborted

---

**Maple [A]** time = 2.257, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx + c))^3}{(fx + e)^2 (a + ia \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)`

[Out] `int(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `-8*f*integrate(1/(-2*I*a*d*f^3*x^3 - 6*I*a*d*e*f^2*x^2 - 6*I*a*d*e^2*f*x - 2*I*a*d*e^3 + 2*(a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c + a*d*e^3*e^c)*e^(d*x)), x) - 8*(4*d*f*x + 4*d*e + (3*d*f*x*e^(4*c) + (3*d*e - 2*f)*e^(4*c))*e^(4*d*x) + (-3*I*d*f*x*e^(3*c) + (-3*I*d*e + 2*I*f)*e^(3*c))*e^(3*d*x) - (5*d*f*x*e^(2*c) + (5*d*e - 2*f)*e^(2*c))*e^(2*d*x) + (I*d*f*x*e^c + (I*d*e - 2*I*f)*e^c)*e^(d*x))/(-8*I*a*d^2*f^3*x^3 - 24*I*a*d^2*e*f^2*x^2 - 24*I*a*d^2*e^2*f*x - 8*I*a*d^2*e^3 + 8*(a*d^2*f^3*x^3*e^(5*c) + 3*a*d^2*e*f^2*x^2*e^(5*c) + 3*a*d^2*e^2*f*x*e^(5*c) + a*d^2*e^3*e^(5*c))*e^(5*d*x) + (-8*I*a*d^2*f^3*x^3*e^(4*c) - 24*I*a*d^2*e*f^2*x^2*e^(4*c) - 24*I*a*d^2*e^2*f*x*e^(4*c) - 8*I*a*d^2*e^3*e^(4*c))*e^(4*d*x) - 16*(a*d^2*f^3*x^3*e^(3*c) + 3*a*d^2*e*f^2*x^2*e^(3*c) + 3*a*d^2*e^2*f*x*e^(3*c) + a*d^2*e^3*e^(3*c))*e^(3*d*x) + (16*I*a*d^2*f^3*x^3*e^(2*c) + 48*I*a*d^2*e*f^2*x^2*e^(2*c) + 48*I*a*d^2*e^2*f*x*e^(2*c) + 16*I*a*d^2*e^3*e^(2*c))*e^(2*d*x) + 8*(a*d^2*f^3*x^3*e^c + 3*a*d^2*e*f^2*x^2*e^c + 3*a*d^2*e^2*f*x*e^c + a*d^2*e^3*e^c)*e^(d*x)) - 8*integrate(1/16*(3*d^2*f^2*x^2 + 3*d^2*e^2 + 4*I*d*e*f - 6*f^2 + (6*d^2*e*f + 4*I*d*f^2)*x)/(a*d^2*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4 + (a*d^2*f^4*x^4*e^c + 4*a*d^2*e*f^3*x^3*e^c + 6*a*d^2*e^2*f^2*x^2*e^c + 4*a*d^2*e^3*f*x*e^c + a*d^2*e^4*e^c)*e^(d*x)), x) - 8*integrate(-1/16*(3*d^2*f^2*x^2 + 3*d^2*e^2 - 4*I*`

$$d*ef - 6*f^2 + (6*d^2*ef - 4*I*d*f^2)*x)/(a*d^2*f^4*x^4 + 4*a*d^2*ef^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4 - (a*d^2*f^4*x^4*e^c + 4*a*d^2*ef^3*x^3*e^c + 6*a*d^2*e^2*f^2*x^2*e^c + 4*a*d^2*e^3*f*x*e^c + a*d^2*e^4*e^c)*e^{(d*x)}, x)$$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^3/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -(4*d*f*x + 4*d*e + (3*d*f*x + 3*d*e - 2*f)*e^{(4*d*x + 4*c)} - (3*I*d*f*x + 3*I*d*e - 2*I*f)*e^{(3*d*x + 3*c)} - (5*d*f*x + 5*d*e - 2*f)*e^{(2*d*x + 2*c)} \\ & - (-I*d*f*x - I*d*e + 2*I*f)*e^{(d*x + c)} - (-I*a*d^2*f^3*x^3 - 3*I*a*d^2*ef^2*x^2 - 3*I*a*d^2*e^2*f*x - I*a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*ef^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^{(5*d*x + 5*c)} \\ & + (-I*a*d^2*f^3*x^3 - 3*I*a*d^2*ef^2*x^2 - 3*I*a*d^2*e^2*f*x - I*a*d^2*e^3)*e^{(4*d*x + 4*c)} - 2*(a*d^2*f^3*x^3 + 3*a*d^2*ef^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^{(3*d*x + 3*c)} \\ & + (2*I*a*d^2*f^3*x^3 + 6*I*a*d^2*ef^2*x^2 + 6*I*a*d^2*e^2*f*x + 2*I*a*d^2*e^3)*e^{(2*d*x + 2*c)} + (a*d^2*f^3*x^3 + 3*a*d^2*ef^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^{(d*x + c)} \\ & ) * integral((8*d*f^2*x + 8*d*ef - (3*d^2*f^2*x^2 + 3*d^2*e^2 + 4*d*ef - 6*f^2 + 2*(3*d^2*ef + 2*d*f^2)*x)*e^{(2*d*x + 2*c)} \\ & + (3*I*d^2*f^2*x^2 + 3*I*d^2*e^2 + 4*I*d*ef - 6*I*f^2 + (6*I*d^2*ef + 4*I*d*f^2)*x)*e^{(d*x + c)}) / (I*a*d^2*f^4*x^4 + 4*I*a*d^2*ef^3*x^3 + 6*I*a*d^2*e^2*f^2*x^2 + 4*I*a*d^2*e^3*f*x + I*a*d^2*e^4 + (a*d^2*f^4*x^4 + 4*a*d^2*ef^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4)*e^{(3*d*x + 3*c)} \\ & + (-I*a*d^2*f^4*x^4 - 4*I*a*d^2*ef^3*x^3 - 6*I*a*d^2*e^2*f^2*x^2 - 4*I*a*d^2*e^3*f*x - I*a*d^2*e^4)*e^{(2*d*x + 2*c)} - (a*d^2*f^4*x^4 + 4*a*d^2*ef^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4)*e^{(d*x + c)}) \\ & , x) / (-I*a*d^2*f^3*x^3 - 3*I*a*d^2*ef^2*x^2 - 3*I*a*d^2*e^2*f*x - I*a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*ef^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^{(5*d*x + 5*c)} \\ & + (-I*a*d^2*f^3*x^3 - 3*I*a*d^2*ef^2*x^2 - 3*I*a*d^2*e^2*f*x - I*a*d^2*e^3)*e^{(4*d*x + 4*c)} - 2*(a*d^2*f^3*x^3 + 3*a*d^2*ef^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^{(3*d*x + 3*c)} \\ & + (2*I*a*d^2*f^3*x^3 + 6*I*a*d^2*ef^2*x^2 + 6*I*a*d^2*e^2*f*x + 2*I*a*d^2*e^3)*e^{(2*d*x + 2*c)} + (a*d^2*f^3*x^3 + 3*a*d^2*ef^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^{(d*x + c)} \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.223 \quad \int \frac{(e+fx)^3 \sinh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=453

$$\frac{6af^2(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}} - \frac{6af^2(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}} - \frac{3af(e+fx)^2\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}} +$$

```
[Out] (e + f*x)^4/(4*b*f) - (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) + (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) - (3*a*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2) + (3*a*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2) + (6*a*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (6*a*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (6*a*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^4) + (6*a*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^4)
```

**Rubi [A]** time = 0.794095, antiderivative size = 453, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$ , Rules used = {5557, 32, 3322, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{6af^2(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}} - \frac{6af^2(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}} - \frac{3af(e+fx)^2\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}} +$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e + f*x)^4/(4*b*f) - (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) + (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) - (3*a*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2) + (3*a*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2) + (6*a*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (6*a*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (6*a*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^4) + (6*a*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^4)
```



$t[a^2 + b^2]*d^4$

### Rule 5557

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Sinh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sinh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

### Rule 3322

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*sin[(e\_.) + (Complex[0, fz\_])\* (f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(-(I\*b) + 2\*a\*E^(-(I\*e) + f\*fz\*x) + I\*b\*E^(2\*(-(I\*e) + f\*fz\*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.)\*(F\_)^(v\_)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[((f + g\*x)^m\*F^u)/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[((f + g\*x)^m\*F^u)/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^(c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.)]\*(f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n]])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f

, g, n}, x] && GtQ[m, 0]

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \sinh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 dx}{b} - \frac{a \int \frac{(e+fx)^3}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{(e+fx)^4}{4bf} - \frac{(2a) \int \frac{e^{c+dx}(e+fx)^3}{-b+2ae^{c+dx}+be^{2(c+dx)}} dx}{b} \\
&= \frac{(e+fx)^4}{4bf} - \frac{(2a) \int \frac{e^{c+dx}(e+fx)^3}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{\sqrt{a^2+b^2}} + \frac{(2a) \int \frac{e^{c+dx}(e+fx)^3}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{\sqrt{a^2+b^2}} \\
&= \frac{(e+fx)^4}{4bf} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{(3af) \int (e+fx)^3 dx}{b\sqrt{a^2+b^2}} \\
&= \frac{(e+fx)^4}{4bf} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} - \frac{3af(e+fx)^3}{b\sqrt{a^2+b^2}} \\
&= \frac{(e+fx)^4}{4bf} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} - \frac{3af(e+fx)^3}{b\sqrt{a^2+b^2}} \\
&= \frac{(e+fx)^4}{4bf} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} - \frac{3af(e+fx)^3}{b\sqrt{a^2+b^2}} \\
&= \frac{(e+fx)^4}{4bf} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} - \frac{3af(e+fx)^3}{b\sqrt{a^2+b^2}} \\
&= \frac{(e+fx)^4}{4bf} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} - \frac{3af(e+fx)^3}{b\sqrt{a^2+b^2}}
\end{aligned}$$

**Mathematica [A]** time = 2.31197, size = 607, normalized size = 1.34

$$a \left( -3d^2 f(e+fx)^2 \text{PolyLog} \left( 2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a} \right) + 3d^2 f(e+fx)^2 \text{PolyLog} \left( 2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} \right) + 6def^2 \text{PolyLog} \left( 3, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a} \right) - 6def^2 \text{PolyLog} \left( 3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(((e + f\*x)^3\*Sinh[c + d\*x])/(a + b\*Sinh[c + d\*x])),x]

[Out] (x\*(4\*e^3 + 6\*e^2\*f\*x + 4\*e\*f^2\*x^2 + f^3\*x^3))/(4\*b) + (a\*(2\*d^3\*e^3\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]] - 3\*d^3\*e^2\*f\*x\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])] - 3\*d^3\*e\*f^2\*x^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])] - d^3\*f^3\*x^3\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]) + 3\*d^3\*e^2\*f\*x\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] + 3\*d^3\*e\*f^2\*x^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] + d^3\*f^3\*x^3\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b\*sqrt(a^2 + b^2))

$$1 + (b \cdot E^{(c + d \cdot x)}) / (a + \sqrt{a^2 + b^2}) - 3 \cdot d^2 \cdot f \cdot (e + f \cdot x)^2 \cdot \text{PolyLog}[2, (b \cdot E^{(c + d \cdot x)}) / (-a + \sqrt{a^2 + b^2})] + 3 \cdot d^2 \cdot f \cdot (e + f \cdot x)^2 \cdot \text{PolyLog}[2, -((b \cdot E^{(c + d \cdot x)}) / (a + \sqrt{a^2 + b^2}))] + 6 \cdot d \cdot e \cdot f^2 \cdot \text{PolyLog}[3, (b \cdot E^{(c + d \cdot x)}) / (-a + \sqrt{a^2 + b^2})] + 6 \cdot d \cdot f^3 \cdot x \cdot \text{PolyLog}[3, (b \cdot E^{(c + d \cdot x)}) / (-a + \sqrt{a^2 + b^2})] - 6 \cdot d \cdot e \cdot f^2 \cdot \text{PolyLog}[3, -((b \cdot E^{(c + d \cdot x)}) / (a + \sqrt{a^2 + b^2}))] - 6 \cdot d \cdot f^3 \cdot x \cdot \text{PolyLog}[3, -((b \cdot E^{(c + d \cdot x)}) / (a + \sqrt{a^2 + b^2}))] - 6 \cdot f^3 \cdot \text{PolyLog}[4, (b \cdot E^{(c + d \cdot x)}) / (-a + \sqrt{a^2 + b^2})] + 6 \cdot f^3 \cdot \text{PolyLog}[4, -((b \cdot E^{(c + d \cdot x)}) / (a + \sqrt{a^2 + b^2}))] / (b \cdot \sqrt{a^2 + b^2} \cdot d^4)$$

**Maple [F]** time = 0.241, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \sinh(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 3.12158, size = 2677, normalized size = 5.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

```
[Out] 1/4*((a^2 + b^2)*d^4*f^3*x^4 + 4*(a^2 + b^2)*d^4*e*f^2*x^3 + 6*(a^2 + b^2)*
d^4*e^2*f*x^2 + 4*(a^2 + b^2)*d^4*e^3*x - 24*a*b*f^3*sqrt((a^2 + b^2)/b^2)*
polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*a*b*f^3*sqrt((a^2 + b^2)/b^2)*polyl
og(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2))/b) - 12*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2*x + a
*b*d^2*e^2*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c
) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) +
12*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2*x + a*b*d^2*e^2*f)*sqrt((a^2 + b^2)/
b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 4*(a*b*d^3*e^3 - 3*a*b*c*d^2*e
^2*f + 3*a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(
d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(a*b*d^
3*e^3 - 3*a*b*c*d^2*e^2*f + 3*a*b*c^2*d*e*f^2 - a*b*c^3*f^3)*sqrt((a^2 + b^
2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^
2) + 2*a) - 4*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*x +
3*a*b*c*d^2*e^2*f - 3*a*b*c^2*d*e*f^2 + a*b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*
log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2
+ 3*a*b*d^3*e^2*f*x + 3*a*b*c*d^2*e^2*f - 3*a*b*c^2*d*e*f^2 + a*b*c^3*f^3)
*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*
x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 24*(a*b*d*f^3*x +
a*b*d*e*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*
x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 24
*(a*b*d*f^3*x + a*b*d*e*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2))/b))/((a^2*b + b^3)*d^4)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \sinh(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

$$3.224 \quad \int \frac{(e+fx)^2 \sinh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=337

$$\frac{2af(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}} + \frac{2af(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2\sqrt{a^2+b^2}} + \frac{2af^2\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}} - \frac{2af^2\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}}$$

```
[Out] (e + f*x)^3/(3*b*f) - (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) + (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) - (2*a*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2) + (2*a*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2) + (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3)
```

**Rubi [A]** time = 0.708701, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {5557, 32, 3322, 2264, 2190, 2531, 2282, 6589}

$$\frac{2af(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}} + \frac{2af(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2\sqrt{a^2+b^2}} + \frac{2af^2\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}} - \frac{2af^2\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e + f*x)^3/(3*b*f) - (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) + (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*Sqrt[a^2 + b^2]*d) - (2*a*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2) + (2*a*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^2) + (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3) - (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*Sqrt[a^2 + b^2]*d^3)
```

**Rule 5557**

```
Int[(((e_.) + (f_.)*(x_.))^(m_.)*Sinh[(c_.) + (d_.)*(x_.)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^n, x_Symbol]]
```

+ d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sinh[c + d\*x]^(n - 1)) / (a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

### Rule 3322

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*sin[(e\_.) + (Complex[0, fz\_])\* (f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(- (I\*b) + 2\*a\*E^(-(I\*e) + f\*fz\*x) + I\*b\*E^(2\*(-I\*e) + f\*fz\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.) \*(F\_)^(v\_)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[ ((f + g\*x)^m\*F^u)/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[ ((f + g\*x)^ m\*F^u)/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

### Rule 2190

Int((((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/ ((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)), x\_Symbol] := Simp [((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Di st[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x) ))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^(c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.)]\*((f\_.) + (g\_.) \*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x) ))^n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f , g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x] , Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi onOfExponentialQ[u, x] && !MatchQ[u, (w\_.)\*((a\_.)\*(v\_)^(n\_.))^(m\_) /; FreeQ[



```
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \sinh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 dx}{b} - \frac{a \int \frac{(e+fx)^2}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{(e+fx)^3}{3bf} - \frac{(2a) \int \frac{e^{c+dx}(e+fx)^2}{-b+2ae^{c+dx}+be^{2(c+dx)}} dx}{b} \\
&= \frac{(e+fx)^3}{3bf} - \frac{(2a) \int \frac{e^{c+dx}(e+fx)^2}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{\sqrt{a^2+b^2}} + \frac{(2a) \int \frac{e^{c+dx}(e+fx)^2}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{\sqrt{a^2+b^2}} \\
&= \frac{(e+fx)^3}{3bf} - \frac{a(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{(2af) \int (e+fx)}{b\sqrt{a^2+b^2}} \\
&= \frac{(e+fx)^3}{3bf} - \frac{a(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} - \frac{2af(e+fx)}{b\sqrt{a^2+b^2}} \\
&= \frac{(e+fx)^3}{3bf} - \frac{a(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} - \frac{2af(e+fx)}{b\sqrt{a^2+b^2}} \\
&= \frac{(e+fx)^3}{3bf} - \frac{a(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} - \frac{2af(e+fx)}{b\sqrt{a^2+b^2}}
\end{aligned}$$

**Mathematica [A]** time = 1.69056, size = 366, normalized size = 1.09

$$a \left( -2df(e+fx) \text{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) + 2df(e+fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) + 2f^2 \text{PolyLog}\left(3, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) - 2f^2 \text{PolyLog}\left(3, \frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2))/(3*b) + (a*(2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]) - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]) + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) - 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]) + 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*f^2*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]))/(b*Sqrt[a^2 + b^2]*d^3)
```

**Maple [F]** time = 0.179, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \sinh(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [C]** time = 2.74246, size = 1913, normalized size = 5.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{3}((a^2 + b^2)d^3f^2x^3 + 3(a^2 + b^2)d^3e*fx^2 + 3(a^2 + b^2)d^3e^2x + 6abf^2\sqrt{(a^2 + b^2)/b^2}\text{polylog}(3, (a\cosh(dx + c) + a\sinh(dx + c) + (b\cosh(dx + c) + b\sinh(dx + c))\sqrt{(a^2 + b^2)/b^2}))/b) - 6abf^2\sqrt{(a^2 + b^2)/b^2}\text{polylog}(3, (a\cosh(dx + c) + a\sinh(dx + c) - (b\cosh(dx + c) + b\sinh(dx + c))\sqrt{(a^2 + b^2)/b^2}))/b) - 6(a*b*d*f^2*x + a*b*d*e*f)\sqrt{(a^2 + b^2)/b^2}\text{dilog}((a\cosh(dx + c) + a\sinh(dx + c) + (b\cosh(dx + c) + b\sinh(dx + c))\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 6(a*b*d*f^2*x + a*b*d*e*f)\sqrt{(a^2 + b^2)/b^2}\text{dilog}((a\cosh(dx + c) + a\sinh(dx + c) - (b\cosh(dx + c) + b\sinh(dx + c))\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 3(a*b*d^2*e^2 - 2a*b*c*d*e*f + a*b*c^2*f^2)\sqrt{(a^2 + b^2)/b^2}\log(2*b*\cosh(dx + c) + 2*b*\sinh(dx + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - 3(a*b*d^2*e^2 - 2a*b*c*d*e*f + a*b*c^2*f^2)\sqrt{(a^2 + b^2)/b^2}\log(2*b*\cosh(dx + c) + 2*b*\sinh(dx + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - 3(a*b*d^2*f^2*x^2 + 2a*b*d^2*e*f*x + 2a*b*c*d*e*f - a*b*c^2*f^2)\sqrt{(a^2 + b^2)/b^2}\log(-(a\cosh(dx + c) + a\sinh(dx + c) + (b\cosh(dx + c) + b\sinh(dx + c))\sqrt{(a^2 + b^2)/b^2} - b)/b) + 3(a*b*d^2*f^2*x^2 + 2a*b*d^2*e*f*x + 2a*b*c*d*e*f - a*b*c^2*f^2)\sqrt{(a^2 + b^2)/b^2}\log(-(a\cosh(dx + c) + a\sinh(dx + c) - (b\cosh(dx + c) + b\sinh(dx + c))\sqrt{(a^2 + b^2)/b^2} - b)/b))/((a^2*b + b^3)*d^3)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \sinh(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

$$3.225 \quad \int \frac{(e+fx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=220

$$-\frac{af \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}} + \frac{af \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2\sqrt{a^2+b^2}} - \frac{a(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{bd\sqrt{a^2+b^2}} + \frac{a(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} + 1\right)}{bd\sqrt{a^2+b^2}}$$

[Out] (e\*x)/b + (f\*x^2)/(2\*b) - (a\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d) + (a\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d) - (a\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^2) + (a\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^2)

**Rubi [A]** time = 0.412666, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5557, 3322, 2264, 2190, 2279, 2391}

$$-\frac{af \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}} + \frac{af \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2\sqrt{a^2+b^2}} - \frac{a(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{bd\sqrt{a^2+b^2}} + \frac{a(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} + 1\right)}{bd\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Sinh[c + d\*x])/(a + b\*Sinh[c + d\*x]), x]

[Out] (e\*x)/b + (f\*x^2)/(2\*b) - (a\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d) + (a\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d) - (a\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^2) + (a\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^2)

### Rule 5557

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Sinh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sinh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\sinh(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx) dx}{b} - \frac{a \int \frac{e+fx}{a+b\sinh(c+dx)} dx}{b} \\
&= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{(2a) \int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}} dx}{b} \\
&= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{(2a) \int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{\sqrt{a^2+b^2}} + \frac{(2a) \int \frac{e^{c+dx}(e+fx)}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{\sqrt{a^2+b^2}} \\
&= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{a(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{(af) \int \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) dx}{b\sqrt{a^2+b^2}d} \\
&= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{a(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{(af) \text{Subst}\left(\int \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right) dx\right)}{b\sqrt{a^2+b^2}d} \\
&= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{a(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} + \frac{a(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} - \frac{af \text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d}
\end{aligned}$$

**Mathematica [A]** time = 0.699166, size = 163, normalized size = 0.74

$$\frac{x(2e+fx)}{2b} - \frac{a \left( f \text{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) - f \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) + d(e+fx) \left( \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right) - \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) \right) \right)}{bd^2\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((e+f\*x)\*Sinh[c+d\*x])/(a+b\*Sinh[c+d\*x]),x]

[Out] (x\*(2\*e+f\*x))/(2\*b) - (a\*(d\*(e+f\*x)\*(Log[1+(b\*E^(c+d\*x))]/(a-Sqrt[a^2+b^2])) - Log[1+(b\*E^(c+d\*x))/(a+Sqrt[a^2+b^2]])) + f\*PolyLog[2, (b\*E^(c+d\*x))/(-a+Sqrt[a^2+b^2])] - f\*PolyLog[2, -((b\*E^(c+d\*x))/(a+Sqrt[a^2+b^2]))])/(b\*Sqrt[a^2+b^2]\*d^2)

**Maple [B]** time = 0.069, size = 440, normalized size = 2.

$$\frac{fx^2}{2b} + \frac{ex}{b} + 2 \frac{ae}{bd\sqrt{a^2+b^2}} \text{Artanh}\left(\frac{1}{2} \frac{2be^{dx+c}+2a}{\sqrt{a^2+b^2}}\right) - \frac{afx}{bd} \ln\left(\left(-be^{dx+c} + \sqrt{a^2+b^2} - a\right)\left(-a + \sqrt{a^2+b^2}\right)^{-1}\right) \frac{1}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/2*f*x^2/b+e*x/b+2*a/b/d*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a
)/(a^2+b^2)^(1/2))-a/b/d*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2
)-a)/(-a+(a^2+b^2)^(1/2)))*x-a/b/d^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a
^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+a/b/d*f/(a^2+b^2)^(1/2)*ln((b*exp(
d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+a/b/d^2*f/(a^2+b^2)^(1/2)*
ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-a/b/d^2*f/(a^2+b
^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+a/b
/d^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(
1/2)))-2*a/b/d^2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2
+b^2)^(1/2))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [B]** time = 2.53014, size = 1243, normalized size = 5.65

$$(a^2 + b^2)d^2fx^2 + 2(a^2 + b^2)d^2ex - 2abf\sqrt{\frac{a^2+b^2}{b^2}}\operatorname{Li}_2\left(\frac{a\cosh(dx+c)+a\sinh(dx+c)+(b\cosh(dx+c)+b\sinh(dx+c))\sqrt{\frac{a^2+b^2}{b^2}-b}}{b} + 1\right) + 2abf\sqrt{\frac{a^2+b^2}{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + b^2)*d^2*f*x^2 + 2*(a^2 + b^2)*d^2*e*x - 2*a*b*f*sqrt((a^2 + b^
2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*a*b*f*sqrt((a^2 + b^2)/b^
2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a*b*d*e - a*b*c*f)*sqrt((a^2
```



```

+ b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2
)/b^2) + 2*a) - 2*(a*b*d*e - a*b*c*f)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*
x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(a*b*d*f*
x + a*b*c*f)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
+ (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*(a*
b*d*f*x + a*b*c*f)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x
+ c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b))/
((a^2*b + b^3)*d^2)

```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \sinh(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)\*sinh(d\*x + c)/(b\*sinh(d\*x + c) + a), x)

$$3.226 \quad \int \frac{\sinh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=54

$$\frac{2a \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd\sqrt{a^2+b^2}} + \frac{x}{b}$$

[Out] x/b + (2\*a\*ArcTanh[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[a^2 + b^2]])/(b\*Sqrt[a^2 + b^2]\*d)

**Rubi [A]** time = 0.0748817, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {2735, 2660, 618, 204}

$$\frac{2a \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd\sqrt{a^2+b^2}} + \frac{x}{b}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d\*x]/(a + b\*Sinh[c + d\*x]),x]

[Out] x/b + (2\*a\*ArcTanh[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[a^2 + b^2]])/(b\*Sqrt[a^2 + b^2]\*d)

#### Rule 2735

Int[((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])/((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*x)/d, x] - Dist[(b\*c - a\*d)/d, Int[1/(c + d\*Sin[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 2660

Int[((a\_) + (b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + 2\*b\*e\*x + a\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

#### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{x}{b} - \frac{a \int \frac{1}{a+b\sinh(c+dx)} dx}{b} \\ &= \frac{x}{b} + \frac{(2ia) \operatorname{Subst}\left(\int \frac{1}{a-2ibx+ax^2} dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{bd} \\ &= \frac{x}{b} - \frac{(4ia) \operatorname{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2} dx, x, -2ib+2a \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{bd} \\ &= \frac{x}{b} + \frac{2a \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d} \end{aligned}$$

**Mathematica [A]** time = 0.105599, size = 64, normalized size = 1.19

$$\frac{2a \tan^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}} + \frac{c}{d} + x$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(Sqrt[-a^2 - b^2]*d))/b
```

**Maple [A]** time = 0.003, size = 87, normalized size = 1.6

$$\frac{1}{bd} \ln \left( \tanh \left( \frac{dx}{2} + \frac{c}{2} \right) + 1 \right) - 2 \frac{a}{bd \sqrt{a^2 + b^2}} \operatorname{Artanh} \left( \frac{1}{2} \frac{2a \tanh(1/2 dx + c/2) - 2b}{\sqrt{a^2 + b^2}} \right) - \frac{1}{bd} \ln \left( \tanh \left( \frac{dx}{2} + \frac{c}{2} \right) - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] 1/d/b\*ln(tanh(1/2\*d\*x+1/2\*c)+1)-2/d\*a/b/(a^2+b^2)^(1/2)\*arctanh(1/2\*(2\*a\*tanh(1/2\*d\*x+1/2\*c)-2\*b)/(a^2+b^2)^(1/2))-1/d/b\*ln(tanh(1/2\*d\*x+1/2\*c)-1)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.47331, size = 473, normalized size = 8.76

$$\frac{(a^2 + b^2)dx + \sqrt{a^2 + b^2}a \log \left( \frac{b^2 \cosh(dx+c)^2 + b^2 \sinh(dx+c)^2 + 2ab \cosh(dx+c) + 2a^2 + b^2 + 2(b^2 \cosh(dx+c) + ab) \sinh(dx+c) + 2\sqrt{a^2 + b^2}(b \cosh(dx+c) + a \sinh(dx+c))}{b \cosh(dx+c)^2 + b \sinh(dx+c)^2 + 2a \cosh(dx+c) + 2(b \cosh(dx+c) + a) \sinh(dx+c) - b} \right)}{(a^2b + b^3)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] ((a^2 + b^2)\*d\*x + sqrt(a^2 + b^2)\*a\*log((b^2\*cosh(d\*x + c)^2 + b^2\*sinh(d\*x + c)^2 + 2\*a\*b\*cosh(d\*x + c) + 2\*a^2 + b^2 + 2\*(b^2\*cosh(d\*x + c) + a\*b)\*sinh(d\*x + c) + 2\*sqrt(a^2 + b^2)\*(b\*cosh(d\*x + c) + b\*sinh(d\*x + c) + a)))/(b\*cosh(d\*x + c)^2 + b\*sinh(d\*x + c)^2 + 2\*a\*cosh(d\*x + c) + 2\*(b\*cosh(d\*x + c) + a)\*sinh(d\*x + c) - b))/((a^2\*b + b^3)\*d)

**Sympy [A]** time = 169.228, size = 371, normalized size = 6.87

$$\left\{ \begin{array}{ll} \infty x & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{x}{b} & \text{for } a = 0 \\ -\frac{b^2 dx \tanh\left(\frac{c}{2} + \frac{dx}{2}\right)}{-b^3 d \tanh\left(\frac{c}{2} + \frac{dx}{2}\right) + ib^2 d \sqrt{b^2}} + \frac{2b^2}{-b^3 d \tanh\left(\frac{c}{2} + \frac{dx}{2}\right) + ib^2 d \sqrt{b^2}} + \frac{ibdx \sqrt{b^2}}{-b^3 d \tanh\left(\frac{c}{2} + \frac{dx}{2}\right) + ib^2 d \sqrt{b^2}} & \text{for } a = -\sqrt{-b^2} \\ \frac{b^2 dx \tanh\left(\frac{c}{2} + \frac{dx}{2}\right)}{b^3 d \tanh\left(\frac{c}{2} + \frac{dx}{2}\right) + ib^2 d \sqrt{b^2}} - \frac{2b^2}{b^3 d \tanh\left(\frac{c}{2} + \frac{dx}{2}\right) + ib^2 d \sqrt{b^2}} + \frac{ibdx \sqrt{b^2}}{b^3 d \tanh\left(\frac{c}{2} + \frac{dx}{2}\right) + ib^2 d \sqrt{b^2}} & \text{for } a = \sqrt{-b^2} \\ \frac{\cosh(c+dx)}{a+d} & \text{for } b = 0 \\ \frac{x \sinh(c)}{a+b \sinh(c)} & \text{for } d = 0 \\ \frac{a \log\left(\tanh\left(\frac{c}{2} + \frac{dx}{2}\right) - \frac{b}{a} - \frac{\sqrt{a^2+b^2}}{a}\right)}{bd\sqrt{a^2+b^2}} - \frac{a \log\left(\tanh\left(\frac{c}{2} + \frac{dx}{2}\right) - \frac{b}{a} + \frac{\sqrt{a^2+b^2}}{a}\right)}{bd\sqrt{a^2+b^2}} + \frac{x}{b} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Piecewise((zoo\*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/b, Eq(a, 0)), (-b\*\*2\*d\*x\*tanh(c/2 + d\*x/2)/(-b\*\*3\*d\*tanh(c/2 + d\*x/2) + I\*b\*\*2\*d\*sqrt(b\*\*2)) + 2\*b\*\*2/(-b\*\*3\*d\*tanh(c/2 + d\*x/2) + I\*b\*\*2\*d\*sqrt(b\*\*2)) + I\*b\*d\*x\*sqrt(b\*\*2)/(-b\*\*3\*d\*tanh(c/2 + d\*x/2) + I\*b\*\*2\*d\*sqrt(b\*\*2)), Eq(a, -sqrt(-b\*\*2))), (b\*\*2\*d\*x\*tanh(c/2 + d\*x/2)/(b\*\*3\*d\*tanh(c/2 + d\*x/2) + I\*b\*\*2\*d\*sqrt(b\*\*2)) - 2\*b\*\*2/(b\*\*3\*d\*tanh(c/2 + d\*x/2) + I\*b\*\*2\*d\*sqrt(b\*\*2)) + I\*b\*d\*x\*sqrt(b\*\*2)/(b\*\*3\*d\*tanh(c/2 + d\*x/2) + I\*b\*\*2\*d\*sqrt(b\*\*2)), Eq(a, sqrt(-b\*\*2))), (cosh(c + d\*x)/(a\*d), Eq(b, 0)), (x\*sinh(c)/(a + b\*sinh(c)), Eq(d, 0)), (a\*log(tanh(c/2 + d\*x/2) - b/a - sqrt(a\*\*2 + b\*\*2)/a)/(b\*d\*sqrt(a\*\*2 + b\*\*2)) - a\*log(tanh(c/2 + d\*x/2) - b/a + sqrt(a\*\*2 + b\*\*2)/a)/(b\*d\*sqrt(a\*\*2 + b\*\*2)) + x/b, True))

**Giac [A]** time = 1.29623, size = 115, normalized size = 2.13

$$-\frac{a \log\left(\frac{|2be^{(dx+c)}+2a-2\sqrt{a^2+b^2}|}{|2be^{(dx+c)}+2a+2\sqrt{a^2+b^2}|}\right)}{\sqrt{a^2+b^2}bd} + \frac{dx+c}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

```
[Out] -a*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) +  
2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b*d) + (d*x + c)/(b*d)
```

$$3.227 \quad \int \frac{\sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=28

$$\text{Unintegrable}\left(\frac{\sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Sinh[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0490424, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Sinh[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Sinh[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [A]** time = 9.2245, size = 0, normalized size = 0.

$$\int \frac{\sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sinh[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Integrate[Sinh[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

---

**Maple [A]** time = 0.065, size = 0, normalized size = 0.

$$\int \frac{\sinh(dx + c)}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(sinh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-2a \int \frac{e^{(dx+c)}}{b^2fx + b^2e - (b^2fxe^{(2c)} + b^2ee^{(2c)})e^{(2dx)} - 2(abfxe^c + abee^c)e^{(dx)}} dx + \frac{\log(fx + e)}{bf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -2\*a\*integrate(-e^(d\*x + c)/(b^2\*f\*x + b^2\*e - (b^2\*f\*x\*e^(2\*c) + b^2\*e\*e^(2\*c))\*e^(2\*d\*x) - 2\*(a\*b\*f\*x\*e^c + a\*b\*e\*e^c)\*e^(d\*x)), x) + log(f\*x + e)/(b\*f)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh(dx + c)}{afx + ae + (bf x + be) \sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] integral(sinh(d\*x + c)/(a\*f\*x + a\*e + (b\*f\*x + b\*e)\*sinh(d\*x + c)), x)

---



**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx + c)}{(fx + e)(b \sinh(dx + c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate(sinh(d\*x + c)/((f\*x + e)\*(b\*sinh(d\*x + c) + a)), x)

$$3.228 \quad \int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=551

$$\frac{6a^2 f^2 (e+fx) \text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d^3 \sqrt{a^2+b^2}} + \frac{6a^2 f^2 (e+fx) \text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2 d^3 \sqrt{a^2+b^2}} + \frac{3a^2 f (e+fx)^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d^2 \sqrt{a^2+b^2}}$$

[Out]  $-(a*(e+f*x)^4)/(4*b^2*f) + (6*f^2*(e+f*x)*\text{Cosh}[c+d*x])/(b*d^3) + ((e+f*x)^3*\text{Cosh}[c+d*x])/(b*d) + (a^2*(e+f*x)^3*\text{Log}[1+(b*E^{(c+d*x)})/(a-\text{Sqrt}[a^2+b^2])])/(b^2*\text{Sqrt}[a^2+b^2]*d) - (a^2*(e+f*x)^3*\text{Log}[1+(b*E^{(c+d*x)})/(a+\text{Sqrt}[a^2+b^2])])/(b^2*\text{Sqrt}[a^2+b^2]*d) + (3*a^2*f*(e+f*x)^2*\text{PolyLog}[2, -((b*E^{(c+d*x)})/(a-\text{Sqrt}[a^2+b^2]))])/(b^2*\text{Sqrt}[a^2+b^2]*d^2) - (3*a^2*f*(e+f*x)^2*\text{PolyLog}[2, -((b*E^{(c+d*x)})/(a+\text{Sqrt}[a^2+b^2]))])/(b^2*\text{Sqrt}[a^2+b^2]*d^2) - (6*a^2*f^2*(e+f*x)*\text{PolyLog}[3, -((b*E^{(c+d*x)})/(a-\text{Sqrt}[a^2+b^2]))])/(b^2*\text{Sqrt}[a^2+b^2]*d^3) + (6*a^2*f^2*(e+f*x)*\text{PolyLog}[3, -((b*E^{(c+d*x)})/(a+\text{Sqrt}[a^2+b^2]))])/(b^2*\text{Sqrt}[a^2+b^2]*d^3) + (6*a^2*f^3*\text{PolyLog}[4, -((b*E^{(c+d*x)})/(a-\text{Sqrt}[a^2+b^2]))])/(b^2*\text{Sqrt}[a^2+b^2]*d^4) - (6*a^2*f^3*\text{PolyLog}[4, -((b*E^{(c+d*x)})/(a+\text{Sqrt}[a^2+b^2]))])/(b^2*\text{Sqrt}[a^2+b^2]*d^4) - (6*f^3*\text{Sinh}[c+d*x])/(b*d^4) - (3*f*(e+f*x)^2*\text{Sinh}[c+d*x])/(b*d^2)$

**Rubi [A]** time = 1.02733, antiderivative size = 551, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$ , Rules used = {5557, 3296, 2637, 32, 3322, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{6a^2 f^2 (e+fx) \text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d^3 \sqrt{a^2+b^2}} + \frac{6a^2 f^2 (e+fx) \text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2 d^3 \sqrt{a^2+b^2}} + \frac{3a^2 f (e+fx)^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d^2 \sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In] Int[((e+f\*x)^3\*Sinh[c+d\*x]^2)/(a+b\*Sinh[c+d\*x]),x]

[Out]  $-(a*(e+f*x)^4)/(4*b^2*f) + (6*f^2*(e+f*x)*\text{Cosh}[c+d*x])/(b*d^3) + ((e+f*x)^3*\text{Cosh}[c+d*x])/(b*d) + (a^2*(e+f*x)^3*\text{Log}[1+(b*E^{(c+d*x)})/(a-\text{Sqrt}[a^2+b^2])])/(b^2*\text{Sqrt}[a^2+b^2]*d) - (a^2*(e+f*x)^3*\text{Log}[1+(b*E^{(c+d*x)})/(a+\text{Sqrt}[a^2+b^2])])/(b^2*\text{Sqrt}[a^2+b^2]*d) + (3*a^2*f*(e+f*x)^2*\text{PolyLog}[2, -((b*E^{(c+d*x)})/(a-\text{Sqrt}[a^2+b^2]))])/(b^2*\text{Sqrt}[a^2+b^2]*d^2) - (3*a^2*f*(e+f*x)^2*\text{PolyLog}[2, -((b*E^{(c+d*x)})/(a+\text{Sqrt}[a^2+b^2]))])/(b^2*\text{Sqrt}[a^2+b^2]*d^2) - (6*a^2*f^2*(e+f*x)*\text{PolyLog}[3,$

$$\begin{aligned} & , -((bE^{(c+dx)})/(a - \sqrt{a^2 + b^2}))/ (b^2 \sqrt{a^2 + b^2} d^3) + (6 \\ & *a^2 f^2 (e + fx) \text{PolyLog}[3, -((bE^{(c+dx)})/(a + \sqrt{a^2 + b^2}))]) / (b \\ & ^2 \sqrt{a^2 + b^2} d^3) + (6a^2 f^3 \text{PolyLog}[4, -((bE^{(c+dx)})/(a - \sqrt{a^2 + b^2}))]) / (b^2 \sqrt{a^2 + b^2} d^4) - (6a^2 f^3 \text{PolyLog}[4, -((bE^{(c+dx)})/(a + \sqrt{a^2 + b^2}))]) / (b^2 \sqrt{a^2 + b^2} d^4) - (6f^3 \text{Sinh}[c + dx]) / (b d^4) - (3f(e + fx)^2 \text{Sinh}[c + dx]) / (b d^2) \end{aligned}$$
Rule 5557

$$\begin{aligned} & \text{Int}[(((e_{.}) + (f_{.})(x_{.}))^{(m_{.})} \text{Sinh}[(c_{.}) + (d_{.})(x_{.})]^{(n_{.})}) / ((a_{.}) + (b_{.}) \\ & \text{Sinh}[(c_{.}) + (d_{.})(x_{.})]), x\_Symbol] \text{:>} \text{Dist}[1/b, \text{Int}[(e + fx)^m \text{Sinh}[c \\ & + dx]^{(n-1)}, x], x] - \text{Dist}[a/b, \text{Int}[(e + fx)^m \text{Sinh}[c + dx]^{(n-1)} \\ & / (a + b \text{Sinh}[c + dx]), x], x] \text{/; FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{IGtQ}\{m, 0\} \\ & \&\& \text{IGtQ}\{n, 0\} \end{aligned}$$
Rule 3296

$$\begin{aligned} & \text{Int}[(((c_{.}) + (d_{.})(x_{.}))^{(m_{.})} \sin[(e_{.}) + (f_{.})(x_{.})]), x\_Symbol] \text{:>} -\text{Simp}[ \\ & ((c + dx)^m \text{Cos}[e + fx]) / f, x] + \text{Dist}[(d m) / f, \text{Int}[(c + dx)^{(m-1)} \text{Cos}[ \\ & e + fx], x], x] \text{/; FreeQ}\{c, d, e, f\}, x\} \&\& \text{GtQ}\{m, 0\} \end{aligned}$$
Rule 2637

$$\begin{aligned} & \text{Int}[\sin[\text{Pi}/2 + (c_{.}) + (d_{.})(x_{.})], x\_Symbol] \text{:>} \text{Simp}[\sin[c + dx] / d, x] \text{/; } \\ & \text{FreeQ}\{c, d\}, x\} \end{aligned}$$
Rule 32

$$\begin{aligned} & \text{Int}[((a_{.}) + (b_{.})(x_{.}))^{(m_{.})}, x\_Symbol] \text{:>} \text{Simp}[(a + b x)^{(m+1)} / (b(m + \\ & 1)), x] \text{/; FreeQ}\{a, b, m\}, x\} \&\& \text{NeQ}\{m, -1\} \end{aligned}$$
Rule 3322

$$\begin{aligned} & \text{Int}[(((c_{.}) + (d_{.})(x_{.}))^{(m_{.})}) / ((a_{.}) + (b_{.}) \sin[(e_{.}) + (\text{Complex}[0, fz_{.}]) * \\ & (f_{.})(x_{.})]), x\_Symbol] \text{:>} \text{Dist}[2, \text{Int}[(c + dx)^m E^{-(I * e) + f * fz * x}) / (- \\ & (I * b) + 2 * a * E^{-(I * e) + f * fz * x}) + I * b * E^{(2 * (-I * e) + f * fz * x))}, x], x] \text{/; F} \\ & \text{reeQ}\{a, b, c, d, e, f, fz\}, x\} \&\& \text{NeQ}\{a^2 - b^2, 0\} \&\& \text{IGtQ}\{m, 0\} \end{aligned}$$
Rule 2264

$$\begin{aligned} & \text{Int}[((F_{.})^{(u_{.})} * ((f_{.}) + (g_{.})(x_{.}))^{(m_{.})}) / ((a_{.}) + (b_{.})(F_{.})^{(u_{.})} + (c_{.}) \\ & * (F_{.})^{(v_{.})}), x\_Symbol] \text{:>} \text{With}\{q = \text{Rt}[b^2 - 4 * a * c, 2]\}, \text{Dist}[(2 * c) / q, \text{Int}[ \\ & ((f + g * x)^m F^u) / (b - q + 2 * c * F^u), x], x] - \text{Dist}[(2 * c) / q, \text{Int}[((f + g * x)^ \\ & m * F^u) / (b + q + 2 * c * F^u), x], x]] \text{/; FreeQ}\{F, a, b, c, f, g\}, x\} \&\& \text{EqQ}\{v, \end{aligned}$$

2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_)]/((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*(a\_) + (b\_)\*(x\_)))^(n\_)]\*((f\_) + (g\_)\*(x\_))^(m\_), x\_Symbol] :> -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n])]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6609

Int[((e\_) + (f\_)\*(x\_))^(m\_)\*PolyLog[n\_, (d\_)\*((F\_)^((c\_)\*(a\_) + (b\_)\*(x\_)))^(p\_)], x\_Symbol] :> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])]/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\* (F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \sinh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^3 \sinh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{(e+fx)^3 \cosh(c+dx)}{bd} - \frac{a \int (e+fx)^3 dx}{b^2} + \frac{a^2 \int \frac{(e+fx)^3}{a+b \sinh(c+dx)} dx}{b^2} - \frac{(3f) \int (e+fx)^2 \cosh(c+dx) dx}{bd} \\
&= -\frac{a(e+fx)^4}{4b^2 f} + \frac{(e+fx)^3 \cosh(c+dx)}{bd} - \frac{3f(e+fx)^2 \sinh(c+dx)}{bd^2} + \frac{(2a^2) \int \frac{e^{c+dx}(e+fx)^3}{-b+2ae^{c+dx}} dx}{b^2} \\
&= -\frac{a(e+fx)^4}{4b^2 f} + \frac{6f^2(e+fx) \cosh(c+dx)}{bd^3} + \frac{(e+fx)^3 \cosh(c+dx)}{bd} - \frac{3f(e+fx)^2 \sinh(c+dx)}{bd^2} \\
&= -\frac{a(e+fx)^4}{4b^2 f} + \frac{6f^2(e+fx) \cosh(c+dx)}{bd^3} + \frac{(e+fx)^3 \cosh(c+dx)}{bd} + \frac{a^2(e+fx)^3 \log\left(\frac{1+e^{c+dx}}{1-e^{c+dx}}\right)}{b^2 \sqrt{a^2+b^2}} \\
&= -\frac{a(e+fx)^4}{4b^2 f} + \frac{6f^2(e+fx) \cosh(c+dx)}{bd^3} + \frac{(e+fx)^3 \cosh(c+dx)}{bd} + \frac{a^2(e+fx)^3 \log\left(\frac{1+e^{c+dx}}{1-e^{c+dx}}\right)}{b^2 \sqrt{a^2+b^2}} \\
&= -\frac{a(e+fx)^4}{4b^2 f} + \frac{6f^2(e+fx) \cosh(c+dx)}{bd^3} + \frac{(e+fx)^3 \cosh(c+dx)}{bd} + \frac{a^2(e+fx)^3 \log\left(\frac{1+e^{c+dx}}{1-e^{c+dx}}\right)}{b^2 \sqrt{a^2+b^2}} \\
&= -\frac{a(e+fx)^4}{4b^2 f} + \frac{6f^2(e+fx) \cosh(c+dx)}{bd^3} + \frac{(e+fx)^3 \cosh(c+dx)}{bd} + \frac{a^2(e+fx)^3 \log\left(\frac{1+e^{c+dx}}{1-e^{c+dx}}\right)}{b^2 \sqrt{a^2+b^2}} \\
&= -\frac{a(e+fx)^4}{4b^2 f} + \frac{6f^2(e+fx) \cosh(c+dx)}{bd^3} + \frac{(e+fx)^3 \cosh(c+dx)}{bd} + \frac{a^2(e+fx)^3 \log\left(\frac{1+e^{c+dx}}{1-e^{c+dx}}\right)}{b^2 \sqrt{a^2+b^2}}
\end{aligned}$$

**Mathematica [A]** time = 2.96971, size = 979, normalized size = 1.78

$$\frac{-a\sqrt{a^2+b^2}f^3x^4d^4 - 4a\sqrt{a^2+b^2}ef^2x^3d^4 - 6a\sqrt{a^2+b^2}e^2fx^2d^4 - 4a\sqrt{a^2+b^2}e^3xd^4 - 8a^2e^3 \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)d^3 + 4b\sqrt{a^2+b^2}e^3x^4d^4}{b^2\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^3\*Sinh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (-4\*a\*Sqrt[a^2 + b^2]\*d^4\*e^3\*x - 6\*a\*Sqrt[a^2 + b^2]\*d^4\*e^2\*f\*x^2 - 4\*a\*Sqrt[a^2 + b^2]\*d^4\*e\*f^2\*x^3 - a\*Sqrt[a^2 + b^2]\*d^4\*f^3\*x^4 - 8\*a^2\*d^3\*e^3\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]] + 4\*b\*Sqrt[a^2 + b^2]\*d^3\*e^3\*x^4)

```

3*Cosh[c + d*x] + 24*b*Sqrt[a^2 + b^2]*d*e*f^2*Cosh[c + d*x] + 12*b*Sqrt[a^
2 + b^2]*d^3*e^2*f*x*Cosh[c + d*x] + 24*b*Sqrt[a^2 + b^2]*d*f^3*x*Cosh[c +
d*x] + 12*b*Sqrt[a^2 + b^2]*d^3*e*f^2*x^2*Cosh[c + d*x] + 4*b*Sqrt[a^2 + b^
2]*d^3*f^3*x^3*Cosh[c + d*x] + 12*a^2*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(
a - Sqrt[a^2 + b^2])] + 12*a^2*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2])] + 4*a^2*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])] - 12*a^2*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
] - 12*a^2*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 4
*a^2*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 12*a^2*d^
2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 12*a^2
*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2
4*a^2*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 24*a^2*d
*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 24*a^2*d*e*f^2*
PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 24*a^2*d*f^3*x*PolyL
og[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 24*a^2*f^3*PolyLog[4, (b
E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 24*a^2*f^3*PolyLog[4, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))] - 12*b*Sqrt[a^2 + b^2]*d^2*e^2*f*Sinh[c + d*x]
- 24*b*Sqrt[a^2 + b^2]*f^3*Sinh[c + d*x] - 24*b*Sqrt[a^2 + b^2]*d^2*e*f^2*x
*Sinh[c + d*x] - 12*b*Sqrt[a^2 + b^2]*d^2*f^3*x^2*Sinh[c + d*x])/(4*b^2*Sqr
t[a^2 + b^2]*d^4)

```

---

**Maple [F]** time = 0.105, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 (\sinh(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

[Out] Exception raised: ValueError

**Fricas [C]** time = 3.19243, size = 5847, normalized size = 10.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\frac{1}{4} \cdot (2 \cdot (a^2 \cdot b + b^3) \cdot d^3 \cdot f^3 \cdot x^3 + 2 \cdot (a^2 \cdot b + b^3) \cdot d^3 \cdot e^3 + 6 \cdot (a^2 \cdot b + b^3) \cdot d^2 \cdot e^2 \cdot f + 12 \cdot (a^2 \cdot b + b^3) \cdot d \cdot e \cdot f^2 + 12 \cdot (a^2 \cdot b + b^3) \cdot f^3 + 6 \cdot ((a^2 \cdot b + b^3) \cdot d^3 \cdot e \cdot f^2 + (a^2 \cdot b + b^3) \cdot d^2 \cdot f^3)) \cdot x^2 + 2 \cdot ((a^2 \cdot b + b^3) \cdot d^3 \cdot f^3 \cdot x^3 + (a^2 \cdot b + b^3) \cdot d^3 \cdot e^3 - 3 \cdot (a^2 \cdot b + b^3) \cdot d^2 \cdot e^2 \cdot f + 6 \cdot (a^2 \cdot b + b^3) \cdot d \cdot e \cdot f^2 - 6 \cdot (a^2 \cdot b + b^3) \cdot f^3 + 3 \cdot ((a^2 \cdot b + b^3) \cdot d^3 \cdot e \cdot f^2 - (a^2 \cdot b + b^3) \cdot d^2 \cdot f^3)) \cdot x^2 + 3 \cdot ((a^2 \cdot b + b^3) \cdot d^3 \cdot e^2 \cdot f - 2 \cdot (a^2 \cdot b + b^3) \cdot d^2 \cdot e \cdot f^2 + 2 \cdot (a^2 \cdot b + b^3) \cdot d \cdot f^3) \cdot x) \cdot \cosh(d \cdot x + c)^2 + 2 \cdot ((a^2 \cdot b + b^3) \cdot d^3 \cdot f^3 \cdot x^3 + (a^2 \cdot b + b^3) \cdot d^3 \cdot e^3 - 3 \cdot (a^2 \cdot b + b^3) \cdot d^2 \cdot e^2 \cdot f + 6 \cdot (a^2 \cdot b + b^3) \cdot d \cdot e \cdot f^2 - 6 \cdot (a^2 \cdot b + b^3) \cdot f^3 + 3 \cdot ((a^2 \cdot b + b^3) \cdot d^3 \cdot e \cdot f^2 - (a^2 \cdot b + b^3) \cdot d^2 \cdot f^3)) \cdot x^2 + 3 \cdot ((a^2 \cdot b + b^3) \cdot d^3 \cdot e^2 \cdot f - 2 \cdot (a^2 \cdot b + b^3) \cdot d^2 \cdot e \cdot f^2 + 2 \cdot (a^2 \cdot b + b^3) \cdot d \cdot f^3) \cdot x) \cdot \sinh(d \cdot x + c)^2 + 12 \cdot ((a^2 \cdot b \cdot d^2 \cdot f^3 \cdot x^2 + 2 \cdot a^2 \cdot b \cdot d^2 \cdot e \cdot f^2 \cdot x + a^2 \cdot b \cdot d^2 \cdot e^2 \cdot f) \cdot \cosh(d \cdot x + c) + (a^2 \cdot b \cdot d^2 \cdot f^3 \cdot x^2 + 2 \cdot a^2 \cdot b \cdot d^2 \cdot e \cdot f^2 \cdot x + a^2 \cdot b \cdot d^2 \cdot e^2 \cdot f) \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2)/b^2} \cdot \operatorname{dilog}((a \cdot \cosh(d \cdot x + c) + a \cdot \sinh(d \cdot x + c) + (b \cdot \cosh(d \cdot x + c) + b \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 12 \cdot ((a^2 \cdot b \cdot d^2 \cdot f^3 \cdot x^2 + 2 \cdot a^2 \cdot b \cdot d^2 \cdot e \cdot f^2 \cdot x + a^2 \cdot b \cdot d^2 \cdot e^2 \cdot f) \cdot \cosh(d \cdot x + c) + (a^2 \cdot b \cdot d^2 \cdot f^3 \cdot x^2 + 2 \cdot a^2 \cdot b \cdot d^2 \cdot e \cdot f^2 \cdot x + a^2 \cdot b \cdot d^2 \cdot e^2 \cdot f) \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2)/b^2} \cdot \operatorname{dilog}((a \cdot \cosh(d \cdot x + c) + a \cdot \sinh(d \cdot x + c) - (b \cdot \cosh(d \cdot x + c) + b \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 4 \cdot ((a^2 \cdot b \cdot d^3 \cdot e^3 - 3 \cdot a^2 \cdot b \cdot c \cdot d^2 \cdot e^2 \cdot f + 3 \cdot a^2 \cdot b \cdot c^2 \cdot d \cdot e \cdot f^2 - a^2 \cdot b \cdot c^3 \cdot f^3) \cdot \cosh(d \cdot x + c) + (a^2 \cdot b \cdot d^3 \cdot e^3 - 3 \cdot a^2 \cdot b \cdot c \cdot d^2 \cdot e^2 \cdot f + 3 \cdot a^2 \cdot b \cdot c^2 \cdot d \cdot e \cdot f^2 - a^2 \cdot b \cdot c^3 \cdot f^3) \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2)/b^2} \cdot \log(2 \cdot b \cdot \cosh(d \cdot x + c) + 2 \cdot b \cdot \sinh(d \cdot x + c) + 2 \cdot b \cdot \sqrt{(a^2 + b^2)/b^2} + 2 \cdot a) + 4 \cdot ((a^2 \cdot b \cdot d^3 \cdot e^3 - 3 \cdot a^2 \cdot b \cdot c \cdot d^2 \cdot e^2 \cdot f + 3 \cdot a^2 \cdot b \cdot c^2 \cdot d \cdot e \cdot f^2 - a^2 \cdot b \cdot c^3 \cdot f^3) \cdot \cosh(d \cdot x + c) + (a^2 \cdot b \cdot d^3 \cdot e^3 - 3 \cdot a^2 \cdot b \cdot c \cdot d^2 \cdot e^2 \cdot f + 3 \cdot a^2 \cdot b \cdot c^2 \cdot d \cdot e \cdot f^2 - a^2 \cdot b \cdot c^3 \cdot f^3) \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2)/b^2} \cdot \log(2 \cdot b \cdot \cosh(d \cdot x + c) + 2 \cdot b \cdot \sinh(d \cdot x + c) - 2 \cdot b \cdot \sqrt{(a^2 + b^2)/b^2} + 2 \cdot a) + 4 \cdot ((a^2 \cdot b \cdot d^3 \cdot f^3 \cdot x^3 + 3 \cdot a^2 \cdot b \cdot d^3 \cdot e \cdot f^2 \cdot x^2 + 3 \cdot a^2 \cdot b \cdot d^3 \cdot e^2 \cdot f \cdot x + 3 \cdot a^2 \cdot b \cdot c \cdot d^2 \cdot e^2 \cdot f - 3 \cdot a^2 \cdot b \cdot c^2 \cdot d \cdot e \cdot f^2 + a^2 \cdot b \cdot c^3 \cdot f^3) \cdot \cosh(d \cdot x + c) + (a^2 \cdot b \cdot d^3 \cdot f^3 \cdot x^3 + 3 \cdot a^2 \cdot b \cdot d^3 \cdot e \cdot f^2 \cdot x^2 + 3 \cdot a^2 \cdot b \cdot d^3 \cdot e^2 \cdot f \cdot x + 3 \cdot a^2 \cdot b \cdot c \cdot d^2 \cdot e^2 \cdot f - 3 \cdot a^2 \cdot b \cdot c^2 \cdot d \cdot e \cdot f^2 + a^2 \cdot b \cdot c^3 \cdot f^3) \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2)/b^2} \cdot \log(-(a \cdot \cosh(d \cdot x + c) + a \cdot \sinh(d \cdot x + c) + (b \cdot \cosh(d \cdot x + c) + b \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2)/b^2} - b)/b) - 4 \cdot ((a^2 \cdot b \cdot d^3 \cdot f^3 \cdot x^3 + 3 \cdot a^2 \cdot b \cdot d^3 \cdot e \cdot f^2 \cdot x^2 +$$

```

3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^
3*f^3)*cosh(d*x + c) + (a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b
*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*s
inh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
- (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 24*(
a^2*b*f^3*cosh(d*x + c) + a^2*b*f^3*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*po
lylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2))/b) - 24*(a^2*b*f^3*cosh(d*x + c) + a^2*b*f^3*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d
*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2
4*((a^2*b*d*f^3*x + a^2*b*d*e*f^2)*cosh(d*x + c) + (a^2*b*d*f^3*x + a^2*b*d
*e*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) +
a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
)/b) + 24*((a^2*b*d*f^3*x + a^2*b*d*e*f^2)*cosh(d*x + c) + (a^2*b*d*f^3*x +
a^2*b*d*e*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b
^2)/b^2))/b) + 6*((a^2*b + b^3)*d^3*e^2*f + 2*(a^2*b + b^3)*d^2*e*f^2 + 2*(
a^2*b + b^3)*d*f^3)*x - ((a^3 + a*b^2)*d^4*f^3*x^4 + 4*(a^3 + a*b^2)*d^4*e*
f^2*x^3 + 6*(a^3 + a*b^2)*d^4*e^2*f*x^2 + 4*(a^3 + a*b^2)*d^4*e^3*x)*cosh(d
*x + c) - ((a^3 + a*b^2)*d^4*f^3*x^4 + 4*(a^3 + a*b^2)*d^4*e*f^2*x^3 + 6*(a
^3 + a*b^2)*d^4*e^2*f*x^2 + 4*(a^3 + a*b^2)*d^4*e^3*x - 4*((a^2*b + b^3)*d^
3*f^3*x^3 + (a^2*b + b^3)*d^3*e^3 - 3*(a^2*b + b^3)*d^2*e^2*f + 6*(a^2*b +
b^3)*d*e*f^2 - 6*(a^2*b + b^3)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^2*b +
b^3)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^2*b + b^3)*d^2*e*f^2
+ 2*(a^2*b + b^3)*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c))/((a^2*b^2 + b^4)*
d^4*cosh(d*x + c) + (a^2*b^2 + b^4)*d^4*sinh(d*x + c))

```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \sinh(dx + c)^2}{b \sinh(dx + c) + a} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

$$3.229 \quad \int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=407

$$\frac{2a^2 f(e+fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d^2 \sqrt{a^2+b^2}} - \frac{2a^2 f(e+fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2 d^2 \sqrt{a^2+b^2}} - \frac{2a^2 f^2 \text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d^3 \sqrt{a^2+b^2}} + \frac{2a^2 f^2 \text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2 d^3 \sqrt{a^2+b^2}}$$

[Out]  $-(a*(e+f*x)^3)/(3*b^2*f) + (2*f^2*\text{Cosh}[c+d*x])/(b*d^3) + ((e+f*x)^2*\text{Cosh}[c+d*x])/(b*d) + (a^2*(e+f*x)^2*\text{Log}[1+(b*E^{(c+d*x)})/(a-\text{Sqrt}[a^2+b^2])])/(b^2*\text{Sqrt}[a^2+b^2]*d) - (a^2*(e+f*x)^2*\text{Log}[1+(b*E^{(c+d*x)})/(a+\text{Sqrt}[a^2+b^2])])/(b^2*\text{Sqrt}[a^2+b^2]*d) + (2*a^2*f*(e+f*x)*\text{PolyLog}[2, -((b*E^{(c+d*x)})/(a-\text{Sqrt}[a^2+b^2]))])/(b^2*\text{Sqrt}[a^2+b^2]*d^2) - (2*a^2*f*(e+f*x)*\text{PolyLog}[2, -((b*E^{(c+d*x)})/(a+\text{Sqrt}[a^2+b^2]))])/(b^2*\text{Sqrt}[a^2+b^2]*d^2) - (2*a^2*f^2*\text{PolyLog}[3, -((b*E^{(c+d*x)})/(a-\text{Sqrt}[a^2+b^2]))])/(b^2*\text{Sqrt}[a^2+b^2]*d^3) + (2*a^2*f^2*\text{PolyLog}[3, -((b*E^{(c+d*x)})/(a+\text{Sqrt}[a^2+b^2]))])/(b^2*\text{Sqrt}[a^2+b^2]*d^3) - (2*f*(e+f*x)*\text{Sinh}[c+d*x])/(b*d^2)$

**Rubi [A]** time = 0.853208, antiderivative size = 407, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {5557, 3296, 2638, 32, 3322, 2264, 2190, 2531, 2282, 6589}

$$\frac{2a^2 f(e+fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d^2 \sqrt{a^2+b^2}} - \frac{2a^2 f(e+fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2 d^2 \sqrt{a^2+b^2}} - \frac{2a^2 f^2 \text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d^3 \sqrt{a^2+b^2}} + \frac{2a^2 f^2 \text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2 d^3 \sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In] Int[((e+f\*x)^2\*Sinh[c+d\*x]^2)/(a+b\*Sinh[c+d\*x]),x]

[Out]  $-(a*(e+f*x)^3)/(3*b^2*f) + (2*f^2*\text{Cosh}[c+d*x])/(b*d^3) + ((e+f*x)^2*\text{Cosh}[c+d*x])/(b*d) + (a^2*(e+f*x)^2*\text{Log}[1+(b*E^{(c+d*x)})/(a-\text{Sqrt}[a^2+b^2])])/(b^2*\text{Sqrt}[a^2+b^2]*d) - (a^2*(e+f*x)^2*\text{Log}[1+(b*E^{(c+d*x)})/(a+\text{Sqrt}[a^2+b^2])])/(b^2*\text{Sqrt}[a^2+b^2]*d) + (2*a^2*f*(e+f*x)*\text{PolyLog}[2, -((b*E^{(c+d*x)})/(a-\text{Sqrt}[a^2+b^2]))])/(b^2*\text{Sqrt}[a^2+b^2]*d^2) - (2*a^2*f*(e+f*x)*\text{PolyLog}[2, -((b*E^{(c+d*x)})/(a+\text{Sqrt}[a^2+b^2]))])/(b^2*\text{Sqrt}[a^2+b^2]*d^2) - (2*a^2*f^2*\text{PolyLog}[3, -((b*E^{(c+d*x)})/(a-\text{Sqrt}[a^2+b^2]))])/(b^2*\text{Sqrt}[a^2+b^2]*d^3) + (2*a^2*f^2*\text{PolyLog}[3, -((b*E^{(c+d*x)})/(a+\text{Sqrt}[a^2+b^2]))])/(b^2*\text{Sqrt}[a^2+b^2]*d^3) - (2*f*(e+f*x)*\text{Sinh}[c+d*x])/(b*d^2)$

Rule 5557

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Sinh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sinh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

Rule 3296

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 3322

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*sin[(e\_.) + (Complex[0, fz\_])\* (f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(-(I\*b) + 2\*a\*E^(-(I\*e) + f\*fz\*x) + I\*b\*E^(2\*(-(I\*e) + f\*fz\*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.)\*(F\_)^(v\_)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x]]

)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*(a\_.) + (b\_.)\*(x\_)))^(n\_.)]\*((f\_.) + (g\_.)\*  
 (x\_)^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)]/((b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \sinh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^2 \sinh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{(e+fx)^2 \cosh(c+dx)}{bd} - \frac{a \int (e+fx)^2 dx}{b^2} + \frac{a^2 \int \frac{(e+fx)^2}{a+b \sinh(c+dx)} dx}{b^2} - \frac{(2f) \int (e+fx) \cosh(c+dx) dx}{bd} \\
&= -\frac{a(e+fx)^3}{3b^2 f} + \frac{(e+fx)^2 \cosh(c+dx)}{bd} - \frac{2f(e+fx) \sinh(c+dx)}{bd^2} + \frac{(2a^2) \int \frac{e^{c+dx}(e+fx)^2}{-b+2ae^{c+dx}+be^{2c+2dx}} dx}{b^2} \\
&= -\frac{a(e+fx)^3}{3b^2 f} + \frac{2f^2 \cosh(c+dx)}{bd^3} + \frac{(e+fx)^2 \cosh(c+dx)}{bd} - \frac{2f(e+fx) \sinh(c+dx)}{bd^2} + \frac{a^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 \sqrt{a^2+b^2} d} \\
&= -\frac{a(e+fx)^3}{3b^2 f} + \frac{2f^2 \cosh(c+dx)}{bd^3} + \frac{(e+fx)^2 \cosh(c+dx)}{bd} + \frac{a^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 \sqrt{a^2+b^2} d} \\
&= -\frac{a(e+fx)^3}{3b^2 f} + \frac{2f^2 \cosh(c+dx)}{bd^3} + \frac{(e+fx)^2 \cosh(c+dx)}{bd} + \frac{a^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 \sqrt{a^2+b^2} d} \\
&= -\frac{a(e+fx)^3}{3b^2 f} + \frac{2f^2 \cosh(c+dx)}{bd^3} + \frac{(e+fx)^2 \cosh(c+dx)}{bd} + \frac{a^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 \sqrt{a^2+b^2} d} \\
&= -\frac{a(e+fx)^3}{3b^2 f} + \frac{2f^2 \cosh(c+dx)}{bd^3} + \frac{(e+fx)^2 \cosh(c+dx)}{bd} + \frac{a^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 \sqrt{a^2+b^2} d}
\end{aligned}$$

**Mathematica [A]** time = 3.03193, size = 453, normalized size = 1.11

$$\frac{3a^2 \left( 2df(e+fx) \text{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) - 2df(e+fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) - 2f^2 \text{PolyLog}\left(3, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) + 2f^2 \text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) - 2d^2 e^2 \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) \right)}{d^3 \sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Sinh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out]  $(-a*x*(3*e^2 + 3*e*f*x + f^2*x^2)) + (3*a^2*(-2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/\sqrt{a^2 + b^2}] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - \sqrt{a^2 + b^2}]] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - \sqrt{a^2 + b^2}]] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + \sqrt{a^2 + b^2}]] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + \sqrt{a^2 + b^2}]] + 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + \sqrt{a^2 + b^2}]] - 2*d*f*(e + f*x)*PolyLog[2, -(b*E$

$$\frac{(c + dx)/(a + \sqrt{a^2 + b^2}) - 2f^2 \text{PolyLog}[3, (bE^{(c + dx)})/(-a + \sqrt{a^2 + b^2})] + 2f^2 \text{PolyLog}[3, -((bE^{(c + dx)})/(a + \sqrt{a^2 + b^2}))]}{(\sqrt{a^2 + b^2}d^3 + (3b \cosh[dx] * ((2f^2 + d^2(e + fx)^2) * \cosh[c] - 2df(e + fx) * \sinh[c]))/d^3 + (3b * (-2df(e + fx) * \cosh[c] + (2f^2 + d^2(e + fx)^2) * \sinh[c]) * \sinh[dx])/d^3)/(3b^2)}$$

**Maple [F]** time = 0.088, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\sinh(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 2.81087, size = 3897, normalized size = 9.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 1/6\*(3\*(a^2\*b + b^3)\*d^2\*f^2\*x^2 + 3\*(a^2\*b + b^3)\*d^2\*e^2 + 6\*(a^2\*b + b^3)\*d\*e\*f + 6\*(a^2\*b + b^3)\*f^2 + 3\*((a^2\*b + b^3)\*d^2\*f^2\*x^2 + (a^2\*b + b^3)

$$\begin{aligned}
& )d^2e^2 - 2*(a^2*b + b^3)*d*ef + 2*(a^2*b + b^3)*f^2 + 2*((a^2*b + b^3)* \\
& d^2*ef - (a^2*b + b^3)*d*f^2)*x)*\cosh(d*x + c)^2 + 3*((a^2*b + b^3)*d^2*f^ \\
& 2*x^2 + (a^2*b + b^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*ef + 2*(a^2*b + b^3)*f^2 \\
& + 2*((a^2*b + b^3)*d^2*ef - (a^2*b + b^3)*d*f^2)*x)*\sinh(d*x + c)^2 + 12* \\
& ((a^2*b*d*f^2*x + a^2*b*d*ef)*\cosh(d*x + c) + (a^2*b*d*f^2*x + a^2*b*d*ef) \\
& )*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x \\
& + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1 \\
& ) - 12*((a^2*b*d*f^2*x + a^2*b*d*ef)*\cosh(d*x + c) + (a^2*b*d*f^2*x + a^2* \\
& b*d*ef)*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2)*\operatorname{dilog}((a*\cosh(d*x + c) + a*si \\
& nh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b \\
& )/b + 1) - 6*((a^2*b*d^2*e^2 - 2*a^2*b*c*d*ef + a^2*b*c^2*f^2)*\cosh(d*x + \\
& c) + (a^2*b*d^2*e^2 - 2*a^2*b*c*d*ef + a^2*b*c^2*f^2)*\sinh(d*x + c))*\sqrt{( \\
& a^2 + b^2)/b^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 \\
& + b^2)/b^2} + 2*a) + 6*((a^2*b*d^2*e^2 - 2*a^2*b*c*d*ef + a^2*b*c^2*f^2)*c \\
& osh(d*x + c) + (a^2*b*d^2*e^2 - 2*a^2*b*c*d*ef + a^2*b*c^2*f^2)*\sinh(d*x + \\
& c))*\sqrt{(a^2 + b^2)/b^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b* \\
& \sqrt{(a^2 + b^2)/b^2} + 2*a) + 6*((a^2*b*d^2*f^2*x^2 + 2*a^2*b*d^2*ef*x + \\
& 2*a^2*b*c*d*ef - a^2*b*c^2*f^2)*\cosh(d*x + c) + (a^2*b*d^2*f^2*x^2 + 2*a^2 \\
& *b*d^2*ef*x + 2*a^2*b*c*d*ef - a^2*b*c^2*f^2)*\sinh(d*x + c))*\sqrt{(a^2 + \\
& b^2)/b^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*si \\
& nh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 6*((a^2*b*d^2*f^2*x^2 + 2*a^2* \\
& b*d^2*ef*x + 2*a^2*b*c*d*ef - a^2*b*c^2*f^2)*\cosh(d*x + c) + (a^2*b*d^2*f \\
& ^2*x^2 + 2*a^2*b*d^2*ef*x + 2*a^2*b*c*d*ef - a^2*b*c^2*f^2)*\sinh(d*x + c) \\
& )*\sqrt{(a^2 + b^2)/b^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d \\
& *x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 12*(a^2*b*f^2*co \\
& sh(d*x + c) + a^2*b*f^2*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2)*\operatorname{polylog}(3, (a* \\
& \cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{( \\
& a^2 + b^2)/b^2))/b) + 12*(a^2*b*f^2*\cosh(d*x + c) + a^2*b*f^2*\sinh(d*x + c \\
& ))*\sqrt{(a^2 + b^2)/b^2)*\operatorname{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b \\
& *\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2))/b) + 6*((a^2*b + b \\
& ^3)*d^2*ef + (a^2*b + b^3)*d*f^2)*x - 2*((a^3 + a*b^2)*d^3*f^2*x^3 + 3*(a^ \\
& 3 + a*b^2)*d^3*ef*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*x)*\cosh(d*x + c) - 2*((a^3 \\
& + a*b^2)*d^3*f^2*x^3 + 3*(a^3 + a*b^2)*d^3*ef*x^2 + 3*(a^3 + a*b^2)*d^3*e \\
& ^2*x - 3*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 - 2*(a^2*b + b^ \\
& 3)*d*ef + 2*(a^2*b + b^3)*f^2 + 2*((a^2*b + b^3)*d^2*ef - (a^2*b + b^3)*d \\
& *f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c))/((a^2*b^2 + b^4)*d^3*\cosh(d*x + c) + \\
& (a^2*b^2 + b^4)*d^3*\sinh(d*x + c))
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \sinh(dx + c)^2}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```



$$3.230 \quad \int \frac{(e+fx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=264

$$\frac{a^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d^2 \sqrt{a^2+b^2}} - \frac{a^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2 d^2 \sqrt{a^2+b^2}} + \frac{a^2(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2 d \sqrt{a^2+b^2}} - \frac{a^2(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2 d \sqrt{a^2+b^2}}$$

[Out]  $-\left(\frac{a e x}{b^2}\right) - \frac{a f x^2}{2 b^2} + \frac{(e+f x) \operatorname{Cosh}[c+d x]}{b d} + \left(a^2 (e+f x) \operatorname{Log}\left[1+\frac{b E^{c+d x}}{a-\sqrt{a^2+b^2}}\right]\right) / \left(b^2 \sqrt{a^2+b^2}\right) d - \left(a^2 (e+f x) \operatorname{Log}\left[1+\frac{b E^{c+d x}}{a+\sqrt{a^2+b^2}}\right]\right) / \left(b^2 \sqrt{a^2+b^2}\right) d + \left(a^2 f \operatorname{PolyLog}\left[2,-\frac{b E^{c+d x}}{a-\sqrt{a^2+b^2}}\right]\right) / \left(b^2 \sqrt{a^2+b^2}\right) d^2 - \left(a^2 f \operatorname{PolyLog}\left[2,-\frac{b E^{c+d x}}{a+\sqrt{a^2+b^2}}\right]\right) / \left(b^2 \sqrt{a^2+b^2}\right) d^2 - \frac{f \operatorname{Sinh}[c+d x]}{b d^2}$

**Rubi [A]** time = 0.486714, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {5557, 3296, 2637, 3322, 2264, 2190, 2279, 2391}

$$\frac{a^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d^2 \sqrt{a^2+b^2}} - \frac{a^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2 d^2 \sqrt{a^2+b^2}} + \frac{a^2(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2 d \sqrt{a^2+b^2}} - \frac{a^2(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2 d \sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{(e+f x) \operatorname{Sinh}[c+d x]^2}{(a+b \operatorname{Sinh}[c+d x])}, x\right]$

[Out]  $-\left(\frac{a e x}{b^2}\right) - \frac{a f x^2}{2 b^2} + \frac{(e+f x) \operatorname{Cosh}[c+d x]}{b d} + \left(a^2 (e+f x) \operatorname{Log}\left[1+\frac{b E^{c+d x}}{a-\sqrt{a^2+b^2}}\right]\right) / \left(b^2 \sqrt{a^2+b^2}\right) d - \left(a^2 (e+f x) \operatorname{Log}\left[1+\frac{b E^{c+d x}}{a+\sqrt{a^2+b^2}}\right]\right) / \left(b^2 \sqrt{a^2+b^2}\right) d + \left(a^2 f \operatorname{PolyLog}\left[2,-\frac{b E^{c+d x}}{a-\sqrt{a^2+b^2}}\right]\right) / \left(b^2 \sqrt{a^2+b^2}\right) d^2 - \left(a^2 f \operatorname{PolyLog}\left[2,-\frac{b E^{c+d x}}{a+\sqrt{a^2+b^2}}\right]\right) / \left(b^2 \sqrt{a^2+b^2}\right) d^2 - \frac{f \operatorname{Sinh}[c+d x]}{b d^2}$

**Rule 5557**

$\operatorname{Int}\left[\frac{(e_{.}) + (f_{.})(x_{.})^{(m_{.})} \operatorname{Sinh}[(c_{.}) + (d_{.})(x_{.})]^{(n_{.})}}{(a_{.}) + (b_{.}) \operatorname{Sinh}[(c_{.}) + (d_{.})(x_{.})]}, x_{\text{Symbol}}\right] := \operatorname{Dist}\left[\frac{1}{b}, \operatorname{Int}[(e+f x)^m \operatorname{Sinh}[c+d x]^{(n-1)}, x], x\right] - \operatorname{Dist}\left[\frac{a}{b}, \operatorname{Int}[(e+f x)^m \operatorname{Sinh}[c+d x]^{(n-1)}, x], x\right]$

$\int \frac{1}{(a + b \sinh[c + dx])^2} dx$ ; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 3296

$\int ((c + dx)^m \cos[e + fx])^n dx$ ; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2637

$\int \sin[\pi/2 + (c + dx)] dx$ ; FreeQ[{c, d}, x]

### Rule 3322

$\int \frac{(c + dx)^m \sin[e + (fz + i)^2 x]}{(a + b \sin[e + (fz + i)^2 x])^2} dx$ ; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 2264

$\int \frac{(f + gx)^m F^u}{(a + b F^u)^2} dx$ ; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

### Rule 2190

$\int \frac{(c + dx)^m \log[1 + (b(F^{g(e+fx)})^n)/a]}{(a + b(F^{g(e+fx)})^n)^2} dx$ ; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2279

$\int \log[a + b(F^{c+dx})^n] dx$ ; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{(e+fx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx) \sinh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
 &= \frac{(e+fx) \cosh(c+dx)}{bd} - \frac{a \int (e+fx) dx}{b^2} + \frac{a^2 \int \frac{e+fx}{a+b \sinh(c+dx)} dx}{b^2} - \frac{f \int \cosh(c+dx) dx}{bd} \\
 &= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx) \cosh(c+dx)}{bd} - \frac{f \sinh(c+dx)}{bd^2} + \frac{(2a^2) \int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}} dx}{b^2} \\
 &= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx) \cosh(c+dx)}{bd} - \frac{f \sinh(c+dx)}{bd^2} + \frac{(2a^2) \int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{b\sqrt{a^2+b^2}} \\
 &= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx) \cosh(c+dx)}{bd} + \frac{a^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d} - \frac{a^2(e+fx) \log}{b^2\sqrt{a^2+b^2}} \\
 &= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx) \cosh(c+dx)}{bd} + \frac{a^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d} - \frac{a^2(e+fx) \log}{b^2\sqrt{a^2+b^2}} \\
 &= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx) \cosh(c+dx)}{bd} + \frac{a^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d} - \frac{a^2(e+fx) \log}{b^2\sqrt{a^2+b^2}}
 \end{aligned}$$

**Mathematica [A]** time = 2.18564, size = 299, normalized size = 1.13

$$\frac{2a^2 \left( f \operatorname{PolyLog}\left(2, \frac{b(\sinh(c+dx)+\cosh(c+dx))}{\sqrt{a^2+b^2}-a}\right) - f \operatorname{PolyLog}\left(2, -\frac{b(\sinh(c+dx)+\cosh(c+dx))}{\sqrt{a^2+b^2}+a}\right) - 2de \tanh^{-1}\left(\frac{a+b \sinh(c+dx)+b \cosh(c+dx)}{\sqrt{a^2+b^2}}\right) + f(c+dx) \log\left(\frac{b(\sinh(c+dx)+\cosh(c+dx))}{a-\sqrt{a^2+b^2}}\right) \right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Sinh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (a\*(c + d\*x)\*(c\*f - d\*(2\*e + f\*x)) + 2\*b\*d\*(e + f\*x)\*Cosh[c + d\*x] + (2\*a^2\*(-2\*d\*e\*ArcTanh[(a + b\*Cosh[c + d\*x] + b\*Sinh[c + d\*x])/Sqrt[a^2 + b^2]] + 2\*c\*f\*ArcTanh[(a + b\*Cosh[c + d\*x] + b\*Sinh[c + d\*x])/Sqrt[a^2 + b^2]] + f\*(c + d\*x)\*Log[1 + (b\*(Cosh[c + d\*x] + Sinh[c + d\*x]))/(a - Sqrt[a^2 + b^2])] - f\*(c + d\*x)\*Log[1 + (b\*(Cosh[c + d\*x] + Sinh[c + d\*x]))/(a + Sqrt[a^2

+ b^2]]) + f\*PolyLog[2, (b\*(Cosh[c + d\*x] + Sinh[c + d\*x]))/(-a + Sqrt[a^2 + b^2])] - f\*PolyLog[2, -((b\*(Cosh[c + d\*x] + Sinh[c + d\*x]))/(a + Sqrt[a^2 + b^2])))]/Sqrt[a^2 + b^2] - 2\*b\*f\*Sinh[c + d\*x]/(2\*b^2\*d^2)

**Maple [B]** time = 0.071, size = 510, normalized size = 1.9

$$-\frac{afx^2}{2b^2} - \frac{aex}{b^2} + \frac{(dfx + de - f)e^{dx+c}}{2d^2b} + \frac{(dfx + de + f)e^{-dx-c}}{2d^2b} - 2\frac{a^2e}{b^2d\sqrt{a^2+b^2}}\text{Artanh}\left(\frac{1}{2}\frac{2be^{dx+c} + 2a}{\sqrt{a^2+b^2}}\right) + \frac{a^2fx}{b^2d}\ln\left(\left(\frac{b\exp(dx+c) + a}{\sqrt{a^2+b^2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] 
$$-1/2*a*f*x^2/b^2 - a*e*x/b^2 + 1/2*(d*f*x+d*e-f)/d^2/b*\exp(d*x+c) + 1/2*(d*f*x+d*e+f)/d^2/b*\exp(-d*x-c) - 2*a^2/b^2/d*e/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) + a^2/b^2/d*f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) * x + a^2/b^2/d^2*f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) * c - a^2/b^2/d*f/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) * x - a^2/b^2/d^2*f/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) * c + a^2/b^2/d^2*f/(a^2+b^2)^{(1/2)}*\text{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) - a^2/b^2/d^2*f/(a^2+b^2)^{(1/2)}*\text{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) + 2*a^2/b^2/d^2*f*c/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.661, size = 2272, normalized size = 8.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] 
$$\frac{1}{2} * ((a^2 * b + b^3) * d * f * x + (a^2 * b + b^3) * d * e + ((a^2 * b + b^3) * d * f * x + (a^2 * b + b^3) * d * e - (a^2 * b + b^3) * f) * \cosh(d * x + c)^2 + ((a^2 * b + b^3) * d * f * x + (a^2 * b + b^3) * d * e - (a^2 * b + b^3) * f) * \sinh(d * x + c)^2 + 2 * (a^2 * b * f * \cosh(d * x + c) + a^2 * b * f * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} * \operatorname{dilog}((a * \cosh(d * x + c) + a * \sinh(d * x + c) + (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b + 1) - 2 * (a^2 * b * f * \cosh(d * x + c) + a^2 * b * f * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} * \operatorname{dilog}((a * \cosh(d * x + c) + a * \sinh(d * x + c) - (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b + 1) - 2 * ((a^2 * b * d * e - a^2 * b * c * f) * \cosh(d * x + c) + (a^2 * b * d * e - a^2 * b * c * f) * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} * \log(2 * b * \cosh(d * x + c) + 2 * b * \sinh(d * x + c) + 2 * b * \sqrt{(a^2 + b^2) / b^2} + 2 * a) + 2 * ((a^2 * b * d * e - a^2 * b * c * f) * \cosh(d * x + c) + (a^2 * b * d * e - a^2 * b * c * f) * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} * \log(2 * b * \cosh(d * x + c) + 2 * b * \sinh(d * x + c) - 2 * b * \sqrt{(a^2 + b^2) / b^2} + 2 * a) + 2 * ((a^2 * b * d * f * x + a^2 * b * c * f) * \cosh(d * x + c) + (a^2 * b * d * f * x + a^2 * b * c * f) * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} * \log(-(a * \cosh(d * x + c) + a * \sinh(d * x + c) + (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b) - 2 * ((a^2 * b * d * f * x + a^2 * b * c * f) * \cosh(d * x + c) + (a^2 * b * d * f * x + a^2 * b * c * f) * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} * \log(-(a * \cosh(d * x + c) + a * \sinh(d * x + c) - (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b) + (a^2 * b + b^3) * f - ((a^3 + a * b^2) * d^2 * f * x^2 + 2 * (a^3 + a * b^2) * d^2 * e * x) * \cosh(d * x + c) - ((a^3 + a * b^2) * d^2 * f * x^2 + 2 * (a^3 + a * b^2) * d^2 * e * x - 2 * ((a^2 * b + b^3) * d * f * x + (a^2 * b + b^3) * d * e - (a^2 * b + b^3) * f) * \cosh(d * x + c)) * \sinh(d * x + c)) / ((a^2 * b^2 + b^4) * d^2 * \cosh(d * x + c) + (a^2 * b^2 + b^4) * d^2 * \sinh(d * x + c))$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \sinh(dx + c)^2}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

$$3.231 \quad \int \frac{\sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=71

$$-\frac{2a^2 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2 d \sqrt{a^2+b^2}} - \frac{ax}{b^2} + \frac{\cosh(c+dx)}{bd}$$

[Out]  $-\left(\frac{a*x}{b^2}\right) - \left(\frac{2*a^2*ArcTanh\left[\frac{b - a*Tanh\left[\frac{c + d*x}{2}\right]}{\sqrt{a^2 + b^2}}\right]}{b^2*\sqrt{a^2 + b^2}*d}\right) + \frac{Cosh[c + d*x]}{b*d}$

**Rubi [A]** time = 0.127736, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2746, 12, 2735, 2660, 618, 204}

$$-\frac{2a^2 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2 d \sqrt{a^2+b^2}} - \frac{ax}{b^2} + \frac{\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d\*x]^2/(a + b\*Sinh[c + d\*x]),x]

[Out]  $-\left(\frac{a*x}{b^2}\right) - \left(\frac{2*a^2*ArcTanh\left[\frac{b - a*Tanh\left[\frac{c + d*x}{2}\right]}{\sqrt{a^2 + b^2}}\right]}{b^2*\sqrt{a^2 + b^2}*d}\right) + \frac{Cosh[c + d*x]}{b*d}$

#### Rule 2746

Int[((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^2/((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> -Simp[(b^2\*Cos[e + f\*x])/(d\*f), x] + Dist[1/d, Int[Simp[a^2\*d - b\*(b\*c - 2\*a\*d)\*Sin[e + f\*x], x]/(c + d\*Sin[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2735

Int[((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])/((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Simp[(b\*x)/d, x] - Dist[(b\*c - a\*d)/d, Int[1/(c + d\*Sin[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0]

### Rule 2660

Int[((a\_.) + (b\_.)\*sin[(c\_.) + (d\_.)\*(x\_.)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + 2\*b\*e\*x + a\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{\sinh^2(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\cosh(c+dx)}{bd} - \frac{\int \frac{a\sinh(c+dx)}{a+b\sinh(c+dx)} dx}{b} \\
 &= \frac{\cosh(c+dx)}{bd} - \frac{a \int \frac{\sinh(c+dx)}{a+b\sinh(c+dx)} dx}{b} \\
 &= -\frac{ax}{b^2} + \frac{\cosh(c+dx)}{bd} + \frac{a^2 \int \frac{1}{a+b\sinh(c+dx)} dx}{b^2} \\
 &= -\frac{ax}{b^2} + \frac{\cosh(c+dx)}{bd} - \frac{(2ia^2) \text{Subst}\left(\int \frac{1}{a-2ibx+ax^2} dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{b^2d} \\
 &= -\frac{ax}{b^2} + \frac{\cosh(c+dx)}{bd} + \frac{(4ia^2) \text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2} dx, x, -2ib + 2a \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{b^2d} \\
 &= -\frac{ax}{b^2} - \frac{2a^2 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}d} + \frac{\cosh(c+dx)}{bd}
 \end{aligned}$$



**Mathematica [A]** time = 0.274514, size = 74, normalized size = 1.04

$$\frac{b \cosh(c + dx) - a \left( -\frac{2a \tan^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} + c + dx \right)}{b^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d\*x]^2/(a + b\*Sinh[c + d\*x]),x]

[Out]  $(-(a*(c + d*x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2])) + b*Cosh[c + d*x])/(b^2*d)$

**Maple [A]** time = 0.03, size = 132, normalized size = 1.9

$$\frac{1}{bd} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} - \frac{a}{db^2} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) + 2 \frac{a^2}{db^2 \sqrt{a^2 + b^2}} \operatorname{Artanh}\left(\frac{1}{2} \frac{2a \tanh(1/2 dx + c/2) - 2b}{\sqrt{a^2 + b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out]  $1/d/b/(\tanh(1/2*d*x+1/2*c)+1)-1/d*a/b^2*\ln(\tanh(1/2*d*x+1/2*c)+1)+2/d*a^2/b^2/(a^2+b^2)^{(1/2)*\operatorname{arctanh}(1/2*(2*a*\tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^{(1/2)})}-1/d/b/(\tanh(1/2*d*x+1/2*c)-1)+1/d*a/b^2*\ln(\tanh(1/2*d*x+1/2*c)-1)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [B]** time = 2.52095, size = 822, normalized size = 11.58

$$2(a^3 + ab^2)dx \cosh(dx + c) - a^2b - b^3 - (a^2b + b^3) \cosh(dx + c)^2 - (a^2b + b^3) \sinh(dx + c)^2 - 2(a^2 \cosh(dx + c) + a$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-1/2*(2*(a^3 + a*b^2)*d*x*\cosh(d*x + c) - a^2*b - b^3 - (a^2*b + b^3)*\cosh(d*x + c)^2 - (a^2*b + b^3)*\sinh(d*x + c)^2 - 2*(a^2*\cosh(d*x + c) + a^2*\sinh(d*x + c))*\sqrt{a^2 + b^2}*\log((b^2*\cosh(d*x + c)^2 + b^2*\sinh(d*x + c)^2 + 2*a*b*\cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*\cosh(d*x + c) + a*b)*\sinh(d*x + c) - 2*\sqrt{a^2 + b^2}*(b*\cosh(d*x + c) + b*\sinh(d*x + c) + a))/(b*\cosh(d*x + c)^2 + b*\sinh(d*x + c)^2 + 2*a*\cosh(d*x + c) + 2*(b*\cosh(d*x + c) + a)*\sinh(d*x + c) - b)) + 2*((a^3 + a*b^2)*d*x - (a^2*b + b^3)*\cosh(d*x + c))*\sinh(d*x + c))/((a^2*b^2 + b^4)*d*\cosh(d*x + c) + (a^2*b^2 + b^4)*d*\sinh(d*x + c))$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [A]** time = 1.16792, size = 161, normalized size = 2.27

$$\frac{a^2 \log\left(\frac{|2be^{(dx+c)} + 2a - 2\sqrt{a^2 + b^2}|}{|2be^{(dx+c)} + 2a + 2\sqrt{a^2 + b^2}|}\right)}{\sqrt{a^2 + b^2}bd} - \frac{(dx + c)a}{b^2d} + \frac{e^{(dx+c)}}{2bd} + \frac{e^{(-dx-c)}}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] a^2*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
+ 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2*d) - (d*x + c)*a/(b^2*d) +
1/2*e^(d*x + c)/(b*d) + 1/2*e^(-d*x - c)/(b*d)
```

$$3.232 \quad \int \frac{\sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=30

$$\text{Unintegrable} \left( \frac{\sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[Sinh[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0755898, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Sinh[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Sinh[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [A]** time = 136.201, size = 0, normalized size = 0.

$$\int \frac{\sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sinh[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Integrate[Sinh[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Maple [A]** time = 0.088, size = 0, normalized size = 0.

$$\int \frac{(\sinh(dx + c))^2}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(sinh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$2a^2 \int -\frac{e^{(dx+c)}}{b^3fx + b^3e - (b^3fxe^{(2c)} + b^3ee^{(2c)})e^{(2dx)} - 2(ab^2fxe^c + ab^2ee^c)e^{(dx)}} dx + \frac{e^{(-c+\frac{de}{f})}E_1\left(\frac{(fx+e)d}{f}\right)}{2bf} - \frac{e^{(c-\frac{de}{f})}E_1\left(-\frac{(fx+e)d}{f}\right)}{2bf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 2\*a^2\*integrate(-e^(d\*x + c)/(b^3\*f\*x + b^3\*e - (b^3\*f\*x\*e^(2\*c) + b^3\*e\*e^(2\*c))\*e^(2\*d\*x) - 2\*(a\*b^2\*f\*x\*e^c + a\*b^2\*e\*e^c)\*e^(d\*x)), x) + 1/2\*e^(-c + d\*e/f)\*exp\_integral\_e(1, (f\*x + e)\*d/f)/(b\*f) - 1/2\*e^(c - d\*e/f)\*exp\_integral\_e(1, -(f\*x + e)\*d/f)/(b\*f) - a\*log(f\*x + e)/(b^2\*f)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh(dx + c)^2}{afx + ae + (bfx + be)\sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] `integral(sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)), x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx + c)^2}{(fx + e)(b \sinh(dx + c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)), x, algorithm="giac")`

[Out] `integrate(sinh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)), x)`

$$3.233 \quad \int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=712

$$\frac{6a^3 f^2 (e+fx) \text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^3 \sqrt{a^2+b^2}} - \frac{6a^3 f^2 (e+fx) \text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3 d^3 \sqrt{a^2+b^2}} - \frac{3a^3 f (e+fx)^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^2 \sqrt{a^2+b^2}}$$

```
[Out] (-3*e*f^2*x)/(4*b*d^2) - (3*f^3*x^2)/(8*b*d^2) + (a^2*(e+f*x)^4)/(4*b^3*f)
) - (e+f*x)^4/(8*b*f) - (6*a*f^2*(e+f*x)*Cosh[c+d*x])/(b^2*d^3) - (a*
(e+f*x)^3*Cosh[c+d*x])/(b^2*d) - (a^3*(e+f*x)^3*Log[1+(b*E^(c+d*x)
)))/(a-Sqrt[a^2+b^2])]/(b^3*Sqrt[a^2+b^2]*d) + (a^3*(e+f*x)^3*Log[1
+(b*E^(c+d*x))/(a+Sqrt[a^2+b^2]])/(b^3*Sqrt[a^2+b^2]*d) - (3*a^3
*f*(e+f*x)^2*PolyLog[2,-((b*E^(c+d*x))/(a-Sqrt[a^2+b^2]))])/(b^3*S
qrt[a^2+b^2]*d^2) + (3*a^3*f*(e+f*x)^2*PolyLog[2,-((b*E^(c+d*x))/(a
+Sqrt[a^2+b^2]))])/(b^3*Sqrt[a^2+b^2]*d^2) + (6*a^3*f^2*(e+f*x)*Poly
Log[3,-((b*E^(c+d*x))/(a-Sqrt[a^2+b^2]))])/(b^3*Sqrt[a^2+b^2]*d^3)
- (6*a^3*f^2*(e+f*x)*PolyLog[3,-((b*E^(c+d*x))/(a+Sqrt[a^2+b^2]))]
)/(b^3*Sqrt[a^2+b^2]*d^3) - (6*a^3*f^3*PolyLog[4,-((b*E^(c+d*x))/(a-
Sqrt[a^2+b^2]))])/(b^3*Sqrt[a^2+b^2]*d^4) + (6*a^3*f^3*PolyLog[4,-((b
*E^(c+d*x))/(a+Sqrt[a^2+b^2]))])/(b^3*Sqrt[a^2+b^2]*d^4) + (6*a*f^3
*Sinh[c+d*x])/(b^2*d^4) + (3*a*f*(e+f*x)^2*Sinh[c+d*x])/(b^2*d^2) + (
3*f^2*(e+f*x)*Cosh[c+d*x]*Sinh[c+d*x])/(4*b*d^3) + ((e+f*x)^3*Cosh[
c+d*x]*Sinh[c+d*x])/(2*b*d) - (3*f^3*Sinh[c+d*x]^2)/(8*b*d^4) - (3*f*
(e+f*x)^2*Sinh[c+d*x]^2)/(4*b*d^2)
```

**Rubi [A]** time = 1.23456, antiderivative size = 712, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 13, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$ , Rules used = {5557, 3311, 32, 3310, 3296, 2637, 3322, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{6a^3 f^2 (e+fx) \text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^3 \sqrt{a^2+b^2}} - \frac{6a^3 f^2 (e+fx) \text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3 d^3 \sqrt{a^2+b^2}} - \frac{3a^3 f (e+fx)^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^2 \sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e+f*x)^3*Sinh[c+d*x]^3)/(a+b*Sinh[c+d*x]),x]
```

```
[Out] (-3*e*f^2*x)/(4*b*d^2) - (3*f^3*x^2)/(8*b*d^2) + (a^2*(e+f*x)^4)/(4*b^3*f)
) - (e+f*x)^4/(8*b*f) - (6*a*f^2*(e+f*x)*Cosh[c+d*x])/(b^2*d^3) - (a*
(e+f*x)^3*Cosh[c+d*x])/(b^2*d) - (a^3*(e+f*x)^3*Log[1+(b*E^(c+d*x)
)))/(a-Sqrt[a^2+b^2])]/(b^3*Sqrt[a^2+b^2]*d) + (a^3*(e+f*x)^3*Log[1
+(b*E^(c+d*x))/(a+Sqrt[a^2+b^2]])/(b^3*Sqrt[a^2+b^2]*d) - (3*a^3
*f*(e+f*x)^2*PolyLog[2,-((b*E^(c+d*x))/(a-Sqrt[a^2+b^2]))])/(b^3*S
qrt[a^2+b^2]*d^2) + (3*a^3*f*(e+f*x)^2*PolyLog[2,-((b*E^(c+d*x))/(a
+Sqrt[a^2+b^2]))])/(b^3*Sqrt[a^2+b^2]*d^2) + (6*a^3*f^2*(e+f*x)*Poly
Log[3,-((b*E^(c+d*x))/(a-Sqrt[a^2+b^2]))])/(b^3*Sqrt[a^2+b^2]*d^3)
- (6*a^3*f^2*(e+f*x)*PolyLog[3,-((b*E^(c+d*x))/(a+Sqrt[a^2+b^2]))]
)/(b^3*Sqrt[a^2+b^2]*d^3) - (6*a^3*f^3*PolyLog[4,-((b*E^(c+d*x))/(a-
Sqrt[a^2+b^2]))])/(b^3*Sqrt[a^2+b^2]*d^4) + (6*a^3*f^3*PolyLog[4,-((b
*E^(c+d*x))/(a+Sqrt[a^2+b^2]))])/(b^3*Sqrt[a^2+b^2]*d^4) + (6*a*f^3
*Sinh[c+d*x])/(b^2*d^4) + (3*a*f*(e+f*x)^2*Sinh[c+d*x])/(b^2*d^2) + (
3*f^2*(e+f*x)*Cosh[c+d*x]*Sinh[c+d*x])/(4*b*d^3) + ((e+f*x)^3*Cosh[
c+d*x]*Sinh[c+d*x])/(2*b*d) - (3*f^3*Sinh[c+d*x]^2)/(8*b*d^4) - (3*f*
(e+f*x)^2*Sinh[c+d*x]^2)/(4*b*d^2)
```

```

)))/(a - Sqrt[a^2 + b^2]))/(b^3*Sqrt[a^2 + b^2]*d) + (a^3*(e + f*x)^3*Log[1
+ (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))/(b^3*Sqrt[a^2 + b^2]*d) - (3*a^3
*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(b^3*S
qrt[a^2 + b^2]*d^2) + (3*a^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))]/(b^3*Sqrt[a^2 + b^2]*d^2) + (6*a^3*f^2*(e + f*x)*Poly
Log[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(b^3*Sqrt[a^2 + b^2]*d^3)
- (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
]/(b^3*Sqrt[a^2 + b^2]*d^3) - (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))]/(b^3*Sqrt[a^2 + b^2]*d^4) + (6*a^3*f^3*PolyLog[4, -((b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(b^3*Sqrt[a^2 + b^2]*d^4) + (6*a*f^3
*Sinh[c + d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*Sinh[c + d*x])/(b^2*d^2) + (
3*f^2*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^3) + ((e + f*x)^3*Cosh[
c + d*x]*Sinh[c + d*x])/(2*b*d) - (3*f^3*Sinh[c + d*x]^2)/(8*b*d^4) - (3*f*
(e + f*x)^2*Sinh[c + d*x]^2)/(4*b*d^2)

```

### Rule 5557

```

Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_
_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sinh[c
+ d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]

```

### Rule 3311

```

Int[(((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbo
l] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Sinh[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sinh[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sinh[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cosh[e + f*x]*(b*Sinh[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

```

### Rule 32

```

Int[(((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

```

### Rule 3310

```

Int[(((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :=
Simp[(d*(b*Sinh[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sinh[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cosh[e + f*x]*(b
*Sinh[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]

```



Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^(c_.)*((a_.) + (b_.
```

```

)*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]

```

### Rule 2282

```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

### Rule 6589

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \sinh^2(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^3 \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{(e+fx)^3 \cosh(c+dx) \sinh(c+dx)}{2bd} - \frac{3f(e+fx)^2 \sinh^2(c+dx)}{4bd^2} - \frac{a \int (e+fx)^3 \sinh(c+dx) dx}{b^2} \\
&= -\frac{(e+fx)^4}{8bf} - \frac{a(e+fx)^3 \cosh(c+dx)}{b^2d} + \frac{3f^2(e+fx) \cosh(c+dx) \sinh(c+dx)}{4bd^3} + \frac{(e+fx)^3}{b^2} \\
&= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{a(e+fx)^3 \cosh(c+dx)}{b^2d} + \frac{3af(e+fx)^2}{b^2} \\
&= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx) \cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^3}{b^2} \\
&= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx) \cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^3}{b^2} \\
&= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx) \cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^3}{b^2} \\
&= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx) \cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^3}{b^2} \\
&= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx) \cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^3}{b^2} \\
&= -\frac{3ef^2x}{4bd^2} - \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} - \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx) \cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^3}{b^2}
\end{aligned}$$

**Mathematica [A]** time = 4.58408, size = 1407, normalized size = 1.98

$$4a^2\sqrt{a^2+b^2}f^3x^4d^4 - 2b^2\sqrt{a^2+b^2}f^3x^4d^4 + 16a^2\sqrt{a^2+b^2}ef^2x^3d^4 - 8b^2\sqrt{a^2+b^2}ef^2x^3d^4 + 24a^2\sqrt{a^2+b^2}e^2fx^2d^4 - 12b^2\sqrt{a^2+b^2}e^2fx^2d^4$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^3\*Sinh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

```
[Out] (16*a^2*Sqrt[a^2 + b^2]*d^4*e^3*x - 8*b^2*Sqrt[a^2 + b^2]*d^4*e^3*x + 24*a^
2*Sqrt[a^2 + b^2]*d^4*e^2*f*x^2 - 12*b^2*Sqrt[a^2 + b^2]*d^4*e^2*f*x^2 + 16
*a^2*Sqrt[a^2 + b^2]*d^4*e*f^2*x^3 - 8*b^2*Sqrt[a^2 + b^2]*d^4*e*f^2*x^3 +
4*a^2*Sqrt[a^2 + b^2]*d^4*f^3*x^4 - 2*b^2*Sqrt[a^2 + b^2]*d^4*f^3*x^4 + 32*
a^3*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 16*a*b*Sqrt[a^2
+ b^2]*d^3*e^3*Cosh[c + d*x] - 96*a*b*Sqrt[a^2 + b^2]*d*e*f^2*Cosh[c + d*x]
- 48*a*b*Sqrt[a^2 + b^2]*d^3*e^2*f*x*Cosh[c + d*x] - 96*a*b*Sqrt[a^2 + b^2
]*d*f^3*x*Cosh[c + d*x] - 48*a*b*Sqrt[a^2 + b^2]*d^3*e*f^2*x^2*Cosh[c + d*x
] - 16*a*b*Sqrt[a^2 + b^2]*d^3*f^3*x^3*Cosh[c + d*x] - 6*b^2*Sqrt[a^2 + b^2
]*d^2*e^2*f*Cosh[2*(c + d*x)] - 3*b^2*Sqrt[a^2 + b^2]*f^3*Cosh[2*(c + d*x)]
- 12*b^2*Sqrt[a^2 + b^2]*d^2*e*f^2*x*Cosh[2*(c + d*x)] - 6*b^2*Sqrt[a^2 +
b^2]*d^2*f^3*x^2*Cosh[2*(c + d*x)] - 48*a^3*d^3*e^2*f*x*Log[1 + (b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2])] - 48*a^3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])] - 16*a^3*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2])] + 48*a^3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2])] + 48*a^3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
])] + 16*a^3*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 4
8*a^3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]
+ 48*a^3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2
]))] + 96*a^3*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] +
96*a^3*d*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 96*a^3*
d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 96*a^3*d*f^3
*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 96*a^3*f^3*PolyLo
g[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 96*a^3*f^3*PolyLog[4, -((b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 48*a*b*Sqrt[a^2 + b^2]*d^2*e^2*f*Sinh
[c + d*x] + 96*a*b*Sqrt[a^2 + b^2]*f^3*Sinh[c + d*x] + 96*a*b*Sqrt[a^2 + b^
2]*d^2*e*f^2*x*Sinh[c + d*x] + 48*a*b*Sqrt[a^2 + b^2]*d^2*f^3*x^2*Sinh[c +
d*x] + 4*b^2*Sqrt[a^2 + b^2]*d^3*e^3*Sinh[2*(c + d*x)] + 6*b^2*Sqrt[a^2 + b
^2]*d*e*f^2*Sinh[2*(c + d*x)] + 12*b^2*Sqrt[a^2 + b^2]*d^3*e^2*f*x*Sinh[2*(
c + d*x)] + 6*b^2*Sqrt[a^2 + b^2]*d*f^3*x*Sinh[2*(c + d*x)] + 12*b^2*Sqrt[a
^2 + b^2]*d^3*e*f^2*x^2*Sinh[2*(c + d*x)] + 4*b^2*Sqrt[a^2 + b^2]*d^3*f^3*x
^3*Sinh[2*(c + d*x)]/(16*b^3*Sqrt[a^2 + b^2]*d^4)
```

---

**Maple [F]** time = 0.099, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 (\sinh(dx + c))^3}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [C]** time = 4.02698, size = 11113, normalized size = 15.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/32*(4*(a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*(a^2*b^2 + b^4)*d^3*e^3 + 6*(a^2*b^2 + b^4)*d^2*e^2*f + 6*(a^2*b^2 + b^4)*d*e*f^2 - (4*(a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*(a^2*b^2 + b^4)*d^3*e^3 - 6*(a^2*b^2 + b^4)*d^2*e^2*f + 6*(a^2*b^2 + b^4)*d*e*f^2 - 3*(a^2*b^2 + b^4)*f^3 + 6*(2*(a^2*b^2 + b^4)*d^3*e*f^2 - (a^2*b^2 + b^4)*d^2*f^3)*x^2 + 6*(2*(a^2*b^2 + b^4)*d^3*e^2*f - 2*(a^2*b^2 + b^4)*d^2*e*f^2 + (a^2*b^2 + b^4)*d*f^3)*x)*cosh(d*x + c)^4 - (4*(a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*(a^2*b^2 + b^4)*d^3*e^3 - 6*(a^2*b^2 + b^4)*d^2*e^2*f + 6*(a^2*b^2 + b^4)*d*e*f^2 - 3*(a^2*b^2 + b^4)*f^3 + 6*(2*(a^2*b^2 + b^4)*d^3*e*f^2 - (a^2*b^2 + b^4)*d^2*f^3)*x^2 + 6*(2*(a^2*b^2 + b^4)*d^3*e^2*f - 2*(a^2*b^2 + b^4)*d^2*e*f^2 + (a^2*b^2 + b^4)*d*f^3)*x)*sinh(d*x + c)^4 + 3*(a^2*b^2 + b^4)*f^3 + 16*((a^3*b + a*b^3)*d^3*f^3*x^3 + (a^3*b + a*b^3)*d^3*e^3 - 3*(a^3*b + a*b^3)*d^2*e^2*f + 6*(a^3*b + a*b^3)*d*e*f^2 - 6*(a^3*b + a*b^3)*f^3 + 3*((a^3*b + a*b^3)*d^3*e*f^2 - (a^3*b + a*b^3)*d^2*f^3)*x^2 + 3*((a^3*b + a*b^3)*d^3*e^2*f - 2*(a^3*b + a*b^3)*d^2*e*f^2 + 2*(a^3*b + a*b^3)*d*f^3)*x)*cosh(d*x + c)^3 + 4*(4*(a^3*b + a*b^3)*d^3*f^3*x^3 + 4*(a^3*b + a*b^3)*d^3*e^3 - 12*(a^3*b + a*b^3)*d^2*e^2*f + 24*(a^3*b + a*b^3)*d*e*f^2 - 24*(a^3*b + a*b^3)*f^3 + 12*((a^3*b + a*b^3)*d^3*e*f^2 - (a^3*b + a*b^3)*d^2*f^3)*x^2 + 12*((a^3*b + a*b^3)*d^3*e^2*f - 2*(a^3*b + a*b^3)*d^2*e*f^2 + 2*(a^3*b + a*b^3)*d*f^3)*x - (4*(a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*(a^2*b^2 + b^4)*d^3*e^3 - 6*(a^2*b^2 + b^4)*d^2*e^2*f + 6*(a^2*b^2 + b^4)*d
```

$$\begin{aligned}
& *e*f^2 - 3*(a^2*b^2 + b^4)*f^3 + 6*(2*(a^2*b^2 + b^4)*d^3*e*f^2 - (a^2*b^2 \\
& + b^4)*d^2*f^3)*x^2 + 6*(2*(a^2*b^2 + b^4)*d^3*e^2*f - 2*(a^2*b^2 + b^4)*d^2 \\
& *e*f^2 + (a^2*b^2 + b^4)*d*f^3)*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 6*(2*( \\
& a^2*b^2 + b^4)*d^3*e*f^2 + (a^2*b^2 + b^4)*d^2*f^3)*x^2 - 4*((2*a^4 + a^2*b \\
& ^2 - b^4)*d^4*f^3*x^4 + 4*(2*a^4 + a^2*b^2 - b^4)*d^4*e*f^2*x^3 + 6*(2*a^4 \\
& + a^2*b^2 - b^4)*d^4*e^2*f*x^2 + 4*(2*a^4 + a^2*b^2 - b^4)*d^4*e^3*x)*\cosh( \\
& d*x + c)^2 - 2*(2*(2*a^4 + a^2*b^2 - b^4)*d^4*f^3*x^4 + 8*(2*a^4 + a^2*b^2 \\
& - b^4)*d^4*e*f^2*x^3 + 12*(2*a^4 + a^2*b^2 - b^4)*d^4*e^2*f*x^2 + 8*(2*a^4 \\
& + a^2*b^2 - b^4)*d^4*e^3*x + 3*(4*(a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*(a^2*b^2 \\
& + b^4)*d^3*e^3 - 6*(a^2*b^2 + b^4)*d^2*e^2*f + 6*(a^2*b^2 + b^4)*d*e*f^2 - \\
& 3*(a^2*b^2 + b^4)*f^3 + 6*(2*(a^2*b^2 + b^4)*d^3*e*f^2 - (a^2*b^2 + b^4)*d^2 \\
& *f^3)*x^2 + 6*(2*(a^2*b^2 + b^4)*d^3*e^2*f - 2*(a^2*b^2 + b^4)*d^2*e*f^2 + \\
& (a^2*b^2 + b^4)*d*f^3)*x)*\cosh(d*x + c)^2 - 24*((a^3*b + a*b^3)*d^3*f^3*x^3 \\
& + (a^3*b + a*b^3)*d^3*e^3 - 3*(a^3*b + a*b^3)*d^2*e^2*f + 6*(a^3*b + a*b^3) \\
& *d*e*f^2 - 6*(a^3*b + a*b^3)*f^3 + 3*((a^3*b + a*b^3)*d^3*e*f^2 - (a^3*b \\
& + a*b^3)*d^2*f^3)*x^2 + 3*((a^3*b + a*b^3)*d^3*e^2*f - 2*(a^3*b + a*b^3)*d^2 \\
& *e*f^2 + 2*(a^3*b + a*b^3)*d*f^3)*x)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 96*( \\
& (a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f)*\cosh(d*x + c)^2 \\
& + 2*(a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f)*\cosh(d*x + \\
& c))*\sinh(d*x + c) + (a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2 \\
& *f)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2})*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh \\
& (d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c)))*\sqrt{(a^2 + b^2)/b^2} - b)/ \\
& b + 1) - 96*((a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f)*\co \\
& sh(d*x + c)^2 + 2*(a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2* \\
& f)*\cosh(d*x + c))*\sinh(d*x + c) + (a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + \\
& a^3*b*d^2*e^2*f)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2})*\operatorname{dilog}((a*\cosh(d*x \\
& + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c)))*\sqrt{(a^2 + b^ \\
& 2)/b^2} - b)/b + 1) - 32*((a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^ \\
& 2*d*e*f^2 - a^3*b*c^3*f^3)*\cosh(d*x + c)^2 + 2*(a^3*b*d^3*e^3 - 3*a^3*b*c*d^ \\
& ^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*\cosh(d*x + c))*\sinh(d*x + c) \\
& + (a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3) \\
& *\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2})*\log(2*b*\cosh(d*x + c) + 2*b*\sinh \\
& (d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 32*((a^3*b*d^3*e^3 - 3*a^3*b \\
& *c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*\cosh(d*x + c)^2 + 2*(a^ \\
& 3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*\co \\
& sh(d*x + c))*\sinh(d*x + c) + (a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b* \\
& c^2*d*e*f^2 - a^3*b*c^3*f^3)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2})*\log(2*b \\
& *\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 32* \\
& ((a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b \\
& *c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*\cosh(d*x + c)^2 + 2*(a^ \\
& 3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^ \\
& ^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*\cosh(d*x + c))*\sinh(d*x + c) \\
& + (a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3 \\
& *b*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*\sinh(d*x + c)^2)*\sqrt{ \\
& (a^2 + b^2)/b^2})*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c)
\end{aligned}$$

$$\begin{aligned}
& ) + b \sinh(dx + c) \sqrt{(a^2 + b^2)/b^2} - b/b) - 32((a^3 b d^3 f^3 x^3 \\
& + 3 a^3 b d^3 e f^2 x^2 + 3 a^3 b d^3 e^2 f x + 3 a^3 b c d^2 e^2 f - 3 a^3 \\
& 3 b c^2 d e f^2 + a^3 b c^3 f^3) \cosh(dx + c)^2 + 2(a^3 b d^3 f^3 x^3 + 3 \\
& a^3 b d^3 e f^2 x^2 + 3 a^3 b d^3 e^2 f x + 3 a^3 b c d^2 e^2 f - 3 a^3 b c \\
& c^2 d e f^2 + a^3 b c^3 f^3) \cosh(dx + c) \sinh(dx + c) + (a^3 b d^3 f^3 x \\
& ^3 + 3 a^3 b d^3 e f^2 x^2 + 3 a^3 b d^3 e^2 f x + 3 a^3 b c d^2 e^2 f - 3 a \\
& a^3 b c^2 d e f^2 + a^3 b c^3 f^3) \sinh(dx + c)^2 \sqrt{(a^2 + b^2)/b^2} * \\
& \log(-(a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c) \\
& )) \sqrt{(a^2 + b^2)/b^2} - b/b) + 192(a^3 b f^3 \cosh(dx + c)^2 + 2 a^3 b f \\
& f^3 \cosh(dx + c) \sinh(dx + c) + a^3 b f^3 \sinh(dx + c)^2) \sqrt{(a^2 + b^ \\
& 2)/b^2} \operatorname{polylog}(4, (a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + \\
& b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2})/b) - 192(a^3 b f^3 \cosh(dx + c)^2 \\
& + 2 a^3 b f^3 \cosh(dx + c) \sinh(dx + c) + a^3 b f^3 \sinh(dx + c)^2) \sqrt{ \\
& (a^2 + b^2)/b^2} \operatorname{polylog}(4, (a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh( \\
& dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2})/b) - 192((a^3 b d f^3 x \\
& + a^3 b d e f^2) \cosh(dx + c)^2 + 2(a^3 b d f^3 x + a^3 b d e f^2) \cosh( \\
& dx + c) \sinh(dx + c) + (a^3 b d f^3 x + a^3 b d e f^2) \sinh(dx + c)^2) \sqrt{ \\
& (a^2 + b^2)/b^2} \operatorname{polylog}(3, (a \cosh(dx + c) + a \sinh(dx + c) + (b \cos \\
& h(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2})/b) + 192((a^3 b d f^3 \\
& x + a^3 b d e f^2) \cosh(dx + c)^2 + 2(a^3 b d f^3 x + a^3 b d e f^2) \cos \\
& h(dx + c) \sinh(dx + c) + (a^3 b d f^3 x + a^3 b d e f^2) \sinh(dx + c)^2) \\
& \sqrt{(a^2 + b^2)/b^2} \operatorname{polylog}(3, (a \cosh(dx + c) + a \sinh(dx + c) - (b \c \\
& osh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2})/b) + 6(2(a^2 b^2 + \\
& b^4) d^3 e^2 f + 2(a^2 b^2 + b^4) d^2 e f^2 + (a^2 b^2 + b^4) d f^3) x + \\
& 16((a^3 b + a b^3) d^3 f^3 x^3 + (a^3 b + a b^3) d^3 e^3 + 3(a^3 b + a b^ \\
& 3) d^2 e^2 f + 6(a^3 b + a b^3) d e f^2 + 6(a^3 b + a b^3) f^3 + 3((a^3 b \\
& b + a b^3) d^3 e f^2 + (a^3 b + a b^3) d^2 f^3) x^2 + 3((a^3 b + a b^3) d^ \\
& 3 e^2 f + 2(a^3 b + a b^3) d^2 e f^2 + 2(a^3 b + a b^3) d f^3) x) \cosh(dx \\
& x + c) + 4(4(a^3 b + a b^3) d^3 f^3 x^3 + 4(a^3 b + a b^3) d^3 e^3 + 12 \\
& (a^3 b + a b^3) d^2 e^2 f + 24(a^3 b + a b^3) d e f^2 + 24(a^3 b + a b^3) \\
& * f^3 - (4(a^2 b^2 + b^4) d^3 f^3 x^3 + 4(a^2 b^2 + b^4) d^3 e^3 - 6(a^2 b \\
& b^2 + b^4) d^2 e^2 f + 6(a^2 b^2 + b^4) d e f^2 - 3(a^2 b^2 + b^4) f^3 + \\
& 6(2(a^2 b^2 + b^4) d^3 e f^2 - (a^2 b^2 + b^4) d^2 f^3) x^2 + 6(2(a^2 b \\
& ^2 + b^4) d^3 e^2 f - 2(a^2 b^2 + b^4) d^2 e f^2 + (a^2 b^2 + b^4) d f^3) * \\
& x) \cosh(dx + c)^3 + 12((a^3 b + a b^3) d^3 e f^2 + (a^3 b + a b^3) d^2 f^ \\
& 3) x^2 + 12((a^3 b + a b^3) d^3 f^3 x^3 + (a^3 b + a b^3) d^3 e^3 - 3(a^3 \\
& * b + a b^3) d^2 e^2 f + 6(a^3 b + a b^3) d e f^2 - 6(a^3 b + a b^3) f^3 + \\
& 3((a^3 b + a b^3) d^3 e f^2 - (a^3 b + a b^3) d^2 f^3) x^2 + 3((a^3 b + \\
& a b^3) d^3 e^2 f - 2(a^3 b + a b^3) d^2 e f^2 + 2(a^3 b + a b^3) d f^3) x \\
& ) \cosh(dx + c)^2 + 12((a^3 b + a b^3) d^3 e^2 f + 2(a^3 b + a b^3) d^2 e \\
& * f^2 + 2(a^3 b + a b^3) d f^3) x - 2((2 a^4 + a^2 b^2 - b^4) d^4 f^3 x^4 \\
& + 4(2 a^4 + a^2 b^2 - b^4) d^4 e f^2 x^3 + 6(2 a^4 + a^2 b^2 - b^4) d^4 e \\
& ^2 f x^2 + 4(2 a^4 + a^2 b^2 - b^4) d^4 e^3 x) \cosh(dx + c) \sinh(dx + c \\
& )) / ((a^2 b^3 + b^5) d^4 \cosh(dx + c)^2 + 2(a^2 b^3 + b^5) d^4 \cosh(dx + \\
& c) \sinh(dx + c) + (a^2 b^3 + b^5) d^4 \sinh(dx + c)^2)
\end{aligned}$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*sinh(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \sinh(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*sinh(d\*x + c)^3/(b\*sinh(d\*x + c) + a), x)



$$3.234 \quad \int \frac{(e+fx)^2 \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=522

$$-\frac{2a^3 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^2 \sqrt{a^2+b^2}} + \frac{2a^3 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3 d^2 \sqrt{a^2+b^2}} + \frac{2a^3 f^2 \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^3 \sqrt{a^2+b^2}} - \frac{2a^3}{b^3 d^3 \sqrt{a^2+b^2}}$$

[Out]  $-(f^2 x)/(4 b d^2) + (a^2 (e + f x)^3)/(3 b^3 f) - (e + f x)^3/(6 b f) - (2 a f^2 \operatorname{Cosh}[c + d x])/(b^2 d^3) - (a (e + f x)^2 \operatorname{Cosh}[c + d x])/(b^2 d) - (a^3 (e + f x)^2 \operatorname{Log}[1 + (b E^{c + d x})/(a - \sqrt{a^2 + b^2})])/(b^3 \sqrt{a^2 + b^2} d) + (a^3 (e + f x)^2 \operatorname{Log}[1 + (b E^{c + d x})/(a + \sqrt{a^2 + b^2})])/(b^3 \sqrt{a^2 + b^2} d) - (2 a^3 f (e + f x) \operatorname{PolyLog}[2, -((b E^{c + d x})/(a - \sqrt{a^2 + b^2}))])/(b^3 \sqrt{a^2 + b^2} d^2) + (2 a^3 f (e + f x) \operatorname{PolyLog}[2, -((b E^{c + d x})/(a + \sqrt{a^2 + b^2}))])/(b^3 \sqrt{a^2 + b^2} d^2) + (2 a^3 f^2 \operatorname{PolyLog}[3, -((b E^{c + d x})/(a - \sqrt{a^2 + b^2}))])/(b^3 \sqrt{a^2 + b^2} d^3) - (2 a^3 f^2 \operatorname{PolyLog}[3, -((b E^{c + d x})/(a + \sqrt{a^2 + b^2}))])/(b^3 \sqrt{a^2 + b^2} d^3) + (2 a f (e + f x) \operatorname{Sinh}[c + d x])/(b^2 d^2) + (f^2 \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x])/(4 b d^3) + ((e + f x)^2 \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x])/(2 b d) - (f (e + f x) \operatorname{Sinh}[c + d x]^2)/(2 b d^2)$

**Rubi [A]** time = 1.04363, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$ , Rules used = {5557, 3311, 32, 2635, 8, 3296, 2638, 3322, 2264, 2190, 2531, 2282, 6589}

$$-\frac{2a^3 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^2 \sqrt{a^2+b^2}} + \frac{2a^3 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3 d^2 \sqrt{a^2+b^2}} + \frac{2a^3 f^2 \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^3 \sqrt{a^2+b^2}} - \frac{2a^3}{b^3 d^3 \sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e + f x)^2 \operatorname{Sinh}[c + d x]^3/(a + b \operatorname{Sinh}[c + d x]), x]$

[Out]  $-(f^2 x)/(4 b d^2) + (a^2 (e + f x)^3)/(3 b^3 f) - (e + f x)^3/(6 b f) - (2 a f^2 \operatorname{Cosh}[c + d x])/(b^2 d^3) - (a (e + f x)^2 \operatorname{Cosh}[c + d x])/(b^2 d) - (a^3 (e + f x)^2 \operatorname{Log}[1 + (b E^{c + d x})/(a - \sqrt{a^2 + b^2})])/(b^3 \sqrt{a^2 + b^2} d) + (a^3 (e + f x)^2 \operatorname{Log}[1 + (b E^{c + d x})/(a + \sqrt{a^2 + b^2})])/(b^3 \sqrt{a^2 + b^2} d) - (2 a^3 f (e + f x) \operatorname{PolyLog}[2, -((b E^{c + d x})/(a - \sqrt{a^2 + b^2}))])/(b^3 \sqrt{a^2 + b^2} d^2) + (2 a^3 f (e + f x) \operatorname{PolyLog}[2, -((b E^{c + d x})/(a + \sqrt{a^2 + b^2}))])/(b^3 \sqrt{a^2 + b^2} d^2) + (2 a^3 f^2 \operatorname{PolyLog}[3, -((b E^{c + d x})/(a - \sqrt{a^2 + b^2}))])/(b^3 \sqrt{a^2 + b^2} d^3) - (2 a^3 f^2 \operatorname{PolyLog}[3, -((b E^{c + d x})/(a + \sqrt{a^2 + b^2}))])/(b^3 \sqrt{a^2 + b^2} d^3) + (2 a f (e + f x) \operatorname{Sinh}[c + d x])/(b^2 d^2) + (f^2 \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x])/(4 b d^3) + ((e + f x)^2 \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x])/(2 b d) - (f (e + f x) \operatorname{Sinh}[c + d x]^2)/(2 b d^2)$

$$\begin{aligned} &^3 \sqrt{a^2 + b^2} d^3 - (2a^3 f^2 \text{PolyLog}[3, -((bE^{(c+dx)})/(a + \sqrt{a^2 + b^2}))]) / (b^3 \sqrt{a^2 + b^2} d^3) + (2af(e+fx) \sinh[c+dx]) / (b^2 d^2) + (f^2 \cosh[c+dx] \sinh[c+dx]) / (4bd^3) + ((e+fx)^2 \cosh[c+dx] \sinh[c+dx]) / (2bd) - (f(e+fx) \sinh[c+dx]^2) / (2bd^2) \end{aligned}$$
Rule 5557

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])^(n_.)/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Sinh[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sinh[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sinh[e + f*x])^n, x], x] - Simp[(b*(c + d*x)^m*Cosh[e + f*x]*(b*Sinh[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] := -Simp[(b*Cosh[c + d*x]*(b*Sinh[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sinh[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cosh[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cosh[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3322

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*sin[(e\_.) + (Complex[0, fz\_])\* (f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(- (I\*b) + 2\*a\*E^(-(I\*e) + f\*fz\*x) + I\*b\*E^(2\*(-(I\*e) + f\*fz\*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.) \*(F\_)^(v\_)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[((f + g\*x)^m\*F^u)/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[((f + g\*x)^ m\*F^u)/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/ ((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)), x\_Symbol] := Simp [((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^(c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.)]\*((f\_.) + (g\_.) \*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\* (F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e + fx)^2 \sinh^3(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\int (e + fx)^2 \sinh^2(c + dx) dx}{b} - \frac{a \int \frac{(e + fx)^2 \sinh^2(c + dx)}{a + b \sinh(c + dx)} dx}{b} \\
&= \frac{(e + fx)^2 \cosh(c + dx) \sinh(c + dx)}{2bd} - \frac{f(e + fx) \sinh^2(c + dx)}{2bd^2} - \frac{a \int (e + fx)^2 \sinh(c + dx) dx}{b^2} \\
&= -\frac{(e + fx)^3}{6bf} - \frac{a(e + fx)^2 \cosh(c + dx)}{b^2d} + \frac{f^2 \cosh(c + dx) \sinh(c + dx)}{4bd^3} + \frac{(e + fx)^2 \cosh(c + dx)}{b^2d} \\
&= -\frac{f^2x}{4bd^2} + \frac{a^2(e + fx)^3}{3b^3f} - \frac{(e + fx)^3}{6bf} - \frac{a(e + fx)^2 \cosh(c + dx)}{b^2d} + \frac{2af(e + fx) \sinh(c + dx)}{b^2d^2} \\
&= -\frac{f^2x}{4bd^2} + \frac{a^2(e + fx)^3}{3b^3f} - \frac{(e + fx)^3}{6bf} - \frac{2af^2 \cosh(c + dx)}{b^2d^3} - \frac{a(e + fx)^2 \cosh(c + dx)}{b^2d} + \frac{2af(e + fx) \sinh(c + dx)}{b^2d^2} \\
&= -\frac{f^2x}{4bd^2} + \frac{a^2(e + fx)^3}{3b^3f} - \frac{(e + fx)^3}{6bf} - \frac{2af^2 \cosh(c + dx)}{b^2d^3} - \frac{a(e + fx)^2 \cosh(c + dx)}{b^2d} - \frac{a^3(e + fx) \sinh(c + dx)}{b^2d^2} \\
&= -\frac{f^2x}{4bd^2} + \frac{a^2(e + fx)^3}{3b^3f} - \frac{(e + fx)^3}{6bf} - \frac{2af^2 \cosh(c + dx)}{b^2d^3} - \frac{a(e + fx)^2 \cosh(c + dx)}{b^2d} - \frac{a^3(e + fx) \sinh(c + dx)}{b^2d^2} \\
&= -\frac{f^2x}{4bd^2} + \frac{a^2(e + fx)^3}{3b^3f} - \frac{(e + fx)^3}{6bf} - \frac{2af^2 \cosh(c + dx)}{b^2d^3} - \frac{a(e + fx)^2 \cosh(c + dx)}{b^2d} - \frac{a^3(e + fx) \sinh(c + dx)}{b^2d^2}
\end{aligned}$$

**Mathematica [A]** time = 4.35971, size = 740, normalized size = 1.42

$$-\frac{48a^3 f(e+fx) \text{PolyLog}\left(2, \frac{be^c+dx}{\sqrt{a^2+b^2}-a}\right)}{d^2 \sqrt{a^2+b^2}} + \frac{48a^3 f(e+fx) \text{PolyLog}\left(2, -\frac{be^c+dx}{\sqrt{a^2+b^2}+a}\right)}{d^2 \sqrt{a^2+b^2}} + \frac{48a^3 f^2 \text{PolyLog}\left(3, \frac{be^c+dx}{\sqrt{a^2+b^2}-a}\right)}{d^3 \sqrt{a^2+b^2}} - \frac{48a^3 f^2 \text{PolyLog}\left(3, -\frac{be^c+dx}{\sqrt{a^2+b^2}+a}\right)}{d^3 \sqrt{a^2+b^2}} + \frac{48a^3 e^2}{d^3 \sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Sinh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] (24\*a^2\*e^2\*x - 12\*b^2\*e^2\*x + 24\*a^2\*e\*f\*x^2 - 12\*b^2\*e\*f\*x^2 + 8\*a^2\*f^2\*x^3 - 4\*b^2\*f^2\*x^3 + (48\*a^3\*e^2\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]\*d) - (24\*a\*b\*e^2\*Cosh[c + d\*x])/d - (48\*a\*b\*f^2\*Cosh[c + d\*x])/d^3 - (48\*a\*b\*e\*f\*x\*Cosh[c + d\*x])/d - (24\*a\*b\*f^2\*x^2\*Cosh[c + d\*x])/d - (6\*b^2\*e\*f\*Cosh[2\*(c + d\*x)]/d^2 - (6\*b^2\*f^2\*x\*Cosh[2\*(c + d\*x)]/d^2 - (48\*a^3\*e\*f\*x\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]\*d) - (24\*a^3\*f^2\*x^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]\*d) + (48\*a^3\*e\*f\*x\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]\*d) + (24\*a^3\*f^2\*x^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]\*d) - (48\*a^3\*f\*(e + f\*x)\*PolyLog[2, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]\*d^2) + (48\*a^3\*f\*(e + f\*x)\*PolyLog[2, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]\*d^2) + (48\*a^3\*f^2\*PolyLog[3, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]\*d^3) - (48\*a^3\*f^2\*PolyLog[3, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]\*d^3) + (48\*a\*b\*e\*f\*Sinh[c + d\*x])/d^2 + (48\*a\*b\*f^2\*x\*Sinh[c + d\*x])/d^2 + (6\*b^2\*e^2\*Sinh[2\*(c + d\*x)]/d + (3\*b^2\*f^2\*Sinh[2\*(c + d\*x)]/d^3 + (12\*b^2\*e\*f\*x\*Sinh[2\*(c + d\*x)]/d + (6\*b^2\*f^2\*x^2\*Sinh[2\*(c + d\*x)]/d)/(24\*b^3)

**Maple [F]** time = 0.086, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\sinh(dx + c))^3}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [C]** time = 3.25167, size = 7156, normalized size = 13.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-1/48*(6*(a^2*b^2 + b^4)*d^2*f^2*x^2 + 6*(a^2*b^2 + b^4)*d^2*e^2 - 3*(2*(a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^2*b^2 + b^4)*d^2*e^2 - 2*(a^2*b^2 + b^4)*d*e*f + (a^2*b^2 + b^4)*f^2 + 2*(2*(a^2*b^2 + b^4)*d^2*e*f - (a^2*b^2 + b^4)*d*f^2)*x)*\cosh(d*x + c)^4 - 3*(2*(a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^2*b^2 + b^4)*d^2*e^2 - 2*(a^2*b^2 + b^4)*d*e*f + (a^2*b^2 + b^4)*f^2 + 2*(2*(a^2*b^2 + b^4)*d^2*e*f - (a^2*b^2 + b^4)*d*f^2)*x)*\sinh(d*x + c)^4 + 6*(a^2*b^2 + b^4)*d*e*f + 24*((a^3*b + a*b^3)*d^2*f^2*x^2 + (a^3*b + a*b^3)*d^2*e^2 - 2*(a^3*b + a*b^3)*d*e*f + 2*(a^3*b + a*b^3)*f^2 + 2*((a^3*b + a*b^3)*d^2*e*f - (a^3*b + a*b^3)*d*f^2)*x)*\cosh(d*x + c)^3 + 12*(2*(a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e^2 - 4*(a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3)*f^2 + 4*((a^3*b + a*b^3)*d^2*e*f - (a^3*b + a*b^3)*d*f^2)*x - (2*(a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^2*b^2 + b^4)*d^2*e^2 - 2*(a^2*b^2 + b^4)*d*e*f + (a^2*b^2 + b^4)*f^2 + 2*(2*(a^2*b^2 + b^4)*d^2*e*f - (a^2*b^2 + b^4)*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 3*(a^2*b^2 + b^4)*f^2 - 8*((2*a^4 + a^2*b^2 - b^4)*d^3*f^2*x^3 + 3*(2*a^4 + a^2*b^2 - b^4)*d^3*e*f*x^2 + 3*(2*a^4 + a^2*b^2 - b^4)*d^3*e^2*x)*\cosh(d*x + c)^2 - 2*(4*(2*a^4 + a^2*b^2 - b^4)*d^3*f^2*x^3 + 12*(2*a^4 + a^2*b^2 - b^4)*d^3*e*f*x^2 + 12*(2*a^4 + a^2*b^2 - b^4)*d^3*e^2*x + 9*(2*(a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^2*b^2 + b^4)*d^2*e^2 - 2*(a^2*b^2 + b^4)*d*e*f + (a^2*b^2 + b^4)*f^2 + 2*(2*(a^2*b^2 + b^4)*d^2*e*f - (a^2*b^2 + b^4)*d*f^2)*x)*\cosh(d*x + c))^2 - 36*((a^3*b + a*b^3)*d^2*f^2*x^2 + (a^3*b + a*b^3)*d^2*e^2 - 2*(a^3*b + a*b^3)*d*e*f + 2*(a^3*b + a*b^3)*f^2 + 2*((a^3*b + a*b^3)*d^2*e*f - (a^3*b + a*b^3)*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 96*((a^3*b*d*f^2*x + a^3*b*d*e*f)*\cosh(d*x + c))^2 + 2*(a^3*b*d*f^2*x + a^3*b*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c) + (a^3*b*d*f^2*x + a^3*b*d*e*f)*\sinh(d*x + c)^2)*\sqrt((a^2 + b^2)/b^2)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 96*((a^3*b*d*f^2*x + a^3*b*d*e*f)*\cosh(d*x + c))^2 + 2*(a^3*b*d*f^2*x + a^3*b*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c) + (a^3*b*d*f^2*x + a^3*b*d*e*f)*\sinh(d*x + c)^2)*\sqrt((a^2 + b^2)/b^2)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 48*((a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*\cosh(d*x + c))^2 + 2*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)$$

$$\begin{aligned}
&^2) * \cosh(d*x + c) * \sinh(d*x + c) + (a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2) * \sinh(d*x + c)^2 * \sqrt{(a^2 + b^2)/b^2} * \log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 48*((a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2) * \cosh(d*x + c)^2 + 2*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2) * \cosh(d*x + c) * \sinh(d*x + c) + (a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2) * \sinh(d*x + c)^2) * \sqrt{(a^2 + b^2)/b^2} * \log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 48*((a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2) * \cosh(d*x + c)^2 + 2*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2) * \cosh(d*x + c) * \sinh(d*x + c) + (a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2) * \sinh(d*x + c)^2) * \sqrt{(a^2 + b^2)/b^2} * \log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c)) * \sqrt{(a^2 + b^2)/b^2} - b)/b) - 48*((a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2) * \cosh(d*x + c)^2 + 2*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2) * \cosh(d*x + c) * \sinh(d*x + c) + (a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2) * \sinh(d*x + c)^2) * \sqrt{(a^2 + b^2)/b^2} * \log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c)) * \sqrt{(a^2 + b^2)/b^2} - b)/b) - 96*(a^3*b*f^2*\cosh(d*x + c)^2 + 2*a^3*b*f^2*\cosh(d*x + c) * \sinh(d*x + c) + a^3*b*f^2*\sinh(d*x + c)^2) * \sqrt{(a^2 + b^2)/b^2} * \text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c)) * \sqrt{(a^2 + b^2)/b^2}))/b) + 96*(a^3*b*f^2*\cosh(d*x + c)^2 + 2*a^3*b*f^2*\cosh(d*x + c) * \sinh(d*x + c) + a^3*b*f^2*\sinh(d*x + c)^2) * \sqrt{(a^2 + b^2)/b^2} * \text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c)) * \sqrt{(a^2 + b^2)/b^2}))/b) + 6*(2*(a^2*b^2 + b^4)*d^2*e*f + (a^2*b^2 + b^4)*d*f^2)*x + 24*((a^3*b + a*b^3)*d^2*f^2*x^2 + (a^3*b + a*b^3)*d^2*e^2 + 2*(a^3*b + a*b^3)*d*e*f + 2*(a^3*b + a*b^3)*f^2 + 2*((a^3*b + a*b^3)*d^2*e*f + (a^3*b + a*b^3)*d*f^2)*x) * \cosh(d*x + c) + 4*(6*(a^3*b + a*b^3)*d^2*f^2*x^2 + 6*(a^3*b + a*b^3)*d^2*e^2 + 12*(a^3*b + a*b^3)*d*e*f - 3*(2*(a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(a^2*b^2 + b^4)*d^2*e^2 - 2*(a^2*b^2 + b^4)*d*e*f + (a^2*b^2 + b^4)*f^2 + 2*(2*(a^2*b^2 + b^4)*d^2*e*f - (a^2*b^2 + b^4)*d*f^2)*x) * \cosh(d*x + c)^3 + 12*(a^3*b + a*b^3)*f^2 + 18*((a^3*b + a*b^3)*d^2*f^2*x^2 + (a^3*b + a*b^3)*d^2*e^2 - 2*(a^3*b + a*b^3)*d*e*f + 2*(a^3*b + a*b^3)*f^2 + 2*((a^3*b + a*b^3)*d^2*e*f - (a^3*b + a*b^3)*d*f^2)*x) * \cosh(d*x + c)^2 + 12*((a^3*b + a*b^3)*d^2*e*f + (a^3*b + a*b^3)*d*f^2)*x - 4*((2*a^4 + a^2*b^2 - b^4)*d^3*f^2*x^3 + 3*(2*a^4 + a^2*b^2 - b^4)*d^3*e*f*x^2 + 3*(2*a^4 + a^2*b^2 - b^4)*d^3*e^2*x) * \cosh(d*x + c) * \sinh(d*x + c) / ((a^2*b^3 + b^5)*d^3*\cosh(d*x + c)^2 + 2*(a^2*b^3 + b^5)*d^3*\cosh(d*x + c) * \sinh(d*x + c) + (a^2*b^3 + b^5)*d^3*\sinh(d*x + c)^2)
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*sinh(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \sinh(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*sinh(d\*x + c)^3/(b\*sinh(d\*x + c) + a), x)



$$3.235 \quad \int \frac{(e+fx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=335

$$-\frac{a^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^2 \sqrt{a^2+b^2}} + \frac{a^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3 d^2 \sqrt{a^2+b^2}} - \frac{a^3 (e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3 d \sqrt{a^2+b^2}} + \frac{a^3 (e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3 d \sqrt{a^2+b^2}}$$

[Out]  $(a^2 e^x)/b^3 - (e^x)/(2b) + (a^2 f x^2)/(2b^3) - (f x^2)/(4b) - (a(e + f x) \operatorname{Cosh}[c + d x])/(b^2 d) - (a^3 (e + f x) \operatorname{Log}[1 + (b E^{c + d x})/(a - \operatorname{Sqrt}[a^2 + b^2])])/(b^3 \operatorname{Sqrt}[a^2 + b^2] d) + (a^3 (e + f x) \operatorname{Log}[1 + (b E^{c + d x})/(a + \operatorname{Sqrt}[a^2 + b^2])])/(b^3 \operatorname{Sqrt}[a^2 + b^2] d) - (a^3 f \operatorname{PolyLog}[2, -((b E^{c + d x})/(a - \operatorname{Sqrt}[a^2 + b^2]))])/(b^3 \operatorname{Sqrt}[a^2 + b^2] d^2) + (a^3 f \operatorname{PolyLog}[2, -((b E^{c + d x})/(a + \operatorname{Sqrt}[a^2 + b^2]))])/(b^3 \operatorname{Sqrt}[a^2 + b^2] d^2) + (a f \operatorname{Sinh}[c + d x])/(b^2 d^2) + ((e + f x) \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x])/(2 b d) - (f \operatorname{Sinh}[c + d x]^2)/(4 b d^2)$

**Rubi [A]** time = 0.593089, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$ , Rules used = {5557, 3310, 3296, 2637, 3322, 2264, 2190, 2279, 2391}

$$-\frac{a^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^2 \sqrt{a^2+b^2}} + \frac{a^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3 d^2 \sqrt{a^2+b^2}} - \frac{a^3 (e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^3 d \sqrt{a^2+b^2}} + \frac{a^3 (e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3 d \sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e + f x) \operatorname{Sinh}[c + d x]^3 / (a + b \operatorname{Sinh}[c + d x]), x]$

[Out]  $(a^2 e^x)/b^3 - (e^x)/(2b) + (a^2 f x^2)/(2b^3) - (f x^2)/(4b) - (a(e + f x) \operatorname{Cosh}[c + d x])/(b^2 d) - (a^3 (e + f x) \operatorname{Log}[1 + (b E^{c + d x})/(a - \operatorname{Sqrt}[a^2 + b^2])])/(b^3 \operatorname{Sqrt}[a^2 + b^2] d) + (a^3 (e + f x) \operatorname{Log}[1 + (b E^{c + d x})/(a + \operatorname{Sqrt}[a^2 + b^2])])/(b^3 \operatorname{Sqrt}[a^2 + b^2] d) - (a^3 f \operatorname{PolyLog}[2, -((b E^{c + d x})/(a - \operatorname{Sqrt}[a^2 + b^2]))])/(b^3 \operatorname{Sqrt}[a^2 + b^2] d^2) + (a^3 f \operatorname{PolyLog}[2, -((b E^{c + d x})/(a + \operatorname{Sqrt}[a^2 + b^2]))])/(b^3 \operatorname{Sqrt}[a^2 + b^2] d^2) + (a f \operatorname{Sinh}[c + d x])/(b^2 d^2) + ((e + f x) \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x])/(2 b d) - (f \operatorname{Sinh}[c + d x]^2)/(4 b d^2)$

### Rule 5557

$\operatorname{Int}[(e + f x)^m \operatorname{Sinh}[c + d x]^n / (a + b \operatorname{Sinh}[c + d x]), x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[1/b, \operatorname{Int}[(e + f x)^m \operatorname{Sinh}[c + d x]^n, x_{\text{Symbol}}]]$

```

+ d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0]

```

### Rule 3310

```

Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
Simp[(d*(b*Sinh[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sinh[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sinh[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]

```

### Rule 3296

```

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

```

### Rule 2637

```

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

```

### Rule 3322

```

Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_]*)
(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

```

### Rule 2264

```

Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]

```

### Rule 2190

```

Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)

```

)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol]  
 :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{(e+fx)\sinh^3(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\sinh^2(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)\sinh^2(c+dx)}{a+b\sinh(c+dx)} dx}{b} \\
 &= \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{f\sinh^2(c+dx)}{4bd^2} - \frac{a \int (e+fx)\sinh(c+dx) dx}{b^2} + \dots \\
 &= -\frac{ex}{2b} - \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} + \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{f\sinh^2(c+dx)}{4bd^2} \\
 &= \frac{a^2ex}{b^3} - \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} - \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} + \frac{af\sinh(c+dx)}{b^2d^2} + \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{2bd} \\
 &= \frac{a^2ex}{b^3} - \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} - \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} + \frac{af\sinh(c+dx)}{b^2d^2} + \frac{(e+fx)\cosh(c+dx)\sinh(c+dx)}{2bd} \\
 &= \frac{a^2ex}{b^3} - \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} - \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} - \frac{a^3(e+fx)\log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}d} + \dots \\
 &= \frac{a^2ex}{b^3} - \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} - \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} - \frac{a^3(e+fx)\log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}d} + \dots \\
 &= \frac{a^2ex}{b^3} - \frac{ex}{2b} + \frac{a^2fx^2}{2b^3} - \frac{fx^2}{4b} - \frac{a(e+fx)\cosh(c+dx)}{b^2d} - \frac{a^3(e+fx)\log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}d} + \dots
 \end{aligned}$$

**Mathematica [A]** time = 2.68747, size = 307, normalized size = 0.92

$$\frac{8a^3 \left( -f \operatorname{PolyLog} \left( 2, \frac{be^{c+dx}}{\sqrt{a^2+b^2-a}} \right) + f \operatorname{PolyLog} \left( 2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2+a}} \right) + 2de \tanh^{-1} \left( \frac{a+be^{c+dx}}{\sqrt{a^2+b^2}} \right) - f(c+dx) \log \left( \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1 \right) + f(c+dx) \log \left( \frac{be^{c+dx}}{\sqrt{a^2+b^2+a}} + 1 \right) - 2cf \tanh^{-1} \left( \frac{a+be^{c+dx}}{\sqrt{a^2+b^2}} \right) \right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Sinh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out]  $(-2*(2*a^2 - b^2)*(c + d*x)*(c*f - d*(2*e + f*x)) - 8*a*b*d*(e + f*x)*\operatorname{Cosh}[c + d*x] - b^2*f*\operatorname{Cosh}[2*(c + d*x)] + (8*a^3*(2*d*e*\operatorname{ArcTanh}[(a + b*E^{(c + d*x)})]/\operatorname{Sqrt}[a^2 + b^2]) - 2*c*f*\operatorname{ArcTanh}[(a + b*E^{(c + d*x)})]/\operatorname{Sqrt}[a^2 + b^2] - f*(c + d*x)*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])] + f*(c + d*x)*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])] - f*\operatorname{PolyLog}[2, (b*E^{(c + d*x)})/(-a + \operatorname{Sqrt}[a^2 + b^2])] + f*\operatorname{PolyLog}[2, -(b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])]/\operatorname{Sqrt}[a^2 + b^2] + 8*a*b*f*\operatorname{Sinh}[c + d*x] + 2*b^2*d*(e + f*x)*\operatorname{Sinh}[2*(c + d*x)])/(8*b^3*d^2)$

**Maple [A]** time = 0.067, size = 589, normalized size = 1.8

$$\frac{a^2 f x^2}{2 b^3} - \frac{f x^2}{4 b} + \frac{a^2 e x}{b^3} - \frac{e x}{2 b} + \frac{(2 d f x + 2 d e - f) e^{2 d x + 2 c}}{16 d^2 b} - \frac{a (d f x + d e - f) e^{d x + c}}{2 b^2 d^2} - \frac{a (d f x + d e + f) e^{-d x - c}}{2 b^2 d^2} - \frac{(2 d f x + 2 d e - f) e^{2 d x + 2 c}}{16 d^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

[Out]  $\frac{1}{2} a^2 f x^2 / b^3 - \frac{1}{4} f x^2 / b + a^2 e x / b^3 - \frac{1}{2} e x / b + \frac{1}{16} (2 d f x + 2 d e - f) / d^2 b \exp(2 d x + 2 c) - \frac{1}{2} a (d f x + d e - f) / b^2 d^2 \exp(d x + c) - \frac{1}{2} a (d f x + d e + f) / b^2 d^2 \exp(-d x - c) - \frac{1}{16} (2 d f x + 2 d e + f) / d^2 b \exp(-2 d x - 2 c) + 2 a^3 / b^3 d e / (a^2 + b^2)^{(1/2)} \operatorname{arctanh}(1/2 * (2 b \exp(d x + c) + 2 a) / (a^2 + b^2)^{(1/2)}) - a^3 / b^3 d f / (a^2 + b^2)^{(1/2)} \ln((-b \exp(d x + c) + (a^2 + b^2)^{(1/2)} - a) / (-a + (a^2 + b^2)^{(1/2)})) * x - a^3 / b^3 d^2 f / (a^2 + b^2)^{(1/2)} \ln((-b \exp(d x + c) + (a^2 + b^2)^{(1/2)} - a) / (-a + (a^2 + b^2)^{(1/2)})) * c + a^3 / b^3 d f / (a^2 + b^2)^{(1/2)} \ln((b \exp(d x + c) + (a^2 + b^2)^{(1/2)} + a) / (a + (a^2 + b^2)^{(1/2)})) * x + a^3 / b^3 d^2 f / (a^2 + b^2)^{(1/2)} \ln((b \exp(d x + c) + (a^2 + b^2)^{(1/2)} + a) / (a + (a^2 + b^2)^{(1/2)})) * c - a^3 / b^3 d^2 f / (a^2 + b^2)^{(1/2)} * \operatorname{dilog}((-b \exp(d x + c) + (a^2 + b^2)^{(1/2)} - a) / (-a + (a^2 + b^2)^{(1/2)})) + a^3 / b^3 d^2 f / (a^2 + b^2)^{(1/2)} * \operatorname{dilog}((b \exp(d x + c) + (a^2 + b^2)^{(1/2)} + a) / (a + (a^2 + b^2)^{(1/2)})) - 2 a^3 / b^3 d^2 f * c / (a^2 + b^2)^{(1/2)} \operatorname{arctanh}(1/2 * (2 b \exp(d x + c) + 2 a) / (a^2 + b^2)^{(1/2)})$

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [B]** time = 2.90955, size = 3992, normalized size = 11.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\frac{1}{16} \left( (2(a^2b^2 + b^4)d^2fx + 2(a^2b^2 + b^4)d^2e - (a^2b^2 + b^4)f) \cosh(dx + c)^4 + (2(a^2b^2 + b^4)d^2fx + 2(a^2b^2 + b^4)d^2e - (a^2b^2 + b^4)f) \sinh(dx + c)^4 - 2(a^2b^2 + b^4)d^2fx - 8((a^3b + ab^3)d^2fx + (a^3b + ab^3)d^2e - (a^3b + ab^3)f) \cosh(dx + c)^3 - 4(2(a^3b + ab^3)d^2fx + 2(a^3b + ab^3)d^2e - 2(a^3b + ab^3)f - (2(a^2b^2 + b^4)d^2fx + 2(a^2b^2 + b^4)d^2e - (a^2b^2 + b^4)f) \cosh(dx + c)) \sinh(dx + c)^3 - 2(a^2b^2 + b^4)d^2e + 4((2a^4 + a^2b^2 - b^4)d^2fx^2 + 2(2a^4 + a^2b^2 - b^4)d^2e) \cosh(dx + c)^2 + 2(2(2a^4 + a^2b^2 - b^4)d^2fx^2 + 4(2a^4 + a^2b^2 - b^4)d^2e) \sinh(dx + c) + a^3b^2 \sinh(dx + c)^2 \sqrt{(a^2 + b^2)/b^2} \operatorname{dilog}((a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 16(a^3b^2 \cosh(dx + c)^2 + 2a^3b^2 \cosh(dx + c) \sinh(dx + c) + a^3b^2 \sinh(dx + c)^2) \sqrt{(a^2 + b^2)/b^2} \operatorname{dilog}((a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 16((a^3b^2d^2e - a^3b^2cd^2f) \cosh(dx + c)^2 + 2(a^3b^2d^2e - a^3b^2cd^2f) \cosh(dx + c) \sinh(dx + c) + (a^3b^2d^2e - a^3b^2cd^2f) \sinh(dx + c)^2) \sqrt{(a^2 + b^2)/b^2} \log(2b \cosh(dx + c) + 2b \sinh(dx + c) + 2b \sqrt{(a^2 + b^2)/b^2} + 2a) - 16((a^3b^2d^2e - a^3b^2cd^2f) \cosh(dx + c)^2 + 2(a^3b^2d^2e - a^3b^2cd^2f) \cosh(dx + c) \sinh(dx + c) + (a^3b^2d^2e - a^3b^2cd^2f) \sinh(dx + c)^2) \sqrt{(a^2 + b^2)/b^2} \log(2b \cosh(dx + c) + 2b \sinh(dx + c) + 2b \sqrt{(a^2 + b^2)/b^2} + 2a) \right)$$

```
*sinh(d*x + c) + (a^3*b*d*e - a^3*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/
b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2)
+ 2*a) - 16*((a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)^2 + 2*(a^3*b*d*f*x + a
^3*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d*f*x + a^3*b*c*f)*sinh(d*x
+ c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 16*((a^3*b
*d*f*x + a^3*b*c*f)*cosh(d*x + c)^2 + 2*(a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x
+ c)*sinh(d*x + c) + (a^3*b*d*f*x + a^3*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 +
b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*s
inh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (a^2*b^2 + b^4)*f - 8*((a^3*b
+ a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e + (a^3*b + a*b^3)*f)*cosh(d*x + c) -
4*(2*(a^3*b + a*b^3)*d*f*x - (2*(a^2*b^2 + b^4)*d*f*x + 2*(a^2*b^2 + b^4)*d
*e - (a^2*b^2 + b^4)*f)*cosh(d*x + c)^3 + 2*(a^3*b + a*b^3)*d*e + 6*((a^3*b
+ a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e - (a^3*b + a*b^3)*f)*cosh(d*x + c)^2
+ 2*(a^3*b + a*b^3)*f - 2*((2*a^4 + a^2*b^2 - b^4)*d^2*f*x^2 + 2*(2*a^4 + a
^2*b^2 - b^4)*d^2*e*x)*cosh(d*x + c))*sinh(d*x + c))/((a^2*b^3 + b^5)*d^2*c
osh(d*x + c)^2 + 2*(a^2*b^3 + b^5)*d^2*cosh(d*x + c)*sinh(d*x + c) + (a^2*b
^3 + b^5)*d^2*sinh(d*x + c)^2)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \sinh(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

$$3.236 \quad \int \frac{\sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=107

$$\frac{2a^3 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3 d \sqrt{a^2+b^2}} + \frac{x(2a^2-b^2)}{2b^3} - \frac{a \cosh(c+dx)}{b^2 d} + \frac{\sinh(c+dx) \cosh(c+dx)}{2bd}$$

[Out]  $((2*a^2 - b^2)*x)/(2*b^3) + (2*a^3*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b^3*Sqrt[a^2 + b^2]*d) - (a*Cosh[c + d*x])/(b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)$

**Rubi [A]** time = 0.220769, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2793, 3023, 2735, 2660, 618, 204}

$$\frac{2a^3 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3 d \sqrt{a^2+b^2}} + \frac{x(2a^2-b^2)}{2b^3} - \frac{a \cosh(c+dx)}{b^2 d} + \frac{\sinh(c+dx) \cosh(c+dx)}{2bd}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sinh}[c + d*x]^3/(a + b*\text{Sinh}[c + d*x]), x]$

[Out]  $((2*a^2 - b^2)*x)/(2*b^3) + (2*a^3*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b^3*Sqrt[a^2 + b^2]*d) - (a*Cosh[c + d*x])/(b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)$

### Rule 2793

$\text{Int}[(a_. + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] := -\text{Simp}[(b^2*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m-2)}*(c + d*\text{Sin}[e + f*x])^{(n+1)})/(d*f*(m+n)), x] + \text{Dist}[1/(d*(m+n)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m-3)}*(c + d*\text{Sin}[e + f*x])^n*\text{Simp}[a^3*d*(m+n) + b^2*(b*c*(m-2) + a*d*(n+1)) - b*(a*b*c - b^2*d*(m+n-1) - 3*a^2*d*(m+n)*\text{Sin}[e + f*x] - b^2*(b*c*(m-1) - a*d*(3*m+2*n-2))*\text{Sin}[e + f*x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{NeQ}[c^2 - d^2, 0] \&\& \text{GtQ}[m, 2] \&\& (\text{IntegerQ}[m] \mid \mid \text{IntegersQ}[2*m, 2*n]) \&\& !( \text{IGtQ}[n, 2] \&\& ( !\text{IntegerQ}[m] \mid \mid (\text{EqQ}[a, 0] \&\&$

NeQ[c, 0]))))

### Rule 3023

Int[((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((A\_.) + (B\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^2), x\_Symbol] :> -Simp[(C\*Cos[e + f\*x]\*(a + b\*Sin[e + f\*x])^(m + 1))/(b\*f\*(m + 2)), x] + Dist[1/(b\*(m + 2)), Int[(a + b\*Sin[e + f\*x])^m\*Simp[A\*b\*(m + 2) + b\*C\*(m + 1) + (b\*B\*(m + 2) - a\*C)\*Sin[e + f\*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

### Rule 2735

Int[((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])/((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(b\*x)/d, x] - Dist[(b\*c - a\*d)/d, Int[1/(c + d\*Sin[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0]

### Rule 2660

Int[((a\_) + (b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] :> With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + 2\*b\*e\*x + a\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rubi steps



$$\begin{aligned}
\int \frac{\sinh^3(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{\int \frac{a+b\sinh(c+dx)+2a\sinh^2(c+dx)}{a+b\sinh(c+dx)} dx}{2b} \\
&= -\frac{a\cosh(c+dx)}{b^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{i\int \frac{-iab+i(2a^2-b^2)\sinh(c+dx)}{a+b\sinh(c+dx)} dx}{2b^2} \\
&= \frac{(2a^2-b^2)x}{2b^3} - \frac{a\cosh(c+dx)}{b^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{a^3\int \frac{1}{a+b\sinh(c+dx)} dx}{b^3} \\
&= \frac{(2a^2-b^2)x}{2b^3} - \frac{a\cosh(c+dx)}{b^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} + \frac{(2ia^3)\text{Subst}\left(\int \frac{1}{a-2ibx+ax^2} dx\right)}{b^3d} \\
&= \frac{(2a^2-b^2)x}{2b^3} - \frac{a\cosh(c+dx)}{b^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{(4ia^3)\text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2} dx\right)}{b^3d} \\
&= \frac{(2a^2-b^2)x}{2b^3} + \frac{2a^3 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3\sqrt{a^2+b^2}d} - \frac{a\cosh(c+dx)}{b^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd}
\end{aligned}$$

**Mathematica [A]** time = 0.305172, size = 101, normalized size = 0.94

$$\frac{-2(b^2-2a^2)(c+dx) - \frac{8a^3 \tan^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} - 4ab \cosh(c+dx) + b^2 \sinh(2(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d\*x]^3/(a + b\*Sinh[c + d\*x]),x]

[Out]  $(-2*(-2*a^2 + b^2)*(c + d*x) - (8*a^3*\text{ArcTan}[(b - a*\text{Tanh}[(c + d*x)/2])/ \text{Sqrt}[-a^2 - b^2]])/\text{Sqrt}[-a^2 - b^2] - 4*a*b*\text{Cosh}[c + d*x] + b^2*\text{Sinh}[2*(c + d*x)])/(4*b^3*d)$

**Maple [B]** time = 0.03, size = 262, normalized size = 2.5

$$-\frac{1}{2bd} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-2} + \frac{1}{2bd} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} - \frac{a}{db^2} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} + \frac{a^2}{db^3} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)`

[Out] 
$$-1/2/d/b/(\tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/b/(\tanh(1/2*d*x+1/2*c)+1)-1/d/b^2/(\tanh(1/2*d*x+1/2*c)+1)*a+1/d/b^3*\ln(\tanh(1/2*d*x+1/2*c)+1)*a^2-1/2/d/b*\ln(\tanh(1/2*d*x+1/2*c)+1)-2/d*a^3/b^3/(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*a*\tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^{(1/2}))+1/2/d/b/(\tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/b/(\tanh(1/2*d*x+1/2*c)-1)+1/d/b^2/(\tanh(1/2*d*x+1/2*c)-1)*a-1/d/b^3*\ln(\tanh(1/2*d*x+1/2*c)-1)*a^2+1/2/d/b*\ln(\tanh(1/2*d*x+1/2*c)-1)$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.42845, size = 1434, normalized size = 13.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & 1/8*(4*(2*a^4 + a^2*b^2 - b^4)*d*x*cosh(d*x + c)^2 + (a^2*b^2 + b^4)*cosh(d*x + c)^4 + (a^2*b^2 + b^4)*sinh(d*x + c)^4 - a^2*b^2 - b^4 - 4*(a^3*b + a*b^3)*cosh(d*x + c)^3 - 4*(a^3*b + a*b^3 - (a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(2*(2*a^4 + a^2*b^2 - b^4)*d*x + 3*(a^2*b^2 + b^4)*cosh(d*x + c)^2 - 6*(a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*x + c)^2)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 4*(a^3*b + a*b^3)*cosh(d*x + c) - 4*(a^3*b + a*b^3 - 2*(2*a^4 + a^2*b^2 - b^4)*d*x*cosh(d*x + c) - (a^2*b^2 + b^4)*cosh(d*x + c)^3 + 3*(a^3*b + \end{aligned}$$

$$a*b^3*\cosh(d*x + c)^2*\sinh(d*x + c)/((a^2*b^3 + b^5)*d*\cosh(d*x + c)^2 + 2*(a^2*b^3 + b^5)*d*\cosh(d*x + c)*\sinh(d*x + c) + (a^2*b^3 + b^5)*d*\sinh(d*x + c)^2)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [A]** time = 1.20252, size = 217, normalized size = 2.03

$$-\frac{a^3 \log\left(\frac{|2be^{(dx+c)}+2a-2\sqrt{a^2+b^2}|}{|2be^{(dx+c)}+2a+2\sqrt{a^2+b^2}|}\right)}{\sqrt{a^2+b^2}b^3d} + \frac{(2a^2-b^2)(dx+c)}{2b^3d} - \frac{(4abe^{(dx+c)}+b^2)e^{(-2dx-2c)}}{8b^3d} + \frac{bde^{(2dx+2c)}-4ade^{(dx+c)}}{8b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out]  $-a^3*\log(\text{abs}(2*b*e^{(d*x + c)} + 2*a - 2*\text{sqrt}(a^2 + b^2))/\text{abs}(2*b*e^{(d*x + c)} + 2*a + 2*\text{sqrt}(a^2 + b^2)))/(\text{sqrt}(a^2 + b^2)*b^3*d) + 1/2*(2*a^2 - b^2)*(d*x + c)/(b^3*d) - 1/8*(4*a*b*e^{(d*x + c)} + b^2)*e^{(-2*d*x - 2*c)}/(b^3*d) + 1/8*(b*d*e^{(2*d*x + 2*c)} - 4*a*d*e^{(d*x + c)})/(b^2*d^2)$

$$3.237 \quad \int \frac{\sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=30

$$\text{Unintegrable} \left( \frac{\sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[Sinh[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0765824, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Sinh[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Sinh[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.002, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[Sinh[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] \$Aborted

---

**Maple [A]** time = 0.091, size = 0, normalized size = 0.

$$\int \frac{(\sinh(dx + c))^3}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)`

[Out] `int(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-2a^3 \int -\frac{e^{(dx+c)}}{b^4fx + b^4e - (b^4fxe^{2c} + b^4ee^{2c})e^{2dx} - 2(ab^3fxe^c + ab^3ee^c)e^{dx}} dx - \frac{e^{(-2c + \frac{2de}{f})} E_1\left(\frac{2(fx+e)d}{f}\right)}{4bf} - \frac{ae^{(-c + \frac{de}{f})} E_1\left(\frac{2(fx+e)d}{f}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `-2*a^3*integrate(-e^(d*x + c)/(b^4*f*x + b^4*e - (b^4*f*x*e^(2*c) + b^4*e*e^(2*c))*e^(2*d*x) - 2*(a*b^3*f*x*e^c + a*b^3*e*e^c)*e^(d*x)), x) - 1/4*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b*f) - 1/2*a*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^2*f) + 1/2*a*e^(c - d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^2*f) - 1/4*e^(2*c - 2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b*f) + 1/2*(2*a^2 - b^2)*log(f*x + e)/(b^3*f)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh(dx + c)^3}{afx + ae + (bfx + be)\sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

```
[Out] integral(sinh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

sage<sub>0</sub>x

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

$$3.238 \quad \int \frac{(e+fx)^3 \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=605

$$\frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}} - \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^3\sqrt{a^2+b^2}} - \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}}$$

[Out]  $(-2*(e + f*x)^3*\operatorname{ArcTanh}[E^{(c + d*x)}])/(a*d) - (b*(e + f*x)^3*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])])/(a*\operatorname{Sqrt}[a^2 + b^2]*d) + (b*(e + f*x)^3*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])/(a*\operatorname{Sqrt}[a^2 + b^2]*d) - (3*f*(e + f*x)^2*\operatorname{PolyLog}[2, -E^{(c + d*x)}])/(a*d^2) + (3*f*(e + f*x)^2*\operatorname{PolyLog}[2, E^{(c + d*x)}])/(a*d^2) - (3*b*f*(e + f*x)^2*\operatorname{PolyLog}[2, -(b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])])/(a*\operatorname{Sqrt}[a^2 + b^2]*d^2) + (3*b*f*(e + f*x)^2*\operatorname{PolyLog}[2, -(b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])/(a*\operatorname{Sqrt}[a^2 + b^2]*d^2) + (6*f^2*(e + f*x)*\operatorname{PolyLog}[3, -E^{(c + d*x)}])/(a*d^3) - (6*f^2*(e + f*x)*\operatorname{PolyLog}[3, E^{(c + d*x)}])/(a*d^3) + (6*b*f^2*(e + f*x)*\operatorname{PolyLog}[3, -(b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])])/(a*\operatorname{Sqrt}[a^2 + b^2]*d^3) - (6*b*f^2*(e + f*x)*\operatorname{PolyLog}[3, -(b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])/(a*\operatorname{Sqrt}[a^2 + b^2]*d^3) - (6*f^3*\operatorname{PolyLog}[4, -E^{(c + d*x)}])/(a*d^4) + (6*f^3*\operatorname{PolyLog}[4, E^{(c + d*x)}])/(a*d^4) - (6*b*f^3*\operatorname{PolyLog}[4, -(b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])])/(a*\operatorname{Sqrt}[a^2 + b^2]*d^4) + (6*b*f^3*\operatorname{PolyLog}[4, -(b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])/(a*\operatorname{Sqrt}[a^2 + b^2]*d^4)$

**Rubi [A]** time = 0.964937, antiderivative size = 605, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$ , Rules used = {5575, 4182, 2531, 6609, 2282, 6589, 3322, 2264, 2190}

$$\frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}} - \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^3\sqrt{a^2+b^2}} - \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\frac{(e + f*x)^3*\operatorname{Csch}[c + d*x]}{a + b*\operatorname{Sinh}[c + d*x]}, x]$

[Out]  $(-2*(e + f*x)^3*\operatorname{ArcTanh}[E^{(c + d*x)}])/(a*d) - (b*(e + f*x)^3*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])])/(a*\operatorname{Sqrt}[a^2 + b^2]*d) + (b*(e + f*x)^3*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])/(a*\operatorname{Sqrt}[a^2 + b^2]*d) - (3*f*(e + f*x)^2*\operatorname{PolyLog}[2, -E^{(c + d*x)}])/(a*d^2) + (3*f*(e + f*x)^2*\operatorname{PolyLog}[2, E^{(c + d*x)}])/(a*d^2) - (3*b*f*(e + f*x)^2*\operatorname{PolyLog}[2, -(b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])])/(a*\operatorname{Sqrt}[a^2 + b^2]*d^2) + (3*b*f*(e + f*x)^2*\operatorname{PolyLog}[2, -(b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])/(a*\operatorname{Sqrt}[a^2 + b^2]*d^2)$

$$\begin{aligned} & [2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))]/(a*\text{Sqrt}[a^2 + b^2]*d^2) + (6 \\ & *f^2*(e + f*x)*\text{PolyLog}[3, -E^{(c + d*x)}])/(a*d^3) - (6*f^2*(e + f*x)*\text{PolyLog} \\ & [3, E^{(c + d*x)}])/(a*d^3) + (6*b*f^2*(e + f*x)*\text{PolyLog}[3, -((b*E^{(c + d*x)}) \\ & / (a - \text{Sqrt}[a^2 + b^2]))]/(a*\text{Sqrt}[a^2 + b^2]*d^3) - (6*b*f^2*(e + f*x)*\text{Poly} \\ & \text{Log}[3, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))]/(a*\text{Sqrt}[a^2 + b^2]*d^3) - \\ & (6*f^3*\text{PolyLog}[4, -E^{(c + d*x)}])/(a*d^4) + (6*f^3*\text{PolyLog}[4, E^{(c + d*x)}]) \\ & / (a*d^4) - (6*b*f^3*\text{PolyLog}[4, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))]/( \\ & a*\text{Sqrt}[a^2 + b^2]*d^4) + (6*b*f^3*\text{PolyLog}[4, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^ \\ & 2 + b^2]))]/(a*\text{Sqrt}[a^2 + b^2]*d^4) \end{aligned}$$

### Rule 5575

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)
*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Csch[c
+ d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a +
b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
GtQ[n, 0]
```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
```



```
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*(F_)[v_]] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_Symbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 3322

```
Int[((c_) + (d_)*(x_))^(m_)/((a_) + (b_)*sin[(e_) + (Complex[0, fz_])*(f_)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u]/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u]/(b + q + 2*c*F^u), x], x] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/ (b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \operatorname{csch}(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^3}{a+b \sinh(c+dx)} dx}{a} \\
&= -\frac{2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{(2b) \int \frac{e^{c+dx}(e+fx)^3}{-b+2ae^{c+dx}+be^{2(c+dx)}} dx}{a} - \frac{(3f) \int (e+fx)^2 \log(1-e^{c+dx})}{ad} \\
&= -\frac{2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{3f(e+fx)^2 \operatorname{Li}_2(-e^{c+dx})}{ad^2} + \frac{3f(e+fx)^2 \operatorname{Li}_2(e^{c+dx})}{ad^2} - \frac{(2b^2)}{ad^2} \\
&= -\frac{2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} \\
&= -\frac{2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} \\
&= -\frac{2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} \\
&= -\frac{2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} \\
&= -\frac{2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} \\
&= -\frac{2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} \\
&= -\frac{2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}
\end{aligned}$$

**Mathematica [A]** time = 3.11956, size = 757, normalized size = 1.25

$$\frac{b\left(-3d^2 f(e+fx)^2 \operatorname{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) + 3d^2 f(e+fx)^2 \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) + 6def^2 \operatorname{PolyLog}\left(3, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) - 6def^2 \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) + 6df^3 x \operatorname{PolyLog}\left(3, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) - 6df^3 x \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)\right)}{a^2 d^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^3\*Csch[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (-2\*d^3\*(e + f\*x)^3\*ArcTanh[Cosh[c + d\*x] + Sinh[c + d\*x]] + (b\*(2\*d^3\*e^3\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]] - 3\*d^3\*e^2\*f\*x\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])] - 3\*d^3\*e\*f^2\*x^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])] - d^3\*f^3\*x^3\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]) + 3\*d^3\*e^2\*f\*x\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] + 3\*d^3\*e\*f^2\*x^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] + d^3\*f^3\*x^3\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^2\*d^3))

```
*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*f^3*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*f^3*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/Sqrt[a^2 + b^2] - 3*f*(d^2*(e + f*x)^2*PolyLog[2, -Cosh[c + d*x] - Sinh[c + d*x]] - 2*d*f*(e + f*x)*PolyLog[3, -Cosh[c + d*x] - Sinh[c + d*x]] + 2*f^2*PolyLog[4, -Cosh[c + d*x] - Sinh[c + d*x]]) + 3*f*(d^2*(e + f*x)^2*PolyLog[2, Cosh[c + d*x] + Sinh[c + d*x]] - 2*d*f*(e + f*x)*PolyLog[3, Cosh[c + d*x] + Sinh[c + d*x]] + 2*f^2*PolyLog[4, Cosh[c + d*x] + Sinh[c + d*x]]))/(a*d^4)
```

**Maple [F]** time = 0.438, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \operatorname{csch}(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [C]** time = 3.05163, size = 3903, normalized size = 6.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-(6*b^2*f^3*\sqrt{(a^2 + b^2)/b^2}*\text{polylog}(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b - 6*b^2*f^3*\sqrt{(a^2 + b^2)/b^2}*\text{polylog}(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b - 6*(a^2 + b^2)*f^3*\text{polylog}(4, \cosh(d*x + c) + \sinh(d*x + c)) + 6*(a^2 + b^2)*f^3*\text{polylog}(4, -\cosh(d*x + c) - \sinh(d*x + c)) + 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*\sqrt{(a^2 + b^2)/b^2}*\text{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*\sqrt{(a^2 + b^2)/b^2}*\text{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*\sqrt{(a^2 + b^2)/b^2}*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*\sqrt{(a^2 + b^2)/b^2}*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + (b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*\sqrt{(a^2 + b^2)/b^2}*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - (b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*\sqrt{(a^2 + b^2)/b^2}*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 6*(b^2*d*f^3*x + b^2*d*e*f^2)*\sqrt{(a^2 + b^2)/b^2}*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b + 6*(b^2*d*f^3*x + b^2*d*e*f^2)*\sqrt{(a^2 + b^2)/b^2}*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b - 3*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*\text{dilog}(\cosh(d*x + c) + \sinh(d*x + c)) + 3*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*\text{dilog}(-\cosh(d*x + c) - \sinh(d*x + c)) + ((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + (a^2 + b^2)*d^3*e^3)*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) - ((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) - ((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*\log(-\cosh(d*x + c) - \sinh(d*x + c) + 1) + 6*((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*\text{polylog}(3, \cosh(d*x + c) + \sinh(d*x + c)) - 6*((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*\text{polylog}(3, -\cosh(d*x + c) - \sinh(d*x + c)))/((a^3 + a*b^2)*d^4)$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.239 \quad \int \frac{(e+fx)^2 \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=433

$$-\frac{2bf(e+fx)\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}} + \frac{2bf(e+fx)\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2\sqrt{a^2+b^2}} + \frac{2bf^2\operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}} - \frac{2bf^2\operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^3\sqrt{a^2+b^2}}$$

[Out]  $(-2*(e + f*x)^2*\operatorname{ArcTanh}[E^{(c + d*x)}])/(a*d) - (b*(e + f*x)^2*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])])/(a*\operatorname{Sqrt}[a^2 + b^2]*d) + (b*(e + f*x)^2*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])/(a*\operatorname{Sqrt}[a^2 + b^2]*d) - (2*f*(e + f*x)*\operatorname{PolyLog}[2, -E^{(c + d*x)}])/(a*d^2) + (2*f*(e + f*x)*\operatorname{PolyLog}[2, E^{(c + d*x)}])/(a*d^2) - (2*b*f*(e + f*x)*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2]))])/(a*\operatorname{Sqrt}[a^2 + b^2]*d^2) + (2*b*f*(e + f*x)*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2]))])/(a*\operatorname{Sqrt}[a^2 + b^2]*d^2) + (2*f^2*\operatorname{PolyLog}[3, -E^{(c + d*x)}])/(a*d^3) - (2*f^2*\operatorname{PolyLog}[3, E^{(c + d*x)}])/(a*d^3) + (2*b*f^2*\operatorname{PolyLog}[3, -((b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2]))])/(a*\operatorname{Sqrt}[a^2 + b^2]*d^3) - (2*b*f^2*\operatorname{PolyLog}[3, -((b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2]))])/(a*\operatorname{Sqrt}[a^2 + b^2]*d^3)$

**Rubi [A]** time = 0.814057, antiderivative size = 433, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {5575, 4182, 2531, 2282, 6589, 3322, 2264, 2190}

$$-\frac{2bf(e+fx)\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}} + \frac{2bf(e+fx)\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2\sqrt{a^2+b^2}} + \frac{2bf^2\operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3\sqrt{a^2+b^2}} - \frac{2bf^2\operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^3\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e + f*x)^2*\operatorname{Csch}[c + d*x]/(a + b*\operatorname{Sinh}[c + d*x]), x]$

[Out]  $(-2*(e + f*x)^2*\operatorname{ArcTanh}[E^{(c + d*x)}])/(a*d) - (b*(e + f*x)^2*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])])/(a*\operatorname{Sqrt}[a^2 + b^2]*d) + (b*(e + f*x)^2*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])/(a*\operatorname{Sqrt}[a^2 + b^2]*d) - (2*f*(e + f*x)*\operatorname{PolyLog}[2, -E^{(c + d*x)}])/(a*d^2) + (2*f*(e + f*x)*\operatorname{PolyLog}[2, E^{(c + d*x)}])/(a*d^2) - (2*b*f*(e + f*x)*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2]))])/(a*\operatorname{Sqrt}[a^2 + b^2]*d^2) + (2*b*f*(e + f*x)*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2]))])/(a*\operatorname{Sqrt}[a^2 + b^2]*d^2) + (2*f^2*\operatorname{PolyLog}[3, -E^{(c + d*x)}])/(a*d^3) - (2*f^2*\operatorname{PolyLog}[3, E^{(c + d*x)}])/(a*d^3) + (2*b*f^2*\operatorname{PolyLog}[3, -((b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2]))])/(a*\operatorname{Sqrt}[a^2 + b^2]*d^3) - (2*b*f^2*\operatorname{PolyLog}[3, -((b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2]))])/(a*\operatorname{Sqrt}[a^2 + b^2]*d^3)$

)/(a\*Sqrt[a^2 + b^2]\*d^3)

### Rule 5575

Int[(Csch[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Csch[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Csch[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 4182

Int[csc[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^(c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 3322

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(-(I\*b) + 2\*a\*E^(-(I\*e) + f\*fz\*x) + I\*b\*E^(2\*(-(I\*e) + f\*fz\*x))), x], x] /; F

reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.)\*(F\_)^(v\_)), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(e + fx)^2 \operatorname{csch}(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\int (e + fx)^2 \operatorname{csch}(c + dx) dx}{a} - \frac{b \int \frac{(e + fx)^2}{a + b \sinh(c + dx)} dx}{a} \\
 &= -\frac{2(e + fx)^2 \tanh^{-1}(e^{c + dx})}{ad} - \frac{(2b) \int \frac{e^{c + dx}(e + fx)^2}{-b + 2ae^{c + dx} + be^{2(c + dx)}} dx}{a} - \frac{(2f) \int (e + fx) \log(1 - e^{c + dx})}{ad} \\
 &= -\frac{2(e + fx)^2 \tanh^{-1}(e^{c + dx})}{ad} - \frac{2f(e + fx) \operatorname{Li}_2(-e^{c + dx})}{ad^2} + \frac{2f(e + fx) \operatorname{Li}_2(e^{c + dx})}{ad^2} - \frac{(2b^2) \int}{ad^2} \\
 &= -\frac{2(e + fx)^2 \tanh^{-1}(e^{c + dx})}{ad} - \frac{b(e + fx)^2 \log\left(1 + \frac{be^{c + dx}}{a - \sqrt{a^2 + b^2}}\right)}{a\sqrt{a^2 + b^2}d} + \frac{b(e + fx)^2 \log\left(1 + \frac{be^{c + dx}}{a + \sqrt{a^2 + b^2}}\right)}{a\sqrt{a^2 + b^2}d} \\
 &= -\frac{2(e + fx)^2 \tanh^{-1}(e^{c + dx})}{ad} - \frac{b(e + fx)^2 \log\left(1 + \frac{be^{c + dx}}{a - \sqrt{a^2 + b^2}}\right)}{a\sqrt{a^2 + b^2}d} + \frac{b(e + fx)^2 \log\left(1 + \frac{be^{c + dx}}{a + \sqrt{a^2 + b^2}}\right)}{a\sqrt{a^2 + b^2}d} \\
 &= -\frac{2(e + fx)^2 \tanh^{-1}(e^{c + dx})}{ad} - \frac{b(e + fx)^2 \log\left(1 + \frac{be^{c + dx}}{a - \sqrt{a^2 + b^2}}\right)}{a\sqrt{a^2 + b^2}d} + \frac{b(e + fx)^2 \log\left(1 + \frac{be^{c + dx}}{a + \sqrt{a^2 + b^2}}\right)}{a\sqrt{a^2 + b^2}d} \\
 &= -\frac{2(e + fx)^2 \tanh^{-1}(e^{c + dx})}{ad} - \frac{b(e + fx)^2 \log\left(1 + \frac{be^{c + dx}}{a - \sqrt{a^2 + b^2}}\right)}{a\sqrt{a^2 + b^2}d} + \frac{b(e + fx)^2 \log\left(1 + \frac{be^{c + dx}}{a + \sqrt{a^2 + b^2}}\right)}{a\sqrt{a^2 + b^2}d}
 \end{aligned}$$



**Mathematica [A]** time = 2.12001, size = 454, normalized size = 1.05

$$\frac{b\left(-2df(e+fx)\text{PolyLog}\left(2,\frac{be^c+dx}{\sqrt{a^2+b^2}-a}\right)+2df(e+fx)\text{PolyLog}\left(2,-\frac{be^c+dx}{\sqrt{a^2+b^2}+a}\right)+2f^2\text{PolyLog}\left(3,\frac{be^c+dx}{\sqrt{a^2+b^2}-a}\right)-2f^2\text{PolyLog}\left(3,-\frac{be^c+dx}{\sqrt{a^2+b^2}+a}\right)+2d^2e^2\tanh^{-1}\left(\frac{a+be^c+dx}{\sqrt{a^2+b^2}}\right)\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Csch[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (d^2\*(e + f\*x)^2\*Log[1 - E^(c + d\*x)] - d^2\*(e + f\*x)^2\*Log[1 + E^(c + d\*x)] - 2\*d\*f\*(e + f\*x)\*PolyLog[2, -E^(c + d\*x)] + 2\*d\*f\*(e + f\*x)\*PolyLog[2, E^(c + d\*x)] + 2\*f^2\*PolyLog[3, -E^(c + d\*x)] - 2\*f^2\*PolyLog[3, E^(c + d\*x)] + (b\*(2\*d^2\*e^2\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]] - 2\*d^2\*e\*f\*x\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]) - d^2\*f^2\*x^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]) + 2\*d^2\*e\*f\*x\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]) + d^2\*f^2\*x^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])] - 2\*d\*f\*(e + f\*x)\*PolyLog[2, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] + 2\*d\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))] + 2\*f^2\*PolyLog[3, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] - 2\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/Sqrt[a^2 + b^2]/(a\*d^3)

**Maple [F]** time = 0.334, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \operatorname{csch}(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*csch(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*csch(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [C]** time = 2.68204, size = 2674, normalized size = 6.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*b^2*f^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b - 2*b^2*f^2*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b - 2*(a^2 + b^2)*f^2*polylog(3, cosh(d*x + c) + sinh(d*x + c)) + 2*(a^2 + b^2)*f^2*polylog(3, -cosh(d*x + c) - sinh(d*x + c)) - 2*(b^2*d*f^2*x + b^2*d*e*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^2*d*f^2*x + b^2*d*e*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + (a^2 + b^2)*d^2*e^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*log(-cosh(d*x + c) - sinh(d*x + c) + 1))/((a^3 + a*b^2)*d^3)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

### 3.240 $\int \frac{(e+fx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)} dx$

**Optimal.** Leaf size=261

$$-\frac{bf\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}} + \frac{bf\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2+a}}\right)}{ad^2\sqrt{a^2+b^2}} - \frac{f\operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{ad^2} + \frac{f\operatorname{PolyLog}\left(2, e^{c+dx}\right)}{ad^2} - \frac{b(e+fx)\operatorname{Log}\left[1 + \frac{bE^{c+dx}}{a+\sqrt{a^2+b^2}}\right]}{ad^2}$$

```
[Out] (-2*(e + f*x)*ArcTanh[E^(c + d*x)]/(a*d) - (b*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(a*Sqrt[a^2 + b^2]*d) + (b*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(a*Sqrt[a^2 + b^2]*d) - (f*PolyLog[2, -E^(c + d*x)]/(a*d^2) + (f*PolyLog[2, E^(c + d*x)]/(a*d^2) - (b*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2)
```

**Rubi [A]** time = 0.459786, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$ , Rules used = {5575, 4182, 2279, 2391, 3322, 2264, 2190}

$$-\frac{bf\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2\sqrt{a^2+b^2}} + \frac{bf\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2+a}}\right)}{ad^2\sqrt{a^2+b^2}} - \frac{f\operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{ad^2} + \frac{f\operatorname{PolyLog}\left(2, e^{c+dx}\right)}{ad^2} - \frac{b(e+fx)\operatorname{Log}\left[1 + \frac{bE^{c+dx}}{a+\sqrt{a^2+b^2}}\right]}{ad^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x])/(a + b*Sinh[c + d*x]), x]
```

```
[Out] (-2*(e + f*x)*ArcTanh[E^(c + d*x)]/(a*d) - (b*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(a*Sqrt[a^2 + b^2]*d) + (b*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(a*Sqrt[a^2 + b^2]*d) - (f*PolyLog[2, -E^(c + d*x)]/(a*d^2) + (f*PolyLog[2, E^(c + d*x)]/(a*d^2) - (b*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2) + (b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*Sqrt[a^2 + b^2]*d^2)
```

#### Rule 5575

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && I
```

GtQ[n, 0]

### Rule 4182

Int[csc[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 3322

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_) + (b\_.)\*sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(-(I\*b) + 2\*a\*E^(-(I\*e) + f\*fz\*x) + I\*b\*E^(2\*(-I\*e) + f\*fz\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.)\*(F\_)^(v\_)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a)/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\operatorname{csch}(c+dx) dx}{a} - \frac{b \int \frac{e+fx}{a+b\sinh(c+dx)} dx}{a} \\
&= -\frac{2(e+fx)\tanh^{-1}(e^{c+dx})}{ad} - \frac{(2b) \int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}} dx}{a} - \frac{f \int \log(1-e^{c+dx}) dx}{ad} + \frac{f \int \log(1+e^{c+dx}) dx}{ad} \\
&= -\frac{2(e+fx)\tanh^{-1}(e^{c+dx})}{ad} - \frac{(2b^2) \int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{a\sqrt{a^2+b^2}} + \frac{(2b^2) \int \frac{e^{c+dx}(e+fx)}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{a\sqrt{a^2+b^2}} \\
&= -\frac{2(e+fx)\tanh^{-1}(e^{c+dx})}{ad} - \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} \\
&= -\frac{2(e+fx)\tanh^{-1}(e^{c+dx})}{ad} - \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} \\
&= -\frac{2(e+fx)\tanh^{-1}(e^{c+dx})}{ad} - \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} + \frac{b(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d}
\end{aligned}$$

**Mathematica [A]** time = 1.95028, size = 306, normalized size = 1.17

$$\frac{b\left(-f\operatorname{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)+f\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)+2de\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)-f(c+dx)\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)+f(c+dx)\log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)-2cf\tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Csch[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (d\*e\*Log[Tanh[(c + d\*x)/2]] - c\*f\*Log[Tanh[(c + d\*x)/2]] + f\*((c + d\*x)\*(Log[1 - E^(-c - d\*x)] - Log[1 + E^(-c - d\*x)]) + PolyLog[2, -E^(-c - d\*x)] - PolyLog[2, E^(-c - d\*x)] + (b\*(2\*d\*e\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]] - 2\*c\*f\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]] - f\*(c + d\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]) + f\*(c + d\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]) - f\*PolyLog[2, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2]]) + f\*PolyLog[2, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])))/Sqrt[a^2 + b^2])/(a\*d^2)

**Maple [B]** time = 0.112, size = 532, normalized size = 2.

$$\frac{e \ln(e^{dx+c} - 1)}{da} + 2 \frac{eb}{da\sqrt{a^2 + b^2}} \operatorname{Artanh}\left(\frac{1}{2} \frac{2be^{dx+c} + 2a}{\sqrt{a^2 + b^2}}\right) - \frac{e \ln(e^{dx+c} + 1)}{da} - \frac{f \operatorname{dilog}(e^{dx+c})}{ad^2} - \frac{bf x}{da} \ln\left(\left(-be^{dx+c} + \sqrt{a^2 + b^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)`

[Out]  $1/d/a*e*\ln(\exp(d*x+c)-1)+2/d*e*b/a/(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-1/d/a*e*\ln(\exp(d*x+c)+1)-1/d^2*f/a*\operatorname{dilog}(\exp(d*x+c))-1/d*f*b/a/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*x-1/d^2*f*b/a/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*c+1/d*f*b/a/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*x+1/d^2*f*b/a/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*c-1/d^2*f*b/a/(a^2+b^2)^{(1/2)}*\operatorname{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))+1/d^2*f*b/a/(a^2+b^2)^{(1/2)}*\operatorname{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})))-1/d/a*\ln(\exp(d*x+c)+1)*f*x-1/d^2*f/a*\operatorname{dilog}(\exp(d*x+c)+1)-1/d^2/a*f*c*\ln(\exp(d*x+c)-1)-2/d^2*f*c*b/a/(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.54721, size = 1621, normalized size = 6.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

```
[Out] -(b^2*f*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - b^2*f
*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (a^2 + b^2)*
f*dilog(cosh(d*x + c) + sinh(d*x + c)) + (a^2 + b^2)*f*dilog(-cosh(d*x + c)
- sinh(d*x + c)) - (b^2*d*e - b^2*c*f)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(
d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^2*d*e
- b^2*c*f)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c)
- 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^2*d*f*x + b^2*c*f)*sqrt((a^2 + b^2)
/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^2*d*f*x + b^2*c*f)*sqrt((a^2 +
b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + ((a^2 + b^2)*d*f*x + (a^2 + b^
2)*d*e)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - ((a^2 + b^2)*d*e - (a^2 +
b^2)*c*f)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - ((a^2 + b^2)*d*f*x + (a^
2 + b^2)*c*f)*log(-cosh(d*x + c) - sinh(d*x + c) + 1))/((a^3 + a*b^2)*d^2)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```



$$3.241 \quad \int \frac{\operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=64

$$\frac{2b \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}} - \frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

[Out]  $-(\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]/(a*d)) + (2*b*\operatorname{ArcTanh}[(b - a*\operatorname{Tanh}[(c + d*x)/2])/ \operatorname{Sqrt}[a^2 + b^2]])/(a*\operatorname{Sqrt}[a^2 + b^2]*d)$

**Rubi [A]** time = 0.0878481, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {2747, 3770, 2660, 618, 204}

$$\frac{2b \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad\sqrt{a^2+b^2}} - \frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Csch}[c + d*x]/(a + b*\operatorname{Sinh}[c + d*x]), x]$

[Out]  $-(\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]/(a*d)) + (2*b*\operatorname{ArcTanh}[(b - a*\operatorname{Tanh}[(c + d*x)/2])/ \operatorname{Sqrt}[a^2 + b^2]])/(a*\operatorname{Sqrt}[a^2 + b^2]*d)$

#### Rule 2747

$\operatorname{Int}[1/(((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)])), x\_Symbol] \rightarrow \operatorname{Dist}[b/(b*c - a*d), \operatorname{Int}[1/(a + b*\sin[e + f*x]), x], x] - \operatorname{Dist}[d/(b*c - a*d), \operatorname{Int}[1/(c + d*\sin[e + f*x]), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0]$

#### Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

#### Rule 2660

```
Int[((a_) + (b_.)*sin[(c_) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

### Rule 618

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int \operatorname{csch}(c+dx) dx}{a} - \frac{b \int \frac{1}{a+b \sinh(c+dx)} dx}{a} \\ &= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(2ib) \operatorname{Subst}\left(\int \frac{1}{a-2ibx+ax^2} dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{ad} \\ &= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} - \frac{(4ib) \operatorname{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2} dx, x, -2ib+2a \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{ad} \\ &= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{2b \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}d} \end{aligned}$$

**Mathematica [A]** time = 0.0823074, size = 69, normalized size = 1.08

$$\frac{\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - \frac{2b \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]), x]
```

[Out]  $((-2*b*ArcTan[(b - a*Tanh[(c + d*x)/2]])/Sqrt[-a^2 - b^2])/Sqrt[-a^2 - b^2] + Log[Tanh[(c + d*x)/2]])/(a*d)$

**Maple [A]** time = 0.003, size = 65, normalized size = 1.

$$-2 \frac{b}{da\sqrt{a^2 + b^2}} \operatorname{Arctanh}\left(\frac{1}{2} \frac{2a \tanh(1/2 dx + c/2) - 2b}{\sqrt{a^2 + b^2}}\right) + \frac{1}{da} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)/(a+b*sinh(d*x+c)),x)`

[Out]  $-2/d*b/a/(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*a*\tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^{(1/2)})+1/d/a*\ln(\tanh(1/2*d*x+1/2*c))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.66225, size = 591, normalized size = 9.23

$$\frac{\sqrt{a^2 + b^2} b \log\left(\frac{b^2 \cosh(dx+c)^2 + b^2 \sinh(dx+c)^2 + 2ab \cosh(dx+c) + 2a^2 + b^2 + 2(b^2 \cosh(dx+c) + ab) \sinh(dx+c) + 2\sqrt{a^2 + b^2}(b \cosh(dx+c) + b \sinh(dx+c) + a)}{b \cosh(dx+c)^2 + b \sinh(dx+c)^2 + 2a \cosh(dx+c) + 2(b \cosh(dx+c) + a) \sinh(dx+c) - b}\right)}{(a^3 + ab^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out]  $(\sqrt{a^2 + b^2})*b*\log((b^2*\cosh(d*x + c))^2 + b^2*\sinh(d*x + c)^2 + 2*a*b*\cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*\cosh(d*x + c) + a*b)*\sinh(d*x + c) + 2*$

$$\sqrt{a^2 + b^2} * (b * \cosh(dx + c) + b * \sinh(dx + c) + a) / (b * \cosh(dx + c)^2 + b * \sinh(dx + c)^2 + 2 * a * \cosh(dx + c) + 2 * (b * \cosh(dx + c) + a) * \sinh(dx + c) - b) - (a^2 + b^2) * \log(\cosh(dx + c) + \sinh(dx + c) + 1) + (a^2 + b^2) * \log(\cosh(dx + c) + \sinh(dx + c) - 1) / ((a^3 + a * b^2) * d)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)/(a+b\*sinh(dx+c)),x)

[Out] Integral(csch(c + dx)/(a + b\*sinh(c + dx)), x)

**Giac [A]** time = 1.58671, size = 144, normalized size = 2.25

$$-\frac{b \log\left(\frac{|2be^{(dx+c)} + 2a - 2\sqrt{a^2+b^2}|}{|2be^{(dx+c)} + 2a + 2\sqrt{a^2+b^2}|}\right)}{\sqrt{a^2 + b^2}ad} - \frac{\log(e^{(dx+c)} + 1)}{ad} + \frac{\log(|e^{(dx+c)} - 1|)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)/(a+b\*sinh(dx+c)),x, algorithm="giac")

[Out] -b\*log(abs(2\*b\*e^(dx + c) + 2\*a - 2\*sqrt(a^2 + b^2))/abs(2\*b\*e^(dx + c) + 2\*a + 2\*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)\*a\*d) - log(e^(dx + c) + 1)/(a\*d) + log(abs(e^(dx + c) - 1))/(a\*d)

$$3.242 \quad \int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=28

$$\text{Unintegrable}\left(\frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Csch[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0487392, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Csch[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Csch[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [A]** time = 5.92489, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Csch[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Integrate[Csch[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

---

**Maple [A]** time = 0.065, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx + c)}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(csch(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx + c)}{(fx + e)(b \sinh(dx + c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] integrate(csch(d\*x + c)/((f\*x + e)\*(b\*sinh(d\*x + c) + a)), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(dx + c)}{afx + ae + (bf x + be) \sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] integral(csch(d\*x + c)/(a\*f\*x + a\*e + (b\*f\*x + b\*e)\*sinh(d\*x + c)), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c + dx)}{(a + b \sinh(c + dx))(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(csch(c + d\*x)/((a + b\*sinh(c + d\*x))\*(e + f\*x)), x)

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.243 \quad \int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=745

$$-\frac{6b^2 f^2 (e+fx) \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2 d^3 \sqrt{a^2+b^2}} + \frac{6b^2 f^2 (e+fx) \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2 d^3 \sqrt{a^2+b^2}} + \frac{3b^2 f (e+fx)^2 \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2 d^2 \sqrt{a^2+b^2}}$$

[Out]  $-\left(\frac{(e+fx)^3}{a*d}\right) + \frac{2*b*(e+fx)^3*\operatorname{ArcTanh}[E^{(c+dx)}]}{(a^2*d)} - \left(\frac{(e+fx)^3*\operatorname{Coth}[c+dx]}{a*d} + \frac{b^2*(e+fx)^3*\operatorname{Log}[1+(b*E^{(c+dx)})/(a-\sqrt{a^2+b^2})]}{(a^2*\sqrt{a^2+b^2}*d)} - \frac{b^2*(e+fx)^3*\operatorname{Log}[1+(b*E^{(c+dx)})/(a+\sqrt{a^2+b^2})]}{(a^2*\sqrt{a^2+b^2}*d)} + \frac{3*f*(e+fx)^2*\operatorname{Log}[1-E^{(2*(c+dx))}]}{(a*d^2)} + \frac{3*b*f*(e+fx)^2*\operatorname{PolyLog}[2, -E^{(c+dx)}]}{(a^2*d^2)} - \frac{3*b*f*(e+fx)^2*\operatorname{PolyLog}[2, E^{(c+dx)}]}{(a^2*d^2)} + \frac{3*b^2*f*(e+fx)^2*\operatorname{PolyLog}[2, -(b*E^{(c+dx)})/(a-\sqrt{a^2+b^2})]}{(a^2*\sqrt{a^2+b^2}*d^2)} - \frac{3*b^2*f*(e+fx)^2*\operatorname{PolyLog}[2, -(b*E^{(c+dx)})/(a+\sqrt{a^2+b^2})]}{(a^2*\sqrt{a^2+b^2}*d^2)} + \frac{3*f^2*(e+fx)*\operatorname{PolyLog}[2, E^{(2*(c+dx))}]}{(a*d^3)} - \frac{6*b*f^2*(e+fx)*\operatorname{PolyLog}[3, -E^{(c+dx)}]}{(a^2*d^3)} + \frac{6*b*f^2*(e+fx)*\operatorname{PolyLog}[3, E^{(c+dx)}]}{(a^2*d^3)} - \frac{6*b^2*f^2*(e+fx)*\operatorname{PolyLog}[3, -(b*E^{(c+dx)})/(a-\sqrt{a^2+b^2})]}{(a^2*\sqrt{a^2+b^2}*d^3)} + \frac{6*b^2*f^2*(e+fx)*\operatorname{PolyLog}[3, -(b*E^{(c+dx)})/(a+\sqrt{a^2+b^2})]}{(a^2*\sqrt{a^2+b^2}*d^3)} - \frac{3*f^3*\operatorname{PolyLog}[3, E^{(2*(c+dx))}]}{(2*a*d^4)} + \frac{6*b*f^3*\operatorname{PolyLog}[4, -E^{(c+dx)}]}{(a^2*d^4)} - \frac{6*b*f^3*\operatorname{PolyLog}[4, E^{(c+dx)}]}{(a^2*d^4)} + \frac{6*b^2*f^3*\operatorname{PolyLog}[4, -(b*E^{(c+dx)})/(a-\sqrt{a^2+b^2})]}{(a^2*\sqrt{a^2+b^2}*d^4)} - \frac{6*b^2*f^3*\operatorname{PolyLog}[4, -(b*E^{(c+dx)})/(a+\sqrt{a^2+b^2})]}{(a^2*\sqrt{a^2+b^2}*d^4)}$

**Rubi [A]** time = 1.31073, antiderivative size = 745, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 11, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$ , Rules used = {5575, 4184, 3716, 2190, 2531, 2282, 6589, 4182, 6609, 3322, 2264}

$$-\frac{6b^2 f^2 (e+fx) \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2 d^3 \sqrt{a^2+b^2}} + \frac{6b^2 f^2 (e+fx) \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2 d^3 \sqrt{a^2+b^2}} + \frac{3b^2 f (e+fx)^2 \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2 d^2 \sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{(e+fx)^3 \operatorname{Csch}[c+dx]^2}{(a+b \operatorname{Sinh}[c+dx])}, x\right]$

[Out]  $-\left(\frac{(e+fx)^3}{a*d}\right) + \frac{2*b*(e+fx)^3*\operatorname{ArcTanh}[E^{(c+dx)}]}{(a^2*d)} - \left(\frac{(e+fx)^3*\operatorname{Coth}[c+dx]}{a*d} + \frac{b^2*(e+fx)^3*\operatorname{Log}[1+(b*E^{(c+dx)})/(a-\sqrt{a^2+b^2})]}{(a^2*\sqrt{a^2+b^2}*d)} - \frac{b^2*(e+fx)^3*\operatorname{Log}[1+(b*E^{(c+dx)})/(a+\sqrt{a^2+b^2})]}{(a^2*\sqrt{a^2+b^2}*d)} + \frac{3*f*(e+fx)^2*\operatorname{Log}[1-E^{(2*(c+dx))}]}{(a*d^2)} + \frac{3*b*f*(e+fx)^2*\operatorname{PolyLog}[2, -E^{(c+dx)}]}{(a^2*d^2)} - \frac{3*b*f*(e+fx)^2*\operatorname{PolyLog}[2, E^{(c+dx)}]}{(a^2*d^2)} + \frac{3*b^2*f*(e+fx)^2*\operatorname{PolyLog}[2, -(b*E^{(c+dx)})/(a-\sqrt{a^2+b^2})]}{(a^2*\sqrt{a^2+b^2}*d^2)} - \frac{3*b^2*f*(e+fx)^2*\operatorname{PolyLog}[2, -(b*E^{(c+dx)})/(a+\sqrt{a^2+b^2})]}{(a^2*\sqrt{a^2+b^2}*d^2)} + \frac{3*f^2*(e+fx)*\operatorname{PolyLog}[2, E^{(2*(c+dx))}]}{(a*d^3)} - \frac{6*b*f^2*(e+fx)*\operatorname{PolyLog}[3, -E^{(c+dx)}]}{(a^2*d^3)} + \frac{6*b*f^2*(e+fx)*\operatorname{PolyLog}[3, E^{(c+dx)}]}{(a^2*d^3)} - \frac{6*b^2*f^2*(e+fx)*\operatorname{PolyLog}[3, -(b*E^{(c+dx)})/(a-\sqrt{a^2+b^2})]}{(a^2*\sqrt{a^2+b^2}*d^3)} + \frac{6*b^2*f^2*(e+fx)*\operatorname{PolyLog}[3, -(b*E^{(c+dx)})/(a+\sqrt{a^2+b^2})]}{(a^2*\sqrt{a^2+b^2}*d^3)} - \frac{3*f^3*\operatorname{PolyLog}[3, E^{(2*(c+dx))}]}{(2*a*d^4)} + \frac{6*b*f^3*\operatorname{PolyLog}[4, -E^{(c+dx)}]}{(a^2*d^4)} - \frac{6*b*f^3*\operatorname{PolyLog}[4, E^{(c+dx)}]}{(a^2*d^4)} + \frac{6*b^2*f^3*\operatorname{PolyLog}[4, -(b*E^{(c+dx)})/(a-\sqrt{a^2+b^2})]}{(a^2*\sqrt{a^2+b^2}*d^4)} - \frac{6*b^2*f^3*\operatorname{PolyLog}[4, -(b*E^{(c+dx)})/(a+\sqrt{a^2+b^2})]}{(a^2*\sqrt{a^2+b^2}*d^4)}$



$$\begin{aligned} & a - \sqrt{a^2 + b^2} \Big] \Big] / (a^2 \sqrt{a^2 + b^2} d) - (b^2 (e + f x)^3 \operatorname{Log}[1 + (b E^{(c + d x)}) / (a + \sqrt{a^2 + b^2})]) / (a^2 \sqrt{a^2 + b^2} d) + (3 f (e + f x)^2 \operatorname{Log}[1 - E^{(2(c + d x))}] / (a d^2) + (3 b f (e + f x)^2 \operatorname{PolyLog}[2, -E^{(c + d x)}] / (a^2 d^2) - (3 b f (e + f x)^2 \operatorname{PolyLog}[2, E^{(c + d x)}] / (a^2 d^2) + (3 b^2 f (e + f x)^2 \operatorname{PolyLog}[2, -((b E^{(c + d x)}) / (a - \sqrt{a^2 + b^2}))]) / (a^2 \sqrt{a^2 + b^2} d^2) - (3 b^2 f (e + f x)^2 \operatorname{PolyLog}[2, -((b E^{(c + d x)}) / (a + \sqrt{a^2 + b^2}))]) / (a^2 \sqrt{a^2 + b^2} d^2) + (3 f^2 (e + f x) \operatorname{PolyLog}[2, E^{(2(c + d x))}] / (a d^3) - (6 b f^2 (e + f x) \operatorname{PolyLog}[3, -E^{(c + d x)}] / (a^2 d^3) + (6 b f^2 (e + f x) \operatorname{PolyLog}[3, E^{(c + d x)}] / (a^2 d^3) - (6 b^2 f^2 (e + f x) \operatorname{PolyLog}[3, -((b E^{(c + d x)}) / (a - \sqrt{a^2 + b^2}))]) / (a^2 \sqrt{a^2 + b^2} d^3) + (6 b^2 f^2 (e + f x) \operatorname{PolyLog}[3, -((b E^{(c + d x)}) / (a + \sqrt{a^2 + b^2}))]) / (a^2 \sqrt{a^2 + b^2} d^3) - (3 f^3 \operatorname{PolyLog}[3, E^{(2(c + d x))}] / (2 a d^4) + (6 b f^3 \operatorname{PolyLog}[4, -E^{(c + d x)}] / (a^2 d^4) - (6 b f^3 \operatorname{PolyLog}[4, E^{(c + d x)}] / (a^2 d^4) + (6 b^2 f^3 \operatorname{PolyLog}[4, -((b E^{(c + d x)}) / (a - \sqrt{a^2 + b^2}))]) / (a^2 \sqrt{a^2 + b^2} d^4) - (6 b^2 f^3 \operatorname{PolyLog}[4, -((b E^{(c + d x)}) / (a + \sqrt{a^2 + b^2}))]) / (a^2 \sqrt{a^2 + b^2} d^4) \end{aligned}$$

### Rule 5575

$$\operatorname{Int}[(\operatorname{Csch}[c] + (d)(x))^n ((e) + (f)(x))^m] / ((a) + (b) \operatorname{Sinh}[c + d x]), x_{\text{Symbol}}] \rightarrow \operatorname{Dist}[1/a, \operatorname{Int}[(e + f x)^m \operatorname{Csch}[c + d x]^n, x], x] - \operatorname{Dist}[b/a, \operatorname{Int}[(e + f x)^m \operatorname{Csch}[c + d x]^{(n-1)} / (a + b \operatorname{Sinh}[c + d x]), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \operatorname{IGtQ}[m, 0] \&\& \operatorname{IGtQ}[n, 0]$$

### Rule 4184

$$\operatorname{Int}[\operatorname{csc}[(e) + (f)(x)]^2 ((c) + (d)(x))^m, x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[(c + d x)^m \operatorname{Cot}[e + f x] / f, x] + \operatorname{Dist}[(d m) / f, \operatorname{Int}[(c + d x)^{(m-1)} \operatorname{Cot}[e + f x], x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x] \&\& \operatorname{GtQ}[m, 0]$$

### Rule 3716

$$\operatorname{Int}[(c + d x)^m \tan[(e) + \operatorname{Pi}(k) + (\operatorname{Complex}[0, fz]) (f)(x)], x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[(I (c + d x)^{(m+1)}) / (d (m + 1)), x] + \operatorname{Dist}[2 I, \operatorname{Int}[(c + d x)^m E^{(2(-I e) + f f z x))} / (E^{(2 I k \operatorname{Pi})} (1 + E^{(2(-I e) + f f z x))} / E^{(2 I k \operatorname{Pi})})], x], x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x] \&\& \operatorname{IntegerQ}[4 k] \&\& \operatorname{IGtQ}[m, 0]$$

### Rule 2190

$$\operatorname{Int}[(F)^{(g)(e) + (f)(x)} ((c) + (d)(x))^m] / ((a) + (b) (F)^{(g)(e) + (f)(x)} ((c) + (d)(x))^n), x_{\text{Symbol}}] \rightarrow \operatorname{Simp}[(c + d x)^m \operatorname{Log}[1 + (b (F^{(g)(e + f x)})^n) / a] / (b f g^n \operatorname{Log}[F]), x] - \operatorname{Di}$$

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_.)*((a_.)*(v_)^(n_.))^m_] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_])*(
f_.)*(x_)]), x_Symbol] := Dist[2, Int[(c + d*x)^m*E^(-(I*e) + f*fz*x)]/(-
```

$(I*b) + 2*a*E^{-(I*e) + f*fz*x} + I*b*E^{(2*(-(I*e) + f*fz*x))}, x], x] /; FreeQ[\{a, b, c, d, e, f, fz\}, x] \&\& NeQ[a^2 - b^2, 0] \&\& IGtQ[m, 0]$

### Rule 2264

$Int[((F_)^(u_)*((f_.) + (g_.)*(x_.))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)), x\_Symbol] :> With[\{q = Rt[b^2 - 4*a*c, 2]\}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b + q + 2*c*F^u), x], x]] /; FreeQ[\{F, a, b, c, f, g\}, x] \&\& EqQ[v, 2*u] \&\& LinearQ[u, x] \&\& NeQ[b^2 - 4*a*c, 0] \&\& IGtQ[m, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \operatorname{csch}^2(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^3 \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\ &= -\frac{(e+fx)^3 \operatorname{coth}(c+dx)}{ad} - \frac{b \int (e+fx)^3 \operatorname{csch}(c+dx) dx}{a^2} + \frac{b^2 \int \frac{(e+fx)^3}{a+b \sinh(c+dx)} dx}{a^2} + \frac{(3f)}{a^2} \\ &= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^2 d} - \frac{(e+fx)^3 \operatorname{coth}(c+dx)}{ad} + \frac{(2b^2) \int \frac{e^{c+dx}(e^{-c+dx}-1)}{-b+2ae^{c+dx}} dx}{a^2} \\ &= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^2 d} - \frac{(e+fx)^3 \operatorname{coth}(c+dx)}{ad} + \frac{3f(e+fx)^2 \log\left(\frac{e^{c+dx}-1}{-b+2ae^{c+dx}}\right)}{ad^2} \\ &= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^2 d} - \frac{(e+fx)^3 \operatorname{coth}(c+dx)}{ad} + \frac{b^2(e+fx)^3 \log\left(\frac{e^{c+dx}-1}{-b+2ae^{c+dx}}\right)}{a^2 \sqrt{a^2+b^2}} \\ &= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^2 d} - \frac{(e+fx)^3 \operatorname{coth}(c+dx)}{ad} + \frac{b^2(e+fx)^3 \log\left(\frac{e^{c+dx}-1}{-b+2ae^{c+dx}}\right)}{a^2 \sqrt{a^2+b^2}} \\ &= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^2 d} - \frac{(e+fx)^3 \operatorname{coth}(c+dx)}{ad} + \frac{b^2(e+fx)^3 \log\left(\frac{e^{c+dx}-1}{-b+2ae^{c+dx}}\right)}{a^2 \sqrt{a^2+b^2}} \\ &= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^2 d} - \frac{(e+fx)^3 \operatorname{coth}(c+dx)}{ad} + \frac{b^2(e+fx)^3 \log\left(\frac{e^{c+dx}-1}{-b+2ae^{c+dx}}\right)}{a^2 \sqrt{a^2+b^2}} \end{aligned}$$

**Mathematica [A]** time = 18.5958, size = 1353, normalized size = 1.82

$$\left(-2e^3 \tanh^{-1}\left(\frac{a+be^{dx}}{\sqrt{a^2+b^2}}\right)d^3 + f^3x^3 \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}} + 1\right)d^3 + 3ef^2x^2 \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}} + 1\right)d^3 + 3e^2fx \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}} + 1\right)d^3 - f^3x^3\right)$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^3\*Csch[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (b^2\*(-2\*d^3\*e^3\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]] + 3\*d^3\*e^2\*f\*x\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])] + 3\*d^3\*e\*f^2\*x^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])] + d^3\*f^3\*x^3\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])] - 3\*d^3\*e^2\*f\*x\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] - 3\*d^3\*e\*f^2\*x^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] - d^3\*f^3\*x^3\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] + 3\*d^2\*f\*(e + f\*x)^2\*PolyLog[2, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] - 3\*d^2\*f\*(e + f\*x)^2\*PolyLog[2, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] - 6\*d\*e\*f^2\*PolyLog[3, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] - 6\*d\*f^3\*x\*PolyLog[3, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] + 6\*d\*e\*f^2\*PolyLog[3, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] + 6\*d\*f^3\*x\*PolyLog[3, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] + 6\*f^3\*PolyLog[4, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] - 6\*f^3\*PolyLog[4, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^2\*Sqrt[a^2 + b^2]\*d^4) - (a\*d^3\*(e + f\*x)^3\*(-1 + Coth[c]) - d^2\*e^2\*(b\*d\*e - 3\*a\*f)\*(d\*x - Log[1 - Cosh[c + d\*x] - Sinh[c + d\*x]]) - 3\*d^2\*e\*f\*(b\*d\*e + 2\*a\*f)\*x\*Log[1 + Cosh[c + d\*x] - Sinh[c + d\*x]] - 3\*d^2\*f^2\*(b\*d\*e + a\*f)\*x^2\*Log[1 + Cosh[c + d\*x] - Sinh[c + d\*x]] - b\*d^3\*f^3\*x^3\*Log[1 + Cosh[c + d\*x] - Sinh[c + d\*x]] + 3\*d^2\*e\*f\*(b\*d\*e - 2\*a\*f)\*x\*Log[1 - Cosh[c + d\*x] + Sinh[c + d\*x]] + 3\*d^2\*f^2\*(b\*d\*e - a\*f)\*x^2\*Log[1 - Cosh[c + d\*x] + Sinh[c + d\*x]] + b\*d^3\*f^3\*x^3\*Log[1 - Cosh[c + d\*x] + Sinh[c + d\*x]] + d^2\*e^2\*(b\*d\*e + 3\*a\*f)\*(d\*x - Log[1 + Cosh[c + d\*x] + Sinh[c + d\*x]]) - 3\*d\*e\*f\*(b\*d\*e - 2\*a\*f)\*PolyLog[2, Cosh[c + d\*x] - Sinh[c + d\*x]] + 3\*d\*e\*f\*(b\*d\*e + 2\*a\*f)\*PolyLog[2, -Cosh[c + d\*x] + Sinh[c + d\*x]] + 6\*f^2\*(-(b\*d\*e) + a\*f)\*(d\*x\*PolyLog[2, Cosh[c + d\*x] - Sinh[c + d\*x]] + PolyLog[3, Cosh[c + d\*x] - Sinh[c + d\*x]]) + 6\*f^2\*(b\*d\*e + a\*f)\*(d\*x\*PolyLog[2, -Cosh[c + d\*x] + Sinh[c + d\*x]] + PolyLog[3, -Cosh[c + d\*x] + Sinh[c + d\*x]]) - 3\*b\*f^3\*(d^2\*x^2\*PolyLog[2, Cosh[c + d\*x] - Sinh[c + d\*x]] + 2\*(d\*x\*PolyLog[3, Cosh[c + d\*x] - Sinh[c + d\*x]] + PolyLog[4, Cosh[c + d\*x] - Sinh[c + d\*x]])) + 3\*b\*f^3\*(d^2\*x^2\*PolyLog[2, -Cosh[c + d\*x] + Sinh[c + d\*x]] + 2\*(d\*x\*PolyLog[3, -Cosh[c + d\*x] + Sinh[c + d\*x]] + PolyLog[4, -Cosh[c + d\*x] + Sinh[c + d\*x]])))/(a^2\*d^4) + (Sech[c/2]\*Sech[c/2 + (d\*x)/2]\*(-(e^3\*Sinh[(d\*x)/2]) - 3\*e^2\*f\*x\*Sinh[(d\*x)/2] - 3\*e\*f^2\*x^2\*Sinh[(d\*x)/2] - f^3\*x^3\*Sinh[(d\*x)/2]))/(2\*a\*d) + (Csch[c/2]\*Csch[c/2 + (d\*x)/2]\*(e^3\*Sinh[(d\*x)/2] + 3\*e^2\*f\*x\*Sinh[(d\*x)/2] + 3\*e\*f^2\*x^2\*Sinh[(d\*x)/2] + f^3\*x^3\*Sinh[(d\*x)/2]))/(2\*a\*d)

---

**Maple [F]** time = 0.692, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 (\operatorname{csch}(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*csch(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*csch(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*csch(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [C]** time = 4.12939, size = 13974, normalized size = 18.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*csch(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-(2*(a^3 + a*b^2)*d^3*e^3 - 6*(a^3 + a*b^2)*c*d^2*e^2*f + 6*(a^3 + a*b^2)*c^2*d*e*f^2 - 2*(a^3 + a*b^2)*c^3*f^3 + 2*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*\cosh(d*x + c)^2 + 4*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c) + 2*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2$$

$$\begin{aligned}
& *f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a \\
& *b^2)*c^3*f^3)*\sinh(d*x + c)^2 + 3*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b \\
& ^3*d^2*e^2*f - (b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*\cosh(d \\
& *x + c)^2 - 2*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*\cosh(d* \\
& x + c)*\sinh(d*x + c) - (b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f \\
& )*\sinh(d*x + c)^2*\sqrt{(a^2 + b^2)/b^2}*dilog((a*\cosh(d*x + c) + a*\sinh(d* \\
& x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + \\
& 1) - 3*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f - (b^3*d^2*f^3 \\
& *x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*\cosh(d*x + c)^2 - 2*(b^3*d^2*f^3* \\
& x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*\cosh(d*x + c)*\sinh(d*x + c) - (b^3 \\
& *d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*\sinh(d*x + c)^2*\sqrt{(a^ \\
& 2 + b^2)/b^2}*dilog((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + \\
& b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - (b^3*d^3*e^3 - 3*b^3* \\
& c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3 - (b^3*d^3*e^3 - 3*b^3*c*d^2* \\
& e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*\cosh(d*x + c)^2 - 2*(b^3*d^3*e^3 - \\
& 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*\cosh(d*x + c)*\sinh(d* \\
& x + c) - (b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3 \\
& )*\sinh(d*x + c)^2*\sqrt{(a^2 + b^2)/b^2}*log(2*b*\cosh(d*x + c) + 2*b*\sinh(d \\
& *x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + (b^3*d^3*e^3 - 3*b^3*c*d^2*e^2 \\
& *f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3 - (b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3 \\
& *b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*\cosh(d*x + c)^2 - 2*(b^3*d^3*e^3 - 3*b^3*c* \\
& d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c) - \\
& (b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*\sinh(d* \\
& x + c)^2*\sqrt{(a^2 + b^2)/b^2}*log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - \\
& 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + (b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 \\
& + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3 - \\
& (b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e \\
& ^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*\cosh(d*x + c)^2 - 2*(b^3*d^3*f^3*x^ \\
& 3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2 \\
& *d*e*f^2 + b^3*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c) - (b^3*d^3*f^3*x^3 + 3* \\
& b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f \\
& ^2 + b^3*c^3*f^3)*\sinh(d*x + c)^2*\sqrt{(a^2 + b^2)/b^2}*log(-(a*\cosh(d*x + \\
& c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2 \\
& )/b^2} - b)/b) - (b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x \\
& + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3 - (b^3*d^3*f^3*x^3 + \\
& 3*b^3*d^3*e*f^2*x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d* \\
& e*f^2 + b^3*c^3*f^3)*\cosh(d*x + c)^2 - 2*(b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2 \\
& *x^2 + 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3* \\
& f^3)*\cosh(d*x + c)*\sinh(d*x + c) - (b^3*d^3*f^3*x^3 + 3*b^3*d^3*e*f^2*x^2 + \\
& 3*b^3*d^3*e^2*f*x + 3*b^3*c*d^2*e^2*f - 3*b^3*c^2*d*e*f^2 + b^3*c^3*f^3)*s \\
& \sinh(d*x + c)^2*\sqrt{(a^2 + b^2)/b^2}*log(-(a*\cosh(d*x + c) + a*\sinh(d*x + \\
& c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 6* \\
& (b^3*f^3*\cosh(d*x + c)^2 + 2*b^3*f^3*\cosh(d*x + c)*\sinh(d*x + c) + b^3*f^3* \\
& \sinh(d*x + c)^2 - b^3*f^3)*\sqrt{(a^2 + b^2)/b^2}*polylog(4, (a*\cosh(d*x + c \\
& ) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/
\end{aligned}$$

$$\begin{aligned}
& b^2)/b) + 6*(b^3*f^3*\cosh(d*x + c)^2 + 2*b^3*f^3*\cosh(d*x + c)*\sinh(d*x + \\
& c) + b^3*f^3*\sinh(d*x + c)^2 - b^3*f^3)*\sqrt{(a^2 + b^2)/b^2)*\text{polylog}(4, (a \\
& *\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{ \\
& ((a^2 + b^2)/b^2))/b) - 6*(b^3*d*f^3*x + b^3*d*e*f^2 - (b^3*d*f^3*x + b^3*d \\
& *e*f^2))*\cosh(d*x + c)^2 - 2*(b^3*d*f^3*x + b^3*d*e*f^2))*\cosh(d*x + c)*\sinh( \\
& d*x + c) - (b^3*d*f^3*x + b^3*d*e*f^2))*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^ \\
& 2)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sin \\
& h(d*x + c))*\sqrt{(a^2 + b^2)/b^2))/b) + 6*(b^3*d*f^3*x + b^3*d*e*f^2 - (b^3 \\
& *d*f^3*x + b^3*d*e*f^2))*\cosh(d*x + c)^2 - 2*(b^3*d*f^3*x + b^3*d*e*f^2))*\cos \\
& h(d*x + c)*\sinh(d*x + c) - (b^3*d*f^3*x + b^3*d*e*f^2))*\sinh(d*x + c)^2)*\sqrt{ \\
& (a^2 + b^2)/b^2)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh( \\
& d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2))/b) - 3*((a^2*b + b^3)*d^ \\
& 2*f^3*x^2 + (a^2*b + b^3)*d^2*e^2*f - 2*(a^3 + a*b^2)*d*e*f^2 - ((a^2*b + b \\
& ^3)*d^2*f^3*x^2 + (a^2*b + b^3)*d^2*e^2*f - 2*(a^3 + a*b^2)*d*e*f^2 + 2*((a \\
& ^2*b + b^3)*d^2*e*f^2 - (a^3 + a*b^2)*d*f^3)*x)*\cosh(d*x + c)^2 - 2*((a^2*b \\
& + b^3)*d^2*f^3*x^2 + (a^2*b + b^3)*d^2*e^2*f - 2*(a^3 + a*b^2)*d*e*f^2 + 2 \\
& *((a^2*b + b^3)*d^2*e*f^2 - (a^3 + a*b^2)*d*f^3)*x)*\cosh(d*x + c)*\sinh(d*x \\
& + c) - ((a^2*b + b^3)*d^2*f^3*x^2 + (a^2*b + b^3)*d^2*e^2*f - 2*(a^3 + a*b^ \\
& 2)*d*e*f^2 + 2*((a^2*b + b^3)*d^2*e*f^2 - (a^3 + a*b^2)*d*f^3)*x)*\sinh(d*x \\
& + c)^2 + 2*((a^2*b + b^3)*d^2*e*f^2 - (a^3 + a*b^2)*d*f^3)*x)*\text{dilog}(\cosh(d*x \\
& + c) + \sinh(d*x + c)) + 3*((a^2*b + b^3)*d^2*f^3*x^2 + (a^2*b + b^3)*d^2* \\
& e^2*f + 2*(a^3 + a*b^2)*d*e*f^2 - ((a^2*b + b^3)*d^2*f^3*x^2 + (a^2*b + b^3 \\
& )*d^2*e^2*f + 2*(a^3 + a*b^2)*d*e*f^2 + 2*((a^2*b + b^3)*d^2*e*f^2 + (a^3 + \\
& a*b^2)*d*f^3)*x)*\cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^2*f^3*x^2 + (a^2*b + \\
& b^3)*d^2*e^2*f + 2*(a^3 + a*b^2)*d*e*f^2 + 2*((a^2*b + b^3)*d^2*e*f^2 + (a \\
& ^3 + a*b^2)*d*f^3)*x)*\cosh(d*x + c)*\sinh(d*x + c) - ((a^2*b + b^3)*d^2*f^3* \\
& x^2 + (a^2*b + b^3)*d^2*e^2*f + 2*(a^3 + a*b^2)*d*e*f^2 + 2*((a^2*b + b^3)* \\
& d^2*e*f^2 + (a^3 + a*b^2)*d*f^3)*x)*\sinh(d*x + c)^2 + 2*((a^2*b + b^3)*d^2* \\
& e*f^2 + (a^3 + a*b^2)*d*f^3)*x)*\text{dilog}(-\cosh(d*x + c) - \sinh(d*x + c)) + ((a \\
& ^2*b + b^3)*d^3*f^3*x^3 + (a^2*b + b^3)*d^3*e^3 + 3*(a^3 + a*b^2)*d^2*e^2*f \\
& + 3*((a^2*b + b^3)*d^3*e*f^2 + (a^3 + a*b^2)*d^2*f^3)*x^2 - ((a^2*b + b^3) \\
& *d^3*f^3*x^3 + (a^2*b + b^3)*d^3*e^3 + 3*(a^3 + a*b^2)*d^2*e^2*f + 3*((a^2*b \\
& + b^3)*d^3*e*f^2 + (a^3 + a*b^2)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f + 2*( \\
& a^3 + a*b^2)*d^2*e*f^2)*x)*\cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^3*f^ \\
& 3*x^3 + (a^2*b + b^3)*d^3*e^3 + 3*(a^3 + a*b^2)*d^2*e^2*f + 3*((a^2*b + b^3) \\
& )*d^3*e*f^2 + (a^3 + a*b^2)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f + 2*( \\
& a^3 + a*b^2)*d^2*e*f^2)*x)*\sinh(d*x + c)^2 + 3*((a^2*b + b^3)*d^3*e^2*f \\
& + 2*(a^3 + a*b^2)*d^2*e*f^2)*x)*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) - (( \\
& a^2*b + b^3)*d^3*e^3 - 3*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d^2*e^2*f + 3*((a^ \\
& 2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 - ((a^2*b + b^3)*c^3 + 3*(a^3 + \\
& a*b^2)*c^2)*f^3 - ((a^2*b + b^3)*d^3*e^3 - 3*(a^3 + a*b^2 + (a^2*b + b^3)* \\
& c)*d^2*e^2*f + 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 - ((a^2*b
\end{aligned}$$

$$\begin{aligned}
& + b^3)c^3 + 3*(a^3 + a*b^2)*c^2)*f^3)*\cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d \\
& ^3*e^3 - 3*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d^2*e^2*f + 3*((a^2*b + b^3)*c^2 \\
& + 2*(a^3 + a*b^2)*c)*d*e*f^2 - ((a^2*b + b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f \\
& ^3)*\cosh(d*x + c)*\sinh(d*x + c) - ((a^2*b + b^3)*d^3*e^3 - 3*(a^3 + a*b^2 + \\
& (a^2*b + b^3)*c)*d^2*e^2*f + 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e \\
& *f^2 - ((a^2*b + b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3)*\sinh(d*x + c)^2)*\log( \\
& \cosh(d*x + c) + \sinh(d*x + c) - 1) - ((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b \\
& + b^3)*c*d^2*e^2*f - 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 + (( \\
& a^2*b + b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - \\
& (a^3 + a*b^2)*d^2*f^3)*x^2 - ((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*c \\
& *d^2*e^2*f - 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 + ((a^2*b + \\
& b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^3 + a \\
& *b^2)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^3 + a*b^2)*d^2*e*f^2 \\
& )*x)*\cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*c*d^2 \\
& *e^2*f - 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 + ((a^2*b + b^3) \\
& *c^3 + 3*(a^3 + a*b^2)*c^2)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^3 + a*b^2 \\
& )*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^3 + a*b^2)*d^2*e*f^2)*x) \\
& *\cosh(d*x + c)*\sinh(d*x + c) - ((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3) \\
& *c*d^2*e^2*f - 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 + ((a^2*b \\
& + b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^3 + \\
& a*b^2)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^3 + a*b^2)*d^2*e*f \\
& ^2)*x)*\sinh(d*x + c)^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^3 + a*b^2)*d^2*e \\
& *f^2)*x)*\log(-\cosh(d*x + c) - \sinh(d*x + c) + 1) + 6*((a^2*b + b^3)*f^3*\cos \\
& h(d*x + c)^2 + 2*(a^2*b + b^3)*f^3*\cosh(d*x + c)*\sinh(d*x + c) + (a^2*b + b \\
& ^3)*f^3*\sinh(d*x + c)^2 - (a^2*b + b^3)*f^3)*\text{polylog}(4, \cosh(d*x + c) + \sin \\
& h(d*x + c)) - 6*((a^2*b + b^3)*f^3*\cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f^3*\co \\
& sh(d*x + c)*\sinh(d*x + c) + (a^2*b + b^3)*f^3*\sinh(d*x + c)^2 - (a^2*b + b \\
& ^3)*f^3)*\text{polylog}(4, -\cosh(d*x + c) - \sinh(d*x + c)) + 6*((a^2*b + b^3)*d*f^3 \\
& *x + (a^2*b + b^3)*d*e*f^2 - (a^3 + a*b^2)*f^3 - ((a^2*b + b^3)*d*f^3*x + ( \\
& a^2*b + b^3)*d*e*f^2 - (a^3 + a*b^2)*f^3)*\cosh(d*x + c)^2 - 2*((a^2*b + b^3) \\
& )*d*f^3*x + (a^2*b + b^3)*d*e*f^2 - (a^3 + a*b^2)*f^3)*\cosh(d*x + c)*\sinh(d \\
& *x + c) - ((a^2*b + b^3)*d*f^3*x + (a^2*b + b^3)*d*e*f^2 - (a^3 + a*b^2)*f^3 \\
& )*\sinh(d*x + c)^2)*\text{polylog}(3, \cosh(d*x + c) + \sinh(d*x + c)) - 6*((a^2*b + \\
& b^3)*d*f^3*x + (a^2*b + b^3)*d*e*f^2 + (a^3 + a*b^2)*f^3 - ((a^2*b + b^3)* \\
& d*f^3*x + (a^2*b + b^3)*d*e*f^2 + (a^3 + a*b^2)*f^3)*\cosh(d*x + c)^2 - 2*(( \\
& a^2*b + b^3)*d*f^3*x + (a^2*b + b^3)*d*e*f^2 + (a^3 + a*b^2)*f^3)*\cosh(d*x \\
& + c)*\sinh(d*x + c) - ((a^2*b + b^3)*d*f^3*x + (a^2*b + b^3)*d*e*f^2 + (a^3 \\
& + a*b^2)*f^3)*\sinh(d*x + c)^2)*\text{polylog}(3, -\cosh(d*x + c) - \sinh(d*x + c)))/ \\
& ((a^4 + a^2*b^2)*d^4*\cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*d^4*\cosh(d*x + c)* \\
& \sinh(d*x + c) + (a^4 + a^2*b^2)*d^4*\sinh(d*x + c)^2 - (a^4 + a^2*b^2)*d^4)
\end{aligned}$$



**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.244 \quad \int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=535

$$\frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2 d^2 \sqrt{a^2+b^2}} - \frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2 d^2 \sqrt{a^2+b^2}} - \frac{2b^2 f^2 \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2 d^3 \sqrt{a^2+b^2}} + \frac{2b^2 f^2 \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2 d^3 \sqrt{a^2+b^2}}$$

[Out]  $-\left(\frac{(e+fx)^2}{a*d}\right) + \frac{(2*b*(e+fx)^2*\operatorname{ArcTanh}[E^{(c+dx)}])}{(a^2*d)} - \left(\frac{(e+fx)^2*\operatorname{Coth}[c+dx]}{a*d} + \frac{(b^2*(e+fx)^2*\operatorname{Log}[1+(b*E^{(c+dx)})/(a-\sqrt{a^2+b^2})])}{(a^2*\sqrt{a^2+b^2}*d)} - \frac{(b^2*(e+fx)^2*\operatorname{Log}[1+(b*E^{(c+dx)})/(a+\sqrt{a^2+b^2})])}{(a^2*\sqrt{a^2+b^2}*d)} + \frac{(2*f*(e+fx)*\operatorname{Log}[1-E^{(2*(c+dx))}])}{(a*d^2)} + \frac{(2*b*f*(e+fx)*\operatorname{PolyLog}[2, -E^{(c+dx)}])}{(a^2*d^2)} - \frac{(2*b*f*(e+fx)*\operatorname{PolyLog}[2, E^{(c+dx)}])}{(a^2*d^2)} + \frac{(2*b^2*f*(e+fx)*\operatorname{PolyLog}[2, -((b*E^{(c+dx)})/(a-\sqrt{a^2+b^2})])]}{(a^2*\sqrt{a^2+b^2}*d^2)} - \frac{(2*b^2*f*(e+fx)*\operatorname{PolyLog}[2, -((b*E^{(c+dx)})/(a+\sqrt{a^2+b^2})])]}{(a^2*\sqrt{a^2+b^2}*d^2)} + \frac{(f^2*\operatorname{PolyLog}[2, E^{(2*(c+dx))}])}{(a*d^3)} - \frac{(2*b*f^2*\operatorname{PolyLog}[3, -E^{(c+dx)}])}{(a^2*d^3)} + \frac{(2*b*f^2*\operatorname{PolyLog}[3, E^{(c+dx)}])}{(a^2*d^3)} - \frac{(2*b^2*f^2*\operatorname{PolyLog}[3, -((b*E^{(c+dx)})/(a-\sqrt{a^2+b^2})])]}{(a^2*\sqrt{a^2+b^2}*d^3)} + \frac{(2*b^2*f^2*\operatorname{PolyLog}[3, -((b*E^{(c+dx)})/(a+\sqrt{a^2+b^2})])]}{(a^2*\sqrt{a^2+b^2}*d^3)}$

**Rubi [A]** time = 1.07963, antiderivative size = 535, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 12, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {5575, 4184, 3716, 2190, 2279, 2391, 4182, 2531, 2282, 6589, 3322, 2264}

$$\frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2 d^2 \sqrt{a^2+b^2}} - \frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2 d^2 \sqrt{a^2+b^2}} - \frac{2b^2 f^2 \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2 d^3 \sqrt{a^2+b^2}} + \frac{2b^2 f^2 \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2 d^3 \sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{(e+fx)^2 \operatorname{Csch}[c+dx]^2}{(a+b \operatorname{Sinh}[c+dx])}, x\right]$

[Out]  $-\left(\frac{(e+fx)^2}{a*d}\right) + \frac{(2*b*(e+fx)^2*\operatorname{ArcTanh}[E^{(c+dx)}])}{(a^2*d)} - \left(\frac{(e+fx)^2*\operatorname{Coth}[c+dx]}{a*d} + \frac{(b^2*(e+fx)^2*\operatorname{Log}[1+(b*E^{(c+dx)})/(a-\sqrt{a^2+b^2})])}{(a^2*\sqrt{a^2+b^2}*d)} - \frac{(b^2*(e+fx)^2*\operatorname{Log}[1+(b*E^{(c+dx)})/(a+\sqrt{a^2+b^2})])}{(a^2*\sqrt{a^2+b^2}*d)} + \frac{(2*f*(e+fx)*\operatorname{Log}[1-E^{(2*(c+dx))}])}{(a*d^2)} + \frac{(2*b*f*(e+fx)*\operatorname{PolyLog}[2, -E^{(c+dx)}])}{(a^2*d^2)} - \frac{(2*b*f*(e+fx)*\operatorname{PolyLog}[2, E^{(c+dx)}])}{(a^2*d^2)} + \frac{(2*b^2*f*(e+fx)*\operatorname{PolyLog}[2, -((b*E^{(c+dx)})/(a-\sqrt{a^2+b^2})])]}{(a^2*\sqrt{a^2+b^2}*d^2)} - \frac{(2*b^2*f*(e+fx)*\operatorname{PolyLog}[2, -((b*E^{(c+dx)})/(a+\sqrt{a^2+b^2})])]}{(a^2*\sqrt{a^2+b^2}*d^2)} + \frac{(f^2*\operatorname{PolyLog}[2, E^{(2*(c+dx))}])}{(a*d^3)} - \frac{(2*b*f^2*\operatorname{PolyLog}[3, -E^{(c+dx)}])}{(a^2*d^3)} + \frac{(2*b*f^2*\operatorname{PolyLog}[3, E^{(c+dx)}])}{(a^2*d^3)} - \frac{(2*b^2*f^2*\operatorname{PolyLog}[3, -((b*E^{(c+dx)})/(a-\sqrt{a^2+b^2})])]}{(a^2*\sqrt{a^2+b^2}*d^3)} + \frac{(2*b^2*f^2*\operatorname{PolyLog}[3, -((b*E^{(c+dx)})/(a+\sqrt{a^2+b^2})])]}{(a^2*\sqrt{a^2+b^2}*d^3)}$

$$\begin{aligned} &^2\text{Sqrt}[a^2 + b^2]*d^2) - (2*b^2*f*(e + f*x)*\text{PolyLog}[2, -((b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b^2]))])/(a^2*\text{Sqrt}[a^2 + b^2]*d^2) + (f^2*\text{PolyLog}[2, E^(2*(c + d*x))])/(a*d^3) - (2*b*f^2*\text{PolyLog}[3, -E^(c + d*x)])/(a^2*d^3) + (2*b*f^2*\text{PolyLog}[3, E^(c + d*x)])/(a^2*d^3) - (2*b^2*f^2*\text{PolyLog}[3, -((b*E^(c + d*x))/(a - \text{Sqrt}[a^2 + b^2]))])/(a^2*\text{Sqrt}[a^2 + b^2]*d^3) + (2*b^2*f^2*\text{PolyLog}[3, -((b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b^2]))])/(a^2*\text{Sqrt}[a^2 + b^2]*d^3) \end{aligned}$$

### Rule 5575

$$\text{Int}[(\text{Csch}[c_.] + (d_.)*(x_.))^n * ((e_.) + (f_.)*(x_.))^m] / ((a_.) + (b_.)*\text{Sinh}[c_. + (d_.)*(x_.)]), x\_Symbol] \rightarrow \text{Dist}[1/a, \text{Int}[(e + f*x)^m * \text{Csch}[c + d*x]^n, x], x] - \text{Dist}[b/a, \text{Int}[(e + f*x)^m * \text{Csch}[c + d*x]^{n-1}]/(a + b*\text{Sinh}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \text{IGtQ}[m, 0] \ \&\& \text{IGtQ}[n, 0]$$

### Rule 4184

$$\text{Int}[\text{csc}[(e_.) + (f_.)*(x_.)]^2 * ((c_.) + (d_.)*(x_.))^m], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m * \text{Cot}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{m-1} * \text{Cot}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \text{GtQ}[m, 0]$$

### Rule 3716

$$\text{Int}[(c_. + (d_.)*(x_.))^m * \tan[(e_.) + \text{Pi}*(k_.) + (\text{Complex}[0, fz\_])*(f_.)*(x_.)]], x\_Symbol] \rightarrow -\text{Simp}[(I*(c + d*x)^{m+1})/(d*(m+1)), x] + \text{Dist}[2*I, \text{Int}[(c + d*x)^m * E^{2*(-I*e + f*fz*x)}]/(E^{2*I*k*Pi}*(1 + E^{2*(-I*e + f*fz*x)})/E^{2*I*k*Pi}), x], x] /; \text{FreeQ}\{c, d, e, f, fz\}, x \ \&\& \text{IntegerQ}[4*k] \ \&\& \text{IGtQ}[m, 0]$$

### Rule 2190

$$\text{Int}[(F_)^((g_.)*((e_.) + (f_.)*(x_.)))^n * ((c_.) + (d_.)*(x_.))^m] / ((a_.) + (b_.)*F_)^((g_.)*((e_.) + (f_.)*(x_.)))^n], x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m * \text{Log}[1 + (b*(F^(g*(e + f*x)))^n)/a] / (b*f*g*n * \text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n * \text{Log}[F]), \text{Int}[(c + d*x)^{m-1} * \text{Log}[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x \ \&\& \text{IGtQ}[m, 0]$$

### Rule 2279

$$\text{Int}[\text{Log}[a_. + (b_.)*F_]^{(e_.)*((c_.) + (d_.)*(x_.))}], x\_Symbol] \rightarrow \text{Dist}[1/(d*e*n * \text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x \ \&\& \text{GtQ}[a, 0]$$

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_)^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.))*((f_.) + (g_.)*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 3322

```
Int[((c_.) + (d_.)*(x_)^(m_.))/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_)^(m_.)))/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u]/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u]/(b + q + 2*c*F^u), x], x]
```

$m * F^u) / (b + q + 2 * c * F^u), x], x]] / ; \text{FreeQ}[\{F, a, b, c, f, g\}, x] \ \&\& \ \text{EqQ}[v, 2 * u] \ \&\& \ \text{LinearQ}[u, x] \ \&\& \ \text{NeQ}[b^2 - 4 * a * c, 0] \ \&\& \ \text{IGtQ}[m, 0]$

### Rubi steps

$$\begin{aligned}
 \int \frac{(e + fx)^2 \operatorname{csch}^2(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\int (e + fx)^2 \operatorname{csch}^2(c + dx) dx}{a} - \frac{b \int \frac{(e + fx)^2 \operatorname{csch}(c + dx)}{a + b \sinh(c + dx)} dx}{a} \\
 &= -\frac{(e + fx)^2 \operatorname{coth}(c + dx)}{ad} - \frac{b \int (e + fx)^2 \operatorname{csch}(c + dx) dx}{a^2} + \frac{b^2 \int \frac{(e + fx)^2}{a + b \sinh(c + dx)} dx}{a^2} + \frac{(2f)}{a^2} \\
 &= -\frac{(e + fx)^2}{ad} + \frac{2b(e + fx)^2 \tanh^{-1}(e^{c+dx})}{a^2 d} - \frac{(e + fx)^2 \operatorname{coth}(c + dx)}{ad} + \frac{(2b^2) \int \frac{e^{c+dx}(e^{-c-dx} - b + 2ae^{c+dx})}{-b + 2ae^{c+dx}} dx}{a^2} \\
 &= -\frac{(e + fx)^2}{ad} + \frac{2b(e + fx)^2 \tanh^{-1}(e^{c+dx})}{a^2 d} - \frac{(e + fx)^2 \operatorname{coth}(c + dx)}{ad} + \frac{2f(e + fx) \log(1)}{ad^2} \\
 &= -\frac{(e + fx)^2}{ad} + \frac{2b(e + fx)^2 \tanh^{-1}(e^{c+dx})}{a^2 d} - \frac{(e + fx)^2 \operatorname{coth}(c + dx)}{ad} + \frac{b^2(e + fx)^2 \log(1)}{a^2 \sqrt{a^2 + b^2}} \\
 &= -\frac{(e + fx)^2}{ad} + \frac{2b(e + fx)^2 \tanh^{-1}(e^{c+dx})}{a^2 d} - \frac{(e + fx)^2 \operatorname{coth}(c + dx)}{ad} + \frac{b^2(e + fx)^2 \log(1)}{a^2 \sqrt{a^2 + b^2}} \\
 &= -\frac{(e + fx)^2}{ad} + \frac{2b(e + fx)^2 \tanh^{-1}(e^{c+dx})}{a^2 d} - \frac{(e + fx)^2 \operatorname{coth}(c + dx)}{ad} + \frac{b^2(e + fx)^2 \log(1)}{a^2 \sqrt{a^2 + b^2}} \\
 &= -\frac{(e + fx)^2}{ad} + \frac{2b(e + fx)^2 \tanh^{-1}(e^{c+dx})}{a^2 d} - \frac{(e + fx)^2 \operatorname{coth}(c + dx)}{ad} + \frac{b^2(e + fx)^2 \log(1)}{a^2 \sqrt{a^2 + b^2}}
 \end{aligned}$$

**Mathematica [A]** time = 16.1467, size = 795, normalized size = 1.49

$$\left(-2e^2 \tanh^{-1}\left(\frac{a + be^{c+dx}}{\sqrt{a^2 + b^2}}\right) d^2 + f^2 x^2 \log\left(\frac{e^{c+dx} b}{a - \sqrt{a^2 + b^2}} + 1\right) d^2 + 2efx \log\left(\frac{e^{c+dx} b}{a - \sqrt{a^2 + b^2}} + 1\right) d^2 - f^2 x^2 \log\left(\frac{e^{c+dx} b}{a + \sqrt{a^2 + b^2}} + 1\right) d^2 - 2ef\right)$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2 \* Csch[c + d\*x]^2) / (a + b \* Sinh[c + d\*x]), x]

[Out] -(((2\*a\*d^2\*(e + f\*x)^2)/(-1 + E^(2\*c)) + 2\*d\*f\*(b\*d\*e - a\*f)\*x\*Log[1 - E^(-c - d\*x)] + b\*d^2\*f^2\*x^2\*Log[1 - E^(-c - d\*x)] - 2\*d\*f\*(b\*d\*e + a\*f)\*x\*Lo

```

g[1 + E^(-c - d*x)] - b*d^2*f^2*x^2*Log[1 + E^(-c - d*x)] - d*e*(b*d*e - 2*
a*f)*(d*x - Log[1 - E^(c + d*x)]) + d*e*(b*d*e + 2*a*f)*(d*x - Log[1 + E^(c
+ d*x)]) + 2*f*(b*d*e + a*f)*PolyLog[2, -E^(-c - d*x)] + 2*f*(-(b*d*e) + a
*f)*PolyLog[2, E^(-c - d*x)] + 2*b*f^2*(d*x*PolyLog[2, -E^(-c - d*x)] + Pol
yLog[3, -E^(-c - d*x)]) - 2*b*f^2*(d*x*PolyLog[2, E^(-c - d*x)] + PolyLog[3
, E^(-c - d*x)])))/(a^2*d^3) + (b^2*(-2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))
/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
)] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 2*d^2*e*
f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c +
d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2]))] - 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^
2 + b^2])] + 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(
a^2*Sqrt[a^2 + b^2]*d^3) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(-(e^2*Sinh[(d*x)
/2]) - 2*e*f*x*Sinh[(d*x)/2] - f^2*x^2*Sinh[(d*x)/2]))/(2*a*d) + (Csch[c/2]
*Csch[c/2 + (d*x)/2]*(e^2*Sinh[(d*x)/2] + 2*e*f*x*Sinh[(d*x)/2] + f^2*x^2*S
inh[(d*x)/2]))/(2*a*d)

```

**Maple [F]** time = 0.743, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\operatorname{csch}(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [C]** time = 3.49746, size = 8581, normalized size = 16.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*(a^3 + a*b^2)*d^2*e^2 - 4*(a^3 + a*b^2)*c*d*e*f + 2*(a^3 + a*b^2)*c^2*f^2 + 2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*cosh(d*x + c)^2 + 4*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + 2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*sinh(d*x + c)^2 + 2*(b^3*d*f^2*x + b^3*d*e*f - (b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)^2 - 2*(b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*f^2*x + b^3*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(b^3*d*f^2*x + b^3*d*e*f - (b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)^2 - 2*(b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*f^2*x + b^3*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2 - (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2 - (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2 - (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2 - (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c)^2 - 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a
```

$$\begin{aligned}
& * \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} \\
& - b/b) + 2*(b^3*f^2*\cosh(dx + c)^2 + 2*b^3*f^2*\cosh(dx + c)*\sinh(dx + c) \\
& + b^3*f^2*\sinh(dx + c)^2 - b^3*f^2)*\sqrt{(a^2 + b^2)/b^2}*polylog(3, (a* \\
& \cosh(dx + c) + a*\sinh(dx + c) + (b*\cosh(dx + c) + b*\sinh(dx + c))*\sqrt{(a^2 + b^2)/b^2}))/b) - 2*(b^3*f^2*\cosh(dx + c)^2 + 2*b^3*f^2*\cosh(dx + c) \\
& *\sinh(dx + c) + b^3*f^2*\sinh(dx + c)^2 - b^3*f^2)*\sqrt{(a^2 + b^2)/b^2}*p \\
& olylog(3, (a*\cosh(dx + c) + a*\sinh(dx + c) - (b*\cosh(dx + c) + b*\sinh(dx \\
& x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) - 2*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3) \\
& )*d*e*f - (a^3 + a*b^2)*f^2 - ((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*e*f \\
& - (a^3 + a*b^2)*f^2)*\cosh(dx + c)^2 - 2*((a^2*b + b^3)*d*f^2*x + (a^2*b + \\
& b^3)*d*e*f - (a^3 + a*b^2)*f^2)*\cosh(dx + c)*\sinh(dx + c) - ((a^2*b + b^3) \\
& )*d*f^2*x + (a^2*b + b^3)*d*e*f - (a^3 + a*b^2)*f^2)*\sinh(dx + c)^2)*dilog \\
& (\cosh(dx + c) + \sinh(dx + c)) + 2*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)* \\
& d*e*f + (a^3 + a*b^2)*f^2 - ((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*e*f + \\
& (a^3 + a*b^2)*f^2)*\cosh(dx + c)^2 - 2*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^ \\
& 3)*d*e*f + (a^3 + a*b^2)*f^2)*\cosh(dx + c)*\sinh(dx + c) - ((a^2*b + b^3)* \\
& d*f^2*x + (a^2*b + b^3)*d*e*f + (a^3 + a*b^2)*f^2)*\sinh(dx + c)^2)*dilog(- \\
& \cosh(dx + c) - \sinh(dx + c)) + ((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3) \\
& )*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f - ((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^ \\
& 3)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (a^3 + a*b^ \\
& 2)*d*f^2)*x)*\cosh(dx + c)^2 - 2*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3) \\
& )*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (a^3 + a*b^2) \\
& )*d*f^2)*x)*\cosh(dx + c)*\sinh(dx + c) - ((a^2*b + b^3)*d^2*f^2*x^2 + (a^2* \\
& b + b^3)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (a^3 \\
& + a*b^2)*d*f^2)*x)*\sinh(dx + c)^2 + 2*((a^2*b + b^3)*d^2*e*f + (a^3 + a*b^ \\
& 2)*d*f^2)*x)*\log(\cosh(dx + c) + \sinh(dx + c) + 1) - ((a^2*b + b^3)*d^2*e^ \\
& 2 - 2*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d*e*f + ((a^2*b + b^3)*c^2 + 2*(a^3 + \\
& a*b^2)*c)*f^2 - ((a^2*b + b^3)*d^2*e^2 - 2*(a^3 + a*b^2 + (a^2*b + b^3)*c) \\
& )*d*e*f + ((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2)*\cosh(dx + c)^2 - 2*( \\
& (a^2*b + b^3)*d^2*e^2 - 2*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d*e*f + ((a^2*b + \\
& b^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2)*\cosh(dx + c)*\sinh(dx + c) - ((a^2*b + \\
& b^3)*d^2*e^2 - 2*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d*e*f + ((a^2*b + b^3)*c^ \\
& 2 + 2*(a^3 + a*b^2)*c)*f^2)*\sinh(dx + c)^2)*\log(\cosh(dx + c) + \sinh(dx + \\
& c) - 1) - ((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*c*d*e*f - ((a^2*b + \\
& b^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2 - ((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b \\
& + b^3)*c*d*e*f - ((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2 + 2*((a^2*b + \\
& b^3)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x)*\cosh(dx + c)^2 - 2*((a^2*b + b^3)*d \\
& ^2*f^2*x^2 + 2*(a^2*b + b^3)*c*d*e*f - ((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2) \\
& )*c)*f^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x)*\cosh(dx + c)* \\
& \sinh(dx + c) - ((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*c*d*e*f - ((a^ \\
& 2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^3 + \\
& a*b^2)*d*f^2)*x)*\sinh(dx + c)^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^3 + a*b^2) \\
& )*d*f^2)*x)*\log(-\cosh(dx + c) - \sinh(dx + c) + 1) - 2*((a^2*b + b^3)*f^2* \\
& \cosh(dx + c)^2 + 2*(a^2*b + b^3)*f^2*\cosh(dx + c)*\sinh(dx + c) + (a^2*b \\
& + b^3)*f^2*\sinh(dx + c)^2 - (a^2*b + b^3)*f^2)*polylog(3, \cosh(dx + c) +
\end{aligned}$$



$$\begin{aligned} & \sinh(dx + c) + 2*((a^2*b + b^3)*f^2*\cosh(dx + c)^2 + 2*(a^2*b + b^3)*f^2 \\ & * \cosh(dx + c)*\sinh(dx + c) + (a^2*b + b^3)*f^2*\sinh(dx + c)^2 - (a^2*b + \\ & b^3)*f^2)*\text{polylog}(3, -\cosh(dx + c) - \sinh(dx + c))/((a^4 + a^2*b^2)*d^3 \\ & * \cosh(dx + c)^2 + 2*(a^4 + a^2*b^2)*d^3*\cosh(dx + c)*\sinh(dx + c) + (a^4 \\ & + a^2*b^2)*d^3*\sinh(dx + c)^2 - (a^4 + a^2*b^2)*d^3) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*csch(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*csch(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.245 \quad \int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=306

$$\frac{b^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2 d^2 \sqrt{a^2+b^2}} - \frac{b^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2 d^2 \sqrt{a^2+b^2}} + \frac{bf \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{a^2 d^2} - \frac{bf \operatorname{PolyLog}\left(2, e^{c+dx}\right)}{a^2 d^2} + \frac{b^2(e+f)}{a^2 d^2}$$

[Out] (2\*b\*(e + f\*x)\*ArcTanh[E^(c + d\*x)]/(a^2\*d) - ((e + f\*x)\*Coth[c + d\*x])/(a\*d) + (b^2\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^2\*Sqrt[a^2 + b^2]\*d) - (b^2\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^2\*Sqrt[a^2 + b^2]\*d) + (f\*Log[Sinh[c + d\*x]])/(a\*d^2) + (b\*f\*PolyLog[2, -E^(c + d\*x)]/(a^2\*d^2) - (b\*f\*PolyLog[2, E^(c + d\*x)]/(a^2\*d^2) + (b^2\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^2\*Sqrt[a^2 + b^2]\*d^2) - (b^2\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^2\*Sqrt[a^2 + b^2]\*d^2))

**Rubi [A]** time = 0.565233, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$ , Rules used = {5575, 4184, 3475, 4182, 2279, 2391, 3322, 2264, 2190}

$$\frac{b^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2 d^2 \sqrt{a^2+b^2}} - \frac{b^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2 d^2 \sqrt{a^2+b^2}} + \frac{bf \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{a^2 d^2} - \frac{bf \operatorname{PolyLog}\left(2, e^{c+dx}\right)}{a^2 d^2} + \frac{b^2(e+f)}{a^2 d^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Csch[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]), x]

[Out] (2\*b\*(e + f\*x)\*ArcTanh[E^(c + d\*x)]/(a^2\*d) - ((e + f\*x)\*Coth[c + d\*x])/(a\*d) + (b^2\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^2\*Sqrt[a^2 + b^2]\*d) - (b^2\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^2\*Sqrt[a^2 + b^2]\*d) + (f\*Log[Sinh[c + d\*x]])/(a\*d^2) + (b\*f\*PolyLog[2, -E^(c + d\*x)]/(a^2\*d^2) - (b\*f\*PolyLog[2, E^(c + d\*x)]/(a^2\*d^2) + (b^2\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^2\*Sqrt[a^2 + b^2]\*d^2) - (b^2\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^2\*Sqrt[a^2 + b^2]\*d^2))

**Rule 5575**

Int[(Csch[(c\_.) + (d\_.)\*(x\_)]^(n\_.))\*((e\_.) + (f\_.)\*(x\_))^(m\_.)]/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Csch[c

+ d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Csch[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

#### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_.)]^2\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] :> -Simp[(((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

#### Rule 4182

Int[csc[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_.)]\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)]]/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

#### Rule 2279

Int[Log[(a\_.) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_.))))^(n\_.)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.)^(n\_.))]/(x\_.), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 3322

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)/((a\_.) + (b\_.)\*sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_.)]), x\_Symbol] :> Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(-(I\*b) + 2\*a\*E^(-(I\*e) + f\*fz\*x) + I\*b\*E^(2\*(-(I\*e) + f\*fz\*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

#### Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.)\*(F\_)^(v\_)), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[

```
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\operatorname{csch}^2(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)} dx}{a} \\
&= -\frac{(e+fx)\operatorname{coth}(c+dx)}{ad} - \frac{b \int (e+fx)\operatorname{csch}(c+dx) dx}{a^2} + \frac{b^2 \int \frac{e+fx}{a+b\sinh(c+dx)} dx}{a^2} + \frac{f \int \operatorname{coth}(c+dx) dx}{a} \\
&= \frac{2b(e+fx)\tanh^{-1}(e^{c+dx})}{a^2d} - \frac{(e+fx)\operatorname{coth}(c+dx)}{ad} + \frac{f \log(\sinh(c+dx))}{ad^2} + \frac{(2b^2) \int \frac{e^c}{-b+2a} dx}{a} \\
&= \frac{2b(e+fx)\tanh^{-1}(e^{c+dx})}{a^2d} - \frac{(e+fx)\operatorname{coth}(c+dx)}{ad} + \frac{f \log(\sinh(c+dx))}{ad^2} + \frac{(2b^3) \int \frac{e^c}{2a-2v} dx}{a^2\sqrt{a^2+b^2}} \\
&= \frac{2b(e+fx)\tanh^{-1}(e^{c+dx})}{a^2d} - \frac{(e+fx)\operatorname{coth}(c+dx)}{ad} + \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} - \frac{b^2(e+fx)}{a^2\sqrt{a^2+b^2}} \\
&= \frac{2b(e+fx)\tanh^{-1}(e^{c+dx})}{a^2d} - \frac{(e+fx)\operatorname{coth}(c+dx)}{ad} + \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} - \frac{b^2(e+fx)}{a^2\sqrt{a^2+b^2}} \\
&= \frac{2b(e+fx)\tanh^{-1}(e^{c+dx})}{a^2d} - \frac{(e+fx)\operatorname{coth}(c+dx)}{ad} + \frac{b^2(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}d} - \frac{b^2(e+fx)}{a^2\sqrt{a^2+b^2}}
\end{aligned}$$

**Mathematica [A]** time = 5.601, size = 405, normalized size = 1.32

$$\frac{2b^2 \left( f \operatorname{PolyLog} \left( 2, \frac{b(\sinh(c+dx)+\cosh(c+dx))}{\sqrt{a^2+b^2}-a} \right) - f \operatorname{PolyLog} \left( 2, -\frac{b(\sinh(c+dx)+\cosh(c+dx))}{\sqrt{a^2+b^2}+a} \right) - 2de \tanh^{-1} \left( \frac{a+b\sinh(c+dx)+b\cosh(c+dx)}{\sqrt{a^2+b^2}} \right) + f(c+dx) \log \left( \frac{b(\sinh(c+dx)+\cosh(c+dx))}{a-\sqrt{a^2+b^2}} \right) \right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(a*d*(e + f*x)*Coth[(c + d*x)/2]) + 2*a*f*Log[Sinh[c + d*x]] - 2*b*d*e*Log[Tanh[(c + d*x)/2]] + 2*b*c*f*Log[Tanh[(c + d*x)/2]] + 2*b*f*(-((c + d*x)*(Log[1 - E^(-c - d*x)] - Log[1 + E^(-c - d*x)])) - PolyLog[2, -E^(-c - d*x)] + PolyLog[2, E^(-c - d*x)])) + (2*b^2*(-2*d*e*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + 2*c*f*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])]) - f*(c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])]) + f*PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -(b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])])/Sqrt[a^2 + b^2] - a*d*(e + f*x)*Tanh[(c + d*x)/2])/(2*a^2*d^2)
```

**Maple [B]** time = 0.105, size = 626, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] -2/d*(f*x+e)/a/(exp(2*d*x+2*c)-1)-2/d^2/a*f*ln(exp(d*x+c))+1/d^2/a*f*ln(exp(d*x+c)-1)+1/d^2/a*f*ln(exp(d*x+c)+1)-1/a^2/d*b*e*ln(exp(d*x+c)-1)-2/a^2/d*b^2*e/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/a^2/d*b*e*ln(exp(d*x+c)+1)+1/a^2/d^2*b*f*c*ln(exp(d*x+c)-1)+2/a^2/d^2*b^2*f*c/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/a^2/d^2*b*f*dilog(exp(d*x+c))+1/a^2/d*b^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/a^2/d^2*b^2*f/(a^2+b^2)^(1/2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-1/a^2/d*b^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-1/a^2/d^2*b^2*f/(a^2+b^2)^(1/2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/a^2/d^2*b^2*f/(a^2+b^2)^(1/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/a^2/d^2*b^2*f/(a^2+b^2)^(1/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/a^2/d*b*f*ln(exp(d*x+c)+1)*x+1/a^2/d^2*b*f*dilog(exp(d*x+c)+1)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [B]** time = 2.89152, size = 4370, normalized size = 14.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*(a^3 + a*b^2)*d*e - 2*(a^3 + a*b^2)*c*f + 2*((a^3 + a*b^2)*d*f*x + (a^3
+ a*b^2)*c*f)*cosh(d*x + c)^2 + 4*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f
)*cosh(d*x + c)*sinh(d*x + c) + 2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)
*sinh(d*x + c)^2 - (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cosh(d*x + c)*sinh(d*x
+ c) + b^3*f*sinh(d*x + c)^2 - b^3*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d
*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b + 1) + (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cosh(d*x + c)*sin
h(d*x + c) + b^3*f*sinh(d*x + c)^2 - b^3*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*
cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2) - b)/b + 1) - (b^3*d*e - b^3*c*f - (b^3*d*e - b^3*c*f)*cos
h(d*x + c)^2 - 2*(b^3*d*e - b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*e
- b^3*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) +
2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d*e - b^3*c*f -
(b^3*d*e - b^3*c*f)*cosh(d*x + c)^2 - 2*(b^3*d*e - b^3*c*f)*cosh(d*x + c)*
sinh(d*x + c) - (b^3*d*e - b^3*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*
log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a
) + (b^3*d*f*x + b^3*c*f - (b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^2 - 2*(b^3*d
*f*x + b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*f*x + b^3*c*f)*sinh(d*
x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^3*d*f
*x + b^3*c*f - (b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^2 - 2*(b^3*d*f*x + b^3*c
*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*f*x + b^3*c*f)*sinh(d*x + c)^2)*sq
rt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + ((a^2*b + b^3)*f*cos
h(d*x + c)^2 + 2*(a^2*b + b^3)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2*b + b^3
)*f*sinh(d*x + c)^2 - (a^2*b + b^3)*f)*dilog(cosh(d*x + c) + sinh(d*x + c))
- ((a^2*b + b^3)*f*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f*cosh(d*x + c)*sinh(
d*x + c) + (a^2*b + b^3)*f*sinh(d*x + c)^2 - (a^2*b + b^3)*f)*dilog(-cosh(d
```

$$\begin{aligned}
& *x + c) - \sinh(dx + c)) + ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e - ((a^2 \\
& *b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^3 + a*b^2)*f)*\cosh(dx + c)^2 - 2* \\
& ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^3 + a*b^2)*f)*\cosh(dx + c)*\sinh(dx + c) - ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^3 + a*b^2)*f)* \\
& \sinh(dx + c)^2 + (a^3 + a*b^2)*f)*\log(\cosh(dx + c) + \sinh(dx + c) + 1) - \\
& ((a^2*b + b^3)*d*e - ((a^2*b + b^3)*d*e - (a^3 + a*b^2 + (a^2*b + b^3)*c)* \\
& f)*\cosh(dx + c)^2 - 2*((a^2*b + b^3)*d*e - (a^3 + a*b^2 + (a^2*b + b^3)*c) \\
& *f)*\cosh(dx + c)*\sinh(dx + c) - ((a^2*b + b^3)*d*e - (a^3 + a*b^2 + (a^2* \\
& b + b^3)*c)*f)*\sinh(dx + c)^2 - (a^3 + a*b^2 + (a^2*b + b^3)*c)*f)*\log(\cos \\
& h(dx + c) + \sinh(dx + c) - 1) - ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*c*f \\
& - ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*c*f)*\cosh(dx + c)^2 - 2*((a^2*b + b \\
& ^3)*d*f*x + (a^2*b + b^3)*c*f)*\cosh(dx + c)*\sinh(dx + c) - ((a^2*b + b^3) \\
& *d*f*x + (a^2*b + b^3)*c*f)*\sinh(dx + c)^2)*\log(-\cosh(dx + c) - \sinh(dx \\
& + c) + 1))/((a^4 + a^2*b^2)*d^2*\cosh(dx + c)^2 + 2*(a^4 + a^2*b^2)*d^2*\cos \\
& h(dx + c)*\sinh(dx + c) + (a^4 + a^2*b^2)*d^2*\sinh(dx + c)^2 - (a^4 + a^2 \\
& *b^2)*d^2)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(dx+c)\*\*2/(a+b\*sinh(dx+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(dx+c)^2/(a+b\*sinh(dx+c)),x, algorithm="giac")

[Out] Timed out

$$3.246 \quad \int \frac{\operatorname{csch}^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=80

$$-\frac{2b^2 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2 d \sqrt{a^2+b^2}} + \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2 d} - \frac{\coth(c+dx)}{ad}$$

[Out] (b\*ArcTanh[Cosh[c + d\*x]])/(a^2\*d) - (2\*b^2\*ArcTanh[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[a^2 + b^2]])/(a^2\*Sqrt[a^2 + b^2]\*d) - Coth[c + d\*x]/(a\*d)

**Rubi [A]** time = 0.147012, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2802, 12, 2747, 3770, 2660, 618, 204}

$$-\frac{2b^2 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2 d \sqrt{a^2+b^2}} + \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2 d} - \frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d\*x]^2/(a + b\*Sinh[c + d\*x]),x]

[Out] (b\*ArcTanh[Cosh[c + d\*x]])/(a^2\*d) - (2\*b^2\*ArcTanh[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[a^2 + b^2]])/(a^2\*Sqrt[a^2 + b^2]\*d) - Coth[c + d\*x]/(a\*d)

### Rule 2802

Int[((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(m\_)\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_), x\_Symbol] :> -Simp[(b^2\*Cos[e + f\*x]\*(a + b\*Sin[e + f\*x])^(m + 1)\*(c + d\*Sin[e + f\*x])^(n + 1))/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 - b^2)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 - b^2)), Int[(a + b\*Sin[e + f\*x])^(m + 1)\*(c + d\*Sin[e + f\*x])^n\*Simp[a\*(b\*c - a\*d)\*(m + 1) + b^2\*d\*(m + n + 2) - (b^2\*c + b\*(b\*c - a\*d)\*(m + 1))\*Sin[e + f\*x] - b^2\*d\*(m + n + 3)\*Sin[e + f\*x]^2, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegerQ[2\*m, 2\*n] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2\*m] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))



Rule 12

$\text{Int}[(a_*)(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{Match} \\ \text{Q}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

Rule 2747

$\text{Int}[1/(((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*\sin[(e_.) + ( \\ f_.)*(x_.)])), x\_Symbol] \rightarrow \text{Dist}[b/(b*c - a*d), \text{Int}[1/(a + b*\sin[e + f*x]), \\ x], x] - \text{Dist}[d/(b*c - a*d), \text{Int}[1/(c + d*\sin[e + f*x]), x], x] /; \text{FreeQ}[\{a \\ , b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 3770

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] \\ /; \text{FreeQ}[\{c, d\}, x]$

Rule 2660

$\text{Int}[((a_.) + (b_.)*\sin[(c_.) + (d_.)*(x_.)])^{(-1)}, x\_Symbol] \rightarrow \text{With}[\{e = \text{Fre} \\ \text{eFactors}[\text{Tan}[(c + d*x)/2], x]\}, \text{Dist}[(2*e)/d, \text{Subst}[\text{Int}[1/(a + 2*b*e*x + a* \\ e^2*x^2), x], x, \text{Tan}[(c + d*x)/2]/e], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[ \\ a^2 - b^2, 0]$

Rule 618

$\text{Int}[((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{I} \\ \text{nt}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}[\{a, b, c\}, \\ x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 204

$\text{Int}[((a_.) + (b_.)*(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt} \\ [-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[ \\ a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)} dx &= -\frac{\operatorname{coth}(c+dx)}{ad} - \frac{\int \frac{b\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)} dx}{a} \\
&= -\frac{\operatorname{coth}(c+dx)}{ad} - \frac{b \int \frac{\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)} dx}{a} \\
&= -\frac{\operatorname{coth}(c+dx)}{ad} - \frac{b \int \operatorname{csch}(c+dx) dx}{a^2} + \frac{b^2 \int \frac{1}{a+b\sinh(c+dx)} dx}{a^2} \\
&= \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2 d} - \frac{\operatorname{coth}(c+dx)}{ad} - \frac{(2ib^2) \operatorname{Subst}\left(\int \frac{1}{a-2ibx+ax^2} dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{a^2 d} \\
&= \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2 d} - \frac{\operatorname{coth}(c+dx)}{ad} + \frac{(4ib^2) \operatorname{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2} dx, x, -2ib+2a \tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^2 d} \\
&= \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2 d} - \frac{2b^2 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2 \sqrt{a^2+b^2} d} - \frac{\operatorname{coth}(c+dx)}{ad}
\end{aligned}$$

**Mathematica [A]** time = 0.734301, size = 100, normalized size = 1.25

$$\frac{2b \left( \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - \frac{2b \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}} \right) + a \tanh\left(\frac{1}{2}(c+dx)\right) + a \operatorname{coth}\left(\frac{1}{2}(c+dx)\right)}{2a^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d\*x]^2/(a + b\*Sinh[c + d\*x]),x]

[Out] -(a\*Coth[(c + d\*x)/2] + 2\*b\*((-2\*b\*ArcTan[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] + Log[Tanh[(c + d\*x)/2]]) + a\*Tanh[(c + d\*x)/2])/((2\*a^2\*d)

**Maple [A]** time = 0.003, size = 105, normalized size = 1.3

$$-\frac{1}{2da} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 2 \frac{b^2}{da^2 \sqrt{a^2+b^2}} \operatorname{Artanh}\left(\frac{1}{2} \frac{2a \tanh(1/2 dx + c/2) - 2b}{\sqrt{a^2+b^2}}\right) - \frac{1}{2da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-1} - \frac{b}{da^2} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)`

[Out] 
$$-1/2/d/a*\tanh(1/2*d*x+1/2*c)+2/d/a^2*b^2/(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*a*\tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^{(1/2)})-1/2/d/a/\tanh(1/2*d*x+1/2*c)-1/d/a^2*b*\ln(\tanh(1/2*d*x+1/2*c))$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.86922, size = 1200, normalized size = 15.

$$2a^3 + 2ab^2 - (b^2 \cosh(dx+c)^2 + 2b^2 \cosh(dx+c) \sinh(dx+c) + b^2 \sinh(dx+c)^2 - b^2) \sqrt{a^2 + b^2} \log\left(\frac{b^2 \cosh(dx+c)^2}{\dots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & -(2*a^3 + 2*a*b^2 - (b^2*\cosh(d*x + c)^2 + 2*b^2*\cosh(d*x + c)*\sinh(d*x + c) \\ & + b^2*\sinh(d*x + c)^2 - b^2)*\sqrt{a^2 + b^2}*\log((b^2*\cosh(d*x + c)^2 + b \\ & ^2*\sinh(d*x + c)^2 + 2*a*b*\cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*\cosh(d*x + \\ & c) + a*b)*\sinh(d*x + c) - 2*\sqrt{a^2 + b^2}*(b*\cosh(d*x + c) + b*\sinh(d*x + \\ & c) + a))/(b*\cosh(d*x + c)^2 + b*\sinh(d*x + c)^2 + 2*a*\cosh(d*x + c) + 2*(b \\ & *\cosh(d*x + c) + a)*\sinh(d*x + c) - b)) + (a^2*b + b^3 - (a^2*b + b^3)*\cosh \\ & (d*x + c)^2 - 2*(a^2*b + b^3)*\cosh(d*x + c)*\sinh(d*x + c) - (a^2*b + b^3)*s \\ & \sinh(d*x + c)^2)*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) - (a^2*b + b^3 - (a^ \\ & 2*b + b^3)*\cosh(d*x + c)^2 - 2*(a^2*b + b^3)*\cosh(d*x + c)*\sinh(d*x + c) - \\ & (a^2*b + b^3)*\sinh(d*x + c)^2)*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1))/((a^ \\ & 4 + a^2*b^2)*d*\cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*d*\cosh(d*x + c)*\sinh(d*x \end{aligned}$$

$$+ c) + (a^4 + a^2*b^2)*d*\sinh(d*x + c)^2 - (a^4 + a^2*b^2)*d$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(csch(c + d\*x)\*\*2/(a + b\*sinh(c + d\*x)), x)

**Giac [A]** time = 1.62952, size = 177, normalized size = 2.21

$$\frac{b^2 \log\left(\frac{|2be^{(dx+c)} + 2a - 2\sqrt{a^2+b^2}|}{|2be^{(dx+c)} + 2a + 2\sqrt{a^2+b^2}|}\right)}{\sqrt{a^2 + b^2}a^2d} + \frac{b \log(e^{(dx+c)} + 1)}{a^2d} - \frac{b \log(|e^{(dx+c)} - 1|)}{a^2d} - \frac{2}{ad(e^{(2dx+2c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] b^2\*log(abs(2\*b\*e^(d\*x + c) + 2\*a - 2\*sqrt(a^2 + b^2))/abs(2\*b\*e^(d\*x + c) + 2\*a + 2\*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)\*a^2\*d) + b\*log(e^(d\*x + c) + 1)/(a^2\*d) - b\*log(abs(e^(d\*x + c) - 1))/(a^2\*d) - 2/(a\*d\*(e^(2\*d\*x + 2\*c) - 1))

$$3.247 \quad \int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Csch[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi [A] time = 0.0771399, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Csch[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Csch[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Mathematica [A] time = 117.797, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Csch[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Integrate[Csch[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Maple [A]** time = 0.879, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx + c))^2}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(csch(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$4b^2 \int -\frac{e^{(dx+c)}}{2(a^2bfx + a^2be - (a^2bfxe^{2c} + a^2bee^{2c})e^{2dx}) - 2(a^3fxe^c + a^3ee^c)e^{(dx)}} dx + \frac{2}{adfx + ade - (adfxe^{2c} + adee^{2c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 4\*b^2\*integrate(-1/2\*e^(d\*x + c)/(a^2\*b\*f\*x + a^2\*b\*e - (a^2\*b\*f\*x\*e^(2\*c) + a^2\*b\*e\*e^(2\*c))\*e^(2\*d\*x) - 2\*(a^3\*f\*x\*e^c + a^3\*e\*e^c)\*e^(d\*x)), x) + 2/(a\*d\*f\*x + a\*d\*e - (a\*d\*f\*x\*e^(2\*c) + a\*d\*e\*e^(2\*c))\*e^(2\*d\*x)) - 4\*integrate(-1/4\*(b\*d\*f\*x + b\*d\*e + a\*f)/(a^2\*d\*f^2\*x^2 + 2\*a^2\*d\*e\*f\*x + a^2\*d\*e^2 - (a^2\*d\*f^2\*x^2\*e^c + 2\*a^2\*d\*e\*f\*x\*e^c + a^2\*d\*e^2\*e^c)\*e^(d\*x)), x) - 4\*integrate(1/4\*(b\*d\*f\*x + b\*d\*e - a\*f)/(a^2\*d\*f^2\*x^2 + 2\*a^2\*d\*e\*f\*x + a^2\*d\*e^2 + (a^2\*d\*f^2\*x^2\*e^c + 2\*a^2\*d\*e\*f\*x\*e^c + a^2\*d\*e^2\*e^c)\*e^(d\*x)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(dx + c)^2}{afx + ae + (bfx + be) \sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] `integral(csch(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)`

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(c + dx)}{(a + b \sinh(c + dx))(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)`

[Out] `Integral(csch(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)`

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")`

[Out] Timed out

$$3.248 \quad \int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=1053

result too large to display

```
[Out] (b*(e + f*x)^3)/(a^2*d) - (6*f^2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^3) +
((e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2*(e + f*x)^3*ArcTanh[E^(c
+ d*x)])/(a^3*d) + (b*(e + f*x)^3*Coth[c + d*x])/(a^2*d) - (3*f*(e + f*x)^2
*Csch[c + d*x])/(2*a*d^2) - ((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x])/(2*a*
d) - (b^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*
Sqrt[a^2 + b^2]*d) + (b^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2])])/(a^3*Sqrt[a^2 + b^2]*d) - (3*b*f*(e + f*x)^2*Log[1 - E^(2*(c + d
*x))])/(a^2*d^2) - (3*f^3*PolyLog[2, -E^(c + d*x)])/(a*d^4) + (3*f*(e + f*x
)^2*PolyLog[2, -E^(c + d*x)])/(2*a*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -
E^(c + d*x)])/(a^3*d^2) + (3*f^3*PolyLog[2, E^(c + d*x)])/(a*d^4) - (3*f*(e
+ f*x)^2*PolyLog[2, E^(c + d*x)])/(2*a*d^2) + (3*b^2*f*(e + f*x)^2*PolyLog
[2, E^(c + d*x)])/(a^3*d^2) - (3*b^3*f*(e + f*x)^2*PolyLog[2, -(b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2])])/(a^3*Sqrt[a^2 + b^2]*d^2) + (3*b^3*f*(e + f*x
)^2*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*Sqrt[a^2 + b
^2]*d^2) - (3*b*f^2*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a^2*d^3) - (3*f
^2*(e + f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (6*b^2*f^2*(e + f*x)*PolyL
og[3, -E^(c + d*x)])/(a^3*d^3) + (3*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/
(a*d^3) - (6*b^2*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a^3*d^3) + (6*b^3*
f^2*(e + f*x)*PolyLog[3, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*Sq
rt[a^2 + b^2]*d^3) - (6*b^3*f^2*(e + f*x)*PolyLog[3, -(b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2])])/(a^3*Sqrt[a^2 + b^2]*d^3) + (3*b*f^3*PolyLog[3, E^(2*(
c + d*x))])/(2*a^2*d^4) + (3*f^3*PolyLog[4, -E^(c + d*x)])/(a*d^4) - (6*b^2
*f^3*PolyLog[4, -E^(c + d*x)])/(a^3*d^4) - (3*f^3*PolyLog[4, E^(c + d*x)])/
(a*d^4) + (6*b^2*f^3*PolyLog[4, E^(c + d*x)])/(a^3*d^4) - (6*b^3*f^3*PolyLo
g[4, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*Sqrt[a^2 + b^2]*d^4) +
(6*b^3*f^3*PolyLog[4, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*Sqrt
[a^2 + b^2]*d^4)
```

**Rubi [A]** time = 1.75162, antiderivative size = 1053, normalized size of antiderivative = 1., number of steps used = 45, number of rules used = 14, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5575, 4186, 4182, 2279, 2391, 2531, 6609, 2282, 6589, 4184, 3716, 2190, 3322, 2264}

$$-\frac{(e+fx)^3 \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^3}{a^3\sqrt{a^2+b^2}d} + \frac{(e+fx)^3 \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^3}{a^3\sqrt{a^2+b^2}d} - \frac{3f(e+fx)^2 \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)b^3}{a^3\sqrt{a^2+b^2}d^2} + \frac{3f(e+fx)^2 \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)b^3}{a^3\sqrt{a^2+b^2}d^2}$$



Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Csch[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] (b\*(e + f\*x)^3)/(a^2\*d) - (6\*f^2\*(e + f\*x)\*ArcTanh[E^(c + d\*x)])/(a\*d^3) + ((e + f\*x)^3\*ArcTanh[E^(c + d\*x)])/(a\*d) - (2\*b^2\*(e + f\*x)^3\*ArcTanh[E^(c + d\*x)])/(a^3\*d) + (b\*(e + f\*x)^3\*Coth[c + d\*x])/(a^2\*d) - (3\*f\*(e + f\*x)^2\*Csch[c + d\*x])/(2\*a\*d^2) - ((e + f\*x)^3\*Coth[c + d\*x]\*Csch[c + d\*x])/(2\*a\*d) - (b^3\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/(a^3\*Sqrt[a^2 + b^2]\*d) + (b^3\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/(a^3\*Sqrt[a^2 + b^2]\*d) - (3\*b\*f\*(e + f\*x)^2\*Log[1 - E^(2\*(c + d\*x))])/(a^2\*d^2) - (3\*f^3\*PolyLog[2, -E^(c + d\*x)])/(a\*d^4) + (3\*f\*(e + f\*x)^2\*PolyLog[2, -E^(c + d\*x)])/(2\*a\*d^2) - (3\*b^2\*f\*(e + f\*x)^2\*PolyLog[2, -E^(c + d\*x)])/(a^3\*d^2) + (3\*f^3\*PolyLog[2, E^(c + d\*x)])/(a\*d^4) - (3\*f\*(e + f\*x)^2\*PolyLog[2, E^(c + d\*x)])/(2\*a\*d^2) + (3\*b^2\*f\*(e + f\*x)^2\*PolyLog[2, E^(c + d\*x)])/(a^3\*d^2) - (3\*b^3\*f\*(e + f\*x)^2\*PolyLog[2, -(b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/(a^3\*Sqrt[a^2 + b^2]\*d^2) + (3\*b^3\*f\*(e + f\*x)^2\*PolyLog[2, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/(a^3\*Sqrt[a^2 + b^2]\*d^2) - (3\*b\*f^2\*(e + f\*x)\*PolyLog[2, E^(2\*(c + d\*x))])/(a^2\*d^3) - (3\*f^2\*(e + f\*x)\*PolyLog[3, -E^(c + d\*x)])/(a\*d^3) + (6\*b^2\*f^2\*(e + f\*x)\*PolyLog[3, -E^(c + d\*x)])/(a^3\*d^3) + (3\*f^2\*(e + f\*x)\*PolyLog[3, E^(c + d\*x)])/(a\*d^3) - (6\*b^2\*f^2\*(e + f\*x)\*PolyLog[3, E^(c + d\*x)])/(a^3\*d^3) + (6\*b^3\*f^2\*(e + f\*x)\*PolyLog[3, -(b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/(a^3\*Sqrt[a^2 + b^2]\*d^3) - (6\*b^3\*f^2\*(e + f\*x)\*PolyLog[3, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/(a^3\*Sqrt[a^2 + b^2]\*d^3) + (3\*b\*f^3\*PolyLog[3, E^(2\*(c + d\*x))])/(2\*a^2\*d^4) + (3\*f^3\*PolyLog[4, -E^(c + d\*x)])/(a\*d^4) - (6\*b^2\*f^3\*PolyLog[4, -E^(c + d\*x)])/(a^3\*d^4) - (3\*f^3\*PolyLog[4, E^(c + d\*x)])/(a\*d^4) + (6\*b^2\*f^3\*PolyLog[4, E^(c + d\*x)])/(a^3\*d^4) - (6\*b^3\*f^3\*PolyLog[4, -(b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/(a^3\*Sqrt[a^2 + b^2]\*d^4) + (6\*b^3\*f^3\*PolyLog[4, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/(a^3\*Sqrt[a^2 + b^2]\*d^4)

### Rule 5575

Int[(Csch[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Csch[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Csch[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 4186

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(b^2\*(c + d\*x)^m\*Cot[e + f\*x]\*(b\*Csc[e + f\*x])^(n - 2))/(f\*(n - 1)), x] + (Dist[(b^2\*d^2\*m\*(m - 1))/(f^2\*(n - 1)\*(n - 2)), Int[(c + d\*x)^(

```
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_.)*((a_.)*(v_)^(n_.))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> -Simp[(((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 3716

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol]
:> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[(((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol]
:> Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 3322

```
Int[(((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol]
:> Dist[2, Int[(((c + d*x)^m*E^(-I*e + f*fz*x))/(-I*b + 2*a*E^(-I*e + f*fz*x) + I*b*E^(2*(-I*e + f*fz*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 2264

```
Int[(((F_)^(u_.)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_.) + (c_.)*(F_)^(v_.)), x_Symbol]
:> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(((f + g*x)^m*F^u)/(b + q + 2*c*F^u), x], x)] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \operatorname{csch}^3(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
 &= -\frac{3f(e+fx)^2 \operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+fx)^3 \operatorname{coth}(c+dx) \operatorname{csch}(c+dx)}{2ad} - \frac{\int (e+fx)^3 \operatorname{csch}(c+dx) dx}{2a} \\
 &= -\frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} + \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} + \frac{b(e+fx)^3 \operatorname{coth}(c+dx)}{a^2d} - \frac{3f}{2a} \\
 &= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} + \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^3 \operatorname{tanh}^{-1}(e^{c+dx})}{a^3d} \\
 &= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} + \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^3 \operatorname{tanh}^{-1}(e^{c+dx})}{a^3d} \\
 &= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} + \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^3 \operatorname{tanh}^{-1}(e^{c+dx})}{a^3d} \\
 &= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} + \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^3 \operatorname{tanh}^{-1}(e^{c+dx})}{a^3d} \\
 &= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} + \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^3 \operatorname{tanh}^{-1}(e^{c+dx})}{a^3d} \\
 &= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} + \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^3 \operatorname{tanh}^{-1}(e^{c+dx})}{a^3d} \\
 &= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} + \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^3 \operatorname{tanh}^{-1}(e^{c+dx})}{a^3d} \\
 &= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} + \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^3 \operatorname{tanh}^{-1}(e^{c+dx})}{a^3d}
 \end{aligned}$$

**Mathematica [B]** time = 45.3315, size = 2800, normalized size = 2.66

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^3\*Csch[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

```

[Out] -(-12*a*b*d^3*e^2*E^(2*c)*f*x - 12*a*b*d^3*e*E^(2*c)*f^2*x^2 - 4*a*b*d^3*E^(2*c)*f^3*x^3 + 2*a^2*d^3*e^3*ArcTanh[E^(c + d*x)] - 4*b^2*d^3*e^3*ArcTanh[E^(c + d*x)] - 2*a^2*d^3*e^3*E^(2*c)*ArcTanh[E^(c + d*x)] + 4*b^2*d^3*e^3*E^(2*c)*ArcTanh[E^(c + d*x)] - 12*a^2*d*e*f^2*ArcTanh[E^(c + d*x)] + 12*a^2*d*e*E^(2*c)*f^2*ArcTanh[E^(c + d*x)] - 3*a^2*d^3*e^2*f*x*Log[1 - E^(c + d*x)] + 6*b^2*d^3*e^2*f*x*Log[1 - E^(c + d*x)] + 3*a^2*d^3*e^2*E^(2*c)*f*x*Log[1 - E^(c + d*x)] - 6*b^2*d^3*e^2*E^(2*c)*f*x*Log[1 - E^(c + d*x)] + 6*a^2*d*f^3*x*Log[1 - E^(c + d*x)] - 6*a^2*d*E^(2*c)*f^3*x*Log[1 - E^(c + d*x)] - 3*a^2*d^3*e*f^2*x^2*Log[1 - E^(c + d*x)] + 6*b^2*d^3*e*f^2*x^2*Log[1 - E^(c + d*x)] + 3*a^2*d^3*e*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] - a^2*d^3*f^3*x^3*Log[1 - E^(c + d*x)] + 2*b^2*d^3*f^3*x^3*Log[1 - E^(c + d*x)] + a^2*d^3*E^(2*c)*f^3*x^3*Log[1 - E^(c + d*x)] - 2*b^2*d^3*E^(2*c)*f^3*x^3*Log[1 - E^(c + d*x)] + 3*a^2*d^3*e^2*f*x*Log[1 + E^(c + d*x)] - 6*b^2*d^3*e^2*f*x*Log[1 + E^(c + d*x)] - 3*a^2*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(c + d*x)] + 6*b^2*d^3*e^2*E^(2*c)*f*x*Log[1 + E^(c + d*x)] - 6*a^2*d*f^3*x*Log[1 + E^(c + d*x)] + 6*a^2*d*E^(2*c)*f^3*x*Log[1 + E^(c + d*x)] + 3*a^2*d^3*e*f^2*x^2*Log[1 + E^(c + d*x)] - 6*b^2*d^3*e*f^2*x^2*Log[1 + E^(c + d*x)] - 3*a^2*d^3*e*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] + 6*b^2*d^3*e*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] + a^2*d^3*f^3*x^3*Log[1 + E^(c + d*x)] - 2*b^2*d^3*f^3*x^3*Log[1 + E^(c + d*x)] - a^2*d^3*E^(2*c)*f^3*x^3*Log[1 + E^(c + d*x)] + 2*b^2*d^3*E^(2*c)*f^3*x^3*Log[1 + E^(c + d*x)] - 6*a*b*d^2*e^2*f*Log[1 - E^(2*(c + d*x))] + 6*a*b*d^2*e^2*E^(2*c)*f*Log[1 - E^(2*(c + d*x))] - 12*a*b*d^2*e*f^2*x*Log[1 - E^(2*(c + d*x))] + 12*a*b*d^2*e*E^(2*c)*f^2*x*Log[1 - E^(2*(c + d*x))] - 6*a*b*d^2*f^3*x^2*Log[1 - E^(2*(c + d*x))] + 6*a*b*d^2*E^(2*c)*f^3*x^2*Log[1 - E^(2*(c + d*x))] - 3*(-1 + E^(2*c))*f*(-2*b^2*d^2*(e + f*x)^2 + a^2*(-2*f^2 + d^2*(e + f*x)^2))*PolyLog[2, -E^(c + d*x)] + 3*(-1 + E^(2*c))*f*(-2*b^2*d^2*(e + f*x)^2 + a^2*(-2*f^2 + d^2*(e + f*x)^2))*PolyLog[2, E^(c + d*x)] - 6*a*b*d*e*f^2*PolyLog[2, E^(2*(c + d*x))] + 6*a*b*d*e*E^(2*c)*f^2*PolyLog[2, E^(2*(c + d*x))] - 6*a*b*d*f^3*x*PolyLog[2, E^(2*(c + d*x))] + 6*a*b*d*E^(2*c)*f^3*x*PolyLog[2, E^(2*(c + d*x))] - 6*a^2*d*e*f^2*PolyLog[3, -E^(c + d*x)] + 12*b^2*d*e*f^2*PolyLog[3, -E^(c + d*x)] + 6*a^2*d*e*E^(2*c)*f^2*PolyLog[3, -E^(c + d*x)] - 12*b^2*d*e*E^(2*c)*f^2*PolyLog[3, -E^(c + d*x)] - 6*a^2*d*f^3*x*PolyLog[3, -E^(c + d*x)] + 12*b^2*d*f^3*x*PolyLog[3, -E^(c + d*x)] + 6*a^2*d*E^(2*c)*f^3*x*PolyLog[3, -E^(c + d*x)] - 12*b^2*d*E^(2*c)*f^3*x*PolyLog[3, -E^(c + d*x)] + 6*a^2*d*e*f^2*PolyLog[3, E^(c + d*x)] - 12*b^2*d*e*f^2*PolyLog[3, E^(c + d*x)] + 12*b^2*d*e*E^(2*c)*f^2*PolyLog[3, E^(c + d*x)] + 6*a^2*d*f^3*x*PolyLog[3, E^(c + d*x)] - 12*b^2*d*f^3*x*PolyLog[3, E^(c + d*x)] - 6*a^2*d*E^(2*c)*f^3*x*PolyLog[3, E^(c + d*x)] + 12*b^2*d*E^(2*c)*f^3*x*PolyLog[3, E^(c + d*x)] + 3*a*b*f^3*PolyLog[3, E^(2*(c + d*x))] - 3*a*b*E^(2*c)*f^3*PolyLog[3, E^(2*(c + d*x))] + 6*a^2*f^3*PolyLog[4, -E^(c + d*x)] - 12*b^2*f^3*PolyLog[4, -E^(c + d*x)] - 6*a^2*E^(2*c)*f^3*PolyLog[4, -E^(c + d*x)] + 12*b^2*E^(2*c)*f^3*PolyLog[4, -E^(c + d*x)] - 6*a^2*f^3*PolyLog[4, E^(c + d*x)] + 12*b^2*f^3*PolyLog[4, E^(c + d*x)] + 6*a^2*E^(2*c)*f^3*PolyLog[4, E^(c + d*x)] - 12

```

```

*b^2*E^(2*c)*f^3*PolyLog[4, E^(c + d*x)]/(2*a^3*d^4*(-1 + E^(2*c))) + (b^3
*(2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 3*d^3*e^2*f*x*Lo
g[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 3*d^3*e*f^2*x^2*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2])] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] +
d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e +
f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*
x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*d*e*f^2*PolyL
og[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*f^3*x*PolyLog[3, (b*E^(
c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))] - 6*d*f^3*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[
a^2 + b^2]))] - 6*f^3*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] +
6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*Sqrt[a^2
+ b^2]*d^4) + (Csch[c]*Csch[c + d*x]^2*(2*b*d*e^3*Cosh[c] + 6*b*d*e^2*f*x*C
osh[c] + 6*b*d*e*f^2*x^2*Cosh[c] + 2*b*d*f^3*x^3*Cosh[c] + 3*a*e^2*f*Cosh[d
*x] + 6*a*e*f^2*x*Cosh[d*x] + 3*a*f^3*x^2*Cosh[d*x] - 3*a*e^2*f*Cosh[2*c +
d*x] - 6*a*e*f^2*x*Cosh[2*c + d*x] - 3*a*f^3*x^2*Cosh[2*c + d*x] - 2*b*d*e^
3*Cosh[c + 2*d*x] - 6*b*d*e^2*f*x*Cosh[c + 2*d*x] - 6*b*d*e*f^2*x^2*Cosh[c
+ 2*d*x] - 2*b*d*f^3*x^3*Cosh[c + 2*d*x] + a*d*e^3*Sinh[d*x] + 3*a*d*e^2*f*
x*Sinh[d*x] + 3*a*d*e*f^2*x^2*Sinh[d*x] + a*d*f^3*x^3*Sinh[d*x] - a*d*e^3*S
inh[2*c + d*x] - 3*a*d*e^2*f*x*Sinh[2*c + d*x] - 3*a*d*e*f^2*x^2*Sinh[2*c +
d*x] - a*d*f^3*x^3*Sinh[2*c + d*x]))/(4*a^2*d^2)

```

**Maple [F]** time = 0.806, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 (\operatorname{csch}(dx + c))^3}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.249 \quad \int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=725

$$-\frac{2b^3 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3 d^2 \sqrt{a^2+b^2}} + \frac{2b^3 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3 d^2 \sqrt{a^2+b^2}} - \frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{a^3 d^2} + \frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{a^3 d^2}$$

[Out] (b\*(e + f\*x)^2)/(a^2\*d) + ((e + f\*x)^2\*ArcTanh[E^(c + d\*x)])/(a\*d) - (2\*b^2\*(e + f\*x)^2\*ArcTanh[E^(c + d\*x)])/(a^3\*d) - (f^2\*ArcTanh[Cosh[c + d\*x]])/(a\*d^3) + (b\*(e + f\*x)^2\*Coth[c + d\*x])/(a^2\*d) - (f\*(e + f\*x)\*Csch[c + d\*x])/(a\*d^2) - ((e + f\*x)^2\*Coth[c + d\*x]\*Csch[c + d\*x])/(2\*a\*d) - (b^3\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^3\*Sqrt[a^2 + b^2]\*d) + (b^3\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^3\*Sqrt[a^2 + b^2]\*d) - (2\*b\*f\*(e + f\*x)\*Log[1 - E^(2\*(c + d\*x))])/(a^2\*d^2) + (f\*(e + f\*x)\*PolyLog[2, -E^(c + d\*x)])/(a\*d^2) - (2\*b^2\*f\*(e + f\*x)\*PolyLog[2, -E^(c + d\*x)])/(a^3\*d^2) - (f\*(e + f\*x)\*PolyLog[2, E^(c + d\*x)])/(a\*d^2) + (2\*b^2\*f\*(e + f\*x)\*PolyLog[2, E^(c + d\*x)])/(a^3\*d^2) - (2\*b^3\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^3\*Sqrt[a^2 + b^2]\*d^2) + (2\*b^3\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^3\*Sqrt[a^2 + b^2]\*d^2) - (b\*f^2\*PolyLog[2, E^(2\*(c + d\*x))])/(a^2\*d^3) - (f^2\*PolyLog[3, -E^(c + d\*x)])/(a\*d^3) + (2\*b^2\*f^2\*PolyLog[3, -E^(c + d\*x)])/(a^3\*d^3) + (f^2\*PolyLog[3, E^(c + d\*x)])/(a\*d^3) - (2\*b^2\*f^2\*PolyLog[3, E^(c + d\*x)])/(a^3\*d^3) + (2\*b^3\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^3\*Sqrt[a^2 + b^2]\*d^3) - (2\*b^3\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^3\*Sqrt[a^2 + b^2]\*d^3)

**Rubi [A]** time = 1.37234, antiderivative size = 725, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 14, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5575, 4186, 3770, 4182, 2531, 2282, 6589, 4184, 3716, 2190, 2279, 2391, 3322, 2264}

$$-\frac{2b^3 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3 d^2 \sqrt{a^2+b^2}} + \frac{2b^3 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3 d^2 \sqrt{a^2+b^2}} - \frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{a^3 d^2} + \frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{a^3 d^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Csch[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] (b\*(e + f\*x)^2)/(a^2\*d) + ((e + f\*x)^2\*ArcTanh[E^(c + d\*x)])/(a\*d) - (2\*b^2\*(e + f\*x)^2\*ArcTanh[E^(c + d\*x)])/(a^3\*d) - (f^2\*ArcTanh[Cosh[c + d\*x]])/(a\*d^3) + (b\*(e + f\*x)^2\*Coth[c + d\*x])/(a^2\*d) - (f\*(e + f\*x)\*Csch[c + d\*x])/(a\*d^2) - ((e + f\*x)^2\*Coth[c + d\*x]\*Csch[c + d\*x])/(2\*a\*d) - (b^3\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^3\*Sqrt[a^2 + b^2]\*d) + (b^3\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^3\*Sqrt[a^2 + b^2]\*d) - (2\*b\*f\*(e + f\*x)\*Log[1 - E^(2\*(c + d\*x))])/(a^2\*d^2) + (f\*(e + f\*x)\*PolyLog[2, -E^(c + d\*x)])/(a\*d^2) - (2\*b^2\*f\*(e + f\*x)\*PolyLog[2, -E^(c + d\*x)])/(a^3\*d^2) - (f\*(e + f\*x)\*PolyLog[2, E^(c + d\*x)])/(a\*d^2) + (2\*b^2\*f\*(e + f\*x)\*PolyLog[2, E^(c + d\*x)])/(a^3\*d^2) - (2\*b^3\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^3\*Sqrt[a^2 + b^2]\*d^2) + (2\*b^3\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^3\*Sqrt[a^2 + b^2]\*d^2) - (b\*f^2\*PolyLog[2, E^(2\*(c + d\*x))])/(a^2\*d^3) - (f^2\*PolyLog[3, -E^(c + d\*x)])/(a\*d^3) + (2\*b^2\*f^2\*PolyLog[3, -E^(c + d\*x)])/(a^3\*d^3) + (f^2\*PolyLog[3, E^(c + d\*x)])/(a\*d^3) - (2\*b^2\*f^2\*PolyLog[3, E^(c + d\*x)])/(a^3\*d^3) + (2\*b^3\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^3\*Sqrt[a^2 + b^2]\*d^3) - (2\*b^3\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^3\*Sqrt[a^2 + b^2]\*d^3)



$$\begin{aligned}
& a*d^3 + (b*(e + f*x)^2*Coth[c + d*x])/(a^2*d) - (f*(e + f*x)*Csch[c + d*x])/(a*d^2) - ((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x])/(2*a*d) - (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*Sqrt[a^2 + b^2]*d) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*Sqrt[a^2 + b^2]*d) - (2*b*f*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a^2*d^2) + (f*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a^3*d^2) - (f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) + (2*b^2*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^3*d^2) - (2*b^3*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*Sqrt[a^2 + b^2]*d^2) + (2*b^3*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*Sqrt[a^2 + b^2]*d^2) - (b*f^2*PolyLog[2, E^(2*(c + d*x))])/(a^2*d^3) - (f^2*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (2*b^2*f^2*PolyLog[3, -E^(c + d*x)])/(a^3*d^3) + (f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) - (2*b^2*f^2*PolyLog[3, E^(c + d*x)])/(a^3*d^3) + (2*b^3*f^2*PolyLog[3, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*Sqrt[a^2 + b^2]*d^3) - (2*b^3*f^2*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*Sqrt[a^2 + b^2]*d^3)
\end{aligned}$$

### Rule 5575

$$\text{Int}[(\text{Csch}[(c\_.) + (d\_.)*(x\_)]^{(n\_)}*((e\_.) + (f\_.)*(x\_))^{(m\_)}]/((a\_.) + (b\_.)*\text{Sinh}[(c\_.) + (d\_.)*(x\_)]), x\_Symbol] \rightarrow \text{Dist}[1/a, \text{Int}[(e + f*x)^m*\text{Csch}[c + d*x]^n, x], x] - \text{Dist}[b/a, \text{Int}[(e + f*x)^m*\text{Csch}[c + d*x]^{(n - 1)}]/(a + b*\text{Sinh}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[n, 0]$$

### Rule 4186

$$\text{Int}[(\text{csc}[(e\_.) + (f\_.)*(x\_)]*(b\_.)^{(n\_)}*((c\_.) + (d\_.)*(x\_))^{(m\_)}], x\_Symbol] \rightarrow -\text{Simp}[(b^2*(c + d*x)^m*\text{Cot}[e + f*x]*(b*\text{Csc}[e + f*x])^{(n - 2)})/(f*(n - 1)), x] + (\text{Dist}[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), \text{Int}[(c + d*x)^{(m - 2)}*(b*\text{Csc}[e + f*x])^{(n - 2)}, x], x] + \text{Dist}[(b^2*(n - 2))/(n - 1), \text{Int}[(c + d*x)^m*(b*\text{Csc}[e + f*x])^{(n - 2)}, x], x] - \text{Simp}[(b^2*d*m*(c + d*x)^{(m - 1)}*(b*\text{Csc}[e + f*x])^{(n - 2)})/(f^2*(n - 1)*(n - 2)), x]) /; \text{FreeQ}\{b, c, d, e, f\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{NeQ}[n, 2] \&\& \text{GtQ}[m, 1]$$

### Rule 3770

$$\text{Int}[\text{csc}[(c\_.) + (d\_.)*(x\_)], x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$$

### Rule 4182

$$\text{Int}[\text{csc}[(e\_.) + (\text{Complex}[0, fz\_])*(f\_.)*(x\_)]*((c\_.) + (d\_.)*(x\_))^{(m\_)}], x\_Symbol] \rightarrow \text{Simp}[(-2*(c + d*x)^m*\text{ArcTanh}[E^{-(I*e) + f*fz*x}])/(f*fz*I), x] + (-\text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 - E^{-(I*e) + f*fz*x}])$$

], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-I\*e) + f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 3716

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]), x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x)))/(E^(2\*I\*k\*Pi)\*(1 + E^(2\*(-I\*e) + f\*fz\*x)))/E^(2\*I\*k\*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))

)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 3322

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_) + (b\_.)\*sin[(e\_.) + (Complex[0, fz\_])\* (f\_.)\*(x\_)]), x\_Symbol] :> Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(-(I\*b) + 2\*a\*E^(-(I\*e) + f\*fz\*x) + I\*b\*E^(2\*(-I\*e) + f\*fz\*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.)\*(F\_)^(v\_)), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \operatorname{csch}^3(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= -\frac{f(e+fx) \operatorname{csch}(c+dx)}{ad^2} - \frac{(e+fx)^2 \operatorname{coth}(c+dx) \operatorname{csch}(c+dx)}{2ad} - \frac{\int (e+fx)^2 \operatorname{csch}(c+dx)}{2a} \\
&= \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} + \frac{b(e+fx)^2 \operatorname{coth}(c+dx)}{a^2d} - \frac{f(e+fx)}{a} \\
&= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3d} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} \\
&= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3d} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} \\
&= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3d} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} \\
&= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3d} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} \\
&= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3d} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} \\
&= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3d} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} \\
&= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3d} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} \\
&= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3d} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} \\
&= \frac{b(e+fx)^2}{a^2d} + \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3d} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3}
\end{aligned}$$

**Mathematica [B]** time = 25.6502, size = 1531, normalized size = 2.11

result too large to display

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Csch[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] (8\*a\*b\*d^2\*e\*E^(2\*c)\*f\*x + 4\*a\*b\*d^2\*E^(2\*c)\*f^2\*x^2 - 2\*a^2\*d^2\*e^2\*ArcTan  
h[E^(c + d\*x)] + 4\*b^2\*d^2\*e^2\*ArcTanh[E^(c + d\*x)] + 2\*a^2\*d^2\*e^2\*E^(2\*c)  
\*ArcTanh[E^(c + d\*x)] - 4\*b^2\*d^2\*e^2\*E^(2\*c)\*ArcTanh[E^(c + d\*x)] + 4\*a^2\*d  
f^2\*ArcTanh[E^(c + d\*x)] - 4\*a^2\*E^(2\*c)\*f^2\*ArcTanh[E^(c + d\*x)] + 2\*a^2\*d  
^2\*e\*f\*x\*Log[1 - E^(c + d\*x)] - 4\*b^2\*d^2\*e\*f\*x\*Log[1 - E^(c + d\*x)] - 2\*a^2  
d^2\*e\*E^(2\*c)\*f\*x\*Log[1 - E^(c + d\*x)] + 4\*b^2\*d^2\*e\*E^(2\*c)\*f\*x\*Log[1 -

$$\begin{aligned}
& E^{(c+dx)} + a^2 d^2 f^2 x^2 \operatorname{Log}[1 - E^{(c+dx)}] - 2b^2 d^2 f^2 x^2 \operatorname{Log}[1 - E^{(c+dx)}] - a^2 d^2 E^{(2c)} f^2 x^2 \operatorname{Log}[1 - E^{(c+dx)}] + 2b^2 d^2 E^{(2c)} f^2 x^2 \operatorname{Log}[1 - E^{(c+dx)}] - 2a^2 d^2 e f x \operatorname{Log}[1 + E^{(c+dx)}] + 4b^2 d^2 e f x \operatorname{Log}[1 + E^{(c+dx)}] + 2a^2 d^2 e E^{(2c)} f x \operatorname{Log}[1 + E^{(c+dx)}] - 4b^2 d^2 e E^{(2c)} f x \operatorname{Log}[1 + E^{(c+dx)}] - a^2 d^2 f^2 x^2 \operatorname{Log}[1 + E^{(c+dx)}] + 2b^2 d^2 f^2 x^2 \operatorname{Log}[1 + E^{(c+dx)}] + a^2 d^2 E^{(2c)} f^2 x^2 \operatorname{Log}[1 + E^{(c+dx)}] - 2b^2 d^2 E^{(2c)} f^2 x^2 \operatorname{Log}[1 + E^{(c+dx)}] + 4a b d e f \operatorname{Log}[1 - E^{(2(c+dx))}] - 4a b d e E^{(2c)} f \operatorname{Log}[1 - E^{(2(c+dx))}] + 4a b d f^2 x \operatorname{Log}[1 - E^{(2(c+dx))}] - 4a b d E^{(2c)} f^2 x \operatorname{Log}[1 - E^{(2(c+dx))}] + 2(a^2 - 2b^2) d (-1 + E^{(2c)}) f (e + f x) \operatorname{PolyLog}[2, -E^{(c+dx)}] - 2(a^2 - 2b^2) d (-1 + E^{(2c)}) f (e + f x) \operatorname{PolyLog}[2, E^{(c+dx)}] + 2a b f^2 \operatorname{PolyLog}[2, E^{(2(c+dx))}] - 2a b E^{(2c)} f^2 \operatorname{PolyLog}[2, E^{(2(c+dx))}] + 2a^2 f^2 \operatorname{PolyLog}[3, -E^{(c+dx)}] - 4b^2 f^2 \operatorname{PolyLog}[3, -E^{(c+dx)}] - 2a^2 E^{(2c)} f^2 \operatorname{PolyLog}[3, -E^{(c+dx)}] + 4b^2 E^{(2c)} f^2 \operatorname{PolyLog}[3, -E^{(c+dx)}] - 2a^2 f^2 \operatorname{PolyLog}[3, E^{(c+dx)}] + 4b^2 f^2 \operatorname{PolyLog}[3, E^{(c+dx)}] + 2a^2 E^{(2c)} f^2 \operatorname{PolyLog}[3, E^{(c+dx)}] - 4b^2 E^{(2c)} f^2 \operatorname{PolyLog}[3, E^{(c+dx)}]) / (2a^3 d^3 (-1 + E^{(2c)})) + (b^3 (2d^2 e^2 \operatorname{ArcTanh}[(a + bE^{(c+dx)})] / \operatorname{Sqrt}[a^2 + b^2]) - 2d^2 e f x \operatorname{Log}[1 + (bE^{(c+dx)}) / (a - \operatorname{Sqrt}[a^2 + b^2])] - d^2 f^2 x^2 \operatorname{Log}[1 + (bE^{(c+dx)}) / (a - \operatorname{Sqrt}[a^2 + b^2])] + 2d^2 e f x \operatorname{Log}[1 + (bE^{(c+dx)}) / (a + \operatorname{Sqrt}[a^2 + b^2])] + d^2 f^2 x^2 \operatorname{Log}[1 + (bE^{(c+dx)}) / (a + \operatorname{Sqrt}[a^2 + b^2])] - 2d f (e + f x) \operatorname{PolyLog}[2, (bE^{(c+dx)}) / (-a + \operatorname{Sqrt}[a^2 + b^2])] + 2d f (e + f x) \operatorname{PolyLog}[2, -((bE^{(c+dx)}) / (a + \operatorname{Sqrt}[a^2 + b^2]))] + 2f^2 \operatorname{PolyLog}[3, (bE^{(c+dx)}) / (-a + \operatorname{Sqrt}[a^2 + b^2])] - 2f^2 \operatorname{PolyLog}[3, -((bE^{(c+dx)}) / (a + \operatorname{Sqrt}[a^2 + b^2]))]) / (a^3 \operatorname{Sqrt}[a^2 + b^2] d^3) + (\operatorname{Csch}[c] \operatorname{Csch}[c + dx]^2 (2b d e^2 \operatorname{Cosh}[c] + 4b d e f x \operatorname{Cosh}[c] + 2b d f^2 x^2 \operatorname{Cosh}[c] + 2a e f \operatorname{Cosh}[dx] + 2a f^2 x \operatorname{Cosh}[dx] - 2a e f \operatorname{Cosh}[2c + dx] - 2a f^2 x \operatorname{Cosh}[2c + dx] - 2b d e^2 \operatorname{Cosh}[c + 2dx] - 4b d e f x \operatorname{Cosh}[c + 2dx] - 2b d f^2 x^2 \operatorname{Cosh}[c + 2dx] + a d e^2 \operatorname{Sinh}[dx] + 2a d e f x \operatorname{Sinh}[dx] + a d f^2 x^2 \operatorname{Sinh}[dx] - a d e^2 \operatorname{Sinh}[2c + dx] - 2a d e f x \operatorname{Sinh}[2c + dx] - a d f^2 x^2 \operatorname{Sinh}[2c + dx])) / (4a^2 d^2)
\end{aligned}$$

**Maple [F]** time = 0.655, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\operatorname{csch}(dx + c))^3}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*csch(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

```
[Out] int((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [C]** time = 4.14245, size = 22066, normalized size = 30.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(a^3*b + a*b^3)*d^2*e^2 - 8*(a^3*b + a*b^3)*c*d*e*f + 4*(a^3*b + a*
b^3)*c^2*f^2 - 4*((a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x
+ 2*(a^3*b + a*b^3)*c*d*e*f - (a^3*b + a*b^3)*c^2*f^2)*cosh(d*x + c)^4 - 4
*((a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x + 2*(a^3*b + a*
b^3)*c*d*e*f - (a^3*b + a*b^3)*c^2*f^2)*sinh(d*x + c)^4 + 2*((a^4 + a^2*b^2
)*d^2*f^2*x^2 + (a^4 + a^2*b^2)*d^2*e^2 + 2*(a^4 + a^2*b^2)*d*e*f + 2*((a^4
+ a^2*b^2)*d^2*e*f + (a^4 + a^2*b^2)*d*f^2)*x)*cosh(d*x + c)^3 + 2*((a^4 +
a^2*b^2)*d^2*f^2*x^2 + (a^4 + a^2*b^2)*d^2*e^2 + 2*(a^4 + a^2*b^2)*d*e*f +
2*((a^4 + a^2*b^2)*d^2*e*f + (a^4 + a^2*b^2)*d*f^2)*x - 8*((a^3*b + a*b^3)
*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x + 2*(a^3*b + a*b^3)*c*d*e*f - (a
^3*b + a*b^3)*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*((a^3*b + a*b^3)*
d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x - (a^3*b + a*b^3)*d^2*e^2 + 4*(a^
3*b + a*b^3)*c*d*e*f - 2*(a^3*b + a*b^3)*c^2*f^2)*cosh(d*x + c)^2 + 2*(2*(a
^3*b + a*b^3)*d^2*f^2*x^2 + 4*(a^3*b + a*b^3)*d^2*e*f*x - 2*(a^3*b + a*b^3)
*d^2*e^2 + 8*(a^3*b + a*b^3)*c*d*e*f - 4*(a^3*b + a*b^3)*c^2*f^2 - 12*((a^3
*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x + 2*(a^3*b + a*b^3)*c
*d*e*f - (a^3*b + a*b^3)*c^2*f^2)*cosh(d*x + c)^2 + 3*((a^4 + a^2*b^2)*d^2*
f^2*x^2 + (a^4 + a^2*b^2)*d^2*e^2 + 2*(a^4 + a^2*b^2)*d*e*f + 2*((a^4 + a^2
*b^2)*d^2*e*f + (a^4 + a^2*b^2)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^2 +
4*(b^4*d*f^2*x + b^4*d*e*f + (b^4*d*f^2*x + b^4*d*e*f)*cosh(d*x + c)^4 + 4*
```

$$\begin{aligned}
& (b^4 d^2 f^2 x + b^4 d e f) \cosh(dx + c) \sinh(dx + c)^3 + (b^4 d^2 f^2 x + b^4 d e f) \sinh(dx + c)^4 - 2(b^4 d^2 f^2 x + b^4 d e f) \cosh(dx + c)^2 - 2(b^4 d^2 f^2 x + b^4 d e f - 3(b^4 d^2 f^2 x + b^4 d e f) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4((b^4 d^2 f^2 x + b^4 d e f) \cosh(dx + c)^3 - (b^4 d^2 f^2 x + b^4 d e f) \cosh(dx + c)) \sinh(dx + c) \sqrt{(a^2 + b^2)/b^2} \operatorname{dilog}((a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 4(b^4 d^2 f^2 x + b^4 d e f + (b^4 d^2 f^2 x + b^4 d e f) \cosh(dx + c)^4 + 4(b^4 d^2 f^2 x + b^4 d e f) \cosh(dx + c) \sinh(dx + c)^3 + (b^4 d^2 f^2 x + b^4 d e f) \sinh(dx + c)^4 - 2(b^4 d^2 f^2 x + b^4 d e f) \cosh(dx + c)^2 - 2(b^4 d^2 f^2 x + b^4 d e f - 3(b^4 d^2 f^2 x + b^4 d e f) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4((b^4 d^2 f^2 x + b^4 d e f) \cosh(dx + c)^3 - (b^4 d^2 f^2 x + b^4 d e f) \cosh(dx + c)) \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} \operatorname{dilog}((a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 2(b^4 d^2 e^2 - 2b^4 c d e f + b^4 c^2 f^2 + (b^4 d^2 e^2 - 2b^4 c d e f + b^4 c^2 f^2) \cosh(dx + c)^4 + 4(b^4 d^2 e^2 - 2b^4 c d e f + b^4 c^2 f^2) \cosh(dx + c) \sinh(dx + c)^3 + (b^4 d^2 e^2 - 2b^4 c d e f + b^4 c^2 f^2) \sinh(dx + c)^4 - 2(b^4 d^2 e^2 - 2b^4 c d e f + b^4 c^2 f^2) \cosh(dx + c)^2 - 2(b^4 d^2 e^2 - 2b^4 c d e f + b^4 c^2 f^2 - 3(b^4 d^2 e^2 - 2b^4 c d e f + b^4 c^2 f^2) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4((b^4 d^2 e^2 - 2b^4 c d e f + b^4 c^2 f^2) \cosh(dx + c)^3 - (b^4 d^2 e^2 - 2b^4 c d e f + b^4 c^2 f^2) \cosh(dx + c)) \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} \log(2b \cosh(dx + c) + 2b \sinh(dx + c) + 2b \sqrt{(a^2 + b^2)/b^2} + 2a) + 2(b^4 d^2 e^2 - 2b^4 c d e f + b^4 c^2 f^2 + (b^4 d^2 e^2 - 2b^4 c d e f + b^4 c^2 f^2) \cosh(dx + c)^4 + 4(b^4 d^2 e^2 - 2b^4 c d e f + b^4 c^2 f^2) \cosh(dx + c) \sinh(dx + c)^3 + (b^4 d^2 e^2 - 2b^4 c d e f + b^4 c^2 f^2) \sinh(dx + c)^4 - 2(b^4 d^2 e^2 - 2b^4 c d e f + b^4 c^2 f^2) \cosh(dx + c)^2 - 2(b^4 d^2 e^2 - 2b^4 c d e f + b^4 c^2 f^2 - 3(b^4 d^2 e^2 - 2b^4 c d e f + b^4 c^2 f^2) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4((b^4 d^2 e^2 - 2b^4 c d e f + b^4 c^2 f^2) \cosh(dx + c)^3 - (b^4 d^2 e^2 - 2b^4 c d e f + b^4 c^2 f^2) \cosh(dx + c)) \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} \log(2b \cosh(dx + c) + 2b \sinh(dx + c) - 2b \sqrt{(a^2 + b^2)/b^2} + 2a) + 2(b^4 d^2 f^2 x^2 + 2b^4 d^2 e f x + 2b^4 c d e f - b^4 c^2 f^2 + (b^4 d^2 f^2 x^2 + 2b^4 d^2 e f x + 2b^4 c d e f - b^4 c^2 f^2) \cosh(dx + c)^4 + 4(b^4 d^2 f^2 x^2 + 2b^4 d^2 e f x + 2b^4 c d e f - b^4 c^2 f^2) \cosh(dx + c) \sinh(dx + c)^3 + (b^4 d^2 f^2 x^2 + 2b^4 d^2 e f x + 2b^4 c d e f - b^4 c^2 f^2) \sinh(dx + c)^4 - 2(b^4 d^2 f^2 x^2 + 2b^4 d^2 e f x + 2b^4 c d e f - b^4 c^2 f^2) \cosh(dx + c)^2 - 2(b^4 d^2 f^2 x^2 + 2b^4 d^2 e f x + 2b^4 c d e f - b^4 c^2 f^2 - 3(b^4 d^2 f^2 x^2 + 2b^4 d^2 e f x + 2b^4 c d e f - b^4 c^2 f^2) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4((b^4 d^2 f^2 x^2 + 2b^4 d^2 e f x + 2b^4 c d e f - b^4 c^2 f^2) \cosh(dx + c)^3 - (b^4 d^2 f^2 x^2 + 2b^4 d^2 e f x + 2b^4 c d e f - b^4 c^2 f^2) \cosh(dx + c)) \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} \log(-(a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b) - 2(b^4 d^2 f^2 x^2 + 2b^4 d^2 e f x + 2b^4 c d e f - b^4 c^2 f^2 +
\end{aligned}$$

$$\begin{aligned}
& (b^4 d^2 f^2 x^2 + 2 b^4 d^2 e f x + 2 b^4 c d e f - b^4 c^2 f^2) \cosh(d x + c)^4 + 4 (b^4 d^2 f^2 x^2 + 2 b^4 d^2 e f x + 2 b^4 c d e f - b^4 c^2 f^2) \cosh(d x + c) \sinh(d x + c)^3 + (b^4 d^2 f^2 x^2 + 2 b^4 d^2 e f x + 2 b^4 c d e f - b^4 c^2 f^2) \sinh(d x + c)^4 - 2 (b^4 d^2 f^2 x^2 + 2 b^4 d^2 e f x + 2 b^4 c d e f - b^4 c^2 f^2) \cosh(d x + c)^2 - 2 (b^4 d^2 f^2 x^2 + 2 b^4 d^2 e f x + 2 b^4 c d e f - b^4 c^2 f^2) \cosh(d x + c)^2 \sinh(d x + c)^2 + 4 ((b^4 d^2 f^2 x^2 + 2 b^4 d^2 e f x + 2 b^4 c d e f - b^4 c^2 f^2) \cosh(d x + c)^3 - (b^4 d^2 f^2 x^2 + 2 b^4 d^2 e f x + 2 b^4 c d e f - b^4 c^2 f^2) \cosh(d x + c)) \sinh(d x + c) \sqrt{(a^2 + b^2)/b^2} \log(- (a \cosh(d x + c) + a \sinh(d x + c) - (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2}) - b)/b) - 4 (b^4 f^2 \cosh(d x + c)^4 + 4 b^4 f^2 \cosh(d x + c) \sinh(d x + c)^3 + b^4 f^2 \sinh(d x + c)^4 - 2 b^4 f^2 \cosh(d x + c)^2 + b^4 f^2 + 2 (3 b^4 f^2 \cosh(d x + c)^2 - b^4 f^2) \sinh(d x + c)^2 + 4 (b^4 f^2 \cosh(d x + c)^3 - b^4 f^2 \cosh(d x + c)) \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2} \operatorname{polylog}(3, (a \cosh(d x + c) + a \sinh(d x + c) + (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2}))/b) + 4 (b^4 f^2 \cosh(d x + c)^4 + 4 b^4 f^2 \cosh(d x + c) \sinh(d x + c)^3 + b^4 f^2 \sinh(d x + c)^4 - 2 b^4 f^2 \cosh(d x + c)^2 + b^4 f^2 + 2 (3 b^4 f^2 \cosh(d x + c)^2 - b^4 f^2) \sinh(d x + c)^2 + 4 (b^4 f^2 \cosh(d x + c)^3 - b^4 f^2 \cosh(d x + c)) \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2} \operatorname{polylog}(3, (a \cosh(d x + c) + a \sinh(d x + c) - (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2}))/b) + 2 ((a^4 + a^2 b^2) d^2 f^2 x^2 + (a^4 + a^2 b^2) d^2 e^2 - 2 (a^4 + a^2 b^2) d e f + 2 ((a^4 + a^2 b^2) d^2 e f - (a^4 + a^2 b^2) d f^2) x) \cosh(d x + c) + 2 ((a^4 - a^2 b^2 - 2 b^4) d f^2 x + ((a^4 - a^2 b^2 - 2 b^4) d f^2 x + (a^4 - a^2 b^2 - 2 b^4) d e f + 2 (a^3 b + a b^3) f^2) \cosh(d x + c)^4 + 4 ((a^4 - a^2 b^2 - 2 b^4) d f^2 x + (a^4 - a^2 b^2 - 2 b^4) d e f + 2 (a^3 b + a b^3) f^2) \cosh(d x + c) \sinh(d x + c)^3 + ((a^4 - a^2 b^2 - 2 b^4) d f^2 x + (a^4 - a^2 b^2 - 2 b^4) d e f + 2 (a^3 b + a b^3) f^2) \sinh(d x + c)^4 + (a^4 - a^2 b^2 - 2 b^4) d e f + 2 (a^3 b + a b^3) f^2 - 2 ((a^4 - a^2 b^2 - 2 b^4) d f^2 x + (a^4 - a^2 b^2 - 2 b^4) d e f + 2 (a^3 b + a b^3) f^2) \cosh(d x + c)^2 \sinh(d x + c)^2 + 4 ((a^4 - a^2 b^2 - 2 b^4) d f^2 x + (a^4 - a^2 b^2 - 2 b^4) d e f + 2 (a^3 b + a b^3) f^2) \cosh(d x + c)^3 - ((a^4 - a^2 b^2 - 2 b^4) d f^2 x + (a^4 - a^2 b^2 - 2 b^4) d e f + 2 (a^3 b + a b^3) f^2) \cosh(d x + c)) \sinh(d x + c) \operatorname{dilog}(\cosh(d x + c) + \sinh(d x + c)) - 2 ((a^4 - a^2 b^2 - 2 b^4) d f^2 x + ((a^4 - a^2 b^2 - 2 b^4) d f^2 x + (a^4 - a^2 b^2 - 2 b^4) d e f - 2 (a^3 b + a b^3) f^2) \cosh(d x + c)^4 + 4 ((a^4 - a^2 b^2 - 2 b^4) d f^2 x + (a^4 - a^2 b^2 - 2 b^4) d e f - 2 (a^3 b + a b^3) f^2) \cosh(d x + c) \sinh(d x + c)^3 + ((a^4 - a^2 b^2 - 2 b^4) d f^2 x + (a^4 - a^2 b^2 - 2 b^4) d e f - 2 (a^3 b + a b^3) f^2) \sinh(d x + c)^4 + (a^4 - a^2 b^2 - 2 b^4) d e f - 2 (a^3 b + a b^3) f^2 - 2 ((a^4 - a^2 b^2 - 2 b^4) d f^2 x + (a^4 - a^2 b^2 - 2 b^4) d e f - 2 (a^3 b + a b^3) f^2) \cosh(d x + c)^2 - 2 ((a^4 - a^2
\end{aligned}$$



$$\begin{aligned}
& *b^2 - 2*b^4)*d*f^2*x + (a^4 - a^2*b^2 - 2*b^4)*d*e*f - 2*(a^3*b + a*b^3)*f \\
& ^2 - 3*((a^4 - a^2*b^2 - 2*b^4)*d*f^2*x + (a^4 - a^2*b^2 - 2*b^4)*d*e*f - 2 \\
& *(a^3*b + a*b^3)*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*((a^4 - a^2*b^2 \\
& - 2*b^4)*d*f^2*x + (a^4 - a^2*b^2 - 2*b^4)*d*e*f - 2*(a^3*b + a*b^3)*f^2)* \\
& \cosh(d*x + c)^3 - ((a^4 - a^2*b^2 - 2*b^4)*d*f^2*x + (a^4 - a^2*b^2 - 2*b^4 \\
& )*d*e*f - 2*(a^3*b + a*b^3)*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\operatorname{dilog}(-\cosh( \\
& d*x + c) - \sinh(d*x + c)) - ((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + (a^4 - a \\
& ^2*b^2 - 2*b^4)*d^2*e^2 + ((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + (a^4 - a^2 \\
& *b^2 - 2*b^4)*d^2*e^2 - 4*(a^3*b + a*b^3)*d*e*f - 2*(a^4 + a^2*b^2)*f^2 + 2 \\
& *((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f - 2*(a^3*b + a*b^3)*d*f^2)*x)*\cosh(d*x + \\
& c)^4 + 4*((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + (a^4 - a^2*b^2 - 2*b^4)*d^2 \\
& *e^2 - 4*(a^3*b + a*b^3)*d*e*f - 2*(a^4 + a^2*b^2)*f^2 + 2*((a^4 - a^2*b^2 \\
& - 2*b^4)*d^2*e*f - 2*(a^3*b + a*b^3)*d*f^2)*x)*\cosh(d*x + c)*\sinh(d*x + c)^ \\
& 3 + ((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + (a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 \\
& - 4*(a^3*b + a*b^3)*d*e*f - 2*(a^4 + a^2*b^2)*f^2 + 2*((a^4 - a^2*b^2 - 2*b \\
& ^4)*d^2*e*f - 2*(a^3*b + a*b^3)*d*f^2)*x))*\sinh(d*x + c)^4 - 4*(a^3*b + a*b^ \\
& 3)*d*e*f - 2*(a^4 + a^2*b^2)*f^2 - 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + \\
& (a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 - 4*(a^3*b + a*b^3)*d*e*f - 2*(a^4 + a^2*b \\
& ^2)*f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f - 2*(a^3*b + a*b^3)*d*f^2)*x)* \\
& \cosh(d*x + c)^2 - 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + (a^4 - a^2*b^2 - \\
& 2*b^4)*d^2*e^2 - 4*(a^3*b + a*b^3)*d*e*f - 2*(a^4 + a^2*b^2)*f^2 - 3*((a^4 \\
& - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + (a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 - 4*(a^3* \\
& b + a*b^3)*d*e*f - 2*(a^4 + a^2*b^2)*f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e \\
& *f - 2*(a^3*b + a*b^3)*d*f^2)*x)*\cosh(d*x + c)^2 + 2*((a^4 - a^2*b^2 - 2*b^ \\
& 4)*d^2*e*f - 2*(a^3*b + a*b^3)*d*f^2)*x))*\sinh(d*x + c)^2 + 2*((a^4 - a^2*b^ \\
& 2 - 2*b^4)*d^2*e*f - 2*(a^3*b + a*b^3)*d*f^2)*x + 4*((a^4 - a^2*b^2 - 2*b^ \\
& 4)*d^2*f^2*x^2 + (a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 - 4*(a^3*b + a*b^3)*d*e*f \\
& - 2*(a^4 + a^2*b^2)*f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f - 2*(a^3*b + a \\
& *b^3)*d*f^2)*x)*\cosh(d*x + c)^3 - ((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + (a \\
& ^4 - a^2*b^2 - 2*b^4)*d^2*e^2 - 4*(a^3*b + a*b^3)*d*e*f - 2*(a^4 + a^2*b^2) \\
& *f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f - 2*(a^3*b + a*b^3)*d*f^2)*x)*\cos \\
& h(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) + ((a^4 - \\
& a^2*b^2 - 2*b^4)*d^2*e^2 + ((a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 + 2*(2*a^3*b + \\
& 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4)*c)*d*e*f - (2*a^4 + 2*a^2*b^2 - (a^4 - a \\
& ^2*b^2 - 2*b^4)*c^2 + 4*(a^3*b + a*b^3)*c)*f^2)*\cosh(d*x + c)^4 + 4*((a^4 - \\
& a^2*b^2 - 2*b^4)*d^2*e^2 + 2*(2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4)* \\
& c)*d*e*f - (2*a^4 + 2*a^2*b^2 - (a^4 - a^2*b^2 - 2*b^4)*c^2 + 4*(a^3*b + a* \\
& b^3)*c)*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^4 - a^2*b^2 - 2*b^4)*d^2*e \\
& ^2 + 2*(2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4)*c)*d*e*f - (2*a^4 + 2*a \\
& ^2*b^2 - (a^4 - a^2*b^2 - 2*b^4)*c^2 + 4*(a^3*b + a*b^3)*c)*f^2)*\sinh(d*x + \\
& c)^4 + 2*(2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4)*c)*d*e*f - (2*a^4 + \\
& 2*a^2*b^2 - (a^4 - a^2*b^2 - 2*b^4)*c^2 + 4*(a^3*b + a*b^3)*c)*f^2 - 2*((a^ \\
& 4 - a^2*b^2 - 2*b^4)*d^2*e^2 + 2*(2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^ \\
& 4)*c)*d*e*f - (2*a^4 + 2*a^2*b^2 - (a^4 - a^2*b^2 - 2*b^4)*c^2 + 4*(a^3*b + \\
& a*b^3)*c)*f^2)*\cosh(d*x + c)^2 - 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 + 2*(2
\end{aligned}$$

$$\begin{aligned}
& *a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4)*c)*d*e*f - (2*a^4 + 2*a^2*b^2 - \\
& (a^4 - a^2*b^2 - 2*b^4)*c^2 + 4*(a^3*b + a*b^3)*c)*f^2 - 3*((a^4 - a^2*b^2 \\
& - 2*b^4)*d^2*e^2 + 2*(2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4)*c)*d*e*f \\
& - (2*a^4 + 2*a^2*b^2 - (a^4 - a^2*b^2 - 2*b^4)*c^2 + 4*(a^3*b + a*b^3)*c)*f \\
& ^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 \\
& + 2*(2*a^3*b + 2*a*b^3 - (a^4 - a^2*b^2 - 2*b^4)*c)*d*e*f - (2*a^4 + 2*a^2* \\
& b^2 - (a^4 - a^2*b^2 - 2*b^4)*c^2 + 4*(a^3*b + a*b^3)*c)*f^2)*\cosh(d*x + c) \\
& ^3 - ((a^4 - a^2*b^2 - 2*b^4)*d^2*e^2 + 2*(2*a^3*b + 2*a*b^3 - (a^4 - a^2*b \\
& ^2 - 2*b^4)*c)*d*e*f - (2*a^4 + 2*a^2*b^2 - (a^4 - a^2*b^2 - 2*b^4)*c^2 + 4 \\
& *(a^3*b + a*b^3)*c)*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \\
& \sinh(d*x + c) - 1) + ((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + 2*(a^4 - a^2*b^ \\
& 2 - 2*b^4)*c*d*e*f + ((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + 2*(a^4 - a^2*b^ \\
& 2 - 2*b^4)*c*d*e*f - ((a^4 - a^2*b^2 - 2*b^4)*c^2 - 4*(a^3*b + a*b^3)*c)*f^ \\
& 2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f + 2*(a^3*b + a*b^3)*d*f^2)*x)*\cosh(d \\
& *x + c)^4 + 4*((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + 2*(a^4 - a^2*b^2 - 2*b \\
& ^4)*c*d*e*f - ((a^4 - a^2*b^2 - 2*b^4)*c^2 - 4*(a^3*b + a*b^3)*c)*f^2 + 2*( \\
& (a^4 - a^2*b^2 - 2*b^4)*d^2*e*f + 2*(a^3*b + a*b^3)*d*f^2)*x)*\cosh(d*x + c) \\
& *\sinh(d*x + c)^3 + ((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + 2*(a^4 - a^2*b^2 \\
& - 2*b^4)*c*d*e*f - ((a^4 - a^2*b^2 - 2*b^4)*c^2 - 4*(a^3*b + a*b^3)*c)*f^2 \\
& + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f + 2*(a^3*b + a*b^3)*d*f^2)*x)*\sinh(d*x \\
& + c)^4 - ((a^4 - a^2*b^2 - 2*b^4)*c^2 - 4*(a^3*b + a*b^3)*c)*f^2 - 2*((a^4 \\
& - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + 2*(a^4 - a^2*b^2 - 2*b^4)*c*d*e*f - ((a^4 \\
& - a^2*b^2 - 2*b^4)*c^2 - 4*(a^3*b + a*b^3)*c)*f^2 + 2*((a^4 - a^2*b^2 - 2* \\
& b^4)*d^2*e*f + 2*(a^3*b + a*b^3)*d*f^2)*x)*\cosh(d*x + c)^2 - 2*((a^4 - a^2* \\
& b^2 - 2*b^4)*d^2*f^2*x^2 + 2*(a^4 - a^2*b^2 - 2*b^4)*c*d*e*f - ((a^4 - a^2* \\
& b^2 - 2*b^4)*c^2 - 4*(a^3*b + a*b^3)*c)*f^2 - 3*((a^4 - a^2*b^2 - 2*b^4)*d^ \\
& 2*f^2*x^2 + 2*(a^4 - a^2*b^2 - 2*b^4)*c*d*e*f - ((a^4 - a^2*b^2 - 2*b^4)*c^ \\
& 2 - 4*(a^3*b + a*b^3)*c)*f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f + 2*(a^3* \\
& b + a*b^3)*d*f^2)*x)*\cosh(d*x + c)^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f + \\
& 2*(a^3*b + a*b^3)*d*f^2)*x)*\sinh(d*x + c)^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d \\
& ^2*e*f + 2*(a^3*b + a*b^3)*d*f^2)*x + 4*((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x \\
& ^2 + 2*(a^4 - a^2*b^2 - 2*b^4)*c*d*e*f - ((a^4 - a^2*b^2 - 2*b^4)*c^2 - 4*( \\
& a^3*b + a*b^3)*c)*f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f + 2*(a^3*b + a*b \\
& ^3)*d*f^2)*x)*\cosh(d*x + c)^3 - ((a^4 - a^2*b^2 - 2*b^4)*d^2*f^2*x^2 + 2*(a \\
& ^4 - a^2*b^2 - 2*b^4)*c*d*e*f - ((a^4 - a^2*b^2 - 2*b^4)*c^2 - 4*(a^3*b + a \\
& *b^3)*c)*f^2 + 2*((a^4 - a^2*b^2 - 2*b^4)*d^2*e*f + 2*(a^3*b + a*b^3)*d*f^2 \\
& )*x)*\cosh(d*x + c))*\sinh(d*x + c))*\log(-\cosh(d*x + c) - \sinh(d*x + c) + 1) \\
& - 2*((a^4 - a^2*b^2 - 2*b^4)*f^2*\cosh(d*x + c)^4 + 4*(a^4 - a^2*b^2 - 2*b^4 \\
& )*f^2*\cosh(d*x + c))*\sinh(d*x + c)^3 + (a^4 - a^2*b^2 - 2*b^4)*f^2*\sinh(d*x \\
& + c)^4 - 2*(a^4 - a^2*b^2 - 2*b^4)*f^2*\cosh(d*x + c)^2 + (a^4 - a^2*b^2 - 2 \\
& *b^4)*f^2 + 2*(3*(a^4 - a^2*b^2 - 2*b^4)*f^2*\cosh(d*x + c)^2 - (a^4 - a^2*b \\
& ^2 - 2*b^4)*f^2)*\sinh(d*x + c)^2 + 4*((a^4 - a^2*b^2 - 2*b^4)*f^2*\cosh(d*x \\
& + c)^3 - (a^4 - a^2*b^2 - 2*b^4)*f^2*\cosh(d*x + c))*\sinh(d*x + c))*\text{polylog}( \\
& 3, \cosh(d*x + c) + \sinh(d*x + c)) + 2*((a^4 - a^2*b^2 - 2*b^4)*f^2*\cosh(d*x \\
& + c)^4 + 4*(a^4 - a^2*b^2 - 2*b^4)*f^2*\cosh(d*x + c))*\sinh(d*x + c)^3 + (a^
\end{aligned}$$

$$\begin{aligned}
& 4 - a^2 b^2 - 2b^4) f^2 \sinh(dx + c)^4 - 2(a^4 - a^2 b^2 - 2b^4) f^2 \cosh(dx + c)^2 + (a^4 - a^2 b^2 - 2b^4) f^2 + 2(3(a^4 - a^2 b^2 - 2b^4) f^2 \cosh(dx + c)^2 - (a^4 - a^2 b^2 - 2b^4) f^2) \sinh(dx + c)^2 + 4((a^4 - a^2 b^2 - 2b^4) f^2 \cosh(dx + c)^3 - (a^4 - a^2 b^2 - 2b^4) f^2 \cosh(dx + c)) \sinh(dx + c) \operatorname{polylog}(3, -\cosh(dx + c) - \sinh(dx + c)) + 2((a^4 + a^2 b^2) d^2 f^2 x^2 + (a^4 + a^2 b^2) d^2 e^2 - 2(a^4 + a^2 b^2) d e f - 8((a^3 b + a b^3) d^2 f^2 x^2 + 2(a^3 b + a b^3) d^2 e f x + 2(a^3 b + a b^3) c d e f - (a^3 b + a b^3) c^2 f^2) \cosh(dx + c)^3 + 3((a^4 + a^2 b^2) d^2 f^2 x^2 + (a^4 + a^2 b^2) d^2 e^2 + 2(a^4 + a^2 b^2) d e f + 2((a^4 + a^2 b^2) d^2 e f + (a^4 + a^2 b^2) d f^2) x) \cosh(dx + c)^2 + 2((a^4 + a^2 b^2) d^2 e f - (a^4 + a^2 b^2) d f^2) x + 4((a^3 b + a b^3) d^2 f^2 x^2 + 2(a^3 b + a b^3) d^2 e f x - (a^3 b + a b^3) d^2 e^2 + 4(a^3 b + a b^3) c d e f - 2(a^3 b + a b^3) c^2 f^2) \cosh(dx + c)) \sinh(dx + c) \\
& ) / ((a^5 + a^3 b^2) d^3 \cosh(dx + c)^4 + 4(a^5 + a^3 b^2) d^3 \cosh(dx + c) \sinh(dx + c)^3 + (a^5 + a^3 b^2) d^3 \sinh(dx + c)^4 - 2(a^5 + a^3 b^2) d^3 \cosh(dx + c)^2 + (a^5 + a^3 b^2) d^3 + 2(3(a^5 + a^3 b^2) d^3 \cosh(dx + c)^2 - (a^5 + a^3 b^2) d^3) \sinh(dx + c)^2 + 4((a^5 + a^3 b^2) d^3 \cosh(dx + c)^3 - (a^5 + a^3 b^2) d^3 \cosh(dx + c)) \sinh(dx + c))
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*csch(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*csch(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.250 \quad \int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=420

$$-\frac{b^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3 d^2 \sqrt{a^2+b^2}} + \frac{b^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3 d^2 \sqrt{a^2+b^2}} - \frac{b^2 f \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{a^3 d^2} + \frac{b^2 f \operatorname{PolyLog}\left(2, e^{c+dx}\right)}{a^3 d^2} + \frac{f \operatorname{PolyLog}\left(2, e^{c+dx}\right)}{a^3 d^2}$$

[Out] ((e + f\*x)\*ArcTanh[E^(c + d\*x)]/(a\*d) - (2\*b^2\*(e + f\*x)\*ArcTanh[E^(c + d\*x)]/(a^3\*d) + (b\*(e + f\*x)\*Coth[c + d\*x])/(a^2\*d) - (f\*Csch[c + d\*x])/(2\*a\*d^2) - ((e + f\*x)\*Coth[c + d\*x]\*Csch[c + d\*x])/(2\*a\*d) - (b^3\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/(a^3\*Sqrt[a^2 + b^2]\*d) + (b^3\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/(a^3\*Sqrt[a^2 + b^2]\*d) - (b\*f\*Log[Sinh[c + d\*x]])/(a^2\*d^2) + (f\*PolyLog[2, -E^(c + d\*x)])/(2\*a\*d^2) - (b^2\*f\*PolyLog[2, -E^(c + d\*x)])/(a^3\*d^2) - (f\*PolyLog[2, E^(c + d\*x)])/(2\*a\*d^2) + (b^2\*f\*PolyLog[2, E^(c + d\*x)])/(a^3\*d^2) - (b^3\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^3\*Sqrt[a^2 + b^2]\*d^2) + (b^3\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^3\*Sqrt[a^2 + b^2]\*d^2)

**Rubi [A]** time = 0.72555, antiderivative size = 420, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {5575, 4185, 4182, 2279, 2391, 4184, 3475, 3322, 2264, 2190}

$$-\frac{b^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3 d^2 \sqrt{a^2+b^2}} + \frac{b^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3 d^2 \sqrt{a^2+b^2}} - \frac{b^2 f \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{a^3 d^2} + \frac{b^2 f \operatorname{PolyLog}\left(2, e^{c+dx}\right)}{a^3 d^2} + \frac{f \operatorname{PolyLog}\left(2, e^{c+dx}\right)}{a^3 d^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Csch[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]), x]

[Out] ((e + f\*x)\*ArcTanh[E^(c + d\*x)]/(a\*d) - (2\*b^2\*(e + f\*x)\*ArcTanh[E^(c + d\*x)]/(a^3\*d) + (b\*(e + f\*x)\*Coth[c + d\*x])/(a^2\*d) - (f\*Csch[c + d\*x])/(2\*a\*d^2) - ((e + f\*x)\*Coth[c + d\*x]\*Csch[c + d\*x])/(2\*a\*d) - (b^3\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/(a^3\*Sqrt[a^2 + b^2]\*d) + (b^3\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/(a^3\*Sqrt[a^2 + b^2]\*d) - (b\*f\*Log[Sinh[c + d\*x]])/(a^2\*d^2) + (f\*PolyLog[2, -E^(c + d\*x)])/(2\*a\*d^2) - (b^2\*f\*PolyLog[2, -E^(c + d\*x)])/(a^3\*d^2) - (f\*PolyLog[2, E^(c + d\*x)])/(2\*a\*d^2) + (b^2\*f\*PolyLog[2, E^(c + d\*x)])/(a^3\*d^2) - (b^3\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^3\*Sqrt[a^2 + b^2]\*d^2) + (b^3\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^3\*Sqrt[a^2 + b^2]\*d^2)

$d^2) + (b^3*f*PolyLog[2, -((b*E^{(c + d*x)})/(a + Sqrt[a^2 + b^2]))]/(a^3*Sqrt[a^2 + b^2]*d^2)$

### Rule 5575

$Int[(Csch[(c_.) + (d_.)*(x_.)]^{(n_.)*((e_.) + (f_.)*(x_.))^{(m_.)}}/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_.)]), x\_Symbol] := Dist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^{(n - 1)})/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] \&\& IGtQ[m, 0] \&\& IGtQ[n, 0]$

### Rule 4185

$Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^{(n_.)*((c_.) + (d_.)*(x_.))}, x\_Symbol] := -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^{(n - 2)})/(f*(n - 1)), x] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^{(n - 2)}, x], x] - Simp[(b^2*d*(b*Csc[e + f*x])^{(n - 2)})/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] \&\& GtQ[n, 1] \&\& NeQ[n, 2]$

### Rule 4182

$Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^{-(I*e) + f*fz*x}]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^{(m - 1)}*Log[1 - E^{-(I*e) + f*fz*x}], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^{(m - 1)}*Log[1 + E^{-(I*e) + f*fz*x}], x], x]) /; FreeQ[{c, d, e, f, fz}, x] \&\& IGtQ[m, 0]$

### Rule 2279

$Int[Log[(a_.) + (b_.)*((F_)^{((e_.)*((c_.) + (d_.)*(x_.)))})^{(n_.)}], x\_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^{(e*(c + d*x)})^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] \&\& GtQ[a, 0]$

### Rule 2391

$Int[Log[(c_.)*((d_.) + (e_.)*(x_.)^{(n_.)})]/(x_.), x\_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] \&\& EqQ[c*d, 1]$

### Rule 4184

$Int[csc[(e_.) + (f_.)*(x_.)]^2*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] := -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^{(m - 1)}*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] \&\& GtQ[m, 0]$

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\operatorname{csch}^3(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\operatorname{csch}^3(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)} dx}{a} \\
&= -\frac{f\operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+fx)\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{\int (e+fx)\operatorname{csch}(c+dx) dx}{2a} - \frac{b \int \frac{(e+fx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)} dx}{a} \\
&= \frac{(e+fx)\tanh^{-1}(e^{c+dx})}{ad} + \frac{b(e+fx)\operatorname{coth}(c+dx)}{a^2d} - \frac{f\operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+fx)\operatorname{coth}(c+dx)}{2a} \\
&= \frac{(e+fx)\tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)\tanh^{-1}(e^{c+dx})}{a^3d} + \frac{b(e+fx)\operatorname{coth}(c+dx)}{a^2d} - \frac{f\operatorname{csch}(c+dx)}{2ad^2} \\
&= \frac{(e+fx)\tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)\tanh^{-1}(e^{c+dx})}{a^3d} + \frac{b(e+fx)\operatorname{coth}(c+dx)}{a^2d} - \frac{f\operatorname{csch}(c+dx)}{2ad^2} \\
&= \frac{(e+fx)\tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)\tanh^{-1}(e^{c+dx})}{a^3d} + \frac{b(e+fx)\operatorname{coth}(c+dx)}{a^2d} - \frac{f\operatorname{csch}(c+dx)}{2ad^2} \\
&= \frac{(e+fx)\tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)\tanh^{-1}(e^{c+dx})}{a^3d} + \frac{b(e+fx)\operatorname{coth}(c+dx)}{a^2d} - \frac{f\operatorname{csch}(c+dx)}{2ad^2} \\
&= \frac{(e+fx)\tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)\tanh^{-1}(e^{c+dx})}{a^3d} + \frac{b(e+fx)\operatorname{coth}(c+dx)}{a^2d} - \frac{f\operatorname{csch}(c+dx)}{2ad^2} \\
&= \frac{(e+fx)\tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)\tanh^{-1}(e^{c+dx})}{a^3d} + \frac{b(e+fx)\operatorname{coth}(c+dx)}{a^2d} - \frac{f\operatorname{csch}(c+dx)}{2ad^2}
\end{aligned}$$

**Mathematica [C]** time = 7.98397, size = 736, normalized size = 1.75

$$\frac{b^3 \left( -f \operatorname{PolyLog} \left( 2, \frac{be^{c+dx}}{\sqrt{a^2+b^2-a}} \right) + f \operatorname{PolyLog} \left( 2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2+a}} \right) + 2de \tanh^{-1} \left( \frac{a+be^{c+dx}}{\sqrt{a^2+b^2}} \right) - f(c+dx) \log \left( \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1 \right) + f(c+dx) \log \left( \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}} + 1 \right) \right)}{a^3 d^2 \sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Csch[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] ((2\*b\*d\*e\*Cosh[(c + d\*x)/2] - a\*f\*Cosh[(c + d\*x)/2] - 2\*b\*c\*f\*Cosh[(c + d\*x)/2] + 2\*b\*f\*(c + d\*x)\*Cosh[(c + d\*x)/2])\*Csch[(c + d\*x)/2])/(4\*a^2\*d^2) + ((-(d\*e) + c\*f - f\*(c + d\*x))\*Csch[(c + d\*x)/2]^2)/(8\*a\*d^2) - (b\*f\*Log[Sinh[c + d\*x]])/(a^2\*d^2) - (e\*Log[Tanh[(c + d\*x)/2]])/(2\*a\*d) + (b^2\*e\*Log[Tanh[(c + d\*x)/2]])/(a^3\*d) + (c\*f\*Log[Tanh[(c + d\*x)/2]])/(2\*a\*d^2) - (b^2\*c\*f\*Log[Tanh[(c + d\*x)/2]])/(a^3\*d^2) + ((I/2)\*f\*(I\*(c + d\*x)\*(Log[1 - E^(-c - d\*x)] - Log[1 + E^(-c - d\*x)]) + I\*(PolyLog[2, -E^(-c - d\*x)] - PolyLog[2, E^(-c - d\*x)])))/(a^3\*d^2)

$$\begin{aligned} & 2, E^{-c-dx}]])))/(a*d^2) - (I*b^2*f*(I*(c+dx)*(Log[1-E^{-c-dx}] \\ & - Log[1+E^{-c-dx}]) + I*(PolyLog[2, -E^{-c-dx}] - PolyLog[2, E^{-c- \\ & - dx]])))/(a^3*d^2) + (b^3*(2*d*e*ArcTanh[(a+bE^{c+dx})/Sqrt[a^2+b \\ & ^2]] - 2*c*f*ArcTanh[(a+bE^{c+dx})/Sqrt[a^2+b^2]] - f*(c+dx)*Log \\ & [1+(bE^{c+dx})/(a-Sqrt[a^2+b^2])] + f*(c+dx)*Log[1+(bE^{c+dx})/ \\ & (a+Sqrt[a^2+b^2])] - f*PolyLog[2, (bE^{c+dx})/(-a+Sqrt[a^2 \\ & + b^2])] + f*PolyLog[2, -(bE^{c+dx})/(a+Sqrt[a^2+b^2])])))/(a^3*S \\ & qrt[a^2+b^2]*d^2) + ((-(d*e) + c*f - f*(c+dx))*Sech[(c+dx)/2]^2)/(8 \\ & *a*d^2) + (Sech[(c+dx)/2]*(2*b*d*e*Sinh[(c+dx)/2] + a*f*Sinh[(c+dx) \\ & ]/2) - 2*b*c*f*Sinh[(c+dx)/2] + 2*b*f*(c+dx)*Sinh[(c+dx)/2]))/(4*a \\ & ^2*d^2) \end{aligned}$$

**Maple [B]** time = 0.162, size = 861, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x)`

[Out] 
$$\begin{aligned} & -(a*d*f*x*\exp(3*d*x+3*c)+a*d*e*\exp(3*d*x+3*c)-2*b*d*f*x*\exp(2*d*x+2*c)+a*d* \\ & f*x*\exp(d*x+c)+a*f*\exp(3*d*x+3*c)-2*b*d*e*\exp(2*d*x+2*c)+a*d*e*\exp(d*x+c)+2 \\ & *b*d*f*x-a*f*\exp(d*x+c)+2*b*d*e)/d^2/a^2/(\exp(2*d*x+2*c)-1)^2-1/a^3/d^2*b^3 \\ & *f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)} \\ & ))*c+1/a^3/d^2*b^3*f/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a \\ & +(a^2+b^2)^{(1/2)}))*c-1/a^3/d*b^2*f*\ln(\exp(d*x+c)+1)*x+2/a^3/d*b^3*e/(a^2+b^ \\ & 2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-2/a^3/d^2*b^3*f* \\ & c/(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-1/a^3/d \\ & *b^3*f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{( \\ & 1/2)}))*x+1/a^3/d*b^3*f/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/ \\ & (a+(a^2+b^2)^{(1/2)}))*x+1/2/d/a*\ln(\exp(d*x+c)+1)*f*x-1/2/d/a*e*\ln(\exp(d*x+c) \\ & -1)+1/2/d/a*e*\ln(\exp(d*x+c)+1)+1/2/d^2/a*f*c*\ln(\exp(d*x+c)-1)-1/a^3/d^2*b^2 \\ & *f*\operatorname{dilog}(\exp(d*x+c))-1/a^3/d^2*b^2*f*\operatorname{dilog}(\exp(d*x+c)+1)+2/a^2/d^2*b*f*\ln(e \\ & xp(d*x+c))+1/2/d^2*f/a*\operatorname{dilog}(\exp(d*x+c))+1/2/d^2*f/a*\operatorname{dilog}(\exp(d*x+c)+1)-1/ \\ & a^2/d^2*b*f*\ln(\exp(d*x+c)-1)-1/a^2/d^2*b*f*\ln(\exp(d*x+c)+1)+1/a^3/d*b^2*e*\ln \\ & (\exp(d*x+c)-1)-1/a^3/d*b^2*e*\ln(\exp(d*x+c)+1)-1/a^3/d^2*b^2*f*c*\ln(\exp(d*x \\ & +c)-1)-1/a^3/d^2*b^3*f/(a^2+b^2)^{(1/2)}*\operatorname{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)} \\ & -a)/(-a+(a^2+b^2)^{(1/2)}))+1/a^3/d^2*b^3*f/(a^2+b^2)^{(1/2)}*\operatorname{dilog}((b*\exp(d*x+ \\ & c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) \end{aligned}$$



**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.99808, size = 10647, normalized size = 25.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\frac{1}{2} * (4 * ((a^3 * b + a * b^3) * d * f * x + (a^3 * b + a * b^3) * c * f) * \cosh(d * x + c)^4 + 4 * ((a^3 * b + a * b^3) * d * f * x + (a^3 * b + a * b^3) * c * f) * \sinh(d * x + c)^4 - 2 * ((a^4 + a^2 * b^2) * d * f * x + (a^4 + a^2 * b^2) * d * e + (a^4 + a^2 * b^2) * f) * \cosh(d * x + c)^3 - 2 * ((a^4 + a^2 * b^2) * d * f * x + (a^4 + a^2 * b^2) * d * e + (a^4 + a^2 * b^2) * f) * \sinh(d * x + c)^3 - 4 * ((a^3 * b + a * b^3) * d * e + 4 * (a^3 * b + a * b^3) * c * f - 4 * ((a^3 * b + a * b^3) * d * f * x - (a^3 * b + a * b^3) * d * e + 2 * (a^3 * b + a * b^3) * c * f) * \cosh(d * x + c)^2 - 2 * (2 * (a^3 * b + a * b^3) * d * f * x - 2 * (a^3 * b + a * b^3) * d * e + 4 * (a^3 * b + a * b^3) * c * f - 12 * ((a^3 * b + a * b^3) * d * f * x + (a^3 * b + a * b^3) * c * f) * \cosh(d * x + c)^2 + 3 * ((a^4 + a^2 * b^2) * d * f * x + (a^4 + a^2 * b^2) * d * e + (a^4 + a^2 * b^2) * f) * \cosh(d * x + c)) * \sinh(d * x + c)^2 - 2 * (b^4 * f * \cosh(d * x + c)^4 + 4 * b^4 * f * \cosh(d * x + c) * \sinh(d * x + c)^3 + b^4 * f * \sinh(d * x + c)^4 - 2 * b^4 * f * \cosh(d * x + c)^2 + b^4 * f + 2 * (3 * b^4 * f * \cosh(d * x + c)^2 - b^4 * f) * \sinh(d * x + c)^2 + 4 * (b^4 * f * \cosh(d * x + c)^3 - b^4 * f * \cosh(d * x + c)) * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} * \operatorname{dilog}((a * \cosh(d * x + c) + a * \sinh(d * x + c) + (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b + 1) + 2 * (b^4 * f * \cosh(d * x + c)^4 + 4 * b^4 * f * \cosh(d * x + c) * \sinh(d * x + c)^3 + b^4 * f * \sinh(d * x + c)^4 - 2 * b^4 * f * \cosh(d * x + c)^2 + b^4 * f + 2 * (3 * b^4 * f * \cosh(d * x + c)^2 - b^4 * f) * \sinh(d * x + c)^2 + 4 * (b^4 * f * \cosh(d * x + c)^3 - b^4 * f * \cosh(d * x + c)) * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} * \operatorname{dilog}((a * \cosh(d * x + c) + a * \sinh(d * x + c) - (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b + 1) + 2 * (b^4 * d * e - b^4 * c * f + (b^4 * d * e - b^4 * c * f) * \cosh(d * x + c)^4 + 4 * (b^4 * d * e - b^4 * c * f) * \cosh(d * x + c) * \sinh(d * x + c)^3 + (b^4 * d * e - b^4 * c * f) * \sinh(d * x + c)^4 - 2 * (b^4 * d * e - b^4 * c * f) * \cosh(d * x + c)^2 - 2 * (b^4 * d * e - b^4 * c * f - 3 * (b^4 * d * e - b^4 * c * f) * \cosh(d * x + c)^2) * \sinh(d * x + c)^2 + 4 * ((b^4 * d * e -$$

$$\begin{aligned}
& b^4*c*f)*\cosh(d*x + c)^3 - (b^4*d*e - b^4*c*f)*\cosh(d*x + c))*\sinh(d*x + c) \\
& ))*\sqrt{(a^2 + b^2)/b^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - 2*(b^4*d*e - b^4*c*f + (b^4*d*e - b^4*c*f)*\cos \\
& h(d*x + c)^4 + 4*(b^4*d*e - b^4*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^4*d \\
& *e - b^4*c*f)*\sinh(d*x + c)^4 - 2*(b^4*d*e - b^4*c*f)*\cosh(d*x + c)^2 - 2*( \\
& b^4*d*e - b^4*c*f - 3*(b^4*d*e - b^4*c*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 \\
& + 4*((b^4*d*e - b^4*c*f)*\cosh(d*x + c)^3 - (b^4*d*e - b^4*c*f)*\cosh(d*x + c) \\
& ))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d* \\
& x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - 2*(b^4*d*f*x + b^4*c*f + (b^4*d \\
& *f*x + b^4*c*f)*\cosh(d*x + c)^4 + 4*(b^4*d*f*x + b^4*c*f)*\cosh(d*x + c)*\sin \\
& h(d*x + c)^3 + (b^4*d*f*x + b^4*c*f)*\sinh(d*x + c)^4 - 2*(b^4*d*f*x + b^4*c \\
& *f)*\cosh(d*x + c)^2 - 2*(b^4*d*f*x + b^4*c*f - 3*(b^4*d*f*x + b^4*c*f)*\cosh \\
& (d*x + c)^2)*\sinh(d*x + c)^2 + 4*((b^4*d*f*x + b^4*c*f)*\cosh(d*x + c)^3 - ( \\
& b^4*d*f*x + b^4*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2)*\log \\
& (-a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c)) \\
& *\sqrt{(a^2 + b^2)/b^2} - b)/b) + 2*(b^4*d*f*x + b^4*c*f + (b^4*d*f*x + b^4*c \\
& *f)*\cosh(d*x + c)^4 + 4*(b^4*d*f*x + b^4*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^ \\
& 3 + (b^4*d*f*x + b^4*c*f)*\sinh(d*x + c)^4 - 2*(b^4*d*f*x + b^4*c*f)*\cosh(d* \\
& x + c)^2 - 2*(b^4*d*f*x + b^4*c*f - 3*(b^4*d*f*x + b^4*c*f)*\cosh(d*x + c)^2) \\
& )*\sinh(d*x + c)^2 + 4*((b^4*d*f*x + b^4*c*f)*\cosh(d*x + c)^3 - (b^4*d*f*x + \\
& b^4*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2)*\log(-a*\cosh( \\
& d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 \\
& + b^2)/b^2} - b)/b) - 2*((a^4 + a^2*b^2)*d*f*x + (a^4 + a^2*b^2)*d*e - (a^4 \\
& + a^2*b^2)*f)*\cosh(d*x + c) - ((a^4 - a^2*b^2 - 2*b^4)*f*\cosh(d*x + c)^4 + \\
& 4*(a^4 - a^2*b^2 - 2*b^4)*f*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^4 - a^2*b^2 \\
& - 2*b^4)*f*\sinh(d*x + c)^4 - 2*(a^4 - a^2*b^2 - 2*b^4)*f*\cosh(d*x + c)^2 + \\
& 2*(3*(a^4 - a^2*b^2 - 2*b^4)*f*\cosh(d*x + c)^2 - (a^4 - a^2*b^2 - 2*b^4)*f \\
& )*\sinh(d*x + c)^2 + (a^4 - a^2*b^2 - 2*b^4)*f + 4*((a^4 - a^2*b^2 - 2*b^4)* \\
& f*\cosh(d*x + c)^3 - (a^4 - a^2*b^2 - 2*b^4)*f*\cosh(d*x + c))*\sinh(d*x + c) \\
& )*\operatorname{dilog}(\cosh(d*x + c) + \sinh(d*x + c)) + ((a^4 - a^2*b^2 - 2*b^4)*f*\cosh(d*x \\
& + c)^4 + 4*(a^4 - a^2*b^2 - 2*b^4)*f*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^4 \\
& - a^2*b^2 - 2*b^4)*f*\sinh(d*x + c)^4 - 2*(a^4 - a^2*b^2 - 2*b^4)*f*\cosh(d*x \\
& + c)^2 + 2*(3*(a^4 - a^2*b^2 - 2*b^4)*f*\cosh(d*x + c)^2 - (a^4 - a^2*b^2 - \\
& 2*b^4)*f)*\sinh(d*x + c)^2 + (a^4 - a^2*b^2 - 2*b^4)*f + 4*((a^4 - a^2*b^2 \\
& - 2*b^4)*f*\cosh(d*x + c)^3 - (a^4 - a^2*b^2 - 2*b^4)*f*\cosh(d*x + c))*\sinh \\
& (d*x + c))*\operatorname{dilog}(-\cosh(d*x + c) - \sinh(d*x + c)) + (((a^4 - a^2*b^2 - 2*b^4) \\
& *d*f*x + (a^4 - a^2*b^2 - 2*b^4)*d*e - 2*(a^3*b + a*b^3)*f)*\cosh(d*x + c)^4 \\
& + 4*((a^4 - a^2*b^2 - 2*b^4)*d*f*x + (a^4 - a^2*b^2 - 2*b^4)*d*e - 2*(a^3*b \\
& + a*b^3)*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^4 - a^2*b^2 - 2*b^4)*d*f*x \\
& + (a^4 - a^2*b^2 - 2*b^4)*d*e - 2*(a^3*b + a*b^3)*f)*\sinh(d*x + c)^4 + (a \\
& ^4 - a^2*b^2 - 2*b^4)*d*f*x + (a^4 - a^2*b^2 - 2*b^4)*d*e - 2*((a^4 - a^2*b \\
& ^2 - 2*b^4)*d*f*x + (a^4 - a^2*b^2 - 2*b^4)*d*e - 2*(a^3*b + a*b^3)*f)*\cosh \\
& (d*x + c)^2 - 2*((a^4 - a^2*b^2 - 2*b^4)*d*f*x + (a^4 - a^2*b^2 - 2*b^4)*d* \\
& e - 3*((a^4 - a^2*b^2 - 2*b^4)*d*f*x + (a^4 - a^2*b^2 - 2*b^4)*d*e - 2*(a^3 \\
& *b + a*b^3)*f)*\cosh(d*x + c)^2 - 2*(a^3*b + a*b^3)*f)*\sinh(d*x + c)^2 - 2*(
\end{aligned}$$

$$\begin{aligned}
& a^3b + ab^3) * f + 4 * ((a^4 - a^2b^2 - 2b^4) * d * f * x + (a^4 - a^2b^2 - 2b^4) * d * e - 2 * (a^3b + ab^3) * f) * \cosh(d * x + c)^3 - ((a^4 - a^2b^2 - 2b^4) * d * f * x + (a^4 - a^2b^2 - 2b^4) * d * e - 2 * (a^3b + ab^3) * f) * \cosh(d * x + c) * \sinh(d * x + c) * \log(\cosh(d * x + c) + \sinh(d * x + c) + 1) - ((a^4 - a^2b^2 - 2b^4) * d * e + (2 * a^3b + 2 * ab^3 - (a^4 - a^2b^2 - 2b^4) * c) * f) * \cosh(d * x + c)^4 + 4 * ((a^4 - a^2b^2 - 2b^4) * d * e + (2 * a^3b + 2 * ab^3 - (a^4 - a^2b^2 - 2b^4) * c) * f) * \cosh(d * x + c) * \sinh(d * x + c)^3 + ((a^4 - a^2b^2 - 2b^4) * d * e + (2 * a^3b + 2 * ab^3 - (a^4 - a^2b^2 - 2b^4) * c) * f) * \sinh(d * x + c)^4 + (a^4 - a^2b^2 - 2b^4) * d * e - 2 * ((a^4 - a^2b^2 - 2b^4) * d * e + (2 * a^3b + 2 * ab^3 - (a^4 - a^2b^2 - 2b^4) * c) * f) * \cosh(d * x + c)^2 - 2 * ((a^4 - a^2b^2 - 2b^4) * d * e - 3 * ((a^4 - a^2b^2 - 2b^4) * d * e + (2 * a^3b + 2 * ab^3 - (a^4 - a^2b^2 - 2b^4) * c) * f) * \cosh(d * x + c)^2 + (2 * a^3b + 2 * ab^3 - (a^4 - a^2b^2 - 2b^4) * c) * f) * \sinh(d * x + c)^2 + (2 * a^3b + 2 * ab^3 - (a^4 - a^2b^2 - 2b^4) * c) * f + 4 * (((a^4 - a^2b^2 - 2b^4) * d * e + (2 * a^3b + 2 * ab^3 - (a^4 - a^2b^2 - 2b^4) * c) * f) * \cosh(d * x + c)^3 - ((a^4 - a^2b^2 - 2b^4) * d * e + (2 * a^3b + 2 * ab^3 - (a^4 - a^2b^2 - 2b^4) * c) * f) * \cosh(d * x + c)) * \sinh(d * x + c) * \log(\cosh(d * x + c) + \sinh(d * x + c) - 1) - (((a^4 - a^2b^2 - 2b^4) * d * f * x + (a^4 - a^2b^2 - 2b^4) * c * f) * \cosh(d * x + c)^4 + 4 * ((a^4 - a^2b^2 - 2b^4) * d * f * x + (a^4 - a^2b^2 - 2b^4) * c * f) * \cosh(d * x + c) * \sinh(d * x + c)^3 + ((a^4 - a^2b^2 - 2b^4) * d * f * x + (a^4 - a^2b^2 - 2b^4) * c * f) * \sinh(d * x + c)^4 + (a^4 - a^2b^2 - 2b^4) * d * f * x + (a^4 - a^2b^2 - 2b^4) * c * f - 2 * ((a^4 - a^2b^2 - 2b^4) * d * f * x + (a^4 - a^2b^2 - 2b^4) * c * f) * \cosh(d * x + c)^2 - 2 * ((a^4 - a^2b^2 - 2b^4) * d * f * x + (a^4 - a^2b^2 - 2b^4) * c * f - 3 * ((a^4 - a^2b^2 - 2b^4) * d * f * x + (a^4 - a^2b^2 - 2b^4) * c * f) * \cosh(d * x + c)^2) * \sinh(d * x + c)^2 + 4 * (((a^4 - a^2b^2 - 2b^4) * d * f * x + (a^4 - a^2b^2 - 2b^4) * c * f) * \cosh(d * x + c)^3 - ((a^4 - a^2b^2 - 2b^4) * d * f * x + (a^4 - a^2b^2 - 2b^4) * c * f) * \cosh(d * x + c)) * \sinh(d * x + c) * \log(-\cosh(d * x + c) - \sinh(d * x + c) + 1) - 2 * ((a^4 + a^2b^2) * d * f * x - 8 * ((a^3b + ab^3) * d * f * x + (a^3b + ab^3) * c * f) * \cosh(d * x + c)^3 + (a^4 + a^2b^2) * d * e + 3 * ((a^4 + a^2b^2) * d * f * x + (a^4 + a^2b^2) * d * e + (a^4 + a^2b^2) * f) * \cosh(d * x + c)^2 - (a^4 + a^2b^2) * f + 4 * ((a^3b + ab^3) * d * f * x - (a^3b + ab^3) * d * e + 2 * (a^3b + ab^3) * c * f) * \cosh(d * x + c)) * \sinh(d * x + c)) / ((a^5 + a^3b^2) * d^2 * \cosh(d * x + c)^4 + 4 * (a^5 + a^3b^2) * d^2 * \cosh(d * x + c) * \sinh(d * x + c)^3 + (a^5 + a^3b^2) * d^2 * \sinh(d * x + c)^4 - 2 * (a^5 + a^3b^2) * d^2 * \cosh(d * x + c)^2 + (a^5 + a^3b^2) * d^2 + 2 * (3 * (a^5 + a^3b^2) * d^2 * \cosh(d * x + c)^2 - (a^5 + a^3b^2) * d^2) * \sinh(d * x + c)^2 + 4 * ((a^5 + a^3b^2) * d^2 * \cosh(d * x + c)^3 - (a^5 + a^3b^2) * d^2 * \cosh(d * x + c)) * \sinh(d * x + c))
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.251 \quad \int \frac{\operatorname{csch}^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=113

$$\frac{2b^3 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3 d \sqrt{a^2+b^2}} + \frac{(a^2-2b^2) \tanh^{-1}(\cosh(c+dx))}{2a^3 d} + \frac{b \coth(c+dx)}{a^2 d} - \frac{\coth(c+dx) \operatorname{csch}(c+dx)}{2ad}$$

[Out]  $((a^2 - 2*b^2)*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/(2*a^3*d) + (2*b^3*\operatorname{ArcTanh}[(b - a*\operatorname{Tanh}[(c + d*x)/2])/ \operatorname{Sqrt}[a^2 + b^2]])/(a^3*\operatorname{Sqrt}[a^2 + b^2]*d) + (b*\operatorname{Coth}[c + d*x])/ (a^2*d) - (\operatorname{Coth}[c + d*x]*\operatorname{Csch}[c + d*x])/(2*a*d)$

**Rubi [A]** time = 0.404259, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2802, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b^3 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3 d \sqrt{a^2+b^2}} + \frac{(a^2-2b^2) \tanh^{-1}(\cosh(c+dx))}{2a^3 d} + \frac{b \coth(c+dx)}{a^2 d} - \frac{\coth(c+dx) \operatorname{csch}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\operatorname{Csch}[c + d*x]^3/(a + b*\operatorname{Sinh}[c + d*x]), x]$

[Out]  $((a^2 - 2*b^2)*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/(2*a^3*d) + (2*b^3*\operatorname{ArcTanh}[(b - a*\operatorname{Tanh}[(c + d*x)/2])/ \operatorname{Sqrt}[a^2 + b^2]])/(a^3*\operatorname{Sqrt}[a^2 + b^2]*d) + (b*\operatorname{Coth}[c + d*x])/ (a^2*d) - (\operatorname{Coth}[c + d*x]*\operatorname{Csch}[c + d*x])/(2*a*d)$

### Rule 2802

$\operatorname{Int}[(a + b*\sin[e + f*x])^m * (c + d*\sin[e + f*x])^n, x\_Symbol] := -\operatorname{Simp}[(b^2*\operatorname{Cos}[e + f*x]*(a + b*\sin[e + f*x])^{m+1} * (c + d*\sin[e + f*x])^{n+1}) / (f*(m+1)*(b*c - a*d)*(a^2 - b^2)), x] + \operatorname{Dist}[1 / ((m+1)*(b*c - a*d)*(a^2 - b^2)), \operatorname{Int}[(a + b*\sin[e + f*x])^{m+1} * (c + d*\sin[e + f*x])^n * \operatorname{Simp}[a*(b*c - a*d)*(m+1) + b^2*d*(m+n+2) - (b^2*c + b*(b*c - a*d)*(m+1))*\sin[e + f*x] - b^2*d*(m+n+3)*\sin[e + f*x]^2, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, n\}, x \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{NeQ}[a^2 - b^2, 0] \&\& \operatorname{NeQ}[c^2 - d^2, 0] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{IntegersQ}[2*m, 2*n] \&\& ((\operatorname{EqQ}[a, 0] \&\& \operatorname{IntegerQ}[m] \&\& \operatorname{!IntegerQ}[n]) || \operatorname{!(IntegerQ}[2*n])$

&& LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]))

### Rule 3055

Int[((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(m\_)\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((A\_.) + (B\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]) + (C\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^2), x\_Symbol] := -Simp[((A\*b^2 - a\*b\*B + a^2\*C)\*Cos[e + f\*x] \*(a + b\*Sin[e + f\*x])^(m + 1)\*(c + d\*Sin[e + f\*x])^(n + 1))/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 - b^2)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 - b^2)), Int[(a + b\*Sin[e + f\*x])^(m + 1)\*(c + d\*Sin[e + f\*x])^n\*Simp[(m + 1)\*(b\*c - a\*d)\*(a\*A - b\*B + a\*C) + d\*(A\*b^2 - a\*b\*B + a^2\*C)\*(m + n + 2) - (c\*(A\*b^2 - a\*b\*B + a^2\*C) + (m + 1)\*(b\*c - a\*d)\*(A\*b - a\*B + b\*C))\*Sin[e + f\*x] - d\*(A\*b^2 - a\*b\*B + a^2\*C)\*(m + n + 3)\*Sin[e + f\*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2\*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))

### Rule 3001

Int[((A\_.) + (B\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])/((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Dist[(A\*b - a\*B)/(b\*c - a\*d), Int[1/(a + b\*Sin[e + f\*x]), x], x] + Dist[(B\*c - A\*d)/(b\*c - a\*d), Int[1/(c + d\*Sin[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b\*c - a\*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

### Rule 2660

Int[((a\_.) + (b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + 2\*b\*e\*x + a\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^3(c+dx)}{a+b \sinh(c+dx)} dx &= -\frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad} + \frac{i \int \frac{\operatorname{csch}^2(c+dx)(2ib+ia \sinh(c+dx)+ib \sinh^2(c+dx))}{a+b \sinh(c+dx)} dx}{2a} \\
&= \frac{b \operatorname{coth}(c+dx)}{a^2d} - \frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{\int \frac{\operatorname{csch}(c+dx)(a^2-2b^2+ab \sinh(c+dx))}{a+b \sinh(c+dx)} dx}{2a^2} \\
&= \frac{b \operatorname{coth}(c+dx)}{a^2d} - \frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{b^3 \int \frac{1}{a+b \sinh(c+dx)} dx}{a^3} - \frac{(a^2-2b^2) \int \operatorname{csch}(c+dx)}{2a^3} \\
&= \frac{(a^2-2b^2) \tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{b \operatorname{coth}(c+dx)}{a^2d} - \frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad} + \frac{(2ib^3) \operatorname{Su}}{2a^3} \\
&= \frac{(a^2-2b^2) \tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{b \operatorname{coth}(c+dx)}{a^2d} - \frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{(4ib^3) \operatorname{Su}}{2a^3} \\
&= \frac{(a^2-2b^2) \tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{2b^3 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3\sqrt{a^2+b^2}d} + \frac{b \operatorname{coth}(c+dx)}{a^2d} - \frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad}
\end{aligned}$$

**Mathematica [A]** time = 1.94352, size = 145, normalized size = 1.28

$$\frac{4(a^2-2b^2) \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + \frac{16b^3 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + a^2 \operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right) + a^2 \operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right) - 4ab \tanh\left(\frac{1}{2}(c+dx)\right)}{8a^3d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d\*x]^3/(a + b\*Sinh[c + d\*x]), x]

[Out] -((16\*b^3\*ArcTan[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 4\*a\*b\*Coth[(c + d\*x)/2] + a^2\*Csch[(c + d\*x)/2]^2 + 4\*(a^2 - 2\*b^2)\*Log[Tanh[(c + d\*x)/2]] + a^2\*Sech[(c + d\*x)/2]^2 - 4\*a\*b\*Tanh[(c + d\*x)/2])/(8\*a^3\*d)

---

**Maple [A]** time = 0.003, size = 164, normalized size = 1.5

$$\frac{1}{8da} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2 + \frac{b}{2da^2} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 2 \frac{b^3}{da^3 \sqrt{a^2 + b^2}} \operatorname{Artanh}\left(\frac{1}{2} \frac{2a \tanh(1/2 dx + c/2) - 2b}{\sqrt{a^2 + b^2}}\right) - \frac{1}{8da} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

[Out] 1/8/d/a\*tanh(1/2\*d\*x+1/2\*c)^2+1/2/d/a^2\*tanh(1/2\*d\*x+1/2\*c)\*b-2/d/a^3\*b^3/(a^2+b^2)^(1/2)\*arctanh(1/2\*(2\*a\*tanh(1/2\*d\*x+1/2\*c)-2\*b)/(a^2+b^2)^(1/2))-1/8/d/a/tanh(1/2\*d\*x+1/2\*c)^2-1/2/d/a\*ln(tanh(1/2\*d\*x+1/2\*c))+1/d/a^3\*ln(tanh(1/2\*d\*x+1/2\*c))\*b^2+1/2/d\*b/a^2/tanh(1/2\*d\*x+1/2\*c)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [B]** time = 3.02522, size = 2844, normalized size = 25.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] -1/2\*(4\*a^3\*b + 4\*a\*b^3 + 2\*(a^4 + a^2\*b^2)\*cosh(d\*x + c)^3 + 2\*(a^4 + a^2\*b^2)\*sinh(d\*x + c)^3 - 4\*(a^3\*b + a\*b^3)\*cosh(d\*x + c)^2 - 2\*(2\*a^3\*b + 2\*a\*b^3 - 3\*(a^4 + a^2\*b^2)\*cosh(d\*x + c))\*sinh(d\*x + c)^2 - 2\*(b^3\*cosh(d\*x + c)^4 + 4\*b^3\*cosh(d\*x + c)\*sinh(d\*x + c)^3 + b^3\*sinh(d\*x + c)^4 - 2\*b^3\*cosh(d\*x + c)^2 + b^3 + 2\*(3\*b^3\*cosh(d\*x + c)^2 - b^3)\*sinh(d\*x + c)^2 + 4\*



```
(b^3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a^4 + a^2*b^2)*cosh(d*x + c) - ((a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^4 + 4*(a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 - a^2*b^2 - 2*b^4)*sinh(d*x + c)^4 + a^4 - a^2*b^2 - 2*b^4 - 2*(a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^2 - 2*(a^4 - a^2*b^2 - 2*b^4 - 3*(a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^3 - (a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^4 + 4*(a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 - a^2*b^2 - 2*b^4)*sinh(d*x + c)^4 + a^4 - a^2*b^2 - 2*b^4 - 2*(a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^2 - 2*(a^4 - a^2*b^2 - 2*b^4 - 3*(a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c)^3 - (a^4 - a^2*b^2 - 2*b^4)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(a^4 + a^2*b^2 + 3*(a^4 + a^2*b^2)*cosh(d*x + c)^2 - 4*(a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^5 + a^3*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^5 + a^3*b^2)*d*sinh(d*x + c)^4 - 2*(a^5 + a^3*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^5 + a^3*b^2)*d*cosh(d*x + c)^2 - (a^5 + a^3*b^2)*d)*sinh(d*x + c)^2 + (a^5 + a^3*b^2)*d + 4*((a^5 + a^3*b^2)*d*cosh(d*x + c)^3 - (a^5 + a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)), x)

[Out] Timed out

**Giac [A]** time = 1.2314, size = 248, normalized size = 2.19

$$-\frac{b^3 \log\left(\frac{|2be^{(dx+c)}+2a-2\sqrt{a^2+b^2}|}{|2be^{(dx+c)}+2a+2\sqrt{a^2+b^2}|}\right)}{\sqrt{a^2+b^2}a^3d} + \frac{(a^2-2b^2)\log(e^{(dx+c)}+1)}{2a^3d} - \frac{(a^2-2b^2)\log(|e^{(dx+c)}-1|)}{2a^3d} - \frac{ae^{(3dx+3c)}-2be^{(2dx+2c)}}{a^2d(e^{(2dx+2c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c)
+ 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^3*d) + 1/2*(a^2 - 2*b^2)*lo
g(e^(d*x + c) + 1)/(a^3*d) - 1/2*(a^2 - 2*b^2)*log(abs(e^(d*x + c) - 1))/(a
^3*d) - (a*e^(3*d*x + 3*c) - 2*b*e^(2*d*x + 2*c) + a*e^(d*x + c) + 2*b)/(a^
2*d*(e^(2*d*x + 2*c) - 1)^2)
```

$$3.252 \quad \int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Optimal. Leaf size=30

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Csch[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0793883, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Csch[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] Defer[Int][Csch[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\operatorname{csch}^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.001, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[Csch[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] \$Aborted

---

**Maple [A]** time = 1.941, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx+c))^3}{(fx+e)(a+b\sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)`

[Out] `int(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `-8*b^3*integrate(-1/4*e^(d*x + c)/(a^3*b*f*x + a^3*b*e - (a^3*b*f*x*e^(2*c) + a^3*b*e*e^(2*c))*e^(2*d*x) - 2*(a^4*f*x*e^c + a^4*e*e^c)*e^(d*x)), x) - (2*b*d*f*x + 2*b*d*e + (a*d*f*x*e^(3*c) + (d*e - f)*a*e^(3*c))*e^(3*d*x) - 2*(b*d*f*x*e^(2*c) + b*d*e*e^(2*c))*e^(2*d*x) + (a*d*f*x*e^c + (d*e + f)*a*e^c)*e^(d*x))/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e^2 + (a^2*d^2*f^2*x^2*e^(4*c) + 2*a^2*d^2*e*f*x*e^(4*c) + a^2*d^2*e^2*e^(4*c))*e^(4*d*x) - 2*(a^2*d^2*f^2*x^2*e^(2*c) + 2*a^2*d^2*e*f*x*e^(2*c) + a^2*d^2*e^2*e^(2*c))*e^(2*d*x)) - 8*integrate(1/16*(2*b^2*d^2*e^2 + 2*a*b*d*e*f - (d^2*e^2 - 2*f^2)*a^2 - (a^2*d^2*f^2 - 2*b^2*d^2*f^2)*x^2 - 2*(a^2*d^2*e*f - 2*b^2*d^2*e*f - a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) - 8*integrate(-1/16*(2*b^2*d^2*e^2 - 2*a*b*d*e*f - (d^2*e^2 - 2*f^2)*a^2 - (a^2*d^2*f^2 - 2*b^2*d^2*f^2)*x^2 - 2*(a^2*d^2*e*f - 2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{csch}(dx+c)^3}{afx+ae+(bfx+be)\sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] integral(csch(d\*x + c)^3/(a\*f\*x + a\*e + (b\*f\*x + b\*e)\*sinh(d\*x + c)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)\*\*3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.253 \quad \int \frac{(e+fx)^3 \cosh(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=139

$$\frac{12if^2(e+fx)\text{PolyLog}\left(3, -ie^{c+dx}\right)}{ad^3} - \frac{6if(e+fx)^2\text{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^2} - \frac{12if^3\text{PolyLog}\left(4, -ie^{c+dx}\right)}{ad^4} - \frac{2i(e+fx)^3 \log\left(\frac{a+ie^{c+dx}}{a-ie^{c+dx}}\right)}{ad}$$

[Out]  $((I/4)*(e + f*x)^4)/(a*f) - ((2*I)*(e + f*x)^3*\text{Log}[1 + I*E^{(c + d*x)}])/(a*d) - ((6*I)*f*(e + f*x)^2*\text{PolyLog}[2, (-I)*E^{(c + d*x)}])/(a*d^2) + ((12*I)*f^2*(e + f*x)*\text{PolyLog}[3, (-I)*E^{(c + d*x)}])/(a*d^3) - ((12*I)*f^3*\text{PolyLog}[4, (-I)*E^{(c + d*x)}])/(a*d^4)$

**Rubi [A]** time = 0.212735, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {5559, 2190, 2531, 6609, 2282, 6589}

$$\frac{12if^2(e+fx)\text{PolyLog}\left(3, -ie^{c+dx}\right)}{ad^3} - \frac{6if(e+fx)^2\text{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^2} - \frac{12if^3\text{PolyLog}\left(4, -ie^{c+dx}\right)}{ad^4} - \frac{2i(e+fx)^3 \log\left(\frac{a+ie^{c+dx}}{a-ie^{c+dx}}\right)}{ad}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(e + f*x)^3*\text{Cosh}[c + d*x]/(a + I*a*\text{Sinh}[c + d*x]), x]$

[Out]  $((I/4)*(e + f*x)^4)/(a*f) - ((2*I)*(e + f*x)^3*\text{Log}[1 + I*E^{(c + d*x)}])/(a*d) - ((6*I)*f*(e + f*x)^2*\text{PolyLog}[2, (-I)*E^{(c + d*x)}])/(a*d^2) + ((12*I)*f^2*(e + f*x)*\text{PolyLog}[3, (-I)*E^{(c + d*x)}])/(a*d^3) - ((12*I)*f^3*\text{PolyLog}[4, (-I)*E^{(c + d*x)}])/(a*d^4)$

### Rule 5559

$\text{Int}[(\text{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.)^{(m_.)}))/((a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_.)]), x\_Symbol] \rightarrow -\text{Simp}[(e + f*x)^{(m + 1)}/(b*f*(m + 1)), x] + \text{Dist}[2, \text{Int}[(e + f*x)^m*E^{(c + d*x)}]/(a + b*E^{(c + d*x)}), x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{EqQ}[a^2 + b^2, 0]$

### Rule 2190

$\text{Int}[(((F_.)^{((g_.)*((e_.) + (f_.)*(x_.)}))^{(n_.)*((c_.) + (d_.)*(x_.)^{(m_.)})})/((a_.) + (b_.)*((F_.)^{((g_.)*((e_.) + (f_.)*(x_.)}))^{(n_.)})), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]/(b*f*g^n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g^n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a], x], x]$

)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(p\_.)], x\_Symbol] := Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x))))^p]]/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x))))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \cosh(c+dx)}{a+ia \sinh(c+dx)} dx &= \frac{i(e+fx)^4}{4af} + 2 \int \frac{e^{c+dx}(e+fx)^3}{a+iae^{c+dx}} dx \\
&= \frac{i(e+fx)^4}{4af} - \frac{2i(e+fx)^3 \log(1+ie^{c+dx})}{ad} + \frac{(6if) \int (e+fx)^2 \log(1+ie^{c+dx}) dx}{ad} \\
&= \frac{i(e+fx)^4}{4af} - \frac{2i(e+fx)^3 \log(1+ie^{c+dx})}{ad} - \frac{6if(e+fx)^2 \text{Li}_2(-ie^{c+dx})}{ad^2} + \frac{(12if^2) \int (e+fx)}{ad^3} \\
&= \frac{i(e+fx)^4}{4af} - \frac{2i(e+fx)^3 \log(1+ie^{c+dx})}{ad} - \frac{6if(e+fx)^2 \text{Li}_2(-ie^{c+dx})}{ad^2} + \frac{12if^2(e+fx) \text{Li}_3}{ad^3} \\
&= \frac{i(e+fx)^4}{4af} - \frac{2i(e+fx)^3 \log(1+ie^{c+dx})}{ad} - \frac{6if(e+fx)^2 \text{Li}_2(-ie^{c+dx})}{ad^2} + \frac{12if^2(e+fx) \text{Li}_3}{ad^3} \\
&= \frac{i(e+fx)^4}{4af} - \frac{2i(e+fx)^3 \log(1+ie^{c+dx})}{ad} - \frac{6if(e+fx)^2 \text{Li}_2(-ie^{c+dx})}{ad^2} + \frac{12if^2(e+fx) \text{Li}_3}{ad^3}
\end{aligned}$$

**Mathematica [A]** time = 0.0779278, size = 118, normalized size = 0.85

$$\frac{i \left( -\frac{24f(d^2(e+fx)^2 \text{PolyLog}(2, -ie^{c+dx}) - 2df(e+fx) \text{PolyLog}(3, -ie^{c+dx}) + 2f^2 \text{PolyLog}(4, -ie^{c+dx}))}{d^4} - \frac{8(e+fx)^3 \log(1+ie^{c+dx})}{d} + \frac{(e+fx)^4}{f} \right)}{4a}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^3\*Cosh[c + d\*x])/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] ((I/4)\*((e + f\*x)^4/f - (8\*(e + f\*x)^3\*Log[1 + I\*E^(c + d\*x)]))/d - (24\*f\*(d^2\*(e + f\*x)^2\*PolyLog[2, (-I)\*E^(c + d\*x)] - 2\*d\*f\*(e + f\*x)\*PolyLog[3, (-I)\*E^(c + d\*x)] + 2\*f^2\*PolyLog[4, (-I)\*E^(c + d\*x)]))/d^4)/a

**Maple [B]** time = 0.118, size = 635, normalized size = 4.6

$$\frac{ief^2x^3}{a} + \frac{6ie^2fcx}{da} - \frac{6ief^2c^2x}{ad^2} - \frac{6ie^2fc \ln(e^{dx+c})}{ad^2} + \frac{6ief^2c^2 \ln(e^{dx+c})}{ad^3} + \frac{6ie^2fc \ln(e^{dx+c} - i)}{ad^2} - \frac{6ief^2c^2 \ln(e^{dx+c} - i)}{ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*cosh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)



```
[Out] I/a*e*f^2*x^3+6*I/a/d*e^2*f*c*x-6*I/a/d^2*e*f^2*c^2*x-6*I/a/d^2*e^2*f*c*ln(
exp(d*x+c))+6*I/a/d^3*e*f^2*c^2*ln(exp(d*x+c))+6*I/a/d^2*e^2*f*c*ln(exp(d*x
+c))-I)-6*I/a/d^3*e*f^2*c^2*ln(exp(d*x+c))-I)-6*I/a/d*e^2*f*ln(1+I*exp(d*x+c)
)*x-6*I/a/d^2*e^2*f*ln(1+I*exp(d*x+c))*c-6*I/a/d*e*f^2*ln(1+I*exp(d*x+c))*x
^2+6*I/a/d^3*e*f^2*ln(1+I*exp(d*x+c))*c^2-12*I/a/d^2*e*f^2*polylog(2,-I*exp
(d*x+c))*x-12*I*f^3*polylog(4,-I*exp(d*x+c))/a/d^4+1/4*I/a*x^4*f^3+3/2*I/a*
e^2*f*x^2-I/a*e^3*x+2*I/a/d^3*f^3*c^3*x+2*I/a/d^4*f^3*c^3*ln(exp(d*x+c))-
4*I/a/d^3*e*f^2*c^3+3*I/a/d^2*e^2*f*c^2-6*I/a/d^2*e^2*f*polylog(2,-I*exp(d*
x+c))+12*I/a/d^3*e*f^2*polylog(3,-I*exp(d*x+c))-2*I/a/d^4*f^3*c^3*ln(exp(d*
x+c))-2*I/a/d^4*f^3*c^3*ln(1+I*exp(d*x+c))-2*I/a/d*f^3*ln(1+I*exp(d*x+c))*x
^3-6*I/a/d^2*f^3*polylog(2,-I*exp(d*x+c))*x^2+12*I/a/d^3*f^3*polylog(3,-I*
exp(d*x+c))*x-2*I/a/d*ln(exp(d*x+c))-I)*e^3+3/2*I/a/d^4*f^3*c^4+2*I/a/d*ln(ex
p(d*x+c))*e^3
```

**Maxima [B]** time = 1.64123, size = 356, normalized size = 2.56

$$\frac{i e^3 \log(i a \sinh(dx + c) + a)}{ad} - \frac{6i \left(dx \log\left(i e^{(dx+c)} + 1\right) + \operatorname{Li}_2\left(-i e^{(dx+c)}\right)\right) e^2 f}{ad^2} - \frac{i \left(f^3 x^4 + 4 e f^2 x^3 + 6 e^2 f x^2\right)}{4 a} - \frac{6i \left(d^2 x^4 + 4 d x^3 + 6 x^2 + 6\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -I*e^3*log(I*a*sinh(d*x + c) + a)/(a*d) - 6*I*(d*x*log(I*e^(d*x + c) + 1) +
dilog(-I*e^(d*x + c)))*e^2*f/(a*d^2) - 1/4*I*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^
2*f*x^2)/a - 6*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x +
c)) - 2*polylog(3, -I*e^(d*x + c)))*e*f^2/(a*d^3) - 2*I*(d^3*x^3*log(I*e^(d
*x + c) + 1) + 3*d^2*x^2*dilog(-I*e^(d*x + c)) - 6*d*x*polylog(3, -I*e^(d*x
+ c)) + 6*polylog(4, -I*e^(d*x + c)))*f^3/(a*d^4) + 1/2*(I*d^4*f^3*x^4 + 4
*I*d^4*e*f^2*x^3 + 6*I*d^4*e^2*f*x^2)/(a*d^4)
```

**Fricas [C]** time = 2.17348, size = 752, normalized size = 5.41

$$\frac{i d^4 f^3 x^4 + 4 i d^4 e f^2 x^3 + 6 i d^4 e^2 f x^2 + 4 i d^4 e^3 x + 8 i c d^3 e^3 - 12 i c^2 d^2 e^2 f + 8 i c^3 d e f^2 - 2 i c^4 f^3 - 48 i f^3 \operatorname{polylog}\left(4, -i e^{(dx+c)}\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(I*d^4*f^3*x^4 + 4*I*d^4*e*f^2*x^3 + 6*I*d^4*e^2*f*x^2 + 4*I*d^4*e^3*x
+ 8*I*c*d^3*e^3 - 12*I*c^2*d^2*e^2*f + 8*I*c^3*d*e*f^2 - 2*I*c^4*f^3 - 48*I
*f^3*polylog(4, -I*e^(d*x + c)) + (-24*I*d^2*f^3*x^2 - 48*I*d^2*e*f^2*x - 2
4*I*d^2*e^2*f)*dilog(-I*e^(d*x + c)) + (-8*I*d^3*e^3 + 24*I*c*d^2*e^2*f - 2
4*I*c^2*d*e*f^2 + 8*I*c^3*f^3)*log(e^(d*x + c) - I) + (-8*I*d^3*f^3*x^3 - 2
4*I*d^3*e*f^2*x^2 - 24*I*d^3*e^2*f*x - 24*I*c*d^2*e^2*f + 24*I*c^2*d*e*f^2
- 8*I*c^3*f^3)*log(I*e^(d*x + c) + 1) + (48*I*d*f^3*x + 48*I*d*e*f^2)*polyl
og(3, -I*e^(d*x + c)))/(a*d^4)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c)}{ia \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

$$3.254 \quad \int \frac{(e+fx)^2 \cosh(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=106

$$-\frac{4if(e+fx)\text{PolyLog}(2, -ie^{c+dx})}{ad^2} + \frac{4if^2\text{PolyLog}(3, -ie^{c+dx})}{ad^3} - \frac{2i(e+fx)^2 \log(1+ie^{c+dx})}{ad} + \frac{i(e+fx)^3}{3af}$$

[Out] ((I/3)\*(e + f\*x)^3)/(a\*f) - ((2\*I)\*(e + f\*x)^2\*Log[1 + I\*E^(c + d\*x)])/(a\*d) - ((4\*I)\*f\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*d^2) + ((4\*I)\*f^2\*PolyLog[3, (-I)\*E^(c + d\*x)])/(a\*d^3)

**Rubi [A]** time = 0.184023, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {5559, 2190, 2531, 2282, 6589}

$$-\frac{4if(e+fx)\text{PolyLog}(2, -ie^{c+dx})}{ad^2} + \frac{4if^2\text{PolyLog}(3, -ie^{c+dx})}{ad^3} - \frac{2i(e+fx)^2 \log(1+ie^{c+dx})}{ad} + \frac{i(e+fx)^3}{3af}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Cosh[c + d\*x])/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] ((I/3)\*(e + f\*x)^3)/(a\*f) - ((2\*I)\*(e + f\*x)^2\*Log[1 + I\*E^(c + d\*x)])/(a\*d) - ((4\*I)\*f\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*d^2) + ((4\*I)\*f^2\*PolyLog[3, (-I)\*E^(c + d\*x)])/(a\*d^3)

#### Rule 5559

Int[(Cosh[(c\_.) + (d\_.)\*(x\_.)]\*((e\_.) + (f\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + Dist[2, Int[((e + f\*x)^m\*E^(c + d\*x))/(a + b\*E^(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]

#### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_.)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_.)))^(n\_.)), x\_Symbol] := Simp[(((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a)/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \cosh(c+dx)}{a+ia \sinh(c+dx)} dx &= \frac{i(e+fx)^3}{3af} + 2 \int \frac{e^{c+dx}(e+fx)^2}{a+iae^{c+dx}} dx \\
&= \frac{i(e+fx)^3}{3af} - \frac{2i(e+fx)^2 \log(1+ie^{c+dx})}{ad} + \frac{(4if) \int (e+fx) \log(1+ie^{c+dx}) dx}{ad} \\
&= \frac{i(e+fx)^3}{3af} - \frac{2i(e+fx)^2 \log(1+ie^{c+dx})}{ad} - \frac{4if(e+fx)\text{Li}_2(-ie^{c+dx})}{ad^2} + \frac{(4if^2) \int \text{Li}_2(-ie^{c+dx})}{ad^2} \\
&= \frac{i(e+fx)^3}{3af} - \frac{2i(e+fx)^2 \log(1+ie^{c+dx})}{ad} - \frac{4if(e+fx)\text{Li}_2(-ie^{c+dx})}{ad^2} + \frac{(4if^2) \text{Subst}\left(\int \frac{\text{Li}_2(-ie^{c+dx})}{x}\right)}{ad} \\
&= \frac{i(e+fx)^3}{3af} - \frac{2i(e+fx)^2 \log(1+ie^{c+dx})}{ad} - \frac{4if(e+fx)\text{Li}_2(-ie^{c+dx})}{ad^2} + \frac{4if^2\text{Li}_3(-ie^{c+dx})}{ad^3}
\end{aligned}$$

**Mathematica [A]** time = 0.0498098, size = 94, normalized size = 0.89

$$\frac{i(-12df^2(e+fx)\text{PolyLog}(2, -ie^{c+dx}) + 12f^3\text{PolyLog}(3, -ie^{c+dx}) + d^2(e+fx)^2(d(e+fx) - 6f \log(1+ie^{c+dx})))}{3ad^3f}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Cosh[c + d\*x])/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] ((I/3)\*(d^2\*(e + f\*x)^2\*(d\*(e + f\*x) - 6\*f\*Log[1 + I\*E^(c + d\*x)]) - 12\*d\*f^2\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)] + 12\*f^3\*PolyLog[3, (-I)\*E^(c + d\*x)]))/(a\*d^3\*f)

**Maple [B]** time = 0.073, size = 393, normalized size = 3.7

$$\frac{-2if^2 \ln(1 + ie^{dx+c})x^2}{da} + \frac{2iefc^2}{ad^2} - \frac{4if^2 \text{polylog}(2, -ie^{dx+c})x}{ad^2} - \frac{2i \ln(e^{dx+c} - i)e^2}{da} - \frac{2if^2c^2x}{ad^2} + \frac{4if^2 \text{polylog}(3, -ie^{dx+c})}{ad^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*cosh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] -2\*I/a/d\*f^2\*ln(1+I\*exp(d\*x+c))\*x^2+2\*I/a/d^2\*e\*f\*c^2-4\*I/a/d^2\*f^2\*polylog(2,-I\*exp(d\*x+c))\*x-2\*I/a/d\*ln(exp(d\*x+c)-I)\*e^2-2\*I/a/d^2\*f^2\*c^2\*x+4\*I\*f^2\*polylog(3,-I\*exp(d\*x+c))/a/d^3-2\*I/a/d^3\*f^2\*c^2\*ln(exp(d\*x+c)-I)+2\*I/a/d\*ln(exp(d\*x+c))\*e^2-I/a\*e^2\*x-4\*I/a/d^2\*e\*f\*c\*ln(exp(d\*x+c))-4\*I/a/d\*e\*f\*ln(1+I\*exp(d\*x+c))\*x+1/3\*I/a\*x^3\*f^2-4\*I/a/d^2\*e\*f\*ln(1+I\*exp(d\*x+c))\*c-4\*I/a/d^2\*e\*f\*polylog(2,-I\*exp(d\*x+c))+2\*I/a/d^3\*f^2\*c^2\*ln(exp(d\*x+c))-4/3\*I/a/d^3\*f^2\*c^3+4\*I/a/d^2\*e\*f\*c\*ln(exp(d\*x+c)-I)+I/a\*e\*f\*x^2+4\*I/a/d\*e\*f\*c\*x+2\*I/a/d^3\*f^2\*c^2\*ln(1+I\*exp(d\*x+c))

**Maxima [A]** time = 1.56212, size = 221, normalized size = 2.08

$$\frac{ie^2 \log(ia \sinh(dx + c) + a)}{ad} - \frac{if^2x^3 + 3iefx^2}{3a} - \frac{4i(dx \log(ie^{(dx+c)} + 1) + \text{Li}_2(-ie^{(dx+c)}))ef}{ad^2} - \frac{2i(d^2x^2 \log(ie^{(dx+c)}))}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -I\*e^2\*log(I\*a\*sinh(d\*x + c) + a)/(a\*d) - 1/3\*(I\*f^2\*x^3 + 3\*I\*e\*f\*x^2)/a - 4\*I\*(d\*x\*log(I\*e^(d\*x + c) + 1) + dilog(-I\*e^(d\*x + c)))\*e\*f/(a\*d^2) - 2\*I\*(d^2\*x^2\*log(I\*e^(d\*x + c) + 1) + 2\*d\*x\*dilog(-I\*e^(d\*x + c)) - 2\*polylog(3, -I\*e^(d\*x + c)))\*f^2/(a\*d^3) + 1/3\*(2\*I\*d^3\*f^2\*x^3 + 6\*I\*d^3\*e\*f\*x^2)/(

$a*d^3)$

**Fricas [C]** time = 2.22861, size = 482, normalized size = 4.55

$$\frac{id^3 f^2 x^3 + 3id^3 e f x^2 + 3id^3 e^2 x + 6icd^2 e^2 - 6ic^2 def + 2ic^3 f^2 + 12i f^2 \operatorname{polylog}(3, -ie^{(dx+c)}) + (-12i df^2 x - 12i def) \operatorname{Li}_2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{3} * (I*d^3*f^2*x^3 + 3*I*d^3*e*f*x^2 + 3*I*d^3*e^2*x + 6*I*c*d^2*e^2 - 6*I*c^2*d*e*f + 2*I*c^3*f^2 + 12*I*f^2*\operatorname{polylog}(3, -I*e^{(d*x + c)}) + (-12*I*d*f^2*x - 12*I*d*e*f)*\operatorname{dilog}(-I*e^{(d*x + c)}) + (-6*I*d^2*e^2 + 12*I*c*d*e*f - 6*I*c^2*f^2)*\log(e^{(d*x + c)} - I) + (-6*I*d^2*f^2*x^2 - 12*I*d^2*e*f*x - 12*I*c*d*e*f + 6*I*c^2*f^2)*\log(I*e^{(d*x + c)} + 1)) / (a*d^3)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e^2 \cosh(c+dx)}{i \sinh(c+dx)+1} dx + \int \frac{f^2 x^2 \cosh(c+dx)}{i \sinh(c+dx)+1} dx + \int \frac{2efx \cosh(c+dx)}{i \sinh(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cosh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)

[Out]  $(\operatorname{Integral}(e^{**2}*\cosh(c + d*x)/(I*\sinh(c + d*x) + 1), x) + \operatorname{Integral}(f^{**2}*x^{**2}*\cosh(c + d*x)/(I*\sinh(c + d*x) + 1), x) + \operatorname{Integral}(2*e*f*x*\cosh(c + d*x)/(I*\sinh(c + d*x) + 1), x))/a$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c)}{i a \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

$$3.255 \quad \int \frac{(e+fx) \cosh(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=73

$$-\frac{2if \operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^2} - \frac{2i(e+fx) \log\left(1+ie^{c+dx}\right)}{ad} + \frac{i(e+fx)^2}{2af}$$

[Out]  $((I/2)*(e + f*x)^2)/(a*f) - ((2*I)*(e + f*x)*\operatorname{Log}[1 + I*E^{(c + d*x)}])/(a*d) - ((2*I)*f*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x)}])/(a*d^2)$

**Rubi [A]** time = 0.111111, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$ , Rules used = {5559, 2190, 2279, 2391}

$$-\frac{2if \operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^2} - \frac{2i(e+fx) \log\left(1+ie^{c+dx}\right)}{ad} + \frac{i(e+fx)^2}{2af}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\frac{(e + f*x)*\operatorname{Cosh}[c + d*x]}{(a + I*a*\operatorname{Sinh}[c + d*x])}, x]$

[Out]  $((I/2)*(e + f*x)^2)/(a*f) - ((2*I)*(e + f*x)*\operatorname{Log}[1 + I*E^{(c + d*x)}])/(a*d) - ((2*I)*f*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x)}])/(a*d^2)$

#### Rule 5559

$\operatorname{Int}[\frac{\operatorname{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))^{(m_.)}}{(a_.) + (b_.)*\operatorname{Sinh}[(c_.) + (d_.)*(x_.)]}, x\_Symbol] \rightarrow -\operatorname{Simp}[(e + f*x)^{(m + 1)}/(b*f*(m + 1)), x] + \operatorname{Dist}[2, \operatorname{Int}[\frac{(e + f*x)^m * E^{(c + d*x)}}{(a + b * E^{(c + d*x)})}, x], x] /;$   
 $\operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \operatorname{IGtQ}[m, 0] \ \&\& \ \operatorname{EqQ}[a^2 + b^2, 0]$

#### Rule 2190

$\operatorname{Int}[\frac{((F_)^{((g_.)*((e_.) + (f_.)*(x_.)))^{(n_.)*((c_.) + (d_.)*(x_.))^{(m_.))}}{(a_.) + (b_.)*((F_)^{((g_.)*((e_.) + (f_.)*(x_.)))^{(n_.))}}, x\_Symbol] \rightarrow \operatorname{Simp}[\frac{(c + d*x)^m * \operatorname{Log}[1 + (b*(F^{(g*(e + f*x))))^n]/a]}{(b*f*g*n*\operatorname{Log}[F])}, x] - \operatorname{Dist}[(d*m)/(b*f*g*n*\operatorname{Log}[F]), \operatorname{Int}[(c + d*x)^{(m - 1)} * \operatorname{Log}[1 + (b*(F^{(g*(e + f*x))))^n]/a], x], x] /;$   
 $\operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \ \&\& \ \operatorname{IGtQ}[m, 0]$

#### Rule 2279



```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \frac{(e + fx) \cosh(c + dx)}{a + ia \sinh(c + dx)} dx &= \frac{i(e + fx)^2}{2af} + 2 \int \frac{e^{c+dx}(e + fx)}{a + iae^{c+dx}} dx \\ &= \frac{i(e + fx)^2}{2af} - \frac{2i(e + fx) \log(1 + ie^{c+dx})}{ad} + \frac{(2if) \int \log(1 + ie^{c+dx}) dx}{ad} \\ &= \frac{i(e + fx)^2}{2af} - \frac{2i(e + fx) \log(1 + ie^{c+dx})}{ad} + \frac{(2if) \text{Subst}\left(\int \frac{\log(1+ix)}{x} dx, x, e^{c+dx}\right)}{ad^2} \\ &= \frac{i(e + fx)^2}{2af} - \frac{2i(e + fx) \log(1 + ie^{c+dx})}{ad} - \frac{2if \text{Li}_2(-ie^{c+dx})}{ad^2} \end{aligned}$$

**Mathematica [A]** time = 0.0258735, size = 66, normalized size = 0.9

$$\frac{i(d(e + fx)(d(e + fx) - 4f \log(1 + ie^{c+dx})) - 4f^2 \text{PolyLog}(2, -ie^{c+dx}))}{2ad^2 f}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((I/2)*(d*(e + f*x)*(d*(e + f*x) - 4*f*Log[1 + I*E^(c + d*x)]) - 4*f^2*PolyLog[2, (-I)*E^(c + d*x)]))/(a*d^2*f)
```

**Maple [B]** time = 0.073, size = 188, normalized size = 2.6

$$\frac{\frac{i}{2}fx^2}{a} - \frac{ieux}{a} - \frac{2i \ln(e^{dx+c} - i)e}{da} + \frac{2i \ln(e^{dx+c})e}{da} + \frac{2ifcx}{da} + \frac{ifc^2}{ad^2} - \frac{2if \ln(1 + ie^{dx+c})x}{da} - \frac{2if \ln(1 + ie^{dx+c})c}{ad^2} - \frac{2if \text{Li}_2(-ie^{dx+c})}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)`

[Out]  $\frac{1}{2}I^2fx^2/a - I^2ex/a - 2I/d/a \ln(\exp(dx+c) - I) + 2I/d/a \ln(\exp(dx+c)) + 2I/d/a f^2cx + I/d^2/a f^2c^2 - 2I/d/a f \ln(1 + I \exp(dx+c)) + 2I/d^2/a f \ln(1 + I \exp(dx+c)) + c - 2I f \operatorname{polylog}(2, -I \exp(dx+c)) + I/d^2/a f^2c \ln(\exp(dx+c) - I) - 2I/d^2/a f^2c \ln(\exp(dx+c))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}f\left(-\frac{ix^2}{a} + 4 \int \frac{x}{ae^{(dx+c)} - ia} dx\right) - \frac{ie \log(ia \sinh(dx+c) + a)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out]  $\frac{1}{2}f(-I^2x^2/a + 4 \operatorname{integrate}(x/(a \cdot e^{(dx+c)} - I \cdot a), x)) - I^2e \log(I \cdot a \sinh(dx+c) + a)/(a \cdot d)$

**Fricas [A]** time = 2.18812, size = 252, normalized size = 3.45

$$\frac{id^2fx^2 + 2id^2ex + 4icde - 2ic^2f - 4if\operatorname{Li}_2(-ie^{(dx+c)}) + (-4ide + 4icf) \log(e^{(dx+c)} - i) + (-4idf x - 4icf) \log(ie^{(dx+c)})}{2ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")`

[Out]  $\frac{1}{2}(I^2d^2fx^2 + 2I^2d^2ex + 4I^2cde - 2I^2c^2f - 4I^2f \operatorname{dilog}(-Ie^{(dx+c)}) + (-4I^2de + 4I^2cf) \log(e^{(dx+c)} - I) + (-4I^2dfx - 4I^2cf) \log(Ie^{(dx+c)} + 1))/(a \cdot d^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e \cosh(c+dx)}{i \sinh(c+dx)+1} dx + \int \frac{fx \cosh(c+dx)}{i \sinh(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] (Integral(e\*cosh(c + d\*x)/(I\*sinh(c + d\*x) + 1), x) + Integral(f\*x\*cosh(c + d\*x)/(I\*sinh(c + d\*x) + 1), x))/a

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \cosh(dx + c)}{i a \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)\*cosh(d\*x + c)/(I\*a\*sinh(d\*x + c) + a), x)

$$3.256 \quad \int \frac{\cosh(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=23

$$-\frac{i \log(-\sinh(c+dx)+i)}{ad}$$

[Out] `((-I)*Log[I - Sinh[c + d*x]])/(a*d)`

**Rubi [A]** time = 0.0273148, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2667, 31}

$$-\frac{i \log(-\sinh(c+dx)+i)}{ad}$$

Antiderivative was successfully verified.

[In] `Int[Cosh[c + d*x]/(a + I*a*Sinh[c + d*x]),x]`

[Out] `((-I)*Log[I - Sinh[c + d*x]])/(a*d)`

#### Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol]
:> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^(p - 1)/2], x], x, b*Sin[e + f*x]], x]
/; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol]
:> Simp[Log[RemoveContent[a + b*x, x]]/b, x]
/; FreeQ[{a, b}, x]
```

#### Rubi steps

$$\int \frac{\cosh(c + dx)}{a + ia \sinh(c + dx)} dx = -\frac{i \operatorname{Subst}\left(\int \frac{1}{a+x} dx, x, ia \sinh(c + dx)\right)}{ad}$$

$$= -\frac{i \log(i - \sinh(c + dx))}{ad}$$

**Mathematica [A]** time = 0.0145465, size = 23, normalized size = 1.

$$-\frac{i \log(-\sinh(c + dx) + i)}{ad}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] ((-I)\*Log[I - Sinh[c + d\*x]])/(a\*d)

**Maple [A]** time = 0.013, size = 23, normalized size = 1.

$$\frac{-i \ln(a + ia \sinh(dx + c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] -I/d\*ln(a+I\*a\*sinh(d\*x+c))/a

**Maxima [A]** time = 1.12375, size = 27, normalized size = 1.17

$$-\frac{i \log(ia \sinh(dx + c) + a)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-I*\log(I*a*\sinh(d*x + c) + a)/(a*d)$

---

**Fricas [A]** time = 2.18425, size = 57, normalized size = 2.48

$$\frac{i dx - 2i \log(e^{(dx+c)} - i)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")`

[Out]  $(I*d*x - 2*I*\log(e^{(d*x + c)} - I))/(a*d)$

---

**Sympy [A]** time = 4.53724, size = 22, normalized size = 0.96

$$\frac{ix}{a} - \frac{2i \log(e^{dx} - ie^{-c})}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x)`

[Out]  $I*x/a - 2*I*\log(\exp(d*x) - I*\exp(-c))/(a*d)$

---

**Giac [A]** time = 1.23608, size = 45, normalized size = 1.96

$$\frac{i(dx+c)}{ad} - \frac{2i \log(i e^{(dx+c)} + 1)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")`

[Out]  $I*(d*x + c)/(a*d) - 2*I*\log(I*e^{(d*x + c)} + 1)/(a*d)$

$$3.257 \quad \int \frac{\cosh(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

**Optimal.** Leaf size=31

$$\text{Unintegrable}\left(\frac{\cosh(c+dx)}{(e+fx)(a+ia \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Cosh[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0491233, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Cosh[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Cosh[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx = \int \frac{\cosh(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

**Mathematica [A]** time = 27.5074, size = 0, normalized size = 0.

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Cosh[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Integrate[Cosh[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

---

**Maple [A]** time = 0.088, size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(fx + e)(a + ia \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] int(cosh(d\*x+c)/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-\frac{i \log(fx + e)}{af} + 2 \int \frac{1}{-iafx - iae + (afxe^c + aee^c)e^{(dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -I\*log(f\*x + e)/(a\*f) + 2\*integrate(1/(-I\*a\*f\*x - I\*a\*e + (a\*f\*x\*e^c + a\*e\*e^c)\*e^(d\*x)), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-ie^{(dx+c)} + 1}{-iafx - iae + (afx + ae)e^{(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] integral((-I\*e^(d\*x + c) + 1)/(-I\*a\*f\*x - I\*a\*e + (a\*f\*x + a\*e)\*e^(d\*x + c)), x)

---



**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(fx+e)(ia \sinh(dx+c)+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate(cosh(d\*x + c)/((f\*x + e)\*(I\*a\*sinh(d\*x + c) + a)), x)

$$3.258 \quad \int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

**Optimal.** Leaf size=31

$$\text{Unintegrable}\left(\frac{\cosh(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Cosh[c + d\*x]/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0492806, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Cosh[c + d\*x]/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Cosh[c + d\*x]/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx = \int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

**Mathematica [A]** time = 32.4266, size = 0, normalized size = 0.

$$\int \frac{\cosh(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Cosh[c + d\*x]/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Integrate[Cosh[c + d\*x]/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

---

**Maple [A]** time = 0.09, size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(fx + e)^2 (a + ia \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)`

[Out] `int(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{i}{af^2x + aef} + 2 \int \frac{1}{-iaf^2x^2 - 2iaefx - ia^2 + (af^2x^2e^c + 2aefxe^c + ae^2e^c)e^{(dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `I/(a*f^2*x + a*e*f) + 2*integrate(1/(-I*a*f^2*x^2 - 2*I*a*e*f*x - I*a*e^2 + (a*f^2*x^2*e^c + 2*a*e*f*x*e^c + a*e^2*e^c)*e^(d*x)), x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{-ie^{(dx+c)} + 1}{-iaf^2x^2 - 2iaefx - ia^2 + (af^2x^2 + 2aefx + ae^2)e^{(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")`

[Out] `integral((-I*e^(d*x + c) + 1)/(-I*a*f^2*x^2 - 2*I*a*e*f*x - I*a*e^2 + (a*f^2*x^2 + 2*a*e*f*x + a*e^2)*e^(d*x + c)), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(f\*x+e)\*\*2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(fx + e)^2 (ia \sinh(dx + c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate(cosh(d\*x + c)/((f\*x + e)^2\*(I\*a\*sinh(d\*x + c) + a)), x)

$$3.259 \quad \int \frac{(e+fx)^3 \cosh^2(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=108

$$-\frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} + \frac{3if(e+fx)^2\sinh(c+dx)}{ad^2} + \frac{6if^3\sinh(c+dx)}{ad^4} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + \frac{(e+fx)^4}{4af}$$

[Out] (e + f\*x)^4/(4\*a\*f) - ((6\*I)\*f^2\*(e + f\*x)\*Cosh[c + d\*x])/(a\*d^3) - (I\*(e + f\*x)^3\*Cosh[c + d\*x])/(a\*d) + ((6\*I)\*f^3\*Sinh[c + d\*x])/(a\*d^4) + ((3\*I)\*f\*(e + f\*x)^2\*Sinh[c + d\*x])/(a\*d^2)

**Rubi [A]** time = 0.163624, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {5563, 32, 3296, 2637}

$$-\frac{6if^2(e+fx)\cosh(c+dx)}{ad^3} + \frac{3if(e+fx)^2\sinh(c+dx)}{ad^2} + \frac{6if^3\sinh(c+dx)}{ad^4} - \frac{i(e+fx)^3\cosh(c+dx)}{ad} + \frac{(e+fx)^4}{4af}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Cosh[c + d\*x]^2)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (e + f\*x)^4/(4\*a\*f) - ((6\*I)\*f^2\*(e + f\*x)\*Cosh[c + d\*x])/(a\*d^3) - (I\*(e + f\*x)^3\*Cosh[c + d\*x])/(a\*d) + ((6\*I)\*f^3\*Sinh[c + d\*x])/(a\*d^4) + ((3\*I)\*f\*(e + f\*x)^2\*Sinh[c + d\*x])/(a\*d^2)

### Rule 5563

Int[(Cosh[(c\_.) + (d\_.)\*(x\_.)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2 + b^2, 0]

### Rule 32

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{(e + fx)^3 \cosh^2(c + dx)}{a + ia \sinh(c + dx)} dx &= -\frac{i \int (e + fx)^3 \sinh(c + dx) dx}{a} + \frac{\int (e + fx)^3 dx}{a} \\ &= \frac{(e + fx)^4}{4af} - \frac{i(e + fx)^3 \cosh(c + dx)}{ad} + \frac{(3if) \int (e + fx)^2 \cosh(c + dx) dx}{ad} \\ &= \frac{(e + fx)^4}{4af} - \frac{i(e + fx)^3 \cosh(c + dx)}{ad} + \frac{3if(e + fx)^2 \sinh(c + dx)}{ad^2} - \frac{(6if^2) \int (e + fx) \sinh(c + dx) dx}{ad^2} \\ &= \frac{(e + fx)^4}{4af} - \frac{6if^2(e + fx) \cosh(c + dx)}{ad^3} - \frac{i(e + fx)^3 \cosh(c + dx)}{ad} + \frac{3if(e + fx)^2 \sinh(c + dx)}{ad^2} \\ &= \frac{(e + fx)^4}{4af} - \frac{6if^2(e + fx) \cosh(c + dx)}{ad^3} - \frac{i(e + fx)^3 \cosh(c + dx)}{ad} + \frac{6if^3 \sinh(c + dx)}{ad^4} + \end{aligned}$$

**Mathematica [A]** time = 0.779149, size = 106, normalized size = 0.98

$$\frac{12if \sinh(c + dx) (d^2(e + fx)^2 + 2f^2) - 4id(e + fx) \cosh(c + dx) (d^2(e + fx)^2 + 6f^2) + d^4x (6e^2fx + 4e^3 + 4ef^2x^2 + f^3x^3)}{4ad^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (d^4*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3) - (4*I)*d*(e + f*x)*(6*f^2 + d^2*(e + f*x)^2)*Cosh[c + d*x] + (12*I)*f*(2*f^2 + d^2*(e + f*x)^2)*Sinh[c + d*x])/(4*a*d^4)
```

**Maple [B]** time = 0.045, size = 448, normalized size = 4.2

$$-\frac{1}{d^4a} \left( id^3e^3 \cosh(dx + c) + 3ic^2f^2ed \cosh(dx + c) + if^3 \left( (dx + c)^3 \cosh(dx + c) - 3(dx + c)^2 \sinh(dx + c) + 6(dx + c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)`

[Out] 
$$-1/d^4/a*(I*d^3*e^3*cosh(d*x+c)+3*I*c^2*f^2*e*d*cosh(d*x+c)+I*f^3*((d*x+c)^3*cosh(d*x+c)-3*(d*x+c)^2*sinh(d*x+c)+6*(d*x+c)*cosh(d*x+c)-6*sinh(d*x+c))-6*I*f^2*c*e*d*((d*x+c)*cosh(d*x+c)-sinh(d*x+c))+3*I*f^2*e*d*((d*x+c)^2*cosh(d*x+c)-2*(d*x+c)*sinh(d*x+c)+2*cosh(d*x+c))+3*I*f*e^2*d^2*((d*x+c)*cosh(d*x+c)-sinh(d*x+c))-3*I*c*f^3*((d*x+c)^2*cosh(d*x+c)-2*(d*x+c)*sinh(d*x+c)+2*cosh(d*x+c))-I*f^3*c^3*cosh(d*x+c)+3*I*c^2*f^3*((d*x+c)*cosh(d*x+c)-sinh(d*x+c))-3*I*c*f*e^2*d^2*cosh(d*x+c)-1/4*f^3*(d*x+c)^4+c*f^3*(d*x+c)^3-f^2*e*d*(d*x+c)^3-3/2*c^2*f^3*(d*x+c)^2+3*c*d*e*f^2*(d*x+c)^2-3/2*d^2*e^2*f*(d*x+c)^2+f^3*c^3*(d*x+c)-3*c^2*f^2*e*d*(d*x+c)+3*c*f*e^2*d^2*(d*x+c)-d^3*e^3*(d*x+c))$$

**Maxima [B]** time = 2.02196, size = 504, normalized size = 4.67

$$\frac{3}{4} e^2 f \left( \frac{4 x e^{(dx+c)}}{a d e^{(dx+c)} - i a d} + \frac{-2 i d^2 x^2 e^c - 2 i d x e^c - (2 i d x e^{(3c)} - 2 i e^{(3c)}) e^{(2dx)} + 2 (d^2 x^2 e^{(2c)} - 3 d x e^{(2c)} + e^{(2c)}) e^{(dx)} - 2 (dx + c)}{a d^2 e^{(dx+2c)} - i a d^2 e^c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out] 
$$\frac{3}{4} e^2 f \left( \frac{4 x e^{(dx+c)}}{a d e^{(dx+c)} - I a d} + (-2 I d^2 x^2 e^c - 2 I d x e^c - (2 I d x e^{(3c)} - 2 I e^{(3c)}) e^{(2dx)} + 2 (d^2 x^2 e^{(2c)} - 3 d x e^{(2c)} + e^{(2c)}) e^{(dx)} - 2 (d x + 1) e^{(-dx)} - 2 I e^c) / (a d^2 e^{(dx+2c)} - I a d^2 e^c) + 1/4 e^3 (4 (d x + c) / (a d) - 2 I e^{(dx+c)} / (a d) - 2 I e^{(-dx-c)} / (a d)) + 1/4 (4 d^3 x^3 e^c - (6 I d^2 x^2 e^{(2c)} - 12 I d x e^{(2c)} + 12 I e^{(2c)}) e^{(dx)} - (6 I d^2 x^2 + 12 I d x + 12 I) e^{(-dx)}) e^2 e^{(-c)} / (a d^3) + 1/4 (d^4 x^4 e^c - (2 I d^3 x^3 e^{(2c)} - 6 I d^2 x^2 e^{(2c)} + 12 I d x e^{(2c)} - 12 I e^{(2c)}) e^{(dx)} - (2 I d^3 x^3 + 6 I d^2 x^2 + 12 I d x + 12 I) e^{(-dx)}) e^3 e^{(-c)} / (a d^4) \right)$$

**Fricas [B]** time = 2.21788, size = 605, normalized size = 5.6

$$\frac{(-2 i d^3 f^3 x^3 - 2 i d^3 e^3 - 6 i d^2 e^2 f - 12 i d e f^2 - 12 i f^3 + (-6 i d^3 e f^2 - 6 i d^2 f^3) x^2 + (-6 i d^3 e^2 f - 12 i d^2 e f^2 - 12 i d f^3) x + (-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(-2*I*d^3*f^3*x^3 - 2*I*d^3*e^3 - 6*I*d^2*e^2*f - 12*I*d*e*f^2 - 12*I*f^3 + (-6*I*d^3*e*f^2 - 6*I*d^2*f^3)*x^2 + (-6*I*d^3*e^2*f - 12*I*d^2*e*f^2 - 12*I*d*f^3)*x + (-2*I*d^3*f^3*x^3 - 2*I*d^3*e^3 + 6*I*d^2*e^2*f - 12*I*d*e*f^2 + 12*I*f^3 + (-6*I*d^3*e*f^2 + 6*I*d^2*f^3)*x^2 + (-6*I*d^3*e^2*f + 12*I*d^2*e*f^2 - 12*I*d*f^3)*x)*e^(2*d*x + 2*c) + (d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2 + 4*d^4*e^3*x)*e^(d*x + c))/e^(-d*x - c)/(a*d^4)
```

**Sympy [A]** time = 2.66624, size = 610, normalized size = 5.65

$$\left\{ \frac{(-2ia^7d^{19}e^3e^{3c}-6ia^7d^{19}e^2fxe^{3c}-6ia^7d^{19}ef^2x^2e^{3c}-2ia^7d^{19}f^3x^3e^{3c}-6ia^7d^{18}e^2fe^{3c}-12ia^7d^{18}ef^2xe^{3c}-6ia^7d^{18}f^3x^2e^{3c}-12ia^7d^{17}ef^2e^{3c}-12ia^7d^{17}f^3xe^{3c}-12ia^7d^{16}e^3e^{3c})e^{-c}}{8a} - \frac{x^3(ief^2e^{2c}-ief^2)e^{-c}}{2a} - \frac{x^2(3ie^2fe^{2c}-3ie^2f)e^{-c}}{4a} - \frac{x(ie^3e^{2c}-ie^3)e^{-c}}{2a} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise(((((-2*I*a**7*d**19*e**3*exp(3*c) - 6*I*a**7*d**19*e**2*f*x*exp(3*c) - 6*I*a**7*d**19*e*f**2*x**2*exp(3*c) - 2*I*a**7*d**19*f**3*x**3*exp(3*c) - 6*I*a**7*d**18*e**2*f*exp(3*c) - 12*I*a**7*d**18*e*f**2*x*exp(3*c) - 6*I*a**7*d**18*f**3*x**2*exp(3*c) - 12*I*a**7*d**17*e*f**2*exp(3*c) - 12*I*a**7*d**17*f**3*x*exp(3*c) - 12*I*a**7*d**16*f**3*exp(3*c)))*exp(-d*x) + (-2*I*a**7*d**19*e**3*exp(5*c) - 6*I*a**7*d**19*e**2*f*x*exp(5*c) - 6*I*a**7*d**19*e*f**2*x**2*exp(5*c) - 2*I*a**7*d**19*f**3*x**3*exp(5*c) + 6*I*a**7*d**18*e**2*f*exp(5*c) + 12*I*a**7*d**18*e*f**2*x*exp(5*c) + 6*I*a**7*d**18*f**3*x**2*exp(5*c) - 12*I*a**7*d**17*e*f**2*exp(5*c) - 12*I*a**7*d**17*f**3*x*exp(5*c) + 12*I*a**7*d**16*f**3*exp(5*c))*exp(d*x))*exp(-4*c)/(4*a**8*d**20), Ne(4*a**8*d**20*exp(4*c), 0)), (-x**4*(I*f**3*exp(2*c) - I*f**3)*exp(-c)/(8*a) - x**3*(I*e*f**2*exp(2*c) - I*e*f**2)*exp(-c)/(2*a) - x**2*(3*I*e**2*f*exp(2*c) - 3*I*e**2*f)*exp(-c)/(4*a) - x*(I*e**3*exp(2*c) - I*e**3)*exp(-c)/(2*a), True)) + e**3*x/a + 3*e**2*f*x**2/(2*a) + e*f**2*x**3/a + f**3*x**4/(4*a)
```



**Giac [B]** time = 1.34559, size = 1084, normalized size = 10.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] 
$$\begin{aligned} & \frac{1}{4} * (d^4 * f^3 * x^4 * e^{(2*d*x + 3*c)} - I * d^4 * f^3 * x^4 * e^{(d*x + 2*c)} - 2 * I * d^3 * f^3 * x^3 * e^{(3*d*x + 4*c)} + 4 * d^4 * f^2 * x^3 * e^{(2*d*x + 3*c + 1)} - 2 * d^3 * f^3 * x^3 * e^{(2*d*x + 3*c)} - 4 * I * d^4 * f^2 * x^3 * e^{(d*x + 2*c + 1)} - 2 * I * d^3 * f^3 * x^3 * e^{(d*x + 2*c)} - 2 * d^3 * f^3 * x^3 * e^c - 6 * I * d^3 * f^2 * x^2 * e^{(3*d*x + 4*c + 1)} + 6 * I * d^2 * f^3 * x^2 * e^{(3*d*x + 4*c)} + 6 * d^4 * f * x^2 * e^{(2*d*x + 3*c + 2)} - 6 * d^3 * f^2 * x^2 * e^{(2*d*x + 3*c + 1)} + 6 * d^2 * f^3 * x^2 * e^{(2*d*x + 3*c)} - 6 * I * d^4 * f * x^2 * e^{(d*x + 2*c + 2)} - 6 * I * d^3 * f^2 * x^2 * e^{(d*x + 2*c + 1)} - 6 * I * d^2 * f^3 * x^2 * e^{(d*x + 2*c)} - 6 * d^3 * f^2 * x^2 * e^{(c + 1)} - 6 * d^2 * f^3 * x^2 * e^c - 6 * I * d^3 * f * x * e^{(3*d*x + 4*c + 2)} + 12 * I * d^2 * f^2 * x * e^{(3*d*x + 4*c + 1)} - 12 * I * d * f^3 * x * e^{(3*d*x + 4*c)} + 4 * d^4 * x * e^{(2*d*x + 3*c + 3)} - 6 * d^3 * f * x * e^{(2*d*x + 3*c + 2)} + 12 * d^2 * f^2 * x * e^{(2*d*x + 3*c + 1)} - 12 * d * f^3 * x * e^{(2*d*x + 3*c)} - 4 * I * d^4 * x * e^{(d*x + 2*c + 3)} - 6 * I * d^3 * f * x * e^{(d*x + 2*c + 2)} - 12 * I * d^2 * f^2 * x * e^{(d*x + 2*c + 1)} - 12 * I * d * f^3 * x * e^{(d*x + 2*c)} - 6 * d^3 * f * x * e^{(c + 2)} - 12 * d^2 * f^2 * x * e^{(c + 1)} - 12 * d * f^3 * x * e^c - 2 * I * d^3 * e^{(3*d*x + 4*c + 3)} + 6 * I * d^2 * f * e^{(3*d*x + 4*c + 2)} - 12 * I * d * f^2 * e^{(3*d*x + 4*c + 1)} + 12 * I * f^3 * e^{(3*d*x + 4*c)} - 2 * d^3 * e^{(2*d*x + 3*c + 3)} + 6 * d^2 * f * e^{(2*d*x + 3*c + 2)} - 12 * d * f^2 * e^{(2*d*x + 3*c + 1)} + 12 * f^3 * e^{(2*d*x + 3*c)} - 2 * I * d^3 * e^{(d*x + 2*c + 3)} - 6 * I * d^2 * f * e^{(d*x + 2*c + 2)} - 12 * I * d * f^2 * e^{(d*x + 2*c + 1)} - 12 * I * f^3 * e^{(d*x + 2*c)} - 2 * d^3 * e^{(c + 3)} - 6 * d^2 * f * e^{(c + 2)} - 12 * d * f^2 * e^{(c + 1)} - 12 * f^3 * e^c) / (a * d^4 * e^{(2*d*x + 3*c)} - I * a * d^4 * e^{(d*x + 2*c)}) \end{aligned}$$

$$3.260 \quad \int \frac{(e+fx)^2 \cosh^2(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=82

$$\frac{2if(e+fx)\sinh(c+dx)}{ad^2} - \frac{2if^2\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2\cosh(c+dx)}{ad} + \frac{(e+fx)^3}{3af}$$

[Out]  $(e + f*x)^3/(3*a*f) - ((2*I)*f^2*Cosh[c + d*x])/(a*d^3) - (I*(e + f*x)^2*Cos$   
 $h[c + d*x])/(a*d) + ((2*I)*f*(e + f*x)*Sinh[c + d*x])/(a*d^2)$

**Rubi [A]** time = 0.126473, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {5563, 32, 3296, 2638}

$$\frac{2if(e+fx)\sinh(c+dx)}{ad^2} - \frac{2if^2\cosh(c+dx)}{ad^3} - \frac{i(e+fx)^2\cosh(c+dx)}{ad} + \frac{(e+fx)^3}{3af}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Cosh[c + d\*x]^2)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out]  $(e + f*x)^3/(3*a*f) - ((2*I)*f^2*Cosh[c + d*x])/(a*d^3) - (I*(e + f*x)^2*Cos$   
 $h[c + d*x])/(a*d) + ((2*I)*f*(e + f*x)*Sinh[c + d*x])/(a*d^2)$

### Rule 5563

Int[(Cosh[(c\_.) + (d\_.)\*(x\_.)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2 + b^2, 0]

### Rule 32

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)], x\_Symbol] := -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[

$e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \ \&\& \ \text{GtQ}[m, 0]$

### Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

### Rubi steps

$$\begin{aligned} \int \frac{(e + fx)^2 \cosh^2(c + dx)}{a + ia \sinh(c + dx)} dx &= -\frac{i \int (e + fx)^2 \sinh(c + dx) dx}{a} + \frac{\int (e + fx)^2 dx}{a} \\ &= \frac{(e + fx)^3}{3af} - \frac{i(e + fx)^2 \cosh(c + dx)}{ad} + \frac{(2if) \int (e + fx) \cosh(c + dx) dx}{ad} \\ &= \frac{(e + fx)^3}{3af} - \frac{i(e + fx)^2 \cosh(c + dx)}{ad} + \frac{2if(e + fx) \sinh(c + dx)}{ad^2} - \frac{(2if^2) \int \sinh(c + dx) dx}{ad^2} \\ &= \frac{(e + fx)^3}{3af} - \frac{2if^2 \cosh(c + dx)}{ad^3} - \frac{i(e + fx)^2 \cosh(c + dx)}{ad} + \frac{2if(e + fx) \sinh(c + dx)}{ad^2} \end{aligned}$$

**Mathematica [A]** time = 0.484384, size = 78, normalized size = 0.95

$$\frac{-3i \cosh(c + dx) (d^2(e + fx)^2 + 2f^2) + 6idf(e + fx) \sinh(c + dx) + d^3x(3e^2 + 3efx + f^2x^2)}{3ad^3}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Cosh[c + d\*x]^2)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (d^3\*x\*(3\*e^2 + 3\*e\*f\*x + f^2\*x^2) - (3\*I)\*(2\*f^2 + d^2\*(e + f\*x)^2)\*Cosh[c + d\*x] + (6\*I)\*d\*f\*(e + f\*x)\*Sinh[c + d\*x])/(3\*a\*d^3)

**Maple [B]** time = 0.042, size = 223, normalized size = 2.7

$$-\frac{1}{d^3 a} \left( if^2 ((dx + c)^2 \cosh(dx + c) - 2(dx + c) \sinh(dx + c) + 2 \cosh(dx + c)) - 2icf^2 ((dx + c) \cosh(dx + c) - \sinh(dx + c)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*cosh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x)

[Out]  $-1/d^3/a*(I*f^2*((d*x+c)^2*\cosh(d*x+c)-2*(d*x+c)*\sinh(d*x+c)+2*\cosh(d*x+c))-2*I*c*f^2*((d*x+c)*\cosh(d*x+c)-\sinh(d*x+c))+2*I*f*e*d*((d*x+c)*\cosh(d*x+c)-\sinh(d*x+c))+I*c^2*f^2*\cosh(d*x+c)-2*I*c*d*f*e*\cosh(d*x+c)+I*d^2*e^2*\cosh(d*x+c)-1/3*f^2*(d*x+c)^3+c*f^2*(d*x+c)^2-e*f*d*(d*x+c)^2-c^2*f^2*(d*x+c)+2*c*d*f*e*(d*x+c)-d^2*e^2*(d*x+c))$

**Maxima [B]** time = 1.59838, size = 366, normalized size = 4.46

$$\frac{1}{2} e^f \left( \frac{4 x e^{(d x+c)}}{a d e^{(d x+c)} - i a d} + \frac{-2 i d^2 x^2 e^c - 2 i d x e^c - (2 i d x e^{(3 c)} - 2 i e^{(3 c)}) e^{(2 d x)} + 2 (d^2 x^2 e^{(2 c)} - 3 d x e^{(2 c)} + e^{(2 c)}) e^{(d x)} - 2 (d x + 1)}{a d^2 e^{(d x+2 c)} - i a d^2 e^c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $1/2*e*f*(4*x*e^{(d*x+c)}/(a*d*e^{(d*x+c)} - I*a*d) + (-2*I*d^2*x^2*e^c - 2*I*d*x*e^c - (2*I*d*x*e^{(3*c)} - 2*I*e^{(3*c)})*e^{(2*d*x)} + 2*(d^2*x^2*e^{(2*c)} - 3*d*x*e^{(2*c)} + e^{(2*c)})*e^{(d*x)} - 2*(d*x+1)*e^{(-d*x)} - 2*I*e^c)/(a*d^2*e^{(d*x+2*c)} - I*a*d^2*e^c) + 1/4*e^2*(4*(d*x+c)/(a*d) - 2*I*e^{(d*x+c)}/(a*d) - 2*I*e^{(-d*x-c)}/(a*d)) + 1/12*(4*d^3*x^3*e^c - (6*I*d^2*x^2*e^{(2*c)} - 12*I*d*x*e^{(2*c)} + 12*I*e^{(2*c)})*e^{(d*x)} - (6*I*d^2*x^2 + 12*I*d*x + 12*I)*e^{(-d*x)})*f^2*e^{(-c)}/(a*d^3)$

**Fricas [B]** time = 2.1649, size = 373, normalized size = 4.55

$$\frac{(-3i d^2 f^2 x^2 - 3i d^2 e^2 - 6i d e f - 6i f^2 + (-6i d^2 e f - 6i d f^2) x + (-3i d^2 f^2 x^2 - 3i d^2 e^2 + 6i d e f - 6i f^2 + (-6i d^2 e f + 6i d f^2) x) e^{(2 d x + 2 c)}}{6 a d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $1/6*(-3*I*d^2*f^2*x^2 - 3*I*d^2*e^2 - 6*I*d*e*f - 6*I*f^2 + (-6*I*d^2*e*f - 6*I*d*f^2)*x + (-3*I*d^2*f^2*x^2 - 3*I*d^2*e^2 + 6*I*d*e*f - 6*I*f^2 + (-6*I*d^2*e*f + 6*I*d*f^2)*x)*e^{(2*d*x+2*c)} + 2*(d^3*f^2*x^3 + 3*d^3*e*f*x^2$

$$+ 3*d^3*e^{2*x}*e^{(d*x + c)}*e^{(-d*x - c)/(a*d^3)}$$

**Sympy [A]** time = 1.94693, size = 376, normalized size = 4.59

$$\left\{ \frac{\left( (-2ia^5d^{11}e^{2c} - 4ia^5d^{11}efxe^{2c} - 2ia^5d^{11}f^2x^2e^{2c} - 4ia^5d^{10}efe^{2c} - 4ia^5d^{10}f^2xe^{2c} - 4ia^5d^9f^2e^{2c})e^{-dx} + (-2ia^5d^{11}e^{2c}e^{4c} - 4ia^5d^{11}efxe^{4c} - 2ia^5d^{11}f^2x^2e^{4c} + 4ia^5d^{10}efe^{4c} + 4ia^5d^{10}f^2xe^{4c} - 4ia^5d^9f^2e^{4c})e^{-c}}{4a^6d^{12}} \right. \\ \left. - \frac{x^3(ief^2e^{2c} - if^2)e^{-c}}{6a} - \frac{x^2(ief^2e^{2c} - if^2)e^{-c}}{2a} - \frac{x(ie^2e^{2c} - ie^2)e^{-c}}{2a} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cosh(d\*x+c)\*\*2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Piecewise(((((-2\*I\*a\*\*5\*d\*\*11\*e\*\*2\*exp(2\*c) - 4\*I\*a\*\*5\*d\*\*11\*e\*f\*x\*exp(2\*c) - 2\*I\*a\*\*5\*d\*\*11\*f\*\*2\*x\*\*2\*exp(2\*c) - 4\*I\*a\*\*5\*d\*\*10\*e\*f\*exp(2\*c) - 4\*I\*a\*\*5\*d\*\*10\*f\*\*2\*x\*exp(2\*c) - 4\*I\*a\*\*5\*d\*\*9\*f\*\*2\*exp(2\*c))\*exp(-d\*x) + (-2\*I\*a\*\*5\*d\*\*11\*e\*\*2\*exp(4\*c) - 4\*I\*a\*\*5\*d\*\*11\*e\*f\*x\*exp(4\*c) - 2\*I\*a\*\*5\*d\*\*11\*f\*\*2\*x\*\*2\*exp(4\*c) + 4\*I\*a\*\*5\*d\*\*10\*e\*f\*exp(4\*c) + 4\*I\*a\*\*5\*d\*\*10\*f\*\*2\*x\*exp(4\*c) - 4\*I\*a\*\*5\*d\*\*9\*f\*\*2\*exp(4\*c))\*exp(d\*x))\*exp(-3\*c)/(4\*a\*\*6\*d\*\*12), Ne(4\*a\*\*6\*d\*\*12\*exp(3\*c), 0)), (-x\*\*3\*(I\*f\*\*2\*exp(2\*c) - I\*f\*\*2)\*exp(-c)/(6\*a) - x\*\*2\*(I\*e\*f\*exp(2\*c) - I\*e\*f)\*exp(-c)/(2\*a) - x\*(I\*e\*\*2\*exp(2\*c) - I\*e\*\*2)\*exp(-c)/(2\*a), True)) + e\*\*2\*x/a + e\*f\*x\*\*2/a + f\*\*2\*x\*\*3/(3\*a)

**Giac [B]** time = 1.2607, size = 648, normalized size = 7.9

$$\frac{2d^3f^2x^3e^{(2dx+3c)} - 2id^3f^2x^3e^{(dx+2c)} - 3id^2f^2x^2e^{(3dx+4c)} + 6d^3fx^2e^{(2dx+3c+1)} - 3d^2f^2x^2e^{(2dx+3c)} - 6id^3fx^2e^{(dx+2c+1)}}{4a^6d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] 1/6\*(2\*d^3\*f^2\*x^3\*e^(2\*d\*x + 3\*c) - 2\*I\*d^3\*f^2\*x^3\*e^(d\*x + 2\*c) - 3\*I\*d^2\*f^2\*x^2\*e^(3\*d\*x + 4\*c) + 6\*d^3\*f\*f\*x^2\*e^(2\*d\*x + 3\*c + 1) - 3\*d^2\*f^2\*x^2\*e^(2\*d\*x + 3\*c) - 6\*I\*d^3\*f\*f\*x^2\*e^(d\*x + 2\*c + 1) - 3\*I\*d^2\*f^2\*x^2\*e^(d\*x + 2\*c) - 3\*d^2\*f^2\*x^2\*e^c - 6\*I\*d^2\*f\*f\*x^2\*e^(3\*d\*x + 4\*c + 1) + 6\*I\*d\*f^2\*x^2\*e^(3\*d\*x + 4\*c) + 6\*d^3\*x^2\*e^(2\*d\*x + 3\*c + 2) - 6\*d^2\*f\*f\*x^2\*e^(2\*d\*x + 3\*c + 1) + 6\*d\*f^2\*x^2\*e^(2\*d\*x + 3\*c) - 6\*I\*d^3\*f\*f\*x^2\*e^(d\*x + 2\*c + 2) - 6\*I\*d^2\*f\*f\*x^2\*e^(d\*x + 2\*c + 1) - 6\*I\*d\*f^2\*x^2\*e^(d\*x + 2\*c) - 6\*d^2\*f\*f\*x^2\*e^(c + 1) - 6\*d\*f^2\*x^2\*e^c - 3\*I\*d^2\*f\*f\*x^2\*e^(3\*d\*x + 4\*c + 2) + 6\*I\*d\*f^2\*x^2\*e^(3\*d\*x + 4\*c + 1) - 6\*I

$$\frac{f^2 e^{3dx + 4c} - 3d^2 e^{2dx + 3c + 2} + 6df e^{2dx + 3c + 1} - 6f^2 e^{2dx + 3c} - 3I d^2 e^{dx + 2c + 2} - 6I d f e^{dx + 2c + 1} - 6I f^2 e^{dx + 2c} - 3d^2 e^{c + 2} - 6df e^{c + 1} - 6f^2 e^c}{a d^3 e^{2dx + 3c} - I a d^3 e^{dx + 2c}}$$

$$3.261 \quad \int \frac{(e+fx) \cosh^2(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=56

$$\frac{if \sinh(c+dx)}{ad^2} - \frac{i(e+fx) \cosh(c+dx)}{ad} + \frac{ex}{a} + \frac{fx^2}{2a}$$

[Out] (e\*x)/a + (f\*x^2)/(2\*a) - (I\*(e + f\*x)\*Cosh[c + d\*x])/(a\*d) + (I\*f\*Sinh[c + d\*x])/(a\*d^2)

**Rubi [A]** time = 0.0715893, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {5563, 3296, 2637}

$$\frac{if \sinh(c+dx)}{ad^2} - \frac{i(e+fx) \cosh(c+dx)}{ad} + \frac{ex}{a} + \frac{fx^2}{2a}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Cosh[c + d\*x]^2)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (e\*x)/a + (f\*x^2)/(2\*a) - (I\*(e + f\*x)\*Cosh[c + d\*x])/(a\*d) + (I\*f\*Sinh[c + d\*x])/(a\*d^2)

### Rule 5563

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] :> Dist[1/a, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2 + b^2, 0]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{(e + fx) \cosh^2(c + dx)}{a + ia \sinh(c + dx)} dx &= -\frac{i \int (e + fx) \sinh(c + dx) dx}{a} + \frac{\int (e + fx) dx}{a} \\ &= \frac{ex}{a} + \frac{fx^2}{2a} - \frac{i(e + fx) \cosh(c + dx)}{ad} + \frac{(if) \int \cosh(c + dx) dx}{ad} \\ &= \frac{ex}{a} + \frac{fx^2}{2a} - \frac{i(e + fx) \cosh(c + dx)}{ad} + \frac{if \sinh(c + dx)}{ad^2} \end{aligned}$$

**Mathematica [A]** time = 0.638456, size = 57, normalized size = 1.02

$$-\frac{(c + dx)(cf - 2de - dfx) + 2id(e + fx) \cosh(c + dx) - 2if \sinh(c + dx)}{2ad^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^2)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] -((c + d*x)*(-2*d*e + c*f - d*f*x) + (2*I)*d*(e + f*x)*Cosh[c + d*x] - (2*I)*f*Sinh[c + d*x])/(2*a*d^2)
```

**Maple [A]** time = 0.04, size = 84, normalized size = 1.5

$$-\frac{1}{ad^2} \left( if((dx + c) \cosh(dx + c) - \sinh(dx + c)) - icf \cosh(dx + c) + ide \cosh(dx + c) - \frac{f(dx + c)^2}{2} + cf(dx + c) - de \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] -1/d^2/a*(I*f*((d*x+c)*cosh(d*x+c)-sinh(d*x+c))-I*c*f*cosh(d*x+c)+I*d*e*cosh(d*x+c)-1/2*f*(d*x+c)^2+c*f*(d*x+c)-d*e*(d*x+c))
```



**Maxima [B]** time = 1.50894, size = 254, normalized size = 4.54

$$\frac{1}{4} f \left( \frac{4xe^{(dx+c)}}{ade^{(dx+c)} - iad} + \frac{-2id^2x^2e^c - 2idxe^c - (2idxe^{(3c)} - 2ie^{(3c)})e^{(2dx)} + 2(d^2x^2e^{(2c)} - 3dxe^{(2c)} + e^{(2c)})e^{(dx)} - 2(dx+1)}{ad^2e^{(dx+2c)} - iad^2e^c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $\frac{1}{4} f \left( \frac{4*x*e^{(d*x+c)}}{a*d*e^{(d*x+c)} - I*a*d} + \frac{(-2*I*d^2*x^2*e^c - 2*I*d*x*e^c - (2*I*d*x*e^{(3*c)} - 2*I*e^{(3*c)})*e^{(2*d*x)} + 2*(d^2*x^2*e^{(2*c)} - 3*d*x*e^{(2*c)} + e^{(2*c)})*e^{(d*x)} - 2*(d*x+1)*e^{(-d*x)} - 2*I*e^c)}{a*d^2*e^{(d*x+2*c)} - I*a*d^2*e^c} \right) + \frac{1}{4} e \left( \frac{4*(d*x+c)}{a*d} - \frac{2*I*e^{(d*x+c)}}{a*d} - \frac{2*I*e^{(-d*x-c)}}{a*d} \right)$

**Fricas [A]** time = 2.15053, size = 178, normalized size = 3.18

$$\frac{(-idfx - ide + (-idfx - ide + if)e^{(2dx+2c)} + (d^2fx^2 + 2d^2ex)e^{(dx+c)} - if)e^{(-dx-c)}}{2ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{2} \left( -I*d*f*x - I*d*e + (-I*d*f*x - I*d*e + I*f)*e^{(2*d*x+2*c)} + (d^2*f*x^2 + 2*d^2*e*x)*e^{(d*x+c)} - I*f*e^{(-d*x-c)} \right) / (a*d^2)$

**Sympy [A]** time = 1.10237, size = 194, normalized size = 3.46

$$\begin{cases} \frac{((-2ia^3d^5ee^c - 2ia^3d^5fxe^c - 2ia^3d^4fe^c)e^{-dx} + (-2ia^3d^5ee^{3c} - 2ia^3d^5fxe^{3c} + 2ia^3d^4fe^{3c})e^{dx})e^{-2c}}{4a^4d^6} & \text{for } 4a^4d^6e^{2c} \neq 0 \\ -\frac{x^2(ife^{2c}-if)e^{-c}}{4a} - \frac{x(iee^{2c}-ie)e^{-c}}{2a} & \text{otherwise} \end{cases} + \frac{ex}{a} + \frac{fx^2}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)\*\*2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Piecewise(((((-2\*I\*a\*\*3\*d\*\*5\*e\*exp(c) - 2\*I\*a\*\*3\*d\*\*5\*f\*x\*exp(c) - 2\*I\*a\*\*3\*d\*\*4\*f\*exp(c))\*exp(-d\*x) + (-2\*I\*a\*\*3\*d\*\*5\*e\*exp(3\*c) - 2\*I\*a\*\*3\*d\*\*5\*f\*x\*e

$x^p(3c) + 2Ia^{**3}d^{**4}f \exp(3c) \exp(dx) \exp(-2c) / (4a^{**4}d^{**6}), \text{Ne}(4$   
 $a^{**4}d^{**6} \exp(2c), 0), (-x^{**2}(If \exp(2c) - If) \exp(-c) / (4a) - x(If$   
 $e \exp(2c) - Ie) \exp(-c) / (2a), \text{True})) + e*x/a + f*x^{**2}/(2*a)$

**Giac [B]** time = 1.16307, size = 312, normalized size = 5.57

$$\frac{d^2 f x^2 e^{(2dx+3c)} - i d^2 f x^2 e^{(dx+2c)} - i d f x e^{(3dx+4c)} + 2 d^2 x e^{(2dx+3c+1)} - d f x e^{(2dx+3c)} - 2i d^2 x e^{(dx+2c+1)} - i d f x e^{(dx+2c)} - d f x e^{(dx+2c)}}{2(ad^2 e^{(2dx+3c)} - i ad^2 e^{(dx+2c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out]  $1/2*(d^2 f x^2 e^{(2dx+3c)} - I d^2 f x^2 e^{(dx+2c)} - I d f x e^{(3dx+4c)} + 2 d^2 x e^{(2dx+3c+1)} - d f x e^{(2dx+3c)} - 2 I d^2 x e^{(dx+2c+1)} - I d f x e^{(dx+2c)} - d f x e^c - I d e^{(3dx+4c+1)} + I f e^{(3dx+4c)} - d e^{(2dx+3c+1)} + f e^{(2dx+3c)} - I d e^{(dx+2c+1)} - I f e^{(dx+2c)} - d e^{(c+1)} - f e^c) / (a d^2 e^{(2dx+3c)} - I a d^2 e^{(dx+2c)})$

$$3.262 \quad \int \frac{\cosh^2(c+dx)}{a+ia \sinh(c+dx)} dx$$

Optimal. Leaf size=22

$$\frac{x}{a} - \frac{i \cosh(c+dx)}{ad}$$

[Out] x/a - (I\*Cosh[c + d\*x])/(a\*d)

**Rubi [A]** time = 0.0431233, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$ , Rules used = {2682, 8}

$$\frac{x}{a} - \frac{i \cosh(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]^2/(a + I\*a\*Sinh[c + d\*x]), x]

[Out] x/a - (I\*Cosh[c + d\*x])/(a\*d)

#### Rule 2682

Int[(cos[(e\_.) + (f\_.)\*(x\_)]\*(g\_.))^p\_]/((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Simp[(g\*(g\*Cos[e + f\*x])^(p - 1))/(b\*f\*(p - 1)), x] + Dist[g^2/a, Int[(g\*Cos[e + f\*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2\*p]

#### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\cosh^2(c+dx)}{a+ia \sinh(c+dx)} dx &= -\frac{i \cosh(c+dx)}{ad} + \frac{\int 1 dx}{a} \\ &= \frac{x}{a} - \frac{i \cosh(c+dx)}{ad} \end{aligned}$$

**Mathematica [B]** time = 0.1447, size = 139, normalized size = 6.32

$$\frac{\cosh^3(c + dx) \left( -i\sqrt{1 + i \sinh(c + dx)} \sinh(c + dx) + \sqrt{1 + i \sinh(c + dx)} - 2\sqrt{1 - i \sinh(c + dx)} \sin^{-1} \left( \frac{\sqrt{1 - i \sinh(c + dx)}}{\sqrt{2}} \right) \right)}{ad\sqrt{1 + i \sinh(c + dx)}(\sinh(c + dx) - i)(\sinh(c + dx) + i)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]^2/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (Cosh[c + d\*x]^3\*(-2\*ArcSin[Sqrt[1 - I\*Sinh[c + d\*x]]/Sqrt[2]]\*Sqrt[1 - I\*Sinh[c + d\*x]] + Sqrt[1 + I\*Sinh[c + d\*x]] - I\*Sqrt[1 + I\*Sinh[c + d\*x]]\*Sinh[c + d\*x]))/(a\*d\*Sqrt[1 + I\*Sinh[c + d\*x]]\*(-I + Sinh[c + d\*x])\*(I + Sinh[c + d\*x])^2)

**Maple [B]** time = 0.044, size = 85, normalized size = 3.9

$$\frac{-i}{da} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} + \frac{1}{da} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) + \frac{i}{da} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^{-1} - \frac{1}{da} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] -I/d/a/(tanh(1/2\*d\*x+1/2\*c)+1)+1/d/a\*ln(tanh(1/2\*d\*x+1/2\*c)+1)+I/d/a/(tanh(1/2\*d\*x+1/2\*c)-1)-1/d/a\*ln(tanh(1/2\*d\*x+1/2\*c)-1)

**Maxima [B]** time = 1.06201, size = 59, normalized size = 2.68

$$\frac{dx + c}{ad} - \frac{i e^{(dx+c)}}{2ad} - \frac{i e^{(-dx-c)}}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] (d\*x + c)/(a\*d) - 1/2\*I\*e^(d\*x + c)/(a\*d) - 1/2\*I\*e^(-d\*x - c)/(a\*d)

**Fricas [A]** time = 2.11257, size = 92, normalized size = 4.18

$$\frac{(2dx e^{(dx+c)} - i e^{(2dx+2c)} - i) e^{(-dx-c)}}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 1/2\*(2\*d\*x\*e^(d\*x + c) - I\*e^(2\*d\*x + 2\*c) - I)\*e^(-d\*x - c)/(a\*d)

**Sympy [A]** time = 0.433829, size = 82, normalized size = 3.73

$$\begin{cases} \frac{(-2iade^{2c}e^{dx}-2iade^{-dx})e^{-c}}{4a^2d^2} & \text{for } 4a^2d^2e^c \neq 0 \\ x \left( -\frac{(ie^{2c}-2e^c-i)e^{-c}}{2a} - \frac{1}{a} \right) & \text{otherwise} \end{cases} + \frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*\*2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Piecewise((( -2\*I\*a\*d\*exp(2\*c)\*exp(d\*x) - 2\*I\*a\*d\*exp(-d\*x))\*exp(-c)/(4\*a\*\*2\*d\*\*2), Ne(4\*a\*\*2\*d\*\*2\*exp(c), 0)), (x\*(-(I\*exp(2\*c) - 2\*exp(c) - I)\*exp(-c)/(2\*a) - 1/a), True)) + x/a

**Giac [A]** time = 1.22428, size = 47, normalized size = 2.14

$$\frac{2dx + 2c - i e^{(dx+c)} - i e^{(-dx-c)}}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] 1/2\*(2\*d\*x + 2\*c - I\*e^(d\*x + c) - I\*e^(-d\*x - c))/(a\*d)

$$3.263 \quad \int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

**Optimal.** Leaf size=76

$$-\frac{i \sinh\left(c - \frac{de}{f}\right) \text{Chi}\left(\frac{de}{f} + dx\right)}{af} - \frac{i \cosh\left(c - \frac{de}{f}\right) \text{Shi}\left(\frac{de}{f} + dx\right)}{af} + \frac{\log(e+fx)}{af}$$

[Out] Log[e + f\*x]/(a\*f) - (I\*CoshIntegral[(d\*e)/f + d\*x]\*Sinh[c - (d\*e)/f])/(a\*f) - (I\*Cosh[c - (d\*e)/f]\*SinhIntegral[(d\*e)/f + d\*x])/(a\*f)

**Rubi [A]** time = 0.204152, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {5563, 31, 3303, 3298, 3301}

$$-\frac{i \sinh\left(c - \frac{de}{f}\right) \text{Chi}\left(\frac{de}{f} + dx\right)}{af} - \frac{i \cosh\left(c - \frac{de}{f}\right) \text{Shi}\left(\frac{de}{f} + dx\right)}{af} + \frac{\log(e+fx)}{af}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]^2/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])),x]

[Out] Log[e + f\*x]/(a\*f) - (I\*CoshIntegral[(d\*e)/f + d\*x]\*Sinh[c - (d\*e)/f])/(a\*f) - (I\*Cosh[c - (d\*e)/f]\*SinhIntegral[(d\*e)/f + d\*x])/(a\*f)

### Rule 5563

Int[(Cosh[(c\_.) + (d\_.)\*(x\_.)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2 + b^2, 0]

### Rule 31

Int[((a\_.) + (b\_.)\*(x\_.))^(-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f,
fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:= Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz},
x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\cosh^2(c+dx)}{(e+fx)(a+ia\sinh(c+dx))} dx &= -\frac{i \int \frac{\sinh(c+dx)}{e+fx} dx}{a} + \frac{\int \frac{1}{e+fx} dx}{a} \\ &= \frac{\log(e+fx)}{af} - \frac{\left(i \cosh\left(c - \frac{de}{f}\right)\right) \int \frac{\sinh\left(\frac{de}{f}+dx\right)}{e+fx} dx}{a} - \frac{\left(i \sinh\left(c - \frac{de}{f}\right)\right) \int \frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx} dx}{a} \\ &= \frac{\log(e+fx)}{af} - \frac{i \operatorname{Chi}\left(\frac{de}{f}+dx\right) \sinh\left(c - \frac{de}{f}\right)}{af} - \frac{i \cosh\left(c - \frac{de}{f}\right) \operatorname{Shi}\left(\frac{de}{f}+dx\right)}{af} \end{aligned}$$

**Mathematica [A]** time = 0.320749, size = 62, normalized size = 0.82

$$\frac{-i \sinh\left(c - \frac{de}{f}\right) \operatorname{Chi}\left(d\left(\frac{e}{f} + x\right)\right) - i \cosh\left(c - \frac{de}{f}\right) \operatorname{Shi}\left(d\left(\frac{e}{f} + x\right)\right) + \log(e+fx)}{af}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/((e + f*x)*(a + I*a*Sinh[c + d*x])),x]
```

```
[Out] (Log[e + f*x] - I*CoshIntegral[d*(e/f + x)]*Sinh[c - (d*e)/f] - I*Cosh[c -
(d*e)/f]*SinhIntegral[d*(e/f + x)])/(a*f)
```

---

**Maple [A]** time = 0.09, size = 103, normalized size = 1.4

$$\frac{\ln(fx + e)}{af} + \frac{i}{2af} e^{\frac{cf-de}{f}} \operatorname{Ei}\left(1, -dx - c - \frac{-cf + de}{f}\right) - \frac{i}{2af} e^{-\frac{cf-de}{f}} \operatorname{Ei}\left(1, dx + c - \frac{cf - de}{f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x)`

[Out] `ln(f*x+e)/a/f+1/2*I/a/f*exp((c*f-d*e)/f)*Ei(1,-d*x-c-(-c*f+d*e)/f)-1/2*I/a/f*exp(-(c*f-d*e)/f)*Ei(1,d*x+c-(c*f-d*e)/f)`

---

**Maxima [A]** time = 1.51494, size = 103, normalized size = 1.36

$$-\frac{i e^{\left(-c + \frac{de}{f}\right)} E_1\left(\frac{(fx+e)d}{f}\right)}{2af} + \frac{i e^{\left(c - \frac{de}{f}\right)} E_1\left(-\frac{(fx+e)d}{f}\right)}{2af} + \frac{\log(fx + e)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `-1/2*I*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(a*f) + 1/2*I*e^(c - d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(a*f) + log(f*x + e)/(a*f)`

---

**Fricas [A]** time = 2.18993, size = 159, normalized size = 2.09

$$\frac{i \operatorname{Ei}\left(-\frac{dfx+de}{f}\right) e^{\left(\frac{de-cf}{f}\right)} - i \operatorname{Ei}\left(\frac{dfx+de}{f}\right) e^{\left(-\frac{de-cf}{f}\right)} + 2 \log\left(\frac{fx+e}{f}\right)}{2af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")`

[Out] `1/2*(I*Ei(-(d*f*x + d*e)/f)*e^((d*e - c*f)/f) - I*Ei((d*f*x + d*e)/f)*e^(-(d*e - c*f)/f) + 2*log((f*x + e)/f))/(a*f)`



---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*\*2/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [A]** time = 1.18928, size = 109, normalized size = 1.43

$$\frac{\left(i \operatorname{Ei}\left(\frac{dfx+de}{f}\right) e^{\left(2c-\frac{de}{f}\right)} - i \operatorname{Ei}\left(-\frac{dfx+de}{f}\right) e^{\left(\frac{de}{f}\right)} - 2e^c \log(ifx+ie)\right) e^{(-c)}}{2af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out]  $-1/2*(I*\operatorname{Ei}((d*f*x + d*e)/f)*e^{(2*c - d*e/f)} - I*\operatorname{Ei}(-(d*f*x + d*e)/f)*e^{(d*e/f)} - 2*e^c*\log(I*f*x + I*e))*e^{(-c)}/(a*f)$

$$3.264 \quad \int \frac{\cosh^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

**Optimal.** Leaf size=103

$$-\frac{id \cosh\left(c - \frac{de}{f}\right) \text{Chi}\left(\frac{de}{f} + dx\right)}{af^2} - \frac{id \sinh\left(c - \frac{de}{f}\right) \text{Shi}\left(\frac{de}{f} + dx\right)}{af^2} + \frac{i \sinh(c+dx)}{af(e+fx)} - \frac{1}{af(e+fx)}$$

[Out]  $-(1/(a*f*(e + f*x))) - (I*d*Cosh[c - (d*e)/f]*CoshIntegral[(d*e)/f + d*x])/(a*f^2) + (I*Sinh[c + d*x])/(a*f*(e + f*x)) - (I*d*Sinh[c - (d*e)/f]*SinhIntegral[(d*e)/f + d*x])/(a*f^2)$

**Rubi [A]** time = 0.220979, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {5563, 32, 3297, 3303, 3298, 3301}

$$-\frac{id \cosh\left(c - \frac{de}{f}\right) \text{Chi}\left(\frac{de}{f} + dx\right)}{af^2} - \frac{id \sinh\left(c - \frac{de}{f}\right) \text{Shi}\left(\frac{de}{f} + dx\right)}{af^2} + \frac{i \sinh(c+dx)}{af(e+fx)} - \frac{1}{af(e+fx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Cosh}[c + d*x]^2/((e + f*x)^2*(a + I*a*Sinh[c + d*x])),x]$

[Out]  $-(1/(a*f*(e + f*x))) - (I*d*Cosh[c - (d*e)/f]*CoshIntegral[(d*e)/f + d*x])/(a*f^2) + (I*Sinh[c + d*x])/(a*f*(e + f*x)) - (I*d*Sinh[c - (d*e)/f]*SinhIntegral[(d*e)/f + d*x])/(a*f^2)$

### Rule 5563

$\text{Int}[(\text{Cosh}[(c_.) + (d_.)*(x_.)]^{(n_.)}*((e_.) + (f_.)*(x_.))^{(m_.)})/((a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_.)]), x\_Symbol] := \text{Dist}[1/a, \text{Int}[(e + f*x)^m*\text{Cosh}[c + d*x]^{(n - 2)}, x], x] + \text{Dist}[1/b, \text{Int}[(e + f*x)^m*\text{Cosh}[c + d*x]^{(n - 2)}*\text{Sinh}[c + d*x], x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, m\}, x\} \ \&\& \ \text{IGtQ}[n, 1] \ \&\& \ \text{EqQ}[a^2 + b^2, 0]$

### Rule 32

$\text{Int}[(a_.) + (b_.)*(x_.)]^{(m_.)}, x\_Symbol] := \text{Simp}[(a + b*x)^{(m + 1)}/(b*(m + 1)), x] /;$   $\text{FreeQ}\{a, b, m\}, x\} \ \&\& \ \text{NeQ}[m, -1]$

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh^2(c + dx)}{(e + fx)^2(a + ia \sinh(c + dx))} dx &= -\frac{i \int \frac{\sinh(c+dx)}{(e+fx)^2} dx}{a} + \frac{\int \frac{1}{(e+fx)^2} dx}{a} \\ &= -\frac{1}{af(e + fx)} + \frac{i \sinh(c + dx)}{af(e + fx)} - \frac{(id) \int \frac{\cosh(c+dx)}{e+fx} dx}{af} \\ &= -\frac{1}{af(e + fx)} + \frac{i \sinh(c + dx)}{af(e + fx)} - \frac{\left(id \cosh\left(c - \frac{de}{f}\right)\right) \int \frac{\cosh\left(\frac{de}{f} + dx\right)}{e+fx} dx}{af} - \frac{\left(id \sinh\left(c - \frac{de}{f}\right)\right)}{af} \\ &= -\frac{1}{af(e + fx)} - \frac{id \cosh\left(c - \frac{de}{f}\right) \operatorname{Chi}\left(\frac{de}{f} + dx\right)}{af^2} + \frac{i \sinh(c + dx)}{af(e + fx)} - \frac{id \sinh\left(c - \frac{de}{f}\right)}{af} \end{aligned}$$

**Mathematica [A]** time = 0.528068, size = 85, normalized size = 0.83

$$\frac{i\left(d(e+fx)\cosh\left(c-\frac{de}{f}\right)\text{Chi}\left(d\left(\frac{e}{f}+x\right)\right)+d(e+fx)\sinh\left(c-\frac{de}{f}\right)\text{Shi}\left(d\left(\frac{e}{f}+x\right)\right)-f(\sinh(c+dx)+i)}{af^2(e+fx)}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]^2/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])),x]

[Out] ((-I)\*(d\*(e + f\*x)\*Cosh[c - (d\*e)/f]\*CoshIntegral[d\*(e/f + x)] - f\*(I + Sinh[c + d\*x]) + d\*(e + f\*x)\*Sinh[c - (d\*e)/f]\*SinhIntegral[d\*(e/f + x)))/(a\*f^2\*(e + f\*x))

**Maple [A]** time = 0.095, size = 164, normalized size = 1.6

$$-\frac{1}{af(fx+e)} + \frac{\frac{i}{2}de^{dx+c}}{af^2}\left(\frac{de}{f}+dx\right)^{-1} + \frac{\frac{i}{2}d}{af^2}e^{\frac{cf-de}{f}}\text{Ei}\left(1,-dx-c-\frac{-cf+de}{f}\right) - \frac{\frac{i}{2}de^{-dx-c}}{af(dfx+de)} + \frac{\frac{i}{2}d}{af^2}e^{-\frac{cf-de}{f}}\text{Ei}\left(1,dx+c-\frac{cf-de}{f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)^2/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] -1/a/f/(f\*x+e)+1/2\*I\*d/a/f^2\*exp(d\*x+c)/(d\*e/f+d\*x)+1/2\*I\*d/a/f^2\*exp((c\*f-d\*e)/f)\*Ei(1,-d\*x-c-(c\*f+d\*e)/f)-1/2\*I/a\*d\*exp(-d\*x-c)/f/(d\*f\*x+d\*e)+1/2\*I/a\*d/f^2\*exp(-(c\*f-d\*e)/f)\*Ei(1,d\*x+c-(c\*f-d\*e)/f)

**Maxima [A]** time = 1.65693, size = 124, normalized size = 1.2

$$-\frac{1}{af^2x+ae f} - \frac{ie^{\left(-c+\frac{de}{f}\right)}E_2\left(\frac{(fx+e)d}{f}\right)}{2(fx+e)af} + \frac{ie^{\left(c-\frac{de}{f}\right)}E_2\left(-\frac{(fx+e)d}{f}\right)}{2(fx+e)af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-1/(a*f^2*x + a*e*f) - 1/2*I*e^{(-c + d*e/f)*\exp\_integral\_e(2, (f*x + e)*d/f)}$   
 $/((f*x + e)*a*f) + 1/2*I*e^{(c - d*e/f)*\exp\_integral\_e(2, -(f*x + e)*d/f)}$   
 $/((f*x + e)*a*f)$

**Fricas [A]** time = 2.16562, size = 278, normalized size = 2.7

$$\frac{\left( i f e^{2dx+2c} + \left( (-i d f x - i d e) \operatorname{Ei} \left( -\frac{d f x + d e}{f} \right) e^{\left( \frac{d e - c f}{f} \right)} + (-i d f x - i d e) \operatorname{Ei} \left( \frac{d f x + d e}{f} \right) e^{\left( -\frac{d e - c f}{f} \right)} - 2 f \right) e^{(d x + c)} - i f \right) e^{(-d x - c)}}{2(a f^3 x + a e f^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")`

[Out]  $1/2*(I*f*e^{(2*d*x + 2*c)} + ((-I*d*f*x - I*d*e)*\operatorname{Ei}(-(d*f*x + d*e)/f)*e^{((d*e - c*f)/f)} + (-I*d*f*x - I*d*e)*\operatorname{Ei}((d*f*x + d*e)/f)*e^{-(d*e - c*f)/f} - 2*f)*e^{(d*x + c)} - I*f)*e^{(-d*x - c)}/(a*f^3*x + a*e*f^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)**2/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)`

[Out] Timed out

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.265 \quad \int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=231

$$-\frac{3if^2(e+fx)\sinh^2(c+dx)}{4ad^3} + \frac{6f^2(e+fx)\sinh(c+dx)}{ad^3} - \frac{3f(e+fx)^2\cosh(c+dx)}{ad^2} + \frac{3if(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{4ad^2}$$

[Out] (((-3\*I)/8)\*f^3\*x)/(a\*d^3) - ((I/4)\*(e + f\*x)^3)/(a\*d) - (6\*f^3\*Cosh[c + d\*x])/(a\*d^4) - (3\*f\*(e + f\*x)^2\*Cosh[c + d\*x])/(a\*d^2) + (6\*f^2\*(e + f\*x)\*Sinh[c + d\*x])/(a\*d^3) + ((e + f\*x)^3\*Sinh[c + d\*x])/(a\*d) + (((3\*I)/8)\*f^3\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(a\*d^4) + (((3\*I)/4)\*f\*(e + f\*x)^2\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(a\*d^2) - (((3\*I)/4)\*f^2\*(e + f\*x)\*Sinh[c + d\*x]^2)/(a\*d^3) - ((I/2)\*(e + f\*x)^3\*Sinh[c + d\*x]^2)/(a\*d)

**Rubi [A]** time = 0.260774, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$ , Rules used = {5563, 3296, 2638, 5446, 3311, 32, 2635, 8}

$$-\frac{3if^2(e+fx)\sinh^2(c+dx)}{4ad^3} + \frac{6f^2(e+fx)\sinh(c+dx)}{ad^3} - \frac{3f(e+fx)^2\cosh(c+dx)}{ad^2} + \frac{3if(e+fx)^2\sinh(c+dx)\cosh(c+dx)}{4ad^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Cosh[c + d\*x]^3)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (((-3\*I)/8)\*f^3\*x)/(a\*d^3) - ((I/4)\*(e + f\*x)^3)/(a\*d) - (6\*f^3\*Cosh[c + d\*x])/(a\*d^4) - (3\*f\*(e + f\*x)^2\*Cosh[c + d\*x])/(a\*d^2) + (6\*f^2\*(e + f\*x)\*Sinh[c + d\*x])/(a\*d^3) + ((e + f\*x)^3\*Sinh[c + d\*x])/(a\*d) + (((3\*I)/8)\*f^3\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(a\*d^4) + (((3\*I)/4)\*f\*(e + f\*x)^2\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(a\*d^2) - (((3\*I)/4)\*f^2\*(e + f\*x)\*Sinh[c + d\*x]^2)/(a\*d^3) - ((I/2)\*(e + f\*x)^3\*Sinh[c + d\*x]^2)/(a\*d)

### Rule 5563

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.)]/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2 + b^2, 0]

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[
{c, d}, x]
```

### Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[
(d*m*(c + d*x)^(m - 1)*(b*Sinh[e + f*x])^n)/(f^2*n^2), x] + (Dist[
(b^2*(n - 1))/n, Int[(c + d*x)^(m - 1)*(b*Sinh[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sinh[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*cos[e + f*x]*(b*Sinh[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] := -Simp[(b*cos[c + d*x]
*(b*sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
]
```

### Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+ia \sinh(c+dx)} dx &= -\frac{i \int (e+fx)^3 \cosh(c+dx) \sinh(c+dx) dx}{a} + \frac{\int (e+fx)^3 \cosh(c+dx) dx}{a} \\
&= \frac{(e+fx)^3 \sinh(c+dx)}{ad} - \frac{i(e+fx)^3 \sinh^2(c+dx)}{2ad} + \frac{(3if) \int (e+fx)^2 \sinh^2(c+dx) dx}{2ad} \\
&= -\frac{3f(e+fx)^2 \cosh(c+dx)}{ad^2} + \frac{(e+fx)^3 \sinh(c+dx)}{ad} + \frac{3if(e+fx)^2 \cosh(c+dx) \sinh(c+dx)}{4ad^2} \\
&= \frac{i(e+fx)^3}{4ad} - \frac{3f(e+fx)^2 \cosh(c+dx)}{ad^2} + \frac{6f^2(e+fx) \sinh(c+dx)}{ad^3} + \frac{(e+fx)^3 \sinh(c+dx)}{ad} \\
&= \frac{3if^3x}{8ad^3} - \frac{i(e+fx)^3}{4ad} - \frac{6f^3 \cosh(c+dx)}{ad^4} - \frac{3f(e+fx)^2 \cosh(c+dx)}{ad^2} + \frac{6f^2(e+fx) \sinh(c+dx)}{ad^3}
\end{aligned}$$

**Mathematica [A]** time = 1.30041, size = 134, normalized size = 0.58

$$\frac{-96f \cosh(c+dx) (d^2(e+fx)^2 + 2f^2) - 4id(e+fx) \cosh(2(c+dx)) (2d^2(e+fx)^2 + 3f^2) + 4 \sinh(c+dx) (8d(e+fx) \cosh(c+dx) - 3f^2)}{32ad^4}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^3\*Cosh[c + d\*x]^3)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (-96\*f\*(2\*f^2 + d^2\*(e + f\*x)^2)\*Cosh[c + d\*x] - (4\*I)\*d\*(e + f\*x)\*(3\*f^2 + 2\*d^2\*(e + f\*x)^2)\*Cosh[2\*(c + d\*x)] + 4\*(8\*d\*(e + f\*x)\*(6\*f^2 + d^2\*(e + f\*x)^2) + (3\*I)\*f\*(f^2 + 2\*d^2\*(e + f\*x)^2)\*Cosh[c + d\*x])\*Sinh[c + d\*x])/(32\*a\*d^4)

**Maple [B]** time = 0.16, size = 429, normalized size = 1.9

$$\frac{-\frac{i}{32} (4 f^3 x^3 d^3 + 12 d^3 e f^2 x^2 + 12 d^3 e^2 f x - 6 d^2 f^3 x^2 + 4 d^3 e^3 - 12 d^2 e f^2 x - 6 d^2 e^2 f + 6 d f^3 x + 6 d e f^2 - 3 f^3) e^{2dx+2c}}{ad^4} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*cosh(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x)

[Out] -1/32\*I\*(4\*d^3\*f^3\*x^3+12\*d^3\*e\*f^2\*x^2+12\*d^3\*e^2\*f\*x-6\*d^2\*f^3\*x^2+4\*d^3\*e^3-12\*d^2\*e\*f^2\*x-6\*d^2\*e^2\*f+6\*d\*f^3\*x+6\*d\*e\*f^2-3\*f^3)/a/d^4\*exp(2\*d\*x+2\*c)+1/2\*(d^3\*f^3\*x^3+3\*d^3\*e\*f^2\*x^2+3\*d^3\*e^2\*f\*x-3\*d^2\*f^3\*x^2+d^3\*e^3-6\*



$$\frac{d^2 e f^2 x - 3 d^2 e^2 f + 6 d f^3 x + 6 d e f^2 - 6 f^3}{a d^4 \exp(d x + c)} - \frac{1}{2} \frac{(d^3 f^3 x^3 + 3 d^3 e f^2 x^2 + 3 d^3 e^2 f x + 3 d^2 f^3 x^2 + d^3 e^3 + 6 d^2 e f^2 x + 3 d^2 e^2 f + 6 d f^3 x + 6 d e f^2 + 6 f^3)}{a d^4 \exp(-d x - c)} - \frac{1}{32} I \frac{(4 d^3 f^3 x^3 + 12 d^3 e f^2 x^2 + 12 d^3 e^2 f x + 6 d^2 f^3 x^2 + 4 d^3 e^3 + 12 d^2 e f^2 x + 6 d^2 e^2 f + 6 d f^3 x + 6 d e f^2 + 3 f^3)}{a d^4 \exp(-2 d x - 2 c)}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
)
```

```
[Out] Exception raised: RuntimeError
```

**Fricas [A]** time = 2.21988, size = 906, normalized size = 3.92

$$\frac{(-4i d^3 f^3 x^3 - 4i d^3 e^3 - 6i d^2 e^2 f - 6i d e f^2 - 3i f^3 + (-12i d^3 e f^2 - 6i d^2 f^3) x^2 + (-12i d^3 e^2 f - 12i d^2 e f^2 - 6i d f^3) x + (-12i d^3 e^3 - 6i d^2 e^2 f - 6i d e f^2 - 3i f^3)) e^{(4d x + 4c)} + 16(d^3 f^3 x^3 + d^3 e^3 - 3d^2 e^2 f + 6d e f^2 - 6f^3 + 3(d^3 e f^2 - d^2 f^3) x^2 + 3(d^3 e^2 f - 2d^2 e f^2 + 2d f^3) x) e^{(3d x + 3c)} - 16(d^3 f^3 x^3 + d^3 e^3 + 3d^2 e^2 f + 6d e f^2 + 6f^3 + 3(d^3 e f^2 + d^2 f^3) x^2 + 3(d^3 e^2 f + 2d^2 e f^2 + 2d f^3) x) e^{(d x + c)}}{(a d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
)
```

```
[Out] 1/32*(-4*I*d^3*f^3*x^3 - 4*I*d^3*e^3 - 6*I*d^2*e^2*f - 6*I*d*e*f^2 - 3*I*f^3 + (-12*I*d^3*e*f^2 - 6*I*d^2*f^3)*x^2 + (-12*I*d^3*e^2*f - 12*I*d^2*e*f^2 - 6*I*d*f^3)*x + (-4*I*d^3*f^3*x^3 - 4*I*d^3*e^3 + 6*I*d^2*e^2*f - 6*I*d*e*f^2 + 3*I*f^3 + (-12*I*d^3*e*f^2 + 6*I*d^2*f^3)*x^2 + (-12*I*d^3*e^2*f + 12*I*d^2*e*f^2 - 6*I*d*f^3)*x)*e^(4*d*x + 4*c) + 16*(d^3*f^3*x^3 + d^3*e^3 - 3*d^2*e^2*f + 6*d*e*f^2 - 6*f^3 + 3*(d^3*e*f^2 - d^2*f^3)*x^2 + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*x)*e^(3*d*x + 3*c) - 16*(d^3*f^3*x^3 + d^3*e^3 + 3*d^2*e^2*f + 6*d*e*f^2 + 6*f^3 + 3*(d^3*e*f^2 + d^2*f^3)*x^2 + 3*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*x)*e^(d*x + c))/(a*d^4)
```

**Sympy [A]** time = 4.77379, size = 1061, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cosh(d\*x+c)\*\*3/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Piecewise(((((-2048\*a\*\*9\*d\*\*27\*e\*\*3\*exp(8\*c) - 6144\*a\*\*9\*d\*\*27\*e\*\*2\*f\*x\*exp(8\*c) - 6144\*a\*\*9\*d\*\*27\*e\*f\*\*2\*x\*\*2\*exp(8\*c) - 2048\*a\*\*9\*d\*\*27\*f\*\*3\*x\*\*3\*exp(8\*c) - 6144\*a\*\*9\*d\*\*26\*e\*\*2\*f\*exp(8\*c) - 12288\*a\*\*9\*d\*\*26\*e\*f\*\*2\*x\*exp(8\*c) - 6144\*a\*\*9\*d\*\*26\*f\*\*3\*x\*\*2\*exp(8\*c) - 12288\*a\*\*9\*d\*\*25\*e\*f\*\*2\*exp(8\*c) - 12288\*a\*\*9\*d\*\*25\*f\*\*3\*x\*exp(8\*c) - 12288\*a\*\*9\*d\*\*24\*f\*\*3\*exp(8\*c))\*exp(-d\*x) + (2048\*a\*\*9\*d\*\*27\*e\*\*3\*exp(10\*c) + 6144\*a\*\*9\*d\*\*27\*e\*\*2\*f\*x\*exp(10\*c) + 6144\*a\*\*9\*d\*\*27\*e\*f\*\*2\*x\*\*2\*exp(10\*c) + 2048\*a\*\*9\*d\*\*27\*f\*\*3\*x\*\*3\*exp(10\*c) - 6144\*a\*\*9\*d\*\*26\*e\*\*2\*f\*exp(10\*c) - 12288\*a\*\*9\*d\*\*26\*e\*f\*\*2\*x\*exp(10\*c) - 6144\*a\*\*9\*d\*\*26\*f\*\*3\*x\*\*2\*exp(10\*c) + 12288\*a\*\*9\*d\*\*25\*e\*f\*\*2\*exp(10\*c) + 12288\*a\*\*9\*d\*\*25\*f\*\*3\*x\*exp(10\*c) - 12288\*a\*\*9\*d\*\*24\*f\*\*3\*exp(10\*c))\*exp(d\*x) + (-512\*I\*a\*\*9\*d\*\*27\*e\*\*3\*exp(7\*c) - 1536\*I\*a\*\*9\*d\*\*27\*e\*\*2\*f\*x\*exp(7\*c) - 1536\*I\*a\*\*9\*d\*\*27\*e\*f\*\*2\*x\*\*2\*exp(7\*c) - 512\*I\*a\*\*9\*d\*\*27\*f\*\*3\*x\*\*3\*exp(7\*c) - 768\*I\*a\*\*9\*d\*\*26\*e\*\*2\*f\*exp(7\*c) - 1536\*I\*a\*\*9\*d\*\*26\*e\*f\*\*2\*x\*exp(7\*c) - 768\*I\*a\*\*9\*d\*\*26\*f\*\*3\*x\*\*2\*exp(7\*c) - 768\*I\*a\*\*9\*d\*\*25\*e\*f\*\*2\*exp(7\*c) - 768\*I\*a\*\*9\*d\*\*25\*f\*\*3\*x\*exp(7\*c) - 384\*I\*a\*\*9\*d\*\*24\*f\*\*3\*exp(7\*c))\*exp(-2\*d\*x) + (-512\*I\*a\*\*9\*d\*\*27\*e\*\*3\*exp(11\*c) - 1536\*I\*a\*\*9\*d\*\*27\*e\*\*2\*f\*x\*exp(11\*c) - 1536\*I\*a\*\*9\*d\*\*27\*e\*f\*\*2\*x\*\*2\*exp(11\*c) - 512\*I\*a\*\*9\*d\*\*27\*f\*\*3\*x\*\*3\*exp(11\*c) + 768\*I\*a\*\*9\*d\*\*26\*e\*\*2\*f\*exp(11\*c) + 1536\*I\*a\*\*9\*d\*\*26\*e\*f\*\*2\*x\*exp(11\*c) + 768\*I\*a\*\*9\*d\*\*26\*f\*\*3\*x\*\*2\*exp(11\*c) - 768\*I\*a\*\*9\*d\*\*25\*e\*f\*\*2\*exp(11\*c) - 768\*I\*a\*\*9\*d\*\*25\*f\*\*3\*x\*exp(11\*c) + 384\*I\*a\*\*9\*d\*\*24\*f\*\*3\*exp(11\*c))\*exp(2\*d\*x))\*exp(-9\*c)/(4096\*a\*\*10\*d\*\*28), Ne(4096\*a\*\*10\*d\*\*28\*exp(9\*c), 0)), (-x\*\*4\*(I\*f\*\*3\*exp(4\*c) - 2\*f\*\*3\*exp(3\*c) - 2\*f\*\*3\*exp(c) - I\*f\*\*3)\*exp(-2\*c)/(16\*a) - x\*\*3\*(I\*e\*f\*\*2\*exp(4\*c) - 2\*e\*f\*\*2\*exp(3\*c) - 2\*e\*f\*\*2\*exp(c) - I\*e\*f\*\*2)\*exp(-2\*c)/(4\*a) - x\*\*2\*(3\*I\*e\*\*2\*f\*exp(4\*c) - 6\*e\*\*2\*f\*exp(3\*c) - 6\*e\*\*2\*f\*exp(c) - 3\*I\*e\*\*2\*f)\*exp(-2\*c)/(8\*a) - x\*(I\*e\*\*3\*exp(4\*c) - 2\*e\*\*3\*exp(3\*c) - 2\*e\*\*3\*exp(c) - I\*e\*\*3)\*exp(-2\*c)/(4\*a), True))

**Giac [B]** time = 1.28698, size = 1357, normalized size = 5.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out]  $(-4*I*d^3*f^3*x^3*e^{(5*d*x + 6*c)} + 12*d^3*f^3*x^3*e^{(4*d*x + 5*c)} - 16*I*d^3*f^3*x^3*e^{(3*d*x + 4*c)} - 16*d^3*f^3*x^3*e^{(2*d*x + 3*c)} + 12*I*d^3*f^3*x^3*e^{(d*x + 2*c)} - 4*d^3*f^3*x^3*e^c - 12*I*d^3*f^2*x^2*e^{(5*d*x + 6*c + 1)} + 6*I*d^2*f^3*x^2*e^{(5*d*x + 6*c)} + 36*d^3*f^2*x^2*e^{(4*d*x + 5*c + 1)} - 42*d^2*f^3*x^2*e^{(4*d*x + 5*c)} - 48*I*d^3*f^2*x^2*e^{(3*d*x + 4*c + 1)} + 48*I*d^2*f^3*x^2*e^{(3*d*x + 4*c)} - 48*d^3*f^2*x^2*e^{(2*d*x + 3*c + 1)} - 48*d^2*f^3*x^2*e^{(2*d*x + 3*c)} + 36*I*d^3*f^2*x^2*e^{(d*x + 2*c + 1)} + 42*I*d^2*f^3*x^2*e^{(d*x + 2*c)} - 12*d^3*f^2*x^2*e^{(c + 1)} - 6*d^2*f^3*x^2*e^c - 12*I*d^3*f*x*e^{(5*d*x + 6*c + 2)} + 12*I*d^2*f^2*x*e^{(5*d*x + 6*c + 1)} - 6*I*d*f^3*x*e^{(5*d*x + 6*c)} + 36*d^3*f*x*e^{(4*d*x + 5*c + 2)} - 84*d^2*f^2*x*e^{(4*d*x + 5*c + 1)} + 90*d*f^3*x*e^{(4*d*x + 5*c)} - 48*I*d^3*f*x*e^{(3*d*x + 4*c + 2)} + 96*I*d^2*f^2*x*e^{(3*d*x + 4*c + 1)} - 96*I*d*f^3*x*e^{(3*d*x + 4*c)} - 48*d^3*f*x*e^{(2*d*x + 3*c + 2)} - 96*d^2*f^2*x*e^{(2*d*x + 3*c + 1)} - 96*d*f^3*x*e^{(2*d*x + 3*c)} + 36*I*d^3*f*x*e^{(d*x + 2*c + 2)} + 84*I*d^2*f^2*x*e^{(d*x + 2*c + 1)} + 90*I*d*f^3*x*e^{(d*x + 2*c)} - 12*d^3*f*x*e^{(c + 2)} - 12*d^2*f^2*x*e^{(c + 1)} - 6*d*f^3*x*e^c - 4*I*d^3*e^{(5*d*x + 6*c + 3)} + 6*I*d^2*f*e^{(5*d*x + 6*c + 2)} - 6*I*d*f^2*e^{(5*d*x + 6*c + 1)} + 3*I*f^3*e^{(5*d*x + 6*c)} + 12*d^3*e^{(4*d*x + 5*c + 3)} - 42*d^2*f*e^{(4*d*x + 5*c + 2)} + 90*d*f^2*e^{(4*d*x + 5*c + 1)} - 93*f^3*e^{(4*d*x + 5*c)} - 16*I*d^3*e^{(3*d*x + 4*c + 3)} + 48*I*d^2*f*e^{(3*d*x + 4*c + 2)} - 96*I*d*f^2*e^{(3*d*x + 4*c + 1)} + 96*I*f^3*e^{(3*d*x + 4*c)} - 16*d^3*e^{(2*d*x + 3*c + 3)} - 48*d^2*f*e^{(2*d*x + 3*c + 2)} - 96*d*f^2*e^{(2*d*x + 3*c + 1)} - 96*f^3*e^{(2*d*x + 3*c)} + 12*I*d^3*e^{(d*x + 2*c + 3)} + 42*I*d^2*f*e^{(d*x + 2*c + 2)} + 90*I*d*f^2*e^{(d*x + 2*c + 1)} + 93*I*f^3*e^{(d*x + 2*c)} - 4*d^3*e^{(c + 3)} - 6*d^2*f*e^{(c + 2)} - 6*d*f^2*e^{(c + 1)} - 3*f^3*e^c)/(32*a*d^4*e^{(3*d*x + 4*c)} - 32*I*a*d^4*e^{(2*d*x + 3*c)})$

$$3.266 \quad \int \frac{(e+fx)^2 \cosh^3(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=171

$$-\frac{2f(e+fx) \cosh(c+dx)}{ad^2} + \frac{if(e+fx) \sinh(c+dx) \cosh(c+dx)}{2ad^2} - \frac{if^2 \sinh^2(c+dx)}{4ad^3} + \frac{2f^2 \sinh(c+dx)}{ad^3} - \frac{i(e+fx)^2 \sinh(c+dx)}{2ad^3}$$

[Out]  $((-I/2)*e*f*x)/(a*d) - ((I/4)*f^2*x^2)/(a*d) - (2*f*(e + f*x)*Cosh[c + d*x])/(a*d^2) + (2*f^2*Sinh[c + d*x])/(a*d^3) + ((e + f*x)^2*Sinh[c + d*x])/(a*d) + ((I/2)*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(a*d^2) - ((I/4)*f^2*Sinh[c + d*x]^2)/(a*d^3) - ((I/2)*(e + f*x)^2*Sinh[c + d*x]^2)/(a*d)$

**Rubi [A]** time = 0.186152, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {5563, 3296, 2637, 5446, 3310}

$$-\frac{2f(e+fx) \cosh(c+dx)}{ad^2} + \frac{if(e+fx) \sinh(c+dx) \cosh(c+dx)}{2ad^2} - \frac{if^2 \sinh^2(c+dx)}{4ad^3} + \frac{2f^2 \sinh(c+dx)}{ad^3} - \frac{i(e+fx)^2 \sinh(c+dx)}{2ad^3}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Cosh[c + d\*x]^3)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out]  $((-I/2)*e*f*x)/(a*d) - ((I/4)*f^2*x^2)/(a*d) - (2*f*(e + f*x)*Cosh[c + d*x])/(a*d^2) + (2*f^2*Sinh[c + d*x])/(a*d^3) + ((e + f*x)^2*Sinh[c + d*x])/(a*d) + ((I/2)*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(a*d^2) - ((I/4)*f^2*Sinh[c + d*x]^2)/(a*d^3) - ((I/2)*(e + f*x)^2*Sinh[c + d*x]^2)/(a*d)$

### Rule 5563

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2 + b^2, 0]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
Simp[(d*(b*Sine + f*x)^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sine + f*x)^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sine + f*x)^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned} \int \frac{(e + fx)^2 \cosh^3(c + dx)}{a + ia \sinh(c + dx)} dx &= -\frac{i \int (e + fx)^2 \cosh(c + dx) \sinh(c + dx) dx}{a} + \frac{\int (e + fx)^2 \cosh(c + dx) dx}{a} \\ &= \frac{(e + fx)^2 \sinh(c + dx)}{ad} - \frac{i(e + fx)^2 \sinh^2(c + dx)}{2ad} + \frac{(if) \int (e + fx) \sinh^2(c + dx) dx}{ad} \\ &= -\frac{2f(e + fx) \cosh(c + dx)}{ad^2} + \frac{(e + fx)^2 \sinh(c + dx)}{ad} + \frac{if(e + fx) \cosh(c + dx) \sinh(c + dx)}{2ad^2} \\ &= -\frac{iefx}{2ad} - \frac{if^2x^2}{4ad} - \frac{2f(e + fx) \cosh(c + dx)}{ad^2} + \frac{2f^2 \sinh(c + dx)}{ad^3} + \frac{(e + fx)^2 \sinh(c + dx)}{ad} \end{aligned}$$

**Mathematica [A]** time = 0.910861, size = 99, normalized size = 0.58

$$\frac{-2i \cosh(2(c + dx)) (2d^2(e + fx)^2 + f^2) + 8 \sinh(c + dx) (2(d^2(e + fx)^2 + 2f^2) + idf(e + fx) \cosh(c + dx)) - 32df(e + fx)^2}{16ad^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (-32*d*f*(e + f*x)*Cosh[c + d*x] - (2*I)*(f^2 + 2*d^2*(e + f*x)^2)*Cosh[2*(
c + d*x)] + 8*(2*(2*f^2 + d^2*(e + f*x)^2) + I*d*f*(e + f*x)*Cosh[c + d*x])
```

\*Sinh[c + d\*x])/(16\*a\*d^3)

**Maple [A]** time = 0.138, size = 241, normalized size = 1.4

$$\frac{-\frac{i}{16} (2 f^2 x^2 d^2 + 4 d^2 e f x + 2 d^2 e^2 - 2 d f^2 x - 2 e f d + f^2) e^{2 d x + 2 c}}{a d^3} + \frac{(f^2 x^2 d^2 + 2 d^2 e f x + d^2 e^2 - 2 d f^2 x - 2 e f d + 2 f^2) e^{d x}}{2 a d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*cosh(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x)

[Out] -1/16\*I\*(2\*d^2\*f^2\*x^2+4\*d^2\*e\*f\*x+2\*d^2\*e^2-2\*d\*f^2\*x-2\*d\*e\*f+f^2)/a/d^3\*exp(2\*d\*x+2\*c)+1/2\*(d^2\*f^2\*x^2+2\*d^2\*e\*f\*x+d^2\*e^2-2\*d\*f^2\*x-2\*d\*e\*f+2\*f^2)/a/d^3\*exp(d\*x+c)-1/2\*(d^2\*f^2\*x^2+2\*d^2\*e\*f\*x+d^2\*e^2+2\*d\*f^2\*x+2\*d\*e\*f+2\*f^2)/a/d^3\*exp(-d\*x-c)-1/16\*I\*(2\*d^2\*f^2\*x^2+4\*d^2\*e\*f\*x+2\*d^2\*e^2+2\*d\*f^2\*x+2\*d\*e\*f+f^2)/a/d^3\*exp(-2\*d\*x-2\*c)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError

**Fricas [A]** time = 2.13009, size = 520, normalized size = 3.04

$$\frac{(-2i d^2 f^2 x^2 - 2i d^2 e^2 - 2i d e f - i f^2 + (-4i d^2 e f - 2i d f^2) x + (-2i d^2 f^2 x^2 - 2i d^2 e^2 + 2i d e f - i f^2 + (-4i d^2 e f + 2i d f^2) x) e^{d x}}{a d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

```
[Out] 1/16*(-2*I*d^2*f^2*x^2 - 2*I*d^2*e^2 - 2*I*d*e*f - I*f^2 + (-4*I*d^2*e*f - 2*I*d*f^2)*x + (-2*I*d^2*f^2*x^2 - 2*I*d^2*e^2 + 2*I*d*e*f - I*f^2 + (-4*I*d^2*e*f + 2*I*d*f^2)*x)*e^(4*d*x + 4*c) + 8*(d^2*f^2*x^2 + d^2*e^2 - 2*d*e*f + 2*f^2 + 2*(d^2*e*f - d*f^2)*x)*e^(3*d*x + 3*c) - 8*(d^2*f^2*x^2 + d^2*e^2 + 2*d*e*f + 2*f^2 + 2*(d^2*e*f + d*f^2)*x)*e^(d*x + c))*e^(-2*d*x - 2*c)/(a*d^3)
```

**Sympy [A]** time = 3.36033, size = 644, normalized size = 3.77

$$\left\{ \frac{((-512a^7d^{17}e^{2e^{6c}} - 1024a^7d^{17}efxe^{6c} - 512a^7d^{17}f^2x^2e^{6c} - 1024a^7d^{16}efe^{6c} - 1024a^7d^{16}f^2xe^{6c} - 1024a^7d^{15}f^2e^{6c})e^{-dx} + (512a^7d^{17}e^{2e^{8c}} + 1024a^7d^{17}efxe^{8c} + 512a^7d^{17}f^2x^2e^{8c})e^{-2c}}{12a}, \frac{x^3(ief^2e^{4c} - 2f^2e^{3c} - 2f^2e^c - if^2)e^{-2c}}{4a}, \frac{x^2(ief^2e^{4c} - 2efe^{3c} - 2efe^c - if^2)e^{-2c}}{4a}, \frac{x(ie^2e^{4c} - 2e^2e^{3c} - 2e^2e^c - ie^2)e^{-2c}}{4a} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Piecewise(((((-512*a**7*d**17*e**2*exp(6*c) - 1024*a**7*d**17*e*f*x*exp(6*c) - 512*a**7*d**17*f**2*x**2*exp(6*c) - 1024*a**7*d**16*e*f*exp(6*c) - 1024*a**7*d**16*f**2*x*exp(6*c) - 1024*a**7*d**15*f**2*exp(6*c))*exp(-d*x) + (512*a**7*d**17*e**2*exp(8*c) + 1024*a**7*d**17*e*f*x*exp(8*c) + 512*a**7*d**17*f**2*x**2*exp(8*c) - 1024*a**7*d**16*e*f*exp(8*c) - 1024*a**7*d**16*f**2*x*exp(8*c) + 1024*a**7*d**15*f**2*exp(8*c))*exp(d*x) + (-128*I*a**7*d**17*e**2*exp(5*c) - 256*I*a**7*d**17*e*f*x*exp(5*c) - 128*I*a**7*d**17*f**2*x**2*exp(5*c) - 128*I*a**7*d**16*e*f*exp(5*c) - 128*I*a**7*d**16*f**2*x*exp(5*c) - 64*I*a**7*d**15*f**2*exp(5*c))*exp(-2*d*x) + (-128*I*a**7*d**17*e**2*exp(9*c) - 256*I*a**7*d**17*e*f*x*exp(9*c) - 128*I*a**7*d**17*f**2*x**2*exp(9*c) + 128*I*a**7*d**16*e*f*exp(9*c) + 128*I*a**7*d**16*f**2*x*exp(9*c) - 64*I*a**7*d**15*f**2*exp(9*c))*exp(2*d*x))*exp(-7*c)/(1024*a**8*d**18), Ne(1024*a**8*d**18*exp(7*c), 0)), (-x**3*(I*f**2*exp(4*c) - 2*f**2*exp(3*c) - 2*f**2*exp(c) - I*f**2)*exp(-2*c)/(12*a) - x**2*(I*e*f*exp(4*c) - 2*e*f*exp(3*c) - 2*e*f*exp(c) - I*e*f)*exp(-2*c)/(4*a) - x*(I*e**2*exp(4*c) - 2*e**2*exp(3*c) - 2*e**2*exp(c) - I*e**2)*exp(-2*c)/(4*a), True))
```

**Giac [B]** time = 1.22192, size = 764, normalized size = 4.47

$$\frac{-2i d^2 f^2 x^2 e^{(5dx+6c)} + 6d^2 f^2 x^2 e^{(4dx+5c)} - 8i d^2 f^2 x^2 e^{(3dx+4c)} - 8d^2 f^2 x^2 e^{(2dx+3c)} + 6i d^2 f^2 x^2 e^{(dx+2c)} - 2d^2 f^2 x^2 e^c - 4i d^2 f^2 x^2 e^{-c}}{12a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] 
$$\begin{aligned} & (-2*I*d^2*f^2*x^2*e^{(5*d*x + 6*c)} + 6*d^2*f^2*x^2*e^{(4*d*x + 5*c)} - 8*I*d^2* \\ & f^2*x^2*e^{(3*d*x + 4*c)} - 8*d^2*f^2*x^2*e^{(2*d*x + 3*c)} + 6*I*d^2*f^2*x^2* \\ & e^{(d*x + 2*c)} - 2*d^2*f^2*x^2*e^c - 4*I*d^2*f*x*e^{(5*d*x + 6*c + 1)} + 2*I*d \\ & f^2*x*e^{(5*d*x + 6*c)} + 12*d^2*f*x*e^{(4*d*x + 5*c + 1)} - 14*d*f^2*x*e^{(4*d \\ & *x + 5*c)} - 16*I*d^2*f*x*e^{(3*d*x + 4*c + 1)} + 16*I*d*f^2*x*e^{(3*d*x + 4*c)} \\ & - 16*d^2*f*x*e^{(2*d*x + 3*c + 1)} - 16*d*f^2*x*e^{(2*d*x + 3*c)} + 12*I*d^2*f \\ & x*e^{(d*x + 2*c + 1)} + 14*I*d*f^2*x*e^{(d*x + 2*c)} - 4*d^2*f*x*e^{(c + 1)} - 2 \\ & *d*f^2*x*e^c - 2*I*d^2*e^{(5*d*x + 6*c + 2)} + 2*I*d*f*e^{(5*d*x + 6*c + 1)} - \\ & I*f^2*e^{(5*d*x + 6*c)} + 6*d^2*e^{(4*d*x + 5*c + 2)} - 14*d*f*e^{(4*d*x + 5*c + \\ & 1)} + 15*f^2*e^{(4*d*x + 5*c)} - 8*I*d^2*e^{(3*d*x + 4*c + 2)} + 16*I*d*f*e^{(3* \\ & d*x + 4*c + 1)} - 16*I*f^2*e^{(3*d*x + 4*c)} - 8*d^2*e^{(2*d*x + 3*c + 2)} - 16* \\ & d*f*e^{(2*d*x + 3*c + 1)} - 16*f^2*e^{(2*d*x + 3*c)} + 6*I*d^2*e^{(d*x + 2*c + 2 \\ & )} + 14*I*d*f*e^{(d*x + 2*c + 1)} + 15*I*f^2*e^{(d*x + 2*c)} - 2*d^2*e^{(c + 2)} - \\ & 2*d*f*e^{(c + 1)} - f^2*e^c)/(16*a*d^3*e^{(3*d*x + 4*c)} - 16*I*a*d^3*e^{(2*d*x \\ & + 3*c)}) \end{aligned}$$



$$3.267 \quad \int \frac{(e+fx) \cosh^3(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=98

$$-\frac{f \cosh(c+dx)}{ad^2} + \frac{if \sinh(c+dx) \cosh(c+dx)}{4ad^2} - \frac{i(e+fx) \sinh^2(c+dx)}{2ad} + \frac{(e+fx) \sinh(c+dx)}{ad} - \frac{ifx}{4ad}$$

[Out]  $((-I/4)*f*x)/(a*d) - (f*Cosh[c + d*x])/(a*d^2) + ((e + f*x)*Sinh[c + d*x])/(a*d) + ((I/4)*f*Cosh[c + d*x]*Sinh[c + d*x])/(a*d^2) - ((I/2)*(e + f*x)*Sinh[c + d*x]^2)/(a*d)$

**Rubi [A]** time = 0.0995435, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {5563, 3296, 2638, 5446, 2635, 8}

$$-\frac{f \cosh(c+dx)}{ad^2} + \frac{if \sinh(c+dx) \cosh(c+dx)}{4ad^2} - \frac{i(e+fx) \sinh^2(c+dx)}{2ad} + \frac{(e+fx) \sinh(c+dx)}{ad} - \frac{ifx}{4ad}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Cosh[c + d\*x]^3)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out]  $((-I/4)*f*x)/(a*d) - (f*Cosh[c + d*x])/(a*d^2) + ((e + f*x)*Sinh[c + d*x])/(a*d) + ((I/4)*f*Cosh[c + d*x]*Sinh[c + d*x])/(a*d^2) - ((I/2)*(e + f*x)*Sinh[c + d*x]^2)/(a*d)$

#### Rule 5563

Int[(Cosh[(c\_.) + (d\_.)\*(x\_.)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2 + b^2, 0]

#### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)], x\_Symbol] := -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*cos[c + d*x]
)*(b*sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

### Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{(e + fx) \cosh^3(c + dx)}{a + ia \sinh(c + dx)} dx &= -\frac{i \int (e + fx) \cosh(c + dx) \sinh(c + dx) dx}{a} + \frac{\int (e + fx) \cosh(c + dx) dx}{a} \\ &= \frac{(e + fx) \sinh(c + dx)}{ad} - \frac{i(e + fx) \sinh^2(c + dx)}{2ad} + \frac{(if) \int \sinh^2(c + dx) dx}{2ad} - \frac{f \int \sinh(c + dx) dx}{ad} \\ &= -\frac{f \cosh(c + dx)}{ad^2} + \frac{(e + fx) \sinh(c + dx)}{ad} + \frac{if \cosh(c + dx) \sinh(c + dx)}{4ad^2} - \frac{i(e + fx) \sinh(c + dx)}{2ad} \\ &= -\frac{ifx}{4ad} - \frac{f \cosh(c + dx)}{ad^2} + \frac{(e + fx) \sinh(c + dx)}{ad} + \frac{if \cosh(c + dx) \sinh(c + dx)}{4ad^2} - \frac{i(e + fx) \sinh(c + dx)}{2ad} \end{aligned}$$

**Mathematica [A]** time = 1.13317, size = 60, normalized size = 0.61

$$\frac{d(e + fx)(4 \sinh(c + dx) - i \cosh(2(c + dx))) + if(\sinh(c + dx) + 4i) \cosh(c + dx)}{4ad^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

[Out]  $(I*f*Cosh[c + d*x]*(4*I + Sinh[c + d*x]) + d*(e + f*x)*((-I)*Cosh[2*(c + d*x)] + 4*Sinh[c + d*x]))/(4*a*d^2)$

**Maple [A]** time = 0.121, size = 113, normalized size = 1.2

$$\frac{-\frac{i}{16}(2dfx + 2de - f)e^{2dx+2c}}{ad^2} + \frac{(dfx + de - f)e^{dx+c}}{2ad^2} - \frac{(dfx + de + f)e^{-dx-c}}{2ad^2} - \frac{\frac{i}{16}(2dfx + 2de + f)e^{-2dx-2c}}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)`

[Out]  $-1/16*I*(2*d*f*x+2*d*e-f)/a/d^2*\exp(2*d*x+2*c)+1/2*(d*f*x+d*e-f)/a/d^2*\exp(d*x+c)-1/2*(d*f*x+d*e+f)/a/d^2*\exp(-d*x-c)-1/16*I*(2*d*f*x+2*d*e+f)/a/d^2*\exp(-2*d*x-2*c)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError

**Fricas [A]** time = 2.14459, size = 242, normalized size = 2.47

$$\frac{(-2idfx - 2ide + (-2idfx - 2ide + if)e^{4dx+4c}) + 8(dfx + de - f)e^{3dx+3c} - 8(dfx + de + f)e^{dx+c} - ife^{(-2dx-2c)}}{16ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*cosh(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")`

[Out]  $1/16*(-2*I*d*f*x - 2*I*d*e + (-2*I*d*f*x - 2*I*d*e + I*f)*e^{(4*d*x + 4*c)} + 8*(d*f*x + d*e - f)*e^{(3*d*x + 3*c)} - 8*(d*f*x + d*e + f)*e^{(d*x + c)} - I*$

$$f) * e^{(-2*d*x - 2*c)} / (a*d^2)$$

**Sympy [A]** time = 2.07587, size = 330, normalized size = 3.37

$$\left\{ \frac{\left( (-512a^5d^9ee^{4c} - 512a^5d^9fxe^{4c} - 512a^5d^8fe^{4c})e^{-dx} + (512a^5d^9ee^{6c} + 512a^5d^9fxe^{6c} - 512a^5d^8fe^{6c})e^{dx} + (-128ia^5d^9ee^{3c} - 128ia^5d^9fxe^{3c} - 64ia^5d^8fe^{3c})e^{-2dx} + (-128ia^5d^9ee^{5c} - 128ia^5d^9fxe^{5c} - 64ia^5d^8fe^{5c})e^{2dx} \right)}{1024a^6d^{10}} - \frac{x^2(ife^{4c} - 2fe^{3c} - 2fe^c - if)e^{-2c}}{8a} - \frac{x(iee^{4c} - 2ee^{3c} - 2ee^c - ie)e^{-2c}}{4a} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)\*\*3/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Piecewise(((((-512\*a\*\*5\*d\*\*9\*e\*exp(4\*c) - 512\*a\*\*5\*d\*\*9\*f\*x\*exp(4\*c) - 512\*a\*\*5\*d\*\*8\*f\*exp(4\*c))\*exp(-d\*x) + (512\*a\*\*5\*d\*\*9\*e\*exp(6\*c) + 512\*a\*\*5\*d\*\*9\*f\*x\*exp(6\*c) - 512\*a\*\*5\*d\*\*8\*f\*exp(6\*c))\*exp(d\*x) + (-128\*I\*a\*\*5\*d\*\*9\*e\*exp(3\*c) - 128\*I\*a\*\*5\*d\*\*9\*f\*x\*exp(3\*c) - 64\*I\*a\*\*5\*d\*\*8\*f\*exp(3\*c))\*exp(-2\*d\*x) + (-128\*I\*a\*\*5\*d\*\*9\*e\*exp(7\*c) - 128\*I\*a\*\*5\*d\*\*9\*f\*x\*exp(7\*c) + 64\*I\*a\*\*5\*d\*\*8\*f\*exp(7\*c))\*exp(2\*d\*x))\*exp(-5\*c)/(1024\*a\*\*6\*d\*\*10), Ne(1024\*a\*\*6\*d\*\*10\*exp(5\*c), 0)), (-x\*\*2\*(I\*f\*exp(4\*c) - 2\*f\*exp(3\*c) - 2\*f\*exp(c) - I\*f)\*exp(-2\*c)/(8\*a) - x\*(I\*e\*exp(4\*c) - 2\*e\*exp(3\*c) - 2\*e\*exp(c) - I\*e)\*exp(-2\*c)/(4\*a), True))

**Giac [B]** time = 1.19529, size = 332, normalized size = 3.39

$$\frac{-2idfxe^{(5dx+6c)} + 6dfxe^{(4dx+5c)} - 8idfxe^{(3dx+4c)} - 8dfxe^{(2dx+3c)} + 6idfxe^{(dx+2c)} - 2dfxe^c - 2ide^{(5dx+6c+1)} + ife^{(5dx+6c)}}{16ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] (-2\*I\*d\*f\*x\*e^(5\*d\*x + 6\*c) + 6\*d\*f\*x\*e^(4\*d\*x + 5\*c) - 8\*I\*d\*f\*x\*e^(3\*d\*x + 4\*c) - 8\*d\*f\*x\*e^(2\*d\*x + 3\*c) + 6\*I\*d\*f\*x\*e^(d\*x + 2\*c) - 2\*d\*f\*x\*e^c - 2\*I\*d\*e^(5\*d\*x + 6\*c + 1) + I\*f\*e^(5\*d\*x + 6\*c) + 6\*d\*e^(4\*d\*x + 5\*c + 1) - 7\*f\*e^(4\*d\*x + 5\*c) - 8\*I\*d\*e^(3\*d\*x + 4\*c + 1) + 8\*I\*f\*e^(3\*d\*x + 4\*c) - 8\*d\*e^(2\*d\*x + 3\*c + 1) - 8\*f\*e^(2\*d\*x + 3\*c) + 6\*I\*d\*e^(d\*x + 2\*c + 1) + 7\*I\*f\*e^(d\*x + 2\*c) - 2\*d\*e^(c + 1) - f\*e^c)/(16\*a\*d^2\*e^(3\*d\*x + 4\*c) - 16\*I\*a\*d^2\*e^(2\*d\*x + 3\*c))

$$3.268 \quad \int \frac{\cosh^3(c+dx)}{a+ia \sinh(c+dx)} dx$$

Optimal. Leaf size=34

$$\frac{\sinh(c+dx)}{ad} - \frac{i \sinh^2(c+dx)}{2ad}$$

[Out] Sinh[c + d\*x]/(a\*d) - ((I/2)\*Sinh[c + d\*x]^2)/(a\*d)

**Rubi [A]** time = 0.047608, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$ , Rules used = {2667}

$$\frac{\sinh(c+dx)}{ad} - \frac{i \sinh^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]^3/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] Sinh[c + d\*x]/(a\*d) - ((I/2)\*Sinh[c + d\*x]^2)/(a\*d)

### Rule 2667

Int[cos[(e\_.) + (f\_.)\*(x\_.)]^(p\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(m\_.), x\_Symbol] :> Dist[1/(b^p\*f), Subst[Int[(a + x)^(m + (p - 1)/2)\*(a - x)^(p - 1/2), x], x, b\*Sin[e + f\*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

### Rubi steps

$$\begin{aligned} \int \frac{\cosh^3(c+dx)}{a+ia \sinh(c+dx)} dx &= -\frac{i \text{Subst}(\int (a-x) dx, x, ia \sinh(c+dx))}{a^3 d} \\ &= \frac{\sinh(c+dx)}{ad} - \frac{i \sinh^2(c+dx)}{2ad} \end{aligned}$$

**Mathematica [A]** time = 0.0491364, size = 28, normalized size = 0.82

$$\frac{(2 - i \sinh(c + dx)) \sinh(c + dx)}{2ad}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]^3/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] ((2 - I\*Sinh[c + d\*x])\*Sinh[c + d\*x])/(2\*a\*d)

**Maple [A]** time = 0.015, size = 29, normalized size = 0.9

$$\frac{\frac{i}{2} (\sinh(dx + c))^2 - \sinh(dx + c)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x)

[Out] -1/d/a\*(1/2\*I\*sinh(d\*x+c)^2-sinh(d\*x+c))

**Maxima [A]** time = 1.16329, size = 81, normalized size = 2.38

$$\frac{i(4i e^{(-dx-c)} + 1)e^{(2dx+2c)}}{8ad} - \frac{i(-4i e^{(-dx-c)} + e^{(-2dx-2c)})}{8ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -1/8\*I\*(4\*I\*e^(-d\*x - c) + 1)\*e^(2\*d\*x + 2\*c)/(a\*d) - 1/8\*I\*(-4\*I\*e^(-d\*x - c) + e^(-2\*d\*x - 2\*c))/(a\*d)

**Fricas [A]** time = 2.08925, size = 120, normalized size = 3.53

$$\frac{(-i e^{(4dx+4c)} + 4 e^{(3dx+3c)} - 4 e^{(dx+c)} - i) e^{(-2dx-2c)}}{8ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{8}*(-I*e^{(4*d*x + 4*c)} + 4*e^{(3*d*x + 3*c)} - 4*e^{(d*x + c)} - I)*e^{(-2*d*x - 2*c)}/(a*d)$

**Sympy [A]** time = 0.766, size = 136, normalized size = 4.

$$\begin{cases} \frac{(-32ia^3d^3e^{5c}e^{2dx}+128a^3d^3e^{4c}e^{dx}-128a^3d^3e^{2c}e^{-dx}-32ia^3d^3e^ce^{-2dx})e^{-3c}}{256a^4d^4} & \text{for } 256a^4d^4e^{3c} \neq 0 \\ -\frac{x(ie^{4c}-2e^{3c}-2e^c-i)e^{-2c}}{4a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*\*3/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Piecewise(((−32\*I\*a\*\*3\*d\*\*3\*exp(5\*c)\*exp(2\*d\*x) + 128\*a\*\*3\*d\*\*3\*exp(4\*c)\*exp(d\*x) − 128\*a\*\*3\*d\*\*3\*exp(2\*c)\*exp(−d\*x) − 32\*I\*a\*\*3\*d\*\*3\*exp(c)\*exp(−2\*d\*x))\*exp(−3\*c)/(256\*a\*\*4\*d\*\*4), Ne(256\*a\*\*4\*d\*\*4\*exp(3\*c), 0)), (−x\*(I\*exp(4\*c) − 2\*exp(3\*c) − 2\*exp(c) − I)\*exp(−2\*c)/(4\*a), True))

**Giac [A]** time = 1.15882, size = 65, normalized size = 1.91

$$\frac{(4e^{(dx+c)} + i)e^{(-2dx-2c)} + ie^{(2dx+2c)} - 4e^{(dx+c)}}{8ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out]  $\frac{-1}{8}*((4*e^{(d*x + c)} + I)*e^{(-2*d*x - 2*c)} + I*e^{(2*d*x + 2*c)} - 4*e^{(d*x + c)})/(a*d)$

$$3.269 \quad \int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

**Optimal.** Leaf size=131

$$-\frac{i \sinh\left(2c - \frac{2de}{f}\right) \text{Chi}\left(\frac{2de}{f} + 2dx\right)}{2af} + \frac{\cosh\left(c - \frac{de}{f}\right) \text{Chi}\left(\frac{de}{f} + dx\right)}{af} + \frac{\sinh\left(c - \frac{de}{f}\right) \text{Shi}\left(\frac{de}{f} + dx\right)}{af} - \frac{i \cosh\left(2c - \frac{2de}{f}\right) \text{Shi}\left(\frac{de}{f} + dx\right)}{2af}$$

[Out] (Cosh[c - (d\*e)/f]\*CoshIntegral[(d\*e)/f + d\*x])/(a\*f) - ((I/2)\*CoshIntegral[(2\*d\*e)/f + 2\*d\*x]\*Sinh[2\*c - (2\*d\*e)/f])/(a\*f) + (Sinh[c - (d\*e)/f]\*SinhIntegral[(d\*e)/f + d\*x])/(a\*f) - ((I/2)\*Cosh[2\*c - (2\*d\*e)/f]\*SinhIntegral[(2\*d\*e)/f + 2\*d\*x])/(a\*f)

**Rubi [A]** time = 0.324126, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {5563, 3303, 3298, 3301, 5448, 12}

$$-\frac{i \sinh\left(2c - \frac{2de}{f}\right) \text{Chi}\left(\frac{2de}{f} + 2dx\right)}{2af} + \frac{\cosh\left(c - \frac{de}{f}\right) \text{Chi}\left(\frac{de}{f} + dx\right)}{af} + \frac{\sinh\left(c - \frac{de}{f}\right) \text{Shi}\left(\frac{de}{f} + dx\right)}{af} - \frac{i \cosh\left(2c - \frac{2de}{f}\right) \text{Shi}\left(\frac{de}{f} + dx\right)}{2af}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]^3/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])),x]

[Out] (Cosh[c - (d\*e)/f]\*CoshIntegral[(d\*e)/f + d\*x])/(a\*f) - ((I/2)\*CoshIntegral[(2\*d\*e)/f + 2\*d\*x]\*Sinh[2\*c - (2\*d\*e)/f])/(a\*f) + (Sinh[c - (d\*e)/f]\*SinhIntegral[(d\*e)/f + d\*x])/(a\*f) - ((I/2)\*Cosh[2\*c - (2\*d\*e)/f]\*SinhIntegral[(2\*d\*e)/f + 2\*d\*x])/(a\*f)

### Rule 5563

Int[(Cosh[(c\_.) + (d\_.)\*(x\_.)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2 + b^2, 0]

### Rule 3303

Int[sin[(e\_.) + (f\_.)\*(x\_.)]/((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] := Dist[Cos[(d\*e - c\*f)/d], Int[Sin[(c\*f)/d + f\*x]/(c + d\*x), x], x] + Dist[Sin[(d\*e - c\*f)



)/d], Int[Cos[(c\*f)/d + f\*x]/(c + d\*x), x], x] /; FreeQ[{c, d, e, f}, x] &&  
NeQ[d\*e - c\*f, 0]

### Rule 3298

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[(I\*SinhIntegral[(c\*f\*fz)/d + f\*fz\*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*e - c\*f\*fz\*I, 0]

### Rule 3301

Int[sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]/((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> Simp[CoshIntegral[(c\*f\*fz)/d + f\*fz\*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d\*(e - Pi/2) - c\*f\*fz\*I, 0]

### Rule 5448

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sinh[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] :> Int[ExpandTrigReduce[(c + d\*x)^m, Sinh[a + b\*x]^n\*Cosh[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] & IGtQ[p, 0]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(c+dx)}{(e+fx)(a+ia\sinh(c+dx))} dx &= -\frac{i \int \frac{\cosh(c+dx)\sinh(c+dx)}{e+fx} dx}{a} + \frac{\int \frac{\cosh(c+dx)}{e+fx} dx}{a} \\
&= -\frac{i \int \frac{\sinh(2c+2dx)}{2(e+fx)} dx}{a} + \frac{\cosh\left(c - \frac{de}{f}\right) \int \frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx} dx}{a} + \frac{\sinh\left(c - \frac{de}{f}\right) \int \frac{\sinh\left(\frac{de}{f}+dx\right)}{e+fx} dx}{a} \\
&= \frac{\cosh\left(c - \frac{de}{f}\right) \text{Chi}\left(\frac{de}{f}+dx\right)}{af} + \frac{\sinh\left(c - \frac{de}{f}\right) \text{Shi}\left(\frac{de}{f}+dx\right)}{af} - \frac{i \int \frac{\sinh(2c+2dx)}{e+fx} dx}{2a} \\
&= \frac{\cosh\left(c - \frac{de}{f}\right) \text{Chi}\left(\frac{de}{f}+dx\right)}{af} + \frac{\sinh\left(c - \frac{de}{f}\right) \text{Shi}\left(\frac{de}{f}+dx\right)}{af} - \frac{\left(i \cosh\left(2c - \frac{2de}{f}\right)\right) \int \frac{\sinh(2c+2dx)}{e+fx} dx}{2a} \\
&= \frac{\cosh\left(c - \frac{de}{f}\right) \text{Chi}\left(\frac{de}{f}+dx\right)}{af} - \frac{i \text{Chi}\left(\frac{2de}{f}+2dx\right) \sinh\left(2c - \frac{2de}{f}\right)}{2af} + \frac{\sinh\left(c - \frac{de}{f}\right) \text{Shi}\left(\frac{de}{f}+dx\right)}{af}
\end{aligned}$$

**Mathematica [A]** time = 0.418634, size = 112, normalized size = 0.85

$$\frac{2 \cosh\left(c - \frac{de}{f}\right) \text{Chi}\left(d\left(\frac{e}{f} + x\right)\right) - i \left(\sinh\left(2c - \frac{2de}{f}\right) \text{Chi}\left(\frac{2d(e+fx)}{f}\right) + 2i \sinh\left(c - \frac{de}{f}\right) \text{Shi}\left(d\left(\frac{e}{f} + x\right)\right) + \cosh\left(2c - \frac{2de}{f}\right) \text{Shi}\left(\frac{2d(e+fx)}{f}\right)\right)}{2af}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]^3/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])),x]

[Out] (2\*Cosh[c - (d\*e)/f]\*CoshIntegral[d\*(e/f + x)] - I\*(CoshIntegral[(2\*d\*(e + f\*x))/f]\*Sinh[2\*c - (2\*d\*e)/f] + (2\*I)\*Sinh[c - (d\*e)/f]\*SinhIntegral[d\*(e/f + x)] + Cosh[2\*c - (2\*d\*e)/f]\*SinhIntegral[(2\*d\*(e + f\*x))/f]))/(2\*a\*f)

**Maple [A]** time = 0.114, size = 180, normalized size = 1.4

$$-\frac{1}{2af} e^{-\frac{cf-de}{f}} \text{Ei}\left(1, dx + c - \frac{cf-de}{f}\right) - \frac{1}{2af} e^{\frac{cf-de}{f}} \text{Ei}\left(1, -dx - c - \frac{-cf+de}{f}\right) + \frac{i}{4af} e^{2\frac{cf-de}{f}} \text{Ei}\left(1, -2dx - 2c - 2\frac{-cf+de}{f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)^3/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x)

[Out]  $-1/2/a/f*\exp(-(c*f-d*e)/f)*Ei(1,d*x+c-(c*f-d*e)/f)-1/2/a/f*\exp((c*f-d*e)/f)*Ei(1,-d*x-c-(-c*f+d*e)/f)+1/4*I/a/f*\exp(2*(c*f-d*e)/f)*Ei(1,-2*d*x-2*c-2*(-c*f+d*e)/f)-1/4*I/a/f*\exp(-2*(c*f-d*e)/f)*Ei(1,2*d*x+2*c-2*(c*f-d*e)/f)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError

**Fricas [A]** time = 2.12151, size = 258, normalized size = 1.97

$$\frac{i Ei\left(-\frac{2(df x+de)}{f}\right) e^{\left(\frac{2(de-cf)}{f}\right)} + 2 Ei\left(-\frac{df x+de}{f}\right) e^{\left(\frac{de-cf}{f}\right)} + 2 Ei\left(\frac{df x+de}{f}\right) e^{\left(-\frac{de-cf}{f}\right)} - i Ei\left(\frac{2(df x+de)}{f}\right) e^{\left(-\frac{2(de-cf)}{f}\right)}}{4 a f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")`

[Out]  $1/4*(I*Ei(-2*(d*f*x + d*e)/f)*e^{(2*(d*e - c*f)/f)} + 2*Ei(-(d*f*x + d*e)/f)*e^{((d*e - c*f)/f)} + 2*Ei((d*f*x + d*e)/f)*e^{-((d*e - c*f)/f)} - I*Ei(2*(d*f*x + d*e)/f)*e^{-2*(d*e - c*f)/f})/(a*f)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)**3/(f*x+e)/(a+I*a*sinh(d*x+c)),x)`

[Out] Timed out

**Giac [A]** time = 1.18696, size = 208, normalized size = 1.59

$$\frac{\left(i \operatorname{Ei}\left(\frac{2(df x + de)}{f}\right) e^{\left(4c - \frac{2de}{f}\right)} - 2 \operatorname{Ei}\left(\frac{df x + de}{f}\right) e^{\left(3c - \frac{de}{f}\right)} - 2 \operatorname{Ei}\left(-\frac{df x + de}{f}\right) e^{\left(c + \frac{de}{f}\right)} - i \operatorname{Ei}\left(-\frac{2(df x + de)}{f}\right) e^{\left(\frac{2de}{f}\right)} + 3i e^{(2c)} \log(fx + e)\right)}{4af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^3/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out]  $-1/4*(I*\operatorname{Ei}(2*(d*f*x + d*e)/f)*e^{(4*c - 2*d*e/f)} - 2*\operatorname{Ei}((d*f*x + d*e)/f)*e^{(3*c - d*e/f)} - 2*\operatorname{Ei}(-(d*f*x + d*e)/f)*e^{(c + d*e/f)} - I*\operatorname{Ei}(-2*(d*f*x + d*e)/f)*e^{(2*d*e/f)} + 3*I*e^{(2*c)}*\log(f*x + e) - 3*I*e^{(2*c)}*\log(I*f*x + I*e))*e^{(-2*c)}/(a*f)$

$$3.270 \quad \int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

**Optimal.** Leaf size=180

$$\frac{d \sinh\left(c - \frac{de}{f}\right) \text{Chi}\left(\frac{de}{f} + dx\right)}{af^2} - \frac{id \cosh\left(2c - \frac{2de}{f}\right) \text{Chi}\left(\frac{2de}{f} + 2dx\right)}{af^2} - \frac{id \sinh\left(2c - \frac{2de}{f}\right) \text{Shi}\left(\frac{2de}{f} + 2dx\right)}{af^2} + \frac{d \cosh\left(c - \frac{de}{f}\right)}{af^2}$$

[Out] -(Cosh[c + d\*x]/(a\*f\*(e + f\*x))) - (I\*d\*Cosh[2\*c - (2\*d\*e)/f]\*CoshIntegral[(2\*d\*e)/f + 2\*d\*x]/(a\*f^2) + (d\*CoshIntegral[(d\*e)/f + d\*x]\*Sinh[c - (d\*e)/f]/(a\*f^2) + ((I/2)\*Sinh[2\*c + 2\*d\*x])/(a\*f\*(e + f\*x)) + (d\*Cosh[c - (d\*e)/f]\*SinhIntegral[(d\*e)/f + d\*x]/(a\*f^2) - (I\*d\*Sinh[2\*c - (2\*d\*e)/f]\*SinhIntegral[(2\*d\*e)/f + 2\*d\*x]/(a\*f^2)

**Rubi [A]** time = 0.391807, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {5563, 3297, 3303, 3298, 3301, 5448, 12}

$$\frac{d \sinh\left(c - \frac{de}{f}\right) \text{Chi}\left(\frac{de}{f} + dx\right)}{af^2} - \frac{id \cosh\left(2c - \frac{2de}{f}\right) \text{Chi}\left(\frac{2de}{f} + 2dx\right)}{af^2} - \frac{id \sinh\left(2c - \frac{2de}{f}\right) \text{Shi}\left(\frac{2de}{f} + 2dx\right)}{af^2} + \frac{d \cosh\left(c - \frac{de}{f}\right)}{af^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]^3/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])),x]

[Out] -(Cosh[c + d\*x]/(a\*f\*(e + f\*x))) - (I\*d\*Cosh[2\*c - (2\*d\*e)/f]\*CoshIntegral[(2\*d\*e)/f + 2\*d\*x]/(a\*f^2) + (d\*CoshIntegral[(d\*e)/f + d\*x]\*Sinh[c - (d\*e)/f]/(a\*f^2) + ((I/2)\*Sinh[2\*c + 2\*d\*x])/(a\*f\*(e + f\*x)) + (d\*Cosh[c - (d\*e)/f]\*SinhIntegral[(d\*e)/f + d\*x]/(a\*f^2) - (I\*d\*Sinh[2\*c - (2\*d\*e)/f]\*SinhIntegral[(2\*d\*e)/f + 2\*d\*x]/(a\*f^2)

### Rule 5563

Int[(Cosh[(c\_.) + (d\_.)\*(x\_.)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.)]/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 1] && EqQ[a^2 + b^2, 0]

### Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

### Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

### Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

### Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

### Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(c+dx)}{(e+fx)^2(a+ia\sinh(c+dx))} dx &= -\frac{i \int \frac{\cosh(c+dx)\sinh(c+dx)}{(e+fx)^2} dx}{a} + \frac{\int \frac{\cosh(c+dx)}{(e+fx)^2} dx}{a} \\
&= -\frac{\cosh(c+dx)}{af(e+fx)} - \frac{i \int \frac{\sinh(2c+2dx)}{2(e+fx)^2} dx}{a} + \frac{d \int \frac{\sinh(c+dx)}{e+fx} dx}{af} \\
&= -\frac{\cosh(c+dx)}{af(e+fx)} - \frac{i \int \frac{\sinh(2c+2dx)}{(e+fx)^2} dx}{2a} + \frac{\left(d \cosh\left(c - \frac{de}{f}\right)\right) \int \frac{\sinh\left(\frac{de}{f}+dx\right)}{e+fx} dx}{af} + \frac{\left(d \sinh\left(c - \frac{de}{f}\right)\right) \int \frac{\cosh\left(\frac{de}{f}+dx\right)}{e+fx} dx}{af} \\
&= -\frac{\cosh(c+dx)}{af(e+fx)} + \frac{d \operatorname{Chi}\left(\frac{de}{f}+dx\right) \sinh\left(c - \frac{de}{f}\right)}{af^2} + \frac{i \sinh(2c+2dx)}{2af(e+fx)} + \frac{d \cosh\left(c - \frac{de}{f}\right) \operatorname{Chi}\left(\frac{de}{f}+dx\right)}{af^2} \\
&= -\frac{\cosh(c+dx)}{af(e+fx)} + \frac{d \operatorname{Chi}\left(\frac{de}{f}+dx\right) \sinh\left(c - \frac{de}{f}\right)}{af^2} + \frac{i \sinh(2c+2dx)}{2af(e+fx)} + \frac{d \cosh\left(c - \frac{de}{f}\right) \operatorname{Chi}\left(\frac{de}{f}+dx\right)}{af^2} \\
&= -\frac{\cosh(c+dx)}{af(e+fx)} - \frac{id \cosh\left(2c - \frac{2de}{f}\right) \operatorname{Chi}\left(\frac{2de}{f}+2dx\right)}{af^2} + \frac{d \operatorname{Chi}\left(\frac{de}{f}+dx\right) \sinh\left(c - \frac{de}{f}\right)}{af^2}
\end{aligned}$$

**Mathematica [A]** time = 0.712969, size = 212, normalized size = 1.18

$$\frac{2d(e+fx) \sinh\left(c - \frac{de}{f}\right) \operatorname{Chi}\left(d\left(\frac{e}{f} + x\right)\right) - 2id(e+fx) \cosh\left(2c - \frac{2de}{f}\right) \operatorname{Chi}\left(\frac{2d(e+fx)}{f}\right) - 2ide \sinh\left(2c - \frac{2de}{f}\right) \operatorname{Shi}\left(\frac{2d(e+fx)}{f}\right)}{af^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]^3/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])),x]

[Out]  $(-2*f*Cosh[c + d*x] - (2*I)*d*(e + f*x)*Cosh[2*c - (2*d*e)/f]*CoshIntegral[(2*d*(e + f*x))/f] + 2*d*(e + f*x)*CoshIntegral[d*(e/f + x)]*Sinh[c - (d*e)/f] + I*f*Sinh[2*(c + d*x)] + 2*d*e*Cosh[c - (d*e)/f]*SinhIntegral[d*(e/f + x)] + 2*d*f*x*Cosh[c - (d*e)/f]*SinhIntegral[d*(e/f + x)] - (2*I)*d*e*Sinh[2*c - (2*d*e)/f]*SinhIntegral[(2*d*(e + f*x))/f] - (2*I)*d*f*x*Sinh[2*c - (2*d*e)/f]*SinhIntegral[(2*d*(e + f*x))/f])/(2*a*f^2*(e + f*x))$

**Maple [A]** time = 0.125, size = 299, normalized size = 1.7

$$-\frac{de^{-dx-c}}{2af(df x + de)} + \frac{d}{2af^2} e^{-\frac{cf-de}{f}} \operatorname{Ei}\left(1, dx + c - \frac{cf-de}{f}\right) - \frac{de^{dx+c}}{2af^2} \left(\frac{de}{f} + dx\right)^{-1} - \frac{d}{2af^2} e^{\frac{cf-de}{f}} \operatorname{Ei}\left(1, -dx - c - \frac{-cf+de}{f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)`

[Out] 
$$-1/2/a*d*\exp(-d*x-c)/f/(d*f*x+d*e)+1/2/a*d/f^2*\exp(-(c*f-d*e)/f)*\text{Ei}(1,d*x+c-(c*f-d*e)/f)-1/2*d/a/f^2*\exp(d*x+c)/(d*e/f+d*x)-1/2*d/a/f^2*\exp((c*f-d*e)/f)*\text{Ei}(1,-d*x-c-(-c*f+d*e)/f)+1/4*I*d/a/f^2*\exp(2*d*x+2*c)/(d*e/f+d*x)+1/2*I*d/a/f^2*\exp(2*(c*f-d*e)/f)*\text{Ei}(1,-2*d*x-2*c-2*(-c*f+d*e)/f)-1/4*I/a*d*\exp(-2*d*x-2*c)/f/(d*f*x+d*e)+1/2*I/a*d/f^2*\exp(-2*(c*f-d*e)/f)*\text{Ei}(1,2*d*x+2*c-2*(c*f-d*e)/f)$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: RuntimeError

**Fricas [A]** time = 2.21271, size = 510, normalized size = 2.83

$$\frac{i f e^{4dx+4c} - 2 f e^{3dx+3c} + \left( (-2i d f x - 2i d e) \text{Ei} \left( -\frac{2(dx+de)}{f} \right) e^{\left( \frac{2(de-cf)}{f} \right)} - 2(dx+de) \text{Ei} \left( -\frac{dx+de}{f} \right) e^{\left( \frac{de-cf}{f} \right)} + 2(dx+de) \right)}{4(a f^3 x + a e f^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")`

[Out] 
$$1/4*(I*f*e^{(4*d*x + 4*c)} - 2*f*e^{(3*d*x + 3*c)} + ((-2*I*d*f*x - 2*I*d*e)*\text{Ei}(-2*(d*f*x + d*e)/f)*e^{(2*(d*e - c*f)/f)} - 2*(d*f*x + d*e)*\text{Ei}(-(d*f*x + d*e)/f)*e^{((d*e - c*f)/f)} + 2*(d*f*x + d*e)*\text{Ei}((d*f*x + d*e)/f)*e^{-(d*e - c*f)/f} + (-2*I*d*f*x - 2*I*d*e)*\text{Ei}(2*(d*f*x + d*e)/f)*e^{(-2*(d*e - c*f)/f)})*e^{(2*d*x + 2*c)} - 2*f*e^{(d*x + c)} - I*f)*e^{(-2*d*x - 2*c)/(a*f^3*x + a*e*f^2)}$$



)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.271 \quad \int \frac{(e+fx)^3 \operatorname{sech}(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=463

$$\frac{3if^2(e+fx)\operatorname{PolyLog}(3,-ie^{c+dx})}{ad^3} - \frac{3if^2(e+fx)\operatorname{PolyLog}(3,ie^{c+dx})}{ad^3} - \frac{3if(e+fx)^2\operatorname{PolyLog}(2,-ie^{c+dx})}{2ad^2} + \frac{3if(e+fx)^2}{ad^3}$$

[Out] (((-3\*I)/2)\*f\*(e + f\*x)^2)/(a\*d^2) - (6\*f^2\*(e + f\*x)\*ArcTan[E^(c + d\*x)])/(a\*d^3) + ((e + f\*x)^3\*ArcTan[E^(c + d\*x)])/(a\*d) + ((3\*I)\*f^2\*(e + f\*x)\*Log[1 + E^(2\*(c + d\*x))])/(a\*d^3) + ((3\*I)\*f^3\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*d^4) - (((3\*I)/2)\*f\*(e + f\*x)^2\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*d^2) - ((3\*I)\*f^3\*PolyLog[2, I\*E^(c + d\*x)])/(a\*d^4) + (((3\*I)/2)\*f\*(e + f\*x)^2\*PolyLog[2, I\*E^(c + d\*x)])/(a\*d^2) + (((3\*I)/2)\*f^3\*PolyLog[2, -E^(2\*(c + d\*x))])/(a\*d^4) + ((3\*I)\*f^2\*(e + f\*x)\*PolyLog[3, (-I)\*E^(c + d\*x)])/(a\*d^3) - ((3\*I)\*f^2\*(e + f\*x)\*PolyLog[3, I\*E^(c + d\*x)])/(a\*d^3) - ((3\*I)\*f^3\*PolyLog[4, (-I)\*E^(c + d\*x)])/(a\*d^4) + ((3\*I)\*f^3\*PolyLog[4, I\*E^(c + d\*x)])/(a\*d^4) + (3\*f\*(e + f\*x)^2\*Sech[c + d\*x])/(2\*a\*d^2) + ((I/2)\*(e + f\*x)^3\*Sech[c + d\*x]^2)/(a\*d) - (((3\*I)/2)\*f\*(e + f\*x)^2\*Tanh[c + d\*x])/(a\*d^2) + ((e + f\*x)^3\*Sech[c + d\*x]\*Tanh[c + d\*x])/(2\*a\*d)

**Rubi [A]** time = 0.483055, antiderivative size = 463, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 13, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$ , Rules used = {5571, 4186, 4180, 2279, 2391, 2531, 6609, 2282, 6589, 5451, 4184, 3718, 2190}

$$\frac{3if^2(e+fx)\operatorname{PolyLog}(3,-ie^{c+dx})}{ad^3} - \frac{3if^2(e+fx)\operatorname{PolyLog}(3,ie^{c+dx})}{ad^3} - \frac{3if(e+fx)^2\operatorname{PolyLog}(2,-ie^{c+dx})}{2ad^2} + \frac{3if(e+fx)^2}{ad^3}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Sech[c + d\*x])/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (((-3\*I)/2)\*f\*(e + f\*x)^2)/(a\*d^2) - (6\*f^2\*(e + f\*x)\*ArcTan[E^(c + d\*x)])/(a\*d^3) + ((e + f\*x)^3\*ArcTan[E^(c + d\*x)])/(a\*d) + ((3\*I)\*f^2\*(e + f\*x)\*Log[1 + E^(2\*(c + d\*x))])/(a\*d^3) + ((3\*I)\*f^3\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*d^4) - (((3\*I)/2)\*f\*(e + f\*x)^2\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*d^2) - ((3\*I)\*f^3\*PolyLog[2, I\*E^(c + d\*x)])/(a\*d^4) + (((3\*I)/2)\*f\*(e + f\*x)^2\*PolyLog[2, I\*E^(c + d\*x)])/(a\*d^2) + (((3\*I)/2)\*f^3\*PolyLog[2, -E^(2\*(c + d\*x))])/(a\*d^4) + ((3\*I)\*f^2\*(e + f\*x)\*PolyLog[3, (-I)\*E^(c + d\*x)])/(a\*d^3) - ((3\*I)\*f^2\*(e + f\*x)\*PolyLog[3, I\*E^(c + d\*x)])/(a\*d^3) - ((3\*I)\*f^3\*PolyLog[4, (-I)\*E^(c + d\*x)])/(a\*d^4) + ((3\*I)\*f^3\*PolyLog[4, I\*E^(c + d\*x)])/(a

$$*d^4) + (3*f*(e + f*x)^2*Sech[c + d*x])/(2*a*d^2) + ((I/2)*(e + f*x)^3*Sech[c + d*x]^2)/(a*d) - (((3*I)/2)*f*(e + f*x)^2*Tanh[c + d*x])/(a*d^2) + ((e + f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(2*a*d)$$

### Rule 5571

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]
```

### Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_.)*((a_.)*(v_)^(n_.))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] := -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
```

```
+ d*x)^m*E^(2*(-(I*e) + f*fz*x))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{(e + fx)^3 \operatorname{sech}(c + dx)}{a + ia \sinh(c + dx)} dx &= -\frac{i \int (e + fx)^3 \operatorname{sech}^2(c + dx) \tanh(c + dx) dx}{a} + \frac{\int (e + fx)^3 \operatorname{sech}^3(c + dx) dx}{a} \\
 &= \frac{3f(e + fx)^2 \operatorname{sech}(c + dx)}{2ad^2} + \frac{i(e + fx)^3 \operatorname{sech}^2(c + dx)}{2ad} + \frac{(e + fx)^3 \operatorname{sech}(c + dx) \tanh(c + dx)}{2ad} \\
 &= -\frac{6f^2(e + fx) \tan^{-1}(e^{c+dx})}{ad^3} + \frac{(e + fx)^3 \tan^{-1}(e^{c+dx})}{ad} + \frac{3f(e + fx)^2 \operatorname{sech}(c + dx)}{2ad^2} + \frac{i(e + fx)^3 \operatorname{sech}^2(c + dx)}{2ad} \\
 &= -\frac{3if(e + fx)^2}{2ad^2} - \frac{6f^2(e + fx) \tan^{-1}(e^{c+dx})}{ad^3} + \frac{(e + fx)^3 \tan^{-1}(e^{c+dx})}{ad} - \frac{3if(e + fx)^2 \operatorname{Li}_2(e^{-c-dx})}{2ad^2} \\
 &= -\frac{3if(e + fx)^2}{2ad^2} - \frac{6f^2(e + fx) \tan^{-1}(e^{c+dx})}{ad^3} + \frac{(e + fx)^3 \tan^{-1}(e^{c+dx})}{ad} + \frac{3if^2(e + fx) \log(e^{-c-dx})}{ad^3} \\
 &= -\frac{3if(e + fx)^2}{2ad^2} - \frac{6f^2(e + fx) \tan^{-1}(e^{c+dx})}{ad^3} + \frac{(e + fx)^3 \tan^{-1}(e^{c+dx})}{ad} + \frac{3if^2(e + fx) \log(e^{-c-dx})}{ad^3} \\
 &= -\frac{3if(e + fx)^2}{2ad^2} - \frac{6f^2(e + fx) \tan^{-1}(e^{c+dx})}{ad^3} + \frac{(e + fx)^3 \tan^{-1}(e^{c+dx})}{ad} + \frac{3if^2(e + fx) \log(e^{-c-dx})}{ad^3}
 \end{aligned}$$

**Mathematica [A]** time = 11.3799, size = 767, normalized size = 1.66

---


$$12(1 + ie^c) f^2 (4f^2 - d^2 e^2) \operatorname{PolyLog}(2, ie^{-c-dx}) - 12(1 + ie^c) f^4 (d^2 x^2 \operatorname{PolyLog}(2, ie^{-c-dx}) + 2(dx \operatorname{PolyLog}(3, ie^{-c-dx})))$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] -((e + f*x)^4/f + (4*(1 - I*E^c)*(e + f*x)^3*Log[1 + I*E^(-c - d*x)])/d + (
(12*I)*(I + E^c)*f*(d^2*(e + f*x)^2*PolyLog[2, (-I)*E^(-c - d*x)] + 2*f*(d*
(e + f*x)*PolyLog[3, (-I)*E^(-c - d*x)] + f*PolyLog[4, (-I)*E^(-c - d*x)]))
)/d^4)/(8*a*(I + E^c)) - ((-12*f^2 + d^2*(e + f*x)^2)^2 + 12*d*(1 + I*E^c)*
f^2*(d^2*e^2 - 4*f^2)*x*Log[1 - I*E^(-c - d*x)] + 12*d^3*e*(1 + I*E^c)*f^3*
x^2*Log[1 - I*E^(-c - d*x)] + 4*d^3*(1 + I*E^c)*f^4*x^3*Log[1 - I*E^(-c - d
*x)] - 4*d*e*(1 + I*E^c)*f*(d^2*e^2 - 12*f^2)*(d*x - Log[I - E^(c + d*x)])
+ 12*(1 + I*E^c)*f^2*(-(d^2*e^2) + 4*f^2)*PolyLog[2, I*E^(-c - d*x)] - 24*d
*e*(1 + I*E^c)*f^3*(d*x*PolyLog[2, I*E^(-c - d*x)] + PolyLog[3, I*E^(-c - d
*x)]) - 12*(1 + I*E^c)*f^4*(d^2*x^2*PolyLog[2, I*E^(-c - d*x)] + 2*(d*x*Pol
yLog[3, I*E^(-c - d*x)] + PolyLog[4, I*E^(-c - d*x)])))/(8*a*d^4*(-I + E^c)
*f) + (x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(8*a*(Cosh[c/2] - I*S
inh[c/2]))*(Cosh[c/2] + I*Sinh[c/2])) + ((I/2)*(e + f*x)^3)/(a*d*(Cosh[c/2 +
(d*x)/2] + I*Sinh[c/2 + (d*x)/2])^2) - ((3*I)*(e^2*f*Sinh[(d*x)/2] + 2*e*f
^2*x*Sinh[(d*x)/2] + f^3*x^2*Sinh[(d*x)/2]))/(a*d^2*(Cosh[c/2] + I*Sinh[c/2
])*(Cosh[c/2 + (d*x)/2] + I*Sinh[c/2 + (d*x)/2]))
```

---

**Maple [B]** time = 0.244, size = 1152, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] 3*I/a/d^2*polylog(2,I*exp(d*x+c))*e*f^2*x-3/2*I/a/d^3*e*f^2*c^2*ln(exp(d*x+
c)-I)+3/2*I/a/d^3*e*f^2*c^2*ln(exp(d*x+c)+I)+3/2*I/a/d^3*ln(1+I*exp(d*x+c))
*c^2*e*f^2-3/2*I/a/d^3*ln(1-I*exp(d*x+c))*c^2*e*f^2+3/2*I/a/d^2*e^2*f*c*ln(
exp(d*x+c)-I)-3/2*I/a/d^2*e^2*f*c*ln(exp(d*x+c)+I)+3/2*I/a/d*ln(1-I*exp(d*x
+c))*e^2*f*x+3/2*I/a/d^2*ln(1-I*exp(d*x+c))*c*e^2*f-3/2*I/a/d*ln(1+I*exp(d*
x+c))*e^2*f*x-3/2*I/a/d^2*ln(1+I*exp(d*x+c))*c*e^2*f-3/2*I/a/d*ln(1+I*exp(d
*x+c))*e*f^2*x^2-3*I/a/d^2*polylog(2,-I*exp(d*x+c))*e*f^2*x+3/2*I/a/d*ln(1-
I*exp(d*x+c))*e*f^2*x^2-3*I*f^3*polylog(4,-I*exp(d*x+c))/a/d^4+3*I*f^3*poly
log(4,I*exp(d*x+c))/a/d^4+1/2*I/a/d*e^3*ln(exp(d*x+c)+I)-3*I/a/d^2*f^3*x^2-
3*I/a/d^4*f^3*c^2+6*I/a/d^4*f^3*polylog(2,-I*exp(d*x+c))-1/2*I/a/d*e^3*ln(e
xp(d*x+c)-I)-3/2*I/a/d^2*f^3*polylog(2,-I*exp(d*x+c))*x^2+6*I/a/d^3*e*f^2*1
n(exp(d*x+c)-I)-6*I/a/d^3*e*f^2*ln(exp(d*x+c))+3*I/a/d^3*e*f^2*polylog(3,-I
*exp(d*x+c))-3*I/a/d^3*e*f^2*polylog(3,I*exp(d*x+c))-6*I/a/d^3*f^3*c*x-6*I/
a/d^4*f^3*c*ln(exp(d*x+c)-I)-3/2*I/a/d^2*e^2*f*polylog(2,-I*exp(d*x+c))+3/2
*I/a/d^2*e^2*f*polylog(2,I*exp(d*x+c))+1/2*I/a/d^4*f^3*c^3*ln(exp(d*x+c)-I)
-1/2*I/a/d^4*f^3*c^3*ln(exp(d*x+c)+I)+6*I/a/d^4*f^3*c*ln(exp(d*x+c))+6*I/a/
d^4*f^3*c*ln(1+I*exp(d*x+c))+1/2*I/a/d^4*f^3*c^3*ln(1-I*exp(d*x+c))-1/2*I/a
```

$$\frac{1}{d^4 f^3 c^3 \ln(1 + I \exp(dx+c))} - \frac{1}{2} \frac{I}{a} \frac{1}{d f^3 \ln(1 + I \exp(dx+c))} x^3 + \frac{6}{a} \frac{1}{d^3 f^3 \ln(1 + I \exp(dx+c))} x + \frac{1}{2} \frac{I}{a} \frac{1}{d f^3 \ln(1 - I \exp(dx+c))} x^3 + \frac{3}{2} \frac{I}{a} \frac{1}{d^2 f^3 \operatorname{polylog}(2, I \exp(dx+c))} x^2 - \frac{3}{a} \frac{I}{d^3 f^3 \operatorname{polylog}(3, I \exp(dx+c))} x + \frac{3}{a} \frac{I}{d^3 f^3 \operatorname{polylog}(3, -I \exp(dx+c))} x + (d f^3 x^3 \exp(dx+c) + 3 d e f^2 x^2 \exp(dx+c) + 3 d e^2 f x \exp(dx+c) + d e^3 \exp(dx+c) + 3 f^3 x^2 \exp(dx+c) - 3 I f^3 x^2 + 6 e f^2 x \exp(dx+c) - 6 I e f^2 x + 3 e^2 f \exp(dx+c) - 3 I e^2 f) / (\exp(dx+c) - I)^2 / d^2 / a$$

**Maxima [A]** time = 2.02883, size = 923, normalized size = 1.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*sech(dx+c)/(a+I\*a\*sinh(dx+c)),x, algorithm="maxima")

[Out] 
$$-\frac{1}{2} e^3 (4 e^{-(dx+c)} / ((4 I a e^{-(dx+c)} + 2 a e^{-(2dx+2c)} - 2 a) * d) + I \log(e^{-(dx+c)} + I) / (a d) - I \log(I e^{-(dx+c)} + 1) / (a d)) + \frac{3}{2} I (d x \log(-I e^{(dx+c)} + 1) + \operatorname{dilog}(I e^{(dx+c)})) e^{2f} / (a d^2) - 6 I e f^2 x / (a d^2) + (-3 I f^3 x^2 - 6 I e f^2 x - 3 I e^2 f + (d f^3 x^3 e^c + 3 e^2 f e^c + 3 (d e f^2 + f^3) x^2 e^c + 3 (d e^2 f + 2 e f^2) x e^c) * e^{(dx)}) / (a d^2 e^{(2dx+2c)} - 2 I a d^2 e^{(dx+c)} - a d^2) - \frac{3}{2} I (d^2 x^2 \log(I e^{(dx+c)} + 1) + 2 d x \operatorname{dilog}(-I e^{(dx+c)}) - 2 \operatorname{polylog}(3, -I e^{(dx+c)})) e f^2 / (a d^3) + \frac{3}{2} I (d^2 x^2 \log(-I e^{(dx+c)} + 1) + 2 d x \operatorname{dilog}(I e^{(dx+c)}) - 2 \operatorname{polylog}(3, I e^{(dx+c)})) e f^2 / (a d^3) + 6 I e f^2 \log(I e^{(dx+c)} + 1) / (a d^3) - \frac{1}{2} I (d^3 x^3 \log(I e^{(dx+c)} + 1) + 3 d^2 x^2 \operatorname{dilog}(-I e^{(dx+c)}) - 6 d x \operatorname{polylog}(3, -I e^{(dx+c)}) + 6 \operatorname{polylog}(4, -I e^{(dx+c)})) f^3 / (a d^4) + \frac{1}{2} I (d^3 x^3 \log(-I e^{(dx+c)} + 1) + 3 d^2 x^2 \operatorname{dilog}(I e^{(dx+c)}) - 6 d x \operatorname{polylog}(3, I e^{(dx+c)}) + 6 \operatorname{polylog}(4, I e^{(dx+c)})) f^3 / (a d^4) - \frac{3}{2} I (d^2 e^2 f - 4 f^3) (d x \log(I e^{(dx+c)} + 1) + \operatorname{dilog}(-I e^{(dx+c)})) / (a d^4) - \frac{1}{8} (I d^4 f^3 x^4 + 4 I d^4 e f^2 x^3 + 6 I d^4 e^2 f x^2) / (a d^4) + \frac{1}{8} (I d^4 f^3 x^4 + 4 I d^4 e f^2 x^3 + (6 I d^2 e^2 f - 24 I f^3) d^2 x^2) / (a d^4)$$

**Fricas [C]** time = 2.45374, size = 3418, normalized size = 7.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*sech(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] (-6\*I\*d^2\*e^2\*f + 12\*I\*c\*d\*e\*f^2 - 6\*I\*c^2\*f^3 + (-3\*I\*d^2\*f^3\*x^2 - 6\*I\*d^2\*e\*f^2\*x - 3\*I\*d^2\*e^2\*f + (3\*I\*d^2\*f^3\*x^2 + 6\*I\*d^2\*e\*f^2\*x + 3\*I\*d^2\*e^2\*f)\*e^(2\*d\*x + 2\*c) + 6\*(d^2\*f^3\*x^2 + 2\*d^2\*e\*f^2\*x + d^2\*e^2\*f)\*e^(d\*x + c))\*dilog(I\*e^(d\*x + c)) + (3\*I\*d^2\*f^3\*x^2 + 6\*I\*d^2\*e\*f^2\*x + 3\*I\*d^2\*e^2\*f - 12\*I\*f^3 + (-3\*I\*d^2\*f^3\*x^2 - 6\*I\*d^2\*e\*f^2\*x - 3\*I\*d^2\*e^2\*f + 12\*I\*f^3)\*e^(2\*d\*x + 2\*c) - 6\*(d^2\*f^3\*x^2 + 2\*d^2\*e\*f^2\*x + d^2\*e^2\*f - 4\*f^3)\*e^(d\*x + c))\*dilog(-I\*e^(d\*x + c)) + (-6\*I\*d^2\*f^3\*x^2 - 12\*I\*d^2\*e\*f^2\*x - 12\*I\*c\*d\*e\*f^2 + 6\*I\*c^2\*f^3)\*e^(2\*d\*x + 2\*c) + 2\*(d^3\*f^3\*x^3 + d^3\*e^3 + 3\*d^2\*e^2\*f - 12\*c\*d\*e\*f^2 + 6\*c^2\*f^3 + 3\*(d^3\*e\*f^2 - d^2\*f^3)\*x^2 + 3\*(d^3\*e^2\*f - 2\*d^2\*e\*f^2)\*x)\*e^(d\*x + c) + (-I\*d^3\*e^3 + 3\*I\*c\*d^2\*e^2\*f - 3\*I\*c^2\*d\*e\*f^2 + I\*c^3\*f^3 + (I\*d^3\*e^3 - 3\*I\*c\*d^2\*e^2\*f + 3\*I\*c^2\*d\*e\*f^2 - I\*c^3\*f^3)\*e^(2\*d\*x + 2\*c) + 2\*(d^3\*e^3 - 3\*c\*d^2\*e^2\*f + 3\*c^2\*d\*e\*f^2 - c^3\*f^3)\*e^(d\*x + c))\*log(e^(d\*x + c) + I) + (I\*d^3\*e^3 - 3\*I\*c\*d^2\*e^2\*f + (3\*I\*c^2 - 12\*I)\*d\*e\*f^2 + (-I\*c^3 + 12\*I\*c)\*f^3 + (-I\*d^3\*e^3 + 3\*I\*c\*d^2\*e^2\*f + (-3\*I\*c^2 + 12\*I)\*d\*e\*f^2 + (I\*c^3 - 12\*I\*c)\*f^3)\*e^(2\*d\*x + 2\*c) - 2\*(d^3\*e^3 - 3\*c\*d^2\*e^2\*f + 3\*(c^2 - 4)\*d\*e\*f^2 - (c^3 - 12\*c)\*f^3)\*e^(d\*x + c))\*log(e^(d\*x + c) - I) + (I\*d^3\*f^3\*x^3 + 3\*I\*d^3\*e\*f^2\*x^2 + 3\*I\*c\*d^2\*e^2\*f - 3\*I\*c^2\*d\*e\*f^2 + (I\*c^3 - 12\*I\*c)\*f^3 + (3\*I\*d^3\*e^2\*f - 12\*I\*d\*f^3)\*x + (-I\*d^3\*f^3\*x^3 - 3\*I\*d^3\*e\*f^2\*x^2 - 3\*I\*c\*d^2\*e^2\*f + 3\*I\*c^2\*d\*e\*f^2 + (-I\*c^3 + 12\*I\*c)\*f^3 + (-3\*I\*d^3\*e^2\*f + 12\*I\*d\*f^3)\*x)\*e^(2\*d\*x + 2\*c) - 2\*(d^3\*f^3\*x^3 + 3\*d^3\*e\*f^2\*x^2 + 3\*c\*d^2\*e^2\*f - 3\*c^2\*d\*e\*f^2 + (c^3 - 12\*c)\*f^3 + 3\*(d^3\*e^2\*f - 4\*d\*f^3)\*x)\*e^(d\*x + c))\*log(I\*e^(d\*x + c) + 1) + (-I\*d^3\*f^3\*x^3 - 3\*I\*d^3\*e\*f^2\*x^2 - 3\*I\*d^3\*e^2\*f\*x - 3\*I\*c\*d^2\*e^2\*f + 3\*I\*c^2\*d\*e\*f^2 - I\*c^3\*f^3 + (I\*d^3\*f^3\*x^3 + 3\*I\*d^3\*e\*f^2\*x^2 + 3\*I\*d^3\*e^2\*f\*x + 3\*I\*c\*d^2\*e^2\*f - 3\*I\*c^2\*d\*e\*f^2 + I\*c^3\*f^3)\*e^(2\*d\*x + 2\*c) + 2\*(d^3\*f^3\*x^3 + 3\*d^3\*e\*f^2\*x^2 + 3\*d^3\*e^2\*f\*x + 3\*c\*d^2\*e^2\*f - 3\*c^2\*d\*e\*f^2 + c^3\*f^3)\*e^(d\*x + c))\*log(-I\*e^(d\*x + c) + 1) + (6\*I\*f^3\*e^(2\*d\*x + 2\*c) + 12\*f^3\*e^(d\*x + c) - 6\*I\*f^3)\*polylog(4, I\*e^(d\*x + c)) + (-6\*I\*f^3\*e^(2\*d\*x + 2\*c) - 12\*f^3\*e^(d\*x + c) + 6\*I\*f^3)\*polylog(4, -I\*e^(d\*x + c)) + (6\*I\*d\*f^3\*x + 6\*I\*d\*e\*f^2 + (-6\*I\*d\*f^3\*x - 6\*I\*d\*e\*f^2)\*e^(2\*d\*x + 2\*c) - 12\*(d\*f^3\*x + d\*e\*f^2)\*e^(d\*x + c))\*polylog(3, I\*e^(d\*x + c)) + (-6\*I\*d\*f^3\*x - 6\*I\*d\*e\*f^2 + (6\*I\*d\*f^3\*x + 6\*I\*d\*e\*f^2)\*e^(2\*d\*x + 2\*c) + 12\*(d\*f^3\*x + d\*e\*f^2)\*e^(d\*x + c))\*polylog(3, -I\*e^(d\*x + c)))/(2\*a\*d^4\*e^(2\*d\*x + 2\*c) - 4\*I\*a\*d^4\*e^(d\*x + c) - 2\*a\*d^4)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((f*x+e)**3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \operatorname{sech}(dx + c)}{ia \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*sech(d*x + c)/(I*a*sinh(d*x + c) + a), x)
```

$$3.272 \quad \int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=268

$$-\frac{if(e+fx)\operatorname{PolyLog}(2,-ie^{c+dx})}{ad^2} + \frac{if(e+fx)\operatorname{PolyLog}(2,ie^{c+dx})}{ad^2} + \frac{if^2\operatorname{PolyLog}(3,-ie^{c+dx})}{ad^3} - \frac{if^2\operatorname{PolyLog}(3,ie^{c+dx})}{ad^3}$$

```
[Out] ((e + f*x)^2*ArcTan[E^(c + d*x)])/(a*d) - (f^2*ArcTan[Sinh[c + d*x]])/(a*d^3) + (I*f^2*Log[Cosh[c + d*x]])/(a*d^3) - (I*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) + (I*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + (I*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^3) - (I*f^2*PolyLog[3, I*E^(c + d*x)])/(a*d^3) + (f*(e + f*x)*Sech[c + d*x])/(a*d^2) + ((I/2)*(e + f*x)^2*Sech[c + d*x]^2)/(a*d) - (I*f*(e + f*x)*Tanh[c + d*x])/(a*d^2) + ((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*a*d)
```

**Rubi [A]** time = 0.263905, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$ , Rules used = {5571, 4186, 3770, 4180, 2531, 2282, 6589, 5451, 4184, 3475}

$$-\frac{if(e+fx)\operatorname{PolyLog}(2,-ie^{c+dx})}{ad^2} + \frac{if(e+fx)\operatorname{PolyLog}(2,ie^{c+dx})}{ad^2} + \frac{if^2\operatorname{PolyLog}(3,-ie^{c+dx})}{ad^3} - \frac{if^2\operatorname{PolyLog}(3,ie^{c+dx})}{ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x])/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] ((e + f*x)^2*ArcTan[E^(c + d*x)])/(a*d) - (f^2*ArcTan[Sinh[c + d*x]])/(a*d^3) + (I*f^2*Log[Cosh[c + d*x]])/(a*d^3) - (I*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) + (I*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + (I*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^3) - (I*f^2*PolyLog[3, I*E^(c + d*x)])/(a*d^3) + (f*(e + f*x)*Sech[c + d*x])/(a*d^2) + ((I/2)*(e + f*x)^2*Sech[c + d*x]^2)/(a*d) - (I*f*(e + f*x)*Tanh[c + d*x])/(a*d^2) + ((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*a*d)
```

### Rule 5571

```
Int[(((e_.) + (f_.)*(x_.))^(m_.)*Sech[(c_.) + (d_.)*(x_.)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && Eq
```

$Q[a^2 + b^2, 0]$

### Rule 4186

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_)]*(b_.))^{\text{(n_.)}}*((c_.) + (d_.)(x_))^{\text{(m_.)}}, x\_Symbol]$   $\rightarrow$   $-\text{Simp}[(b^2*(c + d*x)^m*\text{Cot}[e + f*x]*(b*\text{Csc}[e + f*x])^{(n-2)})/(f*(n-1)), x] + (\text{Dist}[(b^2*d^2*m*(m-1))/(f^2*(n-1)*(n-2)), \text{Int}[(c + d*x)^{(m-2)}*(b*\text{Csc}[e + f*x])^{(n-2)}, x], x] + \text{Dist}[(b^2*(n-2))/(n-1), \text{Int}[(c + d*x)^m*(b*\text{Csc}[e + f*x])^{(n-2)}, x], x] - \text{Simp}[(b^2*d*m*(c + d*x)^{(m-1)}*(b*\text{Csc}[e + f*x])^{(n-2)})/(f^2*(n-1)*(n-2)), x]) /;$   $\text{FreeQ}\{b, c, d, e, f\}, x$  &&  $\text{GtQ}[n, 1]$  &&  $\text{NeQ}[n, 2]$  &&  $\text{GtQ}[m, 1]$

### Rule 3770

$\text{Int}[\text{csc}[(c_.) + (d_.)(x_)], x\_Symbol]$   $\rightarrow$   $-\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /;$   $\text{FreeQ}\{c, d\}, x$

### Rule 4180

$\text{Int}[\text{csc}[(e_.) + \text{Pi}*(k_.) + (\text{Complex}[0, fz\_])*(f_.)(x_)]*((c_.) + (d_.)(x_))^{\text{(m_.)}}, x\_Symbol]$   $\rightarrow$   $\text{Simp}[(-2*(c + d*x)^m*\text{ArcTanh}[E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}])/(f*fz*I), x] + (-\text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 - E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}], x], x] + \text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}], x], x]) /;$   $\text{FreeQ}\{c, d, e, f, fz\}, x$  &&  $\text{IntegerQ}[2*k]$  &&  $\text{IGtQ}[m, 0]$

### Rule 2531

$\text{Int}[\text{Log}[1 + (e_.)*((F_)^{\text{(c_.)*((a_.) + (b_.)(x_))})^{\text{(n_.)}}*(f_.) + (g_.)(x_))^{\text{(m_.)}}, x\_Symbol]$   $\rightarrow$   $-\text{Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n}))/b*c*n*\text{Log}[F], x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{(m-1)}*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n})), x], x] /;$   $\text{FreeQ}\{F, a, b, c, e, f, g, n\}, x$  &&  $\text{GtQ}[m, 0]$

### Rule 2282

$\text{Int}[u_, x\_Symbol]$   $\rightarrow$   $\text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /;$   $\text{FunctionOfExponentialQ}[u, x]$  &&  $!\text{MatchQ}[u, (w_)*((a_.)(v_)^{\text{(n_.)}})^{\text{(m_.)}} /;$   $\text{FreeQ}\{a, m, n\}, x$  &&  $\text{IntegerQ}[m*n]$  &&  $!\text{MatchQ}[u, E^{\text{(c_.)*((a_.) + (b_.)*x)}*(F_) [v_] /;$   $\text{FreeQ}\{a, b, c\}, x$  &&  $\text{InverseFunctionQ}[F[x]]]$

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 5451

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sech[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*Tanh[(a\_.) + (b\_.)\*(x\_)]^(p\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Sech[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Sech[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{(e + fx)^2 \operatorname{sech}(c + dx)}{a + ia \sinh(c + dx)} dx &= -\frac{i \int (e + fx)^2 \operatorname{sech}^2(c + dx) \tanh(c + dx) dx}{a} + \frac{\int (e + fx)^2 \operatorname{sech}^3(c + dx) dx}{a} \\
 &= \frac{f(e + fx) \operatorname{sech}(c + dx)}{ad^2} + \frac{i(e + fx)^2 \operatorname{sech}^2(c + dx)}{2ad} + \frac{(e + fx)^2 \operatorname{sech}(c + dx) \tanh(c + dx)}{2ad} \\
 &= \frac{(e + fx)^2 \tan^{-1}(e^{c+dx})}{ad} - \frac{f^2 \tan^{-1}(\sinh(c + dx))}{ad^3} + \frac{f(e + fx) \operatorname{sech}(c + dx)}{ad^2} + \frac{i(e + fx)^2 \operatorname{sech}^2(c + dx)}{2ad} \\
 &= \frac{(e + fx)^2 \tan^{-1}(e^{c+dx})}{ad} - \frac{f^2 \tan^{-1}(\sinh(c + dx))}{ad^3} + \frac{if^2 \log(\cosh(c + dx))}{ad^3} - \frac{if(e + fx) \operatorname{sech}(c + dx)}{ad} \\
 &= \frac{(e + fx)^2 \tan^{-1}(e^{c+dx})}{ad} - \frac{f^2 \tan^{-1}(\sinh(c + dx))}{ad^3} + \frac{if^2 \log(\cosh(c + dx))}{ad^3} - \frac{if(e + fx) \operatorname{sech}(c + dx)}{ad} \\
 &= \frac{(e + fx)^2 \tan^{-1}(e^{c+dx})}{ad} - \frac{f^2 \tan^{-1}(\sinh(c + dx))}{ad^3} + \frac{if^2 \log(\cosh(c + dx))}{ad^3} - \frac{if(e + fx) \operatorname{sech}(c + dx)}{ad}
 \end{aligned}$$

**Mathematica [A]** time = 11.3621, size = 501, normalized size = 1.87

$$\frac{-6(1+ie^c)ef\text{PolyLog}(2,ie^{-c-dx})-6(1+ie^c)f^2\left(x\text{PolyLog}(2,ie^{-c-dx})+\frac{\text{PolyLog}(3,ie^{-c-dx})}{d}\right)-\frac{3(1+ie^c)(d^2e^2-4f^2)(dx-\log(-e^c+dx+i))}{d}+6(1+ie^c)defx\log(1-ie^{-c-dx})+3}{(e^c-i)d^2}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Sech[c + d\*x])/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] -(((e + f\*x)^3/f + (3\*(1 - I\*E^c)\*(e + f\*x)^2\*Log[1 + I\*E^(-c - d\*x)]))/d + ((6\*I)\*(I + E^c)\*f\*(d\*(e + f\*x)\*PolyLog[2, (-I)\*E^(-c - d\*x)] + f\*PolyLog[3, (-I)\*E^(-c - d\*x)]))/d^3)/(I + E^c) + (3\*(d^2\*e^2 - 4\*f^2)\*x + 3\*d^2\*e\*f\*x^2 + d^2\*f^2\*x^3 + 6\*d\*e\*(1 + I\*E^c)\*f\*x\*Log[1 - I\*E^(-c - d\*x)] + 3\*d\*(1 + I\*E^c)\*f^2\*x^2\*Log[1 - I\*E^(-c - d\*x)] - (3\*(1 + I\*E^c)\*(d^2\*e^2 - 4\*f^2)\*(d\*x - Log[I - E^c(c + d\*x)])))/d - 6\*e\*(1 + I\*E^c)\*f\*PolyLog[2, I\*E^(-c - d\*x)] - 6\*(1 + I\*E^c)\*f^2\*(x\*PolyLog[2, I\*E^(-c - d\*x)] + PolyLog[3, I\*E^(-c - d\*x)]/d))/(d^2\*(-I + E^c)) - x\*(3\*e^2 + 3\*e\*f\*x + f^2\*x^2)\*Sech[c] - ((3\*I)\*(e + f\*x)^2)/(d\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^2) + ((12\*I)\*f\*(e + f\*x)\*Sinh[(d\*x)/2])/(d^2\*(Cosh[c/2] + I\*Sinh[c/2])\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])))/(6\*a)

**Maple [B]** time = 0.132, size = 613, normalized size = 2.3

$$\frac{dx^2f^2e^{dx+c} + 2defxe^{dx+c} + de^2e^{dx+c} - 2if^2x + 2f^2xe^{dx+c} - 2ief + 2efe^{dx+c}}{(e^{dx+c} - i)^2 d^2 a} - \frac{ief \text{polylog}(2, -ie^{dx+c})}{ad^2} - \frac{i \ln(1 + ie^{dx+c})}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*sech(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] (d\*x^2\*f^2\*exp(d\*x+c)+2\*d\*e\*f\*x\*exp(d\*x+c)+d\*e^2\*exp(d\*x+c)-2\*I\*f^2\*x+2\*f^2\*x\*x\*exp(d\*x+c)-2\*I\*e\*f+2\*e\*f\*exp(d\*x+c))/(exp(d\*x+c)-I)^2/d^2/a-I/a/d^2\*e\*f\*polylog(2,-I\*exp(d\*x+c))-I/a/d\*ln(1+I\*exp(d\*x+c))\*e\*f\*x+1/2\*I/a/d\*e^2\*ln(exp(d\*x+c)+I)-1/2\*I/a/d\*e^2\*ln(exp(d\*x+c)-I)-1/2\*I/a/d^3\*c^2\*f^2\*ln(exp(d\*x+c)-I)-I/a/d^2\*e\*f\*c\*ln(exp(d\*x+c)+I)-1/2\*I/a/d^3\*ln(1-I\*exp(d\*x+c))\*c^2\*f^2+I\*f^2\*polylog(3,-I\*exp(d\*x+c))/a/d^3-2\*I/a/d^3\*f^2\*ln(exp(d\*x+c))+1/2\*I/a/d^3\*ln(1+I\*exp(d\*x+c))\*c^2\*f^2+I/a/d^2\*ln(1-I\*exp(d\*x+c))\*e\*f+I/a/d^2\*polylog(2,I\*exp(d\*x+c))\*f^2\*x-1/2\*I/a/d\*ln(1+I\*exp(d\*x+c))\*f^2\*x^2-I/a/d^2\*poly

$$\log(2, -I \exp(dx+c)) * f^2 * x + I/a/d * \ln(1 - I \exp(dx+c)) * e * f * x + 1/2 * I/a/d * \ln(1 - I \exp(dx+c)) * f^2 * x^2 + 1/2 * I/a/d^3 * c^2 * f^2 * \ln(\exp(dx+c) + I) + 2 * I/a/d^3 * f^2 * \ln(\exp(dx+c) - I) - I/a/d^2 * \ln(1 + I \exp(dx+c)) * c * e * f + I/a/d^2 * e * f * c * \ln(\exp(dx+c) - I) - I * f^2 * \text{polylog}(3, I \exp(dx+c)) / a/d^3 + I/a/d^2 * e * f * \text{polylog}(2, I \exp(dx+c))$$

**Maxima [A]** time = 1.99471, size = 522, normalized size = 1.95

$$-\frac{1}{2} e^2 \left( \frac{4e^{(-dx-c)}}{(4i a e^{(-dx-c)} + 2 a e^{(-2dx-2c)} - 2 a) d} + \frac{i \log(e^{(-dx-c)} + i)}{ad} - \frac{i \log(i e^{(-dx-c)} + 1)}{ad} \right) + \frac{-2i f^2 x - 2i e f + (d f^2 x^2 e^c + 2 e f^2 x)}{ad^2 e^{(2dx+2c)} - 2i a d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sech(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$-1/2 * e^2 * (4 * e^{(-d * x - c)} / ((4 * I * a * e^{(-d * x - c)} + 2 * a * e^{(-2 * d * x - 2 * c)} - 2 * a) * d) + I * \log(e^{(-d * x - c)} + I) / (a * d) - I * \log(I * e^{(-d * x - c)} + 1) / (a * d)) + (-2 * I * f^2 * x - 2 * I * e * f + (d * f^2 * x^2 * e^c + 2 * e * f * e^c + 2 * (d * e * f + f^2) * x * e^c) * e^{(d * x)}) / (a * d^2 * e^{(2 * d * x + 2 * c)} - 2 * I * a * d^2 * e^{(d * x + c)} - a * d^2) - I * (d * x * \log(I * e^{(d * x + c)} + 1) + \text{dilog}(-I * e^{(d * x + c)})) * e * f / (a * d^2) + I * (d * x * \log(-I * e^{(d * x + c)} + 1) + \text{dilog}(I * e^{(d * x + c)})) * e * f / (a * d^2) - 2 * I * f^2 * x / (a * d^2) - 1/2 * I * (d^2 * x^2 * \log(I * e^{(d * x + c)} + 1) + 2 * d * x * \text{dilog}(-I * e^{(d * x + c)}) - 2 * \text{polylog}(3, -I * e^{(d * x + c)})) * f^2 / (a * d^3) + 1/2 * I * (d^2 * x^2 * \log(-I * e^{(d * x + c)} + 1) + 2 * d * x * \text{dilog}(I * e^{(d * x + c)}) - 2 * \text{polylog}(3, I * e^{(d * x + c)})) * f^2 / (a * d^3) + 2 * I * f^2 * \log(I * e^{(d * x + c)} + 1) / (a * d^3)$$

**Fricas [C]** time = 2.38792, size = 1958, normalized size = 7.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sech(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$(-4 * I * d * e * f + 4 * I * c * f^2 + (-2 * I * d * f^2 * x - 2 * I * d * e * f + (2 * I * d * f^2 * x + 2 * I * d * e * f) * e^{(2 * d * x + 2 * c)} + 4 * (d * f^2 * x + d * e * f) * e^{(d * x + c)}) * \text{dilog}(I * e^{(d * x + c)}) + (2 * I * d * f^2 * x + 2 * I * d * e * f + (-2 * I * d * f^2 * x - 2 * I * d * e * f) * e^{(2 * d * x + 2 * c)} - 4 * (d * f^2 * x + d * e * f) * e^{(d * x + c)}) * \text{dilog}(-I * e^{(d * x + c)}) + (-4 * I * d * f^2 * x - 4 * I * c * f^2) * e^{(2 * d * x + 2 * c)} + 2 * (d^2 * f^2 * x^2 + d^2 * e^2 + 2 * d * e * f - 4 * c * f^2 + 2 * (d^2 * e * f - d * f^2) * x) * e^{(d * x + c)} + (-I * d^2 * e^2 + 2 * I * c * d * e * f - I * c^2 * f^2$$

```

+ (I*d^2*e^2 - 2*I*c*d*e*f + I*c^2*f^2)*e^(2*d*x + 2*c) + 2*(d^2*e^2 - 2*c*
d*e*f + c^2*f^2)*e^(d*x + c))*log(e^(d*x + c) + I) + (I*d^2*e^2 - 2*I*c*d*e
*f + (I*c^2 - 4*I)*f^2 + (-I*d^2*e^2 + 2*I*c*d*e*f + (-I*c^2 + 4*I)*f^2)*e^
(2*d*x + 2*c) - 2*(d^2*e^2 - 2*c*d*e*f + (c^2 - 4)*f^2)*e^(d*x + c))*log(e^
(d*x + c) - I) + (I*d^2*f^2*x^2 + 2*I*d^2*e*f*x + 2*I*c*d*e*f - I*c^2*f^2 +
(-I*d^2*f^2*x^2 - 2*I*d^2*e*f*x - 2*I*c*d*e*f + I*c^2*f^2)*e^(2*d*x + 2*c)
- 2*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(d*x + c))*log(I*e
^(d*x + c) + 1) + (-I*d^2*f^2*x^2 - 2*I*d^2*e*f*x - 2*I*c*d*e*f + I*c^2*f^2
+ (I*d^2*f^2*x^2 + 2*I*d^2*e*f*x + 2*I*c*d*e*f - I*c^2*f^2)*e^(2*d*x + 2*c
) + 2*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(d*x + c))*log(-I
*e^(d*x + c) + 1) + (-2*I*f^2*e^(2*d*x + 2*c) - 4*f^2*e^(d*x + c) + 2*I*f^2
)*polylog(3, I*e^(d*x + c)) + (2*I*f^2*e^(2*d*x + 2*c) + 4*f^2*e^(d*x + c)
- 2*I*f^2)*polylog(3, -I*e^(d*x + c)))/(2*a*d^3*e^(2*d*x + 2*c) - 4*I*a*d^3
*e^(d*x + c) - 2*a*d^3)

```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*sech(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \operatorname{sech}(dx + c)}{i a \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sech(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*sech(d\*x + c)/(I\*a\*sinh(d\*x + c) + a), x)

$$3.273 \quad \int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=161

$$-\frac{if \operatorname{PolyLog}(2, -ie^{c+dx})}{2ad^2} + \frac{if \operatorname{PolyLog}(2, ie^{c+dx})}{2ad^2} - \frac{if \tanh(c+dx)}{2ad^2} + \frac{f \operatorname{sech}(c+dx)}{2ad^2} + \frac{(e+fx) \tan^{-1}(e^{c+dx})}{ad} + \frac{i(e+fx)}{ad}$$

[Out] ((e + f\*x)\*ArcTan[E^(c + d\*x)])/(a\*d) - ((I/2)\*f\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*d^2) + ((I/2)\*f\*PolyLog[2, I\*E^(c + d\*x)])/(a\*d^2) + (f\*Sech[c + d\*x])/(2\*a\*d^2) + ((I/2)\*(e + f\*x)\*Sech[c + d\*x]^2)/(a\*d) - ((I/2)\*f\*Tanh[c + d\*x])/(a\*d^2) + ((e + f\*x)\*Sech[c + d\*x]\*Tanh[c + d\*x])/(2\*a\*d)

**Rubi [A]** time = 0.142473, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$ , Rules used = {5571, 4185, 4180, 2279, 2391, 5451, 3767, 8}

$$-\frac{if \operatorname{PolyLog}(2, -ie^{c+dx})}{2ad^2} + \frac{if \operatorname{PolyLog}(2, ie^{c+dx})}{2ad^2} - \frac{if \tanh(c+dx)}{2ad^2} + \frac{f \operatorname{sech}(c+dx)}{2ad^2} + \frac{(e+fx) \tan^{-1}(e^{c+dx})}{ad} + \frac{i(e+fx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Sech[c + d\*x])/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] ((e + f\*x)\*ArcTan[E^(c + d\*x)])/(a\*d) - ((I/2)\*f\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*d^2) + ((I/2)\*f\*PolyLog[2, I\*E^(c + d\*x)])/(a\*d^2) + (f\*Sech[c + d\*x])/(2\*a\*d^2) + ((I/2)\*(e + f\*x)\*Sech[c + d\*x]^2)/(a\*d) - ((I/2)\*f\*Tanh[c + d\*x])/(a\*d^2) + ((e + f\*x)\*Sech[c + d\*x]\*Tanh[c + d\*x])/(2\*a\*d)

### Rule 5571

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] :> Dist[1/a, Int[(e + f\*x)^m\*Sech[c + d\*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f\*x)^m\*Sech[c + d\*x]^(n + 1)\*Tanh[c + d\*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]

### Rule 4185

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :> -Simp[(b^2\*(c + d\*x)\*Cot[e + f\*x]\*(b\*Csc[e + f\*x])^(n - 2))/(f\*(n - 1)), x] + (Dist[(b^2\*(n - 2))/(n - 1), Int[(c + d\*x)\*(b\*Csc[e + f\*x])^(n - 2), x]



, x] - Simp[(b^2\*d\*(b\*Csc[e + f\*x])^(n - 2))/(f^2\*(n - 1)\*(n - 2)), x] /;  
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]

### Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 5451

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sech[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*Tanh[(a\_.) + (b\_.)\*(x\_)]^(p\_.), x\_Symbol] :> -Simp[((c + d\*x)^m\*Sech[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Sech[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)} dx &= -\frac{i\int(e+fx)\operatorname{sech}^2(c+dx)\tanh(c+dx)dx}{a} + \frac{\int(e+fx)\operatorname{sech}^3(c+dx)dx}{a} \\
&= \frac{f\operatorname{sech}(c+dx)}{2ad^2} + \frac{i(e+fx)\operatorname{sech}^2(c+dx)}{2ad} + \frac{(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{2ad} + \frac{\int(e+fx)\operatorname{sech}^3(c+dx)dx}{a} \\
&= \frac{(e+fx)\tan^{-1}(e^{c+dx})}{ad} + \frac{f\operatorname{sech}(c+dx)}{2ad^2} + \frac{i(e+fx)\operatorname{sech}^2(c+dx)}{2ad} + \frac{(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{2ad} \\
&= \frac{(e+fx)\tan^{-1}(e^{c+dx})}{ad} + \frac{f\operatorname{sech}(c+dx)}{2ad^2} + \frac{i(e+fx)\operatorname{sech}^2(c+dx)}{2ad} - \frac{if\tanh(c+dx)}{2ad^2} + \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a} \\
&= \frac{(e+fx)\tan^{-1}(e^{c+dx})}{ad} - \frac{if\operatorname{Li}_2(-ie^{c+dx})}{2ad^2} + \frac{if\operatorname{Li}_2(ie^{c+dx})}{2ad^2} + \frac{f\operatorname{sech}(c+dx)}{2ad^2} + \frac{i(e+fx)\operatorname{sech}^2(c+dx)}{2ad} + \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a}
\end{aligned}$$

**Mathematica [B]** time = 2.69126, size = 707, normalized size = 4.39

$$\sqrt{2}f\left(\cosh\left(\frac{1}{2}(c+dx)\right) + i\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2\left(2(-1)^{3/4}(c+dx)^2 + \sqrt{2}\left(-4i\operatorname{PolyLog}\left(2,-ie^{-c-dx}\right) + (4ic + 4idx - 2\pi)\log\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Sech[c + d\*x])/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] ((8\*I)\*d\*(e + f\*x) - 4\*(c + d\*x)\*(c\*f - d\*(2\*e + f\*x))\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^2 - 4\*d\*e\*(c + d\*x - (2\*I)\*Log[Cosh[(c + d\*x)/2] - I\*Sinh[(c + d\*x)/2]])\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^2 + 4\*c\*f\*(c + d\*x - (2\*I)\*Log[Cosh[(c + d\*x)/2] - I\*Sinh[(c + d\*x)/2]])\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^2 - 4\*d\*e\*(c + d\*x + (2\*I)\*Log[Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]])\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^2 + 4\*c\*f\*(c + d\*x + (2\*I)\*Log[Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]])\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^2 + Sqrt[2]\*f\*(2\*(-1)^(3/4)\*(c + d\*x)^2 + Sqrt[2]\*(((4\*I)\*c - 2\*Pi + (4\*I)\*d\*x)\*Log[1 + I\*E^(-c - d\*x)] + Pi\*(-3\*c - 3\*d\*x + 4\*Log[1 + E^(c + d\*x)] - 4\*Log[Cosh[(c + d\*x)/2]] + 2\*Log[-Sin[(Pi - (2\*I)\*(c + d\*x))/4]]) - (4\*I)\*PolyLog[2, (-I)\*E^(-c - d\*x)]))\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^2 + Sqrt[2]\*f\*(-2\*(-1)^(1/4)\*(c + d\*x)^2 + Sqrt[2]\*(-2\*((2\*I)\*c + Pi + (2\*I)\*d\*x)\*Log[1 - I\*E^(-c - d\*x)] + Pi\*(c + d\*x - 4\*Log[1 + E^(c + d\*x)] + 4\*Log[Cosh[(c + d\*x)/2]] + 2\*Log[Sin[(Pi + (2\*I)\*(c + d\*x))/4]]) + (4\*I)\*PolyLog[2, I\*E^(-c - d\*x)]))\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^2 + 16\*f\*Sinh[(c + d\*x)/2]\*((-I)\*Cosh[(c + d\*x)/2] + Sinh[(c + d\*x)/2]))/(16\*d^2\*(a + I\*a\*Sinh[c + d\*x]))

**Maple [A]** time = 0.151, size = 268, normalized size = 1.7

$$\frac{dfxe^{dx+c} + dee^{dx+c} + fe^{dx+c} - if}{(e^{dx+c} - i)^2 d^2 a} - \frac{\frac{i}{2}e \ln(e^{dx+c} - i)}{da} + \frac{\frac{i}{2}e \ln(e^{dx+c} + i)}{da} - \frac{\frac{i}{2}f \ln(1 + ie^{dx+c})x}{da} - \frac{\frac{i}{2}f \ln(1 + ie^{dx+c})c}{ad^2} - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x)`

[Out]  $(d*f*x*\exp(d*x+c)+d*e*\exp(d*x+c)+f*\exp(d*x+c)-I*f)/(\exp(d*x+c)-I)^{2/d^2/a-1}/2*I/a/d*e*\ln(\exp(d*x+c)-I)+1/2*I/a/d*e*\ln(\exp(d*x+c)+I)-1/2*I/a/d*f*\ln(1+I*\exp(d*x+c))*x-1/2*I/a/d^2*f*\ln(1+I*\exp(d*x+c))*c-1/2*I*f*\text{polylog}(2,-I*\exp(d*x+c))/a/d^2+1/2*I/a/d*f*\ln(1-I*\exp(d*x+c))*x+1/2*I/a/d^2*f*\ln(1-I*\exp(d*x+c))*c+1/2*I*f*\text{polylog}(2,I*\exp(d*x+c))/a/d^2+1/2*I/a/d^2*f*c*\ln(\exp(d*x+c)-I)-1/2*I/a/d^2*f*c*\ln(\exp(d*x+c)+I)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2f \left( \frac{(dxe^c + e^c)e^{(dx)} - i}{2ad^2e^{(2dx+2c)} - 4iad^2e^{(dx+c)} - 2ad^2} + \int \frac{x}{4(ae^{(dx+c)} + ia)} dx + \int \frac{x}{4(ae^{(dx+c)} - ia)} dx \right) - \frac{1}{2}e \left( \frac{4e^{(-dx-c)}}{(4iae^{(-dx-c)} + 2ae^{(-2dx-2c)})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*sech(d*x+c)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out]  $2*f*(((d*x*e^c + e^c)*e^{(d*x)} - I)/((2*a*d^2*e^{(2*d*x + 2*c)} - 4*I*a*d^2*e^{(d*x + c)} - 2*a*d^2) + \text{integrate}(1/4*x/(a*e^{(d*x + c)} + I*a), x) + \text{integrate}(1/4*x/(a*e^{(d*x + c)} - I*a), x)) - 1/2*e*(4*e^{(-d*x - c)}/((4*I*a*e^{(-d*x - c)} + 2*a*e^{(-2*d*x - 2*c)} - 2*a)*d) + I*\log(e^{(-d*x - c)} + I)/(a*d) - I*\log(I*e^{(-d*x - c)} + 1)/(a*d))$

**Fricas [B]** time = 2.28914, size = 902, normalized size = 5.6

$$(if e^{(2dx+2c)} + 2fe^{(dx+c)} - if) \text{Li}_2(ie^{(dx+c)}) + (-ife^{(2dx+2c)} - 2fe^{(dx+c)} + if) \text{Li}_2(-ie^{(dx+c)}) + 2(df x + de + f)e^{(dx+c)} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] ((I\*f\*e^(2\*d\*x + 2\*c) + 2\*f\*e^(d\*x + c) - I\*f)\*dilog(I\*e^(d\*x + c)) + (-I\*f\*e^(2\*d\*x + 2\*c) - 2\*f\*e^(d\*x + c) + I\*f)\*dilog(-I\*e^(d\*x + c)) + 2\*(d\*f\*x + d\*e + f)\*e^(d\*x + c) + (-I\*d\*e + I\*c\*f + (I\*d\*e - I\*c\*f)\*e^(2\*d\*x + 2\*c) + 2\*(d\*e - c\*f)\*e^(d\*x + c))\*log(e^(d\*x + c) + I) + (I\*d\*e - I\*c\*f + (-I\*d\*e + I\*c\*f)\*e^(2\*d\*x + 2\*c) - 2\*(d\*e - c\*f)\*e^(d\*x + c))\*log(e^(d\*x + c) - I) + (I\*d\*f\*x + I\*c\*f + (-I\*d\*f\*x - I\*c\*f)\*e^(2\*d\*x + 2\*c) - 2\*(d\*f\*x + c\*f)\*e^(d\*x + c))\*log(I\*e^(d\*x + c) + 1) + (-I\*d\*f\*x - I\*c\*f + (I\*d\*f\*x + I\*c\*f)\*e^(2\*d\*x + 2\*c) + 2\*(d\*f\*x + c\*f)\*e^(d\*x + c))\*log(-I\*e^(d\*x + c) + 1) - 2\*I\*f)/(2\*a\*d^2\*e^(2\*d\*x + 2\*c) - 4\*I\*a\*d^2\*e^(d\*x + c) - 2\*a\*d^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{e \operatorname{sech}(c+dx)}{i \sinh(c+dx)+1} dx + \int \frac{fx \operatorname{sech}(c+dx)}{i \sinh(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] (Integral(e\*sech(c + d\*x)/(I\*sinh(c + d\*x) + 1), x) + Integral(f\*x\*sech(c + d\*x)/(I\*sinh(c + d\*x) + 1), x))/a

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \operatorname{sech}(dx + c)}{i a \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)\*sech(d\*x + c)/(I\*a\*sinh(d\*x + c) + a), x)

$$3.274 \quad \int \frac{\operatorname{sech}(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=42

$$\frac{\tan^{-1}(\sinh(c+dx))}{2ad} + \frac{i}{2d(a+ia \sinh(c+dx))}$$

[Out] ArcTan[Sinh[c + d\*x]]/(2\*a\*d) + (I/2)/(d\*(a + I\*a\*Sinh[c + d\*x]))

**Rubi [A]** time = 0.0566407, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {2667, 44, 206}

$$\frac{\tan^{-1}(\sinh(c+dx))}{2ad} + \frac{i}{2d(a+ia \sinh(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d\*x]/(a + I\*a\*Sinh[c + d\*x]), x]

[Out] ArcTan[Sinh[c + d\*x]]/(2\*a\*d) + (I/2)/(d\*(a + I\*a\*Sinh[c + d\*x]))

#### Rule 2667

Int[cos[(e\_.) + (f\_.)\*(x\_.)]^(p\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(m\_.), x\_Symbol] :> Dist[1/(b^p\*f), Subst[Int[(a + x)^(m + (p - 1)/2)\*(a - x)^(p - 1)/2, x], x, b\*Sin[e + f\*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

#### Rule 44

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rule 206

Int[((a\_.) + (b\_.)\*(x\_.)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{\operatorname{sech}(c+dx)}{a+ia\sinh(c+dx)} dx &= -\frac{(ia) \operatorname{Subst}\left(\int \frac{1}{(a-x)(a+x)^2} dx, x, ia\sinh(c+dx)\right)}{d} \\
 &= -\frac{(ia) \operatorname{Subst}\left(\int \left(\frac{1}{2a(a+x)^2} + \frac{1}{2a(a^2-x^2)}\right) dx, x, ia\sinh(c+dx)\right)}{d} \\
 &= \frac{i}{2d(a+ia\sinh(c+dx))} - \frac{i \operatorname{Subst}\left(\int \frac{1}{a^2-x^2} dx, x, ia\sinh(c+dx)\right)}{2d} \\
 &= \frac{\tan^{-1}(\sinh(c+dx))}{2ad} + \frac{i}{2d(a+ia\sinh(c+dx))}
 \end{aligned}$$

**Mathematica [A]** time = 0.0480522, size = 30, normalized size = 0.71

$$\frac{\tan^{-1}(\sinh(c+dx)) + \frac{1}{\sinh(c+dx)-i}}{2ad}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d\*x]/(a + I\*a\*Sinh[c + d\*x]), x]

[Out] (ArcTan[Sinh[c + d\*x]] + (-I + Sinh[c + d\*x])^(-1))/(2\*a\*d)

**Maple [B]** time = 0.046, size = 91, normalized size = 2.2

$$\frac{i}{2a} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + i\right) - \frac{i}{da} \left(-i + \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-2} - \frac{i}{2a} \ln\left(-i + \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \frac{1}{da} \left(-i + \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d\*x+c)/(a+I\*a\*sinh(d\*x+c)), x)

[Out] 1/2\*I/d/a\*ln(tanh(1/2\*d\*x+1/2\*c)+I)-I/d/a/(-I+tanh(1/2\*d\*x+1/2\*c))^-2-1/2\*I/d/a\*ln(-I+tanh(1/2\*d\*x+1/2\*c))-1/d/a/(-I+tanh(1/2\*d\*x+1/2\*c))

---

**Maxima [B]** time = 1.16804, size = 117, normalized size = 2.79

$$-\frac{2e^{(-dx-c)}}{(4i ae^{(-dx-c)} + 2ae^{(-2dx-2c)} - 2a)d} - \frac{i \log(e^{(-dx-c)} + i)}{2ad} + \frac{i \log(i e^{(-dx-c)} + 1)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-2e^{(-d*x - c)} / ((4*I*a*e^{(-d*x - c)} + 2*a*e^{(-2*d*x - 2*c)} - 2*a)*d) - 1/2 * I * \log(e^{(-d*x - c)} + I) / (a*d) + 1/2 * I * \log(I * e^{(-d*x - c)} + 1) / (a*d)$

---

**Fricas [B]** time = 2.12009, size = 267, normalized size = 6.36

$$\frac{(ie^{(2dx+2c)} + 2e^{(dx+c)} - i) \log(e^{(dx+c)} + i) + (-ie^{(2dx+2c)} - 2e^{(dx+c)} + i) \log(e^{(dx+c)} - i) + 2e^{(dx+c)}}{2ade^{(2dx+2c)} - 4iade^{(dx+c)} - 2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $((I * e^{(2*d*x + 2*c)} + 2 * e^{(d*x + c)} - I) * \log(e^{(d*x + c)} + I) + (-I * e^{(2*d*x + 2*c)} - 2 * e^{(d*x + c)} + I) * \log(e^{(d*x + c)} - I) + 2 * e^{(d*x + c)}) / (2 * a * d * e^{(2*d*x + 2*c)} - 4 * I * a * d * e^{(d*x + c)} - 2 * a * d)$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{sech}(c+dx)}{i \sinh(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Integral(sech(c + d\*x)/(I\*sinh(c + d\*x) + 1), x)/a

---

**Giac [B]** time = 1.15309, size = 147, normalized size = 3.5

$$-\frac{i \log\left(e^{(dx+c)} - e^{(-dx-c)} - 2i\right)}{4ad} + \frac{i \log\left(i e^{(dx+c)} - i e^{(-dx-c)} - 2\right)}{4ad} - \frac{-i e^{(dx+c)} + i e^{(-dx-c)} - 6}{4ad\left(e^{(dx+c)} - e^{(-dx-c)} - 2i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out]  $-\frac{1}{4}I \log\left(\frac{e^{(dx+c)} - e^{(-dx-c)} - 2I}{a*d}\right) + \frac{1}{4}I \log\left(\frac{I e^{(dx+c)} - I e^{(-dx-c)} - 2}{a*d}\right) - \frac{1}{4} \frac{-I e^{(dx+c)} + I e^{(-dx-c)} - 6}{a*d(e^{(dx+c)} - e^{(-dx-c)} - 2I)}$



$$3.275 \quad \int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

**Optimal.** Leaf size=31

$$\text{Unintegrable}\left(\frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Sech[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0496738, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Sech[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Sech[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx = \int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

**Mathematica [A]** time = 65.4834, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sech[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Integrate[Sech[c + d\*x]/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

---

**Maple [A]** time = 0.714, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(dx + c)}{(fx + e)(a + ia \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)`

[Out] `int(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{2((dfxe^c + (de - f)e^c)e^{dx} + if)}{2ad^2f^2x^2 + 4ad^2efx + 2ad^2e^2 - 2(ad^2f^2x^2e^{2c} + 2ad^2efxe^{2c} + ad^2e^2e^{2c})e^{2dx} - (-4iad^2f^2x^2e^c - 8iad^2efxe^c - 4iad^2e^2e^c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `-2*((d*f*x*e^c + (d*e - f)*e^c)*e^(d*x) + I*f)/(2*a*d^2*f^2*x^2 + 4*a*d^2*e*f*x + 2*a*d^2*e^2 - 2*(a*d^2*f^2*x^2*e^(2*c) + 2*a*d^2*e*f*x*e^(2*c) + a*d^2*e^2*e^(2*c))*e^(2*d*x) - (-4*I*a*d^2*f^2*x^2*e^c - 8*I*a*d^2*e*f*x*e^c - 4*I*a*d^2*e^2*e^c)*e^(d*x)) + 2*integrate((d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2 - 4*f^2)/(-4*I*a*d^2*f^3*x^3 - 12*I*a*d^2*e*f^2*x^2 - 12*I*a*d^2*e^2*f*x - 4*I*a*d^2*e^3 + 4*(a*d^2*f^3*x^3*e^c + 3*a*d^2*e*f^2*x^2*e^c + 3*a*d^2*e^2*f*x*e^c + a*d^2*e^3*e^c)*e^(d*x)), x) + 2*integrate(1/(4*I*a*f*x + 4*I*a*e + 4*(a*f*x*e^c + a*e*e^c)*e^(d*x)), x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\frac{(dfx + de - f)e^{(dx+c)} - (ad^2f^2x^2 + 2ad^2efx + ad^2e^2 - (ad^2f^2x^2 + 2ad^2efx + ad^2e^2)e^{2dx+2c}) - (-2iad^2f^2x^2 - 4iad^2efx - 4iad^2e^2e^c)}{ad^2f^2x^2 + 2ad^2efx + ad^2e^2 - (ad^2f^2x^2 + 2ad^2efx + ad^2e^2)e^{2dx+2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $-\left((d*f*x + d*e - f)*e^{(d*x + c)} - (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2 - (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*e^{(2*d*x + 2*c)} - (-2*I*a*d^2*f^2*x^2 - 4*I*a*d^2*e*f*x - 2*I*a*d^2*e^2)*e^{(d*x + c)}\right)*\text{integral}\left(\frac{(-2*I*f^2 + (d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2 - 2*f^2)*e^{(d*x + c)})}{(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3 + (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^2*f*x + a*d^2*e^3)*e^{(2*d*x + 2*c)}), x\right) + I*f)/\left((a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2 - (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*e^{(2*d*x + 2*c)} - (-2*I*a*d^2*f^2*x^2 - 4*I*a*d^2*e*f*x - 2*I*a*d^2*e^2)*e^{(d*x + c)}\right)$

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{sech}(c+dx)}{ie \sinh(c+dx)+e+ifx \sinh(c+dx)+fx} dx$$

$a$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Integral(sech(c + d\*x)/(I\*e\*sinh(c + d\*x) + e + I\*f\*x\*sinh(c + d\*x) + f\*x), x)/a

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{sech}(dx + c)}{(fx + e)(ia \sinh(dx + c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate(sech(d\*x + c)/((f\*x + e)\*(I\*a\*sinh(d\*x + c) + a)), x)

$$3.276 \quad \int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

**Optimal.** Leaf size=31

$$\text{Unintegrable}\left(\frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Sech[c + d\*x]/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0503592, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Sech[c + d\*x]/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Sech[c + d\*x]/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx = \int \frac{\operatorname{sech}(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.007, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[Sech[c + d\*x]/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] \$Aborted

---

**Maple [A]** time = 1.079, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(dx + c)}{(fx + e)^2 (a + ia \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)`

[Out] `int(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{2 \left( (dfxe^c + (de - 2f)e^c)e^{(dx)} + 2if \right)}{2ad^2f^3x^3 + 6ad^2ef^2x^2 + 6ad^2e^2fx + 2ad^2e^3 - 2(ad^2f^3x^3e^{(2c)} + 3ad^2ef^2x^2e^{(2c)} + 3ad^2e^2fxe^{(2c)} + ad^2e^3e^{(2c)})e^{(2dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `-2*((d*f*x*e^c + (d*e - 2*f)*e^c)*e^(d*x) + 2*I*f)/(2*a*d^2*f^3*x^3 + 6*a*d^2*e*f^2*x^2 + 6*a*d^2*e^2*f*x + 2*a*d^2*e^3 - 2*(a*d^2*f^3*x^3*e^(2*c) + 3*a*d^2*e*f^2*x^2*e^(2*c) + 3*a*d^2*e^2*f*x*e^(2*c) + a*d^2*e^3*e^(2*c))*e^(2*d*x) - (-4*I*a*d^2*f^3*x^3*e^c - 12*I*a*d^2*e*f^2*x^2*e^c - 12*I*a*d^2*e^2*f*x*e^c - 4*I*a*d^2*e^3*e^c)*e^(d*x)) + 2*integrate((d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2 - 12*f^2)/(-4*I*a*d^2*f^4*x^4 - 16*I*a*d^2*e*f^3*x^3 - 24*I*a*d^2*e^2*f^2*x^2 - 16*I*a*d^2*e^3*f*x - 4*I*a*d^2*e^4 + 4*(a*d^2*f^4*x^4*e^c + 4*a*d^2*e*f^3*x^3*e^c + 6*a*d^2*e^2*f^2*x^2*e^c + 4*a*d^2*e^3*f*x*e^c + a*d^2*e^4*e^c)*e^(d*x)), x) + 2*integrate(1/(4*I*a*f^2*x^2 + 8*I*a*e*f*x + 4*I*a*e^2 + 4*(a*f^2*x^2*e^c + 2*a*e*f*x*e^c + a*e^2*e^c)*e^(d*x)), x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\frac{(dfx + de - 2f)e^{(dx+c)} - (ad^2f^3x^3 + 3ad^2ef^2x^2 + 3ad^2e^2fx + ad^2e^3 - (ad^2f^3x^3 + 3ad^2ef^2x^2 + 3ad^2e^2fx + ad^2e^3)e^{(2dx)})}{ad^2f^3x^3 + 3ad^2ef^2x^2 + 3ad^2e^2fx + ad^2e^3 - (ad^2f^3x^3 + 3ad^2ef^2x^2 + 3ad^2e^2fx + ad^2e^3)e^{(2dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((d*f*x + d*e - 2*f)*e^(d*x + c) - (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*
a*d^2*e^2*f*x + a*d^2*e^3 - (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d^2*e^
2*f*x + a*d^2*e^3)*e^(2*d*x + 2*c) - (-2*I*a*d^2*f^3*x^3 - 6*I*a*d^2*e*f^2*
x^2 - 6*I*a*d^2*e^2*f*x - 2*I*a*d^2*e^3)*e^(d*x + c))*integral((-6*I*f^2 +
(d^2*f^2*x^2 + 2*d^2*e*f*x + d^2*e^2 - 6*f^2)*e^(d*x + c))/(a*d^2*f^4*x^4 +
4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x + a*d^2*e^4 + (a
*d^2*f^4*x^4 + 4*a*d^2*e*f^3*x^3 + 6*a*d^2*e^2*f^2*x^2 + 4*a*d^2*e^3*f*x +
a*d^2*e^4)*e^(2*d*x + 2*c)), x) + 2*I*f)/(a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2
+ 3*a*d^2*e^2*f*x + a*d^2*e^3 - (a*d^2*f^3*x^3 + 3*a*d^2*e*f^2*x^2 + 3*a*d
^2*e^2*f*x + a*d^2*e^3)*e^(2*d*x + 2*c) - (-2*I*a*d^2*f^3*x^3 - 6*I*a*d^2*
e*f^2*x^2 - 6*I*a*d^2*e^2*f*x - 2*I*a*d^2*e^3)*e^(d*x + c))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)**2/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(dx+c)}{(fx+e)^2 (ia \sinh(dx+c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sech(d*x + c)/((f*x + e)^2*(I*a*sinh(d*x + c) + a)), x)
```

$$3.277 \quad \int \frac{(e+fx)^3 \operatorname{sech}^2(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=450

$$-\frac{f^2(e+fx)\operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^3} + \frac{f^2(e+fx)\operatorname{PolyLog}\left(2, ie^{c+dx}\right)}{ad^3} - \frac{2f^2(e+fx)\operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{ad^3} + \frac{f^3\operatorname{PolyLog}}{ad^3}$$

[Out]  $(2*(e + f*x)^3)/(3*a*d) - (I*f*(e + f*x)^2*\operatorname{ArcTan}[E^{(c + d*x)}])/(a*d^2) + (I*f^3*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(a*d^4) - (2*f*(e + f*x)^2*\operatorname{Log}[1 + E^{(2*(c + d*x))}])/(a*d^2) + (f^3*\operatorname{Log}[\operatorname{Cosh}[c + d*x]])/(a*d^4) - (f^2*(e + f*x)*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x)}])/(a*d^3) + (f^2*(e + f*x)*\operatorname{PolyLog}[2, I*E^{(c + d*x)}])/(a*d^3) - (2*f^2*(e + f*x)*\operatorname{PolyLog}[2, -E^{(2*(c + d*x))}])/(a*d^3) + (f^3*\operatorname{PolyLog}[3, (-I)*E^{(c + d*x)}])/(a*d^4) - (f^3*\operatorname{PolyLog}[3, I*E^{(c + d*x)}])/(a*d^4) + (f^3*\operatorname{PolyLog}[3, -E^{(2*(c + d*x))}])/(a*d^4) - (I*f^2*(e + f*x)*\operatorname{Sech}[c + d*x])/(a*d^3) + (f*(e + f*x)^2*\operatorname{Sech}[c + d*x]^2)/(2*a*d^2) + ((I/3)*(e + f*x)^3*\operatorname{Sech}[c + d*x]^3)/(a*d) - (f^2*(e + f*x)*\operatorname{Tanh}[c + d*x])/(a*d^3) + (2*(e + f*x)^3*\operatorname{Tanh}[c + d*x])/(3*a*d) - ((I/2)*f*(e + f*x)^2*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(a*d^2) + ((e + f*x)^3*\operatorname{Sech}[c + d*x]^2*\operatorname{Tanh}[c + d*x])/(3*a*d)$

**Rubi [A]** time = 0.589366, antiderivative size = 450, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 12, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$ , Rules used = {5571, 4186, 4184, 3475, 3718, 2190, 2531, 2282, 6589, 5451, 3770, 4180}

$$-\frac{f^2(e+fx)\operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^3} + \frac{f^2(e+fx)\operatorname{PolyLog}\left(2, ie^{c+dx}\right)}{ad^3} - \frac{2f^2(e+fx)\operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{ad^3} + \frac{f^3\operatorname{PolyLog}}{ad^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\frac{(e + f*x)^3*\operatorname{Sech}[c + d*x]^2}{(a + I*a*\operatorname{Sinh}[c + d*x])}, x]$

[Out]  $(2*(e + f*x)^3)/(3*a*d) - (I*f*(e + f*x)^2*\operatorname{ArcTan}[E^{(c + d*x)}])/(a*d^2) + (I*f^3*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(a*d^4) - (2*f*(e + f*x)^2*\operatorname{Log}[1 + E^{(2*(c + d*x))}])/(a*d^2) + (f^3*\operatorname{Log}[\operatorname{Cosh}[c + d*x]])/(a*d^4) - (f^2*(e + f*x)*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x)}])/(a*d^3) + (f^2*(e + f*x)*\operatorname{PolyLog}[2, I*E^{(c + d*x)}])/(a*d^3) - (2*f^2*(e + f*x)*\operatorname{PolyLog}[2, -E^{(2*(c + d*x))}])/(a*d^3) + (f^3*\operatorname{PolyLog}[3, (-I)*E^{(c + d*x)}])/(a*d^4) - (f^3*\operatorname{PolyLog}[3, I*E^{(c + d*x)}])/(a*d^4) + (f^3*\operatorname{PolyLog}[3, -E^{(2*(c + d*x))}])/(a*d^4) - (I*f^2*(e + f*x)*\operatorname{Sech}[c + d*x])/(a*d^3) + (f*(e + f*x)^2*\operatorname{Sech}[c + d*x]^2)/(2*a*d^2) + ((I/3)*(e + f*x)^3*\operatorname{Sech}[c + d*x]^3)/(a*d) - (f^2*(e + f*x)*\operatorname{Tanh}[c + d*x])/(a*d^3) + (2*(e + f*x)^3*\operatorname{Tanh}[c + d*x])/(3*a*d) - ((I/2)*f*(e + f*x)^2*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c$

+ d\*x])/(a\*d^2) + ((e + f\*x)^3\*Sech[c + d\*x]^2\*Tanh[c + d\*x])/(3\*a\*d)

### Rule 5571

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Sech[c + d\*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f\*x)^m\*Sech[c + d\*x]^(n + 1)\*Tanh[c + d\*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]

### Rule 4186

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.))^(n\_)\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] := -Simp[(b^2\*(c + d\*x)^m\*Cot[e + f\*x]\*(b\*Csc[e + f\*x])^(n - 2))/(f\*(n - 1)), x] + (Dist[(b^2\*d^2\*m\*(m - 1))/(f^2\*(n - 1)\*(n - 2)), Int[(c + d\*x)^(m - 2)\*(b\*Csc[e + f\*x])^(n - 2), x], x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(c + d\*x)^m\*(b\*Csc[e + f\*x])^(n - 2), x], x] - Simp[(b^2\*d\*m\*(c + d\*x)^(m - 1)\*(b\*Csc[e + f\*x])^(n - 2))/(f^2\*(n - 1)\*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

### Rule 3718

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]), x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x))]/(1 + E^(2\*(-I\*e) + f\*fz\*x)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/ (b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x]]



))<sup>n</sup>)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))<sup>(n\_.)</sup>]\*((f\_.) + (g\_.)\*(x\_))<sup>(m\_.)</sup>, x\_Symbol] := -Simp[((f + g\*x)<sup>m</sup>\*PolyLog[2, -(e\*(F<sup>c\*(a + b\*x)</sup>))<sup>n</sup>])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)<sup>(m - 1)</sup>\*PolyLog[2, -(e\*(F<sup>c\*(a + b\*x)</sup>))<sup>n</sup>], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)<sup>(n\_)</sup>)<sup>(m\_)</sup> /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E<sup>((c\_.)\*((a\_.) + (b\_.)\*x))\* (F\_)</sup>[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))<sup>(p\_.)</sup>]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)<sup>p</sup>]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 5451

Int[((c\_.) + (d\_.)\*(x\_))<sup>(m\_.)</sup>\*Sech[(a\_.) + (b\_.)\*(x\_)]<sup>(n\_.)</sup>\*Tanh[(a\_.) + (b\_.)\*(x\_)]<sup>(p\_.)</sup>, x\_Symbol] := -Simp[((c + d\*x)<sup>m</sup>\*Sech[a + b\*x]<sup>n</sup>)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)<sup>(m - 1)</sup>\*Sech[a + b\*x]<sup>n</sup>, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

### Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))<sup>(m\_.)</sup>, x\_Symbol] := Simp[(-2\*(c + d\*x)<sup>m</sup>\*ArcTanh[E<sup>-(I\*e) + f\*fz\*x</sup>]/E<sup>(I\*k\*Pi)</sup>)]/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)<sup>(m - 1)</sup>\*Log[1 - E<sup>-(I\*e) + f\*fz\*x</sup>]/E<sup>(I\*k\*Pi)</sup>], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)<sup>(m - 1)</sup>\*Log[1 + E<sup>-(I\*e) + f\*fz\*x</sup>]/E<sup>(I\*k\*Pi)</sup>], x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \operatorname{sech}^2(c+dx)}{a+ia \sinh(c+dx)} dx &= -\frac{i \int (e+fx)^3 \operatorname{sech}^3(c+dx) \tanh(c+dx) dx}{a} + \frac{\int (e+fx)^3 \operatorname{sech}^4(c+dx) dx}{a} \\
&= \frac{f(e+fx)^2 \operatorname{sech}^2(c+dx)}{2ad^2} + \frac{i(e+fx)^3 \operatorname{sech}^3(c+dx)}{3ad} + \frac{(e+fx)^3 \operatorname{sech}^2(c+dx) \tanh(c+dx)}{3ad} \\
&= -\frac{if^2(e+fx) \operatorname{sech}(c+dx)}{ad^3} + \frac{f(e+fx)^2 \operatorname{sech}^2(c+dx)}{2ad^2} + \frac{i(e+fx)^3 \operatorname{sech}^3(c+dx)}{3ad} - \frac{f^2(e+fx)}{ad^2} \\
&= \frac{2(e+fx)^3}{3ad} - \frac{if(e+fx)^2 \tan^{-1}(e^{c+dx})}{ad^2} + \frac{if^3 \tan^{-1}(\sinh(c+dx))}{ad^4} + \frac{f^3 \log(\cosh(c+dx))}{ad^4} \\
&= \frac{2(e+fx)^3}{3ad} - \frac{if(e+fx)^2 \tan^{-1}(e^{c+dx})}{ad^2} + \frac{if^3 \tan^{-1}(\sinh(c+dx))}{ad^4} - \frac{2f(e+fx)^2 \log(1+e^{c+dx})}{ad^2} \\
&= \frac{2(e+fx)^3}{3ad} - \frac{if(e+fx)^2 \tan^{-1}(e^{c+dx})}{ad^2} + \frac{if^3 \tan^{-1}(\sinh(c+dx))}{ad^4} - \frac{2f(e+fx)^2 \log(1+e^{c+dx})}{ad^2} \\
&= \frac{2(e+fx)^3}{3ad} - \frac{if(e+fx)^2 \tan^{-1}(e^{c+dx})}{ad^2} + \frac{if^3 \tan^{-1}(\sinh(c+dx))}{ad^4} - \frac{2f(e+fx)^2 \log(1+e^{c+dx})}{ad^2} \\
&= \frac{2(e+fx)^3}{3ad} - \frac{if(e+fx)^2 \tan^{-1}(e^{c+dx})}{ad^2} + \frac{if^3 \tan^{-1}(\sinh(c+dx))}{ad^4} - \frac{2f(e+fx)^2 \log(1+e^{c+dx})}{ad^2}
\end{aligned}$$

**Mathematica [B]** time = 12.2762, size = 1049, normalized size = 2.33

$$\frac{if \left( \frac{(e+fx)^3}{f} + \frac{3(1-ie^c) \log(1+ie^{-c-dx})(e+fx)^2}{d} + \frac{6i(i+e^c)f(d(e+fx)\operatorname{PolyLog}(2,-ie^{-c-dx})+f\operatorname{PolyLog}(3,-ie^{-c-dx}))}{d^3} \right)}{2ad(i+e^c)} + \frac{if(5d^2f^2x^3+15d^2efx^2+\dots)}{2ad(i+e^c)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((e+f\*x)^3\*Sech[c+d\*x]^2)/(a+I\*a\*Sinh[c+d\*x]),x]

[Out] ((-I/2)\*f\*((e+f\*x)^3/f+(3\*(1-I\*E^c)\*(e+f\*x)^2\*Log[1+I\*E^(-c-d\*x)]))/d+((6\*I)\*(I+E^c)\*f\*(d\*(e+f\*x)\*PolyLog[2,(-I)\*E^(-c-d\*x)]+f\*PolyLog[3,(-I)\*E^(-c-d\*x)]))/d^3)/(a\*d\*(I+E^c))+((I/6)\*f\*(3\*(5\*d^2\*e^2-4\*f^2)\*x+15\*d^2\*e\*f\*x^2+5\*d^2\*f^2\*x^3+30\*d\*e\*(1+I\*E^c)\*f\*x\*Log[1-I\*E^(-c-d\*x)]+15\*d\*(1+I\*E^c)\*f^2\*x^2\*Log[1-I\*E^(-c-d\*x)]-(3\*(1+I\*E^c)\*(5\*d^2\*e^2-4\*f^2)\*(d\*x-Log[I-E^c(d+dx)])))/d-30\*e\*(1+I\*E^c)\*f\*PolyLog[2,I\*E^(-c-d\*x)]-30\*(1+I\*E^c)\*f^2\*(x\*PolyLog[2,I\*E^(-c-d\*x)]+PolyLog[3,I\*E^(-c-d\*x)]/d))/(a\*d^3\*(-I+E^c))+e^3\*

$$\begin{aligned} & \text{Sinh}[(d*x)/2] + 3*e^2*f*x*\text{Sinh}[(d*x)/2] + 3*e*f^2*x^2*\text{Sinh}[(d*x)/2] + f^3*x \\ & ^3*\text{Sinh}[(d*x)/2]) / (2*a*d*(\text{Cosh}[c/2] - I*\text{Sinh}[c/2])*(\text{Cosh}[c/2 + (d*x)/2] - I \\ & *\text{Sinh}[c/2 + (d*x)/2])) + (e^3*\text{Sinh}[(d*x)/2] + 3*e^2*f*x*\text{Sinh}[(d*x)/2] + 3*e \\ & *f^2*x^2*\text{Sinh}[(d*x)/2] + f^3*x^3*\text{Sinh}[(d*x)/2]) / (3*a*d*(\text{Cosh}[c/2] + I*\text{Sinh}[ \\ & c/2])*(\text{Cosh}[c/2 + (d*x)/2] + I*\text{Sinh}[c/2 + (d*x)/2])^3) + (I*d*e^3*\text{Cosh}[c/2] \\ & + 3*e^2*f*\text{Cosh}[c/2] + (3*I)*d*e^2*f*x*\text{Cosh}[c/2] + 6*e*f^2*x*\text{Cosh}[c/2] + (3 \\ & *I)*d*e*f^2*x^2*\text{Cosh}[c/2] + 3*f^3*x^2*\text{Cosh}[c/2] + I*d*f^3*x^3*\text{Cosh}[c/2] + d \\ & *e^3*\text{Sinh}[c/2] + (3*I)*e^2*f*\text{Sinh}[c/2] + 3*d*e^2*f*x*\text{Sinh}[c/2] + (6*I)*e*f^ \\ & 2*x*\text{Sinh}[c/2] + 3*d*e*f^2*x^2*\text{Sinh}[c/2] + (3*I)*f^3*x^2*\text{Sinh}[c/2] + d*f^3*x \\ & ^3*\text{Sinh}[c/2]) / (6*a*d^2*(\text{Cosh}[c/2] + I*\text{Sinh}[c/2])*(\text{Cosh}[c/2 + (d*x)/2] + I*S \\ & inh[c/2 + (d*x)/2])^2) + (5*d^2*e^3*\text{Sinh}[(d*x)/2] - 12*e*f^2*\text{Sinh}[(d*x)/2] \\ & + 15*d^2*e^2*f*x*\text{Sinh}[(d*x)/2] - 12*f^3*x*\text{Sinh}[(d*x)/2] + 15*d^2*e*f^2*x^2* \\ & \text{Sinh}[(d*x)/2] + 5*d^2*f^3*x^3*\text{Sinh}[(d*x)/2]) / (6*a*d^3*(\text{Cosh}[c/2] + I*\text{Sinh}[c \\ & /2])*(\text{Cosh}[c/2 + (d*x)/2] + I*\text{Sinh}[c/2 + (d*x)/2])) \end{aligned}$$


---

**Maple [B]** time = 0.18, size = 1001, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)^3*\text{sech}(d*x+c)^2/(a+I*a*\text{sinh}(d*x+c)), x)$

[Out] 
$$\begin{aligned} & -5*f^2/d^2/a*e*\ln(1+I*\exp(d*x+c))*x-5*f^2/d^3/a*e*\ln(1+I*\exp(d*x+c))*c-8*f^ \\ & 2/d^3/a*e*c*\ln(\exp(d*x+c))+5*f^2/d^3/a*e*c*\ln(\exp(d*x+c)-I)+8*f^2/d^2/a*e*c \\ & *x-8/3*f^3/d^4/a*c^3+4/3*f^3/d/a*x^3+3*f^3*\text{polylog}(3, I*\exp(d*x+c))/a/d^4+5* \\ & f^3*\text{polylog}(3, -I*\exp(d*x+c))/a/d^4+1/3*I*(6*I*f^3*x*\exp(2*d*x+2*c)+8*d^2*f^ \\ & 3*x^3*\exp(d*x+c)-3*d*f^3*x^2*\exp(3*d*x+3*c)-4*I*d^2*f^3*x^3+24*d^2*e*f^2*x^ \\ & 2*\exp(d*x+c)-6*d*e*f^2*x*\exp(3*d*x+3*c)+6*I*e*f^2*\exp(2*d*x+2*c)+6*I*e*f^2+ \\ & 24*d^2*e^2*f*x*\exp(d*x+c)-3*d*e^2*f*\exp(3*d*x+3*c)-3*d*f^3*x^2*\exp(d*x+c)-6 \\ & *f^3*x*\exp(3*d*x+3*c)+6*I*f^3*x-4*I*d^2*e^3+8*d^2*e^3*\exp(d*x+c)-6*d*e*f^2* \\ & x*\exp(d*x+c)-6*e*f^2*\exp(3*d*x+3*c)-12*I*d^2*e^2*f*x-3*d*e^2*f*\exp(d*x+c)-6 \\ & *f^3*x*\exp(d*x+c)-12*I*d^2*e*f^2*x^2-6*e*f^2*\exp(d*x+c))/(\exp(d*x+c)+I)/(\exp \\ & (d*x+c)-I)^3/d^3/a-2*f^3/d^4/a*\ln(\exp(d*x+c))+2*f^3/d^4/a*\ln(\exp(d*x+c)-I) \\ & -4*f^3/d^3/a*c^2*x+4*f/d^2/a*\ln(\exp(d*x+c))*e^2-5/2*f^3/d^2/a*\ln(1+I*\exp(d* \\ & x+c))*x^2+5/2*f^3/d^4/a*\ln(1+I*\exp(d*x+c))*c^2+4*f^2/d^3/a*e*c^2-5/2*f/d^2/ \\ & a*\ln(\exp(d*x+c)-I)*e^2-5/2*f^3/d^4/a*c^2*\ln(\exp(d*x+c)-I)+4*f^2/d/a*e*x^2+4 \\ & *f^3/d^4/a*c^2*\ln(\exp(d*x+c))-5*f^3/d^3/a*\text{polylog}(2, -I*\exp(d*x+c))*x-5*f^2/ \\ & d^3/a*e*\text{polylog}(2, -I*\exp(d*x+c))-3/2*f^3/d^2/a*\ln(1-I*\exp(d*x+c))*x^2-3*f^3 \\ & /d^3/a*\text{polylog}(2, I*\exp(d*x+c))*x+3/2*f^3/d^4/a*\ln(1-I*\exp(d*x+c))*c^2-3/2*f \\ & /d^2/a*e^2*\ln(\exp(d*x+c)+I)-3/2*f^3/d^4/a*c^2*\ln(\exp(d*x+c)+I)-3*f^2/d^3/a \\ & e*\text{polylog}(2, I*\exp(d*x+c))+3*f^2/d^3/a*e*c*\ln(\exp(d*x+c)+I)-3*f^2/d^2/a*\ln(1 \end{aligned}$$

$-I*\exp(d*x+c))*e*x-3*f^2/d^3/a*\ln(1-I*\exp(d*x+c))*c*e$

**Maxima [A]** time = 2.5021, size = 984, normalized size = 2.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*e^2*f*(24*(4*I*d*x*e^(4*d*x + 4*c) + (8*d*x*e^(3*c) + e^(3*c))*e^(3*d*x)
) + e^(d*x + c))/(12*I*a*d^2*e^(4*d*x + 4*c) + 24*a*d^2*e^(3*d*x + 3*c) + 2
4*a*d^2*e^(d*x + c) - 12*I*a*d^2) - 3*log((e^(d*x + c) + I)*e^(-c))/(a*d^2)
- 5*log(-I*(I*e^(d*x + c) + 1)*e^(-c))/(a*d^2)) + 4*e^3*(2*e^(-d*x - c)/((
6*a*e^(-d*x - c) + 6*a*e^(-3*d*x - 3*c) - 3*I*a*e^(-4*d*x - 4*c) + 3*I*a)*d
) + I/((6*a*e^(-d*x - c) + 6*a*e^(-3*d*x - 3*c) - 3*I*a*e^(-4*d*x - 4*c) +
3*I*a)*d)) + (4*I*d^2*f^3*x^3 + 12*I*d^2*e*f^2*x^2 - 6*I*f^3*x - 6*I*e*f^2
+ 3*(d*f^3*x^2*e^(3*c) + 2*e*f^2*e^(3*c) + 2*(d*e*f^2 + f^3)*x*e^(3*c))*e^(
3*d*x) + (-6*I*f^3*x*e^(2*c) - 6*I*e*f^2*e^(2*c))*e^(2*d*x) - (8*d^2*f^3*x^
3*e^c - 6*e*f^2*e^c + 3*(8*d^2*e*f^2 - d*f^3)*x^2*e^c - 6*(d*e*f^2 + f^3)*x
*e^c)*e^(d*x))/(3*I*a*d^3*e^(4*d*x + 4*c) + 6*a*d^3*e^(3*d*x + 3*c) + 6*a*d
^3*e^(d*x + c) - 3*I*a*d^3) - 5*(d*x*log(I*e^(d*x + c) + 1) + dilog(-I*e^(d
*x + c)))*e*f^2/(a*d^3) - 3*(d*x*log(-I*e^(d*x + c) + 1) + dilog(I*e^(d*x +
c)))*e*f^2/(a*d^3) - 2*f^3*x/(a*d^3) - 5/2*(d^2*x^2*log(I*e^(d*x + c) + 1)
+ 2*d*x*dilog(-I*e^(d*x + c)) - 2*polylog(3, -I*e^(d*x + c)))*f^3/(a*d^4)
- 3/2*(d^2*x^2*log(-I*e^(d*x + c) + 1) + 2*d*x*dilog(I*e^(d*x + c)) - 2*pol
ylog(3, I*e^(d*x + c)))*f^3/(a*d^4) + 2*f^3*log(e^(d*x + c) - I)/(a*d^4) +
4/3*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2)/(a*d^4)
```

**Fricas [C]** time = 2.41309, size = 3393, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (8*d^3*e^3 - 24*c*d^2*e^2*f + 12*(2*c^2 - 1)*d*e*f^2 - 4*(2*c^3 - 3*c)*f^3
+ (18*d*f^3*x + 18*d*e*f^2 - 18*(d*f^3*x + d*e*f^2)*e^(4*d*x + 4*c) + (36*I
*d*f^3*x + 36*I*d*e*f^2)*e^(3*d*x + 3*c) + (36*I*d*f^3*x + 36*I*d*e*f^2)*e^
(d*x + c))*dilog(I*e^(d*x + c)) + (30*d*f^3*x + 30*d*e*f^2 - 30*(d*f^3*x +
d*e*f^2)*e^(4*d*x + 4*c) + (60*I*d*f^3*x + 60*I*d*e*f^2)*e^(3*d*x + 3*c) +
(60*I*d*f^3*x + 60*I*d*e*f^2)*e^(d*x + c))*dilog(-I*e^(d*x + c)) + 4*(2*d^3
*f^3*x^3 + 6*d^3*e*f^2*x^2 + 6*c*d^2*e^2*f - 6*c^2*d*e*f^2 + (2*c^3 - 3*c)*
f^3 + 3*(2*d^3*e^2*f - d*f^3)*x)*e^(4*d*x + 4*c) + (-16*I*d^3*f^3*x^3 + (-4
8*I*c - 6*I)*d^2*e^2*f + (48*I*c^2 - 12*I)*d*e*f^2 + (-16*I*c^3 + 24*I*c)*f
^3 + (-48*I*d^3*e*f^2 - 6*I*d^2*f^3)*x^2 + (-48*I*d^3*e^2*f - 12*I*d^2*e*f^
2 + 12*I*d*f^3)*x)*e^(3*d*x + 3*c) - 12*(d*f^3*x + d*e*f^2)*e^(2*d*x + 2*c)
+ (-6*I*d^2*f^3*x^2 + 16*I*d^3*e^3 + (-48*I*c - 6*I)*d^2*e^2*f + (48*I*c^2
- 12*I)*d*e*f^2 + (-16*I*c^3 + 24*I*c)*f^3 + (-12*I*d^2*e*f^2 + 12*I*d*f^3
)*x)*e^(d*x + c) + (9*d^2*e^2*f - 18*c*d*e*f^2 + 9*c^2*f^3 - 9*(d^2*e^2*f -
2*c*d*e*f^2 + c^2*f^3)*e^(4*d*x + 4*c) + (18*I*d^2*e^2*f - 36*I*c*d*e*f^2
+ 18*I*c^2*f^3)*e^(3*d*x + 3*c) + (18*I*d^2*e^2*f - 36*I*c*d*e*f^2 + 18*I*c
^2*f^3)*e^(d*x + c))*log(e^(d*x + c) + I) + (15*d^2*e^2*f - 30*c*d*e*f^2 +
3*(5*c^2 - 4)*f^3 - 3*(5*d^2*e^2*f - 10*c*d*e*f^2 + (5*c^2 - 4)*f^3)*e^(4*d
*x + 4*c) + (30*I*d^2*e^2*f - 60*I*c*d*e*f^2 + (30*I*c^2 - 24*I)*f^3)*e^(3*
d*x + 3*c) + (30*I*d^2*e^2*f - 60*I*c*d*e*f^2 + (30*I*c^2 - 24*I)*f^3)*e^(d
*x + c))*log(e^(d*x + c) - I) + (15*d^2*f^3*x^2 + 30*d^2*e*f^2*x + 30*c*d*e
*f^2 - 15*c^2*f^3 - 15*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + 2*c*d*e*f^2 - c^2*f^3
)*e^(4*d*x + 4*c) + (30*I*d^2*f^3*x^2 + 60*I*d^2*e*f^2*x + 60*I*c*d*e*f^2 -
30*I*c^2*f^3)*e^(3*d*x + 3*c) + (30*I*d^2*f^3*x^2 + 60*I*d^2*e*f^2*x + 60*
I*c*d*e*f^2 - 30*I*c^2*f^3)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + (9*d^2*f^
3*x^2 + 18*d^2*e*f^2*x + 18*c*d*e*f^2 - 9*c^2*f^3 - 9*(d^2*f^3*x^2 + 2*d^2*
e*f^2*x + 2*c*d*e*f^2 - c^2*f^3)*e^(4*d*x + 4*c) + (18*I*d^2*f^3*x^2 + 36*I
*d^2*e*f^2*x + 36*I*c*d*e*f^2 - 18*I*c^2*f^3)*e^(3*d*x + 3*c) + (18*I*d^2*f
^3*x^2 + 36*I*d^2*e*f^2*x + 36*I*c*d*e*f^2 - 18*I*c^2*f^3)*e^(d*x + c))*log
(-I*e^(d*x + c) + 1) + (18*f^3*e^(4*d*x + 4*c) - 36*I*f^3*e^(3*d*x + 3*c) -
36*I*f^3*e^(d*x + c) - 18*f^3)*polylog(3, I*e^(d*x + c)) + (30*f^3*e^(4*d*
x + 4*c) - 60*I*f^3*e^(3*d*x + 3*c) - 60*I*f^3*e^(d*x + c) - 30*f^3)*polylo
g(3, -I*e^(d*x + c)))/(6*a*d^4*e^(4*d*x + 4*c) - 12*I*a*d^4*e^(3*d*x + 3*c)
- 12*I*a*d^4*e^(d*x + c) - 6*a*d^4)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*sech(d\*x+c)\*\*2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \operatorname{sech}(dx + c)^2}{ia \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*sech(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*sech(d\*x + c)^2/(I\*a\*sinh(d\*x + c) + a), x)

$$3.278 \quad \int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=325

$$-\frac{f^2 \operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{3ad^3} + \frac{f^2 \operatorname{PolyLog}\left(2, ie^{c+dx}\right)}{3ad^3} - \frac{2f^2 \operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{3ad^3} - \frac{4f(e+fx) \log\left(e^{2(c+dx)} + 1\right)}{3ad^2} - \frac{2if(e$$

[Out]  $(2*(e + f*x)^2)/(3*a*d) - (((2*I)/3)*f*(e + f*x)*\operatorname{ArcTan}[E^{(c + d*x)}])/(a*d^2) - (4*f*(e + f*x)*\operatorname{Log}[1 + E^{(2*(c + d*x)})])/(3*a*d^2) - (f^2*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x)}])/(3*a*d^3) + (f^2*\operatorname{PolyLog}[2, I*E^{(c + d*x)}])/(3*a*d^3) - (2*f^2*\operatorname{PolyLog}[2, -E^{(2*(c + d*x)})])/(3*a*d^3) - ((I/3)*f^2*\operatorname{Sech}[c + d*x])/(a*d^3) + (f*(e + f*x)*\operatorname{Sech}[c + d*x]^2)/(3*a*d^2) + ((I/3)*(e + f*x)^2*\operatorname{Sech}[c + d*x]^3)/(a*d) - (f^2*\operatorname{Tanh}[c + d*x])/(3*a*d^3) + (2*(e + f*x)^2*\operatorname{Tanh}[c + d*x])/(3*a*d) - ((I/3)*f*(e + f*x)*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(a*d^2) + ((e + f*x)^2*\operatorname{Sech}[c + d*x]^2*\operatorname{Tanh}[c + d*x])/(3*a*d)$

**Rubi [A]** time = 0.386601, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$ , Rules used = {5571, 4186, 3767, 8, 4184, 3718, 2190, 2279, 2391, 5451, 4185, 4180}

$$-\frac{f^2 \operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{3ad^3} + \frac{f^2 \operatorname{PolyLog}\left(2, ie^{c+dx}\right)}{3ad^3} - \frac{2f^2 \operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{3ad^3} - \frac{4f(e+fx) \log\left(e^{2(c+dx)} + 1\right)}{3ad^2} - \frac{2if(e$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\frac{(e + f*x)^2*\operatorname{Sech}[c + d*x]^2}{(a + I*a*\operatorname{Sinh}[c + d*x])}, x]$

[Out]  $(2*(e + f*x)^2)/(3*a*d) - (((2*I)/3)*f*(e + f*x)*\operatorname{ArcTan}[E^{(c + d*x)}])/(a*d^2) - (4*f*(e + f*x)*\operatorname{Log}[1 + E^{(2*(c + d*x)})])/(3*a*d^2) - (f^2*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x)}])/(3*a*d^3) + (f^2*\operatorname{PolyLog}[2, I*E^{(c + d*x)}])/(3*a*d^3) - (2*f^2*\operatorname{PolyLog}[2, -E^{(2*(c + d*x)})])/(3*a*d^3) - ((I/3)*f^2*\operatorname{Sech}[c + d*x])/(a*d^3) + (f*(e + f*x)*\operatorname{Sech}[c + d*x]^2)/(3*a*d^2) + ((I/3)*(e + f*x)^2*\operatorname{Sech}[c + d*x]^3)/(a*d) - (f^2*\operatorname{Tanh}[c + d*x])/(3*a*d^3) + (2*(e + f*x)^2*\operatorname{Tanh}[c + d*x])/(3*a*d) - ((I/3)*f*(e + f*x)*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(a*d^2) + ((e + f*x)^2*\operatorname{Sech}[c + d*x]^2*\operatorname{Tanh}[c + d*x])/(3*a*d)$

**Rule 5571**

$\operatorname{Int}[\frac{(e_{.}) + (f_{.})*(x_{.})^{(m_{.})}*\operatorname{Sech}[(c_{.}) + (d_{.})*(x_{.})]^{(n_{.})}}{(a_{.}) + (b_{.})*\operatorname{Sinh}[(c_{.}) + (d_{.})*(x_{.})]}, x_{\text{Symbol}}] := \operatorname{Dist}[1/a, \operatorname{Int}[(e + f*x)^m*\operatorname{Sech}[c + d*x]^{(n + 2)}, x], x] + \operatorname{Dist}[1/b, \operatorname{Int}[(e + f*x)^m*\operatorname{Sech}[c + d*x]^{(n + 1)}*T$

anh[c + d\*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]

### Rule 4186

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] :> -Simp[(b^2\*(c + d\*x)^m\*Cot[e + f\*x]\*(b\*Csc[e + f\*x])^(n - 2))/(f\*(n - 1)), x] + (Dist[(b^2\*d^2\*m\*(m - 1))/(f^2\*(n - 1)\*(n - 2)), Int[(c + d\*x)^(m - 2)\*(b\*Csc[e + f\*x])^(n - 2), x], x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(c + d\*x)^m\*(b\*Csc[e + f\*x])^(n - 2), x], x] - Simp[(b^2\*d\*m\*(c + d\*x)^(m - 1)\*(b\*Csc[e + f\*x])^(n - 2))/(f^2\*(n - 1)\*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_.)]^(n\_.), x\_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

### Rule 8

Int[a\_, x\_Symbol] :> Simp[a\*x, x] /; FreeQ[a, x]

### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_.)]^2\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] :> -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 3718

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*tan[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_.)], x\_Symbol] :> -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x)))/(1 + E^(2\*(-I\*e) + f\*fz\*x)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_.))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_.))^(m\_.))/((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_.))))^(n\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]



Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_)), x_Symbol] :>
-Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/ (f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx)}{a+ia \sinh(c+dx)} dx &= -\frac{i \int (e+fx)^2 \operatorname{sech}^3(c+dx) \tanh(c+dx) dx}{a} + \frac{\int (e+fx)^2 \operatorname{sech}^4(c+dx) dx}{a} \\
&= \frac{f(e+fx) \operatorname{sech}^2(c+dx)}{3ad^2} + \frac{i(e+fx)^2 \operatorname{sech}^3(c+dx)}{3ad} + \frac{(e+fx)^2 \operatorname{sech}^2(c+dx) \tanh(c+dx)}{3ad} \\
&= -\frac{if^2 \operatorname{sech}(c+dx)}{3ad^3} + \frac{f(e+fx) \operatorname{sech}^2(c+dx)}{3ad^2} + \frac{i(e+fx)^2 \operatorname{sech}^3(c+dx)}{3ad} + \frac{2(e+fx)^2 \tanh(c+dx)}{3ad} \\
&= \frac{2(e+fx)^2}{3ad} - \frac{2if(e+fx) \tan^{-1}(e^{c+dx})}{3ad^2} - \frac{if^2 \operatorname{sech}(c+dx)}{3ad^3} + \frac{f(e+fx) \operatorname{sech}^2(c+dx)}{3ad^2} + \frac{i(e+fx)^2 \operatorname{sech}^3(c+dx)}{3ad} \\
&= \frac{2(e+fx)^2}{3ad} - \frac{2if(e+fx) \tan^{-1}(e^{c+dx})}{3ad^2} - \frac{4f(e+fx) \log(1+e^{2(c+dx)})}{3ad^2} - \frac{if^2 \operatorname{sech}(c+dx)}{3ad^3} \\
&= \frac{2(e+fx)^2}{3ad} - \frac{2if(e+fx) \tan^{-1}(e^{c+dx})}{3ad^2} - \frac{4f(e+fx) \log(1+e^{2(c+dx)})}{3ad^2} - \frac{f^2 \operatorname{Li}_2(-ie^{c+dx})}{3ad^3} \\
&= \frac{2(e+fx)^2}{3ad} - \frac{2if(e+fx) \tan^{-1}(e^{c+dx})}{3ad^2} - \frac{4f(e+fx) \log(1+e^{2(c+dx)})}{3ad^2} - \frac{f^2 \operatorname{Li}_2(-ie^{c+dx})}{3ad^3}
\end{aligned}$$

**Mathematica [A]** time = 8.26164, size = 564, normalized size = 1.74

$$12f^2 \operatorname{PolyLog}\left(2, -ie^{-c-dx}\right) + 20f^2 \operatorname{PolyLog}\left(2, ie^{-c-dx}\right) + \frac{d^2 e^2 \sinh(2(c+dx)) - 2id^2 e^2 \cosh(c+dx) + 4id^2 e^2 \cosh(c+2dx) + 2d^2 e f x \sinh(2(c+dx))}{3ad^3}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Sech[c + d\*x]^2)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (((10\*I)\*d\*(e + f\*x)\*(d\*(e + f\*x) + 2\*(1 + I\*E^c))\*f\*Log[1 - I\*E^(-c - d\*x)]) / (-I + E^c) - ((6\*I)\*d\*(e + f\*x)\*(d\*(e + f\*x) + 2\*(1 - I\*E^c))\*f\*Log[1 + I\*E^(-c - d\*x)]) / (I + E^c) + 12\*f^2\*PolyLog[2, (-I)\*E^(-c - d\*x)] + 20\*f^2\*PolyLog[2, I\*E^(-c - d\*x)] + ((-2\*I)\*f^2\*Cosh[c] + 2\*d\*f\*(e + f\*x)\*Cosh[d\*x] - (2\*I)\*d^2\*e^2\*Cosh[c + d\*x] + (4\*I)\*f^2\*Cosh[c + d\*x] - (4\*I)\*d^2\*e\*f\*x\*Cosh[c + d\*x] - (2\*I)\*d^2\*f^2\*x^2\*Cosh[c + d\*x] + 2\*d\*e\*f\*Cosh[2\*c + d\*x] + 2\*d\*f^2\*x\*Cosh[2\*c + d\*x] + (4\*I)\*d^2\*e^2\*Cosh[c + 2\*d\*x] - (2\*I)\*f^2\*Cosh[c + 2\*d\*x] + (8\*I)\*d^2\*e\*f\*x\*Cosh[c + 2\*d\*x] + (4\*I)\*d^2\*f^2\*x^2\*Cosh[c + 2\*d\*x] + 8\*d^2\*e^2\*Sinh[d\*x] - 2\*f^2\*Sinh[d\*x] + 16\*d^2\*e\*f\*x\*Sinh[d\*x] + 8\*d^2\*f^2\*x^2\*Sinh[d\*x] + d^2\*e^2\*Sinh[2\*(c + d\*x)] - 2\*f^2\*Sinh[2\*(c + d\*x)] + 2\*d^2\*e\*f\*x\*Sinh[2\*(c + d\*x)] + d^2\*f^2\*x^2\*Sinh[2\*(c + d\*x)] + 2\*f^2\*Sinh[2\*c + d\*x]) / ((Cosh[c/2] - I\*Sinh[c/2])\*(Cosh[c/2] + I\*Sinh[c/2])\*(Cosh[(c + d\*x)/2] - I\*Sinh[(c + d\*x)/2])\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]))

2])^3)/(12\*a\*d^3)

**Maple [A]** time = 0.154, size = 509, normalized size = 1.6

$$\frac{\frac{2i}{3} \left( -2id^2f^2x^2 + 4d^2x^2f^2e^{dx+c} - df^2xe^{3dx+3c} - 4id^2efx + 8d^2efxe^{dx+c} - defe^{3dx+3c} - 2id^2e^2 + if^2e^{2dx+2c} + 4d^2e^2e^{dx+c} \right)}{(e^{dx+c} + i)(e^{dx+c} - i)^3 d^3 a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*sech(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)), x)

[Out]  $\frac{2}{3}I \cdot (-2I \cdot d^2 \cdot f^2 \cdot x^2 + 4d^2 \cdot x^2 \cdot f^2 \cdot \exp(dx+c) - d \cdot f^2 \cdot x \cdot \exp(3d \cdot x + 3c) - 4I \cdot d^2 \cdot e \cdot f \cdot x + 8d^2 \cdot e \cdot f \cdot x \cdot \exp(dx+c) - d \cdot e \cdot f \cdot \exp(3d \cdot x + 3c) - 2I \cdot d^2 \cdot e^2 + I \cdot f^2 \cdot \exp(2dx+2c) + 4d^2 \cdot e^2 \cdot \exp(dx+c)) / ((\exp(dx+c) + I) \cdot (\exp(dx+c) - I)^3 / d^3 / a - 5/3 \cdot f / d^2 / a \cdot \ln(\exp(dx+c) - I) \cdot e^{-f/d^2/a} \cdot \ln(\exp(dx+c) + I) + 8/3 \cdot f / d^2 / a \cdot \ln(\exp(dx+c)) \cdot e^{5/3 \cdot f^2/d^3/a} \cdot \ln(\exp(dx+c) - I) - 8/3 \cdot f^2/d^3/a \cdot \ln(\exp(dx+c)) + f^2/d^3/a \cdot \ln(\exp(dx+c) + I) + 4/3 \cdot f^2 \cdot x^2/a/d + 8/3 \cdot f^2/d^2/a \cdot x + 4/3 \cdot f^2/d^3/a \cdot c^2 - 5/3 \cdot f^2/d^2/a \cdot \ln(1 + I \cdot \exp(dx+c)) \cdot x - 5/3 \cdot f^2/d^3/a \cdot \ln(1 + I \cdot \exp(dx+c)) \cdot c - 5/3 \cdot f^2 \cdot \text{polylog}(2, -I \cdot \exp(dx+c)) / a/d^3 - f^2/d^2/a \cdot \ln(1 - I \cdot \exp(dx+c)) \cdot x - f^2/d^3/a \cdot \ln(1 - I \cdot \exp(dx+c)) \cdot c - f^2 \cdot \text{polylog}(2, I \cdot \exp(dx+c)) / a/d^3)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$4f^2 \left( \frac{2id^2x^2 + (dxe^{(3c)} + e^{(3c)})e^{(3dx)} - (4d^2x^2e^c - dxe^c - e^c)e^{(dx)} - ie^{(2dx+2c)} - i}{6iad^3e^{(4dx+4c)} + 12ad^3e^{(3dx+3c)} + 12ad^3e^{(dx+c)} - 6iad^3} + i \int \frac{x}{4(ade^{(dx+c)} + iad)} dx - 5i \int \frac{1}{12} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sech(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)), x, algorithm="maxima")

[Out]  $4f^2 \cdot ((2I \cdot d^2 \cdot x^2 + (d \cdot x \cdot e^{(3c)} + e^{(3c)}) \cdot e^{(3d \cdot x)} - (4d^2 \cdot x^2 \cdot e^c - d \cdot x \cdot e^c - e^c) \cdot e^{(dx)} - i \cdot e^{(2dx+2c)} - i) / (6I \cdot a \cdot d^3 \cdot e^{(4d \cdot x + 4c)} + 12 \cdot a \cdot d^3 \cdot e^{(3d \cdot x + 3c)} + 12 \cdot a \cdot d^3 \cdot e^{(d \cdot x + c)} - 6I \cdot a \cdot d^3) + I \cdot \text{integrate}(1/4 \cdot x / (a \cdot d \cdot e^{(d \cdot x + c)} + I \cdot a \cdot d), x) - 5I \cdot \text{integrate}(1/12 \cdot x / (a \cdot d \cdot e^{(d \cdot x + c)} - I \cdot a \cdot d), x)) + 1/3 \cdot e \cdot f \cdot (24 \cdot (4I \cdot d \cdot x \cdot e^{(4d \cdot x + 4c)} + (8d \cdot x \cdot e^{(3c)} + e^{(3c)}) \cdot e^{(3d \cdot x)} + e^{(d \cdot x + c)}) / (12I \cdot a \cdot d^2 \cdot e^{(4d \cdot x + 4c)} + 24 \cdot a \cdot d^2 \cdot e^{(3d \cdot x + 3c)} + 24 \cdot a \cdot d^2 \cdot e^{(d \cdot x + c)} - 12I \cdot a \cdot d^2) + 5/3 \cdot f^2/d^3/a \cdot \ln(\exp(dx+c)) - 5/3 \cdot f^2/d^3/a \cdot \ln(\exp(dx+c)) \cdot c - 5/3 \cdot f^2 \cdot \text{polylog}(2, -I \cdot \exp(dx+c)) / a/d^3 - f^2/d^2/a \cdot \ln(1 - I \cdot \exp(dx+c)) \cdot x - f^2/d^3/a \cdot \ln(1 - I \cdot \exp(dx+c)) \cdot c - f^2 \cdot \text{polylog}(2, I \cdot \exp(dx+c)) / a/d^3)$

$$(3*d*x + 3*c) + 24*a*d^2*e^(d*x + c) - 12*I*a*d^2) - 3*log((e^(d*x + c) + I)*e^(-c))/(a*d^2) - 5*log(-I*(I*e^(d*x + c) + 1)*e^(-c))/(a*d^2)) + 4*e^2*(2*e^(-d*x - c))/((6*a*e^(-d*x - c) + 6*a*e^(-3*d*x - 3*c) - 3*I*a*e^(-4*d*x - 4*c) + 3*I*a)*d) + I/((6*a*e^(-d*x - c) + 6*a*e^(-3*d*x - 3*c) - 3*I*a*e^(-4*d*x - 4*c) + 3*I*a)*d))$$

**Fricas [B]** time = 2.2461, size = 1760, normalized size = 5.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (4*d^2*e^2 - 8*c*d*e*f + 2*(2*c^2 - 1)*f^2 - 2*f^2*e^(2*d*x + 2*c) - (3*f^2*e^(4*d*x + 4*c) - 6*I*f^2*e^(3*d*x + 3*c) - 6*I*f^2*e^(d*x + c) - 3*f^2)*dilog(I*e^(d*x + c)) - (5*f^2*e^(4*d*x + 4*c) - 10*I*f^2*e^(3*d*x + 3*c) - 10*I*f^2*e^(d*x + c) - 5*f^2)*dilog(-I*e^(d*x + c)) + 4*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(4*d*x + 4*c) + (-8*I*d^2*f^2*x^2 + (-16*I*c - 2*I)*d*e*f + (8*I*c^2 - 2*I)*f^2 + (-16*I*d^2*e*f - 2*I*d*f^2)*x)*e^(3*d*x + 3*c) + (8*I*d^2*e^2 + (-16*I*c - 2*I)*d*e*f - 2*I*d*f^2*x + (8*I*c^2 - 2*I)*f^2)*e^(d*x + c) + (3*d*e*f - 3*c*f^2 - 3*(d*e*f - c*f^2)*e^(4*d*x + 4*c) + (6*I*d*e*f - 6*I*c*f^2)*e^(3*d*x + 3*c) + (6*I*d*e*f - 6*I*c*f^2)*e^(d*x + c))*log(e^(d*x + c) + I) + (5*d*e*f - 5*c*f^2 - 5*(d*e*f - c*f^2)*e^(4*d*x + 4*c) + (10*I*d*e*f - 10*I*c*f^2)*e^(3*d*x + 3*c) + (10*I*d*e*f - 10*I*c*f^2)*e^(d*x + c))*log(e^(d*x + c) - I) + (5*d*f^2*x + 5*c*f^2 - 5*(d*f^2*x + c*f^2)*e^(4*d*x + 4*c) + (10*I*d*f^2*x + 10*I*c*f^2)*e^(3*d*x + 3*c) + (10*I*d*f^2*x + 10*I*c*f^2)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + (3*d*f^2*x + 3*c*f^2 - 3*(d*f^2*x + c*f^2)*e^(4*d*x + 4*c) + (6*I*d*f^2*x + 6*I*c*f^2)*e^(3*d*x + 3*c) + (6*I*d*f^2*x + 6*I*c*f^2)*e^(d*x + c))*log(-I*e^(d*x + c) + 1))/(3*a*d^3*e^(4*d*x + 4*c) - 6*I*a*d^3*e^(3*d*x + 3*c) - 6*I*a*d^3*e^(d*x + c) - 3*a*d^3)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*sech(d\*x+c)\*\*2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \operatorname{sech}(dx + c)^2}{ia \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sech(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*sech(d\*x + c)^2/(I\*a\*sinh(d\*x + c) + a), x)

$$3.279 \quad \int \frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=158

$$\frac{f\operatorname{sech}^2(c+dx)}{6ad^2} - \frac{if \tan^{-1}(\sinh(c+dx))}{6ad^2} - \frac{2f \log(\cosh(c+dx))}{3ad^2} - \frac{if \tanh(c+dx)\operatorname{sech}(c+dx)}{6ad^2} + \frac{2(e+fx) \tanh(c+dx)}{3ad}$$

[Out]  $((-I/6)*f*\operatorname{ArcTan}[\operatorname{Sinh}[c+d*x]])/(a*d^2) - (2*f*\operatorname{Log}[\operatorname{Cosh}[c+d*x]])/(3*a*d^2) + (f*\operatorname{Sech}[c+d*x]^2)/(6*a*d^2) + ((I/3)*(e+f*x)*\operatorname{Sech}[c+d*x]^3)/(a*d) + (2*(e+f*x)*\operatorname{Tanh}[c+d*x])/(3*a*d) - ((I/6)*f*\operatorname{Sech}[c+d*x]*\operatorname{Tanh}[c+d*x])/(a*d^2) + ((e+f*x)*\operatorname{Sech}[c+d*x]^2*\operatorname{Tanh}[c+d*x])/(3*a*d)$

**Rubi [A]** time = 0.157548, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$ , Rules used = {5571, 4185, 4184, 3475, 5451, 3768, 3770}

$$\frac{f\operatorname{sech}^2(c+dx)}{6ad^2} - \frac{if \tan^{-1}(\sinh(c+dx))}{6ad^2} - \frac{2f \log(\cosh(c+dx))}{3ad^2} - \frac{if \tanh(c+dx)\operatorname{sech}(c+dx)}{6ad^2} + \frac{2(e+fx) \tanh(c+dx)}{3ad}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e+f*x)*\operatorname{Sech}[c+d*x]^2/(a+I*a*\operatorname{Sinh}[c+d*x]),x]$

[Out]  $((-I/6)*f*\operatorname{ArcTan}[\operatorname{Sinh}[c+d*x]])/(a*d^2) - (2*f*\operatorname{Log}[\operatorname{Cosh}[c+d*x]])/(3*a*d^2) + (f*\operatorname{Sech}[c+d*x]^2)/(6*a*d^2) + ((I/3)*(e+f*x)*\operatorname{Sech}[c+d*x]^3)/(a*d) + (2*(e+f*x)*\operatorname{Tanh}[c+d*x])/(3*a*d) - ((I/6)*f*\operatorname{Sech}[c+d*x]*\operatorname{Tanh}[c+d*x])/(a*d^2) + ((e+f*x)*\operatorname{Sech}[c+d*x]^2*\operatorname{Tanh}[c+d*x])/(3*a*d)$

### Rule 5571

$\operatorname{Int}[(e_+ + (f_+)(x_+))^{m_+} \operatorname{Sech}[(c_+ + (d_+)(x_+))^{n_+}] / ((a_+ + (b_+)(x_+)) \operatorname{Sinh}[(c_+ + (d_+)(x_+))]), x\_Symbol] \rightarrow \operatorname{Dist}[1/a, \operatorname{Int}[(e+f*x)^m \operatorname{Sech}[c+d*x]^{n+2}], x] + \operatorname{Dist}[1/b, \operatorname{Int}[(e+f*x)^m \operatorname{Sech}[c+d*x]^{n+1} \operatorname{Tanh}[c+d*x]], x] /;$  FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]

### Rule 4185

$\operatorname{Int}[(\operatorname{csc}[(e_+ + (f_+)(x_+)]*(b_+))^{n_+} * ((c_+ + (d_+)(x_+))), x\_Symbol] \rightarrow -\operatorname{Simp}[(b^2*(c+d*x)*\operatorname{Cot}[e+f*x]*(b*\operatorname{Csc}[e+f*x])^{n-2})/(f*(n-1)), x] + (\operatorname{Dist}[(b^2*(n-2))/(n-1), \operatorname{Int}[(c+d*x)*(b*\operatorname{Csc}[e+f*x])^{n-2}], x]$

, x] - Simp[(b^2\*d\*(b\*Csc[e + f\*x])^(n - 2))/(f^2\*(n - 1)\*(n - 2)), x] /;  
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]

#### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp  
p[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Co  
t[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d  
\*x], x]]/d, x] /; FreeQ[{c, d}, x]

#### Rule 5451

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sech[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*Tanh[(a\_.) +  
(b\_.)\*(x\_)]^(p\_.), x\_Symbol] :> -Simp[((c + d\*x)^m\*Sech[a + b\*x]^n)/(b\*n),  
x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Sech[a + b\*x]^n, x], x] /; Fre  
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

#### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x  
]\*(b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), I  
nt[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&  
IntegerQ[2\*n]

#### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> -Simp[ArcTanh[Cos[c + d\*x]]/d, x]  
/; FreeQ[{c, d}, x]

#### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)} dx &= -\frac{i\int(e+fx)\operatorname{sech}^3(c+dx)\tanh(c+dx)dx}{a} + \frac{\int(e+fx)\operatorname{sech}^4(c+dx)dx}{a} \\
&= \frac{f\operatorname{sech}^2(c+dx)}{6ad^2} + \frac{i(e+fx)\operatorname{sech}^3(c+dx)}{3ad} + \frac{(e+fx)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{3ad} + \frac{2\int(e+fx)\operatorname{sech}^4(c+dx)dx}{6ad^2} \\
&= \frac{f\operatorname{sech}^2(c+dx)}{6ad^2} + \frac{i(e+fx)\operatorname{sech}^3(c+dx)}{3ad} + \frac{2(e+fx)\tanh(c+dx)}{3ad} - \frac{if\operatorname{sech}(c+dx)\tanh(c+dx)}{6ad^2} \\
&= -\frac{if\tan^{-1}(\sinh(c+dx))}{6ad^2} - \frac{2f\log(\cosh(c+dx))}{3ad^2} + \frac{f\operatorname{sech}^2(c+dx)}{6ad^2} + \frac{i(e+fx)\operatorname{sech}^3(c+dx)}{3ad}
\end{aligned}$$

**Mathematica [A]** time = 1.13355, size = 194, normalized size = 1.23

$$\frac{2d(e+fx)(\cosh(2(c+dx)) - 2i\sinh(c+dx)) + \cosh(c+dx)\left(-i\sinh(c+dx)\left(2f\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right) - 4if\log(\cosh(c+dx))\right)}{6ad^2(\sinh(c+dx) - i)\left(\cosh\left(\frac{1}{2}(c+dx)\right) - i\sinh\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Sech[c + d\*x]^2)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (2\*d\*(e + f\*x)\*(Cosh[2\*(c + d\*x)] - (2\*I)\*Sinh[c + d\*x]) + Cosh[c + d\*x]\*(-(d\*e) - I\*f + c\*f - 2\*f\*ArcTan[Tanh[(c + d\*x)/2]] + (4\*I)\*f\*Log[Cosh[c + d\*x]]) - I\*(d\*e - c\*f + 2\*f\*ArcTan[Tanh[(c + d\*x)/2]] - (4\*I)\*f\*Log[Cosh[c + d\*x]])\*Sinh[c + d\*x))/(6\*a\*d^2\*(Cosh[(c + d\*x)/2] - I\*Sinh[(c + d\*x)/2])\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])\*(-I + Sinh[c + d\*x]))

**Maple [A]** time = 0.164, size = 143, normalized size = 0.9

$$\frac{4fx}{3da} + \frac{4cf}{3ad^2} - \frac{\frac{i}{3}\left(-8dfxe^{dx+c} + fe^{3dx+3c} - 8dee^{dx+c} + fe^{dx+c} + 4idfx + 4ide\right)}{\left(e^{dx+c} + i\right)\left(e^{dx+c} - i\right)^3 d^2 a} - \frac{5f\ln\left(e^{dx+c} - i\right)}{6ad^2} - \frac{f\ln\left(e^{dx+c} + i\right)}{2ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)\*sech(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] 4/3\*f\*x/a/d+4/3\*f/a/d^2\*c-1/3\*I\*(-8\*d\*f\*x\*exp(d\*x+c)+f\*exp(3\*d\*x+3\*c)-8\*d\*e\*exp(d\*x+c)+f\*exp(d\*x+c)+4\*I\*d\*f\*x+4\*I\*d\*e)/(exp(d\*x+c)+I)/(exp(d\*x+c)-I)^3



$$/d^2/a-5/6*f/a/d^2*\ln(\exp(d*x+c)-I)-1/2*f/a/d^2*\ln(\exp(d*x+c)+I)$$

**Maxima [A]** time = 1.33948, size = 339, normalized size = 2.15

$$\frac{1}{6} f \left( \frac{24 (4i dx e^{4dx+4c}) + (8 dx e^{3c} + e^{3c}) e^{3dx} + e^{dx+c}}{12i ad^2 e^{4dx+4c} + 24 ad^2 e^{3dx+3c} + 24 ad^2 e^{dx+c} - 12i ad^2} - \frac{3 \log((e^{dx+c} + i) e^{-c})}{ad^2} - \frac{5 \log(-i (i e^{dx+c} + 1) e^{-c})}{ad^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 1/6\*f\*(24\*(4\*I\*d\*x\*e^(4\*d\*x + 4\*c) + (8\*d\*x\*e^(3\*c) + e^(3\*c))\*e^(3\*d\*x) + e^(d\*x + c))/(12\*I\*a\*d^2\*e^(4\*d\*x + 4\*c) + 24\*a\*d^2\*e^(3\*d\*x + 3\*c) + 24\*a\*d^2\*e^(d\*x + c) - 12\*I\*a\*d^2) - 3\*log((e^(d\*x + c) + I)\*e^(-c))/(a\*d^2) - 5\*log(-I\*(I\*e^(d\*x + c) + 1)\*e^(-c))/(a\*d^2) + 4\*e\*(2\*e^(-d\*x - c)/((6\*a\*e^(-d\*x - c) + 6\*a\*e^(-3\*d\*x - 3\*c) - 3\*I\*a\*e^(-4\*d\*x - 4\*c) + 3\*I\*a)\*d) + I/((6\*a\*e^(-d\*x - c) + 6\*a\*e^(-3\*d\*x - 3\*c) - 3\*I\*a\*e^(-4\*d\*x - 4\*c) + 3\*I\*a)\*d))

**Fricas [A]** time = 2.29769, size = 527, normalized size = 3.34

$$\frac{8 d f x e^{4 d x+4 c} + 8 d e + (-16 i d f x - 2 i f) e^{3 d x+3 c} + (16 i d e - 2 i f) e^{d x+c} - (3 f e^{4 d x+4 c} - 6 i f e^{3 d x+3 c} - 6 i f e^{d x+c} - 3 f e^{d x+c})}{6 a d^2 e^{4 d x+4 c} - 12 i a d^2 e^{3 d x+3 c} - 12 i a d^2 e^{d x+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] (8\*d\*f\*x\*e^(4\*d\*x + 4\*c) + 8\*d\*e + (-16\*I\*d\*f\*x - 2\*I\*f)\*e^(3\*d\*x + 3\*c) + (16\*I\*d\*e - 2\*I\*f)\*e^(d\*x + c) - (3\*f\*e^(4\*d\*x + 4\*c) - 6\*I\*f\*e^(3\*d\*x + 3\*c) - 6\*I\*f\*e^(d\*x + c) - 3\*f)\*log(e^(d\*x + c) + I) - (5\*f\*e^(4\*d\*x + 4\*c) - 10\*I\*f\*e^(3\*d\*x + 3\*c) - 10\*I\*f\*e^(d\*x + c) - 5\*f)\*log(e^(d\*x + c) - I))/(6\*a\*d^2\*e^(4\*d\*x + 4\*c) - 12\*I\*a\*d^2\*e^(3\*d\*x + 3\*c) - 12\*I\*a\*d^2\*e^(d\*x + c) - 6\*a\*d^2)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(d\*x+c)\*\*2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [A]** time = 1.21864, size = 352, normalized size = 2.23

$$8dfxe^{4dx+4c} - 16idfxe^{(3dx+3c)} - 3fe^{(4dx+4c)} \log(e^{(dx+c)} + i) + 6ife^{(3dx+3c)} \log(e^{(dx+c)} + i) + 6ife^{(dx+c)} \log(e^{(dx+c)} + i)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] (8\*d\*f\*x\*e^(4\*d\*x + 4\*c) - 16\*I\*d\*f\*x\*e^(3\*d\*x + 3\*c) - 3\*f\*e^(4\*d\*x + 4\*c) \*log(e^(d\*x + c) + I) + 6\*I\*f\*e^(3\*d\*x + 3\*c)\*log(e^(d\*x + c) + I) + 6\*I\*f\*e^(d\*x + c)\*log(e^(d\*x + c) + I) - 5\*f\*e^(4\*d\*x + 4\*c)\*log(e^(d\*x + c) - I) + 10\*I\*f\*e^(3\*d\*x + 3\*c)\*log(e^(d\*x + c) - I) + 10\*I\*f\*e^(d\*x + c)\*log(e^(d\*x + c) - I) + 8\*d\*e - 2\*I\*f\*e^(3\*d\*x + 3\*c) + 16\*I\*d\*e^(d\*x + c + 1) - 2\*I\*f\*e^(d\*x + c) + 3\*f\*log(e^(d\*x + c) + I) + 5\*f\*log(e^(d\*x + c) - I))/(6\*a\*d^2\*e^(4\*d\*x + 4\*c) - 12\*I\*a\*d^2\*e^(3\*d\*x + 3\*c) - 12\*I\*a\*d^2\*e^(d\*x + c) - 6\*a\*d^2)

$$3.280 \quad \int \frac{\operatorname{sech}^2(c+dx)}{a+ia \sinh(c+dx)} dx$$

Optimal. Leaf size=47

$$\frac{2 \tanh(c+dx)}{3ad} + \frac{\operatorname{isech}(c+dx)}{3d(a+ia \sinh(c+dx))}$$

[Out] ((I/3)\*Sech[c + d\*x])/(d\*(a + I\*a\*Sinh[c + d\*x])) + (2\*Tanh[c + d\*x])/(3\*a\*d)

**Rubi [A]** time = 0.054996, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2672, 3767, 8}

$$\frac{2 \tanh(c+dx)}{3ad} + \frac{\operatorname{isech}(c+dx)}{3d(a+ia \sinh(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d\*x]^2/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] ((I/3)\*Sech[c + d\*x])/(d\*(a + I\*a\*Sinh[c + d\*x])) + (2\*Tanh[c + d\*x])/(3\*a\*d)

### Rule 2672

Int[(cos[(e\_.) + (f\_.)\*(x\_)]\*(g\_.))^(p\_)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(m\_), x\_Symbol] :> Simp[(b\*(g\*Cos[e + f\*x])^(p + 1)\*(a + b\*Sin[e + f\*x])^m)/(a\*f\*g\*Simplify[2\*m + p + 1]), x] + Dist[Simplify[m + p + 1]/(a\*Simplify[2\*m + p + 1]), Int[(g\*Cos[e + f\*x])^p\*(a + b\*Sin[e + f\*x])^(m + 1), x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simplify[m + p + 1], 0] && NeQ[2\*m + p + 1, 0] && !IGtQ[m, 0]

### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}^2(c+dx)}{a+ia\sinh(c+dx)} dx &= \frac{i\operatorname{sech}(c+dx)}{3d(a+ia\sinh(c+dx))} + \frac{2 \int \operatorname{sech}^2(c+dx) dx}{3a} \\ &= \frac{i\operatorname{sech}(c+dx)}{3d(a+ia\sinh(c+dx))} + \frac{(2i) \operatorname{Subst}\left(\int 1 dx, x, -i \tanh(c+dx)\right)}{3ad} \\ &= \frac{i\operatorname{sech}(c+dx)}{3d(a+ia\sinh(c+dx))} + \frac{2 \tanh(c+dx)}{3ad} \end{aligned}$$

**Mathematica [A]** time = 0.0461272, size = 47, normalized size = 1.

$$\frac{\operatorname{sech}(c+dx)(\cosh(2(c+dx)) - 2i \sinh(c+dx))}{3ad(\sinh(c+dx) - i)}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d\*x]^2/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (Sech[c + d\*x]\*(Cosh[2\*(c + d\*x)] - (2\*I)\*Sinh[c + d\*x]))/(3\*a\*d\*(-I + Sinh[c + d\*x]))

**Maple [A]** time = 0.049, size = 75, normalized size = 1.6

$$2 \frac{1}{da} \left( \frac{1}{4} (\tanh(1/2 dx + c/2) + i)^{-1} - \frac{1}{3} (-i + \tanh(1/2 dx + c/2))^{-3} + \frac{i/2}{(-i + \tanh(1/2 dx + c/2))^2} + \frac{3}{4} (-i + \tanh(1/2 dx + c/2)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] 2/d/a\*(1/4/(tanh(1/2\*d\*x+1/2\*c)+I)-1/3/(-I+tanh(1/2\*d\*x+1/2\*c))^3+1/2\*I/(-I+tanh(1/2\*d\*x+1/2\*c))^2+3/4/(-I+tanh(1/2\*d\*x+1/2\*c)))

**Maxima [B]** time = 1.18232, size = 140, normalized size = 2.98

$$\frac{8e^{(-dx-c)}}{(6ae^{(-dx-c)} + 6ae^{(-3dx-3c)} - 3iae^{(-4dx-4c)} + 3ia)d} + \frac{4i}{(6ae^{(-dx-c)} + 6ae^{(-3dx-3c)} - 3iae^{(-4dx-4c)} + 3ia)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $8e^{(-d*x - c)/((6*a*e^{(-d*x - c)} + 6*a*e^{(-3*d*x - 3*c)} - 3*I*a*e^{(-4*d*x - 4*c)} + 3*I*a)*d)} + 4*I/((6*a*e^{(-d*x - c)} + 6*a*e^{(-3*d*x - 3*c)} - 3*I*a*e^{(-4*d*x - 4*c)} + 3*I*a)*d)$

**Fricas [A]** time = 2.07062, size = 144, normalized size = 3.06

$$-\frac{4(-2ie^{(dx+c)} - 1)}{3ade^{(4dx+4c)} - 6iade^{(3dx+3c)} - 6iade^{(dx+c)} - 3ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $-4*(-2*I*e^{(d*x + c)} - 1)/(3*a*d*e^{(4*d*x + 4*c)} - 6*I*a*d*e^{(3*d*x + 3*c)} - 6*I*a*d*e^{(d*x + c)} - 3*a*d)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{sech}^2(c+dx)}{i \sinh(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)\*\*2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Integral(sech(c + d\*x)\*\*2/(I\*sinh(c + d\*x) + 1), x)/a

**Giac [A]** time = 1.15949, size = 81, normalized size = 1.72

$$\frac{1}{2ad(i e^{(dx+c)} - 1)} - \frac{-3i e^{(2dx+2c)} - 12 e^{(dx+c)} + 5i}{6ad(e^{(dx+c)} - i)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] 1/2/(a\*d\*(I\*e^(d\*x + c) - 1)) - 1/6\*(-3\*I\*e^(2\*d\*x + 2\*c) - 12\*e^(d\*x + c) + 5\*I)/(a\*d\*(e^(d\*x + c) - I)^3)

$$3.281 \quad \int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Optimal. Leaf size=33

$$\text{Unintegrable} \left( \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[Sech[c + d\*x]^2/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi [A] time = 0.0780557, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Sech[c + d\*x]^2/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Sech[c + d\*x]^2/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx = \int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Mathematica [A] time = 122.852, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sech[c + d\*x]^2/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Integrate[Sech[c + d\*x]^2/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

**Maple [A]** time = 0.248, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(dx + c))^2}{(fx + e)(a + ia \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d\*x+c)^2/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] int(sech(d\*x+c)^2/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-4if \int \frac{1}{8i adf^2x^2 + 16i adefx + 8i ade^2 + 8(adf^2x^2e^c + 2 adefxe^c + ade^2e^c)e^{(dx)}} dx - \frac{1}{12 ad^3 f^3 x^3 + 36 ad^3 ef^2 x^2 + 36 ad^3 e^2 f x + 12 ad^3 e^3 + 36 ad^3 e^2 f x + 12 ad^3 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)^2/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -4\*I\*f\*integrate(1/(8\*I\*a\*d\*f^2\*x^2 + 16\*I\*a\*d\*e\*f\*x + 8\*I\*a\*d\*e^2 + 8\*(a\*d\*f^2\*x^2\*e^c + 2\*a\*d\*e\*f\*x\*e^c + a\*d\*e^2\*e^c)\*e^(d\*x)), x) - 4\*(4\*d^2\*f^2\*x^2 + 8\*d^2\*e\*f\*x + 4\*d^2\*e^2 - 2\*f^2\*e^(2\*d\*x + 2\*c) - 2\*f^2 + (I\*d\*f^2\*x\*e^(3\*c) + (I\*d\*e\*f - 2\*I\*f^2)\*e^(3\*c))\*e^(3\*d\*x) + (8\*I\*d^2\*f^2\*x^2\*e^c + (16\*I\*d^2\*e\*f + I\*d\*f^2)\*x\*e^c + (8\*I\*d^2\*e^2 + I\*d\*e\*f - 2\*I\*f^2)\*e^c)\*e^(d\*x))/(12\*a\*d^3\*f^3\*x^3 + 36\*a\*d^3\*e\*f^2\*x^2 + 36\*a\*d^3\*e^2\*f\*x + 12\*a\*d^3\*e^3 - 12\*(a\*d^3\*f^3\*x^3\*e^(4\*c) + 3\*a\*d^3\*e\*f^2\*x^2\*e^(4\*c) + 3\*a\*d^3\*e^2\*f\*x\*e^(4\*c) + a\*d^3\*e^3\*e^(4\*c))\*e^(4\*d\*x) - (-24\*I\*a\*d^3\*f^3\*x^3\*e^(3\*c) - 72\*I\*a\*d^3\*e\*f^2\*x^2\*e^(3\*c) - 72\*I\*a\*d^3\*e^2\*f\*x\*e^(3\*c) - 24\*I\*a\*d^3\*e^3\*e^(3\*c))\*e^(3\*d\*x) - (-24\*I\*a\*d^3\*f^3\*x^3\*e^c - 72\*I\*a\*d^3\*e\*f^2\*x^2\*e^c - 72\*I\*a\*d^3\*e^2\*f\*x\*e^c - 24\*I\*a\*d^3\*e^3\*e^c)\*e^(d\*x)) - 4\*integrate((5\*d^2\*f^3\*x^2 + 10\*d^2\*e\*f^2\*x + 5\*d^2\*e^2\*f - 12\*f^3)/(24\*a\*d^3\*f^4\*x^4 + 96\*a\*d^3\*e\*f^3\*x^3 + 144\*a\*d^3\*e^2\*f^2\*x^2 + 96\*a\*d^3\*e^3\*f\*x + 24\*a\*d^3\*e^4 + (24\*I\*a\*d^3\*f^4\*x^4\*e^c + 96\*I\*a\*d^3\*e\*f^3\*x^3\*e^c + 144\*I\*a\*d^3\*e^2\*f^2\*x^2\*e^c + 96\*I\*a\*d^3\*e^3\*f\*x\*e^c + 24\*I\*a\*d^3\*e^4\*e^c)\*e^(d\*x)), x)



---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)^2/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-(4*d^2*f^2*x^2 + 8*d^2*e*f*x + 4*d^2*e^2 - 2*f^2*e^{(2*d*x + 2*c)} - 2*f^2 + (I*d*f^2*x + I*d*e*f - 2*I*f^2)*e^{(3*d*x + 3*c)} + (8*I*d^2*f^2*x^2 + 8*I*d^2*e^2 + I*d*e*f - 2*I*f^2 + (16*I*d^2*e*f + I*d*f^2)*x)*e^{(d*x + c)} - (3*a*d^3*f^3*x^3 + 9*a*d^3*e*f^2*x^2 + 9*a*d^3*e^2*f*x + 3*a*d^3*e^3 - 3*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*e^{(4*d*x + 4*c)} - (-6*I*a*d^3*f^3*x^3 - 18*I*a*d^3*e*f^2*x^2 - 18*I*a*d^3*e^2*f*x - 6*I*a*d^3*e^3)*e^{(3*d*x + 3*c)} - (-6*I*a*d^3*f^3*x^3 - 18*I*a*d^3*e*f^2*x^2 - 18*I*a*d^3*e^2*f*x - 6*I*a*d^3*e^3)*e^{(d*x + c)})*integral(-1/3*(4*d^2*f^3*x^2 + 8*d^2*e*f^2*x + 4*d^2*e^2*f - 6*f^3 - (I*d^2*f^3*x^2 + 2*I*d^2*e*f^2*x + I*d^2*e^2*f - 6*I*f^3)*e^{(d*x + c)})/(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4 + (a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*e^{(2*d*x + 2*c)}), x))/(3*a*d^3*f^3*x^3 + 9*a*d^3*e*f^2*x^2 + 9*a*d^3*e^2*f*x + 3*a*d^3*e^3 - 3*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*e^{(4*d*x + 4*c)} - (-6*I*a*d^3*f^3*x^3 - 18*I*a*d^3*e*f^2*x^2 - 18*I*a*d^3*e^2*f*x - 6*I*a*d^3*e^3)*e^{(3*d*x + 3*c)} - (-6*I*a*d^3*f^3*x^3 - 18*I*a*d^3*e*f^2*x^2 - 18*I*a*d^3*e^2*f*x - 6*I*a*d^3*e^3)*e^{(d*x + c)})$$

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(c+dx)}{ie \sinh(c+dx)+e+ifx \sinh(c+dx)+fx} dx$$

a

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)\*\*2/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Integral(sech(c + d\*x)\*\*2/(I\*e\*sinh(c + d\*x) + e + I\*f\*x\*sinh(c + d\*x) + f\*x), x)/a

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(dx+c)^2}{(fx+e)(ia \sinh(dx+c)+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sech(d*x + c)^2/((f*x + e)*(I*a*sinh(d*x + c) + a)), x)
```

$$3.282 \quad \int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Sech[c + d\*x]^2/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0767881, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Sech[c + d\*x]^2/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Sech[c + d\*x]^2/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx = \int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.02, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[Sech[c + d\*x]^2/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] \$Aborted

---

**Maple [A]** time = 1.753, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(dx+c))^2}{(fx+e)^2 (a+ia \sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d\*x+c)^2/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] int(sech(d\*x+c)^2/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)^2/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-4*I*f*\operatorname{integrate}\left(\frac{1}{(4*I*a*d*f^3*x^3 + 12*I*a*d*e*f^2*x^2 + 12*I*a*d*e^2*f*x + 4*I*a*d*e^3 + 4*(a*d*f^3*x^3*e^c + 3*a*d*e*f^2*x^2*e^c + 3*a*d*e^2*f*x*e^c + a*d*e^3*e^c)*e^{(d*x)}), x\right) - 4*(2*d^2*f^2*x^2 + 4*d^2*e*f*x + 2*d^2*e^2 - 3*f^2*e^{(2*d*x + 2*c)} - 3*f^2 + (I*d*f^2*x*e^{(3*c)} + (I*d*e*f - 3*I*f^2)*e^{(3*c)}))*e^{(3*d*x)} + (4*I*d^2*f^2*x^2*e^c + (8*I*d^2*e*f + I*d*f^2)*x*e^c + (4*I*d^2*e^2 + I*d*e*f - 3*I*f^2)*e^c)*e^{(d*x)} / (6*a*d^3*f^4*x^4 + 24*a*d^3*e*f^3*x^3 + 36*a*d^3*e^2*f^2*x^2 + 24*a*d^3*e^3*f*x + 6*a*d^3*e^4 - 6*(a*d^3*f^4*x^4*e^{(4*c)} + 4*a*d^3*e*f^3*x^3*e^{(4*c)} + 6*a*d^3*e^2*f^2*x^2*e^{(4*c)} + 4*a*d^3*e^3*f*x*e^{(4*c)} + a*d^3*e^4*e^{(4*c)}))*e^{(4*d*x)} - (-12*I*a*d^3*f^4*x^4*e^{(3*c)} - 48*I*a*d^3*e*f^3*x^3*e^{(3*c)} - 72*I*a*d^3*e^2*f^2*x^2*e^{(3*c)} - 48*I*a*d^3*e^3*f*x*e^{(3*c)} - 12*I*a*d^3*e^4*e^{(3*c)}))*e^{(3*d*x)} - (-12*I*a*d^3*f^4*x^4*e^c - 48*I*a*d^3*e*f^3*x^3*e^c - 72*I*a*d^3*e^2*f^2*x^2*e^c - 48*I*a*d^3*e^3*f*x*e^c - 12*I*a*d^3*e^4*e^c)*e^{(d*x)} - 4*\operatorname{integrate}\left(\frac{5*d^2*f^3*x^2 + 10*d^2*e*f^2*x + 5*d^2*e^2*f - 24*f^3}{(12*a*d^3*f^5*x^5 + 60*a*d^3*e*f^4*x^4 + 120*a*d^3*e^2*f^3*x^3 + 120*a*d^3*e^3*f^2*x^2 + 60*a*d^3*e^4*f*x + 12*a*d^3*e^5 + (12*I*a*d^3*f^5*x^5*e^c + 60*I*a*d^3*e*f^4*x^4*e^c + 120*I*a*d^3*e^2*f^3*x^3*e^c + 120*I*a*d^3*e^3*f^2*x^2*e^c + 60*I*a*d^3*e^4*f*x*e^c + 12*I*a*d^3*e^5*e^c)*e^{(d*x)}), x\right)$

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)^2/(f\*x+e)^2/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-(4*d^2*f^2*x^2 + 8*d^2*e*f*x + 4*d^2*e^2 - 6*f^2*e^{(2*d*x + 2*c)} - 6*f^2 + (2*I*d*f^2*x + 2*I*d*e*f - 6*I*f^2)*e^{(3*d*x + 3*c)} + (8*I*d^2*f^2*x^2 + 8*I*d^2*e^2 + 2*I*d*e*f - 6*I*f^2 + (16*I*d^2*e*f + 2*I*d*f^2)*x)*e^{(d*x + c)} - (3*a*d^3*f^4*x^4 + 12*a*d^3*e*f^3*x^3 + 18*a*d^3*e^2*f^2*x^2 + 12*a*d^3*e^3*f*x + 3*a*d^3*e^4 - 3*(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*e^{(4*d*x + 4*c)} - (-6*I*a*d^3*f^4*x^4 - 24*I*a*d^3*e*f^3*x^3 - 36*I*a*d^3*e^2*f^2*x^2 - 24*I*a*d^3*e^3*f*x - 6*I*a*d^3*e^4)*e^{(3*d*x + 3*c)} - (-6*I*a*d^3*f^4*x^4 - 24*I*a*d^3*e*f^3*x^3 - 36*I*a*d^3*e^2*f^2*x^2 - 24*I*a*d^3*e^3*f*x - 6*I*a*d^3*e^4)*e^{(d*x + c)}) *integral(-1/3*(8*d^2*f^3*x^2 + 16*d^2*e*f^2*x + 8*d^2*e^2*f - 24*f^3 - (2*I*d^2*f^3*x^2 + 4*I*d^2*e*f^2*x + 2*I*d^2*e^2*f - 24*I*f^3)*e^{(d*x + c)})/(a*d^3*f^5*x^5 + 5*a*d^3*e*f^4*x^4 + 10*a*d^3*e^2*f^3*x^3 + 10*a*d^3*e^3*f^2*x^2 + 5*a*d^3*e^4*f*x + a*d^3*e^5 + (a*d^3*f^5*x^5 + 5*a*d^3*e*f^4*x^4 + 10*a*d^3*e^2*f^3*x^3 + 10*a*d^3*e^3*f^2*x^2 + 5*a*d^3*e^4*f*x + a*d^3*e^5)*e^{(2*d*x + 2*c)}), x)/(3*a*d^3*f^4*x^4 + 12*a*d^3*e*f^3*x^3 + 18*a*d^3*e^2*f^2*x^2 + 12*a*d^3*e^3*f*x + 3*a*d^3*e^4 - 3*(a*d^3*f^4*x^4 + 4*a*d^3*e*f^3*x^3 + 6*a*d^3*e^2*f^2*x^2 + 4*a*d^3*e^3*f*x + a*d^3*e^4)*e^{(4*d*x + 4*c)} - (-6*I*a*d^3*f^4*x^4 - 24*I*a*d^3*e*f^3*x^3 - 36*I*a*d^3*e^2*f^2*x^2 - 24*I*a*d^3*e^3*f*x - 6*I*a*d^3*e^4)*e^{(3*d*x + 3*c)} - (-6*I*a*d^3*f^4*x^4 - 24*I*a*d^3*e*f^3*x^3 - 36*I*a*d^3*e^2*f^2*x^2 - 24*I*a*d^3*e^3*f*x - 6*I*a*d^3*e^4)*e^{(d*x + c)})$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)\*\*2/(f\*x+e)\*\*2/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^2/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")`

[Out] Timed out

$$3.283 \quad \int \frac{(e+fx)^3 \operatorname{sech}^3(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=667

$$\frac{9if^2(e+fx)\operatorname{PolyLog}(3, -ie^{c+dx})}{4ad^3} - \frac{9if^2(e+fx)\operatorname{PolyLog}(3, ie^{c+dx})}{4ad^3} - \frac{9if(e+fx)^2\operatorname{PolyLog}(2, -ie^{c+dx})}{8ad^2} + \frac{9if(e+fx)^2\operatorname{PolyLog}(2, ie^{c+dx})}{8ad^2}$$

[Out]  $((-I/2)*f*(e + f*x)^2)/(a*d^2) - (5*f^2*(e + f*x)*\operatorname{ArcTan}[E^(c + d*x)])/(a*d^3) + (3*(e + f*x)^3*\operatorname{ArcTan}[E^(c + d*x)])/(4*a*d) + (I*f^2*(e + f*x)*\operatorname{Log}[1 + E^(2*(c + d*x))])/(a*d^3) + (((5*I)/2)*f^3*\operatorname{PolyLog}[2, (-I)*E^(c + d*x)])/(a*d^4) - (((9*I)/8)*f*(e + f*x)^2*\operatorname{PolyLog}[2, (-I)*E^(c + d*x)])/(a*d^2) - (((5*I)/2)*f^3*\operatorname{PolyLog}[2, I*E^(c + d*x)])/(a*d^4) + (((9*I)/8)*f*(e + f*x)^2*\operatorname{PolyLog}[2, I*E^(c + d*x)])/(a*d^2) + ((I/2)*f^3*\operatorname{PolyLog}[2, -E^(2*(c + d*x))])/(a*d^4) + (((9*I)/4)*f^2*(e + f*x)*\operatorname{PolyLog}[3, (-I)*E^(c + d*x)])/(a*d^3) - (((9*I)/4)*f^2*(e + f*x)*\operatorname{PolyLog}[3, I*E^(c + d*x)])/(a*d^3) - (((9*I)/4)*f^3*\operatorname{PolyLog}[4, (-I)*E^(c + d*x)])/(a*d^4) + (((9*I)/4)*f^3*\operatorname{PolyLog}[4, I*E^(c + d*x)])/(a*d^4) - (f^3*\operatorname{Sech}[c + d*x])/(4*a*d^4) + (9*f*(e + f*x)^2*\operatorname{Sech}[c + d*x])/(8*a*d^2) - ((I/4)*f^2*(e + f*x)*\operatorname{Sech}[c + d*x]^2)/(a*d^3) + (f*(e + f*x)^2*\operatorname{Sech}[c + d*x]^3)/(4*a*d^2) + ((I/4)*(e + f*x)^3*\operatorname{Sech}[c + d*x]^4)/(a*d) + ((I/4)*f^3*\operatorname{Tanh}[c + d*x])/(a*d^4) - ((I/2)*f*(e + f*x)^2*\operatorname{Tanh}[c + d*x])/(a*d^2) - (f^2*(e + f*x)*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(4*a*d^3) + (3*(e + f*x)^3*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(8*a*d) - ((I/4)*f*(e + f*x)^2*\operatorname{Sech}[c + d*x]^2*\operatorname{Tanh}[c + d*x])/(a*d^2) + ((e + f*x)^3*\operatorname{Sech}[c + d*x]^3*\operatorname{Tanh}[c + d*x])/(4*a*d)$

**Rubi [A]** time = 0.71112, antiderivative size = 667, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 16, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.516$ , Rules used = {5571, 4186, 4185, 4180, 2279, 2391, 2531, 6609, 2282, 6589, 5451, 3767, 8, 4184, 3718, 2190}

$$\frac{9if^2(e+fx)\operatorname{PolyLog}(3, -ie^{c+dx})}{4ad^3} - \frac{9if^2(e+fx)\operatorname{PolyLog}(3, ie^{c+dx})}{4ad^3} - \frac{9if(e+fx)^2\operatorname{PolyLog}(2, -ie^{c+dx})}{8ad^2} + \frac{9if(e+fx)^2\operatorname{PolyLog}(2, ie^{c+dx})}{8ad^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\frac{(e + f*x)^3*\operatorname{Sech}[c + d*x]^3}{(a + I*a*\operatorname{Sinh}[c + d*x])}, x]$

[Out]  $((-I/2)*f*(e + f*x)^2)/(a*d^2) - (5*f^2*(e + f*x)*\operatorname{ArcTan}[E^(c + d*x)])/(a*d^3) + (3*(e + f*x)^3*\operatorname{ArcTan}[E^(c + d*x)])/(4*a*d) + (I*f^2*(e + f*x)*\operatorname{Log}[1 + E^(2*(c + d*x))])/(a*d^3) + (((5*I)/2)*f^3*\operatorname{PolyLog}[2, (-I)*E^(c + d*x)])/(a*d^4) - (((9*I)/8)*f*(e + f*x)^2*\operatorname{PolyLog}[2, (-I)*E^(c + d*x)])/(a*d^2) -$

$$\begin{aligned} &(((5*I)/2)*f^3*PolyLog[2, I*E^(c + d*x)])/(a*d^4) + (((9*I)/8)*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)]/(a*d^2) + ((I/2)*f^3*PolyLog[2, -E^(2*(c + d*x))])/(a*d^4) + (((9*I)/4)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)]/(a*d^3) - (((9*I)/4)*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)]/(a*d^3) - (((9*I)/4)*f^3*PolyLog[4, (-I)*E^(c + d*x)]/(a*d^4) + (((9*I)/4)*f^3*PolyLog[4, I*E^(c + d*x)]/(a*d^4) - (f^3*Sech[c + d*x])/(4*a*d^4) + (9*f*(e + f*x)^2*Sech[c + d*x])/(8*a*d^2) - ((I/4)*f^2*(e + f*x)*Sech[c + d*x]^2)/(a*d^3) + (f*(e + f*x)^2*Sech[c + d*x]^3)/(4*a*d^2) + ((I/4)*(e + f*x)^3*Sech[c + d*x]^4)/(a*d) + ((I/4)*f^3*Tanh[c + d*x])/(a*d^4) - ((I/2)*f*(e + f*x)^2*Tanh[c + d*x])/(a*d^2) - (f^2*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(4*a*d^3) + (3*(e + f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(8*a*d) - ((I/4)*f*(e + f*x)^2*Sech[c + d*x]^2*Tanh[c + d*x])/(a*d^2) + ((e + f*x)^3*Sech[c + d*x]^3*Tanh[c + d*x])/(4*a*d) \end{aligned}$$

### Rule 5571

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(n + 1)*Tanh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]
```

### Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

### Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] := -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*(c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
```



- E<sup>-(I\*e) + f\*fz\*x)/E<sup>(I\*k\*Pi)</sup>], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)<sup>(m - 1)</sup>\*Log[1 + E<sup>-(I\*e) + f\*fz\*x)/E<sup>(I\*k\*Pi)</sup>], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]</sup></sup>

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))<sup>(n\_.)</sup>], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F<sup>(e\*(c + d\*x))</sup>)<sup>n</sup>], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)<sup>(n\_.)</sup>)]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x<sup>n</sup>)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))<sup>(n\_.)</sup>]\*((f\_.) + (g\_.)\*(x\_)<sup>(m\_.)</sup>), x\_Symbol] :> -Simp[((f + g\*x)<sup>m</sup>\*PolyLog[2, -(e\*(F<sup>(c\*(a + b\*x))</sup>)<sup>n</sup>)]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)<sup>(m - 1)</sup>\*PolyLog[2, -(e\*(F<sup>(c\*(a + b\*x))</sup>)<sup>n</sup>]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_)<sup>(m\_.)</sup>)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))<sup>(p\_.)</sup>], x\_Symbol] :> Simp[((e + f\*x)<sup>m</sup>\*PolyLog[n + 1, d\*(F<sup>(c\*(a + b\*x))</sup>)<sup>p</sup>]/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)<sup>(m - 1)</sup>\*PolyLog[n + 1, d\*(F<sup>(c\*(a + b\*x))</sup>)<sup>p</sup>], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_.)\*((a\_.)\*(v\_)<sup>(n\_)</sup>)<sup>(m\_)</sup> /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E<sup>((c\_.)\*((a\_.) + (b\_.)\*x))\* (F\_)</sup>[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_)<sup>(p\_.)</sup>)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)<sup>p</sup>]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

Rule 5451

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sech[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*Tanh[(a\_.) + (b\_.)\*(x\_)]^(p\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Sech[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Sech[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3718

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)], x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x)))/(1 + E^(2\*(-I\*e) + f\*fz\*x)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \operatorname{sech}^3(c+dx)}{a+ia \sinh(c+dx)} dx &= -\frac{i \int (e+fx)^3 \operatorname{sech}^4(c+dx) \tanh(c+dx) dx}{a} + \frac{\int (e+fx)^3 \operatorname{sech}^5(c+dx) dx}{a} \\
&= \frac{f(e+fx)^2 \operatorname{sech}^3(c+dx)}{4ad^2} + \frac{i(e+fx)^3 \operatorname{sech}^4(c+dx)}{4ad} + \frac{(e+fx)^3 \operatorname{sech}^3(c+dx) \tanh(c+dx)}{4ad} \\
&= -\frac{f^3 \operatorname{sech}(c+dx)}{4ad^4} + \frac{9f(e+fx)^2 \operatorname{sech}(c+dx)}{8ad^2} - \frac{if^2(e+fx) \operatorname{sech}^2(c+dx)}{4ad^3} + \frac{f(e+fx)^2}{4ad} \\
&= -\frac{5f^2(e+fx) \tan^{-1}(e^{c+dx})}{ad^3} + \frac{3(e+fx)^3 \tan^{-1}(e^{c+dx})}{4ad} - \frac{f^3 \operatorname{sech}(c+dx)}{4ad^4} + \frac{9f(e+fx)}{8ad} \\
&= -\frac{if(e+fx)^2}{2ad^2} - \frac{5f^2(e+fx) \tan^{-1}(e^{c+dx})}{ad^3} + \frac{3(e+fx)^3 \tan^{-1}(e^{c+dx})}{4ad} - \frac{9if(e+fx)^2 \operatorname{Li}_2(-e^{-c-dx})}{8ad^2} \\
&= -\frac{if(e+fx)^2}{2ad^2} - \frac{5f^2(e+fx) \tan^{-1}(e^{c+dx})}{ad^3} + \frac{3(e+fx)^3 \tan^{-1}(e^{c+dx})}{4ad} + \frac{if^2(e+fx) \log(-e^{-c-dx})}{ad} \\
&= -\frac{if(e+fx)^2}{2ad^2} - \frac{5f^2(e+fx) \tan^{-1}(e^{c+dx})}{ad^3} + \frac{3(e+fx)^3 \tan^{-1}(e^{c+dx})}{4ad} + \frac{if^2(e+fx) \log(-e^{-c-dx})}{ad} \\
&= -\frac{if(e+fx)^2}{2ad^2} - \frac{5f^2(e+fx) \tan^{-1}(e^{c+dx})}{ad^3} + \frac{3(e+fx)^3 \tan^{-1}(e^{c+dx})}{4ad} + \frac{if^2(e+fx) \log(-e^{-c-dx})}{ad}
\end{aligned}$$

**Mathematica [B]** time = 13.394, size = 1804, normalized size = 2.7

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^3\*Sech[c + d\*x]^3)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out]  $(-3*(4*d^2*e*(d^2*e^2 - 4*f^2)*x + 2*d^2*f*(3*d^2*e^2 - 4*f^2)*x^2 + 4*d^4*e*f^2*x^3 + d^4*f^3*x^4 + 4*d*(1 - I*E^c)*f*(3*d^2*e^2 - 4*f^2)*x*\operatorname{Log}[1 + I*E^{(-c - d*x)}] + 12*d^3*e*(1 - I*E^c)*f^2*x^2*\operatorname{Log}[1 + I*E^{(-c - d*x)}] + 4*d^3*(1 - I*E^c)*f^3*x^3*\operatorname{Log}[1 + I*E^{(-c - d*x)}] + (4*I)*d*e*(I + E^c)*(d^2*e^2 - 4*f^2)*(d*x - \operatorname{Log}[I + E^{(c + d*x)}]) + 4*(1 - I*E^c)*f*(-3*d^2*e^2 + 4*f^2)*\operatorname{PolyLog}[2, (-I)*E^{(-c - d*x)}] + (24*I)*d*e*(I + E^c)*f^2*(d*x*\operatorname{PolyLog}[2, (-I)*E^{(-c - d*x)}] + \operatorname{PolyLog}[3, (-I)*E^{(-c - d*x)}]) + (12*I)*(I + E^c)*f^3*(d^2*x^2*\operatorname{PolyLog}[2, (-I)*E^{(-c - d*x)}] + 2*(d*x*\operatorname{PolyLog}[3, (-I)*E^{(-c - d*x)}] + \operatorname{PolyLog}[4, (-I)*E^{(-c - d*x)}])))/(32*a*d^4*(I + E^c)) - ((28*f^2 - 3*d^2*(e + f*x)^2)^2 + 12*d*(1 + I*E^c)*f^2*(9*d^2*e^2 - 28*f^2)*x*\operatorname{Log}[1 - I*E^{(-c - d*x)}] + 108*d^3*e*(1 + I*E^c)*f^3*x^2*\operatorname{Log}[1 - I*E^{(-c - d*x)}] + 36*d^3*(1 + I*E^c)*f^4*x^3*\operatorname{Log}[1 - I*E^{(-c - d*x)}] - 12*d*e*(1 + I*E^c)*f*(3*d^2*e^2 - 28*f^2)*(d*x - \operatorname{Log}[I - E^{(c + d*x)}]) + 12*(1 + I*E^c)*f^2*(-9*d$

$$\begin{aligned}
& ^2e^2 + 28f^2) * \text{PolyLog}[2, I * E^{-c - d * x}] - 216 * d * e * (1 + I * E^c) * f^3 * (d * x * \\
& \text{PolyLog}[2, I * E^{-c - d * x}] + \text{PolyLog}[3, I * E^{-c - d * x}]) - 108 * (1 + I * E^c) * \\
& f^4 * (d^2 * x^2 * \text{PolyLog}[2, I * E^{-c - d * x}] + 2 * (d * x * \text{PolyLog}[3, I * E^{-c - d * x}] \\
& + \text{PolyLog}[4, I * E^{-c - d * x}]))) / (96 * a * d^4 * (-I + E^c) * f) + ((3 * e^3 * x * \text{Cosh}[c] \\
& ] / (4 * a) + (3 * e^3 * x * \text{Sinh}[c] / (4 * a)) / (1 + \text{Cosh}[2 * c] + \text{Sinh}[2 * c]) + ((9 * e^2 * f \\
& * x^2 * \text{Cosh}[c] / (8 * a) + (9 * e^2 * f * x^2 * \text{Sinh}[c] / (8 * a)) / (1 + \text{Cosh}[2 * c] + \text{Sinh}[2 * \\
& c]) + ((3 * e * f^2 * x^3 * \text{Cosh}[c] / (4 * a) + (3 * e * f^2 * x^3 * \text{Sinh}[c] / (4 * a)) / (1 + \text{Cosh} \\
& [2 * c] + \text{Sinh}[2 * c]) + ((3 * f^3 * x^4 * \text{Cosh}[c] / (16 * a) + (3 * f^3 * x^4 * \text{Sinh}[c] / (16 * \\
& a)) / (1 + \text{Cosh}[2 * c] + \text{Sinh}[2 * c]) - ((I / 8) * (e^3 + 3 * e^2 * f * x + 3 * e * f^2 * x^2 + f \\
& ^3 * x^3)) / (a * d * (\text{Cosh}[c / 2 + (d * x) / 2] - I * \text{Sinh}[c / 2 + (d * x) / 2]))^2 + (((3 * I) / 4) \\
& * (e^2 * f * \text{Sinh}[(d * x) / 2] + 2 * e * f^2 * x * \text{Sinh}[(d * x) / 2] + f^3 * x^2 * \text{Sinh}[(d * x) / 2])) / ( \\
& a * d^2 * (\text{Cosh}[c / 2] - I * \text{Sinh}[c / 2]) * (\text{Cosh}[c / 2 + (d * x) / 2] - I * \text{Sinh}[c / 2 + (d * x) / 2 \\
& ])) + ((I / 8) * (e^3 + 3 * e^2 * f * x + 3 * e * f^2 * x^2 + f^3 * x^3)) / (a * d * (\text{Cosh}[c / 2 + (d \\
& * x) / 2] + I * \text{Sinh}[c / 2 + (d * x) / 2]))^4 - ((I / 4) * (e^2 * f * \text{Sinh}[(d * x) / 2] + 2 * e * f^2 * \\
& x * \text{Sinh}[(d * x) / 2] + f^3 * x^2 * \text{Sinh}[(d * x) / 2])) / (a * d^2 * (\text{Cosh}[c / 2] + I * \text{Sinh}[c / 2]) * \\
& (\text{Cosh}[c / 2 + (d * x) / 2] + I * \text{Sinh}[c / 2 + (d * x) / 2]))^3 + ((2 * I) * d^2 * e^3 * \text{Cosh}[c / 2] \\
& + d * e^2 * f * \text{Cosh}[c / 2] - (2 * I) * e * f^2 * \text{Cosh}[c / 2] + (6 * I) * d^2 * e^2 * f * x * \text{Cosh}[c / 2] \\
& + 2 * d * e * f^2 * x * \text{Cosh}[c / 2] - (2 * I) * f^3 * x * \text{Cosh}[c / 2] + (6 * I) * d^2 * e * f^2 * x^2 * \text{Cosh} \\
& [c / 2] + d * f^3 * x^2 * \text{Cosh}[c / 2] + (2 * I) * d^2 * f^3 * x^3 * \text{Cosh}[c / 2] - 2 * d^2 * e^3 * \text{Sinh}[c \\
& / 2] - I * d * e^2 * f * \text{Sinh}[c / 2] + 2 * e * f^2 * \text{Sinh}[c / 2] - 6 * d^2 * e^2 * f * x * \text{Sinh}[c / 2] - ( \\
& 2 * I) * d * e * f^2 * x * \text{Sinh}[c / 2] + 2 * f^3 * x * \text{Sinh}[c / 2] - 6 * d^2 * e * f^2 * x^2 * \text{Sinh}[c / 2] - \\
& I * d * f^3 * x^2 * \text{Sinh}[c / 2] - 2 * d^2 * f^3 * x^3 * \text{Sinh}[c / 2]) / (8 * a * d^3 * (\text{Cosh}[c / 2] + I * \text{Si} \\
& nh[c / 2]) * (\text{Cosh}[c / 2 + (d * x) / 2] + I * \text{Sinh}[c / 2 + (d * x) / 2]))^2 - ((I / 4) * (7 * d^2 * e \\
& ^2 * f * \text{Sinh}[(d * x) / 2] - 2 * f^3 * \text{Sinh}[(d * x) / 2] + 14 * d^2 * e * f^2 * x * \text{Sinh}[(d * x) / 2] + 7 \\
& * d^2 * f^3 * x^2 * \text{Sinh}[(d * x) / 2])) / (a * d^4 * (\text{Cosh}[c / 2] + I * \text{Sinh}[c / 2]) * (\text{Cosh}[c / 2 + ( \\
& d * x) / 2] + I * \text{Sinh}[c / 2 + (d * x) / 2]))
\end{aligned}$$

**Maple [B]** time = 0.259, size = 2026, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f * x + e)^3 * \text{sech}(d * x + c)^3 / (a + I * a * \text{sinh}(d * x + c)), x)$

[Out]  $\begin{aligned}
& 9/8 * I / a / d^2 * \ln(1 - I * \exp(d * x + c)) * c * e^2 * f - 9/8 * I / a / d * \ln(1 + I * \exp(d * x + c)) * e^2 * f * x \\
& - 9/8 * I / a / d^2 * \ln(1 + I * \exp(d * x + c)) * c * e^2 * f + 9/8 * I / a / d^2 * e^2 * f * c * \ln(\exp(d * x + c) - I \\
& ) + 9/8 * I / a / d^3 * \ln(1 + I * \exp(d * x + c)) * c^2 * e * f^2 - 9/8 * I / a / d^3 * e * f^2 * c^2 * \ln(\exp(d * x \\
& + c) - I) + 9/8 * I / a / d * \ln(1 - I * \exp(d * x + c)) * e * f^2 * x^2 + 9/4 * I / a / d^2 * \text{polylog}(2, I * \exp(d \\
& * x + c)) * e * f^2 * x + 9/8 * I / a / d * \ln(1 - I * \exp(d * x + c)) * e^2 * f * x - 9/4 * I * f^3 * \text{polylog}(4, -I * \\
& \exp(d * x + c)) / a / d^4 + 9/4 * I * f^3 * \text{polylog}(4, I * \exp(d * x + c)) / a / d^4 - I / a / d^4 * f^3 * c^2 + 3 \\
& / 8 * I / a / d * e^3 * \ln(\exp(d * x + c) + I) + 7/2 * I / a / d^4 * f^3 * \text{polylog}(2, -I * \exp(d * x + c)) - 3/2 *
\end{aligned}$

$$\begin{aligned} & I/a/d^4*f^3*polylog(2, I*exp(d*x+c))-3/8*I/a/d*e^3*ln(exp(d*x+c)-I)-I/a/d^2* \\ & f^3*x^2-2*I/a/d^3*e*f^2*ln(exp(d*x+c))-3/2*I/a/d^3*f^3*ln(1-I*exp(d*x+c))*x \\ & -3/2*I/a/d^4*f^3*ln(1-I*exp(d*x+c))*c-3/8*I/a/d*f^3*ln(1+I*exp(d*x+c))*x^3- \\ & 3/8*I/a/d^4*f^3*ln(1+I*exp(d*x+c))*c^3-9/8*I/a/d^2*f^3*polylog(2, -I*exp(d*x \\ & +c))*x^2+9/4*I/a/d^3*f^3*polylog(3, -I*exp(d*x+c))*x+3/8*I/a/d*f^3*ln(1-I*ex \\ & p(d*x+c))*x^3+3/8*I/a/d^4*f^3*ln(1-I*exp(d*x+c))*c^3+9/8*I/a/d^2*f^3*polylo \\ & g(2, I*exp(d*x+c))*x^2-9/4*I/a/d^3*f^3*polylog(3, I*exp(d*x+c))*x+7/2*I/a/d^3 \\ & *f^3*ln(1+I*exp(d*x+c))*x+7/2*I/a/d^4*f^3*ln(1+I*exp(d*x+c))*c+9/4*I/a/d^3* \\ & e*f^2*polylog(3, -I*exp(d*x+c))-9/4*I/a/d^3*e*f^2*polylog(3, I*exp(d*x+c))+7/ \\ & 2*I/a/d^3*e*f^2*ln(exp(d*x+c)-I)-3/2*I/a/d^3*e*f^2*ln(exp(d*x+c)+I)+2*I/a/d \\ & ^4*f^3*c*ln(exp(d*x+c))-9/8*I/a/d^2*e^2*f*polylog(2, -I*exp(d*x+c))+9/8*I/a/ \\ & d^2*e^2*f*polylog(2, I*exp(d*x+c))-7/2*I/a/d^4*f^3*c*ln(exp(d*x+c)-I)+3/2*I/ \\ & a/d^4*f^3*c*ln(exp(d*x+c)+I)+3/8*I/a/d^4*f^3*c^3*ln(exp(d*x+c)-I)-3/8*I/a/d \\ & ^4*f^3*c^3*ln(exp(d*x+c)+I)-2*I/a/d^3*f^3*c*x+9/8*I/a/d^3*e*f^2*c^2*ln(exp( \\ & d*x+c)+I)-9/8*I/a/d^3*ln(1-I*exp(d*x+c))*c^2*e*f^2-9/8*I/a/d*ln(1+I*exp(d*x \\ & +c))*e*f^2*x^2-9/4*I/a/d^2*polylog(2, -I*exp(d*x+c))*e*f^2*x-9/8*I/a/d^2*e^2 \\ & *f*c*ln(exp(d*x+c)+I)+1/4*(2*I*f^3-22*I*d^2*f^3*x^2*exp(2*d*x+2*c)-22*I*d^2 \\ & *e^2*f*exp(2*d*x+2*c)+9*d^3*e*f^2*x^2*exp(5*d*x+5*c)+9*d^3*e^2*f*x*exp(5*d* \\ & x+5*c)+18*d^2*e*f^2*x*exp(5*d*x+5*c)+16*d^2*e*f^2*x*exp(3*d*x+3*c)-8*I*d^2* \\ & e*f^2*x-44*I*d^2*e*f^2*x*exp(2*d*x+2*c)-d^2*f^3*x^2*exp(d*x+c)-d^2*e^2*f*ex \\ & p(d*x+c)+3*d^3*f^3*x^3*exp(d*x+c)-2*d*f^3*x*exp(d*x+c)-2*d*e*f^2*exp(d*x+c) \\ & +6*I*d^3*f^3*x^3*exp(2*d*x+2*c)-6*I*d^3*f^3*x^3*exp(4*d*x+4*c)-18*I*d^2*f^3 \\ & *x^2*exp(4*d*x+4*c)-18*I*d^2*e^2*f*exp(4*d*x+4*c)-18*I*d^3*e^2*f*x*exp(4*d* \\ & x+4*c)+18*I*d^3*e*f^2*x^2*exp(2*d*x+2*c)-18*I*d^3*e*f^2*x^2*exp(4*d*x+4*c)+ \\ & 18*I*d^3*e^2*f*x*exp(2*d*x+2*c)-36*I*d^2*e*f^2*x*exp(4*d*x+4*c)+3*d^3*e^3*ex \\ & xp(5*d*x+5*c)+2*I*f^3*exp(4*d*x+4*c)+4*I*f^3*exp(2*d*x+2*c)+2*d^3*e^3*exp(3 \\ & *d*x+3*c)+9*d^3*e*f^2*x^2*exp(d*x+c)+9*d^3*e^2*f*x*exp(d*x+c)-2*d^2*e*f^2*x \\ & *exp(d*x+c)-2*f^3*exp(d*x+c)+3*d^3*e^3*exp(d*x+c)-4*I*d^2*e^2*f-4*I*d^2*f^3 \\ & *x^2-2*f^3*exp(5*d*x+5*c)-4*f^3*exp(3*d*x+3*c)+6*d^3*e*f^2*x^2*exp(3*d*x+3* \\ & c)+6*d^3*e^2*f*x*exp(3*d*x+3*c)+2*d^3*f^3*x^3*exp(3*d*x+3*c)+6*I*d^3*e^3*ex \\ & p(2*d*x+2*c)-6*I*d^3*e^3*exp(4*d*x+4*c)+9*d^2*e^2*f*exp(5*d*x+5*c)+8*d^2*e^ \\ & 2*f*exp(3*d*x+3*c)-2*d*f^3*x*exp(5*d*x+5*c)-4*d*f^3*x*exp(3*d*x+3*c)-2*d*e* \\ & f^2*exp(5*d*x+5*c)-4*d*e*f^2*exp(3*d*x+3*c)+3*d^3*f^3*x^3*exp(5*d*x+5*c)+9* \\ & d^2*f^3*x^2*exp(5*d*x+5*c)+8*d^2*f^3*x^2*exp(3*d*x+3*c))/(exp(d*x+c)+I)^2/( \\ & exp(d*x+c)-I)^4/d^4/a \end{aligned}$$


---

**Maxima [B]** time = 6.25254, size = 1800, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*sech(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima"

)

```
[Out] -1/8*e^3*(64*(3*e^(-d*x - c) - 6*I*e^(-2*d*x - 2*c) + 2*e^(-3*d*x - 3*c) +
6*I*e^(-4*d*x - 4*c) + 3*e^(-5*d*x - 5*c)))/((64*I*a*e^(-d*x - c) - 32*a*e^(-
2*d*x - 2*c) + 128*I*a*e^(-3*d*x - 3*c) + 32*a*e^(-4*d*x - 4*c) + 64*I*a*e
^(-5*d*x - 5*c) + 32*a*e^(-6*d*x - 6*c) - 32*a)*d) + 3*I*log(e^(-d*x - c) +
I)/(a*d) - 3*I*log(e^(-d*x - c) - I)/(a*d)) - 2*I*e*f^2*x/(a*d^2) + (-4*I*
d^2*f^3*x^2 - 8*I*d^2*e*f^2*x - 4*I*d^2*e^2*f + 2*I*f^3 + (3*d^3*f^3*x^3*e^
(5*c) + 9*(d^3*e*f^2 + d^2*f^3)*x^2*e^(5*c) + (9*d^3*e^2*f + 18*d^2*e*f^2 -
2*d*f^3)*x*e^(5*c) + (9*d^2*e^2*f - 2*d*e*f^2 - 2*f^3)*e^(5*c))*e^(5*d*x)
+ (-6*I*d^3*f^3*x^3*e^(4*c) + (-18*I*d^3*e*f^2 - 18*I*d^2*f^3)*x^2*e^(4*c)
+ (-18*I*d^3*e^2*f - 36*I*d^2*e*f^2)*x*e^(4*c) + (-18*I*d^2*e^2*f + 2*I*f^3
)*e^(4*c))*e^(4*d*x) + 2*(d^3*f^3*x^3*e^(3*c) + (3*d^3*e*f^2 + 4*d^2*f^3)*x
^2*e^(3*c) + (3*d^3*e^2*f + 8*d^2*e*f^2 - 2*d*f^3)*x*e^(3*c) + 2*(2*d^2*e^2
*f - d*e*f^2 - f^3)*e^(3*c))*e^(3*d*x) + (6*I*d^3*f^3*x^3*e^(2*c) + (18*I*d
^3*e*f^2 - 22*I*d^2*f^3)*x^2*e^(2*c) + (18*I*d^3*e^2*f - 44*I*d^2*e*f^2)*x*
e^(2*c) + (-22*I*d^2*e^2*f + 4*I*f^3)*e^(2*c))*e^(2*d*x) + (3*d^3*f^3*x^3*e
^c + (9*d^3*e*f^2 - d^2*f^3)*x^2*e^c + (9*d^3*e^2*f - 2*d^2*e*f^2 - 2*d*f^3
)*x*e^c - (d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*e^c)*e^(d*x))/(4*a*d^4*e^(6*d*x +
6*c) - 8*I*a*d^4*e^(5*d*x + 5*c) + 4*a*d^4*e^(4*d*x + 4*c) - 16*I*a*d^4*e^
(3*d*x + 3*c) - 4*a*d^4*e^(2*d*x + 2*c) - 8*I*a*d^4*e^(d*x + c) - 4*a*d^4)
- 9/8*I*(d^2*x^2*log(I*e^(d*x + c) + 1) + 2*d*x*dilog(-I*e^(d*x + c)) - 2*p
olylog(3, -I*e^(d*x + c)))*e*f^2/(a*d^3) + 9/8*I*(d^2*x^2*log(-I*e^(d*x + c
) + 1) + 2*d*x*dilog(I*e^(d*x + c)) - 2*polylog(3, I*e^(d*x + c)))*e*f^2/(a
*d^3) + 7/2*I*e*f^2*log(I*e^(d*x + c) + 1)/(a*d^3) - 3/2*I*e*f^2*log(I*e^(d
*x + c) - 1)/(a*d^3) - 3/8*I*(d^3*x^3*log(I*e^(d*x + c) + 1) + 3*d^2*x^2*di
log(-I*e^(d*x + c)) - 6*d*x*polylog(3, -I*e^(d*x + c)) + 6*polylog(4, -I*e^
(d*x + c)))*f^3/(a*d^4) + 3/8*I*(d^3*x^3*log(-I*e^(d*x + c) + 1) + 3*d^2*x^
2*dilog(I*e^(d*x + c)) - 6*d*x*polylog(3, I*e^(d*x + c)) + 6*polylog(4, I*e
^(d*x + c)))*f^3/(a*d^4) - 1/8*I*(9*d^2*e^2*f - 28*f^3)*(d*x*log(I*e^(d*x +
c) + 1) + dilog(-I*e^(d*x + c)))/(a*d^4) + 3/8*I*(3*d^2*e^2*f - 4*f^3)*(d*
x*log(-I*e^(d*x + c) + 1) + dilog(I*e^(d*x + c)))/(a*d^4) - 1/32*(3*I*d^4*f
^3*x^4 + 12*I*d^4*e*f^2*x^3 + (18*I*d^2*e^2*f - 24*I*f^3)*d^2*x^2)/(a*d^4)
+ 1/32*(3*I*d^4*f^3*x^4 + 12*I*d^4*e*f^2*x^3 + (18*I*d^2*e^2*f - 56*I*f^3)*
d^2*x^2)/(a*d^4)
```

---

**Fricas [C]** time = 2.89648, size = 9095, normalized size = 13.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*sech(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas"

)

```
[Out] (-8*I*d^2*e^2*f + 16*I*c*d*e*f^2 + (-8*I*c^2 + 4*I)*f^3 + (-9*I*d^2*f^3*x^2
- 18*I*d^2*e*f^2*x - 9*I*d^2*e^2*f + 12*I*f^3 + (9*I*d^2*f^3*x^2 + 18*I*d^
2*e*f^2*x + 9*I*d^2*e^2*f - 12*I*f^3))*e^(6*d*x + 6*c) + 6*(3*d^2*f^3*x^2 +
6*d^2*e*f^2*x + 3*d^2*e^2*f - 4*f^3))*e^(5*d*x + 5*c) + (9*I*d^2*f^3*x^2 + 1
8*I*d^2*e*f^2*x + 9*I*d^2*e^2*f - 12*I*f^3))*e^(4*d*x + 4*c) + 12*(3*d^2*f^3
*x^2 + 6*d^2*e*f^2*x + 3*d^2*e^2*f - 4*f^3))*e^(3*d*x + 3*c) + (-9*I*d^2*f^3
*x^2 - 18*I*d^2*e*f^2*x - 9*I*d^2*e^2*f + 12*I*f^3))*e^(2*d*x + 2*c) + 6*(3*
d^2*f^3*x^2 + 6*d^2*e*f^2*x + 3*d^2*e^2*f - 4*f^3))*e^(d*x + c))*dilog(I*e^(
d*x + c)) + (9*I*d^2*f^3*x^2 + 18*I*d^2*e*f^2*x + 9*I*d^2*e^2*f - 28*I*f^3
+ (-9*I*d^2*f^3*x^2 - 18*I*d^2*e*f^2*x - 9*I*d^2*e^2*f + 28*I*f^3))*e^(6*d*x
+ 6*c) - 2*(9*d^2*f^3*x^2 + 18*d^2*e*f^2*x + 9*d^2*e^2*f - 28*f^3))*e^(5*d*
x + 5*c) + (-9*I*d^2*f^3*x^2 - 18*I*d^2*e*f^2*x - 9*I*d^2*e^2*f + 28*I*f^3)
*e^(4*d*x + 4*c) - 4*(9*d^2*f^3*x^2 + 18*d^2*e*f^2*x + 9*d^2*e^2*f - 28*f^3
))*e^(3*d*x + 3*c) + (9*I*d^2*f^3*x^2 + 18*I*d^2*e*f^2*x + 9*I*d^2*e^2*f - 2
8*I*f^3))*e^(2*d*x + 2*c) - 2*(9*d^2*f^3*x^2 + 18*d^2*e*f^2*x + 9*d^2*e^2*f
- 28*f^3))*e^(d*x + c))*dilog(-I*e^(d*x + c)) + (-8*I*d^2*f^3*x^2 - 16*I*d^2
*e*f^2*x - 16*I*c*d*e*f^2 + 8*I*c^2*f^3))*e^(6*d*x + 6*c) + 2*(3*d^3*f^3*x^3
+ 3*d^3*e^3 + 9*d^2*e^2*f - 2*(8*c + 1)*d*e*f^2 + 2*(4*c^2 - 1)*f^3 + (9*d
^3*e*f^2 + d^2*f^3))*x^2 + (9*d^3*e^2*f + 2*d^2*e*f^2 - 2*d*f^3))*x)*e^(5*d*x
+ 5*c) + (-12*I*d^3*f^3*x^3 - 12*I*d^3*e^3 - 36*I*d^2*e^2*f - 16*I*c*d*e*f
^2 + (8*I*c^2 + 4*I)*f^3 + (-36*I*d^3*e*f^2 - 44*I*d^2*f^3))*x^2 + (-36*I*d^
3*e^2*f - 88*I*d^2*e*f^2))*x)*e^(4*d*x + 4*c) + 4*(d^3*f^3*x^3 + d^3*e^3 + 4
*d^2*e^2*f - 2*(8*c + 1)*d*e*f^2 + 2*(4*c^2 - 1)*f^3 + (3*d^3*e*f^2 - 4*d^2
*f^3))*x^2 + (3*d^3*e^2*f - 8*d^2*e*f^2 - 2*d*f^3))*x)*e^(3*d*x + 3*c) + (12*
I*d^3*f^3*x^3 + 12*I*d^3*e^3 - 44*I*d^2*e^2*f + 16*I*c*d*e*f^2 + (-8*I*c^2
+ 8*I)*f^3 + (36*I*d^3*e*f^2 - 36*I*d^2*f^3))*x^2 + (36*I*d^3*e^2*f - 72*I*d
^2*e*f^2))*x)*e^(2*d*x + 2*c) + 2*(3*d^3*f^3*x^3 + 3*d^3*e^3 - d^2*e^2*f - 2
*(8*c + 1)*d*e*f^2 + 2*(4*c^2 - 1)*f^3 + 9*(d^3*e*f^2 - d^2*f^3))*x^2 + (9*d
^3*e^2*f - 18*d^2*e*f^2 - 2*d*f^3))*x)*e^(d*x + c) + (-3*I*d^3*e^3 + 9*I*c*d
^2*e^2*f + (-9*I*c^2 + 12*I)*d*e*f^2 + (3*I*c^3 - 12*I*c)*f^3 + (3*I*d^3*e^
3 - 9*I*c*d^2*e^2*f + (9*I*c^2 - 12*I)*d*e*f^2 + (-3*I*c^3 + 12*I*c)*f^3))*e
^(6*d*x + 6*c) + 6*(d^3*e^3 - 3*c*d^2*e^2*f + (3*c^2 - 4)*d*e*f^2 - (c^3 -
4*c)*f^3))*e^(5*d*x + 5*c) + (3*I*d^3*e^3 - 9*I*c*d^2*e^2*f + (9*I*c^2 - 12*
I)*d*e*f^2 + (-3*I*c^3 + 12*I*c)*f^3))*e^(4*d*x + 4*c) + 12*(d^3*e^3 - 3*c*d
^2*e^2*f + (3*c^2 - 4)*d*e*f^2 - (c^3 - 4*c)*f^3))*e^(3*d*x + 3*c) + (-3*I*d
^3*e^3 + 9*I*c*d^2*e^2*f + (-9*I*c^2 + 12*I)*d*e*f^2 + (3*I*c^3 - 12*I*c)*f
^3))*e^(2*d*x + 2*c) + 6*(d^3*e^3 - 3*c*d^2*e^2*f + (3*c^2 - 4)*d*e*f^2 - (c
^3 - 4*c)*f^3))*e^(d*x + c))*log(e^(d*x + c) + I) + (3*I*d^3*e^3 - 9*I*c*d^2
*e^2*f + (9*I*c^2 - 28*I)*d*e*f^2 + (-3*I*c^3 + 28*I*c)*f^3 + (-3*I*d^3*e^3
+ 9*I*c*d^2*e^2*f + (-9*I*c^2 + 28*I)*d*e*f^2 + (3*I*c^3 - 28*I*c)*f^3))*e
^(6*d*x + 6*c) - 2*(3*d^3*e^3 - 9*c*d^2*e^2*f + (9*c^2 - 28)*d*e*f^2 - (3*c^
3 - 28*c)*f^3))*e^(5*d*x + 5*c) + (-3*I*d^3*e^3 + 9*I*c*d^2*e^2*f + (-9*I*c^
```

$$\begin{aligned}
& 2 + 28*I)*d*e*f^2 + (3*I*c^3 - 28*I*c)*f^3)*e^{(4*d*x + 4*c)} - 4*(3*d^3*e^3 \\
& - 9*c*d^2*e^2*f + (9*c^2 - 28)*d*e*f^2 - (3*c^3 - 28*c)*f^3)*e^{(3*d*x + 3*c)} \\
& ) + (3*I*d^3*e^3 - 9*I*c*d^2*e^2*f + (9*I*c^2 - 28*I)*d*e*f^2 + (-3*I*c^3 + \\
& 28*I*c)*f^3)*e^{(2*d*x + 2*c)} - 2*(3*d^3*e^3 - 9*c*d^2*e^2*f + (9*c^2 - 28) \\
& *d*e*f^2 - (3*c^3 - 28*c)*f^3)*e^{(d*x + c))*\log(e^{(d*x + c)} - I) + (3*I*d^3 \\
& *f^3*x^3 + 9*I*d^3*e*f^2*x^2 + 9*I*c*d^2*e^2*f - 9*I*c^2*d*e*f^2 + (3*I*c^3 \\
& - 28*I*c)*f^3 + (9*I*d^3*e^2*f - 28*I*d*f^3)*x + (-3*I*d^3*f^3*x^3 - 9*I*d \\
& ^3*e*f^2*x^2 - 9*I*c*d^2*e^2*f + 9*I*c^2*d*e*f^2 + (-3*I*c^3 + 28*I*c)*f^3 \\
& + (-9*I*d^3*e^2*f + 28*I*d*f^3)*x)*e^{(6*d*x + 6*c)} - 2*(3*d^3*f^3*x^3 + 9*d \\
& ^3*e*f^2*x^2 + 9*c*d^2*e^2*f - 9*c^2*d*e*f^2 + (3*c^3 - 28*c)*f^3 + (9*d^3*e \\
& e^2*f - 28*d*f^3)*x)*e^{(5*d*x + 5*c)} + (-3*I*d^3*f^3*x^3 - 9*I*d^3*e*f^2*x^ \\
& 2 - 9*I*c*d^2*e^2*f + 9*I*c^2*d*e*f^2 + (-3*I*c^3 + 28*I*c)*f^3 + (-9*I*d^3 \\
& *e^2*f + 28*I*d*f^3)*x)*e^{(4*d*x + 4*c)} - 4*(3*d^3*f^3*x^3 + 9*d^3*e*f^2*x^ \\
& 2 + 9*c*d^2*e^2*f - 9*c^2*d*e*f^2 + (3*c^3 - 28*c)*f^3 + (9*d^3*e^2*f - 28* \\
& d*f^3)*x)*e^{(3*d*x + 3*c)} + (3*I*d^3*f^3*x^3 + 9*I*d^3*e*f^2*x^2 + 9*I*c*d^ \\
& 2*e^2*f - 9*I*c^2*d*e*f^2 + (3*I*c^3 - 28*I*c)*f^3 + (9*I*d^3*e^2*f - 28*I* \\
& d*f^3)*x)*e^{(2*d*x + 2*c)} - 2*(3*d^3*f^3*x^3 + 9*d^3*e*f^2*x^2 + 9*c*d^2*e^ \\
& 2*f - 9*c^2*d*e*f^2 + (3*c^3 - 28*c)*f^3 + (9*d^3*e^2*f - 28*d*f^3)*x)*e^{(d \\
& *x + c))*\log(I*e^{(d*x + c)} + 1) + (-3*I*d^3*f^3*x^3 - 9*I*d^3*e*f^2*x^2 - 9 \\
& *I*c*d^2*e^2*f + 9*I*c^2*d*e*f^2 + (-3*I*c^3 + 12*I*c)*f^3 + (-9*I*d^3*e^2*f \\
& + 12*I*d*f^3)*x + (3*I*d^3*f^3*x^3 + 9*I*d^3*e*f^2*x^2 + 9*I*c*d^2*e^2*f \\
& - 9*I*c^2*d*e*f^2 + (3*I*c^3 - 12*I*c)*f^3 + (9*I*d^3*e^2*f - 12*I*d*f^3)*x) \\
& )*e^{(6*d*x + 6*c)} + 6*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*c*d^2*e^2*f - 3*c^ \\
& 2*d*e*f^2 + (c^3 - 4*c)*f^3 + (3*d^3*e^2*f - 4*d*f^3)*x)*e^{(5*d*x + 5*c)} + \\
& (3*I*d^3*f^3*x^3 + 9*I*d^3*e*f^2*x^2 + 9*I*c*d^2*e^2*f - 9*I*c^2*d*e*f^2 + \\
& (3*I*c^3 - 12*I*c)*f^3 + (9*I*d^3*e^2*f - 12*I*d*f^3)*x)*e^{(4*d*x + 4*c)} + \\
& 12*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + (c^3 - \\
& 4*c)*f^3 + (3*d^3*e^2*f - 4*d*f^3)*x)*e^{(3*d*x + 3*c)} + (-3*I*d^3*f^3*x^3 - \\
& 9*I*d^3*e*f^2*x^2 - 9*I*c*d^2*e^2*f + 9*I*c^2*d*e*f^2 + (-3*I*c^3 + 12*I*c) \\
& )*f^3 + (-9*I*d^3*e^2*f + 12*I*d*f^3)*x)*e^{(2*d*x + 2*c)} + 6*(d^3*f^3*x^3 + \\
& 3*d^3*e*f^2*x^2 + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + (c^3 - 4*c)*f^3 + (3*d^3 \\
& *e^2*f - 4*d*f^3)*x)*e^{(d*x + c))*\log(-I*e^{(d*x + c)} + 1) + (18*I*f^3*e^{(6* \\
& d*x + 6*c)} + 36*f^3*e^{(5*d*x + 5*c)} + 18*I*f^3*e^{(4*d*x + 4*c)} + 72*f^3*e^{( \\
& 3*d*x + 3*c)} - 18*I*f^3*e^{(2*d*x + 2*c)} + 36*f^3*e^{(d*x + c)} - 18*I*f^3)*po \\
& lylog(4, I*e^{(d*x + c)}) + (-18*I*f^3*e^{(6*d*x + 6*c)} - 36*f^3*e^{(5*d*x + 5* \\
& c)} - 18*I*f^3*e^{(4*d*x + 4*c)} - 72*f^3*e^{(3*d*x + 3*c)} + 18*I*f^3*e^{(2*d*x \\
& + 2*c)} - 36*f^3*e^{(d*x + c)} + 18*I*f^3)*polylog(4, -I*e^{(d*x + c)}) + (18*I* \\
& d*f^3*x + 18*I*d*e*f^2 + (-18*I*d*f^3*x - 18*I*d*e*f^2))*e^{(6*d*x + 6*c)} - 3 \\
& 6*(d*f^3*x + d*e*f^2)*e^{(5*d*x + 5*c)} + (-18*I*d*f^3*x - 18*I*d*e*f^2)*e^{(4 \\
& *d*x + 4*c)} - 72*(d*f^3*x + d*e*f^2)*e^{(3*d*x + 3*c)} + (18*I*d*f^3*x + 18*I \\
& *d*e*f^2)*e^{(2*d*x + 2*c)} - 36*(d*f^3*x + d*e*f^2)*e^{(d*x + c))*polylog(3, \\
& I*e^{(d*x + c)}) + (-18*I*d*f^3*x - 18*I*d*e*f^2 + (18*I*d*f^3*x + 18*I*d*e*f \\
& ^2))*e^{(6*d*x + 6*c)} + 36*(d*f^3*x + d*e*f^2)*e^{(5*d*x + 5*c)} + (18*I*d*f^3*x \\
& + 18*I*d*e*f^2)*e^{(4*d*x + 4*c)} + 72*(d*f^3*x + d*e*f^2)*e^{(3*d*x + 3*c)} \\
& + (-18*I*d*f^3*x - 18*I*d*e*f^2)*e^{(2*d*x + 2*c)} + 36*(d*f^3*x + d*e*f^2)*e
\end{aligned}$$



$$\begin{aligned} & \text{polylog}(3, -Ie^{(dx+c)}) / (8a^4d^4e^{(6dx+6c)} - 16Ia^4d^4e^{(5dx+5c)} + 8a^4d^4e^{(4dx+4c)} - 32Ia^4d^4e^{(3dx+3c)} - \\ & 8a^4d^4e^{(2dx+2c)} - 16Ia^4d^4e^{(dx+c)} - 8a^4d^4) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*sech(d\*x+c)\*\*3/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \operatorname{sech}(dx+c)^3}{ia \sinh(dx+c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*sech(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*sech(d\*x + c)^3/(I\*a\*sinh(d\*x + c) + a), x)

$$3.284 \quad \int \frac{(e+fx)^2 \operatorname{sech}^3(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=423

$$-\frac{3if(e+fx)\operatorname{PolyLog}(2,-ie^{c+dx})}{4ad^2} + \frac{3if(e+fx)\operatorname{PolyLog}(2,ie^{c+dx})}{4ad^2} + \frac{3if^2\operatorname{PolyLog}(3,-ie^{c+dx})}{4ad^3} - \frac{3if^2\operatorname{PolyLog}(3,ie^{c+dx})}{4ad^3}$$

```
[Out] (3*(e + f*x)^2*ArcTan[E^(c + d*x)])/(4*a*d) - (5*f^2*ArcTan[Sinh[c + d*x]])
/(6*a*d^3) + ((I/3)*f^2*Log[Cosh[c + d*x]])/(a*d^3) - (((3*I)/4)*f*(e + f*x)
)*PolyLog[2, (-I)*E^(c + d*x)]/(a*d^2) + (((3*I)/4)*f*(e + f*x)*PolyLog[2,
I*E^(c + d*x)]/(a*d^2) + (((3*I)/4)*f^2*PolyLog[3, (-I)*E^(c + d*x)]/(a*
d^3) - (((3*I)/4)*f^2*PolyLog[3, I*E^(c + d*x)]/(a*d^3) + (3*f*(e + f*x)*S
ech[c + d*x])/(4*a*d^2) - ((I/12)*f^2*Sech[c + d*x]^2)/(a*d^3) + (f*(e + f*
x)*Sech[c + d*x]^3)/(6*a*d^2) + ((I/4)*(e + f*x)^2*Sech[c + d*x]^4)/(a*d) -
((I/3)*f*(e + f*x)*Tanh[c + d*x])/(a*d^2) - (f^2*Sech[c + d*x]*Tanh[c + d*
x])/(12*a*d^3) + (3*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(8*a*d) - ((I/
6)*f*(e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(a*d^2) + ((e + f*x)^2*Sech[c
+ d*x]^3*Tanh[c + d*x])/(4*a*d)
```

**Rubi [A]** time = 0.39833, antiderivative size = 423, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$ , Rules used = {5571, 4186, 3768, 3770, 4180, 2531, 2282, 6589, 5451, 4185, 4184, 3475}

$$-\frac{3if(e+fx)\operatorname{PolyLog}(2,-ie^{c+dx})}{4ad^2} + \frac{3if(e+fx)\operatorname{PolyLog}(2,ie^{c+dx})}{4ad^2} + \frac{3if^2\operatorname{PolyLog}(3,-ie^{c+dx})}{4ad^3} - \frac{3if^2\operatorname{PolyLog}(3,ie^{c+dx})}{4ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x]^3)/(a + I*a*Sinh[c + d*x]),x]
```

```
[Out] (3*(e + f*x)^2*ArcTan[E^(c + d*x)])/(4*a*d) - (5*f^2*ArcTan[Sinh[c + d*x]])
/(6*a*d^3) + ((I/3)*f^2*Log[Cosh[c + d*x]])/(a*d^3) - (((3*I)/4)*f*(e + f*x)
)*PolyLog[2, (-I)*E^(c + d*x)]/(a*d^2) + (((3*I)/4)*f*(e + f*x)*PolyLog[2,
I*E^(c + d*x)]/(a*d^2) + (((3*I)/4)*f^2*PolyLog[3, (-I)*E^(c + d*x)]/(a*
d^3) - (((3*I)/4)*f^2*PolyLog[3, I*E^(c + d*x)]/(a*d^3) + (3*f*(e + f*x)*S
ech[c + d*x])/(4*a*d^2) - ((I/12)*f^2*Sech[c + d*x]^2)/(a*d^3) + (f*(e + f*
x)*Sech[c + d*x]^3)/(6*a*d^2) + ((I/4)*(e + f*x)^2*Sech[c + d*x]^4)/(a*d) -
((I/3)*f*(e + f*x)*Tanh[c + d*x])/(a*d^2) - (f^2*Sech[c + d*x]*Tanh[c + d*
x])/(12*a*d^3) + (3*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(8*a*d) - ((I/
6)*f*(e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(a*d^2) + ((e + f*x)^2*Sech[c
```

$$+ d*x]^3*\text{Tanh}[c + d*x]/(4*a*d)$$

### Rule 5571

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Sech[c + d\*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f\*x)^m\*Sech[c + d\*x]^(n + 1)\*Tanh[c + d\*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]

### Rule 4186

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(b^2\*(c + d\*x)^m\*Cot[e + f\*x]\*(b\*Csc[e + f\*x])^(n - 2))/(f\*(n - 1)), x] + (Dist[(b^2\*d^2\*m\*(m - 1))/(f^2\*(n - 1)\*(n - 2)), Int[(c + d\*x)^(m - 2)\*(b\*Csc[e + f\*x])^(n - 2), x], x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(c + d\*x)^m\*(b\*Csc[e + f\*x])^(n - 2), x], x] - Simp[(b^2\*d\*m\*(c + d\*x)^(m - 1)\*(b\*Csc[e + f\*x])^(n - 2))/(f^2\*(n - 1)\*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

### Rule 3768

Int[(csc[(c\_.) + (d\_.)\*(x\_)]\*(b\_.))^(n\_.), x\_Symbol] := -Simp[(b\*Cos[c + d\*x]\*(b\*Csc[c + d\*x])^(n - 1))/(d\*(n - 1)), x] + Dist[(b^2\*(n - 2))/(n - 1), Int[(b\*Csc[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

### Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x

))<sup>n</sup>)]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)<sup>(m - 1)</sup>\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))<sup>n</sup>], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :=> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 5451

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sech[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*Tanh[(a\_.) + (b\_.)\*(x\_)]^(p\_.), x\_Symbol] :=> -Simp[((c + d\*x)^m\*Sech[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Sech[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

### Rule 4185

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.))^(n\_)\*((c\_.) + (d\_.)\*(x\_)), x\_Symbol] :=> -Simp[(b^2\*(c + d\*x)\*Cot[e + f\*x]\*(b\*Csc[e + f\*x])^(n - 2))/(f\*(n - 1)), x] + (Dist[(b^2\*(n - 2))/(n - 1), Int[(c + d\*x)\*(b\*Csc[e + f\*x])^(n - 2), x], x] - Simp[(b^2\*d\*(b\*Csc[e + f\*x])^(n - 2))/(f^2\*(n - 1)\*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]

### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :=> -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :=> -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \operatorname{sech}^3(c+dx)}{a+ia \sinh(c+dx)} dx &= -\frac{i \int (e+fx)^2 \operatorname{sech}^4(c+dx) \tanh(c+dx) dx}{a} + \frac{\int (e+fx)^2 \operatorname{sech}^5(c+dx) dx}{a} \\
&= \frac{f(e+fx) \operatorname{sech}^3(c+dx)}{6ad^2} + \frac{i(e+fx)^2 \operatorname{sech}^4(c+dx)}{4ad} + \frac{(e+fx)^2 \operatorname{sech}^3(c+dx) \tanh(c+dx)}{4ad} \\
&= \frac{3f(e+fx) \operatorname{sech}(c+dx)}{4ad^2} - \frac{if^2 \operatorname{sech}^2(c+dx)}{12ad^3} + \frac{f(e+fx) \operatorname{sech}^3(c+dx)}{6ad^2} + \frac{i(e+fx)^2 \operatorname{sech}^4(c+dx)}{4ad} \\
&= \frac{3(e+fx)^2 \tan^{-1}(e^{c+dx})}{4ad} - \frac{5f^2 \tan^{-1}(\sinh(c+dx))}{6ad^3} + \frac{3f(e+fx) \operatorname{sech}(c+dx)}{4ad^2} - \frac{if^2 \operatorname{sech}^2(c+dx)}{12ad^3} \\
&= \frac{3(e+fx)^2 \tan^{-1}(e^{c+dx})}{4ad} - \frac{5f^2 \tan^{-1}(\sinh(c+dx))}{6ad^3} + \frac{if^2 \log(\cosh(c+dx))}{3ad^3} - \frac{3if(e+fx) \operatorname{sech}(c+dx)}{4ad} \\
&= \frac{3(e+fx)^2 \tan^{-1}(e^{c+dx})}{4ad} - \frac{5f^2 \tan^{-1}(\sinh(c+dx))}{6ad^3} + \frac{if^2 \log(\cosh(c+dx))}{3ad^3} - \frac{3if(e+fx) \operatorname{sech}(c+dx)}{4ad} \\
&= \frac{3(e+fx)^2 \tan^{-1}(e^{c+dx})}{4ad} - \frac{5f^2 \tan^{-1}(\sinh(c+dx))}{6ad^3} + \frac{if^2 \log(\cosh(c+dx))}{3ad^3} - \frac{3if(e+fx) \operatorname{sech}(c+dx)}{4ad}
\end{aligned}$$

**Mathematica [B]** time = 12.9861, size = 1284, normalized size = 3.04

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Sech[c + d\*x]^3)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] -((9\*d^2\*e^2 - 28\*f^2)\*x + 9\*d^2\*e\*f\*x^2 + 3\*d^2\*f^2\*x^3 + 18\*d\*e\*(1 + I\*E^c)\*f\*x\*Log[1 - I\*E^(-c - d\*x)] + 9\*d\*(1 + I\*E^c)\*f^2\*x^2\*Log[1 - I\*E^(-c - d\*x)] - ((1 + I\*E^c)\*(9\*d^2\*e^2 - 28\*f^2)\*(d\*x - Log[I - E^(c + d\*x)]))/d - 18\*e\*(1 + I\*E^c)\*f\*PolyLog[2, I\*E^(-c - d\*x)] - 18\*(1 + I\*E^c)\*f^2\*(x\*PolyLog[2, I\*E^(-c - d\*x)] + PolyLog[3, I\*E^(-c - d\*x)]/d))/(24\*a\*d^2\*(-I + E^c)) - (3\*d^2\*e^2\*x - 4\*f^2\*x + 3\*d^2\*e\*f\*x^2 + d^2\*f^2\*x^3 - (6\*I)\*d\*e\*f\*x\*Log[1 + I\*Cosh[c + d\*x] - I\*Sinh[c + d\*x]]\*(I + Cosh[c] + Sinh[c]) - (3\*I)\*d\*f^2\*x^2\*Log[1 + I\*Cosh[c + d\*x] - I\*Sinh[c + d\*x]]\*(I + Cosh[c] + Sinh[c]) + (I\*(3\*d^2\*e^2 - 4\*f^2)\*(d\*x - Log[I + Cosh[c + d\*x] + Sinh[c + d\*x]])\*(I + Cosh[c] + Sinh[c]))/d + (6\*I)\*e\*f\*PolyLog[2, (-I)\*(Cosh[c + d\*x] - Sinh[c + d\*x])]\*(I + Cosh[c] + Sinh[c]) + ((6\*I)\*f^2\*(d\*x\*PolyLog[2, (-I)\*(Cosh[c + d\*x] - Sinh[c + d\*x])]) + PolyLog[3, (-I)\*(Cosh[c + d\*x] - Sinh[c + d\*x])])\*(I + Cosh[c] + Sinh[c])/d)/(8\*a\*d^2\*(I + Cosh[c] + Sinh[c])) + ((3\*e^2\*x\*Cosh[c])/(4\*a) + (3\*e^2\*x\*Sinh[c])/(4\*a))/(1 + Cosh[2\*c] + Sinh[2\*c]) +

$$\begin{aligned} & \left( \frac{(3efx^2 \cosh[c])}{(4a)} + \frac{(3efx^2 \sinh[c])}{(4a)} \right) / (1 + \cosh[2c] + \sinh[2c]) \\ & + \left( \frac{(f^2 x^3 \cosh[c])}{(4a)} + \frac{(f^2 x^3 \sinh[c])}{(4a)} \right) / (1 + \cosh[2c] + \sinh[2c]) \\ & - \left( \frac{(I/8)(e^2 + 2efx + f^2 x^2)}{(a d (\cosh[c/2 + (d*x)/2] - I \sinh[c/2 + (d*x)/2])^2)} \right) \\ & + \left( \frac{(I/2)(ef \sinh[(d*x)/2] + f^2 x \sinh[(d*x)/2])}{(a d^2 (\cosh[c/2 - I \sinh[c/2]) (\cosh[c/2 + (d*x)/2] - I \sinh[c/2 + (d*x)/2])} \right) \\ & + \left( \frac{(I/8)(e^2 + 2efx + f^2 x^2)}{(a d (\cosh[c/2 + (d*x)/2] + I \sinh[c/2 + (d*x)/2])^4} \right) \\ & - \left( \frac{(I/6)(ef \sinh[(d*x)/2] + f^2 x \sinh[(d*x)/2])}{(a d^2 (\cosh[c/2 + I \sinh[c/2]) (\cosh[c/2 + (d*x)/2] + I \sinh[c/2 + (d*x)/2])^3} \right) \\ & + \left( \frac{(3I) d^2 e^2 \cosh[c/2] + d e f \cosh[c/2] - I f^2 \cosh[c/2] + (6I) d^2 e f x \cosh[c/2] + d f^2 x \cosh[c/2] + (3I) d^2 f^2 x^2 \cosh[c/2] - 3 d^2 e^2 \sinh[c/2] - I d e f \sinh[c/2] + f^2 \sinh[c/2] - 6 d^2 e f x \sinh[c/2] - I d f^2 x \sinh[c/2] - 3 d^2 f^2 x^2 \sinh[c/2]}{(12 a d^3 (\cosh[c/2] + I \sinh[c/2]) (\cosh[c/2 + (d*x)/2] + I \sinh[c/2 + (d*x)/2])^2} \right) \\ & - \left( \frac{((7I)/6)(ef \sinh[(d*x)/2] + f^2 x \sinh[(d*x)/2])}{(a d^2 (\cosh[c/2] + I \sinh[c/2]) (\cosh[c/2 + (d*x)/2] + I \sinh[c/2 + (d*x)/2])} \right) \end{aligned}$$

**Maple [B]** time = 0.208, size = 1044, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*sech(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x)

[Out]  $\frac{1}{12} \left( -2f^2 \exp(5dx+5c) - 4f^2 \exp(3dx+3c) - 2f^2 \exp(dx+c) - 44I d e f \exp(2dx+2c) + 12d^2 e f x \exp(3dx+3c) + 18d^2 e f x \exp(5dx+5c) + 6d^2 f^2 x^2 \exp(3dx+3c) + 9d^2 f^2 x^2 \exp(5dx+5c) + 18d f^2 x \exp(5dx+5c) + 9d^2 x^2 f^2 \exp(dx+c) - 2d f^2 x \exp(dx+c) - 2d e f \exp(dx+c) + 16d f^2 x \exp(3dx+3c) + 16d e f \exp(3dx+3c) - 36I d e f \exp(4dx+4c) - 18I d^2 f^2 x^2 \exp(4dx+4c) + 18I d^2 f^2 x^2 \exp(2dx+2c) - 44I d f^2 x \exp(2dx+2c) + 36I d^2 e f x \exp(2dx+2c) - 36I d^2 e f x \exp(4dx+4c) - 8I d f e - 8I d f^2 x + 6d^2 e^2 \exp(3dx+3c) + 9d^2 e^2 \exp(5dx+5c) + 18d^2 e f x \exp(dx+c) - 36I d f^2 x \exp(4dx+4c) + 9d^2 e^2 \exp(dx+c) + 18d e f \exp(5dx+5c) - 18I d^2 e^2 \exp(4dx+4c) + 18I d^2 e^2 \exp(2dx+2c) \right) / (\exp(dx+c) + I)^2 / (\exp(dx+c) - I)^4 / d^3 / a - 3/4 I / a / d^2 e f c \ln(\exp(dx+c) + I) - 3/8 I / a / d \ln(1 + I \exp(dx+c)) f^2 x^2 - 3/4 I / a / d^2 \operatorname{polylog}(2, -I \exp(dx+c)) f^2 x + 3/4 I / a / d^2 e f c \ln(\exp(dx+c) - I) + 3/8 I / a / d^3 \ln(1 + I \exp(dx+c)) c^2 f^2 + 3/8 I / a / d \ln(1 - I \exp(dx+c)) f^2 x^2 - 3/8 I / a / d e^2 \ln(\exp(dx+c) - I) - 3/4 I / a / d \ln(1 + I \exp(dx+c)) e f x + 3/4 I / a / d^2 \operatorname{polylog}(2, I \exp(dx+c)) f^2 x - 3/8 I / a / d^3 c^2 f^2 \ln(\exp(dx+c) - I) - 3/4 I f^2 \operatorname{polylog}(3, I \exp(dx+c)) / a / d^3 + 3/4 I / a / d^2 \ln(1 - I \exp(dx+c)) c e f - 3/4 I / a / d^2 \ln(1 + I \exp(dx+c)) c e f + 3/4 I f^2 \operatorname{polylog}(3, -I \exp(dx+c)) / a / d^3 + 3/4 I / a / d^2 e f \operatorname{polylog}(2, I \exp(dx+c))$

$$\begin{aligned} & *x+c)) - 2/3 * I/a/d^3 * f^2 * \ln(\exp(d*x+c)) - 3/4 * I/a/d^2 * e*f * \text{polylog}(2, -I*\exp(d*x+c)) \\ & - 3/8 * I/a/d^3 * \ln(1-I*\exp(d*x+c)) * c^2 * f^2 + 3/8 * I/a/d * e^2 * \ln(\exp(d*x+c)+I) + 7 \\ & /6 * I/a/d^3 * f^2 * \ln(\exp(d*x+c)-I) + 3/8 * I/a/d^3 * c^2 * f^2 * \ln(\exp(d*x+c)+I) - 1/2 * I/a/d^3 * f^2 * \ln(\exp(d*x+c)+I) \\ & + 3/4 * I/a/d * \ln(1-I*\exp(d*x+c)) * e*f*x \end{aligned}$$

**Maxima [B]** time = 3.73354, size = 1081, normalized size = 2.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sech(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/8 * e^2 * (64 * (3 * e^{(-d*x - c)} - 6 * I * e^{(-2*d*x - 2*c)} + 2 * e^{(-3*d*x - 3*c)} + 6 * I * e^{(-4*d*x - 4*c)} + 3 * e^{(-5*d*x - 5*c)}) / ((64 * I * a * e^{(-d*x - c)} - 32 * a * e^{(-2*d*x - 2*c)} + 128 * I * a * e^{(-3*d*x - 3*c)} + 32 * a * e^{(-4*d*x - 4*c)} + 64 * I * a * e^{(-5*d*x - 5*c)} + 32 * a * e^{(-6*d*x - 6*c)} - 32 * a) * d) + 3 * I * \log(e^{(-d*x - c)} + I) / (a * d) - 3 * I * \log(e^{(-d*x - c)} - I) / (a * d)) + (-8 * I * d * f^2 * x - 8 * I * d * e * f + (9 * d^2 * f^2 * x^2 * e^{(5*c)} + 18 * (d^2 * e * f + d * f^2) * x * e^{(5*c)} + 2 * (9 * d * e * f - f^2) * e^{(5*c)}) * e^{(5*d*x)} + (-18 * I * d^2 * f^2 * x^2 * e^{(4*c)} - 36 * I * d * e * f * e^{(4*c)} + (-36 * I * d^2 * e * f - 36 * I * d * f^2) * x * e^{(4*c)}) * e^{(4*d*x)} + 2 * (3 * d^2 * f^2 * x^2 * e^{(3*c)} + 2 * (3 * d^2 * e * f + 4 * d * f^2) * x * e^{(3*c)} + 2 * (4 * d * e * f - f^2) * e^{(3*c)}) * e^{(3*d*x)} + (18 * I * d^2 * f^2 * x^2 * e^{(2*c)} - 44 * I * d * e * f * e^{(2*c)} + (36 * I * d^2 * e * f - 44 * I * d * f^2) * x * e^{(2*c)}) * e^{(2*d*x)} + (9 * d^2 * f^2 * x^2 * e^c + 2 * (9 * d^2 * e * f - d * f^2) * x * e^c - 2 * (d * e * f + f^2) * e^c) * e^{(d*x)}) / (12 * a * d^3 * e^{(6*d*x + 6*c)} - 24 * I * a * d^3 * e^{(5*d*x + 5*c)} + 12 * a * d^3 * e^{(4*d*x + 4*c)} - 48 * I * a * d^3 * e^{(3*d*x + 3*c)} - 12 * a * d^3 * e^{(2*d*x + 2*c)} - 24 * I * a * d^3 * e^{(d*x + c)} - 12 * a * d^3) - 3/4 * I * (d*x * \log(I * e^{(d*x + c)} + 1) + \text{dilog}(-I * e^{(d*x + c)})) * e*f / (a*d^2) + 3/4 * I * (d*x * \log(-I * e^{(d*x + c)} + 1) + \text{dilog}(I * e^{(d*x + c)})) * e*f / (a*d^2) - 2/3 * I * f^2 * x / (a*d^2) - 3/8 * I * (d^2 * x^2 * \log(I * e^{(d*x + c)} + 1) + 2 * d * x * \text{dilog}(-I * e^{(d*x + c)}) - 2 * \text{polylog}(3, -I * e^{(d*x + c)})) * f^2 / (a*d^3) + 3/8 * I * (d^2 * x^2 * \log(-I * e^{(d*x + c)} + 1) + 2 * d * x * \text{dilog}(I * e^{(d*x + c)}) - 2 * \text{polylog}(3, I * e^{(d*x + c)})) * f^2 / (a*d^3) + 7/6 * I * f^2 * \log(I * e^{(d*x + c)} + 1) / (a*d^3) - 1/2 * I * f^2 * \log(I * e^{(d*x + c)} - 1) / (a*d^3) \end{aligned}$$

**Fricas [C]** time = 2.77009, size = 5145, normalized size = 12.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sech(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $(-16*I*d*e*f + 16*I*c*f^2 + (-18*I*d*f^2*x - 18*I*d*e*f + (18*I*d*f^2*x + 18*I*d*e*f)*e^{(6*d*x + 6*c)} + 36*(d*f^2*x + d*e*f)*e^{(5*d*x + 5*c)} + (18*I*d*f^2*x + 18*I*d*e*f)*e^{(4*d*x + 4*c)} + 72*(d*f^2*x + d*e*f)*e^{(3*d*x + 3*c)} + (-18*I*d*f^2*x - 18*I*d*e*f)*e^{(2*d*x + 2*c)} + 36*(d*f^2*x + d*e*f)*e^{(d*x + c)})*dilog(I*e^{(d*x + c)}) + (18*I*d*f^2*x + 18*I*d*e*f + (-18*I*d*f^2*x - 18*I*d*e*f)*e^{(6*d*x + 6*c)} - 36*(d*f^2*x + d*e*f)*e^{(5*d*x + 5*c)} + (-18*I*d*f^2*x - 18*I*d*e*f)*e^{(4*d*x + 4*c)} - 72*(d*f^2*x + d*e*f)*e^{(3*d*x + 3*c)} + (18*I*d*f^2*x + 18*I*d*e*f)*e^{(2*d*x + 2*c)} - 36*(d*f^2*x + d*e*f)*e^{(d*x + c)})*dilog(-I*e^{(d*x + c)}) + (-16*I*d*f^2*x - 16*I*c*f^2)*e^{(6*d*x + 6*c)} + 2*(9*d^2*f^2*x^2 + 9*d^2*e^2 + 18*d*e*f - 2*(8*c + 1)*f^2 + 2*(9*d^2*e*f + d*f^2)*x)*e^{(5*d*x + 5*c)} + (-36*I*d^2*f^2*x^2 - 36*I*d^2*e^2 - 72*I*d*e*f - 16*I*c*f^2 + (-72*I*d^2*e*f - 88*I*d*f^2)*x)*e^{(4*d*x + 4*c)} + 4*(3*d^2*f^2*x^2 + 3*d^2*e^2 + 8*d*e*f - 2*(8*c + 1)*f^2 + 2*(3*d^2*e*f - 4*d*f^2)*x)*e^{(3*d*x + 3*c)} + (36*I*d^2*f^2*x^2 + 36*I*d^2*e^2 - 88*I*d*e*f + 16*I*c*f^2 + (72*I*d^2*e*f - 72*I*d*f^2)*x)*e^{(2*d*x + 2*c)} + 2*(9*d^2*f^2*x^2 + 9*d^2*e^2 - 2*d*e*f - 2*(8*c + 1)*f^2 + 18*(d^2*e*f - d*f^2)*x)*e^{(d*x + c)} + (-9*I*d^2*e^2 + 18*I*c*d*e*f + (-9*I*c^2 + 12*I)*f^2 + (9*I*d^2*e^2 - 18*I*c*d*e*f + (9*I*c^2 - 12*I)*f^2)*e^{(6*d*x + 6*c)} + 6*(3*d^2*e^2 - 6*c*d*e*f + (3*c^2 - 4)*f^2)*e^{(5*d*x + 5*c)} + (9*I*d^2*e^2 - 18*I*c*d*e*f + (9*I*c^2 - 12*I)*f^2)*e^{(4*d*x + 4*c)} + 12*(3*d^2*e^2 - 6*c*d*e*f + (3*c^2 - 4)*f^2)*e^{(3*d*x + 3*c)} + (-9*I*d^2*e^2 + 18*I*c*d*e*f + (-9*I*c^2 + 12*I)*f^2)*e^{(2*d*x + 2*c)} + 6*(3*d^2*e^2 - 6*c*d*e*f + (3*c^2 - 4)*f^2)*e^{(d*x + c)})*log(e^{(d*x + c)} + I) + (9*I*d^2*e^2 - 18*I*c*d*e*f + (9*I*c^2 - 28*I)*f^2 + (-9*I*d^2*e^2 + 18*I*c*d*e*f + (-9*I*c^2 + 28*I)*f^2)*e^{(6*d*x + 6*c)} - 2*(9*d^2*e^2 - 18*c*d*e*f + (9*c^2 - 28)*f^2)*e^{(5*d*x + 5*c)} + (-9*I*d^2*e^2 + 18*I*c*d*e*f + (-9*I*c^2 + 28*I)*f^2)*e^{(4*d*x + 4*c)} - 4*(9*d^2*e^2 - 18*c*d*e*f + (9*c^2 - 28)*f^2)*e^{(3*d*x + 3*c)} + (9*I*d^2*e^2 - 18*I*c*d*e*f + (9*I*c^2 - 28*I)*f^2)*e^{(2*d*x + 2*c)} - 2*(9*d^2*e^2 - 18*c*d*e*f + (9*c^2 - 28)*f^2)*e^{(d*x + c)})*log(e^{(d*x + c)} - I) + (9*I*d^2*f^2*x^2 + 18*I*d^2*e*f*x + 18*I*c*d*e*f - 9*I*c^2*f^2 + (-9*I*d^2*f^2*x^2 - 18*I*d^2*e*f*x - 18*I*c*d*e*f + 9*I*c^2*f^2)*e^{(6*d*x + 6*c)} - 18*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^{(5*d*x + 5*c)} + (-9*I*d^2*f^2*x^2 - 18*I*d^2*e*f*x - 18*I*c*d*e*f + 9*I*c^2*f^2)*e^{(4*d*x + 4*c)} - 36*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^{(3*d*x + 3*c)} + (9*I*d^2*f^2*x^2 + 18*I*d^2*e*f*x + 18*I*c*d*e*f - 9*I*c^2*f^2)*e^{(2*d*x + 2*c)} - 18*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^{(d*x + c)})*log(I*e^{(d*x + c)} + 1) + (-9*I*d^2*f^2*x^2 - 18*I*d^2*e*f*x - 18*I*c*d*e*f + 9*I*c^2*f^2 + (9*I*d^2*f^2*x^2 + 18*I*d^2*e*f*x + 18*I*c*d*e*f - 9*I*c^2*f^2)*e^{(6*d*x + 6*c)} + 18*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^{(5*d*x + 5*c)} + (9*I*d^2*f^2*x^2 + 18*I*d^2*e*f*x + 18*I*c*d*e*f - 9*I*c^2*f^2)*e^{(4*d*x + 4*c)} + 36*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^{(3*d*x + 3*c)}$



```

+ (-9*I*d^2*f^2*x^2 - 18*I*d^2*e*f*x - 18*I*c*d*e*f + 9*I*c^2*f^2)*e^(2*d*x
+ 2*c) + 18*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*e^(d*x + c))
*log(-I*e^(d*x + c) + 1) + (-18*I*f^2*e^(6*d*x + 6*c) - 36*f^2*e^(5*d*x + 5
*c) - 18*I*f^2*e^(4*d*x + 4*c) - 72*f^2*e^(3*d*x + 3*c) + 18*I*f^2*e^(2*d*x
+ 2*c) - 36*f^2*e^(d*x + c) + 18*I*f^2)*polylog(3, I*e^(d*x + c)) + (18*I
f^2*e^(6*d*x + 6*c) + 36*f^2*e^(5*d*x + 5*c) + 18*I*f^2*e^(4*d*x + 4*c) + 7
2*f^2*e^(3*d*x + 3*c) - 18*I*f^2*e^(2*d*x + 2*c) + 36*f^2*e^(d*x + c) - 18*
I*f^2)*polylog(3, -I*e^(d*x + c)))/(24*a*d^3*e^(6*d*x + 6*c) - 48*I*a*d^3*e
^(5*d*x + 5*c) + 24*a*d^3*e^(4*d*x + 4*c) - 96*I*a*d^3*e^(3*d*x + 3*c) - 24
*a*d^3*e^(2*d*x + 2*c) - 48*I*a*d^3*e^(d*x + c) - 24*a*d^3)

```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)**3/(a+I*a*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \operatorname{sech}(dx + c)^3}{ia \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sech(d*x + c)^3/(I*a*sinh(d*x + c) + a), x)
```

$$3.285 \quad \int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=233

$$-\frac{3if\operatorname{PolyLog}(2, -ie^{c+dx})}{8ad^2} + \frac{3if\operatorname{PolyLog}(2, ie^{c+dx})}{8ad^2} + \frac{if \tanh^3(c+dx)}{12ad^2} - \frac{if \tanh(c+dx)}{4ad^2} + \frac{f\operatorname{sech}^3(c+dx)}{12ad^2} + \frac{3f\operatorname{sech}}{8a}$$

[Out] (3\*(e + f\*x)\*ArcTan[E^(c + d\*x)])/(4\*a\*d) - (((3\*I)/8)\*f\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*d^2) + (((3\*I)/8)\*f\*PolyLog[2, I\*E^(c + d\*x)])/(a\*d^2) + (3\*f\*Sech[c + d\*x])/(8\*a\*d^2) + (f\*Sech[c + d\*x]^3)/(12\*a\*d^2) + ((I/4)\*(e + f\*x)\*Sech[c + d\*x]^4)/(a\*d) - ((I/4)\*f\*Tanh[c + d\*x])/(a\*d^2) + (3\*(e + f\*x)\*Sech[c + d\*x]\*Tanh[c + d\*x])/(8\*a\*d) + ((e + f\*x)\*Sech[c + d\*x]^3\*Tanh[c + d\*x])/(4\*a\*d) + ((I/12)\*f\*Tanh[c + d\*x]^3)/(a\*d^2)

**Rubi [A]** time = 0.189825, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$ , Rules used = {5571, 4185, 4180, 2279, 2391, 5451, 3767}

$$-\frac{3if\operatorname{PolyLog}(2, -ie^{c+dx})}{8ad^2} + \frac{3if\operatorname{PolyLog}(2, ie^{c+dx})}{8ad^2} + \frac{if \tanh^3(c+dx)}{12ad^2} - \frac{if \tanh(c+dx)}{4ad^2} + \frac{f\operatorname{sech}^3(c+dx)}{12ad^2} + \frac{3f\operatorname{sech}}{8a}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Sech[c + d\*x]^3)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (3\*(e + f\*x)\*ArcTan[E^(c + d\*x)])/(4\*a\*d) - (((3\*I)/8)\*f\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*d^2) + (((3\*I)/8)\*f\*PolyLog[2, I\*E^(c + d\*x)])/(a\*d^2) + (3\*f\*Sech[c + d\*x])/(8\*a\*d^2) + (f\*Sech[c + d\*x]^3)/(12\*a\*d^2) + ((I/4)\*(e + f\*x)\*Sech[c + d\*x]^4)/(a\*d) - ((I/4)\*f\*Tanh[c + d\*x])/(a\*d^2) + (3\*(e + f\*x)\*Sech[c + d\*x]\*Tanh[c + d\*x])/(8\*a\*d) + ((e + f\*x)\*Sech[c + d\*x]^3\*Tanh[c + d\*x])/(4\*a\*d) + ((I/12)\*f\*Tanh[c + d\*x]^3)/(a\*d^2)

### Rule 5571

Int[(((e\_.) + (f\_.)\*(x\_.))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_.)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Sech[c + d\*x]^(n + 2), x], x] + Dist[1/b, Int[(e + f\*x)^m\*Sech[c + d\*x]^(n + 1)\*Tanh[c + d\*x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && EqQ[a^2 + b^2, 0]

### Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :=
-Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^ (m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] := -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+ia\sinh(c+dx)} dx &= -\frac{i\int(e+fx)\operatorname{sech}^4(c+dx)\tanh(c+dx)dx}{a} + \frac{\int(e+fx)\operatorname{sech}^5(c+dx)dx}{a} \\
&= \frac{f\operatorname{sech}^3(c+dx)}{12ad^2} + \frac{i(e+fx)\operatorname{sech}^4(c+dx)}{4ad} + \frac{(e+fx)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4ad} + \frac{3\int(e+fx)\operatorname{sech}^5(c+dx)dx}{12ad^2} \\
&= \frac{3f\operatorname{sech}(c+dx)}{8ad^2} + \frac{f\operatorname{sech}^3(c+dx)}{12ad^2} + \frac{i(e+fx)\operatorname{sech}^4(c+dx)}{4ad} + \frac{3(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{8ad} \\
&= \frac{3(e+fx)\tan^{-1}(e^{c+dx})}{4ad} + \frac{3f\operatorname{sech}(c+dx)}{8ad^2} + \frac{f\operatorname{sech}^3(c+dx)}{12ad^2} + \frac{i(e+fx)\operatorname{sech}^4(c+dx)}{4ad} - \frac{3(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{8ad} \\
&= \frac{3(e+fx)\tan^{-1}(e^{c+dx})}{4ad} + \frac{3f\operatorname{sech}(c+dx)}{8ad^2} + \frac{f\operatorname{sech}^3(c+dx)}{12ad^2} + \frac{i(e+fx)\operatorname{sech}^4(c+dx)}{4ad} - \frac{3(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{8ad} \\
&= \frac{3(e+fx)\tan^{-1}(e^{c+dx})}{4ad} - \frac{3i\operatorname{Li}_2(-ie^{c+dx})}{8ad^2} + \frac{3i\operatorname{Li}_2(ie^{c+dx})}{8ad^2} + \frac{3f\operatorname{sech}(c+dx)}{8ad^2} + \frac{f\operatorname{sech}^3(c+dx)}{12ad^2}
\end{aligned}$$

**Mathematica [B]** time = 6.66753, size = 1290, normalized size = 5.54

result too large to display

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Sech[c + d\*x]^3)/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] ((I/24)\*(6\*d\*e - I\*f - 6\*c\*f + 6\*f\*(c + d\*x)))/(d^2\*(a + I\*a\*Sinh[c + d\*x])) + ((I/8)\*(d\*e - c\*f + f\*(c + d\*x)))/(d^2\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^2\*(a + I\*a\*Sinh[c + d\*x])) + (3\*(c + d\*x)\*(2\*d\*e - 2\*c\*f + f\*(c + d\*x))\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^2)/(16\*d^2\*(a + I\*a\*Sinh[c + d\*x])) + (((3\*I)/8)\*e\*((I/2)\*(c + d\*x) + Log[Cosh[(c + d\*x)/2] - I\*Sinh[(c + d\*x)/2]])\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^2)/(d\*(a + I\*a\*Sinh[c + d\*x])) - (((3\*I)/8)\*c\*f\*((I/2)\*(c + d\*x) + Log[Cosh[(c + d\*x)/2] - I\*Sinh[(c + d\*x)/2]])\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^2)/(d^2\*(a + I\*a\*Sinh[c + d\*x])) - (((3\*I)/8)\*e\*((-I/2)\*(c + d\*x) + Log[Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]])\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^2)/(d\*(a + I\*a\*Sinh[c + d\*x])) + (((3\*I)/8)\*c\*f\*((-I/2)\*(c + d\*x) + Log[Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2]])\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^2)/(d^2\*(a + I\*a\*Sinh[c + d\*x])) + (3\*f\*(-(c + d\*x)^2/(4\*E^((I/4)\*Pi)) - ((3\*Pi\*(c + d\*x))/4 - Pi\*Log[1 + E^(c + d\*x)] - 2\*(-Pi/4 + (I/2)\*(c + d\*x))\*Log[1 - E^((2\*I)\*(-Pi/4 + (I/2)\*(c + d\*x)))] + Pi\*Log[Cosh[(c + d\*x)/2]] - (Pi\*Log[-Sin[Pi/4 - (I/2)\*(c + d\*x)]]))/2 + I\*PolyLog[2, E^((2\*I)\*(-Pi/4 + (I/2)\*(c + d\*x)))]/Sqrt[2])\*(Cosh[(c + d\*x)/2] + I\*Sinh[(c + d\*x)/2])^2)/(4\*Sqrt[2]\*d^2\*(a + I\*a\*Sinh[c + d\*x])) + (3\*f\*(-(E^((I/4)\*Pi)\*(c + d\*x)^2)/4 +

$$\begin{aligned} & \left( \frac{\pi(c+dx)}{4} - \pi \log[1 + E^{(c+dx)}] - 2\left(\frac{\pi}{4} + \frac{I}{2}(c+dx)\right) \cdot \right. \\ & \log[1 - E^{((2I)(\pi/4 + (I/2)(c+dx)))] + \pi \log[\cosh[(c+dx)/2]] + \left. \left( \pi \log[\sin[\pi/4 + (I/2)(c+dx)]] \right) / 2 + I \cdot \text{PolyLog}[2, E^{((2I)(\pi/4 + (I/2)(c+dx)))] \right] / \sqrt{2} \cdot \left( \cosh[(c+dx)/2] + I \sinh[(c+dx)/2] \right)^2 / \left( 4 \sqrt{2} \cdot d^2 \cdot (a + I \sinh[c+dx]) \right) - \left( \frac{I}{8} \cdot (d \cdot e - c \cdot f + f \cdot (c+dx)) \cdot \left( \cosh[(c+dx)/2] + I \sinh[(c+dx)/2] \right)^2 / \left( d^2 \cdot \left( \cosh[(c+dx)/2] - I \sinh[(c+dx)/2] \right)^2 \cdot (a + I \sinh[c+dx]) \right) - \left( \frac{I}{12} \cdot f \cdot \sinh[(c+dx)/2] \right) / \left( d^2 \cdot \left( \cosh[(c+dx)/2] + I \sinh[(c+dx)/2] \right) \cdot (a + I \sinh[c+dx]) \right) - \left( \frac{7I}{12} \cdot f \cdot \left( \cosh[(c+dx)/2] + I \sinh[(c+dx)/2] \right) \cdot \sinh[(c+dx)/2] \right) / \left( d^2 \cdot (a + I \sinh[c+dx]) \right) + \left( \frac{I}{4} \cdot f \cdot \left( \cosh[(c+dx)/2] + I \sinh[(c+dx)/2] \right) \right) \cdot \sinh[(c+dx)/2] / \left( d^2 \cdot \left( \cosh[(c+dx)/2] - I \sinh[(c+dx)/2] \right) \cdot (a + I \sinh[c+dx]) \right) \end{aligned}$$

**Maple [B]** time = 0.213, size = 445, normalized size = 1.9

$$\frac{9 f e^{5dx+5c} + 8 f e^{3dx+3c} - f e^{dx+c} - 18 i f e^{4dx+4c} - 18 i d f x e^{4dx+4c} - 18 i d e e^{4dx+4c} - 22 i f e^{2dx+2c} + 6 d e e^{3dx+3c} + 9 d f x e^{2dx+2c}}{12 (e^{dx+c} + i)^2 (e^{dx+c} - i)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)\*sech(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x)

[Out]  $\frac{1}{12} \cdot (9 \cdot f \cdot \exp(5 \cdot d \cdot x + 5 \cdot c) + 8 \cdot f \cdot \exp(3 \cdot d \cdot x + 3 \cdot c) - f \cdot \exp(d \cdot x + c) - 18 \cdot I \cdot f \cdot \exp(4 \cdot d \cdot x + 4 \cdot c) - 18 \cdot I \cdot d \cdot f \cdot x \cdot \exp(4 \cdot d \cdot x + 4 \cdot c) - 18 \cdot I \cdot d \cdot e \cdot \exp(4 \cdot d \cdot x + 4 \cdot c) - 22 \cdot I \cdot f \cdot \exp(2 \cdot d \cdot x + 2 \cdot c) + 6 \cdot d \cdot e \cdot \exp(3 \cdot d \cdot x + 3 \cdot c) + 9 \cdot d \cdot f \cdot x \cdot \exp(5 \cdot d \cdot x + 5 \cdot c) + 6 \cdot d \cdot f \cdot x \cdot \exp(3 \cdot d \cdot x + 3 \cdot c) + 18 \cdot I \cdot d \cdot f \cdot x \cdot \exp(2 \cdot d \cdot x + 2 \cdot c) - 4 \cdot I \cdot f + 9 \cdot d \cdot f \cdot x \cdot \exp(d \cdot x + c) + 18 \cdot I \cdot d \cdot e \cdot \exp(2 \cdot d \cdot x + 2 \cdot c) + 9 \cdot d \cdot e \cdot \exp(5 \cdot d \cdot x + 5 \cdot c) + 9 \cdot d \cdot e \cdot \exp(d \cdot x + c)) / (\exp(d \cdot x + c) + I)^2 / (\exp(d \cdot x + c) - I)^4 / d^2 / a - 3/8 \cdot I / a / d \cdot e \cdot \ln(\exp(d \cdot x + c) - I) + 3/8 \cdot I / a / d \cdot e \cdot \ln(\exp(d \cdot x + c) + I) - 3/8 \cdot I / a / d \cdot f \cdot \ln(1 + I \cdot \exp(d \cdot x + c)) \cdot x - 3/8 \cdot I / a / d^2 \cdot f \cdot \ln(1 + I \cdot \exp(d \cdot x + c)) \cdot c - 3/8 \cdot I \cdot f \cdot \text{polylog}(2, -I \cdot \exp(d \cdot x + c)) / a / d^2 + 3/8 \cdot I / a / d \cdot f \cdot \ln(1 - I \cdot \exp(d \cdot x + c)) \cdot x + 3/8 \cdot I / a / d^2 \cdot f \cdot \ln(1 - I \cdot \exp(d \cdot x + c)) \cdot c + 3/8 \cdot I \cdot f \cdot \text{polylog}(2, I \cdot \exp(d \cdot x + c)) / a / d^2 + 3/8 \cdot I / a / d^2 \cdot f \cdot c \cdot \ln(\exp(d \cdot x + c) - I) - 3/8 \cdot I / a / d^2 \cdot f \cdot c \cdot \ln(\exp(d \cdot x + c) + I)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$8 f \left( \frac{9 (d x e^{(5c)} + e^{(5c)}) e^{(5dx)} + (-18 i d x e^{(4c)} - 18 i e^{(4c)}) e^{(4dx)} + 2 (3 d x e^{(3c)} + 4 e^{(3c)}) e^{(3dx)} + (18 i d x e^{(2c)} - 22 i e^{(2c)}) e^{(2dx)}}{96 a d^2 e^{(6dx+6c)} - 192 i a d^2 e^{(5dx+5c)} + 96 a d^2 e^{(4dx+4c)} - 384 i a d^2 e^{(3dx+3c)} - 96 a d^2 e^{(2dx+2c)} - 192 i a d^2 e^{(dx+c)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 8*f*((9*(d*x*e^(5*c) + e^(5*c))*e^(5*d*x) + (-18*I*d*x*e^(4*c) - 18*I*e^(4*c))
*e^(4*d*x) + 2*(3*d*x*e^(3*c) + 4*e^(3*c))*e^(3*d*x) + (18*I*d*x*e^(2*c)
- 22*I*e^(2*c))*e^(2*d*x) + (9*d*x*e^c - e^c)*e^(d*x) - 4*I)/(96*a*d^2*e^(
6*d*x + 6*c) - 192*I*a*d^2*e^(5*d*x + 5*c) + 96*a*d^2*e^(4*d*x + 4*c) - 384
*I*a*d^2*e^(3*d*x + 3*c) - 96*a*d^2*e^(2*d*x + 2*c) - 192*I*a*d^2*e^(d*x +
c) - 96*a*d^2) + 3*integrate(1/64*x/(a*e^(d*x + c) + I*a), x) + 3*integrate
(1/64*x/(a*e^(d*x + c) - I*a), x) - 1/8*e*(64*(3*e^(-d*x - c) - 6*I*e^(-2*
d*x - 2*c) + 2*e^(-3*d*x - 3*c) + 6*I*e^(-4*d*x - 4*c) + 3*e^(-5*d*x - 5*c)
)/((64*I*a*e^(-d*x - c) - 32*a*e^(-2*d*x - 2*c) + 128*I*a*e^(-3*d*x - 3*c)
+ 32*a*e^(-4*d*x - 4*c) + 64*I*a*e^(-5*d*x - 5*c) + 32*a*e^(-6*d*x - 6*c) -
32*a)*d) + 3*I*log(e^(-d*x - c) + I)/(a*d) - 3*I*log(e^(-d*x - c) - I)/(a*
d))
```

---

**Fricas [B]** time = 2.44546, size = 2423, normalized size = 10.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((9*I*f*e^(6*d*x + 6*c) + 18*f*e^(5*d*x + 5*c) + 9*I*f*e^(4*d*x + 4*c) + 36
*f*e^(3*d*x + 3*c) - 9*I*f*e^(2*d*x + 2*c) + 18*f*e^(d*x + c) - 9*I*f)*dilo
g(I*e^(d*x + c)) + (-9*I*f*e^(6*d*x + 6*c) - 18*f*e^(5*d*x + 5*c) - 9*I*f*e
^(4*d*x + 4*c) - 36*f*e^(3*d*x + 3*c) + 9*I*f*e^(2*d*x + 2*c) - 18*f*e^(d*x
+ c) + 9*I*f)*dilog(-I*e^(d*x + c)) + 18*(d*f*x + d*e + f)*e^(5*d*x + 5*c)
+ (-36*I*d*f*x - 36*I*d*e - 36*I*f)*e^(4*d*x + 4*c) + 4*(3*d*f*x + 3*d*e +
4*f)*e^(3*d*x + 3*c) + (36*I*d*f*x + 36*I*d*e - 44*I*f)*e^(2*d*x + 2*c) +
2*(9*d*f*x + 9*d*e - f)*e^(d*x + c) + (-9*I*d*e + 9*I*c*f + (9*I*d*e - 9*I*
c*f)*e^(6*d*x + 6*c) + 18*(d*e - c*f)*e^(5*d*x + 5*c) + (9*I*d*e - 9*I*c*f)
*e^(4*d*x + 4*c) + 36*(d*e - c*f)*e^(3*d*x + 3*c) + (-9*I*d*e + 9*I*c*f)*e^
(2*d*x + 2*c) + 18*(d*e - c*f)*e^(d*x + c))*log(e^(d*x + c) + I) + (9*I*d*e
- 9*I*c*f + (-9*I*d*e + 9*I*c*f)*e^(6*d*x + 6*c) - 18*(d*e - c*f)*e^(5*d*x
+ 5*c) + (-9*I*d*e + 9*I*c*f)*e^(4*d*x + 4*c) - 36*(d*e - c*f)*e^(3*d*x +
3*c) + (9*I*d*e - 9*I*c*f)*e^(2*d*x + 2*c) - 18*(d*e - c*f)*e^(d*x + c))*lo
g(e^(d*x + c) - I) + (9*I*d*f*x + 9*I*c*f + (-9*I*d*f*x - 9*I*c*f)*e^(6*d*x
+ 6*c) - 18*(d*f*x + c*f)*e^(5*d*x + 5*c) + (-9*I*d*f*x - 9*I*c*f)*e^(4*d*
x + 4*c) - 36*(d*f*x + c*f)*e^(3*d*x + 3*c) + (9*I*d*f*x + 9*I*c*f)*e^(2*d*
x + 2*c) - 18*(d*f*x + c*f)*e^(d*x + c))*log(I*e^(d*x + c) + 1) + (-9*I*d*f
*x - 9*I*c*f + (9*I*d*f*x + 9*I*c*f)*e^(6*d*x + 6*c) + 18*(d*f*x + c*f)*e^(
```

$5dx + 5c) + (9I*df*x + 9I*c*f)*e^{(4dx + 4c)} + 36*(df*x + c*f)*e^{(3dx + 3c)} + (-9I*df*x - 9I*c*f)*e^{(2dx + 2c)} + 18*(df*x + c*f)*e^{(dx + c)} * \log(-I*e^{(dx + c)} + 1) - 8*I*f)/(24*a*d^2*e^{(6dx + 6c)} - 48*I*a*d^2*e^{(5dx + 5c)} + 24*a*d^2*e^{(4dx + 4c)} - 96*I*a*d^2*e^{(3dx + 3c)} - 24*a*d^2*e^{(2dx + 2c)} - 48*I*a*d^2*e^{(dx + c)} - 24*a*d^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(dx+c)\*\*3/(a+I\*a\*sinh(dx+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \operatorname{sech}(dx + c)^3}{i a \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(dx+c)^3/(a+I\*a\*sinh(dx+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)\*sech(dx + c)^3/(I\*a\*sinh(dx + c) + a), x)

$$3.286 \quad \int \frac{\operatorname{sech}^3(c+dx)}{a+ia \sinh(c+dx)} dx$$

**Optimal.** Leaf size=91

$$\frac{ia}{8d(a+ia \sinh(c+dx))^2} - \frac{i}{8d(a-ia \sinh(c+dx))} + \frac{i}{4d(a+ia \sinh(c+dx))} + \frac{3 \tan^{-1}(\sinh(c+dx))}{8ad}$$

[Out] (3\*ArcTan[Sinh[c + d\*x]])/(8\*a\*d) - (I/8)/(d\*(a - I\*a\*Sinh[c + d\*x])) + ((I/8)\*a)/(d\*(a + I\*a\*Sinh[c + d\*x])^2) + (I/4)/(d\*(a + I\*a\*Sinh[c + d\*x]))

**Rubi [A]** time = 0.0811465, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {2667, 44, 206}

$$\frac{ia}{8d(a+ia \sinh(c+dx))^2} - \frac{i}{8d(a-ia \sinh(c+dx))} + \frac{i}{4d(a+ia \sinh(c+dx))} + \frac{3 \tan^{-1}(\sinh(c+dx))}{8ad}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d\*x]^3/(a + I\*a\*Sinh[c + d\*x]),x]

[Out] (3\*ArcTan[Sinh[c + d\*x]])/(8\*a\*d) - (I/8)/(d\*(a - I\*a\*Sinh[c + d\*x])) + ((I/8)\*a)/(d\*(a + I\*a\*Sinh[c + d\*x])^2) + (I/4)/(d\*(a + I\*a\*Sinh[c + d\*x]))

### Rule 2667

Int[cos[(e\_.) + (f\_.)\*(x\_)]^(p\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(m\_.), x\_Symbol] :> Dist[1/(b^p\*f), Subst[Int[(a + x)^(m + (p - 1)/2)\*(a - x)^((p - 1)/2), x], x, b\*Sin[e + f\*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] || !IntegerQ[m + 1/2])

### Rule 44

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

### Rule 206



```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}^3(c+dx)}{a+ia \sinh(c+dx)} dx &= -\frac{(ia^3) \operatorname{Subst}\left(\int \frac{1}{(a-x)^2(a+x)^3} dx, x, ia \sinh(c+dx)\right)}{d} \\ &= -\frac{(ia^3) \operatorname{Subst}\left(\int \left(\frac{1}{8a^3(a-x)^2} + \frac{1}{4a^2(a+x)^3} + \frac{1}{4a^3(a+x)^2} + \frac{3}{8a^3(a^2-x^2)}\right) dx, x, ia \sinh(c+dx)\right)}{d} \\ &= -\frac{i}{8d(a-ia \sinh(c+dx))} + \frac{ia}{8d(a+ia \sinh(c+dx))^2} + \frac{i}{4d(a+ia \sinh(c+dx))} - \frac{(3i) \operatorname{Subst}\left(\int \frac{1}{8a^3(a-x)^2} dx, x, ia \sinh(c+dx)\right)}{d} \\ &= \frac{3 \tan^{-1}(\sinh(c+dx))}{8ad} - \frac{i}{8d(a-ia \sinh(c+dx))} + \frac{ia}{8d(a+ia \sinh(c+dx))^2} + \frac{i}{4d(a+ia \sinh(c+dx))} \end{aligned}$$

**Mathematica [A]** time = 0.10488, size = 101, normalized size = 1.11

$$\frac{\operatorname{sech}^2(c+dx) \left(3 \sinh^3(c+dx) \tan^{-1}(\sinh(c+dx)) + \sinh^2(c+dx) \left(3 - 3i \tan^{-1}(\sinh(c+dx))\right) + 3 \sinh(c+dx) \left(\tan^{-1}(\sinh(c+dx)) - i\right)\right)}{8ad(\sinh(c+dx) - i)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + I*a*Sinh[c + d*x]), x]
```

```
[Out] (Sech[c + d*x]^2*(2 - (3*I)*ArcTan[Sinh[c + d*x]] + 3*(-I + ArcTan[Sinh[c + d*x]])*Sinh[c + d*x] + (3 - (3*I)*ArcTan[Sinh[c + d*x]])*Sinh[c + d*x]^2 + 3*ArcTan[Sinh[c + d*x]]*Sinh[c + d*x]^3)/(8*a*d*(-I + Sinh[c + d*x]))
```

**Maple [B]** time = 0.056, size = 180, normalized size = 2.

$$\frac{i}{4} \frac{d}{da} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + i \right)^{-2} + \frac{3i}{8} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + i\right) - \frac{1}{4} \frac{d}{da} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + i \right)^{-1} + \frac{i}{2} \frac{d}{da} \left( -i + \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^{-4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x)`

[Out]  $\frac{1}{4} \frac{I}{d} \frac{a}{a} (\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+I)^2 + \frac{3}{8} \frac{I}{d} \frac{a}{a} \ln(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+I) - \frac{1}{4} \frac{I}{d} \frac{a}{a} (\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+I) + \frac{1}{2} \frac{I}{d} \frac{a}{a} (-I+\tanh(\frac{1}{2}d*x+\frac{1}{2}c))^4 - \frac{3}{8} \frac{I}{d} \frac{a}{a} \ln(-I+\tanh(\frac{1}{2}d*x+\frac{1}{2}c)) - \frac{3}{2} \frac{I}{d} \frac{a}{a} (-I+\tanh(\frac{1}{2}d*x+\frac{1}{2}c))^2 + \frac{1}{d} \frac{a}{a} (-I+\tanh(\frac{1}{2}d*x+\frac{1}{2}c))^3 - \frac{1}{d} \frac{a}{a} (-I+\tanh(\frac{1}{2}d*x+\frac{1}{2}c))$

**Maxima [B]** time = 1.14857, size = 243, normalized size = 2.67

$$\frac{8 \left( 3e^{(-dx-c)} - 6ie^{(-2dx-2c)} + 2e^{(-3dx-3c)} + 6ie^{(-4dx-4c)} + 3e^{(-5dx-5c)} \right)}{(64iae^{(-dx-c)} - 32ae^{(-2dx-2c)} + 128iae^{(-3dx-3c)} + 32ae^{(-4dx-4c)} + 64iae^{(-5dx-5c)} + 32ae^{(-6dx-6c)} - 32a)d} - \frac{3i \log(e^{(-dx-c)} + I)}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out]  $-8 \cdot (3e^{-dx-c} - 6Ie^{-2dx-2c} + 2e^{-3dx-3c} + 6Ie^{-4dx-4c} + 3e^{-5dx-5c}) / ((64Iae^{-dx-c} - 32ae^{-2dx-2c} + 128Iae^{-3dx-3c} + 32ae^{-4dx-4c} + 64Iae^{-5dx-5c} + 32ae^{-6dx-6c} - 32a) \cdot d) - \frac{3}{8} \frac{I \cdot \log(e^{-dx-c} + I)}{a \cdot d} + \frac{3}{8} \frac{I \cdot \log(e^{-dx-c} - I)}{a \cdot d}$

**Fricas [B]** time = 2.10379, size = 763, normalized size = 8.38

$$\frac{(3ie^{(6dx+6c)} + 6e^{(5dx+5c)} + 3ie^{(4dx+4c)} + 12e^{(3dx+3c)} - 3ie^{(2dx+2c)} + 6e^{(dx+c)} - 3i) \log(e^{(dx+c)} + i) + (-3ie^{(6dx+6c)} - 6e^{(5dx+5c)} - 3ie^{(4dx+4c)} - 12e^{(3dx+3c)} + 3ie^{(2dx+2c)} + 6e^{(dx+c)} - 3i) \log(e^{(dx+c)} - i)}{8ade^{(6dx+6c)} - 16iade^{(5dx+5c)} + 8ade^{(4dx+4c)} - 16iade^{(3dx+3c)} + 8ade^{(2dx+2c)} - 6ade^{(dx+c)} - 3i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^3/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")`

[Out]  $((3Ie^{(6dx+6c)} + 6e^{(5dx+5c)} + 3Ie^{(4dx+4c)} + 12e^{(3dx+3c)} - 3Ie^{(2dx+2c)} + 6e^{(dx+c)} - 3I) \cdot \log(e^{(dx+c)} + I) + (-3Ie^{(6dx+6c)} - 6e^{(5dx+5c)} - 3Ie^{(4dx+4c)} - 12e^{(3dx+3c)} + 3Ie^{(2dx+2c)} + 6e^{(dx+c)} - 3I) \cdot \log(e^{(dx+c)} - I) + 6e^{(5dx+5c)} - 12Ie^{(4dx+4c)} + 4e^{(3dx+3c)} + 12Ie^{(2dx+2c)} + 6e^{(dx+c)}) / (8a \cdot d \cdot e^{(6dx+6c)} - 16I \cdot a \cdot d \cdot e^{(5dx+5c)} + 8a \cdot d \cdot e^{(4dx+4c)} - 16I \cdot a \cdot d \cdot e^{(3dx+3c)} - 8a \cdot d \cdot e^{(2dx+2c)} - 6a \cdot d \cdot e^{(dx+c)} - 3i)$

$$+ 2*c) - 16*I*a*d*e^(d*x + c) - 8*a*d)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{sech}^3(c+dx)}{i \sinh(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)\*\*3/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Integral(sech(c + d\*x)\*\*3/(I\*sinh(c + d\*x) + 1), x)/a

**Giac [B]** time = 1.19123, size = 250, normalized size = 2.75

$$\frac{3i \log(-ie^{(dx+c)} + ie^{(-dx-c)} + 2)}{16 ad} - \frac{3i \log(-ie^{(dx+c)} + ie^{(-dx-c)} - 2)}{16 ad} + \frac{3e^{(dx+c)} - 3e^{(-dx-c)} + 10i}{16 ad(i e^{(dx+c)} - i e^{(-dx-c)} - 2)} - \frac{-9i(e^{(dx+c)} - e^{(-dx-c)})}{32 a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)^3/(a+I\*a\*sinh(d\*x+c)),x, algorithm="giac")

[Out] 3/16\*I\*log(-I\*e^(d\*x + c) + I\*e^(-d\*x - c) + 2)/(a\*d) - 3/16\*I\*log(-I\*e^(d\*x + c) + I\*e^(-d\*x - c) - 2)/(a\*d) + 1/16\*(3\*e^(d\*x + c) - 3\*e^(-d\*x - c) + 10\*I)/(a\*d\*(I\*e^(d\*x + c) - I\*e^(-d\*x - c) - 2)) - 1/32\*(-9\*I\*(e^(d\*x + c) - e^(-d\*x - c))^2 - 52\*e^(d\*x + c) + 52\*e^(-d\*x - c) + 84\*I)/(a\*d\*(e^(d\*x + c) - e^(-d\*x - c) - 2\*I)^2)

$$3.287 \quad \int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Sech[c + d\*x]^3/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0796316, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Sech[c + d\*x]^3/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Sech[c + d\*x]^3/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx = \int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

**Mathematica [A]** time = 115.857, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sech[c + d\*x]^3/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Integrate[Sech[c + d\*x]^3/((e + f\*x)\*(a + I\*a\*Sinh[c + d\*x])), x]

**Maple [A]** time = 1.718, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(dx + c))^3}{(fx + e)(a + ia \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d\*x+c)^3/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)), x)

[Out] int(sech(d\*x+c)^3/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)^3/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)), x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -8*(4*I*d^2*f^3*x^2 + 8*I*d^2*e*f^2*x + 4*I*d^2*e^2*f - 6*I*f^3 + (9*d^3*f^3*x^3*e^{(5*c)} + 9*(3*d^3*e*f^2 - d^2*f^3)*x^2*e^{(5*c)} + (27*d^3*e^2*f - 18*d^2*e*f^2 - 2*d*f^3)*x*e^{(5*c)} + (9*d^3*e^3 - 9*d^2*e^2*f - 2*d*e*f^2 + 6*f^3)*e^{(5*c)})*e^{(5*d*x)} \\ & + (-18*I*d^3*f^3*x^3*e^{(4*c)} + (-54*I*d^3*e*f^2 + 18*I*d^2*f^3)*x^2*e^{(4*c)} + (-54*I*d^3*e^2*f + 36*I*d^2*e*f^2)*x*e^{(4*c)} + (-18*I*d^3*e^3 + 18*I*d^2*e^2*f - 6*I*f^3)*e^{(4*c)})*e^{(4*d*x)} \\ & + 2*(3*d^3*f^3*x^3*e^{(3*c)} + (9*d^3*e*f^2 - 4*d^2*f^3)*x^2*e^{(3*c)} + (9*d^3*e^2*f - 8*d^2*e*f^2 - 2*d*f^3)*x*e^{(3*c)} + (3*d^3*e^3 - 4*d^2*e^2*f - 2*d*e*f^2 + 6*f^3)*e^{(3*c)})*e^{(3*d*x)} \\ & + (18*I*d^3*f^3*x^3*e^{(2*c)} + (54*I*d^3*e*f^2 + 22*I*d^2*f^3)*x^2*e^{(2*c)} + (54*I*d^3*e^2*f + 44*I*d^2*e*f^2)*x*e^{(2*c)} + (18*I*d^3*e^3 + 22*I*d^2*e^2*f - 12*I*f^3)*e^{(2*c)})*e^{(2*d*x)} \\ & + (9*d^3*f^3*x^3*e^c + (27*d^3*e*f^2 + d^2*f^3)*x^2*e^c + (27*d^3*e^2*f + 2*d^2*e*f^2 - 2*d*f^3)*x*e^c + (9*d^3*e^3 + d^2*e^2*f - 2*d*e*f^2 + 6*f^3)*e^c)*e^{(d*x)} \\ & )/(96*a*d^4*f^4*x^4 + 384*a*d^4*e*f^3*x^3 + 576*a*d^4*e^2*f^2*x^2 + 384*a*d^4*e^3*f*x + 96*a*d^4*e^4 - 96*(a*d^4*f^4*x^4*e^{(6*c)} + 4*a*d^4*e*f^3*x^3*e^{(6*c)} + 6*a*d^4*e^2*f^2*x^2*e^{(6*c)} + 4*a*d^4*e^3*f*x*e^{(6*c)} + a*d^4*e^4*e^{(6*c)})*e^{(6*d*x)} \\ & - (-192*I*a*d^4*f^4*x^4*e^{(5*c)} - 768*I*a*d^4*e*f^3*x^3*e^{(5*c)} - 1152*I*a*d^4*e^2*f^2*x^2*e^{(5*c)} - 768*I*a*d^4*e^3*f*x*e^{(5*c)} - 192*I*a*d^4 \end{aligned}$$

```

*e^4*e^(5*c))*e^(5*d*x) - 96*(a*d^4*f^4*x^4*e^(4*c) + 4*a*d^4*e*f^3*x^3*e^(
4*c) + 6*a*d^4*e^2*f^2*x^2*e^(4*c) + 4*a*d^4*e^3*f*x*e^(4*c) + a*d^4*e^4*e^
(4*c))*e^(4*d*x) - (-384*I*a*d^4*f^4*x^4*e^(3*c) - 1536*I*a*d^4*e*f^3*x^3*e
^(3*c) - 2304*I*a*d^4*e^2*f^2*x^2*e^(3*c) - 1536*I*a*d^4*e^3*f*x*e^(3*c) -
384*I*a*d^4*e^4*e^(3*c))*e^(3*d*x) + 96*(a*d^4*f^4*x^4*e^(2*c) + 4*a*d^4*e*
f^3*x^3*e^(2*c) + 6*a*d^4*e^2*f^2*x^2*e^(2*c) + 4*a*d^4*e^3*f*x*e^(2*c) + a
*d^4*e^4*e^(2*c))*e^(2*d*x) - (-192*I*a*d^4*f^4*x^4*e^c - 768*I*a*d^4*e*f^3
*x^3*e^c - 1152*I*a*d^4*e^2*f^2*x^2*e^c - 768*I*a*d^4*e^3*f*x*e^c - 192*I*a
*d^4*e^4*e^c)*e^(d*x) + 8*integrate((9*d^4*f^4*x^4 + 36*d^4*e*f^3*x^3 + 9*
d^4*e^4 - 28*d^2*e^2*f^2 + 48*f^4 + 2*(27*d^4*e^2*f^2 - 14*d^2*f^4)*x^2 + 4
*(9*d^4*e^3*f - 14*d^2*e*f^3)*x)/(-192*I*a*d^4*f^5*x^5 - 960*I*a*d^4*e*f^4*
x^4 - 1920*I*a*d^4*e^2*f^3*x^3 - 1920*I*a*d^4*e^3*f^2*x^2 - 960*I*a*d^4*e^4
*f*x - 192*I*a*d^4*e^5 + 192*(a*d^4*f^5*x^5*e^c + 5*a*d^4*e*f^4*x^4*e^c + 1
0*a*d^4*e^2*f^3*x^3*e^c + 10*a*d^4*e^3*f^2*x^2*e^c + 5*a*d^4*e^4*f*x*e^c +
a*d^4*e^5*e^c)*e^(d*x)), x) + 8*integrate((3*d^2*f^2*x^2 + 6*d^2*e*f*x + 3*
d^2*e^2 - 4*f^2)/(64*I*a*d^2*f^3*x^3 + 192*I*a*d^2*e*f^2*x^2 + 192*I*a*d^2*
e^2*f*x + 64*I*a*d^2*e^3 + 64*(a*d^2*f^3*x^3*e^c + 3*a*d^2*e*f^2*x^2*e^c +
3*a*d^2*e^2*f*x*e^c + a*d^2*e^3*e^c)*e^(d*x)), x)

```

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="fricas")
```

```

[Out] -(4*I*d^2*f^3*x^2 + 8*I*d^2*e*f^2*x + 4*I*d^2*e^2*f - 6*I*f^3 + (9*d^3*f^3*
x^3 + 9*d^3*e^3 - 9*d^2*e^2*f - 2*d*e*f^2 + 6*f^3 + 9*(3*d^3*e*f^2 - d^2*f^
3)*x^2 + (27*d^3*e^2*f - 18*d^2*e*f^2 - 2*d*f^3)*x)*e^(5*d*x + 5*c) + (-18*
I*d^3*f^3*x^3 - 18*I*d^3*e^3 + 18*I*d^2*e^2*f - 6*I*f^3 + (-54*I*d^3*e*f^2
+ 18*I*d^2*f^3)*x^2 + (-54*I*d^3*e^2*f + 36*I*d^2*e*f^2)*x)*e^(4*d*x + 4*c)
+ 2*(3*d^3*f^3*x^3 + 3*d^3*e^3 - 4*d^2*e^2*f - 2*d*e*f^2 + 6*f^3 + (9*d^3*
e*f^2 - 4*d^2*f^3)*x^2 + (9*d^3*e^2*f - 8*d^2*e*f^2 - 2*d*f^3)*x)*e^(3*d*x
+ 3*c) + (18*I*d^3*f^3*x^3 + 18*I*d^3*e^3 + 22*I*d^2*e^2*f - 12*I*f^3 + (54
*I*d^3*e*f^2 + 22*I*d^2*f^3)*x^2 + (54*I*d^3*e^2*f + 44*I*d^2*e*f^2)*x)*e^(
2*d*x + 2*c) + (9*d^3*f^3*x^3 + 9*d^3*e^3 + d^2*e^2*f - 2*d*e*f^2 + 6*f^3 +
(27*d^3*e*f^2 + d^2*f^3)*x^2 + (27*d^3*e^2*f + 2*d^2*e*f^2 - 2*d*f^3)*x)*e
^(d*x + c) - (12*a*d^4*f^4*x^4 + 48*a*d^4*e*f^3*x^3 + 72*a*d^4*e^2*f^2*x^2
+ 48*a*d^4*e^3*f*x + 12*a*d^4*e^4 - 12*(a*d^4*f^4*x^4 + 4*a*d^4*e*f^3*x^3 +
6*a*d^4*e^2*f^2*x^2 + 4*a*d^4*e^3*f*x + a*d^4*e^4)*e^(6*d*x + 6*c) - (-24*
I*a*d^4*f^4*x^4 - 96*I*a*d^4*e*f^3*x^3 - 144*I*a*d^4*e^2*f^2*x^2 - 96*I*a*d

```

$$\begin{aligned}
&^4e^3f*x - 24*I*a*d^4e^4)*e^{(5*d*x + 5*c)} - 12*(a*d^4f^4*x^4 + 4*a*d^4* \\
&e*f^3*x^3 + 6*a*d^4e^2f^2*x^2 + 4*a*d^4e^3f*x + a*d^4e^4)*e^{(4*d*x + 4 \\
&*c)} - (-48*I*a*d^4f^4*x^4 - 192*I*a*d^4e*f^3*x^3 - 288*I*a*d^4e^2f^2*x^ \\
&2 - 192*I*a*d^4e^3f*x - 48*I*a*d^4e^4)*e^{(3*d*x + 3*c)} + 12*(a*d^4f^4*x \\
&^4 + 4*a*d^4e*f^3*x^3 + 6*a*d^4e^2f^2*x^2 + 4*a*d^4e^3f*x + a*d^4e^4) \\
&*e^{(2*d*x + 2*c)} - (-24*I*a*d^4f^4*x^4 - 96*I*a*d^4e*f^3*x^3 - 144*I*a*d^ \\
&4e^2f^2*x^2 - 96*I*a*d^4e^3f*x - 24*I*a*d^4e^4)*e^{(d*x + c)})*integral( \\
&1/12*(-8*I*d^2f^4*x^2 - 16*I*d^2e*f^3*x - 8*I*d^2e^2f^2 + 24*I*f^4 + (9 \\
&*d^4f^4*x^4 + 36*d^4e*f^3*x^3 + 9*d^4e^4 - 20*d^2e^2f^2 + 24*f^4 + 2*( \\
&27*d^4e^2f^2 - 10*d^2f^4)*x^2 + 4*(9*d^4e^3f - 10*d^2e*f^3)*x)*e^{(d*x \\
&+ c)})/(a*d^4f^5*x^5 + 5*a*d^4e*f^4*x^4 + 10*a*d^4e^2f^3*x^3 + 10*a*d^4 \\
&e^3f^2*x^2 + 5*a*d^4e^4f*x + a*d^4e^5 + (a*d^4f^5*x^5 + 5*a*d^4e*f^4 \\
&*x^4 + 10*a*d^4e^2f^3*x^3 + 10*a*d^4e^3f^2*x^2 + 5*a*d^4e^4f*x + a*d^ \\
&4e^5)*e^{(2*d*x + 2*c)}), x))/(12*a*d^4f^4*x^4 + 48*a*d^4e*f^3*x^3 + 72*a* \\
&d^4e^2f^2*x^2 + 48*a*d^4e^3f*x + 12*a*d^4e^4 - 12*(a*d^4f^4*x^4 + 4*a \\
&*d^4e*f^3*x^3 + 6*a*d^4e^2f^2*x^2 + 4*a*d^4e^3f*x + a*d^4e^4)*e^{(6*d*x \\
&x + 6*c)} - (-24*I*a*d^4f^4*x^4 - 96*I*a*d^4e*f^3*x^3 - 144*I*a*d^4e^2f^ \\
&2*x^2 - 96*I*a*d^4e^3f*x - 24*I*a*d^4e^4)*e^{(5*d*x + 5*c)} - 12*(a*d^4f^ \\
&4*x^4 + 4*a*d^4e*f^3*x^3 + 6*a*d^4e^2f^2*x^2 + 4*a*d^4e^3f*x + a*d^4e \\
&^4)*e^{(4*d*x + 4*c)} - (-48*I*a*d^4f^4*x^4 - 192*I*a*d^4e*f^3*x^3 - 288*I* \\
&a*d^4e^2f^2*x^2 - 192*I*a*d^4e^3f*x - 48*I*a*d^4e^4)*e^{(3*d*x + 3*c)} + \\
&12*(a*d^4f^4*x^4 + 4*a*d^4e*f^3*x^3 + 6*a*d^4e^2f^2*x^2 + 4*a*d^4e^3* \\
&f*x + a*d^4e^4)*e^{(2*d*x + 2*c)} - (-24*I*a*d^4f^4*x^4 - 96*I*a*d^4e*f^3* \\
&x^3 - 144*I*a*d^4e^2f^2*x^2 - 96*I*a*d^4e^3f*x - 24*I*a*d^4e^4)*e^{(d*x \\
&+ c)})
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)\*\*3/(f\*x+e)/(a+I\*a\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(f*x+e)/(a+I*a*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```



$$3.288 \quad \int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

Optimal. Leaf size=33

$$\text{Unintegrable}\left(\frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Sech[c + d\*x]^3/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0811407, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Sech[c + d\*x]^3/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Sech[c + d\*x]^3/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx = \int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)^2(a+ia \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.034, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[Sech[c + d\*x]^3/((e + f\*x)^2\*(a + I\*a\*Sinh[c + d\*x])), x]

[Out] \$Aborted

---

**Maple [A]** time = 2.361, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(dx + c))^3}{(fx + e)^2 (a + ia \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)`

[Out] `int(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `-8*(8*I*d^2*f^3*x^2 + 16*I*d^2*e*f^2*x + 8*I*d^2*e^2*f - 24*I*f^3 + 3*(3*d^3*f^3*x^3*e^(5*c) + 3*(3*d^3*e*f^2 - 2*d^2*f^3)*x^2*e^(5*c) + (9*d^3*e^2*f - 12*d^2*e*f^2 - 2*d*f^3)*x*e^(5*c) + (3*d^3*e^3 - 6*d^2*e^2*f - 2*d*e*f^2 + 8*f^3)*e^(5*c))*e^(5*d*x) + (-18*I*d^3*f^3*x^3*e^(4*c) + (-54*I*d^3*e*f^2 + 36*I*d^2*f^3)*x^2*e^(4*c) + (-54*I*d^3*e^2*f + 72*I*d^2*e*f^2)*x*e^(4*c) + (-18*I*d^3*e^3 + 36*I*d^2*e^2*f - 24*I*f^3)*e^(4*c))*e^(4*d*x) + 2*(3*d^3*f^3*x^3*e^(3*c) + (9*d^3*e*f^2 - 8*d^2*f^3)*x^2*e^(3*c) + (9*d^3*e^2*f - 16*d^2*e*f^2 - 6*d*f^3)*x*e^(3*c) + (3*d^3*e^3 - 8*d^2*e^2*f - 6*d*e*f^2 + 24*f^3)*e^(3*c))*e^(3*d*x) + (18*I*d^3*f^3*x^3*e^(2*c) + (54*I*d^3*e*f^2 + 44*I*d^2*f^3)*x^2*e^(2*c) + (54*I*d^3*e^2*f + 88*I*d^2*e*f^2)*x*e^(2*c) + (18*I*d^3*e^3 + 44*I*d^2*e^2*f - 48*I*f^3)*e^(2*c))*e^(2*d*x) + (9*d^3*f^3*x^3*e^c + (27*d^3*e*f^2 + 2*d^2*f^3)*x^2*e^c + (27*d^3*e^2*f + 4*d^2*e*f^2 - 6*d*f^3)*x*e^c + (9*d^3*e^3 + 2*d^2*e^2*f - 6*d*e*f^2 + 24*f^3)*e^c)*e^(d*x))/(96*a*d^4*f^5*x^5 + 480*a*d^4*e*f^4*x^4 + 960*a*d^4*e^2*f^3*x^3 + 960*a*d^4*e^3*f^2*x^2 + 480*a*d^4*e^4*f*x + 96*a*d^4*e^5 - 96*(a*d^4*f^5*x^5*e^(6*c) + 5*a*d^4*e*f^4*x^4*e^(6*c) + 10*a*d^4*e^2*f^3*x^3*e^(6*c) + 10*a*d^4*e^3*f^2*x^2*e^(6*c) + 5*a*d^4*e^4*f*x*e^(6*c) + a*d^4*e^5*e^(6*c))*e^(6*d*x) - (-192*I*a*d^4*f^5*x^5*e^(5*c) - 960*I*a*d^4*e*f^4*x^4*e^(5*c) - 1920*I*a*d^4*e^2*f^3*x^3*e^(5*c) - 1920*I*a*d^4*e^3*f^2*x^2*e^(5*c) - 960*I*a*d^4*`

$$\begin{aligned}
& e^{4fx}e^{5c} - 192Iad^4e^5e^{5c})e^{5dx} - 96(a^4f^5x^5e^{4c} + 5a^4d^4ef^4x^4e^{4c} + 10a^4d^4e^2f^3x^3e^{4c} + 10a^4d^4e^3f^2x^2e^{4c} + 5a^4d^4e^4f^2xe^{4c} + a^4d^4e^5e^{4c})e^{4dx} \\
& - (-384Iad^4f^5x^5e^{3c} - 1920Iad^4ef^4x^4e^{3c} - 3840Iad^4e^2f^3x^3e^{3c} - 3840Iad^4e^3f^2x^2e^{3c} - 1920Iad^4e^4f^2xe^{3c} - 384Iad^4e^5e^{3c})e^{3dx} + 96(a^4f^5x^5e^{2c} + 5a^4d^4ef^4x^4e^{2c} + 10a^4d^4e^2f^3x^3e^{2c} + 10a^4d^4e^3f^2x^2e^{2c} + 5a^4d^4e^4f^2xe^{2c} + a^4d^4e^5e^{2c})e^{2dx} \\
& - (-192Iad^4f^5x^5e^c - 960Iad^4ef^4x^4e^c - 1920Iad^4e^2f^3x^3e^c - 1920Iad^4e^3f^2x^2e^c - 960Iad^4e^4f^2xe^c - 192Iad^4e^5e^c)e^{dx}) + 8\int(3d^4f^4x^4 + 12d^4ef^3x^3 + 3d^4e^4 - 28d^2e^2f^2 + 80f^4 + 2(9d^4e^2f^2 - 14d^2f^4)x^2 + 4(3d^4e^3f - 14d^2ef^3)x)/(-64Iad^4f^6x^6 - 384Iad^4ef^5x^5 - 960Iad^4e^2f^4x^4 - 1280Iad^4e^3f^3x^3 - 960Iad^4e^4f^2x^2 - 384Iad^4e^5f^2x - 64Iad^4e^6 + 64(a^4f^6x^6e^c + 6a^4d^4ef^5x^5e^c + 15a^4d^4e^2f^4x^4e^c + 20a^4d^4e^3f^3x^3e^c + 15a^4d^4e^4f^2x^2e^c + 6a^4d^4e^5f^2xe^c + a^4d^4e^6e^c)e^{dx}), x) + 8\int(3(d^2f^2x^2 + 2d^2ef^2x + d^2e^2 - 4f^2)/(64Iad^2f^4x^4 + 256Iad^2ef^3x^3 + 384Iad^2e^2f^2x^2 + 256Iad^2e^3f^2x + 64Iad^2e^4 + 64(a^2d^2f^4x^4e^c + 4a^2d^2ef^3x^3e^c + 6a^2d^2e^2f^2x^2e^c + 4a^2d^2e^3f^2xe^c + a^2d^2e^4e^c)e^{dx}), x)
\end{aligned}$$


---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(dx+c)^3/(f*x+e)^2/(a+I*a*sinh(dx+c)),x, algorithm="fricas")
```

```
[Out] -(8*I*d^2*f^3*x^2 + 16*I*d^2*e*f^2*x + 8*I*d^2*e^2*f - 24*I*f^3 + 3*(3*d^3*f^3*x^3 + 3*d^3*e^3 - 6*d^2*e^2*f - 2*d*e*f^2 + 8*f^3 + 3*(3*d^3*e*f^2 - 2*d^2*f^3)*x^2 + (9*d^3*e^2*f - 12*d^2*e*f^2 - 2*d*f^3)*x)*e^(5*d*x + 5*c) + (-18*I*d^3*f^3*x^3 - 18*I*d^3*e^3 + 36*I*d^2*e^2*f - 24*I*f^3 + (-54*I*d^3*e*f^2 + 36*I*d^2*f^3)*x^2 + (-54*I*d^3*e^2*f + 72*I*d^2*e*f^2)*x)*e^(4*d*x + 4*c) + 2*(3*d^3*f^3*x^3 + 3*d^3*e^3 - 8*d^2*e^2*f - 6*d*e*f^2 + 24*f^3 + (9*d^3*e*f^2 - 8*d^2*f^3)*x^2 + (9*d^3*e^2*f - 16*d^2*e*f^2 - 6*d*f^3)*x)*e^(3*d*x + 3*c) + (18*I*d^3*f^3*x^3 + 18*I*d^3*e^3 + 44*I*d^2*e^2*f - 48*I*f^3 + (54*I*d^3*e*f^2 + 44*I*d^2*f^3)*x^2 + (54*I*d^3*e^2*f + 88*I*d^2*e*f^2)*x)*e^(2*d*x + 2*c) + (9*d^3*f^3*x^3 + 9*d^3*e^3 + 2*d^2*e^2*f - 6*d*e*f^2
```

$$\begin{aligned}
& + 24f^3 + (27d^3e^2f^2 + 2d^2f^3)x^2 + (27d^3e^2f + 4d^2e^2f^2 - 6d^2f^3)x)e^{(dx + c)} - (12a^4d^4f^5x^5 + 60a^4d^4e^2f^4x^4 + 120a^4d^4e^2f^3x^3 + 120a^4d^4e^3f^2x^2 + 60a^4d^4e^4fx + 12a^4d^4e^5 - 12(a^4d^4f^5x^5 + 5a^4d^4e^2f^4x^4 + 10a^4d^4e^2f^3x^3 + 10a^4d^4e^3f^2x^2 + 5a^4d^4e^4fx + a^4d^4e^5))e^{(6dx + 6c)} - (-24Ia^4d^4f^5x^5 - 120Ia^4d^4e^2f^4x^4 - 240Ia^4d^4e^2f^3x^3 - 240Ia^4d^4e^3f^2x^2 - 120Ia^4d^4e^4fx - 24Ia^4d^4e^5)e^{(5dx + 5c)} - 12(a^4d^4f^5x^5 + 5a^4d^4e^2f^4x^4 + 10a^4d^4e^2f^3x^3 + 10a^4d^4e^3f^2x^2 + 5a^4d^4e^4fx + a^4d^4e^5)e^{(4dx + 4c)} - (-48Ia^4d^4f^5x^5 - 240Ia^4d^4e^2f^4x^4 - 480Ia^4d^4e^2f^3x^3 - 480Ia^4d^4e^3f^2x^2 - 240Ia^4d^4e^4fx - 48Ia^4d^4e^5)e^{(3dx + 3c)} + 12(a^4d^4f^5x^5 + 5a^4d^4e^2f^4x^4 + 10a^4d^4e^2f^3x^3 + 10a^4d^4e^3f^2x^2 + 5a^4d^4e^4fx + a^4d^4e^5)e^{(2dx + 2c)} - (-24Ia^4d^4f^5x^5 - 120Ia^4d^4e^2f^4x^4 - 240Ia^4d^4e^2f^3x^3 - 240Ia^4d^4e^3f^2x^2 - 120Ia^4d^4e^4fx - 24Ia^4d^4e^5)e^{(dx + c)}) \int (1/4 * (-8I^2d^2f^4x^2 - 16I^2d^2e^2f^3x - 8I^2d^2e^2f^2 + 40I^2f^4 + (3d^4f^4x^4 + 12d^4e^2f^3x^3 + 3d^4e^4 - 20d^2e^2f^2 + 40f^4 + 2(9d^4e^2f^2 - 10d^2f^4))x^2 + 4(3d^4e^3f - 10d^2e^2f^3)x)e^{(dx + c)}) / (a^4d^4f^6x^6 + 6a^4d^4e^2f^5x^5 + 15a^4d^4e^2f^4x^4 + 20a^4d^4e^3f^3x^3 + 15a^4d^4e^4f^2x^2 + 6a^4d^4e^5fx + a^4d^4e^6))e^{(2dx + 2c)}, x) / (12a^4d^4f^5x^5 + 60a^4d^4e^2f^4x^4 + 120a^4d^4e^2f^3x^3 + 120a^4d^4e^3f^2x^2 + 60a^4d^4e^4fx + 12a^4d^4e^5 - 12(a^4d^4f^5x^5 + 5a^4d^4e^2f^4x^4 + 10a^4d^4e^2f^3x^3 + 10a^4d^4e^3f^2x^2 + 5a^4d^4e^4fx + a^4d^4e^5))e^{(6dx + 6c)} - (-24Ia^4d^4f^5x^5 - 120Ia^4d^4e^2f^4x^4 - 240Ia^4d^4e^2f^3x^3 - 240Ia^4d^4e^3f^2x^2 - 120Ia^4d^4e^4fx - 24Ia^4d^4e^5)e^{(5dx + 5c)} - 12(a^4d^4f^5x^5 + 5a^4d^4e^2f^4x^4 + 10a^4d^4e^2f^3x^3 + 10a^4d^4e^3f^2x^2 + 5a^4d^4e^4fx + a^4d^4e^5)e^{(4dx + 4c)} - (-48Ia^4d^4f^5x^5 - 240Ia^4d^4e^2f^4x^4 - 480Ia^4d^4e^2f^3x^3 - 480Ia^4d^4e^3f^2x^2 - 240Ia^4d^4e^4fx - 48Ia^4d^4e^5)e^{(3dx + 3c)} + 12(a^4d^4f^5x^5 + 5a^4d^4e^2f^4x^4 + 10a^4d^4e^2f^3x^3 + 10a^4d^4e^3f^2x^2 + 5a^4d^4e^4fx + a^4d^4e^5)e^{(2dx + 2c)} - (-24Ia^4d^4f^5x^5 - 120Ia^4d^4e^2f^4x^4 - 240Ia^4d^4e^2f^3x^3 - 240Ia^4d^4e^3f^2x^2 - 120Ia^4d^4e^4fx - 24Ia^4d^4e^5)e^{(dx + c)})
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)\*\*3/(f\*x+e)\*\*2/(a+I\*a\*sinh(dx+c)),x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^3/(f*x+e)^2/(a+I*a*sinh(d*x+c)),x, algorithm="giac")`

[Out] Timed out

$$3.289 \quad \int \frac{(e+fx)^3 \cosh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=356

$$-\frac{6f^2(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3} - \frac{6f^2(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3} + \frac{3f(e+fx)^2\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2} + \dots$$

[Out]  $-(e+fx)^4/(4b^2f) + ((e+fx)^3 \text{Log}[1 + (bE^{c+dx})/(a - \text{Sqrt}[a^2 + b^2])])/(bd) + ((e+fx)^3 \text{Log}[1 + (bE^{c+dx})/(a + \text{Sqrt}[a^2 + b^2])])/(bd) + (3f(e+fx)^2 \text{PolyLog}[2, -((bE^{c+dx})/(a - \text{Sqrt}[a^2 + b^2]))])/(bd^2) + (3f(e+fx)^2 \text{PolyLog}[2, -((bE^{c+dx})/(a + \text{Sqrt}[a^2 + b^2]))])/(bd^2) - (6f^2(e+fx) \text{PolyLog}[3, -((bE^{c+dx})/(a - \text{Sqrt}[a^2 + b^2]))])/(bd^3) - (6f^2(e+fx) \text{PolyLog}[3, -((bE^{c+dx})/(a + \text{Sqrt}[a^2 + b^2]))])/(bd^3) + (6f^3 \text{PolyLog}[4, -((bE^{c+dx})/(a - \text{Sqrt}[a^2 + b^2]))])/(bd^4) + (6f^3 \text{PolyLog}[4, -((bE^{c+dx})/(a + \text{Sqrt}[a^2 + b^2]))])/(bd^4)$

**Rubi [A]** time = 0.482998, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {5561, 2190, 2531, 6609, 2282, 6589}

$$-\frac{6f^2(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3} - \frac{6f^2(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3} + \frac{3f(e+fx)^2\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2} + \dots$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(e+fx)^3 \text{Cosh}[c+dx]/(a+b \text{Sinh}[c+dx]), x]$

[Out]  $-(e+fx)^4/(4b^2f) + ((e+fx)^3 \text{Log}[1 + (bE^{c+dx})/(a - \text{Sqrt}[a^2 + b^2])])/(bd) + ((e+fx)^3 \text{Log}[1 + (bE^{c+dx})/(a + \text{Sqrt}[a^2 + b^2])])/(bd) + (3f(e+fx)^2 \text{PolyLog}[2, -((bE^{c+dx})/(a - \text{Sqrt}[a^2 + b^2]))])/(bd^2) + (3f(e+fx)^2 \text{PolyLog}[2, -((bE^{c+dx})/(a + \text{Sqrt}[a^2 + b^2]))])/(bd^2) - (6f^2(e+fx) \text{PolyLog}[3, -((bE^{c+dx})/(a - \text{Sqrt}[a^2 + b^2]))])/(bd^3) - (6f^2(e+fx) \text{PolyLog}[3, -((bE^{c+dx})/(a + \text{Sqrt}[a^2 + b^2]))])/(bd^3) + (6f^3 \text{PolyLog}[4, -((bE^{c+dx})/(a - \text{Sqrt}[a^2 + b^2]))])/(bd^4) + (6f^3 \text{PolyLog}[4, -((bE^{c+dx})/(a + \text{Sqrt}[a^2 + b^2]))])/(bd^4)$

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \cosh(c+dx)}{a+b \sinh(c+dx)} dx &= -\frac{(e+fx)^4}{4bf} + \int \frac{e^{c+dx}(e+fx)^3}{a-\sqrt{a^2+b^2}+be^{c+dx}} dx + \int \frac{e^{c+dx}(e+fx)^3}{a+\sqrt{a^2+b^2}+be^{c+dx}} dx \\
&= -\frac{(e+fx)^4}{4bf} + \frac{(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} - \frac{(3f) \int (e+fx)^2 \cosh(c+dx)}{b} \\
&= -\frac{(e+fx)^4}{4bf} + \frac{(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} + \frac{3f(e+fx)^2 \int \cosh(c+dx)}{b} \\
&= -\frac{(e+fx)^4}{4bf} + \frac{(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} + \frac{3f(e+fx)^2 \int \cosh(c+dx)}{b} \\
&= -\frac{(e+fx)^4}{4bf} + \frac{(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} + \frac{3f(e+fx)^2 \int \cosh(c+dx)}{b} \\
&= -\frac{(e+fx)^4}{4bf} + \frac{(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} + \frac{3f(e+fx)^2 \int \cosh(c+dx)}{b} \\
&= -\frac{(e+fx)^4}{4bf} + \frac{(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} + \frac{3f(e+fx)^2 \int \cosh(c+dx)}{b}
\end{aligned}$$

**Mathematica [A]** time = 0.149016, size = 329, normalized size = 0.92

$$\frac{12f\left(d^2(e+fx)^2 \text{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) - 2df(e+fx) \text{PolyLog}\left(3, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) + 2f^2 \text{PolyLog}\left(4, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)\right)}{d^4} + \frac{12f\left(d^2(e+fx)^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) - 2df(e+fx) \text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) + 2f^2 \text{PolyLog}\left(4, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)\right)}{d^4}$$

4b

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^3\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (-((e + f\*x)^4/f) + (4\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]))/d + (4\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]))/d + (12\*f\*(d^2\*(e + f\*x)^2\*PolyLog[2, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] - 2\*d\*f\*(e + f\*x)\*PolyLog[3, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] + 2\*f^2\*PolyLog[4, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])])/d^4 + (12\*f\*(d^2\*(e + f\*x)^2\*PolyLog[2, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] - 2\*d\*f\*(e + f\*x)\*PolyLog[3, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] + 2\*f^2\*PolyLog[4, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/d^4)/(4\*b)



**Maple [F]** time = 0.259, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{e^3 \log(b \sinh(dx + c) + a)}{bd} + \frac{f^3 x^4 + 4ef^2 x^3 + 6e^2 f x^2}{4b} - \int \frac{2(bf^3 x^3 + 3bef^2 x^2 + 3be^2 f x - (af^3 x^3 e^c + 3aef^2 x^2 e^c + 2a^2 e^3 x e^c))}{b^2 e^{2dx+2c} + 2abe^{dx+c} - b^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] e^3\*log(b\*sinh(d\*x + c) + a)/(b\*d) + 1/4\*(f^3\*x^4 + 4\*e\*f^2\*x^3 + 6\*e^2\*f\*x^2)/b - integrate(-2\*(b\*f^3\*x^3 + 3\*b\*e\*f^2\*x^2 + 3\*b\*e^2\*f\*x - (a\*f^3\*x^3\*e^c + 3\*a\*e\*f^2\*x^2\*e^c + 3\*a\*e^2\*f\*x\*e^c))\*e^(d\*x))/(b^2\*e^(2\*d\*x + 2\*c) + 2\*a\*b\*e^(d\*x + c) - b^2), x)

**Fricas [C]** time = 2.47518, size = 2128, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] -1/4\*(d^4\*f^3\*x^4 + 4\*d^4\*e\*f^2\*x^3 + 6\*d^4\*e^2\*f\*x^2 + 4\*d^4\*e^3\*x - 24\*f^3\*polylog(4, (a\*cosh(d\*x + c) + a\*sinh(d\*x + c) + (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2)))/b - 24\*f^3\*polylog(4, (a\*cosh(d\*x + c) + a\*sinh(d\*x + c) - (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2)))/b - 12\*(d^2\*f^3\*x^2 + 2\*d^2\*e\*f^2\*x + d^2\*e^2\*f)\*dilog((a\*cosh(d\*x + c) + a\*sinh(d\*x + c) + (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b

$$\begin{aligned} &^2) - b)/b + 1) - 12*(d^2*f^3*x^2 + 2*d^2*e*f^2*x + d^2*e^2*f)*dilog((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2) - b)/b + 1) - 4*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2) + 2*a) - 4*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2) + 2*a) - 4*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2) - b)/b) - 4*(d^3*f^3*x^3 + 3*d^3*e*f^2*x^2 + 3*d^3*e^2*f*x + 3*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2) - b)/b) + 24*(d*f^3*x + d*e*f^2)*polylog(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2) )/b) + 24*(d*f^3*x + d*e*f^2)*polylog(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2) )/b))/(b*d^4) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*cosh(d\*x + c)/(b\*sinh(d\*x + c) + a), x)

$$3.290 \quad \int \frac{(e+fx)^2 \cosh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=264

$$\frac{2f(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2} + \frac{2f(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2} - \frac{2f^2\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3} - \frac{2f^2\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3}$$

[Out]  $-(e + f*x)^3/(3*b*f) + ((e + f*x)^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])])/(b*d) + ((e + f*x)^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(b*d) + (2*f*(e + f*x)*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(b*d^2) + (2*f*(e + f*x)*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(b*d^2) - (2*f^2*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(b*d^3) - (2*f^2*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(b*d^3)$

**Rubi [A]** time = 0.411303, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$ , Rules used = {5561, 2190, 2531, 2282, 6589}

$$\frac{2f(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2} + \frac{2f(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2} - \frac{2f^2\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3} - \frac{2f^2\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(e + f*x)^2*\text{Cosh}[c + d*x]}{(a + b*\text{Sinh}[c + d*x])}, x]$

[Out]  $-(e + f*x)^3/(3*b*f) + ((e + f*x)^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])])/(b*d) + ((e + f*x)^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(b*d) + (2*f*(e + f*x)*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(b*d^2) + (2*f*(e + f*x)*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(b*d^2) - (2*f^2*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(b*d^3) - (2*f^2*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(b*d^3)$

**Rule 5561**

$\text{Int}[(\text{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))^{(m_.)})/((a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_.)]), x\_Symbol] := -\text{Simp}[(e + f*x)^{(m + 1)}/(b*f*(m + 1)), x] + (\text{Int}[\frac{(e + f*x)^m*E^{(c + d*x)}}{(a - \text{Rt}[a^2 + b^2, 2] + b*E^{(c + d*x)})}, x] + \text{Int}[\frac{(e + f*x)^m*E^{(c + d*x)}}{(a + \text{Rt}[a^2 + b^2, 2] + b*E^{(c + d*x)})}, x])$

, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*(a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \cosh(c+dx)}{a+b \sinh(c+dx)} dx &= -\frac{(e+fx)^3}{3bf} + \int \frac{e^{c+dx}(e+fx)^2}{a-\sqrt{a^2+b^2}+be^{c+dx}} dx + \int \frac{e^{c+dx}(e+fx)^2}{a+\sqrt{a^2+b^2}+be^{c+dx}} dx \\
&= -\frac{(e+fx)^3}{3bf} + \frac{(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} - \frac{(2f) \int (e+fx)}{d} \\
&= -\frac{(e+fx)^3}{3bf} + \frac{(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} + \frac{2f(e+fx)}{d} \\
&= -\frac{(e+fx)^3}{3bf} + \frac{(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} + \frac{2f(e+fx)}{d} \\
&= -\frac{(e+fx)^3}{3bf} + \frac{(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd} + \frac{(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{bd} + \frac{2f(e+fx)}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.145766, size = 244, normalized size = 0.92

$$\frac{6f\left(d(e+fx)\text{PolyLog}\left(2,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)-f\text{PolyLog}\left(3,\frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right)\right)}{d^3} + \frac{6f\left(d(e+fx)\text{PolyLog}\left(2,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)-f\text{PolyLog}\left(3,-\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)\right)}{d^3} + \frac{3(e+fx)^2 \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d}$$


---

3b

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (-((e + f\*x)^3/f) + (3\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]))/d + (3\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]))/d + (6\*f\*(d\*(e + f\*x)\*PolyLog[2, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] - f\*PolyLog[3, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])])/d^3 + (6\*f\*(d\*(e + f\*x)\*PolyLog[2, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] - f\*PolyLog[3, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/d^3)/(3\*b)

**Maple [F]** time = 0.204, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)`

[Out] `int((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{e^2 \log(b \sinh(dx + c) + a)}{bd} + \frac{f^2 x^3 + 3efx^2}{3b} - \int -\frac{2(bf^2x^2 + 2befx - (af^2x^2e^c + 2aefxe^c)e^{(dx)})}{b^2e^{(2dx+2c)} + 2abe^{(dx+c)} - b^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `e^2*log(b*sinh(d*x + c) + a)/(b*d) + 1/3*(f^2*x^3 + 3*e*f*x^2)/b - integrate(-2*(b*f^2*x^2 + 2*b*e*f*x - (a*f^2*x^2*e^c + 2*a*e*f*x*e^c)*e^(d*x))/(b^2*e^(2*d*x + 2*c) + 2*a*b*e^(d*x + c) - b^2), x)`

**Fricas [C]** time = 2.31073, size = 1504, normalized size = 5.7

$$d^3 f^2 x^3 + 3 d^3 e f x^2 + 3 d^3 e^2 x + 6 f^2 \operatorname{polylog}\left(3, \frac{a \cosh(dx+c) + a \sinh(dx+c) + (b \cosh(dx+c) + b \sinh(dx+c)) \sqrt{\frac{a^2+b^2}{b^2}}}{b}\right) + 6 f^2 \operatorname{polylog}\left(3, \frac{a \cosh(dx+c) + a \sinh(dx+c) + (b \cosh(dx+c) + b \sinh(dx+c)) \sqrt{\frac{a^2+b^2}{b^2}}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] `-1/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^2 + 3*d^3*e^2*x + 6*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*(d*f^2*x + d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 6*(d*f^2*x + d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*(d^2*e^2 - 2*c*d*e*f + c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(d^2*e^2 - 2*c*d*e*f + c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f -`

$$c^2 f^2 \log(-(a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b) - 3(d^2 f^2 x^2 + 2d^2 e f x + 2c d e f - c^2 f^2) \log(-(a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b)) / (b d^3)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*cosh(d\*x + c)/(b\*sinh(d\*x + c) + a), x)

$$3.291 \quad \int \frac{(e+fx) \cosh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=170

$$\frac{f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2} + \frac{f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2} + \frac{(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{bd} + \frac{(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} + 1\right)}{bd} - \frac{(e+fx)^2}{2bf}$$

[Out]  $-(e + f*x)^2/(2*b*f) + ((e + f*x)*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])])/(b*d) + ((e + f*x)*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])/(b*d) + (f*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2]))])/(b*d^2) + (f*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2]))])/(b*d^2)$

**Rubi [A]** time = 0.234474, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {5561, 2190, 2279, 2391}

$$\frac{f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2} + \frac{f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2} + \frac{(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{bd} + \frac{(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} + 1\right)}{bd} - \frac{(e+fx)^2}{2bf}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e + f*x)*\operatorname{Cosh}[c + d*x]/(a + b*\operatorname{Sinh}[c + d*x]), x]$

[Out]  $-(e + f*x)^2/(2*b*f) + ((e + f*x)*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])])/(b*d) + ((e + f*x)*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])/(b*d) + (f*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2]))])/(b*d^2) + (f*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2]))])/(b*d^2)$

### Rule 5561

$\operatorname{Int}[(\operatorname{Cosh}[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^{(m_.)})/((a_.) + (b_.)*\operatorname{Sinh}[(c_.) + (d_.)*(x_)]), x\_Symbol] \rightarrow -\operatorname{Simp}[(e + f*x)^{(m + 1)}/(b*f*(m + 1)), x] + (\operatorname{Int}[(e + f*x)^m*E^{(c + d*x)}/(a - \operatorname{Rt}[a^2 + b^2, 2] + b*E^{(c + d*x)}), x] + \operatorname{Int}[(e + f*x)^m*E^{(c + d*x)}/(a + \operatorname{Rt}[a^2 + b^2, 2] + b*E^{(c + d*x)}), x]) /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \operatorname{IGtQ}[m, 0] \ \&\& \operatorname{NeQ}[a^2 + b^2, 0]$

### Rule 2190

$\operatorname{Int}[(F_.)^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)}*((c_.) + (d_.)*(x_))^{(m_.)})}/((a_.) + (b_.)*(F_.)^{((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)})}, x\_Symbol] \rightarrow \operatorname{Simp}$



```

[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

### Rule 2279

```

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

```

### Rule 2391

```

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(e + fx) \cosh(c + dx)}{a + b \sinh(c + dx)} dx &= -\frac{(e + fx)^2}{2bf} + \int \frac{e^{c+dx}(e + fx)}{a - \sqrt{a^2 + b^2} + be^{c+dx}} dx + \int \frac{e^{c+dx}(e + fx)}{a + \sqrt{a^2 + b^2} + be^{c+dx}} dx \\
&= -\frac{(e + fx)^2}{2bf} + \frac{(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{bd} + \frac{(e + fx) \log\left(1 + \frac{be^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{bd} - \frac{f \int \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{bd} \\
&= -\frac{(e + fx)^2}{2bf} + \frac{(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{bd} + \frac{(e + fx) \log\left(1 + \frac{be^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{bd} - \frac{f \operatorname{Subst}\left(\int \frac{\log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{x} dx\right)}{bd} \\
&= -\frac{(e + fx)^2}{2bf} + \frac{(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{bd} + \frac{(e + fx) \log\left(1 + \frac{be^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{bd} + \frac{f \operatorname{Li}_2\left(-\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{bd^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0348309, size = 157, normalized size = 0.92

$$\frac{2f^2 \operatorname{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2 + b^2} - a}\right) + 2f^2 \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right) - d(e + fx) \left(-2f \log\left(\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}} + 1\right) - 2f \log\left(\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a} + 1\right)\right)}{2bd^2 f}$$

Antiderivative was successfully verified.

```

[In] Integrate[((e + f*x)*Cosh[c + d*x])/(a + b*Sinh[c + d*x]),x]

```

```

[Out] (-(d*(e + f*x)*(d*e + d*f*x - 2*f*Log[1 + (b*E^(c + d*x))]/(a - Sqrt[a^2 + b^2])) - 2*f*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])) + 2*f^2*PolyLo

```

$g[2, (b \cdot E^{(c + d \cdot x)}) / (-a + \text{Sqrt}[a^2 + b^2])] + 2 \cdot f^2 \cdot \text{PolyLog}[2, -((b \cdot E^{(c + d \cdot x)}) / (a + \text{Sqrt}[a^2 + b^2]))] / (2 \cdot b \cdot d^2 \cdot f)$

**Maple [B]** time = 0.077, size = 412, normalized size = 2.4

$$-\frac{fx^2}{2b} + \frac{ex}{b} - \frac{cf \ln\left(\frac{be^{2dx+2c} + 2ae^{dx+c} - b}{d^2b}\right)}{d^2b} + 2 \frac{cf \ln\left(\frac{e^{dx+c}}{d^2b}\right)}{d^2b} + \frac{e \ln\left(\frac{be^{2dx+2c} + 2ae^{dx+c} - b}{bd}\right)}{bd} - 2 \frac{e \ln\left(\frac{e^{dx+c}}{bd}\right)}{bd} + \frac{fx}{bd} \ln\left(\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x)`

[Out]  $-1/2 \cdot f \cdot x^2 / b + e \cdot x / b - 1/d^2 \cdot b \cdot f \cdot c \cdot \ln(b \cdot \exp(2 \cdot d \cdot x + 2 \cdot c) + 2 \cdot a \cdot \exp(d \cdot x + c) - b) + 2/d^2 \cdot b \cdot f \cdot c \cdot \ln(\exp(d \cdot x + c)) + 1/d \cdot b \cdot e \cdot \ln(b \cdot \exp(2 \cdot d \cdot x + 2 \cdot c) + 2 \cdot a \cdot \exp(d \cdot x + c) - b) - 2/d \cdot b \cdot e \cdot \ln(\exp(d \cdot x + c)) + 1/d \cdot b \cdot f \cdot \ln((-b \cdot \exp(d \cdot x + c) + (a^2 + b^2)^{(1/2)} - a) / (-a + (a^2 + b^2)^{(1/2)})) \cdot x + 1/d^2 \cdot b \cdot f \cdot \ln((-b \cdot \exp(d \cdot x + c) + (a^2 + b^2)^{(1/2)} - a) / (-a + (a^2 + b^2)^{(1/2)})) \cdot c + 1/d \cdot b \cdot f \cdot \ln((b \cdot \exp(d \cdot x + c) + (a^2 + b^2)^{(1/2)} + a) / (a + (a^2 + b^2)^{(1/2)})) \cdot x + 1/d^2 \cdot b \cdot f \cdot \ln((b \cdot \exp(d \cdot x + c) + (a^2 + b^2)^{(1/2)} + a) / (a + (a^2 + b^2)^{(1/2)})) \cdot c + 1/d^2 \cdot b \cdot f \cdot \text{dilog}((b \cdot \exp(d \cdot x + c) + (a^2 + b^2)^{(1/2)} + a) / (a + (a^2 + b^2)^{(1/2)})) + 1/d^2 \cdot b \cdot f \cdot \text{dilog}((-b \cdot \exp(d \cdot x + c) + (a^2 + b^2)^{(1/2)} - a) / (-a + (a^2 + b^2)^{(1/2)})) - 2/d \cdot b \cdot f \cdot c \cdot x - 1/d^2 \cdot b \cdot f \cdot c^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} f \left( \frac{x^2}{b} - \int \frac{4(axe^{(dx+c)} - bx)}{b^2 e^{(2dx+2c)} + 2abe^{(dx+c)} - b^2} dx \right) + \frac{e \log(b \sinh(dx+c) + a)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out]  $1/2 \cdot f \cdot (x^2 / b - \text{integrate}(4 \cdot (a \cdot x \cdot e^{(d \cdot x + c)} - b \cdot x) / (b^2 \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + 2 \cdot a \cdot b \cdot e^{(d \cdot x + c)} - b^2), x)) + e \cdot \log(b \cdot \sinh(d \cdot x + c) + a) / (b \cdot d)$

**Fricas [B]** time = 2.16192, size = 964, normalized size = 5.67

$$d^2 f x^2 + 2 d^2 e x - 2 f \operatorname{Li}_2 \left( \frac{a \cosh(dx+c) + a \sinh(dx+c) + (b \cosh(dx+c) + b \sinh(dx+c)) \sqrt{\frac{a^2+b^2}{b^2} - b}}{b} + 1 \right) - 2 f \operatorname{Li}_2 \left( \frac{a \cosh(dx+c) + a \sinh(dx+c) - (b \cosh(dx+c) + b \sinh(dx+c)) \sqrt{\frac{a^2+b^2}{b^2} - b}}{b} + 1 \right)$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-1/2*(d^2*f*x^2 + 2*d^2*e*x - 2*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(d*e - c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(d*e - c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(d*f*x + c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(d*f*x + c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b)))/(b*d^2)$$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \cosh(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*cosh(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \cosh(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

$$3.292 \quad \int \frac{\cosh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=18

$$\frac{\log(a + b \sinh(c + dx))}{bd}$$

[Out] Log[a + b\*Sinh[c + d\*x]]/(b\*d)

**Rubi [A]** time = 0.0273205, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2668, 31}

$$\frac{\log(a + b \sinh(c + dx))}{bd}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]/(a + b\*Sinh[c + d\*x]),x]

[Out] Log[a + b\*Sinh[c + d\*x]]/(b\*d)

#### Rule 2668

Int[cos[(e\_.) + (f\_.)\*(x\_)]^(p\_.)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(m\_.), x\_Symbol] :> Dist[1/(b^p\*f), Subst[Int[(a + x)^m\*(b^2 - x^2)^((p - 1)/2), x], x, b\*Sin[e + f\*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{\cosh(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x} dx, x, b \sinh(c + dx)\right)}{bd} \\ &= \frac{\log(a + b \sinh(c + dx))}{bd} \end{aligned}$$

**Mathematica [A]** time = 0.0069378, size = 18, normalized size = 1.

$$\frac{\log(a + b \sinh(c + dx))}{bd}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]/(a + b\*Sinh[c + d\*x]),x]

[Out] Log[a + b\*Sinh[c + d\*x]]/(b\*d)

---

**Maple [A]** time = 0.002, size = 19, normalized size = 1.1

$$\frac{\ln(a + b \sinh(dx + c))}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] ln(a+b\*sinh(d\*x+c))/b/d

---

**Maxima [A]** time = 1.15693, size = 24, normalized size = 1.33

$$\frac{\log(b \sinh(dx + c) + a)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] log(b\*sinh(d\*x + c) + a)/(b\*d)

---

**Fricas [B]** time = 2.04264, size = 104, normalized size = 5.78

$$\frac{dx - \log\left(\frac{2(b \sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out]  $-(d*x - \log(2*(b*\sinh(d*x + c) + a)/(\cosh(d*x + c) - \sinh(d*x + c))))/(b*d)$

**Sympy [A]** time = 1.09728, size = 41, normalized size = 2.28

$$\begin{cases} \frac{x \cosh(c)}{a} & \text{for } b = 0 \wedge d = 0 \\ \frac{x \cosh(c)}{a+b \sinh(c)} & \text{for } d = 0 \\ \frac{\sinh(c+dx)}{\sinh(c+dx)} & \text{for } b = 0 \\ \frac{\log\left(\frac{a^d}{b} + \sinh(c+dx)\right)}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)),x)`

[Out] `Piecewise((x*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (x*cosh(c)/(a + b*sinh(c)), Eq(d, 0)), (sinh(c + d*x)/(a*d), Eq(b, 0)), (log(a/b + sinh(c + d*x))/(b*d), True))`

**Giac [A]** time = 1.1611, size = 45, normalized size = 2.5

$$\frac{\log\left(\left|b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 2a\right|\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")`

[Out]  $\log(\text{abs}(b*(e^{(d*x + c)} - e^{(-d*x - c)}) + 2*a))/(b*d)$

$$3.293 \quad \int \frac{\cosh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=28

$$\text{Unintegrable} \left( \frac{\cosh(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[Cosh[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0471313, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Cosh[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Cosh[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\cosh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [A]** time = 12.8653, size = 0, normalized size = 0.

$$\int \frac{\cosh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Cosh[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Integrate[Cosh[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]



---

**Maple [A]** time = 0.197, size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)`

[Out] `int(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{\log(fx + e)}{bf} - \frac{1}{2} \int \frac{4(ae^{(dx+c)} - b)}{b^2fx + b^2e - (b^2fxe^{(2c)} + b^2ee^{(2c)})e^{(2dx)} - 2(abfxe^c + abee^c)e^{(dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `log(f*x + e)/(b*f) - 1/2*integrate(-4*(a*e^(d*x + c) - b)/(b^2*f*x + b^2*e - (b^2*f*x*e^(2*c) + b^2*e*e^(2*c))*e^(2*d*x) - 2*(a*b*f*x*e^c + a*b*e*e^c)*e^(d*x)), x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx + c)}{afx + ae + (bf*x + be)\sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] `integral(cosh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(fx + e)(b \sinh(dx + c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate(cosh(d\*x + c)/((f\*x + e)\*(b\*sinh(d\*x + c) + a)), x)

$$3.294 \quad \int \frac{(e+fx)^3 \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=527

$$\frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3} + \frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^3} + \frac{3f\sqrt{a^2+b^2}(e+fx)^2}{b^2d^3}$$

```
[Out] -(a*(e + f*x)^4)/(4*b^2*f) + (6*f^2*(e + f*x)*Cosh[c + d*x])/(b*d^3) + ((e + f*x)^3*Cosh[c + d*x])/(b*d) + (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*d) + (3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^2) - (3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^2) - (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^3) + (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^3) + (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^4) - (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^4) - (6*f^3*Sinh[c + d*x])/(b*d^4) - (3*f*(e + f*x)^2*Sinh[c + d*x])/(b*d^2)
```

**Rubi [A]** time = 0.908079, antiderivative size = 527, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$ , Rules used = {5565, 32, 3296, 2637, 3322, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3} + \frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^3} + \frac{3f\sqrt{a^2+b^2}(e+fx)^2}{b^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(a*(e + f*x)^4)/(4*b^2*f) + (6*f^2*(e + f*x)*Cosh[c + d*x])/(b*d^3) + ((e + f*x)^3*Cosh[c + d*x])/(b*d) + (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*d) + (3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^2) - (3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^2) - (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^3) + (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^3) + (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*d^4) - (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*d^4) - (6*f^3*Sinh[c + d*x])/(b*d^4) - (3*f*(e + f*x)^2*Sinh[c + d*x])/(b*d^2)
```

```

*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(b^2*d^3)
+ (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(b^2*d^4) - (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(b^2*d^4) - (6*f^3*Sinh[c + d*x])/(b*d^4) - (3*f*(e + f*x)^2*Sinh[c + d*x])/(b*d^2)

```

### Rule 5565

```

Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_))*((e_.) + (f_.)*(x_))^(m_.)]/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Dist[a/b^2, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

```

### Rule 32

```

Int[((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

```

### Rule 3296

```

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

```

### Rule 2637

```

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

```

### Rule 3322

```

Int[((c_.) + (d_.)*(x_))^(m_.)]/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol] := Dist[2, Int[(c + d*x)^m*E^(-(I*e) + f*fz*x))/(-(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

```

### Rule 2264

```

Int[((F_)^(u_))*((f_.) + (g_.)*(x_))^(m_.)]/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,

```

2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_)]/((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_)\*((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_)))^(n\_))]\*(f\_) + (g\_)\*(x\_)]^(m\_), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 6609

Int[((e\_) + (f\_)\*(x\_))^(m\_)\*PolyLog[n\_, (d\_)\*((F\_)^((c\_)\*((a\_) + (b\_)\*(x\_)))^(p\_)], x\_Symbol] := Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\* (F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_) ]/((d\_) + (e\_)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx &= -\frac{a \int (e+fx)^3 dx}{b^2} + \frac{\int (e+fx)^3 \sinh(c+dx) dx}{b} + \frac{(a^2+b^2) \int \frac{(e+fx)^3}{a+b \sinh(c+dx)} dx}{b^2} \\
&= -\frac{a(e+fx)^4}{4b^2 f} + \frac{(e+fx)^3 \cosh(c+dx)}{bd} + \frac{(2(a^2+b^2)) \int \frac{e^{c+dx}(e+fx)^3}{-b+2ae^{c+dx}+be^{2(c+dx)}} dx}{b^2} - \frac{(3f) \int (e+fx)^3 dx}{b^2} \\
&= -\frac{a(e+fx)^4}{4b^2 f} + \frac{(e+fx)^3 \cosh(c+dx)}{bd} - \frac{3f(e+fx)^2 \sinh(c+dx)}{bd^2} + \frac{(2\sqrt{a^2+b^2}) \int \frac{(e+fx)^3}{2a-2b \sinh(c+dx)}}{b} \\
&= -\frac{a(e+fx)^4}{4b^2 f} + \frac{6f^2(e+fx) \cosh(c+dx)}{bd^3} + \frac{(e+fx)^3 \cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}(e+fx)^3}{b^2} \\
&= -\frac{a(e+fx)^4}{4b^2 f} + \frac{6f^2(e+fx) \cosh(c+dx)}{bd^3} + \frac{(e+fx)^3 \cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}(e+fx)^3}{b^2} \\
&= -\frac{a(e+fx)^4}{4b^2 f} + \frac{6f^2(e+fx) \cosh(c+dx)}{bd^3} + \frac{(e+fx)^3 \cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}(e+fx)^3}{b^2} \\
&= -\frac{a(e+fx)^4}{4b^2 f} + \frac{6f^2(e+fx) \cosh(c+dx)}{bd^3} + \frac{(e+fx)^3 \cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}(e+fx)^3}{b^2} \\
&= -\frac{a(e+fx)^4}{4b^2 f} + \frac{6f^2(e+fx) \cosh(c+dx)}{bd^3} + \frac{(e+fx)^3 \cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}(e+fx)^3}{b^2} \\
&= -\frac{a(e+fx)^4}{4b^2 f} + \frac{6f^2(e+fx) \cosh(c+dx)}{bd^3} + \frac{(e+fx)^3 \cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}(e+fx)^3}{b^2}
\end{aligned}$$

**Mathematica [A]** time = 3.28391, size = 933, normalized size = 1.77

$$af^3x^4d^4 + 4aef^2x^3d^4 + 6ae^2fx^2d^4 + 4ae^3xd^4 + 8\sqrt{a^2+b^2}e^3 \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)d^3 - 4be^3 \cosh(c+dx)d^3 - 4bf^3x^3 \cosh(c+dx)$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^3\*Cosh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] -(4\*a\*d^4\*e^3\*x + 6\*a\*d^4\*e^2\*f\*x^2 + 4\*a\*d^4\*e\*f^2\*x^3 + a\*d^4\*f^3\*x^4 + 8\*sqrt[a^2 + b^2]\*d^3\*e^3\*ArcTanh[(a + b\*E^(c + d\*x))/sqrt[a^2 + b^2]] - 4\*b\*d^3\*e^3\*Cosh[c + d\*x] - 24\*b\*d\*e\*f^2\*Cosh[c + d\*x] - 12\*b\*d^3\*e^2\*f\*x\*Cosh[c + d\*x] - 24\*b\*d\*f^3\*x\*Cosh[c + d\*x] - 12\*b\*d^3\*e\*f^2\*x^2\*Cosh[c + d\*x] - 4\*b\*d^3\*f^3\*x^3\*Cosh[c + d\*x] - 12\*sqrt[a^2 + b^2]\*d^3\*e^2\*f\*x\*Log[1 + (b\*E^(c + d\*x))/(a - sqrt[a^2 + b^2])] - 12\*sqrt[a^2 + b^2]\*d^3\*e\*f^2\*x^2\*Log[1 + (b\*E^(c + d\*x))/(a - sqrt[a^2 + b^2])] - 4\*sqrt[a^2 + b^2]\*d^3\*f^3\*x^3\*

```

Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 12*Sqrt[a^2 + b^2]*d^3*e^2
*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 12*Sqrt[a^2 + b^2]*d^
3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 4*Sqrt[a^2 + b
^2]*d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 12*Sqrt[a^
2 + b^2]*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]
)] + 12*Sqrt[a^2 + b^2]*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]))] + 24*Sqrt[a^2 + b^2]*d*e*f^2*PolyLog[3, (b*E^(c + d*x))
/(-a + Sqrt[a^2 + b^2])] + 24*Sqrt[a^2 + b^2]*d*f^3*x*PolyLog[3, (b*E^(c +
d*x))/(-a + Sqrt[a^2 + b^2])] - 24*Sqrt[a^2 + b^2]*d*e*f^2*PolyLog[3, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 24*Sqrt[a^2 + b^2]*d*f^3*x*PolyLog[3
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 24*Sqrt[a^2 + b^2]*f^3*PolyLo
g[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 24*Sqrt[a^2 + b^2]*f^3*PolyL
og[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 12*b*d^2*e^2*f*Sinh[c + d
*x] + 24*b*f^3*Sinh[c + d*x] + 24*b*d^2*e*f^2*x*Sinh[c + d*x] + 12*b*d^2*f^
3*x^2*Sinh[c + d*x])/(4*b^2*d^4)

```

**Maple [F]** time = 0.269, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 (\cosh(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [C]** time = 3.06394, size = 4691, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\frac{1}{4} \cdot (2 \cdot b \cdot d^3 \cdot f^3 \cdot x^3 + 2 \cdot b \cdot d^3 \cdot e^3 + 6 \cdot b \cdot d^2 \cdot e^2 \cdot f + 12 \cdot b \cdot d \cdot e \cdot f^2 + 12 \cdot b \cdot f^3 + 6 \cdot (b \cdot d^3 \cdot e \cdot f^2 + b \cdot d^2 \cdot f^3) \cdot x^2 + 2 \cdot (b \cdot d^3 \cdot f^3 \cdot x^3 + b \cdot d^3 \cdot e^3 - 3 \cdot b \cdot d^2 \cdot e^2 \cdot f + 6 \cdot b \cdot d \cdot e \cdot f^2 - 6 \cdot b \cdot f^3 + 3 \cdot (b \cdot d^3 \cdot e \cdot f^2 - b \cdot d^2 \cdot f^3) \cdot x^2 + 3 \cdot (b \cdot d^3 \cdot e^2 \cdot f - 2 \cdot b \cdot d^2 \cdot e \cdot f^2 + 2 \cdot b \cdot d \cdot f^3) \cdot x) \cdot \cosh(d \cdot x + c)^2 + 2 \cdot (b \cdot d^3 \cdot f^3 \cdot x^3 + b \cdot d^3 \cdot e^3 - 3 \cdot b \cdot d^2 \cdot e^2 \cdot f + 6 \cdot b \cdot d \cdot e \cdot f^2 - 6 \cdot b \cdot f^3 + 3 \cdot (b \cdot d^3 \cdot e \cdot f^2 - b \cdot d^2 \cdot f^3) \cdot x^2 + 3 \cdot (b \cdot d^3 \cdot e^2 \cdot f - 2 \cdot b \cdot d^2 \cdot e \cdot f^2 + 2 \cdot b \cdot d \cdot f^3) \cdot x) \cdot \sinh(d \cdot x + c)^2 + 12 \cdot ((b \cdot d^2 \cdot f^3 \cdot x^2 + 2 \cdot b \cdot d^2 \cdot e \cdot f^2 \cdot x + b \cdot d^2 \cdot e^2 \cdot f) \cdot \cosh(d \cdot x + c) + (b \cdot d^2 \cdot f^3 \cdot x^2 + 2 \cdot b \cdot d^2 \cdot e \cdot f^2 \cdot x + b \cdot d^2 \cdot e^2 \cdot f) \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2) / b^2} \cdot \operatorname{dilog}((a \cdot \cosh(d \cdot x + c) + a \cdot \sinh(d \cdot x + c) + (b \cdot \cosh(d \cdot x + c) + b \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2) / b^2} - b) / b + 1) - 12 \cdot ((b \cdot d^2 \cdot f^3 \cdot x^2 + 2 \cdot b \cdot d^2 \cdot e \cdot f^2 \cdot x + b \cdot d^2 \cdot e^2 \cdot f) \cdot \cosh(d \cdot x + c) + (b \cdot d^2 \cdot f^3 \cdot x^2 + 2 \cdot b \cdot d^2 \cdot e \cdot f^2 \cdot x + b \cdot d^2 \cdot e^2 \cdot f) \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2) / b^2} \cdot \operatorname{dilog}((a \cdot \cosh(d \cdot x + c) + a \cdot \sinh(d \cdot x + c) - (b \cdot \cosh(d \cdot x + c) + b \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2) / b^2} - b) / b + 1) - 4 \cdot ((b \cdot d^3 \cdot e^3 - 3 \cdot b \cdot c \cdot d^2 \cdot e^2 \cdot f + 3 \cdot b \cdot c^2 \cdot d \cdot e \cdot f^2 - b \cdot c^3 \cdot f^3) \cdot \cosh(d \cdot x + c) + (b \cdot d^3 \cdot e^3 - 3 \cdot b \cdot c \cdot d^2 \cdot e^2 \cdot f + 3 \cdot b \cdot c^2 \cdot d \cdot e \cdot f^2 - b \cdot c^3 \cdot f^3) \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2) / b^2} \cdot \log(2 \cdot b \cdot \cosh(d \cdot x + c) + 2 \cdot b \cdot \sinh(d \cdot x + c) + 2 \cdot b \cdot \sqrt{(a^2 + b^2) / b^2} + 2 \cdot a) + 4 \cdot ((b \cdot d^3 \cdot e^3 - 3 \cdot b \cdot c \cdot d^2 \cdot e^2 \cdot f + 3 \cdot b \cdot c^2 \cdot d \cdot e \cdot f^2 - b \cdot c^3 \cdot f^3) \cdot \cosh(d \cdot x + c) + (b \cdot d^3 \cdot e^3 - 3 \cdot b \cdot c \cdot d^2 \cdot e^2 \cdot f + 3 \cdot b \cdot c^2 \cdot d \cdot e \cdot f^2 - b \cdot c^3 \cdot f^3) \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2) / b^2} \cdot \log(2 \cdot b \cdot \cosh(d \cdot x + c) + 2 \cdot b \cdot \sinh(d \cdot x + c) - 2 \cdot b \cdot \sqrt{(a^2 + b^2) / b^2} + 2 \cdot a) + 4 \cdot ((b \cdot d^3 \cdot f^3 \cdot x^3 + 3 \cdot b \cdot d^3 \cdot e \cdot f^2 \cdot x^2 + 3 \cdot b \cdot d^3 \cdot e^2 \cdot f \cdot x + 3 \cdot b \cdot c \cdot d^2 \cdot e^2 \cdot f - 3 \cdot b \cdot c^2 \cdot d \cdot e \cdot f^2 + b \cdot c^3 \cdot f^3) \cdot \cosh(d \cdot x + c) + (b \cdot d^3 \cdot f^3 \cdot x^3 + 3 \cdot b \cdot d^3 \cdot e \cdot f^2 \cdot x^2 + 3 \cdot b \cdot d^3 \cdot e^2 \cdot f \cdot x + 3 \cdot b \cdot c \cdot d^2 \cdot e^2 \cdot f - 3 \cdot b \cdot c^2 \cdot d \cdot e \cdot f^2 + b \cdot c^3 \cdot f^3) \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2) / b^2} \cdot \log(-(a \cdot \cosh(d \cdot x + c) + a \cdot \sinh(d \cdot x + c) + (b \cdot \cosh(d \cdot x + c) + b \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2) / b^2} - b) / b) - 4 \cdot ((b \cdot d^3 \cdot f^3 \cdot x^3 + 3 \cdot b \cdot d^3 \cdot e \cdot f^2 \cdot x^2 + 3 \cdot b \cdot d^3 \cdot e^2 \cdot f \cdot x + 3 \cdot b \cdot c \cdot d^2 \cdot e^2 \cdot f - 3 \cdot b \cdot c^2 \cdot d \cdot e \cdot f^2 + b \cdot c^3 \cdot f^3) \cdot \cosh(d \cdot x + c) + (b \cdot d^3 \cdot f^3 \cdot x^3 + 3 \cdot b \cdot d^3 \cdot e \cdot f^2 \cdot x^2 + 3 \cdot b \cdot d^3 \cdot e^2 \cdot f \cdot x + 3 \cdot b \cdot c \cdot d^2 \cdot e^2 \cdot f - 3 \cdot b \cdot c^2 \cdot d \cdot e \cdot f^2 + b \cdot c^3 \cdot f^3) \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2) / b^2} \cdot \log(-(a \cdot \cosh(d \cdot x + c) + a \cdot \sinh(d \cdot x + c) - (b \cdot \cosh(d \cdot x + c) + b \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2) / b^2} - b) / b) + 24 \cdot (b \cdot f^3 \cdot \cosh(d \cdot x + c) + b \cdot f^3 \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2) / b^2} \cdot \operatorname{polylog}(4, (a \cdot \cosh(d \cdot x + c) + a \cdot \sinh(d \cdot x + c) + (b \cdot \cosh(d \cdot x + c) + b \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2) / b^2}) / b) - 24 \cdot (b \cdot f^3 \cdot \cosh(d \cdot x + c) + b \cdot f^3 \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2) / b^2} \cdot \operatorname{polylog}(4, (a \cdot \cosh(d \cdot x + c) + a \cdot \sinh(d \cdot x + c) - (b \cdot \cosh(d \cdot x + c) + b \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2) / b^2}) / b) - 24 \cdot ((b \cdot d \cdot f^3 \cdot x + b \cdot d \cdot e \cdot f^2) \cdot \cosh(d \cdot x + c) + (b \cdot d \cdot f^3 \cdot x + b \cdot d \cdot e \cdot f^2) \cdot \sinh(d \cdot x + c)) \cdot \sqrt{(a^2 + b^2)}$$



```

/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*((b*d*f^3*x + b*d*e*f^2)*cosh
(d*x + c) + (b*d*f^3*x + b*d*e*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*po
lylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2))/b) + 6*(b*d^3*e^2*f + 2*b*d^2*e*f^2 + 2*b*d*f
^3)*x - (a*d^4*f^3*x^4 + 4*a*d^4*e*f^2*x^3 + 6*a*d^4*e^2*f*x^2 + 4*a*d^4*e
^3*x)*cosh(d*x + c) - (a*d^4*f^3*x^4 + 4*a*d^4*e*f^2*x^3 + 6*a*d^4*e^2*f*x^2
+ 4*a*d^4*e^3*x - 4*(b*d^3*f^3*x^3 + b*d^3*e^3 - 3*b*d^2*e^2*f + 6*b*d*e*f
^2 - 6*b*f^3 + 3*(b*d^3*e*f^2 - b*d^2*f^3))*x^2 + 3*(b*d^3*e^2*f - 2*b*d^2*e
*f^2 + 2*b*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c))/(b^2*d^4*cosh(d*x + c) +
b^2*d^4*sinh(d*x + c))

```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cosh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c)^2}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*cosh(d\*x + c)^2/(b\*sinh(d\*x + c) + a), x)

$$3.295 \quad \int \frac{(e+fx)^2 \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=389

$$\frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2} - \frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2} - \frac{2f^2\sqrt{a^2+b^2}\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3}$$

[Out]  $-(a*(e + f*x)^3)/(3*b^2*f) + (2*f^2*\text{Cosh}[c + d*x])/(b*d^3) + ((e + f*x)^2*\text{Cosh}[c + d*x])/(b*d) + (\text{Sqrt}[a^2 + b^2]*(e + f*x)^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])])/(b^2*d) - (\text{Sqrt}[a^2 + b^2]*(e + f*x)^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(b^2*d) + (2*\text{Sqrt}[a^2 + b^2]*f*(e + f*x))*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))]/(b^2*d^2) - (2*\text{Sqrt}[a^2 + b^2]*f*(e + f*x))*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))]/(b^2*d^2) - (2*\text{Sqrt}[a^2 + b^2]*f^2*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))]/(b^2*d^3) + (2*\text{Sqrt}[a^2 + b^2]*f^2*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))]/(b^2*d^3) - (2*f*(e + f*x))*\text{Sinh}[c + d*x])/(b*d^2)$

**Rubi [A]** time = 0.785279, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {5565, 32, 3296, 2638, 3322, 2264, 2190, 2531, 2282, 6589}

$$\frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2} - \frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2} - \frac{2f^2\sqrt{a^2+b^2}\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Cosh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]), x]

[Out]  $-(a*(e + f*x)^3)/(3*b^2*f) + (2*f^2*\text{Cosh}[c + d*x])/(b*d^3) + ((e + f*x)^2*\text{Cosh}[c + d*x])/(b*d) + (\text{Sqrt}[a^2 + b^2]*(e + f*x)^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])])/(b^2*d) - (\text{Sqrt}[a^2 + b^2]*(e + f*x)^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(b^2*d) + (2*\text{Sqrt}[a^2 + b^2]*f*(e + f*x))*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))]/(b^2*d^2) - (2*\text{Sqrt}[a^2 + b^2]*f*(e + f*x))*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))]/(b^2*d^2) - (2*\text{Sqrt}[a^2 + b^2]*f^2*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))]/(b^2*d^3) + (2*\text{Sqrt}[a^2 + b^2]*f^2*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))]/(b^2*d^3) - (2*f*(e + f*x))*\text{Sinh}[c + d*x])/(b*d^2)$

Rule 5565

Int[(Cosh[c\_.] + (d\_.)\*(x\_.)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.)]/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := -Dist[a/b^2, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

Rule 32

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 3296

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)], x\_Symbol] := -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_.)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3322

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)/((a\_.) + (b\_.)\*sin[(e\_.) + (Complex[0, fz\_] \* (f\_.)\*(x\_.))]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(-(I\*b) + 2\*a\*E^(-(I\*e) + f\*fz\*x) + I\*b\*E^(2\*(-(I\*e) + f\*fz\*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 2264

Int[((F\_)^(u\_.)\*((f\_.) + (g\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_.) + (c\_.)\*(F\_)^(v\_.)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[((f + g\*x)^m\*F^u)/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[((f + g\*x)^m\*F^u)/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_.)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_.)))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Di

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^(n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :=> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx &= -\frac{a \int (e+fx)^2 dx}{b^2} + \frac{\int (e+fx)^2 \sinh(c+dx) dx}{b} + \frac{(a^2+b^2) \int \frac{(e+fx)^2}{a+b \sinh(c+dx)} dx}{b^2} \\
&= -\frac{a(e+fx)^3}{3b^2 f} + \frac{(e+fx)^2 \cosh(c+dx)}{bd} + \frac{(2(a^2+b^2)) \int \frac{e^{c+dx}(e+fx)^2}{-b+2ae^{c+dx}+be^{2(c+dx)}} dx}{b^2} - \frac{(2f) \int}{b} \\
&= -\frac{a(e+fx)^3}{3b^2 f} + \frac{(e+fx)^2 \cosh(c+dx)}{bd} - \frac{2f(e+fx) \sinh(c+dx)}{bd^2} + \frac{(2\sqrt{a^2+b^2}) \int \frac{1}{2a-}}{b} \\
&= -\frac{a(e+fx)^3}{3b^2 f} + \frac{2f^2 \cosh(c+dx)}{bd^3} + \frac{(e+fx)^2 \cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}(e+fx)^2 \log(1)}{b^2 d} \\
&= -\frac{a(e+fx)^3}{3b^2 f} + \frac{2f^2 \cosh(c+dx)}{bd^3} + \frac{(e+fx)^2 \cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}(e+fx)^2 \log(1)}{b^2 d} \\
&= -\frac{a(e+fx)^3}{3b^2 f} + \frac{2f^2 \cosh(c+dx)}{bd^3} + \frac{(e+fx)^2 \cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}(e+fx)^2 \log(1)}{b^2 d} \\
&= -\frac{a(e+fx)^3}{3b^2 f} + \frac{2f^2 \cosh(c+dx)}{bd^3} + \frac{(e+fx)^2 \cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}(e+fx)^2 \log(1)}{b^2 d}
\end{aligned}$$

**Mathematica [A]** time = 2.91874, size = 447, normalized size = 1.15

$$\frac{3\sqrt{a^2+b^2} \left( -2df(e+fx) \text{PolyLog} \left( 2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a} \right) + 2df(e+fx) \text{PolyLog} \left( 2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} \right) + 2f^2 \text{PolyLog} \left( 3, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a} \right) \right)}{b^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Cosh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out]  $-(a*d^3*x*(3*e^2 + 3*e*f*x + f^2*x^2) + 3*\text{Sqrt}[a^2 + b^2]*(2*d^2*e^2*\text{ArcTan}[\frac{(a + b*E^{(c + d*x)})}{\text{Sqrt}[a^2 + b^2]}] - 2*d^2*e*f*x*\text{Log}[1 + \frac{(b*E^{(c + d*x)})}{(a - \text{Sqrt}[a^2 + b^2])}] - d^2*f^2*x^2*\text{Log}[1 + \frac{(b*E^{(c + d*x)})}{(a - \text{Sqrt}[a^2 + b^2])}] + 2*d^2*e*f*x*\text{Log}[1 + \frac{(b*E^{(c + d*x)})}{(a + \text{Sqrt}[a^2 + b^2])}] + d^2*f^2*x^2*\text{Log}[1 + \frac{(b*E^{(c + d*x)})}{(a + \text{Sqrt}[a^2 + b^2])}] - 2*d*f*(e + f*x)*\text{PolyLog}[2, \frac{(b*E^{(c + d*x)})}{(-a + \text{Sqrt}[a^2 + b^2])}] + 2*d*f*(e + f*x)*\text{PolyLog}[2, -\frac{(b*E^{(c + d*x)})}{(a + \text{Sqrt}[a^2 + b^2])}]) + 2*f^2*\text{PolyLog}[3, \frac{(b*E^{(c + d*x)})}{(-a + \text{Sqrt}[a^2 + b^2])}] - 2*f^2*\text{PolyLog}[3, -\frac{(b*E^{(c + d*x)})}{(a + \text{Sqrt}[a^2 + b^2])}]) - 3*b*\text{Cosh}[d*x]*((2*f^2 + d^2*(e + f*x)^2)*\text{Cosh}[c] - 2*d*f*(e + f*x)*\text{Sinh}[c]) + 3*b*(2*d*f*(e + f*x)*\text{Cosh}[c] - (2*f^2 + d^2*(e + f*x)^2)*\text{Sinh}[c])$

$x)^2 * \text{Sinh}[c]) * \text{Sinh}[d*x]) / (3*b^2*d^3)$

**Maple [F]** time = 0.23, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\cosh(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)`

[Out] `int((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [C]** time = 2.75005, size = 3163, normalized size = 8.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out]  $\frac{1}{6} * (3*b*d^2*f^2*x^2 + 3*b*d^2*e^2 + 6*b*d*e*f + 6*b*f^2 + 3*(b*d^2*f^2*x^2 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*f - b*d*f^2)*x)*\cosh(d*x + c)^2 + 3*(b*d^2*f^2*x^2 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*f - b*d*f^2)*x)*\sinh(d*x + c)^2 + 12*((b*d*f^2*x + b*d*e*f)*\cosh(d*x + c) + (b*d*f^2*x + b*d*e*f)*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} * \text{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)$

```

)/b^2) - b)/b + 1) - 12*((b*d*f^2*x + b*d*e*f)*cosh(d*x + c) + (b*d*f^2*x +
b*d*e*f)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*s
inh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
b)/b + 1) - 6*((b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cosh(d*x + c) + (b*d^2
*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*
b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6*
((b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*cosh(d*x + c) + (b*d^2*e^2 - 2*b*c*d
*e*f + b*c^2*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c
) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6*((b*d^2*f^2*x^
2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*cosh(d*x + c) + (b*d^2*f^2*x^2
+ 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 6*((b*d^2*f^2*x^2 + 2*b*d^2*e*f*x
+ 2*b*c*d*e*f - b*c^2*f^2)*cosh(d*x + c) + (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x +
2*b*c*d*e*f - b*c^2*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh
(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
+ b^2)/b^2) - b)/b) - 12*(b*f^2*cosh(d*x + c) + b*f^2*sinh(d*x + c))*sqrt(
(a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*
x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 12*(b*f^2*cosh(d*x +
c) + b*f^2*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c)
+ a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2))/b) + 6*(b*d^2*e*f + b*d*f^2)*x - 2*(a*d^3*f^2*x^3 + 3*a*d^3*e*f*x^2 +
3*a*d^3*e^2*x)*cosh(d*x + c) - 2*(a*d^3*f^2*x^3 + 3*a*d^3*e*f*x^2 + 3*a*d^3
*e^2*x - 3*(b*d^2*f^2*x^2 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*f
- b*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))/(b^2*d^3*cosh(d*x + c) + b^2*d^
3*sinh(d*x + c))

```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cosh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c)^2}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```



$$3.296 \quad \int \frac{(e+fx) \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=252

$$\frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2} - \frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2} + \frac{\sqrt{a^2+b^2}(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2d} - \frac{\sqrt{a^2+b^2}}{b^2d}$$

[Out] -((a\*e\*x)/b^2) - (a\*f\*x^2)/(2\*b^2) + ((e + f\*x)\*Cosh[c + d\*x])/(b\*d) + (Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/(b^2\*d) - (Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/(b^2\*d) + (Sqrt[a^2 + b^2]\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^2\*d^2) - (Sqrt[a^2 + b^2]\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^2\*d^2) - (f\*Sinh[c + d\*x])/(b\*d^2)

**Rubi [A]** time = 0.442924, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {5565, 3296, 2637, 3322, 2264, 2190, 2279, 2391}

$$\frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2} - \frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2} + \frac{\sqrt{a^2+b^2}(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^2d} - \frac{\sqrt{a^2+b^2}}{b^2d}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Cosh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]), x]

[Out] -((a\*e\*x)/b^2) - (a\*f\*x^2)/(2\*b^2) + ((e + f\*x)\*Cosh[c + d\*x])/(b\*d) + (Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/(b^2\*d) - (Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/(b^2\*d) + (Sqrt[a^2 + b^2]\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^2\*d^2) - (Sqrt[a^2 + b^2]\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^2\*d^2) - (f\*Sinh[c + d\*x])/(b\*d^2)

### Rule 5565

Int[(Cosh[(c\_.) + (d\_.)\*(x\_.)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.)]/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] :> -Dist[a/b^2, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]

&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[  
((c + d\*x)^m\*cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*cos[  
e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /;  
FreeQ[{c, d}, x]

### Rule 3322

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*sin[(e\_.) + (Complex[0, fz\_]\*)  
(f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(-  
(I\*b) + 2\*a\*E^(-(I\*e) + f\*fz\*x) + I\*b\*E^(2\*(-I\*e) + f\*fz\*x))), x], x] /; F  
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.)  
\*(F\_)^(v\_)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[  
((f + g\*x)^m\*F^u)/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[  
((f + g\*x)^m\*F^u)/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,  
2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/  
((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)), x\_Symbol] := Simp  
[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Di  
st[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))  
)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_.) + (b\_.)\*((F\_)^(e\_.)\*((c\_.) + (d\_.)\*(x\_)))^(n\_.)], x\_Symbol]  
:= Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x))  
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{(e+fx) \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx &= -\frac{a \int (e+fx) dx}{b^2} + \frac{\int (e+fx) \sinh(c+dx) dx}{b} + \frac{(a^2+b^2) \int \frac{e+fx}{a+b \sinh(c+dx)} dx}{b^2} \\
 &= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx) \cosh(c+dx)}{bd} + \frac{(2(a^2+b^2)) \int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}} dx}{b^2} - \frac{f \int \cosh(c+dx) dx}{b} \\
 &= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx) \cosh(c+dx)}{bd} - \frac{f \sinh(c+dx)}{bd^2} + \frac{(2\sqrt{a^2+b^2}) \int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{b} \\
 &= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx) \cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} - \frac{\sqrt{a^2+b^2}}{b} \\
 &= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx) \cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} - \frac{\sqrt{a^2+b^2}}{b} \\
 &= -\frac{aex}{b^2} - \frac{afx^2}{2b^2} + \frac{(e+fx) \cosh(c+dx)}{bd} + \frac{\sqrt{a^2+b^2}(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d} - \frac{\sqrt{a^2+b^2}}{b}
 \end{aligned}$$

**Mathematica [A]** time = 1.95755, size = 258, normalized size = 1.02

$$\frac{2\sqrt{a^2+b^2} \left( f \text{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) - f \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) - 2de \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) + f(c+dx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right) \right)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Cosh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]), x]

[Out] (a\*(c + d\*x)\*(c\*f - d\*(2\*e + f\*x)) + 2\*b\*d\*(e + f\*x)\*Cosh[c + d\*x] + 2\*sqrt[a^2 + b^2]\*(-2\*d\*e\*ArcTanh[(a + b\*E^(c + d\*x))/sqrt[a^2 + b^2]] + 2\*c\*f\*ArcTanh[(a + b\*E^(c + d\*x))/sqrt[a^2 + b^2]] + f\*(c + d\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - sqrt[a^2 + b^2])] - f\*(c + d\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + sqrt[a^2 + b^2])] + f\*PolyLog[2, (b\*E^(c + d\*x))/(-a + sqrt[a^2 + b^2])] - f\*PolyLog[2, -(b\*E^(c + d\*x))/(a + sqrt[a^2 + b^2])]) - 2\*b\*f\*Sinh[c + d\*x])/(2\*b^2\*d^2)

---

**Maple [B]** time = 0.089, size = 901, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)*\cosh(d*x+c)^2/(a+b*\sinh(d*x+c)),x)$

[Out] 
$$\begin{aligned} & -1/2*a*f*x^2/b^2-a*e*x/b^2+1/2*(d*f*x+d*e-f)/d^2/b*\exp(d*x+c)+1/2*(d*f*x+d* \\ & e+f)/d^2/b*\exp(-d*x-c)-2*a^2/b^2/d*e/(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d \\ & *x+c)+2*a)/(a^2+b^2)^{(1/2)})-2/d*e/(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+ \\ & c)+2*a)/(a^2+b^2)^{(1/2)})+a^2/b^2/d*f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2 \\ & +b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*x+a^2/b^2/d^2*f/(a^2+b^2)^{(1/2)}*\ln((-b \\ & *\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*c-a^2/b^2/d*f/(a^2+b^2 \\ & )^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*x-a^2/b^2/ \\ & d^2*f/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)} \\ & ))*c+a^2/b^2/d^2*f/(a^2+b^2)^{(1/2)}*\operatorname{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a) \\ & /(-a+(a^2+b^2)^{(1/2)}))-a^2/b^2/d^2*f/(a^2+b^2)^{(1/2)}*\operatorname{dilog}((b*\exp(d*x+c)+(a \\ & ^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))+1/d*f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+ \\ & c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*x+1/d^2*f/(a^2+b^2)^{(1/2)}*\ln((- \\ & b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*c-1/d*f/(a^2+b^2)^{(1/ \\ & 2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*x-1/d^2*f/(a^2+ \\ & b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*c+1/d^2 \\ & *f/(a^2+b^2)^{(1/2)}*\operatorname{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1 \\ & /2)}))-1/d^2*f/(a^2+b^2)^{(1/2)}*\operatorname{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^ \\ & 2+b^2)^{(1/2)}))+2*a^2/b^2/d^2*f*c/(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c \\ & )+2*a)/(a^2+b^2)^{(1/2)})+2/d^2*f*c/(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+ \\ & c)+2*a)/(a^2+b^2)^{(1/2)}) \end{aligned}$$

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x+e)*\cosh(d*x+c)^2/(a+b*\sinh(d*x+c)),x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

---

**Fricas [B]** time = 2.32499, size = 1872, normalized size = 7.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & \frac{1}{2} * (b*d*f*x + b*d*e + (b*d*f*x + b*d*e - b*f) * \cosh(d*x + c)^2 + (b*d*f*x + \\ & b*d*e - b*f) * \sinh(d*x + c)^2 + 2 * (b*f * \cosh(d*x + c) + b*f * \sinh(d*x + c)) * \sqrt{\frac{a^2 + b^2}{b^2}} * \operatorname{dilog}((a * \cosh(d*x + c) + a * \sinh(d*x + c) + (b * \cosh(d*x + c) + b * \sinh(d*x + c)) * \sqrt{\frac{a^2 + b^2}{b^2}} - b) / b + 1) - 2 * (b*f * \cosh(d*x + c) + b*f * \sinh(d*x + c)) * \sqrt{\frac{a^2 + b^2}{b^2}} * \operatorname{dilog}((a * \cosh(d*x + c) + a * \sinh(d*x + c) - (b * \cosh(d*x + c) + b * \sinh(d*x + c)) * \sqrt{\frac{a^2 + b^2}{b^2}} - b) / b + 1) - 2 * ((b*d*e - b*c*f) * \cosh(d*x + c) + (b*d*e - b*c*f) * \sinh(d*x + c)) * \sqrt{\frac{a^2 + b^2}{b^2}} * \log(2 * b * \cosh(d*x + c) + 2 * b * \sinh(d*x + c) + 2 * b * \sqrt{\frac{a^2 + b^2}{b^2}} + 2 * a) + 2 * ((b*d*e - b*c*f) * \cosh(d*x + c) + (b*d*e - b*c*f) * \sinh(d*x + c)) * \sqrt{\frac{a^2 + b^2}{b^2}} * \log(2 * b * \cosh(d*x + c) + 2 * b * \sinh(d*x + c) - 2 * b * \sqrt{\frac{a^2 + b^2}{b^2}} + 2 * a) + 2 * ((b*d*f*x + b*c*f) * \cosh(d*x + c) + (b*d*f*x + b*c*f) * \sinh(d*x + c)) * \sqrt{\frac{a^2 + b^2}{b^2}} * \log(-(a * \cosh(d*x + c) + a * \sinh(d*x + c) + (b * \cosh(d*x + c) + b * \sinh(d*x + c)) * \sqrt{\frac{a^2 + b^2}{b^2}} - b) / b) - 2 * ((b*d*f*x + b*c*f) * \cosh(d*x + c) + (b*d*f*x + b*c*f) * \sinh(d*x + c)) * \sqrt{\frac{a^2 + b^2}{b^2}} * \log(-(a * \cosh(d*x + c) + a * \sinh(d*x + c) - (b * \cosh(d*x + c) + b * \sinh(d*x + c)) * \sqrt{\frac{a^2 + b^2}{b^2}} - b) / b) + b*f - (a*d^2*f*x^2 + 2*a*d^2*e*x) * \cosh(d*x + c) - (a*d^2*f*x^2 + 2*a*d^2*e*x - 2 * (b*d*f*x + b*d*e - b*f) * \cosh(d*x + c)) * \sinh(d*x + c)) / (b^2*d^2*\cosh(d*x + c) + b^2*d^2*\sinh(d*x + c)) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \cosh(dx + c)^2}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

$$3.297 \quad \int \frac{\cosh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=68

$$-\frac{2\sqrt{a^2+b^2} \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2 d} - \frac{ax}{b^2} + \frac{\cosh(c+dx)}{bd}$$

[Out] -((a\*x)/b^2) - (2\*Sqrt[a^2 + b^2]\*ArcTanh[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[a^2 + b^2]])/(b^2\*d) + Cosh[c + d\*x]/(b\*d)

**Rubi [A]** time = 0.116962, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {2695, 2735, 2660, 618, 204}

$$-\frac{2\sqrt{a^2+b^2} \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^2 d} - \frac{ax}{b^2} + \frac{\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]^2/(a + b\*Sinh[c + d\*x]),x]

[Out] -((a\*x)/b^2) - (2\*Sqrt[a^2 + b^2]\*ArcTanh[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[a^2 + b^2]])/(b^2\*d) + Cosh[c + d\*x]/(b\*d)

#### Rule 2695

Int[(cos[(e\_.) + (f\_.)\*(x\_.)]\*(g\_.))^(p\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(m\_.), x\_Symbol] :> Simp[(g\*(g\*Cos[e + f\*x])^(p - 1)\*(a + b\*Sin[e + f\*x])^(m + 1))/(b\*f\*(m + p)), x] + Dist[(g^2\*(p - 1))/(b\*(m + p)), Int[(g\*Cos[e + f\*x])^(p - 2)\*(a + b\*Sin[e + f\*x])^m\*(b + a\*Sin[e + f\*x]), x], x] /; FreeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && IntegersQ[2\*m, 2\*p]

#### Rule 2735

Int[((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])/((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Simp[(b\*x)/d, x] - Dist[(b\*c - a\*d)/d, Int[1/(c + d\*Sin[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0]

Rule 2660

Int[((a\_) + (b\_)\*sin[(c\_) + (d\_)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + 2\*b\*e\*x + a\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 618

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^2(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\cosh(c + dx)}{bd} + \frac{i \int \frac{-ib + ia \sinh(c + dx)}{a + b \sinh(c + dx)} dx}{b} \\
 &= -\frac{ax}{b^2} + \frac{\cosh(c + dx)}{bd} + \frac{(a^2 + b^2) \int \frac{1}{a + b \sinh(c + dx)} dx}{b^2} \\
 &= -\frac{ax}{b^2} + \frac{\cosh(c + dx)}{bd} - \frac{(2i(a^2 + b^2)) \text{Subst}\left(\int \frac{1}{a - 2ibx + ax^2} dx, x, \tan\left(\frac{1}{2}(ic + idx)\right)\right)}{b^2 d} \\
 &= -\frac{ax}{b^2} + \frac{\cosh(c + dx)}{bd} + \frac{(4i(a^2 + b^2)) \text{Subst}\left(\int \frac{1}{-4(a^2 + b^2) - x^2} dx, x, -2ib + 2a \tan\left(\frac{1}{2}(ic + idx)\right)\right)}{b^2 d} \\
 &= -\frac{ax}{b^2} - \frac{2\sqrt{a^2 + b^2} \tanh^{-1}\left(\frac{b - a \tanh\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a^2 + b^2}}\right)}{b^2 d} + \frac{\cosh(c + dx)}{bd}
 \end{aligned}$$

**Mathematica [C]** time = 2.08619, size = 492, normalized size = 7.24

$$\cosh(c + dx) \left( 2\sqrt{b^2(a - ib)} \sqrt{1 + i \sinh(c + dx)} \tanh^{-1} \left( \frac{\sqrt{a - ib} \sqrt{-\frac{b(\sinh(c + dx) + i)}{a - ib}}}{\sqrt{a + ib} \sqrt{-\frac{b(\sinh(c + dx) - i)}{a + ib}}} \right) + \sqrt{a + ib} \left( \sqrt{b^2} \sqrt{\frac{b(1 + i \sinh(c + dx))}{b - ia}} \left( \sqrt{a - ib} \sqrt{1 + i \sinh(c + dx)} \right) \right) \right)$$


---


$$b\sqrt{b^2 d} \sqrt{a - ib} \sqrt{a + ib} \sqrt{1 + i \sinh(c + dx)}$$



Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]^2/(a + b\*Sinh[c + d\*x]),x]

[Out] (Cosh[c + d\*x]\*(2\*(a - I\*b)\*Sqrt[b^2]\*ArcTanh[(Sqrt[a - I\*b]\*Sqrt[-((b\*(I + Sinh[c + d\*x]))/(a - I\*b))])]/(Sqrt[a + I\*b]\*Sqrt[-((b\*(-I + Sinh[c + d\*x]))/(a + I\*b))])]\*Sqrt[1 + I\*Sinh[c + d\*x]] + Sqrt[a + I\*b]\*((-2\*I)\*Sqrt[a - I\*b]\*b\*ArcTan[(Sqrt[(-I)\*b]\*Sqrt[-((b\*(I + Sinh[c + d\*x]))/(a - I\*b))])]/(Sqrt[I\*b]\*Sqrt[-((b\*(-I + Sinh[c + d\*x]))/(a + I\*b))])]\*Sqrt[1 + I\*Sinh[c + d\*x]] + Sqrt[b^2]\*Sqrt[(b\*(1 + I\*Sinh[c + d\*x]))/((-I)\*a + b)]\*(-2\*(-1)^(3/4))\*Sqrt[b]\*ArcSin[(-1)^(1/4)\*Sqrt[a - I\*b]\*Sqrt[-((b\*(I + Sinh[c + d\*x]))/(a - I\*b))])]/(Sqrt[2]\*Sqrt[b])) + Sqrt[a - I\*b]\*Sqrt[1 + I\*Sinh[c + d\*x]]\*Sqrt[-((b\*(I + Sinh[c + d\*x]))/(a - I\*b))])]/(Sqrt[a - I\*b]\*Sqrt[a + I\*b]\*b\*Sqrt[b^2]\*d\*Sqrt[1 + I\*Sinh[c + d\*x]]\*Sqrt[-((b\*(-I + Sinh[c + d\*x]))/(a + I\*b))])\*Sqrt[-((b\*(I + Sinh[c + d\*x]))/(a - I\*b))])]

**Maple [B]** time = 0.039, size = 174, normalized size = 2.6

$$\frac{1}{bd} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} - \frac{a}{db^2} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) - \frac{1}{bd} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^{-1} + \frac{a}{db^2} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] 1/d/b/(tanh(1/2\*d\*x+1/2\*c)+1)-1/d\*a/b^2\*ln(tanh(1/2\*d\*x+1/2\*c)+1)-1/d/b/(tanh(1/2\*d\*x+1/2\*c)-1)+1/d\*a/b^2\*ln(tanh(1/2\*d\*x+1/2\*c)-1)+2/d\*a^2/b^2/(a^2+b^2)^(1/2)\*arctanh(1/2\*(2\*a\*tanh(1/2\*d\*x+1/2\*c)-2\*b)/(a^2+b^2)^(1/2))+2/d/(a^2+b^2)^(1/2)\*arctanh(1/2\*(2\*a\*tanh(1/2\*d\*x+1/2\*c)-2\*b)/(a^2+b^2)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.16567, size = 684, normalized size = 10.06

$$\frac{2 a d x \cosh (d x+c)-b \cosh (d x+c)^2-b \sinh (d x+c)^2-2 \sqrt{a^2+b^2}(\cosh (d x+c)+\sinh (d x+c)) \log \left(\frac{b^2 \cosh (d x+c)^2+b^2 \sinh (d x+c)^2+2 a b \cosh (d x+c)+2 a^2+b^2}{2\left(b^2 d \cosh (d x+c)+b^2 d \sinh (d x+c)\right)}\right)}{2\left(b^2 d \cosh (d x+c)+b^2 d \sinh (d x+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-1/2*(2*a*d*x*cosh(d*x+c)-b*cosh(d*x+c)^2-b*sinh(d*x+c)^2-2*sqrt(a^2+b^2)*(cosh(d*x+c)+sinh(d*x+c))*log((b^2*cosh(d*x+c)^2+b^2*sinh(d*x+c)^2+2*a*b*cosh(d*x+c)+2*a^2+b^2+2*(b^2*cosh(d*x+c)+a*b)*sinh(d*x+c)-2*sqrt(a^2+b^2)*(b*cosh(d*x+c)+b*sinh(d*x+c)+a))/(b*cosh(d*x+c)^2+b*sinh(d*x+c)^2+2*a*cosh(d*x+c)+2*(b*cosh(d*x+c)+a)*sinh(d*x+c)-b))+2*(a*d*x-b*cosh(d*x+c))*sinh(d*x+c)-b)/(b^2*d*cosh(d*x+c)+b^2*d*sinh(d*x+c))$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [A]** time = 1.18774, size = 157, normalized size = 2.31

$$-\frac{(d x+c) a}{b^2 d}+\frac{e^{(d x+c)}}{2 b d}+\frac{e^{(-d x-c)}}{2 b d}+\frac{\sqrt{a^2+b^2} \log \left(\frac{\left|2 b e^{(d x+c)}+2 a-2 \sqrt{a^2+b^2}\right|}{\left|2 b e^{(d x+c)}+2 a+2 \sqrt{a^2+b^2}\right|}\right)}{b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -(d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)/(b*d) + 1/2*e^(-d*x - c)/(b*d) + sqrt(a^2 + b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(b^2*d)
```

$$3.298 \quad \int \frac{\cosh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=30

$$\text{Unintegrable} \left( \frac{\cosh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[Cosh[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0748446, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Cosh[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Cosh[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\cosh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [A]** time = 25.0941, size = 0, normalized size = 0.

$$\int \frac{\cosh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Cosh[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Integrate[Cosh[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Maple [A]** time = 0.111, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx + c))^2}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)), x)

[Out] int(cosh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$2(a^2e^c + b^2e^c) \int -\frac{e^{(dx)}}{b^3fx + b^3e - (b^3fxe^{(2c)} + b^3ee^{(2c)})e^{(2dx)} - 2(ab^2fxe^c + ab^2ee^c)e^{(dx)}} dx + \frac{e^{(-c + \frac{de}{f})} E_1\left(\frac{(fx+e)d}{f}\right)}{2bf} - \frac{e^{(c - \frac{de}{f})} E_1\left(\frac{(fx+e)d}{f}\right)}{2bf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)), x, algorithm="maxima")

[Out] 2\*(a^2\*e^c + b^2\*e^c)\*integrate(-e^(d\*x)/(b^3\*f\*x + b^3\*e - (b^3\*f\*x\*e^(2\*c) + b^3\*e\*e^(2\*c))\*e^(2\*d\*x) - 2\*(a\*b^2\*f\*x\*e^c + a\*b^2\*e\*e^c)\*e^(d\*x)), x) + 1/2\*e^(-c + d\*e/f)\*exp\_integral\_e(1, (f\*x + e)\*d/f)/(b\*f) - 1/2\*e^(c - d\*e/f)\*exp\_integral\_e(1, -(f\*x + e)\*d/f)/(b\*f) - a\*log(f\*x + e)/(b^2\*f)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx + c)^2}{afx + ae + (bfx + be) \sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)), x, algorithm="fricas")

[Out] `integral(cosh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)), x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)^2}{(fx + e)(b \sinh(dx + c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)), x, algorithm="giac")`

[Out] `integrate(cosh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)), x)`

$$3.299 \quad \int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=642

$$\frac{6f^2(a^2+b^2)(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3} - \frac{6f^2(a^2+b^2)(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^3} + \frac{3f(a^2+b^2)(e+fx)}{b^3d^3}$$

```
[Out] (3*f^3*x)/(8*b*d^3) + (e + f*x)^3/(4*b*d) - ((a^2 + b^2)*(e + f*x)^4)/(4*b^3*f) + (6*a*f^3*Cosh[c + d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*Cosh[c + d*x])/(b^2*d^2) + ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) + ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*d) + (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^2) + (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) - (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^3) - (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^3) + (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^4) + (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^4) - (6*a*f^2*(e + f*x)*Sinh[c + d*x])/(b^2*d^3) - (a*(e + f*x)^3*Sinh[c + d*x])/(b^2*d) - (3*f^3*Cosh[c + d*x]*Sinh[c + d*x])/(8*b*d^4) - (3*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^2) + (3*f^2*(e + f*x)*Sinh[c + d*x]^2)/(4*b*d^3) + ((e + f*x)^3*Sinh[c + d*x]^2)/(2*b*d)
```

**Rubi [A]** time = 0.784937, antiderivative size = 642, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 14, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5565, 3296, 2638, 5446, 3311, 32, 2635, 8, 5561, 2190, 2531, 6609, 2282, 6589}

$$\frac{6f^2(a^2+b^2)(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3} - \frac{6f^2(a^2+b^2)(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^3} + \frac{3f(a^2+b^2)(e+fx)}{b^3d^3}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Cosh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

```
[Out] (3*f^3*x)/(8*b*d^3) + (e + f*x)^3/(4*b*d) - ((a^2 + b^2)*(e + f*x)^4)/(4*b^3*f) + (6*a*f^3*Cosh[c + d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*Cosh[c + d*x])/(b^2*d^2) + ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) + ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*d)
```

$$\frac{\sqrt{a^2 + b^2}}{b^3 d} + \frac{(3(a^2 + b^2) f (e + f x)^2 \text{PolyLog}[2, -(b E^{(c + d x)}) / (a - \sqrt{a^2 + b^2})])}{b^3 d^2} + \frac{(3(a^2 + b^2) f (e + f x)^2 \text{PolyLog}[2, -(b E^{(c + d x)}) / (a + \sqrt{a^2 + b^2})])}{b^3 d^2} - \frac{(6(a^2 + b^2) f^2 (e + f x) \text{PolyLog}[3, -(b E^{(c + d x)}) / (a - \sqrt{a^2 + b^2})])}{b^3 d^3} - \frac{(6(a^2 + b^2) f^2 (e + f x) \text{PolyLog}[3, -(b E^{(c + d x)}) / (a + \sqrt{a^2 + b^2})])}{b^3 d^3} + \frac{(6(a^2 + b^2) f^3 \text{PolyLog}[4, -(b E^{(c + d x)}) / (a - \sqrt{a^2 + b^2})])}{b^3 d^4} + \frac{(6(a^2 + b^2) f^3 \text{PolyLog}[4, -(b E^{(c + d x)}) / (a + \sqrt{a^2 + b^2})])}{b^3 d^4} - \frac{(6 a f^2 (e + f x) \text{Sinh}[c + d x])}{b^2 d^3} - \frac{(a (e + f x)^3 \text{Sinh}[c + d x])}{b^2 d} - \frac{(3 f^3 \text{Cosh}[c + d x] \text{Sinh}[c + d x])}{8 b d^4} - \frac{(3 f (e + f x)^2 \text{Cosh}[c + d x] \text{Sinh}[c + d x])}{4 b d^2} + \frac{(3 f^2 (e + f x) \text{Sinh}[c + d x]^2)}{4 b d^3} + \frac{(e + f x)^3 \text{Sinh}[c + d x]^2}{2 b d}$$

### Rule 5565

$$\text{Int}[(\text{Cosh}[(c_.) + (d_.)(x_.)]^{(n_.)} * ((e_.) + (f_.)(x_.))^{(m_.)}) / ((a_.) + (b_.) * \text{Sinh}[(c_.) + (d_.)(x_.)]), x\_Symbol] := -\text{Dist}[a/b^2, \text{Int}[(e + f x)^m \text{Cosh}[c + d x]^{(n - 2)}, x], x] + (\text{Dist}[1/b, \text{Int}[(e + f x)^m \text{Cosh}[c + d x]^{(n - 2)} * \text{Sinh}[c + d x], x], x] + \text{Dist}[(a^2 + b^2)/b^2, \text{Int}[(e + f x)^m \text{Cosh}[c + d x]^{(n - 2)} / (a + b \text{Sinh}[c + d x]), x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{IGtQ}[n, 1] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{IGtQ}[m, 0]$$

### Rule 3296

$$\text{Int}[(c_.) + (d_.)(x_.)]^{(m_.)} * \sin[(e_.) + (f_.)(x_.)], x\_Symbol] := -\text{Simp}[(c + d x)^m \text{Cos}[e + f x] / f, x] + \text{Dist}[(d m) / f, \text{Int}[(c + d x)^{(m - 1)} \text{Cos}[e + f x], x], x] /; \text{FreeQ}\{c, d, e, f, x\} \&\& \text{GtQ}[m, 0]$$

### Rule 2638

$$\text{Int}[\sin[(c_.) + (d_.)(x_.)], x\_Symbol] := -\text{Simp}[\text{Cos}[c + d x] / d, x] /; \text{FreeQ}\{c, d, x\}$$

### Rule 5446

$$\text{Int}[\text{Cosh}[(a_.) + (b_.)(x_.)] * ((c_.) + (d_.)(x_.))^{(m_.)} * \text{Sinh}[(a_.) + (b_.)(x_.)]^{(n_.)}, x\_Symbol] := \text{Simp}[(c + d x)^m \text{Sinh}[a + b x]^{(n + 1)} / (b (n + 1)), x] - \text{Dist}[(d m) / (b (n + 1)), \text{Int}[(c + d x)^{(m - 1)} \text{Sinh}[a + b x]^{(n + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, n, x\} \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[n, -1]$$

### Rule 3311

$$\text{Int}[(c_.) + (d_.)(x_.)]^{(m_.)} * ((b_.) * \sin[(e_.) + (f_.)(x_.)])^{(n_.)}, x\_Symbol] := \text{Simp}[(d m (c + d x)^{(m - 1)} (b \text{Sin}[e + f x])^n) / (f^2 n^2), x] + (\text{Dist}[(b^2 (n - 1)) / n, \text{Int}[(c + d x)^m (b \text{Sin}[e + f x])^{(n - 2)}, x], x] - \text{Dist}[($$



$d^2 m(m-1)/(f^2 n^2)$ ,  $\text{Int}[(c + d*x)^{(m-2)}*(b*\text{Sin}[e + f*x])^n, x]$ ,  $x]$   
 $-\text{Simp}[(b*(c + d*x)^m*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(n-1)})/(f*n), x] /;$   
 $\text{FreeQ}\{b, c, d, e, f\}, x\} \&\& \text{GtQ}[n, 1] \&\& \text{GtQ}[m, 1]$

Rule 32

$\text{Int}[(a + b*x)^m, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m+1)}/(b*(m + 1)), x] /;$   
 $\text{FreeQ}\{a, b, m\}, x\} \&\& \text{NeQ}[m, -1]$

Rule 2635

$\text{Int}[(b*\text{sin}[c + d*x])^n, x\_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /;$   
 $\text{FreeQ}\{b, c, d\}, x\} \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 8

$\text{Int}[a, x\_Symbol] \rightarrow \text{Simp}[a*x, x] /;$   
 $\text{FreeQ}[a, x]$

Rule 5561

$\text{Int}[(\text{Cosh}[c + d*x])^m*((e + f*x)^n)/((a + b*\text{Sin}[c + d*x]), x\_Symbol] \rightarrow -\text{Simp}[(e + f*x)^{(m+1)}/(b*f*(m + 1)), x] + (\text{Int}[(e + f*x)^m*\text{E}^{(c + d*x)}/(a - \text{Rt}[a^2 + b^2, 2] + b*\text{E}^{(c + d*x)}), x] + \text{Int}[(e + f*x)^m*\text{E}^{(c + d*x)}/(a + \text{Rt}[a^2 + b^2, 2] + b*\text{E}^{(c + d*x)}), x]) /;$   
 $\text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[a^2 + b^2, 0]$

Rule 2190

$\text{Int}[(F^{(g*(e + f*x))})^n*((c + d*x)^m)/((a + b*(F^{(g*(e + f*x))})^n), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x))})^n)/a]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x))})^n)/a], x], x] /;$   
 $\text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0]$

Rule 2531

$\text{Int}[\text{Log}[1 + (e + f*x)*(F^{(c*(a + b*x))})^n], x\_Symbol] \rightarrow -\text{Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x))})^n)]/(b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{(m-1)}*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x))})^n)], x], x] /;$   
 $\text{FreeQ}\{F, a, b, c, e, f, g, n\}, x\} \&\& \text{GtQ}[m, 0]$

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx &= -\frac{a \int (e+fx)^3 \cosh(c+dx) dx}{b^2} + \frac{\int (e+fx)^3 \cosh(c+dx) \sinh(c+dx) dx}{b} + \frac{(a^2+b^2)}{b} \\
&= -\frac{(a^2+b^2)(e+fx)^4}{4b^3 f} - \frac{a(e+fx)^3 \sinh(c+dx)}{b^2 d} + \frac{(e+fx)^3 \sinh^2(c+dx)}{2bd} + \frac{(a^2+b^2)}{b} \\
&= -\frac{(a^2+b^2)(e+fx)^4}{4b^3 f} + \frac{3af(e+fx)^2 \cosh(c+dx)}{b^2 d^2} + \frac{(a^2+b^2)(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3 d} \\
&= \frac{(e+fx)^3}{4bd} - \frac{(a^2+b^2)(e+fx)^4}{4b^3 f} + \frac{3af(e+fx)^2 \cosh(c+dx)}{b^2 d^2} + \frac{(a^2+b^2)(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3 d} \\
&= \frac{3f^3 x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{(a^2+b^2)(e+fx)^4}{4b^3 f} + \frac{6af^3 \cosh(c+dx)}{b^2 d^4} + \frac{3af(e+fx)^2 \cosh(c+dx)}{b^2 d^2} \\
&= \frac{3f^3 x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{(a^2+b^2)(e+fx)^4}{4b^3 f} + \frac{6af^3 \cosh(c+dx)}{b^2 d^4} + \frac{3af(e+fx)^2 \cosh(c+dx)}{b^2 d^2} \\
&= \frac{3f^3 x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{(a^2+b^2)(e+fx)^4}{4b^3 f} + \frac{6af^3 \cosh(c+dx)}{b^2 d^4} + \frac{3af(e+fx)^2 \cosh(c+dx)}{b^2 d^2}
\end{aligned}$$

**Mathematica [B]** time = 23.9018, size = 10263, normalized size = 15.99

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^3\*Cosh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] Result too large to show

**Maple [F]** time = 0.28, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 (\cosh(dx+c))^3}{a+b \sinh(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/8*e^3*((4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) - 8*(a^2 + b^2)*(d
*x + c)/(b^3*d) - (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d) - 8*(a^2
+ b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d) + 1/32*(8*(
a^2*d^4*f^3*e^(2*c) + b^2*d^4*f^3*e^(2*c))*x^4 + 32*(a^2*d^4*e*f^2*e^(2*c)
+ b^2*d^4*e*f^2*e^(2*c))*x^3 + 48*(a^2*d^4*e^2*f*e^(2*c) + b^2*d^4*e^2*f*e
(2*c))*x^2 + (4*b^2*d^3*f^3*x^3*e^(4*c) + 6*(2*d^3*e*f^2 - d^2*f^3)*b^2*x^2
*e^(4*c) + 6*(2*d^3*e^2*f - 2*d^2*e*f^2 + d*f^3)*b^2*x*e^(4*c) - 3*(2*d^2*e
^2*f - 2*d*e*f^2 + f^3)*b^2*e^(4*c))*e^(2*d*x) - 16*(a*b*d^3*f^3*x^3*e^(3*c
) + 3*(d^3*e*f^2 - d^2*f^3)*a*b*x^2*e^(3*c) + 3*(d^3*e^2*f - 2*d^2*e*f^2 +
2*d*f^3)*a*b*x*e^(3*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a*b*e^(3*c))*e^(
d*x) + 16*(a*b*d^3*f^3*x^3*e^c + 3*(d^3*e*f^2 + d^2*f^3)*a*b*x^2*e^c + 3*(d
^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*a*b*x*e^c + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*
f^3)*a*b*e^c)*e^(-d*x) + (4*b^2*d^3*f^3*x^3 + 6*(2*d^3*e*f^2 + d^2*f^3)*b^2
*x^2 + 6*(2*d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*b^2*x + 3*(2*d^2*e^2*f + 2*d*e
*f^2 + f^3)*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^4) - integrate(-2*((a^2*b*f^3
+ b^3*f^3)*x^3 + 3*(a^2*b*e*f^2 + b^3*e*f^2)*x^2 + 3*(a^2*b*e^2*f + b^3*e^2
*f)*x - ((a^3*f^3*e^c + a*b^2*f^3*e^c)*x^3 + 3*(a^3*e*f^2*e^c + a*b^2*e*f^2
*e^c)*x^2 + 3*(a^3*e^2*f*e^c + a*b^2*e^2*f*e^c)*x)*e^(d*x))/(b^4*e^(2*d*x +
2*c) + 2*a*b^3*e^(d*x + c) - b^4), x)
```

**Fricas [C]** time = 3.06476, size = 9852, normalized size = 15.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 + 6*b^2*d^2*e^2*f + 6*b^2*d*e*f^2 +
3*b^2*f^3 + (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b^2*d
*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d^3*e
^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c)^4 + (4*b^2*d^3*f^3*x^3
+ 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d
^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*
f^3)*x)*sinh(d*x + c)^4 - 16*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 - 3*a*b*d^2*e^2
*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(a
*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x + c)^3 - 4*(4*a*b
*d^3*f^3*x^3 + 4*a*b*d^3*e^3 - 12*a*b*d^2*e^2*f + 24*a*b*d*e*f^2 - 24*a*b*f
^3 + 12*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 12*(a*b*d^3*e^2*f - 2*a*b*d^2*e
*f^2 + 2*a*b*d*f^3)*x - (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*
f + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(
2*b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c))*sinh(d*x +
c)^3 + 6*(2*b^2*d^3*e*f^2 + b^2*d^2*f^3)*x^2 - 8*((a^2 + b^2)*d^4*f^3*x^4
+ 4*(a^2 + b^2)*d^4*e*f^2*x^3 + 6*(a^2 + b^2)*d^4*e^2*f*x^2 + 4*(a^2 + b^2)
*d^4*e^3*x + 8*(a^2 + b^2)*c*d^3*e^3 - 12*(a^2 + b^2)*c^2*d^2*e^2*f + 8*(a^
2 + b^2)*c^3*d*e*f^2 - 2*(a^2 + b^2)*c^4*f^3)*cosh(d*x + c)^2 - 2*(4*(a^2 +
b^2)*d^4*f^3*x^4 + 16*(a^2 + b^2)*d^4*e*f^2*x^3 + 24*(a^2 + b^2)*d^4*e^2*f
*x^2 + 16*(a^2 + b^2)*d^4*e^3*x + 32*(a^2 + b^2)*c*d^3*e^3 - 48*(a^2 + b^2)
*c^2*d^2*e^2*f + 32*(a^2 + b^2)*c^3*d*e*f^2 - 8*(a^2 + b^2)*c^4*f^3 - 3*(4*
b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b^2*d*e*f^2 - 3*b^2*f
^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d^3*e^2*f - 2*b^2*d^2
*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c)^2 + 24*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3
- 3*a*b*d^2*e^2*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 - a*b*d^2*
f^3)*x^2 + 3*(a*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x +
c))*sinh(d*x + c)^2 + 6*(2*b^2*d^3*e^2*f + 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x +
16*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 + 3*a*b*d^2*e^2*f + 6*a*b*d*e*f^2 + 6*a*
b*f^3 + 3*(a*b*d^3*e*f^2 + a*b*d^2*f^3)*x^2 + 3*(a*b*d^3*e^2*f + 2*a*b*d^2*
e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x + c) + 96*(((a^2 + b^2)*d^2*f^3*x^2 + 2*(a
^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*cosh(d*x + c)^2 + 2*((a^2 +
b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*cosh(
d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2
*x + (a^2 + b^2)*d^2*e^2*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)
/b + 1) + 96*(((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 +
b^2)*d^2*e^2*f)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^
2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + ((a^2
+ b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*si
nh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c)
+ b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 32*(((a^2 + b^2)*d^3
*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*
c^3*f^3)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2
*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*cosh(d*x + c)*sinh(d*
x + c) + ((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c
```

$$\begin{aligned}
& ^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*\sinh(d*x + c)^2)*\log(2*b*\cosh(d*x + c) + \\
& 2*b*\sinh(d*x + c) + 2*b*\sqrt{((a^2 + b^2)/b^2) + 2*a}) + 32*(((a^2 + b^2)*d^3 \\
& *e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)* \\
& c^3*f^3)*\cosh(d*x + c)^2 + 2*((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2 \\
& *f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*\cosh(d*x + c)*\sinh(d* \\
& x + c) + ((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c \\
& ^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*\sinh(d*x + c)^2)*\log(2*b*\cosh(d*x + c) + \\
& 2*b*\sinh(d*x + c) - 2*b*\sqrt{((a^2 + b^2)/b^2) + 2*a}) + 32*(((a^2 + b^2)*d^3 \\
& *f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 \\
& + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*\cosh \\
& (d*x + c)^2 + 2*((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3* \\
& (a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e \\
& *f^2 + (a^2 + b^2)*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c) + ((a^2 + b^2)*d^3*f \\
& ^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 \\
& + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*\sinh( \\
& d*x + c)^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b* \\
& \sinh(d*x + c)))*\sqrt{((a^2 + b^2)/b^2) - b}/b) + 32*(((a^2 + b^2)*d^3*f^3*x^3 \\
& + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)* \\
& c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*\cosh(d*x + c \\
& )^2 + 2*((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b \\
& ^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + ( \\
& a^2 + b^2)*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c) + ((a^2 + b^2)*d^3*f^3*x^3 \\
& + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c \\
& *d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*\sinh(d*x + c) \\
& ^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x \\
& + c)))*\sqrt{((a^2 + b^2)/b^2) - b}/b) + 192*(((a^2 + b^2)*f^3*\cosh(d*x + c)^2 \\
& + 2*(a^2 + b^2)*f^3*\cosh(d*x + c)*\sinh(d*x + c) + (a^2 + b^2)*f^3*\sinh(d*x \\
& + c)^2)*\text{polylog}(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + \\
& b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2)}/b) + 192*(((a^2 + b^2)*f^3*\cosh(d*x \\
& + c)^2 + 2*(a^2 + b^2)*f^3*\cosh(d*x + c)*\sinh(d*x + c) + (a^2 + b^2)*f^3*s \\
& \sinh(d*x + c)^2)*\text{polylog}(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x \\
& + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2)}/b) - 192*(((a^2 + b^2)*d*f^ \\
& 3*x + (a^2 + b^2)*d*e*f^2)*\cosh(d*x + c)^2 + 2*((a^2 + b^2)*d*f^3*x + (a^2 \\
& + b^2)*d*e*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + ((a^2 + b^2)*d*f^3*x + (a^2 + \\
& b^2)*d*e*f^2)*\sinh(d*x + c)^2)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + \\
& c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2)}/b) - 192*((( \\
& (a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*\cosh(d*x + c)^2 + 2*((a^2 + b^2) \\
& *d*f^3*x + (a^2 + b^2)*d*e*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + ((a^2 + b^2)* \\
& d*f^3*x + (a^2 + b^2)*d*e*f^2)*\sinh(d*x + c)^2)*\text{polylog}(3, (a*\cosh(d*x + c) \\
& + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b \\
& ^2)}/b) + 4*(4*a*b*d^3*f^3*x^3 + 4*a*b*d^3*e^3 + 12*a*b*d^2*e^2*f + 24*a*b* \\
& d*e*f^2 + 24*a*b*f^3 + (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f \\
& + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2 \\
& *b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*\cosh(d*x + c)^3 + 12*(a*b* \\
& d^3*e*f^2 + a*b*d^2*f^3)*x^2 - 12*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 - 3*a*b*d^
\end{aligned}$$

$$2e^{2f} + 6abd^2ef^2 - 6ab^2f^3 + 3(ab^3d^3ef^2 - ab^2d^2f^3)x^2 + 3(ab^3d^3e^2f - 2ab^2d^2e^2f^2 + 2ab^2d^2f^3)x \cosh(dx + c)^2 + 12(ab^3d^3e^2f + 2ab^2d^2e^2f^2 + 2ab^2d^2f^3)x - 4((a^2 + b^2)d^4f^3x^4 + 4(a^2 + b^2)d^4e^2f^2x^3 + 6(a^2 + b^2)d^4e^2fx^2 + 4(a^2 + b^2)d^4e^3x + 8(a^2 + b^2)c^2d^3e^3 - 12(a^2 + b^2)c^2d^2e^2f + 8(a^2 + b^2)c^3d^2ef^2 - 2(a^2 + b^2)c^4f^3) \cosh(dx + c) \sinh(dx + c) / (b^3d^4 \cosh(dx + c)^2 + 2b^3d^4 \cosh(dx + c) \sinh(dx + c) + b^3d^4 \sinh(dx + c)^2)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cosh(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*cosh(d\*x + c)^3/(b\*sinh(d\*x + c) + a), x)

$$3.300 \quad \int \frac{(e+fx)^2 \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=477

$$\frac{2f(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2} + \frac{2f(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2} - \frac{2f^2(a^2+b^2)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}$$

[Out] (e\*f\*x)/(2\*b\*d) + (f^2\*x^2)/(4\*b\*d) - ((a^2 + b^2)\*(e + f\*x)^3)/(3\*b^3\*f) + (2\*a\*f\*(e + f\*x)\*Cosh[c + d\*x])/(b^2\*d^2) + ((a^2 + b^2)\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b^3\*d) + ((a^2 + b^2)\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b^3\*d) + (2\*(a^2 + b^2)\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^3\*d^2) + (2\*(a^2 + b^2)\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^3\*d^2) - (2\*(a^2 + b^2)\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^3\*d^3) - (2\*(a^2 + b^2)\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^3\*d^3) - (2\*a\*f^2\*Sinh[c + d\*x])/(b^2\*d^3) - (a\*(e + f\*x)^2\*Sinh[c + d\*x])/(b^2\*d) - (f\*(e + f\*x)\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(2\*b\*d^2) + (f^2\*Sinh[c + d\*x]^2)/(4\*b\*d^3) + ((e + f\*x)^2\*Sinh[c + d\*x]^2)/(2\*b\*d)

**Rubi [A]** time = 0.642494, antiderivative size = 477, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 10, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$ , Rules used = {5565, 3296, 2637, 5446, 3310, 5561, 2190, 2531, 2282, 6589}

$$\frac{2f(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2} + \frac{2f(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2} - \frac{2f^2(a^2+b^2)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Cosh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] (e\*f\*x)/(2\*b\*d) + (f^2\*x^2)/(4\*b\*d) - ((a^2 + b^2)\*(e + f\*x)^3)/(3\*b^3\*f) + (2\*a\*f\*(e + f\*x)\*Cosh[c + d\*x])/(b^2\*d^2) + ((a^2 + b^2)\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b^3\*d) + ((a^2 + b^2)\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b^3\*d) + (2\*(a^2 + b^2)\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^3\*d^2) + (2\*(a^2 + b^2)\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^3\*d^2) - (2\*(a^2 + b^2)\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^3\*d^3) - (2\*(a^2 + b^2)\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^3\*d^3) - (2\*a\*f^2\*Sinh[c + d\*x])/(b^2\*d^3) - (a\*(e + f\*x)^2\*Sinh[c + d\*x])/(b^2\*d) - (f\*(e + f\*x)\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(2\*b\*d^2) + (f^2\*Sinh[c + d\*x]^2)/(4\*b\*d^3) + ((e + f\*x)^2\*Sinh[c + d\*x]^2)/(2\*b\*d)



$$- (a*(e + f*x)^2*\text{Sinh}[c + d*x])/(b^2*d) - (f*(e + f*x)*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(2*b*d^2) + (f^2*\text{Sinh}[c + d*x]^2)/(4*b*d^3) + ((e + f*x)^2*\text{Sinh}[c + d*x]^2)/(2*b*d)$$

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] := -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)
]*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
Simp[(d*(b*Sinh[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sinh[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sinh[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]
]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
```

, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*(a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx &= -\frac{a \int (e+fx)^2 \cosh(c+dx) dx}{b^2} + \frac{\int (e+fx)^2 \cosh(c+dx) \sinh(c+dx) dx}{b} + \frac{(a^2+b^2)}{b^2} \\
&= -\frac{(a^2+b^2)(e+fx)^3}{3b^3f} - \frac{a(e+fx)^2 \sinh(c+dx)}{b^2d} + \frac{(e+fx)^2 \sinh^2(c+dx)}{2bd} + \frac{(a^2+b^2)}{b^2} \\
&= -\frac{(a^2+b^2)(e+fx)^3}{3b^3f} + \frac{2af(e+fx) \cosh(c+dx)}{b^2d^2} + \frac{(a^2+b^2)(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d} \\
&= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} - \frac{(a^2+b^2)(e+fx)^3}{3b^3f} + \frac{2af(e+fx) \cosh(c+dx)}{b^2d^2} + \frac{(a^2+b^2)(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d} \\
&= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} - \frac{(a^2+b^2)(e+fx)^3}{3b^3f} + \frac{2af(e+fx) \cosh(c+dx)}{b^2d^2} + \frac{(a^2+b^2)(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d} \\
&= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} - \frac{(a^2+b^2)(e+fx)^3}{3b^3f} + \frac{2af(e+fx) \cosh(c+dx)}{b^2d^2} + \frac{(a^2+b^2)(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d}
\end{aligned}$$

**Mathematica [B]** time = 17.6291, size = 3021, normalized size = 6.33

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Cosh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] -((a^2 + b^2)\*(6\*e^2\*E^(2\*c)\*x + 6\*e\*E^(2\*c)\*f\*x^2 + 2\*E^(2\*c)\*f^2\*x^3 + (6\*a\*Sqrt[a^2 + b^2]\*e^2\*ArcTan[(a + b\*E^(c + d\*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]\*d) + (6\*a\*Sqrt[-(a^2 + b^2)^2]\*e^2\*E^(2\*c)\*ArcTan[(a + b\*E^(c + d\*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)\*d) - (6\*a\*Sqrt[-(a^2 + b^2)^2]\*e^2\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)\*d) + (6\*a\*Sqrt[-(a^2 + b^2)^2]\*e^2\*E^(2\*c)\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)\*d) + (3\*e^2\*Log[2\*a\*E^(c + d\*x) + b\*(-1 + E^(2\*(c + d\*x)))]/d - (3\*e^2\*E^(2\*c)\*Log[2\*a\*E^(c + d\*x) + b\*(-1 + E^(2\*(c + d\*x)))]/d + (6\*e\*f\*x\*Log[1 + (b\*E^(2\*c + d\*x))/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)]])/d - (6\*e\*E^(2\*c)\*f\*x\*Log[1 + (b\*E^(2\*c + d\*x))/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)]])/d + (3\*f^2\*x^2\*Log[1 + (b\*E^(2\*c + d\*x))/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)]])/d - (3\*E^(2\*c)\*f^2\*x^2\*Log[1 + (b\*E^(2\*c + d\*x))/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)]])/d + (6\*e\*f\*x\*Log[1 + (b\*E^(2\*c + d\*x))/(a\*E^c + Sqrt[(a^2 + b^2)\*E^(2\*c)]])/d - (6\*e\*E^(2\*c)\*f\*x\*Log[1 + (b\*

$$\begin{aligned}
& E^{(2*c + d*x)} / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}]) / d + (3 * f^2 * x^2 * \text{Log}[1 + \\
& (b * E^{(2*c + d*x)}) / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}])]) / d - (3 * E^{(2*c)} * f^2 * \\
& x^2 * \text{Log}[1 + (b * E^{(2*c + d*x)}) / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}])]) / d - (6 \\
& * (-1 + E^{(2*c)}) * f * (e + f * x) * \text{PolyLog}[2, -((b * E^{(2*c + d*x)}) / (a * E^c - \text{Sqrt}[(a \\
& ^2 + b^2) * E^{(2*c)}])]) / d^2 - (6 * (-1 + E^{(2*c)}) * f * (e + f * x) * \text{PolyLog}[2, -((b * \\
& E^{(2*c + d*x)}) / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}])]) / d^2 - (6 * f^2 * \text{PolyLog}[ \\
& 3, -((b * E^{(2*c + d*x)}) / (a * E^c - \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}])]) / d^3 + (6 * E^{(2 \\
& *c)} * f^2 * \text{PolyLog}[3, -((b * E^{(2*c + d*x)}) / (a * E^c - \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}])]) \\
& ]) / d^3 - (6 * f^2 * \text{PolyLog}[3, -((b * E^{(2*c + d*x)}) / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{ \\
& (2*c)}])]) / d^3 + (6 * E^{(2*c)} * f^2 * \text{PolyLog}[3, -((b * E^{(2*c + d*x)}) / (a * E^c + \text{Sqr \\
& t}[(a^2 + b^2) * E^{(2*c)}])]) / d^3) / (3 * b^3 * (-1 + E^{(2*c)})) + \text{Csch}[c] * (\text{Cosh}[2*c \\
& + 2*d*x] / (96 * b^3 * d^3) - \text{Sinh}[2*c + 2*d*x] / (96 * b^3 * d^3)) * (-24 * a * b * d^2 * e^2 * C \\
& osh[d*x] - 48 * a * b * d * e * f * \text{Cosh}[d*x] - 48 * a * b * f^2 * \text{Cosh}[d*x] - 48 * a * b * d^2 * e * f * x \\
& * \text{Cosh}[d*x] - 48 * a * b * d * f^2 * x * \text{Cosh}[d*x] - 24 * a * b * d^2 * f^2 * x^2 * \text{Cosh}[d*x] + 24 * a \\
& * b * d^2 * e^2 * \text{Cosh}[2*c + d*x] + 48 * a * b * d * e * f * \text{Cosh}[2*c + d*x] + 48 * a * b * f^2 * \text{Cosh} \\
& [2*c + d*x] + 48 * a * b * d^2 * e * f * x * \text{Cosh}[2*c + d*x] + 48 * a * b * d * f^2 * x * \text{Cosh}[2*c + \\
& d*x] + 24 * a * b * d^2 * f^2 * x^2 * \text{Cosh}[2*c + d*x] + 48 * a^2 * d^3 * e^2 * x * \text{Cosh}[c + 2*d*x] \\
& ] + 48 * b^2 * d^3 * e^2 * x * \text{Cosh}[c + 2*d*x] + 48 * a^2 * d^3 * e * f * x^2 * \text{Cosh}[c + 2*d*x] + \\
& 48 * b^2 * d^3 * e * f * x^2 * \text{Cosh}[c + 2*d*x] + 16 * a^2 * d^3 * f^2 * x^3 * \text{Cosh}[c + 2*d*x] + \\
& 16 * b^2 * d^3 * f^2 * x^3 * \text{Cosh}[c + 2*d*x] + 48 * a^2 * d^3 * e^2 * x * \text{Cosh}[3*c + 2*d*x] + 4 \\
& 8 * b^2 * d^3 * e^2 * x * \text{Cosh}[3*c + 2*d*x] + 48 * a^2 * d^3 * e * f * x^2 * \text{Cosh}[3*c + 2*d*x] + \\
& 48 * b^2 * d^3 * e * f * x^2 * \text{Cosh}[3*c + 2*d*x] + 16 * a^2 * d^3 * f^2 * x^3 * \text{Cosh}[3*c + 2*d*x] \\
& + 16 * b^2 * d^3 * f^2 * x^3 * \text{Cosh}[3*c + 2*d*x] + 24 * a * b * d^2 * e^2 * \text{Cosh}[2*c + 3*d*x] \\
& - 48 * a * b * d * e * f * \text{Cosh}[2*c + 3*d*x] + 48 * a * b * f^2 * \text{Cosh}[2*c + 3*d*x] + 48 * a * b * d^ \\
& 2 * e * f * x * \text{Cosh}[2*c + 3*d*x] - 48 * a * b * d * f^2 * x * \text{Cosh}[2*c + 3*d*x] + 24 * a * b * d^2 * f \\
& ^2 * x^2 * \text{Cosh}[2*c + 3*d*x] - 24 * a * b * d^2 * e^2 * \text{Cosh}[4*c + 3*d*x] + 48 * a * b * d * e * f * \\
& \text{Cosh}[4*c + 3*d*x] - 48 * a * b * f^2 * \text{Cosh}[4*c + 3*d*x] - 48 * a * b * d^2 * e * f * x * \text{Cosh}[4* \\
& c + 3*d*x] + 48 * a * b * d * f^2 * x * \text{Cosh}[4*c + 3*d*x] - 24 * a * b * d^2 * f^2 * x^2 * \text{Cosh}[4*c \\
& + 3*d*x] - 6 * b^2 * d^2 * e^2 * \text{Cosh}[3*c + 4*d*x] + 6 * b^2 * d * e * f * \text{Cosh}[3*c + 4*d*x] \\
& - 3 * b^2 * f^2 * \text{Cosh}[3*c + 4*d*x] - 12 * b^2 * d^2 * e * f * x * \text{Cosh}[3*c + 4*d*x] + 6 * b^2 * \\
& d * f^2 * x * \text{Cosh}[3*c + 4*d*x] - 6 * b^2 * d^2 * f^2 * x^2 * \text{Cosh}[3*c + 4*d*x] + 6 * b^2 * d^ \\
& 2 * e^2 * \text{Cosh}[5*c + 4*d*x] - 6 * b^2 * d * e * f * \text{Cosh}[5*c + 4*d*x] + 3 * b^2 * f^2 * \text{Cosh}[5* \\
& c + 4*d*x] + 12 * b^2 * d^2 * e * f * x * \text{Cosh}[5*c + 4*d*x] - 6 * b^2 * d * f^2 * x * \text{Cosh}[5*c + \\
& 4*d*x] + 6 * b^2 * d^2 * f^2 * x^2 * \text{Cosh}[5*c + 4*d*x] + 12 * b^2 * d^2 * e^2 * \text{Sinh}[c] + 12 * \\
& b^2 * d * e * f * \text{Sinh}[c] + 6 * b^2 * f^2 * \text{Sinh}[c] + 24 * b^2 * d^2 * e * f * x * \text{Sinh}[c] + 12 * b^2 * d \\
& * f^2 * x * \text{Sinh}[c] + 12 * b^2 * d^2 * f^2 * x^2 * \text{Sinh}[c] - 24 * a * b * d^2 * e^2 * \text{Sinh}[d*x] - 48 \\
& * a * b * d * e * f * \text{Sinh}[d*x] - 48 * a * b * f^2 * \text{Sinh}[d*x] - 48 * a * b * d^2 * e * f * x * \text{Sinh}[d*x] - \\
& 48 * a * b * d * f^2 * x * \text{Sinh}[d*x] - 24 * a * b * d^2 * f^2 * x^2 * \text{Sinh}[d*x] + 24 * a * b * d^2 * e^2 * \text{Si} \\
& nh[2*c + d*x] + 48 * a * b * d * e * f * \text{Sinh}[2*c + d*x] + 48 * a * b * f^2 * \text{Sinh}[2*c + d*x] + \\
& 48 * a * b * d^2 * e * f * x * \text{Sinh}[2*c + d*x] + 48 * a * b * d * f^2 * x * \text{Sinh}[2*c + d*x] + 24 * a * b \\
& * d^2 * f^2 * x^2 * \text{Sinh}[2*c + d*x] + 48 * a^2 * d^3 * e^2 * x * \text{Sinh}[c + 2*d*x] + 48 * b^2 * d^ \\
& 3 * e^2 * x * \text{Sinh}[c + 2*d*x] + 48 * a^2 * d^3 * e * f * x^2 * \text{Sinh}[c + 2*d*x] + 48 * b^2 * d^3 * e \\
& * f * x^2 * \text{Sinh}[c + 2*d*x] + 16 * a^2 * d^3 * f^2 * x^3 * \text{Sinh}[c + 2*d*x] + 16 * b^2 * d^3 * f^ \\
& 2 * x^3 * \text{Sinh}[c + 2*d*x] + 48 * a^2 * d^3 * e^2 * x * \text{Sinh}[3*c + 2*d*x] + 48 * b^2 * d^3 * e^2 \\
& * x * \text{Sinh}[3*c + 2*d*x] + 48 * a^2 * d^3 * e * f * x^2 * \text{Sinh}[3*c + 2*d*x] + 48 * b^2 * d^3 * e
\end{aligned}$$

$f*x^2*\text{Sinh}[3*c + 2*d*x] + 16*a^2*d^3*f^2*x^3*\text{Sinh}[3*c + 2*d*x] + 16*b^2*d^3*f^2*x^3*\text{Sinh}[3*c + 2*d*x] + 24*a*b*d^2*e^2*\text{Sinh}[2*c + 3*d*x] - 48*a*b*d*e*f*\text{Sinh}[2*c + 3*d*x] + 48*a*b*f^2*\text{Sinh}[2*c + 3*d*x] + 48*a*b*d^2*e*f*x*\text{Sinh}[2*c + 3*d*x] - 48*a*b*d*f^2*x*\text{Sinh}[2*c + 3*d*x] + 24*a*b*d^2*f^2*x^2*\text{Sinh}[2*c + 3*d*x] - 24*a*b*d^2*e^2*\text{Sinh}[4*c + 3*d*x] + 48*a*b*d*e*f*\text{Sinh}[4*c + 3*d*x] - 48*a*b*f^2*\text{Sinh}[4*c + 3*d*x] - 48*a*b*d^2*e*f*x*\text{Sinh}[4*c + 3*d*x] + 48*a*b*d*f^2*x*\text{Sinh}[4*c + 3*d*x] - 24*a*b*d^2*f^2*x^2*\text{Sinh}[4*c + 3*d*x] - 6*b^2*d^2*e^2*\text{Sinh}[3*c + 4*d*x] + 6*b^2*d*e*f*\text{Sinh}[3*c + 4*d*x] - 3*b^2*f^2*\text{Sinh}[3*c + 4*d*x] - 12*b^2*d^2*e*f*x*\text{Sinh}[3*c + 4*d*x] + 6*b^2*d*f^2*x*\text{Sinh}[3*c + 4*d*x] - 6*b^2*d^2*f^2*x^2*\text{Sinh}[3*c + 4*d*x] + 6*b^2*d^2*e^2*\text{Sinh}[5*c + 4*d*x] - 6*b^2*d*e*f*\text{Sinh}[5*c + 4*d*x] + 3*b^2*f^2*\text{Sinh}[5*c + 4*d*x] + 12*b^2*d^2*e*f*x*\text{Sinh}[5*c + 4*d*x] - 6*b^2*d*f^2*x*\text{Sinh}[5*c + 4*d*x] + 6*b^2*d^2*f^2*x^2*\text{Sinh}[5*c + 4*d*x])$

**Maple [F]** time = 0.234, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\cosh(dx + c))^3}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*cosh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*cosh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8}e^2 \left( \frac{(4ae^{(-dx-c)} - b)e^{(2dx+2c)}}{b^2d} - \frac{8(a^2 + b^2)(dx + c)}{b^3d} - \frac{4ae^{(-dx-c)} + be^{(-2dx-2c)}}{b^2d} - \frac{8(a^2 + b^2) \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)})}{b^3d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-1/8*e^2*((4*a*e^{(-d*x - c)} - b)*e^{(2*d*x + 2*c)}/(b^2*d) - 8*(a^2 + b^2)*(d*x + c)/(b^3*d) - (4*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)})/(b^2*d) - 8*(a^2 + b^2)*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/(b^3*d)) + 1/48*(16*(a^2*d^3*f^2*e^{(2*c)} + b^2*d^3*f^2*e^{(2*c)})*x^3 + 48*(a^2*d^3*e*f*e^{(2*c)} + b^2*d^3*e*f*e^{(2*c)})*x^2 + 3*(2*b^2*d^2*f^2*x^2*e^{(4*c)} + 2*(2*d^2*e*f - d$

```

*f^2)*b^2*x*e^(4*c) - (2*d*e*f - f^2)*b^2*e^(4*c))*e^(2*d*x) - 24*(a*b*d^2*
f^2*x^2*e^(3*c) + 2*(d^2*e*f - d*f^2)*a*b*x*e^(3*c) - 2*(d*e*f - f^2)*a*b*e
^(3*c))*e^(d*x) + 24*(a*b*d^2*f^2*x^2*e^c + 2*(d^2*e*f + d*f^2)*a*b*x*e^c +
2*(d*e*f + f^2)*a*b*e^c)*e^(-d*x) + 3*(2*b^2*d^2*f^2*x^2 + 2*(2*d^2*e*f +
d*f^2)*b^2*x + (2*d*e*f + f^2)*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^3) - integr
ate(-2*((a^2*b*f^2 + b^3*f^2)*x^2 + 2*(a^2*b*e*f + b^3*e*f)*x - ((a^3*f^2*e
^c + a*b^2*f^2*e^c)*x^2 + 2*(a^3*e*f*e^c + a*b^2*e*f*e^c)*x)*e^(d*x))/(b^4*
e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x)

```

**Fricas [C]** time = 2.64666, size = 6356, normalized size = 13.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```

[Out] 1/48*(6*b^2*d^2*f^2*x^2 + 6*b^2*d^2*e^2 + 6*b^2*d*e*f + 3*(2*b^2*d^2*f^2*x^
2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x
)*cosh(d*x + c)^4 + 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^
2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*sinh(d*x + c)^4 + 3*b^2*f^2 - 24*(
a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f -
a*b*d*f^2)*x)*cosh(d*x + c)^3 - 12*(2*a*b*d^2*f^2*x^2 + 2*a*b*d^2*e^2 - 4*a
*b*d*e*f + 4*a*b*f^2 + 4*(a*b*d^2*e*f - a*b*d*f^2)*x - (2*b^2*d^2*f^2*x^2 +
2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*c
osh(d*x + c))*sinh(d*x + c)^3 - 16*((a^2 + b^2)*d^3*f^2*x^3 + 3*(a^2 + b^2)
*d^3*e*f*x^2 + 3*(a^2 + b^2)*d^3*e^2*x + 6*(a^2 + b^2)*c*d^2*e^2 - 6*(a^2 +
b^2)*c^2*d*e*f + 2*(a^2 + b^2)*c^3*f^2)*cosh(d*x + c)^2 - 2*(8*(a^2 + b^2)
*d^3*f^2*x^3 + 24*(a^2 + b^2)*d^3*e*f*x^2 + 24*(a^2 + b^2)*d^3*e^2*x + 48*(
a^2 + b^2)*c*d^2*e^2 - 48*(a^2 + b^2)*c^2*d*e*f + 16*(a^2 + b^2)*c^3*f^2 -
9*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2
*e*f - b^2*d*f^2)*x)*cosh(d*x + c)^2 + 36*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 -
2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*cosh(d*x + c))*sin
h(d*x + c)^2 + 6*(2*b^2*d^2*e*f + b^2*d*f^2)*x + 24*(a*b*d^2*f^2*x^2 + a*b*
d^2*e^2 + 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f + a*b*d*f^2)*x)*cosh(d*x
+ c) + 96*(((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c)^2 + 2*(
(a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + ((a^
2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x +
c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)
/b^2) - b)/b + 1) + 96*(((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x
+ c)^2 + 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*cosh(d*x + c)*sinh(d*x
+ c) + ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*sinh(d*x + c)^2)*dilog((a

```

```

*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b + 1) + 48*(((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*
d*e*f + (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)^2 + 2*(((a^2 + b^2)*d^2*e^2 - 2*(
a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + ((a
^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*sinh(d*x +
c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2
) + 2*a) + 48*(((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c
^2*f^2)*cosh(d*x + c)^2 + 2*(((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f +
(a^2 + b^2)*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*d^2*e^2 - 2
*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*sinh(d*x + c)^2)*log(2*b*cosh(d
*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 48*(((a^2
+ b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2
+ b^2)*c^2*f^2)*cosh(d*x + c)^2 + 2*(((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^
2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)*s
inh(d*x + c) + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2
+ b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c
) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/
b^2) - b)/b) + 48*(((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(
a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*cosh(d*x + c)^2 + 2*(((a^2 + b^2)*
d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)
*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 +
b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*sinh(d*x + c
)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*
x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 96*(((a^2 + b^2)*f^2*cosh(d*x + c)^2
+ 2*(a^2 + b^2)*f^2*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*f^2*sinh(d*x
+ c)^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 96*(((a^2 + b^2)*f^2*cosh(d*x
+ c)^2 + 2*(a^2 + b^2)*f^2*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*f^2*si
nh(d*x + c)^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 4*(6*a*b*d^2*f^2*x^2 +
6*a*b*d^2*e^2 + 12*a*b*d*e*f + 12*a*b*f^2 + 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^
2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*cosh(d*x +
c)^3 - 18*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*
b*d^2*e*f - a*b*d*f^2)*x)*cosh(d*x + c)^2 + 12*(a*b*d^2*e*f + a*b*d*f^2)*x
- 8*(((a^2 + b^2)*d^3*f^2*x^3 + 3*(a^2 + b^2)*d^3*e*f*x^2 + 3*(a^2 + b^2)*d^
3*e^2*x + 6*(a^2 + b^2)*c*d^2*e^2 - 6*(a^2 + b^2)*c^2*d*e*f + 2*(a^2 + b^2)
*c^3*f^2)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d^3*cosh(d*x + c)^2 + 2*b^3*d^
3*cosh(d*x + c)*sinh(d*x + c) + b^3*d^3*sinh(d*x + c)^2)

```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cosh(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*cosh(d\*x + c)^3/(b\*sinh(d\*x + c) + a), x)



$$3.301 \quad \int \frac{(e+fx) \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=298

$$\frac{f(a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^3 d^2} + \frac{f(a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right)}{b^3 d^2} + \frac{(a^2 + b^2)(e + fx) \log\left(\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}} + 1\right)}{b^3 d} + \dots$$

[Out] (f\*x)/(4\*b\*d) - ((a^2 + b^2)\*(e + f\*x)^2)/(2\*b^3\*f) + (a\*f\*Cosh[c + d\*x])/((b^2\*d^2) + ((a^2 + b^2)\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])))/(b^3\*d) + ((a^2 + b^2)\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])))/(b^3\*d) + ((a^2 + b^2)\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^3\*d^2) + ((a^2 + b^2)\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^3\*d^2) - (a\*(e + f\*x)\*Sinh[c + d\*x])/(b^2\*d) - (f\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(4\*b\*d^2) + ((e + f\*x)\*Sinh[c + d\*x]^2)/(2\*b\*d)

**Rubi [A]** time = 0.357755, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {5565, 3296, 2638, 5446, 2635, 8, 5561, 2190, 2279, 2391}

$$\frac{f(a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^3 d^2} + \frac{f(a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right)}{b^3 d^2} + \frac{(a^2 + b^2)(e + fx) \log\left(\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}} + 1\right)}{b^3 d} + \dots$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Cosh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] (f\*x)/(4\*b\*d) - ((a^2 + b^2)\*(e + f\*x)^2)/(2\*b^3\*f) + (a\*f\*Cosh[c + d\*x])/((b^2\*d^2) + ((a^2 + b^2)\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])))/(b^3\*d) + ((a^2 + b^2)\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])))/(b^3\*d) + ((a^2 + b^2)\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^3\*d^2) + ((a^2 + b^2)\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^3\*d^2) - (a\*(e + f\*x)\*Sinh[c + d\*x])/(b^2\*d) - (f\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(4\*b\*d^2) + ((e + f\*x)\*Sinh[c + d\*x]^2)/(2\*b\*d)

**Rule 5565**

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]^(n\_))\*((e\_.) + (f\_.)\*(x\_))^(m\_.)]/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] :> -Dist[a/b^2, Int[(e + f\*x)^m\*Cosh

$[c + d*x]^{(n - 2)}, x], x] + (\text{Dist}[1/b, \text{Int}[(e + f*x)^m * \text{Cosh}[c + d*x]^{(n - 2)} * \text{Sinh}[c + d*x], x], x] + \text{Dist}[(a^2 + b^2)/b^2, \text{Int}[(e + f*x)^m * \text{Cosh}[c + d*x]^{(n - 2)}] / (a + b * \text{Sinh}[c + d*x]), x], x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[n, 1] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{IGtQ}[m, 0]$

### Rule 3296

$\text{Int}[(c + d*x)^m * \sin[e + f*x], x\_Symbol] := -\text{Simp}[(c + d*x)^m * \cos[e + f*x] / f, x] + \text{Dist}[(d*m) / f, \text{Int}[(c + d*x)^{(m - 1)} * \cos[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$

### Rule 2638

$\text{Int}[\sin[c + d*x], x\_Symbol] := -\text{Simp}[\cos[c + d*x] / d, x] /; \text{FreeQ}\{c, d\}, x]$

### Rule 5446

$\text{Int}[\text{Cosh}[a + b*x] * (c + d*x)^m * \text{Sinh}[a + b*x]^{(n - 1)}, x\_Symbol] := \text{Simp}[(c + d*x)^m * \text{Sinh}[a + b*x]^n / (b*(n + 1)), x] - \text{Dist}[(d*m) / (b*(n + 1)), \text{Int}[(c + d*x)^{(m - 1)} * \text{Sinh}[a + b*x]^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[n, -1]$

### Rule 2635

$\text{Int}[(b * \sin[c + d*x])^n, x\_Symbol] := -\text{Simp}[b * \cos[c + d*x] * (b * \sin[c + d*x])^{(n - 1)} / (d*n), x] + \text{Dist}[b^2 * (n - 1) / n, \text{Int}[(b * \sin[c + d*x])^{(n - 2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

### Rule 8

$\text{Int}[a, x\_Symbol] := \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

### Rule 5561

$\text{Int}[(\text{Cosh}[c + d*x])^m * (e + f*x)^n / ((a + b * \text{Sinh}[c + d*x])^m), x\_Symbol] := -\text{Simp}[(e + f*x)^{m + 1} / (b*f*(m + 1)), x] + (\text{Int}[(e + f*x)^m * E^{(c + d*x)} / (a - \text{Rt}[a^2 + b^2, 2] + b * E^{(c + d*x)}), x] + \text{Int}[(e + f*x)^m * E^{(c + d*x)} / (a + \text{Rt}[a^2 + b^2, 2] + b * E^{(c + d*x)}), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[a^2 + b^2, 0]$

### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned} \int \frac{(e + fx) \cosh^3(c + dx)}{a + b \sinh(c + dx)} dx &= -\frac{a \int (e + fx) \cosh(c + dx) dx}{b^2} + \frac{\int (e + fx) \cosh(c + dx) \sinh(c + dx) dx}{b} + \frac{(a^2 + b^2) \int \frac{1}{a - \sqrt{a^2 + b^2} \sinh(c + dx)} dx}{b} \\ &= -\frac{(a^2 + b^2)(e + fx)^2}{2b^3 f} - \frac{a(e + fx) \sinh(c + dx)}{b^2 d} + \frac{(e + fx) \sinh^2(c + dx)}{2bd} + \frac{(a^2 + b^2) \int \frac{1}{a - \sqrt{a^2 + b^2} \sinh(c + dx)} dx}{b} \\ &= -\frac{(a^2 + b^2)(e + fx)^2}{2b^3 f} + \frac{af \cosh(c + dx)}{b^2 d^2} + \frac{(a^2 + b^2)(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^3 d} + \frac{(a^2 + b^2) \int \frac{1}{a - \sqrt{a^2 + b^2} \sinh(c + dx)} dx}{b} \\ &= \frac{fx}{4bd} - \frac{(a^2 + b^2)(e + fx)^2}{2b^3 f} + \frac{af \cosh(c + dx)}{b^2 d^2} + \frac{(a^2 + b^2)(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^3 d} + \frac{(a^2 + b^2) \int \frac{1}{a - \sqrt{a^2 + b^2} \sinh(c + dx)} dx}{b} \\ &= \frac{fx}{4bd} - \frac{(a^2 + b^2)(e + fx)^2}{2b^3 f} + \frac{af \cosh(c + dx)}{b^2 d^2} + \frac{(a^2 + b^2)(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^3 d} + \frac{(a^2 + b^2) \int \frac{1}{a - \sqrt{a^2 + b^2} \sinh(c + dx)} dx}{b} \end{aligned}$$

**Mathematica [A]** time = 1.3663, size = 251, normalized size = 0.84

$$\frac{8(a^2 + b^2) \left( f \operatorname{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2 + b^2} - a}\right) + f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right) + f(c + dx) \log\left(\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}} + 1\right) + f(c + dx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2 + b^2}}\right) \right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (8*a*b*f*Cosh[c + d*x] + 2*b^2*d*(e + f*x)*Cosh[2*(c + d*x)] + 8*(a^2 + b^2)*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]) + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]) - 8*a*b*d*(e + f*x)*Sinh[c + d*x] - b^2*f*Sinh[2*(c + d*x)]/(8*b^3*d^2)
```

**Maple [B]** time = 0.11, size = 975, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/2*f*x^2/b+1/16*(2*d*f*x+2*d*e-f)/d^2/b*exp(2*d*x+2*c)+1/16*(2*d*f*x+2*d*e+f)/d^2/b*exp(-2*d*x-2*c)-1/d^2/b*f*c^2+1/d/b*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d/b*e*ln(exp(d*x+c))+1/d^2/b*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d^2/b*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/2*a^2*f*x^2/b^3-1/b^3/d^2*a^2*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/b^3/d^2*a^2*f*c*ln(exp(d*x+c))+1/b^3/d*a^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/b^3/d^2*a^2*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/b^3/d*a^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/b^3/d^2*a^2*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/b^3/d*a^2*f*c*x+a^2*e*x/b^3+e*x/b-1/b^3/d^2*a^2*f*c^2+1/b^3/d^2*a^2*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/b^3/d^2*a^2*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/b^3/d*a^2*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/b^3/d*a^2*e*ln(exp(d*x+c))-2/d/b*f*c*x-1/d^2/b*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d^2/b*f*c*ln(exp(d*x+c))+1/d/b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/d^2/b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/d/b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/d^2/b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/2*a*(d*f*x+d*e+f)/b^2/d^2*exp(-d*x-c)-1/2*a*(d*f*x+d*e-f)/b^2/d^2*exp(d*x+c)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{8} e^{\int \left( \frac{(4ae^{-dx-c} - b)e^{(2dx+2c)}}{b^2d} - \frac{8(a^2 + b^2)(dx + c)}{b^3d} - \frac{4ae^{-dx-c} + be^{(-2dx-2c)}}{b^2d} - \frac{8(a^2 + b^2) \log(-2ae^{-dx-c} + be^{(-2dx-2c)})}{b^3d} \right) dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$-1/8 * e^{((4*a*e^{-d*x - c} - b)*e^{(2*d*x + 2*c)})/(b^2*d) - 8*(a^2 + b^2)*(d*x + c)/(b^3*d) - (4*a*e^{-d*x - c} + b*e^{(-2*d*x - 2*c)})/(b^2*d) - 8*(a^2 + b^2)*\log(-2*a*e^{-d*x - c} + b*e^{(-2*d*x - 2*c)} - b)/(b^3*d)} + 1/16 * f * ((8*(a^2*d^2*e^{(2*c)} + b^2*d^2*e^{(2*c)}) * x^2 + (2*b^2*d*x*e^{(4*c)} - b^2*e^{(4*c)}) * e^{(2*d*x)} - 8*(a*b*d*x*e^{(3*c)} - a*b*e^{(3*c)}) * e^{(d*x)} + 8*(a*b*d*x*e^c + a*b*e^c) * e^{(-d*x)} + (2*b^2*d*x + b^2) * e^{(-2*d*x)}) * e^{(-2*c)}) / (b^3*d^2) - 2 * \int \text{egrate}(16*((a^3*e^c + a*b^2*e^c) * x * e^{(d*x)} - (a^2*b + b^3) * x) / (b^4 * e^{(2*d*x + 2*c)} + 2*a*b^3 * e^{(d*x + c)} - b^4), x))$$

**Fricas [B]** time = 2.36882, size = 3507, normalized size = 11.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$1/16 * (2*b^2*d*f*x + (2*b^2*d*f*x + 2*b^2*d*e - b^2*f) * \cosh(d*x + c)^4 + (2*b^2*d*f*x + 2*b^2*d*e - b^2*f) * \sinh(d*x + c)^4 + 2*b^2*d*e - 8*(a*b*d*f*x + a*b*d*e - a*b*f) * \cosh(d*x + c)^3 - 4*(2*a*b*d*f*x + 2*a*b*d*e - 2*a*b*f - (2*b^2*d*f*x + 2*b^2*d*e - b^2*f) * \cosh(d*x + c)) * \sinh(d*x + c)^3 + b^2*f - 8*((a^2 + b^2) * d^2 * f * x^2 + 2*(a^2 + b^2) * d^2 * e * x + 4*(a^2 + b^2) * c * d * e - 2*(a^2 + b^2) * c^2 * f) * \cosh(d*x + c)^2 - 2*(4*(a^2 + b^2) * d^2 * f * x^2 + 8*(a^2 + b^2) * d^2 * e * x + 16*(a^2 + b^2) * c * d * e - 8*(a^2 + b^2) * c^2 * f - 3*(2*b^2*d*f*x + 2*b^2*d*e - b^2*f) * \cosh(d*x + c)^2 + 12*(a*b*d*f*x + a*b*d*e - a*b*f) * \cosh(d*x + c) * \sinh(d*x + c)^2 + 8*(a*b*d*f*x + a*b*d*e + a*b*f) * \cosh(d*x + c) + 16*((a^2 + b^2) * f * \cosh(d*x + c)^2 + 2*(a^2 + b^2) * f * \cosh(d*x + c) * \sinh(d*x + c) + (a^2 + b^2) * f * \sinh(d*x + c)^2) * \text{dilog}((a * \cosh(d*x + c) + a * \sinh(d*x + c) + (b * \cosh(d*x + c) + b * \sinh(d*x + c)) * \sqrt{(a^2 + b^2)/b^2} - b) / b + 1) + 16*((a^2 + b^2) * f * \cosh(d*x + c)^2 + 2*(a^2 + b^2) * f * \cosh(d*x + c) * \sinh(d*x + c) + (a^2 + b^2) * f * \sinh(d*x + c)^2) * \text{dilog}((a * \cosh(d*x + c) + a * \sinh(d*x + c) - (b * \cosh(d*x + c) + b * \sinh(d*x + c)) * \sqrt{(a^2 + b^2)/b^2} - b) /$$

```

b + 1) + 16*(((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)^2 + 2*((a^2
+ b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*d*
e - (a^2 + b^2)*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x
+ c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*(((a^2 + b^2)*d*e - (a^2 + b^2
)*c*f)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c
)*sinh(d*x + c) + ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*sinh(d*x + c)^2)*log(
2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) +
16*(((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*
d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*d*f*x +
(a^2 + b^2)*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
+ (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 16*(((
a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^2 + 2*((a^2 + b^2)*d*f*x
+ (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 + b^2)*d*f*x + (a^2
+ b^2)*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(2*a*b*d
*f*x + 2*a*b*d*e + (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^3 + 2*a*
b*f - 6*(a*b*d*f*x + a*b*d*e - a*b*f)*cosh(d*x + c)^2 - 4*((a^2 + b^2)*d^2*
f*x^2 + 2*(a^2 + b^2)*d^2*e*x + 4*(a^2 + b^2)*c*d*e - 2*(a^2 + b^2)*c^2*f)*
cosh(d*x + c))*sinh(d*x + c))/(b^3*d^2*cosh(d*x + c)^2 + 2*b^3*d^2*cosh(d*x
+ c)*sinh(d*x + c) + b^3*d^2*sinh(d*x + c)^2)

```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \cosh(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

```
[Out] integrate((f*x + e)*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

$$3.302 \quad \int \frac{\cosh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=59

$$\frac{(a^2 + b^2) \log(a + b \sinh(c + dx))}{b^3 d} - \frac{a \sinh(c + dx)}{b^2 d} + \frac{\sinh^2(c + dx)}{2bd}$$

[Out] ((a^2 + b^2)\*Log[a + b\*Sinh[c + d\*x]])/(b^3\*d) - (a\*Sinh[c + d\*x])/(b^2\*d) + Sinh[c + d\*x]^2/(2\*b\*d)

**Rubi [A]** time = 0.0692951, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2668, 697}

$$\frac{(a^2 + b^2) \log(a + b \sinh(c + dx))}{b^3 d} - \frac{a \sinh(c + dx)}{b^2 d} + \frac{\sinh^2(c + dx)}{2bd}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d\*x]^3/(a + b\*Sinh[c + d\*x]),x]

[Out] ((a^2 + b^2)\*Log[a + b\*Sinh[c + d\*x]])/(b^3\*d) - (a\*Sinh[c + d\*x])/(b^2\*d) + Sinh[c + d\*x]^2/(2\*b\*d)

### Rule 2668

Int[cos[(e\_.) + (f\_.)\*(x\_.)]^(p\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(m\_.), x\_Symbol] :> Dist[1/(b^p\*f), Subst[Int[(a + x)^m\*(b^2 - x^2)^((p - 1)/2), x], x, b\*Sin[e + f\*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

### Rule 697

Int[((d\_.) + (e\_.)\*(x\_.))^(m\_.)\*((a\_.) + (c\_.)\*(x\_.)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IGtQ[p, 0]

### Rubi steps



$$\begin{aligned}
\int \frac{\cosh^3(c + dx)}{a + b \sinh(c + dx)} dx &= -\frac{\text{Subst}\left(\int \frac{-b^2 - x^2}{a+x} dx, x, b \sinh(c + dx)\right)}{b^3 d} \\
&= -\frac{\text{Subst}\left(\int \left(a - x + \frac{-a^2 - b^2}{a+x}\right) dx, x, b \sinh(c + dx)\right)}{b^3 d} \\
&= \frac{(a^2 + b^2) \log(a + b \sinh(c + dx))}{b^3 d} - \frac{a \sinh(c + dx)}{b^2 d} + \frac{\sinh^2(c + dx)}{2bd}
\end{aligned}$$

**Mathematica [A]** time = 0.0505462, size = 53, normalized size = 0.9

$$-\frac{(a^2 + b^2) \log(a + b \sinh(c + dx)) + ab \sinh(c + dx) - \frac{1}{2} b^2 \sinh^2(c + dx)}{b^3 d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d\*x]^3/(a + b\*Sinh[c + d\*x]),x]

[Out] -((-((a^2 + b^2)\*Log[a + b\*Sinh[c + d\*x]]) + a\*b\*Sinh[c + d\*x] - (b^2\*Sinh[c + d\*x]^2)/2)/(b^3\*d))

**Maple [B]** time = 0.043, size = 291, normalized size = 4.9

$$\frac{1}{2bd} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-2} - \frac{1}{2bd} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} + \frac{a}{db^2} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} - \frac{a^2}{db^3} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

[Out] 1/2/d/b/(tanh(1/2\*d\*x+1/2\*c)+1)^2-1/2/d/b/(tanh(1/2\*d\*x+1/2\*c)+1)+1/d/b^2/(tanh(1/2\*d\*x+1/2\*c)+1)\*a-1/d/b^3\*ln(tanh(1/2\*d\*x+1/2\*c)+1)\*a^2-1/d/b\*ln(tanh(1/2\*d\*x+1/2\*c)+1)+1/2/d/b/(tanh(1/2\*d\*x+1/2\*c)-1)^2+1/2/d/b/(tanh(1/2\*d\*x+1/2\*c)-1)+1/d/b^2/(tanh(1/2\*d\*x+1/2\*c)-1)\*a-1/d/b^3\*ln(tanh(1/2\*d\*x+1/2\*c)-1)\*a^2-1/d/b\*ln(tanh(1/2\*d\*x+1/2\*c)-1)+1/d/b^3\*ln(tanh(1/2\*d\*x+1/2\*c)^2\*a-2\*tanh(1/2\*d\*x+1/2\*c)\*b-a)\*a^2+1/d/b\*ln(tanh(1/2\*d\*x+1/2\*c)^2\*a-2\*tanh(1/2\*d\*x+1/2\*c)\*b-a)

---

**Maxima [B]** time = 1.16017, size = 171, normalized size = 2.9

$$\frac{(4ae^{(-dx-c)} - b)e^{(2dx+2c)}}{8b^2d} + \frac{(a^2 + b^2)(dx + c)}{b^3d} + \frac{4ae^{(-dx-c)} + be^{(-2dx-2c)}}{8b^2d} + \frac{(a^2 + b^2) \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -1/8\*(4\*a\*e^(-d\*x - c) - b)\*e^(2\*d\*x + 2\*c)/(b^2\*d) + (a^2 + b^2)\*(d\*x + c)/(b^3\*d) + 1/8\*(4\*a\*e^(-d\*x - c) + b\*e^(-2\*d\*x - 2\*c))/(b^2\*d) + (a^2 + b^2)\*log(-2\*a\*e^(-d\*x - c) + b\*e^(-2\*d\*x - 2\*c) - b)/(b^3\*d)

---

**Fricas [B]** time = 2.27073, size = 848, normalized size = 14.37

$$b^2 \cosh(dx + c)^4 + b^2 \sinh(dx + c)^4 - 8(a^2 + b^2)dx \cosh(dx + c)^2 - 4ab \cosh(dx + c)^3 + 4(b^2 \cosh(dx + c) - ab) \sinh(dx + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 1/8\*(b^2\*cosh(d\*x + c)^4 + b^2\*sinh(d\*x + c)^4 - 8\*(a^2 + b^2)\*d\*x\*cosh(d\*x + c)^2 - 4\*a\*b\*cosh(d\*x + c)^3 + 4\*(b^2\*cosh(d\*x + c) - a\*b)\*sinh(d\*x + c)^3 + 4\*a\*b\*cosh(d\*x + c) + 2\*(3\*b^2\*cosh(d\*x + c)^2 - 4\*(a^2 + b^2)\*d\*x - 6\*a\*b\*cosh(d\*x + c))\*sinh(d\*x + c)^2 + b^2 + 8\*((a^2 + b^2)\*cosh(d\*x + c)^2 + 2\*(a^2 + b^2)\*cosh(d\*x + c)\*sinh(d\*x + c) + (a^2 + b^2)\*sinh(d\*x + c)^2)\*log(2\*(b\*sinh(d\*x + c) + a)/(cosh(d\*x + c) - sinh(d\*x + c))) + 4\*(b^2\*cosh(d\*x + c)^3 - 4\*(a^2 + b^2)\*d\*x\*cosh(d\*x + c) - 3\*a\*b\*cosh(d\*x + c)^2 + a\*b)\*sinh(d\*x + c))/(b^3\*d\*cosh(d\*x + c)^2 + 2\*b^3\*d\*cosh(d\*x + c)\*sinh(d\*x + c) + b^3\*d\*sinh(d\*x + c)^2)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [A]** time = 1.16287, size = 128, normalized size = 2.17

$$\frac{(a^2 + b^2) \log\left(\left|b(e^{dx+c}) - e^{(-dx-c)}\right| + 2a\right)}{b^3 d} + \frac{bd(e^{dx+c})^2 - 4ad(e^{dx+c}) - e^{(-dx-c)}}{8b^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] (a^2 + b^2)\*log(abs(b\*(e^(d\*x + c) - e^(-d\*x - c)) + 2\*a))/(b^3\*d) + 1/8\*(b\*d\*(e^(d\*x + c) - e^(-d\*x - c))^2 - 4\*a\*d\*(e^(d\*x + c) - e^(-d\*x - c)))/(b^2\*d^2)

$$3.303 \quad \int \frac{\cosh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=30

$$\text{Unintegrable} \left( \frac{\cosh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[Cosh[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0768147, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Cosh[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Cosh[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\cosh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [A]** time = 81.9985, size = 0, normalized size = 0.

$$\int \frac{\cosh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Cosh[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Integrate[Cosh[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Maple [A]** time = 0.234, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx + c))^3}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(cosh(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{e^{\left(-2c + \frac{2de}{f}\right)} E_1\left(\frac{2(fx+e)d}{f}\right)}{4bf} + \frac{ae^{\left(-c + \frac{de}{f}\right)} E_1\left(\frac{(fx+e)d}{f}\right)}{2b^2f} + \frac{ae^{\left(c - \frac{de}{f}\right)} E_1\left(-\frac{(fx+e)d}{f}\right)}{2b^2f} - \frac{e^{\left(2c - \frac{2de}{f}\right)} E_1\left(-\frac{2(fx+e)d}{f}\right)}{4bf} + \frac{(a^2 + b^2) \log(fx + e)}{b^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 1/4\*e^(-2\*c + 2\*d\*e/f)\*exp\_integral\_e(1, 2\*(f\*x + e)\*d/f)/(b\*f) + 1/2\*a\*e^(-c + d\*e/f)\*exp\_integral\_e(1, (f\*x + e)\*d/f)/(b^2\*f) + 1/2\*a\*e^(c - d\*e/f)\*exp\_integral\_e(1, -(f\*x + e)\*d/f)/(b^2\*f) - 1/4\*e^(2\*c - 2\*d\*e/f)\*exp\_integral\_e(1, -2\*(f\*x + e)\*d/f)/(b\*f) + (a^2 + b^2)\*log(f\*x + e)/(b^3\*f) - 1/8\*integrate(16\*(a^2\*b + b^3 - (a^3\*e^c + a\*b^2\*e^c))\*e^(d\*x))/(b^4\*f\*x + b^4\*e - (b^4\*f\*x\*e^(2\*c) + b^4\*e\*e^(2\*c))\*e^(2\*d\*x) - 2\*(a\*b^3\*f\*x\*e^c + a\*b^3\*e^c)\*e^(d\*x)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx + c)^3}{afx + ae + (bf x + be) \sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

sage<sub>0</sub>x

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

$$3.304 \quad \int \frac{(e+fx)^3 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=786

$$\frac{6iaf^2(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^3(a^2+b^2)} - \frac{6iaf^2(e+fx)\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{d^3(a^2+b^2)} - \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^3(a^2+b^2)} - \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^3(a^2+b^2)}$$

```
[Out] (2*a*(e + f*x)^3*ArcTan[E^(c + d*x)]/((a^2 + b^2)*d) + (b*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) + (b*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) - (b*(e + f*x)^3*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d) - ((3*I)*a*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)]/((a^2 + b^2)*d^2) + ((3*I)*a*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)]/((a^2 + b^2)*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))]/(2*(a^2 + b^2)*d^2) + ((6*I)*a*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)]/((a^2 + b^2)*d^3) - ((6*I)*a*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)]/((a^2 + b^2)*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^3) + (3*b*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))]/(2*(a^2 + b^2)*d^3) - ((6*I)*a*f^3*PolyLog[4, (-I)*E^(c + d*x)]/((a^2 + b^2)*d^4) + ((6*I)*a*f^3*PolyLog[4, I*E^(c + d*x)]/((a^2 + b^2)*d^4) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^4) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^4) - (3*b*f^3*PolyLog[4, -E^(2*(c + d*x))]/(4*(a^2 + b^2)*d^4)
```

**Rubi [A]** time = 1.45704, antiderivative size = 786, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 10, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {5573, 5561, 2190, 2531, 6609, 2282, 6589, 6742, 4180, 3718}

$$\frac{6iaf^2(e+fx)\operatorname{PolyLog}\left(3,-ie^{c+dx}\right)}{d^3(a^2+b^2)} - \frac{6iaf^2(e+fx)\operatorname{PolyLog}\left(3,ie^{c+dx}\right)}{d^3(a^2+b^2)} - \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^3(a^2+b^2)} - \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3,\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^3(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*(e + f*x)^3*ArcTan[E^(c + d*x)]/((a^2 + b^2)*d) + (b*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) + (b*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) - (b*(e + f*x)^3*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d) - ((3*I)*a*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)]/((a^2 + b^2)*d^2) + ((3*I)*a*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)]/((a^2 + b^2)*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))]/(2*(a^2 + b^2)*d^2) + ((6*I)*a*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)]/((a^2 + b^2)*d^3) - ((6*I)*a*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)]/((a^2 + b^2)*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^3) + (3*b*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))]/(2*(a^2 + b^2)*d^3) - ((6*I)*a*f^3*PolyLog[4, (-I)*E^(c + d*x)]/((a^2 + b^2)*d^4) + ((6*I)*a*f^3*PolyLog[4, I*E^(c + d*x)]/((a^2 + b^2)*d^4) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^4) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^4) - (3*b*f^3*PolyLog[4, -E^(2*(c + d*x))]/(4*(a^2 + b^2)*d^4)
```

```

3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]/((a^2 + b^2)*d) - (b*(e +
f*x)^3*Log[1 + E^(2*(c + d*x))]/((a^2 + b^2)*d) - ((3*I)*a*f*(e + f*x)^2*
PolyLog[2, (-I)*E^(c + d*x)]/((a^2 + b^2)*d^2) + ((3*I)*a*f*(e + f*x)^2*Po
lyLog[2, I*E^(c + d*x)]/((a^2 + b^2)*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2,
-((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)*d^2) + (3*b*f*(e +
f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 + b^2)*d
^2) - (3*b*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))]/(2*(a^2 + b^2)*d^2)
+ ((6*I)*a*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)]/((a^2 + b^2)*d^3) -
((6*I)*a*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)]/((a^2 + b^2)*d^3) - (6*b*
f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 +
b^2)*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))]/((a^2 + b^2)*d^3) + (3*b*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x
))]/(2*(a^2 + b^2)*d^3) - ((6*I)*a*f^3*PolyLog[4, (-I)*E^(c + d*x)]/((a^2
+ b^2)*d^4) + ((6*I)*a*f^3*PolyLog[4, I*E^(c + d*x)]/((a^2 + b^2)*d^4) +
(6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)
*d^4) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/((a^
2 + b^2)*d^4) - (3*b*f^3*PolyLog[4, -E^(2*(c + d*x))]/(4*(a^2 + b^2)*d^4)

```

### Rule 5573

```

Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]

```

### Rule 5561

```

Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

```

### Rule 2190

```

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g^n*Log[F]), x] - Di
st[(d*m)/(b*f*g^n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

### Rule 2531

```

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)

```



```

*(x_)^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

```

### Rule 6609

```

Int[((e_) + (f_)*(x_))^(m_)*PolyLog[n_, (d_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(p_)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

```

### Rule 2282

```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

### Rule 6589

```

Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

```

### Rule 6742

```

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

```

### Rule 4180

```

Int[csc[(e_) + Pi*(k_) + (Complex[0, fz_])*(f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]

```

### Rule 3718

```

Int[((c_) + (d_)*(x_))^(m_)*tan[(e_) + (Complex[0, fz_])*(f_)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c

```

+ d\*x)^m\*E^(2\*(-(I\*e) + f\*fz\*x)))/(1 + E^(2\*(-(I\*e) + f\*fz\*x))), x], x] /;  
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(e+fx)^3 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \operatorname{sech}(c+dx)(a-b \sinh(c+dx)) dx}{a^2+b^2} + \frac{b^2 \int \frac{(e+fx)^3 \cosh(c+dx)}{a+b \sinh(c+dx)} dx}{a^2+b^2} \\
 &= -\frac{b(e+fx)^4}{4(a^2+b^2)f} + \frac{\int (a(e+fx)^3 \operatorname{sech}(c+dx) - b(e+fx)^3 \tanh(c+dx)) dx}{a^2+b^2} + \frac{b^2 \int \frac{e^{c+dx}}{a-\sqrt{a^2+b^2}} dx}{a^2+b^2} \\
 &= -\frac{b(e+fx)^4}{4(a^2+b^2)f} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{a \int (e+fx)^3 dx}{a^2+b^2} \\
 &= \frac{2a(e+fx)^3 \tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
 &= \frac{2a(e+fx)^3 \tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
 &= \frac{2a(e+fx)^3 \tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
 &= \frac{2a(e+fx)^3 \tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
 &= \frac{2a(e+fx)^3 \tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
 &= \frac{2a(e+fx)^3 \tan^{-1}\left(e^{c+dx}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d}
 \end{aligned}$$

**Mathematica [B]** time = 28.6787, size = 10644, normalized size = 13.54

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^3\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] Result too large to show

**Maple [F]** time = 0.351, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \operatorname{sech}(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-e^3 \left( \frac{2a \arctan(e^{(-dx-c)})}{(a^2 + b^2)d} - \frac{b \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{(a^2 + b^2)d} + \frac{b \log(e^{(-2dx-2c)} + 1)}{(a^2 + b^2)d} \right) + \int \frac{4f^3x}{(b(e^{(dx+c)} - e^{(-dx-c)}) + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-e^3 * (2*a*\arctan(e^{(-d*x - c)}) / ((a^2 + b^2)*d) - b*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b) / ((a^2 + b^2)*d) + b*\log(e^{(-2*d*x - 2*c)} + 1) / ((a^2 + b^2)*d)) + \operatorname{integrate}(4*f^3*x^3 / ((b*(e^{(d*x + c)} - e^{(-d*x - c)}) + 2*a)*(e^{(d*x + c)} + e^{(-d*x - c)})) + 12*e*f^2*x^2 / ((b*(e^{(d*x + c)} - e^{(-d*x - c)}) + 2*a)*(e^{(d*x + c)} + e^{(-d*x - c)})) + 12*e^2*f*x / ((b*(e^{(d*x + c)} - e^{(-d*x - c)}) + 2*a)*(e^{(d*x + c)} + e^{(-d*x - c)})), x)$

**Fricas [C]** time = 3.03533, size = 4177, normalized size = 5.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] (6\*b\*f^3\*polylog(4, (a\*cosh(d\*x + c) + a\*sinh(d\*x + c) + (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2))/b) + 6\*b\*f^3\*polylog(4, (a\*cosh(d\*x + c) + a\*sinh(d\*x + c) - (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2))/b) + 3\*(b\*d^2\*f^3\*x^2 + 2\*b\*d^2\*e\*f^2\*x + b\*d^2\*e^2\*f)\*dilog((a\*cosh(d\*x + c) + a\*sinh(d\*x + c) + (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 3\*(b\*d^2\*f^3\*x^2 + 2\*b\*d^2\*e\*f^2\*x + b\*d^2\*e^2\*f)\*dilog((a\*cosh(d\*x + c) + a\*sinh(d\*x + c) - (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (3\*I\*a\*d^2\*f^3\*x^2 - 3\*b\*d^2\*f^3\*x^2 + 6\*I\*a\*d^2\*e\*f^2\*x - 6\*b\*d^2\*e\*f^2\*x + 3\*I\*a\*d^2\*e^2\*f - 3\*b\*d^2\*e^2\*f)\*dilog(I\*cosh(d\*x + c) + I\*sinh(d\*x + c)) + (-3\*I\*a\*d^2\*f^3\*x^2 - 3\*b\*d^2\*f^3\*x^2 - 6\*I\*a\*d^2\*e\*f^2\*x - 6\*b\*d^2\*e\*f^2\*x - 3\*I\*a\*d^2\*e^2\*f - 3\*b\*d^2\*e^2\*f)\*dilog(-I\*cosh(d\*x + c) - I\*sinh(d\*x + c)) + (b\*d^3\*e^3 - 3\*b\*c\*d^2\*e^2\*f + 3\*b\*c^2\*d\*e\*f^2 - b\*c^3\*f^3)\*log(2\*b\*cosh(d\*x + c) + 2\*b\*sinh(d\*x + c) + 2\*b\*sqrt((a^2 + b^2)/b^2) + 2\*a) + (b\*d^3\*e^3 - 3\*b\*c\*d^2\*e^2\*f + 3\*b\*c^2\*d\*e\*f^2 - b\*c^3\*f^3)\*log(2\*b\*cosh(d\*x + c) + 2\*b\*sinh(d\*x + c) - 2\*b\*sqrt((a^2 + b^2)/b^2) + 2\*a) + (b\*d^3\*f^3\*x^3 + 3\*b\*d^3\*e\*f^2\*x^2 + 3\*b\*d^3\*e^2\*f\*x + 3\*b\*c\*d^2\*e^2\*f - 3\*b\*c^2\*d\*e\*f^2 + b\*c^3\*f^3)\*log(-(a\*cosh(d\*x + c) + a\*sinh(d\*x + c) + (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2) - b)/b) + (b\*d^3\*f^3\*x^3 + 3\*b\*d^3\*e\*f^2\*x^2 + 3\*b\*d^3\*e^2\*f\*x + 3\*b\*c\*d^2\*e^2\*f - 3\*b\*c^2\*d\*e\*f^2 + b\*c^3\*f^3)\*log(-(a\*cosh(d\*x + c) + a\*sinh(d\*x + c) - (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2) - b)/b) + (I\*a\*d^3\*e^3 - b\*d^3\*e^3 - 3\*I\*a\*c\*d^2\*e^2\*f + 3\*b\*c\*d^2\*e^2\*f + 3\*I\*a\*c^2\*d\*e\*f^2 - 3\*b\*c^2\*d\*e\*f^2 - I\*a\*c^3\*f^3 + b\*c^3\*f^3)\*log(cosh(d\*x + c) + sinh(d\*x + c) + I) + (-I\*a\*d^3\*e^3 - b\*d^3\*e^3 + 3\*I\*a\*c\*d^2\*e^2\*f + 3\*b\*c\*d^2\*e^2\*f - 3\*I\*a\*c^2\*d\*e\*f^2 - 3\*b\*c^2\*d\*e\*f^2 + I\*a\*c^3\*f^3 + b\*c^3\*f^3)\*log(cosh(d\*x + c) + sinh(d\*x + c) - I) + (-I\*a\*d^3\*f^3\*x^3 - b\*d^3\*f^3\*x^3 - 3\*I\*a\*d^3\*e\*f^2\*x^2 - 3\*b\*d^3\*e\*f^2\*x^2 - 3\*I\*a\*d^3\*e^2\*f\*x - 3\*b\*d^3\*e^2\*f\*x - 3\*I\*a\*c\*d^2\*e^2\*f - 3\*b\*c\*d^2\*e^2\*f + 3\*I\*a\*c^2\*d\*e\*f^2 + 3\*b\*c^2\*d\*e\*f^2 - I\*a\*c^3\*f^3 - b\*c^3\*f^3)\*log(I\*cosh(d\*x + c) + I\*sinh(d\*x + c) + 1) + (I\*a\*d^3\*f^3\*x^3 - b\*d^3\*f^3\*x^3 + 3\*I\*a\*d^3\*e\*f^2\*x^2 - 3\*b\*d^3\*e\*f^2\*x^2 + 3\*I\*a\*d^3\*e^2\*f\*x - 3\*b\*d^3\*e^2\*f\*x + 3\*I\*a\*c\*d^2\*e^2\*f - 3\*b\*c\*d^2\*e^2\*f - 3\*I\*a\*c^2\*d\*e\*f^2 + 3\*b\*c^2\*d\*e\*f^2 + I\*a\*c^3\*f^3 - b\*c^3\*f^3)\*log(-I\*cosh(d\*x + c) - I\*sinh(d\*x + c) + 1) - 6\*(-I\*a\*f^3 + b\*f^3)\*polylog(4, I\*cosh(d\*x + c) + I\*sinh(d\*x + c)) - 6\*(I\*a\*f^3 + b\*f^3)\*polylog(4, -I\*cosh(d\*x + c) - I\*sinh(d\*x + c)) - 6\*(b\*d\*f^3\*x + b\*d\*e\*f^2)\*polylog(3, (a\*cosh(d\*x + c) + a\*sinh(d\*x + c) + (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2))/b) - 6\*(b\*d\*f^3\*x + b\*d\*e\*f^2)\*polylog(3, (a\*cosh(d\*x + c) + a\*sinh(d\*x + c) - (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2))/b) + (-6\*I\*a\*d\*f^3\*x + 6\*b\*d\*f^3\*x - 6\*I\*a\*d\*e\*f^2 + 6\*b\*d\*e\*f^2)\*polylog(3, I\*cosh(d\*x + c) + I\*sinh(d\*x + c)) + (6\*I\*a\*d\*f^3\*x + 6\*b\*d\*f^3\*x + 6\*I\*a\*d\*e\*f^2 + 6\*b\*d\*e\*f^2)\*polylog(3, -I\*cosh(d\*x + c) - I\*sinh(d\*x + c)))/((a^2 + b^2)\*d^4)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^3 \operatorname{sech}(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*\*3\*sech(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \operatorname{sech}(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*sech(d\*x + c)/(b\*sinh(d\*x + c) + a), x)

$$3.305 \quad \int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=558

$$-\frac{2iaf(e+fx)\operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{d^2(a^2+b^2)} + \frac{2iaf(e+fx)\operatorname{PolyLog}\left(2, ie^{c+dx}\right)}{d^2(a^2+b^2)} + \frac{2bf(e+fx)\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)} + \frac{2bf(e+fx)\operatorname{PolyLog}\left(2, \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)}$$

[Out] (2\*a\*(e + f\*x)^2\*ArcTan[E^(c + d\*x)]/((a^2 + b^2)\*d) + (b\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])]/((a^2 + b^2)\*d) + (b\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])]/((a^2 + b^2)\*d) - (b\*(e + f\*x)^2\*Log[1 + E^(2\*(c + d\*x))]/((a^2 + b^2)\*d) - ((2\*I)\*a\*f\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)]/((a^2 + b^2)\*d^2) + ((2\*I)\*a\*f\*(e + f\*x)\*PolyLog[2, I\*E^(c + d\*x)]/((a^2 + b^2)\*d^2) + (2\*b\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)\*d^2) + (2\*b\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 + b^2)\*d^2) - (b\*f\*(e + f\*x)\*PolyLog[2, -E^(2\*(c + d\*x))]/((a^2 + b^2)\*d^2) + ((2\*I)\*a\*f^2\*PolyLog[3, (-I)\*E^(c + d\*x)]/((a^2 + b^2)\*d^3) - ((2\*I)\*a\*f^2\*PolyLog[3, I\*E^(c + d\*x)]/((a^2 + b^2)\*d^3) - (2\*b\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)\*d^3) - (2\*b\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 + b^2)\*d^3) + (b\*f^2\*PolyLog[3, -E^(2\*(c + d\*x))]/(2\*(a^2 + b^2)\*d^3)

**Rubi [A]** time = 1.03445, antiderivative size = 558, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 9, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$ , Rules used = {5573, 5561, 2190, 2531, 2282, 6589, 6742, 4180, 3718}

$$-\frac{2iaf(e+fx)\operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{d^2(a^2+b^2)} + \frac{2iaf(e+fx)\operatorname{PolyLog}\left(2, ie^{c+dx}\right)}{d^2(a^2+b^2)} + \frac{2bf(e+fx)\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)} + \frac{2bf(e+fx)\operatorname{PolyLog}\left(2, \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]), x]

[Out] (2\*a\*(e + f\*x)^2\*ArcTan[E^(c + d\*x)]/((a^2 + b^2)\*d) + (b\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])]/((a^2 + b^2)\*d) + (b\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])]/((a^2 + b^2)\*d) - (b\*(e + f\*x)^2\*Log[1 + E^(2\*(c + d\*x))]/((a^2 + b^2)\*d) - ((2\*I)\*a\*f\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)]/((a^2 + b^2)\*d^2) + ((2\*I)\*a\*f\*(e + f\*x)\*PolyLog[2, I\*E^(c + d\*x)]/((a^2 + b^2)\*d^2) + (2\*b\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)\*d^2) + (2\*b\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 + b^2)\*d^2) - (b\*f\*(e + f\*x)\*PolyLog[2, -E^(2\*(c + d\*x))]/((a^2 + b^2)\*d^2) + ((2\*I)\*a\*f^2\*PolyLog[3, (-I)\*E^(c + d\*x)]/((a^2 + b^2)\*d^3) - ((2\*I)\*a\*f^2\*PolyLog[3, I\*E^(c + d\*x)]/((a^2 + b^2)\*d^3) - (2\*b\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)\*d^3) - (2\*b\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 + b^2)\*d^3) + (b\*f^2\*PolyLog[3, -E^(2\*(c + d\*x))]/(2\*(a^2 + b^2)\*d^3)

$$\frac{(e^{c+dx})/(a - \sqrt{a^2 + b^2})}{(a^2 + b^2)d^2} + \frac{(2bf(e+fx) \operatorname{PolyLog}[2, -(bE^{c+dx})/(a + \sqrt{a^2 + b^2})])}{(a^2 + b^2)d^2} - \frac{(bf(e+fx) \operatorname{PolyLog}[2, -E^{2(c+dx)}])}{(a^2 + b^2)d^2} + \frac{((2I)af^2 \operatorname{PolyLog}[3, (-I)E^{c+dx}])}{(a^2 + b^2)d^3} - \frac{((2I)af^2 \operatorname{PolyLog}[3, IE^{c+dx}])}{(a^2 + b^2)d^3} - \frac{(2bf^2 \operatorname{PolyLog}[3, -(bE^{c+dx})/(a - \sqrt{a^2 + b^2})])}{(a^2 + b^2)d^3} - \frac{(2bf^2 \operatorname{PolyLog}[3, -(bE^{c+dx})/(a + \sqrt{a^2 + b^2})])}{(a^2 + b^2)d^3} + \frac{(bf^2 \operatorname{PolyLog}[3, -E^{2(c+dx)}])}{2(a^2 + b^2)d^3}$$

### Rule 5573

$$\operatorname{Int}[\frac{(e + f x)^m \operatorname{Sech}[c + d x]^n}{(a + b \operatorname{Sinh}[c + d x])}, x] := \operatorname{Dist}[\frac{b^2}{a^2 + b^2}, \operatorname{Int}[\frac{(e + f x)^m \operatorname{Sech}[c + d x]^{n-2}}{(a + b \operatorname{Sinh}[c + d x])}, x] + \operatorname{Dist}[\frac{1}{a^2 + b^2}, \operatorname{Int}[\frac{(e + f x)^m \operatorname{Sech}[c + d x]^n (a - b \operatorname{Sinh}[c + d x])}{(a + b \operatorname{Sinh}[c + d x])}, x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, x\} \&\& \operatorname{IGtQ}[m, 0] \&\& \operatorname{NeQ}[a^2 + b^2, 0] \&\& \operatorname{IGtQ}[n, 0]$$

### Rule 5561

$$\operatorname{Int}[\frac{\operatorname{Cosh}[c + d x] (e + f x)^m}{(a + b \operatorname{Sinh}[c + d x])}, x] := -\operatorname{Simp}[\frac{(e + f x)^{m+1}}{b f (m+1)}, x] + \operatorname{Int}[\frac{(e + f x)^m E^{c+dx}}{(a - \operatorname{Rt}[a^2 + b^2, 2] + b E^{c+dx})}, x] + \operatorname{Int}[\frac{(e + f x)^m E^{c+dx}}{(a + \operatorname{Rt}[a^2 + b^2, 2] + b E^{c+dx})}, x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, x\} \&\& \operatorname{IGtQ}[m, 0] \&\& \operatorname{NeQ}[a^2 + b^2, 0]$$

### Rule 2190

$$\operatorname{Int}[\frac{(F^{(g_1)(e_1 + f_1 x)})^{n_1} ((c_1 + d_1 x)^{m_1})}{(a_1 + b_1 (F^{(g_1)(e_1 + f_1 x)})^{n_1})}, x] := \operatorname{Simp}[\frac{(c + d x)^m \operatorname{Log}[1 + (b F^{g(e+fx)})^n / a]}{b f g^n \operatorname{Log}[F]}, x] - \operatorname{Dist}[\frac{d^m}{b f g^n \operatorname{Log}[F]}, \operatorname{Int}[\frac{(c + d x)^{m-1} \operatorname{Log}[1 + (b F^{g(e+fx)})^n]}{a}], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, f, g, n, x\} \&\& \operatorname{IGtQ}[m, 0]$$

### Rule 2531

$$\operatorname{Int}[\operatorname{Log}[1 + (e_1) (F^{(c_1)(a_1 + b_1 x)})^{n_1}] (f_1 + g_1 x)^{m_1}, x] := -\operatorname{Simp}[\frac{(f + g x)^m \operatorname{PolyLog}[2, -(e (F^{c(a+bx)})^n)]}{b c^n \operatorname{Log}[F]}, x] + \operatorname{Dist}[\frac{(g^m)}{b c^n \operatorname{Log}[F]}, \operatorname{Int}[\frac{(f + g x)^{m-1} \operatorname{PolyLog}[2, -(e (F^{c(a+bx)})^n)]}{b c^n \operatorname{Log}[F]}, x] /; \operatorname{FreeQ}\{F, a, b, c, e, f, g, n, x\} \&\& \operatorname{GtQ}[m, 0]$$

### Rule 2282

$$\operatorname{Int}[u, x] := \operatorname{With}\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \operatorname{Funci}$$

```

onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

### Rule 6589

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]

```

### Rule 6742

```

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]

```

### Rule 4180

```

Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^m_], x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]

```

### Rule 3718

```

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

```

### Rubi steps



$$\begin{aligned}
\int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \operatorname{sech}(c+dx)(a-b \sinh(c+dx)) dx}{a^2+b^2} + \frac{b^2 \int \frac{(e+fx)^2 \cosh(c+dx)}{a+b \sinh(c+dx)} dx}{a^2+b^2} \\
&= -\frac{b(e+fx)^3}{3(a^2+b^2)f} + \frac{\int (a(e+fx)^2 \operatorname{sech}(c+dx) - b(e+fx)^2 \tanh(c+dx)) dx}{a^2+b^2} + \frac{b^2 \int \frac{e^{c+dx}}{a-\sqrt{a^2+b^2}} dx}{a^2} \\
&= -\frac{b(e+fx)^3}{3(a^2+b^2)f} + \frac{b(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{a \int (e^{c+dx}) dx}{(a^2+b^2)d} \\
&= \frac{2a(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d} + \frac{b(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
&= \frac{2a(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d} + \frac{b(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
&= \frac{2a(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d} + \frac{b(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
&= \frac{2a(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d} + \frac{b(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
&= \frac{2a(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d} + \frac{b(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} + \frac{b(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d}
\end{aligned}$$

**Mathematica [B]** time = 21.5492, size = 1639, normalized size = 2.94

result too large to display

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (12\*b\*d^3\*e^2\*E^(2\*c)\*x - 12\*b\*d^3\*e^2\*(1 + E^(2\*c))\*x - 12\*b\*d^3\*e\*f\*x^2 - 4\*b\*d^3\*f^2\*x^3 + 12\*a\*d^2\*e^2\*(1 + E^(2\*c))\*ArcTan[E^(c + d\*x)] + 6\*b\*d^2\*e^2\*(1 + E^(2\*c))\*(2\*d\*x - Log[1 + E^(2\*(c + d\*x))]) + (12\*I)\*a\*d\*e\*(1 + E^(2\*c))\*f\*(d\*x\*(Log[1 - I\*E^(c + d\*x)] - Log[1 + I\*E^(c + d\*x)]) - PolyLog[2, (-I)\*E^(c + d\*x)] + PolyLog[2, I\*E^(c + d\*x)]) + 6\*b\*d\*e\*(1 + E^(2\*c))\*f\*(2\*d\*x\*(d\*x - Log[1 + E^(2\*(c + d\*x))]) - PolyLog[2, -E^(2\*(c + d\*x))]) +

```
(6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log[1 +
I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2, I*E
^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*x)])
+ b*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))]) - 6*
d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))])/((6*(a^2
+ b^2)*d^3*(1 + E^(2*c))) - (b*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E
^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a
^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c
)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*
a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((
-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b
*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(
c + d*x) + b*(-1 + E^(2*(c + d*x)))]/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x
) + b*(-1 + E^(2*(c + d*x)))]/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^
c - Sqrt[(a^2 + b^2)*E^(2*c)]]])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d
*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]]])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c
+ d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]]])/d - (3*E^(2*c)*f^2*x^2*Log[1
+ (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]]])/d + (6*e*f*x*Log
[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]]])/d - (6*e*E^(2*
c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]]])/d +
(3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]]])/
d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*
E^(2*c)]]])/d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x)
)/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]])))/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x
)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]])))/d^2
- (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)
]])))/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2
+ b^2)*E^(2*c)]])))/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + S
qrt[(a^2 + b^2)*E^(2*c)]])))/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c +
d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]])))/d^3)/(3*(a^2 + b^2)*(-1 + E^(
2*c))) + (b*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Csch[c/2]*Sech[c/2]*Sech[c])/((6*(
a^2 + b^2))
```

---

**Maple [F]** time = 0.267, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \operatorname{sech}(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out]  $\int (f*x+e)^2*\operatorname{sech}(d*x+c)/(a+b*\sinh(d*x+c)),x$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-e^2 \left( \frac{2a \arctan(e^{(-dx-c)})}{(a^2 + b^2)d} - \frac{b \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{(a^2 + b^2)d} + \frac{b \log(e^{(-2dx-2c)} + 1)}{(a^2 + b^2)d} \right) + \int \frac{4f^2x}{(b(e^{(dx+c)} - e^{(-dx-c)}) + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((f*x+e)^2*\operatorname{sech}(d*x+c)/(a+b*\sinh(d*x+c)),x, \text{algorithm}="maxima")$

[Out]  $-e^2*(2*a*\arctan(e^{(-d*x - c)})/((a^2 + b^2)*d) - b*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/((a^2 + b^2)*d) + b*\log(e^{(-2*d*x - 2*c)} + 1)/((a^2 + b^2)*d)) + \operatorname{integrate}(4*f^2*x^2/((b*(e^{(d*x + c)} - e^{(-d*x - c)}) + 2*a)*(e^{(d*x + c)} + e^{(-d*x - c)})) + 8*e*f*x/((b*(e^{(d*x + c)} - e^{(-d*x - c)}) + 2*a)*(e^{(d*x + c)} + e^{(-d*x - c)})), x)$

**Fricas [C]** time = 2.61712, size = 2745, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((f*x+e)^2*\operatorname{sech}(d*x+c)/(a+b*\sinh(d*x+c)),x, \text{algorithm}="fricas")$

[Out]  $-(2*b*f^2*\operatorname{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b + 2*b*f^2*\operatorname{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b - 2*(b*d*f^2*x + b*d*e*f)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 2*(b*d*f^2*x + b*d*e*f)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - (2*I*a*d*f^2*x - 2*b*d*f^2*x + 2*I*a*d*e*f - 2*b*d*e*f)*\operatorname{dilog}(I*\cosh(d*x + c) + I*\sinh(d*x + c)) - (-2*I*a*d*f^2*x - 2*b*d*f^2*x - 2*I*a*d*e*f - 2*b*d*e*f)*\operatorname{dilog}(-I*\cosh(d*x + c) - I*\sinh(d*x + c)) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c)$

) + (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2) - b)/b) - (b\*d^2\*f^2\*x^2 + 2\*b\*d^2\*e\*f\*x + 2\*b\*c\*d\*e\*f - b\*c^2\*f^2)\*log(-(a\*cosh(d\*x + c) + a\*sinh(d\*x + c) - (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2) - b)/b) - (I\*a\*d^2\*e^2 - b\*d^2\*e^2 - 2\*I\*a\*c\*d\*e\*f + 2\*b\*c\*d\*e\*f + I\*a\*c^2\*f^2 - b\*c^2\*f^2)\*log(cosh(d\*x + c) + sinh(d\*x + c) + I) - (-I\*a\*d^2\*e^2 - b\*d^2\*e^2 + 2\*I\*a\*c\*d\*e\*f + 2\*b\*c\*d\*e\*f - I\*a\*c^2\*f^2 - b\*c^2\*f^2)\*log(cosh(d\*x + c) + sinh(d\*x + c) - I) - (-I\*a\*d^2\*f^2\*x^2 - b\*d^2\*f^2\*x^2 - 2\*I\*a\*d^2\*e\*f\*x - 2\*b\*d^2\*e\*f\*x - 2\*I\*a\*c\*d\*e\*f - 2\*b\*c\*d\*e\*f + I\*a\*c^2\*f^2 + b\*c^2\*f^2)\*log(I\*cosh(d\*x + c) + I\*sinh(d\*x + c) + 1) - (I\*a\*d^2\*f^2\*x^2 - b\*d^2\*f^2\*x^2 + 2\*I\*a\*d^2\*e\*f\*x - 2\*b\*d^2\*e\*f\*x + 2\*I\*a\*c\*d\*e\*f - 2\*b\*c\*d\*e\*f - I\*a\*c^2\*f^2 + b\*c^2\*f^2)\*log(-I\*cosh(d\*x + c) - I\*sinh(d\*x + c) + 1) + 2\*(I\*a\*f^2 - b\*f^2)\*polylog(3, I\*cosh(d\*x + c) + I\*sinh(d\*x + c)) + 2\*(-I\*a\*f^2 - b\*f^2)\*polylog(3, -I\*cosh(d\*x + c) - I\*sinh(d\*x + c)))/((a^2 + b^2)\*d^3)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^2 \operatorname{sech}(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*\*2\*sech(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \operatorname{sech}(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*sech(d\*x + c)/(b\*sinh(d\*x + c) + a), x)

### 3.306 $\int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx$

**Optimal.** Leaf size=334

$$-\frac{iaf\operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{d^2(a^2+b^2)} + \frac{iaf\operatorname{PolyLog}\left(2, ie^{c+dx}\right)}{d^2(a^2+b^2)} + \frac{bf\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)} + \frac{bf\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2(a^2+b^2)} - \frac{bfP}{d^2(a^2+b^2)}$$

```
[Out] (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) + (b*(e + f*x)*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]/((a^2 + b^2)*d) + (b*(e + f*x)*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]/((a^2 + b^2)*d) - (b*(e + f*x)*
Log[1 + E^(2*(c + d*x))]/((a^2 + b^2)*d) - (I*a*f*PolyLog[2, (-I)*E^(c + d
*x)])/((a^2 + b^2)*d^2) + (I*a*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^
2) + (b*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2
)*d^2) + (b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 +
b^2)*d^2) - (b*f*PolyLog[2, -E^(2*(c + d*x))]/(2*(a^2 + b^2)*d^2)
```

**Rubi [A]** time = 0.59629, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 8, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5573, 5561, 2190, 2279, 2391, 6742, 4180, 3718}

$$-\frac{iaf\operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{d^2(a^2+b^2)} + \frac{iaf\operatorname{PolyLog}\left(2, ie^{c+dx}\right)}{d^2(a^2+b^2)} + \frac{bf\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)} + \frac{bf\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2(a^2+b^2)} - \frac{bfP}{d^2(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) + (b*(e + f*x)*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]/((a^2 + b^2)*d) + (b*(e + f*x)*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]/((a^2 + b^2)*d) - (b*(e + f*x)*
Log[1 + E^(2*(c + d*x))]/((a^2 + b^2)*d) - (I*a*f*PolyLog[2, (-I)*E^(c + d
*x)])/((a^2 + b^2)*d^2) + (I*a*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^
2) + (b*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2
)*d^2) + (b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 +
b^2)*d^2) - (b*f*PolyLog[2, -E^(2*(c + d*x))]/(2*(a^2 + b^2)*d^2)
```

#### Rule 5573

```
Int[(((e_.) + (f_.)*(x_.))^(m_.)*Sech[(c_.) + (d_.)*(x_.)]^(n_.))/((a_.) + (b_.
)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e +
```

$f*x)^m*\text{Sech}[c + d*x]^{(n - 2)}/(a + b*\text{Sinh}[c + d*x]), x, x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(e + f*x)^m*\text{Sech}[c + d*x]^n*(a - b*\text{Sinh}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{IGtQ}[n, 0]$

### Rule 5561

$\text{Int}[(\text{Cosh}[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^{(m_.)})/((a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_)]), x\_Symbol] :> -\text{Simp}[(e + f*x)^{(m + 1)}/(b*f*(m + 1)), x] + (\text{Int}[(e + f*x)^m*\text{E}^{(c + d*x)}/(a - \text{Rt}[a^2 + b^2, 2] + b*\text{E}^{(c + d*x)}), x] + \text{Int}[(e + f*x)^m*\text{E}^{(c + d*x)}/(a + \text{Rt}[a^2 + b^2, 2] + b*\text{E}^{(c + d*x)}), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[a^2 + b^2, 0]$

### Rule 2190

$\text{Int}[(\text{F}^{((g_.)*((e_.) + (f_.)*(x_)))})^{(n_.)}*((c_.) + (d_.)*(x_))^{(m_.)})/((a_.) + (b_.)*(\text{F}^{((g_.)*((e_.) + (f_.)*(x_)))})^{(n_.)}), x\_Symbol] :> \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(\text{F}^{(g*(e + f*x))})^n)/a]]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + (b*(\text{F}^{(g*(e + f*x))})^n)/a]], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0]$

### Rule 2279

$\text{Int}[\text{Log}[(a_.) + (b_.)*(\text{F}^{((e_.)*((c_.) + (d_.)*(x_)))})^{(n_.)})], x\_Symbol] :> \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (\text{F}^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \&\& \text{GtQ}[a, 0]$

### Rule 2391

$\text{Int}[\text{Log}[(c_.)*((d_.) + (e_.)*(x_))^{(n_.)})]/(x_), x\_Symbol] :> -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x\} \&\& \text{EqQ}[c*d, 1]$

### Rule 6742

$\text{Int}[u_, x\_Symbol] :> \text{With}\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]$

### Rule 4180

$\text{Int}[\text{csc}[(e_.) + \text{Pi}*(k_.) + (\text{Complex}[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^{(m_.)}, x\_Symbol] :> \text{Simp}[(-2*(c + d*x)^m*\text{ArcTanh}[\text{E}^{-(I*e) + f*fz*x}/\text{E}^{(I*k*Pi)}])/(f*fz*I), x] + (-\text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 - \text{E}^{-(I*e) + f*fz*x}/\text{E}^{(I*k*Pi)}], x], x] + \text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + \text{E}^{-(I*e) + f*fz*x}/\text{E}^{(I*k*Pi)}], x], x]) /; \text{FreeQ}\{c, d, e, f, fz\}, x\} \&\& \text{IntegerQ}[2*k] \&\& \text{IGtQ}[m, 0]$

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e + fx)\operatorname{sech}(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\int (e + fx)\operatorname{sech}(c + dx)(a - b \sinh(c + dx)) dx}{a^2 + b^2} + \frac{b^2 \int \frac{(e + fx) \cosh(c + dx)}{a + b \sinh(c + dx)} dx}{a^2 + b^2} \\
&= -\frac{b(e + fx)^2}{2(a^2 + b^2)f} + \frac{\int (a(e + fx)\operatorname{sech}(c + dx) - b(e + fx) \tanh(c + dx)) dx}{a^2 + b^2} + \frac{b^2 \int \frac{e^{c+dx}(e + fx)}{a - \sqrt{a^2 + b^2}} dx}{a^2 + b^2} \\
&= -\frac{b(e + fx)^2}{2(a^2 + b^2)f} + \frac{b(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{(a^2 + b^2)d} + \frac{b(e + fx) \log\left(1 + \frac{be^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{(a^2 + b^2)d} + \frac{a \int (e + fx) dx}{a^2 + b^2} \\
&= \frac{2a(e + fx) \tan^{-1}\left(e^{c+dx}\right)}{(a^2 + b^2)d} + \frac{b(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{(a^2 + b^2)d} + \frac{b(e + fx) \log\left(1 + \frac{be^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{(a^2 + b^2)d} \\
&= \frac{2a(e + fx) \tan^{-1}\left(e^{c+dx}\right)}{(a^2 + b^2)d} + \frac{b(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{(a^2 + b^2)d} + \frac{b(e + fx) \log\left(1 + \frac{be^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{(a^2 + b^2)d} \\
&= \frac{2a(e + fx) \tan^{-1}\left(e^{c+dx}\right)}{(a^2 + b^2)d} + \frac{b(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{(a^2 + b^2)d} + \frac{b(e + fx) \log\left(1 + \frac{be^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{(a^2 + b^2)d} \\
&= \frac{2a(e + fx) \tan^{-1}\left(e^{c+dx}\right)}{(a^2 + b^2)d} + \frac{b(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{(a^2 + b^2)d} + \frac{b(e + fx) \log\left(1 + \frac{be^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{(a^2 + b^2)d}
\end{aligned}$$

**Mathematica [A]** time = 2.73318, size = 439, normalized size = 1.31

$$\frac{2bf \operatorname{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2 + b^2} - a}\right) + 2bf \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right) - 2iaf \operatorname{PolyLog}(2, -i(\sinh(c + dx) + \cosh(c + dx))) + 2iaf \operatorname{PolyLog}(2, i(\sinh(c + dx) + \cosh(c + dx)))}{a^2 + b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Sech[c + d*x])/(a + b*Sinh[c + d*x]), x]
```

```
[Out] (2*b*c*d*e - 2*b*c^2*f + 2*b*d^2*e*x - 2*b*c*d*f*x + 4*a*d*e*ArcTan[Cosh[c
+ d*x] + Sinh[c + d*x]] + 4*a*d*f*x*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] +
2*b*c*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*b*d*f*x*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*b*c*f*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2])] + 2*b*d*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2])] + 2*b*d*e*Log[a + b*Sinh[c + d*x]] - 2*b*c*f*Log[a + b*Sinh[c + d*x]]
- 2*b*d*e*Log[1 + Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] - 2*b*d*f*x*Log[1
+ Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] + 2*b*f*PolyLog[2, (b*E^(c + d*x)
)/(-a + Sqrt[a^2 + b^2])] + 2*b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))] - (2*I)*a*f*PolyLog[2, (-I)*(Cosh[c + d*x] + Sinh[c + d*x])] +
(2*I)*a*f*PolyLog[2, I*(Cosh[c + d*x] + Sinh[c + d*x])] - b*f*PolyLog[2, -C
osh[2*(c + d*x)] - Sinh[2*(c + d*x)]]/(2*(a^2 + b^2)*d^2)
```

**Maple [B]** time = 0.128, size = 954, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] -2/d*e/(2*a^2+2*b^2)*b*ln(1+exp(2*d*x+2*c))+4/d*e/(2*a^2+2*b^2)*a*arctan(ex
p(d*x+c))+2/d*e*b/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d*f
/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*b*x-2/d^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+
c))*b*c+2*I/d^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*c-2*I/d^2*f/(2*a^2+2*b
^2)*dilog(1+I*exp(d*x+c))*a-2/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*b*x-2/d^
2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*b*c+2*I/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d
*x+c))*a*x+2*I/d^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))*a-2/d^2*f/(2*a^2+2
*b^2)*dilog(1+I*exp(d*x+c))*b-2*I/d*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*a*x-
2/d^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))*b-2*I/d^2*f/(2*a^2+2*b^2)*ln(1+
I*exp(d*x+c))*a*c+2/d*f*b/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a
)/(-a+(a^2+b^2)^(1/2)))*x+2/d^2*f*b/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^
2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+2/d*f*b/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+
(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+2/d^2*f*b/(2*a^2+2*b^2)*ln((b*exp
(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/d^2*f*b/(2*a^2+2*b^2)*d
ilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+2/d^2*f*b/(2*a
^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+2/d^2
*f*c/(2*a^2+2*b^2)*b*ln(1+exp(2*d*x+2*c))-4/d^2*f*c/(2*a^2+2*b^2)*a*arctan(
exp(d*x+c))-2/d^2*f*c*b/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)
```



**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-e \left( \frac{2a \arctan(e^{-dx-c})}{(a^2 + b^2)d} - \frac{b \log(-2ae^{-dx-c} + be^{-2dx-2c} - b)}{(a^2 + b^2)d} + \frac{b \log(e^{-2dx-2c} + 1)}{(a^2 + b^2)d} \right) + 2f \int \frac{2}{(b(e^{dx+c} - e^{-dx-c}) + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -e\*(2\*a\*arctan(e^(-d\*x - c))/((a^2 + b^2)\*d) - b\*log(-2\*a\*e^(-d\*x - c) + b\*e^(-2\*d\*x - 2\*c) - b)/((a^2 + b^2)\*d) + b\*log(e^(-2\*d\*x - 2\*c) + 1)/((a^2 + b^2)\*d)) + 2\*f\*integrate(2\*x/((b\*(e^(d\*x + c) - e^(-d\*x - c)) + 2\*a)\*(e^(d\*x + c) + e^(-d\*x - c))), x)

**Fricas [A]** time = 2.41935, size = 1547, normalized size = 4.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] (b\*f\*dilog((a\*cosh(d\*x + c) + a\*sinh(d\*x + c) + (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + b\*f\*dilog((a\*cosh(d\*x + c) + a\*sinh(d\*x + c) - (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (I\*a\*f - b\*f)\*dilog(I\*cosh(d\*x + c) + I\*sinh(d\*x + c)) + (-I\*a\*f - b\*f)\*dilog(-I\*cosh(d\*x + c) - I\*sinh(d\*x + c)) + (b\*d\*e - b\*c\*f)\*log(2\*b\*cosh(d\*x + c) + 2\*b\*sinh(d\*x + c) + 2\*b\*sqrt((a^2 + b^2)/b^2) + 2\*a) + (b\*d\*e - b\*c\*f)\*log(2\*b\*cosh(d\*x + c) + 2\*b\*sinh(d\*x + c) - 2\*b\*sqrt((a^2 + b^2)/b^2) + 2\*a) + (b\*d\*f\*x + b\*c\*f)\*log(-(a\*cosh(d\*x + c) + a\*sinh(d\*x + c) + (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2) - b)/b) + (b\*d\*f\*x + b\*c\*f)\*log(-(a\*cosh(d\*x + c) + a\*sinh(d\*x + c) - (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2) - b)/b) + (I\*a\*d\*e - b\*d\*e - I\*a\*c\*f + b\*c\*f)\*log(cosh(d\*x + c) + sinh(d\*x + c) + I) + (-I\*a\*d\*e - b\*d\*e + I\*a\*c\*f + b\*c\*f)\*log(cosh(d\*x + c) + sinh(d\*x + c) - I) + (-I\*a\*d\*f\*x - b\*d\*f\*x - I\*a\*c\*f - b\*c\*f)\*log(I\*cosh(d\*x + c) + I\*sinh(d\*x + c) + 1) + (I\*a\*d\*f\*x - b\*d\*f\*x + I\*a\*c\*f - b\*c\*f)\*log(-I\*cosh(d\*x + c) - I\*sinh(d\*x + c) + 1) /((a^2 + b^2)\*d^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \operatorname{sech}(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*sech(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \operatorname{sech}(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)\*sech(d\*x + c)/(b\*sinh(d\*x + c) + a), x)

$$3.307 \quad \int \frac{\operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=69

$$\frac{b \log(a + b \sinh(c + dx))}{d(a^2 + b^2)} + \frac{a \tan^{-1}(\sinh(c + dx))}{d(a^2 + b^2)} - \frac{b \log(\cosh(c + dx))}{d(a^2 + b^2)}$$

[Out] (a\*ArcTan[Sinh[c + d\*x]])/((a^2 + b^2)\*d) - (b\*Log[Cosh[c + d\*x]])/((a^2 + b^2)\*d) + (b\*Log[a + b\*Sinh[c + d\*x]])/((a^2 + b^2)\*d)

**Rubi [A]** time = 0.0698349, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$ , Rules used = {2668, 706, 31, 635, 204, 260}

$$\frac{b \log(a + b \sinh(c + dx))}{d(a^2 + b^2)} + \frac{a \tan^{-1}(\sinh(c + dx))}{d(a^2 + b^2)} - \frac{b \log(\cosh(c + dx))}{d(a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d\*x]/(a + b\*Sinh[c + d\*x]),x]

[Out] (a\*ArcTan[Sinh[c + d\*x]])/((a^2 + b^2)\*d) - (b\*Log[Cosh[c + d\*x]])/((a^2 + b^2)\*d) + (b\*Log[a + b\*Sinh[c + d\*x]])/((a^2 + b^2)\*d)

### Rule 2668

Int[cos[(e\_.) + (f\_.)\*(x\_.)]^(p\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(m\_.), x\_Symbol] :> Dist[1/(b^p\*f), Subst[Int[(a + x)^m\*(b^2 - x^2)^((p - 1)/2), x], x, b\*Sine[e + f\*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

### Rule 706

Int[1/(((d\_.) + (e\_.)\*(x\_.))\*((a\_.) + (c\_.)\*(x\_.)^2)), x\_Symbol] :> Dist[e^2/(c\*d^2 + a\*e^2), Int[1/(d + e\*x), x], x] + Dist[1/(c\*d^2 + a\*e^2), Int[(c\*d - c\*e\*x)/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c\*d^2 + a\*e^2, 0]

### Rule 31

```
Int[((a_) + (b_.)*(x_))^( -1), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^( -1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx &= -\frac{b \operatorname{Subst}\left(\int \frac{1}{(a+x)(-b^2-x^2)} dx, x, b \sinh(c+dx)\right)}{d} \\ &= \frac{b \operatorname{Subst}\left(\int \frac{1}{a+x} dx, x, b \sinh(c+dx)\right)}{(a^2+b^2)d} + \frac{b \operatorname{Subst}\left(\int \frac{-a+x}{-b^2-x^2} dx, x, b \sinh(c+dx)\right)}{(a^2+b^2)d} \\ &= \frac{b \log(a+b \sinh(c+dx))}{(a^2+b^2)d} + \frac{b \operatorname{Subst}\left(\int \frac{x}{-b^2-x^2} dx, x, b \sinh(c+dx)\right)}{(a^2+b^2)d} - \frac{(ab) \operatorname{Subst}\left(\int \frac{1}{-b^2-x^2} dx, x, b \sinh(c+dx)\right)}{(a^2+b^2)d} \\ &= \frac{a \tan^{-1}(\sinh(c+dx))}{(a^2+b^2)d} - \frac{b \log(\cosh(c+dx))}{(a^2+b^2)d} + \frac{b \log(a+b \sinh(c+dx))}{(a^2+b^2)d} \end{aligned}$$

**Mathematica [A]** time = 0.0963634, size = 114, normalized size = 1.65

$$\frac{b \left( \left( \sqrt{-b^2} - a \right) \log \left( \sqrt{-b^2} - b \sinh(c+dx) \right) - 2\sqrt{-b^2} \log(a+b \sinh(c+dx)) + \left( a + \sqrt{-b^2} \right) \log \left( \sqrt{-b^2} + b \sinh(c+dx) \right) \right)}{2\sqrt{-b^2}d(a^2+b^2)}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d\*x]/(a + b\*Sinh[c + d\*x]),x]

[Out]  $-(b*((-a + \sqrt{-b^2})*\text{Log}[\sqrt{-b^2} - b*\text{Sinh}[c + d*x]] - 2*\sqrt{-b^2}*\text{Log}[a + b*\text{Sinh}[c + d*x]] + (a + \sqrt{-b^2})*\text{Log}[\sqrt{-b^2} + b*\text{Sinh}[c + d*x]]))/(2*\sqrt{-b^2}*(a^2 + b^2)*d)$

**Maple [A]** time = 0.004, size = 100, normalized size = 1.5

$$\frac{b}{d(a^2 + b^2)} \ln \left( \left( \tanh \left( \frac{dx}{2} + \frac{c}{2} \right) \right)^2 a - 2 \tanh \left( \frac{1}{2} dx + \frac{c}{2} \right) b - a \right) - \frac{b}{d(a^2 + b^2)} \ln \left( \left( \tanh \left( \frac{dx}{2} + \frac{c}{2} \right) \right)^2 + 1 \right) + 2 \frac{a \arctan \left( \tanh \left( \frac{dx}{2} + \frac{c}{2} \right) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out]  $1/d*b/(a^2+b^2)*\ln(\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)*b-a)-1/d/(a^2+b^2)*b*\ln(\tanh(1/2*d*x+1/2*c)^2+1)+2/d/(a^2+b^2)*a*\arctan(\tanh(1/2*d*x+1/2*c))$

**Maxima [A]** time = 1.6696, size = 128, normalized size = 1.86

$$-\frac{2a \arctan(e^{-dx-c})}{(a^2 + b^2)d} + \frac{b \log(-2ae^{-dx-c} + be^{-2dx-2c} - b)}{(a^2 + b^2)d} - \frac{b \log(e^{-2dx-2c} + 1)}{(a^2 + b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-2*a*\arctan(e^{(-d*x - c)})/((a^2 + b^2)*d) + b*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/((a^2 + b^2)*d) - b*\log(e^{(-2*d*x - 2*c)} + 1)/((a^2 + b^2)*d)$

**Fricas [A]** time = 2.15555, size = 247, normalized size = 3.58

$$\frac{2a \arctan(\cosh(dx + c) + \sinh(dx + c)) + b \log\left(\frac{2(b \sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right) - b \log\left(\frac{2 \cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{(a^2 + b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $(2*a*\arctan(\cosh(d*x + c) + \sinh(d*x + c)) + b*\log(2*(b*\sinh(d*x + c) + a)/(\cosh(d*x + c) - \sinh(d*x + c))) - b*\log(2*\cosh(d*x + c)/(\cosh(d*x + c) - \sinh(d*x + c))))/((a^2 + b^2)*d)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(sech(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

**Giac [A]** time = 1.14886, size = 171, normalized size = 2.48

$$\frac{b^2 \log\left(\left|b(e^{dx+c}) - e^{(-dx-c)}\right| + 2a\right)}{a^2bd + b^3d} + \frac{\left(\pi + 2 \arctan\left(\frac{1}{2}\left(e^{(2dx+2c)} - 1\right)e^{(-dx-c)}\right)\right)a}{2(a^2d + b^2d)} - \frac{b \log\left(\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4\right)}{2(a^2d + b^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out]  $b^2*\log(\operatorname{abs}(b*(e^{(d*x + c)} - e^{(-d*x - c)}) + 2*a))/(a^2*b*d + b^3*d) + 1/2*(\pi + 2*\arctan(1/2*(e^{(2*d*x + 2*c)} - 1)*e^{(-d*x - c)}))*a/(a^2*d + b^2*d) - 1/2*b*\log((e^{(d*x + c)} - e^{(-d*x - c)})^2 + 4)/(a^2*d + b^2*d)$

$$3.308 \quad \int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=28

$$\text{Unintegrable}\left(\frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Sech[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0489457, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Sech[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Sech[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [A]** time = 18.4726, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sech[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Integrate[Sech[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

---

**Maple [A]** time = 0.074, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(dx + c)}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(sech(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(dx + c)}{(fx + e)(b \sinh(dx + c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] integrate(sech(d\*x + c)/((f\*x + e)\*(b\*sinh(d\*x + c) + a)), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{sech}(dx + c)}{afx + ae + (bf x + be) \sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] integral(sech(d\*x + c)/(a\*f\*x + a\*e + (b\*f\*x + b\*e)\*sinh(d\*x + c)), x)

---



**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(c + dx)}{(a + b \sinh(c + dx))(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(sech(c + d\*x)/((a + b\*sinh(c + d\*x))\*(e + f\*x)), x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(dx + c)}{(fx + e)(b \sinh(dx + c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate(sech(d\*x + c)/((f\*x + e)\*(b\*sinh(d\*x + c) + a)), x)

$$3.309 \quad \int \frac{(e+fx)^3 \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=780

$$\frac{6ibf^2(e+fx)\operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{d^3(a^2+b^2)} - \frac{6ibf^2(e+fx)\operatorname{PolyLog}\left(2, ie^{c+dx}\right)}{d^3(a^2+b^2)} - \frac{3af^2(e+fx)\operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{d^3(a^2+b^2)} - \frac{6b^2f^2(e+fx)\operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{d^3(a^2+b^2)}$$

[Out] (a\*(e + f\*x)^3)/((a^2 + b^2)\*d) - (6\*b\*f\*(e + f\*x)^2\*ArcTan[E^(c + d\*x)])/((a^2 + b^2)\*d^2) + (b^2\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d) - (b^2\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d) - (3\*a\*f\*(e + f\*x)^2\*Log[1 + E^(2\*(c + d\*x))])/((a^2 + b^2)\*d^2) + ((6\*I)\*b\*f^2\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)])/((a^2 + b^2)\*d^3) - ((6\*I)\*b\*f^2\*(e + f\*x)\*PolyLog[2, I\*E^(c + d\*x)])/((a^2 + b^2)\*d^3) + (3\*b^2\*f\*(e + f\*x)^2\*PolyLog[2, -(b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d^2) - (3\*b^2\*f\*(e + f\*x)^2\*PolyLog[2, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d^2) - (3\*a\*f^2\*(e + f\*x)\*PolyLog[2, -E^(2\*(c + d\*x))])/((a^2 + b^2)\*d^3) - ((6\*I)\*b\*f^3\*PolyLog[3, (-I)\*E^(c + d\*x)])/((a^2 + b^2)\*d^4) + ((6\*I)\*b\*f^3\*PolyLog[3, I\*E^(c + d\*x)])/((a^2 + b^2)\*d^4) - (6\*b^2\*f^2\*(e + f\*x)\*PolyLog[3, -(b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d^3) + (6\*b^2\*f^2\*(e + f\*x)\*PolyLog[3, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d^3) + (3\*a\*f^3\*PolyLog[3, -E^(2\*(c + d\*x))])/((2\*(a^2 + b^2)\*d^4) + (6\*b^2\*f^3\*PolyLog[4, -(b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d^4) - (6\*b^2\*f^3\*PolyLog[4, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d^4) + (b\*(e + f\*x)^3\*Sech[c + d\*x])/((a^2 + b^2)\*d) + (a\*(e + f\*x)^3\*Tanh[c + d\*x])/((a^2 + b^2)\*d)

**Rubi [A]** time = 1.68822, antiderivative size = 780, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 13, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$ , Rules used = {5573, 3322, 2264, 2190, 2531, 6609, 2282, 6589, 6742, 4184, 3718, 5451, 4180}

$$\frac{6ibf^2(e+fx)\operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{d^3(a^2+b^2)} - \frac{6ibf^2(e+fx)\operatorname{PolyLog}\left(2, ie^{c+dx}\right)}{d^3(a^2+b^2)} - \frac{3af^2(e+fx)\operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{d^3(a^2+b^2)} - \frac{6b^2f^2(e+fx)\operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{d^3(a^2+b^2)}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]), x]

```
[Out] (a*(e + f*x)^3)/((a^2 + b^2)*d) - (6*b*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d^2) + (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (3*a*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d^2) + ((6*I)*b*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) - ((6*I)*b*f^2*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^3) + (3*b^2*f*(e + f*x)^2*PolyLog[2, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d^2) - (3*a*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2)*d^3) - ((6*I)*b*f^3*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^4) + ((6*I)*b*f^3*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)*d^4) - (6*b^2*f^2*(e + f*x)*PolyLog[3, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d^3) + (6*b^2*f^2*(e + f*x)*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d^3) + (3*a*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^4) + (6*b^2*f^3*PolyLog[4, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d^4) - (6*b^2*f^3*PolyLog[4, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d^4) + (b*(e + f*x)^3*Sech[c + d*x])/((a^2 + b^2)*d) + (a*(e + f*x)^3*Tanh[c + d*x])/((a^2 + b^2)*d)
```

### Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

### Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])* (f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*(a_) + (b_)*(x_)))^(n_)]*(f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_) + (f_)*(x_))^(m_)*PolyLog[n_, (d_)*((F_)^((c_)*(a_) + (b_
)*(x_)))^(p_)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 4184

```
Int[csc[(e_) + (f_)*(x_)]^2*((c_) + (d_)*(x_))^(m_), x_Symbol] := -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
```

$t[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{GtQ}[m, 0]$

### Rule 3718

$\text{Int}[\text{((c_.) + (d_.)*(x_))}^{(m_.)} \tan[(e_.) + (\text{Complex}[0, fz\_])*(f_.)*(x\_)], x\_Symbol] \text{ :> } -\text{Simp}[(I*(c + d*x)^{(m + 1)})/(d*(m + 1)), x] + \text{Dist}[2*I, \text{Int}[\text{((c + d*x)}^m * E^{(2*(-(I*e) + f*fz*x))})/(1 + E^{(2*(-(I*e) + f*fz*x))})}, x], x] /; \text{FreeQ}\{c, d, e, f, fz\}, x\} \ \&\& \ \text{IGtQ}[m, 0]$

### Rule 5451

$\text{Int}[\text{((c_.) + (d_.)*(x_))}^{(m_.)} \text{Sech}[(a_.) + (b_.)*(x_)]^{(n_.)} \text{Tanh}[(a_.) + (b_.)*(x_)]^{(p_.)}, x\_Symbol] \text{ :> } -\text{Simp}[\text{((c + d*x)}^m * \text{Sech}[a + b*x]^n)/(b*n), x] + \text{Dist}[(d*m)/(b*n), \text{Int}[(c + d*x)^{(m - 1)} * \text{Sech}[a + b*x]^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x\} \ \&\& \ \text{EqQ}[p, 1] \ \&\& \ \text{GtQ}[m, 0]$

### Rule 4180

$\text{Int}[\text{csc}[(e_.) + \text{Pi}*(k_.) + (\text{Complex}[0, fz\_])*(f_.)*(x_)] * \text{((c_.) + (d_.)*(x_))}^{(m_.)}, x\_Symbol] \text{ :> } \text{Simp}[\text{(-2*(c + d*x)}^m * \text{ArcTanh}[E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}])/(f*fz*I), x] + \text{(-Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{(m - 1)} * \text{Log}[1 - E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}], x], x] + \text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{(m - 1)} * \text{Log}[1 + E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}], x], x]) /; \text{FreeQ}\{c, d, e, f, fz\}, x\} \ \&\& \ \text{IntegerQ}[2*k] \ \&\& \ \text{IGtQ}[m, 0]$

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \operatorname{sech}^2(c+dx)(a-b \sinh(c+dx)) dx}{a^2+b^2} + \frac{b^2 \int \frac{(e+fx)^3}{a+b \sinh(c+dx)} dx}{a^2+b^2} \\
&= \frac{\int (a(e+fx)^3 \operatorname{sech}^2(c+dx) - b(e+fx)^3 \operatorname{sech}(c+dx) \tanh(c+dx)) dx}{a^2+b^2} + \frac{(2b^2) \int \frac{e^{c+dx}}{-b+2ae^{c+dx}} dx}{a^2+b^2} \\
&= \frac{(2b^3) \int \frac{e^{c+dx}(e+fx)^3}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{(a^2+b^2)^{3/2}} - \frac{(2b^3) \int \frac{e^{c+dx}(e+fx)^3}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{(a^2+b^2)^{3/2}} + \frac{a \int (e+fx)^3 \operatorname{sech}^2(c+dx) dx}{a^2+b^2} \\
&= \frac{b^2(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} - \frac{b^2(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} + \frac{b(e+fx)^3 \operatorname{sech}(c+dx)}{(a^2+b^2) d} \\
&= \frac{a(e+fx)^3}{(a^2+b^2) d} - \frac{6bf(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2) d^2} + \frac{b^2(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} - \frac{b^2(e+fx)^3}{(a^2+b^2) d} \\
&= \frac{a(e+fx)^3}{(a^2+b^2) d} - \frac{6bf(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2) d^2} + \frac{b^2(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} - \frac{b^2(e+fx)^3}{(a^2+b^2) d} \\
&= \frac{a(e+fx)^3}{(a^2+b^2) d} - \frac{6bf(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2) d^2} + \frac{b^2(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} - \frac{b^2(e+fx)^3}{(a^2+b^2) d} \\
&= \frac{a(e+fx)^3}{(a^2+b^2) d} - \frac{6bf(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2) d^2} + \frac{b^2(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} - \frac{b^2(e+fx)^3}{(a^2+b^2) d}
\end{aligned}$$

**Mathematica [A]** time = 13.8837, size = 1143, normalized size = 1.47

$$\left(-2e^3 \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) d^3 + f^3 x^3 \log\left(\frac{e^{c+dx} b}{a-\sqrt{a^2+b^2}} + 1\right) d^3 + 3ef^2 x^2 \log\left(\frac{e^{c+dx} b}{a-\sqrt{a^2+b^2}} + 1\right) d^3 + 3e^2 f x \log\left(\frac{e^{c+dx} b}{a-\sqrt{a^2+b^2}} + 1\right) d^3 - f^3\right)$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^3\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

```
[Out] -(f*(-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e*f*x^2 + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] - 6*a*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*d*e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]) + (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*x)]) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))]))/(2*(a^2 + b^2)*d^4*(1 + E^(2*c))) + (b^2*(-2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 6*d*f^3*x*PolyLog[3, (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 6*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*d*f^3*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*f^3*PolyLog[4, (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(a^2 + b^2)^(3/2)*d^4) + (Sech[c]*Sech[c + d*x]*(b*e^3*Cosh[c] + 3*b*e^2*f*x*Cosh[c] + 3*b*e*f^2*x^2*Cosh[c] + b*f^3*x^3*Cosh[c] + a*e^3*Sinh[d*x] + 3*a*e^2*f*x*Sinh[d*x] + 3*a*e*f^2*x^2*Sinh[d*x] + a*f^3*x^3*Sinh[d*x]))/((a^2 + b^2)*d)
```

---

**Maple [F]** time = 0.705, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 (\operatorname{sech}(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [C]** time = 4.21643, size = 14519, normalized size = 18.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(a^3 + a*b^2)*d^3*e^3 - 12*(a^3 + a*b^2)*c*d^2*e^2*f + 12*(a^3 + a*b^2)*c^2*d*e*f^2 - 4*(a^3 + a*b^2)*c^3*f^3 - 4*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*cosh(d*x + c)^2 - 4*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*sinh(d*x + c)^2 - 6*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f + (b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*cosh(d*x + c)^2 + 2*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 6*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f + (b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*cosh(d*x + c)^2 + 2*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3 + (b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*cosh(d*x + c)^2 + 2*(b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d^3*e^3 - 3*b^3*c*d^2*e^2*f + 3*b^3*c^2*d*e*f^2 - b^3*c^3*f^3)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(
```



$$\begin{aligned}
& b^3 d^3 e^3 - 3 b^3 c d^2 e^2 f + 3 b^3 c^2 d e f^2 - b^3 c^3 f^3 + (b^3 d^3 e^3 - 3 b^3 c d^2 e^2 f + 3 b^3 c^2 d e f^2 - b^3 c^3 f^3) \cosh(d x + c)^2 + 2 (b^3 d^3 e^3 - 3 b^3 c d^2 e^2 f + 3 b^3 c^2 d e f^2 - b^3 c^3 f^3) \cosh(d x + c) \sinh(d x + c) + (b^3 d^3 e^3 - 3 b^3 c d^2 e^2 f + 3 b^3 c^2 d e f^2 - b^3 c^3 f^3) \sinh(d x + c)^2 \sqrt{(a^2 + b^2)/b^2} \log(2 b \cosh(d x + c) + 2 b \sinh(d x + c) - 2 b \sqrt{(a^2 + b^2)/b^2} + 2 a) - 2 (b^3 d^3 e^3 - 3 b^3 c d^2 e^2 f + 3 b^3 c^2 d e f^2 - b^3 c^3 f^3) \cosh(d x + c)^2 + 2 (b^3 d^3 e^3 - 3 b^3 c d^2 e^2 f + 3 b^3 c^2 d e f^2 - b^3 c^3 f^3) \cosh(d x + c) \sinh(d x + c) + (b^3 d^3 e^3 - 3 b^3 c d^2 e^2 f + 3 b^3 c^2 d e f^2 - b^3 c^3 f^3) \sinh(d x + c)^2 \sqrt{(a^2 + b^2)/b^2} \log(-a \cosh(d x + c) + a \sinh(d x + c) + (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b) + 2 (b^3 d^3 e^3 - 3 b^3 c d^2 e^2 f + 3 b^3 c^2 d e f^2 - b^3 c^3 f^3) \cosh(d x + c)^2 + 2 (b^3 d^3 e^3 - 3 b^3 c d^2 e^2 f + 3 b^3 c^2 d e f^2 - b^3 c^3 f^3) \cosh(d x + c) \sinh(d x + c) + (b^3 d^3 e^3 - 3 b^3 c d^2 e^2 f + 3 b^3 c^2 d e f^2 - b^3 c^3 f^3) \sinh(d x + c)^2 \sqrt{(a^2 + b^2)/b^2} \log(-a \cosh(d x + c) + a \sinh(d x + c) - (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b) - 12 (b^3 f^3 \cosh(d x + c)^2 + 2 b^3 f^3 \cosh(d x + c) \sinh(d x + c) + b^3 f^3 \sinh(d x + c)^2 + b^3 f^3) \sqrt{(a^2 + b^2)/b^2} \operatorname{polylog}(4, (a \cosh(d x + c) + a \sinh(d x + c) + (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2}))/b) + 12 (b^3 f^3 \cosh(d x + c)^2 + 2 b^3 f^3 \cosh(d x + c) \sinh(d x + c) + b^3 f^3 \sinh(d x + c)^2 + b^3 f^3) \sqrt{(a^2 + b^2)/b^2} \operatorname{polylog}(4, (a \cosh(d x + c) + a \sinh(d x + c) - (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2}))/b) + 12 (b^3 d f^3 x + b^3 d e f^2 + (b^3 d f^3 x + b^3 d e f^2) \cosh(d x + c)^2 + 2 (b^3 d f^3 x + b^3 d e f^2) \cosh(d x + c) \sinh(d x + c) + (b^3 d f^3 x + b^3 d e f^2) \sinh(d x + c)^2) \sqrt{(a^2 + b^2)/b^2} \operatorname{polylog}(3, (a \cosh(d x + c) + a \sinh(d x + c) + (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2}))/b) - 12 (b^3 d f^3 x + b^3 d e f^2 + (b^3 d f^3 x + b^3 d e f^2) \cosh(d x + c)^2 + 2 (b^3 d f^3 x + b^3 d e f^2) \cosh(d x + c) \sinh(d x + c) + (b^3 d f^3 x + b^3 d e f^2) \sinh(d x + c)^2) \sqrt{(a^2 + b^2)/b^2} \operatorname{polylog}(3, (a \cosh(d x + c) + a \sinh(d x + c) - (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2}))/b) - 4 ((a^2 b + b^3) d^3 f^3 x^3 + 3 (a^2 b + b^3) d^3 e f^2 x^2 + 3 (a^2 b + b^3) d^3 e^2 f x + (a^2 b + b^3) d^3 e^3) \cosh(d x + c) + (12 (a^3 + a b^2) d f^3 x + 12 I (a^2 b + b^3) d f^3 x + 12 (a^3 + a b^2) d e f^2 + 12 I (a^2 b + b^3) d e f^2 + (12 (a^3 + a b^2) d f^3 x + 12 I (a^2 b + b^3) d f^3 x + 12 (a^3 + a b^2) d e f^2 + 12 I (a^2 b + b^3) d e f^2) \cosh(d x + c)^2 + (24 (a^3 + a b^2) d f^3 x + 24 I (a^2 b + b^3) d f^3 x + 24 (a^3 + a b^2) d e f^2 + 24 I (a^2 b + b^3) d e f^2) \cosh(d x + c) \sinh(d
\end{aligned}$$

$$\begin{aligned}
& *x + c) + (12*(a^3 + a*b^2)*d*f^3*x + 12*I*(a^2*b + b^3)*d*f^3*x + 12*(a^3 \\
& + a*b^2)*d*e*f^2 + 12*I*(a^2*b + b^3)*d*e*f^2)*\sinh(d*x + c)^2)*\operatorname{dilog}(I*\cos \\
& h(d*x + c) + I*\sinh(d*x + c)) + (12*(a^3 + a*b^2)*d*f^3*x - 12*I*(a^2*b + b^3) \\
& *d*f^3*x + 12*(a^3 + a*b^2)*d*e*f^2 - 12*I*(a^2*b + b^3)*d*e*f^2 + (12*( \\
& a^3 + a*b^2)*d*f^3*x - 12*I*(a^2*b + b^3)*d*f^3*x + 12*(a^3 + a*b^2)*d*e*f^2 \\
& - 12*I*(a^2*b + b^3)*d*e*f^2)*\cosh(d*x + c)^2 + (24*(a^3 + a*b^2)*d*f^3*x \\
& - 24*I*(a^2*b + b^3)*d*f^3*x + 24*(a^3 + a*b^2)*d*e*f^2 - 24*I*(a^2*b + b^3) \\
& *d*e*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (12*(a^3 + a*b^2)*d*f^3*x - 12*I* \\
& (a^2*b + b^3)*d*f^3*x + 12*(a^3 + a*b^2)*d*e*f^2 - 12*I*(a^2*b + b^3)*d*e*f \\
& ^2)*\sinh(d*x + c)^2)*\operatorname{dilog}(-I*\cosh(d*x + c) - I*\sinh(d*x + c)) + (6*(a^3 + \\
& a*b^2)*d^2*e^2*f + 6*I*(a^2*b + b^3)*d^2*e^2*f - 12*(a^3 + a*b^2)*c*d*e*f^2 \\
& - 12*I*(a^2*b + b^3)*c*d*e*f^2 + 6*(a^3 + a*b^2)*c^2*f^3 + 6*I*(a^2*b + b^3) \\
& *c^2*f^3 + (6*(a^3 + a*b^2)*d^2*e^2*f + 6*I*(a^2*b + b^3)*d^2*e^2*f - 12* \\
& (a^3 + a*b^2)*c*d*e*f^2 - 12*I*(a^2*b + b^3)*c*d*e*f^2 + 6*(a^3 + a*b^2)*c^2 \\
& *f^3 + 6*I*(a^2*b + b^3)*c^2*f^3)*\cosh(d*x + c)^2 + (12*(a^3 + a*b^2)*d^2*e \\
& ^2*f + 12*I*(a^2*b + b^3)*d^2*e^2*f - 24*(a^3 + a*b^2)*c*d*e*f^2 - 24*I*(a \\
& ^2*b + b^3)*c*d*e*f^2 + 12*(a^3 + a*b^2)*c^2*f^3 + 12*I*(a^2*b + b^3)*c^2*f \\
& ^3)*\cosh(d*x + c)*\sinh(d*x + c) + (6*(a^3 + a*b^2)*d^2*e^2*f + 6*I*(a^2*b + \\
& b^3)*d^2*e^2*f - 12*(a^3 + a*b^2)*c*d*e*f^2 - 12*I*(a^2*b + b^3)*c*d*e*f^2 \\
& + 6*(a^3 + a*b^2)*c^2*f^3 + 6*I*(a^2*b + b^3)*c^2*f^3)*\sinh(d*x + c)^2)*\log \\
& (\cosh(d*x + c) + \sinh(d*x + c) + I) + (6*(a^3 + a*b^2)*d^2*e^2*f - 6*I*(a^2*b \\
& + b^3)*d^2*e^2*f - 12*(a^3 + a*b^2)*c*d*e*f^2 + 12*I*(a^2*b + b^3)*c*d* \\
& e*f^2 + 6*(a^3 + a*b^2)*c^2*f^3 - 6*I*(a^2*b + b^3)*c^2*f^3 + (6*(a^3 + a*b \\
& ^2)*d^2*e^2*f - 6*I*(a^2*b + b^3)*d^2*e^2*f - 12*(a^3 + a*b^2)*c*d*e*f^2 + \\
& 12*I*(a^2*b + b^3)*c*d*e*f^2 + 6*(a^3 + a*b^2)*c^2*f^3 - 6*I*(a^2*b + b^3)* \\
& c^2*f^3)*\cosh(d*x + c)^2 + (12*(a^3 + a*b^2)*d^2*e^2*f - 12*I*(a^2*b + b^3) \\
& *d^2*e^2*f - 24*(a^3 + a*b^2)*c*d*e*f^2 + 24*I*(a^2*b + b^3)*c*d*e*f^2 + 12 \\
& *(a^3 + a*b^2)*c^2*f^3 - 12*I*(a^2*b + b^3)*c^2*f^3)*\cosh(d*x + c)*\sinh(d*x \\
& + c) + (6*(a^3 + a*b^2)*d^2*e^2*f - 6*I*(a^2*b + b^3)*d^2*e^2*f - 12*(a^3 \\
& + a*b^2)*c*d*e*f^2 + 12*I*(a^2*b + b^3)*c*d*e*f^2 + 6*(a^3 + a*b^2)*c^2*f^3 \\
& - 6*I*(a^2*b + b^3)*c^2*f^3)*\sinh(d*x + c)^2)*\log(\cosh(d*x + c) + \sinh(d*x \\
& + c) - I) + (6*(a^3 + a*b^2)*d^2*f^3*x^2 - 6*I*(a^2*b + b^3)*d^2*f^3*x^2 + \\
& 12*(a^3 + a*b^2)*d^2*e*f^2*x - 12*I*(a^2*b + b^3)*d^2*e*f^2*x + 12*(a^3 + \\
& a*b^2)*c*d*e*f^2 - 12*I*(a^2*b + b^3)*c*d*e*f^2 - 6*(a^3 + a*b^2)*c^2*f^3 + \\
& 6*I*(a^2*b + b^3)*c^2*f^3 + (6*(a^3 + a*b^2)*d^2*f^3*x^2 - 6*I*(a^2*b + b^3) \\
& *d^2*f^3*x^2 + 12*(a^3 + a*b^2)*d^2*e*f^2*x - 12*I*(a^2*b + b^3)*d^2*e*f^2 \\
& *x + 12*(a^3 + a*b^2)*c*d*e*f^2 - 12*I*(a^2*b + b^3)*c*d*e*f^2 - 6*(a^3 + \\
& a*b^2)*c^2*f^3 + 6*I*(a^2*b + b^3)*c^2*f^3)*\cosh(d*x + c)^2 + (12*(a^3 + a* \\
& b^2)*d^2*f^3*x^2 - 12*I*(a^2*b + b^3)*d^2*f^3*x^2 + 24*(a^3 + a*b^2)*d^2*e* \\
& f^2*x - 24*I*(a^2*b + b^3)*d^2*e*f^2*x + 24*(a^3 + a*b^2)*c*d*e*f^2 - 24*I* \\
& (a^2*b + b^3)*c*d*e*f^2 - 12*(a^3 + a*b^2)*c^2*f^3 + 12*I*(a^2*b + b^3)*c^2 \\
& *f^3)*\cosh(d*x + c)*\sinh(d*x + c) + (6*(a^3 + a*b^2)*d^2*f^3*x^2 - 6*I*(a^2 \\
& *b + b^3)*d^2*f^3*x^2 + 12*(a^3 + a*b^2)*d^2*e*f^2*x - 12*I*(a^2*b + b^3)*d \\
& ^2*e*f^2*x + 12*(a^3 + a*b^2)*c*d*e*f^2 - 12*I*(a^2*b + b^3)*c*d*e*f^2 - 6* \\
& (a^3 + a*b^2)*c^2*f^3 + 6*I*(a^2*b + b^3)*c^2*f^3)*\sinh(d*x + c)^2)*\log(I*c
\end{aligned}$$

```

osh(d*x + c) + I*sinh(d*x + c) + 1) + (6*(a^3 + a*b^2)*d^2*f^3*x^2 + 6*I*(a
^2*b + b^3)*d^2*f^3*x^2 + 12*(a^3 + a*b^2)*d^2*e*f^2*x + 12*I*(a^2*b + b^3)
*d^2*e*f^2*x + 12*(a^3 + a*b^2)*c*d*e*f^2 + 12*I*(a^2*b + b^3)*c*d*e*f^2 -
6*(a^3 + a*b^2)*c^2*f^3 - 6*I*(a^2*b + b^3)*c^2*f^3 + (6*(a^3 + a*b^2)*d^2*
f^3*x^2 + 6*I*(a^2*b + b^3)*d^2*f^3*x^2 + 12*(a^3 + a*b^2)*d^2*e*f^2*x + 12
*I*(a^2*b + b^3)*d^2*e*f^2*x + 12*(a^3 + a*b^2)*c*d*e*f^2 + 12*I*(a^2*b + b
^3)*c*d*e*f^2 - 6*(a^3 + a*b^2)*c^2*f^3 - 6*I*(a^2*b + b^3)*c^2*f^3)*cosh(d
*x + c)^2 + (12*(a^3 + a*b^2)*d^2*f^3*x^2 + 12*I*(a^2*b + b^3)*d^2*f^3*x^2
+ 24*(a^3 + a*b^2)*d^2*e*f^2*x + 24*I*(a^2*b + b^3)*d^2*e*f^2*x + 24*(a^3 +
a*b^2)*c*d*e*f^2 + 24*I*(a^2*b + b^3)*c*d*e*f^2 - 12*(a^3 + a*b^2)*c^2*f^3
- 12*I*(a^2*b + b^3)*c^2*f^3)*cosh(d*x + c)*sinh(d*x + c) + (6*(a^3 + a*b^
2)*d^2*f^3*x^2 + 6*I*(a^2*b + b^3)*d^2*f^3*x^2 + 12*(a^3 + a*b^2)*d^2*e*f^2
*x + 12*I*(a^2*b + b^3)*d^2*e*f^2*x + 12*(a^3 + a*b^2)*c*d*e*f^2 + 12*I*(a^
2*b + b^3)*c*d*e*f^2 - 6*(a^3 + a*b^2)*c^2*f^3 - 6*I*(a^2*b + b^3)*c^2*f^3)
*sinh(d*x + c)^2)*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) - 12*((a^3 +
a*b^2)*f^3 + I*(a^2*b + b^3)*f^3 + ((a^3 + a*b^2)*f^3 + I*(a^2*b + b^3)*f^3
)*cosh(d*x + c)^2 + 2*((a^3 + a*b^2)*f^3 + I*(a^2*b + b^3)*f^3)*cosh(d*x +
c)*sinh(d*x + c) + ((a^3 + a*b^2)*f^3 + I*(a^2*b + b^3)*f^3)*sinh(d*x + c)^
2)*polylog(3, I*cosh(d*x + c) + I*sinh(d*x + c)) - (12*(a^3 + a*b^2)*f^3 -
12*I*(a^2*b + b^3)*f^3 + 12*((a^3 + a*b^2)*f^3 - I*(a^2*b + b^3)*f^3)*cosh(
d*x + c)^2 + 24*((a^3 + a*b^2)*f^3 - I*(a^2*b + b^3)*f^3)*cosh(d*x + c)*sin
h(d*x + c) + 12*((a^3 + a*b^2)*f^3 - I*(a^2*b + b^3)*f^3)*sinh(d*x + c)^2)*
polylog(3, -I*cosh(d*x + c) - I*sinh(d*x + c)) - 4*((a^2*b + b^3)*d^3*f^3*x
^3 + 3*(a^2*b + b^3)*d^3*e*f^2*x^2 + 3*(a^2*b + b^3)*d^3*e^2*f*x + (a^2*b +
b^3)*d^3*e^3 + 2*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^
2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*
b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*cosh(d*x + c))*sinh(d*x + c))/((a
^4 + 2*a^2*b^2 + b^4)*d^4*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^4*c
osh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*d^4*sinh(d*x + c)^2 +
(a^4 + 2*a^2*b^2 + b^4)*d^4)

```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*sech(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.310 \quad \int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=548

$$\frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2 (a^2+b^2)^{3/2}} - \frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2 (a^2+b^2)^{3/2}} - \frac{2b^2 f^2 \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^3 (a^2+b^2)^{3/2}} + \frac{2b^2 f^2 \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^3 (a^2+b^2)^{3/2}}$$

[Out]  $(a*(e+f*x)^2)/((a^2+b^2)*d) - (4*b*f*(e+f*x)*\operatorname{ArcTan}[E^{(c+d*x)}])/((a^2+b^2)*d^2) + (b^2*(e+f*x)^2*\operatorname{Log}[1+(b*E^{(c+d*x)})/(a-\operatorname{Sqrt}[a^2+b^2])])/((a^2+b^2)^{(3/2)*d} - (b^2*(e+f*x)^2*\operatorname{Log}[1+(b*E^{(c+d*x)})/(a+\operatorname{Sqrt}[a^2+b^2])])/((a^2+b^2)^{(3/2)*d} - (2*a*f*(e+f*x)*\operatorname{Log}[1+E^{(2*(c+d*x))}])/((a^2+b^2)*d^2) + ((2*I)*b*f^2*\operatorname{PolyLog}[2, (-I)*E^{(c+d*x)}])/((a^2+b^2)*d^3) - ((2*I)*b*f^2*\operatorname{PolyLog}[2, I*E^{(c+d*x)}])/((a^2+b^2)*d^3) + (2*b^2*f*(e+f*x)*\operatorname{PolyLog}[2, -((b*E^{(c+d*x)})/(a-\operatorname{Sqrt}[a^2+b^2])])/((a^2+b^2)^{(3/2)*d^2} - (2*b^2*f*(e+f*x)*\operatorname{PolyLog}[2, -((b*E^{(c+d*x)})/(a+\operatorname{Sqrt}[a^2+b^2])])])/((a^2+b^2)^{(3/2)*d^2} - (a*f^2*\operatorname{PolyLog}[2, -E^{(2*(c+d*x))}])/((a^2+b^2)*d^3) - (2*b^2*f^2*\operatorname{PolyLog}[3, -((b*E^{(c+d*x)})/(a-\operatorname{Sqrt}[a^2+b^2])])])/((a^2+b^2)^{(3/2)*d^3} + (2*b^2*f^2*\operatorname{PolyLog}[3, -((b*E^{(c+d*x)})/(a+\operatorname{Sqrt}[a^2+b^2])])])/((a^2+b^2)^{(3/2)*d^3} + (b*(e+f*x)^2*\operatorname{Sech}[c+d*x])/((a^2+b^2)*d) + (a*(e+f*x)^2*\operatorname{Tanh}[c+d*x])/((a^2+b^2)*d)$

**Rubi [A]** time = 1.29548, antiderivative size = 548, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 14, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5573, 3322, 2264, 2190, 2531, 2282, 6589, 6742, 4184, 3718, 2279, 2391, 5451, 4180}

$$\frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2 (a^2+b^2)^{3/2}} - \frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2 (a^2+b^2)^{3/2}} - \frac{2b^2 f^2 \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^3 (a^2+b^2)^{3/2}} + \frac{2b^2 f^2 \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^3 (a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e+f*x)^2*\operatorname{Sech}[c+d*x]^2/(a+b*\operatorname{Sinh}[c+d*x]),x]$

[Out]  $(a*(e+f*x)^2)/((a^2+b^2)*d) - (4*b*f*(e+f*x)*\operatorname{ArcTan}[E^{(c+d*x)}])/((a^2+b^2)*d^2) + (b^2*(e+f*x)^2*\operatorname{Log}[1+(b*E^{(c+d*x)})/(a-\operatorname{Sqrt}[a^2+b^2])])/((a^2+b^2)^{(3/2)*d} - (b^2*(e+f*x)^2*\operatorname{Log}[1+(b*E^{(c+d*x)})/(a+\operatorname{Sqrt}[a^2+b^2])])/((a^2+b^2)^{(3/2)*d} - (2*a*f*(e+f*x)*\operatorname{Log}[1+E^{(2*(c+d*x))}])/((a^2+b^2)*d^2) + ((2*I)*b*f^2*\operatorname{PolyLog}[2, (-I)*E^{(c+d*x)}])/((a^2+b^2)*d^3) - ((2*I)*b*f^2*\operatorname{PolyLog}[2, I*E^{(c+d*x)}])/((a^2+b^2)*d^3) + (2*b^2*f*(e+f*x)*\operatorname{PolyLog}[2, -((b*E^{(c+d*x)})/(a-\operatorname{Sqrt}[a^2+b^2])])/((a^2+b^2)^{(3/2)*d^2} - (2*b^2*f*(e+f*x)*\operatorname{PolyLog}[2, -((b*E^{(c+d*x)})/(a+\operatorname{Sqrt}[a^2+b^2])])])/((a^2+b^2)^{(3/2)*d^2} - (a*f^2*\operatorname{PolyLog}[2, -E^{(2*(c+d*x))}])/((a^2+b^2)*d^3) - (2*b^2*f^2*\operatorname{PolyLog}[3, -((b*E^{(c+d*x)})/(a-\operatorname{Sqrt}[a^2+b^2])])])/((a^2+b^2)^{(3/2)*d^3} + (2*b^2*f^2*\operatorname{PolyLog}[3, -((b*E^{(c+d*x)})/(a+\operatorname{Sqrt}[a^2+b^2])])])/((a^2+b^2)^{(3/2)*d^3} + (b*(e+f*x)^2*\operatorname{Sech}[c+d*x])/((a^2+b^2)*d) + (a*(e+f*x)^2*\operatorname{Tanh}[c+d*x])/((a^2+b^2)*d)$

$$\begin{aligned} & /((a^2 + b^2)*d^3) - ((2*I)*b*f^2*PolyLog[2, I*E^(c + d*x)]/((a^2 + b^2)*d \\ & ^3) + (2*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]) \\ & )])/((a^2 + b^2)^(3/2)*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x) \\ & ))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d^2) - (a*f^2*PolyLog[2, -E^ \\ & (2*(c + d*x))]/((a^2 + b^2)*d^3) - (2*b^2*f^2*PolyLog[3, -((b*E^(c + d*x)) \\ & )/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d^3) + (2*b^2*f^2*PolyLog[3, - \\ & ((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d^3) + (b*(e + \\ & f*x)^2*Sech[c + d*x])/((a^2 + b^2)*d) + (a*(e + f*x)^2*Tanh[c + d*x])/((a^ \\ & 2 + b^2)*d) \end{aligned}$$

### Rule 5573

$$\begin{aligned} & \text{Int}[((e_.) + (f_.)*(x_.))^(m_.)*\text{Sech}[(c_.) + (d_.)*(x_.)]^(n_.)/((a_.) + (b_. \\ & )*\text{Sinh}[(c_.) + (d_.)*(x_.)]), x\_Symbol] \text{ :> } \text{Dist}[b^2/(a^2 + b^2), \text{Int}[((e + \\ & f*x)^m*\text{Sech}[c + d*x]^(n - 2))/(a + b*\text{Sinh}[c + d*x]), x], x] + \text{Dist}[1/(a^2 + \\ & b^2), \text{Int}[(e + f*x)^m*\text{Sech}[c + d*x]^n*(a - b*\text{Sinh}[c + d*x]), x], x] /; \text{Fre} \\ & \text{eQ}\{a, b, c, d, e, f, x\} \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{NeQ}[a^2 + b^2, 0] \ \&\& \ \text{IGtQ}[n, 0] \end{aligned}$$

### Rule 3322

$$\begin{aligned} & \text{Int}[((c_.) + (d_.)*(x_.))^(m_.)/((a_.) + (b_.)*\sin[(e_.) + (\text{Complex}[0, fz\_]) * \\ & (f_.)*(x_.)]), x\_Symbol] \text{ :> } \text{Dist}[2, \text{Int}[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(- \\ & (I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; \text{F} \\ & \text{reeQ}\{a, b, c, d, e, f, fz, x\} \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{IGtQ}[m, 0] \end{aligned}$$

### Rule 2264

$$\begin{aligned} & \text{Int}[((F_)^(u_)*((f_.) + (g_.)*(x_.))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.) \\ & *(F_)^(v_)), x\_Symbol] \text{ :> } \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c)/q, \text{Int}[ \\ & ((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - \text{Dist}[(2*c)/q, \text{Int}[((f + g*x)^ \\ & m*F^u)/(b + q + 2*c*F^u), x], x] /; \text{FreeQ}\{F, a, b, c, f, g, x\} \ \&\& \ \text{EqQ}[v, \\ & 2*u] \ \&\& \ \text{LinearQ}[u, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IGtQ}[m, 0] \end{aligned}$$

### Rule 2190

$$\begin{aligned} & \text{Int}[(((F_)^(g_.)*((e_.) + (f_.)*(x_.)))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.))/ \\ & ((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_.)))^(n_.)), x\_Symbol] \text{ :> } \text{Simp} \\ & [((c + d*x)^m*\text{Log}[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*\text{Log}[F]), x] - \text{Di} \\ & \text{st}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^(m - 1)*\text{Log}[1 + (b*(F^(g*(e + f*x) \\ & ))^n)/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n, x\} \ \&\& \ \text{IGtQ}[m, 0] \end{aligned}$$

### Rule 2531

$$\begin{aligned} & \text{Int}[\text{Log}[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_.)))]^(n_.)*((f_.) + (g_.) \\ & *(x_.))^(m_.), x\_Symbol] \text{ :> } -\text{Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F^(c*(a + b*x \end{aligned}$$

```

)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]

```

### Rule 2282

```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

### Rule 6589

```

Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]

```

### Rule 6742

```

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]

```

### Rule 4184

```

Int[csc[(e_) + (f_)*(x_)]^2*((c_) + (d_)*(x_))^(m_), x_Symbol] := -Simp
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

```

### Rule 3718

```

Int[((c_) + (d_)*(x_))^(m_)*tan[(e_) + (Complex[0, fz_])*(f_)*(x_)], x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

```

### Rule 2279

```

Int[Log[(a_) + (b_)*(F_)^((e_)*((c_) + (d_)*(x_)))]^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 5451

```
Int[((c_.) + (d_.)*(x_)^(m_.))*Sech[(a_.) + (b_.)*(x_)^(n_.)*Tanh[(a_.) + (b_.)*(x_)^(p_.)], x_Symbol] := -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n), x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_)^(m_.)), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \operatorname{sech}^2(c+dx)(a-b \sinh(c+dx)) dx}{a^2+b^2} + \frac{b^2 \int \frac{(e+fx)^2}{a+b \sinh(c+dx)} dx}{a^2+b^2} \\
&= \frac{\int (a(e+fx)^2 \operatorname{sech}^2(c+dx) - b(e+fx)^2 \operatorname{sech}(c+dx) \tanh(c+dx)) dx}{a^2+b^2} + \frac{(2b^2) \int \frac{e^{-c-dx}}{-b+2a \sinh(c+dx)} dx}{a^2+b^2} \\
&= \frac{(2b^3) \int \frac{e^{c+dx}(e+fx)^2}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{(a^2+b^2)^{3/2}} - \frac{(2b^3) \int \frac{e^{c+dx}(e+fx)^2}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{(a^2+b^2)^{3/2}} + \frac{a \int (e+fx)^2 \operatorname{sech}^2(c+dx) dx}{a^2+b^2} \\
&= \frac{b^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} - \frac{b^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} + \frac{b(e+fx)^2 \operatorname{sech}(c+dx)}{(a^2+b^2) d} \\
&= \frac{a(e+fx)^2}{(a^2+b^2) d} - \frac{4bf(e+fx) \tan^{-1}(e^{c+dx})}{(a^2+b^2) d^2} + \frac{b^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} - \frac{b^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} \\
&= \frac{a(e+fx)^2}{(a^2+b^2) d} - \frac{4bf(e+fx) \tan^{-1}(e^{c+dx})}{(a^2+b^2) d^2} + \frac{b^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} - \frac{b^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} \\
&= \frac{a(e+fx)^2}{(a^2+b^2) d} - \frac{4bf(e+fx) \tan^{-1}(e^{c+dx})}{(a^2+b^2) d^2} + \frac{b^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} - \frac{b^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d}
\end{aligned}$$

**Mathematica [A]** time = 8.35974, size = 905, normalized size = 1.65

$$\frac{\left(-2e^2 \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) d^2 + f^2 x^2 \log\left(\frac{e^{c+dx} b}{a-\sqrt{a^2+b^2}} + 1\right) d^2 + 2efx \log\left(\frac{e^{c+dx} b}{a-\sqrt{a^2+b^2}} + 1\right) d^2 - f^2 x^2 \log\left(\frac{e^{c+dx} b}{a+\sqrt{a^2+b^2}} + 1\right) d^2 - 2efx \log\left(\frac{e^{c+dx} b}{a+\sqrt{a^2+b^2}} + 1\right) d^2\right)}{(a^2+b^2)^{3/2} d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]), x]

```
[Out] (b^2*(-2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x
*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]) + d^2*f^2*x^2*Log[1 + (b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2]]) - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]]) - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]]) + 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]])
- 2*d*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) - 2*
f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]]) + 2*f^2*PolyLog[3, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 + b^2)^(3/2)*d^3) - (2*a*e
*f*Sech[c]*(Cosh[c]*Log[Cosh[c]*Cosh[d*x] + Sinh[c]*Sinh[d*x]] - d*x*Sinh[c
]))/((a^2 + b^2)*d^2*(Cosh[c]^2 - Sinh[c]^2)) - (4*b*e*f*ArcTan[(Sinh[c] +
Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]]/((a^2 + b^2)*d^2*Sqrt[
Cosh[c]^2 - Sinh[c]^2]) + (a*f^2*Csch[c]*(-(d^2*x^2)/E^ArcTanh[Coth[c]]) +
(I*Coth[c]*(-(d*x*(-Pi + (2*I)*ArcTanh[Coth[c]])) - Pi*Log[1 + E^(2*d*x)]
- 2*(I*d*x + I*ArcTanh[Coth[c]])*Log[1 - E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]
]]))]) + Pi*Log[Cosh[d*x]] + (2*I)*ArcTanh[Coth[c]]*Log[I*Sinh[d*x] + ArcTanh
[Coth[c]]]) + I*PolyLog[2, E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]]))])/Sqrt[1
- Coth[c]^2])*Sech[c])/((a^2 + b^2)*d^3*Sqrt[Csch[c]^2*(-Cosh[c]^2 + Sinh[
c]^2)]) - (2*b*f^2*(((I)*Csch[c]*(I*(d*x + ArcTanh[Coth[c]]))*(Log[1 - E^(-
(d*x) - ArcTanh[Coth[c]])] - Log[1 + E^(-(d*x) - ArcTanh[Coth[c]])]) + I*(P
olyLog[2, -E^(-(d*x) - ArcTanh[Coth[c]])] - PolyLog[2, E^(-(d*x) - ArcTanh[
Coth[c]])])))/Sqrt[1 - Coth[c]^2] - (2*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)
/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]]*ArcTanh[Coth[c]]/Sqrt[Cosh[c]^2 - Sinh[c
]^2]))/((a^2 + b^2)*d^3) + (Sech[c]*Sech[c + d*x]*(b*e^2*Cosh[c] + 2*b*e*f*
x*Cosh[c] + b*f^2*x^2*Cosh[c] + a*e^2*Sinh[d*x] + 2*a*e*f*x*Sinh[d*x] + a*f
^2*x^2*Sinh[d*x]))/((a^2 + b^2)*d)
```

**Maple [F]** time = 0.539, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\operatorname{sech}(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [C]** time = 3.29049, size = 8370, normalized size = 15.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(a^3 + a*b^2)*d^2*e^2 - 8*(a^3 + a*b^2)*c*d*e*f + 4*(a^3 + a*b^2)*c^2*f^2 - 4*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*cosh(d*x + c)^2 - 4*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*sinh(d*x + c)^2 - 4*(b^3*d*f^2*x + b^3*d*e*f + (b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)^2 + 2*(b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d*f^2*x + b^3*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 4*(b^3*d*f^2*x + b^3*d*e*f + (b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)^2 + 2*(b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d*f^2*x + b^3*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2 + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)^2 + 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2 + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)^2 + 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2 + (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c)^2 + 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 4*(b^3*d*f^2*x + b^3*d*e*f + (b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)^2 + 2*(b^3*d*f^2*x + b^3*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d*f^2*x + b^3*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2 + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)^2 + 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2 + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)^2 + 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a)
```

$$\begin{aligned}
& 2)/b^2) - b)/b) + 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2 + (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*\cosh(d*x + c)^2 + 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*\sinh(d*x + c)^2)*\sqrt{((a^2 + b^2)/b^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c)))*\sqrt{((a^2 + b^2)/b^2) - b)/b) + 4*(b^3*f^2*\cosh(d*x + c)^2 + 2*b^3*f^2*\cosh(d*x + c)*\sinh(d*x + c) + b^3*f^2*\sinh(d*x + c)^2 + b^3*f^2)*\sqrt{((a^2 + b^2)/b^2)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c)))*\sqrt{((a^2 + b^2)/b^2)))/b) - 4*(b^3*f^2*\cosh(d*x + c)^2 + 2*b^3*f^2*\cosh(d*x + c)*\sinh(d*x + c) + b^3*f^2*\sinh(d*x + c)^2 + b^3*f^2)*\sqrt{((a^2 + b^2)/b^2)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c)))*\sqrt{((a^2 + b^2)/b^2)))/b) - 4*((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*d^2*e*f*x + (a^2*b + b^3)*d^2*e^2)*\cosh(d*x + c) + 4*((a^3 + a*b^2)*f^2 + I*(a^2*b + b^3)*f^2 + ((a^3 + a*b^2)*f^2 + I*(a^2*b + b^3)*f^2)*\cosh(d*x + c)^2 + 2*((a^3 + a*b^2)*f^2 + I*(a^2*b + b^3)*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + ((a^3 + a*b^2)*f^2 + I*(a^2*b + b^3)*f^2)*\sinh(d*x + c)^2)*\text{dilog}(I*\cosh(d*x + c) + I*\sinh(d*x + c)) + (4*(a^3 + a*b^2)*f^2 - 4*I*(a^2*b + b^3)*f^2 + 4*((a^3 + a*b^2)*f^2 - I*(a^2*b + b^3)*f^2)*\cosh(d*x + c)^2 + 8*((a^3 + a*b^2)*f^2 - I*(a^2*b + b^3)*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + 4*((a^3 + a*b^2)*f^2 - I*(a^2*b + b^3)*f^2)*\sinh(d*x + c)^2)*\text{dilog}(-I*\cosh(d*x + c) - I*\sinh(d*x + c)) + (4*(a^3 + a*b^2)*d*e*f + 4*I*(a^2*b + b^3)*d*e*f - 4*(a^3 + a*b^2)*c*f^2 - 4*I*(a^2*b + b^3)*c*f^2 + (4*(a^3 + a*b^2)*d*e*f + 4*I*(a^2*b + b^3)*d*e*f - 4*(a^3 + a*b^2)*c*f^2 - 4*I*(a^2*b + b^3)*c*f^2)*\cosh(d*x + c)^2 + (8*(a^3 + a*b^2)*d*e*f + 8*I*(a^2*b + b^3)*d*e*f - 8*(a^3 + a*b^2)*c*f^2 - 8*I*(a^2*b + b^3)*c*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (4*(a^3 + a*b^2)*d*e*f + 4*I*(a^2*b + b^3)*d*e*f - 4*(a^3 + a*b^2)*c*f^2 - 4*I*(a^2*b + b^3)*c*f^2)*\sinh(d*x + c)^2)*\log(\cosh(d*x + c) + \sinh(d*x + c) + I) + (4*(a^3 + a*b^2)*d*e*f - 4*I*(a^2*b + b^3)*d*e*f - 4*(a^3 + a*b^2)*c*f^2 + 4*I*(a^2*b + b^3)*c*f^2 + (4*(a^3 + a*b^2)*d*e*f - 4*I*(a^2*b + b^3)*d*e*f - 4*(a^3 + a*b^2)*c*f^2 + 4*I*(a^2*b + b^3)*c*f^2)*\cosh(d*x + c)^2 + (8*(a^3 + a*b^2)*d*e*f - 8*I*(a^2*b + b^3)*d*e*f - 8*(a^3 + a*b^2)*c*f^2 + 8*I*(a^2*b + b^3)*c*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (4*(a^3 + a*b^2)*d*e*f - 4*I*(a^2*b + b^3)*d*e*f - 4*(a^3 + a*b^2)*c*f^2 + 4*I*(a^2*b + b^3)*c*f^2)*\sinh(d*x + c)^2)*\log(\cosh(d*x + c) + \sinh(d*x + c) - I) + (4*(a^3 + a*b^2)*d*f^2*x - 4*I*(a^2*b + b^3)*d*f^2*x + 4*(a^3 + a*b^2)*c*f^2 - 4*I*(a^2*b + b^3)*c*f^2 + (4*(a^3 + a*b^2)*d*f^2*x - 4*I*(a^2*b + b^3)*d*f^2*x + 4*(a^3 + a*b^2)*c*f^2 - 4*I*(a^2*b + b^3)*c*f^2)*\cosh(d*x + c)^2 + (8*(a^3 + a*b^2)*d*f^2*x - 8*I*(a^2*b + b^3)*d*f^2*x + 8*(a^3 + a*b^2)*c*f^2 - 8*I*(a^2*b + b^3)*c*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (4*(a^3 + a*b^2)*d*f^2*x - 4*I*(a^2*b + b^3)*d*f^2*x + 4*(a^3 + a*b^2)*c*f^2 - 4*I*(a^2*b + b^3)*c*f^2)*\sinh(d*x + c)^2)*\log(I*\cosh(d*x + c) + I*\sinh(d*x + c) + 1) + (4*(a^3 + a*b^2)*d*f^2*x + 4*I*(a^2*b + b^3)*d*f^2*x + 4*(a^3 + a*b^2)*c*f^2 + 4*I*(a^2*b + b^3)*c*f^2 + (4*(a^3 + a*b^2)*d*f^2*x + 4*I*(a^2*b + b^3)*d*f^2*x + 4*(a^3 + a*b^2)*c*f^2 + 4*I*(a^2*b + b^3)*c*f^2)
\end{aligned}$$

)\*cosh(d\*x + c)^2 + (8\*(a^3 + a\*b^2)\*d\*f^2\*x + 8\*I\*(a^2\*b + b^3)\*d\*f^2\*x + 8\*(a^3 + a\*b^2)\*c\*f^2 + 8\*I\*(a^2\*b + b^3)\*c\*f^2)\*cosh(d\*x + c)\*sinh(d\*x + c) + (4\*(a^3 + a\*b^2)\*d\*f^2\*x + 4\*I\*(a^2\*b + b^3)\*d\*f^2\*x + 4\*(a^3 + a\*b^2)\*c\*f^2 + 4\*I\*(a^2\*b + b^3)\*c\*f^2)\*sinh(d\*x + c)^2\*log(-I\*cosh(d\*x + c) - I\*sinh(d\*x + c) + 1) - 4\*((a^2\*b + b^3)\*d^2\*f^2\*x^2 + 2\*(a^2\*b + b^3)\*d^2\*e\*f\*x + (a^2\*b + b^3)\*d^2\*e^2 + 2\*((a^3 + a\*b^2)\*d^2\*f^2\*x^2 + 2\*(a^3 + a\*b^2)\*d^2\*e\*f\*x + 2\*(a^3 + a\*b^2)\*c\*d\*e\*f - (a^3 + a\*b^2)\*c^2\*f^2)\*cosh(d\*x + c)\*sinh(d\*x + c))/((a^4 + 2\*a^2\*b^2 + b^4)\*d^3\*cosh(d\*x + c)^2 + 2\*(a^4 + 2\*a^2\*b^2 + b^4)\*d^3\*cosh(d\*x + c)\*sinh(d\*x + c) + (a^4 + 2\*a^2\*b^2 + b^4)\*d^3\*sinh(d\*x + c)^2 + (a^4 + 2\*a^2\*b^2 + b^4)\*d^3)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^2 \operatorname{sech}^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*sech(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*\*2\*sech(c + d\*x)\*\*2/(a + b\*sinh(c + d\*x)), x)

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.311 \quad \int \frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=295

$$\frac{b^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2 (a^2+b^2)^{3/2}} - \frac{b^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2+a}}\right)}{d^2 (a^2+b^2)^{3/2}} - \frac{bf \tan^{-1}(\sinh(c+dx))}{d^2 (a^2+b^2)} - \frac{af \log(\cosh(c+dx))}{d^2 (a^2+b^2)} + \frac{b^2(e+fx)}{a}$$

[Out] -((b\*f\*ArcTan[Sinh[c + d\*x]])/((a^2 + b^2)\*d^2)) + (b^2\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d) - (b^2\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d) - (a\*f\*Log[Cosh[c + d\*x]])/((a^2 + b^2)\*d^2) + (b^2\*f\*PolyLog[2, -(b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d^2) - (b^2\*f\*PolyLog[2, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d^2) + (b\*(e + f\*x)\*Sech[c + d\*x])/((a^2 + b^2)\*d) + (a\*(e + f\*x)\*Tanh[c + d\*x])/((a^2 + b^2)\*d)

**Rubi [A]** time = 0.71703, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$ , Rules used = {5573, 3322, 2264, 2190, 2279, 2391, 6742, 4184, 3475, 5451, 3770}

$$\frac{b^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2 (a^2+b^2)^{3/2}} - \frac{b^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2+a}}\right)}{d^2 (a^2+b^2)^{3/2}} - \frac{bf \tan^{-1}(\sinh(c+dx))}{d^2 (a^2+b^2)} - \frac{af \log(\cosh(c+dx))}{d^2 (a^2+b^2)} + \frac{b^2(e+fx)}{a}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] -((b\*f\*ArcTan[Sinh[c + d\*x]])/((a^2 + b^2)\*d^2)) + (b^2\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d) - (b^2\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d) - (a\*f\*Log[Cosh[c + d\*x]])/((a^2 + b^2)\*d^2) + (b^2\*f\*PolyLog[2, -(b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d^2) - (b^2\*f\*PolyLog[2, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d^2) + (b\*(e + f\*x)\*Sech[c + d\*x])/((a^2 + b^2)\*d) + (a\*(e + f\*x)\*Tanh[c + d\*x])/((a^2 + b^2)\*d)

Rule 5573

```
Int[(((e_.) + (f_.)*(x_.))^(m_.)*Sech[(c_.) + (d_.)*(x_.)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

### Rule 3322

```
Int[(((c_.) + (d_.)*(x_.))^(m_.))/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])* (f_.)*(x_.)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_.))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_.)))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.))/((a_) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_.)))^(n_.)), x_Symbol] := Simp[(c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_.)))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_.)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[
((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] := -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps



$$\begin{aligned}
\int \frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx)) dx}{a^2+b^2} + \frac{b^2 \int \frac{e+fx}{a+b\sinh(c+dx)} dx}{a^2+b^2} \\
&= \frac{\int (a(e+fx)\operatorname{sech}^2(c+dx) - b(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)) dx}{a^2+b^2} + \frac{(2b^2) \int \frac{e+dx}{-b+2ae^{c+dx}}}{a^2+b^2} \\
&= \frac{(2b^3) \int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{(a^2+b^2)^{3/2}} - \frac{(2b^3) \int \frac{e^{c+dx}(e+fx)}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{(a^2+b^2)^{3/2}} + \frac{a \int (e+fx)\operatorname{sech}^2(c+dx)}{a^2+b^2} \\
&= \frac{b^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} - \frac{b^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} + \frac{b(e+fx)\operatorname{sech}(c+dx)}{(a^2+b^2) d} \\
&= -\frac{bf \tan^{-1}(\sinh(c+dx))}{(a^2+b^2) d^2} + \frac{b^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} - \frac{b^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} \\
&= -\frac{bf \tan^{-1}(\sinh(c+dx))}{(a^2+b^2) d^2} + \frac{b^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} - \frac{b^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d}
\end{aligned}$$

**Mathematica [A]** time = 3.16199, size = 284, normalized size = 0.96

$$\frac{b^2 \left( f \operatorname{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) - f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) - 2de \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) + f(c+dx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right) - f(c+dx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right) + 2cf \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) \right)}{(a^2+b^2)^{3/2} d^2}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] ((-2\*b\*f\*ArcTan[Tanh[(c + d\*x)/2]])/(a^2 + b^2) - (a\*f\*Log[Cosh[c + d\*x]])/(a^2 + b^2) + (b^2\*(-2\*d\*e\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]] + 2\*c\*f\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]] + f\*(c + d\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]) - f\*(c + d\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]) + f\*PolyLog[2, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] - f\*PolyLog[2, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])]))/(a^2 + b^2)^(3/2) + (d\*(e + f\*x)\*Sech[c + d\*x]\*(b + a\*Sinh[c + d\*x]))/(a^2 + b^2))/d^2

**Maple [B]** time = 0.169, size = 1928, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)*\text{sech}(d*x+c)^2/(a+b*\sinh(d*x+c)),x)$

[Out] 
$$\begin{aligned} & -2/d^2/(a^2+b^2)^{(3/2)}*b^2*f/(2*a^2+2*b^2)*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)} \\ & +a)/(a+(a^2+b^2)^{(1/2)})) * a^2*c+2/d^2/(a^2+b^2)^{(3/2)}*b^2*f*c/(2*a^2+2*b^2)* \\ & \text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) * a^2+2/d/(a^2+b^2)^{(3/2)}*b \\ & ^2*f/(2*a^2+2*b^2)*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)} \\ & )) * a^2*x-2/d/(a^2+b^2)^{(3/2)}*b^2*f/(2*a^2+2*b^2)*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) * a^2*x+2/d^2/(a^2+b^2)^{(3/2)}*b^2*f/(2*a^2+2*b^2) \\ & *\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) * a^2*c+2/d^2/ \\ & (a^2+b^2)*a^3*f/(2*a^2+2*b^2)*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)-4/d^2/( \\ & a^2+b^2)*b^3*f/(2*a^2+2*b^2)*\arctan(\exp(d*x+c))+2/d^2/(a^2+b^2)^{(5/2)}*a^4*f \\ & *\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-1/2/d^2/(a^2+b^2)^2*a*b^ \\ & 2*f*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)-2/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^ \\ & 2)*\ln(1+\exp(2*d*x+2*c))+2/d^2/(a^2+b^2)^{(3/2)}*b^4*f/(2*a^2+2*b^2)*\text{arctanh}(1 \\ & /2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-4/d^2/(a^2+b^2)*a^2*b*f/(2*a^2+2*b \\ & ^2)*\arctan(\exp(d*x+c))+2/d^2/(a^2+b^2)^{(5/2)}*a^2*b^2*f*\text{arctanh}(1/2*(2*b*\exp \\ & (d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-4/d^2/(a^2+b^2)^{(1/2)}*a^2*f/(2*a^2+2*b^2)*\text{arc} \\ & \text{tanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-2/d/(a^2+b^2)^{(3/2)}*b^4*e/(2 \\ & *a^2+2*b^2)*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-2/d/(a^2+b^2) \\ & ^{(1/2)}*b^2*e/(2*a^2+2*b^2)*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)} \\ & )-2/d^2/(a^2+b^2)*b^2*f/(2*a^2+2*b^2)*a*\ln(1+\exp(2*d*x+2*c))+1/d^2/(a^2+b^2) \\ & )*b^2*f/(2*a^2+2*b^2)*a*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)-2/d^2/(a^2+b^ \\ & 2)^{(1/2)}*b^2*f/(2*a^2+2*b^2)*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/ \\ & 2)})-2/d^2/(a^2+b^2)^{(3/2)}*b^4*f/(2*a^2+2*b^2)*\text{dilog}((b*\exp(d*x+c)+(a^2+b^2) \\ & ^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))+2/d^2/(a^2+b^2)^{(3/2)}*b^4*f/(2*a^2+2*b^2)*\text{di} \\ & \text{log}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))+2/d^2/(a^2+b^2) \\ & ^{(3/2)}*b^2*f/(2*a^2+2*b^2)*\text{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2 \\ & +b^2)^{(1/2)})) * a^2+2/d^2/(a^2+b^2)^{(3/2)}*b^4*f*c/(2*a^2+2*b^2)*\text{arctanh}(1/2*( \\ & 2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+2/d/(a^2+b^2)^{(3/2)}*b^4*f/(2*a^2+2*b^2) \\ & )*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) * x-2/d/(a^2+b^2) \\ & ^{(3/2)}*b^4*f/(2*a^2+2*b^2)*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2) \\ & ^{(1/2)})) * x-2/d/(a^2+b^2)^{(3/2)}*b^2*e/(2*a^2+2*b^2)*\text{arctanh}(1/2*(2*b*\exp(d* \\ & x+c)+2*a)/(a^2+b^2)^{(1/2)}) * a^2+2/d^2/(a^2+b^2)^{(3/2)}*b^4*f/(2*a^2+2*b^2)*\ln \\ & ((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) * c-2/d^2/(a^2+b^2)^ \\ & (3/2)*b^4*f/(2*a^2+2*b^2)*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^ \\ & (1/2))) * c+2/d^2/(a^2+b^2)^{(1/2)}*b^2*f*c/(2*a^2+2*b^2)*\text{arctanh}(1/2*(2*b*\exp( \\ & d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+2/d^2/(a^2+b^2)^{(3/2)}*b^2*f/(2*a^2+2*b^2)*\text{arct} \\ & \text{anh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) * a^2-2/d^2/(a^2+b^2)^{(3/2)}*b^2 \end{aligned}$$

```
*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))
)*a^2-1/d^2/(a^2+b^2)^2*a^3*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d^2/(
a^2+b^2)*a*f*ln(exp(d*x+c))-2*(f*x+e)*(-b*exp(d*x+c)+a)/d/(a^2+b^2)/(1+exp(
2*d*x+2*c))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [B]** time = 2.48013, size = 3164, normalized size = 10.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*(a^3 + a*b^2)*d*f*x*cosh(d*x + c)^2 + 2*(a^3 + a*b^2)*d*f*x*sinh(d*x + c)
)^2 - 2*(a^3 + a*b^2)*d*e + (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cosh(d*x + c)*
sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 + b^3*f)*sqrt((a^2 + b^2)/b^2)*dilog(
(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cosh(d*x
+ c)*sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 + b^3*f)*sqrt((a^2 + b^2)/b^2)*
dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*d*e - b^3*c*f + (b^3*d*e - b^3
*c*f)*cosh(d*x + c)^2 + 2*(b^3*d*e - b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) +
(b^3*d*e - b^3*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*
x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d*e -
b^3*c*f + (b^3*d*e - b^3*c*f)*cosh(d*x + c)^2 + 2*(b^3*d*e - b^3*c*f)*cosh(
d*x + c)*sinh(d*x + c) + (b^3*d*e - b^3*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b
^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b
^2) + 2*a) + (b^3*d*f*x + b^3*c*f + (b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^2 +
2*(b^3*d*f*x + b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b^3*d*f*x + b^3*c*f
)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x
```

+ c) + (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2) - b)/b) - (b^3\*d\*f\*x + b^3\*c\*f + (b^3\*d\*f\*x + b^3\*c\*f)\*cosh(d\*x + c)^2 + 2\*(b^3\*d\*f\*x + b^3\*c\*f)\*cosh(d\*x + c)\*sinh(d\*x + c) + (b^3\*d\*f\*x + b^3\*c\*f)\*sinh(d\*x + c)^2)\*sqrt((a^2 + b^2)/b^2)\*log(-(a\*cosh(d\*x + c) + a\*sinh(d\*x + c) - (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2) - b)/b) - 2\*((a^2\*b + b^3)\*f\*cosh(d\*x + c)^2 + 2\*(a^2\*b + b^3)\*f\*cosh(d\*x + c)\*sinh(d\*x + c) + (a^2\*b + b^3)\*f\*sinh(d\*x + c)^2 + (a^2\*b + b^3)\*f)\*arctan(cosh(d\*x + c) + sinh(d\*x + c)) + 2\*((a^2\*b + b^3)\*d\*f\*x + (a^2\*b + b^3)\*d\*e)\*cosh(d\*x + c) - ((a^3 + a\*b^2)\*f\*cosh(d\*x + c)^2 + 2\*(a^3 + a\*b^2)\*f\*cosh(d\*x + c)\*sinh(d\*x + c) + (a^3 + a\*b^2)\*f\*sinh(d\*x + c)^2 + (a^3 + a\*b^2)\*f)\*log(2\*cosh(d\*x + c)/(cosh(d\*x + c) - sinh(d\*x + c))) + 2\*(2\*(a^3 + a\*b^2)\*d\*f\*x\*cosh(d\*x + c) + (a^2\*b + b^3)\*d\*f\*x + (a^2\*b + b^3)\*d\*e)\*sinh(d\*x + c))/((a^4 + 2\*a^2\*b^2 + b^4)\*d^2\*cosh(d\*x + c)^2 + 2\*(a^4 + 2\*a^2\*b^2 + b^4)\*d^2\*cosh(d\*x + c)\*sinh(d\*x + c) + (a^4 + 2\*a^2\*b^2 + b^4)\*d^2\*sinh(d\*x + c)^2 + (a^4 + 2\*a^2\*b^2 + b^4)\*d^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \operatorname{sech}^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*sech(c + d\*x)\*\*2/(a + b\*sinh(c + d\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \operatorname{sech}(dx + c)^2}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)\*sech(d\*x + c)^2/(b\*sinh(d\*x + c) + a), x)

$$3.312 \quad \int \frac{\operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx$$

Optimal. Leaf size=77

$$\frac{\operatorname{sech}(c+dx)(a \sinh(c+dx)+b)}{d(a^2+b^2)} - \frac{2b^2 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}$$

[Out]  $(-2*b^2*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*d) + (Sech[c + d*x]*(b + a*Sinh[c + d*x]))/((a^2 + b^2)*d)$

**Rubi [A]** time = 0.0996727, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {2696, 12, 2660, 618, 204}

$$\frac{\operatorname{sech}(c+dx)(a \sinh(c+dx)+b)}{d(a^2+b^2)} - \frac{2b^2 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d\*x]^2/(a + b\*Sinh[c + d\*x]),x]

[Out]  $(-2*b^2*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*d) + (Sech[c + d*x]*(b + a*Sinh[c + d*x]))/((a^2 + b^2)*d)$

### Rule 2696

Int[(cos[(e\_.) + (f\_.)\*(x\_.)]\*(g\_.))^(p\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(m\_.), x\_Symbol] :> Simp[((g\*Cos[e + f\*x])^(p + 1)\*(a + b\*Sin[e + f\*x])^(m + 1)\*(b - a\*Sin[e + f\*x]))/(f\*g\*(a^2 - b^2)\*(p + 1)), x] + Dist[1/(g^2\*(a^2 - b^2)\*(p + 1)), Int[(g\*Cos[e + f\*x])^(p + 2)\*(a + b\*Sin[e + f\*x])^m\*(a^2\*(p + 2) - b^2\*(m + p + 2) + a\*b\*(m + p + 3)\*Sin[e + f\*x]), x], x] /; FreeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[2\*m, 2\*p]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 2660

Int[((a\_) + (b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + 2\*b\*e\*x + a\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{\operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\operatorname{sech}(c+dx)(b+a \sinh(c+dx))}{(a^2+b^2)d} + \frac{\int \frac{b^2}{a+b \sinh(c+dx)} dx}{a^2+b^2} \\
 &= \frac{\operatorname{sech}(c+dx)(b+a \sinh(c+dx))}{(a^2+b^2)d} + \frac{b^2 \int \frac{1}{a+b \sinh(c+dx)} dx}{a^2+b^2} \\
 &= \frac{\operatorname{sech}(c+dx)(b+a \sinh(c+dx))}{(a^2+b^2)d} - \frac{(2ib^2) \operatorname{Subst}\left(\int \frac{1}{a-2ibx+ax^2} dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{(a^2+b^2)d} \\
 &= \frac{\operatorname{sech}(c+dx)(b+a \sinh(c+dx))}{(a^2+b^2)d} + \frac{(4ib^2) \operatorname{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2} dx, x, -2ib+2a \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{(a^2+b^2)d} \\
 &= -\frac{2b^2 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d} + \frac{\operatorname{sech}(c+dx)(b+a \sinh(c+dx))}{(a^2+b^2)d}
 \end{aligned}$$

**Mathematica [A]** time = 0.172899, size = 104, normalized size = 1.35

$$\frac{a\sqrt{-a^2 - b^2} \tanh(c + dx) + b\sqrt{-a^2 - b^2} \operatorname{sech}(c + dx) + 2b^2 \tan^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2 - b^2}}\right)}{d(-a^2 - b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d\*x]^2/(a + b\*Sinh[c + d\*x]), x]

[Out] -((2\*b^2\*ArcTan[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[-a^2 - b^2]] + b\*Sqrt[-a^2 - b^2]\*Sech[c + d\*x] + a\*Sqrt[-a^2 - b^2]\*Tanh[c + d\*x])/((-a^2 - b^2)^(3/2)\*d))

**Maple [A]** time = 0.003, size = 90, normalized size = 1.2

$$\frac{1}{d} \left( 2 \frac{b^2}{(a^2 + b^2)^{3/2}} \operatorname{Arctanh} \left( \frac{1}{2} \frac{2a \tanh(1/2 dx + c/2) - 2b}{\sqrt{a^2 + b^2}} \right) - 2 \frac{-a \tanh(1/2 dx + c/2) - b}{(a^2 + b^2) \left( (\tanh(1/2 dx + c/2))^2 + 1 \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)), x)

[Out] 1/d\*(2\*b^2/(a^2+b^2)^(3/2)\*arctanh(1/2\*(2\*a\*tanh(1/2\*d\*x+1/2\*c)-2\*b)/(a^2+b^2)^(1/2))-2/(a^2+b^2)\*(-a\*tanh(1/2\*d\*x+1/2\*c)-b)/(tanh(1/2\*d\*x+1/2\*c)^2+1))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)), x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [B]** time = 2.15788, size = 884, normalized size = 11.48

$$\frac{2a^3 + 2ab^2 - (b^2 \cosh(dx+c)^2 + 2b^2 \cosh(dx+c) \sinh(dx+c) + b^2 \sinh(dx+c)^2 + b^2) \sqrt{a^2 + b^2} \log\left(\frac{b^2 \cosh(dx+c)^2 + b^2 \sinh(dx+c)^2 + 2ab \cosh(dx+c) \sinh(dx+c) + a^2 + b^2}{(a^4 + 2a^2b^2 + b^4)d \cosh(dx+c)^2 + 2(a^4 + 2a^2b^2 + b^4)d \sinh(dx+c)^2 + (a^4 + 2a^2b^2 + b^4)d}\right)}{(a^4 + 2a^2b^2 + b^4)d \cosh(dx+c)^2 + 2(a^4 + 2a^2b^2 + b^4)d \sinh(dx+c)^2 + (a^4 + 2a^2b^2 + b^4)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-(2a^3 + 2ab^2 - (b^2 \cosh(dx+c)^2 + 2b^2 \cosh(dx+c) \sinh(dx+c) + b^2 \sinh(dx+c)^2 + b^2) \sqrt{a^2 + b^2} \log((b^2 \cosh(dx+c)^2 + b^2 \sinh(dx+c)^2 + 2ab \cosh(dx+c) \sinh(dx+c) + a^2 + b^2) / (b^2 \cosh(dx+c)^2 + b^2 \sinh(dx+c)^2 + 2ab \cosh(dx+c) \sinh(dx+c) + a^2 + b^2)) - 2 \sqrt{a^2 + b^2} (b \cosh(dx+c) + b \sinh(dx+c) + a)) / (b \cosh(dx+c)^2 + b \sinh(dx+c)^2 + 2a \cosh(dx+c) + 2(b \cosh(dx+c) + a) \sinh(dx+c) - b) - 2(a^2b + b^3) \cosh(dx+c) - 2(a^2b + b^3) \sinh(dx+c)) / ((a^4 + 2a^2b^2 + b^4) d \cosh(dx+c)^2 + 2(a^4 + 2a^2b^2 + b^4) d \sinh(dx+c)^2 + (a^4 + 2a^2b^2 + b^4) d)$$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(sech(c + d\*x)\*\*2/(a + b\*sinh(c + d\*x)), x)

---

**Giac [A]** time = 1.23397, size = 165, normalized size = 2.14

$$-\frac{b^2 \log\left(\frac{|-2be^{(dx+c)} - 2a - 2\sqrt{a^2+b^2}|}{|-2be^{(dx+c)} - 2a + 2\sqrt{a^2+b^2}|}\right)}{(a^2d + b^2d)\sqrt{a^2 + b^2}} + \frac{2(b e^{(dx+c)} - a)}{(a^2d + b^2d)(e^{2dx+2c} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -b^2*log(abs(-2*b*e^(d*x + c) - 2*a - 2*sqrt(a^2 + b^2))/abs(-2*b*e^(d*x + c) - 2*a + 2*sqrt(a^2 + b^2)))/((a^2*d + b^2*d)*sqrt(a^2 + b^2)) + 2*(b*e^(d*x + c) - a)/((a^2*d + b^2*d)*(e^(2*d*x + 2*c) + 1))
```

$$3.313 \quad \int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=30

$$\text{Unintegrable} \left( \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[Sech[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0769621, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Sech[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Sech[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [A]** time = 68.3938, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sech[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Integrate[Sech[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Maple [A]** time = 0.567, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(dx + c))^2}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)), x)

[Out] int(sech(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$4b^2 \int \frac{e^{(dx+c)}}{2(a^2be + b^3e + (a^2bf + b^3f)x - (a^2bee^{2c} + b^3ee^{2c} + (a^2bfe^{2c} + b^3fe^{2c})x)e^{2dx} - 2(a^3ee^c + ab^2ee^c + (a^3f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)), x, algorithm="maxima")

[Out] 4\*b^2\*integrate(-1/2\*e^(d\*x + c)/(a^2\*b\*e + b^3\*e + (a^2\*b\*f + b^3\*f)\*x - (a^2\*b\*e\*e^(2\*c) + b^3\*e\*e^(2\*c) + (a^2\*b\*f\*e^(2\*c) + b^3\*f\*e^(2\*c))\*x)\*e^(2\*d\*x) - 2\*(a^3\*e\*e^c + a\*b^2\*e\*e^c + (a^3\*f\*e^c + a\*b^2\*f\*e^c)\*x)\*e^(d\*x), x) + 2\*(b\*e^(d\*x + c) - a)/(a^2\*d\*e + b^2\*d\*e + (a^2\*d\*f + b^2\*d\*f)\*x + (a^2\*d\*e\*e^(2\*c) + b^2\*d\*e\*e^(2\*c) + (a^2\*d\*f\*e^(2\*c) + b^2\*d\*f\*e^(2\*c))\*x)\*e^(2\*d\*x)) + 4\*integrate(1/2\*(b\*f\*e^(d\*x + c) - a\*f)/(a^2\*d\*e^2 + b^2\*d\*e^2 + (a^2\*d\*f^2 + b^2\*d\*f^2)\*x^2 + 2\*(a^2\*d\*e\*f + b^2\*d\*e\*f)\*x + (a^2\*d\*e^2\*e^(2\*c) + b^2\*d\*e^2\*e^(2\*c) + (a^2\*d\*f^2\*e^(2\*c) + b^2\*d\*f^2\*e^(2\*c))\*x^2 + 2\*(a^2\*d\*e\*f\*e^(2\*c) + b^2\*d\*e\*f\*e^(2\*c))\*x)\*e^(2\*d\*x)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{sech}(dx + c)^2}{afx + ae + (bfx + be) \sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sech(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(c + dx)}{(a + b \sinh(c + dx))(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sech(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.314 \quad \int \frac{(e+fx)^2 \operatorname{sech}^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=928

result too large to display

```
[Out] (2*a*b^2*(e + f*x)^2*ArcTan[E^(c + d*x)])/((a^2 + b^2)^2*d) + (a*(e + f*x)^2*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (a*f^2*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d^3) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (b^3*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)^2*d) + (b*f^2*Log[Cosh[c + d*x]])/((a^2 + b^2)*d^3) - ((2*I)*a*b^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - (I*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) + ((2*I)*a*b^2*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + (I*a*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) + (2*b^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) + (2*b^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (b^3*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2)^2*d^2) + ((2*I)*a*b^2*f^2*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^3) + (I*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) - ((2*I)*a*b^2*f^2*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)^2*d^3) - (I*a*f^2*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)*d^3) - (2*b^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) - (2*b^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^3) + (b^3*f^2*PolyLog[3, -E^(2*(c + d*x))])/((2*(a^2 + b^2)^2*d^3) + (a*f*(e + f*x)*Sech[c + d*x])/((a^2 + b^2)*d^2) + (b*(e + f*x)^2*Sech[c + d*x]^2)/(2*(a^2 + b^2)*d) - (b*f*(e + f*x)*Tanh[c + d*x])/((a^2 + b^2)*d^2) + (a*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*(a^2 + b^2)*d)
```

**Rubi [A]** time = 1.77192, antiderivative size = 928, normalized size of antiderivative = 1., number of steps used = 39, number of rules used = 14, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5573, 5561, 2190, 2531, 2282, 6589, 6742, 4180, 3718, 4186, 3770, 5451, 4184, 3475}

$$\frac{(e+fx)^2 \log\left(\frac{e^{c+dx} b}{a-\sqrt{a^2+b^2}} + 1\right) b^3}{(a^2+b^2)^2 d} + \frac{(e+fx)^2 \log\left(\frac{e^{c+dx} b}{a+\sqrt{a^2+b^2}} + 1\right) b^3}{(a^2+b^2)^2 d} - \frac{(e+fx)^2 \log(1+e^{2(c+dx)}) b^3}{(a^2+b^2)^2 d} + \frac{2f(e+fx) \operatorname{PolyLog}}{(a^2+b^2)}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Sech[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] (2\*a\*b^2\*(e + f\*x)^2\*ArcTan[E^(c + d\*x)]/((a^2 + b^2)^2\*d) + (a\*(e + f\*x)^2\*ArcTan[E^(c + d\*x)]/((a^2 + b^2)\*d) - (a\*f^2\*ArcTan[Sinh[c + d\*x]]/((a^2 + b^2)\*d^3) + (b^3\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/((a^2 + b^2)^2\*d) + (b^3\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/((a^2 + b^2)^2\*d) - (b^3\*(e + f\*x)^2\*Log[1 + E^(2\*(c + d\*x))]/((a^2 + b^2)^2\*d) + (b\*f^2\*Log[Cosh[c + d\*x]]/((a^2 + b^2)\*d^3) - ((2\*I)\*a\*b^2\*f\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)]/((a^2 + b^2)^2\*d^2) - (I\*a\*f\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)]/((a^2 + b^2)\*d^2) + ((2\*I)\*a\*b^2\*f\*(e + f\*x)\*PolyLog[2, I\*E^(c + d\*x)]/((a^2 + b^2)^2\*d^2) + (I\*a\*f\*(e + f\*x)\*PolyLog[2, I\*E^(c + d\*x)]/((a^2 + b^2)\*d^2) + (2\*b^3\*f\*(e + f\*x)\*PolyLog[2, -(b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/((a^2 + b^2)^2\*d^2) + (2\*b^3\*f\*(e + f\*x)\*PolyLog[2, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/((a^2 + b^2)^2\*d^2) - (b^3\*f\*(e + f\*x)\*PolyLog[2, -E^(2\*(c + d\*x))]/((a^2 + b^2)^2\*d^2) + ((2\*I)\*a\*b^2\*f^2\*PolyLog[3, (-I)\*E^(c + d\*x)]/((a^2 + b^2)^2\*d^3) + (I\*a\*f^2\*PolyLog[3, (-I)\*E^(c + d\*x)]/((a^2 + b^2)\*d^3) - ((2\*I)\*a\*b^2\*f^2\*PolyLog[3, I\*E^(c + d\*x)]/((a^2 + b^2)^2\*d^3) - (I\*a\*f^2\*PolyLog[3, I\*E^(c + d\*x)]/((a^2 + b^2)\*d^3) - (2\*b^3\*f^2\*PolyLog[3, -(b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/((a^2 + b^2)^2\*d^3) - (2\*b^3\*f^2\*PolyLog[3, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/((a^2 + b^2)^2\*d^3) + (b^3\*f^2\*PolyLog[3, -E^(2\*(c + d\*x))]/(2\*(a^2 + b^2)^2\*d^3) + (a\*f\*(e + f\*x)\*Sech[c + d\*x])/((a^2 + b^2)\*d^2) + (b\*(e + f\*x)^2\*Sech[c + d\*x]^2)/(2\*(a^2 + b^2)\*d) - (b\*f\*(e + f\*x)\*Tanh[c + d\*x])/((a^2 + b^2)\*d^2) + (a\*(e + f\*x)^2\*Sech[c + d\*x]\*Tanh[c + d\*x])/(2\*(a^2 + b^2)\*d)

### Rule 5573

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f\*x)^m\*Sech[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f\*x)^m\*Sech[c + d\*x]^n\*(a - b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]

### Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*(f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 4180

```
Int[csc[(e_) + Pi*(k_) + (Complex[0, fz_])*(f_)*(x_)]*((c_) + (d_)*(x_
))^m_], x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)]]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol]
:> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))]/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

### Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) + (b_.)*(x_)]^(p_), x_Symbol]
:> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n), x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(e+fx)^2 \operatorname{sech}^3(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \operatorname{sech}^3(c+dx)(a-b \sinh(c+dx)) dx}{a^2+b^2} + \frac{b^2 \int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx}{a^2+b^2} \\
&= \frac{b^2 \int (e+fx)^2 \operatorname{sech}(c+dx)(a-b \sinh(c+dx)) dx}{(a^2+b^2)^2} + \frac{b^4 \int \frac{(e+fx)^2 \cosh(c+dx)}{a+b \sinh(c+dx)} dx}{(a^2+b^2)^2} + \frac{\int (a(e+fx)^2 \operatorname{sech}(c+dx) - b(e+fx)^2 \tanh(c+dx)) dx}{(a^2+b^2)^2} \\
&= -\frac{b^3(e+fx)^3}{3(a^2+b^2)^2 f} + \frac{b^2 \int (a(e+fx)^2 \operatorname{sech}(c+dx) - b(e+fx)^2 \tanh(c+dx)) dx}{(a^2+b^2)^2} + \frac{b^4 \int \frac{(e+fx)^2 \cosh(c+dx)}{a+b \sinh(c+dx)} dx}{(a^2+b^2)^2} \\
&= -\frac{b^3(e+fx)^3}{3(a^2+b^2)^2 f} + \frac{b^3(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} + \frac{b^3(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} + \frac{b^4 \int \frac{(e+fx)^2 \cosh(c+dx)}{a+b \sinh(c+dx)} dx}{(a^2+b^2)^2} \\
&= \frac{2ab^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} + \frac{a(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2) d} - \frac{af^2 \tan^{-1}(\sinh(c+dx))}{(a^2+b^2) d^3} + \frac{b^4 \int \frac{(e+fx)^2 \cosh(c+dx)}{a+b \sinh(c+dx)} dx}{(a^2+b^2)^2} \\
&= \frac{2ab^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} + \frac{a(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2) d} - \frac{af^2 \tan^{-1}(\sinh(c+dx))}{(a^2+b^2) d^3} + \frac{b^4 \int \frac{(e+fx)^2 \cosh(c+dx)}{a+b \sinh(c+dx)} dx}{(a^2+b^2)^2} \\
&= \frac{2ab^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} + \frac{a(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2) d} - \frac{af^2 \tan^{-1}(\sinh(c+dx))}{(a^2+b^2) d^3} + \frac{b^4 \int \frac{(e+fx)^2 \cosh(c+dx)}{a+b \sinh(c+dx)} dx}{(a^2+b^2)^2} \\
&= \frac{2ab^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} + \frac{a(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2) d} - \frac{af^2 \tan^{-1}(\sinh(c+dx))}{(a^2+b^2) d^3} + \frac{b^4 \int \frac{(e+fx)^2 \cosh(c+dx)}{a+b \sinh(c+dx)} dx}{(a^2+b^2)^2}
\end{aligned}$$

**Mathematica [B]** time = 31.5229, size = 3368, normalized size = 3.63

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Sech[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out]  $-(-12*b^3*d^3*e^{2*c}*x + 12*a^2*b*d*e^{2*c})*f^2*x + 12*b^3*d*e^{2*c}*f^2*x - 12*b^3*d^3*e^{2*c}*f*x^2 - 4*b^3*d^3*e^{2*c}*f^2*x^3 - 6*a^3*d^2*e$

$$\begin{aligned}
& ^2\text{ArcTan}[E^{(c + d*x)}] - 18*a*b^2*d^2*e^2*\text{ArcTan}[E^{(c + d*x)}] - 6*a^3*d^2*e \\
& ^2*E^{(2*c)}*\text{ArcTan}[E^{(c + d*x)}] - 18*a*b^2*d^2*e^2*E^{(2*c)}*\text{ArcTan}[E^{(c + d*x)}] \\
& ) + 12*a^3*f^2*\text{ArcTan}[E^{(c + d*x)}] + 12*a*b^2*f^2*\text{ArcTan}[E^{(c + d*x)}] + 12 \\
& *a^3*E^{(2*c)}*f^2*\text{ArcTan}[E^{(c + d*x)}] + 12*a*b^2*E^{(2*c)}*f^2*\text{ArcTan}[E^{(c + d \\
& *x)}] - (6*I)*a^3*d^2*e*f*x*\text{Log}[1 - I*E^{(c + d*x)}] - (18*I)*a*b^2*d^2*e*f*x* \\
& \text{Log}[1 - I*E^{(c + d*x)}] - (6*I)*a^3*d^2*e*E^{(2*c)}*f*x*\text{Log}[1 - I*E^{(c + d*x)}] \\
& - (18*I)*a*b^2*d^2*e*E^{(2*c)}*f*x*\text{Log}[1 - I*E^{(c + d*x)}] - (3*I)*a^3*d^2*f^ \\
& 2*x^2*\text{Log}[1 - I*E^{(c + d*x)}] - (9*I)*a*b^2*d^2*f^2*x^2*\text{Log}[1 - I*E^{(c + d*x)} \\
& ) - (3*I)*a^3*d^2*E^{(2*c)}*f^2*x^2*\text{Log}[1 - I*E^{(c + d*x)}] - (9*I)*a*b^2*d^2 \\
& *E^{(2*c)}*f^2*x^2*\text{Log}[1 - I*E^{(c + d*x)}] + (6*I)*a^3*d^2*e*f*x*\text{Log}[1 + I*E^{( \\
& c + d*x)}] + (18*I)*a*b^2*d^2*e*f*x*\text{Log}[1 + I*E^{(c + d*x)}] + (6*I)*a^3*d^2*e \\
& *E^{(2*c)}*f*x*\text{Log}[1 + I*E^{(c + d*x)}] + (18*I)*a*b^2*d^2*e*E^{(2*c)}*f*x*\text{Log}[1 \\
& + I*E^{(c + d*x)}] + (3*I)*a^3*d^2*f^2*x^2*\text{Log}[1 + I*E^{(c + d*x)}] + (9*I)*a*b \\
& ^2*d^2*f^2*x^2*\text{Log}[1 + I*E^{(c + d*x)}] + (3*I)*a^3*d^2*E^{(2*c)}*f^2*x^2*\text{Log}[1 \\
& + I*E^{(c + d*x)}] + (9*I)*a*b^2*d^2*E^{(2*c)}*f^2*x^2*\text{Log}[1 + I*E^{(c + d*x)}] \\
& + 6*b^3*d^2*e^2*\text{Log}[1 + E^{(2*(c + d*x))}] + 6*b^3*d^2*e^2*E^{(2*c)}*\text{Log}[1 + E^{ \\
& (2*(c + d*x))}] - 6*a^2*b*f^2*\text{Log}[1 + E^{(2*(c + d*x))}] - 6*b^3*f^2*\text{Log}[1 + E \\
& ^{2*(c + d*x)}] - 6*a^2*b*E^{(2*c)}*f^2*\text{Log}[1 + E^{(2*(c + d*x))}] - 6*b^3*E^{(2 \\
& *c)}*f^2*\text{Log}[1 + E^{(2*(c + d*x))}] + 12*b^3*d^2*e*f*x*\text{Log}[1 + E^{(2*(c + d*x))} \\
& ] + 12*b^3*d^2*e*E^{(2*c)}*f*x*\text{Log}[1 + E^{(2*(c + d*x))}] + 6*b^3*d^2*f^2*x^2*L \\
& og[1 + E^{(2*(c + d*x))}] + 6*b^3*d^2*E^{(2*c)}*f^2*x^2*\text{Log}[1 + E^{(2*(c + d*x))} \\
& ] + (6*I)*a*(a^2 + 3*b^2)*d*(1 + E^{(2*c)})*f*(e + f*x)*\text{PolyLog}[2, (-I)*E^{(c \\
& + d*x)}] - (6*I)*a*(a^2 + 3*b^2)*d*(1 + E^{(2*c)})*f*(e + f*x)*\text{PolyLog}[2, I*E^{ \\
& (c + d*x)}] + 6*b^3*d*e*f*\text{PolyLog}[2, -E^{(2*(c + d*x))}] + 6*b^3*d*e*E^{(2*c)}*f \\
& *\text{PolyLog}[2, -E^{(2*(c + d*x))}] + 6*b^3*d*f^2*x*\text{PolyLog}[2, -E^{(2*(c + d*x))}] \\
& + 6*b^3*d*E^{(2*c)}*f^2*x*\text{PolyLog}[2, -E^{(2*(c + d*x))}] - (6*I)*a^3*f^2*\text{PolyLo \\
& g}[3, (-I)*E^{(c + d*x)}] - (18*I)*a*b^2*f^2*\text{PolyLog}[3, (-I)*E^{(c + d*x)}] - (6 \\
& *I)*a^3*E^{(2*c)}*f^2*\text{PolyLog}[3, (-I)*E^{(c + d*x)}] - (18*I)*a*b^2*E^{(2*c)}*f^2 \\
& *\text{PolyLog}[3, (-I)*E^{(c + d*x)}] + (6*I)*a^3*f^2*\text{PolyLog}[3, I*E^{(c + d*x)}] + ( \\
& 18*I)*a*b^2*f^2*\text{PolyLog}[3, I*E^{(c + d*x)}] + (6*I)*a^3*E^{(2*c)}*f^2*\text{PolyLog}[3 \\
& , I*E^{(c + d*x)}] + (18*I)*a*b^2*E^{(2*c)}*f^2*\text{PolyLog}[3, I*E^{(c + d*x)}] - 3*b \\
& ^3*f^2*\text{PolyLog}[3, -E^{(2*(c + d*x))}] - 3*b^3*E^{(2*c)}*f^2*\text{PolyLog}[3, -E^{(2*(c \\
& + d*x))}]/(6*(a^2 + b^2)^2*d^3*(1 + E^{(2*c)})) - (b^3*(6*e^2*E^{(2*c)}*x + 6* \\
& e*E^{(2*c)}*f*x^2 + 2*E^{(2*c)}*f^2*x^3 + (6*a*\text{Sqrt}[a^2 + b^2]*e^2*\text{ArcTan}[(a + \\
& b*E^{(c + d*x)})/\text{Sqrt}[-a^2 - b^2])]/(\text{Sqrt}[-(a^2 + b^2)^2]*d) + (6*a*\text{Sqrt}[-(a^ \\
& 2 + b^2)^2]*e^2*E^{(2*c)}*\text{ArcTan}[(a + b*E^{(c + d*x)})/\text{Sqrt}[-a^2 - b^2])]/((a^2 \\
& + b^2)^(3/2)*d) - (6*a*\text{Sqrt}[-(a^2 + b^2)^2]*e^2*\text{ArcTanh}[(a + b*E^{(c + d*x)} \\
& )/\text{Sqrt}[a^2 + b^2])]/((-a^2 - b^2)^(3/2)*d) + (6*a*\text{Sqrt}[-(a^2 + b^2)^2]*e^2* \\
& E^{(2*c)}*\text{ArcTanh}[(a + b*E^{(c + d*x)})/\text{Sqrt}[a^2 + b^2])]/((-a^2 - b^2)^(3/2)*d \\
& ) + (3*e^2*\text{Log}[2*a*E^{(c + d*x)} + b*(-1 + E^{(2*(c + d*x))})])/d - (3*e^2*E^{(2 \\
& *c)}*\text{Log}[2*a*E^{(c + d*x)} + b*(-1 + E^{(2*(c + d*x))})])/d + (6*e*f*x*\text{Log}[1 + ( \\
& b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d - (6*e*E^{(2*c)}*f*x \\
& *\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d + (3*f^2 \\
& *x^2*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d - (3 \\
& *E^{(2*c)}*f^2*x^2*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}
\end{aligned}$$

$$\begin{aligned} & \dots ])/d + (6*e*f*x*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d - (6*e*E^{(2*c)}*f*x*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d + (3*f^2*x^2*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d - (3*E^{(2*c)}*f^2*x^2*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d - (6*(-1 + E^{(2*c)})*f*(e + f*x)*PolyLog[2, -((b*E^{(2*c + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^2 - (6*(-1 + E^{(2*c)})*f*(e + f*x)*PolyLog[2, -((b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^2 - (6*f^2*PolyLog[3, -((b*E^{(2*c + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^3 + (6*E^{(2*c)}*f^2*PolyLog[3, -((b*E^{(2*c + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^3 - (6*f^2*PolyLog[3, -((b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^3 + (6*E^{(2*c)}*f^2*PolyLog[3, -((b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^3)/(3*(a^2 + b^2)^2*(-1 + E^{(2*c)})) + (Csch[c]*Sech[c]*Sech[c + d*x]^2*(-6*a^2*b*e*f - 6*b^3*e*f + 12*b^3*d^2*e^2*x - 6*a^2*b*f^2*x - 6*b^3*f^2*x + 12*b^3*d^2*e*f*x^2 + 4*b^3*d^2*f^2*x^3 + 6*a^2*b*e*f*Cosh[2*c] + 6*b^3*e*f*Cosh[2*c] + 6*a^2*b*f^2*x*Cosh[2*c] + 6*b^3*f^2*x*Cosh[2*c] + 6*a^2*b*e*f*Cosh[2*d*x] + 6*b^3*e*f*Cosh[2*d*x] + 6*a^2*b*f^2*x*Cosh[2*d*x] + 6*b^3*f^2*x*Cosh[2*d*x] - 3*a^3*d*e^2*Cosh[c - d*x] - 3*a*b^2*d*e^2*Cosh[c - d*x] - 6*a^3*d*e*f*x*Cosh[c - d*x] - 6*a*b^2*d*e*f*x*Cosh[c - d*x] - 3*a^3*d*f^2*x^2*Cosh[c - d*x] - 3*a*b^2*d*f^2*x^2*Cosh[c - d*x] + 3*a^3*d*e^2*Cosh[3*c + d*x] + 3*a*b^2*d*e^2*Cosh[3*c + d*x] + 6*a^3*d*e*f*x*Cosh[3*c + d*x] + 6*a*b^2*d*e*f*x*Cosh[3*c + d*x] + 3*a^3*d*f^2*x^2*Cosh[3*c + d*x] + 3*a*b^2*d*f^2*x^2*Cosh[3*c + d*x] - 6*a^2*b*e*f*Cosh[2*c + 2*d*x] - 6*b^3*e*f*Cosh[2*c + 2*d*x] + 12*b^3*d^2*e^2*x*Cosh[2*c + 2*d*x] - 6*a^2*b*f^2*x*Cosh[2*c + 2*d*x] - 6*b^3*f^2*x*Cosh[2*c + 2*d*x] + 12*b^3*d^2*e*f*x^2*Cosh[2*c + 2*d*x] + 4*b^3*d^2*f^2*x^3*Cosh[2*c + 2*d*x] + 6*a^2*b*d*e^2*Sinh[2*c] + 6*b^3*d*e^2*Sinh[2*c] + 12*a^2*b*d*e*f*x*Sinh[2*c] + 12*b^3*d*e*f*x*Sinh[2*c] + 6*a^2*b*d*f^2*x^2*Sinh[2*c] + 6*b^3*d*f^2*x^2*Sinh[2*c] + 6*a^3*e*f*Sinh[c - d*x] + 6*a*b^2*e*f*Sinh[c - d*x] + 6*a^3*f^2*x*Sinh[c - d*x] + 6*a*b^2*f^2*x*Sinh[c - d*x] + 6*a^3*e*f*Sinh[3*c + d*x] + 6*a*b^2*e*f*Sinh[3*c + d*x] + 6*a^3*f^2*x*Sinh[3*c + d*x] + 6*a*b^2*f^2*x*Sinh[3*c + d*x]))/(24*(a^2 + b^2)^2*d^2)
\end{aligned}$$

**Maple [F]** time = 0.477, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\operatorname{sech}(dx + c))^3}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*sech(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

[Out]  $\int ((f*x+e)^2*\operatorname{sech}(d*x+c)^3/(a+b*\sinh(d*x+c)), x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((f*x+e)^2*\operatorname{sech}(d*x+c)^3/(a+b*\sinh(d*x+c)), x, \text{algorithm}="maxima")$

[Out]  $a^3*d^2*f^2*\operatorname{integrate}(x^2*e^{(d*x+c)}/(a^4*d^2*e^{(2*d*x+2*c)} + 2*a^2*b^2*d^2*e^{(2*d*x+2*c)} + b^4*d^2*e^{(2*d*x+2*c)} + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 3*a*b^2*d^2*f^2*\operatorname{integrate}(x^2*e^{(d*x+c)}/(a^4*d^2*e^{(2*d*x+2*c)} + 2*a^2*b^2*d^2*e^{(2*d*x+2*c)} + b^4*d^2*e^{(2*d*x+2*c)} + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*b^3*d^2*f^2*\operatorname{integrate}(x^2/(a^4*d^2*e^{(2*d*x+2*c)} + 2*a^2*b^2*d^2*e^{(2*d*x+2*c)} + b^4*d^2*e^{(2*d*x+2*c)} + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*a^3*d^2*e*f*\operatorname{integrate}(x*e^{(d*x+c)}/(a^4*d^2*e^{(2*d*x+2*c)} + 2*a^2*b^2*d^2*e^{(2*d*x+2*c)} + b^4*d^2*e^{(2*d*x+2*c)} + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 6*a*b^2*d^2*e*f*\operatorname{integrate}(x*e^{(d*x+c)}/(a^4*d^2*e^{(2*d*x+2*c)} + 2*a^2*b^2*d^2*e^{(2*d*x+2*c)} + b^4*d^2*e^{(2*d*x+2*c)} + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 4*b^3*d^2*e*f*\operatorname{integrate}(x/(a^4*d^2*e^{(2*d*x+2*c)} + 2*a^2*b^2*d^2*e^{(2*d*x+2*c)} + b^4*d^2*e^{(2*d*x+2*c)} + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - a^2*b*f^2*(2*(d*x+c)/((a^4+2*a^2*b^2+b^4)*d^3) - \log(e^{(2*d*x+2*c)}+1)/((a^4+2*a^2*b^2+b^4)*d^3)) - b^3*f^2*(2*(d*x+c)/((a^4+2*a^2*b^2+b^4)*d^3) - \log(e^{(2*d*x+2*c)}+1)/((a^4+2*a^2*b^2+b^4)*d^3)) + (b^3*\log(-2*a*e^{(-d*x-c)} + b*e^{(-2*d*x-2*c)} - b)/((a^4+2*a^2*b^2+b^4)*d) - b^3*\log(e^{(-2*d*x-2*c)}+1)/((a^4+2*a^2*b^2+b^4)*d) - (a^3+3*a*b^2)*\arctan(e^{(-d*x-c)})/((a^4+2*a^2*b^2+b^4)*d) + (a*e^{(-d*x-c)} + 2*b*e^{(-2*d*x-2*c)} - a*e^{(-3*d*x-3*c)})/((a^2+b^2+2*(a^2+b^2))*e^{(-2*d*x-2*c)} + (a^2+b^2)*e^{(-4*d*x-4*c)})*d)*e^2 - 2*a^3*f^2*\arctan(e^{(d*x+c)})/((a^4+2*a^2*b^2+b^4)*d^3) - 2*a*b^2*f^2*\arctan(e^{(d*x+c)})/((a^4+2*a^2*b^2+b^4)*d^3) + (2*b*f^2*x + 2*b*e*f + (a*d*f^2*x^2*e^{(3*c)} + 2*a*e*f*e^{(3*c)} + 2*(d*e*f + f^2)*a*x*e^{(3*c)})*e^{(3*d*x)} + 2*(b*d*f^2*x^2*e^{(2*c)} + b*e*f*e^{(2*c)} + (2*d*e*f + f^2)*b*x*e^{(2*c)})*e^{(2*d*x)} - (a*d*f^2*x^2*e^c - 2*a*e*f*e^c + 2*(d*e*f - f^2)*a*x*e^c)*e^{(d*x)})/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^{(4*c)} + b^2*d^2*e^{(4*c)})*e^{(4*d*x)} + 2*(a^2*d^2*e^{(2*c)} + b^2*d^2*e^{(2*c)})*e^{(2*d*x)}) - \operatorname{integrate}(2*(b^4*f^2*x^2 + 2*b^4*e*f*x - (a*b^3*f^2*x^2*e^c + 2*a*b^3*e*f*x*e^c)*e^{(d*x)})/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b*e^{(2*c)} + 2*a^2*b^3*e^{(2*c)} + b^5*e^{(2*c)}))*e^{(2*d*x)} - 2*(a^5*e^c + 2*a^3*b^2*e^c + a*b^4*e^c)*e^{(d*x)}, x)$

**Fricas [C]** time = 5.2455, size = 24035, normalized size = 25.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sech(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/2*(4*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*\cosh(d*x + c)^4 + 4*( \\ & (a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*\sinh(d*x + c)^4 - 4*(a^2*b + b \\ & ^3)*d*e*f + 4*(a^2*b + b^3)*c*f^2 - 2*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a \\ & *b^2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^3 + a*b^2)*d^2*e*f + (a^3 + a \\ & *b^2)*d*f^2)*x)*\cosh(d*x + c)^3 - 2*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a*b \\ & ^2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^3 + a*b^2)*d^2*e*f + (a^3 + a*b \\ & ^2)*d*f^2)*x - 8*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*\cosh(d*x + c \\ & ))*\sinh(d*x + c)^3 - 4*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 + \\ & (a^2*b + b^3)*d*e*f - 2*(a^2*b + b^3)*c*f^2 + (2*(a^2*b + b^3)*d^2*e*f - ( \\ & a^2*b + b^3)*d*f^2)*x)*\cosh(d*x + c)^2 - 2*(2*(a^2*b + b^3)*d^2*f^2*x^2 + 2 \\ & *(a^2*b + b^3)*d^2*e^2 + 2*(a^2*b + b^3)*d*e*f - 4*(a^2*b + b^3)*c*f^2 - 12 \\ & *((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*\cosh(d*x + c)^2 + 2*(2*(a^2* \\ & b + b^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x + 3*((a^3 + a*b^2)*d^2*f^2*x^2 + \\ & (a^3 + a*b^2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^3 + a*b^2)*d^2*e*f + \\ & (a^3 + a*b^2)*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 2*((a^3 + a*b^2)*d \\ & ^2*f^2*x^2 + (a^3 + a*b^2)*d^2*e^2 - 2*(a^3 + a*b^2)*d*e*f + 2*((a^3 + a*b^ \\ & 2)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x)*\cosh(d*x + c) - 4*(b^3*d*f^2*x + b^3*d \\ & *e*f + (b^3*d*f^2*x + b^3*d*e*f)*\cosh(d*x + c))^4 + 4*(b^3*d*f^2*x + b^3*d*e \\ & *f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^3*d*f^2*x + b^3*d*e*f)*\sinh(d*x + c) \\ & ^4 + 2*(b^3*d*f^2*x + b^3*d*e*f)*\cosh(d*x + c)^2 + 2*(b^3*d*f^2*x + b^3*d*e \\ & *f + 3*(b^3*d*f^2*x + b^3*d*e*f)*\cosh(d*x + c))^2*\sinh(d*x + c)^2 + 4*((b^3 \\ & *d*f^2*x + b^3*d*e*f)*\cosh(d*x + c))^3 + (b^3*d*f^2*x + b^3*d*e*f)*\cosh(d*x \\ & + c))*\sinh(d*x + c))*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x \\ & + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 4*(b^3*d*f^2*x \\ & + b^3*d*e*f + (b^3*d*f^2*x + b^3*d*e*f)*\cosh(d*x + c))^4 + 4*(b^3*d*f^2*x + \\ & b^3*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^3*d*f^2*x + b^3*d*e*f)*\sinh( \\ & d*x + c)^4 + 2*(b^3*d*f^2*x + b^3*d*e*f)*\cosh(d*x + c)^2 + 2*(b^3*d*f^2*x + \\ & b^3*d*e*f + 3*(b^3*d*f^2*x + b^3*d*e*f)*\cosh(d*x + c))^2*\sinh(d*x + c)^2 + \\ & 4*((b^3*d*f^2*x + b^3*d*e*f)*\cosh(d*x + c))^3 + (b^3*d*f^2*x + b^3*d*e*f)*c \\ & \cosh(d*x + c))*\sinh(d*x + c))*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b* \\ & \cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + (4*b^3 \\ & *d*f^2*x + 4*b^3*d*e*f - 2*I*(a^3 + 3*a*b^2)*d*f^2*x + (4*b^3*d*f^2*x + 4*b \\ & ^3*d*e*f - 2*I*(a^3 + 3*a*b^2)*d*f^2*x - 2*I*(a^3 + 3*a*b^2)*d*e*f)*\cosh(d* \\ & x + c))^4 + (16*b^3*d*f^2*x + 16*b^3*d*e*f - 8*I*(a^3 + 3*a*b^2)*d*f^2*x - 8 \\ & *I*(a^3 + 3*a*b^2)*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (4*b^3*d*f^2*x + \end{aligned}$$

$$\begin{aligned}
& 4*b^3*d*e*f - 2*I*(a^3 + 3*a*b^2)*d*f^2*x - 2*I*(a^3 + 3*a*b^2)*d*e*f)*\sinh \\
& (d*x + c)^4 - 2*I*(a^3 + 3*a*b^2)*d*e*f + (8*b^3*d*f^2*x + 8*b^3*d*e*f - 4* \\
& I*(a^3 + 3*a*b^2)*d*f^2*x - 4*I*(a^3 + 3*a*b^2)*d*e*f)*\cosh(d*x + c)^2 + (8 \\
& *b^3*d*f^2*x + 8*b^3*d*e*f - 4*I*(a^3 + 3*a*b^2)*d*f^2*x - 4*I*(a^3 + 3*a*b \\
& ^2)*d*e*f + (24*b^3*d*f^2*x + 24*b^3*d*e*f - 12*I*(a^3 + 3*a*b^2)*d*f^2*x - \\
& 12*I*(a^3 + 3*a*b^2)*d*e*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + ((16*b^3*d* \\
& f^2*x + 16*b^3*d*e*f - 8*I*(a^3 + 3*a*b^2)*d*f^2*x - 8*I*(a^3 + 3*a*b^2)*d* \\
& e*f)*\cosh(d*x + c)^3 + (16*b^3*d*f^2*x + 16*b^3*d*e*f - 8*I*(a^3 + 3*a*b^2) \\
& *d*f^2*x - 8*I*(a^3 + 3*a*b^2)*d*e*f)*\cosh(d*x + c))*\sinh(d*x + c))*\operatorname{dilog}(I \\
& *\cosh(d*x + c) + I*\sinh(d*x + c)) + (4*b^3*d*f^2*x + 4*b^3*d*e*f + 2*I*(a^3 \\
& + 3*a*b^2)*d*f^2*x + (4*b^3*d*f^2*x + 4*b^3*d*e*f + 2*I*(a^3 + 3*a*b^2)*d* \\
& f^2*x + 2*I*(a^3 + 3*a*b^2)*d*e*f)*\cosh(d*x + c)^4 + (16*b^3*d*f^2*x + 16*b \\
& ^3*d*e*f + 8*I*(a^3 + 3*a*b^2)*d*f^2*x + 8*I*(a^3 + 3*a*b^2)*d*e*f)*\cosh(d* \\
& x + c)*\sinh(d*x + c)^3 + (4*b^3*d*f^2*x + 4*b^3*d*e*f + 2*I*(a^3 + 3*a*b^2) \\
& *d*f^2*x + 2*I*(a^3 + 3*a*b^2)*d*e*f)*\sinh(d*x + c)^4 + 2*I*(a^3 + 3*a*b^2) \\
& *d*e*f + (8*b^3*d*f^2*x + 8*b^3*d*e*f + 4*I*(a^3 + 3*a*b^2)*d*f^2*x + 4*I*( \\
& a^3 + 3*a*b^2)*d*e*f)*\cosh(d*x + c)^2 + (8*b^3*d*f^2*x + 8*b^3*d*e*f + 4*I* \\
& (a^3 + 3*a*b^2)*d*f^2*x + 4*I*(a^3 + 3*a*b^2)*d*e*f + (24*b^3*d*f^2*x + 24* \\
& b^3*d*e*f + 12*I*(a^3 + 3*a*b^2)*d*f^2*x + 12*I*(a^3 + 3*a*b^2)*d*e*f)*\cosh \\
& (d*x + c)^2)*\sinh(d*x + c)^2 + ((16*b^3*d*f^2*x + 16*b^3*d*e*f + 8*I*(a^3 + \\
& 3*a*b^2)*d*f^2*x + 8*I*(a^3 + 3*a*b^2)*d*e*f)*\cosh(d*x + c)^3 + (16*b^3*d* \\
& f^2*x + 16*b^3*d*e*f + 8*I*(a^3 + 3*a*b^2)*d*f^2*x + 8*I*(a^3 + 3*a*b^2)*d* \\
& e*f)*\cosh(d*x + c))*\sinh(d*x + c))*\operatorname{dilog}(-I*\cosh(d*x + c) - I*\sinh(d*x + c) \\
& ) - 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2 + (b^3*d^2*e^2 - 2*b^3*c*d* \\
& e*f + b^3*c^2*f^2)*\cosh(d*x + c))^4 + 4*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c \\
& ^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3 \\
& *c^2*f^2)*\sinh(d*x + c)^4 + 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*c \\
& osh(d*x + c)^2 + 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2 + 3*(b^3*d^2* \\
& e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((b \\
& ^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*\cosh(d*x + c))^3 + (b^3*d^2*e^2 - \\
& 2*b^3*c*d*e*f + b^3*c^2*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2*b*\cosh(d*x \\
& + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - 2*(b^3*d^2*e \\
& ^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2 + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f \\
& ^2)*\cosh(d*x + c))^4 + 4*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*\cosh(d* \\
& x + c)*\sinh(d*x + c)^3 + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*\sinh(d \\
& *x + c)^4 + 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*\cosh(d*x + c)^2 + \\
& 2*(b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2 + 3*(b^3*d^2*e^2 - 2*b^3*c*d* \\
& e*f + b^3*c^2*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((b^3*d^2*e^2 - 2*b \\
& ^3*c*d*e*f + b^3*c^2*f^2)*\cosh(d*x + c))^3 + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + \\
& b^3*c^2*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh \\
& (d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - 2*(b^3*d^2*f^2*x^2 + 2*b^3*d \\
& ^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2 + (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x \\
& + 2*b^3*c*d*e*f - b^3*c^2*f^2)*\cosh(d*x + c))^4 + 4*(b^3*d^2*f^2*x^2 + 2*b^ \\
& 3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + \\
& (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*\sinh(d*x
\end{aligned}$$

$$\begin{aligned}
& + c)^4 + 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2) \\
& )*\cosh(d*x + c)^2 + 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - \\
& b^3*c^2*f^2 + 3*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2 \\
& *f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*((b^3*d^2*f^2*x^2 + 2*b^3*d^2*e \\
& *f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*\cosh(d*x + c)^3 + (b^3*d^2*f^2*x^2 + 2* \\
& b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*\cosh(d*x + c))*\sinh(d*x + c))* \\
& \log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c \\
& ))*\sqrt((a^2 + b^2)/b^2) - b)/b) - 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2 \\
& *b^3*c*d*e*f - b^3*c^2*f^2 + (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d \\
& *e*f - b^3*c^2*f^2)*\cosh(d*x + c)^4 + 4*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x \\
& + 2*b^3*c*d*e*f - b^3*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^3*d^2*f^2 \\
& *x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*\sinh(d*x + c)^4 + 2*( \\
& b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*\cosh(d*x + \\
& c)^2 + 2*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2 \\
& + 3*(b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*\cosh( \\
& d*x + c)^2)*\sinh(d*x + c)^2 + 4*((b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3 \\
& *c*d*e*f - b^3*c^2*f^2)*\cosh(d*x + c)^3 + (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f* \\
& x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(-(a*\cosh \\
& (d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt((a^2 \\
& + b^2)/b^2) - b)/b) + (2*b^3*d^2*e^2 - 4*b^3*c*d*e*f - I*(a^3 + 3*a*b^2)*d \\
& ^2*e^2 + 2*I*(a^3 + 3*a*b^2)*c*d*e*f + (2*b^3*d^2*e^2 - 4*b^3*c*d*e*f - I*( \\
& a^3 + 3*a*b^2)*d^2*e^2 + 2*I*(a^3 + 3*a*b^2)*c*d*e*f + 2*(b^3*c^2 - a^2*b - \\
& b^3)*f^2 + I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2)*\cosh(d*x + c)^4 \\
& + (8*b^3*d^2*e^2 - 16*b^3*c*d*e*f - 4*I*(a^3 + 3*a*b^2)*d^2*e^2 + 8*I*(a^3 \\
& + 3*a*b^2)*c*d*e*f + 8*(b^3*c^2 - a^2*b - b^3)*f^2 + 4*I*(2*a^3 + 2*a*b^2 - \\
& (a^3 + 3*a*b^2)*c^2)*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (2*b^3*d^2*e^2 - \\
& 4*b^3*c*d*e*f - I*(a^3 + 3*a*b^2)*d^2*e^2 + 2*I*(a^3 + 3*a*b^2)*c*d*e*f + \\
& 2*(b^3*c^2 - a^2*b - b^3)*f^2 + I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f \\
& ^2)*\sinh(d*x + c)^4 + 2*(b^3*c^2 - a^2*b - b^3)*f^2 + I*(2*a^3 + 2*a*b^2 - \\
& (a^3 + 3*a*b^2)*c^2)*f^2 + (4*b^3*d^2*e^2 - 8*b^3*c*d*e*f - 2*I*(a^3 + 3*a* \\
& b^2)*d^2*e^2 + 4*I*(a^3 + 3*a*b^2)*c*d*e*f + 4*(b^3*c^2 - a^2*b - b^3)*f^2 \\
& + 2*I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2)*\cosh(d*x + c)^2 + (4*b^3 \\
& *d^2*e^2 - 8*b^3*c*d*e*f - 2*I*(a^3 + 3*a*b^2)*d^2*e^2 + 4*I*(a^3 + 3*a*b^2 \\
& )*c*d*e*f + 4*(b^3*c^2 - a^2*b - b^3)*f^2 + 2*I*(2*a^3 + 2*a*b^2 - (a^3 + 3 \\
& *a*b^2)*c^2)*f^2 + (12*b^3*d^2*e^2 - 24*b^3*c*d*e*f - 6*I*(a^3 + 3*a*b^2)*d \\
& ^2*e^2 + 12*I*(a^3 + 3*a*b^2)*c*d*e*f + 12*(b^3*c^2 - a^2*b - b^3)*f^2 + 6* \\
& I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2)*\cosh(d*x + c)^2*\sinh(d*x + \\
& c)^2 + ((8*b^3*d^2*e^2 - 16*b^3*c*d*e*f - 4*I*(a^3 + 3*a*b^2)*d^2*e^2 + 8*I \\
& *(a^3 + 3*a*b^2)*c*d*e*f + 8*(b^3*c^2 - a^2*b - b^3)*f^2 + 4*I*(2*a^3 + 2*a \\
& *b^2 - (a^3 + 3*a*b^2)*c^2)*f^2)*\cosh(d*x + c)^3 + (8*b^3*d^2*e^2 - 16*b^3* \\
& c*d*e*f - 4*I*(a^3 + 3*a*b^2)*d^2*e^2 + 8*I*(a^3 + 3*a*b^2)*c*d*e*f + 8*(b^ \\
& 3*c^2 - a^2*b - b^3)*f^2 + 4*I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2) \\
& *\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + I) + (2* \\
& b^3*d^2*e^2 - 4*b^3*c*d*e*f + I*(a^3 + 3*a*b^2)*d^2*e^2 - 2*I*(a^3 + 3*a*b^ \\
& 2)*c*d*e*f + (2*b^3*d^2*e^2 - 4*b^3*c*d*e*f + I*(a^3 + 3*a*b^2)*d^2*e^2 - 2
\end{aligned}$$

$$\begin{aligned}
& *I*(a^3 + 3*a*b^2)*c*d*e*f + 2*(b^3*c^2 - a^2*b - b^3)*f^2 - I*(2*a^3 + 2*a \\
& *b^2 - (a^3 + 3*a*b^2)*c^2)*f^2)*\cosh(d*x + c)^4 + (8*b^3*d^2*e^2 - 16*b^3* \\
& c*d*e*f + 4*I*(a^3 + 3*a*b^2)*d^2*e^2 - 8*I*(a^3 + 3*a*b^2)*c*d*e*f + 8*(b^ \\
& 3*c^2 - a^2*b - b^3)*f^2 - 4*I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2) \\
& *\cosh(d*x + c)*\sinh(d*x + c)^3 + (2*b^3*d^2*e^2 - 4*b^3*c*d*e*f + I*(a^3 + \\
& 3*a*b^2)*d^2*e^2 - 2*I*(a^3 + 3*a*b^2)*c*d*e*f + 2*(b^3*c^2 - a^2*b - b^3)* \\
& f^2 - I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2)*\sinh(d*x + c)^4 + 2*(b \\
& ^3*c^2 - a^2*b - b^3)*f^2 - I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2 + \\
& (4*b^3*d^2*e^2 - 8*b^3*c*d*e*f + 2*I*(a^3 + 3*a*b^2)*d^2*e^2 - 4*I*(a^3 + \\
& 3*a*b^2)*c*d*e*f + 4*(b^3*c^2 - a^2*b - b^3)*f^2 - 2*I*(2*a^3 + 2*a*b^2 - ( \\
& a^3 + 3*a*b^2)*c^2)*f^2)*\cosh(d*x + c)^2 + (4*b^3*d^2*e^2 - 8*b^3*c*d*e*f + \\
& 2*I*(a^3 + 3*a*b^2)*d^2*e^2 - 4*I*(a^3 + 3*a*b^2)*c*d*e*f + 4*(b^3*c^2 - a \\
& ^2*b - b^3)*f^2 - 2*I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2 + (12*b^3 \\
& *d^2*e^2 - 24*b^3*c*d*e*f + 6*I*(a^3 + 3*a*b^2)*d^2*e^2 - 12*I*(a^3 + 3*a*b \\
& ^2)*c*d*e*f + 12*(b^3*c^2 - a^2*b - b^3)*f^2 - 6*I*(2*a^3 + 2*a*b^2 - (a^3 \\
& + 3*a*b^2)*c^2)*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + ((8*b^3*d^2*e^2 - 1 \\
& 6*b^3*c*d*e*f + 4*I*(a^3 + 3*a*b^2)*d^2*e^2 - 8*I*(a^3 + 3*a*b^2)*c*d*e*f + \\
& 8*(b^3*c^2 - a^2*b - b^3)*f^2 - 4*I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2 \\
& )*f^2)*\cosh(d*x + c)^3 + (8*b^3*d^2*e^2 - 16*b^3*c*d*e*f + 4*I*(a^3 + 3*a*b \\
& ^2)*d^2*e^2 - 8*I*(a^3 + 3*a*b^2)*c*d*e*f + 8*(b^3*c^2 - a^2*b - b^3)*f^2 - \\
& 4*I*(2*a^3 + 2*a*b^2 - (a^3 + 3*a*b^2)*c^2)*f^2)*\cosh(d*x + c))*\sinh(d*x + \\
& c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - I) + (2*b^3*d^2*f^2*x^2 + 4*b^3*d^ \\
& 2*e*f*x + 4*b^3*c*d*e*f - 2*b^3*c^2*f^2 + I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 + 2 \\
& *I*(a^3 + 3*a*b^2)*d^2*e*f*x + 2*I*(a^3 + 3*a*b^2)*c*d*e*f - I*(a^3 + 3*a*b \\
& ^2)*c^2*f^2 + (2*b^3*d^2*f^2*x^2 + 4*b^3*d^2*e*f*x + 4*b^3*c*d*e*f - 2*b^3* \\
& c^2*f^2 + I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 + 2*I*(a^3 + 3*a*b^2)*d^2*e*f*x + 2 \\
& *I*(a^3 + 3*a*b^2)*c*d*e*f - I*(a^3 + 3*a*b^2)*c^2*f^2)*\cosh(d*x + c)^4 + ( \\
& 8*b^3*d^2*f^2*x^2 + 16*b^3*d^2*e*f*x + 16*b^3*c*d*e*f - 8*b^3*c^2*f^2 + 4*I \\
& *(a^3 + 3*a*b^2)*d^2*f^2*x^2 + 8*I*(a^3 + 3*a*b^2)*d^2*e*f*x + 8*I*(a^3 + 3 \\
& *a*b^2)*c*d*e*f - 4*I*(a^3 + 3*a*b^2)*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^ \\
& 3 + (2*b^3*d^2*f^2*x^2 + 4*b^3*d^2*e*f*x + 4*b^3*c*d*e*f - 2*b^3*c^2*f^2 + \\
& I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 + 2*I*(a^3 + 3*a*b^2)*d^2*e*f*x + 2*I*(a^3 + \\
& 3*a*b^2)*c*d*e*f - I*(a^3 + 3*a*b^2)*c^2*f^2)*\sinh(d*x + c)^4 + (4*b^3*d^2* \\
& f^2*x^2 + 8*b^3*d^2*e*f*x + 8*b^3*c*d*e*f - 4*b^3*c^2*f^2 + 2*I*(a^3 + 3*a* \\
& b^2)*d^2*f^2*x^2 + 4*I*(a^3 + 3*a*b^2)*d^2*e*f*x + 4*I*(a^3 + 3*a*b^2)*c*d* \\
& e*f - 2*I*(a^3 + 3*a*b^2)*c^2*f^2)*\cosh(d*x + c)^2 + (4*b^3*d^2*f^2*x^2 + 8 \\
& *b^3*d^2*e*f*x + 8*b^3*c*d*e*f - 4*b^3*c^2*f^2 + 2*I*(a^3 + 3*a*b^2)*d^2*f^ \\
& 2*x^2 + 4*I*(a^3 + 3*a*b^2)*d^2*e*f*x + 4*I*(a^3 + 3*a*b^2)*c*d*e*f - 2*I*( \\
& a^3 + 3*a*b^2)*c^2*f^2 + (12*b^3*d^2*f^2*x^2 + 24*b^3*d^2*e*f*x + 24*b^3*c* \\
& d*e*f - 12*b^3*c^2*f^2 + 6*I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 + 12*I*(a^3 + 3*a* \\
& b^2)*d^2*e*f*x + 12*I*(a^3 + 3*a*b^2)*c*d*e*f - 6*I*(a^3 + 3*a*b^2)*c^2*f^2 \\
& )*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + ((8*b^3*d^2*f^2*x^2 + 16*b^3*d^2*e*f*x \\
& + 16*b^3*c*d*e*f - 8*b^3*c^2*f^2 + 4*I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 + 8*I*( \\
& a^3 + 3*a*b^2)*d^2*e*f*x + 8*I*(a^3 + 3*a*b^2)*c*d*e*f - 4*I*(a^3 + 3*a*b^2 \\
& )*c^2*f^2)*\cosh(d*x + c)^3 + (8*b^3*d^2*f^2*x^2 + 16*b^3*d^2*e*f*x + 16*b^3
\end{aligned}$$



$$\begin{aligned}
& *c*d*e*f - 8*b^3*c^2*f^2 + 4*I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 + 8*I*(a^3 + 3*a \\
& *b^2)*d^2*e*f*x + 8*I*(a^3 + 3*a*b^2)*c*d*e*f - 4*I*(a^3 + 3*a*b^2)*c^2*f^2 \\
& )*\cosh(d*x + c))*\sinh(d*x + c))*\log(I*\cosh(d*x + c) + I*\sinh(d*x + c) + 1) \\
& + (2*b^3*d^2*f^2*x^2 + 4*b^3*d^2*e*f*x + 4*b^3*c*d*e*f - 2*b^3*c^2*f^2 - I* \\
& (a^3 + 3*a*b^2)*d^2*f^2*x^2 - 2*I*(a^3 + 3*a*b^2)*d^2*e*f*x - 2*I*(a^3 + 3* \\
& a*b^2)*c*d*e*f + I*(a^3 + 3*a*b^2)*c^2*f^2 + (2*b^3*d^2*f^2*x^2 + 4*b^3*d^2 \\
& *e*f*x + 4*b^3*c*d*e*f - 2*b^3*c^2*f^2 - I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 - 2* \\
& I*(a^3 + 3*a*b^2)*d^2*e*f*x - 2*I*(a^3 + 3*a*b^2)*c*d*e*f + I*(a^3 + 3*a*b^ \\
& 2)*c^2*f^2)*\cosh(d*x + c)^4 + (8*b^3*d^2*f^2*x^2 + 16*b^3*d^2*e*f*x + 16*b^ \\
& 3*c*d*e*f - 8*b^3*c^2*f^2 - 4*I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 - 8*I*(a^3 + 3* \\
& a*b^2)*d^2*e*f*x - 8*I*(a^3 + 3*a*b^2)*c*d*e*f + 4*I*(a^3 + 3*a*b^2)*c^2*f^ \\
& 2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (2*b^3*d^2*f^2*x^2 + 4*b^3*d^2*e*f*x + 4 \\
& *b^3*c*d*e*f - 2*b^3*c^2*f^2 - I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 - 2*I*(a^3 + 3 \\
& *a*b^2)*d^2*e*f*x - 2*I*(a^3 + 3*a*b^2)*c*d*e*f + I*(a^3 + 3*a*b^2)*c^2*f^2 \\
& )*\sinh(d*x + c)^4 + (4*b^3*d^2*f^2*x^2 + 8*b^3*d^2*e*f*x + 8*b^3*c*d*e*f - \\
& 4*b^3*c^2*f^2 - 2*I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 - 4*I*(a^3 + 3*a*b^2)*d^2*e \\
& *f*x - 4*I*(a^3 + 3*a*b^2)*c*d*e*f + 2*I*(a^3 + 3*a*b^2)*c^2*f^2)*\cosh(d*x \\
& + c)^2 + (4*b^3*d^2*f^2*x^2 + 8*b^3*d^2*e*f*x + 8*b^3*c*d*e*f - 4*b^3*c^2*f \\
& ^2 - 2*I*(a^3 + 3*a*b^2)*d^2*f^2*x^2 - 4*I*(a^3 + 3*a*b^2)*d^2*e*f*x - 4*I* \\
& (a^3 + 3*a*b^2)*c*d*e*f + 2*I*(a^3 + 3*a*b^2)*c^2*f^2 + (12*b^3*d^2*f^2*x^2 \\
& + 24*b^3*d^2*e*f*x + 24*b^3*c*d*e*f - 12*b^3*c^2*f^2 - 6*I*(a^3 + 3*a*b^2) \\
& *d^2*f^2*x^2 - 12*I*(a^3 + 3*a*b^2)*d^2*e*f*x - 12*I*(a^3 + 3*a*b^2)*c*d*e* \\
& f + 6*I*(a^3 + 3*a*b^2)*c^2*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + ((8*b^3 \\
& *d^2*f^2*x^2 + 16*b^3*d^2*e*f*x + 16*b^3*c*d*e*f - 8*b^3*c^2*f^2 - 4*I*(a^3 \\
& + 3*a*b^2)*d^2*f^2*x^2 - 8*I*(a^3 + 3*a*b^2)*d^2*e*f*x - 8*I*(a^3 + 3*a*b^ \\
& 2)*c*d*e*f + 4*I*(a^3 + 3*a*b^2)*c^2*f^2)*\cosh(d*x + c)^3 + (8*b^3*d^2*f^2* \\
& x^2 + 16*b^3*d^2*e*f*x + 16*b^3*c*d*e*f - 8*b^3*c^2*f^2 - 4*I*(a^3 + 3*a*b^ \\
& 2)*d^2*f^2*x^2 - 8*I*(a^3 + 3*a*b^2)*d^2*e*f*x - 8*I*(a^3 + 3*a*b^2)*c*d*e* \\
& f + 4*I*(a^3 + 3*a*b^2)*c^2*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(-I*\cosh( \\
& d*x + c) - I*\sinh(d*x + c) + 1) + 4*(b^3*f^2*\cosh(d*x + c)^4 + 4*b^3*f^2*co \\
& sh(d*x + c)*\sinh(d*x + c)^3 + b^3*f^2*\sinh(d*x + c)^4 + 2*b^3*f^2*\cosh(d*x \\
& + c)^2 + b^3*f^2 + 2*(3*b^3*f^2*\cosh(d*x + c)^2 + b^3*f^2)*\sinh(d*x + c)^2 \\
& + 4*(b^3*f^2*\cosh(d*x + c)^3 + b^3*f^2*\cosh(d*x + c))*\sinh(d*x + c))*polylo \\
& g(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c \\
& ))*\sqrt{(a^2 + b^2)/b^2}))/b) + 4*(b^3*f^2*\cosh(d*x + c)^4 + 4*b^3*f^2*\cosh( \\
& d*x + c)*\sinh(d*x + c)^3 + b^3*f^2*\sinh(d*x + c)^4 + 2*b^3*f^2*\cosh(d*x + c \\
& )^2 + b^3*f^2 + 2*(3*b^3*f^2*\cosh(d*x + c)^2 + b^3*f^2)*\sinh(d*x + c)^2 + 4 \\
& *(b^3*f^2*\cosh(d*x + c)^3 + b^3*f^2*\cosh(d*x + c))*\sinh(d*x + c))*polylog(3 \\
& , (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))* \\
& \sqrt{(a^2 + b^2)/b^2}))/b) - (4*b^3*f^2 + (4*b^3*f^2 - 2*I*(a^3 + 3*a*b^2)*f \\
& ^2)*\cosh(d*x + c)^4 + (16*b^3*f^2 - 8*I*(a^3 + 3*a*b^2)*f^2)*\cosh(d*x + c)* \\
& \sinh(d*x + c)^3 + (4*b^3*f^2 - 2*I*(a^3 + 3*a*b^2)*f^2)*\sinh(d*x + c)^4 - 2 \\
& *I*(a^3 + 3*a*b^2)*f^2 + (8*b^3*f^2 - 4*I*(a^3 + 3*a*b^2)*f^2)*\cosh(d*x + c \\
& )^2 + (8*b^3*f^2 - 4*I*(a^3 + 3*a*b^2)*f^2 + (24*b^3*f^2 - 12*I*(a^3 + 3*a* \\
& b^2)*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + ((16*b^3*f^2 - 8*I*(a^3 + 3*a*
\end{aligned}$$

```

b^2)*f^2)*cosh(d*x + c)^3 + (16*b^3*f^2 - 8*I*(a^3 + 3*a*b^2)*f^2)*cosh(d*x
+ c))*sinh(d*x + c))*polylog(3, I*cosh(d*x + c) + I*sinh(d*x + c)) - (4*b^
3*f^2 + (4*b^3*f^2 + 2*I*(a^3 + 3*a*b^2)*f^2)*cosh(d*x + c)^4 + (16*b^3*f^2
+ 8*I*(a^3 + 3*a*b^2)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*b^3*f^2 + 2*
I*(a^3 + 3*a*b^2)*f^2)*sinh(d*x + c)^4 + 2*I*(a^3 + 3*a*b^2)*f^2 + (8*b^3*f
^2 + 4*I*(a^3 + 3*a*b^2)*f^2)*cosh(d*x + c)^2 + (8*b^3*f^2 + 4*I*(a^3 + 3*a
*b^2)*f^2 + (24*b^3*f^2 + 12*I*(a^3 + 3*a*b^2)*f^2)*cosh(d*x + c)^2)*sinh(d
*x + c)^2 + ((16*b^3*f^2 + 8*I*(a^3 + 3*a*b^2)*f^2)*cosh(d*x + c)^3 + (16*b
^3*f^2 + 8*I*(a^3 + 3*a*b^2)*f^2)*cosh(d*x + c))*sinh(d*x + c))*polylog(3,
-I*cosh(d*x + c) - I*sinh(d*x + c)) + 2*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 +
a*b^2)*d^2*e^2 - 2*(a^3 + a*b^2)*d*e*f + 8*((a^2*b + b^3)*d*f^2*x + (a^2*b
+ b^3)*c*f^2)*cosh(d*x + c)^3 - 3*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a*b^
2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^3 + a*b^2)*d^2*e*f + (a^3 + a*b^
2)*d*f^2)*x)*cosh(d*x + c)^2 + 2*((a^3 + a*b^2)*d^2*e*f - (a^3 + a*b^2)*d*f
^2)*x - 4*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 + (a^2*b + b^3
)*d*e*f - 2*(a^2*b + b^3)*c*f^2 + (2*(a^2*b + b^3)*d^2*e*f - (a^2*b + b^3)*
d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d
*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)*sinh(d*x + c)^3 + (
a^4 + 2*a^2*b^2 + b^4)*d^3*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*
cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^3 + 2*(3*(a^4 + 2*a^2*b^2 + b^4
)*d^3*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^3)*sinh(d*x + c)^2 + 4*((
a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d^3*co
sh(d*x + c))*sinh(d*x + c))

```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \operatorname{sech}(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*sech(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

$$3.315 \quad \int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=560

$$\frac{b^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2 (a^2+b^2)^2} + \frac{b^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2 (a^2+b^2)^2} - \frac{b^3 f \operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{2d^2 (a^2+b^2)^2} - \frac{iab^2 f \operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{d^2 (a^2+b^2)^2} +$$

[Out] (2\*a\*b^2\*(e + f\*x)\*ArcTan[E^(c + d\*x)])/((a^2 + b^2)^2\*d) + (a\*(e + f\*x)\*ArcTan[E^(c + d\*x)])/((a^2 + b^2)\*d) + (b^3\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^2\*d) + (b^3\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2\*d) - (b^3\*(e + f\*x)\*Log[1 + E^(2\*(c + d\*x))])/((a^2 + b^2)^2\*d) - (I\*a\*b^2\*f\*PolyLog[2, (-I)\*E^(c + d\*x)])/((a^2 + b^2)^2\*d^2) - ((I/2)\*a\*f\*PolyLog[2, (-I)\*E^(c + d\*x)])/((a^2 + b^2)\*d^2) + (I\*a\*b^2\*f\*PolyLog[2, I\*E^(c + d\*x)])/((a^2 + b^2)^2\*d^2) + ((I/2)\*a\*f\*PolyLog[2, I\*E^(c + d\*x)])/((a^2 + b^2)\*d^2) + (b^3\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2\*d^2) + (b^3\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2\*d^2) - (b^3\*f\*PolyLog[2, -E^(2\*(c + d\*x))])/((2\*(a^2 + b^2)^2\*d^2) + (a\*f\*Sech[c + d\*x])/(2\*(a^2 + b^2)\*d^2) + (b\*(e + f\*x)\*Sech[c + d\*x]^2)/(2\*(a^2 + b^2)\*d) - (b\*f\*Tanh[c + d\*x])/(2\*(a^2 + b^2)\*d^2) + (a\*(e + f\*x)\*Sech[c + d\*x]\*Tanh[c + d\*x])/(2\*(a^2 + b^2)\*d)

**Rubi [A]** time = 0.938346, antiderivative size = 560, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 12, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$ , Rules used = {5573, 5561, 2190, 2279, 2391, 6742, 4180, 3718, 4185, 5451, 3767, 8}

$$\frac{b^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2 (a^2+b^2)^2} + \frac{b^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2 (a^2+b^2)^2} - \frac{b^3 f \operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{2d^2 (a^2+b^2)^2} - \frac{iab^2 f \operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{d^2 (a^2+b^2)^2} +$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Sech[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]), x]

[Out] (2\*a\*b^2\*(e + f\*x)\*ArcTan[E^(c + d\*x)])/((a^2 + b^2)^2\*d) + (a\*(e + f\*x)\*ArcTan[E^(c + d\*x)])/((a^2 + b^2)\*d) + (b^3\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^2\*d) + (b^3\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2\*d) - (b^3\*(e + f\*x)\*Log[1 + E^(2\*(c + d\*x))])/((a^2 + b^2)^2\*d) - (I\*a\*b^2\*f\*PolyLog[2, (-I)\*E^(c + d\*x)])/((a^2 + b^2)^2\*d^2) - ((I/2)\*a\*f\*PolyLog[2, (-I)\*E^(c + d\*x)])/((a^2 +

$$b^2)d^2) + (I*a*b^2*f*PolyLog[2, I*E^(c + d*x)]/((a^2 + b^2)^2*d^2) + ((I/2)*a*f*PolyLog[2, I*E^(c + d*x)]/((a^2 + b^2)*d^2) + (b^3*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)^2*d^2) + (b^3*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 + b^2)^2*d^2) - (b^3*f*PolyLog[2, -E^(2*(c + d*x))]/(2*(a^2 + b^2)^2*d^2) + (a*f*Sech[c + d*x])/(2*(a^2 + b^2)*d^2) + (b*(e + f*x)*Sech[c + d*x]^2)/(2*(a^2 + b^2)*d) - (b*f*Tanh[c + d*x])/(2*(a^2 + b^2)*d^2) + (a*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*(a^2 + b^2)*d)$$

### Rule 5573

```
Int[(((e_.) + (f_.)*(x_.))^(m_.)*Sech[(c_.) + (d_.)*(x_.)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^(n*(a - b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_.))))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_.))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_.))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_.)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^m_., x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[(c
+ d*x)^m*E^(2*(-I*e) + f*fz*x))/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_)), x_Symbol] :=
-Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] := -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\operatorname{sech}^3(c+dx)(a-b\sinh(c+dx)) dx}{a^2+b^2} + \frac{b^2 \int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx}{a^2+b^2} \\
 &= \frac{b^2 \int (e+fx)\operatorname{sech}(c+dx)(a-b\sinh(c+dx)) dx}{(a^2+b^2)^2} + \frac{b^4 \int \frac{(e+fx)\cosh(c+dx)}{a+b\sinh(c+dx)} dx}{(a^2+b^2)^2} + \frac{\int (a(e+fx) - b^2 \frac{e^{c+dx}}{a-\sqrt{a^2+b^2}}) dx}{(a^2+b^2)^2} \\
 &= -\frac{b^3(e+fx)^2}{2(a^2+b^2)^2 f} + \frac{b^2 \int (a(e+fx)\operatorname{sech}(c+dx) - b(e+fx)\tanh(c+dx)) dx}{(a^2+b^2)^2} + \frac{b^4 \int \frac{e^{c+dx}}{a-\sqrt{a^2+b^2}} dx}{(a^2+b^2)^2} \\
 &= -\frac{b^3(e+fx)^2}{2(a^2+b^2)^2 f} + \frac{b^3(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} + \frac{b^3(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} + \frac{af \operatorname{sech}(c+dx)}{2(a^2+b^2)} \\
 &= \frac{2ab^2(e+fx) \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} + \frac{a(e+fx) \tan^{-1}(e^{c+dx})}{(a^2+b^2) d} + \frac{b^3(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} + \frac{b^3(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} \\
 &= \frac{2ab^2(e+fx) \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} + \frac{a(e+fx) \tan^{-1}(e^{c+dx})}{(a^2+b^2) d} + \frac{b^3(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} + \frac{b^3(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} \\
 &= \frac{2ab^2(e+fx) \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} + \frac{a(e+fx) \tan^{-1}(e^{c+dx})}{(a^2+b^2) d} + \frac{b^3(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} + \frac{b^3(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} \\
 &= \frac{2ab^2(e+fx) \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} + \frac{a(e+fx) \tan^{-1}(e^{c+dx})}{(a^2+b^2) d} + \frac{b^3(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d} + \frac{b^3(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2 d}
 \end{aligned}$$

**Mathematica [A]** time = 7.07565, size = 588, normalized size = 1.05

$$\frac{-iaf(a^2+3b^2)\operatorname{PolyLog}\left(2, -ie^{c+dx}\right) + iaf(a^2+3b^2)\operatorname{PolyLog}\left(2, ie^{c+dx}\right) + 2b^3 f \operatorname{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) + 2b^3 f \operatorname{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Sech[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]), x]

```
[Out] (2*b^3*d*e*(c + d*x) - 2*b^3*c*f*(c + d*x) + 2*a^3*d*e*ArcTan[E^(c + d*x)]
+ 6*a*b^2*d*e*ArcTan[E^(c + d*x)] - 2*a^3*c*f*ArcTan[E^(c + d*x)] - 6*a*b^2
*c*f*ArcTan[E^(c + d*x)] + I*a^3*f*(c + d*x)*Log[1 - I*E^(c + d*x)] + (3*I)
*a*b^2*f*(c + d*x)*Log[1 - I*E^(c + d*x)] - I*a^3*f*(c + d*x)*Log[1 + I*E^(
c + d*x)] - (3*I)*a*b^2*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + 2*b^3*f*(c + d
*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*b^3*f*(c + d*x)*Log[
1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*b^3*d*e*Log[1 + E^(2*(c + d*
x))] + 2*b^3*c*f*Log[1 + E^(2*(c + d*x))] - 2*b^3*f*(c + d*x)*Log[1 + E^(2*
(c + d*x))] + 2*b^3*d*e*Log[a + b*Sinh[c + d*x]] - 2*b^3*c*f*Log[a + b*Sinh
[c + d*x]] - I*a*(a^2 + 3*b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] + I*a*(a^2 +
3*b^2)*f*PolyLog[2, I*E^(c + d*x)] + 2*b^3*f*PolyLog[2, (b*E^(c + d*x))/(-a
+ Sqrt[a^2 + b^2])] + 2*b^3*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))] - b^3*f*PolyLog[2, -E^(2*(c + d*x))] + (a^2 + b^2)*d*(e + f*x)*Sec
h[c + d*x]^2*(b + a*Sinh[c + d*x]) + (a^2 + b^2)*f*Sech[c + d*x]*(a - b*Sin
h[c + d*x]))/(2*(a^2 + b^2)^2*d^2)
```

---

**Maple [B]** time = 0.176, size = 2051, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] 2/(a^2+b^2)/d^2*b^3*f/(2*a^2+2*b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/
(a+(a^2+b^2)^(1/2)))-2/(a^2+b^2)/d*b^3*e/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))
+2/(a^2+b^2)/d*b^3*e/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/
(a^2+b^2)/d*a^3*e/(2*a^2+2*b^2)*arctan(exp(d*x+c))-2/(a^2+b^2)/d^2*b^3*f/(2
*a^2+2*b^2)*dilog(1+I*exp(d*x+c))-2/(a^2+b^2)/d^2*b^3*f/(2*a^2+2*b^2)*dilog
(1-I*exp(d*x+c))+2/(a^2+b^2)/d^2*b^3*f/(2*a^2+2*b^2)*dilog((-b*exp(d*x+c)+(
a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/(a^2+b^2)/d*b^3*f/(2*a^2+2*b^2)*l
n(1+I*exp(d*x+c))*x-2/(a^2+b^2)/d^2*b^3*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*
c-2/(a^2+b^2)/d*b^3*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x-2/(a^2+b^2)/d^2*b^
3*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c+2/(a^2+b^2)/d^2*b^3*f*c/(2*a^2+2*b^2
)*ln(1+exp(2*d*x+2*c))-2/(a^2+b^2)/d^2*b^3*f*c/(2*a^2+2*b^2)*ln(b*exp(2*d*x
+2*c)+2*a*exp(d*x+c)-b)+I/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*dilog(1-I*exp(d
*x+c))-2/(a^2+b^2)/d^2*a^3*f*c/(2*a^2+2*b^2)*arctan(exp(d*x+c))-I/(a^2+b^2)
/d^2*a^3*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))+1/(a^2+b^2)^(3/2)/d^2*a*b^3*
f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+
b^2)^(3/2)/d^2*a^3*b*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^
2+b^2)^(1/2))-1/(a^2+b^2)^(1/2)/d^2*a*b*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*
exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+3*I/(a^2+b^2)/d^2*a*b^2*f/(2*a^2+2*b^2)*ln
```



$$\begin{aligned}
& (1-I\exp(d*x+c))*c-3*I/(a^2+b^2)/d*a*b^2*f/(2*a^2+2*b^2)*\ln(1+I\exp(d*x+c)) \\
& *x-3*I/(a^2+b^2)/d^2*a*b^2*f/(2*a^2+2*b^2)*\ln(1+I\exp(d*x+c))*c+3*I/(a^2+b^2) \\
& /d*a*b^2*f/(2*a^2+2*b^2)*\ln(1-I\exp(d*x+c))*x-3*I/(a^2+b^2)/d^2*a*b^2*f/(2*a^2+2*b^2) \\
& *dilog(1+I\exp(d*x+c))+3*I/(a^2+b^2)/d^2*a*b^2*f/(2*a^2+2*b^2)*dilog(1-I\exp(d*x+c)) \\
& -I/(a^2+b^2)/d*a^3*f/(2*a^2+2*b^2)*\ln(1+I\exp(d*x+c))*x-I/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2) \\
& *\ln(1+I\exp(d*x+c))*c-1/(a^2+b^2)^(3/2)/d*a*b^3*e/(2*a^2+2*b^2)*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^(1/2)) \\
& -1/(a^2+b^2)^(3/2)/d*a^3*b*e/(2*a^2+2*b^2)*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^(1/2)) \\
& /d*a*b^2*f*c/(2*a^2+2*b^2)*\operatorname{arctan}(\exp(d*x+c))+I/(a^2+b^2)/d*a^3*f/(2*a^2+2*b^2)*\ln(1-I\exp(d*x+c)) \\
& *x+I/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*\ln(1-I\exp(d*x+c))*c+1/(a^2+b^2)^(1/2)/d*a*b*e/(2*a^2+2*b^2) \\
& *\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+6/(a^2+b^2)/d*a*b^2*e/(2*a^2+2*b^2) \\
& *\operatorname{arctan}(\exp(d*x+c))+2/(a^2+b^2)/d*b^3*f/(2*a^2+2*b^2)*\ln((-b*\exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))) \\
& *x+2/(a^2+b^2)/d^2*b^3*f/(2*a^2+2*b^2)*\ln((-b*\exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))) \\
& *c+2/(a^2+b^2)/d*b^3*f/(2*a^2+2*b^2)*\ln((b*\exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))) \\
& *x+2/(a^2+b^2)/d^2*b^3*f/(2*a^2+2*b^2)*\ln((b*\exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))) \\
& *c+(a*d*f*x*\exp(3*d*x+3*c)+a*d*e*\exp(3*d*x+3*c)+2*b*d*f*x*\exp(2*d*x+2*c)-a*d*f*x*\exp(d*x+c)+a*f*\exp(3*d*x+3*c)+2*b \\
& *d*e*\exp(2*d*x+2*c)-a*d*e*\exp(d*x+c)+b*f*\exp(2*d*x+2*c)+a*f*\exp(d*x+c)+b*f)/d^2/(a^2+b^2)/(1+\exp(2*d*x+2*c))^2
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left( \frac{b^3 \log(-2ae^{-dx-c}) + be^{(-2dx-2c)} - b}{(a^4 + 2a^2b^2 + b^4)d} - \frac{b^3 \log(e^{(-2dx-2c)} + 1)}{(a^4 + 2a^2b^2 + b^4)d} - \frac{(a^3 + 3ab^2) \arctan(e^{-dx-c})}{(a^4 + 2a^2b^2 + b^4)d} + \frac{ae^{-dx-c} + 2b^2}{(a^2 + b^2 + 2(a^2 + b^2))} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $(b^3 \log(-2*a*e^{-d*x-c}) + b*e^{(-2*d*x-2*c)} - b)/((a^4 + 2*a^2*b^2 + b^4)*d) - b^3 \log(e^{(-2*d*x-2*c)} + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^3 + 3*a*b^2)*\arctan(e^{(-d*x-c)})/((a^4 + 2*a^2*b^2 + b^4)*d) + (a*e^{(-d*x-c)} + 2*b*e^{(-2*d*x-2*c)} - a*e^{(-3*d*x-3*c)})/((a^2 + b^2 + 2*(a^2 + b^2))*e^{(-2*d*x-2*c)} + (a^2 + b^2)*e^{(-4*d*x-4*c)})*d)*e + f*((a*d*x*e^{(3*c)} + a*e^{(3*c)})*e^{(3*d*x)} + (2*b*d*x*e^{(2*c)} + b*e^{(2*c)})*e^{(2*d*x)} - (a*d*x*e^c - a*e^c)*e^{(d*x)} + b)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^{(4*c)} + b^2*d^2*e^{(4*c)})*e^{(4*d*x)} + 2*(a^2*d^2*e^{(2*c)} + b^2*d^2*e^{(2*c)})*e^{(2*d*x)}) - 8*\integrate(-1/4*(a*b^3*x*e^{(d*x+c)} - b^4*x)/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b*e^{(2*c)} + 2*a^2*b^3*e^{(2*c)} + b^5*e^{(2*c)})*e^{(2*d*x)} - 2*(a^5*e^c + 2*a^3*b^2*e^c + a*b^4*e^c)*e^{(d*x)}), x) + 8*\integrate(1/8*(2*b^3*x + (a^3*e^c + 3$

```
*a*b^2*e^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c) + 2*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x)
```

**Fricas [B]** time = 3.51499, size = 11385, normalized size = 20.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a*b^2)*f)*cosh(d*x + c)^3 + 2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a*b^2)*f)*sinh(d*x + c)^3 + 2*(2*(a^2*b + b^3)*d*f*x + 2*(a^2*b + b^3)*d*e + (a^2*b + b^3)*f)*cosh(d*x + c)^2 + 2*(2*(a^2*b + b^3)*d*f*x + 2*(a^2*b + b^3)*d*e + (a^2*b + b^3)*f) + 3*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a*b^2)*f)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a^2*b + b^3)*f - 2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e - (a^3 + a*b^2)*f)*cosh(d*x + c) + 2*(b^3*f*cosh(d*x + c)^4 + 4*b^3*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*f*sinh(d*x + c)^4 + 2*b^3*f*cosh(d*x + c)^2 + b^3*f + 2*(3*b^3*f*cosh(d*x + c)^2 + b^3*f)*sinh(d*x + c)^2 + 4*(b^3*f*cosh(d*x + c)^3 + b^3*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(b^3*f*cosh(d*x + c)^4 + 4*b^3*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*f*sinh(d*x + c)^4 + 2*b^3*f*cosh(d*x + c)^2 + b^3*f + 2*(3*b^3*f*cosh(d*x + c)^2 + b^3*f)*sinh(d*x + c)^2 + 4*(b^3*f*cosh(d*x + c)^3 + b^3*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - ((2*b^3*f - I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)^4 + (8*b^3*f - 4*I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*b^3*f - I*(a^3 + 3*a*b^2)*f)*sinh(d*x + c)^4 + 2*b^3*f + (4*b^3*f - 2*I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)^2 + (4*b^3*f + (12*b^3*f - 6*I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)^2 - 2*I*(a^3 + 3*a*b^2)*f)*sinh(d*x + c)^2 - I*(a^3 + 3*a*b^2)*f + ((8*b^3*f - 4*I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)^3 + (8*b^3*f - 4*I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - ((2*b^3*f + I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)^4 + (8*b^3*f + 4*I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*b^3*f + I*(a^3 + 3*a*b^2)*f)*sinh(d*x + c)^4 + 2*b^3*f + (4*b^3*f + 2*I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)^2 + (4*b^3*f + (12*b^3*f + 6*I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)^2 + 2*I*(a^3 + 3*a*b^2)*f)*sinh(d*x + c)^2 + I*(a^3 + 3*a*b^2)*f + ((8*b^3*f + 4*I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c)^3 + (8*b^3*f + 4*I*(a^3 + 3*a*b^2)*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + 2*(b^3*d*e - b^3*c*f + (b^3*d*e - b^3*c*f)*cosh(d*x + c)^4 + 4*(b^3*d
```

$$\begin{aligned}
& *e - b^3*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^3*d*e - b^3*c*f)*\sinh(d*x \\
& + c)^4 + 2*(b^3*d*e - b^3*c*f)*\cosh(d*x + c)^2 + 2*(b^3*d*e - b^3*c*f + 3*( \\
& b^3*d*e - b^3*c*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((b^3*d*e - b^3*c*f \\
& )*\cosh(d*x + c)^3 + (b^3*d*e - b^3*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2 \\
& *b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 2 \\
& *(b^3*d*e - b^3*c*f + (b^3*d*e - b^3*c*f)*\cosh(d*x + c)^4 + 4*(b^3*d*e - b^ \\
& 3*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^3*d*e - b^3*c*f)*\sinh(d*x + c)^4 \\
& + 2*(b^3*d*e - b^3*c*f)*\cosh(d*x + c)^2 + 2*(b^3*d*e - b^3*c*f + 3*(b^3*d*e \\
& - b^3*c*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((b^3*d*e - b^3*c*f)*\cosh( \\
& d*x + c)^3 + (b^3*d*e - b^3*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2*b*\cosh \\
& (d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 2*(b^3*d \\
& *f*x + b^3*c*f + (b^3*d*f*x + b^3*c*f)*\cosh(d*x + c)^4 + 4*(b^3*d*f*x + b^3 \\
& *c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^3*d*f*x + b^3*c*f)*\sinh(d*x + c)^4 \\
& + 2*(b^3*d*f*x + b^3*c*f)*\cosh(d*x + c)^2 + 2*(b^3*d*f*x + b^3*c*f + 3*(b^ \\
& 3*d*f*x + b^3*c*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((b^3*d*f*x + b^3*c \\
& *f)*\cosh(d*x + c)^3 + (b^3*d*f*x + b^3*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(- \\
& (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c) \\
& )*\sqrt{(a^2 + b^2)/b^2} - b)/b) + 2*(b^3*d*f*x + b^3*c*f + (b^3*d*f*x + b^3 \\
& *c*f)*\cosh(d*x + c)^4 + 4*(b^3*d*f*x + b^3*c*f)*\cosh(d*x + c)*\sinh(d*x + c) \\
& ^3 + (b^3*d*f*x + b^3*c*f)*\sinh(d*x + c)^4 + 2*(b^3*d*f*x + b^3*c*f)*\cosh(d \\
& *x + c)^2 + 2*(b^3*d*f*x + b^3*c*f + 3*(b^3*d*f*x + b^3*c*f)*\cosh(d*x + c)^ \\
& 2)*\sinh(d*x + c)^2 + 4*((b^3*d*f*x + b^3*c*f)*\cosh(d*x + c)^3 + (b^3*d*f*x \\
& + b^3*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x \\
& + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - \\
& (2*b^3*d*e - 2*b^3*c*f + (2*b^3*d*e - 2*b^3*c*f - I*(a^3 + 3*a*b^2)*d*e + \\
& I*(a^3 + 3*a*b^2)*c*f)*\cosh(d*x + c)^4 + (8*b^3*d*e - 8*b^3*c*f - 4*I*(a^3 \\
& + 3*a*b^2)*d*e + 4*I*(a^3 + 3*a*b^2)*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ( \\
& 2*b^3*d*e - 2*b^3*c*f - I*(a^3 + 3*a*b^2)*d*e + I*(a^3 + 3*a*b^2)*c*f)*\sinh \\
& (d*x + c)^4 - I*(a^3 + 3*a*b^2)*d*e + I*(a^3 + 3*a*b^2)*c*f + (4*b^3*d*e - \\
& 4*b^3*c*f - 2*I*(a^3 + 3*a*b^2)*d*e + 2*I*(a^3 + 3*a*b^2)*c*f)*\cosh(d*x + c \\
& )^2 + (4*b^3*d*e - 4*b^3*c*f - 2*I*(a^3 + 3*a*b^2)*d*e + 2*I*(a^3 + 3*a*b^2 \\
& )*c*f + (12*b^3*d*e - 12*b^3*c*f - 6*I*(a^3 + 3*a*b^2)*d*e + 6*I*(a^3 + 3*a \\
& *b^2)*c*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + ((8*b^3*d*e - 8*b^3*c*f - 4*I \\
& *(a^3 + 3*a*b^2)*d*e + 4*I*(a^3 + 3*a*b^2)*c*f)*\cosh(d*x + c)^3 + (8*b^3*d* \\
& e - 8*b^3*c*f - 4*I*(a^3 + 3*a*b^2)*d*e + 4*I*(a^3 + 3*a*b^2)*c*f)*\cosh(d*x \\
& + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + I) - (2*b^3*d*e - \\
& 2*b^3*c*f + (2*b^3*d*e - 2*b^3*c*f + I*(a^3 + 3*a*b^2)*d*e - I*(a^3 + 3*a* \\
& b^2)*c*f)*\cosh(d*x + c)^4 + (8*b^3*d*e - 8*b^3*c*f + 4*I*(a^3 + 3*a*b^2)*d* \\
& e - 4*I*(a^3 + 3*a*b^2)*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (2*b^3*d*e - 2 \\
& *b^3*c*f + I*(a^3 + 3*a*b^2)*d*e - I*(a^3 + 3*a*b^2)*c*f)*\sinh(d*x + c)^4 + \\
& I*(a^3 + 3*a*b^2)*d*e - I*(a^3 + 3*a*b^2)*c*f + (4*b^3*d*e - 4*b^3*c*f + 2 \\
& *I*(a^3 + 3*a*b^2)*d*e - 2*I*(a^3 + 3*a*b^2)*c*f)*\cosh(d*x + c)^2 + (4*b^3* \\
& d*e - 4*b^3*c*f + 2*I*(a^3 + 3*a*b^2)*d*e - 2*I*(a^3 + 3*a*b^2)*c*f + (12*b \\
& ^3*d*e - 12*b^3*c*f + 6*I*(a^3 + 3*a*b^2)*d*e - 6*I*(a^3 + 3*a*b^2)*c*f)*\co \\
& sh(d*x + c)^2)*\sinh(d*x + c)^2 + ((8*b^3*d*e - 8*b^3*c*f + 4*I*(a^3 + 3*a*b
\end{aligned}$$

$$\begin{aligned}
&^2)*d*e - 4*I*(a^3 + 3*a*b^2)*c*f)*\cosh(d*x + c)^3 + (8*b^3*d*e - 8*b^3*c*f \\
&+ 4*I*(a^3 + 3*a*b^2)*d*e - 4*I*(a^3 + 3*a*b^2)*c*f)*\cosh(d*x + c))*\sinh(d \\
&*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - I) - (2*b^3*d*f*x + 2*b^3*c*f \\
&+ (2*b^3*d*f*x + 2*b^3*c*f + I*(a^3 + 3*a*b^2)*d*f*x + I*(a^3 + 3*a*b^2)*c* \\
&f)*\cosh(d*x + c)^4 + (8*b^3*d*f*x + 8*b^3*c*f + 4*I*(a^3 + 3*a*b^2)*d*f*x + \\
&4*I*(a^3 + 3*a*b^2)*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (2*b^3*d*f*x + 2* \\
&b^3*c*f + I*(a^3 + 3*a*b^2)*d*f*x + I*(a^3 + 3*a*b^2)*c*f)*\sinh(d*x + c)^4 \\
&+ I*(a^3 + 3*a*b^2)*d*f*x + I*(a^3 + 3*a*b^2)*c*f + (4*b^3*d*f*x + 4*b^3*c* \\
&f + 2*I*(a^3 + 3*a*b^2)*d*f*x + 2*I*(a^3 + 3*a*b^2)*c*f)*\cosh(d*x + c)^2 + \\
&(4*b^3*d*f*x + 4*b^3*c*f + 2*I*(a^3 + 3*a*b^2)*d*f*x + 2*I*(a^3 + 3*a*b^2)* \\
&c*f + (12*b^3*d*f*x + 12*b^3*c*f + 6*I*(a^3 + 3*a*b^2)*d*f*x + 6*I*(a^3 + 3 \\
&a*b^2)*c*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + ((8*b^3*d*f*x + 8*b^3*c*f + \\
&4*I*(a^3 + 3*a*b^2)*d*f*x + 4*I*(a^3 + 3*a*b^2)*c*f)*\cosh(d*x + c)^3 + (8* \\
&b^3*d*f*x + 8*b^3*c*f + 4*I*(a^3 + 3*a*b^2)*d*f*x + 4*I*(a^3 + 3*a*b^2)*c*f \\
&)*\cosh(d*x + c))*\sinh(d*x + c))*\log(I*\cosh(d*x + c) + I*\sinh(d*x + c) + 1) \\
&- (2*b^3*d*f*x + 2*b^3*c*f + (2*b^3*d*f*x + 2*b^3*c*f - I*(a^3 + 3*a*b^2)*d \\
&*f*x - I*(a^3 + 3*a*b^2)*c*f)*\cosh(d*x + c)^4 + (8*b^3*d*f*x + 8*b^3*c*f - \\
&4*I*(a^3 + 3*a*b^2)*d*f*x - 4*I*(a^3 + 3*a*b^2)*c*f)*\cosh(d*x + c)*\sinh(d*x \\
&+ c)^3 + (2*b^3*d*f*x + 2*b^3*c*f - I*(a^3 + 3*a*b^2)*d*f*x - I*(a^3 + 3*a \\
&b^2)*c*f)*\sinh(d*x + c)^4 - I*(a^3 + 3*a*b^2)*d*f*x - I*(a^3 + 3*a*b^2)*c* \\
&f + (4*b^3*d*f*x + 4*b^3*c*f - 2*I*(a^3 + 3*a*b^2)*d*f*x - 2*I*(a^3 + 3*a*b \\
&^2)*c*f)*\cosh(d*x + c)^2 + (4*b^3*d*f*x + 4*b^3*c*f - 2*I*(a^3 + 3*a*b^2)*d \\
&*f*x - 2*I*(a^3 + 3*a*b^2)*c*f + (12*b^3*d*f*x + 12*b^3*c*f - 6*I*(a^3 + 3* \\
&a*b^2)*d*f*x - 6*I*(a^3 + 3*a*b^2)*c*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + \\
&((8*b^3*d*f*x + 8*b^3*c*f - 4*I*(a^3 + 3*a*b^2)*d*f*x - 4*I*(a^3 + 3*a*b^2) \\
&)*c*f)*\cosh(d*x + c)^3 + (8*b^3*d*f*x + 8*b^3*c*f - 4*I*(a^3 + 3*a*b^2)*d*f* \\
&x - 4*I*(a^3 + 3*a*b^2)*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(-I*\cosh(d*x \\
&+ c) - I*\sinh(d*x + c) + 1) - 2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e - \\
&3*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a*b^2)*f)*\cosh(d*x + c) \\
&^2 - (a^3 + a*b^2)*f - 2*(2*(a^2*b + b^3)*d*f*x + 2*(a^2*b + b^3)*d*e + (a^ \\
&2*b + b^3)*f)*\cosh(d*x + c))*\sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^2*co \\
&sh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d^2*\cosh(d*x + c)*\sinh(d*x + c)^3 \\
&+ (a^4 + 2*a^2*b^2 + b^4)*d^2*\sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)* \\
&d^2*\cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2 + 2*(3*(a^4 + 2*a^2*b^2 + \\
&b^4)*d^2*\cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2)*\sinh(d*x + c)^2 + \\
&4*((a^4 + 2*a^2*b^2 + b^4)*d^2*\cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d^ \\
&2*\cosh(d*x + c))*\sinh(d*x + c))
\end{aligned}$$


---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \operatorname{sech}^3(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)`

[Out] `Integral((e + f*x)*sech(c + d*x)**3/(a + b*sinh(c + d*x)), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \operatorname{sech}(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")`

[Out] `integrate((f*x + e)*sech(d*x + c)^3/(b*sinh(d*x + c) + a), x)`

$$3.316 \quad \int \frac{\operatorname{sech}^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=119

$$\frac{b^3 \log(a + b \sinh(c + dx))}{d(a^2 + b^2)^2} + \frac{a(a^2 + 3b^2) \tan^{-1}(\sinh(c + dx))}{2d(a^2 + b^2)^2} - \frac{b^3 \log(\cosh(c + dx))}{d(a^2 + b^2)^2} + \frac{\operatorname{sech}^2(c + dx)(a \sinh(c + dx) + 1)}{2d(a^2 + b^2)}$$

[Out] (a\*(a^2 + 3\*b^2)\*ArcTan[Sinh[c + d\*x]])/(2\*(a^2 + b^2)^2\*d) - (b^3\*Log[Cosh[c + d\*x]])/((a^2 + b^2)^2\*d) + (b^3\*Log[a + b\*Sinh[c + d\*x]])/((a^2 + b^2)^2\*d) + (Sech[c + d\*x]^2\*(b + a\*Sinh[c + d\*x]))/(2\*(a^2 + b^2)\*d)

**Rubi [A]** time = 0.143604, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2668, 741, 801, 635, 203, 260}

$$\frac{b^3 \log(a + b \sinh(c + dx))}{d(a^2 + b^2)^2} + \frac{a(a^2 + 3b^2) \tan^{-1}(\sinh(c + dx))}{2d(a^2 + b^2)^2} - \frac{b^3 \log(\cosh(c + dx))}{d(a^2 + b^2)^2} + \frac{\operatorname{sech}^2(c + dx)(a \sinh(c + dx) + 1)}{2d(a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d\*x]^3/(a + b\*Sinh[c + d\*x]),x]

[Out] (a\*(a^2 + 3\*b^2)\*ArcTan[Sinh[c + d\*x]])/(2\*(a^2 + b^2)^2\*d) - (b^3\*Log[Cosh[c + d\*x]])/((a^2 + b^2)^2\*d) + (b^3\*Log[a + b\*Sinh[c + d\*x]])/((a^2 + b^2)^2\*d) + (Sech[c + d\*x]^2\*(b + a\*Sinh[c + d\*x]))/(2\*(a^2 + b^2)\*d)

### Rule 2668

Int[cos[(e\_.) + (f\_.)\*(x\_)]^(p\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(m\_.), x\_Symbol] :> Dist[1/(b^p\*f), Subst[Int[(a + x)^m\*(b^2 - x^2)^((p - 1)/2), x], x, b\*Sin[e + f\*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

### Rule 741

Int[((d\_.) + (e\_.)\*(x\_))^(m\_.)\*((a\_.) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> -Simp[((d + e\*x)^(m + 1)\*(a + c\*d\*x)\*(a + c\*x^2)^(p + 1))/(2\*a\*(p + 1)\*(c\*d^2 + a\*e^2)), x] + Dist[1/(2\*a\*(p + 1)\*(c\*d^2 + a\*e^2)), Int[(d + e\*x)^m\*Simp[c\*d^2\*(2\*p + 3) + a\*e^2\*(m + 2\*p + 3) + c\*e\*d\*(m + 2\*p + 4)\*x, x]\*(a + c\*x^2)^(p + 1), x], x]

$2)^{(p+1)}, x], x] /; \text{FreeQ}[\{a, c, d, e, m\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{IntQuadraticQ}[a, 0, c, d, e, m, p, x]$

### Rule 801

$\text{Int}[(((d_.) + (e_.)*(x_.))^{(m_.)}*((f_.) + (g_.)*(x_.)))/((a_.) + (c_.)*(x_.)^2), x\_Symbol] \ :> \ \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(f + g*x)/(a + c*x^2), x], x] /; \text{FreeQ}[\{a, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{IntegerQ}[m]$

### Rule 635

$\text{Int}[((d_.) + (e_.)*(x_.))/((a_.) + (c_.)*(x_.)^2), x\_Symbol] \ :> \ \text{Dist}[d, \text{Int}[1/(a + c*x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c*x^2), x], x] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{!NiceSqrtQ}[-(a*c)]$

### Rule 203

$\text{Int}[((a_.) + (b_.)*(x_.)^2)^{-1}, x\_Symbol] \ :> \ \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

### Rule 260

$\text{Int}[(x_.)^{(m_.)}/((a_.) + (b_.)*(x_.)^{(n_.)}), x\_Symbol] \ :> \ \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]]/(b*n), x] /; \text{FreeQ}[\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

### Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{b^3 \operatorname{Subst}\left(\int \frac{1}{(a+x)(-b^2-x^2)^2} dx, x, b\sinh(c+dx)\right)}{d} \\
&= \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2(a^2+b^2)d} - \frac{b \operatorname{Subst}\left(\int \frac{a^2+2b^2+ax}{(a+x)(-b^2-x^2)} dx, x, b\sinh(c+dx)\right)}{2(a^2+b^2)d} \\
&= \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2(a^2+b^2)d} - \frac{b \operatorname{Subst}\left(\int \left(-\frac{2b^2}{(a^2+b^2)(a+x)} + \frac{-a^3-3ab^2+2b^2x}{(a^2+b^2)(b^2+x^2)}\right) dx, x, b\sinh(c+dx)\right)}{2(a^2+b^2)d} \\
&= \frac{b^3 \log(a+b\sinh(c+dx))}{(a^2+b^2)^2 d} + \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2(a^2+b^2)d} - \frac{b \operatorname{Subst}\left(\int \frac{-a^3-3ab^2+2b^2x}{b^2+x^2} dx, x, b\sinh(c+dx)\right)}{2(a^2+b^2)d} \\
&= \frac{b^3 \log(a+b\sinh(c+dx))}{(a^2+b^2)^2 d} + \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2(a^2+b^2)d} - \frac{b^3 \operatorname{Subst}\left(\int \frac{x}{b^2+x^2} dx, x, b\sinh(c+dx)\right)}{(a^2+b^2)^2 d} \\
&= \frac{a(a^2+3b^2)\tan^{-1}(\sinh(c+dx))}{2(a^2+b^2)^2 d} - \frac{b^3 \log(\cosh(c+dx))}{(a^2+b^2)^2 d} + \frac{b^3 \log(a+b\sinh(c+dx))}{(a^2+b^2)^2 d} + \frac{\operatorname{sech}^2(c+dx)(b+a\sinh(c+dx))}{2(a^2+b^2)d}
\end{aligned}$$

**Mathematica [A]** time = 0.188339, size = 104, normalized size = 0.87

$$\frac{b(a^2+b^2)\operatorname{sech}^2(c+dx) + 2a(a^2+3b^2)\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + a(a^2+b^2)\tanh(c+dx)\operatorname{sech}(c+dx) + 2b^3(\log(a+b\sinh(c+dx)) - \log(\cosh(c+dx)))}{2d(a^2+b^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d\*x]^3/(a + b\*Sinh[c + d\*x]), x]

[Out] (2\*a\*(a^2 + 3\*b^2)\*ArcTan[Tanh[(c + d\*x)/2]] + 2\*b^3\*(-Log[Cosh[c + d\*x]] + Log[a + b\*Sinh[c + d\*x]]) + b\*(a^2 + b^2)\*Sech[c + d\*x]^2 + a\*(a^2 + b^2)\*Sech[c + d\*x]\*Tanh[c + d\*x])/(2\*(a^2 + b^2)^2\*d)

**Maple [B]** time = 0.003, size = 468, normalized size = 3.9

$$\frac{b^3}{d(a^4 + 2a^2b^2 + b^4)} \ln\left(\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2 a - 2 \tanh\left(\frac{1}{2}dx + \frac{c}{2}\right)b - a\right) - \frac{a^3}{d(a^4 + 2a^2b^2 + b^4)} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)`

[Out]  $\frac{1}{d} \frac{b^3}{(a^4 + 2a^2b^2 + b^4)} \ln(\tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 a - 2 \tanh(\frac{1}{2}dx + \frac{1}{2}c) * b - a) - \frac{1}{d} \frac{b^3}{(a^4 + 2a^2b^2 + b^4)} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1)^2} \tanh(\frac{1}{2}dx + \frac{1}{2}c) * a^3 - \frac{1}{d} \frac{b^3}{(a^4 + 2a^2b^2 + b^4)} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1)^2} \tanh(\frac{1}{2}dx + \frac{1}{2}c) * a^2 b - \frac{2}{d} \frac{b^3}{(a^4 + 2a^2b^2 + b^4)} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1)^2} \tanh(\frac{1}{2}dx + \frac{1}{2}c) * a^2 b^2 - \frac{2}{d} \frac{b^3}{(a^4 + 2a^2b^2 + b^4)} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1)^2} \tanh(\frac{1}{2}dx + \frac{1}{2}c) * a^2 b^3 + \frac{1}{d} \frac{b^3}{(a^4 + 2a^2b^2 + b^4)} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1)^2} \tanh(\frac{1}{2}dx + \frac{1}{2}c) * a^3 + \frac{1}{d} \frac{b^3}{(a^4 + 2a^2b^2 + b^4)} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1)^2} \tanh(\frac{1}{2}dx + \frac{1}{2}c) * a^2 b^2 - \frac{1}{d} \frac{b^3}{(a^4 + 2a^2b^2 + b^4)} \ln(\tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1) + \frac{1}{d} \frac{b^3}{(a^4 + 2a^2b^2 + b^4)} \arctan(\tanh(\frac{1}{2}dx + \frac{1}{2}c)) * a^3 + \frac{3}{d} \frac{b^3}{(a^4 + 2a^2b^2 + b^4)} \arctan(\tanh(\frac{1}{2}dx + \frac{1}{2}c)) * a^2 b^2$

**Maxima [A]** time = 1.96289, size = 292, normalized size = 2.45

$$\frac{b^3 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{(a^4 + 2a^2b^2 + b^4)d} - \frac{b^3 \log(e^{(-2dx-2c)} + 1)}{(a^4 + 2a^2b^2 + b^4)d} - \frac{(a^3 + 3ab^2) \arctan(e^{(-dx-c)})}{(a^4 + 2a^2b^2 + b^4)d} + \frac{ae^{(-dx-c)} + 2}{(a^2 + b^2 + 2(a^2 + b^2))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out]  $b^3 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b) / ((a^4 + 2a^2b^2 + b^4) * d) - b^3 \log(e^{(-2dx-2c)} + 1) / ((a^4 + 2a^2b^2 + b^4) * d) - (a^3 + 3a^2b) \arctan(e^{(-dx-c)}) / ((a^4 + 2a^2b^2 + b^4) * d) + (ae^{(-dx-c)} + 2 * b * e^{(-2dx-2c)} - ae^{(-3dx-3c)}) / ((a^2 + b^2 + 2 * (a^2 + b^2)) * e^{(-2dx-2c)} + (a^2 + b^2) * e^{(-4dx-4c)}) * d$

**Fricas [B]** time = 2.43699, size = 2215, normalized size = 18.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

```
[Out] ((a^3 + a*b^2)*cosh(d*x + c)^3 + (a^3 + a*b^2)*sinh(d*x + c)^3 + 2*(a^2*b +
b^3)*cosh(d*x + c)^2 + (2*a^2*b + 2*b^3 + 3*(a^3 + a*b^2)*cosh(d*x + c))*s
inh(d*x + c)^2 + ((a^3 + 3*a*b^2)*cosh(d*x + c)^4 + 4*(a^3 + 3*a*b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a*b^2)*sinh(d*x + c)^4 + a^3 + 3*a*b^2
+ 2*(a^3 + 3*a*b^2)*cosh(d*x + c)^2 + 2*(a^3 + 3*a*b^2 + 3*(a^3 + 3*a*b^2)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3 + 3*a*b^2)*cosh(d*x + c)^3 + (a^
3 + 3*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x
+ c)) - (a^3 + a*b^2)*cosh(d*x + c) + (b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x
+ c)*sinh(d*x + c)^3 + b^3*sinh(d*x + c)^4 + 2*b^3*cosh(d*x + c)^2 + b^3 +
2*(3*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 +
b^3*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x +
c) - sinh(d*x + c))) - (b^3*cosh(d*x + c)^4 + 4*b^3*cosh(d*x + c)*sinh(d*x
+ c)^3 + b^3*sinh(d*x + c)^4 + 2*b^3*cosh(d*x + c)^2 + b^3 + 2*(3*b^3*cosh(
d*x + c)^2 + b^3)*sinh(d*x + c)^2 + 4*(b^3*cosh(d*x + c)^3 + b^3*cosh(d*x +
c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) -
(a^3 + a*b^2 - 3*(a^3 + a*b^2)*cosh(d*x + c)^2 - 4*(a^2*b + b^3)*cosh(d*x +
c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^4 + 4*(a^4 + 2
*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d
*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4
+ 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d)*sinh(d*x
+ c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d + 4*((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x
+ c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c))*sinh(d*x + c))
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

**Giac [B]** time = 1.36663, size = 400, normalized size = 3.36

$$\frac{b^4 \log\left(\left|b(e^{(dx+c)} - e^{(-dx-c)}) + 2a\right|\right)}{a^4bd + 2a^2b^3d + b^5d} - \frac{b^3 \log\left(\left((e^{(dx+c)} - e^{(-dx-c)})^2 + 4\right)\right)}{2(a^4d + 2a^2b^2d + b^4d)} + \frac{\left(\pi + 2 \arctan\left(\frac{1}{2}(e^{(2dx+2c)} - 1)e^{(-dx-c)}\right)\right)(a^3 + 3b^3)}{4(a^4d + 2a^2b^2d + b^4d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] 
$$b^4 \log(\operatorname{abs}(b(e^{dx+c}) - e^{-dx-c}) + 2a)) / (a^4 b d + 2a^2 b^3 d + b^5 d) - 1/2 b^3 \log((e^{dx+c} - e^{-dx-c})^2 + 4) / (a^4 d + 2a^2 b^2 d + b^4 d) + 1/4 (\pi + 2 \arctan(1/2 (e^{2dx+2c} - 1) e^{-dx-c})) (a^3 + 3ab^2) / (a^4 d + 2a^2 b^2 d + b^4 d) + 1/2 (b^3 (e^{dx+c} - e^{-dx-c})^2 + 2a^3 (e^{dx+c} - e^{-dx-c}) + 2ab^2 (e^{dx+c} - e^{-dx-c}) + 4a^2 b + 8b^3) / ((a^4 d + 2a^2 b^2 d + b^4 d) ((e^{dx+c} - e^{-dx-c})^2 + 4))$$

$$3.317 \quad \int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Optimal. Leaf size=30

$$\text{Unintegrable} \left( \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[Sech[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0768256, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Sech[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Sech[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [A]** time = 133.342, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Sech[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Integrate[Sech[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Maple [A]** time = 1.058, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(dx + c))^3}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)), x)

[Out] int(sech(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)), x, algorithm="maxima")

[Out]  $-(b*f - (a*d*f*x*e^{(3*c)} + (d*e - f)*a*e^{(3*c)})e^{(3*d*x)} - (2*b*d*f*x*e^{(2*c)} + (2*d*e - f)*b*e^{(2*c)})e^{(2*d*x)} + (a*d*f*x*e^c + (d*e + f)*a*e^c)e^{(d*x)}) / (a^2*d^2*e^2 + b^2*d^2*e^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*(a^2*d^2*e*f + b^2*d^2*e*f)*x + (a^2*d^2*e^2*e^{(4*c)} + b^2*d^2*e^2*e^{(4*c)} + (a^2*d^2*f^2*e^{(4*c)} + b^2*d^2*f^2*e^{(4*c)})*x^2 + 2*(a^2*d^2*e*f*e^{(4*c)} + b^2*d^2*e*f*e^{(4*c)})*x)*e^{(4*d*x)} + 2*(a^2*d^2*e^2*e^{(2*c)} + b^2*d^2*e^2*e^{(2*c)} + (a^2*d^2*f^2*e^{(2*c)} + b^2*d^2*f^2*e^{(2*c)})*x^2 + 2*(a^2*d^2*e*f*e^{(2*c)} + b^2*d^2*e*f*e^{(2*c)})*x)*e^{(2*d*x)}) + 8*\integrate(1/8*(2*b^3*d^2*f^2*x^2 + 4*b^3*d^2*e*f*x - 2*a^2*b*f^2 + 2*(d^2*e^2 - f^2)*b^3 + ((d^2*e^2 - 2*f^2)*a^3*e^c + (3*d^2*e^2 - 2*f^2)*a*b^2*e^c + (a^3*d^2*f^2*e^c + 3*a*b^2*d^2*f^2*e^c)*x^2 + 2*(a^3*d^2*e*f*e^c + 3*a*b^2*d^2*e*f*e^c)*x)*e^{(d*x)}) / (a^4*d^2*e^3 + 2*a^2*b^2*d^2*e^3 + b^4*d^2*e^3 + (a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*e*f^2 + 2*a^2*b^2*d^2*e*f^2 + b^4*d^2*e*f^2)*x^2 + 3*(a^4*d^2*e^2*f + 2*a^2*b^2*d^2*e^2*f + b^4*d^2*e^2*f)*x + (a^4*d^2*e^3*e^{(2*c)} + 2*a^2*b^2*d^2*e^3*e^{(2*c)} + b^4*d^2*e^3*e^{(2*c)} + (a^4*d^2*f^3*e^{(2*c)} + 2*a^2*b^2*d^2*f^3*e^{(2*c)} + b^4*d^2*f^3*e^{(2*c)})*x^3 + 3*(a^4*d^2*e*f^2*e^{(2*c)} + 2*a^2*b^2*d^2*e*f^2*e^{(2*c)} + b^4*d^2*e*f^2*e^{(2*c)})*x^2 + 3*(a^4*d^2*e^2*f*e^{(2*c)} + 2*a^2*b^2*d^2*e^2*f*e^{(2*c)} + b^4*d^2*e^2*f*e^{(2*c)} + b^4*d^2*$

$e^{2*f*e^{(2*c)}}*x)*e^{(2*d*x)}, x) - 8*\text{integrate}(-1/4*(a*b^3*e^{(d*x + c)} - b^4)/(a^4*b*e + 2*a^2*b^3*e + b^5*e + (a^4*b*f + 2*a^2*b^3*f + b^5*f)*x - (a^4*b*e*e^{(2*c)} + 2*a^2*b^3*e*e^{(2*c)} + b^5*e*e^{(2*c)} + (a^4*b*f*e^{(2*c)} + 2*a^2*b^3*f*e^{(2*c)} + b^5*f*e^{(2*c)})*x)*e^{(2*d*x)} - 2*(a^5*e*e^c + 2*a^3*b^2*e*e^c + a*b^4*e*e^c + (a^5*f*e^c + 2*a^3*b^2*f*e^c + a*b^4*f*e^c)*x)*e^{(d*x)}), x)$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{sech}(dx + c)^3}{afx + ae + (bf x + be) \sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] `integral(sech(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)`

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{sech}^3(c + dx)}{(a + b \sinh(c + dx))(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)`

[Out] `Integral(sech(c + d*x)**3/((a + b*sinh(c + d*x))*(e + f*x)), x)`

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")`

[Out] Timed out

$$3.318 \quad \int \frac{x^m \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

Optimal. Leaf size=26

$$\text{Unintegrable} \left( \frac{x^m \cosh^3(c+dx)}{a+b \sinh(c+dx)}, x \right)$$

[Out] Unintegrable[(x^m\*Cosh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]), x]

**Rubi [A]** time = 0.0616135, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{x^m \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

Verification is Not applicable to the result.

[In] Int[(x^m\*Cosh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]), x]

[Out] Defer[Int] [(x^m\*Cosh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]), x]

Rubi steps

$$\int \frac{x^m \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx = \int \frac{x^m \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Mathematica [A]** time = 11.3477, size = 0, normalized size = 0.

$$\int \frac{x^m \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^m\*Cosh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]), x]

[Out] Integrate[(x^m\*Cosh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]), x]

---

**Maple [A]** time = 0.072, size = 0, normalized size = 0.

$$\int \frac{x^m (\cosh(dx + c))^3}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*cosh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

[Out] int(x^m\*cosh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \cosh(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cosh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] integrate(x^m\*cosh(d\*x + c)^3/(b\*sinh(d\*x + c) + a), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m \cosh(dx + c)^3}{b \sinh(dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cosh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] integral(x^m\*cosh(d\*x + c)^3/(b\*sinh(d\*x + c) + a), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \cosh^3(c + dx)}{a + b \sinh(c + dx)} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*cosh(d*x+c)**3/(a+b*sinh(d*x+c)),x)`

[Out] `Integral(x**m*cosh(c + d*x)**3/(a + b*sinh(c + d*x)), x)`

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \cosh(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*cosh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")`

[Out] `integrate(x^m*cosh(d*x + c)^3/(b*sinh(d*x + c) + a), x)`

$$3.319 \quad \int \frac{x^m \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=26

$$\text{Unintegrable}\left(\frac{x^m \cosh^2(c+dx)}{a+b \sinh(c+dx)}, x\right)$$

[Out] Unintegrable[(x^m\*Cosh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]), x]

**Rubi [A]** time = 0.0617755, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{x^m \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

Verification is Not applicable to the result.

[In] Int[(x^m\*Cosh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]), x]

[Out] Defer[Int] [(x^m\*Cosh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]), x]

Rubi steps

$$\int \frac{x^m \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx = \int \frac{x^m \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Mathematica [A]** time = 8.57976, size = 0, normalized size = 0.

$$\int \frac{x^m \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^m\*Cosh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]), x]

[Out] Integrate[(x^m\*Cosh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]), x]

---

**Maple [A]** time = 0.115, size = 0, normalized size = 0.

$$\int \frac{x^m (\cosh(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*cosh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] int(x^m\*cosh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \cosh(dx + c)^2}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cosh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] integrate(x^m\*cosh(d\*x + c)^2/(b\*sinh(d\*x + c) + a), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m \cosh(dx + c)^2}{b \sinh(dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cosh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] integral(x^m\*cosh(d\*x + c)^2/(b\*sinh(d\*x + c) + a), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \cosh^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*cosh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(x\*\*m\*cosh(c + d\*x)\*\*2/(a + b\*sinh(c + d\*x)), x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \cosh(dx + c)^2}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cosh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate(x^m\*cosh(d\*x + c)^2/(b\*sinh(d\*x + c) + a), x)

$$3.320 \quad \int \frac{x^m \cosh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=24

$$\text{Unintegrable} \left( \frac{x^m \cosh(c+dx)}{a+b \sinh(c+dx)}, x \right)$$

[Out] Unintegrable[(x^m\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x]), x]

**Rubi [A]** time = 0.0382973, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{x^m \cosh(c+dx)}{a+b \sinh(c+dx)} dx$$

Verification is Not applicable to the result.

[In] Int[(x^m\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x]), x]

[Out] Defer[Int] [(x^m\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x]), x]

Rubi steps

$$\int \frac{x^m \cosh(c+dx)}{a+b \sinh(c+dx)} dx = \int \frac{x^m \cosh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Mathematica [A]** time = 5.52251, size = 0, normalized size = 0.

$$\int \frac{x^m \cosh(c+dx)}{a+b \sinh(c+dx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^m\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x]), x]

[Out] Integrate[(x^m\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x]), x]

---

**Maple [A]** time = 0.068, size = 0, normalized size = 0.

$$\int \frac{x^m \cosh(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int(x^m\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{x e^{(2dx + m \log(x) + 2c)}}{b(m+1)e^{(2dx+2c)} + 2a(m+1)e^{(dx+c)} - b(m+1)} - \frac{1}{2} \int \frac{2(2adxe^{(3dx+3c)} - 2a(m+1)e^{(dx+c)} + b(m+1) - b^2(m+1)e^{(4dx+4c)} + 4ab(m+1)e^{(3dx+3c)} - 4ab(m+1)e^{(dx+c)} + b^2(m+1)e^{(2dx+2c)})}{b^2(m+1)e^{(4dx+4c)} + 4ab(m+1)e^{(3dx+3c)} - 4ab(m+1)e^{(dx+c)} + b^2(m+1)e^{(2dx+2c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] x\*e^(2\*d\*x + m\*log(x) + 2\*c)/(b\*(m + 1)\*e^(2\*d\*x + 2\*c) + 2\*a\*(m + 1)\*e^(d\*x + c) - b\*(m + 1)) - 1/2\*integrate(2\*(2\*a\*d\*x\*e^(3\*d\*x + 3\*c) - 2\*a\*(m + 1)\*e^(d\*x + c) + b\*(m + 1) - (2\*b\*d\*x\*e^(2\*c) + b\*(m + 1)\*e^(2\*c))\*e^(2\*d\*x) \*x^m/(b^2\*(m + 1)\*e^(4\*d\*x + 4\*c) + 4\*a\*b\*(m + 1)\*e^(3\*d\*x + 3\*c) - 4\*a\*b\*(m + 1)\*e^(d\*x + c) + b^2\*(m + 1) + 2\*(2\*a^2\*(m + 1)\*e^(2\*c) - b^2\*(m + 1)\*e^(2\*c))\*e^(2\*d\*x)), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^m \cosh(dx + c)}{b \sinh(dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] integral(x^m\*cosh(d\*x + c)/(b\*sinh(d\*x + c) + a), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \cosh(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*m\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(x\*\*m\*cosh(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^m \cosh(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate(x^m\*cosh(d\*x + c)/(b\*sinh(d\*x + c) + a), x)

$$3.321 \quad \int \frac{(e+fx) \cosh(c+dx)}{(a+b \sinh(c+dx))^2} dx$$

**Optimal.** Leaf size=74

$$-\frac{2f \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}} - \frac{e+fx}{bd(a+b \sinh(c+dx))}$$

[Out]  $(-2*f*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b*Sqrt[a^2 + b^2]*d^2) - (e + f*x)/(b*d*(a + b*Sinh[c + d*x]))$

**Rubi [A]** time = 0.0729192, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {5464, 2660, 618, 204}

$$-\frac{2f \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}} - \frac{e+fx}{bd(a+b \sinh(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x])^2,x]

[Out]  $(-2*f*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b*Sqrt[a^2 + b^2]*d^2) - (e + f*x)/(b*d*(a + b*Sinh[c + d*x]))$

#### Rule 5464

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Simp[((e + f\*x)^m\*(a + b\*Sinh[c + d\*x])^(n + 1))/(b\*d\*(n + 1)), x] - Dist[(f\*m)/(b\*d\*(n + 1)), Int[(e + f\*x)^(m - 1)\*(a + b\*Sinh[c + d\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

#### Rule 2660

Int[((a\_.) + (b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] :> With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + 2\*b\*e\*x + a\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[



$a^2 - b^2, 0]$

### Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

### Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

### Rubi steps

$$\begin{aligned} \int \frac{(e + fx) \cosh(c + dx)}{(a + b \sinh(c + dx))^2} dx &= -\frac{e + fx}{bd(a + b \sinh(c + dx))} + \frac{f \int \frac{1}{a + b \sinh(c + dx)} dx}{bd} \\ &= -\frac{e + fx}{bd(a + b \sinh(c + dx))} - \frac{(2if) \operatorname{Subst}\left(\int \frac{1}{a - 2ibx + ax^2} dx, x, \tan\left(\frac{1}{2}(ic + idx)\right)\right)}{bd^2} \\ &= -\frac{e + fx}{bd(a + b \sinh(c + dx))} + \frac{(4if) \operatorname{Subst}\left(\int \frac{1}{-4(a^2 + b^2) - x^2} dx, x, -2ib + 2a \tan\left(\frac{1}{2}(ic + idx)\right)\right)}{bd^2} \\ &= -\frac{2f \tanh^{-1}\left(\frac{b - a \tanh\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a^2 + b^2}}\right)}{b\sqrt{a^2 + b^2}d^2} - \frac{e + fx}{bd(a + b \sinh(c + dx))} \end{aligned}$$

**Mathematica [A]** time = 0.503442, size = 78, normalized size = 1.05

$$\frac{2f \tan^{-1}\left(\frac{b - a \tanh\left(\frac{1}{2}(c + dx)\right)}{\sqrt{-a^2 - b^2}}\right)}{\sqrt{-a^2 - b^2}} - \frac{d(e + fx)}{a + b \sinh(c + dx)}{bd^2}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x])^2,x]

[Out]  $((2*f*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - (d*(e + f*x))/(a + b*Sinh[c + d*x]))/(b*d^2)$

**Maple [B]** time = 0.214, size = 164, normalized size = 2.2

$$-2 \frac{(fx + e)e^{dx+c}}{bd(b e^{2dx+2c} + 2ae^{dx+c} - b)} + \frac{f}{d^2b} \ln \left( e^{dx+c} + \frac{1}{b} (a\sqrt{a^2 + b^2} - a^2 - b^2) \frac{1}{\sqrt{a^2 + b^2}} \right) \frac{1}{\sqrt{a^2 + b^2}} - \frac{f}{d^2b} \ln \left( e^{dx+c} + \frac{1}{b} (a\sqrt{a^2 + b^2} + a^2 + b^2) \frac{1}{\sqrt{a^2 + b^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)`

[Out]  $-2*(f*x+e)/b/d*\exp(d*x+c)/(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)+1/(a^2+b^2)^{(1/2)}*f/d^2/b*\ln(\exp(d*x+c)+(a*(a^2+b^2)^{(1/2)}-a^2-b^2)/(a^2+b^2)^{(1/2)})/b)-1/(a^2+b^2)^{(1/2)}*f/d^2/b*\ln(\exp(d*x+c)+(a*(a^2+b^2)^{(1/2)}+a^2+b^2)/(a^2+b^2)^{(1/2)})/b)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.1182, size = 1015, normalized size = 13.72

$$\frac{(bf \cosh(dx + c)^2 + bf \sinh(dx + c)^2 + 2af \cosh(dx + c) - bf + 2(bf \cosh(dx + c) + af) \sinh(dx + c))\sqrt{a^2 + b^2} \log\left(\frac{e^{dx+c} + \frac{1}{b}(a\sqrt{a^2+b^2} - a^2 - b^2)}{\sqrt{a^2+b^2}}\right) + (bf \cosh(dx + c) - bf + 2(bf \cosh(dx + c) + af) \sinh(dx + c))\sqrt{a^2 + b^2} \log\left(\frac{e^{dx+c} + \frac{1}{b}(a\sqrt{a^2+b^2} + a^2 + b^2)}{\sqrt{a^2+b^2}}\right)}{(a^2b^2 + b^4)d^2 \cosh(dx + c)^2 + (a^2b^2 + b^4)d^2 \sinh(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")`

```
[Out] ((b*f*cosh(d*x + c)^2 + b*f*sinh(d*x + c)^2 + 2*a*f*cosh(d*x + c) - b*f + 2
*(b*f*cosh(d*x + c) + a*f)*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*cosh(d*x
+ c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*
cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b
*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x
+ c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*((a^2 + b^2)*d*f*x +
(a^2 + b^2)*d*e)*cosh(d*x + c) - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*d*e)*s
inh(d*x + c))/((a^2*b^2 + b^4)*d^2*cosh(d*x + c)^2 + (a^2*b^2 + b^4)*d^2*si
nh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d^2*cosh(d*x + c) - (a^2*b^2 + b^4)*d^2 +
2*((a^2*b^2 + b^4)*d^2*cosh(d*x + c) + (a^3*b + a*b^3)*d^2)*sinh(d*x + c))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \cosh(dx + c)}{(b \sinh(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

$$3.322 \quad \int \frac{(e+fx)^2 \cosh(c+dx)}{(a+b \sinh(c+dx))^2} dx$$

**Optimal.** Leaf size=234

$$\frac{2f^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}} - \frac{2f^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}} + \frac{2f(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{bd^2\sqrt{a^2+b^2}} - \frac{2f(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2\sqrt{a^2+b^2}}$$

[Out] (2\*f\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d^2) - (2\*f\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d^2) + (2\*f^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^3) - (2\*f^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^3) - (e + f\*x)^2/(b\*d\*(a + b\*Sinh[c + d\*x]))

**Rubi [A]** time = 0.440379, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {5464, 3322, 2264, 2190, 2279, 2391}

$$\frac{2f^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}} - \frac{2f^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}} + \frac{2f(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{bd^2\sqrt{a^2+b^2}} - \frac{2f(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x])^2,x]

[Out] (2\*f\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d^2) - (2\*f\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d^2) + (2\*f^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^3) - (2\*f^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^3) - (e + f\*x)^2/(b\*d\*(a + b\*Sinh[c + d\*x]))

**Rule 5464**

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Simp[((e + f\*x)^m\*(a + b\*Sinh[c + d\*x])^(n + 1))/(b\*d\*(n + 1)), x] - Dist[(f\*m)/(b\*d\*(n + 1)), Int[(e + f\*x)^(m - 1)\*(a + b\*Sinh[c + d\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :=> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :=> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:=> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :=> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \cosh(c+dx)}{(a+b \sinh(c+dx))^2} dx &= -\frac{(e+fx)^2}{bd(a+b \sinh(c+dx))} + \frac{(2f) \int \frac{e+fx}{a+b \sinh(c+dx)} dx}{bd} \\
&= -\frac{(e+fx)^2}{bd(a+b \sinh(c+dx))} + \frac{(4f) \int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}} dx}{bd} \\
&= -\frac{(e+fx)^2}{bd(a+b \sinh(c+dx))} + \frac{(4f) \int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{\sqrt{a^2+b^2}d} - \frac{(4f) \int \frac{e^{c+dx}(e+fx)}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{\sqrt{a^2+b^2}d} \\
&= \frac{2f(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{2f(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{(e+fx)^2}{bd(a+b \sinh(c+dx))} \\
&= \frac{2f(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{2f(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{(e+fx)^2}{bd(a+b \sinh(c+dx))} \\
&= \frac{2f(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{2f(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} + \frac{2f^2 \text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} - \frac{(e+fx)^2}{bd(a+b \sinh(c+dx))}
\end{aligned}$$

**Mathematica [A]** time = 1.37844, size = 175, normalized size = 0.75

$$\frac{2f \left( f \text{PolyLog} \left( 2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a} \right) - f \text{PolyLog} \left( 2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} \right) + d(e+fx) \left( \log \left( \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1 \right) - \log \left( \frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} + 1 \right) \right) \right)}{bd^3 \sqrt{a^2+b^2}} - \frac{(e+fx)^2}{bd(a+b \sinh(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x])^2,x]

[Out] (2\*f\*(d\*(e + f\*x)\*(Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]) - Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]) + f\*PolyLog[2, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2]]) - f\*PolyLog[2, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])]))/(b\*Sqrt[a^2 + b^2]\*d^3) - (e + f\*x)^2/(b\*d\*(a + b\*Sinh[c + d\*x]))

**Maple [B]** time = 0.19, size = 491, normalized size = 2.1

$$-2 \frac{(x^2 f^2 + 2 e f x + e^2) e^{dx+c}}{bd (be^{2dx+2c} + 2 ae^{dx+c} - b)} - 4 \frac{ef}{d^2 b \sqrt{a^2 + b^2}} \text{Artanh} \left( 1/2 \frac{2 be^{dx+c} + 2 a}{\sqrt{a^2 + b^2}} \right) + 2 \frac{f^2 x}{d^2 b \sqrt{a^2 + b^2}} \ln \left( \frac{-be^{dx+c} + \sqrt{a^2 + b^2}}{-a + \sqrt{a^2 + b^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)^2*\cosh(d*x+c)/(a+b*\sinh(d*x+c))^2,x)$

[Out] 
$$\begin{aligned} & -2*(f^2*x^2+2*e*f*x+e^2)/b/d*\exp(d*x+c)/(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b) \\ & -4*f/b/d^2*e/(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) \\ & +2*f^2/b/d^2/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) \\ & *x+2*f^2/b/d^3/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) \\ & *c-2*f^2/b/d^2/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) \\ & *x-2*f^2/b/d^3/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) \\ & *c+2*f^2/b/d^3/(a^2+b^2)^{(1/2)}*\operatorname{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) \\ & -2*f^2/b/d^3/(a^2+b^2)^{(1/2)}*\operatorname{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) \\ & +4*f^2/b/d^3*c/(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x+e)^2*\cosh(d*x+c)/(a+b*\sinh(d*x+c))^2,x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.40127, size = 3195, normalized size = 13.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x+e)^2*\cosh(d*x+c)/(a+b*\sinh(d*x+c))^2,x, \text{algorithm}="fricas")$

[Out] 
$$\begin{aligned} & 2*((b^2*f^2*\cosh(d*x+c)^2 + b^2*f^2*\sinh(d*x+c)^2 + 2*a*b*f^2*\cosh(d*x+c) \\ & - b^2*f^2 + 2*(b^2*f^2*\cosh(d*x+c) + a*b*f^2)*\sinh(d*x+c))*\sqrt{(a^2 + b^2)/b^2} \\ & *\operatorname{dilog}((a*\cosh(d*x+c) + a*\sinh(d*x+c) + (b*\cosh(d*x+c) + b*\sinh(d*x+c)) \\ & *\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - (b^2*f^2*\cosh(d*x+c)^2 + b^2*f^2*\sinh(d*x+c)^2 \\ & + 2*a*b*f^2*\cosh(d*x+c) - b^2*f^2 + 2*(b^2 \end{aligned}$$

```

*f^2*cosh(d*x + c) + a*b*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a
*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b + 1) + (b^2*d*e*f - b^2*c*f^2 - (b^2*d*e*f - b^2*c
*f^2)*cosh(d*x + c)^2 - (b^2*d*e*f - b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*
e*f - a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*e*f - a*b*c*f^2 + (b^2*d*e*f - b^
2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d
*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d*e*f
- b^2*c*f^2 - (b^2*d*e*f - b^2*c*f^2)*cosh(d*x + c)^2 - (b^2*d*e*f - b^2*c
*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*e*f - a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*
e*f - a*b*c*f^2 + (b^2*d*e*f - b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sq
rt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^
2 + b^2)/b^2) + 2*a) - (b^2*d*f^2*x + b^2*c*f^2 - (b^2*d*f^2*x + b^2*c*f^2)
*cosh(d*x + c)^2 - (b^2*d*f^2*x + b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*f^2
*x + a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*f^2*x + a*b*c*f^2 + (b^2*d*f^2*x +
b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b) + (b^2*d*f^2*x + b^2*c*f^2 - (b^2*d*f^2*x + b^2*c*f^2
)*cosh(d*x + c)^2 - (b^2*d*f^2*x + b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*f^
2*x + a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*f^2*x + a*b*c*f^2 + (b^2*d*f^2*x
+ b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cos
h(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b) - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x
+ (a^2 + b^2)*d^2*e^2)*cosh(d*x + c) - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 +
b^2)*d^2*e*f*x + (a^2 + b^2)*d^2*e^2)*sinh(d*x + c))/((a^2*b^2 + b^4)*d^3*
cosh(d*x + c)^2 + (a^2*b^2 + b^4)*d^3*sinh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d
^3*cosh(d*x + c) - (a^2*b^2 + b^4)*d^3 + 2*((a^2*b^2 + b^4)*d^3*cosh(d*x +
c) + (a^3*b + a*b^3)*d^3)*sinh(d*x + c))

```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))\*\*2,x)

[Out] Timed out

---



**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c)}{(b \sinh(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

$$3.323 \quad \int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b \sinh(c+dx))^2} dx$$

**Optimal.** Leaf size=348

$$\frac{6f^2(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}} - \frac{6f^2(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}} - \frac{6f^3\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4\sqrt{a^2+b^2}} + \frac{6f^3\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^4}$$

[Out] (3\*f\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d^2) - (3\*f\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d^2) + (6\*f^2\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^3) - (6\*f^2\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^3) - (6\*f^3\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^4) + (6\*f^3\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^4) - (e + f\*x)^3/(b\*d\*(a + b\*Sinh[c + d\*x]))

**Rubi [A]** time = 0.739712, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$ , Rules used = {5464, 3322, 2264, 2190, 2531, 2282, 6589}

$$\frac{6f^2(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}} - \frac{6f^2(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}} - \frac{6f^3\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4\sqrt{a^2+b^2}} + \frac{6f^3\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^4}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x])^2,x]

[Out] (3\*f\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d^2) - (3\*f\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d^2) + (6\*f^2\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^3) - (6\*f^2\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^3) - (6\*f^3\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^4) + (6\*f^3\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^4) - (e + f\*x)^3/(b\*d\*(a + b\*Sinh[c + d\*x]))

**Rule 5464**

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Simp[((e + f\*x)^m\*(a + b\*Sinh[c +

$d*x]^{(n+1)}/(b*d*(n+1)), x] - \text{Dist}[(f*m)/(b*d*(n+1)), \text{Int}[(e+f*x)^{(m-1)*(a+b*\text{Sinh}[c+d*x])}^{(n+1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[n, -1]$

### Rule 3322

$\text{Int}[(c + d*x)^{(m_1)}/(a + b*\sin[e + \text{Complex}[0, fz]]*(f_1*x))], x\_Symbol] :> \text{Dist}[2, \text{Int}[(c + d*x)^m * E^{-(I*e + f*fz*x)} / (-I*b + 2*a * E^{-(I*e + f*fz*x)} + I*b * E^{(2*(-I*e + f*fz*x))}), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, fz\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[m, 0]$

### Rule 2264

$\text{Int}[(F_1)^{(u_1)}*((f_1) + (g_1)*(x_1))^{(m_1)}/(a + b*(F_1)^{(u_1)} + (c_1)*(F_1)^{(v_1)})], x\_Symbol] :> \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c)/q, \text{Int}[(f + g*x)^m * F^u / (b - q + 2*c * F^u), x], x] - \text{Dist}[(2*c)/q, \text{Int}[(f + g*x)^m * F^u / (b + q + 2*c * F^u), x], x]] /; \text{FreeQ}\{F, a, b, c, f, g\}, x] \&\& \text{EqQ}[v, 2*u] \&\& \text{LinearQ}[u, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[m, 0]$

### Rule 2190

$\text{Int}[(F_1)^{(g_1)*((e_1) + (f_1)*(x_1))}^{(n_1)}*((c_1) + (d_1)*(x_1))^{(m_1)}/(a + b*(F_1)^{(g_1)*((e_1) + (f_1)*(x_1))}^{(n_1)})], x\_Symbol] :> \text{Simp}[(c + d*x)^m * \text{Log}[1 + (b*(F_1)^{(g_1)*((e_1) + (f_1)*(x_1))}^{(n_1)})/a] / (b*f*g*n * \text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n * \text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)} * \text{Log}[1 + (b*(F_1)^{(g_1)*((e_1) + (f_1)*(x_1))}^{(n_1)})/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

### Rule 2531

$\text{Int}[\text{Log}[1 + (e_1)*((F_1)^{(c_1)*((a_1) + (b_1)*(x_1))}^{(n_1)}) * ((f_1) + (g_1)*(x_1))^{(m_1)}], x\_Symbol] :> -\text{Simp}[(f + g*x)^m * \text{PolyLog}[2, -(e*(F_1)^{(c_1)*((a_1) + (b_1)*(x_1))}^{(n_1)})]^{(m_1)}) / (b*c*n * \text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n * \text{Log}[F]), \text{Int}[(f + g*x)^{(m-1)} * \text{PolyLog}[2, -(e*(F_1)^{(c_1)*((a_1) + (b_1)*(x_1))}^{(n_1)})]^{(m_1)}], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$

### Rule 2282

$\text{Int}[u, x\_Symbol] :> \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \&\& \text{!MatchQ}[u, (w_1)*((a_1)*(v_1)^{(n_1)})^{(m_1)}] /; \text{FreeQ}\{a, m, n\}, x] \&\& \text{IntegerQ}[m*n] \&\& \text{!MatchQ}[u, E^{(c_1)*((a_1) + (b_1)*x)} * (F_1)[v_1] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{InverseFunctionQ}[F[x]]]$

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:= Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b \sinh(c+dx))^2} dx &= -\frac{(e+fx)^3}{bd(a+b \sinh(c+dx))} + \frac{(3f) \int \frac{(e+fx)^2}{a+b \sinh(c+dx)} dx}{bd} \\
&= -\frac{(e+fx)^3}{bd(a+b \sinh(c+dx))} + \frac{(6f) \int \frac{e^{c+dx}(e+fx)^2}{-b+2ae^{c+dx}+be^{2(c+dx)}} dx}{bd} \\
&= -\frac{(e+fx)^3}{bd(a+b \sinh(c+dx))} + \frac{(6f) \int \frac{e^{c+dx}(e+fx)^2}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{\sqrt{a^2+b^2}d} - \frac{(6f) \int \frac{e^{c+dx}(e+fx)^2}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{\sqrt{a^2+b^2}d} \\
&= \frac{3f(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{3f(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{(e+fx)^3}{bd(a+b \sinh(c+dx))} \\
&= \frac{3f(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{3f(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} + \frac{6f^2(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} \\
&= \frac{3f(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{3f(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} + \frac{6f^2(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} \\
&= \frac{3f(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{3f(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} + \frac{6f^2(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3}
\end{aligned}$$

**Mathematica [A]** time = 2.56684, size = 368, normalized size = 1.06

$$3f \left( 2df(e+fx) \text{PolyLog} \left( 2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a} \right) - 2df(e+fx) \text{PolyLog} \left( 2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} \right) - 2f^2 \text{PolyLog} \left( 3, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a} \right) + 2f^2 \text{PolyLog} \left( 3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2, x]
```

```
[Out] (3*f*(-2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]) + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]) - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) + 2*d^2*e*f*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]))/(b^2*d^3)
```

+ Sqrt[a^2 + b^2]] - d^2\*f^2\*x^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] + 2\*d\*f\*(e + f\*x)\*PolyLog[2, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] - 2\*d\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))] - 2\*f^2\*PolyLog[3, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] + 2\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])))]/(b\*Sqrt[a^2 + b^2]\*d^4) - (e + f\*x)^3/(b\*d\*(a + b\*Sinh[c + d\*x]))

**Maple [F]** time = 0.32, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c)}{(a + b \sinh(dx + c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^2,x)

[Out] int((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^2,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 2.78683, size = 5462, normalized size = 15.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned}
& -(6*(b^2*d*f^3*x + b^2*d*e*f^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*\cosh(d*x + c))^2 \\
& - (b^2*d*f^3*x + b^2*d*e*f^2)*\sinh(d*x + c)^2 - 2*(a*b*d*f^3*x + a*b*d*e*f^2) \\
& *\cosh(d*x + c) - 2*(a*b*d*f^3*x + a*b*d*e*f^2 + (b^2*d*f^3*x + b^2*d*e*f^2) \\
& *\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}*\operatorname{dilog}((a*\cosh(d*x \\
& + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} \\
& - b)/b + 1) - 6*(b^2*d*f^3*x + b^2*d*e*f^2 - (b^2*d*f^3*x + b^2*d*e*f^2) \\
& *\cosh(d*x + c))^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*\sinh(d*x + c)^2 - 2*(a \\
& *b*d*f^3*x + a*b*d*e*f^2)*\cosh(d*x + c) - 2*(a*b*d*f^3*x + a*b*d*e*f^2 + (b^2 \\
& *d*f^3*x + b^2*d*e*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} \\
& )*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x \\
& + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 3*(b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 \\
& + b^2*c^2*f^3 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*\cosh(d*x \\
& + c))^2 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*\sinh(d*x + c)^2 - \\
& 2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3)*\cosh(d*x + c) - 2*(a*b*d^2 \\
& *e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3 + (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 \\
& + b^2*c^2*f^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}*\log(2*b* \\
& \cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 3*(b \\
& ^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3 - (b^2*d^2*e^2*f - 2*b^2*c*d*e \\
& *f^2 + b^2*c^2*f^3)*\cosh(d*x + c))^2 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2 \\
& *c^2*f^3)*\sinh(d*x + c)^2 - 2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3) \\
& *\cosh(d*x + c) - 2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3 + (b^2 \\
& *d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} \\
& )*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) \\
& + 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3 - (b^2*d^2*f^3*x^2 \\
& + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*\cosh(d*x + c))^2 - (b^2*d^2*f^3*x^2 \\
& + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*\sinh(d*x + c)^2 - 2*(a*b*d^2*f^3*x^2 \\
& + 2*a*b*d^2*e*f^2*x + 2*a*b*c*d*e*f^2 - a*b*c^2*f^3)*\cosh(d*x + c) - 2*(a*b*d^2*f^3 \\
& *x^2 + 2*a*b*d^2*e*f^2*x + 2*a*b*c*d*e*f^2 - a*b*c^2*f^3 + (b^2*d^2*f^3*x^2 \\
& + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*\cosh(d*x + c))*\sinh(d*x \\
& + c))*\sqrt{(a^2 + b^2)/b^2}*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x \\
& + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2 \\
& *e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3 - (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x \\
& + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*\cosh(d*x + c))^2 - (b^2*d^2*f^3*x^2 + 2*b^2*d^2 \\
& *e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*\sinh(d*x + c)^2 - 2*(a*b*d^2*f^3*x^2 \\
& + 2*a*b*d^2*e*f^2*x + 2*a*b*c*d*e*f^2 - a*b*c^2*f^3)*\cosh(d*x + c) - 2*(a*b*d^2 \\
& *f^3*x^2 + 2*a*b*d^2*e*f^2*x + 2*a*b*c*d*e*f^2 - a*b*c^2*f^3 + (b^2*d^2*f^3*x^2 \\
& + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*\cosh(d*x + c))*\sinh(d*x \\
& + c))*\sqrt{(a^2 + b^2)/b^2}*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x \\
& + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + 6*(b^2*f^3*\cosh(d*x + c))^2 \\
& + b^2*f^3*\sinh(d*x + c)^2 + 2*a*b*f^3*\cosh(d*x + c) - b^2*f^3 + 2*(b^2*f^3*\cosh(d*x + c) \\
& + a*b*f^3)*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}*\operatorname{polylog}(3, (a*\cosh(d*x + c) \\
& + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} \\
& ))/b) - 6*(b^2*f^3*\cosh(d*x + c))^2 + b^2*f^3*\sinh(d*x + c)^2 + 2*a*b*f^3*
\end{aligned}$$

```

cosh(d*x + c) - b^2*f^3 + 2*(b^2*f^3*cosh(d*x + c) + a*b*f^3)*sinh(d*x + c)
)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*((a^2 + b^2)
*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + (a
^2 + b^2)*d^3*e^3)*cosh(d*x + c) + 2*((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b
^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + (a^2 + b^2)*d^3*e^3)*sinh(d*
x + c))/((a^2*b^2 + b^4)*d^4*cosh(d*x + c)^2 + (a^2*b^2 + b^4)*d^4*sinh(d*x
+ c)^2 + 2*(a^3*b + a*b^3)*d^4*cosh(d*x + c) - (a^2*b^2 + b^4)*d^4 + 2*((a
^2*b^2 + b^4)*d^4*cosh(d*x + c) + (a^3*b + a*b^3)*d^4)*sinh(d*x + c))

```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c)}{(b \sinh(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*cosh(d\*x + c)/(b\*sinh(d\*x + c) + a)^2, x)

$$3.324 \quad \int \frac{(e+fx) \cosh(c+dx)}{(a+b \sinh(c+dx))^2} dx$$

**Optimal.** Leaf size=74

$$-\frac{2f \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}} - \frac{e+fx}{bd(a+b \sinh(c+dx))}$$

[Out]  $(-2*f*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b*Sqrt[a^2 + b^2]*d^2) - (e + f*x)/(b*d*(a + b*Sinh[c + d*x]))$

**Rubi [A]** time = 0.0700089, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {5464, 2660, 618, 204}

$$-\frac{2f \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2\sqrt{a^2+b^2}} - \frac{e+fx}{bd(a+b \sinh(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x])^2,x]

[Out]  $(-2*f*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(b*Sqrt[a^2 + b^2]*d^2) - (e + f*x)/(b*d*(a + b*Sinh[c + d*x]))$

#### Rule 5464

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Simp[((e + f\*x)^m\*(a + b\*Sinh[c + d\*x])^(n + 1))/(b\*d\*(n + 1)), x] - Dist[(f\*m)/(b\*d\*(n + 1)), Int[(e + f\*x)^(m - 1)\*(a + b\*Sinh[c + d\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

#### Rule 2660

Int[((a\_.) + (b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] :> With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + 2\*b\*e\*x + a\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[



$a^2 - b^2, 0]$

### Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

### Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

### Rubi steps

$$\begin{aligned} \int \frac{(e + fx) \cosh(c + dx)}{(a + b \sinh(c + dx))^2} dx &= -\frac{e + fx}{bd(a + b \sinh(c + dx))} + \frac{f \int \frac{1}{a + b \sinh(c + dx)} dx}{bd} \\ &= -\frac{e + fx}{bd(a + b \sinh(c + dx))} - \frac{(2if) \operatorname{Subst}\left(\int \frac{1}{a - 2ibx + ax^2} dx, x, \tan\left(\frac{1}{2}(ic + idx)\right)\right)}{bd^2} \\ &= -\frac{e + fx}{bd(a + b \sinh(c + dx))} + \frac{(4if) \operatorname{Subst}\left(\int \frac{1}{-4(a^2 + b^2) - x^2} dx, x, -2ib + 2a \tan\left(\frac{1}{2}(ic + idx)\right)\right)}{bd^2} \\ &= -\frac{2f \tanh^{-1}\left(\frac{b - a \tanh\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a^2 + b^2}}\right)}{b\sqrt{a^2 + b^2}d^2} - \frac{e + fx}{bd(a + b \sinh(c + dx))} \end{aligned}$$

**Mathematica [A]** time = 0.396371, size = 78, normalized size = 1.05

$$\frac{2f \tan^{-1}\left(\frac{b - a \tanh\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2}} - \frac{d(e + fx)}{a + b \sinh(c + dx)}{bd^2}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x])^2,x]

[Out]  $((2*f*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - (d*(e + f*x))/(a + b*Sinh[c + d*x]))/(b*d^2)$

**Maple [B]** time = 0., size = 164, normalized size = 2.2

$$-2 \frac{(fx + e)e^{dx+c}}{bd(be^{2dx+2c} + 2ae^{dx+c} - b)} + \frac{f}{d^2b} \ln\left(e^{dx+c} + \frac{1}{b}\left(a\sqrt{a^2 + b^2} - a^2 - b^2\right)\frac{1}{\sqrt{a^2 + b^2}}\right) \frac{1}{\sqrt{a^2 + b^2}} - \frac{f}{d^2b} \ln\left(e^{dx+c} + \frac{1}{b}\left(a\sqrt{a^2 + b^2} + a^2 + b^2\right)\frac{1}{\sqrt{a^2 + b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x)`

[Out]  $-2*(f*x+e)/b/d*\exp(d*x+c)/(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)+1/(a^2+b^2)^{(1/2)}*f/d^2/b*\ln(\exp(d*x+c)+(a*(a^2+b^2)^{(1/2)}-a^2-b^2)/(a^2+b^2)^{(1/2)})/b)-1/(a^2+b^2)^{(1/2)}*f/d^2/b*\ln(\exp(d*x+c)+(a*(a^2+b^2)^{(1/2)}+a^2+b^2)/(a^2+b^2)^{(1/2)})/b)$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.60907, size = 1015, normalized size = 13.72

$$\frac{(bf \cosh(dx + c)^2 + bf \sinh(dx + c)^2 + 2af \cosh(dx + c) - bf + 2(bf \cosh(dx + c) + af) \sinh(dx + c))\sqrt{a^2 + b^2} \log\left(\frac{e^{dx+c} + \frac{1}{b}(a\sqrt{a^2 + b^2} - a^2 - b^2)}{\sqrt{a^2 + b^2}}\right) + (bf \cosh(dx + c) - bf + 2(bf \cosh(dx + c) + af) \sinh(dx + c))\sqrt{a^2 + b^2} \log\left(\frac{e^{dx+c} + \frac{1}{b}(a\sqrt{a^2 + b^2} + a^2 + b^2)}{\sqrt{a^2 + b^2}}\right)}{(a^2b^2 + b^4)d^2 \cosh(dx + c)^2 + (a^2b^2 + b^4)d^2 \sinh(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="fricas")`

```
[Out] ((b*f*cosh(d*x + c)^2 + b*f*sinh(d*x + c)^2 + 2*a*f*cosh(d*x + c) - b*f + 2
*(b*f*cosh(d*x + c) + a*f)*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*cosh(d*x
+ c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*
cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b
*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x
+ c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*((a^2 + b^2)*d*f*x +
(a^2 + b^2)*d*e)*cosh(d*x + c) - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*d*e)*s
inh(d*x + c))/((a^2*b^2 + b^4)*d^2*cosh(d*x + c)^2 + (a^2*b^2 + b^4)*d^2*si
nh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d^2*cosh(d*x + c) - (a^2*b^2 + b^4)*d^2 +
2*((a^2*b^2 + b^4)*d^2*cosh(d*x + c) + (a^3*b + a*b^3)*d^2)*sinh(d*x + c))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \cosh(dx + c)}{(b \sinh(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

$$3.325 \quad \int \frac{(e+fx)^2 \cosh(c+dx)}{(a+b \sinh(c+dx))^2} dx$$

**Optimal.** Leaf size=234

$$\frac{2f^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}} - \frac{2f^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}} + \frac{2f(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{bd^2\sqrt{a^2+b^2}} - \frac{2f(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2\sqrt{a^2+b^2}}$$

[Out] (2\*f\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d^2) - (2\*f\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d^2) + (2\*f^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^3) - (2\*f^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^3) - (e + f\*x)^2/(b\*d\*(a + b\*Sinh[c + d\*x]))

**Rubi [A]** time = 0.439988, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$ , Rules used = {5464, 3322, 2264, 2190, 2279, 2391}

$$\frac{2f^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}} - \frac{2f^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}} + \frac{2f(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{bd^2\sqrt{a^2+b^2}} - \frac{2f(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x])^2,x]

[Out] (2\*f\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d^2) - (2\*f\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d^2) + (2\*f^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^3) - (2\*f^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^3) - (e + f\*x)^2/(b\*d\*(a + b\*Sinh[c + d\*x]))

**Rule 5464**

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Simp[((e + f\*x)^m\*(a + b\*Sinh[c + d\*x])^(n + 1))/(b\*d\*(n + 1)), x] - Dist[(f\*m)/(b\*d\*(n + 1)), Int[(e + f\*x)^(m - 1)\*(a + b\*Sinh[c + d\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_] *
(f_.)*(x_))], x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :=> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] :=> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:=> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :=> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \cosh(c+dx)}{(a+b \sinh(c+dx))^2} dx &= -\frac{(e+fx)^2}{bd(a+b \sinh(c+dx))} + \frac{(2f) \int \frac{e+fx}{a+b \sinh(c+dx)} dx}{bd} \\
&= -\frac{(e+fx)^2}{bd(a+b \sinh(c+dx))} + \frac{(4f) \int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}} dx}{bd} \\
&= -\frac{(e+fx)^2}{bd(a+b \sinh(c+dx))} + \frac{(4f) \int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{\sqrt{a^2+b^2}d} - \frac{(4f) \int \frac{e^{c+dx}(e+fx)}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{\sqrt{a^2+b^2}d} \\
&= \frac{2f(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{2f(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{(e+fx)^2}{bd(a+b \sinh(c+dx))} \\
&= \frac{2f(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{2f(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{(e+fx)^2}{bd(a+b \sinh(c+dx))} \\
&= \frac{2f(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{2f(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} + \frac{2f^2 \text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} - \frac{(e+fx)^2}{bd(a+b \sinh(c+dx))}
\end{aligned}$$

**Mathematica [A]** time = 0.550565, size = 175, normalized size = 0.75

$$\frac{2f \left( f \text{PolyLog} \left( 2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a} \right) - f \text{PolyLog} \left( 2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} \right) + d(e+fx) \left( \log \left( \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1 \right) - \log \left( \frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} + 1 \right) \right) \right)}{bd^3 \sqrt{a^2+b^2}} - \frac{(e+fx)^2}{bd(a+b \sinh(c+dx))}$$

Antiderivative was successfully verified.

[In] Integrate[((e+f\*x)^2\*Cosh[c+d\*x])/(a+b\*Sinh[c+d\*x])^2,x]

[Out] (2\*f\*(d\*(e+f\*x)\*(Log[1+(b\*E^(c+d\*x))/(a-Sqrt[a^2+b^2]])]-Log[1+(b\*E^(c+d\*x))/(a+Sqrt[a^2+b^2]])))+f\*PolyLog[2,(b\*E^(c+d\*x))/(-a+Sqrt[a^2+b^2]])-f\*PolyLog[2,-((b\*E^(c+d\*x))/(a+Sqrt[a^2+b^2]))]))/(b\*Sqrt[a^2+b^2]\*d^3)-(e+f\*x)^2/(b\*d\*(a+b\*Sinh[c+d\*x]))

**Maple [B]** time = 0.001, size = 491, normalized size = 2.1

$$-2 \frac{(x^2 f^2 + 2 e f x + e^2) e^{dx+c}}{bd (be^{2dx+2c} + 2ae^{dx+c} - b)} - 4 \frac{ef}{d^2 b \sqrt{a^2+b^2}} \text{Artanh} \left( \frac{1}{2} \frac{2be^{dx+c} + 2a}{\sqrt{a^2+b^2}} \right) + 2 \frac{f^2 x}{d^2 b \sqrt{a^2+b^2}} \ln \left( \frac{-be^{dx+c} + \sqrt{a^2+b^2}}{-a + \sqrt{a^2+b^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)^2*\cosh(d*x+c)/(a+b*\sinh(d*x+c))^2,x)$

[Out] 
$$\begin{aligned} & -2*(f^2*x^2+2*e*f*x+e^2)/b/d*\exp(d*x+c)/(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b) \\ & -4*f/b/d^2*e/(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) \\ & +2*f^2/b/d^2/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) \\ & *x+2*f^2/b/d^3/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) \\ & *c-2*f^2/b/d^2/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) \\ & *x-2*f^2/b/d^3/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) \\ & *c+2*f^2/b/d^3/(a^2+b^2)^{(1/2)}*\operatorname{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) \\ & -2*f^2/b/d^3/(a^2+b^2)^{(1/2)}*\operatorname{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) \\ & +4*f^2/b/d^3*c/(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x+e)^2*\cosh(d*x+c)/(a+b*\sinh(d*x+c))^2,x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.93246, size = 3195, normalized size = 13.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x+e)^2*\cosh(d*x+c)/(a+b*\sinh(d*x+c))^2,x, \text{algorithm}="fricas")$

[Out] 
$$\begin{aligned} & 2*((b^2*f^2*\cosh(d*x+c)^2 + b^2*f^2*\sinh(d*x+c)^2 + 2*a*b*f^2*\cosh(d*x+c) \\ & - b^2*f^2 + 2*(b^2*f^2*\cosh(d*x+c) + a*b*f^2)*\sinh(d*x+c))*\sqrt{(a^2 + b^2)/b^2} \\ & *\operatorname{dilog}((a*\cosh(d*x+c) + a*\sinh(d*x+c) + (b*\cosh(d*x+c) + b*\sinh(d*x+c)) \\ & *\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - (b^2*f^2*\cosh(d*x+c)^2 + b^2*f^2*\sinh(d*x+c)^2 \\ & + 2*a*b*f^2*\cosh(d*x+c) - b^2*f^2 + 2*(b^2 \end{aligned}$$

```

*f^2*cosh(d*x + c) + a*b*f^2)*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a
*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt
((a^2 + b^2)/b^2) - b)/b + 1) + (b^2*d*e*f - b^2*c*f^2 - (b^2*d*e*f - b^2*c
*f^2)*cosh(d*x + c)^2 - (b^2*d*e*f - b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*
e*f - a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*e*f - a*b*c*f^2 + (b^2*d*e*f - b^
2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d
*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (b^2*d*e*f
- b^2*c*f^2 - (b^2*d*e*f - b^2*c*f^2)*cosh(d*x + c)^2 - (b^2*d*e*f - b^2*c
*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*e*f - a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*
e*f - a*b*c*f^2 + (b^2*d*e*f - b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sq
rt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^
2 + b^2)/b^2) + 2*a) - (b^2*d*f^2*x + b^2*c*f^2 - (b^2*d*f^2*x + b^2*c*f^2)
*cosh(d*x + c)^2 - (b^2*d*f^2*x + b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*f^2
*x + a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*f^2*x + a*b*c*f^2 + (b^2*d*f^2*x +
b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b) + (b^2*d*f^2*x + b^2*c*f^2 - (b^2*d*f^2*x + b^2*c*f^2
)*cosh(d*x + c)^2 - (b^2*d*f^2*x + b^2*c*f^2)*sinh(d*x + c)^2 - 2*(a*b*d*f^
2*x + a*b*c*f^2)*cosh(d*x + c) - 2*(a*b*d*f^2*x + a*b*c*f^2 + (b^2*d*f^2*x
+ b^2*c*f^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*log(-(a*cos
h(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2) - b)/b) - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x
+ (a^2 + b^2)*d^2*e^2)*cosh(d*x + c) - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 +
b^2)*d^2*e*f*x + (a^2 + b^2)*d^2*e^2)*sinh(d*x + c))/((a^2*b^2 + b^4)*d^3*
cosh(d*x + c)^2 + (a^2*b^2 + b^4)*d^3*sinh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d
^3*cosh(d*x + c) - (a^2*b^2 + b^4)*d^3 + 2*((a^2*b^2 + b^4)*d^3*cosh(d*x +
c) + (a^3*b + a*b^3)*d^3)*sinh(d*x + c))

```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))\*\*2,x)

[Out] Timed out



**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c)}{(b \sinh(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)/(a+b*sinh(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)/(b*sinh(d*x + c) + a)^2, x)
```

$$3.326 \quad \int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b \sinh(c+dx))^2} dx$$

**Optimal.** Leaf size=348

$$\frac{6f^2(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}} - \frac{6f^2(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}} - \frac{6f^3\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4\sqrt{a^2+b^2}} + \frac{6f^3\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^4}$$

[Out] (3\*f\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d^2) - (3\*f\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d^2) + (6\*f^2\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^3) - (6\*f^2\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^3) - (6\*f^3\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^4) + (6\*f^3\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^4) - (e + f\*x)^3/(b\*d\*(a + b\*Sinh[c + d\*x]))

**Rubi [A]** time = 0.73091, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$ , Rules used = {5464, 3322, 2264, 2190, 2531, 2282, 6589}

$$\frac{6f^2(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3\sqrt{a^2+b^2}} - \frac{6f^2(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3\sqrt{a^2+b^2}} - \frac{6f^3\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4\sqrt{a^2+b^2}} + \frac{6f^3\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^4}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x])^2,x]

[Out] (3\*f\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d^2) - (3\*f\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b\*Sqrt[a^2 + b^2]\*d^2) + (6\*f^2\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^3) - (6\*f^2\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^3) - (6\*f^3\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^4) + (6\*f^3\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*Sqrt[a^2 + b^2]\*d^4) - (e + f\*x)^3/(b\*d\*(a + b\*Sinh[c + d\*x]))

**Rule 5464**

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Simp[((e + f\*x)^m\*(a + b\*Sinh[c +

$d*x]^{(n+1)}/(b*d*(n+1)), x] - \text{Dist}[(f*m)/(b*d*(n+1)), \text{Int}[(e+f*x)^{(m-1)*(a+b*\text{Sinh}[c+d*x])^{(n+1)}, x}], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[n, -1]$

### Rule 3322

$\text{Int}[(c + d*x)^{(m_1)}/((a + b*\sin[e + \text{Complex}[0, fz]]*(f_1)*x)], x\_Symbol] :> \text{Dist}[2, \text{Int}[(c + d*x)^m * E^{-(I*e + f*fz*x)} / (-I*b + 2*a * E^{-(I*e + f*fz*x)} + I*b * E^{(2*(-I*e + f*fz*x))}), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, fz\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[m, 0]$

### Rule 2264

$\text{Int}[(F_1)^{(u_1)}*((f_1) + (g_1)*x)^{(m_1)}/((a_1) + (b_1)*(F_1)^{(u_1)} + (c_1)*(F_1)^{(v_1)}), x\_Symbol] :> \text{With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c)/q, \text{Int}[(f + g*x)^m * F^u / (b - q + 2*c * F^u), x], x] - \text{Dist}[(2*c)/q, \text{Int}[(f + g*x)^m * F^u / (b + q + 2*c * F^u), x], x]] /; \text{FreeQ}\{F, a, b, c, f, g\}, x] \&\& \text{EqQ}[v, 2*u] \&\& \text{LinearQ}[u, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[m, 0]$

### Rule 2190

$\text{Int}[(F_1)^{(g_1)*((e_1) + (f_1)*x))^{(n_1)}*((c_1) + (d_1)*x)^{(m_1)}/((a_1) + (b_1)*(F_1)^{(g_1)*((e_1) + (f_1)*x))^{(n_1)}), x\_Symbol] :> \text{Simp}[(c + d*x)^m * \text{Log}[1 + (b*(F_1)^{(g_1)*((e_1) + (f_1)*x)})^n] / a] / (b*f*g*n * \text{Log}[F]), x] - \text{Dist}[(d*m) / (b*f*g*n * \text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)} * \text{Log}[1 + (b*(F_1)^{(g_1)*((e_1) + (f_1)*x)})^n] / a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

### Rule 2531

$\text{Int}[\text{Log}[1 + (e_1)*((F_1)^{(c_1)*((a_1) + (b_1)*x))^{(n_1)}]*((f_1) + (g_1)*x)^{(m_1)}, x\_Symbol] :> -\text{Simp}[(f + g*x)^m * \text{PolyLog}[2, -(e*(F_1)^{(c_1)*((a_1) + (b_1)*x)})^n] / (b*c*n * \text{Log}[F]), x] + \text{Dist}[(g*m) / (b*c*n * \text{Log}[F]), \text{Int}[(f + g*x)^{(m-1)} * \text{PolyLog}[2, -(e*(F_1)^{(c_1)*((a_1) + (b_1)*x)})^n], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$

### Rule 2282

$\text{Int}[u, x\_Symbol] :> \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \&\& !\text{MatchQ}[u, (w_1)*((a_1)*(v_1)^{(n_1)})^{(m_1)} /; \text{FreeQ}\{a, m, n\}, x] \&\& \text{IntegerQ}[m*n] \&\& !\text{MatchQ}[u, E^{(c_1)*((a_1) + (b_1)*x)}*(F_1)[v_1] /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{InverseFunctionQ}[F[x]]]$

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:= Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b \sinh(c+dx))^2} dx &= -\frac{(e+fx)^3}{bd(a+b \sinh(c+dx))} + \frac{(3f) \int \frac{(e+fx)^2}{a+b \sinh(c+dx)} dx}{bd} \\
&= -\frac{(e+fx)^3}{bd(a+b \sinh(c+dx))} + \frac{(6f) \int \frac{e^{c+dx}(e+fx)^2}{-b+2ae^{c+dx}+be^{2(c+dx)}} dx}{bd} \\
&= -\frac{(e+fx)^3}{bd(a+b \sinh(c+dx))} + \frac{(6f) \int \frac{e^{c+dx}(e+fx)^2}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{\sqrt{a^2+b^2}d} - \frac{(6f) \int \frac{e^{c+dx}(e+fx)^2}{2a+2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{\sqrt{a^2+b^2}d} \\
&= \frac{3f(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{3f(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{(e+fx)^3}{bd(a+b \sinh(c+dx))} \\
&= \frac{3f(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{3f(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} + \frac{6f^2(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} \\
&= \frac{3f(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{3f(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} + \frac{6f^2(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3} \\
&= \frac{3f(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} - \frac{3f(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^2} + \frac{6f^2(e+fx)\text{Li}_2\left(-\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}d^3}
\end{aligned}$$

**Mathematica [A]** time = 0.388116, size = 368, normalized size = 1.06

$$3f \left( 2df(e+fx) \text{PolyLog} \left( 2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a} \right) - 2df(e+fx) \text{PolyLog} \left( 2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} \right) - 2f^2 \text{PolyLog} \left( 3, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a} \right) + 2f^2 \text{PolyLog} \left( 3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^2, x]
```

```
[Out] (3*f*(-2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]) + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]) - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) - 2*f^2*PolyLog[3, (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*d^3)
```

+ Sqrt[a^2 + b^2]] - d^2\*f^2\*x^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] + 2\*d\*f\*(e + f\*x)\*PolyLog[2, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] - 2\*d\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))] - 2\*f^2\*PolyLog[3, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] + 2\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])))]/(b\*Sqrt[a^2 + b^2]\*d^4) - (e + f\*x)^3/(b\*d\*(a + b\*Sinh[c + d\*x]))

**Maple [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c)}{(a + b \sinh(dx + c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^2,x)

[Out] int((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^2,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 2.89884, size = 5462, normalized size = 15.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned}
& -(6*(b^2*d*f^3*x + b^2*d*e*f^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*\cosh(d*x + c))^2 \\
& - (b^2*d*f^3*x + b^2*d*e*f^2)*\sinh(d*x + c)^2 - 2*(a*b*d*f^3*x + a*b*d*e*f^2) \\
& *\cosh(d*x + c) - 2*(a*b*d*f^3*x + a*b*d*e*f^2 + (b^2*d*f^3*x + b^2*d*e*f^2) \\
& *\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}*\operatorname{dilog}((a*\cosh(d*x \\
& + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} \\
& - b)/b + 1) - 6*(b^2*d*f^3*x + b^2*d*e*f^2 - (b^2*d*f^3*x + b^2*d*e*f^2) \\
& *\cosh(d*x + c))^2 - (b^2*d*f^3*x + b^2*d*e*f^2)*\sinh(d*x + c)^2 - 2*(a \\
& *b*d*f^3*x + a*b*d*e*f^2)*\cosh(d*x + c) - 2*(a*b*d*f^3*x + a*b*d*e*f^2 + (b^2 \\
& *d*f^3*x + b^2*d*e*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} \\
& )*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x \\
& + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 3*(b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 \\
& + b^2*c^2*f^3 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*\cosh(d*x \\
& + c))^2 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*\sinh(d*x + c)^2 - \\
& 2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3)*\cosh(d*x + c) - 2*(a*b*d^2 \\
& *e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3 + (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 \\
& + b^2*c^2*f^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}*\log(2*b* \\
& \cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 3*(b \\
& ^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3 - (b^2*d^2*e^2*f - 2*b^2*c*d*e \\
& *f^2 + b^2*c^2*f^3)*\cosh(d*x + c))^2 - (b^2*d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2 \\
& *c^2*f^3)*\sinh(d*x + c)^2 - 2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3) \\
& *\cosh(d*x + c) - 2*(a*b*d^2*e^2*f - 2*a*b*c*d*e*f^2 + a*b*c^2*f^3 + (b^2 \\
& *d^2*e^2*f - 2*b^2*c*d*e*f^2 + b^2*c^2*f^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} \\
& )*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) \\
& + 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3 - (b^2*d^2*f^3*x^2 \\
& + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*\cosh(d*x + c))^2 - (b^2*d^2*f^3*x^2 \\
& + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*\sinh(d*x + c)^2 - 2*(a*b*d^2*f^3*x^2 \\
& + 2*a*b*d^2*e*f^2*x + 2*a*b*c*d*e*f^2 - a*b*c^2*f^3)*\cosh(d*x + c) - 2*(a*b*d^2*f^3 \\
& *x^2 + 2*a*b*d^2*e*f^2*x + 2*a*b*c*d*e*f^2 - a*b*c^2*f^3 + (b^2*d^2*f^3*x^2 \\
& + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*\cosh(d*x + c))*\sinh(d*x + c) \\
& )*\sqrt{(a^2 + b^2)/b^2}*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) \\
& + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2 \\
& *e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3 - (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x \\
& + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*\cosh(d*x + c))^2 - (b^2*d^2*f^3*x^2 + 2*b^2*d^2 \\
& *e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*\sinh(d*x + c)^2 - 2*(a*b*d^2*f^3*x^2 \\
& + 2*a*b*d^2*e*f^2*x + 2*a*b*c*d*e*f^2 - a*b*c^2*f^3)*\cosh(d*x + c) - 2*(a*b*d^2 \\
& *f^3*x^2 + 2*a*b*d^2*e*f^2*x + 2*a*b*c*d*e*f^2 - a*b*c^2*f^3 + (b^2*d^2*f^3*x^2 \\
& + 2*b^2*d^2*e*f^2*x + 2*b^2*c*d*e*f^2 - b^2*c^2*f^3)*\cosh(d*x + c))*\sinh(d*x + c) \\
& )*\sqrt{(a^2 + b^2)/b^2}*\log(- (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) \\
& + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + 6*(b^2*f^3*\cosh(d*x + c))^2 + b^2*f^3*\sinh \\
& (d*x + c)^2 + 2*a*b*f^3*\cosh(d*x + c) - b^2*f^3 + 2*(b^2*f^3*\cosh(d*x + c) \\
& + a*b*f^3)*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}*\operatorname{polylog}(3, (a*\cosh(d*x + c) \\
& + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} \\
& ))/b) - 6*(b^2*f^3*\cosh(d*x + c))^2 + b^2*f^3*\sinh(d*x + c)^2 + 2*a*b*f^3*
\end{aligned}$$

$$\begin{aligned} & \cosh(dx + c) - b^2 f^3 + 2(b^2 f^3 \cosh(dx + c) + a b f^3) \sinh(dx + c) \\ & ) \sqrt{(a^2 + b^2)/b^2} \operatorname{polylog}(3, (a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2})/b) + 2((a^2 + b^2) \\ & * d^3 f^3 x^3 + 3(a^2 + b^2) d^3 e f^2 x^2 + 3(a^2 + b^2) d^3 e^2 f x + (a^2 + b^2) d^3 e^3) \cosh(dx + c) + 2((a^2 + b^2) d^3 f^3 x^3 + 3(a^2 + b^2) \\ & d^3 e f^2 x^2 + 3(a^2 + b^2) d^3 e^2 f x + (a^2 + b^2) d^3 e^3) \sinh(dx + c) / ((a^2 b^2 + b^4) d^4 \cosh(dx + c)^2 + (a^2 b^2 + b^4) d^4 \sinh(dx + c)^2 + 2(a^3 b + a b^3) d^4 \cosh(dx + c) - (a^2 b^2 + b^4) d^4 + 2((a^2 b^2 + b^4) d^4 \cosh(dx + c) + (a^3 b + a b^3) d^4) \sinh(dx + c)) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c)}{(b \sinh(dx + c) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^2,x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*cosh(d\*x + c)/(b\*sinh(d\*x + c) + a)^2, x)

$$3.327 \quad \int \frac{(e+fx) \cosh(c+dx)}{(a+b \sinh(c+dx))^3} dx$$

**Optimal.** Leaf size=112

$$-\frac{af \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2 (a^2+b^2)^{3/2}} - \frac{f \cosh(c+dx)}{2d^2 (a^2+b^2)(a+b \sinh(c+dx))} - \frac{e+fx}{2bd(a+b \sinh(c+dx))^2}$$

[Out]  $-\left(\frac{a*f*ArcTanh\left[\frac{b-a*\Tanh\left[\frac{c+d*x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{b*(a^2+b^2)^{3/2}*d^2}\right) - \frac{e+f*x}{2*b*d*(a+b*\Sinh[c+d*x])^2} - \frac{f*Cosh[c+d*x]}{2*(a^2+b^2)*d^2*(a+b*\Sinh[c+d*x])}$

**Rubi [A]** time = 0.0983786, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5464, 2664, 12, 2660, 618, 204}

$$-\frac{af \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2 (a^2+b^2)^{3/2}} - \frac{f \cosh(c+dx)}{2d^2 (a^2+b^2)(a+b \sinh(c+dx))} - \frac{e+fx}{2bd(a+b \sinh(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x])^3,x]

[Out]  $-\left(\frac{a*f*ArcTanh\left[\frac{b-a*\Tanh\left[\frac{c+d*x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{b*(a^2+b^2)^{3/2}*d^2}\right) - \frac{e+f*x}{2*b*d*(a+b*\Sinh[c+d*x])^2} - \frac{f*Cosh[c+d*x]}{2*(a^2+b^2)*d^2*(a+b*\Sinh[c+d*x])}$

#### Rule 5464

Int[Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.)\*((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Simp[((e + f\*x)^m\*(a + b\*Sinh[c + d\*x])^(n + 1))/(b\*d\*(n + 1)), x] - Dist[(f\*m)/(b\*d\*(n + 1)), Int[(e + f\*x)^(m - 1)\*(a + b\*Sinh[c + d\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

#### Rule 2664

Int[((a\_.) + (b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_.), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x]\*(a + b\*Sinh[c + d\*x])^(n + 1))/(d\*(n + 1)\*(a^2 - b^2)), x] + Dist[1



$$\int \frac{(n+1)(a^2 - b^2)}{(a + b \sin[c + dx])^{n+1}} \operatorname{Simp}[a(n+1) - b(n+2)\sin[c + dx], x] \, dx$$
 /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2\*n]

### Rule 12

$$\int (a_1 u_1) (a_2 v_1) \, dx$$
 /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 2660

$$\int ((a_1) + (b_1) \sin[(c_1) + (d_1)x])^{-1} \, dx$$
 /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

### Rule 618

$$\int ((a_1) + (b_1)x + (c_1)x^2)^{-1} \, dx$$
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

$$\int ((a_1) + (b_1)x^2)^{-1} \, dx$$
 /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
\int \frac{(e + fx) \cosh(c + dx)}{(a + b \sinh(c + dx))^3} dx &= -\frac{e + fx}{2bd(a + b \sinh(c + dx))^2} + \frac{f \int \frac{1}{(a + b \sinh(c + dx))^2} dx}{2bd} \\
&= -\frac{e + fx}{2bd(a + b \sinh(c + dx))^2} - \frac{f \cosh(c + dx)}{2(a^2 + b^2) d^2 (a + b \sinh(c + dx))} + \frac{f \int \frac{a}{a + b \sinh(c + dx)} dx}{2b(a^2 + b^2) d} \\
&= -\frac{e + fx}{2bd(a + b \sinh(c + dx))^2} - \frac{f \cosh(c + dx)}{2(a^2 + b^2) d^2 (a + b \sinh(c + dx))} + \frac{(af) \int \frac{1}{a + b \sinh(c + dx)} dx}{2b(a^2 + b^2) d} \\
&= -\frac{e + fx}{2bd(a + b \sinh(c + dx))^2} - \frac{f \cosh(c + dx)}{2(a^2 + b^2) d^2 (a + b \sinh(c + dx))} - \frac{(iaf) \operatorname{Subst} \left( \int \frac{1}{a - 2ibx + ax^2} dx \right)}{b(a^2 + b^2)} \\
&= -\frac{e + fx}{2bd(a + b \sinh(c + dx))^2} - \frac{f \cosh(c + dx)}{2(a^2 + b^2) d^2 (a + b \sinh(c + dx))} + \frac{(2iaf) \operatorname{Subst} \left( \int \frac{1}{-4(a^2 + b^2)} dx \right)}{b(a^2 + b^2)} \\
&= -\frac{af \tanh^{-1} \left( \frac{b - a \tanh\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a^2 + b^2}} \right)}{b(a^2 + b^2)^{3/2} d^2} - \frac{e + fx}{2bd(a + b \sinh(c + dx))^2} - \frac{f \cosh(c + dx)}{2(a^2 + b^2) d^2 (a + b \sinh(c + dx))}
\end{aligned}$$

**Mathematica [A]** time = 1.16218, size = 112, normalized size = 1.

$$-\frac{2af \tan^{-1} \left( \frac{b - a \tanh\left(\frac{1}{2}(c + dx)\right)}{\sqrt{-a^2 - b^2}} \right)}{(-a^2 - b^2)^{3/2}} + \frac{d(e + fx)}{(a + b \sinh(c + dx))^2} + \frac{f \cosh(c + dx)}{(a^2 + b^2)(a + b \sinh(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x])^3,x]

[Out] -((f\*Cosh[c + d\*x])/((a^2 + b^2)\*(a + b\*Sinh[c + d\*x])) + ((2\*a\*f\*ArcTan[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) + (d\*(e + f\*x))/(a + b\*Sinh[c + d\*x])^2)/b)/(2\*d^2)

**Maple [B]** time = 0.319, size = 308, normalized size = 2.8

$$\frac{2a^2 d f x e^{2dx+2c} + 2b^2 d f x e^{2dx+2c} + 2a^2 d e e^{2dx+2c} - ab f e^{3dx+3c} + 2b^2 d e e^{2dx+2c} - 2a^2 f e^{2dx+2c} + b^2 f e^{2dx+2c} + 3fae^{dx+2c}}{d^2 b (be^{2dx+2c} + 2ae^{dx+c} - b)^2 (a^2 + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)`

[Out] 
$$-1/b*(2*a^2*d*f*x*exp(2*d*x+2*c)+2*b^2*d*f*x*exp(2*d*x+2*c)+2*a^2*d*e*exp(2*d*x+2*c)-a*b*f*exp(3*d*x+3*c)+2*b^2*d*e*exp(2*d*x+2*c)-2*a^2*f*exp(2*d*x+2*c)+b^2*f*exp(2*d*x+2*c)+3*f*a*exp(d*x+c)*b-f*b^2)/d^2/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)^2/(a^2+b^2)+1/2/(a^2+b^2)^{(3/2)}*f*a/d^2/b*\ln(\exp(d*x+c)+(a*(a^2+b^2)^{(3/2)}-a^4-2*a^2*b^2-b^4)/(a^2+b^2)^{(3/2)}/b)-1/2/(a^2+b^2)^{(3/2)}*f*a/d^2/b*\ln(\exp(d*x+c)+(a*(a^2+b^2)^{(3/2)}+a^4+2*a^2*b^2+b^4)/(a^2+b^2)^{(3/2)}/b))$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.3144, size = 2811, normalized size = 25.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & 1/2*(2*(a^3*b + a*b^3)*f*cosh(d*x + c)^3 + 2*(a^3*b + a*b^3)*f*sinh(d*x + c)^3 - 6*(a^3*b + a*b^3)*f*cosh(d*x + c) - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*e - (2*a^4 + a^2*b^2 - b^4)*f)*cosh(d*x + c)^2 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*e - 3*(a^3*b + a*b^3)*f*cosh(d*x + c) - (2*a^4 + a^2*b^2 - b^4)*f)*sinh(d*x + c)^2 + (a*b^2*f*cosh(d*x + c)^4 + a*b^2*f*sinh(d*x + c)^4 + 4*a^2*b*f*cosh(d*x + c)^3 - 4*a^2*b*f*cosh(d*x + c) + a*b^2*f + 2*(2*a^3 - a*b^2)*f*cosh(d*x + c)^2 + 4*(a*b^2*f*cosh(d*x + c) + a^2*b*f)*sinh(d*x + c)^3 + 2*(3*a*b^2*f*cosh(d*x + c)^2 + 6*a^2*b*f*cosh(d*x + c) + (2*a^3 - a*b^2)*f)*sinh(d*x \end{aligned}$$

```

+ c)^2 + 4*(a*b^2*f*cosh(d*x + c)^3 + 3*a^2*b*f*cosh(d*x + c)^2 - a^2*b*f +
(2*a^3 - a*b^2)*f*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*c
osh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 +
2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x +
c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*c
osh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a^2*b^2 + b^
4)*f + 2*(3*(a^3*b + a*b^3)*f*cosh(d*x + c)^2 - 3*(a^3*b + a*b^3)*f - 2*(2*
(a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*e - (2*a^4 + a^
2*b^2 - b^4)*f)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b^3 + 2*a^2*b^5 + b^7)*
d^2*cosh(d*x + c)^4 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*sinh(d*x + c)^4 + 4*(
a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2*cosh(d*x + c)^3 + 2*(2*a^6*b + 3*a^4*b^3 -
b^7)*d^2*cosh(d*x + c)^2 - 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2*cosh(d*x +
c) + 4*((a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c) + (a^5*b^2 + 2*a^3*b^
4 + a*b^6)*d^2)*sinh(d*x + c)^3 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^2 + 2*(3*(a
^4*b^3 + 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c)^2 + 6*(a^5*b^2 + 2*a^3*b^4 + a*
b^6)*d^2*cosh(d*x + c) + (2*a^6*b + 3*a^4*b^3 - b^7)*d^2)*sinh(d*x + c)^2 +
4*((a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^
4 + a*b^6)*d^2*cosh(d*x + c)^2 + (2*a^6*b + 3*a^4*b^3 - b^7)*d^2*cosh(d*x +
c) - (a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2)*sinh(d*x + c))

```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \cosh(dx + c)}{(b \sinh(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((f\*x + e)\*cosh(d\*x + c)/(b\*sinh(d\*x + c) + a)^3, x)

$$3.328 \quad \int \frac{(e+fx)^2 \cosh(c+dx)}{(a+b \sinh(c+dx))^3} dx$$

**Optimal.** Leaf size=306

$$\frac{af^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3 (a^2+b^2)^{3/2}} - \frac{af^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3 (a^2+b^2)^{3/2}} + \frac{af(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{bd^2 (a^2+b^2)^{3/2}} - \frac{af(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2 (a^2+b^2)^{3/2}}$$

```
[Out] (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)^(3/2)*d^2) - (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)^(3/2)*d^2) + (f^2*Log[a + b*Sinh[c + d*x]])/(b*(a^2 + b^2)*d^3) + (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) - (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) - (e + f*x)^2/(2*b*d*(a + b*Sinh[c + d*x])^2) - (f*(e + f*x)*Cosh[c + d*x])/((a^2 + b^2)*d^2*(a + b*Sinh[c + d*x]))
```

**Rubi [A]** time = 0.519948, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$ , Rules used = {5464, 3324, 3322, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{af^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3 (a^2+b^2)^{3/2}} - \frac{af^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3 (a^2+b^2)^{3/2}} + \frac{af(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{bd^2 (a^2+b^2)^{3/2}} - \frac{af(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2 (a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)^(3/2)*d^2) - (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)^(3/2)*d^2) + (f^2*Log[a + b*Sinh[c + d*x]])/(b*(a^2 + b^2)*d^3) + (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) - (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) - (e + f*x)^2/(2*b*d*(a + b*Sinh[c + d*x])^2) - (f*(e + f*x)*Cosh[c + d*x])/((a^2 + b^2)*d^2*(a + b*Sinh[c + d*x]))
```

Rule 5464

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] := Simp[((e + f*x)^m*(a + b*Sinh[c +
d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] := Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sinh[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sinh[e + f*x]), x],
x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
+ b*Sinh[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

### Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_]*)
(f_.)*(x_)])], x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2190

```
Int((((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

Int[Log[(c\_.)\*(d\_.) + (e\_.)\*(x\_)^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 2668

Int[cos[(e\_.) + (f\_.)\*(x\_)^(p\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(m\_.), x\_Symbol] := Dist[1/(b^p\*f), Subst[Int[(a + x)^m\*(b^2 - x^2)^((p - 1)/2), x], x, b\*Sin[e + f\*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

Rule 31

Int[((a\_.) + (b\_.)\*(x\_))^(n\_.), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{(e + fx)^2 \cosh(c + dx)}{(a + b \sinh(c + dx))^3} dx &= -\frac{(e + fx)^2}{2bd(a + b \sinh(c + dx))^2} + \frac{f \int \frac{e + fx}{(a + b \sinh(c + dx))^2} dx}{bd} \\
 &= -\frac{(e + fx)^2}{2bd(a + b \sinh(c + dx))^2} - \frac{f(e + fx) \cosh(c + dx)}{(a^2 + b^2) d^2 (a + b \sinh(c + dx))} + \frac{(af) \int \frac{e + fx}{a + b \sinh(c + dx)} dx}{b(a^2 + b^2) d} \\
 &= -\frac{(e + fx)^2}{2bd(a + b \sinh(c + dx))^2} - \frac{f(e + fx) \cosh(c + dx)}{(a^2 + b^2) d^2 (a + b \sinh(c + dx))} + \frac{(2af) \int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}} dx}{b(a^2 + b^2) d} \\
 &= \frac{f^2 \log(a + b \sinh(c + dx))}{b(a^2 + b^2) d^3} - \frac{(e + fx)^2}{2bd(a + b \sinh(c + dx))^2} - \frac{f(e + fx) \cosh(c + dx)}{(a^2 + b^2) d^2 (a + b \sinh(c + dx))} \\
 &= \frac{af(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b(a^2 + b^2)^{3/2} d^2} - \frac{af(e + fx) \log\left(1 + \frac{be^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{b(a^2 + b^2)^{3/2} d^2} + \frac{f^2 \log(a + b \sinh(c + dx))}{b(a^2 + b^2) d^3} \\
 &= \frac{af(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b(a^2 + b^2)^{3/2} d^2} - \frac{af(e + fx) \log\left(1 + \frac{be^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{b(a^2 + b^2)^{3/2} d^2} + \frac{f^2 \log(a + b \sinh(c + dx))}{b(a^2 + b^2) d^3} \\
 &= \frac{af(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b(a^2 + b^2)^{3/2} d^2} - \frac{af(e + fx) \log\left(1 + \frac{be^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{b(a^2 + b^2)^{3/2} d^2} + \frac{f^2 \log(a + b \sinh(c + dx))}{b(a^2 + b^2) d^3}
 \end{aligned}$$

**Mathematica [B]** time = 16.4123, size = 623, normalized size = 2.04

$$2e^c f \frac{\left( a(e^{2c}-1)f \left( \text{PolyLog}\left(2, -\frac{be^{2c+dx}}{ae^c - \sqrt{e^{2c}(a^2+b^2)}}\right) - \text{PolyLog}\left(2, -\frac{be^{2c+dx}}{\sqrt{e^{2c}(a^2+b^2)} + ae^c}\right) + dx \left( \log\left(\frac{be^{2c+dx}}{ae^c - \sqrt{e^{2c}(a^2+b^2)}} + 1\right) - \log\left(\frac{be^{2c+dx}}{\sqrt{e^{2c}(a^2+b^2)} + ae^c} + 1\right) \right) \right)}{2d\sqrt{e^{2c}(a^2+b^2)}} - \frac{ae^{-c}(e^{2c}-1)e \tanh\left(\frac{dx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} \right)}{b(e^{2c} - 1)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x])^3, x]

[Out] 
$$\frac{(f^2 x \text{Coth}[c])/(b(a^2 + b^2)d^2) + (2E^c f (-E^c f x) + (-1 + E^{2c}) f x)/E^c - (a e (-1 + E^{2c}) \text{ArcTanh}[(a + b E^{c + d x})/\sqrt{a^2 + b^2}]) / (\sqrt{a^2 + b^2} E^c) + (a(-1 + E^{2c}) f \text{ArcTanh}[(a + b E^{c + d x})/\sqrt{a^2 + b^2}]) / (\sqrt{a^2 + b^2} d E^c) + ((-1 + E^{2c}) f (-2x + (2a \text{ArcTan}[(a + b E^{c + d x})/\sqrt{-a^2 - b^2}]) / (\sqrt{-a^2 - b^2} d) + \text{Log}[2 a E^{c + d x} + b(-1 + E^{2(c + d x)})]) / d) / (2 E^c) + (a(-1 + E^{2c}) f (d x (\text{Log}[1 + (b E^{2c + d x}) / (a E^c - \sqrt{(a^2 + b^2) E^{2c}})]) - \text{Log}[1 + (b E^{2c + d x}) / (a E^c + \sqrt{(a^2 + b^2) E^{2c}})])) + \text{PolyLog}[2, -(b E^{2c + d x}) / (a E^c - \sqrt{(a^2 + b^2) E^{2c}})]) - \text{PolyLog}[2, -(b E^{2c + d x}) / (a E^c + \sqrt{(a^2 + b^2) E^{2c}})]) / (2 d \sqrt{(a^2 + b^2) E^{2c}})) / (b(a^2 + b^2) d^2 (-1 + E^{2c})) - (f^2 x \text{Cosh}[c] \text{Csch}[c/2] \text{Sech}[c/2]) / (2 b(a^2 + b^2) d^2) - (e + f x)^2 / (2 b d (a + b \text{Sinh}[c + d x])^2) + (\text{Csch}[c/2] \text{Sech}[c/2] (a e f \text{Cosh}[c] + a f^2 x \text{Cosh}[c] + b e f \text{Sinh}[d x] + b f^2 x \text{Sinh}[d x])) / (2 b(a^2 + b^2) d^2 (a + b \text{Sinh}[c + d x]))$$

**Maple [B]** time = 0.3, size = 805, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3, x)

[Out] 
$$-2/b(a^2 d f^2 x^2 \exp(2 d x + 2 c) + b^2 d f^2 x^2 \exp(2 d x + 2 c) + 2 a^2 d e f x \exp(2 d x + 2 c) - a b f^2 x \exp(3 d x + 3 c) + 2 b^2 d e f x \exp(2 d x + 2 c) + a^2 d e^2 \exp(2 d x + 2 c) - 2 a^2 f^2 x \exp(2 d x + 2 c) - a b e f \exp(3 d x + 3 c) + b^2 d e^2 \exp(2 d x + 2 c) + b^2 f^2 x \exp(2 d x + 2 c) - 2 a^2 e f \exp(2 d x + 2 c) + 3 a b f^2 x \exp(d x + c) + b^2 e f \exp(2 d x + 2 c) + 3 a b e f \exp(d x + c) - b^2 f^2 x - b^2 e f) / d^2 / (a^2 + b^2) / (b \exp(2 d x + 2 c) + 2 a \exp(d x + c) - b)^2 + 1 / b f^2 / d^3 / (a^2 + b^2)$$



$$2+b^2)*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)-2/b*f^2/d^3/(a^2+b^2)*\ln(\exp(d*x+c))-2/b*f/d^2/(a^2+b^2)^{(3/2)}*a*e*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2}))+1/b*f^2/d^2/(a^2+b^2)^{(3/2)}*a*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*x+1/b*f^2/d^3/(a^2+b^2)^{(3/2)}*a*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*c-1/b*f^2/d^2/(a^2+b^2)^{(3/2)}*a*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*x-1/b*f^2/d^3/(a^2+b^2)^{(3/2)}*a*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*c+1/b*f^2/d^3/(a^2+b^2)^{(3/2)}*a*\operatorname{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))-1/b*f^2/d^3/(a^2+b^2)^{(3/2)}*a*\operatorname{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))+2/b*f^2/d^3/(a^2+b^2)^{(3/2)}*a*c*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2}))$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 3.17122, size = 11416, normalized size = 37.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3,x, algorithm="fricas")

[Out] 
$$-(2*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*\cosh(d*x + c)^4 + 2*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*\sinh(d*x + c)^4 - 2*(a^2*b^2 + b^4)*d*e*f + 2*(a^2*b^2 + b^4)*c*f^2 + 2*(3*(a^3*b + a*b^3)*d*f^2*x - (a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3)*c*f^2)*\cosh(d*x + c)^3 + 2*(3*(a^3*b + a*b^3)*d*f^2*x - (a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3)*c*f^2 + 4*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2*e^2 - (2*a^4 + a^2*b^2 - b^4)*d*e*f + 2*(2*a^4 + a^2*b^2 - b^4)*c*f^2 + (2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f + (2*a^4 + a^2*b^2 - b^4)*d*f^2)*x)*\cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2*e^2 - (2*a^4 + a^2*b^2 - b^4)*d*e*f + 2*(2*a^4 + a^2*b^2 - b^4)*c*f^2 + (2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f + (2*a^4 + a^2*b^2 - b^4)*d*f^2)*x)*\sinh(d*x + c)^2$$

$$\begin{aligned}
& 4)d^2e^2 - (2a^4 + a^2b^2 - b^4)d^2ef + 2(2a^4 + a^2b^2 - b^4)cf^2 \\
& + 6((a^2b^2 + b^4)d^2f^2x + (a^2b^2 + b^4)cf^2)\cosh(dx + c)^2 + \\
& 2(a^4 + 2a^2b^2 + b^4)d^2ef + (2a^4 + a^2b^2 - b^4)d^2f^2x + 3(3 \\
& (a^3b + ab^3)d^2f^2x - (a^3b + ab^3)d^2ef + 4(a^3b + ab^3)cf^2) \\
& \cosh(dx + c)\sinh(dx + c)^2 - (ab^3f^2\cosh(dx + c)^4 + ab^3f^2\sinh(dx + c)^4 \\
& + 4a^2b^2f^2\cosh(dx + c)^3 - 4a^2b^2f^2\cosh(dx + c) \\
& + ab^3f^2 + 2(2a^3b - ab^3)f^2\cosh(dx + c)^2 + 4(ab^3f^2\cosh(dx + c) \\
& + a^2b^2f^2)\sinh(dx + c)^3 + 2(3ab^3f^2\cosh(dx + c)^2 + \\
& 6a^2b^2f^2\cosh(dx + c) + (2a^3b - ab^3)f^2)\sinh(dx + c)^2 + 4(ab^3f^2\cosh(dx + c)^3 \\
& + 3a^2b^2f^2\cosh(dx + c)^2 - a^2b^2f^2 + (2a^3b - ab^3)f^2\cosh(dx + c))\sinh(dx + c) \\
& )\sqrt{(a^2 + b^2)/b^2}\operatorname{dilog}((a\cosh(dx + c) + a\sinh(dx + c) + (b\cosh(dx + c) + b\sinh(dx + c)) \\
& )\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + (ab^3f^2\cosh(dx + c)^4 + ab^3f^2\sinh(dx + c)^4 \\
& + 4a^2b^2f^2\cosh(dx + c)^3 - 4a^2b^2f^2\cosh(dx + c) + ab^3f^2 + 2(2a^3b - ab^3)f^2\cosh(dx + c)^2 \\
& + 4(ab^3f^2\cosh(dx + c) + a^2b^2f^2)\sinh(dx + c)^3 + 2(3ab^3f^2\cosh(dx + c)^2 \\
& + 6a^2b^2f^2\cosh(dx + c) + (2a^3b - ab^3)f^2)\sinh(dx + c)^2 + 4(ab^3f^2\cosh(dx + c)^3 \\
& + 3a^2b^2f^2\cosh(dx + c)^2 - a^2b^2f^2 + (2a^3b - ab^3)f^2\cosh(dx + c))\sinh(dx + c) \\
& )\sqrt{(a^2 + b^2)/b^2}\operatorname{dilog}((a\cosh(dx + c) + a\sinh(dx + c) - (b\cosh(dx + c) + b\sinh(dx + c)) \\
& )\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - (ab^3d^2f^2x + ab^3cf^2 + (ab^3d^2f^2x + ab^3cf^2) \\
& )\cosh(dx + c)^4 + (ab^3d^2f^2x + ab^3cf^2)\sinh(dx + c)^4 + 4(a^2b^2d^2f^2x + a^2b^2cf^2) \\
& )\cosh(dx + c)^3 + 4(a^2b^2d^2f^2x + a^2b^2cf^2 + (ab^3d^2f^2x + ab^3cf^2)\cosh(dx + c)) \\
& )\sinh(dx + c)^3 + 2((2a^3b - ab^3)d^2f^2x + (2a^3b - ab^3)cf^2)\cosh(dx + c)^2 \\
& + 2((2a^3b - ab^3)d^2f^2x + (2a^3b - ab^3)cf^2 + 3(ab^3d^2f^2x + ab^3cf^2)\cosh(dx + c)^2 \\
& + 6(a^2b^2d^2f^2x + a^2b^2cf^2)\cosh(dx + c))\sinh(dx + c)^2 - 4(a^2b^2d^2f^2x + a^2b^2cf^2) \\
& )\cosh(dx + c) - 4(a^2b^2d^2f^2x + a^2b^2cf^2 - (ab^3d^2f^2x + ab^3cf^2)\cosh(dx + c)^3 \\
& - 3(a^2b^2d^2f^2x + a^2b^2cf^2)\cosh(dx + c)^2 - ((2a^3b - ab^3)d^2f^2x + (2a^3b - ab^3)cf^2) \\
& )\cosh(dx + c)\sinh(dx + c)\sqrt{(a^2 + b^2)/b^2}\log(-(a\cosh(dx + c) + a\sinh(dx + c) \\
& + (b\cosh(dx + c) + b\sinh(dx + c))\sqrt{(a^2 + b^2)/b^2} - b)/b) \\
& + (ab^3d^2f^2x + ab^3cf^2 + (ab^3d^2f^2x + ab^3cf^2)\cosh(dx + c)^4 + (ab^3d^2f^2x + ab^3cf^2) \\
& )\sinh(dx + c)^4 + 4(a^2b^2d^2f^2x + a^2b^2cf^2)\cosh(dx + c)^3 + 4(a^2b^2d^2f^2x + a^2b^2cf^2 \\
& + (ab^3d^2f^2x + ab^3cf^2)\cosh(dx + c))\sinh(dx + c)^3 + 2((2a^3b - ab^3)d^2f^2x \\
& + (2a^3b - ab^3)cf^2)\cosh(dx + c)^2 + 2((2a^3b - ab^3)d^2f^2x + (2a^3b - ab^3)cf^2 \\
& + 3(ab^3d^2f^2x + ab^3cf^2)\cosh(dx + c)^2 + 6(a^2b^2d^2f^2x + a^2b^2cf^2)\cosh(dx + c) \\
& )\sinh(dx + c)^2 - 4(a^2b^2d^2f^2x + a^2b^2cf^2)\cosh(dx + c) - 4(a^2b^2d^2f^2x + a^2b^2cf^2 \\
& - (ab^3d^2f^2x + ab^3cf^2)\cosh(dx + c)^3 - 3(a^2b^2d^2f^2x + a^2b^2cf^2)\cosh(dx + c)^2 \\
& - ((2a^3b - ab^3)d^2f^2x + (2a^3b - ab^3)cf^2)\cosh(dx + c))\sinh(dx + c)\sqrt{(a^2 + b^2)/b^2} \\
& )\log(-(a\cosh(dx + c) + a\sinh(dx + c) - (b\cosh(dx + c) + b\sinh(dx + c))\sqrt{(a^2 + b^2)/b^2} - b)/b)
\end{aligned}$$

$$\begin{aligned}
& *x + c)) * \sqrt{(a^2 + b^2)/b^2} - b)/b) - 2*((a^3*b + a*b^3)*d*f^2*x - 3*(a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3)*c*f^2)*\cosh(d*x + c) - ((a^2*b^2 + b^4)*f^2*\cosh(d*x + c)^4 + (a^2*b^2 + b^4)*f^2*\sinh(d*x + c)^4 + 4*(a^3*b + a*b^3)*f^2*\cosh(d*x + c)^3 + 2*(2*a^4 + a^2*b^2 - b^4)*f^2*\cosh(d*x + c)^2 - 4*(a^3*b + a*b^3)*f^2*\cosh(d*x + c) + 4*((a^2*b^2 + b^4)*f^2*\cosh(d*x + c) + (a^3*b + a*b^3)*f^2)*\sinh(d*x + c)^3 + (a^2*b^2 + b^4)*f^2 + 2*(3*(a^2*b^2 + b^4)*f^2*\cosh(d*x + c)^2 + 6*(a^3*b + a*b^3)*f^2*\cosh(d*x + c) + (2*a^4 + a^2*b^2 - b^4)*f^2)*\sinh(d*x + c)^2 + 4*((a^2*b^2 + b^4)*f^2*\cosh(d*x + c)^3 + 3*(a^3*b + a*b^3)*f^2*\cosh(d*x + c)^2 + (2*a^4 + a^2*b^2 - b^4)*f^2*\cosh(d*x + c) - (a^3*b + a*b^3)*f^2)*\sinh(d*x + c) - (a*b^3*d*e*f - a*b^3*c*f^2 + (a*b^3*d*e*f - a*b^3*c*f^2)*\cosh(d*x + c)^4 + (a*b^3*d*e*f - a*b^3*c*f^2)*\sinh(d*x + c)^4 + 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*\cosh(d*x + c)^3 + 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2 + (a*b^3*d*e*f - a*b^3*c*f^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2)*\cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2 + 3*(a*b^3*d*e*f - a*b^3*c*f^2)*\cosh(d*x + c)^2 + 6*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*\cosh(d*x + c))*\sinh(d*x + c)^2 - 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*\cosh(d*x + c) - 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2 - (a*b^3*d*e*f - a*b^3*c*f^2)^2)*\cosh(d*x + c)^3 - 3*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*\cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - ((a^2*b^2 + b^4)*f^2*\cosh(d*x + c)^4 + (a^2*b^2 + b^4)*f^2*\sinh(d*x + c)^4 + 4*(a^3*b + a*b^3)*f^2*\cosh(d*x + c)^3 + 2*(2*a^4 + a^2*b^2 - b^4)*f^2*\cosh(d*x + c)^2 - 4*(a^3*b + a*b^3)*f^2*\cosh(d*x + c) + 4*((a^2*b^2 + b^4)*f^2*\cosh(d*x + c) + (a^3*b + a*b^3)*f^2)*\sinh(d*x + c)^3 + (a^2*b^2 + b^4)*f^2 + 2*(3*(a^2*b^2 + b^4)*f^2*\cosh(d*x + c)^2 + 6*(a^3*b + a*b^3)*f^2*\cosh(d*x + c) + (2*a^4 + a^2*b^2 - b^4)*f^2)*\sinh(d*x + c)^2 + 4*((a^2*b^2 + b^4)*f^2*\cosh(d*x + c)^3 + 3*(a^3*b + a*b^3)*f^2*\cosh(d*x + c)^2 + (2*a^4 + a^2*b^2 - b^4)*f^2*\cosh(d*x + c) - (a^3*b + a*b^3)*f^2)*\sinh(d*x + c) + (a*b^3*d*e*f - a*b^3*c*f^2 + (a*b^3*d*e*f - a*b^3*c*f^2)*\cosh(d*x + c)^4 + (a*b^3*d*e*f - a*b^3*c*f^2)*\sinh(d*x + c)^4 + 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*\cosh(d*x + c)^3 + 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2 + (a*b^3*d*e*f - a*b^3*c*f^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2)*\cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2 + 3*(a*b^3*d*e*f - a*b^3*c*f^2)*\cosh(d*x + c)^2 + 6*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*\cosh(d*x + c))*\sinh(d*x + c)^2 - 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*\cosh(d*x + c) - 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2 - (a*b^3*d*e*f - a*b^3*c*f^2)*\cosh(d*x + c)^3 - 3*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*\cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - 2*((a^3*b + a*b^3)*d*f^2*x - 3*(a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3)*c*f^2 - 4*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*\cosh(d*x + c)^3 - 3*(3*(a^3*b + a*b^3)*d*f^2*x - (a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3)*c*f^2)*\cosh(d*x + c)^2 - 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + (a^4 +
\end{aligned}$$

$$2a^2b^2 + b^4)d^2e^2 - (2a^4 + a^2b^2 - b^4)d*ef + 2*(2a^4 + a^2b^2 - b^4)*c*f^2 + (2*(a^4 + 2a^2b^2 + b^4)d^2*ef + (2a^4 + a^2b^2 - b^4)d*f^2)*x)*\cosh(dx + c))*\sinh(dx + c))/((a^4*b^3 + 2a^2*b^5 + b^7)*d^3*\cosh(dx + c)^4 + (a^4*b^3 + 2a^2*b^5 + b^7)*d^3*\sinh(dx + c)^4 + 4*(a^5*b^2 + 2a^3*b^4 + a*b^6)*d^3*\cosh(dx + c)^3 + 2*(2a^6*b + 3a^4*b^3 - b^7)*d^3*\cosh(dx + c)^2 - 4*(a^5*b^2 + 2a^3*b^4 + a*b^6)*d^3*\cosh(dx + c) + (a^4*b^3 + 2a^2*b^5 + b^7)*d^3 + 4*((a^4*b^3 + 2a^2*b^5 + b^7)*d^3*\cosh(dx + c) + (a^5*b^2 + 2a^3*b^4 + a*b^6)*d^3)*\sinh(dx + c)^3 + 2*(3*(a^4*b^3 + 2a^2*b^5 + b^7)*d^3*\cosh(dx + c)^2 + 6*(a^5*b^2 + 2a^3*b^4 + a*b^6)*d^3*\cosh(dx + c) + (2a^6*b + 3a^4*b^3 - b^7)*d^3)*\sinh(dx + c)^2 + 4*((a^4*b^3 + 2a^2*b^5 + b^7)*d^3*\cosh(dx + c)^3 + 3*(a^5*b^2 + 2a^3*b^4 + a*b^6)*d^3*\cosh(dx + c)^2 + (2a^6*b + 3a^4*b^3 - b^7)*d^3*\cosh(dx + c) - (a^5*b^2 + 2a^3*b^4 + a*b^6)*d^3)*\sinh(dx + c))$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c)}{(b \sinh(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*cosh(d\*x + c)/(b\*sinh(d\*x + c) + a)^3, x)

$$3.329 \quad \int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b \sinh(c+dx))^3} dx$$

**Optimal.** Leaf size=631

$$\frac{3af^2(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3(a^2+b^2)^{3/2}} - \frac{3af^2(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3(a^2+b^2)^{3/2}} + \frac{3f^3\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4(a^2+b^2)} + \frac{3f^3\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^4(a^2+b^2)}$$

[Out]  $(-3*f*(e + f*x)^2)/(2*b*(a^2 + b^2)*d^2) + (3*f^2*(e + f*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])])/(b*(a^2 + b^2)*d^3) + (3*a*f*(e + f*x)^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])])/(2*b*(a^2 + b^2)^{(3/2)}*d^2) + (3*f^2*(e + f*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(b*(a^2 + b^2)*d^3) - (3*a*f*(e + f*x)^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(2*b*(a^2 + b^2)^{(3/2)}*d^2) + (3*f^3*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4) + (3*a*f^2*(e + f*x)*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)^{(3/2)}*d^3) + (3*f^3*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4) - (3*a*f^2*(e + f*x)*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)^{(3/2)}*d^3) - (3*a*f^3*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)^{(3/2)}*d^4) + (3*a*f^3*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)^{(3/2)}*d^4) - (e + f*x)^3/(2*b*d*(a + b*\text{Sinh}[c + d*x])^2) - (3*f*(e + f*x)^2*\text{Cosh}[c + d*x])/(2*(a^2 + b^2)*d^2*(a + b*\text{Sinh}[c + d*x]))$

**Rubi [A]** time = 1.09479, antiderivative size = 631, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$ , Rules used = {5464, 3324, 3322, 2264, 2190, 2531, 2282, 6589, 5561, 2279, 2391}

$$\frac{3af^2(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3(a^2+b^2)^{3/2}} - \frac{3af^2(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3(a^2+b^2)^{3/2}} + \frac{3f^3\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4(a^2+b^2)} + \frac{3f^3\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^4(a^2+b^2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(e + f*x)^3*\text{Cosh}[c + d*x]}{(a + b*\text{Sinh}[c + d*x])^3}, x]$

[Out]  $(-3*f*(e + f*x)^2)/(2*b*(a^2 + b^2)*d^2) + (3*f^2*(e + f*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])])/(b*(a^2 + b^2)*d^3) + (3*a*f*(e + f*x)^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])])/(2*b*(a^2 + b^2)^{(3/2)}*d^2) + (3*f^2*(e + f*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(b*(a^2 + b^2)*d^3) - (3*a*f*(e + f*x)^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(2*b*(a^2 + b^2)^{(3/2)}*d^2) + (3*f^3*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4) + (3*a*f^2*(e + f*x)*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)^{(3/2)}*d^3) + (3*f^3*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4) - (3*a*f^2*(e + f*x)*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)^{(3/2)}*d^3) - (3*a*f^3*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)^{(3/2)}*d^4) + (3*a*f^3*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)^{(3/2)}*d^4) - (e + f*x)^3/(2*b*d*(a + b*\text{Sinh}[c + d*x])^2) - (3*f*(e + f*x)^2*\text{Cosh}[c + d*x])/(2*(a^2 + b^2)*d^2*(a + b*\text{Sinh}[c + d*x]))$

$$\begin{aligned} & 2])]/(2*b*(a^2 + b^2)^{(3/2)*d^2} + (3*f^3*PolyLog[2, -((b*E^{(c + d*x)})/(a \\ & - Sqrt[a^2 + b^2]))]/(b*(a^2 + b^2)*d^4) + (3*a*f^2*(e + f*x)*PolyLog[2, - \\ & ((b*E^{(c + d*x)})/(a - Sqrt[a^2 + b^2]))]/(b*(a^2 + b^2)^{(3/2)*d^3} + (3*f^ \\ & 3*PolyLog[2, -((b*E^{(c + d*x)})/(a + Sqrt[a^2 + b^2]))]/(b*(a^2 + b^2)*d^4) \\ & - (3*a*f^2*(e + f*x)*PolyLog[2, -((b*E^{(c + d*x)})/(a + Sqrt[a^2 + b^2]))] \\ & )/(b*(a^2 + b^2)^{(3/2)*d^3} - (3*a*f^3*PolyLog[3, -((b*E^{(c + d*x)})/(a - Sqr \\ & t[a^2 + b^2]))]/(b*(a^2 + b^2)^{(3/2)*d^4} + (3*a*f^3*PolyLog[3, -((b*E^{(c \\ & + d*x)})/(a + Sqrt[a^2 + b^2]))]/(b*(a^2 + b^2)^{(3/2)*d^4} - (e + f*x)^{3/2} \\ & *b*d*(a + b*Sinh[c + d*x])^2 - (3*f*(e + f*x)^2*Cosh[c + d*x])/(2*(a^2 + b \\ & ^2)*d^2*(a + b*Sinh[c + d*x])) \end{aligned}$$

#### Rule 5464

$$\begin{aligned} & \text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))^{(m_.)*((a_.) + (b_.)*\text{Sinh}[ \\ & (c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \text{:> Simp}[(e + f*x)^m*(a + b*\text{Sinh}[c + \\ & d*x])^{(n + 1)}/(b*d*(n + 1)), x] - \text{Dist}[(f*m)/(b*d*(n + 1)), \text{Int}[(e + f*x)^ \\ & (m - 1)*(a + b*\text{Sinh}[c + d*x])^{(n + 1)}, x], x] \text{/; FreeQ}\{a, b, c, d, e, f, n \\ & \}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[n, -1] \end{aligned}$$

#### Rule 3324

$$\begin{aligned} & \text{Int}[(c_.) + (d_.)*(x_.))^{(m_.)/((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2, x\_ \\ & Symbol] \text{:> Simp}[(b*(c + d*x)^m*\text{Cos}[e + f*x])/(f*(a^2 - b^2)*(a + b*\text{Sin}[e + \\ & f*x])), x] + (\text{Dist}[a/(a^2 - b^2), \text{Int}[(c + d*x)^m/(a + b*\text{Sin}[e + f*x]), x], \\ & x] - \text{Dist}[(b*d*m)/(f*(a^2 - b^2)), \text{Int}[(c + d*x)^{(m - 1)*\text{Cos}[e + f*x])/(a \\ & + b*\text{Sin}[e + f*x]), x], x]) \text{/; FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[a^2 - b^ \\ & 2, 0] \&\& \text{IGtQ}[m, 0] \end{aligned}$$

#### Rule 3322

$$\begin{aligned} & \text{Int}[(c_.) + (d_.)*(x_.))^{(m_.)/((a_.) + (b_.)*\sin[(e_.) + (\text{Complex}[0, fz\_]) * \\ & (f_.)*(x_.)]), x\_Symbol] \text{:> Dist}[2, \text{Int}[(c + d*x)^m*E^{-(I*e) + f*fz*x})/(- \\ & (I*b) + 2*a*E^{-(I*e) + f*fz*x} + I*b*E^{(2*(-I*e) + f*fz*x))}, x], x] \text{/; F \\ & reeQ}\{a, b, c, d, e, f, fz\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[m, 0] \end{aligned}$$

#### Rule 2264

$$\begin{aligned} & \text{Int}[(F_.)^{(u_.)*((f_.) + (g_.)*(x_.))^{(m_.)/((a_.) + (b_.)*(F_.)^{(u_.) + (c_.) \\ & *(F_.)^{(v_.)}, x\_Symbol] \text{:> With}\{q = \text{Rt}[b^2 - 4*a*c, 2]\}, \text{Dist}[(2*c)/q, \text{Int} \\ & [(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - \text{Dist}[(2*c)/q, \text{Int}[(f + g*x)^ \\ & m*F^u/(b + q + 2*c*F^u), x], x]) \text{/; FreeQ}\{F, a, b, c, f, g\}, x] \&\& \text{EqQ}[v, \\ & 2*u] \&\& \text{LinearQ}[u, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[m, 0] \end{aligned}$$

#### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*(f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 5561

```
Int[(Cosh[(c_) + (d_)*(x_)]*((e_) + (f_)*(x_))^(m_))/((a_) + (b_)*Sin
h[(c_) + (d_)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b \sinh(c+dx))^3} dx &= -\frac{(e+fx)^3}{2bd(a+b \sinh(c+dx))^2} + \frac{(3f) \int \frac{(e+fx)^2}{(a+b \sinh(c+dx))^2} dx}{2bd} \\
&= -\frac{(e+fx)^3}{2bd(a+b \sinh(c+dx))^2} - \frac{3f(e+fx)^2 \cosh(c+dx)}{2(a^2+b^2)d^2(a+b \sinh(c+dx))} + \frac{(3af) \int \frac{(e+fx)^2}{a+b \sinh(c+dx)} dx}{2b(a^2+b^2)d} \\
&= -\frac{3f(e+fx)^2}{2b(a^2+b^2)d^2} - \frac{(e+fx)^3}{2bd(a+b \sinh(c+dx))^2} - \frac{3f(e+fx)^2 \cosh(c+dx)}{2(a^2+b^2)d^2(a+b \sinh(c+dx))} + \frac{(3af)}{2b(a^2+b^2)d} \\
&= -\frac{3f(e+fx)^2}{2b(a^2+b^2)d^2} + \frac{3f^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3} + \frac{3f^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3} - \frac{3af}{2b(a^2+b^2)d} \\
&= -\frac{3f(e+fx)^2}{2b(a^2+b^2)d^2} + \frac{3f^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3} + \frac{3af(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b(a^2+b^2)^{3/2}d^2} + \frac{3af}{2b(a^2+b^2)d} \\
&= -\frac{3f(e+fx)^2}{2b(a^2+b^2)d^2} + \frac{3f^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3} + \frac{3af(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b(a^2+b^2)^{3/2}d^2} + \frac{3af}{2b(a^2+b^2)d} \\
&= -\frac{3f(e+fx)^2}{2b(a^2+b^2)d^2} + \frac{3f^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3} + \frac{3af(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b(a^2+b^2)^{3/2}d^2} + \frac{3af}{2b(a^2+b^2)d}
\end{aligned}$$

**Mathematica [B]** time = 24.6796, size = 5753, normalized size = 9.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```



[Out] Result too large to show

**Maple [F]** time = 0.43, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c)}{(a + b \sinh(dx + c))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3,x)

[Out] int((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3,x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 4.17084, size = 24868, normalized size = 39.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3,x, algorithm="fricas")

[Out]  $\frac{1}{2} * (6 * (a^2 * b^2 + b^4) * d^2 * e^2 * f - 12 * (a^2 * b^2 + b^4) * c * d * e * f^2 + 6 * (a^2 * b^2 + b^4) * c^2 * f^3 - 6 * ((a^2 * b^2 + b^4) * d^2 * f^3 * x^2 + 2 * (a^2 * b^2 + b^4) * d^2 * e * f^2 * x + 2 * (a^2 * b^2 + b^4) * c * d * e * f^2 - (a^2 * b^2 + b^4) * c^2 * f^3) * \cosh(d * x + c)^4 - 6 * ((a^2 * b^2 + b^4) * d^2 * f^3 * x^2 + 2 * (a^2 * b^2 + b^4) * d^2 * e * f^2 * x + 2 * (a^2 * b^2 + b^4) * c * d * e * f^2 - (a^2 * b^2 + b^4) * c^2 * f^3) * \sinh(d * x + c)^4 - 6 * (3 *$

$$\begin{aligned}
& (a^3*b + a*b^3)*d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d^2*e*f^2*x - (a^3*b + a*b^3)*d^2*e^2*f + 8*(a^3*b + a*b^3)*c*d*e*f^2 - 4*(a^3*b + a*b^3)*c^2*f^3)*\cosh(d*x + c)^3 - 6*(3*(a^3*b + a*b^3)*d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d^2*e*f^2*x - (a^3*b + a*b^3)*d^2*e^2*f + 8*(a^3*b + a*b^3)*c*d*e*f^2 - 4*(a^3*b + a*b^3)*c^2*f^3 + 4*((a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^2*b^2 + b^4)*d^2*e*f^2*x + 2*(a^2*b^2 + b^4)*c*d*e*f^2 - (a^2*b^2 + b^4)*c^2*f^3)*\cosh(d*x + c))^*\sinh(d*x + c)^3 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*e^3 - 3*(2*a^4 + a^2*b^2 - b^4)*d^2*e^2*f + 12*(2*a^4 + a^2*b^2 - b^4)*c*d*e*f^2 - 6*(2*a^4 + a^2*b^2 - b^4)*c^2*f^3 + 3*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*e*f^2 + (2*a^4 + a^2*b^2 - b^4)*d^2*f^3)*x^2 + 6*((a^4 + 2*a^2*b^2 + b^4)*d^3*e^2*f + (2*a^4 + a^2*b^2 - b^4)*d^2*e*f^2)*x)*\cosh(d*x + c)^2 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*e^3 - 3*(2*a^4 + a^2*b^2 - b^4)*d^2*e^2*f + 12*(2*a^4 + a^2*b^2 - b^4)*c*d*e*f^2 - 6*(2*a^4 + a^2*b^2 - b^4)*c^2*f^3 + 3*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*e*f^2 + (2*a^4 + a^2*b^2 - b^4)*d^2*f^3)*x^2 + 18*((a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^2*b^2 + b^4)*d^2*e*f^2*x + 2*(a^2*b^2 + b^4)*c*d*e*f^2 - (a^2*b^2 + b^4)*c^2*f^3)*\cosh(d*x + c)^2 + 6*((a^4 + 2*a^2*b^2 + b^4)*d^3*e^2*f + (2*a^4 + a^2*b^2 - b^4)*d^2*e*f^2)*x + 9*(3*(a^3*b + a*b^3)*d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d^2*e*f^2*x - (a^3*b + a*b^3)*d^2*e^2*f + 8*(a^3*b + a*b^3)*c*d*e*f^2 - 4*(a^3*b + a*b^3)*c^2*f^3)*\cosh(d*x + c))^*\sinh(d*x + c)^2 - 6*(a*b^3*f^3*\cosh(d*x + c)^4 + a*b^3*f^3*\sinh(d*x + c)^4 + 4*a^2*b^2*f^3*\cosh(d*x + c)^3 - 4*a^2*b^2*f^3*\cosh(d*x + c) + a*b^3*f^3 + 2*(2*a^3*b - a*b^3)*f^3*\cosh(d*x + c)^2 + 4*(a*b^3*f^3*\cosh(d*x + c) + a^2*b^2*f^3)*\sinh(d*x + c)^3 + 2*(3*a*b^3*f^3*\cosh(d*x + c)^2 + 6*a^2*b^2*f^3*\cosh(d*x + c) + (2*a^3*b - a*b^3)*f^3)*\sinh(d*x + c)^2 + 4*(a*b^3*f^3*\cosh(d*x + c)^3 + 3*a^2*b^2*f^3*\cosh(d*x + c)^2 - a^2*b^2*f^3 + (2*a^3*b - a*b^3)*f^3*\cosh(d*x + c))^*\sinh(d*x + c))^*\sqrt((a^2 + b^2)/b^2)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt((a^2 + b^2)/b^2)))/b) + 6*((a^3*b + a*b^3)*d^2*f^3*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f^2*x - 3*(a^3*b + a*b^3)*d^2*e^2*f + 8*(a^3*b + a*b^3)*c*d*e*f^2 - 4*(a^3*b + a*b^3)*c^2*f^3)*\cosh(d*x + c) + 6*((a^2*b^2 + b^4)*f^3*\cosh(d*x + c)^4 + (a^2*b^2 + b^4)*f^3*\sinh(d*x + c)^4 + 4*(a^3*b + a*b^3)*f^3*\cosh(d*x + c)^3 + 2*(2*a^4 + a^2*b^2 - b^4)*f^3*\cosh(d*x + c)^2 - 4*(a^3*b + a*b^3)*f^3*\cosh(d*x + c) + (a^2*b^2 + b^4)*f^3 + 4*((a^2*b^2 + b^4)*f^3*\cosh(d*x + c) + (a^3*b + a*b^3)*f^3)*\sinh(d*x + c)^3 + 2*(3*(a^2*b^2 + b^4)*f^3*\cosh(d*x + c)^2 + 6*(a^3*b + a*b^3)*f^3*\cosh(d*x + c) + (2*a^4 + a^2*b^2 - b^4)*f^3)*\sinh(d*x + c)^2 + 4*((a^2*b^2 + b^4)*f^3*\cosh(d*x + c)^3 + 3*(a^3*b + a*b^3)
\end{aligned}$$

$$\begin{aligned}
& f^3 \cosh(dx + c)^2 + (2a^4 + a^2b^2 - b^4) f^3 \cosh(dx + c) - (a^3b + \\
& ab^3) f^3 \sinh(dx + c) + (ab^3 d f^3 x + ab^3 d e f^2 + (ab^3 d f^3 x \\
& + ab^3 d e f^2) \cosh(dx + c)^4 + (ab^3 d f^3 x + ab^3 d e f^2) \sinh(dx \\
& + c)^4 + 4(a^2 b^2 d f^3 x + a^2 b^2 d e f^2) \cosh(dx + c)^3 + 4(a^2 b^2 d \\
& f^3 x + a^2 b^2 d e f^2 + (ab^3 d f^3 x + ab^3 d e f^2) \cosh(dx + \\
& c)) \sinh(dx + c)^3 + 2((2a^3 b - ab^3) d f^3 x + (2a^3 b - ab^3) d e \\
& f^2) \cosh(dx + c)^2 + 2((2a^3 b - ab^3) d f^3 x + (2a^3 b - ab^3) d e \\
& f^2 + 3(ab^3 d f^3 x + ab^3 d e f^2) \cosh(dx + c)^2 + 6(a^2 b^2 d f^3 \\
& x + a^2 b^2 d e f^2) \cosh(dx + c)) \sinh(dx + c)^2 - 4(a^2 b^2 d f^3 x + \\
& a^2 b^2 d e f^2) \cosh(dx + c) - 4(a^2 b^2 d f^3 x + a^2 b^2 d e f^2 - (a \\
& b^3 d f^3 x + ab^3 d e f^2) \cosh(dx + c)^3 - 3(a^2 b^2 d f^3 x + a^2 b^2 \\
& d e f^2) \cosh(dx + c)^2 - ((2a^3 b - ab^3) d f^3 x + (2a^3 b - ab^3) \\
& d e f^2) \cosh(dx + c)) \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2}) \operatorname{dilog}((a \cos \\
& h(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 \\
& + b^2)/b^2}) - b)/b + 1) + 6((a^2 b^2 + b^4) f^3 \cosh(dx + c)^4 + (a^2 b \\
& ^2 + b^4) f^3 \sinh(dx + c)^4 + 4(a^3 b + ab^3) f^3 \cosh(dx + c)^3 + 2( \\
& 2a^4 + a^2 b^2 - b^4) f^3 \cosh(dx + c)^2 - 4(a^3 b + ab^3) f^3 \cosh(dx \\
& + c) + (a^2 b^2 + b^4) f^3 + 4((a^2 b^2 + b^4) f^3 \cosh(dx + c) + (a^3 b \\
& + ab^3) f^3) \sinh(dx + c)^3 + 2(3(a^2 b^2 + b^4) f^3 \cosh(dx + c)^2 + \\
& 6(a^3 b + ab^3) f^3 \cosh(dx + c) + (2a^4 + a^2 b^2 - b^4) f^3) \sinh(dx \\
& + c)^2 + 4((a^2 b^2 + b^4) f^3 \cosh(dx + c)^3 + 3(a^3 b + ab^3) f^3 \c \\
& osh(dx + c)^2 + (2a^4 + a^2 b^2 - b^4) f^3 \cosh(dx + c) - (a^3 b + ab^3 \\
& ) f^3) \sinh(dx + c) - (ab^3 d f^3 x + ab^3 d e f^2 + (ab^3 d f^3 x + a \\
& b^3 d e f^2) \cosh(dx + c)^4 + (ab^3 d f^3 x + ab^3 d e f^2) \sinh(dx + c \\
& )^4 + 4(a^2 b^2 d f^3 x + a^2 b^2 d e f^2) \cosh(dx + c)^3 + 4(a^2 b^2 d \\
& f^3 x + a^2 b^2 d e f^2 + (ab^3 d f^3 x + ab^3 d e f^2) \cosh(dx + c)) \si \\
& nh(dx + c)^3 + 2((2a^3 b - ab^3) d f^3 x + (2a^3 b - ab^3) d e f^2) \c \\
& osh(dx + c)^2 + 2((2a^3 b - ab^3) d f^3 x + (2a^3 b - ab^3) d e f^2 + \\
& 3(ab^3 d f^3 x + ab^3 d e f^2) \cosh(dx + c)^2 + 6(a^2 b^2 d f^3 x + a \\
& ^2 b^2 d e f^2) \cosh(dx + c)) \sinh(dx + c)^2 - 4(a^2 b^2 d f^3 x + a^2 b \\
& ^2 d e f^2) \cosh(dx + c) - 4(a^2 b^2 d f^3 x + a^2 b^2 d e f^2 - (ab^3 d \\
& f^3 x + ab^3 d e f^2) \cosh(dx + c)^3 - 3(a^2 b^2 d f^3 x + a^2 b^2 d e \\
& f^2) \cosh(dx + c)^2 - ((2a^3 b - ab^3) d f^3 x + (2a^3 b - ab^3) d e \\
& f^2) \cosh(dx + c)) \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2}) \operatorname{dilog}((a \cosh(dx \\
& + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^ \\
& 2)/b^2}) - b)/b + 1) + 3(2(a^2 b^2 + b^4) d e f^2 - 2(a^2 b^2 + b^4) c f^ \\
& 3 + 2((a^2 b^2 + b^4) d e f^2 - (a^2 b^2 + b^4) c f^3) \cosh(dx + c)^4 + 2 \\
& ((a^2 b^2 + b^4) d e f^2 - (a^2 b^2 + b^4) c f^3) \sinh(dx + c)^4 + 8((a^ \\
& 3 b + ab^3) d e f^2 - (a^3 b + ab^3) c f^3) \cosh(dx + c)^3 + 8((a^3 b + \\
& ab^3) d e f^2 - (a^3 b + ab^3) c f^3 + ((a^2 b^2 + b^4) d e f^2 - (a^2 b \\
& ^2 + b^4) c f^3) \cosh(dx + c)) \sinh(dx + c)^3 + 4((2a^4 + a^2 b^2 - b^4 \\
& ) d e f^2 - (2a^4 + a^2 b^2 - b^4) c f^3) \cosh(dx + c)^2 + 4((2a^4 + a^ \\
& 2 b^2 - b^4) d e f^2 - (2a^4 + a^2 b^2 - b^4) c f^3 + 3((a^2 b^2 + b^4) d \\
& e f^2 - (a^2 b^2 + b^4) c f^3) \cosh(dx + c)^2 + 6((a^3 b + ab^3) d e f^ \\
& 2 - (a^3 b + ab^3) c f^3) \cosh(dx + c)) \sinh(dx + c)^2 - 8((a^3 b + ab
\end{aligned}$$

$$\begin{aligned}
&^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3)*\cosh(d*x + c) - 8*((a^3*b + a*b^3)*d*e \\
&*f^2 - (a^3*b + a*b^3)*c*f^3 - ((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c \\
&*f^3)*\cosh(d*x + c)^3 - 3*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3) \\
&*cosh(d*x + c)^2 - ((2*a^4 + a^2*b^2 - b^4)*d*e*f^2 - (2*a^4 + a^2*b^2 - b^ \\
&4)*c*f^3)*\cosh(d*x + c))*\sinh(d*x + c) - (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f \\
&^2 + a*b^3*c^2*f^3 + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)* \\
&cosh(d*x + c)^4 + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*\sin \\
&h(d*x + c)^4 + 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3) \\
&)*cosh(d*x + c)^3 + 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^ \\
&2*f^3 + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cosh(d*x + c) \\
&)*\sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d \\
&*e*f^2 + (2*a^3*b - a*b^3)*c^2*f^3)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3) \\
&*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d*e*f^2 + (2*a^3*b - a*b^3)*c^2*f^3 + 3* \\
&(a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cosh(d*x + c)^2 + 6*( \\
&a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*cosh(d*x + c))*s \\
&inh(d*x + c)^2 - 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f \\
&^3)*cosh(d*x + c) - 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^ \\
&2*f^3 - (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cosh(d*x + c) \\
&^3 - 3*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*cosh(d*x \\
&+ c)^2 - ((2*a^3*b - a*b^3)*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d*e*f^2 + (2 \\
&a^3*b - a*b^3)*c^2*f^3)*cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} \\
&))*\log(2*b*cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + \\
&2*a) + 3*(2*(a^2*b^2 + b^4)*d*e*f^2 - 2*(a^2*b^2 + b^4)*c*f^3 + 2*((a^2*b^2 \\
&+ b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*cosh(d*x + c)^4 + 2*((a^2*b^2 + b^ \\
&4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*\sinh(d*x + c)^4 + 8*((a^3*b + a*b^3)*d* \\
&*e*f^2 - (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c)^3 + 8*((a^3*b + a*b^3)*d*e*f^2 \\
&- (a^3*b + a*b^3)*c*f^3 + ((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3) \\
&)*cosh(d*x + c))*\sinh(d*x + c)^3 + 4*((2*a^4 + a^2*b^2 - b^4)*d*e*f^2 - (2* \\
&a^4 + a^2*b^2 - b^4)*c*f^3)*cosh(d*x + c)^2 + 4*((2*a^4 + a^2*b^2 - b^4)*d* \\
&*e*f^2 - (2*a^4 + a^2*b^2 - b^4)*c*f^3 + 3*((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b \\
&^2 + b^4)*c*f^3)*cosh(d*x + c)^2 + 6*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b + a \\
&b^3)*c*f^3)*cosh(d*x + c))*\sinh(d*x + c)^2 - 8*((a^3*b + a*b^3)*d*e*f^2 - ( \\
&a^3*b + a*b^3)*c*f^3)*cosh(d*x + c) - 8*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b + \\
&a*b^3)*c*f^3 - ((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*cosh(d*x \\
&+ c)^3 - 3*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3)*cosh(d*x + c)^ \\
&2 - ((2*a^4 + a^2*b^2 - b^4)*d*e*f^2 - (2*a^4 + a^2*b^2 - b^4)*c*f^3)*cosh( \\
&d*x + c))*\sinh(d*x + c) + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2* \\
&f^3 + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cosh(d*x + c)^4 \\
&+ (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*\sinh(d*x + c)^4 + \\
&4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*cosh(d*x + c) \\
&^3 + 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3 + (a*b^3* \\
&d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*cosh(d*x + c))*\sinh(d*x + c) \\
&^3 + 2*((2*a^3*b - a*b^3)*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d*e*f^2 + (2*a^ \\
&3*b - a*b^3)*c^2*f^3)*cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d^2*e^2*f - 2* \\
&(2*a^3*b - a*b^3)*c*d*e*f^2 + (2*a^3*b - a*b^3)*c^2*f^3 + 3*(a*b^3*d^2*e^2*
\end{aligned}$$

$$\begin{aligned}
& f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*\cosh(d*x + c)^2 + 6*(a^2*b^2*d^2*e^2 \\
& *f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 \\
& - 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*\cosh(d*x + \\
& c) - 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3 - (a*b^3* \\
& d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*\cosh(d*x + c))^3 - 3*(a^2*b^2 \\
& *d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*\cosh(d*x + c)^2 - ((2*a \\
& ^3*b - a*b^3)*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d*e*f^2 + (2*a^3*b - a*b^3) \\
& *c^2*f^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2})*\log(2*b*\cosh \\
& (d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 3*(2*(a^ \\
& 2*b^2 + b^4)*d*f^3*x + 2*(a^2*b^2 + b^4)*c*f^3 + 2*((a^2*b^2 + b^4)*d*f^3*x \\
& + (a^2*b^2 + b^4)*c*f^3)*\cosh(d*x + c))^4 + 2*((a^2*b^2 + b^4)*d*f^3*x + (a \\
& ^2*b^2 + b^4)*c*f^3)*\sinh(d*x + c))^4 + 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b \\
& + a*b^3)*c*f^3)*\cosh(d*x + c))^3 + 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b \\
& ^3)*c*f^3 + ((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3)*\cosh(d*x + c) \\
& )*\sinh(d*x + c))^3 + 4*((2*a^4 + a^2*b^2 - b^4)*d*f^3*x + (2*a^4 + a^2*b^2 - \\
& b^4)*c*f^3)*\cosh(d*x + c))^2 + 4*((2*a^4 + a^2*b^2 - b^4)*d*f^3*x + (2*a^4 \\
& + a^2*b^2 - b^4)*c*f^3 + 3*((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3 \\
& )*\cosh(d*x + c))^2 + 6*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3)*\cos \\
& h(d*x + c))*\sinh(d*x + c))^2 - 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)* \\
& c*f^3)*\cosh(d*x + c) - 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3 - \\
& ((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3)*\cosh(d*x + c))^3 - 3*((a^ \\
& 3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3)*\cosh(d*x + c))^2 - ((2*a^4 + a \\
& ^2*b^2 - b^4)*d*f^3*x + (2*a^4 + a^2*b^2 - b^4)*c*f^3)*\cosh(d*x + c))*\sinh( \\
& d*x + c) + (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a \\
& *b^3*c^2*f^3 + (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 \\
& - a*b^3*c^2*f^3)*\cosh(d*x + c))^4 + (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2* \\
& x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3)*\sinh(d*x + c))^4 + 4*(a^2*b^2*d^2*f^3 \\
& *x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*\cosh( \\
& d*x + c))^3 + 4*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d \\
& *e*f^2 - a^2*b^2*c^2*f^3 + (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b \\
& ^3*c*d*e*f^2 - a*b^3*c^2*f^3)*\cosh(d*x + c))*\sinh(d*x + c))^3 + 2*((2*a^3*b \\
& - a*b^3)*d^2*f^3*x^2 + 2*(2*a^3*b - a*b^3)*d^2*e*f^2*x + 2*(2*a^3*b - a*b^3) \\
& )*c*d*e*f^2 - (2*a^3*b - a*b^3)*c^2*f^3)*\cosh(d*x + c))^2 + 2*((2*a^3*b - a \\
& b^3)*d^2*f^3*x^2 + 2*(2*a^3*b - a*b^3)*d^2*e*f^2*x + 2*(2*a^3*b - a*b^3)*c \\
& *d*e*f^2 - (2*a^3*b - a*b^3)*c^2*f^3 + 3*(a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e* \\
& f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3)*\cosh(d*x + c))^2 + 6*(a^2*b^2*d^2 \\
& *f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*\c \\
& osh(d*x + c))*\sinh(d*x + c))^2 - 4*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^ \\
& 2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*\cosh(d*x + c) - 4*(a^2*b^2*d^2 \\
& *f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3 - \\
& (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^ \\
& 3)*\cosh(d*x + c))^3 - 3*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2 \\
& *b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*\cosh(d*x + c))^2 - ((2*a^3*b - a*b^3)*d^2* \\
& f^3*x^2 + 2*(2*a^3*b - a*b^3)*d^2*e*f^2*x + 2*(2*a^3*b - a*b^3)*c*d*e*f^2 - \\
& (2*a^3*b - a*b^3)*c^2*f^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/}
\end{aligned}$$

$$\begin{aligned}
& b^2)) * \log(-(a * \cosh(dx + c) + a * \sinh(dx + c) + (b * \cosh(dx + c) + b * \sinh(dx \\
& * x + c))) * \sqrt{((a^2 + b^2)/b^2) - b}/b) + 3 * (2 * (a^2 * b^2 + b^4) * d * f^3 * x + 2 * ( \\
& a^2 * b^2 + b^4) * c * f^3 + 2 * ((a^2 * b^2 + b^4) * d * f^3 * x + (a^2 * b^2 + b^4) * c * f^3) * \\
& \cosh(dx + c)^4 + 2 * ((a^2 * b^2 + b^4) * d * f^3 * x + (a^2 * b^2 + b^4) * c * f^3) * \sinh( \\
& dx + c)^4 + 8 * ((a^3 * b + a * b^3) * d * f^3 * x + (a^3 * b + a * b^3) * c * f^3) * \cosh(dx + \\
& c)^3 + 8 * ((a^3 * b + a * b^3) * d * f^3 * x + (a^3 * b + a * b^3) * c * f^3 + ((a^2 * b^2 + b^ \\
& 4) * d * f^3 * x + (a^2 * b^2 + b^4) * c * f^3) * \cosh(dx + c)) * \sinh(dx + c)^3 + 4 * ((2 * \\
& a^4 + a^2 * b^2 - b^4) * d * f^3 * x + (2 * a^4 + a^2 * b^2 - b^4) * c * f^3) * \cosh(dx + c) \\
& ^2 + 4 * ((2 * a^4 + a^2 * b^2 - b^4) * d * f^3 * x + (2 * a^4 + a^2 * b^2 - b^4) * c * f^3 + 3 \\
& * ((a^2 * b^2 + b^4) * d * f^3 * x + (a^2 * b^2 + b^4) * c * f^3) * \cosh(dx + c)^2 + 6 * ((a^ \\
& 3 * b + a * b^3) * d * f^3 * x + (a^3 * b + a * b^3) * c * f^3) * \cosh(dx + c)) * \sinh(dx + c) \\
& ^2 - 8 * ((a^3 * b + a * b^3) * d * f^3 * x + (a^3 * b + a * b^3) * c * f^3) * \cosh(dx + c) - 8 * ( \\
& (a^3 * b + a * b^3) * d * f^3 * x + (a^3 * b + a * b^3) * c * f^3 - ((a^2 * b^2 + b^4) * d * f^3 * x \\
& + (a^2 * b^2 + b^4) * c * f^3) * \cosh(dx + c)^3 - 3 * ((a^3 * b + a * b^3) * d * f^3 * x + (a^ \\
& 3 * b + a * b^3) * c * f^3) * \cosh(dx + c)^2 - ((2 * a^4 + a^2 * b^2 - b^4) * d * f^3 * x + (2 \\
& * a^4 + a^2 * b^2 - b^4) * c * f^3) * \cosh(dx + c)) * \sinh(dx + c) - (a * b^3 * d^2 * f^3 * \\
& x^2 + 2 * a * b^3 * d^2 * e * f^2 * x + 2 * a * b^3 * c * d * e * f^2 - a * b^3 * c^2 * f^3 + (a * b^3 * d^2 * \\
& f^3 * x^2 + 2 * a * b^3 * d^2 * e * f^2 * x + 2 * a * b^3 * c * d * e * f^2 - a * b^3 * c^2 * f^3) * \cosh(dx \\
& + c)^4 + (a * b^3 * d^2 * f^3 * x^2 + 2 * a * b^3 * d^2 * e * f^2 * x + 2 * a * b^3 * c * d * e * f^2 - a \\
& b^3 * c^2 * f^3) * \sinh(dx + c)^4 + 4 * (a^2 * b^2 * d^2 * f^3 * x^2 + 2 * a^2 * b^2 * d^2 * e * f^2 \\
& * x + 2 * a^2 * b^2 * c * d * e * f^2 - a^2 * b^2 * c^2 * f^3) * \cosh(dx + c)^3 + 4 * (a^2 * b^2 * d^ \\
& 2 * f^3 * x^2 + 2 * a^2 * b^2 * d^2 * e * f^2 * x + 2 * a^2 * b^2 * c * d * e * f^2 - a^2 * b^2 * c^2 * f^3 + \\
& (a * b^3 * d^2 * f^3 * x^2 + 2 * a * b^3 * d^2 * e * f^2 * x + 2 * a * b^3 * c * d * e * f^2 - a * b^3 * c^2 * f \\
& ^3) * \cosh(dx + c)) * \sinh(dx + c)^3 + 2 * ((2 * a^3 * b - a * b^3) * d^2 * f^3 * x^2 + 2 * ( \\
& 2 * a^3 * b - a * b^3) * d^2 * e * f^2 * x + 2 * (2 * a^3 * b - a * b^3) * c * d * e * f^2 - (2 * a^3 * b - a \\
& * b^3) * c^2 * f^3) * \cosh(dx + c)^2 + 2 * ((2 * a^3 * b - a * b^3) * d^2 * f^3 * x^2 + 2 * (2 * a^ \\
& 3 * b - a * b^3) * d^2 * e * f^2 * x + 2 * (2 * a^3 * b - a * b^3) * c * d * e * f^2 - (2 * a^3 * b - a * b^3) \\
& ) * c^2 * f^3 + 3 * (a * b^3 * d^2 * f^3 * x^2 + 2 * a * b^3 * d^2 * e * f^2 * x + 2 * a * b^3 * c * d * e * f^2 \\
& - a * b^3 * c^2 * f^3) * \cosh(dx + c)^2 + 6 * (a^2 * b^2 * d^2 * f^3 * x^2 + 2 * a^2 * b^2 * d^2 * e \\
& * f^2 * x + 2 * a^2 * b^2 * c * d * e * f^2 - a^2 * b^2 * c^2 * f^3) * \cosh(dx + c)) * \sinh(dx + c \\
& )^2 - 4 * (a^2 * b^2 * d^2 * f^3 * x^2 + 2 * a^2 * b^2 * d^2 * e * f^2 * x + 2 * a^2 * b^2 * c * d * e * f^2 \\
& - a^2 * b^2 * c^2 * f^3) * \cosh(dx + c) - 4 * (a^2 * b^2 * d^2 * f^3 * x^2 + 2 * a^2 * b^2 * d^2 * e \\
& * f^2 * x + 2 * a^2 * b^2 * c * d * e * f^2 - a^2 * b^2 * c^2 * f^3 - (a * b^3 * d^2 * f^3 * x^2 + 2 * a * b \\
& ^3 * d^2 * e * f^2 * x + 2 * a * b^3 * c * d * e * f^2 - a * b^3 * c^2 * f^3) * \cosh(dx + c)^3 - 3 * (a^ \\
& 2 * b^2 * d^2 * f^3 * x^2 + 2 * a^2 * b^2 * d^2 * e * f^2 * x + 2 * a^2 * b^2 * c * d * e * f^2 - a^2 * b^2 * c \\
& ^2 * f^3) * \cosh(dx + c)^2 - ((2 * a^3 * b - a * b^3) * d^2 * f^3 * x^2 + 2 * (2 * a^3 * b - a * b \\
& ^3) * d^2 * e * f^2 * x + 2 * (2 * a^3 * b - a * b^3) * c * d * e * f^2 - (2 * a^3 * b - a * b^3) * c^2 * f^3 \\
& ) * \cosh(dx + c)) * \sinh(dx + c)) * \sqrt{((a^2 + b^2)/b^2) * \log(-(a * \cosh(dx + c) \\
& + a * \sinh(dx + c) - (b * \cosh(dx + c) + b * \sinh(dx + c))) * \sqrt{((a^2 + b^2)/ \\
& b^2) - b}/b) + 2 * (3 * (a^3 * b + a * b^3) * d^2 * f^3 * x^2 + 6 * (a^3 * b + a * b^3) * d^2 * e * f \\
& ^2 * x - 9 * (a^3 * b + a * b^3) * d^2 * e^2 * f + 24 * (a^3 * b + a * b^3) * c * d * e * f^2 - 12 * (a^3 \\
& * b + a * b^3) * c^2 * f^3 - 12 * ((a^2 * b^2 + b^4) * d^2 * f^3 * x^2 + 2 * (a^2 * b^2 + b^4) * d \\
& ^2 * e * f^2 * x + 2 * (a^2 * b^2 + b^4) * c * d * e * f^2 - (a^2 * b^2 + b^4) * c^2 * f^3) * \cosh(dx \\
& + c)^3 - 9 * (3 * (a^3 * b + a * b^3) * d^2 * f^3 * x^2 + 6 * (a^3 * b + a * b^3) * d^2 * e * f^2 * x \\
& - (a^3 * b + a * b^3) * d^2 * e^2 * f + 8 * (a^3 * b + a * b^3) * c * d * e * f^2 - 4 * (a^3 * b + a * b
\end{aligned}$$

$$\begin{aligned} &^3)*c^2*f^3)*\cosh(d*x + c)^2 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 2 \\ &*(a^4 + 2*a^2*b^2 + b^4)*d^3*e^3 - 3*(2*a^4 + a^2*b^2 - b^4)*d^2*e^2*f + 12 \\ &*(2*a^4 + a^2*b^2 - b^4)*c*d*e*f^2 - 6*(2*a^4 + a^2*b^2 - b^4)*c^2*f^3 + 3* \\ &(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*e*f^2 + (2*a^4 + a^2*b^2 - b^4)*d^2*f^3)*x^2 \\ &+ 6*((a^4 + 2*a^2*b^2 + b^4)*d^3*e^2*f + (2*a^4 + a^2*b^2 - b^4)*d^2*e*f^2 \\ &)*x)*\cosh(d*x + c))*\sinh(d*x + c))/((a^4*b^3 + 2*a^2*b^5 + b^7)*d^4*\cosh(d* \\ &x + c)^4 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^4*\sinh(d*x + c)^4 + 4*(a^5*b^2 + 2 \\ &*a^3*b^4 + a*b^6)*d^4*\cosh(d*x + c)^3 + 2*(2*a^6*b + 3*a^4*b^3 - b^7)*d^4*c \\ &osh(d*x + c)^2 - 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^4*\cosh(d*x + c) + (a^4*b \\ &^3 + 2*a^2*b^5 + b^7)*d^4 + 4*((a^4*b^3 + 2*a^2*b^5 + b^7)*d^4*\cosh(d*x + c \\ &)) + (a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^4)*\sinh(d*x + c)^3 + 2*(3*(a^4*b^3 + 2* \\ &a^2*b^5 + b^7)*d^4*\cosh(d*x + c)^2 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^4*co \\ &sh(d*x + c) + (2*a^6*b + 3*a^4*b^3 - b^7)*d^4)*\sinh(d*x + c)^2 + 4*((a^4*b^ \\ &3 + 2*a^2*b^5 + b^7)*d^4*\cosh(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)* \\ &d^4*\cosh(d*x + c)^2 + (2*a^6*b + 3*a^4*b^3 - b^7)*d^4*\cosh(d*x + c) - (a^5* \\ &b^2 + 2*a^3*b^4 + a*b^6)*d^4)*\sinh(d*x + c)) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c)}{(b \sinh(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*cosh(d\*x + c)/(b\*sinh(d\*x + c) + a)^3, x)

$$3.330 \quad \int \frac{(e+fx) \cosh(c+dx)}{(a+b \sinh(c+dx))^3} dx$$

**Optimal.** Leaf size=112

$$-\frac{af \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2 (a^2+b^2)^{3/2}} - \frac{f \cosh(c+dx)}{2d^2 (a^2+b^2)(a+b \sinh(c+dx))} - \frac{e+fx}{2bd(a+b \sinh(c+dx))^2}$$

[Out]  $-\left(\frac{a*f*ArcTanh\left[\frac{b-a*\Tanh\left[\frac{c+d*x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{b*(a^2+b^2)^{3/2}*d^2}\right) - \frac{e+f*x}{2*b*d*(a+b*\Sinh[c+d*x])^2} - \frac{f*Cosh[c+d*x]}{2*(a^2+b^2)*d^2*(a+b*\Sinh[c+d*x])}$

**Rubi [A]** time = 0.0951033, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5464, 2664, 12, 2660, 618, 204}

$$-\frac{af \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd^2 (a^2+b^2)^{3/2}} - \frac{f \cosh(c+dx)}{2d^2 (a^2+b^2)(a+b \sinh(c+dx))} - \frac{e+fx}{2bd(a+b \sinh(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x])^3,x]

[Out]  $-\left(\frac{a*f*ArcTanh\left[\frac{b-a*\Tanh\left[\frac{c+d*x}{2}\right]}{\sqrt{a^2+b^2}}\right]}{b*(a^2+b^2)^{3/2}*d^2}\right) - \frac{e+f*x}{2*b*d*(a+b*\Sinh[c+d*x])^2} - \frac{f*Cosh[c+d*x]}{2*(a^2+b^2)*d^2*(a+b*\Sinh[c+d*x])}$

#### Rule 5464

Int[Cosh[(c\_.) + (d\_.)\*(x\_.)]\*((e\_.) + (f\_.)\*(x\_.))^(m\_.)\*((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Simp[((e + f\*x)^m\*(a + b\*Sinh[c + d\*x])^(n + 1))/(b\*d\*(n + 1)), x] - Dist[(f\*m)/(b\*d\*(n + 1)), Int[(e + f\*x)^(m - 1)\*(a + b\*Sinh[c + d\*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

#### Rule 2664

Int[((a\_.) + (b\_.)\*sin[(c\_.) + (d\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> -Simp[(b\*Cos[c + d\*x]\*(a + b\*Sinh[c + d\*x])^(n + 1))/(d\*(n + 1)\*(a^2 - b^2)), x] + Dist[1



$$\int \frac{(n+1)(a^2 - b^2)}{(a + b \sin[c + dx])^{n+1}} \operatorname{Simp}[a(n+1) - b(n+2)\sin[c + dx], x] \, dx$$
 /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2\*n]

### Rule 12

$$\int (a_1 u) (x_{\text{Symbol}}) \, dx \rightarrow \operatorname{Dist}[a, \int u, x] \, dx$$
 /; FreeQ[a, x] && !MatchQ[u, (b\_)(v\_)] /; FreeQ[b, x]

### Rule 2660

$$\int ((a_1) + (b_1) \sin[(c_1) + (d_1)x])^{-1} (x_{\text{Symbol}}) \, dx \rightarrow \operatorname{With}\{e = \operatorname{FreeFactors}[\operatorname{Tan}[(c + dx)/2], x]\}, \operatorname{Dist}[(2e)/d, \operatorname{Subst}[\int 1/(a + 2be^x + ae^{2x^2}), x], x, \operatorname{Tan}[(c + dx)/2]/e], x] \, dx$$
 /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

### Rule 618

$$\int ((a_1) + (b_1)x + (c_1)x^2)^{-1} (x_{\text{Symbol}}) \, dx \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\int 1/\operatorname{Simp}[b^2 - 4ac - x^2, x], x], x, b + 2cx], x] \, dx$$
 /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4ac, 0]

### Rule 204

$$\int ((a_1) + (b_1)x^2)^{-1} (x_{\text{Symbol}}) \, dx \rightarrow -\operatorname{Simp}[\operatorname{ArcTan}[(Rt[-b, 2]x)/Rt[-a, 2]]/(Rt[-a, 2]Rt[-b, 2]), x] \, dx$$
 /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
\int \frac{(e + fx) \cosh(c + dx)}{(a + b \sinh(c + dx))^3} dx &= -\frac{e + fx}{2bd(a + b \sinh(c + dx))^2} + \frac{f \int \frac{1}{(a + b \sinh(c + dx))^2} dx}{2bd} \\
&= -\frac{e + fx}{2bd(a + b \sinh(c + dx))^2} - \frac{f \cosh(c + dx)}{2(a^2 + b^2)d^2(a + b \sinh(c + dx))} + \frac{f \int \frac{a}{a + b \sinh(c + dx)} dx}{2b(a^2 + b^2)d} \\
&= -\frac{e + fx}{2bd(a + b \sinh(c + dx))^2} - \frac{f \cosh(c + dx)}{2(a^2 + b^2)d^2(a + b \sinh(c + dx))} + \frac{(af) \int \frac{1}{a + b \sinh(c + dx)} dx}{2b(a^2 + b^2)d} \\
&= -\frac{e + fx}{2bd(a + b \sinh(c + dx))^2} - \frac{f \cosh(c + dx)}{2(a^2 + b^2)d^2(a + b \sinh(c + dx))} - \frac{(iaf) \operatorname{Subst}\left(\int \frac{1}{a - 2ibx + ax^2}\right)}{b(a^2 + b^2)} \\
&= -\frac{e + fx}{2bd(a + b \sinh(c + dx))^2} - \frac{f \cosh(c + dx)}{2(a^2 + b^2)d^2(a + b \sinh(c + dx))} + \frac{(2iaf) \operatorname{Subst}\left(\int \frac{1}{-4(a^2 + b^2)}\right)}{b(a^2 + b^2)} \\
&= -\frac{af \tanh^{-1}\left(\frac{b - a \tanh\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a^2 + b^2}}\right)}{b(a^2 + b^2)^{3/2}d^2} - \frac{e + fx}{2bd(a + b \sinh(c + dx))^2} - \frac{f \cosh(c + dx)}{2(a^2 + b^2)d^2(a + b \sinh(c + dx))}
\end{aligned}$$

**Mathematica [A]** time = 0.373767, size = 112, normalized size = 1.

$$-\frac{2af \tan^{-1}\left(\frac{b - a \tanh\left(\frac{1}{2}(c + dx)\right)}{\sqrt{-a^2 - b^2}}\right)}{(-a^2 - b^2)^{3/2}} + \frac{d(e + fx)}{(a + b \sinh(c + dx))^2} + \frac{f \cosh(c + dx)}{(a^2 + b^2)(a + b \sinh(c + dx))}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x])^3,x]

[Out] -((f\*Cosh[c + d\*x])/((a^2 + b^2)\*(a + b\*Sinh[c + d\*x])) + ((2\*a\*f\*ArcTan[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[-a^2 - b^2]])/(-a^2 - b^2)^(3/2) + (d\*(e + f\*x))/(a + b\*Sinh[c + d\*x])^2)/b)/(2\*d^2)

**Maple [B]** time = 0., size = 308, normalized size = 2.8

$$\frac{2a^2dfxe^{2dx+2c} + 2b^2dfxe^{2dx+2c} + 2a^2dee^{2dx+2c} - abfe^{3dx+3c} + 2b^2dee^{2dx+2c} - 2a^2fe^{2dx+2c} + b^2fe^{2dx+2c} + 3fae^{dx+c}}{d^2b\left(be^{2dx+2c} + 2ae^{dx+c} - b\right)^2(a^2 + b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x)`

[Out] 
$$-1/b*(2*a^2*d*f*x*exp(2*d*x+2*c)+2*b^2*d*f*x*exp(2*d*x+2*c)+2*a^2*d*e*exp(2*d*x+2*c)-a*b*f*exp(3*d*x+3*c)+2*b^2*d*e*exp(2*d*x+2*c)-2*a^2*f*exp(2*d*x+2*c)+b^2*f*exp(2*d*x+2*c)+3*f*a*exp(d*x+c)*b-f*b^2)/d^2/(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)^2/(a^2+b^2)+1/2/(a^2+b^2)^{(3/2)}*f*a/d^2/b*\ln(\exp(d*x+c)+(a*(a^2+b^2)^{(3/2)}-a^4-2*a^2*b^2-b^4)/(a^2+b^2)^{(3/2)}/b)-1/2/(a^2+b^2)^{(3/2)}*f*a/d^2/b*\ln(\exp(d*x+c)+(a*(a^2+b^2)^{(3/2)}+a^4+2*a^2*b^2+b^4)/(a^2+b^2)^{(3/2)}/b))$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.21918, size = 2811, normalized size = 25.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*cosh(d*x+c)/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")`

[Out] 
$$1/2*(2*(a^3*b + a*b^3)*f*\cosh(d*x + c)^3 + 2*(a^3*b + a*b^3)*f*\sinh(d*x + c)^3 - 6*(a^3*b + a*b^3)*f*\cosh(d*x + c) - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*e - (2*a^4 + a^2*b^2 - b^4)*f)*\cosh(d*x + c)^2 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*e - 3*(a^3*b + a*b^3)*f*\cosh(d*x + c) - (2*a^4 + a^2*b^2 - b^4)*f)*\sinh(d*x + c)^2 + (a*b^2*f*\cosh(d*x + c)^4 + a*b^2*f*\sinh(d*x + c)^4 + 4*a^2*b*f*\cosh(d*x + c)^3 - 4*a^2*b*f*\cosh(d*x + c) + a*b^2*f + 2*(2*a^3 - a*b^2)*f*\cosh(d*x + c)^2 + 4*(a*b^2*f*\cosh(d*x + c) + a^2*b*f)*\sinh(d*x + c)^3 + 2*(3*a*b^2*f*\cosh(d*x + c)^2 + 6*a^2*b*f*\cosh(d*x + c) + (2*a^3 - a*b^2)*f)*\sinh(d*x$$

$$\begin{aligned}
& + c)^2 + 4*(a*b^2*f*cosh(d*x + c)^3 + 3*a^2*b*f*cosh(d*x + c)^2 - a^2*b*f + \\
& (2*a^3 - a*b^2)*f*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + b^2)*log((b^2*c \\
& osh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + \\
& 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + \\
& c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*c \\
& sh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a^2*b^2 + b^ \\
& 4)*f + 2*(3*(a^3*b + a*b^3)*f*cosh(d*x + c)^2 - 3*(a^3*b + a*b^3)*f - 2*(2* \\
& (a^4 + 2*a^2*b^2 + b^4)*d*f*x + 2*(a^4 + 2*a^2*b^2 + b^4)*d*e - (2*a^4 + a^ \\
& 2*b^2 - b^4)*f)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b^3 + 2*a^2*b^5 + b^7)* \\
& d^2*cosh(d*x + c)^4 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*sinh(d*x + c)^4 + 4*( \\
& a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2*cosh(d*x + c)^3 + 2*(2*a^6*b + 3*a^4*b^3 - \\
& b^7)*d^2*cosh(d*x + c)^2 - 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2*cosh(d*x + \\
& c) + 4*((a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c) + (a^5*b^2 + 2*a^3*b^ \\
& 4 + a*b^6)*d^2)*sinh(d*x + c)^3 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^2 + 2*(3*(a \\
& ^4*b^3 + 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c)^2 + 6*(a^5*b^2 + 2*a^3*b^4 + a* \\
& b^6)*d^2*cosh(d*x + c) + (2*a^6*b + 3*a^4*b^3 - b^7)*d^2)*sinh(d*x + c)^2 + \\
& 4*((a^4*b^3 + 2*a^2*b^5 + b^7)*d^2*cosh(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^ \\
& 4 + a*b^6)*d^2*cosh(d*x + c)^2 + (2*a^6*b + 3*a^4*b^3 - b^7)*d^2*cosh(d*x + \\
& c) - (a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^2)*sinh(d*x + c))
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \cosh(dx + c)}{(b \sinh(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((f\*x + e)\*cosh(d\*x + c)/(b\*sinh(d\*x + c) + a)^3, x)

$$3.331 \quad \int \frac{(e+fx)^2 \cosh(c+dx)}{(a+b \sinh(c+dx))^3} dx$$

**Optimal.** Leaf size=306

$$\frac{af^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3 (a^2+b^2)^{3/2}} - \frac{af^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3 (a^2+b^2)^{3/2}} + \frac{af(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{bd^2 (a^2+b^2)^{3/2}} - \frac{af(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2 (a^2+b^2)^{3/2}}$$

```
[Out] (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)^(3/2)*d^2) - (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)^(3/2)*d^2) + (f^2*Log[a + b*Sinh[c + d*x]])/(b*(a^2 + b^2)*d^3) + (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) - (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) - (e + f*x)^2/(2*b*d*(a + b*Sinh[c + d*x])^2) - (f*(e + f*x)*Cosh[c + d*x])/((a^2 + b^2)*d^2*(a + b*Sinh[c + d*x]))
```

**Rubi [A]** time = 0.521416, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 9, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$ , Rules used = {5464, 3324, 3322, 2264, 2190, 2279, 2391, 2668, 31}

$$\frac{af^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3 (a^2+b^2)^{3/2}} - \frac{af^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3 (a^2+b^2)^{3/2}} + \frac{af(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{bd^2 (a^2+b^2)^{3/2}} - \frac{af(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2 (a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```

```
[Out] (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)^(3/2)*d^2) - (a*f*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)^(3/2)*d^2) + (f^2*Log[a + b*Sinh[c + d*x]])/(b*(a^2 + b^2)*d^3) + (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) - (a*f^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) - (e + f*x)^2/(2*b*d*(a + b*Sinh[c + d*x])^2) - (f*(e + f*x)*Cosh[c + d*x])/((a^2 + b^2)*d^2*(a + b*Sinh[c + d*x]))
```

Rule 5464

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)]^(n_.), x_Symbol] := Simp[((e + f*x)^m*(a + b*Sinh[c +
d*x])^(n + 1))/(b*d*(n + 1)), x] - Dist[(f*m)/(b*d*(n + 1)), Int[(e + f*x)^
(m - 1)*(a + b*Sinh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n
}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### Rule 3324

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2, x_
Symbol] := Simp[(b*(c + d*x)^m*Cos[e + f*x])/(f*(a^2 - b^2)*(a + b*Sinh[e +
f*x])), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sinh[e + f*x]), x],
x] - Dist[(b*d*m)/(f*(a^2 - b^2)), Int[((c + d*x)^(m - 1)*Cos[e + f*x])/(a
+ b*Sinh[e + f*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

### Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_]*)
(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2190

```
Int((((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

Int[Log[(c\_.)\*(d\_.) + (e\_.)\*(x\_)^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 2668

Int[cos[(e\_.) + (f\_.)\*(x\_)^(p\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(m\_.), x\_Symbol] := Dist[1/(b^p\*f), Subst[Int[(a + x)^m\*(b^2 - x^2)^((p - 1)/2), x], x, b\*Sin[e + f\*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

Rule 31

Int[((a\_.) + (b\_.)\*(x\_))^(n\_.), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned}
\int \frac{(e + fx)^2 \cosh(c + dx)}{(a + b \sinh(c + dx))^3} dx &= -\frac{(e + fx)^2}{2bd(a + b \sinh(c + dx))^2} + \frac{f \int \frac{e + fx}{(a + b \sinh(c + dx))^2} dx}{bd} \\
&= -\frac{(e + fx)^2}{2bd(a + b \sinh(c + dx))^2} - \frac{f(e + fx) \cosh(c + dx)}{(a^2 + b^2) d^2 (a + b \sinh(c + dx))} + \frac{(af) \int \frac{e + fx}{a + b \sinh(c + dx)} dx}{b(a^2 + b^2) d} \\
&= -\frac{(e + fx)^2}{2bd(a + b \sinh(c + dx))^2} - \frac{f(e + fx) \cosh(c + dx)}{(a^2 + b^2) d^2 (a + b \sinh(c + dx))} + \frac{(2af) \int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}} dx}{b(a^2 + b^2) d} \\
&= \frac{f^2 \log(a + b \sinh(c + dx))}{b(a^2 + b^2) d^3} - \frac{(e + fx)^2}{2bd(a + b \sinh(c + dx))^2} - \frac{f(e + fx) \cosh(c + dx)}{(a^2 + b^2) d^2 (a + b \sinh(c + dx))} \\
&= \frac{af(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b(a^2 + b^2)^{3/2} d^2} - \frac{af(e + fx) \log\left(1 + \frac{be^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{b(a^2 + b^2)^{3/2} d^2} + \frac{f^2 \log(a + b \sinh(c + dx))}{b(a^2 + b^2) d^3} \\
&= \frac{af(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b(a^2 + b^2)^{3/2} d^2} - \frac{af(e + fx) \log\left(1 + \frac{be^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{b(a^2 + b^2)^{3/2} d^2} + \frac{f^2 \log(a + b \sinh(c + dx))}{b(a^2 + b^2) d^3} \\
&= \frac{af(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b(a^2 + b^2)^{3/2} d^2} - \frac{af(e + fx) \log\left(1 + \frac{be^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{b(a^2 + b^2)^{3/2} d^2} + \frac{f^2 \log(a + b \sinh(c + dx))}{b(a^2 + b^2) d^3}
\end{aligned}$$

**Mathematica [B]** time = 7.10572, size = 623, normalized size = 2.04

$$2e^c f \frac{\left( a(e^{2c}-1)f \left( \text{PolyLog}\left(2, -\frac{be^{2c+dx}}{ae^c - \sqrt{e^{2c}(a^2+b^2)}}\right) - \text{PolyLog}\left(2, -\frac{be^{2c+dx}}{\sqrt{e^{2c}(a^2+b^2)} + ae^c}\right) + dx \left( \log\left(\frac{be^{2c+dx}}{ae^c - \sqrt{e^{2c}(a^2+b^2)}} + 1\right) - \log\left(\frac{be^{2c+dx}}{\sqrt{e^{2c}(a^2+b^2)} + ae^c} + 1\right) \right) \right)}{2d\sqrt{e^{2c}(a^2+b^2)}} - \frac{ae^{-c}(e^{2c}-1)e \tanh\left(\frac{dx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} \right)}{b(e^{2c} - 1)}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Cosh[c + d\*x])/(a + b\*Sinh[c + d\*x])^3, x]

[Out] 
$$\frac{(f^2 x \text{Coth}[c])/(b(a^2 + b^2)d^2) + (2E^c f (-E^c f x) + (-1 + E^{2c}) f x)/E^c - (a e (-1 + E^{2c}) \text{ArcTanh}[(a + b E^{c + d x})/\sqrt{a^2 + b^2}]) / (\sqrt{a^2 + b^2} E^c) + (a(-1 + E^{2c}) f \text{ArcTanh}[(a + b E^{c + d x})/\sqrt{a^2 + b^2}]) / (\sqrt{a^2 + b^2} d E^c) + ((-1 + E^{2c}) f (-2x + (2a \text{ArcTan}[(a + b E^{c + d x})/\sqrt{-a^2 - b^2}]) / (\sqrt{-a^2 - b^2} d) + \text{Log}[2 a E^{c + d x} + b(-1 + E^{2(c + d x)})]) / d) / (2 E^c) + (a(-1 + E^{2c}) f (d x (\text{Log}[1 + (b E^{2c + d x}) / (a E^c - \sqrt{(a^2 + b^2) E^{2c}})]) - \text{Log}[1 + (b E^{2c + d x}) / (a E^c + \sqrt{(a^2 + b^2) E^{2c}})]) + \text{PolyLog}[2, -(b E^{2c + d x}) / (a E^c - \sqrt{(a^2 + b^2) E^{2c}})]) - \text{PolyLog}[2, -(b E^{2c + d x}) / (a E^c + \sqrt{(a^2 + b^2) E^{2c}})]))) / (2 d \sqrt{(a^2 + b^2) E^{2c}}) / (b(a^2 + b^2) d^2 (-1 + E^{2c})) - (f^2 x \text{Cosh}[c] \text{Csch}[c/2] \text{Sech}[c/2]) / (2 b(a^2 + b^2) d^2) - (e + f x)^2 / (2 b d (a + b \text{Sinh}[c + d x])^2) + (\text{Csch}[c/2] \text{Sech}[c/2] (a e f \text{Cosh}[c] + a f^2 x \text{Cosh}[c] + b e f \text{Sinh}[d x] + b f^2 x \text{Sinh}[d x])) / (2 b(a^2 + b^2) d^2 (a + b \text{Sinh}[c + d x]))$$

**Maple [B]** time = 0., size = 805, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3, x)

[Out] 
$$-2/b(a^2 d f^2 x^2 \exp(2 d x + 2 c) + b^2 d f^2 x^2 \exp(2 d x + 2 c) + 2 a^2 d e f x \exp(2 d x + 2 c) - a b f^2 x \exp(3 d x + 3 c) + 2 b^2 d e f x \exp(2 d x + 2 c) + a^2 d e^2 \exp(2 d x + 2 c) - 2 a^2 f^2 x \exp(2 d x + 2 c) - a b e f \exp(3 d x + 3 c) + b^2 d e^2 \exp(2 d x + 2 c) + b^2 f^2 x \exp(2 d x + 2 c) - 2 a^2 e f \exp(2 d x + 2 c) + 3 a b f^2 x \exp(d x + c) + b^2 e f \exp(2 d x + 2 c) + 3 a b e f \exp(d x + c) - b^2 f^2 x - b^2 e f) / d^2 / (a^2 + b^2) / (b \exp(2 d x + 2 c) + 2 a \exp(d x + c) - b)^2 + 1 / b f^2 / d^3 / (a^2 + b^2)$$



$$2+b^2)*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)-2/b*f^2/d^3/(a^2+b^2)*\ln(\exp(d*x+c))-2/b*f/d^2/(a^2+b^2)^{(3/2)}*a*e*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2}))+1/b*f^2/d^2/(a^2+b^2)^{(3/2)}*a*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*x+1/b*f^2/d^3/(a^2+b^2)^{(3/2)}*a*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*c-1/b*f^2/d^2/(a^2+b^2)^{(3/2)}*a*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*x-1/b*f^2/d^3/(a^2+b^2)^{(3/2)}*a*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*c+1/b*f^2/d^3/(a^2+b^2)^{(3/2)}*a*\operatorname{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))-1/b*f^2/d^3/(a^2+b^2)^{(3/2)}*a*\operatorname{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))+2/b*f^2/d^3/(a^2+b^2)^{(3/2)}*a*c*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2}))$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 3.04368, size = 11416, normalized size = 37.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3,x, algorithm="fricas")

[Out] 
$$-(2*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*\cosh(d*x + c)^4 + 2*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*\sinh(d*x + c)^4 - 2*(a^2*b^2 + b^4)*d*e*f + 2*(a^2*b^2 + b^4)*c*f^2 + 2*(3*(a^3*b + a*b^3)*d*f^2*x - (a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3)*c*f^2)*\cosh(d*x + c)^3 + 2*(3*(a^3*b + a*b^3)*d*f^2*x - (a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3)*c*f^2 + 4*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2*e^2 - (2*a^4 + a^2*b^2 - b^4)*d*e*f + 2*(2*a^4 + a^2*b^2 - b^4)*c*f^2 + (2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f + (2*a^4 + a^2*b^2 - b^4)*d*f^2)*x)*\cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2*e^2 - (2*a^4 + a^2*b^2 - b^4)*d*e*f + 2*(2*a^4 + a^2*b^2 - b^4)*c*f^2 + (2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f + (2*a^4 + a^2*b^2 - b^4)*d*f^2)*x)*\sinh(d*x + c)^2$$

$$\begin{aligned}
& 4)d^2e^2 - (2a^4 + a^2b^2 - b^4)d^2ef + 2(2a^4 + a^2b^2 - b^4)cf^2 \\
& + 6((a^2b^2 + b^4)d^2f^2x + (a^2b^2 + b^4)cf^2)\cosh(dx + c)^2 + (2(a^4 + 2a^2b^2 + b^4)d^2ef + (2a^4 + a^2b^2 - b^4)d^2f^2)x + 3(3 \\
& (a^3b + ab^3)d^2f^2x - (a^3b + ab^3)d^2ef + 4(a^3b + ab^3)cf^2) \\
& \cosh(dx + c)\sinh(dx + c)^2 - (ab^3f^2\cosh(dx + c)^4 + ab^3f^2\sinh(dx + c)^4 + 4a^2b^2f^2\cosh(dx + c)^3 - 4a^2b^2f^2\cosh(dx + c) \\
& + ab^3f^2 + 2(2a^3b - ab^3)f^2\cosh(dx + c)^2 + 4(ab^3f^2\cosh(dx + c) + a^2b^2f^2)\sinh(dx + c)^3 + 2(3ab^3f^2\cosh(dx + c)^2 + \\
& 6a^2b^2f^2\cosh(dx + c) + (2a^3b - ab^3)f^2)\sinh(dx + c)^2 + 4(ab^3f^2\cosh(dx + c)^3 + 3a^2b^2f^2\cosh(dx + c)^2 - a^2b^2f^2 + (2 \\
& a^3b - ab^3)f^2\cosh(dx + c)\sinh(dx + c))\sqrt{(a^2 + b^2)/b^2}\operatorname{dilog}((a\cosh(dx + c) + a\sinh(dx + c) + (b\cosh(dx + c) + b\sinh(dx + c)) \\
& \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + (ab^3f^2\cosh(dx + c)^4 + ab^3f^2\sinh(dx + c)^4 + 4a^2b^2f^2\cosh(dx + c)^3 - 4a^2b^2f^2\cosh(dx + c) \\
& + ab^3f^2 + 2(2a^3b - ab^3)f^2\cosh(dx + c)^2 + 4(ab^3f^2\cosh(dx + c) + a^2b^2f^2)\sinh(dx + c)^3 + 2(3ab^3f^2\cosh(dx + c)^2 + \\
& 6a^2b^2f^2\cosh(dx + c) + (2a^3b - ab^3)f^2)\sinh(dx + c)^2 + 4(ab^3f^2\cosh(dx + c)^3 + 3a^2b^2f^2\cosh(dx + c)^2 - a^2b^2f^2 + \\
& (2a^3b - ab^3)f^2\cosh(dx + c)\sinh(dx + c))\sqrt{(a^2 + b^2)/b^2}\operatorname{dilog}((a\cosh(dx + c) + a\sinh(dx + c) - (b\cosh(dx + c) + b\sinh(dx + c)) \\
& \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - (ab^3d^2f^2x + ab^3cf^2 + (ab^3d^2f^2x + ab^3cf^2)\cosh(dx + c)^4 + (ab^3d^2f^2x + ab^3cf^2)\sinh(dx + c)^4 + 4(a^2b^2d^2f^2x + a^2b^2cf^2)\cosh(dx + c)^3 + 4(a^2b^2d^2f^2x + a^2b^2cf^2 + (ab^3d^2f^2x + ab^3cf^2)\cosh(dx + c))\sinh(dx + c)^3 + 2((2a^3b - ab^3)d^2f^2x + (2a^3b - ab^3)cf^2)\cosh(dx + c)^2 + 2((2a^3b - ab^3)d^2f^2x + (2a^3b - ab^3)cf^2 + 3(ab^3d^2f^2x + ab^3cf^2)\cosh(dx + c)^2 + 6(a^2b^2d^2f^2x + a^2b^2cf^2)\cosh(dx + c))\sinh(dx + c)^2 - 4(a^2b^2d^2f^2x + a^2b^2cf^2)\cosh(dx + c) - 4(a^2b^2d^2f^2x + a^2b^2cf^2 - (ab^3d^2f^2x + ab^3cf^2)\cosh(dx + c)^3 - 3(a^2b^2d^2f^2x + a^2b^2cf^2)\cosh(dx + c)^2 - ((2a^3b - ab^3)d^2f^2x + (2a^3b - ab^3)cf^2)\cosh(dx + c)\sinh(dx + c))\sqrt{(a^2 + b^2)/b^2}\log(-(a\cosh(dx + c) + a\sinh(dx + c) + (b\cosh(dx + c) + b\sinh(dx + c))\sqrt{(a^2 + b^2)/b^2} - b)/b) + (ab^3d^2f^2x + ab^3cf^2 + (ab^3d^2f^2x + ab^3cf^2)\cosh(dx + c)^4 + (ab^3d^2f^2x + ab^3cf^2)\sinh(dx + c)^4 + 4(a^2b^2d^2f^2x + a^2b^2cf^2)\cosh(dx + c)^3 + 4(a^2b^2d^2f^2x + a^2b^2cf^2 + (ab^3d^2f^2x + ab^3cf^2)\cosh(dx + c))\sinh(dx + c)^3 + 2((2a^3b - ab^3)d^2f^2x + (2a^3b - ab^3)cf^2)\cosh(dx + c)^2 + 2((2a^3b - ab^3)d^2f^2x + (2a^3b - ab^3)cf^2 + 3(ab^3d^2f^2x + ab^3cf^2)\cosh(dx + c)^2 + 6(a^2b^2d^2f^2x + a^2b^2cf^2)\cosh(dx + c))\sinh(dx + c)^2 - 4(a^2b^2d^2f^2x + a^2b^2cf^2)\cosh(dx + c) - 4(a^2b^2d^2f^2x + a^2b^2cf^2 - (ab^3d^2f^2x + ab^3cf^2)\cosh(dx + c)^3 - 3(a^2b^2d^2f^2x + a^2b^2cf^2)\cosh(dx + c)^2 - ((2a^3b - ab^3)d^2f^2x + (2a^3b - ab^3)cf^2)\cosh(dx + c)\sinh(dx + c))\sqrt{(a^2 + b^2)/b^2}\log(-(a\cosh(dx + c) + a\sinh(dx + c) - (b\cosh(dx + c) + b\sinh(dx + c))\sqrt{(a^2 + b^2)/b^2} - b)/b)
\end{aligned}$$

$$\begin{aligned}
& *x + c)) * \sqrt{(a^2 + b^2)/b^2} - b)/b) - 2*((a^3*b + a*b^3)*d*f^2*x - 3*(a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3)*c*f^2)*\cosh(d*x + c) - ((a^2*b^2 + b^4)*f^2*\cosh(d*x + c)^4 + (a^2*b^2 + b^4)*f^2*\sinh(d*x + c)^4 + 4*(a^3*b + a*b^3)*f^2*\cosh(d*x + c)^3 + 2*(2*a^4 + a^2*b^2 - b^4)*f^2*\cosh(d*x + c)^2 - 4*(a^3*b + a*b^3)*f^2*\cosh(d*x + c) + 4*((a^2*b^2 + b^4)*f^2*\cosh(d*x + c) + (a^3*b + a*b^3)*f^2)*\sinh(d*x + c)^3 + (a^2*b^2 + b^4)*f^2 + 2*(3*(a^2*b^2 + b^4)*f^2*\cosh(d*x + c)^2 + 6*(a^3*b + a*b^3)*f^2*\cosh(d*x + c) + (2*a^4 + a^2*b^2 - b^4)*f^2)*\sinh(d*x + c)^2 + 4*((a^2*b^2 + b^4)*f^2*\cosh(d*x + c)^3 + 3*(a^3*b + a*b^3)*f^2*\cosh(d*x + c)^2 + (2*a^4 + a^2*b^2 - b^4)*f^2*2*\cosh(d*x + c) - (a^3*b + a*b^3)*f^2)*\sinh(d*x + c) - (a*b^3*d*e*f - a*b^3*c*f^2 + (a*b^3*d*e*f - a*b^3*c*f^2)*\cosh(d*x + c)^4 + (a*b^3*d*e*f - a*b^3*c*f^2)*\sinh(d*x + c)^4 + 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*\cosh(d*x + c)^3 + 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2 + (a*b^3*d*e*f - a*b^3*c*f^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2)*\cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2 + 3*(a*b^3*d*e*f - a*b^3*c*f^2)*\cosh(d*x + c)^2 + 6*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*\cosh(d*x + c))*\sinh(d*x + c)^2 - 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*\cosh(d*x + c) - 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2 - (a*b^3*d*e*f - a*b^3*c*f^2)^2)*\cosh(d*x + c)^3 - 3*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*\cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2})*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - ((a^2*b^2 + b^4)*f^2*\cosh(d*x + c)^4 + (a^2*b^2 + b^4)*f^2*\sinh(d*x + c)^4 + 4*(a^3*b + a*b^3)*f^2*\cosh(d*x + c)^3 + 2*(2*a^4 + a^2*b^2 - b^4)*f^2*\cosh(d*x + c)^2 - 4*(a^3*b + a*b^3)*f^2*\cosh(d*x + c) + 4*((a^2*b^2 + b^4)*f^2*\cosh(d*x + c) + (a^3*b + a*b^3)*f^2)*\sinh(d*x + c)^3 + (a^2*b^2 + b^4)*f^2 + 2*(3*(a^2*b^2 + b^4)*f^2*\cosh(d*x + c)^2 + 6*(a^3*b + a*b^3)*f^2*\cosh(d*x + c) + (2*a^4 + a^2*b^2 - b^4)*f^2)*\sinh(d*x + c)^2 + 4*((a^2*b^2 + b^4)*f^2*\cosh(d*x + c)^3 + 3*(a^3*b + a*b^3)*f^2*2*\cosh(d*x + c)^2 + (2*a^4 + a^2*b^2 - b^4)*f^2*\cosh(d*x + c) - (a^3*b + a*b^3)*f^2)*\sinh(d*x + c) + (a*b^3*d*e*f - a*b^3*c*f^2 + (a*b^3*d*e*f - a*b^3*c*f^2)*\cosh(d*x + c)^4 + (a*b^3*d*e*f - a*b^3*c*f^2)*\sinh(d*x + c)^4 + 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*\cosh(d*x + c)^3 + 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2 + (a*b^3*d*e*f - a*b^3*c*f^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2)*\cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2 + 3*(a*b^3*d*e*f - a*b^3*c*f^2)*\cosh(d*x + c)^2 + 6*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*\cosh(d*x + c))*\sinh(d*x + c)^2 - 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*\cosh(d*x + c) - 4*(a^2*b^2*d*e*f - a^2*b^2*c*f^2 - (a*b^3*d*e*f - a*b^3*c*f^2)*\cosh(d*x + c)^3 - 3*(a^2*b^2*d*e*f - a^2*b^2*c*f^2)*\cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d*e*f - (2*a^3*b - a*b^3)*c*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2})*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - 2*((a^3*b + a*b^3)*d*f^2*x - 3*(a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3)*c*f^2 - 4*((a^2*b^2 + b^4)*d*f^2*x + (a^2*b^2 + b^4)*c*f^2)*\cosh(d*x + c)^3 - 3*(3*(a^3*b + a*b^3)*d*f^2*x - (a^3*b + a*b^3)*d*e*f + 4*(a^3*b + a*b^3)*c*f^2)*\cosh(d*x + c)^2 - 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f^2*x^2 + (a^4 +
\end{aligned}$$

$$2a^2b^2 + b^4)d^2e^2 - (2a^4 + a^2b^2 - b^4)d*ef + 2*(2a^4 + a^2b^2 - b^4)*c*f^2 + (2*(a^4 + 2a^2b^2 + b^4)d^2*ef + (2a^4 + a^2b^2 - b^4)d*f^2)*x)*\cosh(dx + c))*\sinh(dx + c))/((a^4*b^3 + 2a^2*b^5 + b^7)*d^3*\cosh(dx + c)^4 + (a^4*b^3 + 2a^2*b^5 + b^7)*d^3*\sinh(dx + c)^4 + 4*(a^5*b^2 + 2a^3*b^4 + a*b^6)*d^3*\cosh(dx + c)^3 + 2*(2a^6*b + 3a^4*b^3 - b^7)*d^3*\cosh(dx + c)^2 - 4*(a^5*b^2 + 2a^3*b^4 + a*b^6)*d^3*\cosh(dx + c) + (a^4*b^3 + 2a^2*b^5 + b^7)*d^3 + 4*((a^4*b^3 + 2a^2*b^5 + b^7)*d^3*\cosh(dx + c) + (a^5*b^2 + 2a^3*b^4 + a*b^6)*d^3)*\sinh(dx + c)^3 + 2*(3*(a^4*b^3 + 2a^2*b^5 + b^7)*d^3*\cosh(dx + c)^2 + 6*(a^5*b^2 + 2a^3*b^4 + a*b^6)*d^3*\cosh(dx + c) + (2a^6*b + 3a^4*b^3 - b^7)*d^3)*\sinh(dx + c)^2 + 4*((a^4*b^3 + 2a^2*b^5 + b^7)*d^3*\cosh(dx + c)^3 + 3*(a^5*b^2 + 2a^3*b^4 + a*b^6)*d^3*\cosh(dx + c)^2 + (2a^6*b + 3a^4*b^3 - b^7)*d^3*\cosh(dx + c) - (a^5*b^2 + 2a^3*b^4 + a*b^6)*d^3)*\sinh(dx + c))$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c)}{(b \sinh(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*cosh(d\*x + c)/(b\*sinh(d\*x + c) + a)^3, x)

$$3.332 \quad \int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b \sinh(c+dx))^3} dx$$

**Optimal.** Leaf size=631

$$\frac{3af^2(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3(a^2+b^2)^{3/2}} - \frac{3af^2(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3(a^2+b^2)^{3/2}} + \frac{3f^3\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4(a^2+b^2)} + \frac{3f^3\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^4(a^2+b^2)}$$

[Out]  $(-3*f*(e + f*x)^2)/(2*b*(a^2 + b^2)*d^2) + (3*f^2*(e + f*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])])/(b*(a^2 + b^2)*d^3) + (3*a*f*(e + f*x)^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])])/(2*b*(a^2 + b^2)^{(3/2)}*d^2) + (3*f^2*(e + f*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(b*(a^2 + b^2)*d^3) - (3*a*f*(e + f*x)^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(2*b*(a^2 + b^2)^{(3/2)}*d^2) + (3*f^3*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4) + (3*a*f^2*(e + f*x)*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)^{(3/2)}*d^3) + (3*f^3*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4) - (3*a*f^2*(e + f*x)*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)^{(3/2)}*d^3) - (3*a*f^3*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)^{(3/2)}*d^4) + (3*a*f^3*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)^{(3/2)}*d^4) - (e + f*x)^3/(2*b*d*(a + b*\text{Sinh}[c + d*x])^2) - (3*f*(e + f*x)^2*\text{Cosh}[c + d*x])/(2*(a^2 + b^2)*d^2*(a + b*\text{Sinh}[c + d*x]))$

**Rubi [A]** time = 1.08746, antiderivative size = 631, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$ , Rules used = {5464, 3324, 3322, 2264, 2190, 2531, 2282, 6589, 5561, 2279, 2391}

$$\frac{3af^2(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^3(a^2+b^2)^{3/2}} - \frac{3af^2(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^3(a^2+b^2)^{3/2}} + \frac{3f^3\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^4(a^2+b^2)} + \frac{3f^3\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^4(a^2+b^2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(e + f*x)^3*\text{Cosh}[c + d*x]}{(a + b*\text{Sinh}[c + d*x])^3}, x]$

[Out]  $(-3*f*(e + f*x)^2)/(2*b*(a^2 + b^2)*d^2) + (3*f^2*(e + f*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])])/(b*(a^2 + b^2)*d^3) + (3*a*f*(e + f*x)^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])])/(2*b*(a^2 + b^2)^{(3/2)}*d^2) + (3*f^2*(e + f*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(b*(a^2 + b^2)*d^3) - (3*a*f*(e + f*x)^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(2*b*(a^2 + b^2)^{(3/2)}*d^2) + (3*f^3*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4) + (3*a*f^2*(e + f*x)*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)^{(3/2)}*d^3) + (3*f^3*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)*d^4) - (3*a*f^2*(e + f*x)*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)^{(3/2)}*d^3) - (3*a*f^3*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)^{(3/2)}*d^4) + (3*a*f^3*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)^{(3/2)}*d^4) - (e + f*x)^3/(2*b*d*(a + b*\text{Sinh}[c + d*x])^2) - (3*f*(e + f*x)^2*\text{Cosh}[c + d*x])/(2*(a^2 + b^2)*d^2*(a + b*\text{Sinh}[c + d*x]))$

$$\begin{aligned} & 2] ] ] / (2 * b * (a^2 + b^2)^{(3/2)} * d^2) + (3 * f^3 * \text{PolyLog}[2, -((b * E^{(c + d * x)}) / (a - \text{Sqrt}[a^2 + b^2]))] / (b * (a^2 + b^2) * d^4) + (3 * a * f^2 * (e + f * x) * \text{PolyLog}[2, -((b * E^{(c + d * x)}) / (a - \text{Sqrt}[a^2 + b^2]))] / (b * (a^2 + b^2)^{(3/2)} * d^3) + (3 * f^3 * \text{PolyLog}[2, -((b * E^{(c + d * x)}) / (a + \text{Sqrt}[a^2 + b^2]))] / (b * (a^2 + b^2) * d^4) - (3 * a * f^2 * (e + f * x) * \text{PolyLog}[2, -((b * E^{(c + d * x)}) / (a + \text{Sqrt}[a^2 + b^2]))] / (b * (a^2 + b^2)^{(3/2)} * d^3) - (3 * a * f^3 * \text{PolyLog}[3, -((b * E^{(c + d * x)}) / (a - \text{Sqrt}[a^2 + b^2]))] / (b * (a^2 + b^2)^{(3/2)} * d^4) + (3 * a * f^3 * \text{PolyLog}[3, -((b * E^{(c + d * x)}) / (a + \text{Sqrt}[a^2 + b^2]))] / (b * (a^2 + b^2)^{(3/2)} * d^4) - (e + f * x)^3 / (2 * b * d * (a + b * \text{Sinh}[c + d * x])^2) - (3 * f * (e + f * x)^2 * \text{Cosh}[c + d * x]) / (2 * (a^2 + b^2) * d^2 * (a + b * \text{Sinh}[c + d * x])) \end{aligned}$$
Rule 5464

$$\text{Int}[\text{Cosh}[(c_.) + (d_.) * (x_.)] * ((e_.) + (f_.) * (x_.))^{(m_.)} * ((a_.) + (b_.) * \text{Sinh}[(c_.) + (d_.) * (x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(e + f * x)^m * (a + b * \text{Sinh}[c + d * x])^{(n + 1)} / (b * d * (n + 1)), x] - \text{Dist}[(f * m) / (b * d * (n + 1)), \text{Int}[(e + f * x)^{(m - 1)} * (a + b * \text{Sinh}[c + d * x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[n, -1]$$
Rule 3324

$$\text{Int}[(c_.) + (d_.) * (x_.))^{(m_.)} / ((a_.) + (b_.) * \sin[(e_.) + (f_.) * (x_.)])^2, x\_Symbol] \rightarrow \text{Simp}[(b * (c + d * x)^m * \text{Cos}[e + f * x]) / (f * (a^2 - b^2) * (a + b * \sin[e + f * x])), x] + (\text{Dist}[a / (a^2 - b^2), \text{Int}[(c + d * x)^m / (a + b * \sin[e + f * x]), x], x] - \text{Dist}[(b * d * m) / (f * (a^2 - b^2)), \text{Int}[(c + d * x)^{(m - 1)} * \text{Cos}[e + f * x]) / (a + b * \sin[e + f * x]), x], x]) /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[m, 0]$$
Rule 3322

$$\text{Int}[(c_.) + (d_.) * (x_.))^{(m_.)} / ((a_.) + (b_.) * \sin[(e_.) + (\text{Complex}[0, fz\_]) * (f_.) * (x_.)]), x\_Symbol] \rightarrow \text{Dist}[2, \text{Int}[(c + d * x)^m * E^{-((I * e) + f * fz * x)} / (- (I * b) + 2 * a * E^{-((I * e) + f * fz * x)} + I * b * E^{(2 * (- (I * e) + f * fz * x))}), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, fz\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{IGtQ}[m, 0]$$
Rule 2264

$$\text{Int}[(F_.)^{(u_.)} * ((f_.) + (g_.) * (x_.))^{(m_.)} / ((a_.) + (b_.) * (F_.)^{(u_.)} + (c_.) * (F_.)^{(v_.)}), x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[b^2 - 4 * a * c, 2]\}, \text{Dist}[(2 * c) / q, \text{Int}[(f + g * x)^m * F^u / (b - q + 2 * c * F^u), x], x] - \text{Dist}[(2 * c) / q, \text{Int}[(f + g * x)^m * F^u / (b + q + 2 * c * F^u), x], x]) /; \text{FreeQ}[\{F, a, b, c, f, g\}, x] \&\& \text{EqQ}[v, 2 * u] \&\& \text{LinearQ}[u, x] \&\& \text{NeQ}[b^2 - 4 * a * c, 0] \&\& \text{IGtQ}[m, 0]$$
Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*(f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 5561

```
Int[(Cosh[(c_) + (d_)*(x_)]*((e_) + (f_)*(x_))^(m_))/((a_) + (b_)*Sin
h[(c_) + (d_)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \cosh(c+dx)}{(a+b \sinh(c+dx))^3} dx &= -\frac{(e+fx)^3}{2bd(a+b \sinh(c+dx))^2} + \frac{(3f) \int \frac{(e+fx)^2}{(a+b \sinh(c+dx))^2} dx}{2bd} \\
&= -\frac{(e+fx)^3}{2bd(a+b \sinh(c+dx))^2} - \frac{3f(e+fx)^2 \cosh(c+dx)}{2(a^2+b^2)d^2(a+b \sinh(c+dx))} + \frac{(3af) \int \frac{(e+fx)^2}{a+b \sinh(c+dx)} dx}{2b(a^2+b^2)d} \\
&= -\frac{3f(e+fx)^2}{2b(a^2+b^2)d^2} - \frac{(e+fx)^3}{2bd(a+b \sinh(c+dx))^2} - \frac{3f(e+fx)^2 \cosh(c+dx)}{2(a^2+b^2)d^2(a+b \sinh(c+dx))} + \frac{(3af)}{2b(a^2+b^2)d} \\
&= -\frac{3f(e+fx)^2}{2b(a^2+b^2)d^2} + \frac{3f^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3} + \frac{3f^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3} - \frac{3af}{2b(a^2+b^2)d} \\
&= -\frac{3f(e+fx)^2}{2b(a^2+b^2)d^2} + \frac{3f^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3} + \frac{3af(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b(a^2+b^2)^{3/2}d^2} + \frac{3af}{2b(a^2+b^2)d} \\
&= -\frac{3f(e+fx)^2}{2b(a^2+b^2)d^2} + \frac{3f^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3} + \frac{3af(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b(a^2+b^2)^{3/2}d^2} + \frac{3af}{2b(a^2+b^2)d} \\
&= -\frac{3f(e+fx)^2}{2b(a^2+b^2)d^2} + \frac{3f^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)d^3} + \frac{3af(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{2b(a^2+b^2)^{3/2}d^2} + \frac{3af}{2b(a^2+b^2)d}
\end{aligned}$$

**Mathematica [B]** time = 7.56222, size = 5753, normalized size = 9.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x])/(a + b*Sinh[c + d*x])^3,x]
```



[Out] Result too large to show

---

**Maple [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c)}{(a + b \sinh(dx + c))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3,x)

[Out] int((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3,x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [C]** time = 4.70474, size = 24868, normalized size = 39.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3,x, algorithm="fricas")

[Out]  $\frac{1}{2}*(6*(a^2*b^2 + b^4)*d^2*e^2*f - 12*(a^2*b^2 + b^4)*c*d*e*f^2 + 6*(a^2*b^2 + b^4)*c^2*f^3 - 6*((a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^2*b^2 + b^4)*d^2*e*f^2*x + 2*(a^2*b^2 + b^4)*c*d*e*f^2 - (a^2*b^2 + b^4)*c^2*f^3)*\cosh(dx + c)^4 - 6*((a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^2*b^2 + b^4)*d^2*e*f^2*x + 2*(a^2*b^2 + b^4)*c*d*e*f^2 - (a^2*b^2 + b^4)*c^2*f^3)*\sinh(dx + c)^4 - 6*(3*$

$$\begin{aligned}
& (a^3b + a^2b^2)d^2f^3x^2 + 6(a^3b + a^2b^2)d^2e^2f^2x - (a^3b + a^2b^2)d^2e^2f + 8(a^3b + a^2b^2)cd^2e^2f^2 - 4(a^3b + a^2b^2)c^2f^3)\cosh(dx + c)^3 - 6(3(a^3b + a^2b^2)d^2f^3x^2 + 6(a^3b + a^2b^2)d^2e^2f^2x - (a^3b + a^2b^2)d^2e^2f + 8(a^3b + a^2b^2)cd^2e^2f^2 - 4(a^3b + a^2b^2)c^2f^3 + 4((a^2b^2 + b^4)d^2f^3x^2 + 2(a^2b^2 + b^4)d^2e^2f^2x + 2(a^2b^2 + b^4)cd^2e^2f^2 - (a^2b^2 + b^4)c^2f^3)\cosh(dx + c))\sinh(dx + c)^3 - 2(2(a^4 + 2a^2b^2 + b^4)d^3f^3x^3 + 2(a^4 + 2a^2b^2 + b^4)d^3e^3 - 3(2a^4 + a^2b^2 - b^4)d^2e^2f + 12(2a^4 + a^2b^2 - b^4)cd^2e^2f^2 - 6(2a^4 + a^2b^2 - b^4)c^2f^3 + 3(2(a^4 + 2a^2b^2 + b^4)d^3e^2f^2 + (2a^4 + a^2b^2 - b^4)d^2e^2f^2)x)\cosh(dx + c)^2 - 2(2(a^4 + 2a^2b^2 + b^4)d^3f^3x^3 + 2(a^4 + 2a^2b^2 + b^4)d^3e^3 - 3(2a^4 + a^2b^2 - b^4)d^2e^2f + 12(2a^4 + a^2b^2 - b^4)cd^2e^2f^2 - 6(2a^4 + a^2b^2 - b^4)c^2f^3 + 3(2(a^4 + 2a^2b^2 + b^4)d^3e^2f^2 + (2a^4 + a^2b^2 - b^4)d^2e^2f^2)x + 9(3(a^3b + a^2b^2)d^2f^3x^2 + 6(a^3b + a^2b^2)d^2e^2f^2x - (a^3b + a^2b^2)d^2e^2f + 8(a^3b + a^2b^2)cd^2e^2f^2 - 4(a^3b + a^2b^2)c^2f^3)\cosh(dx + c))\sinh(dx + c)^2 - 6(a^2b^3f^3\cosh(dx + c)^4 + a^2b^3f^3\sinh(dx + c)^4 + 4a^2b^2f^3\cosh(dx + c)^3 - 4a^2b^2f^3\sinh(dx + c)^3 + a^2b^3f^3 + 2(2a^3b - a^2b^3)f^3\cosh(dx + c)^2 + 4(a^2b^3f^3\cosh(dx + c) + a^2b^2f^3\sinh(dx + c))^3 + 2(3a^2b^3f^3\cosh(dx + c)^2 + 6a^2b^2f^3\cosh(dx + c) + (2a^3b - a^2b^3)f^3)\sinh(dx + c)^2 + 4(a^2b^3f^3\cosh(dx + c)^3 + 3a^2b^2f^3\cosh(dx + c)^2 - a^2b^2f^3 + (2a^3b - a^2b^3)f^3\cosh(dx + c))\sinh(dx + c))\sqrt{(a^2 + b^2)/b^2})\text{polylog}(3, (a\cosh(dx + c) + a\sinh(dx + c) + (b\cosh(dx + c) + b\sinh(dx + c))\sqrt{(a^2 + b^2)/b^2}))/b) + 6((a^3b + a^2b^2)d^2f^3x^2 + 2(a^3b + a^2b^2)d^2e^2f^2x - 3(a^3b + a^2b^2)d^2e^2f + 8(a^3b + a^2b^2)cd^2e^2f^2 - 4(a^3b + a^2b^2)c^2f^3)\cosh(dx + c) + 6((a^2b^2 + b^4)f^3\cosh(dx + c)^4 + (a^2b^2 + b^4)f^3\sinh(dx + c)^4 + 4(a^3b + a^2b^2)f^3\cosh(dx + c)^3 + 2(2a^4 + a^2b^2 - b^4)f^3\cosh(dx + c)^2 - 4(a^3b + a^2b^2)f^3\cosh(dx + c) + (a^2b^2 + b^4)f^3 + 4((a^2b^2 + b^4)f^3\cosh(dx + c) + (a^3b + a^2b^2)f^3)\sinh(dx + c))^3 + 2(3(a^2b^2 + b^4)f^3\cosh(dx + c)^2 + 6(a^3b + a^2b^2)f^3\cosh(dx + c) + (2a^4 + a^2b^2 - b^4)f^3)\sinh(dx + c)^2 + 4((a^2b^2 + b^4)f^3\cosh(dx + c)^3 + 3(a^3b + a^2b^2)
\end{aligned}$$



$$\begin{aligned}
&^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3)*\cosh(d*x + c) - 8*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3 - ((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*\cosh(d*x + c)^3 - 3*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3)*\cosh(d*x + c)^2 - ((2*a^4 + a^2*b^2 - b^4)*d*e*f^2 - (2*a^4 + a^2*b^2 - b^4)*c*f^3)*\cosh(d*x + c))*\sinh(d*x + c) - (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*\cosh(d*x + c)^4 + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*\sinh(d*x + c)^4 + 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*\cosh(d*x + c)^3 + 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3 + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d*e*f^2 + (2*a^3*b - a*b^3)*c^2*f^3)*\cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d*e*f^2 + (2*a^3*b - a*b^3)*c^2*f^3 + 3*(a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*\cosh(d*x + c)^2 + 6*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 - 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*\cosh(d*x + c) - 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3 - (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*\cosh(d*x + c))^3 - 3*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*\cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d*e*f^2 + (2*a^3*b - a*b^3)*c^2*f^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt((a^2 + b^2)/b^2))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt((a^2 + b^2)/b^2) + 2*a) + 3*(2*(a^2*b^2 + b^4)*d*e*f^2 - 2*(a^2*b^2 + b^4)*c*f^3 + 2*((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*\cosh(d*x + c)^4 + 2*((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*\sinh(d*x + c)^4 + 8*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3)*\cosh(d*x + c)^3 + 8*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3 + ((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*((2*a^4 + a^2*b^2 - b^4)*d*e*f^2 - (2*a^4 + a^2*b^2 - b^4)*c*f^3)*\cosh(d*x + c)^2 + 4*((2*a^4 + a^2*b^2 - b^4)*d*e*f^2 - (2*a^4 + a^2*b^2 - b^4)*c*f^3 + 3*((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*\cosh(d*x + c)^2 + 6*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 - 8*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3)*\cosh(d*x + c) - 8*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3 - ((a^2*b^2 + b^4)*d*e*f^2 - (a^2*b^2 + b^4)*c*f^3)*\cosh(d*x + c))^3 - 3*((a^3*b + a*b^3)*d*e*f^2 - (a^3*b + a*b^3)*c*f^3)*\cosh(d*x + c)^2 - ((2*a^4 + a^2*b^2 - b^4)*d*e*f^2 - (2*a^4 + a^2*b^2 - b^4)*c*f^3)*\cosh(d*x + c))*\sinh(d*x + c) + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3 + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*\cosh(d*x + c))^4 + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*\sinh(d*x + c)^4 + 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*\cosh(d*x + c)^3 + 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3 + (a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*((2*a^3*b - a*b^3)*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d*e*f^2 + (2*a^3*b - a*b^3)*c^2*f^3)*\cosh(d*x + c)^2 + 2*((2*a^3*b - a*b^3)*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d*e*f^2 + (2*a^3*b - a*b^3)*c^2*f^3 + 3*(a*b^3*d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + (2*a^3*b - a*b^3)*c^2*f^3)
\end{aligned}$$

$$\begin{aligned}
& f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*\cosh(d*x + c)^2 + 6*(a^2*b^2*d^2*e^2 \\
& *f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 \\
& - 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*\cosh(d*x + \\
& c) - 4*(a^2*b^2*d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3 - (a*b^3* \\
& d^2*e^2*f - 2*a*b^3*c*d*e*f^2 + a*b^3*c^2*f^3)*\cosh(d*x + c))^3 - 3*(a^2*b^2 \\
& *d^2*e^2*f - 2*a^2*b^2*c*d*e*f^2 + a^2*b^2*c^2*f^3)*\cosh(d*x + c)^2 - ((2*a \\
& ^3*b - a*b^3)*d^2*e^2*f - 2*(2*a^3*b - a*b^3)*c*d*e*f^2 + (2*a^3*b - a*b^3) \\
& *c^2*f^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2})*\log(2*b*\cosh \\
& (d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 3*(2*(a^ \\
& 2*b^2 + b^4)*d*f^3*x + 2*(a^2*b^2 + b^4)*c*f^3 + 2*((a^2*b^2 + b^4)*d*f^3*x \\
& + (a^2*b^2 + b^4)*c*f^3)*\cosh(d*x + c)^4 + 2*((a^2*b^2 + b^4)*d*f^3*x + (a \\
& ^2*b^2 + b^4)*c*f^3)*\sinh(d*x + c)^4 + 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b \\
& + a*b^3)*c*f^3)*\cosh(d*x + c)^3 + 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b \\
& ^3)*c*f^3 + ((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3)*\cosh(d*x + c) \\
& )*\sinh(d*x + c)^3 + 4*((2*a^4 + a^2*b^2 - b^4)*d*f^3*x + (2*a^4 + a^2*b^2 - \\
& b^4)*c*f^3)*\cosh(d*x + c)^2 + 4*((2*a^4 + a^2*b^2 - b^4)*d*f^3*x + (2*a^4 \\
& + a^2*b^2 - b^4)*c*f^3 + 3*((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3 \\
& )*\cosh(d*x + c)^2 + 6*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3)*\cos \\
& h(d*x + c))*\sinh(d*x + c)^2 - 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)* \\
& c*f^3)*\cosh(d*x + c) - 8*((a^3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3 - \\
& ((a^2*b^2 + b^4)*d*f^3*x + (a^2*b^2 + b^4)*c*f^3)*\cosh(d*x + c))^3 - 3*((a^ \\
& 3*b + a*b^3)*d*f^3*x + (a^3*b + a*b^3)*c*f^3)*\cosh(d*x + c)^2 - ((2*a^4 + a \\
& ^2*b^2 - b^4)*d*f^3*x + (2*a^4 + a^2*b^2 - b^4)*c*f^3)*\cosh(d*x + c))*\sinh( \\
& d*x + c) + (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a \\
& *b^3*c^2*f^3 + (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 \\
& - a*b^3*c^2*f^3)*\cosh(d*x + c))^4 + (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2* \\
& x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3)*\sinh(d*x + c))^4 + 4*(a^2*b^2*d^2*f^3 \\
& *x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*\cosh( \\
& d*x + c)^3 + 4*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d \\
& *e*f^2 - a^2*b^2*c^2*f^3 + (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b \\
& ^3*c*d*e*f^2 - a*b^3*c^2*f^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*((2*a^3*b \\
& - a*b^3)*d^2*f^3*x^2 + 2*(2*a^3*b - a*b^3)*d^2*e*f^2*x + 2*(2*a^3*b - a*b^3) \\
& )*c*d*e*f^2 - (2*a^3*b - a*b^3)*c^2*f^3)*\cosh(d*x + c)^2 + 2*((2*a^3*b - a* \\
& b^3)*d^2*f^3*x^2 + 2*(2*a^3*b - a*b^3)*d^2*e*f^2*x + 2*(2*a^3*b - a*b^3)*c* \\
& d*e*f^2 - (2*a^3*b - a*b^3)*c^2*f^3 + 3*(a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e* \\
& f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^3)*\cosh(d*x + c))^2 + 6*(a^2*b^2*d^2 \\
& *f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*\c \\
& osh(d*x + c))*\sinh(d*x + c)^2 - 4*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^ \\
& 2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*\cosh(d*x + c) - 4*(a^2*b^2*d^2 \\
& *f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2*b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3 - \\
& (a*b^3*d^2*f^3*x^2 + 2*a*b^3*d^2*e*f^2*x + 2*a*b^3*c*d*e*f^2 - a*b^3*c^2*f^ \\
& 3)*\cosh(d*x + c))^3 - 3*(a^2*b^2*d^2*f^3*x^2 + 2*a^2*b^2*d^2*e*f^2*x + 2*a^2 \\
& *b^2*c*d*e*f^2 - a^2*b^2*c^2*f^3)*\cosh(d*x + c)^2 - ((2*a^3*b - a*b^3)*d^2* \\
& f^3*x^2 + 2*(2*a^3*b - a*b^3)*d^2*e*f^2*x + 2*(2*a^3*b - a*b^3)*c*d*e*f^2 - \\
& (2*a^3*b - a*b^3)*c^2*f^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/}
\end{aligned}$$

$$\begin{aligned}
& b^2)) * \log(- (a * \cosh(dx + c) + a * \sinh(dx + c) + (b * \cosh(dx + c) + b * \sinh(dx \\
& * x + c)) * \sqrt{((a^2 + b^2)/b^2) - b}/b) + 3 * (2 * (a^2 * b^2 + b^4) * d * f^3 * x + 2 * ( \\
& a^2 * b^2 + b^4) * c * f^3 + 2 * ((a^2 * b^2 + b^4) * d * f^3 * x + (a^2 * b^2 + b^4) * c * f^3) * \\
& \cosh(dx + c)^4 + 2 * ((a^2 * b^2 + b^4) * d * f^3 * x + (a^2 * b^2 + b^4) * c * f^3) * \sinh( \\
& dx + c)^4 + 8 * ((a^3 * b + a * b^3) * d * f^3 * x + (a^3 * b + a * b^3) * c * f^3) * \cosh(dx + \\
& c)^3 + 8 * ((a^3 * b + a * b^3) * d * f^3 * x + (a^3 * b + a * b^3) * c * f^3 + ((a^2 * b^2 + b^4) \\
& * d * f^3 * x + (a^2 * b^2 + b^4) * c * f^3) * \cosh(dx + c)) * \sinh(dx + c)^3 + 4 * ((2 * \\
& a^4 + a^2 * b^2 - b^4) * d * f^3 * x + (2 * a^4 + a^2 * b^2 - b^4) * c * f^3) * \cosh(dx + c) \\
& ^2 + 4 * ((2 * a^4 + a^2 * b^2 - b^4) * d * f^3 * x + (2 * a^4 + a^2 * b^2 - b^4) * c * f^3 + 3 \\
& * ((a^2 * b^2 + b^4) * d * f^3 * x + (a^2 * b^2 + b^4) * c * f^3) * \cosh(dx + c)^2 + 6 * ((a^3 \\
& * b + a * b^3) * d * f^3 * x + (a^3 * b + a * b^3) * c * f^3) * \cosh(dx + c)) * \sinh(dx + c)^2 \\
& - 8 * ((a^3 * b + a * b^3) * d * f^3 * x + (a^3 * b + a * b^3) * c * f^3) * \cosh(dx + c) - 8 * ( \\
& (a^3 * b + a * b^3) * d * f^3 * x + (a^3 * b + a * b^3) * c * f^3 - ((a^2 * b^2 + b^4) * d * f^3 * x \\
& + (a^2 * b^2 + b^4) * c * f^3) * \cosh(dx + c)^3 - 3 * ((a^3 * b + a * b^3) * d * f^3 * x + (a^3 \\
& * b + a * b^3) * c * f^3) * \cosh(dx + c)^2 - ((2 * a^4 + a^2 * b^2 - b^4) * d * f^3 * x + (2 \\
& * a^4 + a^2 * b^2 - b^4) * c * f^3) * \cosh(dx + c)) * \sinh(dx + c) - (a * b^3 * d^2 * f^3 * \\
& x^2 + 2 * a * b^3 * d^2 * e * f^2 * x + 2 * a * b^3 * c * d * e * f^2 - a * b^3 * c^2 * f^3 + (a * b^3 * d^2 * \\
& f^3 * x^2 + 2 * a * b^3 * d^2 * e * f^2 * x + 2 * a * b^3 * c * d * e * f^2 - a * b^3 * c^2 * f^3) * \cosh(dx \\
& + c)^4 + (a * b^3 * d^2 * f^3 * x^2 + 2 * a * b^3 * d^2 * e * f^2 * x + 2 * a * b^3 * c * d * e * f^2 - a \\
& b^3 * c^2 * f^3) * \sinh(dx + c)^4 + 4 * (a^2 * b^2 * d^2 * f^3 * x^2 + 2 * a^2 * b^2 * d^2 * e * f^2 \\
& * x + 2 * a^2 * b^2 * c * d * e * f^2 - a^2 * b^2 * c^2 * f^3) * \cosh(dx + c)^3 + 4 * (a^2 * b^2 * d^2 \\
& * f^3 * x^2 + 2 * a^2 * b^2 * d^2 * e * f^2 * x + 2 * a^2 * b^2 * c * d * e * f^2 - a^2 * b^2 * c^2 * f^3 + \\
& (a * b^3 * d^2 * f^3 * x^2 + 2 * a * b^3 * d^2 * e * f^2 * x + 2 * a * b^3 * c * d * e * f^2 - a * b^3 * c^2 * f \\
& ^3) * \cosh(dx + c)) * \sinh(dx + c)^3 + 2 * ((2 * a^3 * b - a * b^3) * d^2 * f^3 * x^2 + 2 * ( \\
& 2 * a^3 * b - a * b^3) * d^2 * e * f^2 * x + 2 * (2 * a^3 * b - a * b^3) * c * d * e * f^2 - (2 * a^3 * b - a \\
& * b^3) * c^2 * f^3) * \cosh(dx + c)^2 + 2 * ((2 * a^3 * b - a * b^3) * d^2 * f^3 * x^2 + 2 * (2 * a^3 \\
& * b - a * b^3) * d^2 * e * f^2 * x + 2 * (2 * a^3 * b - a * b^3) * c * d * e * f^2 - (2 * a^3 * b - a * b^3) \\
& * c^2 * f^3 + 3 * (a * b^3 * d^2 * f^3 * x^2 + 2 * a * b^3 * d^2 * e * f^2 * x + 2 * a * b^3 * c * d * e * f^2 \\
& - a * b^3 * c^2 * f^3) * \cosh(dx + c)^2 + 6 * (a^2 * b^2 * d^2 * f^3 * x^2 + 2 * a^2 * b^2 * d^2 * e \\
& * f^2 * x + 2 * a^2 * b^2 * c * d * e * f^2 - a^2 * b^2 * c^2 * f^3) * \cosh(dx + c)) * \sinh(dx + c) \\
& ^2 - 4 * (a^2 * b^2 * d^2 * f^3 * x^2 + 2 * a^2 * b^2 * d^2 * e * f^2 * x + 2 * a^2 * b^2 * c * d * e * f^2 \\
& - a^2 * b^2 * c^2 * f^3) * \cosh(dx + c) - 4 * (a^2 * b^2 * d^2 * f^3 * x^2 + 2 * a^2 * b^2 * d^2 * e \\
& * f^2 * x + 2 * a^2 * b^2 * c * d * e * f^2 - a^2 * b^2 * c^2 * f^3 - (a * b^3 * d^2 * f^3 * x^2 + 2 * a * b \\
& ^3 * d^2 * e * f^2 * x + 2 * a * b^3 * c * d * e * f^2 - a * b^3 * c^2 * f^3) * \cosh(dx + c)^3 - 3 * (a^2 \\
& * b^2 * d^2 * f^3 * x^2 + 2 * a^2 * b^2 * d^2 * e * f^2 * x + 2 * a^2 * b^2 * c * d * e * f^2 - a^2 * b^2 * c \\
& ^2 * f^3) * \cosh(dx + c)^2 - ((2 * a^3 * b - a * b^3) * d^2 * f^3 * x^2 + 2 * (2 * a^3 * b - a * b \\
& ^3) * d^2 * e * f^2 * x + 2 * (2 * a^3 * b - a * b^3) * c * d * e * f^2 - (2 * a^3 * b - a * b^3) * c^2 * f^3 \\
& ) * \cosh(dx + c)) * \sinh(dx + c)) * \sqrt{((a^2 + b^2)/b^2) * \log(- (a * \cosh(dx + c) \\
& + a * \sinh(dx + c) - (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{((a^2 + b^2)/ \\
& b^2) - b}/b) + 2 * (3 * (a^3 * b + a * b^3) * d^2 * f^3 * x^2 + 6 * (a^3 * b + a * b^3) * d^2 * e * f \\
& ^2 * x - 9 * (a^3 * b + a * b^3) * d^2 * e^2 * f + 24 * (a^3 * b + a * b^3) * c * d * e * f^2 - 12 * (a^3 \\
& * b + a * b^3) * c^2 * f^3 - 12 * ((a^2 * b^2 + b^4) * d^2 * f^3 * x^2 + 2 * (a^2 * b^2 + b^4) * d \\
& ^2 * e * f^2 * x + 2 * (a^2 * b^2 + b^4) * c * d * e * f^2 - (a^2 * b^2 + b^4) * c^2 * f^3) * \cosh(dx \\
& + c)^3 - 9 * (3 * (a^3 * b + a * b^3) * d^2 * f^3 * x^2 + 6 * (a^3 * b + a * b^3) * d^2 * e * f^2 * x \\
& - (a^3 * b + a * b^3) * d^2 * e^2 * f + 8 * (a^3 * b + a * b^3) * c * d * e * f^2 - 4 * (a^3 * b + a * b
\end{aligned}$$

$$\begin{aligned} &^3)*c^2*f^3)*\cosh(d*x + c)^2 - 2*(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 2 \\ &*(a^4 + 2*a^2*b^2 + b^4)*d^3*e^3 - 3*(2*a^4 + a^2*b^2 - b^4)*d^2*e^2*f + 12 \\ &*(2*a^4 + a^2*b^2 - b^4)*c*d*e*f^2 - 6*(2*a^4 + a^2*b^2 - b^4)*c^2*f^3 + 3* \\ &(2*(a^4 + 2*a^2*b^2 + b^4)*d^3*e*f^2 + (2*a^4 + a^2*b^2 - b^4)*d^2*f^3)*x^2 \\ &+ 6*((a^4 + 2*a^2*b^2 + b^4)*d^3*e^2*f + (2*a^4 + a^2*b^2 - b^4)*d^2*e*f^2 \\ &)*x)*\cosh(d*x + c))*\sinh(d*x + c))/((a^4*b^3 + 2*a^2*b^5 + b^7)*d^4*\cosh(d* \\ &x + c)^4 + (a^4*b^3 + 2*a^2*b^5 + b^7)*d^4*\sinh(d*x + c)^4 + 4*(a^5*b^2 + 2 \\ &*a^3*b^4 + a*b^6)*d^4*\cosh(d*x + c)^3 + 2*(2*a^6*b + 3*a^4*b^3 - b^7)*d^4*c \\ &osh(d*x + c)^2 - 4*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^4*\cosh(d*x + c) + (a^4*b \\ &^3 + 2*a^2*b^5 + b^7)*d^4 + 4*((a^4*b^3 + 2*a^2*b^5 + b^7)*d^4*\cosh(d*x + c \\ &)) + (a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^4)*\sinh(d*x + c)^3 + 2*(3*(a^4*b^3 + 2* \\ &a^2*b^5 + b^7)*d^4*\cosh(d*x + c)^2 + 6*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*d^4*co \\ &sh(d*x + c) + (2*a^6*b + 3*a^4*b^3 - b^7)*d^4)*\sinh(d*x + c)^2 + 4*((a^4*b^ \\ &3 + 2*a^2*b^5 + b^7)*d^4*\cosh(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)* \\ &d^4*\cosh(d*x + c)^2 + (2*a^6*b + 3*a^4*b^3 - b^7)*d^4*\cosh(d*x + c) - (a^5* \\ &b^2 + 2*a^3*b^4 + a*b^6)*d^4)*\sinh(d*x + c)) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c)}{(b \sinh(dx + c) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)/(a+b\*sinh(d\*x+c))^3,x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*cosh(d\*x + c)/(b\*sinh(d\*x + c) + a)^3, x)

$$3.333 \quad \int \frac{(e+fx)^3 \cosh(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=448

$$\frac{6af^2(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3} + \frac{6af^2(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^3} - \frac{3af(e+fx)^2\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}$$

[Out] (a\*(e + f\*x)^4)/(4\*b^2\*f) - (6\*f^3\*Cosh[c + d\*x])/(b\*d^4) - (3\*f\*(e + f\*x)^2\*Cosh[c + d\*x])/(b\*d^2) - (a\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b^2\*d) - (a\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b^2\*d) - (3\*a\*f\*(e + f\*x)^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^2\*d^2) - (3\*a\*f\*(e + f\*x)^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^2\*d^2) + (6\*a\*f^2\*(e + f\*x)\*PolyLog[3, -(b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b^2\*d^3) + (6\*a\*f^2\*(e + f\*x)\*PolyLog[3, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b^2\*d^3) - (6\*a\*f^3\*PolyLog[4, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^2\*d^4) - (6\*a\*f^3\*PolyLog[4, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^2\*d^4) + (6\*f^2\*(e + f\*x)\*Sinh[c + d\*x])/(b\*d^3) + ((e + f\*x)^3\*Sinh[c + d\*x])/(b\*d)

**Rubi [A]** time = 0.646703, antiderivative size = 448, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$ , Rules used = {5579, 3296, 2638, 5561, 2190, 2531, 6609, 2282, 6589}

$$\frac{6af^2(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3} + \frac{6af^2(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^3} - \frac{3af(e+fx)^2\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (a\*(e + f\*x)^4)/(4\*b^2\*f) - (6\*f^3\*Cosh[c + d\*x])/(b\*d^4) - (3\*f\*(e + f\*x)^2\*Cosh[c + d\*x])/(b\*d^2) - (a\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b^2\*d) - (a\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b^2\*d) - (3\*a\*f\*(e + f\*x)^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^2\*d^2) - (3\*a\*f\*(e + f\*x)^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^2\*d^2) + (6\*a\*f^2\*(e + f\*x)\*PolyLog[3, -(b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b^2\*d^3) + (6\*a\*f^2\*(e + f\*x)\*PolyLog[3, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b^2\*d^3) - (6\*a\*f^3\*PolyLog[4, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^2\*d^4) - (6\*a\*f^3\*PolyLog[4, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^2\*d^4) + (6\*f^2\*(e +



$f*x)*\text{Sinh}[c + d*x]/(b*d^3) + ((e + f*x)^3*\text{Sinh}[c + d*x])/(b*d)$

### Rule 5579

$\text{Int}[(\text{Cosh}[c_.] + (d_.)*(x_)]^{(p_.)}*((e_.) + (f_.)*(x_))^{(m_.)}*\text{Sinh}[(c_.) + (d_.)*(x_)]^{(n_.)})/((a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_)]), x\_Symbol] :> \text{Dist}[1/b, \text{Int}[(e + f*x)^m*\text{Cosh}[c + d*x]^p*\text{Sinh}[c + d*x]^{(n - 1)}, x], x] - \text{Dist}[a/b, \text{Int}[(e + f*x)^m*\text{Cosh}[c + d*x]^p*\text{Sinh}[c + d*x]^{(n - 1)})/(a + b*\text{Sinh}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \text{IGtQ}[m, 0] \ \&\& \text{IGtQ}[n, 0] \ \&\& \text{IGtQ}[p, 0]$

### Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_)]^{(m_.)}*\sin[(e_.) + (f_.)*(x_)], x\_Symbol] :> -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m - 1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \text{GtQ}[m, 0]$

### Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_)], x\_Symbol] :> -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}\{c, d\}, x]$

### Rule 5561

$\text{Int}[(\text{Cosh}[c_.] + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^{(m_.)})/((a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_)]), x\_Symbol] :> -\text{Simp}[(e + f*x)^{(m + 1)}/(b*f*(m + 1)), x] + (\text{Int}[(e + f*x)^m*\text{E}^{(c + d*x)})/(a - \text{Rt}[a^2 + b^2, 2] + b*\text{E}^{(c + d*x)}), x] + \text{Int}[(e + f*x)^m*\text{E}^{(c + d*x)})/(a + \text{Rt}[a^2 + b^2, 2] + b*\text{E}^{(c + d*x)}), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \text{IGtQ}[m, 0] \ \&\& \text{NeQ}[a^2 + b^2, 0]$

### Rule 2190

$\text{Int}[(F_.)^{((g_.)*((e_.) + (f_.)*(x_)))}((c_.) + (d_.)*(x_))^{(m_.)})/((a_.) + (b_.)*(F_.)^{((g_.)*((e_.) + (f_.)*(x_)))})), x\_Symbol] :> \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x \ \&\& \text{IGtQ}[m, 0]$

### Rule 2531

$\text{Int}[\text{Log}[1 + (e_.)*(F_.)^{((c_.)*((a_.) + (b_.)*(x_)))}]), x\_Symbol] :> -\text{Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n})]/(b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{(m - 1)}*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n})], x], x] /; \text{FreeQ}\{F, a, b, c, e, f$

, g, n}, x] && GtQ[m, 0]

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \cosh(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \cosh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^3 \cosh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{a(e+fx)^4}{4b^2 f} + \frac{(e+fx)^3 \sinh(c+dx)}{bd} - \frac{a \int \frac{e^{c+dx}(e+fx)^3}{a-\sqrt{a^2+b^2}+be^{c+dx}} dx}{b} - \frac{a \int \frac{e^{c+dx}(e+fx)^3}{a+\sqrt{a^2+b^2}+be^{c+dx}} dx}{b} \\
&= \frac{a(e+fx)^4}{4b^2 f} - \frac{3f(e+fx)^2 \cosh(c+dx)}{bd^2} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2 d} \\
&= \frac{a(e+fx)^4}{4b^2 f} - \frac{3f(e+fx)^2 \cosh(c+dx)}{bd^2} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2 d} \\
&= \frac{a(e+fx)^4}{4b^2 f} - \frac{6f^3 \cosh(c+dx)}{bd^4} - \frac{3f(e+fx)^2 \cosh(c+dx)}{bd^2} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2 d} \\
&= \frac{a(e+fx)^4}{4b^2 f} - \frac{6f^3 \cosh(c+dx)}{bd^4} - \frac{3f(e+fx)^2 \cosh(c+dx)}{bd^2} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2 d} \\
&= \frac{a(e+fx)^4}{4b^2 f} - \frac{6f^3 \cosh(c+dx)}{bd^4} - \frac{3f(e+fx)^2 \cosh(c+dx)}{bd^2} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2 d}
\end{aligned}$$

**Mathematica [B]** time = 20.9593, size = 10378, normalized size = 23.17

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^3\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x  
]

[Out] Result too large to show

**Maple [F]** time = 0.198, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 \cosh(dx+c) \sinh(dx+c)}{a+b \sinh(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)`

[Out] `int((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2} e^3 \left( \frac{2(dx+c)a}{b^2d} - \frac{e^{(dx+c)}}{bd} + \frac{e^{(-dx-c)}}{bd} + \frac{2a \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{b^2d} \right) - \frac{(ad^4 f^3 x^4 e^c + 4ad^4 e f^2 x^3 e^c + 6ad^4 e^2 f x^2 e^c + 4ad^4 e^3 x e^c + 6ad^4 e^4 e^c)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `-1/2*e^3*(2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d) + 2*a*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^2*d)) - 1/4*(a*d^4*f^3*x^4*e^c + 4*a*d^4*e*f^2*x^3*e^c + 6*a*d^4*e^2*f*x^2*e^c - 2*(b*d^3*f^3*x^3*e^(2*c) + 3*(d^3*e*f^2 - d^2*f^3)*b*x^2*e^(2*c) + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b*x*e^(2*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*b*e^(2*c))*e^(d*x) + 2*(b*d^3*f^3*x^3 + 3*(d^3*e*f^2 + d^2*f^3)*b*x^2 + 3*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*b*x + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*b)*e^(-d*x) + integrate(-2*(a*b*f^3*x^3 + 3*a*b*e*f^2*x^2 + 3*a*b*e^2*f*x - (a^2*f^3*x^3*e^c + 3*a^2*e*f^2*x^2*e^c + 3*a^2*e^2*f*x*e^c)*e^(d*x))/(b^3*e^(2*d*x + 2*c) + 2*a*b^2*e^(d*x + c) - b^3), x)`

**Fricas [C]** time = 2.77144, size = 4582, normalized size = 10.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] `-1/4*(2*b*d^3*f^3*x^3 + 2*b*d^3*e^3 + 6*b*d^2*e^2*f + 12*b*d*e*f^2 + 12*b*f^3 + 6*(b*d^3*e*f^2 + b*d^2*f^3)*x^2 - 2*(b*d^3*f^3*x^3 + b*d^3*e^3 - 3*b*d^2*e^2*f + 6*b*d*e*f^2 - 6*b*f^3 + 3*(b*d^3*e*f^2 - b*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f - 2*b*d^2*e*f^2 + 2*b*d*f^3)*x)*cosh(d*x + c)^2 - 2*(b*d^3*f^3*x^3 + b*d^3*e^3 - 3*b*d^2*e^2*f + 6*b*d*e*f^2 - 6*b*f^3 + 3*(b*d^3*e*f^2 - b*d^2*f^3)*x)*sinh(d*x + c)^2`

$$\begin{aligned}
& ^2*f^3)*x^2 + 3*(b*d^3*e^2*f - 2*b*d^2*e*f^2 + 2*b*d*f^3)*x)*\sinh(d*x + c)^2 \\
& + 6*(b*d^3*e^2*f + 2*b*d^2*e*f^2 + 2*b*d*f^3)*x - (a*d^4*f^3*x^4 + 4*a*d^4 \\
& *e*f^2*x^3 + 6*a*d^4*e^2*f*x^2 + 4*a*d^4*e^3*x + 8*a*c*d^3*e^3 - 12*a*c^2*d^2 \\
& *e^2*f + 8*a*c^3*d*e*f^2 - 2*a*c^4*f^3)*\cosh(d*x + c) + 12*((a*d^2*f^3*x^2 \\
& + 2*a*d^2*e*f^2*x + a*d^2*e^2*f)*\cosh(d*x + c) + (a*d^2*f^3*x^2 + 2*a*d^2 \\
& *e*f^2*x + a*d^2*e^2*f)*\sinh(d*x + c))*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x \\
& + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + \\
& 1) + 12*((a*d^2*f^3*x^2 + 2*a*d^2*e*f^2*x + a*d^2*e^2*f)*\cosh(d*x + c) + (a \\
& *d^2*f^3*x^2 + 2*a*d^2*e*f^2*x + a*d^2*e^2*f)*\sinh(d*x + c))*\operatorname{dilog}((a*\cosh( \\
& d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 \\
& + b^2)/b^2} - b)/b + 1) + 4*((a*d^3*e^3 - 3*a*c*d^2*e^2*f + 3*a*c^2*d*e*f^2 \\
& - a*c^3*f^3)*\cosh(d*x + c) + (a*d^3*e^3 - 3*a*c*d^2*e^2*f + 3*a*c^2*d*e*f^2 \\
& - a*c^3*f^3)*\sinh(d*x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2 \\
& *b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 4*((a*d^3*e^3 - 3*a*c*d^2*e^2*f + 3*a*c^2 \\
& *d*e*f^2 - a*c^3*f^3)*\cosh(d*x + c) + (a*d^3*e^3 - 3*a*c*d^2*e^2*f + 3*a*c^2 \\
& *d*e*f^2 - a*c^3*f^3)*\sinh(d*x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x \\
& + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 4*((a*d^3*f^3*x^3 + 3*a*d^3*e*f^2 \\
& *x^2 + 3*a*d^3*e^2*f*x + 3*a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*\cos \\
& h(d*x + c) + (a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + 3*a*c*d^2 \\
& *e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*\sinh(d*x + c))*\log(-(a*\cosh(d*x + c) \\
& ) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/ \\
& b^2} - b)/b) + 4*((a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + 3* \\
& a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*\cosh(d*x + c) + (a*d^3*f^3*x^3 \\
& + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + 3*a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 \\
& + a*c^3*f^3)*\sinh(d*x + c))*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*co \\
& sh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + 24*(a*f^3*co \\
& sh(d*x + c) + a*f^3*\sinh(d*x + c))*\operatorname{polylog}(4, (a*\cosh(d*x + c) + a*\sinh(d*x \\
& + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) + 24* \\
& (a*f^3*\cosh(d*x + c) + a*f^3*\sinh(d*x + c))*\operatorname{polylog}(4, (a*\cosh(d*x + c) + a \\
& *\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/ \\
& /b) - 24*((a*d*f^3*x + a*d*e*f^2)*\cosh(d*x + c) + (a*d*f^3*x + a*d*e*f^2)*\s \\
& inh(d*x + c))*\operatorname{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + \\
& c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) - 24*((a*d*f^3*x + a*d*e*f \\
& ^2)*\cosh(d*x + c) + (a*d*f^3*x + a*d*e*f^2)*\sinh(d*x + c))*\operatorname{polylog}(3, (a*co \\
& sh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a \\
& ^2 + b^2)/b^2}))/b) - (a*d^4*f^3*x^4 + 4*a*d^4*e*f^2*x^3 + 6*a*d^4*e^2*f*x^2 \\
& + 4*a*d^4*e^3*x + 8*a*c*d^3*e^3 - 12*a*c^2*d^2*e^2*f + 8*a*c^3*d*e*f^2 - 2 \\
& *a*c^4*f^3 + 4*(b*d^3*f^3*x^3 + b*d^3*e^3 - 3*b*d^2*e^2*f + 6*b*d*e*f^2 - 6 \\
& *b*f^3 + 3*(b*d^3*e*f^2 - b*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f - 2*b*d^2*e*f^2 + \\
& 2*b*d*f^3)*x)*\cosh(d*x + c))*\sinh(d*x + c))/(b^2*d^4*\cosh(d*x + c) + b^2*d \\
& ^4*\sinh(d*x + c))
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cosh(d\*x+c)\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c) \sinh(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*cosh(d\*x + c)\*sinh(d\*x + c)/(b\*sinh(d\*x + c) + a), x)

$$3.334 \quad \int \frac{(e+fx)^2 \cosh(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=330

$$\frac{2af(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2} - \frac{2af(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2} + \frac{2af^2\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3} + \frac{2af^2\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^3}$$

[Out] (a\*(e + f\*x)^3)/(3\*b^2\*f) - (2\*f\*(e + f\*x)\*Cosh[c + d\*x])/(b\*d^2) - (a\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b^2\*d) - (a\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b^2\*d) - (2\*a\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^2\*d^2) - (2\*a\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^2\*d^2) + (2\*a\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^2\*d^3) + (2\*a\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^2\*d^3) + (2\*f^2\*Sinh[c + d\*x])/(b\*d^3) + ((e + f\*x)^2\*Sinh[c + d\*x])/(b\*d)

**Rubi [A]** time = 0.549545, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5579, 3296, 2637, 5561, 2190, 2531, 2282, 6589}

$$\frac{2af(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^2} - \frac{2af(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^2} + \frac{2af^2\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2d^3} + \frac{2af^2\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2d^3}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (a\*(e + f\*x)^3)/(3\*b^2\*f) - (2\*f\*(e + f\*x)\*Cosh[c + d\*x])/(b\*d^2) - (a\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b^2\*d) - (a\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b^2\*d) - (2\*a\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^2\*d^2) - (2\*a\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^2\*d^2) + (2\*a\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^2\*d^3) + (2\*a\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^2\*d^3) + (2\*f^2\*Sinh[c + d\*x])/(b\*d^3) + ((e + f\*x)^2\*Sinh[c + d\*x])/(b\*d)

**Rule 5579**

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]^(n\_.)]/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := D

```
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)])*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
```



```
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \cosh(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \cosh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^2 \cosh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{a(e+fx)^3}{3b^2 f} + \frac{(e+fx)^2 \sinh(c+dx)}{bd} - \frac{a \int \frac{e^{c+dx}(e+fx)^2}{a-\sqrt{a^2+b^2}+be^{c+dx}} dx}{b} - \frac{a \int \frac{e^{c+dx}(e+fx)^2}{a+\sqrt{a^2+b^2}+be^{c+dx}} dx}{b} \\
&= \frac{a(e+fx)^3}{3b^2 f} - \frac{2f(e+fx) \cosh(c+dx)}{bd^2} - \frac{a(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d} - \frac{a(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2 d} \\
&= \frac{a(e+fx)^3}{3b^2 f} - \frac{2f(e+fx) \cosh(c+dx)}{bd^2} - \frac{a(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d} - \frac{a(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2 d} \\
&= \frac{a(e+fx)^3}{3b^2 f} - \frac{2f(e+fx) \cosh(c+dx)}{bd^2} - \frac{a(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d} - \frac{a(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2 d} \\
&= \frac{a(e+fx)^3}{3b^2 f} - \frac{2f(e+fx) \cosh(c+dx)}{bd^2} - \frac{a(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d} - \frac{a(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2 d}
\end{aligned}$$

**Mathematica [B]** time = 14.2168, size = 1301, normalized size = 3.94

$$\frac{1}{2} \left( 2a \left( 2e^{2c} f^2 x^3 + 6e^{2c} f x^2 - \frac{3e^{2c} f^2 \log\left(\frac{e^{2c+dx} b}{ae^c - \sqrt{(a^2+b^2)e^{2c}} + 1}\right) x^2}{d} + \frac{3f^2 \log\left(\frac{e^{2c+dx} b}{ae^c - \sqrt{(a^2+b^2)e^{2c}} + 1}\right) x^2}{d} - \frac{3e^{2c} f^2 \log\left(\frac{e^{2c+dx} b}{e^c a + \sqrt{(a^2+b^2)e^{2c}} + 1}\right) x^2}{d} + \frac{3f^2 \log\left(\frac{e^{2c+dx} b}{e^c a + \sqrt{(a^2+b^2)e^{2c}} + 1}\right) x^2}{d} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((2*a*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))]/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))]/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]]])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]]])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]]])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]]])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]]])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]]])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]]])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]]])/d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]])))/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]])))/d^2 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]])))/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]])))/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]])))/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]])))/d^3)/(3*b^2*(-1 + E^(2*c))) - (a*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Cosh[c]*Csch[c/2]*Sech[c/2])/(3*b^2) + (2*Cosh[d*x]*(-2*d*e*f*Cosh[c] - 2*d*f^2*x*Cosh[c] + d^2*e^2*Sinh[c] + 2*f^2*Sinh[c] + 2*d^2*e*f*x*Sinh[c] + d^2*f^2*x^2*Sinh[c]))/(b*d^3) + (2*(d^2*e^2*Cosh[c] + 2*f^2*Cosh[c] + 2*d^2*e*f*x*Cosh[c] + d^2*f^2*x^2*Cosh[c] - 2*d*e*f*Sinh[c] - 2*d*f^2*x*Sinh[c])*Sinh[d*x])/(b*d^3))/2
```

---

**Maple [F]** time = 0.126, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c) \sinh(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

[Out]  $\int ((f*x+e)^2*\cosh(d*x+c)*\sinh(d*x+c)/(a+b*\sinh(d*x+c)),x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}e^2\left(\frac{2(dx+c)a}{b^2d} - \frac{e^{(dx+c)}}{bd} + \frac{e^{(-dx-c)}}{bd} + \frac{2a\log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{b^2d}\right) - \frac{(2ad^3f^2x^3e^c + 6ad^3efx^2e^c - 3(bd^2f$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x+e)^2*\cosh(d*x+c)*\sinh(d*x+c)/(a+b*\sinh(d*x+c)),x, \text{algorithm}="maxima")$

[Out]  $-1/2*e^2*(2*(d*x + c)*a/(b^2*d) - e^{(d*x + c)/(b*d)} + e^{(-d*x - c)/(b*d)} + 2*a*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/(b^2*d)) - 1/6*(2*a*d^3*f^2*x^3*e^c + 6*a*d^3*e*f*x^2*e^c - 3*(b*d^2*f^2*x^2*e^{(2*c)} + 2*(d^2*e*f - d*f^2)*b*x*e^{(2*c)} - 2*(d*e*f - f^2)*b*e^{(2*c)})*e^{(d*x)} + 3*(b*d^2*f^2*x^2 + 2*(d^2*e*f + d*f^2)*b*x + 2*(d*e*f + f^2)*b)*e^{(-d*x)}*e^{(-c)}/(b^2*d^3) + \text{integrate}(-2*(a*b*f^2*x^2 + 2*a*b*e*f*x - (a^2*f^2*x^2*e^c + 2*a^2*e*f*x*e^c)*e^{(d*x)})/(b^3*e^{(2*d*x + 2*c)} + 2*a*b^2*e^{(d*x + c)} - b^3), x)$

**Fricas [C]** time = 2.47519, size = 3051, normalized size = 9.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x+e)^2*\cosh(d*x+c)*\sinh(d*x+c)/(a+b*\sinh(d*x+c)),x, \text{algorithm}="fricas")$

[Out]  $-1/6*(3*b*d^2*f^2*x^2 + 3*b*d^2*e^2 + 6*b*d*e*f + 6*b*f^2 - 3*(b*d^2*f^2*x^2 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*f - b*d*f^2)*x)*\cosh(d*x + c)^2 - 3*(b*d^2*f^2*x^2 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*f - b*d*f^2)*x)*\sinh(d*x + c)^2 + 6*(b*d^2*e*f + b*d*f^2)*x - 2*(a*d^3*f^2*x^3 + 3*a*d^3*e*f*x^2 + 3*a*d^3*e^2*x + 6*a*c*d^2*e^2 - 6*a*c^2*d*e*f + 2*a*c^3*f^2)*\cosh(d*x + c) + 12*((a*d*f^2*x + a*d*e*f)*\cosh(d*x + c) + (a*d*f^2*x + a*d*e*f)*\sinh(d*x + c))*\text{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\text{sqrt}((a^2 + b^2)/b^2) - b)/b + 1) + 12*((a*d*f^2*x + a*d*e*f)*\cosh(d*x + c) + (a*d*f^2*x + a*d*e*f)*\sinh(d*x + c))*\text{dilo}$

```

g((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 6*((a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2
)*cosh(d*x + c) + (a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2)*sinh(d*x + c))*log(
2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) +
6*((a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2)*cosh(d*x + c) + (a*d^2*e^2 - 2*a*c
*d*e*f + a*c^2*f^2)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c
) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 6*((a*d^2*f^2*x^2 + 2*a*d^2*e*f*x +
2*a*c*d*e*f - a*c^2*f^2)*cosh(d*x + c) + (a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + 2
*a*c*d*e*f - a*c^2*f^2)*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x +
c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 6
*((a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*cosh(d*x + c) +
(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*sinh(d*x + c))*l
og(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c)
))*sqrt((a^2 + b^2)/b^2) - b)/b) - 12*(a*f^2*cosh(d*x + c) + a*f^2*sinh(d*x
+ c))*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*
sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 12*(a*f^2*cosh(d*x + c) + a*f^2*
sinh(d*x + c))*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x
+ c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*(a*d^3*f^2*x^3 + 3*a*
d^3*e*f*x^2 + 3*a*d^3*e^2*x + 6*a*c*d^2*e^2 - 6*a*c^2*d*e*f + 2*a*c^3*f^2 +
3*(b*d^2*f^2*x^2 + b*d^2*e^2 - 2*b*d*e*f + 2*b*f^2 + 2*(b*d^2*e*f - b*d*f^
2)*x)*cosh(d*x + c))*sinh(d*x + c))/(b^2*d^3*cosh(d*x + c) + b^2*d^3*sinh(d
*x + c))

```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cosh(d\*x+c)\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c) \sinh(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

$$3.335 \quad \int \frac{(e+fx) \cosh(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=212

$$\frac{af \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d^2} - \frac{af \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2+a}}\right)}{b^2 d^2} - \frac{a(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{b^2 d} - \frac{a(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2+a}} + 1\right)}{b^2 d}$$

[Out] (a\*(e + f\*x)^2)/(2\*b^2\*f) - (f\*Cosh[c + d\*x])/(b\*d^2) - (a\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b^2\*d) - (a\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b^2\*d) - (a\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^2\*d^2) - (a\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^2\*d^2) + ((e + f\*x)\*Sinh[c + d\*x])/(b\*d)

**Rubi [A]** time = 0.312137, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$ , Rules used = {5579, 3296, 2638, 5561, 2190, 2279, 2391}

$$\frac{af \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d^2} - \frac{af \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2+a}}\right)}{b^2 d^2} - \frac{a(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{b^2 d} - \frac{a(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2+a}} + 1\right)}{b^2 d}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(a + b\*Sinh[c + d\*x]), x]

[Out] (a\*(e + f\*x)^2)/(2\*b^2\*f) - (f\*Cosh[c + d\*x])/(b\*d^2) - (a\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b^2\*d) - (a\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b^2\*d) - (a\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^2\*d^2) - (a\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^2\*d^2) + ((e + f\*x)\*Sinh[c + d\*x])/(b\*d)

**Rule 5579**

Int[(Cosh[(c\_.) + (d\_.)\*(x\_.)]^(p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]^(n\_.)]/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] :> Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^p\*Sinh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Cosh[c + d\*x]^p\*Sinh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

**Rule 3296**

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)])*((e_.) + (f_.)*(x_))^(m_.)/((a_.) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx) \cosh(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx) \cosh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx) \cosh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{a(e+fx)^2}{2b^2 f} + \frac{(e+fx) \sinh(c+dx)}{bd} - \frac{a \int \frac{e^{c+dx}(e+fx)}{a-\sqrt{a^2+b^2+be^{c+dx}}} dx}{b} - \frac{a \int \frac{e^{c+dx}(e+fx)}{a+\sqrt{a^2+b^2+be^{c+dx}}} dx}{b} \\
&= \frac{a(e+fx)^2}{2b^2 f} - \frac{f \cosh(c+dx)}{bd^2} - \frac{a(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d} - \frac{a(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2 d} \\
&= \frac{a(e+fx)^2}{2b^2 f} - \frac{f \cosh(c+dx)}{bd^2} - \frac{a(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d} - \frac{a(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2 d} \\
&= \frac{a(e+fx)^2}{2b^2 f} - \frac{f \cosh(c+dx)}{bd^2} - \frac{a(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d} - \frac{a(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^2 d}
\end{aligned}$$

**Mathematica [A]** time = 1.05742, size = 206, normalized size = 0.97

$$\frac{-a \left( f \text{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) + f \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) + f(c+dx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right) + f(c+dx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} + 1\right) \right)}{b^2 d^2}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out]  $(-(b*f*Cosh[c + d*x]) - a*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]) + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]]) + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]) + b*d*(e + f*x)*Sinh[c + d*x])/(b^2*d^2)$

**Maple [B]** time = 0.064, size = 483, normalized size = 2.3

$$\frac{afx^2}{2b^2} - \frac{aex}{b^2} + \frac{(dfx + de - f)e^{dx+c}}{2d^2b} - \frac{(dfx + de + f)e^{-dx-c}}{2d^2b} + \frac{afc \ln(b e^{2dx+2c} + 2ae^{dx+c} - b)}{b^2 d^2} - 2 \frac{afc \ln(e^{dx+c})}{b^2 d^2} - \frac{ae \ln(e^{dx+c})}{b^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)`

[Out]  $\frac{1}{2}a^2f^2x^2/b^2 - a^2ex/b^2 + \frac{1}{2}(df^2x + d^2e - f^2)/d^2/b \exp(dx+c) - \frac{1}{2}(df^2x + d^2e + f^2)/d^2/b \exp(-dx-c) + a/b^2/d^2f^2c \ln(b \exp(2dx+2c) + 2a \exp(dx+c) - b) - 2a/b^2/d^2f^2c \ln(\exp(dx+c)) - a/b^2/d^2f^2c \ln(b \exp(2dx+2c) + 2a \exp(dx+c) - b) + 2a/b^2/d^2f^2c \ln(\exp(dx+c)) - a/b^2/d^2f^2c \ln((-b \exp(dx+c) + (a^2+b^2)^{1/2}) - a) / (-a + (a^2+b^2)^{1/2}) * x - a/b^2/d^2f^2c \ln((-b \exp(dx+c) + (a^2+b^2)^{1/2}) - a) / (-a + (a^2+b^2)^{1/2}) * c - a/b^2/d^2f^2c \ln((b \exp(dx+c) + (a^2+b^2)^{1/2}) + a) / (a + (a^2+b^2)^{1/2}) * x - a/b^2/d^2f^2c \ln((b \exp(dx+c) + (a^2+b^2)^{1/2}) + a) / (a + (a^2+b^2)^{1/2}) * c - a/b^2/d^2f^2c \operatorname{dilog}((b \exp(dx+c) + (a^2+b^2)^{1/2}) + a) / (a + (a^2+b^2)^{1/2}) - a/b^2/d^2f^2c \operatorname{dilog}((-b \exp(dx+c) + (a^2+b^2)^{1/2}) - a) / (-a + (a^2+b^2)^{1/2}) + 2a/b^2/d^2f^2cx + a/b^2/d^2f^2c^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}e^{\left(\frac{2(dx+c)a}{b^2d} - \frac{e^{(dx+c)}}{bd} + \frac{e^{(-dx-c)}}{bd} + \frac{2a \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{b^2d}\right)} - \frac{1}{4}f^2 \left(\frac{2(ad^2x^2e^c - (bdxe^{(2c)} - be^{(2c)})e^{(d)}}{b^2d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out]  $-1/2e^*(2*(d*x + c)*a/(b^2*d) - e^{(d*x + c)/(b*d)} + e^{(-d*x - c)/(b*d)} + 2*a*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/(b^2*d)) - 1/4*f*(2*(a*d^2*x^2*e^c - (b*d*x*e^{(2*c)} - b*e^{(2*c)})*e^{(d*x)} + (b*d*x + b)*e^{(-d*x)})*e^{(-c)/(b^2*d^2)} - \operatorname{integrate}(8*(a^2*x*e^{(d*x + c)} - a*b*x)/(b^3*e^{(2*d*x + 2*c)} + 2*a*b^2*e^{(d*x + c)} - b^3), x))$

**Fricas [B]** time = 2.27905, size = 1760, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out]  $-1/2*(b*d*f*x + b*d*e - (b*d*f*x + b*d*e - b*f)*\cosh(d*x + c)^2 - (b*d*f*x + b*d*e - b*f)*\sinh(d*x + c)^2 + b*f - (a*d^2*f*x^2 + 2*a*d^2*e*x + 4*a*c*d$

```
*e - 2*a*c^2*f)*cosh(d*x + c) + 2*(a*f*cosh(d*x + c) + a*f*sinh(d*x + c))*d
ilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a*f*cosh(d*x + c) + a*f*sinh(d*x
+ c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(
d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*((a*d*e - a*c*f)*cosh(d*x +
c) + (a*d*e - a*c*f)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((a*d*e - a*c*f)*cosh(d*x + c) +
(a*d*e - a*c*f)*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) -
2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*((a*d*f*x + a*c*f)*cosh(d*x + c) + (a
*d*f*x + a*c*f)*sinh(d*x + c))*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*((a*d*f
*x + a*c*f)*cosh(d*x + c) + (a*d*f*x + a*c*f)*sinh(d*x + c))*log(-(a*cosh(
d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b) - (a*d^2*f*x^2 + 2*a*d^2*e*x + 4*a*c*d*e - 2*a*c^2*f + 2
*(b*d*f*x + b*d*e - b*f)*cosh(d*x + c))*sinh(d*x + c))/(b^2*d^2*cosh(d*x +
c) + b^2*d^2*sinh(d*x + c))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \cosh(dx + c) \sinh(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

$$3.336 \quad \int \frac{\cosh(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=34

$$\frac{\sinh(c+dx)}{bd} - \frac{a \log(a+b \sinh(c+dx))}{b^2d}$$

[Out]  $-\left(\frac{a \log[a + b \sinh[c + d x]]}{b^2 d}\right) + \frac{\sinh[c + d x]}{b d}$

**Rubi [A]** time = 0.0569536, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$ , Rules used = {2833, 12, 43}

$$\frac{\sinh(c+dx)}{bd} - \frac{a \log(a+b \sinh(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Cosh}[c + d x] * \text{Sinh}[c + d x]) / (a + b * \text{Sinh}[c + d x]), x]$

[Out]  $-\left(\frac{a \log[a + b \sinh[c + d x]]}{b^2 d}\right) + \frac{\sinh[c + d x]}{b d}$

### Rule 2833

$\text{Int}[\cos[(e_.) + (f_.)(x_.)] * ((a_.) + (b_.) * \sin[(e_.) + (f_.)(x_.)])^{(m_.)} * ((c_.) + (d_.) * \sin[(e_.) + (f_.)(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[1/(b*f), \text{Subst}[\text{Int}[(a + x)^m * (c + (d*x)/b)^n, x], x, b * \text{Sin}[e + f*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x]$

### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

### Rule 43

$\text{Int}[((a_.) + (b_.)(x_.))^{(m_.)} * ((c_.) + (d_.)(x_.))^{(n_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(c+dx)\sinh(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{x}{b(a+x)} dx, x, b\sinh(c+dx)\right)}{bd} \\
&= \frac{\text{Subst}\left(\int \frac{x}{a+x} dx, x, b\sinh(c+dx)\right)}{b^2d} \\
&= \frac{\text{Subst}\left(\int \left(1 - \frac{a}{a+x}\right) dx, x, b\sinh(c+dx)\right)}{b^2d} \\
&= -\frac{a \log(a+b\sinh(c+dx))}{b^2d} + \frac{\sinh(c+dx)}{bd}
\end{aligned}$$

**Mathematica [A]** time = 0.0386967, size = 33, normalized size = 0.97

$$-\frac{\frac{a \log(a+b\sinh(c+dx))}{b^2} - \frac{\sinh(c+dx)}{b}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cosh[c + d\*x]\*Sinh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] -(((a\*Log[a + b\*Sinh[c + d\*x]])/b^2 - Sinh[c + d\*x]/b)/d)

**Maple [A]** time = 0.013, size = 35, normalized size = 1.

$$-\frac{a \ln(a+b\sinh(dx+c))}{b^2d} + \frac{\sinh(dx+c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] -a\*ln(a+b\*sinh(d\*x+c))/b^2/d+sinh(d\*x+c)/b/d

**Maxima [B]** time = 1.07533, size = 112, normalized size = 3.29

$$-\frac{(dx+c)a}{b^2d} + \frac{e^{(dx+c)}}{2bd} - \frac{e^{(-dx-c)}}{2bd} - \frac{a \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-(d*x + c)*a/(b^2*d) + 1/2*e^{(d*x + c)/(b*d)} - 1/2*e^{-(d*x - c)/(b*d)} - a*\log(-2*a*e^{-(d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/(b^2*d)$

**Fricas [B]** time = 2.12315, size = 354, normalized size = 10.41

$$\frac{2 \, a \, d \, x \, \cosh(dx + c) + b \, \cosh(dx + c)^2 + b \, \sinh(dx + c)^2 - 2 \, (a \, \cosh(dx + c) + a \, \sinh(dx + c)) \log\left(\frac{2 \, (b \, \sinh(dx + c) + a)}{\cosh(dx + c) - \sinh(dx + c)}\right)}{2 \, (b^2 \, d \, \cosh(dx + c) + b^2 \, d \, \sinh(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $1/2*(2*a*d*x*cosh(d*x + c) + b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 - 2*(a*cosh(d*x + c) + a*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(a*d*x + b*cosh(d*x + c))*sinh(d*x + c) - b)/(b^2*d*cosh(d*x + c) + b^2*d*sinh(d*x + c))$

**Sympy [A]** time = 1.47607, size = 65, normalized size = 1.91

$$\begin{cases} \frac{x \sinh(c) \cosh(c)}{a+b \sinh(c)} & \text{for } b = 0 \wedge d = 0 \\ \frac{x \sinh(c) \cosh(c)}{a+b \sinh(c)} & \text{for } d = 0 \\ \frac{\sinh^2(c+dx)}{2ad} & \text{for } b = 0 \\ -\frac{a \log\left(\frac{a}{b} + \sinh(c+dx)\right)}{b^2d} + \frac{\sinh(c+dx)}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Piecewise((x\*sinh(c)\*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (x\*sinh(c)\*cosh(c)/(a + b\*sinh(c)), Eq(d, 0)), (sinh(c + d\*x)\*\*2/(2\*a\*d), Eq(b, 0)), (-a\*log(a/b + sinh(c + d\*x))/(b\*\*2\*d) + sinh(c + d\*x)/(b\*d), True))

---

**Giac [A]** time = 1.14879, size = 84, normalized size = 2.47

$$\frac{e^{(dx+c)} - e^{(-dx-c)}}{2bd} - \frac{a \log\left(\left|b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 2a\right|\right)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(e^(d*x + c) - e^(-d*x - c))/(b*d) - a*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(b^2*d)
```

$$3.337 \quad \int \frac{\cosh(c+dx) \sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable} \left( \frac{\sinh(c+dx) \cosh(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[(Cosh[c + d\*x]\*Sinh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0571966, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cosh(c+dx) \sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Cosh[c + d\*x]\*Sinh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] Defer[Int] [(Cosh[c + d\*x]\*Sinh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\cosh(c+dx) \sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\cosh(c+dx) \sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [A]** time = 73.8462, size = 0, normalized size = 0.

$$\int \frac{\cosh(c+dx) \sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cosh[c + d\*x]\*Sinh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] Integrate[(Cosh[c + d\*x]\*Sinh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Maple [A]** time = 0.118, size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c) \sinh(dx + c)}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)\*sinh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x)

[Out] int(cosh(d\*x+c)\*sinh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-\frac{e^{(-c+\frac{de}{f})}E_1\left(\frac{(fx+e)d}{f}\right)}{2bf} - \frac{e^{(c-\frac{de}{f})}E_1\left(-\frac{(fx+e)d}{f}\right)}{2bf} - \frac{a \log(fx + e)}{b^2f} + \frac{1}{4} \int -\frac{8(a^2e^{(dx+c)} - ab)}{b^3fx + b^3e - (b^3fxe^{(2c)} + b^3ee^{(2c)})e^{(2dx)} - 2(ab^2f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*sinh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x, algorithm="maxima")

[Out] -1/2\*e^(-c + d\*e/f)\*exp\_integral\_e(1, (f\*x + e)\*d/f)/(b\*f) - 1/2\*e^(c - d\*e/f)\*exp\_integral\_e(1, -(f\*x + e)\*d/f)/(b\*f) - a\*log(f\*x + e)/(b^2\*f) + 1/4\*integrate(-8\*(a^2\*e^(d\*x + c) - a\*b)/(b^3\*f\*x + b^3\*e - (b^3\*f\*x\*e^(2\*c) + b^3\*e\*e^(2\*c)))\*e^(2\*d\*x) - 2\*(a\*b^2\*f\*x\*e^c + a\*b^2\*e\*e^c)\*e^(d\*x)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx + c) \sinh(dx + c)}{afx + ae + (bfx + be) \sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] integral(cosh(d*x + c)*sinh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
+ c)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)\sinh(dx+c)}{(fx+e)(b\sinh(dx+c)+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] integrate(cosh(d*x + c)*sinh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

$$3.338 \quad \int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=696

$$\frac{6af^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3} - \frac{6af^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^3} - \frac{3af\sqrt{a^2+b^2}(e+fx)^2}{b}$$

```
[Out] (3*e*f^2*x)/(4*b*d^2) + (3*f^3*x^2)/(8*b*d^2) + (a^2*(e + f*x)^4)/(4*b^3*f)
+ (e + f*x)^4/(8*b*f) - (6*a*f^2*(e + f*x)*Cosh[c + d*x])/(b^2*d^3) - (a*(
e + f*x)^3*Cosh[c + d*x])/(b^2*d) - (3*f^3*Cosh[c + d*x]^2)/(8*b*d^4) - (3*
f*(e + f*x)^2*Cosh[c + d*x]^2)/(4*b*d^2) - (a*Sqrt[a^2 + b^2]*(e + f*x)^3*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) + (a*Sqrt[a^2 + b^2]
*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*d) - (3*a
*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2]))])/(b^3*d^2) + (3*a*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) + (6*a*Sqrt[a^2 + b^2]*f^2*(e
+ f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^3) - (
6*a*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/(b^3*d^3) - (6*a*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*
x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^4) + (6*a*Sqrt[a^2 + b^2]*f^3*PolyLog[4
, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^4) + (6*a*f^3*Sinh[c +
d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*Sinh[c + d*x])/(b^2*d^2) + (3*f^2*(e +
f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^3) + ((e + f*x)^3*Cosh[c + d*x]*S
inh[c + d*x])/(2*b*d)
```

**Rubi [A]** time = 1.12957, antiderivative size = 696, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 14, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {5579, 3311, 32, 3310, 5565, 3296, 2637, 3322, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{6af^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3} - \frac{6af^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^3} - \frac{3af\sqrt{a^2+b^2}(e+fx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (3*e*f^2*x)/(4*b*d^2) + (3*f^3*x^2)/(8*b*d^2) + (a^2*(e + f*x)^4)/(4*b^3*f)
+ (e + f*x)^4/(8*b*f) - (6*a*f^2*(e + f*x)*Cosh[c + d*x])/(b^2*d^3) - (a*(
e + f*x)^3*Cosh[c + d*x])/(b^2*d) - (3*f^3*Cosh[c + d*x]^2)/(8*b*d^4) - (3*
```

$$\begin{aligned}
& f*(e + f*x)^2*\text{Cosh}[c + d*x]^2/(4*b*d^2) - (a*\text{Sqrt}[a^2 + b^2]*(e + f*x)^3*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])])/(b^3*d) + (a*\text{Sqrt}[a^2 + b^2] \\
& *(e + f*x)^3*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(b^3*d) - (3*a \\
& *\text{Sqrt}[a^2 + b^2]*f*(e + f*x)^2*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + \\
& b^2]))]/(b^3*d^2) + (3*a*\text{Sqrt}[a^2 + b^2]*f*(e + f*x)^2*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])]) \\
& )/(b^3*d^2) + (6*a*\text{Sqrt}[a^2 + b^2]*f^2*(e + f*x)*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])]) \\
& )/(b^3*d^3) - (6*a*\text{Sqrt}[a^2 + b^2]*f^2*(e + f*x)*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])]) \\
& )/(b^3*d^3) - (6*a*\text{Sqrt}[a^2 + b^2]*f^3*\text{PolyLog}[4, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])]) \\
& )/(b^3*d^4) + (6*a*\text{Sqrt}[a^2 + b^2]*f^3*\text{PolyLog}[4, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])]) \\
& )/(b^3*d^4) + (6*a*f^3*\text{Sinh}[c + d*x])/(b^2*d^4) + (3*a*f*(e + f*x)^2*\text{Sinh}[c + d*x])/(b^2*d^2) + (3*f^2*(e + \\
& f*x)*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(4*b*d^3) + ((e + f*x)^3*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(2*b*d)
\end{aligned}$$

### Rule 5579

$$\begin{aligned}
& \text{Int}[(\text{Cosh}[(c_.) + (d_.)*(x_.)]^{(p_.)*((e_.) + (f_.)*(x_.))^{(m_.)*\text{Sinh}[(c_.) + \\
& (d_.)*(x_.)]^{(n_.)}})/(a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_.)]), x\_Symbol] \text{ :> } D \\
& \text{ist}[1/b, \text{Int}[(e + f*x)^m*\text{Cosh}[c + d*x]^p*\text{Sinh}[c + d*x]^{(n - 1)}, x], x] - D \\
& \text{ist}[a/b, \text{Int}[(e + f*x)^m*\text{Cosh}[c + d*x]^p*\text{Sinh}[c + d*x]^{(n - 1)}]/(a + b*\text{Sinh} \\
& [c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{IGtQ}\{m, 0\} \&\& \text{IGtQ}\{n, \\
& 0\} \&\& \text{IGtQ}\{p, 0\}
\end{aligned}$$

### Rule 3311

$$\begin{aligned}
& \text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)*((b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}, x\_Symbol] \text{ :> } \text{Simp}[(d*m*(c + d*x)^{(m - 1)}*(b*\sin[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist} \\
& [(b^2*(n - 1))/n, \text{Int}[(c + d*x)^m*(b*\sin[e + f*x])^{(n - 2)}, x], x] - \text{Dist}[( \\
& d^2*m*(m - 1)/(f^2*n^2), \text{Int}[(c + d*x)^{(m - 2)}*(b*\sin[e + f*x])^n, x], x] \\
& - \text{Simp}[(b*(c + d*x)^m*\text{Cos}[e + f*x]*(b*\sin[e + f*x])^{(n - 1)}]/(f*n), x]) /; \\
& \text{FreeQ}\{b, c, d, e, f\}, x\} \&\& \text{GtQ}\{n, 1\} \&\& \text{GtQ}\{m, 1\}
\end{aligned}$$

### Rule 32

$$\text{Int}[(a_.) + (b_.)*(x_.)]^{(m_.)}, x\_Symbol] \text{ :> } \text{Simp}[(a + b*x)^{(m + 1)}/(b*(m + 1)), x] /; \text{FreeQ}\{a, b, m\}, x\} \&\& \text{NeQ}\{m, -1\}$$

### Rule 3310

$$\begin{aligned}
& \text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)*((b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}, x\_Symbol] \text{ :> } \\
& \text{Simp}[(d*(b*\sin[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n - 1))/n, \text{Int}[(c + \\
& d*x)*(b*\sin[e + f*x])^{(n - 2)}, x], x] - \text{Simp}[(b*(c + d*x)*\text{Cos}[e + f*x]*(b \\
& *\sin[e + f*x])^{(n - 1)}]/(f*n), x]) /; \text{FreeQ}\{b, c, d, e, f\}, x\} \&\& \text{GtQ}\{n, 1\}
\end{aligned}$$

]

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)
)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] := -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)
]*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*(f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \cosh^2(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^3 \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= -\frac{3f(e+fx)^2 \cosh^2(c+dx)}{4bd^2} + \frac{(e+fx)^3 \cosh(c+dx) \sinh(c+dx)}{2bd} + \frac{a^2 \int (e+fx)^3 \cosh^2(c+dx) dx}{b^2d} \\
&= \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{a(e+fx)^3 \cosh(c+dx)}{b^2d} - \frac{3f^3 \cosh^2(c+dx)}{8bd^4} - \frac{3f^2(e+fx) \cosh(c+dx) \sinh(c+dx)}{4bd^3} \\
&= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{a(e+fx)^3 \cosh(c+dx)}{b^2d} - \frac{3f^2(e+fx) \cosh(c+dx) \sinh(c+dx)}{4bd^3} \\
&= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx) \cosh(c+dx)}{b^2d^3} - \frac{3f^2(e+fx) \cosh(c+dx) \sinh(c+dx)}{4bd^3} \\
&= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx) \cosh(c+dx)}{b^2d^3} - \frac{3f^2(e+fx) \cosh(c+dx) \sinh(c+dx)}{4bd^3} \\
&= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx) \cosh(c+dx)}{b^2d^3} - \frac{3f^2(e+fx) \cosh(c+dx) \sinh(c+dx)}{4bd^3} \\
&= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx) \cosh(c+dx)}{b^2d^3} - \frac{3f^2(e+fx) \cosh(c+dx) \sinh(c+dx)}{4bd^3} \\
&= \frac{3ef^2x}{4bd^2} + \frac{3f^3x^2}{8bd^2} + \frac{a^2(e+fx)^4}{4b^3f} + \frac{(e+fx)^4}{8bf} - \frac{6af^2(e+fx) \cosh(c+dx)}{b^2d^3} - \frac{3f^2(e+fx) \cosh(c+dx) \sinh(c+dx)}{4bd^3}
\end{aligned}$$

**Mathematica [C]** time = 14.6937, size = 2961, normalized size = 4.25

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e^3*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/(Sqrt[-a^2 - b^2]*d))/(4*b) + (3*e^2*f*(x^2 + (2*a*((I*Pi*ArcTanh[(-b + a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (2*((-I)*c + ArcCos[((-I)*a)/b])*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2
```

$$\begin{aligned}
& - b^2]] + ((-2*I)*c + \text{Pi} - (2*I)*d*x)*\text{ArcTanh}[\frac{(a - I*b)*\text{Tan}[\frac{(2*I)*c + \text{Pi} + (2*I)*d*x}{4}}{\sqrt{-a^2 - b^2}}] - (\text{ArcCos}[\frac{(-I)*a}{b}] + (2*I)*\text{ArcTanh}[\frac{(a + I*b)*\text{Cot}[\frac{(2*I)*c + \text{Pi} + (2*I)*d*x}{4}}{\sqrt{-a^2 - b^2}}])*\text{Log}[\frac{(I*a + b)*(a + I*(b + \sqrt{-a^2 - b^2}))*(-I + \text{Cot}[\frac{(2*I)*c + \text{Pi} + (2*I)*d*x}{4}])}{(b*(I*a + b + I*\sqrt{-a^2 - b^2})*\text{Cot}[\frac{(2*I)*c + \text{Pi} + (2*I)*d*x}{4}])}] - (\text{ArcCos}[\frac{(-I)*a}{b}] - (2*I)*\text{ArcTanh}[\frac{(a + I*b)*\text{Cot}[\frac{(2*I)*c + \text{Pi} + (2*I)*d*x}{4}}{\sqrt{-a^2 - b^2}}])*\text{Log}[\frac{(I*a + b)*(I*a - b + \sqrt{-a^2 - b^2})*(I + \text{Cot}[\frac{(2*I)*c + \text{Pi} + (2*I)*d*x}{4}])}{(b*(a - I*b + \sqrt{-a^2 - b^2})*\text{Cot}[\frac{(2*I)*c + \text{Pi} + (2*I)*d*x}{4}])}] + (\text{ArcCos}[\frac{(-I)*a}{b}] - (2*I)*\text{ArcTanh}[\frac{(a + I*b)*\text{Cot}[\frac{(2*I)*c + \text{Pi} + (2*I)*d*x}{4}}{\sqrt{-a^2 - b^2}}]) - (2*I)*\text{ArcTanh}[\frac{(a - I*b)*\text{Tan}[\frac{(2*I)*c + \text{Pi} + (2*I)*d*x}{4}}{\sqrt{-a^2 - b^2}}])*\text{Log}[-\frac{((-1)^{3/4}*\sqrt{-a^2 - b^2}*E^{-(c/2 - (d*x)/2)})}{(\sqrt{2}*\sqrt{(-I)*b}*\sqrt{a + b*\text{Sinh}[c + d*x]})}] + (\text{ArcCos}[\frac{(-I)*a}{b}] + (2*I)*(\text{ArcTanh}[\frac{(a + I*b)*\text{Cot}[\frac{(2*I)*c + \text{Pi} + (2*I)*d*x}{4}}{\sqrt{-a^2 - b^2}}] + \text{ArcTanh}[\frac{(a - I*b)*\text{Tan}[\frac{(2*I)*c + \text{Pi} + (2*I)*d*x}{4}}{\sqrt{-a^2 - b^2}}]))*\text{Log}[\frac{((-1)^{1/4}*\sqrt{-a^2 - b^2}*E^{(c + d*x)/2})}{(\sqrt{2}*\sqrt{(-I)*b}*\sqrt{a + b*\text{Sinh}[c + d*x]})}] + I*(\text{PolyLog}[2, \frac{(I*a + \sqrt{-a^2 - b^2})*(I*a + b - I*\sqrt{-a^2 - b^2})*\text{Cot}[\frac{(2*I)*c + \text{Pi} + (2*I)*d*x}{4}])}{(b*(I*a + b + I*\sqrt{-a^2 - b^2})*\text{Cot}[\frac{(2*I)*c + \text{Pi} + (2*I)*d*x}{4}])}] - \text{PolyLog}[2, \frac{(a + I*\sqrt{-a^2 - b^2})*(-a + I*b + \sqrt{-a^2 - b^2})*\text{Cot}[\frac{(2*I)*c + \text{Pi} + (2*I)*d*x}{4}])}{(b*(I*a + b + I*\sqrt{-a^2 - b^2})*\text{Cot}[\frac{(2*I)*c + \text{Pi} + (2*I)*d*x}{4}])}])]/\sqrt{-a^2 - b^2})/d^2)/(8*b) + (e*f^2*(x^3 - (3*a*(d^2*x^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \sqrt{a^2 + b^2})]) - d^2*x^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \sqrt{a^2 + b^2})]) + 2*d*x*\text{PolyLog}[2, \frac{(b*E^{(c + d*x)})}{(-a + \sqrt{a^2 + b^2})}] - 2*d*x*\text{PolyLog}[2, -\frac{(b*E^{(c + d*x)})}{(a + \sqrt{a^2 + b^2})}]) - 2*\text{PolyLog}[3, \frac{(b*E^{(c + d*x)})}{(-a + \sqrt{a^2 + b^2})}] + 2*\text{PolyLog}[3, -\frac{(b*E^{(c + d*x)})}{(a + \sqrt{a^2 + b^2})}])]/(\sqrt{a^2 + b^2}*d^3))/(4*b) + (f^3*(x^4 - (4*a*(d^3*x^3*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \sqrt{a^2 + b^2})]) - d^3*x^3*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \sqrt{a^2 + b^2})]) + 3*d^2*x^2*\text{PolyLog}[2, \frac{(b*E^{(c + d*x)})}{(-a + \sqrt{a^2 + b^2})}] - 3*d^2*x^2*\text{PolyLog}[2, -\frac{(b*E^{(c + d*x)})}{(a + \sqrt{a^2 + b^2})}]) - 6*d*x*\text{PolyLog}[3, \frac{(b*E^{(c + d*x)})}{(-a + \sqrt{a^2 + b^2})}] + 6*d*x*\text{PolyLog}[3, -\frac{(b*E^{(c + d*x)})}{(a + \sqrt{a^2 + b^2})}]) + 6*\text{PolyLog}[4, \frac{(b*E^{(c + d*x)})}{(-a + \sqrt{a^2 + b^2})}] - 6*\text{PolyLog}[4, -\frac{(b*E^{(c + d*x)})}{(a + \sqrt{a^2 + b^2})}])]/(\sqrt{a^2 + b^2}*d^4))/(16*b) + (e*f^2*(2*(4*a^2 + b^2)*x^3 - (6*a*(4*a^2 + 3*b^2)*(d^2*x^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \sqrt{a^2 + b^2})]) - d^2*x^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \sqrt{a^2 + b^2})]) + 2*d*x*\text{PolyLog}[2, \frac{(b*E^{(c + d*x)})}{(-a + \sqrt{a^2 + b^2})}] - 2*d*x*\text{PolyLog}[2, -\frac{(b*E^{(c + d*x)})}{(a + \sqrt{a^2 + b^2})}]) - 2*\text{PolyLog}[3, \frac{(b*E^{(c + d*x)})}{(-a + \sqrt{a^2 + b^2})}] + 2*\text{PolyLog}[3, -\frac{(b*E^{(c + d*x)})}{(a + \sqrt{a^2 + b^2})}])]/(\sqrt{a^2 + b^2}*d^3) - (24*a*b*\text{Cosh}[d*x]*((2 + d^2*x^2)*\text{Cosh}[c] - 2*d*x*\text{Sinh}[c]))/d^3 + (3*b^2*\text{Cosh}[2*d*x]*(-2*d*x*\text{Cosh}[2*c] + (1 + 2*d^2*x^2)*\text{Sinh}[2*c]))/d^3 - (24*a*b*(-2*d*x*\text{Cosh}[c] + (2 + d^2*x^2)*\text{Sinh}[c])*\text{Sinh}[d*x])/d^3 + (3*b^2*((1 + 2*d^2*x^2)*\text{Cosh}[2*c] - 2*d*x*\text{Sinh}[2*c])*\text{Sinh}[2*d*x])/d^3)/(8*b^3) + (f^3*((4*a^2 + b^2)*x^4 - (4*a*(4*a^2 + 3*b^2)*(d^3*x^3*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \sqrt{a^2 + b^2})]) - d^3*x^3*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \sqrt{a^2 + b^2})]) +
\end{aligned}$$

$$\begin{aligned}
& 3*d^2*x^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 3*d^2*x^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])))]/(Sqrt[a^2 + b^2]*d^4) - (16*a*b*Cosh[d*x]*(d*x*(6 + d^2*x^2)*Cosh[c] - 3*(2 + d^2*x^2)*Sinh[c]))/d^4 + (b^2*Cosh[2*d*x]*(-3*(1 + 2*d^2*x^2)*Cosh[2*c] + 2*d*x*(3 + 2*d^2*x^2)*Sinh[2*c]))/d^4 - (16*a*b*(-3*(2 + d^2*x^2)*Cosh[c] + d*x*(6 + d^2*x^2)*Sinh[c])*Sinh[d*x])/d^4 + (b^2*(2*d*x*(3 + 2*d^2*x^2)*Cosh[2*c] - 3*(1 + 2*d^2*x^2)*Sinh[2*c])*Sinh[2*d*x])/d^4)/(16*b^3) + (e^3*((4*a^2 + b^2)*(c + d*x) - (2*a*(4*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 4*a*b*Cosh[c + d*x] + b^2*Sinh[2*(c + d*x)])/(4*b^3*d) + (3*e^2*f*((4*a^2 + b^2)*(-c + d*x)*(c + d*x) - 8*a*b*d*x*Cosh[c + d*x] - b^2*Cosh[2*(c + d*x)] - (2*a*(4*a^2 + 3*b^2)*(2*c*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + (c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])]) - (c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])]) + PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])] - PolyLog[2, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])))]/Sqrt[a^2 + b^2] + 8*a*b*Sinh[c + d*x] + 2*b^2*d*x*Sinh[2*(c + d*x)]))/(8*b^3*d^2)
\end{aligned}$$

**Maple [F]** time = 0.191, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 (\cosh(dx + c))^2 \sinh(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*cosh(d\*x+c)^2\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*cosh(d\*x+c)^2\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)^2\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")



[Out] Exception raised: ValueError

---

**Fricas [C]** time = 3.10445, size = 8699, normalized size = 12.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)^2\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/32*(4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 + 6*b^2*d^2*e^2*f + 6*b^2*d*e*f^2 \\ & + 3*b^2*f^3 - (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b^2*d \\ & *e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d^3* \\ & e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*\cosh(d*x + c)^4 - (4*b^2*d^3*f^3*x^3 \\ & + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3* \\ & e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d \\ & *f^3)*x)*\sinh(d*x + c)^4 + 16*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 - 3*a*b*d^2*e^2* \\ & f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*( \\ & a*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*\cosh(d*x + c)^3 + 4*(4*a* \\ & b*d^3*f^3*x^3 + 4*a*b*d^3*e^3 - 12*a*b*d^2*e^2*f + 24*a*b*d*e*f^2 - 24*a*b* \\ & f^3 + 12*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 12*(a*b*d^3*e^2*f - 2*a*b*d^2* \\ & e*f^2 + 2*a*b*d*f^3)*x - (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2* \\ & *f + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6* \\ & (2*b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*\cosh(d*x + c))*\sinh(d*x \\ & + c)^3 + 6*(2*b^2*d^3*e*f^2 + b^2*d^2*f^3)*x^2 - 4*((2*a^2 + b^2)*d^4*f^3*x \\ & ^4 + 4*(2*a^2 + b^2)*d^4*e*f^2*x^3 + 6*(2*a^2 + b^2)*d^4*e^2*f*x^2 + 4*(2*a \\ & ^2 + b^2)*d^4*e^3*x)*\cosh(d*x + c)^2 - 2*(2*(2*a^2 + b^2)*d^4*f^3*x^4 + 8*( \\ & 2*a^2 + b^2)*d^4*e*f^2*x^3 + 12*(2*a^2 + b^2)*d^4*e^2*f*x^2 + 8*(2*a^2 + b^2) \\ & *d^4*e^3*x + 3*(4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f + 6*b \\ & ^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^2*d \\ & ^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*\cosh(d*x + c)^2 - 24*(a*b*d^3*f^3 \\ & *x^3 + a*b*d^3*e^3 - 3*a*b*d^2*e^2*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b* \\ & d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(a*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d \\ & *f^3)*x)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 96*((a*b*d^2*f^3*x^2 + 2*a*b*d^2* \\ & e*f^2*x + a*b*d^2*e^2*f)*\cosh(d*x + c)^2 + 2*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e \\ & *f^2*x + a*b*d^2*e^2*f)*\cosh(d*x + c))*\sinh(d*x + c) + (a*b*d^2*f^3*x^2 + 2* \\ & a*b*d^2*e*f^2*x + a*b*d^2*e^2*f)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2}*dilog \\ & ((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c)) \\ & *\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 96*((a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2 \\ & *x + a*b*d^2*e^2*f)*\cosh(d*x + c)^2 + 2*(a*b*d^2*f^3*x^2 + 2*a*b*d^2*e*f^2* \\ & x + a*b*d^2*e^2*f)*\cosh(d*x + c))*\sinh(d*x + c) + (a*b*d^2*f^3*x^2 + 2*a*b*d \end{aligned}$$

$$\begin{aligned}
& \left( a^2 e^{2fx} + a b d^2 e^{2f} \right) \sinh(dx + c)^2 \sqrt{(a^2 + b^2)/b^2} \operatorname{dilog} \left( \frac{a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2}}{(a^2 + b^2)/b^2} - \frac{b}{b + 1} \right) \\
& - 32 \left( (a b d^3 e^3 - 3 a^2 b c d^2 e^{2f} + 3 a^2 b c^2 d e^{2f} - a b c^3 f^3) \cosh(dx + c)^2 + 2 (a b d^3 e^3 - 3 a^2 b c d^2 e^{2f} + 3 a^2 b c^2 d e^{2f} - a b c^3 f^3) \cosh(dx + c) \sinh(dx + c) + (a b d^3 e^3 - 3 a^2 b c d^2 e^{2f} + 3 a^2 b c^2 d e^{2f} - a b c^3 f^3) \sinh(dx + c)^2 \right) \sqrt{(a^2 + b^2)/b^2} \\
& \log(2 b \cosh(dx + c) + 2 b \sinh(dx + c) + 2 b \sqrt{(a^2 + b^2)/b^2} + 2 a) + 32 \left( (a b d^3 e^3 - 3 a^2 b c d^2 e^{2f} + 3 a^2 b c^2 d e^{2f} - a b c^3 f^3) \cosh(dx + c)^2 + 2 (a b d^3 e^3 - 3 a^2 b c d^2 e^{2f} + 3 a^2 b c^2 d e^{2f} - a b c^3 f^3) \cosh(dx + c) \sinh(dx + c) + (a b d^3 e^3 - 3 a^2 b c d^2 e^{2f} + 3 a^2 b c^2 d e^{2f} - a b c^3 f^3) \sinh(dx + c)^2 \right) \sqrt{(a^2 + b^2)/b^2} \\
& \log(2 b \cosh(dx + c) + 2 b \sinh(dx + c) - 2 b \sqrt{(a^2 + b^2)/b^2} + 2 a) + 32 \left( (a b d^3 f^3 x^3 + 3 a^2 b d^3 e^{2f} x^2 + 3 a^2 b d^3 e^{2f} x + 3 a^2 b c d^2 e^{2f} - 3 a^2 b c^2 d e^{2f} + a b c^3 f^3) \cosh(dx + c)^2 + 2 (a b d^3 f^3 x^3 + 3 a^2 b d^3 e^{2f} x^2 + 3 a^2 b d^3 e^{2f} x + 3 a^2 b c d^2 e^{2f} - 3 a^2 b c^2 d e^{2f} + a b c^3 f^3) \cosh(dx + c) \sinh(dx + c) + (a b d^3 f^3 x^3 + 3 a^2 b d^3 e^{2f} x^2 + 3 a^2 b d^3 e^{2f} x + 3 a^2 b c d^2 e^{2f} - 3 a^2 b c^2 d e^{2f} + a b c^3 f^3) \sinh(dx + c)^2 \right) \sqrt{(a^2 + b^2)/b^2} \\
& \log(-a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - \frac{b}{b}) - 32 \left( (a b d^3 f^3 x^3 + 3 a^2 b d^3 e^{2f} x^2 + 3 a^2 b d^3 e^{2f} x + 3 a^2 b c d^2 e^{2f} - 3 a^2 b c^2 d e^{2f} + a b c^3 f^3) \cosh(dx + c)^2 + 2 (a b d^3 f^3 x^3 + 3 a^2 b d^3 e^{2f} x^2 + 3 a^2 b d^3 e^{2f} x + 3 a^2 b c d^2 e^{2f} - 3 a^2 b c^2 d e^{2f} + a b c^3 f^3) \cosh(dx + c) \sinh(dx + c) + (a b d^3 f^3 x^3 + 3 a^2 b d^3 e^{2f} x^2 + 3 a^2 b d^3 e^{2f} x + 3 a^2 b c d^2 e^{2f} - 3 a^2 b c^2 d e^{2f} + a b c^3 f^3) \sinh(dx + c)^2 \right) \sqrt{(a^2 + b^2)/b^2} \\
& \log(-a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - \frac{b}{b}) + 192 \left( a b f^3 \cosh(dx + c)^2 + 2 a^2 b f^3 \cosh(dx + c) \sinh(dx + c) + a b f^3 \sinh(dx + c)^2 \right) \sqrt{(a^2 + b^2)/b^2} \operatorname{polylog} \left( 4, \frac{a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2}}{b} \right) \\
& - 192 \left( a b f^3 \cosh(dx + c)^2 + 2 a^2 b f^3 \cosh(dx + c) \sinh(dx + c) + a b f^3 \sinh(dx + c)^2 \right) \sqrt{(a^2 + b^2)/b^2} \operatorname{polylog} \left( 4, \frac{a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2}}{b} \right) \\
& - 192 \left( (a b d f^3 x + a b d e^{2f}) \cosh(dx + c)^2 + 2 (a b d f^3 x + a b d e^{2f}) \cosh(dx + c) \sinh(dx + c) + (a b d f^3 x + a b d e^{2f}) \sinh(dx + c)^2 \right) \sqrt{(a^2 + b^2)/b^2} \operatorname{polylog} \left( 3, \frac{a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2}}{b} \right) \\
& + 192 \left( (a b d f^3 x + a b d e^{2f}) \cosh(dx + c)^2 + 2 (a b d f^3 x + a b d e^{2f}) \cosh(dx + c) \sinh(dx + c) + (a b d f^3 x + a b d e^{2f}) \sinh(dx + c)^2 \right) \sqrt{(a^2 + b^2)/b^2} \operatorname{polylog} \left( 3, \frac{a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2}}{b} \right) \\
& + 6 \left( 2 b^2 d^3 e^{2f} + 2 b^2 d^2 e^{2f} + b^2 d f^3 \right) x + 16 \left( a b d^3 f^3 x^3 + a b d^3 e^3 + 3 a^2 b d^2 e^{2f} + 6 a^2 b d e^{2f} + 6 a^2 b f^3 + 3 (a b d^3 e^{2f} + a b d^2 f^3) x^2 + 3 (a b d^3 e^{2f} + 2 a b d^2 e^{2f} + 2 a b d f^3) x \right) \cosh(dx + c) \\
& + 4 \left( 4 a b d^3 f^3 x^3 + 4 a b d^3 e^3 + 12 a b d^2 e^{2f} + 24 a b d e^{2f} \right) \sinh(dx + c)
\end{aligned}$$

```
*f^2 + 24*a*b*f^3 - (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e^2*f +
6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 + 6*(2*b^
2*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c)^3 + 12*(a*b*d^3
*e*f^2 + a*b*d^2*f^3)*x^2 + 12*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 - 3*a*b*d^2*e
^2*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*
(a*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x + c)^2 + 12*(a*
b*d^3*e^2*f + 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x - 2*((2*a^2 + b^2)*d^4*f^3*x
^4 + 4*(2*a^2 + b^2)*d^4*e*f^2*x^3 + 6*(2*a^2 + b^2)*d^4*e^2*f*x^2 + 4*(2*a
^2 + b^2)*d^4*e^3*x)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d^4*cosh(d*x + c)^2
+ 2*b^3*d^4*cosh(d*x + c)*sinh(d*x + c) + b^3*d^4*sinh(d*x + c)^2)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*cosh(d*x+c)**2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c)^2 \sinh(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

$$3.339 \quad \int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=510

$$\frac{2af\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2} + \frac{2af\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2} + \frac{2af^2\sqrt{a^2+b^2}\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}$$

[Out] (f^2\*x)/(4\*b\*d^2) + (a^2\*(e + f\*x)^3)/(3\*b^3\*f) + (e + f\*x)^3/(6\*b\*f) - (2\*a\*f^2\*Cosh[c + d\*x])/(b^2\*d^3) - (a\*(e + f\*x)^2\*Cosh[c + d\*x])/(b^2\*d) - (f\*(e + f\*x)\*Cosh[c + d\*x]^2)/(2\*b\*d^2) - (a\*Sqrt[a^2 + b^2]\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]))/(b^3\*d) + (a\*Sqrt[a^2 + b^2]\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]))/(b^3\*d) - (2\*a\*Sqrt[a^2 + b^2]\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^3\*d^2) + (2\*a\*Sqrt[a^2 + b^2]\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^3\*d^2) + (2\*a\*Sqrt[a^2 + b^2]\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^3\*d^3) - (2\*a\*Sqrt[a^2 + b^2]\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^3\*d^3) + (2\*a\*f\*(e + f\*x)\*Sinh[c + d\*x])/(b^2\*d^2) + (f^2\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(4\*b\*d^3) + ((e + f\*x)^2\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(2\*b\*d)

**Rubi [A]** time = 0.962293, antiderivative size = 510, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 14, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {5579, 3311, 32, 2635, 8, 5565, 3296, 2638, 3322, 2264, 2190, 2531, 2282, 6589}

$$\frac{2af\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2} + \frac{2af\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2} + \frac{2af^2\sqrt{a^2+b^2}\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^3}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Cosh[c + d\*x]^2\*Sinh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (f^2\*x)/(4\*b\*d^2) + (a^2\*(e + f\*x)^3)/(3\*b^3\*f) + (e + f\*x)^3/(6\*b\*f) - (2\*a\*f^2\*Cosh[c + d\*x])/(b^2\*d^3) - (a\*(e + f\*x)^2\*Cosh[c + d\*x])/(b^2\*d) - (f\*(e + f\*x)\*Cosh[c + d\*x]^2)/(2\*b\*d^2) - (a\*Sqrt[a^2 + b^2]\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]))/(b^3\*d) + (a\*Sqrt[a^2 + b^2]\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]))/(b^3\*d) - (2\*a\*Sqrt[a^2 + b^2]\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^3\*d^2) + (2\*a\*Sqrt[a^2 + b^2]\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^3\*d^2) + (2\*a\*Sqrt[a^2 + b^2]\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^3\*d^3) - (2\*a\*Sqrt[a^2 + b^2]\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^3\*d^3) + (2\*a\*f\*(e + f\*x)\*Sinh[c + d\*x])/(b^2\*d^2) + (f^2\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(4\*b\*d^3) + ((e + f\*x)^2\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(2\*b\*d)

$$\begin{aligned} & , -((bE^{(c+dx)})/(a - \sqrt{a^2 + b^2}))/b^3d^3 - (2a\sqrt{a^2 + b^2} \\ & *f^2\text{PolyLog}[3, -(bE^{(c+dx)})/(a + \sqrt{a^2 + b^2})])/b^3d^3 + (2 \\ & *a*f*(e + fx)*\text{Sinh}[c + dx])/b^2d^2 + (f^2\text{Cosh}[c + dx]*\text{Sinh}[c + dx]) \\ & / (4*b*d^3) + ((e + fx)^2\text{Cosh}[c + dx]*\text{Sinh}[c + dx])/(2*b*d) \end{aligned}$$
Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
(d_.)*(x_)]^(n_.)]/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
0] && IGtQ[p, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:= Simp[(d*m*(c + d*x)^(m - 1)*(b*Sinh[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sinh[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sinh[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cosh[e + f*x]*(b*Sinh[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_.), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_.), x_Symbol] := -Simp[(b*Cosh[c + d*x]
*(b*Sinh[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sinh[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)]/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)
]*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
```

```
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \cosh^2(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^2 \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\ &= -\frac{f(e+fx) \cosh^2(c+dx)}{2bd^2} + \frac{(e+fx)^2 \cosh(c+dx) \sinh(c+dx)}{2bd} + \frac{a^2 \int (e+fx)^2 \cosh^2(c+dx) dx}{2bd^2} \\ &= \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{a(e+fx)^2 \cosh(c+dx)}{b^2d} - \frac{f(e+fx) \cosh^2(c+dx)}{2bd^2} \\ &= \frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{a(e+fx)^2 \cosh(c+dx)}{b^2d} - \frac{f(e+fx) \cosh^2(c+dx)}{2bd^2} \\ &= \frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2 \cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^2 \cosh(c+dx)}{b^2d} \\ &= \frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2 \cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^2 \cosh(c+dx)}{b^2d} \\ &= \frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2 \cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^2 \cosh(c+dx)}{b^2d} \\ &= \frac{f^2x}{4bd^2} + \frac{a^2(e+fx)^3}{3b^3f} + \frac{(e+fx)^3}{6bf} - \frac{2af^2 \cosh(c+dx)}{b^2d^3} - \frac{a(e+fx)^2 \cosh(c+dx)}{b^2d} \end{aligned}$$





,  $-\left(\frac{b e^{(c+dx)}}{a + \sqrt{a^2 + b^2}}\right) / \left(\sqrt{a^2 + b^2} d^3\right) - (24 a b \cosh[dx] \left((2 + d^2 x^2) \cosh[c] - 2 d x \sinh[c]\right) / d^3 + (3 b^2 \cosh[2 d x] \left(-2 d x \cosh[2 c] + (1 + 2 d^2 x^2) \sinh[2 c]\right) / d^3 - (24 a b \left(-2 d x \cosh[c] + (2 + d^2 x^2) \sinh[c]\right) \sinh[dx] / d^3 + (3 b^2 \left((1 + 2 d^2 x^2) \cosh[2 c] - 2 d x \sinh[2 c]\right) \sinh[2 d x] / d^3) / (24 b^3) + (e^{2 \left((4 a^2 + b^2)(c+dx) - (2 a (4 a^2 + 3 b^2) \operatorname{ArcTan}[(b - a \operatorname{Tanh}[(c+dx)/2]) / \sqrt{-a^2 - b^2}])} / \sqrt{-a^2 - b^2} - 4 a b \cosh[c+dx] + b^2 \sinh[2(c+dx)])\right) / (4 b^3 d) + (e^{f \left((4 a^2 + b^2)(-c+dx)(c+dx) - 8 a b d x \cosh[c+dx] - b^2 \cosh[2(c+dx)] - (2 a (4 a^2 + 3 b^2) (2 c \operatorname{ArcTan}[(a + b \cosh[c+dx] + b \sinh[c+dx]) / \sqrt{a^2 + b^2}]) + (c+dx) \log[1 + (b(\cosh[c+dx] + \sinh[c+dx])) / (a - \sqrt{a^2 + b^2})]) - (c+dx) \log[1 + (b(\cosh[c+dx] + \sinh[c+dx])) / (a + \sqrt{a^2 + b^2})]) + \operatorname{PolyLog}[2, (b(\cosh[c+dx] + \sinh[c+dx])) / (-a + \sqrt{a^2 + b^2})] - \operatorname{PolyLog}[2, -(b(\cosh[c+dx] + \sinh[c+dx])) / (a + \sqrt{a^2 + b^2})])\right) / \sqrt{a^2 + b^2} + 8 a b \sinh[c+dx] + 2 b^2 d x \sinh[2(c+dx)]) / (4 b^3 d^2)$

**Maple [F]** time = 0.145, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\cosh(dx + c))^2 \sinh(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*cosh(d\*x+c)^2\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*cosh(d\*x+c)^2\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)^2\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 2.71597, size = 5658, normalized size = 11.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)^2\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/48*(6*b^2*d^2*f^2*x^2 + 6*b^2*d^2*e^2 + 6*b^2*d*e*f - 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*\cosh(d*x + c)^4 - 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*\sinh(d*x + c)^4 + 3*b^2*f^2 + 24*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*\cosh(d*x + c)^3 + 12*(2*a*b*d^2*f^2*x^2 + 2*a*b*d^2*e^2 - 4*a*b*d*e*f + 4*a*b*f^2 + 4*(a*b*d^2*e*f - a*b*d*f^2)*x - (2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 8*((2*a^2 + b^2)*d^3*f^2*x^3 + 3*(2*a^2 + b^2)*d^3*e*f*x^2 + 3*(2*a^2 + b^2)*d^3*e^2*x)*\cosh(d*x + c)^2 - 2*(4*(2*a^2 + b^2)*d^3*f^2*x^3 + 12*(2*a^2 + b^2)*d^3*e*f*x^2 + 12*(2*a^2 + b^2)*d^3*e^2*x + 9*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*\cosh(d*x + c)^2 - 36*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 96*((a*b*d*f^2*x + a*b*d*e*f)*\cosh(d*x + c)^2 + 2*(a*b*d*f^2*x + a*b*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c) + (a*b*d*f^2*x + a*b*d*e*f)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2}*dilog((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 96*((a*b*d*f^2*x + a*b*d*e*f)*\cosh(d*x + c)^2 + 2*(a*b*d*f^2*x + a*b*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c) + (a*b*d*f^2*x + a*b*d*e*f)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2}*dilog((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 48*((a*b*d^2*e^2 - 2*a*b*c*d*e*f + a*b*c^2*f^2)*\cosh(d*x + c)^2 + 2*(a*b*d^2*e^2 - 2*a*b*c*d*e*f + a*b*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (a*b*d^2*e^2 - 2*a*b*c*d*e*f + a*b*c^2*f^2)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2}*log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 48*((a*b*d^2*e^2 - 2*a*b*c*d*e*f + a*b*c^2*f^2)*\cosh(d*x + c)^2 + 2*(a*b*d^2*e^2 - 2*a*b*c*d*e*f + a*b*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (a*b*d^2*e^2 - 2*a*b*c*d*e*f + a*b*c^2*f^2)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2}*log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 48*((a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2)*\cosh(d*x + c)^2 + 2*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2}*log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 48*((a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2)*\cosh(d*x + c)^2 + 2*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2}*log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) \end{aligned}$$

$$\begin{aligned} & \left( \frac{a^2 + b^2}{b^2} - b \right) / b - 48 \left( (a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2) * \cosh(dx + c)^2 + 2*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2) * \cosh(dx + c) * \sinh(dx + c) + (a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2) * \sinh(dx + c)^2 \right) * \sqrt{\left( \frac{a^2 + b^2}{b^2} \right) * \log\left( -(a * \cosh(dx + c) + a * \sinh(dx + c) - (b * \cosh(dx + c) + b * \sinh(dx + c))) * \sqrt{\left( \frac{a^2 + b^2}{b^2} \right) - b} / b \right) - 96 * (a*b*f^2 * \cosh(dx + c)^2 + 2*a*b*f^2 * \cosh(dx + c) * \sinh(dx + c) + a*b*f^2 * \sinh(dx + c)^2) * \sqrt{\left( \frac{a^2 + b^2}{b^2} \right) * \text{polylog}\left( 3, (a * \cosh(dx + c) + a * \sinh(dx + c) + (b * \cosh(dx + c) + b * \sinh(dx + c))) * \sqrt{\left( \frac{a^2 + b^2}{b^2} \right)} / b \right) + 96 * (a*b*f^2 * \cosh(dx + c)^2 + 2*a*b*f^2 * \cosh(dx + c) * \sinh(dx + c) + a*b*f^2 * \sinh(dx + c)^2) * \sqrt{\left( \frac{a^2 + b^2}{b^2} \right) * \text{polylog}\left( 3, (a * \cosh(dx + c) + a * \sinh(dx + c) - (b * \cosh(dx + c) + b * \sinh(dx + c))) * \sqrt{\left( \frac{a^2 + b^2}{b^2} \right)} / b \right) + 6 * (2*b^2*d^2*e*f + b^2*d*f^2) * x + 24 * (a*b*d^2*f^2*x^2 + a*b*d^2*e^2 + 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f + a*b*d*f^2) * x) * \cosh(dx + c) + 4 * (6*a*b*d^2*f^2*x^2 + 6*a*b*d^2*e^2 + 12*a*b*d*e*f + 12*a*b*f^2 - 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2) * x) * \cosh(dx + c)^3 + 18*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2) * x) * \cosh(dx + c)^2 + 12*(a*b*d^2*e*f + a*b*d*f^2) * x - 4*((2*a^2 + b^2) * d^3*f^2*x^3 + 3*(2*a^2 + b^2) * d^3*e*f*x^2 + 3*(2*a^2 + b^2) * d^3*e^2*x) * \cosh(dx + c) * \sinh(dx + c)) / (b^3*d^3 * \cosh(dx + c)^2 + 2*b^3*d^3 * \cosh(dx + c) * \sinh(dx + c) + b^3*d^3 * \sinh(dx + c)^2)} \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cosh(dx+c)\*\*2\*sinh(dx+c)/(a+b\*sinh(dx+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c)^2 \sinh(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

$$3.340 \quad \int \frac{(e+fx) \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=327

$$\frac{af\sqrt{a^2+b^2}\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2} + \frac{af\sqrt{a^2+b^2}\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2} - \frac{a\sqrt{a^2+b^2}(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{b^3d} +$$

```
[Out] (a^2*e*x)/b^3 + (e*x)/(2*b) + (a^2*f*x^2)/(2*b^3) + (f*x^2)/(4*b) - (a*(e +
f*x)*Cosh[c + d*x])/(b^2*d) - (f*Cosh[c + d*x]^2)/(4*b*d^2) - (a*Sqrt[a^2
+ b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) +
(a*Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])
)/(b^3*d) - (a*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2]))])/(b^3*d^2) + (a*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(b^3*d^2) + (a*f*Sinh[c + d*x])/(b^2*d^2) + ((e + f
*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)
```

**Rubi [A]** time = 0.550004, antiderivative size = 327, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {5579, 3310, 5565, 3296, 2637, 3322, 2264, 2190, 2279, 2391}

$$\frac{af\sqrt{a^2+b^2}\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3d^2} + \frac{af\sqrt{a^2+b^2}\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3d^2} - \frac{a\sqrt{a^2+b^2}(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{b^3d} +$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*e*x)/b^3 + (e*x)/(2*b) + (a^2*f*x^2)/(2*b^3) + (f*x^2)/(4*b) - (a*(e +
f*x)*Cosh[c + d*x])/(b^2*d) - (f*Cosh[c + d*x]^2)/(4*b*d^2) - (a*Sqrt[a^2
+ b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) +
(a*Sqrt[a^2 + b^2]*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])
)/(b^3*d) - (a*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2]))])/(b^3*d^2) + (a*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]))])/(b^3*d^2) + (a*f*Sinh[c + d*x])/(b^2*d^2) + ((e + f
*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)
```

**Rule 5579**

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
(d_.)*(x_)]^(n_.)]/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := D
```

```

ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
0] && IGtQ[p, 0]

```

### Rule 3310

```

Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
Simp[(d*(b*Sinh[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sinh[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sinh[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]

```

### Rule 5565

```

Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_))*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

```

### Rule 3296

```

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

```

### Rule 2637

```

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

```

### Rule 3322

```

Int[((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_]
*(f_.)*(x_))]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

```

### Rule 2264

```

Int[((F_)^(u_))*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.
)*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^

```

$m \cdot F^u / (b + q + 2 \cdot c \cdot F^u), x], x]] /;$  FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_)]/((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_)))^(n\_))], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx) \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx) \cosh^2(c+dx) dx}{b} - \frac{a \int \frac{(e+fx) \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= -\frac{f \cosh^2(c+dx)}{4bd^2} + \frac{(e+fx) \cosh(c+dx) \sinh(c+dx)}{2bd} + \frac{a^2 \int (e+fx) dx}{b^3} \\
&= \frac{a^2 ex}{b^3} + \frac{ex}{2b} + \frac{a^2 fx^2}{2b^3} + \frac{fx^2}{4b} - \frac{a(e+fx) \cosh(c+dx)}{b^2 d} - \frac{f \cosh^2(c+dx)}{4bd^2} + \dots \\
&= \frac{a^2 ex}{b^3} + \frac{ex}{2b} + \frac{a^2 fx^2}{2b^3} + \frac{fx^2}{4b} - \frac{a(e+fx) \cosh(c+dx)}{b^2 d} - \frac{f \cosh^2(c+dx)}{4bd^2} + \dots \\
&= \frac{a^2 ex}{b^3} + \frac{ex}{2b} + \frac{a^2 fx^2}{2b^3} + \frac{fx^2}{4b} - \frac{a(e+fx) \cosh(c+dx)}{b^2 d} - \frac{f \cosh^2(c+dx)}{4bd^2} - \dots \\
&= \frac{a^2 ex}{b^3} + \frac{ex}{2b} + \frac{a^2 fx^2}{2b^3} + \frac{fx^2}{4b} - \frac{a(e+fx) \cosh(c+dx)}{b^2 d} - \frac{f \cosh^2(c+dx)}{4bd^2} - \dots \\
&= \frac{a^2 ex}{b^3} + \frac{ex}{2b} + \frac{a^2 fx^2}{2b^3} + \frac{fx^2}{4b} - \frac{a(e+fx) \cosh(c+dx)}{b^2 d} - \frac{f \cosh^2(c+dx)}{4bd^2} - \dots
\end{aligned}$$

**Mathematica [C]** time = 4.61866, size = 1549, normalized size = 4.74

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (2*b^2*e*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]]
)/(Sqrt[-a^2 - b^2]*d) + b^2*f*(x^2 + (2*a*((I*Pi*ArcTanh[(-b + a*Tanh[(c
+ d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (2*((-I)*c + ArcCos[((-I)*a)
/b])*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]
] + ((-2*I)*c + Pi - (2*I)*d*x)*ArcTanh[((a - I*b)*Tan[((2*I)*c + Pi + (2*I
)*d*x)/4])/Sqrt[-a^2 - b^2]] - (ArcCos[((-I)*a)/b] + (2*I)*ArcTanh[((a + I*
b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]))*Log[((I*a + b)*(a
+ I*(b + Sqrt[-a^2 - b^2]))*(-I + Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I
*a + b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))] - (ArcCos[
((-I)*a)/b] - (2*I)*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/S
qrt[-a^2 - b^2]))*Log[((I*a + b)*(I*a - b + Sqrt[-a^2 - b^2]))*(I + Cot[((2*
```



$$\begin{aligned}
& I)c + \text{Pi} + (2*I)*d*x)/4)))/(b*(a - I*b + \text{Sqrt}[-a^2 - b^2]*\text{Cot}[(2*I)*c + \text{Pi} \\
& + (2*I)*d*x)/4])) + (\text{ArcCos}[((-I)*a)/b] - (2*I)*\text{ArcTanh}[(a + I*b)*\text{Cot}[(2*I)*c + \text{Pi} \\
& + (2*I)*d*x)/4])/\text{Sqrt}[-a^2 - b^2]] - (2*I)*\text{ArcTanh}[(a - I*b)*\text{Tan}[(2*I)*c + \text{Pi} + (2*I)*d*x)/4]) \\
& / \text{Sqrt}[-a^2 - b^2]] * \text{Log}[-((-1)^{(3/4)} * \text{Sqrt}[-a^2 - b^2]) * E^{(-c/2 - (d*x)/2)} / (\text{Sqrt}[2] * \text{Sqrt}[(-I)*b] * \text{Sqrt}[a + b * \text{Sinh}[c + \\
& d*x]])] + (\text{ArcCos}[((-I)*a)/b] + (2*I)*(\text{ArcTanh}[(a + I*b)*\text{Cot}[(2*I)*c + \text{Pi} + (2*I)*d*x)/4]) \\
& / \text{Sqrt}[-a^2 - b^2]] + \text{ArcTanh}[(a - I*b)*\text{Tan}[(2*I)*c + \text{Pi} + (2*I)*d*x)/4]) / \text{Sqrt}[-a^2 - b^2]])) * \text{Log}[-((-1)^{(1/4)} * \text{Sqrt}[-a^2 - b^2]) * E^{(c + d*x)/2}) / (\text{Sqrt}[2] * \text{Sqrt}[(-I)*b] * \text{Sqrt}[a + b * \text{Sinh}[c + d*x]])] + I * (\text{PolyLog}[2, ((I*a + \text{Sqrt}[-a^2 - b^2]) * (I*a + b - I * \text{Sqrt}[-a^2 - b^2]) * \text{Cot}[(2*I)*c + \text{Pi} + (2*I)*d*x)/4])) / (b * (I*a + b + I * \text{Sqrt}[-a^2 - b^2]) * \text{Cot}[(2*I)*c + \text{Pi} + (2*I)*d*x)/4])) - \text{PolyLog}[2, ((a + I * \text{Sqrt}[-a^2 - b^2]) * (-a + I*b + \text{Sqrt}[-a^2 - b^2]) * \text{Cot}[(2*I)*c + \text{Pi} + (2*I)*d*x)/4])) / (b * (I*a + b + I * \text{Sqrt}[-a^2 - b^2]) * \text{Cot}[(2*I)*c + \text{Pi} + (2*I)*d*x)/4]))] / \text{Sqrt}[-a^2 - b^2]] / d^2) + (2 * e * ((4 * a^2 + b^2) * (c + d*x) - (2 * a * (4 * a^2 + 3 * b^2) * \text{ArcTan}[(b - a * \text{Tanh}[(c + d*x) / 2]) / \text{Sqrt}[-a^2 - b^2]]) / \text{Sqrt}[-a^2 - b^2] - 4 * a * b * \text{Cosh}[c + d*x] + b^2 * \text{Sinh}[2 * (c + d*x)])) / d + (f * ((4 * a^2 + b^2) * (-c + d*x) * (c + d*x) - 8 * a * b * d * x * \text{Cosh}[c + d*x] - b^2 * \text{Cosh}[2 * (c + d*x)] - (2 * a * (4 * a^2 + 3 * b^2) * (2 * c * \text{ArcTanh}[(a + b * \text{Cosh}[c + d*x] + b * \text{Sinh}[c + d*x]) / \text{Sqrt}[a^2 + b^2]]) + (c + d*x) * \text{Log}[1 + (b * (\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x])) / (a - \text{Sqrt}[a^2 + b^2])]) - (c + d*x) * \text{Log}[1 + (b * (\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x])) / (a + \text{Sqrt}[a^2 + b^2])]) + \text{PolyLog}[2, (b * (\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x])) / (-a + \text{Sqrt}[a^2 + b^2])]) - \text{PolyLog}[2, -(b * (\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x])) / (a + \text{Sqrt}[a^2 + b^2])])) / \text{Sqrt}[a^2 + b^2] + 8 * a * b * \text{Sinh}[c + d*x] + 2 * b^2 * d * x * \text{Sinh}[2 * (c + d*x)])) / d^2) / (8 * b^3)
\end{aligned}$$

**Maple [B]** time = 0.082, size = 1012, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)*\cosh(d*x+c)^2*\sinh(d*x+c)/(a+b*\sinh(d*x+c)),x)$

[Out]  $\frac{1}{2}a^2f*x^2/b^3+1/4f*x^2/b+a^2e*x/b^3+1/2e*x/b+1/16*(2*d*f*x+2*d*e-f)/d^2/b*\exp(2*d*x+2*c)-1/2a*(d*f*x+d*e-f)/b^2/d^2*\exp(d*x+c)-1/2a*(d*f*x+d*e+f)/b^2/d^2*\exp(-d*x-c)-1/16*(2*d*f*x+2*d*e+f)/d^2/b*\exp(-2*d*x-2*c)+2*a^3/b^3/d*e/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+2*a/b/d*e/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-a^3/b^3/d*f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})))*x-a^3/b^3/d^2*f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})))*c+a^3/b^3/d*f/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})))*x+a^3/b^3/d^2*f/(a^2+b^2)^{(1/2)}*\ln$

$$\begin{aligned} & ((b \exp(dx+c) + (a^2+b^2)^{1/2} + a) / (a + (a^2+b^2)^{1/2})) * c - a^3/b^3/d^2 * f / (a^2 + b^2)^{1/2} * \operatorname{dilog}((-b \exp(dx+c) + (a^2+b^2)^{1/2} - a) / (-a + (a^2+b^2)^{1/2})) + a^3/b^3/d^2 * f / (a^2+b^2)^{1/2} * \operatorname{dilog}((b \exp(dx+c) + (a^2+b^2)^{1/2} + a) / (a + (a^2+b^2)^{1/2})) - a/b/d * f / (a^2+b^2)^{1/2} * \ln((-b \exp(dx+c) + (a^2+b^2)^{1/2} - a) / (-a + (a^2+b^2)^{1/2})) * x - a/b/d^2 * f / (a^2+b^2)^{1/2} * \ln((-b \exp(dx+c) + (a^2+b^2)^{1/2} - a) / (-a + (a^2+b^2)^{1/2})) * c + a/b/d * f / (a^2+b^2)^{1/2} * \ln((b \exp(dx+c) + (a^2+b^2)^{1/2} + a) / (a + (a^2+b^2)^{1/2})) * x + a/b/d^2 * f / (a^2+b^2)^{1/2} * \ln((b \exp(dx+c) + (a^2+b^2)^{1/2} + a) / (a + (a^2+b^2)^{1/2})) * c - a/b/d^2 * f / (a^2+b^2)^{1/2} * \operatorname{dilog}((-b \exp(dx+c) + (a^2+b^2)^{1/2} - a) / (-a + (a^2+b^2)^{1/2})) + a/b/d^2 * f / (a^2+b^2)^{1/2} * \operatorname{dilog}((b \exp(dx+c) + (a^2+b^2)^{1/2} + a) / (a + (a^2+b^2)^{1/2})) - 2 * a^3/b^3/d^2 * f * c / (a^2+b^2)^{1/2} * \operatorname{arctanh}(1/2 * (2 * b \exp(dx+c) + 2 * a) / (a^2 + b^2)^{1/2}) - 2 * a/b/d^2 * f * c / (a^2+b^2)^{1/2} * \operatorname{arctanh}(1/2 * (2 * b \exp(dx+c) + 2 * a) / (a^2 + b^2)^{1/2}) \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(dx+c)^2\*sinh(dx+c)/(a+b\*sinh(dx+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.38246, size = 3200, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(dx+c)^2\*sinh(dx+c)/(a+b\*sinh(dx+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/16 * (2 * b^2 * d * f * x - (2 * b^2 * d * f * x + 2 * b^2 * d * e - b^2 * f) * \cosh(dx + c)^4 - (2 * b^2 * d * f * x + 2 * b^2 * d * e - b^2 * f) * \sinh(dx + c)^4 + 2 * b^2 * d * e + 8 * (a * b * d * f * x + a * b * d * e - a * b * f) * \cosh(dx + c)^3 + 4 * (2 * a * b * d * f * x + 2 * a * b * d * e - 2 * a * b * f - (2 * b^2 * d * f * x + 2 * b^2 * d * e - b^2 * f) * \cosh(dx + c)) * \sinh(dx + c)^3 + b^2 * f - 4 * ((2 * a^2 + b^2) * d^2 * f * x^2 + 2 * (2 * a^2 + b^2) * d^2 * e * x) * \cosh(dx + c)^2 - 2 * (2 * (2 * a^2 + b^2) * d^2 * f * x^2 + 4 * (2 * a^2 + b^2) * d^2 * e * x + 3 * (2 * b^2 * d * f * x + 2 * b \end{aligned}$$

$$\begin{aligned}
& ^2*d*e - b^2*f)*\cosh(d*x + c)^2 - 12*(a*b*d*f*x + a*b*d*e - a*b*f)*\cosh(d*x \\
& + c))*\sinh(d*x + c)^2 + 16*(a*b*f*\cosh(d*x + c)^2 + 2*a*b*f*\cosh(d*x + c)* \\
& \sinh(d*x + c) + a*b*f*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2)*\operatorname{dilog}((a*\cosh( \\
& d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 \\
& + b^2)/b^2) - b)/b + 1) - 16*(a*b*f*\cosh(d*x + c)^2 + 2*a*b*f*\cosh(d*x + c) \\
& *\sinh(d*x + c) + a*b*f*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2)*\operatorname{dilog}((a*\cosh \\
& (d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 \\
& + b^2)/b^2) - b)/b + 1) - 16*((a*b*d*e - a*b*c*f)*\cosh(d*x + c)^2 + 2*(a*b \\
& *d*e - a*b*c*f)*\cosh(d*x + c)*\sinh(d*x + c) + (a*b*d*e - a*b*c*f)*\sinh(d*x \\
& + c)^2)*\sqrt{(a^2 + b^2)/b^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2 \\
& *b*\sqrt{(a^2 + b^2)/b^2) + 2*a) + 16*((a*b*d*e - a*b*c*f)*\cosh(d*x + c)^2 + \\
& 2*(a*b*d*e - a*b*c*f)*\cosh(d*x + c)*\sinh(d*x + c) + (a*b*d*e - a*b*c*f)*\si \\
& nh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + \\
& c) - 2*b*\sqrt{(a^2 + b^2)/b^2) + 2*a) + 16*((a*b*d*f*x + a*b*c*f)*\cosh(d*x \\
& + c)^2 + 2*(a*b*d*f*x + a*b*c*f)*\cosh(d*x + c)*\sinh(d*x + c) + (a*b*d*f*x \\
& + a*b*c*f)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2)*\log(-(a*\cosh(d*x + c) + a \\
& *\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2) \\
& - b)/b) - 16*((a*b*d*f*x + a*b*c*f)*\cosh(d*x + c)^2 + 2*(a*b*d*f*x + a*b*c* \\
& f)*\cosh(d*x + c)*\sinh(d*x + c) + (a*b*d*f*x + a*b*c*f)*\sinh(d*x + c)^2)*\sqrt{ \\
& (a^2 + b^2)/b^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + \\
& c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2) - b)/b) + 8*(a*b*d*f*x + a*b*d* \\
& e + a*b*f)*\cosh(d*x + c) + 4*(2*a*b*d*f*x + 2*a*b*d*e - (2*b^2*d*f*x + 2*b^ \\
& 2*d*e - b^2*f)*\cosh(d*x + c)^3 + 2*a*b*f + 6*(a*b*d*f*x + a*b*d*e - a*b*f)* \\
& \cosh(d*x + c)^2 - 2*((2*a^2 + b^2)*d^2*f*x^2 + 2*(2*a^2 + b^2)*d^2*e*x)*\cos \\
& h(d*x + c))*\sinh(d*x + c))/(b^3*d^2*\cosh(d*x + c)^2 + 2*b^3*d^2*\cosh(d*x + \\
& c)*\sinh(d*x + c) + b^3*d^2*\sinh(d*x + c)^2)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)\*\*2\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \cosh(dx + c)^2 \sinh(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^2*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)^2*sinh(d*x + c)/(b*sinh(d*x + c) + a), x)
```

$$3.341 \quad \int \frac{\cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=95

$$\frac{2a\sqrt{a^2+b^2} \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3d} + \frac{x(2a^2+b^2)}{2b^3} - \frac{\cosh(c+dx)(2a-b \sinh(c+dx))}{2b^2d}$$

[Out]  $((2*a^2 + b^2)*x)/(2*b^3) + (2*a*\text{Sqrt}[a^2 + b^2]*\text{ArcTanh}[(b - a*\text{Tanh}[(c + d*x)/2]]/\text{Sqrt}[a^2 + b^2])/(b^3*d) - (\text{Cosh}[c + d*x]*(2*a - b*\text{Sinh}[c + d*x]))/(2*b^2*d)$

**Rubi [A]** time = 0.180593, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$ , Rules used = {2865, 2735, 2660, 618, 204}

$$\frac{2a\sqrt{a^2+b^2} \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3d} + \frac{x(2a^2+b^2)}{2b^3} - \frac{\cosh(c+dx)(2a-b \sinh(c+dx))}{2b^2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Cosh}[c + d*x]^2*\text{Sinh}[c + d*x])/(a + b*\text{Sinh}[c + d*x]),x]$

[Out]  $((2*a^2 + b^2)*x)/(2*b^3) + (2*a*\text{Sqrt}[a^2 + b^2]*\text{ArcTanh}[(b - a*\text{Tanh}[(c + d*x)/2]]/\text{Sqrt}[a^2 + b^2])/(b^3*d) - (\text{Cosh}[c + d*x]*(2*a - b*\text{Sinh}[c + d*x]))/(2*b^2*d)$

### Rule 2865

$\text{Int}[(\cos[(e_.) + (f_.)*(x_.)]*(g_.))^{(p_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]), x\_Symbol] \rightarrow \text{Simp}[(g*(g*\text{Cos}[e + f*x])^{(p-1)}*(a + b*\text{Sin}[e + f*x])^{(m+1)}*(b*c*(m+p+1) - a*d*(p + b*d*(m+p)*\text{Sin}[e + f*x]))/(b^2*f*(m+p)*(m+p+1)), x] + \text{Dist}[(g^{2*(p-1)})/(b^2*(m+p)*(m+p+1)), \text{Int}[(g*\text{Cos}[e + f*x])^{(p-2)}*(a + b*\text{Sin}[e + f*x])^m*\text{Simp}[b*(a*d*m + b*c*(m+p+1)) + (a*b*c*(m+p+1) - d*(a^2*p - b^2*(m+p))]*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{GtQ}[p, 1] \&\& \text{NeQ}[m + p, 0] \&\& \text{NeQ}[m + p + 1, 0] \&\& \text{IntegerQ}[2*m]$

Rule 2735

Int[((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])/((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] := Simp[(b\*x)/d, x] - Dist[(b\*c - a\*d)/d, Int[1/(c + d\*Sin[e + f\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b\*c - a\*d, 0]

Rule 2660

Int[((a\_) + (b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + 2\*b\*e\*x + a\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx &= -\frac{\cosh(c+dx)(2a-b \sinh(c+dx))}{2b^2d} + \frac{i \int \frac{iab-i(2a^2+b^2) \sinh(c+dx)}{a+b \sinh(c+dx)} dx}{2b^2} \\
 &= \frac{(2a^2+b^2)x}{2b^3} - \frac{\cosh(c+dx)(2a-b \sinh(c+dx))}{2b^2d} - \frac{(a(a^2+b^2)) \int \frac{1}{a+b \sinh(c+dx)} dx}{b^3} \\
 &= \frac{(2a^2+b^2)x}{2b^3} - \frac{\cosh(c+dx)(2a-b \sinh(c+dx))}{2b^2d} + \frac{(2ia(a^2+b^2)) \text{Subst}\left(\int \frac{1}{a-2ibx+b^2} dx\right)}{b^3d} \\
 &= \frac{(2a^2+b^2)x}{2b^3} - \frac{\cosh(c+dx)(2a-b \sinh(c+dx))}{2b^2d} - \frac{(4ia(a^2+b^2)) \text{Subst}\left(\int \frac{1}{-4(a^2+b^2)-2ibx+b^2} dx\right)}{b^3d} \\
 &= \frac{(2a^2+b^2)x}{2b^3} + \frac{2a\sqrt{a^2+b^2} \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^3d} - \frac{\cosh(c+dx)(2a-b \sinh(c+dx))}{2b^2d}
 \end{aligned}$$

**Mathematica [A]** time = 0.427519, size = 109, normalized size = 1.15

$$\frac{8a\sqrt{-a^2 - b^2} \tan^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right) + 4a^2c + 4a^2dx - 4ab \cosh(c+dx) + b^2 \sinh(2(c+dx)) + 2b^2c + 2b^2dx}{4b^3d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cosh[c + d\*x]^2\*Sinh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (4\*a^2\*c + 2\*b^2\*c + 4\*a^2\*d\*x + 2\*b^2\*d\*x + 8\*a\*Sqrt[-a^2 - b^2]\*ArcTan[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[-a^2 - b^2]] - 4\*a\*b\*Cosh[c + d\*x] + b^2\*Sinh[2\*(c + d\*x)])/(4\*b^3\*d)

**Maple [B]** time = 0.035, size = 260, normalized size = 2.7

$$-\frac{1}{2bd} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-2} + \frac{1}{2bd} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} - \frac{a}{db^2} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} + \frac{a^2}{db^3} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)^2\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] -1/2/d/b/(tanh(1/2\*d\*x+1/2\*c)+1)^2+1/2/d/b/(tanh(1/2\*d\*x+1/2\*c)+1)-1/d/b^2/(tanh(1/2\*d\*x+1/2\*c)+1)\*a+1/d/b^3\*ln(tanh(1/2\*d\*x+1/2\*c)+1)\*a^2+1/2/d/b\*ln(tanh(1/2\*d\*x+1/2\*c)+1)+1/2/d/b/(tanh(1/2\*d\*x+1/2\*c)-1)^2+1/2/d/b/(tanh(1/2\*d\*x+1/2\*c)-1)+1/d/b^2/(tanh(1/2\*d\*x+1/2\*c)-1)\*a-1/d/b^3\*ln(tanh(1/2\*d\*x+1/2\*c)-1)\*a^2-1/2/d/b\*ln(tanh(1/2\*d\*x+1/2\*c)-1)-2/d\*a\*(a^2+b^2)^(1/2)/b^3\*arctanh(1/2\*(2\*a\*tanh(1/2\*d\*x+1/2\*c)-2\*b)/(a^2+b^2)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.27919, size = 1150, normalized size = 12.11

$$b^2 \cosh(dx + c)^4 + b^2 \sinh(dx + c)^4 + 4(2a^2 + b^2)dx \cosh(dx + c)^2 - 4ab \cosh(dx + c)^3 + 4(b^2 \cosh(dx + c) - ab) \sinh(dx + c)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\frac{1}{8}(b^2 \cosh(dx + c)^4 + b^2 \sinh(dx + c)^4 + 4(2a^2 + b^2)dx \cosh(dx + c)^2 - 4ab \cosh(dx + c)^3 + 4(b^2 \cosh(dx + c) - ab) \sinh(dx + c)^3 - 4ab \cosh(dx + c) + 2(3b^2 \cosh(dx + c)^2 + 2(2a^2 + b^2)dx \sinh(dx + c) - 6ab \cosh(dx + c)) \sinh(dx + c)^2 + 8(a \cosh(dx + c)^2 + 2a \cosh(dx + c) \sinh(dx + c) + a \sinh(dx + c)^2) \sqrt{a^2 + b^2} \log((b^2 \cosh(dx + c)^2 + b^2 \sinh(dx + c)^2 + 2ab \cosh(dx + c) + 2a^2 + b^2 + 2(b^2 \cosh(dx + c) + ab) \sinh(dx + c) + 2 \sqrt{a^2 + b^2} (b \cosh(dx + c) + b \sinh(dx + c) + a)) / (b \cosh(dx + c)^2 + b \sinh(dx + c)^2 + 2a \cosh(dx + c) + 2(b \cosh(dx + c) + a) \sinh(dx + c) - b)) - b^2 + 4(b^2 \cosh(dx + c)^3 + 2(2a^2 + b^2)dx \cosh(dx + c) - 3ab \cosh(dx + c)^2 - ab) \sinh(dx + c)) / (b^3 d \cosh(dx + c)^2 + 2b^3 d \cosh(dx + c) \sinh(dx + c) + b^3 d \sinh(dx + c)^2)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*\*2\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out



**Giac [A]** time = 1.15216, size = 223, normalized size = 2.35

$$\frac{(2a^2 + b^2)(dx + c)}{2b^3d} - \frac{(4abe^{(dx+c)} + b^2)e^{(-2dx-2c)}}{8b^3d} - \frac{(a^3 + ab^2) \log\left(\frac{|2be^{(dx+c)} + 2a - 2\sqrt{a^2+b^2}|}{|2be^{(dx+c)} + 2a + 2\sqrt{a^2+b^2}|}\right)}{\sqrt{a^2 + b^2}b^3d} + \frac{bde^{(2dx+2c)} - 4ade^{(dx+c)}}{8b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] 1/2\*(2\*a^2 + b^2)\*(d\*x + c)/(b^3\*d) - 1/8\*(4\*a\*b\*e^(d\*x + c) + b^2)\*e^(-2\*d\*x - 2\*c)/(b^3\*d) - (a^3 + a\*b^2)\*log(abs(2\*b\*e^(d\*x + c) + 2\*a - 2\*sqrt(a^2 + b^2)))/abs(2\*b\*e^(d\*x + c) + 2\*a + 2\*sqrt(a^2 + b^2))/(sqrt(a^2 + b^2)\*b^3\*d) + 1/8\*(b\*d\*e^(2\*d\*x + 2\*c) - 4\*a\*d\*e^(d\*x + c))/(b^2\*d^2)

$$3.342 \quad \int \frac{\cosh^2(c+dx) \sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable} \left( \frac{\sinh(c+dx) \cosh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[(Cosh[c + d\*x]^2\* Sinh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x]))], x]

**Rubi [A]** time = 0.0815182, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cosh^2(c+dx) \sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Cosh[c + d\*x]^2\* Sinh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int] [(Cosh[c + d\*x]^2\* Sinh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x]))], x]

Rubi steps

$$\int \frac{\cosh^2(c+dx) \sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\cosh^2(c+dx) \sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.001, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Cosh[c + d\*x]^2\* Sinh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] \$Aborted

**Maple [A]** time = 0.109, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx+c))^2 \sinh(dx+c)}{(fx+e)(a+b\sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)`

[Out] `int(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-2(a^3e^c + ab^2e^c) \int \frac{e^{(dx)}}{b^4fx + b^4e - (b^4fxe^{(2c)} + b^4ee^{(2c)})e^{(2dx)} - 2(ab^3fxe^c + ab^3ee^c)e^{(dx)}} dx - \frac{e^{(-2c + \frac{2de}{f})} E_1\left(\frac{2(fx+e)d}{f}\right)}{4bf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `-2*(a^3*e^c + a*b^2*e^c)*integrate(-e^(d*x)/(b^4*f*x + b^4*e - (b^4*f*x*e^(2*c) + b^4*e*e^(2*c))*e^(2*d*x) - 2*(a*b^3*f*x*e^c + a*b^3*e*e^c)*e^(d*x)), x) - 1/4*e^(-2*c + 2*d*e/f)*exp_integral_e(1, 2*(f*x + e)*d/f)/(b*f) - 1/2*a*e^(-c + d*e/f)*exp_integral_e(1, (f*x + e)*d/f)/(b^2*f) + 1/2*a*e^(c - d*e/f)*exp_integral_e(1, -(f*x + e)*d/f)/(b^2*f) - 1/4*e^(2*c - 2*d*e/f)*exp_integral_e(1, -2*(f*x + e)*d/f)/(b*f) + 1/2*(2*a^2 + b^2)*log(f*x + e)/(b^3*f)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx+c)^2 \sinh(dx+c)}{afx + ae + (bfx + be) \sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^2*sinh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^2 \sinh(dx+c)}{(fx+e)(b \sinh(dx+c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^2*sinh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

$$3.343 \quad \int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=864

$$\frac{a(a^2+b^2)(e+fx)^4}{4b^4f} - \frac{a \sinh^2(c+dx)(e+fx)^3}{2b^2d} - \frac{a(a^2+b^2) \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)(e+fx)^3}{b^4d} - \frac{a(a^2+b^2) \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)(e+fx)^3}{b^4d}$$

```
[Out] (-3*a*f^3*x)/(8*b^2*d^3) - (a*(e+f*x)^3)/(4*b^2*d) + (a*(a^2+b^2)*(e+f*x)^4)/(4*b^4*f) - (6*a^2*f^3*Cosh[c+d*x])/(b^3*d^4) - (40*f^3*Cosh[c+d*x])/(9*b*d^4) - (3*a^2*f*(e+f*x)^2*Cosh[c+d*x])/(b^3*d^2) - (2*f*(e+f*x)^2*Cosh[c+d*x])/(b*d^2) - (2*f^3*Cosh[c+d*x]^3)/(27*b*d^4) - (f*(e+f*x)^2*Cosh[c+d*x]^3)/(3*b*d^2) - (a*(a^2+b^2)*(e+f*x)^3*Log[1+(b*E^(c+d*x))/(a-Sqrt[a^2+b^2]])/(b^4*d) - (a*(a^2+b^2)*(e+f*x)^3*Log[1+(b*E^(c+d*x))/(a+Sqrt[a^2+b^2]])/(b^4*d) - (3*a*(a^2+b^2)*f*(e+f*x)^2*PolyLog[2,-((b*E^(c+d*x))/(a-Sqrt[a^2+b^2]))])/(b^4*d^2) - (3*a*(a^2+b^2)*f*(e+f*x)^2*PolyLog[2,-((b*E^(c+d*x))/(a+Sqrt[a^2+b^2]))])/(b^4*d^2) + (6*a*(a^2+b^2)*f^2*(e+f*x)*PolyLog[3,-((b*E^(c+d*x))/(a-Sqrt[a^2+b^2]))])/(b^4*d^3) + (6*a*(a^2+b^2)*f^2*(e+f*x)*PolyLog[3,-((b*E^(c+d*x))/(a+Sqrt[a^2+b^2]))])/(b^4*d^3) - (6*a*(a^2+b^2)*f^3*PolyLog[4,-((b*E^(c+d*x))/(a-Sqrt[a^2+b^2]))])/(b^4*d^4) - (6*a*(a^2+b^2)*f^3*PolyLog[4,-((b*E^(c+d*x))/(a+Sqrt[a^2+b^2]))])/(b^4*d^4) + (6*a^2*f^2*(e+f*x)*Sinh[c+d*x])/(b^3*d^3) + (40*f^2*(e+f*x)*Sinh[c+d*x])/(9*b*d^3) + (a^2*(e+f*x)^3*Sinh[c+d*x])/(b^3*d) + (2*(e+f*x)^3*Sinh[c+d*x])/(3*b*d) + (3*a*f^3*Cosh[c+d*x]*Sinh[c+d*x])/(8*b^2*d^4) + (3*a*f*(e+f*x)^2*Cosh[c+d*x]*Sinh[c+d*x])/(4*b^2*d^2) + (2*f^2*(e+f*x)*Cosh[c+d*x]^2*Sinh[c+d*x])/(9*b*d^3) + ((e+f*x)^3*Cosh[c+d*x]^2*Sinh[c+d*x])/(3*b*d) - (3*a*f^2*(e+f*x)*Sinh[c+d*x]^2)/(4*b^2*d^3) - (a*(e+f*x)^3*Sinh[c+d*x]^2)/(2*b^2*d)
```

**Rubi [A]** time = 1.11732, antiderivative size = 864, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 16, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$ , Rules used = {5579, 3311, 3296, 2638, 3310, 5565, 5446, 32, 2635, 8, 5561, 2190, 2531, 6609, 2282, 6589}

$$\frac{a(a^2+b^2)(e+fx)^4}{4b^4f} - \frac{a \sinh^2(c+dx)(e+fx)^3}{2b^2d} - \frac{a(a^2+b^2) \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)(e+fx)^3}{b^4d} - \frac{a(a^2+b^2) \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)(e+fx)^3}{b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[((e+f*x)^3*Cosh[c+d*x]^3*Sinh[c+d*x])/(a+b*Sinh[c+d*x]),x]
```

```
[Out] (-3*a*f^3*x)/(8*b^2*d^3) - (a*(e + f*x)^3)/(4*b^2*d) + (a*(a^2 + b^2)*(e + f*x)^4)/(4*b^4*f) - (6*a^2*f^3*Cosh[c + d*x])/(b^3*d^4) - (40*f^3*Cosh[c + d*x])/(9*b*d^4) - (3*a^2*f*(e + f*x)^2*Cosh[c + d*x])/(b^3*d^2) - (2*f*(e + f*x)^2*Cosh[c + d*x])/(b*d^2) - (2*f^3*Cosh[c + d*x]^3)/(27*b*d^4) - (f*(e + f*x)^2*Cosh[c + d*x]^3)/(3*b*d^2) - (a*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) - (3*a*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^2) - (3*a*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^2) + (6*a*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^3) + (6*a*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^3) - (6*a*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^4) - (6*a*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^4) + (6*a^2*f^2*(e + f*x)*Sinh[c + d*x])/(b^3*d^3) + (40*f^2*(e + f*x)*Sinh[c + d*x])/(9*b*d^3) + (a^2*(e + f*x)^3*Sinh[c + d*x])/(b^3*d) + (2*(e + f*x)^3*Sinh[c + d*x])/(3*b*d) + (3*a*f^3*Cosh[c + d*x]*Sinh[c + d*x])/(8*b^2*d^4) + (3*a*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^2*d^2) + (2*f^2*(e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(9*b*d^3) + ((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b*d) - (3*a*f^2*(e + f*x)*Sinh[c + d*x]^2)/(4*b^2*d^3) - (a*(e + f*x)^3*Sinh[c + d*x]^2)/(2*b^2*d)
```

### Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Sinh[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sinh[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sinh[e + f*x])^n, x], x] - Simp[(b*(c + d*x)^m*Cosh[e + f*x]*(b*Sinh[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cosh[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cosh[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
Simp[(d*(b*Ssin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Ssin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Ssin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_))*((e_.) + (f_.)*(x_))^(m_.)/((a_.) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x
]*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589



```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e + fx)^3 \cosh^3(c + dx) \sinh(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\int (e + fx)^3 \cosh^3(c + dx) dx}{b} - \frac{a \int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= -\frac{f(e + fx)^2 \cosh^3(c + dx)}{3bd^2} + \frac{(e + fx)^3 \cosh^2(c + dx) \sinh(c + dx)}{3bd} + \frac{a^2 \int (e + fx)^3 \cosh^3(c + dx) dx}{b^2} \\
&= \frac{a(a^2 + b^2)(e + fx)^4}{4b^4 f} - \frac{2f^3 \cosh^3(c + dx)}{27bd^4} - \frac{f(e + fx)^2 \cosh^3(c + dx)}{3bd^2} + \frac{a^2 \int (e + fx)^3 \cosh^3(c + dx) dx}{b^2} \\
&= \frac{a(a^2 + b^2)(e + fx)^4}{4b^4 f} - \frac{3a^2 f(e + fx)^2 \cosh(c + dx)}{b^3 d^2} - \frac{2f(e + fx)^2 \cosh(c + dx)}{bd^2} + \frac{a^2 \int (e + fx)^3 \cosh^3(c + dx) dx}{b^2} \\
&= -\frac{a(e + fx)^3}{4b^2 d} + \frac{a(a^2 + b^2)(e + fx)^4}{4b^4 f} - \frac{4f^3 \cosh(c + dx)}{9bd^4} - \frac{3a^2 f(e + fx)^2}{b^3 d^2} + \frac{a^2 \int (e + fx)^3 \cosh^3(c + dx) dx}{b^2} \\
&= -\frac{3af^3 x}{8b^2 d^3} - \frac{a(e + fx)^3}{4b^2 d} + \frac{a(a^2 + b^2)(e + fx)^4}{4b^4 f} - \frac{6a^2 f^3 \cosh(c + dx)}{b^3 d^4} - \frac{40a^2 \int (e + fx)^3 \cosh^3(c + dx) dx}{b^2} \\
&= -\frac{3af^3 x}{8b^2 d^3} - \frac{a(e + fx)^3}{4b^2 d} + \frac{a(a^2 + b^2)(e + fx)^4}{4b^4 f} - \frac{6a^2 f^3 \cosh(c + dx)}{b^3 d^4} - \frac{40a^2 \int (e + fx)^3 \cosh^3(c + dx) dx}{b^2} \\
&= -\frac{3af^3 x}{8b^2 d^3} - \frac{a(e + fx)^3}{4b^2 d} + \frac{a(a^2 + b^2)(e + fx)^4}{4b^4 f} - \frac{6a^2 f^3 \cosh(c + dx)}{b^3 d^4} - \frac{40a^2 \int (e + fx)^3 \cosh^3(c + dx) dx}{b^2}
\end{aligned}$$

**Mathematica [B]** time = 51.0765, size = 7460, normalized size = 8.63

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]), x]
```

```
[Out] Result too large to show
```

---

**Maple [F]** time = 0.24, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 (\cosh(dx + c))^3 \sinh(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*cosh(d\*x+c)^3\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*cosh(d\*x+c)^3\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)^3\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/24*e^3*((3*a*b*e^{-d*x - c} - b^2 - 3*(4*a^2 + 3*b^2)*e^{-2*d*x - 2*c})* \\ & e^{(3*d*x + 3*c)/(b^3*d) + 24*(a^3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^{-2* \\ & d*x - 2*c} + b^2*e^{-3*d*x - 3*c} + 3*(4*a^2 + 3*b^2)*e^{-d*x - c}))/b^3*d \\ & + 24*(a^3 + a*b^2)*\log(-2*a*e^{-d*x - c} + b*e^{-2*d*x - 2*c} - b)/b^4*d \\ & - 1/864*(216*(a^3*d^4*f^3*e^{(3*c)} + a*b^2*d^4*f^3*e^{(3*c)})*x^4 + 864*(a^3*d^4*e^2*f*e^{(3*c)} + a*b^2*d^4*e^2*f*e^{(3*c)})*x^3 + 1296*(a^3*d^4*e^2*f*e^{(3*c)} + a*b^2*d^4*e^2*f*e^{(3*c)})*x^2 - 4*(9*b^3*d^3*f^3*x^3*e^{(6*c)} + 9*(3*d^3*e*f^2 - d^2*f^3)*b^3*x^2*e^{(6*c)} + 3*(9*d^3*e^2*f - 6*d^2*e*f^2 + 2*d*f^3)*b^3*x*e^{(6*c)} - (9*d^2*e^2*f - 6*d*e*f^2 + 2*f^3)*b^3*e^{(6*c)})*e^{(3*d*x)} + 27*(4*a*b^2*d^3*f^3*x^3*e^{(5*c)} + 6*(2*d^3*e*f^2 - d^2*f^3)*a*b^2*x^2*e^{(5*c)} + 6*(2*d^3*e^2*f - 2*d^2*e*f^2 + d*f^3)*a*b^2*x*e^{(5*c)} - 3*(2*d^2*e^2*f - 2*d*e*f^2 + f^3)*a*b^2*e^{(5*c)})*e^{(2*d*x)} + 108*(12*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a^2*b*e^{(4*c)} + 9*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*b^3*e^{(4*c)} - (4*a^2*b*d^3*f^3*e^{(4*c)} + 3*b^3*d^3*f^3*e^{(4*c)})*x^3 - 3*(4*(d^3*e*f^2 - d^2*f^3)*a^2*b*e^{(4*c)} + 3*(d^3*e*f^2 - d^2*f^3)*b^3*e^{(4*c)})*x^2 - 3*(4*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*a^2*b*e^{(4*c)} + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b^3*e^{(4*c)})*x)*e^{(d*x)} + 108*(12*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*a^2*b*e^{(2*c)} + 9*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*b^3*e^{(2*c)} + (4*a^2*b*d^3*f^3*e^{(2*c)} + 3*b^3*d^3*f^3*e^{(2*c)})*x^3 + 3*(4*(d^3*e*f^2 + d^2*f^3)*a^2*b*e^{(2*c)} + 3*(d^3*e*f^2 + d^2*f^3)*b^3*e^{(2*c)})*x^2 + 3*(4*(d^3*e \\ & \end{aligned}$$

$$\begin{aligned} &^2*f + 2*d^2*e*f^2 + 2*d*f^3)*a^2*b*e^{(2*c)} + 3*(d^3*e^2*f + 2*d^2*e*f^2 + \\ &2*d*f^3)*b^3*e^{(2*c)})*x)*e^{(-d*x)} + 27*(4*a*b^2*d^3*f^3*x^3*e^c + 6*(2*d^3* \\ &e*f^2 + d^2*f^3)*a*b^2*x^2*e^c + 6*(2*d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*a*b^ \\ &2*x*e^c + 3*(2*d^2*e^2*f + 2*d*e*f^2 + f^3)*a*b^2*e^c)*e^{(-2*d*x)} + 4*(9*b^ \\ &3*d^3*f^3*x^3 + 9*(3*d^3*e*f^2 + d^2*f^3)*b^3*x^2 + 3*(9*d^3*e^2*f + 6*d^2* \\ &e*f^2 + 2*d*f^3)*b^3*x + (9*d^2*e^2*f + 6*d*e*f^2 + 2*f^3)*b^3)*e^{(-3*d*x)} \\ &)*e^{(-3*c)}/(b^4*d^4) + \text{integrate}(-2*((a^3*b*f^3 + a*b^3*f^3)*x^3 + 3*(a^3*b* \\ &e*f^2 + a*b^3*e*f^2)*x^2 + 3*(a^3*b*e^2*f + a*b^3*e^2*f)*x - ((a^4*f^3*e^c \\ &+ a^2*b^2*f^3*e^c)*x^3 + 3*(a^4*e*f^2*e^c + a^2*b^2*e*f^2*e^c)*x^2 + 3*(a^4 \\ &*e^2*f*e^c + a^2*b^2*e^2*f*e^c)*x)*e^{(d*x)})/(b^5*e^{(2*d*x + 2*c)} + 2*a*b^4* \\ &e^{(d*x + c)} - b^5), x) \end{aligned}$$

**Fricas [C]** time = 3.58887, size = 17256, normalized size = 19.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] 
$$\begin{aligned} &-1/864*(36*b^3*d^3*f^3*x^3 + 36*b^3*d^3*e^3 + 36*b^3*d^2*e^2*f + 24*b^3*d*e* \\ &f^2 - 4*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f \\ &^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f \\ &- 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*\cosh(d*x + c)^6 - 4*(9*b^3*d^3*f^3*x^3 \\ &+ 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d \\ &^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3* \\ &d*f^3)*x)*\sinh(d*x + c)^6 + 8*b^3*f^3 + 27*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d \\ &^3*e^3 - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3 \\ &*e*f^2 - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a* \\ &b^2*d*f^3)*x)*\cosh(d*x + c)^5 + 3*(36*a*b^2*d^3*f^3*x^3 + 36*a*b^2*d^3*e^3 \\ &- 54*a*b^2*d^2*e^2*f + 54*a*b^2*d*e*f^2 - 27*a*b^2*f^3 + 54*(2*a*b^2*d^3*e* \\ &f^2 - a*b^2*d^2*f^3)*x^2 + 54*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^ \\ &2*d*f^3)*x - 8*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3 \\ &*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3 \\ &*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 - \\ &108*((4*a^2*b + 3*b^3)*d^3*f^3*x^3 + (4*a^2*b + 3*b^3)*d^3*e^3 - 3*(4*a^2* \\ &b + 3*b^3)*d^2*e^2*f + 6*(4*a^2*b + 3*b^3)*d*e*f^2 - 6*(4*a^2*b + 3*b^3)*f^ \\ &3 + 3*((4*a^2*b + 3*b^3)*d^3*e*f^2 - (4*a^2*b + 3*b^3)*d^2*f^3)*x^2 + 3*((4 \\ &*a^2*b + 3*b^3)*d^3*e^2*f - 2*(4*a^2*b + 3*b^3)*d^2*e*f^2 + 2*(4*a^2*b + 3* \\ &b^3)*d*f^3)*x)*\cosh(d*x + c)^4 - 3*(36*(4*a^2*b + 3*b^3)*d^3*f^3*x^3 + 36*( \\ &4*a^2*b + 3*b^3)*d^3*e^3 - 108*(4*a^2*b + 3*b^3)*d^2*e^2*f + 216*(4*a^2*b + \end{aligned}$$

$$\begin{aligned}
& 3*b^3)*d*e*f^2 - 216*(4*a^2*b + 3*b^3)*f^3 + 108*((4*a^2*b + 3*b^3)*d^3*e* \\
& f^2 - (4*a^2*b + 3*b^3)*d^2*f^3)*x^2 + 20*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^ \\
& 3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3* \\
& d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*\cosh( \\
& d*x + c)^2 + 108*((4*a^2*b + 3*b^3)*d^3*e^2*f - 2*(4*a^2*b + 3*b^3)*d^2*e*f \\
& ^2 + 2*(4*a^2*b + 3*b^3)*d*f^3)*x - 45*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e \\
& ^3 - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f \\
& ^2 - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2* \\
& d*f^3)*x)*\cosh(d*x + c))*\sinh(d*x + c)^4 - 216*((a^3 + a*b^2)*d^4*f^3*x^4 + \\
& 4*(a^3 + a*b^2)*d^4*e*f^2*x^3 + 6*(a^3 + a*b^2)*d^4*e^2*f*x^2 + 4*(a^3 + a \\
& *b^2)*d^4*e^3*x + 8*(a^3 + a*b^2)*c*d^3*e^3 - 12*(a^3 + a*b^2)*c^2*d^2*e^2* \\
& f + 8*(a^3 + a*b^2)*c^3*d*e*f^2 - 2*(a^3 + a*b^2)*c^4*f^3)*\cosh(d*x + c)^3 \\
& - 2*(108*(a^3 + a*b^2)*d^4*f^3*x^4 + 432*(a^3 + a*b^2)*d^4*e*f^2*x^3 + 648* \\
& (a^3 + a*b^2)*d^4*e^2*f*x^2 + 432*(a^3 + a*b^2)*d^4*e^3*x + 864*(a^3 + a*b^ \\
& 2)*c*d^3*e^3 - 1296*(a^3 + a*b^2)*c^2*d^2*e^2*f + 864*(a^3 + a*b^2)*c^3*d*e \\
& *f^2 - 216*(a^3 + a*b^2)*c^4*f^3 + 40*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - \\
& 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2* \\
& f^3)*x^2 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*\cosh(d*x \\
& + c)^3 - 135*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 - 6*a*b^2*d^2*e^2*f + 6 \\
& *a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 - a*b^2*d^2*f^3)*x^2 + \\
& 6*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x)*\cosh(d*x + c)^2 \\
& + 216*((4*a^2*b + 3*b^3)*d^3*f^3*x^3 + (4*a^2*b + 3*b^3)*d^3*e^3 - 3*(4*a^2 \\
& *b + 3*b^3)*d^2*e^2*f + 6*(4*a^2*b + 3*b^3)*d*e*f^2 - 6*(4*a^2*b + 3*b^3)*f \\
& ^3 + 3*((4*a^2*b + 3*b^3)*d^3*e*f^2 - (4*a^2*b + 3*b^3)*d^2*f^3)*x^2 + 3*(( \\
& 4*a^2*b + 3*b^3)*d^3*e^2*f - 2*(4*a^2*b + 3*b^3)*d^2*e*f^2 + 2*(4*a^2*b + 3 \\
& *b^3)*d*f^3)*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 36*(3*b^3*d^3*e*f^2 + b^3* \\
& d^2*f^3)*x^2 + 108*((4*a^2*b + 3*b^3)*d^3*f^3*x^3 + (4*a^2*b + 3*b^3)*d^3*e \\
& ^3 + 3*(4*a^2*b + 3*b^3)*d^2*e^2*f + 6*(4*a^2*b + 3*b^3)*d*e*f^2 + 6*(4*a^2 \\
& *b + 3*b^3)*f^3 + 3*((4*a^2*b + 3*b^3)*d^3*e*f^2 + (4*a^2*b + 3*b^3)*d^2*f^ \\
& 3)*x^2 + 3*((4*a^2*b + 3*b^3)*d^3*e^2*f + 2*(4*a^2*b + 3*b^3)*d^2*e*f^2 + 2 \\
& *(4*a^2*b + 3*b^3)*d*f^3)*x)*\cosh(d*x + c)^2 + 6*(18*(4*a^2*b + 3*b^3)*d^3* \\
& f^3*x^3 + 18*(4*a^2*b + 3*b^3)*d^3*e^3 + 54*(4*a^2*b + 3*b^3)*d^2*e^2*f + 1 \\
& 08*(4*a^2*b + 3*b^3)*d*e*f^2 - 10*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^ \\
& 3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3) \\
& *x^2 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*\cosh(d*x + c) \\
& ^4 + 108*(4*a^2*b + 3*b^3)*f^3 + 45*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 \\
& - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 \\
& - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2*d*f \\
& ^3)*x)*\cosh(d*x + c)^3 + 54*((4*a^2*b + 3*b^3)*d^3*e*f^2 + (4*a^2*b + 3*b^3 \\
& )*d^2*f^3)*x^2 - 108*((4*a^2*b + 3*b^3)*d^3*f^3*x^3 + (4*a^2*b + 3*b^3)*d^3 \\
& *e^3 - 3*(4*a^2*b + 3*b^3)*d^2*e^2*f + 6*(4*a^2*b + 3*b^3)*d*e*f^2 - 6*(4*a \\
& ^2*b + 3*b^3)*f^3 + 3*((4*a^2*b + 3*b^3)*d^3*e*f^2 - (4*a^2*b + 3*b^3)*d^2* \\
& f^3)*x^2 + 3*((4*a^2*b + 3*b^3)*d^3*e^2*f - 2*(4*a^2*b + 3*b^3)*d^2*e*f^2 + \\
& 2*(4*a^2*b + 3*b^3)*d*f^3)*x)*\cosh(d*x + c)^2 + 54*((4*a^2*b + 3*b^3)*d^3* \\
& e^2*f + 2*(4*a^2*b + 3*b^3)*d^2*e*f^2 + 2*(4*a^2*b + 3*b^3)*d*f^3)*x - 108*
\end{aligned}$$

$$\begin{aligned}
& ((a^3 + a*b^2)*d^4*f^3*x^4 + 4*(a^3 + a*b^2)*d^4*e*f^2*x^3 + 6*(a^3 + a*b^2) \\
& )*d^4*e^2*f*x^2 + 4*(a^3 + a*b^2)*d^4*e^3*x + 8*(a^3 + a*b^2)*c*d^3*e^3 - 1 \\
& 2*(a^3 + a*b^2)*c^2*d^2*e^2*f + 8*(a^3 + a*b^2)*c^3*d*e*f^2 - 2*(a^3 + a*b^ \\
& 2)*c^4*f^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 12*(9*b^3*d^3*e^2*f + 6*b^3*d^ \\
& 2*e*f^2 + 2*b^3*d*f^3)*x + 27*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 + 6*a* \\
& b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 + 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 + a*b^ \\
& 2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f + 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x) \\
& *\cosh(d*x + c) + 2592*(((a^3 + a*b^2)*d^2*f^3*x^2 + 2*(a^3 + a*b^2)*d^2*e*f \\
& ^2*x + (a^3 + a*b^2)*d^2*e^2*f)*\cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d^2*f^3*x \\
& ^2 + 2*(a^3 + a*b^2)*d^2*e*f^2*x + (a^3 + a*b^2)*d^2*e^2*f)*\cosh(d*x + c)^ \\
& 2*\sinh(d*x + c) + 3*((a^3 + a*b^2)*d^2*f^3*x^2 + 2*(a^3 + a*b^2)*d^2*e*f^2*x \\
& + (a^3 + a*b^2)*d^2*e^2*f)*\cosh(d*x + c)*\sinh(d*x + c)^2 + ((a^3 + a*b^2) \\
& *d^2*f^3*x^2 + 2*(a^3 + a*b^2)*d^2*e*f^2*x + (a^3 + a*b^2)*d^2*e^2*f)*\sinh( \\
& d*x + c)^3)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b \\
& *\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2) - b}/b + 1) + 2592*(((a^3 + a*b^2)*d^ \\
& 2*f^3*x^2 + 2*(a^3 + a*b^2)*d^2*e*f^2*x + (a^3 + a*b^2)*d^2*e^2*f)*\cosh(d*x \\
& + c)^3 + 3*((a^3 + a*b^2)*d^2*f^3*x^2 + 2*(a^3 + a*b^2)*d^2*e*f^2*x + (a^3 \\
& + a*b^2)*d^2*e^2*f)*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*((a^3 + a*b^2)*d^2*f \\
& ^3*x^2 + 2*(a^3 + a*b^2)*d^2*e*f^2*x + (a^3 + a*b^2)*d^2*e^2*f)*\cosh(d*x + \\
& c)*\sinh(d*x + c)^2 + ((a^3 + a*b^2)*d^2*f^3*x^2 + 2*(a^3 + a*b^2)*d^2*e*f^2 \\
& *x + (a^3 + a*b^2)*d^2*e^2*f)*\sinh(d*x + c)^3)*\operatorname{dilog}((a*\cosh(d*x + c) + a*s \\
& \sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2) - \\
& b}/b + 1) + 864*(((a^3 + a*b^2)*d^3*e^3 - 3*(a^3 + a*b^2)*c*d^2*e^2*f + 3*( \\
& a^3 + a*b^2)*c^2*d*e*f^2 - (a^3 + a*b^2)*c^3*f^3)*\cosh(d*x + c)^3 + 3*((a^3 \\
& + a*b^2)*d^3*e^3 - 3*(a^3 + a*b^2)*c*d^2*e^2*f + 3*(a^3 + a*b^2)*c^2*d*e*f \\
& ^2 - (a^3 + a*b^2)*c^3*f^3)*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*((a^3 + a*b^2) \\
& )*d^3*e^3 - 3*(a^3 + a*b^2)*c*d^2*e^2*f + 3*(a^3 + a*b^2)*c^2*d*e*f^2 - (a^ \\
& 3 + a*b^2)*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c)^2 + ((a^3 + a*b^2)*d^3*e^3 \\
& - 3*(a^3 + a*b^2)*c*d^2*e^2*f + 3*(a^3 + a*b^2)*c^2*d*e*f^2 - (a^3 + a*b^2) \\
& *c^3*f^3)*\sinh(d*x + c)^3)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b* \\
& \sqrt{((a^2 + b^2)/b^2) + 2*a}) + 864*(((a^3 + a*b^2)*d^3*e^3 - 3*(a^3 + a*b^2) \\
& )*c*d^2*e^2*f + 3*(a^3 + a*b^2)*c^2*d*e*f^2 - (a^3 + a*b^2)*c^3*f^3)*\cosh(d \\
& *x + c)^3 + 3*((a^3 + a*b^2)*d^3*e^3 - 3*(a^3 + a*b^2)*c*d^2*e^2*f + 3*(a^3 \\
& + a*b^2)*c^2*d*e*f^2 - (a^3 + a*b^2)*c^3*f^3)*\cosh(d*x + c)^2*\sinh(d*x + c \\
& ) + 3*((a^3 + a*b^2)*d^3*e^3 - 3*(a^3 + a*b^2)*c*d^2*e^2*f + 3*(a^3 + a*b^2) \\
& )*c^2*d*e*f^2 - (a^3 + a*b^2)*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c)^2 + ((a^ \\
& 3 + a*b^2)*d^3*e^3 - 3*(a^3 + a*b^2)*c*d^2*e^2*f + 3*(a^3 + a*b^2)*c^2*d*e* \\
& f^2 - (a^3 + a*b^2)*c^3*f^3)*\sinh(d*x + c)^3)*\log(2*b*\cosh(d*x + c) + 2*b*s \\
& \sinh(d*x + c) - 2*b*\sqrt{((a^2 + b^2)/b^2) + 2*a}) + 864*(((a^3 + a*b^2)*d^3*f \\
& ^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a \\
& ^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f \\
& ^3)*\cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e* \\
& f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^ \\
& 3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*\cosh(d*x + c)^2*\sinh(d*x + \\
& c) + 3*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3
\end{aligned}$$

$$\begin{aligned}
& + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d* \\
& e*f^2 + (a^3 + a*b^2)*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c)^2 + ((a^3 + a*b^ \\
& 2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f* \\
& x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^ \\
& 2)*c^3*f^3)*\sinh(d*x + c)^3*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*c \\
& osh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + 864*(((a^3 \\
& + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3* \\
& e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 \\
& + a*b^2)*c^3*f^3)*\cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + \\
& a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2 \\
& *e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*\cosh(d*x + c) \\
& ^2*\sinh(d*x + c) + 3*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2 \\
& *x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + \\
& a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c)^2 \\
& + ((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b \\
& ^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 \\
& + (a^3 + a*b^2)*c^3*f^3)*\sinh(d*x + c)^3*\log(-(a*\cosh(d*x + c) + a*\sinh(d \\
& *x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) \\
& + 5184*(((a^3 + a*b^2)*f^3*\cosh(d*x + c)^3 + 3*(a^3 + a*b^2)*f^3*\cosh(d*x + \\
& c)^2*\sinh(d*x + c) + 3*(a^3 + a*b^2)*f^3*\cosh(d*x + c)*\sinh(d*x + c)^2 + ( \\
& a^3 + a*b^2)*f^3*\sinh(d*x + c)^3)*\text{polylog}(4, (a*\cosh(d*x + c) + a*\sinh(d*x \\
& + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) + 5184 \\
& *(((a^3 + a*b^2)*f^3*\cosh(d*x + c)^3 + 3*(a^3 + a*b^2)*f^3*\cosh(d*x + c)^2*s \\
& \sinh(d*x + c) + 3*(a^3 + a*b^2)*f^3*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a^3 + a \\
& *b^2)*f^3*\sinh(d*x + c)^3)*\text{polylog}(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - \\
& (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) - 5184*(((a^3 \\
& + a*b^2)*d*f^3*x + (a^3 + a*b^2)*d*e*f^2)*\cosh(d*x + c)^3 + 3*((a^3 + a*b^ \\
& 2)*d*f^3*x + (a^3 + a*b^2)*d*e*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*((a^3 \\
& + a*b^2)*d*f^3*x + (a^3 + a*b^2)*d*e*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^2 + \\
& ((a^3 + a*b^2)*d*f^3*x + (a^3 + a*b^2)*d*e*f^2)*\sinh(d*x + c)^3)*\text{polylog}(3, \\
& (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))* \\
& \sqrt{(a^2 + b^2)/b^2}))/b) - 5184*(((a^3 + a*b^2)*d*f^3*x + (a^3 + a*b^2)*d*e \\
& *f^2)*\cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d*f^3*x + (a^3 + a*b^2)*d*e*f^2)*c \\
& osh(d*x + c)^2*\sinh(d*x + c) + 3*((a^3 + a*b^2)*d*f^3*x + (a^3 + a*b^2)*d*e \\
& *f^2)*\cosh(d*x + c)*\sinh(d*x + c)^2 + ((a^3 + a*b^2)*d*f^3*x + (a^3 + a*b^2 \\
& )*d*e*f^2)*\sinh(d*x + c)^3)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - \\
& (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) + 3*(36*a*b^ \\
& 2*d^3*f^3*x^3 + 36*a*b^2*d^3*e^3 + 54*a*b^2*d^2*e^2*f + 54*a*b^2*d*e*f^2 + \\
& 27*a*b^2*f^3 - 8*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b \\
& ^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d \\
& ^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*\cosh(d*x + c)^5 + 45*(4*a*b^2* \\
& d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 - 3*a*b \\
& ^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f - \\
& 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x)*\cosh(d*x + c)^4 - 144*(((4*a^2*b + 3*b^ \\
& 3)*d^3*f^3*x^3 + (4*a^2*b + 3*b^3)*d^3*e^3 - 3*(4*a^2*b + 3*b^3)*d^2*e^2*f
\end{aligned}$$

$$\begin{aligned}
& + 6*(4*a^2*b + 3*b^3)*d*e*f^2 - 6*(4*a^2*b + 3*b^3)*f^3 + 3*((4*a^2*b + 3*b^3)*d^3*e*f^2 - (4*a^2*b + 3*b^3)*d^2*f^3)*x^2 + 3*((4*a^2*b + 3*b^3)*d^3*e^2*f - 2*(4*a^2*b + 3*b^3)*d^2*e*f^2 + 2*(4*a^2*b + 3*b^3)*d*f^3)*x)*\cosh(d*x + c)^3 + 54*(2*a*b^2*d^3*e*f^2 + a*b^2*d^2*f^3)*x^2 - 216*((a^3 + a*b^2)*d^4*f^3*x^4 + 4*(a^3 + a*b^2)*d^4*e*f^2*x^3 + 6*(a^3 + a*b^2)*d^4*e^2*f*x^2 + 4*(a^3 + a*b^2)*d^4*e^3*x + 8*(a^3 + a*b^2)*c*d^3*e^3 - 12*(a^3 + a*b^2)*c^2*d^2*e^2*f + 8*(a^3 + a*b^2)*c^3*d*e*f^2 - 2*(a^3 + a*b^2)*c^4*f^3)*\cosh(d*x + c)^2 + 54*(2*a*b^2*d^3*e^2*f + 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x + 72*((4*a^2*b + 3*b^3)*d^3*f^3*x^3 + (4*a^2*b + 3*b^3)*d^3*e^3 + 3*(4*a^2*b + 3*b^3)*d^2*e^2*f + 6*(4*a^2*b + 3*b^3)*d*e*f^2 + 6*(4*a^2*b + 3*b^3)*f^3 + 3*((4*a^2*b + 3*b^3)*d^3*e*f^2 + (4*a^2*b + 3*b^3)*d^2*f^3)*x^2 + 3*((4*a^2*b + 3*b^3)*d^3*e^2*f + 2*(4*a^2*b + 3*b^3)*d^2*e*f^2 + 2*(4*a^2*b + 3*b^3)*d*f^3)*x)*\cosh(d*x + c))*\sinh(d*x + c))/(b^4*d^4*\cosh(d*x + c)^3 + 3*b^4*d^4*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*b^4*d^4*\cosh(d*x + c)*\sinh(d*x + c))^2 + b^4*d^4*\sinh(d*x + c)^3)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cosh(d\*x+c)\*\*3\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c)^3 \sinh(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)^3\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*cosh(d\*x + c)^3\*sinh(d\*x + c)/(b\*sinh(d\*x + c) + a), x)

$$3.344 \quad \int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=636

$$\frac{2af(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2} - \frac{2af(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^2} + \frac{2af^2(a^2+b^2)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2} - \frac{2af^2(a^2+b^2)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^2} + \frac{2af^2(a^2+b^2)\text{PolyLog}\left(3, \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2} - \frac{2af^2(a^2+b^2)\text{PolyLog}\left(3, \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}$$

```
[Out] -(a*e*f*x)/(2*b^2*d) - (a*f^2*x^2)/(4*b^2*d) + (a*(a^2 + b^2)*(e + f*x)^3)/(3*b^4*f) - (2*a^2*f*(e + f*x)*Cosh[c + d*x])/(b^3*d^2) - (4*f*(e + f*x)*Cosh[c + d*x])/(3*b*d^2) - (2*f*(e + f*x)*Cosh[c + d*x]^3)/(9*b*d^2) - (a*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) - (2*a*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^2) - (2*a*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^2) + (2*a*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^3) + (2*a*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^3) + (2*a^2*f^2*Sinh[c + d*x])/(b^3*d^3) + (14*f^2*Sinh[c + d*x])/(9*b*d^3) + (a^2*(e + f*x)^2*Sinh[c + d*x])/(b^3*d) + (2*(e + f*x)^2*Sinh[c + d*x])/(3*b*d) + (a*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d^2) + ((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b*d) - (a*f^2*Sinh[c + d*x]^2)/(4*b^2*d^3) - (a*(e + f*x)^2*Sinh[c + d*x]^2)/(2*b^2*d) + (2*f^2*Sinh[c + d*x]^3)/(27*b*d^3)
```

**Rubi [A]** time = 0.875195, antiderivative size = 636, normalized size of antiderivative = 1., number of steps used = 23, number of rules used = 13, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$ , Rules used = {5579, 3311, 3296, 2637, 2633, 5565, 5446, 3310, 5561, 2190, 2531, 2282, 6589}

$$\frac{2af(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2} - \frac{2af(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^2} + \frac{2af^2(a^2+b^2)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2} - \frac{2af^2(a^2+b^2)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^2} + \frac{2af^2(a^2+b^2)\text{PolyLog}\left(3, \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{b^4d^2} - \frac{2af^2(a^2+b^2)\text{PolyLog}\left(3, \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(a*e*f*x)/(2*b^2*d) - (a*f^2*x^2)/(4*b^2*d) + (a*(a^2 + b^2)*(e + f*x)^3)/(3*b^4*f) - (2*a^2*f*(e + f*x)*Cosh[c + d*x])/(b^3*d^2) - (4*f*(e + f*x)*Cosh[c + d*x])/(3*b*d^2) - (2*f*(e + f*x)*Cosh[c + d*x]^3)/(9*b*d^2) - (a*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) - (2*a*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^2) - (2*a*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^2) + (2*a*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^3) + (2*a*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^3) + (2*a^2*f^2*Sinh[c + d*x])/(b^3*d^3) + (14*f^2*Sinh[c + d*x])/(9*b*d^3) + (a^2*(e + f*x)^2*Sinh[c + d*x])/(b^3*d) + (2*(e + f*x)^2*Sinh[c + d*x])/(3*b*d) + (a*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d^2) + ((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b*d) - (a*f^2*Sinh[c + d*x]^2)/(4*b^2*d^3) - (a*(e + f*x)^2*Sinh[c + d*x]^2)/(2*b^2*d) + (2*f^2*Sinh[c + d*x]^3)/(27*b*d^3)
```



```
) - (a*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]
)])/ (b^4*d) - (2*a*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]))])/ (b^4*d^2) - (2*a*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/ (b^4*d^2) + (2*a*(a^2 + b^2)*f^2*
PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/ (b^4*d^3) + (2*a*(a^2
+ b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/ (b^4*d^3)
+ (2*a^2*f^2*Sinh[c + d*x])/ (b^3*d^3) + (14*f^2*Sinh[c + d*x])/ (9*b*d^3) +
(a^2*(e + f*x)^2*Sinh[c + d*x])/ (b^3*d) + (2*(e + f*x)^2*Sinh[c + d*x])/ (3
*b*d) + (a*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/ (2*b^2*d^2) + ((e + f*x
)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/ (3*b*d) - (a*f^2*Sinh[c + d*x]^2)/ (4*b^2
*d^3) - (a*(e + f*x)^2*Sinh[c + d*x]^2)/ (2*b^2*d) + (2*f^2*Sinh[c + d*x]^3)
/ (27*b*d^3)
```

### Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_.)]^(p_.)*((e_.) + (f_.)*(x_.))^(m_.)*Sinh[(c_.) +
(d_.)*(x_.)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
0] && IGtQ[p, 0]
```

### Rule 3311

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbo
l] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Sinh[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sinh[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sinh[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cosh[e + f*x]*(b*Sinh[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

### Rule 3296

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := -Simp[
((c + d*x)^m*Cosh[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cosh
[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sinh[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_.)]^(n_.), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
```

nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d\*x]], x] /; FreeQ[{c, d}, x]  
&& IGtQ[(n - 1)/2, 0]

### Rule 5565

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)])], x\_Symbol] := -Dist[a/b^2, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]  
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

### Rule 5446

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sinh[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Simp[((c + d\*x)^m\*Sinh[a + b\*x]^(n + 1))/(b\*(n + 1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*Sinh[a + b\*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

### Rule 3310

Int[((c\_.) + (d\_.)\*(x\_))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(d\*(b\*Sinh[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)\*(b\*Sinh[e + f\*x])^(n - 2), x], x] - Simp[(b\*(c + d\*x)\*Cos[e + f\*x]\*(b\*Sinh[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

### Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)])], x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)], x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a)]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \cosh^3(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^2 \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= -\frac{2f(e+fx) \cosh^3(c+dx)}{9bd^2} + \frac{(e+fx)^2 \cosh^2(c+dx) \sinh(c+dx)}{3bd} + \frac{a^2 \int (e+fx)^2 \cosh^3(c+dx) dx}{b^3d} \\
&= \frac{a(a^2+b^2)(e+fx)^3}{3b^4f} - \frac{2f(e+fx) \cosh^3(c+dx)}{9bd^2} + \frac{a^2(e+fx)^2 \sinh(c+dx)}{b^3d} \\
&= \frac{a(a^2+b^2)(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx) \cosh(c+dx)}{b^3d^2} - \frac{4f(e+fx) \cosh(c+dx)}{3bd^2} \\
&= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a(a^2+b^2)(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx) \cosh(c+dx)}{b^3d^2} - \frac{4f(e+fx) \cosh(c+dx)}{3bd^2} \\
&= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a(a^2+b^2)(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx) \cosh(c+dx)}{b^3d^2} - \frac{4f(e+fx) \cosh(c+dx)}{3bd^2} \\
&= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a(a^2+b^2)(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx) \cosh(c+dx)}{b^3d^2} - \frac{4f(e+fx) \cosh(c+dx)}{3bd^2}
\end{aligned}$$

**Mathematica [B]** time = 16.7223, size = 3509, normalized size = 5.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (f^2*(2*a*x^3*(-1 + Coth[c]) - 2*a*x^3*Coth[c] - (6*a*b^2*(d^2*x^2*Log[1 + ((a - Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*d*x*PolyLog[2, ((-a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*PolyLog[3, ((-a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b]))/(Sqrt[a^2 + b^2]*(-a + Sqrt[a^2 + b^2])*d^3) - (6*a*b^2*(d^2*x^2*Log[1 + ((a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*d*x*PolyLog[2, ((a + Sqrt[a^2 + b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b] - 2*PolyLog[3, ((a + Sqrt[a^2 + b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b]))/(Sqrt[a^2 + b^2]*(a + Sqrt[a^2 + b^2])*d^3) + (6*a^2*(d^2*x^2*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])]) + 2*d*x*PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])])
```

$$\begin{aligned}
& [c + d*x))/(-a + \text{Sqrt}[a^2 + b^2])] - 2*\text{PolyLog}[3, (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(-a + \text{Sqrt}[a^2 + b^2]))]/(\text{Sqrt}[a^2 + b^2]*d^3) - (6*a^2*(d^2*x \\
& ^2*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a + \text{Sqrt}[a^2 + b^2])] + 2*d \\
& *x*\text{PolyLog}[2, -((b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a + \text{Sqrt}[a^2 + b^2]))] \\
& - 2*\text{PolyLog}[3, -((b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a + \text{Sqrt}[a^2 + b^2]) \\
& ))]/(\text{Sqrt}[a^2 + b^2]*d^3) + (6*b*\text{Cosh}[d*x]*(-2*d*x*\text{Cosh}[c] + (2 + d^2*x^2) \\
& *\text{Sinh}[c]))/d^3 + (6*b*((2 + d^2*x^2)*\text{Cosh}[c] - 2*d*x*\text{Sinh}[c])*\text{Sinh}[d*x])/d^ \\
& 3))/((12*b^2) - (e^2*((a*\text{Log}[a + b*\text{Sinh}[c + d*x]])/b^2 - \text{Sinh}[c + d*x]/b))/ \\
& (2*d) + (e*f*(-(b*\text{Cosh}[c + d*x]) - a*(-(c + d*x)^2/2 + (c + d*x)*\text{Log}[1 + (b* \\
& E^(c + d*x))/(a - \text{Sqrt}[a^2 + b^2])) + (c + d*x)*\text{Log}[1 + (b*E^(c + d*x))/(a \\
& + \text{Sqrt}[a^2 + b^2])) - c*\text{Log}[a + b*\text{Sinh}[c + d*x]] + \text{PolyLog}[2, (b*E^(c + d*x) \\
& ))/(-a + \text{Sqrt}[a^2 + b^2])) + \text{PolyLog}[2, -((b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b \\
& ^2])))] + b*d*x*\text{Sinh}[c + d*x))/(b^2*d^2) + (e^2*(-3*a*(2*a^2 + b^2)*\text{Log}[a \\
& + b*\text{Sinh}[c + d*x]] + 3*b*(2*a^2 + b^2)*\text{Sinh}[c + d*x] - 3*a*b^2*\text{Sinh}[c + d*x] \\
& ]^2 + 2*b^3*\text{Sinh}[c + d*x]^3))/(6*b^4*d) + (e*f*(-18*b*(4*a^2 + b^2)*\text{Cosh}[c \\
& + d*x] - 18*a*b^2*d*x*\text{Cosh}[2*(c + d*x)] - 2*b^3*\text{Cosh}[3*(c + d*x)] - 36*a*(2 \\
& *a^2 + b^2)*(-(c + d*x)^2/2 + (c + d*x)*\text{Log}[1 + (b*E^(c + d*x))/(a - \text{Sqrt}[a \\
& ^2 + b^2])) + (c + d*x)*\text{Log}[1 + (b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b^2])) - c* \\
& \text{Log}[a + b*\text{Sinh}[c + d*x]] + \text{PolyLog}[2, (b*E^(c + d*x))/(-a + \text{Sqrt}[a^2 + b^2] \\
& )) + \text{PolyLog}[2, -((b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b^2])))] + 18*b*(4*a^2 + \\
& b^2)*d*x*\text{Sinh}[c + d*x] + 9*a*b^2*\text{Sinh}[2*(c + d*x)] + 6*b^3*d*x*\text{Sinh}[3*(c + \\
& d*x)]))/((36*b^4*d^2) + (f^2*((2*a*(2*a^2 + b^2)*(-1 + \text{Coth}[c]))*(2*x^3 + (6* \\
& a*(d^2*x^2*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a - \text{Sqrt}[a^2 + b^2] \\
& )) + 2*d*x*\text{PolyLog}[2, (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(-a + \text{Sqrt}[a^2 + \\
& b^2]))] - 2*\text{PolyLog}[3, (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(-a + \text{Sqrt}[a^2 + \\
& b^2]))]*\text{Sinh}[c]*(\text{Cosh}[c] + \text{Sinh}[c]))/(\text{Sqrt}[a^2 + b^2]*d^3) - (3*b^2*(d^2*x^ \\
& 2*\text{Log}[1 + ((a - \text{Sqrt}[a^2 + b^2])*(\text{Cosh}[c + d*x] - \text{Sinh}[c + d*x]))/b] - 2*d* \\
& x*\text{PolyLog}[2, ((-a + \text{Sqrt}[a^2 + b^2])*(\text{Cosh}[c + d*x] - \text{Sinh}[c + d*x]))/b] - \\
& 2*\text{PolyLog}[3, ((-a + \text{Sqrt}[a^2 + b^2])*(\text{Cosh}[c + d*x] - \text{Sinh}[c + d*x]))/b))*(- \\
& 1 + \text{Cosh}[2*c] + \text{Sinh}[2*c]))/(\text{Sqrt}[a^2 + b^2]*(-a + \text{Sqrt}[a^2 + b^2])*d^3) - \\
& (3*b^2*(d^2*x^2*\text{Log}[1 + ((a + \text{Sqrt}[a^2 + b^2])*(\text{Cosh}[c + d*x] - \text{Sinh}[c + d \\
& *x]))/b] - 2*d*x*\text{PolyLog}[2, ((a + \text{Sqrt}[a^2 + b^2])*(-\text{Cosh}[c + d*x] + \text{Sinh}[c \\
& + d*x]))/b] - 2*\text{PolyLog}[3, ((a + \text{Sqrt}[a^2 + b^2])*(-\text{Cosh}[c + d*x] + \text{Sinh}[c \\
& + d*x]))/b))*(-1 + \text{Cosh}[2*c] + \text{Sinh}[2*c]))/(\text{Sqrt}[a^2 + b^2]*(a + \text{Sqrt}[a^2 \\
& + b^2])*d^3) - (3*a*(d^2*x^2*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a \\
& + \text{Sqrt}[a^2 + b^2])) + 2*d*x*\text{PolyLog}[2, -((b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x] \\
& ))/(a + \text{Sqrt}[a^2 + b^2]))] - 2*\text{PolyLog}[3, -((b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d* \\
& x]))/(a + \text{Sqrt}[a^2 + b^2])))]*(-1 + \text{Cosh}[2*c] + \text{Sinh}[2*c]))/(\text{Sqrt}[a^2 + b^2 \\
& ]*d^3)))/(3*b^4) + \text{Csch}[c]*(\text{Cosh}[3*c + 3*d*x]/(108*b^4*d^3) - \text{Sinh}[3*c + 3* \\
& d*x]/(108*b^4*d^3))*(27*a*b^2*\text{Cosh}[d*x] + 54*a*b^2*d*x*\text{Cosh}[d*x] + 54*a*b^2 \\
& *d^2*x^2*\text{Cosh}[d*x] - 27*a*b^2*\text{Cosh}[2*c + d*x] - 54*a*b^2*d*x*\text{Cosh}[2*c + d*x] \\
& ] - 54*a*b^2*d^2*x^2*\text{Cosh}[2*c + d*x] + 432*a^2*b*\text{Cosh}[c + 2*d*x] + 108*b^3* \\
& \text{Cosh}[c + 2*d*x] + 432*a^2*b*d*x*\text{Cosh}[c + 2*d*x] + 108*b^3*d*x*\text{Cosh}[c + 2*d* \\
& x] + 216*a^2*b*d^2*x^2*\text{Cosh}[c + 2*d*x] + 54*b^3*d^2*x^2*\text{Cosh}[c + 2*d*x] - 4 \\
& 32*a^2*b*\text{Cosh}[3*c + 2*d*x] - 108*b^3*\text{Cosh}[3*c + 2*d*x] - 432*a^2*b*d*x*\text{Cosh}
\end{aligned}$$

$$\begin{aligned}
& [3*c + 2*d*x] - 108*b^3*d*x*Cosh[3*c + 2*d*x] - 216*a^2*b*d^2*x^2*Cosh[3*c \\
& + 2*d*x] - 54*b^3*d^2*x^2*Cosh[3*c + 2*d*x] - 144*a^3*d^3*x^3*Cosh[2*c + 3* \\
& d*x] - 72*a*b^2*d^3*x^3*Cosh[2*c + 3*d*x] - 144*a^3*d^3*x^3*Cosh[4*c + 3*d* \\
& x] - 72*a*b^2*d^3*x^3*Cosh[4*c + 3*d*x] - 432*a^2*b*Cosh[3*c + 4*d*x] - 108 \\
& *b^3*Cosh[3*c + 4*d*x] + 432*a^2*b*d*x*Cosh[3*c + 4*d*x] + 108*b^3*d*x*Cosh \\
& [3*c + 4*d*x] - 216*a^2*b*d^2*x^2*Cosh[3*c + 4*d*x] - 54*b^3*d^2*x^2*Cosh[3 \\
& *c + 4*d*x] + 432*a^2*b*Cosh[5*c + 4*d*x] + 108*b^3*Cosh[5*c + 4*d*x] - 432 \\
& *a^2*b*d*x*Cosh[5*c + 4*d*x] - 108*b^3*d*x*Cosh[5*c + 4*d*x] + 216*a^2*b*d^ \\
& 2*x^2*Cosh[5*c + 4*d*x] + 54*b^3*d^2*x^2*Cosh[5*c + 4*d*x] + 27*a*b^2*Cosh[ \\
& 4*c + 5*d*x] - 54*a*b^2*d*x*Cosh[4*c + 5*d*x] + 54*a*b^2*d^2*x^2*Cosh[4*c + \\
& 5*d*x] - 27*a*b^2*Cosh[6*c + 5*d*x] + 54*a*b^2*d*x*Cosh[6*c + 5*d*x] - 54* \\
& a*b^2*d^2*x^2*Cosh[6*c + 5*d*x] - 4*b^3*Cosh[5*c + 6*d*x] + 12*b^3*d*x*Cosh \\
& [5*c + 6*d*x] - 18*b^3*d^2*x^2*Cosh[5*c + 6*d*x] + 4*b^3*Cosh[7*c + 6*d*x] \\
& - 12*b^3*d*x*Cosh[7*c + 6*d*x] + 18*b^3*d^2*x^2*Cosh[7*c + 6*d*x] - 8*b^3*S \\
& inh[c] - 24*b^3*d*x*Sinh[c] - 36*b^3*d^2*x^2*Sinh[c] + 27*a*b^2*Sinh[d*x] + \\
& 54*a*b^2*d*x*Sinh[d*x] + 54*a*b^2*d^2*x^2*Sinh[d*x] - 27*a*b^2*Sinh[2*c + \\
& d*x] - 54*a*b^2*d*x*Sinh[2*c + d*x] - 54*a*b^2*d^2*x^2*Sinh[2*c + d*x] + 43 \\
& 2*a^2*b*Sinh[c + 2*d*x] + 108*b^3*Sinh[c + 2*d*x] + 432*a^2*b*d*x*Sinh[c + \\
& 2*d*x] + 108*b^3*d*x*Sinh[c + 2*d*x] + 216*a^2*b*d^2*x^2*Sinh[c + 2*d*x] + \\
& 54*b^3*d^2*x^2*Sinh[c + 2*d*x] - 432*a^2*b*Sinh[3*c + 2*d*x] - 108*b^3*Sinh \\
& [3*c + 2*d*x] - 432*a^2*b*d*x*Sinh[3*c + 2*d*x] - 108*b^3*d*x*Sinh[3*c + 2* \\
& d*x] - 216*a^2*b*d^2*x^2*Sinh[3*c + 2*d*x] - 54*b^3*d^2*x^2*Sinh[3*c + 2*d* \\
& x] - 144*a^3*d^3*x^3*Sinh[2*c + 3*d*x] - 72*a*b^2*d^3*x^3*Sinh[2*c + 3*d*x] \\
& - 144*a^3*d^3*x^3*Sinh[4*c + 3*d*x] - 72*a*b^2*d^3*x^3*Sinh[4*c + 3*d*x] - \\
& 432*a^2*b*Sinh[3*c + 4*d*x] - 108*b^3*Sinh[3*c + 4*d*x] + 432*a^2*b*d*x*Si \\
& nh[3*c + 4*d*x] + 108*b^3*d*x*Sinh[3*c + 4*d*x] - 216*a^2*b*d^2*x^2*Sinh[3* \\
& c + 4*d*x] - 54*b^3*d^2*x^2*Sinh[3*c + 4*d*x] + 432*a^2*b*Sinh[5*c + 4*d*x] \\
& + 108*b^3*Sinh[5*c + 4*d*x] - 432*a^2*b*d*x*Sinh[5*c + 4*d*x] - 108*b^3*d* \\
& x*Sinh[5*c + 4*d*x] + 216*a^2*b*d^2*x^2*Sinh[5*c + 4*d*x] + 54*b^3*d^2*x^2* \\
& Sinh[5*c + 4*d*x] + 27*a*b^2*Sinh[4*c + 5*d*x] - 54*a*b^2*d*x*Sinh[4*c + 5* \\
& d*x] + 54*a*b^2*d^2*x^2*Sinh[4*c + 5*d*x] - 27*a*b^2*Sinh[6*c + 5*d*x] + 54 \\
& *a*b^2*d*x*Sinh[6*c + 5*d*x] - 54*a*b^2*d^2*x^2*Sinh[6*c + 5*d*x] - 4*b^3*S \\
& inh[5*c + 6*d*x] + 12*b^3*d*x*Sinh[5*c + 6*d*x] - 18*b^3*d^2*x^2*Sinh[5*c + \\
& 6*d*x] + 4*b^3*Sinh[7*c + 6*d*x] - 12*b^3*d*x*Sinh[7*c + 6*d*x] + 18*b^3*d \\
& ^2*x^2*Sinh[7*c + 6*d*x])))/8
\end{aligned}$$


---

**Maple [F]** time = 0.191, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\cosh(dx + c))^3 \sinh(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)^2*\cosh(d*x+c)^3*\sinh(d*x+c)/(a+b*\sinh(d*x+c)),x)$

[Out]  $\text{int}((f*x+e)^2*\cosh(d*x+c)^3*\sinh(d*x+c)/(a+b*\sinh(d*x+c)),x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x+e)^2*\cosh(d*x+c)^3*\sinh(d*x+c)/(a+b*\sinh(d*x+c)),x, \text{algorithm}="maxima")$

[Out] 
$$\begin{aligned} & -1/24*e^2*((3*a*b*e^{(-d*x - c)} - b^2 - 3*(4*a^2 + 3*b^2)*e^{(-2*d*x - 2*c)}) * \\ & e^{(3*d*x + 3*c)}/(b^3*d) + 24*(a^3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^{(-2 \\ & *d*x - 2*c)} + b^2*e^{(-3*d*x - 3*c)} + 3*(4*a^2 + 3*b^2)*e^{(-d*x - c)})/(b^3*d \\ & ) + 24*(a^3 + a*b^2)*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/(b^4*d \\ & )) - 1/432*(144*(a^3*d^3*f^2*e^{(3*c)} + a*b^2*d^3*f^2*e^{(3*c)})*x^3 + 432*(a^ \\ & 3*d^3*e*f*e^{(3*c)} + a*b^2*d^3*e*f*e^{(3*c)})*x^2 - 2*(9*b^3*d^2*f^2*x^2*e^{(6* \\ & c)} + 6*(3*d^2*e*f - d*f^2)*b^3*x*e^{(6*c)} - 2*(3*d*e*f - f^2)*b^3*e^{(6*c)})*e \\ & ^{(3*d*x)} + 27*(2*a*b^2*d^2*f^2*x^2*e^{(5*c)} + 2*(2*d^2*e*f - d*f^2)*a*b^2*x* \\ & e^{(5*c)} - (2*d*e*f - f^2)*a*b^2*e^{(5*c)})*e^{(2*d*x)} + 54*(8*(d*e*f - f^2)*a^ \\ & 2*b*e^{(4*c)} + 6*(d*e*f - f^2)*b^3*e^{(4*c)} - (4*a^2*b*d^2*f^2*e^{(4*c)} + 3*b^ \\ & 3*d^2*f^2*e^{(4*c)})*x^2 - 2*(4*(d^2*e*f - d*f^2)*a^2*b*e^{(4*c)} + 3*(d^2*e*f \\ & - d*f^2)*b^3*e^{(4*c)})*x)*e^{(d*x)} + 54*(8*(d*e*f + f^2)*a^2*b*e^{(2*c)} + 6*(d \\ & *e*f + f^2)*b^3*e^{(2*c)} + (4*a^2*b*d^2*f^2*e^{(2*c)} + 3*b^3*d^2*f^2*e^{(2*c)}) \\ & *x^2 + 2*(4*(d^2*e*f + d*f^2)*a^2*b*e^{(2*c)} + 3*(d^2*e*f + d*f^2)*b^3*e^{(2* \\ & c)})*x)*e^{(-d*x)} + 27*(2*a*b^2*d^2*f^2*x^2*e^c + 2*(2*d^2*e*f + d*f^2)*a*b^2 \\ & *x*e^c + (2*d*e*f + f^2)*a*b^2*e^c)*e^{(-2*d*x)} + 2*(9*b^3*d^2*f^2*x^2 + 6*( \\ & 3*d^2*e*f + d*f^2)*b^3*x + 2*(3*d*e*f + f^2)*b^3)*e^{(-3*d*x)})*e^{(-3*c)}/(b^4 \\ & *d^3) + \text{integrate}(-2*((a^3*b*f^2 + a*b^3*f^2)*x^2 + 2*(a^3*b*e*f + a*b^3*e* \\ & f)*x - ((a^4*f^2*e^c + a^2*b^2*f^2*e^c)*x^2 + 2*(a^4*e*f*e^c + a^2*b^2*e*f* \\ & e^c)*x)*e^{(d*x)})/(b^5*e^{(2*d*x + 2*c)} + 2*a*b^4*e^{(d*x + c)} - b^5), x) \end{aligned}$$

**Fricas [C]** time = 3.04855, size = 10886, normalized size = 17.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)^3\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-1/432*(18*b^3*d^2*f^2*x^2 + 18*b^3*d^2*e^2 - 2*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*\cosh(d*x + c)^6 - 2*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*\sinh(d*x + c)^6 + 12*b^3*d*e*f + 27*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*\cosh(d*x + c)^5 + 3*(18*a*b^2*d^2*f^2*x^2 + 18*a*b^2*d^2*e^2 - 18*a*b^2*d*e*f + 9*a*b^2*f^2 + 18*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x - 4*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 4*b^3*f^2 - 54*((4*a^2*b + 3*b^3)*d^2*f^2*x^2 + (4*a^2*b + 3*b^3)*d^2*e^2 - 2*(4*a^2*b + 3*b^3)*d*e*f + 2*(4*a^2*b + 3*b^3)*f^2 + 2*((4*a^2*b + 3*b^3)*d^2*e*f - (4*a^2*b + 3*b^3)*d*f^2)*x)*\cosh(d*x + c)^4 - 3*(18*(4*a^2*b + 3*b^3)*d^2*f^2*x^2 + 18*(4*a^2*b + 3*b^3)*d^2*e^2 - 36*(4*a^2*b + 3*b^3)*d*e*f + 36*(4*a^2*b + 3*b^3)*f^2 + 10*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*\cosh(d*x + c)^2 + 36*((4*a^2*b + 3*b^3)*d^2*e*f - (4*a^2*b + 3*b^3)*d*f^2)*x - 45*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^4 - 144*((a^3 + a*b^2)*d^3*f^2*x^3 + 3*(a^3 + a*b^2)*d^3*e*f*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*x + 6*(a^3 + a*b^2)*c*d^2*e^2 - 6*(a^3 + a*b^2)*c^2*d*e*f + 2*(a^3 + a*b^2)*c^3*f^2)*\cosh(d*x + c)^3 - 2*(72*(a^3 + a*b^2)*d^3*f^2*x^3 + 216*(a^3 + a*b^2)*d^3*e*f*x^2 + 216*(a^3 + a*b^2)*d^3*e^2*x + 432*(a^3 + a*b^2)*c*d^2*e^2 - 432*(a^3 + a*b^2)*c^2*d*e*f + 144*(a^3 + a*b^2)*c^3*f^2 + 20*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*\cosh(d*x + c)^3 - 135*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*\cosh(d*x + c)^2 + 108*((4*a^2*b + 3*b^3)*d^2*f^2*x^2 + (4*a^2*b + 3*b^3)*d^2*e^2 - 2*(4*a^2*b + 3*b^3)*d*e*f + 2*(4*a^2*b + 3*b^3)*f^2 + 2*((4*a^2*b + 3*b^3)*d^2*e*f - (4*a^2*b + 3*b^3)*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 54*((4*a^2*b + 3*b^3)*d^2*f^2*x^2 + (4*a^2*b + 3*b^3)*d^2*e^2 + 2*(4*a^2*b + 3*b^3)*d*e*f + 2*(4*a^2*b + 3*b^3)*f^2 + 2*((4*a^2*b + 3*b^3)*d^2*e*f + (4*a^2*b + 3*b^3)*d*f^2)*x)*\cosh(d*x + c)^2 + 6*(9*(4*a^2*b + 3*b^3)*d^2*f^2*x^2 + 9*(4*a^2*b + 3*b^3)*d^2*e^2 - 5*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*\cosh(d*x + c)^4 + 18*(4*a^2*b + 3*b^3)*d*e*f + 45*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*\cosh(d*x + c)^3 + 18*(4*a^2*b + 3*b^3)*f^2 - 54*((4*a^2*b + 3*b^3)*d^2*f^2*x^2 + (4*a^2*b + 3*b^3)*d^2*e^2 - 2*(4*a^2*b + 3*b^3)*d*e*f + 2*(4*a^2*b + 3*b^3)*f^2 + 2*((4*a^2*b + 3*b^3)*d^2*e*f - (4*a^2*b + 3*b^3)*d*f^2)*x)*\cosh(d*x + c)^2 + 18*((4*a^2*b + 3*b^3)*d^2*e*f + (4*a^2*b + 3*b^3)*d*f^2)*x - 72*((a^3 + a*b^2)*d^3*f^2*x^3 + 3*(a^3 + a*b^2)*d^3*e*f*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*x + 6*(a^3 + a*b^2)*c*d^2$$



$$\begin{aligned}
& 2e^2 - 6*(a^3 + a*b^2)*c^2*d*e*f + 2*(a^3 + a*b^2)*c^3*f^2)*\cosh(d*x + c)) \\
& * \sinh(d*x + c)^2 + 12*(3*b^3*d^2*e*f + b^3*d*f^2)*x + 27*(2*a*b^2*d^2*f^2*x \\
& ^2 + 2*a*b^2*d^2*e^2 + 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f + a*b \\
& ^2*d*f^2)*x)*\cosh(d*x + c) + 864*(((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*d* \\
& e*f)*\cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*d*e*f)*\cosh \\
& (d*x + c)^2*\sinh(d*x + c) + 3*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*d*e*f) \\
& *\cosh(d*x + c)*\sinh(d*x + c)^2 + ((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*d*e \\
& *f)*\sinh(d*x + c)^3)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x \\
& + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2) - b}/b + 1) + 864*(((a^3 + a \\
& *b^2)*d*f^2*x + (a^3 + a*b^2)*d*e*f)*\cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d*f \\
& ^2*x + (a^3 + a*b^2)*d*e*f)*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*((a^3 + a*b^2) \\
& )*d*f^2*x + (a^3 + a*b^2)*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c)^2 + ((a^3 + a* \\
& b^2)*d*f^2*x + (a^3 + a*b^2)*d*e*f)*\sinh(d*x + c)^3)*\operatorname{dilog}((a*\cosh(d*x + c) \\
& + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b \\
& ^2) - b}/b + 1) + 432*(((a^3 + a*b^2)*d^2*e^2 - 2*(a^3 + a*b^2)*c*d*e*f + ( \\
& a^3 + a*b^2)*c^2*f^2)*\cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d^2*e^2 - 2*(a^3 + \\
& a*b^2)*c*d*e*f + (a^3 + a*b^2)*c^2*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c) + 3* \\
& ((a^3 + a*b^2)*d^2*e^2 - 2*(a^3 + a*b^2)*c*d*e*f + (a^3 + a*b^2)*c^2*f^2)*\c \\
& osh(d*x + c)*\sinh(d*x + c)^2 + ((a^3 + a*b^2)*d^2*e^2 - 2*(a^3 + a*b^2)*c*d \\
& *e*f + (a^3 + a*b^2)*c^2*f^2)*\sinh(d*x + c)^3)*\log(2*b*\cosh(d*x + c) + 2*b* \\
& \sinh(d*x + c) + 2*b*\sqrt{((a^2 + b^2)/b^2) + 2*a}) + 432*(((a^3 + a*b^2)*d^2* \\
& e^2 - 2*(a^3 + a*b^2)*c*d*e*f + (a^3 + a*b^2)*c^2*f^2)*\cosh(d*x + c)^3 + 3* \\
& ((a^3 + a*b^2)*d^2*e^2 - 2*(a^3 + a*b^2)*c*d*e*f + (a^3 + a*b^2)*c^2*f^2)*\c \\
& osh(d*x + c)^2*\sinh(d*x + c) + 3*((a^3 + a*b^2)*d^2*e^2 - 2*(a^3 + a*b^2)*c \\
& *d*e*f + (a^3 + a*b^2)*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^2 + ((a^3 + a*b \\
& ^2)*d^2*e^2 - 2*(a^3 + a*b^2)*c*d*e*f + (a^3 + a*b^2)*c^2*f^2)*\sinh(d*x + c \\
& )^3)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{((a^2 + b^2)/b^2) \\
& + 2*a}) + 432*(((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a \\
& ^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*\cosh(d*x + c)^3 + 3*((a^3 + a* \\
& b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a \\
& ^3 + a*b^2)*c^2*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*((a^3 + a*b^2)*d^2*f \\
& ^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2) \\
& )*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^2 + ((a^3 + a*b^2)*d^2*f^2*x^2 + 2*( \\
& a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*\s \\
& inh(d*x + c)^3)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) \\
& + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2) - b}/b) + 432*(((a^3 + a*b^2)*d^2* \\
& f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^ \\
& 2)*c^2*f^2)*\cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2) \\
& )*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*\cosh(d*x + c \\
& )^2*\sinh(d*x + c) + 3*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f* \\
& x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x \\
& + c)^2 + ((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + \\
& a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*\sinh(d*x + c)^3)*\log(-(a*\cosh(d*x \\
& + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^ \\
& 2)/b^2) - b}/b) - 864*(((a^3 + a*b^2)*f^2*\cosh(d*x + c)^3 + 3*(a^3 + a*b^2)*
\end{aligned}$$

```
f^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3 + a*b^2)*f^2*cosh(d*x + c)*sinh(
d*x + c)^2 + (a^3 + a*b^2)*f^2*sinh(d*x + c)^3)*polylog(3, (a*cosh(d*x + c)
+ a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b
^2))/b) - 864*((a^3 + a*b^2)*f^2*cosh(d*x + c)^3 + 3*(a^3 + a*b^2)*f^2*cosh
(d*x + c)^2*sinh(d*x + c) + 3*(a^3 + a*b^2)*f^2*cosh(d*x + c)*sinh(d*x + c)
^2 + (a^3 + a*b^2)*f^2*sinh(d*x + c)^3)*polylog(3, (a*cosh(d*x + c) + a*sin
h(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b)
+ 3*(18*a*b^2*d^2*f^2*x^2 + 18*a*b^2*d^2*e^2 + 18*a*b^2*d*e*f - 4*(9*b^3*d^
2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^
3*d*f^2)*x)*cosh(d*x + c)^5 + 9*a*b^2*f^2 + 45*(2*a*b^2*d^2*f^2*x^2 + 2*a*b
^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*
x)*cosh(d*x + c)^4 - 72*((4*a^2*b + 3*b^3)*d^2*f^2*x^2 + (4*a^2*b + 3*b^3)*
d^2*e^2 - 2*(4*a^2*b + 3*b^3)*d*e*f + 2*(4*a^2*b + 3*b^3)*f^2 + 2*((4*a^2*b
+ 3*b^3)*d^2*e*f - (4*a^2*b + 3*b^3)*d*f^2)*x)*cosh(d*x + c)^3 - 144*((a^3
+ a*b^2)*d^3*f^2*x^3 + 3*(a^3 + a*b^2)*d^3*e*f*x^2 + 3*(a^3 + a*b^2)*d^3*e
^2*x + 6*(a^3 + a*b^2)*c*d^2*e^2 - 6*(a^3 + a*b^2)*c^2*d*e*f + 2*(a^3 + a*b
^2)*c^3*f^2)*cosh(d*x + c)^2 + 18*(2*a*b^2*d^2*e*f + a*b^2*d*f^2)*x + 36*((
4*a^2*b + 3*b^3)*d^2*f^2*x^2 + (4*a^2*b + 3*b^3)*d^2*e^2 + 2*(4*a^2*b + 3*b
^3)*d*e*f + 2*(4*a^2*b + 3*b^3)*f^2 + 2*((4*a^2*b + 3*b^3)*d^2*e*f + (4*a^2
*b + 3*b^3)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))/(b^4*d^3*cosh(d*x + c)^
3 + 3*b^4*d^3*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^4*d^3*cosh(d*x + c)*sinh(
d*x + c)^2 + b^4*d^3*sinh(d*x + c)^3)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cosh(d\*x+c)\*\*3\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c)^3 \sinh(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm  
m="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)^3*sinh(d*x + c)/(b*sinh(d*x + c) + a),  
x)
```

$$3.345 \quad \int \frac{(e+fx) \cosh^3(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=400

$$\frac{af(a^2+b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2} - \frac{af(a^2+b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^2} - \frac{a^2f \cosh(c+dx)}{b^3d^2} - \frac{a(a^2+b^2)(e+fx)}{b^4}$$

[Out]  $-(a*f*x)/(4*b^2*d) + (a*(a^2 + b^2)*(e + f*x)^2)/(2*b^4*f) - (a^2*f*\operatorname{Cosh}[c + d*x])/(b^3*d^2) - (2*f*\operatorname{Cosh}[c + d*x])/(3*b*d^2) - (f*\operatorname{Cosh}[c + d*x]^3)/(9*b*d^2) - (a*(a^2 + b^2)*(e + f*x)*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])])/(b^4*d) - (a*(a^2 + b^2)*(e + f*x)*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])/(b^4*d) - (a*(a^2 + b^2)*f*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2]))])/(b^4*d^2) - (a*(a^2 + b^2)*f*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2]))])/(b^4*d^2) + (a^2*(e + f*x)*\operatorname{Sinh}[c + d*x])/(b^3*d) + (2*(e + f*x)*\operatorname{Sinh}[c + d*x])/(3*b*d) + (a*f*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x])/(4*b^2*d^2) + ((e + f*x)*\operatorname{Cosh}[c + d*x]^2*\operatorname{Sinh}[c + d*x])/(3*b*d) - (a*(e + f*x)*\operatorname{Sinh}[c + d*x]^2)/(2*b^2*d)$

**Rubi [A]** time = 0.484925, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5579, 3310, 3296, 2638, 5565, 5446, 2635, 8, 5561, 2190, 2279, 2391}

$$\frac{af(a^2+b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4d^2} - \frac{af(a^2+b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4d^2} - \frac{a^2f \cosh(c+dx)}{b^3d^2} - \frac{a(a^2+b^2)(e+fx)}{b^4}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e + f*x)*\operatorname{Cosh}[c + d*x]^3*\operatorname{Sinh}[c + d*x]/(a + b*\operatorname{Sinh}[c + d*x]), x]$

[Out]  $-(a*f*x)/(4*b^2*d) + (a*(a^2 + b^2)*(e + f*x)^2)/(2*b^4*f) - (a^2*f*\operatorname{Cosh}[c + d*x])/(b^3*d^2) - (2*f*\operatorname{Cosh}[c + d*x])/(3*b*d^2) - (f*\operatorname{Cosh}[c + d*x]^3)/(9*b*d^2) - (a*(a^2 + b^2)*(e + f*x)*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])])/(b^4*d) - (a*(a^2 + b^2)*(e + f*x)*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])/(b^4*d) - (a*(a^2 + b^2)*f*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2]))])/(b^4*d^2) - (a*(a^2 + b^2)*f*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2]))])/(b^4*d^2) + (a^2*(e + f*x)*\operatorname{Sinh}[c + d*x])/(b^3*d) + (2*(e + f*x)*\operatorname{Sinh}[c + d*x])/(3*b*d) + (a*f*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x])/(4*b^2*d^2) + ((e + f*x)*\operatorname{Cosh}[c + d*x]^2*\operatorname{Sinh}[c + d*x])/(3*b*d) - (a*(e + f*x)*\operatorname{Sinh}[c + d*x]^2)/(2*b^2*d)$

Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_.)]^(p_.)*((e_.) + (f_.)*(x_.))^(m_.)*Sinh[(c_.) +
(d_.)*(x_.)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
0] && IGtQ[p, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_.))*((b_.)*sin[(e_.) + (f_.)*(x_.)])^(n_), x_Symbol] :=>
Simp[(d*(b*Sinh[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sinh[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sinh[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] :=> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] :=> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_.)]^(n_.)*((e_.) + (f_.)*(x_.))^(m_.))/((a_.) + (b_.
)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] :=> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)
*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.)*Sinh[(a_.) + (b_.)*
(x_.)]^(n_.), x_Symbol] :=> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx) \cosh^3(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx) \cosh^3(c+dx) dx}{b} - \frac{a \int \frac{(e+fx) \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= -\frac{f \cosh^3(c+dx)}{9bd^2} + \frac{(e+fx) \cosh^2(c+dx) \sinh(c+dx)}{3bd} + \frac{a^2 \int (e+fx) \cosh^3(c+dx) dx}{b^3} \\
&= \frac{a(a^2+b^2)(e+fx)^2}{2b^4f} - \frac{f \cosh^3(c+dx)}{9bd^2} + \frac{a^2(e+fx) \sinh(c+dx)}{b^3d} + \frac{2(e+fx) \cosh^3(c+dx)}{9bd^2} \\
&= \frac{a(a^2+b^2)(e+fx)^2}{2b^4f} - \frac{a^2f \cosh(c+dx)}{b^3d^2} - \frac{2f \cosh(c+dx)}{3bd^2} - \frac{f \cosh^3(c+dx)}{9bd^2} \\
&= -\frac{afx}{4b^2d} + \frac{a(a^2+b^2)(e+fx)^2}{2b^4f} - \frac{a^2f \cosh(c+dx)}{b^3d^2} - \frac{2f \cosh(c+dx)}{3bd^2} - \frac{f \cosh^3(c+dx)}{9bd^2} \\
&= -\frac{afx}{4b^2d} + \frac{a(a^2+b^2)(e+fx)^2}{2b^4f} - \frac{a^2f \cosh(c+dx)}{b^3d^2} - \frac{2f \cosh(c+dx)}{3bd^2} - \frac{f \cosh^3(c+dx)}{9bd^2}
\end{aligned}$$

**Mathematica [A]** time = 3.43536, size = 551, normalized size = 1.38

$$36b^2f \left( a \left( \text{PolyLog} \left( 2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a} \right) + \text{PolyLog} \left( 2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} \right) + (c+dx) \log \left( \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1 \right) + (c+dx) \log \left( \frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} + 1 \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Cosh[c + d\*x]^3\*Sinh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x  
]

[Out] -(-36\*b^2\*d\*e\*(-(a\*Log[a + b\*Sinh[c + d\*x]]) + b\*Sinh[c + d\*x]) + 36\*b^2\*f\*(b\*Cosh[c + d\*x] + a\*(-(c + d\*x)^2/2 + (c + d\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]) + (c + d\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]) - c\*Log[a + b\*Sinh[c + d\*x]] + PolyLog[2, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2]]) + PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))]) - b\*d\*x\*Sinh[c + d\*x]) + 12\*d\*e\*(3\*a\*(2\*a^2 + b^2)\*Log[a + b\*Sinh[c + d\*x]] - 3\*b\*(2\*a^2 + b^2)\*Sinh[c + d\*x] + 3\*a\*b^2\*Sinh[c + d\*x]^2 - 2\*b^3\*Sinh[c + d\*x]^3) + f\*(18\*b\*(4\*a^2 + b^2)\*Cosh[c + d\*x] + 18\*a\*b^2\*d\*x\*Cosh[2\*(c + d\*x)] + 2\*b^3\*Cosh[3\*(c + d\*x)] + 36\*a\*(2\*a^2 + b^2)\*(-(c + d\*x)^2/2 + (c + d\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]) + (c + d\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]) - c\*Log[a + b\*Sinh[c + d\*x]] + PolyLog[2, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2]]) + PolyLog[2, -((b\*E^(c + d\*x))/(a + S

$$\sqrt[3]{a^2 + b^2}}) - 18*b*(4*a^2 + b^2)*d*x*\text{Sinh}[c + d*x] - 9*a*b^2*\text{Sinh}[2*(c + d*x)] - 6*b^3*d*x*\text{Sinh}[3*(c + d*x)])/(72*b^4*d^2)$$

**Maple [B]** time = 0.099, size = 1102, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x)`

[Out] 
$$\begin{aligned} & -1/16*a*(2*d*f*x+2*d*e-f)/b^2/d^2*\exp(2*d*x+2*c)-1/8*(4*a^2+3*b^2)*(d*f*x+d \\ & *e+f)/b^3/d^2*\exp(-d*x-c)-1/16*a*(2*d*f*x+2*d*e+f)/b^2/d^2*\exp(-2*d*x-2*c)+ \\ & 1/2*a^3*f*x^2/b^4+1/2*a*f*x^2/b^2-a^3/b^4/d*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^( \\ & 1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-a^3/b^4/d^2*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^( \\ & 1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-a^3/b^4/d*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^(1/2) \\ & )+a)/(a+(a^2+b^2)^(1/2)))*x-a^3/b^4/d^2*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^(1/2)+ \\ & a)/(a+(a^2+b^2)^(1/2)))*c+a^3/b^4/d^2*f*c*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c) \\ & )-b)-2*a^3/b^4/d^2*f*c*\ln(\exp(d*x+c))+2*a^3/b^4/d*f*c*x-a^3*e*x/b^4-a*e*x/b \\ & ^2+a/b^2/d^2*f*c*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)-2*a/b^2/d^2*f*c*\ln(\exp(d*x+c))-a/b^2/d^2*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))) \\ & )*x-a/b^2/d^2*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2) \\ & ))*c-a/b^2/d^2*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-a \\ & /b^2/d^2*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2*a/b \\ & ^2/d^2*f*c*x-1/72*(3*d*f*x+3*d*e+f)/d^2/b*\exp(-3*d*x-3*c)+1/72*(3*d*f*x+3*d*e \\ & -f)/d^2/b*\exp(3*d*x+3*c)+1/8*(4*a^2*d*f*x+3*b^2*d*f*x+4*a^2*d*e+3*b^2*d*e-4 \\ & *a^2*f-3*b^2*f)/b^3/d^2*\exp(d*x+c)+a/b^2/d^2*f*c^2-a/b^2/d*e*\ln(b*\exp(2*d*x \\ & +2*c)+2*a*\exp(d*x+c)-b)+2*a/b^2/d*e*\ln(\exp(d*x+c))-a/b^2/d^2*f*dilog((b*\exp \\ & (d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-a/b^2/d^2*f*dilog((-b*\exp(d \\ & *x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+a^3/b^4/d^2*f*c^2-a^3/b^4/d^2 \\ & *f*dilog((b*\exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-a^3/b^4/d^2 \\ & *f*dilog((-b*\exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-a^3/b^4/d^2 \\ & *e*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)+2*a^3/b^4/d*e*\ln(\exp(d*x+c)) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{24}e^{\left(\frac{(3abe^{-dx-c}) - b^2 - 3(4a^2 + 3b^2)e^{(-2dx-2c)})e^{(3dx+3c)}}{b^3d} + \frac{24(a^3 + ab^2)(dx+c)}{b^4d} + \frac{3abe^{(-2dx-2c)} + b^2e^{(-3dx-3c)} + 3}{b^3d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -1/24*e*((3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 + 3*b^2)*e^(-2*d*x - 2*c))*e^
(3*d*x + 3*c)/(b^3*d) + 24*(a^3 + a*b^2)*(d*x + c)/(b^4*d) + (3*a*b*e^(-2*d
*x - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*(4*a^2 + 3*b^2)*e^(-d*x - c))/(b^3*d)
+ 24*(a^3 + a*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^4*d))
- 1/144*f*((72*(a^3*d^2*e^(3*c) + a*b^2*d^2*e^(3*c))*x^2 - 2*(3*b^3*d*x*e^
(6*c) - b^3*e^(6*c))*e^(3*d*x) + 9*(2*a*b^2*d*x*e^(5*c) - a*b^2*e^(5*c))*e^
(2*d*x) + 18*(4*a^2*b*e^(4*c) + 3*b^3*e^(4*c) - (4*a^2*b*d*e^(4*c) + 3*b^3*
d*e^(4*c))*x)*e^(d*x) + 18*(4*a^2*b*e^(2*c) + 3*b^3*e^(2*c) + (4*a^2*b*d*e^
(2*c) + 3*b^3*d*e^(2*c))*x)*e^(-d*x) + 9*(2*a*b^2*d*x*e^c + a*b^2*e^c)*e^(-
2*d*x) + 2*(3*b^3*d*x + b^3)*e^(-3*d*x))*e^(-3*c)/(b^4*d^2) - 9*integrate(3
2*((a^4*e^c + a^2*b^2*e^c)*x*e^(d*x) - (a^3*b + a*b^3)*x)/(b^5*e^(2*d*x + 2
*c) + 2*a*b^4*e^(d*x + c) - b^5), x))
```

---

**Fricas [B]** time = 2.56898, size = 5828, normalized size = 14.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/144*(2*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^6 + 2*(3*b^3*d*f*x
+ 3*b^3*d*e - b^3*f)*sinh(d*x + c)^6 - 6*b^3*d*f*x - 9*(2*a*b^2*d*f*x + 2*
a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^5 - 3*(6*a*b^2*d*f*x + 6*a*b^2*d*e - 3*a
*b^2*f - 4*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c))*sinh(d*x + c)^5
- 6*b^3*d*e + 18*((4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b + 3*b^3)*d*e - (4*a^2
*b + 3*b^3)*f)*cosh(d*x + c)^4 + 3*(6*(4*a^2*b + 3*b^3)*d*f*x + 6*(4*a^2*b
+ 3*b^3)*d*e + 10*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^2 - 6*(4*
a^2*b + 3*b^3)*f - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)
)*sinh(d*x + c)^4 - 2*b^3*f + 72*((a^3 + a*b^2)*d^2*f*x^2 + 2*(a^3 + a*b^2)
*d^2*e*x + 4*(a^3 + a*b^2)*c*d*e - 2*(a^3 + a*b^2)*c^2*f)*cosh(d*x + c)^3 +
2*(36*(a^3 + a*b^2)*d^2*f*x^2 + 72*(a^3 + a*b^2)*d^2*e*x + 144*(a^3 + a*b^
2)*c*d*e - 72*(a^3 + a*b^2)*c^2*f + 20*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*co
sh(d*x + c)^3 - 45*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^2
+ 36*((4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b + 3*b^3)*d*e - (4*a^2*b + 3*b^3)*f
)*cosh(d*x + c))*sinh(d*x + c)^3 - 18*((4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b +
3*b^3)*d*e + (4*a^2*b + 3*b^3)*f)*cosh(d*x + c)^2 + 6*(5*(3*b^3*d*f*x + 3*
b^3*d*e - b^3*f)*cosh(d*x + c)^4 - 3*(4*a^2*b + 3*b^3)*d*f*x - 15*(2*a*b^2*
```

$$\begin{aligned}
& d*f*x + 2*a*b^2*d*e - a*b^2*f)*\cosh(d*x + c)^3 - 3*(4*a^2*b + 3*b^3)*d*e + \\
& 18*((4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b + 3*b^3)*d*e - (4*a^2*b + 3*b^3)*f)* \\
& \cosh(d*x + c)^2 - 3*(4*a^2*b + 3*b^3)*f + 36*((a^3 + a*b^2)*d^2*f*x^2 + 2*( \\
& a^3 + a*b^2)*d^2*e*x + 4*(a^3 + a*b^2)*c*d*e - 2*(a^3 + a*b^2)*c^2*f)*\cosh( \\
& d*x + c))*\sinh(d*x + c)^2 - 9*(2*a*b^2*d*f*x + 2*a*b^2*d*e + a*b^2*f)*\cosh( \\
& d*x + c) - 144*((a^3 + a*b^2)*f*\cosh(d*x + c)^3 + 3*(a^3 + a*b^2)*f*\cosh(d* \\
& x + c)^2*\sinh(d*x + c) + 3*(a^3 + a*b^2)*f*\cosh(d*x + c)*\sinh(d*x + c)^2 + \\
& (a^3 + a*b^2)*f*\sinh(d*x + c)^3)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + \\
& (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 14 \\
& 4*((a^3 + a*b^2)*f*\cosh(d*x + c)^3 + 3*(a^3 + a*b^2)*f*\cosh(d*x + c)^2*\sinh \\
& (d*x + c) + 3*(a^3 + a*b^2)*f*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a^3 + a*b^2) \\
& *f*\sinh(d*x + c)^3)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x \\
& + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 144*(((a^3 + a* \\
& b^2)*d*e - (a^3 + a*b^2)*c*f)*\cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d*e - (a^3 \\
& + a*b^2)*c*f)*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*((a^3 + a*b^2)*d*e - (a^3 \\
& + a*b^2)*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^2 + ((a^3 + a*b^2)*d*e - (a^3 + a \\
& *b^2)*c*f)*\sinh(d*x + c)^3)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b \\
& *\sqrt{(a^2 + b^2)/b^2} + 2*a) - 144*(((a^3 + a*b^2)*d*e - (a^3 + a*b^2)*c*f \\
& )*\cosh(d*x + c)^3 + 3*((a^3 + a*b^2)*d*e - (a^3 + a*b^2)*c*f)*\cosh(d*x + c) \\
& ^2*\sinh(d*x + c) + 3*((a^3 + a*b^2)*d*e - (a^3 + a*b^2)*c*f)*\cosh(d*x + c)* \\
& \sinh(d*x + c)^2 + ((a^3 + a*b^2)*d*e - (a^3 + a*b^2)*c*f)*\sinh(d*x + c)^3)* \\
& \log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) \\
& ) - 144*(((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*\cosh(d*x + c)^3 + 3*((a^ \\
& 3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*((a \\
& ^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^2 + ((a^ \\
& 3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*\sinh(d*x + c)^3)*\log(-(a*\cosh(d*x + c) \\
& ) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/ \\
& b^2} - b)/b) - 144*(((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*\cosh(d*x + c) \\
& ^3 + 3*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*\cosh(d*x + c)^2*\sinh(d*x + \\
& c) + 3*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*\cosh(d*x + c)*\sinh(d*x + \\
& c)^2 + ((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*c*f)*\sinh(d*x + c)^3)*\log(-(a*c \\
& osh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{( \\
& a^2 + b^2)/b^2} - b)/b) - 3*(6*a*b^2*d*f*x - 4*(3*b^3*d*f*x + 3*b^3*d*e - b \\
& ^3*f)*\cosh(d*x + c)^5 + 6*a*b^2*d*e + 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b \\
& ^2*f)*\cosh(d*x + c)^4 + 3*a*b^2*f - 24*((4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b \\
& + 3*b^3)*d*e - (4*a^2*b + 3*b^3)*f)*\cosh(d*x + c)^3 - 72*((a^3 + a*b^2)*d^2 \\
& *f*x^2 + 2*(a^3 + a*b^2)*d^2*e*x + 4*(a^3 + a*b^2)*c*d*e - 2*(a^3 + a*b^2)* \\
& c^2*f)*\cosh(d*x + c)^2 + 12*((4*a^2*b + 3*b^3)*d*f*x + (4*a^2*b + 3*b^3)*d* \\
& e + (4*a^2*b + 3*b^3)*f)*\cosh(d*x + c))*\sinh(d*x + c))/(b^4*d^2*\cosh(d*x + \\
& c)^3 + 3*b^4*d^2*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*b^4*d^2*\cosh(d*x + c)*\si \\
& nh(d*x + c)^2 + b^4*d^2*\sinh(d*x + c)^3)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)\*\*3\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \cosh(dx + c)^3 \sinh(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^3\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)\*cosh(d\*x + c)^3\*sinh(d\*x + c)/(b\*sinh(d\*x + c) + a), x)

$$3.346 \quad \int \frac{\cosh^3(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=85

$$\frac{(a^2 + b^2) \sinh(c + dx)}{b^3 d} - \frac{a(a^2 + b^2) \log(a + b \sinh(c + dx))}{b^4 d} - \frac{a \sinh^2(c + dx)}{2b^2 d} + \frac{\sinh^3(c + dx)}{3bd}$$

[Out]  $-\left(\frac{a(a^2 + b^2) \operatorname{Log}[a + b \operatorname{Sinh}[c + d*x]]}{b^4*d}\right) + \left(\frac{(a^2 + b^2) \operatorname{Sinh}[c + d*x]}{b^3*d}\right) - \frac{a \operatorname{Sinh}[c + d*x]^2}{2*b^2*d} + \frac{\operatorname{Sinh}[c + d*x]^3}{3*b*d}$

**Rubi [A]** time = 0.12166, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2837, 12, 772}

$$\frac{(a^2 + b^2) \sinh(c + dx)}{b^3 d} - \frac{a(a^2 + b^2) \log(a + b \sinh(c + dx))}{b^4 d} - \frac{a \sinh^2(c + dx)}{2b^2 d} + \frac{\sinh^3(c + dx)}{3bd}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Cosh}[c + d*x]^3 \operatorname{Sinh}[c + d*x]) / (a + b \operatorname{Sinh}[c + d*x]), x]$

[Out]  $-\left(\frac{a(a^2 + b^2) \operatorname{Log}[a + b \operatorname{Sinh}[c + d*x]]}{b^4*d}\right) + \left(\frac{(a^2 + b^2) \operatorname{Sinh}[c + d*x]}{b^3*d}\right) - \frac{a \operatorname{Sinh}[c + d*x]^2}{2*b^2*d} + \frac{\operatorname{Sinh}[c + d*x]^3}{3*b*d}$

### Rule 2837

$\operatorname{Int}[\cos[(e_.) + (f_.)*(x_.)]^{(p_.)} * ((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}) * ((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[1/(b^p * f), \operatorname{Subst}[\operatorname{Int}[(a + x)^m * (c + (d*x)/b)^n * (b^2 - x^2)^{(p-1)/2}], x], x, b * \operatorname{Sin}[e + f*x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f, m, n\}, x \&\& \operatorname{IntegerQ}[(p-1)/2] \&\& \operatorname{NeQ}[a^2 - b^2, 0]$

### Rule 12

$\operatorname{Int}[(a_)*(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$   $\operatorname{FreeQ}[a, x] \&\& \operatorname{!MatchQ}[u, (b_)*(v_)] /;$   $\operatorname{FreeQ}[b, x]$

### Rule 772

$\operatorname{Int}(((d_.) + (e_.)*(x_.))^{(m_.)} * ((f_.) + (g_.)*(x_.)) * ((a_.) + (c_.)*(x_.)^2)^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(d + e*x)^m * (f + g*x) * (a + c*x^2)^p,$

$x], x] /; \text{FreeQ}\{a, c, d, e, f, g, m\}, x] \&\& \text{IGtQ}[p, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{\cosh^3(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx &= -\frac{\text{Subst}\left(\int \frac{x(-b^2-x^2)}{b(a+x)} dx, x, b \sinh(c+dx)\right)}{b^3 d} \\ &= -\frac{\text{Subst}\left(\int \frac{x(-b^2-x^2)}{a+x} dx, x, b \sinh(c+dx)\right)}{b^4 d} \\ &= -\frac{\text{Subst}\left(\int \left(-a^2\left(1+\frac{b^2}{a^2}\right) + ax - x^2 + \frac{a(a^2+b^2)}{a+x}\right) dx, x, b \sinh(c+dx)\right)}{b^4 d} \\ &= -\frac{a(a^2+b^2) \log(a+b \sinh(c+dx))}{b^4 d} + \frac{(a^2+b^2) \sinh(c+dx)}{b^3 d} - \frac{a \sinh^2(c+dx)}{2b^2 d} + \end{aligned}$$

**Mathematica [A]** time = 0.164234, size = 75, normalized size = 0.88

$$\frac{6b(a^2+b^2) \sinh(c+dx) - 6a(a^2+b^2) \log(a+b \sinh(c+dx)) - 3ab^2 \sinh^2(c+dx) + 2b^3 \sinh^3(c+dx)}{6b^4 d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cosh[c + d\*x]^3\*Sinh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (-6\*a\*(a^2 + b^2)\*Log[a + b\*Sinh[c + d\*x]] + 6\*b\*(a^2 + b^2)\*Sinh[c + d\*x] - 3\*a\*b^2\*Sinh[c + d\*x]^2 + 2\*b^3\*Sinh[c + d\*x]^3)/(6\*b^4\*d)

**Maple [B]** time = 0.039, size = 428, normalized size = 5.

$$-\frac{1}{3bd} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-3} + \frac{1}{2bd} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-2} - \frac{a}{2db^2} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-2} - \frac{a^2}{db^3} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)^3\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

```
[Out] -1/3/d/b/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^2*a-1/d/b^3/(tanh(1/2*d*x+1/2*c)+1)*a^2+1/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)*a-1/d/b/(tanh(1/2*d*x+1/2*c)+1)+1/d*a^3/b^4*ln(tanh(1/2*d*x+1/2*c)+1)+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/3/d/b/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^2*a-1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2-1/d/b^3/(tanh(1/2*d*x+1/2*c)-1)*a^2-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a-1/d/b/(tanh(1/2*d*x+1/2*c)-1)+1/d*a^3/b^4*ln(tanh(1/2*d*x+1/2*c)-1)+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/d*a^3/b^4*ln(tanh(1/2*d*x+1/2*c))^2*a-2*tanh(1/2*d*x+1/2*c)*b-a)-1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c))^2*a-2*tanh(1/2*d*x+1/2*c)*b-a)
```

**Maxima [B]** time = 1.06732, size = 247, normalized size = 2.91

$$\frac{(3abe^{-dx-c} - b^2 - 3(4a^2 + 3b^2)e^{-2dx-2c})e^{3dx+3c}}{24b^3d} - \frac{(a^3 + ab^2)(dx + c)}{b^4d} - \frac{3abe^{-2dx-2c} + b^2e^{-3dx-3c} + 3(4a^2 + 3b^2)e^{-4dx-4c}}{24b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/24*(3*a*b*e^(-d*x - c) - b^2 - 3*(4*a^2 + 3*b^2)*e^(-2*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) - (a^3 + a*b^2)*(d*x + c)/(b^4*d) - 1/24*(3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*d*x - 3*c) + 3*(4*a^2 + 3*b^2)*e^(-d*x - c))/(b^3*d) - (a^3 + a*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^4*d)
```

**Fricas [B]** time = 2.20706, size = 1613, normalized size = 18.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/24*(b^3*cosh(d*x + c)^6 + b^3*sinh(d*x + c)^6 - 3*a*b^2*cosh(d*x + c)^5 + 24*(a^3 + a*b^2)*d*x*cosh(d*x + c)^3 + 3*(2*b^3*cosh(d*x + c) - a*b^2)*sinh(d*x + c)^5 + 3*(4*a^2*b + 3*b^3)*cosh(d*x + c)^4 + 3*(5*b^3*cosh(d*x + c)^2 - 5*a*b^2*cosh(d*x + c) + 4*a^2*b + 3*b^3)*sinh(d*x + c)^4 - 3*a*b^2*cosh(d*x + c) + 2*(10*b^3*cosh(d*x + c)^3 - 15*a*b^2*cosh(d*x + c)^2 + 12*(a^3
```

$$\begin{aligned}
& + a*b^2)*d*x + 6*(4*a^2*b + 3*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 - b^3 - \\
& 3*(4*a^2*b + 3*b^3)*\cosh(d*x + c)^2 + 3*(5*b^3*\cosh(d*x + c)^4 - 10*a*b^2*c \\
& \cosh(d*x + c)^3 + 24*(a^3 + a*b^2)*d*x*\cosh(d*x + c) - 4*a^2*b - 3*b^3 + 6*( \\
& 4*a^2*b + 3*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - 24*((a^3 + a*b^2)*\cosh( \\
& d*x + c)^3 + 3*(a^3 + a*b^2)*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*(a^3 + a*b^2 \\
& )*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a^3 + a*b^2)*\sinh(d*x + c)^3)*\log(2*(b*s \\
& \sinh(d*x + c) + a)/(\cosh(d*x + c) - \sinh(d*x + c))) + 3*(2*b^3*\cosh(d*x + c) \\
& ^5 - 5*a*b^2*\cosh(d*x + c)^4 + 24*(a^3 + a*b^2)*d*x*\cosh(d*x + c)^2 + 4*(4* \\
& a^2*b + 3*b^3)*\cosh(d*x + c)^3 - a*b^2 - 2*(4*a^2*b + 3*b^3)*\cosh(d*x + c)) \\
& *\sinh(d*x + c))/(b^4*d*\cosh(d*x + c)^3 + 3*b^4*d*\cosh(d*x + c)^2*\sinh(d*x + \\
& c) + 3*b^4*d*\cosh(d*x + c)*\sinh(d*x + c)^2 + b^4*d*\sinh(d*x + c)^3)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*\*3\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [A]** time = 1.15208, size = 215, normalized size = 2.53

$$\frac{(a^3 + ab^2) \log\left(\left|b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 2a\right|\right)}{b^4 d} + \frac{b^2 d^2 \left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 - 3abd^2 \left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 12a^2 d^2 \left(e^{(dx+c)} - e^{(-dx-c)}\right)}{24b^3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^3\*sinh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out]  $-(a^3 + a*b^2)*\log(\text{abs}(b*(e^{(d*x + c)} - e^{(-d*x - c)}) + 2*a))/(b^4*d) + 1/2$   
 $4*(b^2*d^2*(e^{(d*x + c)} - e^{(-d*x - c)})^3 - 3*a*b*d^2*(e^{(d*x + c)} - e^{(-d*x - c)})^2 + 12*a^2*d^2*(e^{(d*x + c)} - e^{(-d*x - c)}) + 12*b^2*d^2*(e^{(d*x + c)} - e^{(-d*x - c)}))/(b^3*d^3)$

$$3.347 \quad \int \frac{\cosh^3(c+dx) \sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable} \left( \frac{\sinh(c+dx) \cosh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[(Cosh[c + d\*x]^3\*Sinh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x]))], x]

**Rubi [A]** time = 0.0842915, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cosh^3(c+dx) \sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Cosh[c + d\*x]^3\*Sinh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] Defer[Int] [(Cosh[c + d\*x]^3\*Sinh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x]))], x]

Rubi steps

$$\int \frac{\cosh^3(c+dx) \sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\cosh^3(c+dx) \sinh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.001, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Cosh[c + d\*x]^3\*Sinh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]



[Out] \$Aborted

**Maple [A]** time = 0.155, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx+c))^3 \sinh(dx+c)}{(fx+e)(a+b \sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)^3\*sinh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(cosh(d\*x+c)^3\*sinh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{e^{\left(-3c+\frac{3de}{f}\right)} E_1\left(\frac{3(fx+e)d}{f}\right)}{8bf} - \frac{ae^{\left(-2c+\frac{2de}{f}\right)} E_1\left(\frac{2(fx+e)d}{f}\right)}{4b^2f} + \frac{ae^{\left(2c-\frac{2de}{f}\right)} E_1\left(-\frac{2(fx+e)d}{f}\right)}{4b^2f} - \frac{e^{\left(3c-\frac{3de}{f}\right)} E_1\left(-\frac{3(fx+e)d}{f}\right)}{8bf} - \frac{(4a^2+3b^2) \log(fx+e)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^3\*sinh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-1/8*e^{(-3*c + 3*d*e/f)*exp\_integral\_e(1, 3*(f*x + e)*d/f)/(b*f)} - 1/4*a*e^{(-2*c + 2*d*e/f)*exp\_integral\_e(1, 2*(f*x + e)*d/f)/(b^2*f)} + 1/4*a*e^{(2*c - 2*d*e/f)*exp\_integral\_e(1, -2*(f*x + e)*d/f)/(b^2*f)} - 1/8*e^{(3*c - 3*d*e/f)*exp\_integral\_e(1, -3*(f*x + e)*d/f)/(b*f)} - 1/8*(4*a^2 + 3*b^2)*e^{(-c + d*e/f)*exp\_integral\_e(1, (f*x + e)*d/f)/(b^3*f)} - 1/8*(4*a^2*e^c + 3*b^2*e^c)*e^{(-d*e/f)*exp\_integral\_e(1, -(f*x + e)*d/f)/(b^3*f)} - (a^3 + a*b^2)*\log(f*x + e)/(b^4*f) + 1/16*integrate(32*(a^3*b + a*b^3 - (a^4*e^c + a^2*b^2*e^c)*e^{(d*x)})/(b^5*f*x + b^5*e - (b^5*f*x*e^{(2*c)} + b^5*e*e^{(2*c)})*e^{(2*d*x)} - 2*(a*b^4*f*x*e^c + a*b^4*e*e^c)*e^{(d*x)}), x)$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx+c)^3 \sinh(dx+c)}{afx+ae+(bfx+be) \sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^3*sinh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^3 \sinh(dx+c)}{(fx+e)(b \sinh(dx+c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^3*sinh(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

$$3.348 \quad \int \frac{(e+fx)^3 \tanh(c+dx)}{a+b \sinh(c+dx)} dx$$

Optimal. Leaf size=1021

result too large to display

```
[Out] (2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b*(a^2 + b^2)*d) - (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])]/((a^2 + b^2)*d) - (a*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])]/((a^2 + b^2)*d) + (a*(e + f*x)^3*Log[1 + E^(2*(c + d*x))]/((a^2 + b^2)*d) - ((3*I)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + ((3*I)*a^2*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) + ((3*I)*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - ((3*I)*a^2*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (3*a*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)*d^2) - (3*a*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 + b^2)*d^2) + (3*a*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))]/(2*(a^2 + b^2)*d^2) + ((6*I)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((6*I)*a^2*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(b*d^3) + ((6*I)*a^2*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^3) + (6*a*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)*d^3) + (6*a*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 + b^2)*d^3) - (3*a*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))]/(2*(a^2 + b^2)*d^3) - ((6*I)*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(b*d^4) + ((6*I)*a^2*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^4) + ((6*I)*f^3*PolyLog[4, I*E^(c + d*x)])/(b*d^4) - ((6*I)*a^2*f^3*PolyLog[4, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^4) - (6*a*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)*d^4) - (6*a*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 + b^2)*d^4) + (3*a*f^3*PolyLog[4, -E^(2*(c + d*x))]/(4*(a^2 + b^2)*d^4)
```

**Rubi [A]** time = 1.32959, antiderivative size = 1021, normalized size of antiderivative = 1., number of steps used = 39, number of rules used = 11, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$ , Rules used = {5567, 4180, 2531, 6609, 2282, 6589, 5573, 5561, 2190, 6742, 3718}

$$-\frac{6i \operatorname{PolyLog}\left(4, -ie^{c+dx}\right) f^3}{bd^4} + \frac{6ia^2 \operatorname{PolyLog}\left(4, -ie^{c+dx}\right) f^3}{b(a^2 + b^2)d^4} + \frac{6i \operatorname{PolyLog}\left(4, ie^{c+dx}\right) f^3}{bd^4} - \frac{6ia^2 \operatorname{PolyLog}\left(4, ie^{c+dx}\right) f^3}{b(a^2 + b^2)d^4}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (2\*(e + f\*x)^3\*ArcTan[E^(c + d\*x)]/(b\*d) - (2\*a^2\*(e + f\*x)^3\*ArcTan[E^(c + d\*x)]/(b\*(a^2 + b^2)\*d) - (a\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])]/((a^2 + b^2)\*d) - (a\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])]/((a^2 + b^2)\*d) + (a\*(e + f\*x)^3\*Log[1 + E^(2\*(c + d\*x))]/((a^2 + b^2)\*d) - ((3\*I)\*f\*(e + f\*x)^2\*PolyLog[2, (-I)\*E^(c + d\*x)]/(b\*d^2) + ((3\*I)\*a^2\*f\*(e + f\*x)^2\*PolyLog[2, (-I)\*E^(c + d\*x)]/(b\*(a^2 + b^2)\*d^2) + ((3\*I)\*f\*(e + f\*x)^2\*PolyLog[2, I\*E^(c + d\*x)]/(b\*d^2) - ((3\*I)\*a^2\*f\*(e + f\*x)^2\*PolyLog[2, I\*E^(c + d\*x)]/(b\*(a^2 + b^2)\*d^2) - (3\*a\*f\*(e + f\*x)^2\*PolyLog[2, -(b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])]/((a^2 + b^2)\*d^2) - (3\*a\*f\*(e + f\*x)^2\*PolyLog[2, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])]/((a^2 + b^2)\*d^2) + (3\*a\*f\*(e + f\*x)^2\*PolyLog[2, -E^(2\*(c + d\*x))]/(2\*(a^2 + b^2)\*d^2) + ((6\*I)\*f^2\*(e + f\*x)\*PolyLog[3, (-I)\*E^(c + d\*x)]/(b\*d^3) - ((6\*I)\*a^2\*f^2\*(e + f\*x)\*PolyLog[3, (-I)\*E^(c + d\*x)]/(b\*(a^2 + b^2)\*d^3) - ((6\*I)\*f^2\*(e + f\*x)\*PolyLog[3, I\*E^(c + d\*x)]/(b\*d^3) + ((6\*I)\*a^2\*f^2\*(e + f\*x)\*PolyLog[3, I\*E^(c + d\*x)]/(b\*(a^2 + b^2)\*d^3) + (6\*a\*f^2\*(e + f\*x)\*PolyLog[3, -(b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])]/((a^2 + b^2)\*d^3) + (6\*a\*f^2\*(e + f\*x)\*PolyLog[3, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])]/((a^2 + b^2)\*d^3) - (3\*a\*f^2\*(e + f\*x)\*PolyLog[3, -E^(2\*(c + d\*x))]/(2\*(a^2 + b^2)\*d^3) - ((6\*I)\*f^3\*PolyLog[4, (-I)\*E^(c + d\*x)]/(b\*d^4) + ((6\*I)\*a^2\*f^3\*PolyLog[4, (-I)\*E^(c + d\*x)]/(b\*(a^2 + b^2)\*d^4) + ((6\*I)\*f^3\*PolyLog[4, I\*E^(c + d\*x)]/(b\*d^4) - ((6\*I)\*a^2\*f^3\*PolyLog[4, I\*E^(c + d\*x)]/(b\*(a^2 + b^2)\*d^4) - (6\*a\*f^3\*PolyLog[4, -(b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])]/((a^2 + b^2)\*d^4) - (6\*a\*f^3\*PolyLog[4, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])]/((a^2 + b^2)\*d^4) + (3\*a\*f^3\*PolyLog[4, -E^(2\*(c + d\*x))]/(4\*(a^2 + b^2)\*d^4))

### Rule 5567

Int[(((e\_) + (f\_)\*(x\_))^(m\_)\*Tanh[(c\_) + (d\_)\*(x\_)]^(n\_))/((a\_) + (b\_)\*Sinh[(c\_) + (d\_)\*(x\_)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Sech[c + d\*x]\*Tanh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sech[c + d\*x]\*Tanh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 4180

Int[csc[(e\_) + Pi\*(k\_) + (Complex[0, fz\_])\*(f\_)\*(x\_)]\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)]]/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*(a_.) + (b_.)
*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
```

, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

### Rule 2190

Int[(((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_))\*((c\_) + (d\_)\*(x\_))^(m\_)] /  
 ((a\_) + (b\_)\*((F\_)^((g\_)\*(e\_) + (f\_)\*(x\_)))^(n\_)), x\_Symbol] :> Simp  
 [((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]) / (b\*f\*g\*n\*Log[F]), x] - Di  
 st[(d\*m) / (b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)  
 ))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 6742

Int[u\_, x\_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]  
 ]

### Rule 3718

Int[((c\_) + (d\_)\*(x\_))^(m\_)\*tan[(e\_) + (Complex[0, fz\_] \* (f\_)\*(x\_)]], x  
 \_Symbol] :> -Simp[(I\*(c + d\*x)^(m + 1)) / (d\*(m + 1)), x] + Dist[2\*I, Int[((c  
 + d\*x)^m \* E^(2\*(-(I\*e) + f\*fz\*x))] / (1 + E^(2\*(-(I\*e) + f\*fz\*x))), x], x] /;  
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \tanh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \operatorname{sech}(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^3 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{a \int (e+fx)^3 \operatorname{sech}(c+dx)(a-b \sinh(c+dx)) dx}{b(a^2+b^2)} - \frac{(ab) \int \frac{(e+fx)^3 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx}{b(a^2+b^2)} \\
&= \frac{a(e+fx)^4}{4(a^2+b^2)f} + \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{3if(e+fx)^2 \operatorname{Li}_2(-ie^{c+dx})}{bd^2} + \frac{3if(e+fx)^2 \operatorname{Li}_2(-ie^{c+dx})}{bd^2} \\
&= \frac{a(e+fx)^4}{4(a^2+b^2)f} + \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
&= \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
&= \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
&= \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
&= \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
&= \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
&= \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
&= \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
&= \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} - \frac{a(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d}
\end{aligned}$$

**Mathematica [B]** time = 25.5273, size = 2333, normalized size = 2.29

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^3\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

```

[Out] (8*b*Sqrt[-(a^2 + b^2)^2]*d^3*e^3*ArcTan[E^(c + d*x)] - 8*a^2*Sqrt[a^2 + b^
2]*d^3*e^3*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]] - 8*a^2*Sqrt[-a^2 -
b^2]*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + (12*I)*b*Sqrt[
-(a^2 + b^2)^2]*d^3*e^2*f*x*Log[1 - I*E^(c + d*x)] + (12*I)*b*Sqrt[-(a^2 +
b^2)^2]*d^3*e*f^2*x^2*Log[1 - I*E^(c + d*x)] + (4*I)*b*Sqrt[-(a^2 + b^2)^2]
*d^3*f^3*x^3*Log[1 - I*E^(c + d*x)] - (12*I)*b*Sqrt[-(a^2 + b^2)^2]*d^3*e^2
*f*x*Log[1 + I*E^(c + d*x)] - (12*I)*b*Sqrt[-(a^2 + b^2)^2]*d^3*e*f^2*x^2*L
og[1 + I*E^(c + d*x)] - (4*I)*b*Sqrt[-(a^2 + b^2)^2]*d^3*f^3*x^3*Log[1 + I*
E^(c + d*x)] + 4*a*Sqrt[-(a^2 + b^2)^2]*d^3*e^3*Log[1 + E^(2*(c + d*x))] +
12*a*Sqrt[-(a^2 + b^2)^2]*d^3*e^2*f*x*Log[1 + E^(2*(c + d*x))] + 12*a*Sqrt[
-(a^2 + b^2)^2]*d^3*e*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 4*a*Sqrt[-(a^2 + b
^2)^2]*d^3*f^3*x^3*Log[1 + E^(2*(c + d*x))] - 4*a*Sqrt[-(a^2 + b^2)^2]*d^3*
e^3*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))] - 12*a*Sqrt[-(a^2 + b^2
)^2]*d^3*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c
)])] - 12*a*Sqrt[-(a^2 + b^2)^2]*d^3*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a
*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 4*a*Sqrt[-(a^2 + b^2)^2]*d^3*f^3*x^3*L
og[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])] - 12*a*Sqrt[
-(a^2 + b^2)^2]*d^3*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b
^2)*E^(2*c)])] - 12*a*Sqrt[-(a^2 + b^2)^2]*d^3*e*f^2*x^2*Log[1 + (b*E^(2*c
+ d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - 4*a*Sqrt[-(a^2 + b^2)^2]*d^3
*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])] - (
12*I)*b*Sqrt[-(a^2 + b^2)^2]*d^2*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)]
+ (12*I)*b*Sqrt[-(a^2 + b^2)^2]*d^2*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)
] + 6*a*Sqrt[-(a^2 + b^2)^2]*d^2*e^2*f*PolyLog[2, -E^(2*(c + d*x))] + 12*a*
Sqrt[-(a^2 + b^2)^2]*d^2*e*f^2*x*PolyLog[2, -E^(2*(c + d*x))] + 6*a*Sqrt[-(
a^2 + b^2)^2]*d^2*f^3*x^2*PolyLog[2, -E^(2*(c + d*x))] - 12*a*Sqrt[-(a^2 +
b^2)^2]*d^2*e^2*f*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*
E^(2*c)]))] - 24*a*Sqrt[-(a^2 + b^2)^2]*d^2*e*f^2*x*PolyLog[2, -((b*E^(2*c
+ d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] - 12*a*Sqrt[-(a^2 + b^2)^2]*d
^2*f^3*x^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)
]))] - 12*a*Sqrt[-(a^2 + b^2)^2]*d^2*e^2*f*PolyLog[2, -((b*E^(2*c + d*x))/(
a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 24*a*Sqrt[-(a^2 + b^2)^2]*d^2*e*f^2*
x*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] - 12
*a*Sqrt[-(a^2 + b^2)^2]*d^2*f^3*x^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c +
Sqrt[(a^2 + b^2)*E^(2*c)]))] + (24*I)*b*Sqrt[-(a^2 + b^2)^2]*d*e*f^2*PolyL
og[3, (-I)*E^(c + d*x)] + (24*I)*b*Sqrt[-(a^2 + b^2)^2]*d*f^3*x*PolyLog[3,
(-I)*E^(c + d*x)] - (24*I)*b*Sqrt[-(a^2 + b^2)^2]*d*e*f^2*PolyLog[3, I*E^(c
+ d*x)] - (24*I)*b*Sqrt[-(a^2 + b^2)^2]*d*f^3*x*PolyLog[3, I*E^(c + d*x)]
- 6*a*Sqrt[-(a^2 + b^2)^2]*d*e*f^2*PolyLog[3, -E^(2*(c + d*x))] - 6*a*Sqrt[
-(a^2 + b^2)^2]*d*f^3*x*PolyLog[3, -E^(2*(c + d*x))] + 24*a*Sqrt[-(a^2 + b^
2)^2]*d*e*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)]))] + 24*a*Sqrt[-(a^2 + b^2)^2]*d*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/
(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))] + 24*a*Sqrt[-(a^2 + b^2)^2]*d*e*f^2*P
olyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))] + 24*a*
Sqrt[-(a^2 + b^2)^2]*d*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(

```



$$a^2 + b^2)E^{(2c)}]] - (24I)*b*sqrt[-(a^2 + b^2)^2]*f^3*PolyLog[4, (-I)*E^{(c + dx)}] + (24I)*b*sqrt[-(a^2 + b^2)^2]*f^3*PolyLog[4, I*E^{(c + dx)}] + 3*a*sqrt[-(a^2 + b^2)^2]*f^3*PolyLog[4, -E^{(2*(c + dx))}] - 24*a*sqrt[-(a^2 + b^2)^2]*f^3*PolyLog[4, -((bE^{(2c + dx)})/(aE^c - sqrt[(a^2 + b^2)*E^{(2c)}]))] - 24*a*sqrt[-(a^2 + b^2)^2]*f^3*PolyLog[4, -((bE^{(2c + dx)})/(aE^c + sqrt[(a^2 + b^2)*E^{(2c)}])))]/(4*sqrt[-a^2 - b^2]*(a^2 + b^2)^(3/2)*d^4)$$

**Maple [F]** time = 0.353, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \tanh(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-e^3 \left( \frac{2b \arctan(e^{-dx-c})}{(a^2 + b^2)d} + \frac{a \log(-2ae^{-dx-c} + be^{-2dx-2c} - b)}{(a^2 + b^2)d} - \frac{a \log(e^{-2dx-2c} + 1)}{(a^2 + b^2)d} \right) + \int \frac{2f^3 x^3 (e^{dx+c})}{(b(e^{dx+c}) - e^{-dx-c}) + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-e^3*(2*b*arctan(e^{-d*x - c})/((a^2 + b^2)*d) + a*\log(-2*a*e^{-d*x - c} + b*e^{-2*d*x - 2*c} - b)/((a^2 + b^2)*d) - a*\log(e^{-2*d*x - 2*c} + 1)/((a^2 + b^2)*d) + integrate(2*f^3*x^3*(e^{d*x + c} - e^{-d*x - c}))/((b*(e^{d*x + c} - e^{-d*x - c}) + 2*a)*(e^{d*x + c} + e^{-d*x - c})) + 6*e*f^2*x^2*(e^{d*x + c} - e^{-d*x - c}))/((b*(e^{d*x + c} - e^{-d*x - c}) + 2*a)*(e^{d*x + c} + e^{-d*x - c})) + 6*e^2*f*x*(e^{d*x + c} - e^{-d*x - c}))/((b*(e^{d*x + c} - e^{-d*x - c}) + 2*a)*(e^{d*x + c} + e^{-d*x - c})), x)$

**Fricas [C]** time = 3.12044, size = 4177, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-(6*a*f^3*\text{polylog}(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) + 6*a*f^3*\text{polylog}(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) + 3*(a*d^2*f^3*x^2 + 2*a*d^2*e*f^2*x + a*d^2*e^2*f)*\text{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 3*(a*d^2*f^3*x^2 + 2*a*d^2*e*f^2*x + a*d^2*e^2*f)*\text{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - (3*a*d^2*f^3*x^2 + 3*I*b*d^2*f^3*x^2 + 6*a*d^2*e*f^2*x + 6*I*b*d^2*e*f^2*x + 3*a*d^2*e^2*f + 3*I*b*d^2*e^2*f)*\text{dilog}(I*\cosh(d*x + c) + I*\sinh(d*x + c)) - (3*a*d^2*f^3*x^2 - 3*I*b*d^2*f^3*x^2 + 6*a*d^2*e*f^2*x - 6*I*b*d^2*e*f^2*x + 3*a*d^2*e^2*f - 3*I*b*d^2*e^2*f)*\text{dilog}(-I*\cosh(d*x + c) - I*\sinh(d*x + c)) + (a*d^3*e^3 - 3*a*c*d^2*e^2*f + 3*a*c^2*d*e*f^2 - a*c^3*f^3)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + (a*d^3*e^3 - 3*a*c*d^2*e^2*f + 3*a*c^2*d*e*f^2 - a*c^3*f^3)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + (a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + 3*a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + (a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + 3*a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - (a*d^3*e^3 + I*b*d^3*e^3 - 3*a*c*d^2*e^2*f - 3*I*b*c*d^2*e^2*f + 3*a*c^2*d*e*f^2 + 3*I*b*c^2*d*e*f^2 - a*c^3*f^3 - I*b*c^3*f^3)*\log(\cosh(d*x + c) + \sinh(d*x + c) + I) - (a*d^3*e^3 - I*b*d^3*e^3 - 3*a*c*d^2*e^2*f + 3*I*b*c*d^2*e^2*f + 3*a*c^2*d*e*f^2 - 3*I*b*c^2*d*e*f^2 - a*c^3*f^3 + I*b*c^3*f^3)*\log(\cosh(d*x + c) + \sinh(d*x + c) - I) - (a*d^3*f^3*x^3 - I*b*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 - 3*I*b*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x - 3*I*b*d^3*e^2*f*x + 3*a*c*d^2*e^2*f - 3*I*b*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + 3*I*b*c^2*d*e*f^2 + a*c^3*f^3 - I*b*c^3*f^3)*\log(I*\cosh(d*x + c) + I*\sinh(d*x + c) + 1) - (a*d^3*f^3*x^3 + I*b*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*I*b*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + 3*I*b*d^3*e^2*f*x + 3*a*c*d^2*e^2*f + 3*I*b*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 - 3*I*b*c^2*d*e*f^2 + a*c^3*f^3 + I*b*c^3*f^3)*\log(-I*\cosh(d*x + c) - I*\sinh(d*x + c) + 1) - (6*a*f^3 + 6*I*b*f^3)*\text{polylog}(4, I*\cosh(d*x + c) + I*\sinh(d*x + c)) - (6*a*f^3 - 6*I*b*f^3)*\text{polylog}(4, -I*\cosh(d*x + c) - I*\sinh(d*x + c)) - 6*(a*d*f^3*x + a*d*e*f^2)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b)$$

$$\begin{aligned} & \sqrt{(a^2 + b^2)/b^2})/b) - 6*(a*d*f^3*x + a*d*e*f^2)*\text{polylog}(3, (a*\cosh(d*x + c) \\ & + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\text{sqrt}((a^2 + b^2)/b^2))/b) + (6*a*d*f^3*x + 6*I*b*d*f^3*x + 6*a*d*e*f^2 + 6*I*b*d*e*f^2)*\text{polylo} \\ & \text{g}(3, I*\cosh(d*x + c) + I*\sinh(d*x + c)) + (6*a*d*f^3*x - 6*I*b*d*f^3*x + 6* \\ & a*d*e*f^2 - 6*I*b*d*e*f^2)*\text{polylog}(3, -I*\cosh(d*x + c) - I*\sinh(d*x + c))/ \\ & ((a^2 + b^2)*d^4) \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^3 \tanh(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*\*3\*tanh(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.349 \quad \int \frac{(e+fx)^2 \tanh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=716

$$\frac{2ia^2 f(e+fx) \text{PolyLog}\left(2, -ie^{c+dx}\right)}{bd^2(a^2+b^2)} - \frac{2ia^2 f(e+fx) \text{PolyLog}\left(2, ie^{c+dx}\right)}{bd^2(a^2+b^2)} - \frac{2af(e+fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)} - \frac{2af(e+fx) \text{PolyLog}\left(2, \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)}$$

```
[Out] (2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b*(a^2 + b^2)*d) - (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) - (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) + (a*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d) - ((2*I)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + ((2*I)*a^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)]/(b*(a^2 + b^2)*d^2) + ((2*I)*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/(b*d^2) - ((2*I)*a^2*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/(b*(a^2 + b^2)*d^2) - (2*a*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)*d^2) - (2*a*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d^2) + (a*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2)*d^2) + ((2*I)*f^2*PolyLog[3, (-I)*E^(c + d*x)]/(b*d^3) - ((2*I)*a^2*f^2*PolyLog[3, (-I)*E^(c + d*x)]/(b*(a^2 + b^2)*d^3) - ((2*I)*f^2*PolyLog[3, I*E^(c + d*x)]/(b*d^3) + ((2*I)*a^2*f^2*PolyLog[3, I*E^(c + d*x)]/(b*(a^2 + b^2)*d^3) + (2*a*f^2*PolyLog[3, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)*d^3) + (2*a*f^2*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d^3) - (a*f^2*PolyLog[3, -E^(2*(c + d*x))])/((2*(a^2 + b^2)*d^3))
```

**Rubi [A]** time = 1.06746, antiderivative size = 716, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 10, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$ , Rules used = {5567, 4180, 2531, 2282, 6589, 5573, 5561, 2190, 6742, 3718}

$$\frac{2ia^2 f(e+fx) \text{PolyLog}\left(2, -ie^{c+dx}\right)}{bd^2(a^2+b^2)} - \frac{2ia^2 f(e+fx) \text{PolyLog}\left(2, ie^{c+dx}\right)}{bd^2(a^2+b^2)} - \frac{2af(e+fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)} - \frac{2af(e+fx) \text{PolyLog}\left(2, \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b*(a^2 + b^2)*d) - (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) - (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d) + (a*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)*d) - ((2*I)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + ((2*I)*a^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)]/(b*(a^2 + b^2)*d^2) + ((2*I)*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/(b*d^2) - ((2*I)*a^2*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/(b*(a^2 + b^2)*d^2) - (2*a*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)*d^2) - (2*a*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d^2) + (a*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((a^2 + b^2)*d^2) + ((2*I)*f^2*PolyLog[3, (-I)*E^(c + d*x)]/(b*d^3) - ((2*I)*a^2*f^2*PolyLog[3, (-I)*E^(c + d*x)]/(b*(a^2 + b^2)*d^3) - ((2*I)*f^2*PolyLog[3, I*E^(c + d*x)]/(b*d^3) + ((2*I)*a^2*f^2*PolyLog[3, I*E^(c + d*x)]/(b*(a^2 + b^2)*d^3) + (2*a*f^2*PolyLog[3, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)*d^3) + (2*a*f^2*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)*d^3) - (a*f^2*PolyLog[3, -E^(2*(c + d*x))])/((2*(a^2 + b^2)*d^3))
```

```

rt[a^2 + b^2]]]/((a^2 + b^2)*d) - (a*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]])]/((a^2 + b^2)*d) + (a*(e + f*x)^2*Log[1 + E^(2*(c + d
*x))]/((a^2 + b^2)*d) - ((2*I)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)]/(
b*d^2) + ((2*I)*a^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)]/(b*(a^2 + b^2
)*d^2) + ((2*I)*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/(b*d^2) - ((2*I)*a^2
*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/(b*(a^2 + b^2)*d^2) - (2*a*f*(e + f
*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)*d^2)
- (2*a*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(
(a^2 + b^2)*d^2) + (a*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))]/((a^2 + b^2
)*d^2) + ((2*I)*f^2*PolyLog[3, (-I)*E^(c + d*x)]/(b*d^3) - ((2*I)*a^2*f^2*
PolyLog[3, (-I)*E^(c + d*x)]/(b*(a^2 + b^2)*d^3) - ((2*I)*f^2*PolyLog[3, I
*E^(c + d*x)]/(b*d^3) + ((2*I)*a^2*f^2*PolyLog[3, I*E^(c + d*x)]/(b*(a^2
+ b^2)*d^3) + (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]
)/((a^2 + b^2)*d^3) + (2*a*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))]/((a^2 + b^2)*d^3) - (a*f^2*PolyLog[3, -E^(2*(c + d*x))]/(2*(a^2
+ b^2)*d^3)

```

### Rule 5567

```

Int[(((e_.) + (f_.)*(x_.))^(m_.)*Tanh[(c_.) + (d_.)*(x_.)]^(n_.))/((a_.) + (b_
.)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sech[c
+ d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

```

### Rule 4180

```

Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_.)]*((c_.) + (d_.)*(x_
.))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)]]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]

```

### Rule 2531

```

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_.)))^(n_.)]*((f_.) + (g_.)
*(x_.))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]

```

### Rule 2282

```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]

```

```
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

### Rule 3718

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))]/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /;
```

FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(e+fx)^2 \tanh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \operatorname{sech}(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
 &= \frac{2(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{a \int (e+fx)^2 \operatorname{sech}(c+dx)(a-b \sinh(c+dx)) dx}{b(a^2+b^2)} - \frac{(ab) \int \frac{(e+fx)^2}{a+b \sinh(c+dx)} dx}{b(a^2+b^2)} \\
 &= \frac{a(e+fx)^3}{3(a^2+b^2)f} + \frac{2(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{2if(e+fx)\operatorname{Li}_2(-ie^{c+dx})}{bd^2} + \frac{2if(e+fx)\operatorname{Li}_2\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2} \\
 &= \frac{a(e+fx)^3}{3(a^2+b^2)f} + \frac{2(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{a(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} - \frac{a(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
 &= \frac{2(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} - \frac{a(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
 &= \frac{2(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} - \frac{a(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
 &= \frac{2(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} - \frac{a(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
 &= \frac{2(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} - \frac{a(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
 &= \frac{2(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} - \frac{a(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
 &= \frac{2(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} - \frac{a(e+fx)^2 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d}
 \end{aligned}$$

**Mathematica [B]** time = 19.14, size = 1640, normalized size = 2.29

result too large to display

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

```

[Out] (-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e*f*x^2
+ 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] - 6*a*d^
2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*d*e*(1 +
E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - PolyLog
[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1 + E^(2*c))*
f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]) +
(6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log[1
+ I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2, I*
E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*x)]
) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))]) - 6
*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))])/(6*(a^
2 + b^2)*d^3*(1 + E^(2*c))) + (a*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E
^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-
a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*
c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6
*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/(
(-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a +
b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^
(c + d*x) + b*(-1 + E^(2*(c + d*x)))]/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*
x) + b*(-1 + E^(2*(c + d*x)))]/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))]/(a*E
^c - Sqrt[(a^2 + b^2)*E^(2*c)]))/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c +
d*x))]/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))/d + (3*f^2*x^2*Log[1 + (b*E^(2*
c + d*x))]/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))/d - (3*E^(2*c)*f^2*x^2*Log[
1 + (b*E^(2*c + d*x))]/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))/d + (6*e*f*x*Lo
g[1 + (b*E^(2*c + d*x))]/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))/d - (6*e*E^(2
*c)*f*x*Log[1 + (b*E^(2*c + d*x))]/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))/d +
(3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))]/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))
/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))]/(a*E^c + Sqrt[(a^2 + b^2)
*E^(2*c)]))/d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x)
))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*
x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d^
2 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)
]])])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^
2 + b^2)*E^(2*c)])])/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c +
Sqrt[(a^2 + b^2)*E^(2*c)])])/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c +
d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d^3)/(3*(a^2 + b^2)*(-1 + E^
(2*c))) - (a*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Csch[c/2]*Sech[c/2]*Sech[c])/(6*
(a^2 + b^2))

```



**Maple [F]** time = 0.285, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \tanh(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-e^2 \left( \frac{2b \arctan(e^{(-dx-c)})}{(a^2 + b^2)d} + \frac{a \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{(a^2 + b^2)d} - \frac{a \log(e^{(-2dx-2c)} + 1)}{(a^2 + b^2)d} \right) + \int \frac{2f^2x^2(e^{(dx+c)})}{(b(e^{(dx+c)} - e^{(-dx-c)}) + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-e^2 * (2*b*arctan(e^{(-d*x - c)}) / ((a^2 + b^2)*d) + a*log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b) / ((a^2 + b^2)*d) - a*log(e^{(-2*d*x - 2*c)} + 1) / ((a^2 + b^2)*d)) + integrate(2*f^2*x^2*(e^{(d*x + c)} - e^{(-d*x - c)}) / ((b*(e^{(d*x + c)} - e^{(-d*x - c)}) + 2*a)*(e^{(d*x + c)} + e^{(-d*x - c)})) + 4*e*f*x*(e^{(d*x + c)} - e^{(-d*x - c)}) / ((b*(e^{(d*x + c)} - e^{(-d*x - c)}) + 2*a)*(e^{(d*x + c)} + e^{(-d*x - c)})), x)$

**Fricas [C]** time = 2.70121, size = 2743, normalized size = 3.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $(2*a*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b + 2*a*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +$

```

b^2)/b^2))/b) - 2*(a*d*f^2*x + a*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x
+ c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b +
1) - 2*(a*d*f^2*x + a*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (2*a*d
*f^2*x + 2*I*b*d*f^2*x + 2*a*d*e*f + 2*I*b*d*e*f)*dilog(I*cosh(d*x + c) + I
*sinh(d*x + c)) + (2*a*d*f^2*x - 2*I*b*d*f^2*x + 2*a*d*e*f - 2*I*b*d*e*f)*d
ilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - (a*d^2*e^2 - 2*a*c*d*e*f + a*c^2
*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2)
+ 2*a) - (a*d^2*e^2 - 2*a*c*d*e*f + a*c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b
*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (a*d^2*f^2*x^2 + 2*a*d^
2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
+ (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (a*d^
2*f^2*x^2 + 2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*log(-(a*cosh(d*x + c)
+ a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^
2) - b)/b) + (a*d^2*e^2 + I*b*d^2*e^2 - 2*a*c*d*e*f - 2*I*b*c*d*e*f + a*c^2
*f^2 + I*b*c^2*f^2)*log(cosh(d*x + c) + sinh(d*x + c) + I) + (a*d^2*e^2 - I
*b*d^2*e^2 - 2*a*c*d*e*f + 2*I*b*c*d*e*f + a*c^2*f^2 - I*b*c^2*f^2)*log(cos
h(d*x + c) + sinh(d*x + c) - I) + (a*d^2*f^2*x^2 - I*b*d^2*f^2*x^2 + 2*a*d^
2*e*f*x - 2*I*b*d^2*e*f*x + 2*a*c*d*e*f - 2*I*b*c*d*e*f - a*c^2*f^2 + I*b*c
^2*f^2)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) + (a*d^2*f^2*x^2 + I*b*d
^2*f^2*x^2 + 2*a*d^2*e*f*x + 2*I*b*d^2*e*f*x + 2*a*c*d*e*f + 2*I*b*c*d*e*f
- a*c^2*f^2 - I*b*c^2*f^2)*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) - (2
*a*f^2 + 2*I*b*f^2)*polylog(3, I*cosh(d*x + c) + I*sinh(d*x + c)) - (2*a*f^
2 - 2*I*b*f^2)*polylog(3, -I*cosh(d*x + c) - I*sinh(d*x + c))/((a^2 + b^2)
*d^3)

```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^2 \tanh(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*\*2\*tanh(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.350 \quad \int \frac{(e+fx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=421

$$\frac{ia^2 f \text{PolyLog}\left(2, -ie^{c+dx}\right)}{bd^2 (a^2 + b^2)} - \frac{ia^2 f \text{PolyLog}\left(2, ie^{c+dx}\right)}{bd^2 (a^2 + b^2)} - \frac{af \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2 (a^2 + b^2)} - \frac{af \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2 (a^2 + b^2)} + \frac{af \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2 (a^2 + b^2)}$$

```
[Out] (2*(e + f*x)*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*(e + f*x)*ArcTan[E^(c + d*
x)])/(b*(a^2 + b^2)*d) - (a*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])])/((a^2 + b^2)*d) - (a*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])])/((a^2 + b^2)*d) + (a*(e + f*x)*Log[1 + E^(2*(c + d*x))])/((a
^2 + b^2)*d) - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + (I*a^2*f*PolyLo
g[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) + (I*f*PolyLog[2, I*E^(c + d*x)
])/ (b*d^2) - (I*a^2*f*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (a*f
*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2) -
(a*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2)
+ (a*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^2)
```

**Rubi [A]** time = 0.596636, antiderivative size = 421, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 9, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5567, 4180, 2279, 2391, 5573, 5561, 2190, 6742, 3718}

$$\frac{ia^2 f \text{PolyLog}\left(2, -ie^{c+dx}\right)}{bd^2 (a^2 + b^2)} - \frac{ia^2 f \text{PolyLog}\left(2, ie^{c+dx}\right)}{bd^2 (a^2 + b^2)} - \frac{af \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2 (a^2 + b^2)} - \frac{af \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2 (a^2 + b^2)} + \frac{af \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2 (a^2 + b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Tanh[c + d*x])/(a + b*Sinh[c + d*x]), x]
```

```
[Out] (2*(e + f*x)*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*(e + f*x)*ArcTan[E^(c + d*
x)])/(b*(a^2 + b^2)*d) - (a*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2])])/((a^2 + b^2)*d) - (a*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])])/((a^2 + b^2)*d) + (a*(e + f*x)*Log[1 + E^(2*(c + d*x))])/((a
^2 + b^2)*d) - (I*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + (I*a^2*f*PolyLo
g[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) + (I*f*PolyLog[2, I*E^(c + d*x)
])/ (b*d^2) - (I*a^2*f*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (a*f
*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2) -
(a*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)*d^2)
+ (a*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^2)
```

Rule 5567

Int[(((e\_.) + (f\_.)\*(x\_.))^(m\_.)\*Tanh[(c\_.) + (d\_.)\*(x\_.)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Sech[c + d\*x]\*Tanh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sech[c + d\*x]\*Tanh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_.)]\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

Rule 2279

Int[Log[(a\_.) + (b\_.)\*((F\_)^(e\_.)\*((c\_.) + (d\_.)\*(x\_.)))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 5573

Int[(((e\_.) + (f\_.)\*(x\_.))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_.)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f\*x)^m\*Sech[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f\*x)^m\*Sech[c + d\*x]^n\*(a - b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]

Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_.)]\*((e\_.) + (f\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

Rule 2190

```

Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^((n_)*((c_) + (d_)*(x_)^((m_)))/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^((n_))), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

### Rule 6742

```

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]

```

### Rule 3718

```

Int[((c_) + (d_)*(x_)^((m_))*tan[(e_) + (Complex[0, fz_])*(f_)*(x_)]), x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-I*e) + f*fz*x))/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\operatorname{sech}(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx}{b} \\
&= \frac{2(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{a \int (e+fx)\operatorname{sech}(c+dx)(a-b\sinh(c+dx)) dx}{b(a^2+b^2)} - \frac{(ab) \int \frac{(e+fx)}{a+b\sinh(c+dx)} dx}{a^2+b^2} \\
&= \frac{a(e+fx)^2}{2(a^2+b^2)f} + \frac{2(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{a \int (a(e+fx)\operatorname{sech}(c+dx) - b(e+fx)\tanh(c+dx)) dx}{b(a^2+b^2)} \\
&= \frac{a(e+fx)^2}{2(a^2+b^2)f} + \frac{2(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{a(e+fx)\log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} - \frac{a(e+fx)\log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
&= \frac{2(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2(e+fx)\tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} - \frac{a(e+fx)\log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} - \frac{a(e+fx)\log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
&= \frac{2(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2(e+fx)\tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} - \frac{a(e+fx)\log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} - \frac{a(e+fx)\log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
&= \frac{2(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2(e+fx)\tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} - \frac{a(e+fx)\log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} - \frac{a(e+fx)\log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} \\
&= \frac{2(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2(e+fx)\tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} - \frac{a(e+fx)\log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d} - \frac{a(e+fx)\log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)d}
\end{aligned}$$

**Mathematica [A]** time = 2.63113, size = 438, normalized size = 1.04

$$-2af\operatorname{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) - 2af\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) + af\operatorname{PolyLog}(2, -\sinh(2(c+dx)) - \cosh(2(c+dx))) - 2ib$$

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]), x]

[Out] (-2\*a\*c\*d\*e + 2\*a\*c^2\*f - 2\*a\*d^2\*e\*x + 2\*a\*c\*d\*f\*x + 4\*b\*d\*e\*ArcTan[Cosh[c + d\*x] + Sinh[c + d\*x]] + 4\*b\*d\*f\*x\*ArcTan[Cosh[c + d\*x] + Sinh[c + d\*x]] - 2\*a\*c\*f\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])] - 2\*a\*d\*f\*x\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])] - 2\*a\*c\*f\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] - 2\*a\*d\*f\*x\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])]

$$\begin{aligned} & (a + \sqrt{a^2 + b^2})] - 2*a*d*f*x*Log[1 + (b*E^(c + d*x))/(a + \sqrt{a^2 + b^2})] - 2*a*d*e*Log[a + b*\sinh[c + d*x]] + 2*a*c*f*Log[a + b*\sinh[c + d*x]] \\ & + 2*a*d*e*Log[1 + \cosh[2*(c + d*x)] + \sinh[2*(c + d*x)]] + 2*a*d*f*x*Log[1 + \cosh[2*(c + d*x)] + \sinh[2*(c + d*x)]] - 2*a*f*PolyLog[2, (b*E^(c + d*x))/(-a + \sqrt{a^2 + b^2})] \\ & - 2*a*f*PolyLog[2, -(b*E^(c + d*x))/(a + \sqrt{a^2 + b^2})] - (2*I)*b*f*PolyLog[2, (-I)*(Cosh[c + d*x] + Sinh[c + d*x])] + (2*I)*b*f*PolyLog[2, I*(Cosh[c + d*x] + Sinh[c + d*x])] + a*f*PolyLog[2, -Cosh[2*(c + d*x)] - Sinh[2*(c + d*x)]]/(2*(a^2 + b^2)*d^2) \end{aligned}$$

**Maple [B]** time = 0.154, size = 1287, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)`

[Out] 
$$\begin{aligned} & 2/d^2*f/(2*a^2+2*b^2)*\ln(1-I*\exp(d*x+c))*a*c-2/d*f/(2*a^2+2*b^2)*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) \\ & *a*x-2/d^2*f/(2*a^2+2*b^2)*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) \\ & *a*c-2/d*f/(2*a^2+2*b^2)*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) \\ & *a*x-2/d^2*f/(2*a^2+2*b^2)*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) \\ & *a*c+2/d*f/(2*a^2+2*b^2)*\ln(1+I*\exp(d*x+c))*a*x-4/d^2*f*c/(2*a^2+2*b^2)*b*\arctan(\exp(d*x+c))+2/d^2*f*c/(2*a^2+2*b^2)*a*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)-2/d^2*f*c/(2*a^2+2*b^2)*(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-2/d^2*f*c/(2*a^2+2*b^2)*a*\ln(1+\exp(2*d*x+2*c))-2/d*e/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})*a^2+2*I/d^2*f/(2*a^2+2*b^2)*\operatorname{dilog}(1-I*\exp(d*x+c))*b-2*I/d^2*f/(2*a^2+2*b^2)*\operatorname{dilog}(1+I*\exp(d*x+c))*b+2/d^2*f/(2*a^2+2*b^2)*\ln(1+I*\exp(d*x+c))*a*c+2/d*f/(2*a^2+2*b^2)*\ln(1-I*\exp(d*x+c))*a*x+2/d^2*f*c/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})*a^2+2*I/d*f/(2*a^2+2*b^2)*b*x+2*I/d^2*f/(2*a^2+2*b^2)*\ln(1-I*\exp(d*x+c))*b*c-2*I/d*f/(2*a^2+2*b^2)*\ln(1+I*\exp(d*x+c))*b*x-2*I/d^2*f/(2*a^2+2*b^2)*\ln(1+I*\exp(d*x+c))*b*c-2/d/(a^2+b^2)^{(1/2)}*b^2*e/(2*a^2+2*b^2)*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+2/d^2/(a^2+b^2)^{(1/2)}*b^2*f*c/(2*a^2+2*b^2)*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+2/d^2*f/(2*a^2+2*b^2)*\operatorname{dilog}(1-I*\exp(d*x+c))*a-2/d^2*f/(2*a^2+2*b^2)*\operatorname{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) \\ & *a-2/d^2*f/(2*a^2+2*b^2)*\operatorname{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) \\ & *a+2/d*e/(2*a^2+2*b^2)*a*\ln(1+\exp(2*d*x+2*c))+4/d*e/(2*a^2+2*b^2)*b*\arctan(\exp(d*x+c))-2/d*e/(2*a^2+2*b^2)*a*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)+2/d*e/(2*a^2+2*b^2)*(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+2/d^2*f/(2*a^2+2*b^2)*\operatorname{dilog} \end{aligned}$$



$(1+I*\exp(d*x+c))*a$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-e^{\left(\frac{2b \arctan(e^{-dx-c})}{(a^2+b^2)d} + \frac{a \log(-2ae^{-dx-c} + be^{-2dx-2c}) - b}{(a^2+b^2)d} - \frac{a \log(e^{-2dx-2c} + 1)}{(a^2+b^2)d}\right)} + f \int \frac{2x(e^{dx+c}) - 2x(e^{-dx-c})}{(b(e^{dx+c}) - e^{-dx-c}) + 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-e*(2*b*\arctan(e^{-d*x - c})/((a^2 + b^2)*d) + a*\log(-2*a*e^{-d*x - c} + b*e^{-2*d*x - 2*c}) - b)/((a^2 + b^2)*d) - a*\log(e^{-2*d*x - 2*c} + 1)/((a^2 + b^2)*d) + f*\integrate(2*x*(e^{d*x + c} - e^{-d*x - c})/((b*(e^{d*x + c}) - e^{-d*x - c}) + 2*a)*(e^{d*x + c} + e^{-d*x - c})), x$

**Fricas [A]** time = 2.46716, size = 1544, normalized size = 3.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $-(a*f*dilog((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + a*f*dilog((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - (a*f + I*b*f)*dilog(I*\cosh(d*x + c) + I*\sinh(d*x + c)) - (a*f - I*b*f)*dilog(-I*\cosh(d*x + c) - I*\sinh(d*x + c)) + (a*d*e - a*c*f)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + (a*d*e - a*c*f)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + (a*d*f*x + a*c*f)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + (a*d*f*x + a*c*f)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - (a*d*e + I*b*d*e - a*c*f - I*b*c*f)*\log(\cosh(d*x + c) + \sinh(d*x + c) + I) - (a*d*e - I*b*d*e - a*c*f + I*b*c*f)*\log(\cosh(d*x + c) + \sinh(d*x + c) - I) - (a*d*f*x - I*b*d*f*x + a*c*f - I*b*c*f)*\log(I*\cosh(d*x + c) + I*\sinh(d*x + c) + 1) - (a*d*f*x + I*b*d*f*x + a*c*f + I*b*c*f)*\log(-I*\cosh(d*x + c) - I*\sinh(d*x + c) + 1))/$

$((a^2 + b^2)*d^2)$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \tanh(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*tanh(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.351 \quad \int \frac{\tanh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=69

$$-\frac{a \log(a + b \sinh(c + dx))}{d(a^2 + b^2)} + \frac{b \tan^{-1}(\sinh(c + dx))}{d(a^2 + b^2)} + \frac{a \log(\cosh(c + dx))}{d(a^2 + b^2)}$$

[Out] (b\*ArcTan[Sinh[c + d\*x]])/((a^2 + b^2)\*d) + (a\*Log[Cosh[c + d\*x]])/((a^2 + b^2)\*d) - (a\*Log[a + b\*Sinh[c + d\*x]])/((a^2 + b^2)\*d)

**Rubi [A]** time = 0.0769974, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {2721, 801, 635, 203, 260}

$$-\frac{a \log(a + b \sinh(c + dx))}{d(a^2 + b^2)} + \frac{b \tan^{-1}(\sinh(c + dx))}{d(a^2 + b^2)} + \frac{a \log(\cosh(c + dx))}{d(a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d\*x]/(a + b\*Sinh[c + d\*x]),x]

[Out] (b\*ArcTan[Sinh[c + d\*x]])/((a^2 + b^2)\*d) + (a\*Log[Cosh[c + d\*x]])/((a^2 + b^2)\*d) - (a\*Log[a + b\*Sinh[c + d\*x]])/((a^2 + b^2)\*d)

#### Rule 2721

Int[((a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)])^(m\_)\*tan[(e\_) + (f\_)\*(x\_)]^(p\_), x\_Symbol] := Dist[1/f, Subst[Int[(x^p\*(a + x)^m)/(b^2 - x^2)^(p + 1)/2], x], x, b\*Sin[e + f\*x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

#### Rule 801

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Int[ExpandIntegrand[((d + e\*x)^m\*(f + g\*x))/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[m]

#### Rule 635

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}

}, x] && !NiceSqrtQ[-(a\*c)]

### Rule 203

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{\tanh(c + dx)}{a + b \sinh(c + dx)} dx &= -\frac{\text{Subst}\left(\int \frac{x}{(a+x)(-b^2-x^2)} dx, x, b \sinh(c + dx)\right)}{d} \\
 &= -\frac{\text{Subst}\left(\int \left(\frac{a}{(a^2+b^2)(a+x)} + \frac{-b^2-ax}{(a^2+b^2)(b^2+x^2)}\right) dx, x, b \sinh(c + dx)\right)}{d} \\
 &= -\frac{a \log(a + b \sinh(c + dx))}{(a^2 + b^2) d} - \frac{\text{Subst}\left(\int \frac{-b^2-ax}{b^2+x^2} dx, x, b \sinh(c + dx)\right)}{(a^2 + b^2) d} \\
 &= -\frac{a \log(a + b \sinh(c + dx))}{(a^2 + b^2) d} + \frac{a \text{Subst}\left(\int \frac{x}{b^2+x^2} dx, x, b \sinh(c + dx)\right)}{(a^2 + b^2) d} + \frac{b^2 \text{Subst}\left(\int \frac{1}{b^2+x^2} dx, x, b \sinh(c + dx)\right)}{(a^2 + b^2) d} \\
 &= \frac{b \tan^{-1}(\sinh(c + dx))}{(a^2 + b^2) d} + \frac{a \log(\cosh(c + dx))}{(a^2 + b^2) d} - \frac{a \log(a + b \sinh(c + dx))}{(a^2 + b^2) d}
 \end{aligned}$$

**Mathematica [A]** time = 0.0767782, size = 51, normalized size = 0.74

$$\frac{a(\log(\cosh(c + dx)) - \log(a + b \sinh(c + dx))) + 2b \tan^{-1}\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right)}{d(a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d\*x]/(a + b\*Sinh[c + d\*x]),x]

[Out]  $(2*b*ArcTan[Tanh[(c + d*x)/2]] + a*(Log[Cosh[c + d*x]] - Log[a + b*Sinh[c + d*x]]))/((a^2 + b^2)*d)$

**Maple [A]** time = 0.002, size = 113, normalized size = 1.6

$$-2 \frac{a \ln\left(\left(\tanh\left(\frac{1}{2} dx + \frac{c}{2}\right)\right)^2 a - 2 \tanh\left(\frac{1}{2} dx + \frac{c}{2}\right) b - a\right)}{d(2a^2 + 2b^2)} + 2 \frac{a \ln\left(\left(\tanh\left(\frac{1}{2} dx + \frac{c}{2}\right)\right)^2 + 1\right)}{d(2a^2 + 2b^2)} + 4 \frac{b \arctan\left(\tanh\left(\frac{1}{2} dx + \frac{c}{2}\right)\right)}{d(2a^2 + 2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(d*x+c)/(a+b*sinh(d*x+c)),x)`

[Out]  $-2/d*a/(2*a^2+2*b^2)*\ln(\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)*b-a)+2/d/(2*a^2+2*b^2)*a*\ln(\tanh(1/2*d*x+1/2*c)^2+1)+4/d/(2*a^2+2*b^2)*b*\arctan(\tanh(1/2*d*x+1/2*c))$

**Maxima [A]** time = 1.60528, size = 128, normalized size = 1.86

$$-\frac{2b \arctan\left(e^{(-dx-c)}\right)}{(a^2 + b^2)d} - \frac{a \log\left(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b\right)}{(a^2 + b^2)d} + \frac{a \log\left(e^{(-2dx-2c)} + 1\right)}{(a^2 + b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out]  $-2*b*\arctan(e^{(-d*x - c)})/((a^2 + b^2)*d) - a*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/((a^2 + b^2)*d) + a*\log(e^{(-2*d*x - 2*c)} + 1)/((a^2 + b^2)*d)$

**Fricas [A]** time = 2.13869, size = 247, normalized size = 3.58

$$\frac{2b \arctan(\cosh(dx+c) + \sinh(dx+c)) - a \log\left(\frac{2(b \sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right) + a \log\left(\frac{2 \cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{(a^2 + b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*b*arctan(cosh(d*x + c) + sinh(d*x + c)) - a*log(2*(b*sinh(d*x + c) + a)/
(cosh(d*x + c) - sinh(d*x + c))) + a*log(2*cosh(d*x + c)/(cosh(d*x + c) - s
inh(d*x + c))))/((a^2 + b^2)*d)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

**Giac [A]** time = 1.44589, size = 115, normalized size = 1.67

$$\frac{\frac{2b \arctan(e^{(dx+c)})}{a^2+b^2} + \frac{a \log(e^{(2dx+2c)+1})}{a^2+b^2} - \frac{a \log(|be^{(2dx+2c)}+2ae^{(dx+c)}-b|)}{a^2+b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (2*b*arctan(e^(d*x + c))/(a^2 + b^2) + a*log(e^(2*d*x + 2*c) + 1)/(a^2 + b^
2) - a*log(abs(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - b))/(a^2 + b^2))/d
```

$$3.352 \quad \int \frac{\tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=28

$$\text{Unintegrable}\left(\frac{\tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x\right)$$

[Out] Unintegrable[Tanh[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0476321, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Tanh[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Tanh[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [A]** time = 16.8269, size = 0, normalized size = 0.

$$\int \frac{\tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Tanh[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Integrate[Tanh[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

---

**Maple [A]** time = 0.244, size = 0, normalized size = 0.

$$\int \frac{\tanh(dx + c)}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(tanh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(dx + c)}{(fx + e)(b \sinh(dx + c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] integrate(tanh(d\*x + c)/((f\*x + e)\*(b\*sinh(d\*x + c) + a)), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tanh(dx + c)}{afx + ae + (bf x + be) \sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] integral(tanh(d\*x + c)/(a\*f\*x + a\*e + (b\*f\*x + b\*e)\*sinh(d\*x + c)), x)

---



**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(c + dx)}{(a + b \sinh(c + dx))(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(tanh(c + d\*x)/((a + b\*sinh(c + d\*x))\*(e + f\*x)), x)

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.353 \quad \int \frac{(e+fx)^3 \operatorname{sech}(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$$

Optimal. Leaf size=917

result too large to display

```
[Out] (e + f*x)^3/(b*d) - (a^2*(e + f*x)^3)/(b*(a^2 + b^2)*d) + (6*a*f*(e + f*x)^2*ArcTan[E^(c + d*x)]/((a^2 + b^2)*d^2) - (a*b*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*d) + (a*b*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*d) - (3*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))]/(b*d^2) + (3*a^2*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))]/(b*(a^2 + b^2)*d^2) - ((6*I)*a*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)]/((a^2 + b^2)*d^3) + ((6*I)*a*f^2*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/((a^2 + b^2)*d^3) - (3*a*b*f*(e + f*x)^2*PolyLog[2, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*d^2) + (3*a*b*f*(e + f*x)^2*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*d^2) - (3*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))]/(b*d^3) + (3*a^2*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))]/(b*(a^2 + b^2)*d^3) + ((6*I)*a*f^3*PolyLog[3, (-I)*E^(c + d*x)]/((a^2 + b^2)*d^4) - ((6*I)*a*f^3*PolyLog[3, I*E^(c + d*x)]/((a^2 + b^2)*d^4) + (6*a*b*f^2*(e + f*x)*PolyLog[3, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*d^3) - (6*a*b*f^2*(e + f*x)*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*d^3) + (3*f^3*PolyLog[3, -E^(2*(c + d*x))]/(2*b*d^4) - (3*a^2*f^3*PolyLog[3, -E^(2*(c + d*x))]/(2*b*(a^2 + b^2)*d^4) - (6*a*b*f^3*PolyLog[4, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*d^4) + (6*a*b*f^3*PolyLog[4, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*d^4) - (a*(e + f*x)^3*Sech[c + d*x])/((a^2 + b^2)*d) + ((e + f*x)^3*Tanh[c + d*x])/(b*d) - (a^2*(e + f*x)^3*Tanh[c + d*x])/(b*(a^2 + b^2)*d)
```

**Rubi [A]** time = 1.71208, antiderivative size = 917, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 14, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {5583, 4184, 3718, 2190, 2531, 2282, 6589, 5573, 3322, 2264, 6609, 6742, 5451, 4180}

$$\frac{6ia \operatorname{PolyLog}\left(3, -ie^{c+dx}\right) f^3}{(a^2 + b^2) d^4} - \frac{6ia \operatorname{PolyLog}\left(3, ie^{c+dx}\right) f^3}{(a^2 + b^2) d^4} + \frac{3 \operatorname{PolyLog}\left(3, -e^{2(c+dx)}\right) f^3}{2bd^4} - \frac{3a^2 \operatorname{PolyLog}\left(3, -e^{2(c+dx)}\right) f^3}{2b(a^2 + b^2) d^4}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]), x]
```

```
[Out] (e + f*x)^3/(b*d) - (a^2*(e + f*x)^3)/(b*(a^2 + b^2)*d) + (6*a*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d^2) - (a*b*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (a*b*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d) - (3*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/((b*d^2) + (3*a^2*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/((b*(a^2 + b^2)*d^2) - ((6*I)*a*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) + ((6*I)*a*f^2*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^3) - (3*a*b*f*(e + f*x)^2*PolyLog[2, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d^2) + (3*a*b*f*(e + f*x)^2*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d^2) - (3*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((b*d^3) + (3*a^2*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/((b*(a^2 + b^2)*d^3) + ((6*I)*a*f^3*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^4) - ((6*I)*a*f^3*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)*d^4) + (6*a*b*f^2*(e + f*x)*PolyLog[3, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d^3) - (6*a*b*f^2*(e + f*x)*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d^3) + (3*f^3*PolyLog[3, -E^(2*(c + d*x))])/((2*b*d^4) - (3*a^2*f^3*PolyLog[3, -E^(2*(c + d*x))])/((2*b*(a^2 + b^2)*d^4) - (6*a*b*f^3*PolyLog[4, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d^4) + (6*a*b*f^3*PolyLog[4, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)*d^4) - (a*(e + f*x)^3*Sech[c + d*x])/((a^2 + b^2)*d) + ((e + f*x)^3*Tanh[c + d*x])/((b*d) - (a^2*(e + f*x)^3*Tanh[c + d*x])/((b*(a^2 + b^2)*d)
```

### Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[(((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 3718

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))]/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*(a_) + (b_)*(x_)))^(n_)]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5573

```
Int[(((e_) + (f_)*(x_))^(m_))*Sech[(c_) + (d_)*(x_)]^(n_)]/((a_) + (b_
)*Sinh[(c_) + (d_)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[(((c_) + (d_)*(x_))^(m_)]/((a_) + (b_)*sin[(e_) + (Complex[0, fz_]
*(f_)*(x_))]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^(c_.)*((a_.) + (b_.)
*(x_))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] := -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^m_., x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \operatorname{sech}(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \operatorname{sech}^2(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^3 \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{(e+fx)^3 \tanh(c+dx)}{bd} - \frac{a \int (e+fx)^3 \operatorname{sech}^2(c+dx)(a-b \sinh(c+dx)) dx}{b(a^2+b^2)} \\
&= \frac{(e+fx)^3}{bd} + \frac{(e+fx)^3 \tanh(c+dx)}{bd} - \frac{a \int (a(e+fx)^3 \operatorname{sech}^2(c+dx) - b(e+fx)^3 \sinh(c+dx)) dx}{b(a^2+b^2)} \\
&= \frac{(e+fx)^3}{bd} - \frac{3f(e+fx)^2 \log(1+e^{2(c+dx)})}{bd^2} + \frac{(e+fx)^3 \tanh(c+dx)}{bd} - \frac{(2ab^2) \int (e+fx)^3 \sinh(c+dx) dx}{b(a^2+b^2)} \\
&= \frac{(e+fx)^3}{bd} - \frac{ab(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} + \frac{ab(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} \\
&= \frac{(e+fx)^3}{bd} - \frac{a^2(e+fx)^3}{b(a^2+b^2)d} + \frac{6af(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d^2} - \frac{ab(e+fx)^3 \log(1+e^{2(c+dx)})}{(a^2+b^2)^{3/2} d} \\
&= \frac{(e+fx)^3}{bd} - \frac{a^2(e+fx)^3}{b(a^2+b^2)d} + \frac{6af(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d^2} - \frac{ab(e+fx)^3 \log(1+e^{2(c+dx)})}{(a^2+b^2)^{3/2} d} \\
&= \frac{(e+fx)^3}{bd} - \frac{a^2(e+fx)^3}{b(a^2+b^2)d} + \frac{6af(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d^2} - \frac{ab(e+fx)^3 \log(1+e^{2(c+dx)})}{(a^2+b^2)^{3/2} d} \\
&= \frac{(e+fx)^3}{bd} - \frac{a^2(e+fx)^3}{b(a^2+b^2)d} + \frac{6af(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d^2} - \frac{ab(e+fx)^3 \log(1+e^{2(c+dx)})}{(a^2+b^2)^{3/2} d}
\end{aligned}$$

**Mathematica [A]** time = 13.2951, size = 1143, normalized size = 1.25

$$\frac{f(-4bf^2x^3d^3 - 12befx^2d^3 + 12be^2e^{2c}xd^3 - 12be^2(1+e^{2c})xd^3 + 12ae^2(1+e^{2c})\tan^{-1}(e^{c+dx})d^2 + 6be^2(1+e^{2c})(2dx-1))}{(a^2+b^2)^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (f*(12*b*d^3*e^2*E^(2*c)*x - 12*b*d^3*e^2*(1 + E^(2*c))*x - 12*b*d^3*e*f*x^2 - 4*b*d^3*f^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] + 6*b*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*a*d*e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + 6*b*d*e*(1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]) + (6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*x)]) + b*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))]))/(2*(a^2 + b^2)*d^4*(1 + E^(2*c))) + (a*b*(2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] + 6*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*f^3*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*f^3*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^4) + (Sech[c]*Sech[c + d*x]*(-(a*e^3*Cosh[c]) - 3*a*e^2*f*x*Cosh[c] - 3*a*e*f^2*x^2*Cosh[c] - a*f^3*x^3*Cosh[c] + b*e^3*Sinh[d*x] + 3*b*e^2*f*x*Sinh[d*x] + 3*b*e*f^2*x^2*Sinh[d*x] + b*f^3*x^3*Sinh[d*x]))/((a^2 + b^2)*d)
```

---

**Maple [F]** time = 0.728, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \operatorname{sech}(dx + c) \tanh(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [C]** time = 4.40787, size = 14924, normalized size = 16.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(a^2*b + b^3)*d^3*e^3 - 12*(a^2*b + b^3)*c*d^2*e^2*f + 12*(a^2*b + b^3)*c^2*d*e*f^2 - 4*(a^2*b + b^3)*c^3*f^3 - 4*((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*d^3*e*f^2*x^2 + 3*(a^2*b + b^3)*d^3*e^2*f*x + 3*(a^2*b + b^3)*c*d^2*e^2*f - 3*(a^2*b + b^3)*c^2*d*e*f^2 + (a^2*b + b^3)*c^3*f^3)*cosh(d*x + c)^2 - 4*((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*d^3*e*f^2*x^2 + 3*(a^2*b + b^3)*d^3*e^2*f*x + 3*(a^2*b + b^3)*c*d^2*e^2*f - 3*(a^2*b + b^3)*c^2*d*e*f^2 + (a^2*b + b^3)*c^3*f^3)*sinh(d*x + c)^2 + 6*(a*b^2*d^2*f^3*x^2 + 2*a*b^2*d^2*e*f^2*x + a*b^2*d^2*e^2*f + (a*b^2*d^2*f^3*x^2 + 2*a*b^2*d^2*e*f^2*x + a*b^2*d^2*e^2*f)*cosh(d*x + c)^2 + 2*(a*b^2*d^2*f^3*x^2 + 2*a*b^2*d^2*e*f^2*x + a*b^2*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d^2*f^3*x^2 + 2*a*b^2*d^2*e*f^2*x + a*b^2*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 6*(a*b^2*d^2*f^3*x^2 + 2*a*b^2*d^2*e*f^2*x + a*b^2*d^2*e^2*f + (a*b^2*d^2*f^3*x^2 + 2*a*b^2*d^2*e*f^2*x + a*b^2*d^2*e^2*f)*cosh(d*x + c)^2 + 2*(a*b^2*d^2*f^3*x^2 + 2*a*b^2*d^2*e*f^2*x + a*b^2*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d^2*f^3*x^2 + 2*a*b^2*d^2*e*f^2*x + a*b^2*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b
```



$$\begin{aligned}
& * \sinh(dx + c) * \sqrt{(a^2 + b^2)/b^2} - b/b + 1) - 2*(a*b^2*d^3*e^3 - 3*a* \\
& b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c^3*f^3 + (a*b^2*d^3*e^3 - 3* \\
& a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c^3*f^3) * \cosh(dx + c)^2 + \\
& 2*(a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c^3*f^3) * \cosh(dx + c) * \sinh(dx + c) + (a*b^2*d^3*e^3 - 3*a* \\
& b^2*c^2*d*e*f^2 - a*b^2*c^3*f^3) * \sinh(dx + c)^2 * \sqrt{(a^2 + b^2)/b^2} * \log(2*b * \cosh(dx + c) + 2*b * \sinh(dx + c) + 2*b * \sqrt{(a^2 + b^2)/b^2} + 2*a) \\
& + 2*(a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c^3*f^3 + (a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c^3*f^3) * \cosh(dx + c)^2 + 2*(a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c^3*f^3) * \cosh(dx + c) * \sinh(dx + c) + (a*b^2*d^3*e^3 - 3*a*b^2*c*d^2*e^2*f + 3*a*b^2*c^2*d*e*f^2 - a*b^2*c^3*f^3) * \sinh(dx + c)^2) * \sqrt{(a^2 + b^2)/b^2} * \log(2*b * \cosh(dx + c) + 2*b * \sinh(dx + c) - 2*b * \sqrt{(a^2 + b^2)/b^2} + 2*a) + 2*(a*b^2*d^3*f^3*x^3 + 3*a*b^2*d^3*e*f^2*x^2 + 3*a*b^2*d^3*e^2*f*x + 3*a*b^2*c*d^2*e^2*f - 3*a*b^2*c^2*d*e*f^2 + a*b^2*c^3*f^3) * \cosh(dx + c)^2 + 2*(a*b^2*d^3*f^3*x^3 + 3*a*b^2*d^3*e*f^2*x^2 + 3*a*b^2*d^3*e^2*f*x + 3*a*b^2*c*d^2*e^2*f - 3*a*b^2*c^2*d*e*f^2 + a*b^2*c^3*f^3) * \cosh(dx + c) * \sinh(dx + c) + (a*b^2*d^3*f^3*x^3 + 3*a*b^2*d^3*e*f^2*x^2 + 3*a*b^2*d^3*e^2*f*x + 3*a*b^2*c*d^2*e^2*f - 3*a*b^2*c^2*d*e*f^2 + a*b^2*c^3*f^3) * \sinh(dx + c)^2) * \sqrt{(a^2 + b^2)/b^2} * \log(-(a * \cosh(dx + c) + a * \sinh(dx + c) + (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2} - b)/b) - 2*(a*b^2*d^3*f^3*x^3 + 3*a*b^2*d^3*e*f^2*x^2 + 3*a*b^2*d^3*e^2*f*x + 3*a*b^2*c*d^2*e^2*f - 3*a*b^2*c^2*d*e*f^2 + a*b^2*c^3*f^3) * \cosh(dx + c)^2 + 2*(a*b^2*d^3*f^3*x^3 + 3*a*b^2*d^3*e*f^2*x^2 + 3*a*b^2*d^3*e^2*f*x + 3*a*b^2*c*d^2*e^2*f - 3*a*b^2*c^2*d*e*f^2 + a*b^2*c^3*f^3) * \cosh(dx + c) * \sinh(dx + c) + (a*b^2*d^3*f^3*x^3 + 3*a*b^2*d^3*e*f^2*x^2 + 3*a*b^2*d^3*e^2*f*x + 3*a*b^2*c*d^2*e^2*f - 3*a*b^2*c^2*d*e*f^2 + a*b^2*c^3*f^3) * \sinh(dx + c)^2) * \sqrt{(a^2 + b^2)/b^2} * \log(-(a * \cosh(dx + c) + a * \sinh(dx + c) - (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2} - b)/b) + 12*(a*b^2*f^3 * \cosh(dx + c)^2 + 2*a*b^2*f^3 * \cosh(dx + c) * \sinh(dx + c) + a*b^2*f^3 * \sinh(dx + c)^2 + a*b^2*f^3) * \sqrt{(a^2 + b^2)/b^2} * \text{polylog}(4, (a * \cosh(dx + c) + a * \sinh(dx + c) + (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2}))/b) - 12*(a*b^2*f^3 * \cosh(dx + c)^2 + 2*a*b^2*f^3 * \cosh(dx + c) * \sinh(dx + c) + a*b^2*f^3 * \sinh(dx + c)^2 + a*b^2*f^3) * \sqrt{(a^2 + b^2)/b^2} * \text{polylog}(4, (a * \cosh(dx + c) + a * \sinh(dx + c) - (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2}))/b) - 12*(a*b^2*d*f^3*x + a*b^2*d*e*f^2 + (a*b^2*d*f^3*x + a*b^2*d*e*f^2) * \cosh(dx + c)^2 + 2*(a*b^2*d*f^3*x + a*b^2*d*e*f^2) * \sinh(dx + c)^2) * \sqrt{(a^2 + b^2)/b^2} * \text{polylog}(3, (a * \cosh(dx + c) + a * \sinh(dx + c) + (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2}))/b) + 12*(a*b^2*d*f^3*x + a*b^2*d*e*f^2 + (a*b^2*d*f^3*x + a*b^2*d*e*f^2) * \cosh(dx + c)^2 + 2*(a*b^2*d*f^3*x + a*b^2*d*e*f^2) * \cosh(dx + c) *
\end{aligned}$$

$$\begin{aligned}
& \sinh(dx + c) + (a*b^2*d*f^3*x + a*b^2*d*e*f^2)*\sinh(dx + c)^2*\sqrt{(a^2 + b^2)/b^2}*\text{polylog}(3, (a*\cosh(dx + c) + a*\sinh(dx + c) - (b*\cosh(dx + c) + b*\sinh(dx + c))*\sqrt{(a^2 + b^2)/b^2}))/b) + 4*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + (a^3 + a*b^2)*d^3*e^3)*\cosh(dx + c) - (12*I*(a^3 + a*b^2)*d*f^3*x - 12*(a^2*b + b^3)*d*f^3*x + 12*I*(a^3 + a*b^2)*d*e*f^2 - 12*(a^2*b + b^3)*d*e*f^2 + (12*I*(a^3 + a*b^2)*d*f^3*x - 12*(a^2*b + b^3)*d*f^3*x + 12*I*(a^3 + a*b^2)*d*e*f^2 - 12*(a^2*b + b^3)*d*e*f^2)*\cosh(dx + c)^2 + (24*I*(a^3 + a*b^2)*d*f^3*x - 24*(a^2*b + b^3)*d*f^3*x + 24*I*(a^3 + a*b^2)*d*e*f^2 - 24*(a^2*b + b^3)*d*e*f^2)*\cosh(dx + c)*\sinh(dx + c) + (12*I*(a^3 + a*b^2)*d*f^3*x - 12*(a^2*b + b^3)*d*f^3*x + 12*I*(a^3 + a*b^2)*d*e*f^2 - 12*(a^2*b + b^3)*d*e*f^2)*\sinh(dx + c)^2*\text{dilog}(I*\cosh(dx + c) + I*\sinh(dx + c)) - (-12*I*(a^3 + a*b^2)*d*f^3*x - 12*(a^2*b + b^3)*d*f^3*x - 12*I*(a^3 + a*b^2)*d*e*f^2 - 12*(a^2*b + b^3)*d*e*f^2 + (-12*I*(a^3 + a*b^2)*d*f^3*x - 12*(a^2*b + b^3)*d*f^3*x - 12*I*(a^3 + a*b^2)*d*e*f^2 - 12*(a^2*b + b^3)*d*e*f^2)*\cosh(dx + c)^2 + (-24*I*(a^3 + a*b^2)*d*f^3*x - 24*(a^2*b + b^3)*d*f^3*x - 24*I*(a^3 + a*b^2)*d*e*f^2 - 24*(a^2*b + b^3)*d*e*f^2)*\cosh(dx + c)*\sinh(dx + c) + (-12*I*(a^3 + a*b^2)*d*f^3*x - 12*(a^2*b + b^3)*d*f^3*x - 12*I*(a^3 + a*b^2)*d*e*f^2 - 12*(a^2*b + b^3)*d*e*f^2)*\sinh(dx + c)^2*\text{dilog}(-I*\cosh(dx + c) - I*\sinh(dx + c)) - (6*I*(a^3 + a*b^2)*d^2*e^2*f - 6*(a^2*b + b^3)*d^2*e^2*f - 12*I*(a^3 + a*b^2)*c*d*e*f^2 + 12*(a^2*b + b^3)*c*d*e*f^2 + 6*I*(a^3 + a*b^2)*c^2*f^3 - 6*(a^2*b + b^3)*c^2*f^3 + (6*I*(a^3 + a*b^2)*d^2*e^2*f - 6*(a^2*b + b^3)*d^2*e^2*f - 12*I*(a^3 + a*b^2)*c*d*e*f^2 + 12*(a^2*b + b^3)*c*d*e*f^2 + 6*I*(a^3 + a*b^2)*c^2*f^3 - 6*(a^2*b + b^3)*c^2*f^3)*\cosh(dx + c)^2 + (12*I*(a^3 + a*b^2)*d^2*e^2*f - 12*(a^2*b + b^3)*d^2*e^2*f - 24*I*(a^3 + a*b^2)*c*d*e*f^2 + 24*(a^2*b + b^3)*c*d*e*f^2 + 12*I*(a^3 + a*b^2)*c^2*f^3 - 12*(a^2*b + b^3)*c^2*f^3)*\cosh(dx + c)*\sinh(dx + c) + (6*I*(a^3 + a*b^2)*d^2*e^2*f - 6*(a^2*b + b^3)*d^2*e^2*f - 12*I*(a^3 + a*b^2)*c*d*e*f^2 + 12*(a^2*b + b^3)*c*d*e*f^2 + 6*I*(a^3 + a*b^2)*c^2*f^3 - 6*(a^2*b + b^3)*c^2*f^3)*\sinh(dx + c)^2*\log(\cosh(dx + c) + \sinh(dx + c) + I) - (-6*I*(a^3 + a*b^2)*d^2*e^2*f - 6*(a^2*b + b^3)*d^2*e^2*f + 12*I*(a^3 + a*b^2)*c*d*e*f^2 + 12*(a^2*b + b^3)*c*d*e*f^2 - 6*I*(a^3 + a*b^2)*c^2*f^3 - 6*(a^2*b + b^3)*c^2*f^3 + (-6*I*(a^3 + a*b^2)*d^2*e^2*f - 6*(a^2*b + b^3)*d^2*e^2*f + 12*I*(a^3 + a*b^2)*c*d*e*f^2 + 12*(a^2*b + b^3)*c*d*e*f^2 - 6*I*(a^3 + a*b^2)*c^2*f^3 - 6*(a^2*b + b^3)*c^2*f^3)*\cosh(dx + c)^2 + (-12*I*(a^3 + a*b^2)*d^2*e^2*f - 12*(a^2*b + b^3)*d^2*e^2*f + 24*I*(a^3 + a*b^2)*c*d*e*f^2 + 24*(a^2*b + b^3)*c*d*e*f^2 - 12*I*(a^3 + a*b^2)*c^2*f^3 - 12*(a^2*b + b^3)*c^2*f^3)*\cosh(dx + c)*\sinh(dx + c) + (-6*I*(a^3 + a*b^2)*d^2*e^2*f - 6*(a^2*b + b^3)*d^2*e^2*f + 12*I*(a^3 + a*b^2)*c*d*e*f^2 + 12*(a^2*b + b^3)*c*d*e*f^2 - 6*I*(a^3 + a*b^2)*c^2*f^3 - 6*(a^2*b + b^3)*c^2*f^3)*\sinh(dx + c)^2*\log(\cosh(dx + c) + \sinh(dx + c) - I) - (-6*I*(a^3 + a*b^2)*d^2*f^3*x^2 - 6*(a^2*b + b^3)*d^2*f^3*x^2 - 12*I*(a^3 + a*b^2)*d^2*e*f^2*x - 12*(a^2*b + b^3)*d^2*e*f^2*x - 12*I*(a^3 + a*b^2)*c*d*e*f^2 - 12*(a^2*b + b^3)*c*d*e*f^2 + 6*I*(a^3 + a*b^2)*c^2*f^3 + 6*(a^2*b + b^3)*c^2*f^3 + (-6*I*(a^3 + a*b^2)*d^2*f^3*x^2 - 6*(a^2*b + b^3)*d^2*f^3*x^2 - 12*I*(a^3 + a*b^2)
\end{aligned}$$

$$\begin{aligned}
& )d^2*ef^2*x - 12*(a^2*b + b^3)*d^2*ef^2*x - 12*I*(a^3 + a*b^2)*c*d*ef^2 \\
& - 12*(a^2*b + b^3)*c*d*ef^2 + 6*I*(a^3 + a*b^2)*c^2*f^3 + 6*(a^2*b + b^3) \\
& *c^2*f^3)*\cosh(d*x + c)^2 + (-12*I*(a^3 + a*b^2)*d^2*f^3*x^2 - 12*(a^2*b + \\
& b^3)*d^2*f^3*x^2 - 24*I*(a^3 + a*b^2)*d^2*ef^2*x - 24*(a^2*b + b^3)*d^2*ef^2*x \\
& - 24*I*(a^3 + a*b^2)*c*d*ef^2 - 24*(a^2*b + b^3)*c*d*ef^2 + 12*I*(a \\
& ^3 + a*b^2)*c^2*f^3 + 12*(a^2*b + b^3)*c^2*f^3)*\cosh(d*x + c)*\sinh(d*x + c) \\
& + (-6*I*(a^3 + a*b^2)*d^2*f^3*x^2 - 6*(a^2*b + b^3)*d^2*f^3*x^2 - 12*I*(a^ \\
& 3 + a*b^2)*d^2*ef^2*x - 12*(a^2*b + b^3)*d^2*ef^2*x - 12*I*(a^3 + a*b^2)* \\
& c*d*ef^2 - 12*(a^2*b + b^3)*c*d*ef^2 + 6*I*(a^3 + a*b^2)*c^2*f^3 + 6*(a^2 \\
& *b + b^3)*c^2*f^3)*\sinh(d*x + c)^2*\log(I*\cosh(d*x + c) + I*\sinh(d*x + c) + \\
& 1) - (6*I*(a^3 + a*b^2)*d^2*f^3*x^2 - 6*(a^2*b + b^3)*d^2*f^3*x^2 + 12*I*( \\
& a^3 + a*b^2)*d^2*ef^2*x - 12*(a^2*b + b^3)*d^2*ef^2*x + 12*I*(a^3 + a*b^2 \\
& )*c*d*ef^2 - 12*(a^2*b + b^3)*c*d*ef^2 - 6*I*(a^3 + a*b^2)*c^2*f^3 + 6*(a \\
& ^2*b + b^3)*c^2*f^3 + (6*I*(a^3 + a*b^2)*d^2*f^3*x^2 - 6*(a^2*b + b^3)*d^2* \\
& f^3*x^2 + 12*I*(a^3 + a*b^2)*d^2*ef^2*x - 12*(a^2*b + b^3)*d^2*ef^2*x + 1 \\
& 2*I*(a^3 + a*b^2)*c*d*ef^2 - 12*(a^2*b + b^3)*c*d*ef^2 - 6*I*(a^3 + a*b^2 \\
& )*c^2*f^3 + 6*(a^2*b + b^3)*c^2*f^3)*\cosh(d*x + c)^2 + (12*I*(a^3 + a*b^2)* \\
& d^2*f^3*x^2 - 12*(a^2*b + b^3)*d^2*f^3*x^2 + 24*I*(a^3 + a*b^2)*d^2*ef^2*x \\
& - 24*(a^2*b + b^3)*d^2*ef^2*x + 24*I*(a^3 + a*b^2)*c*d*ef^2 - 24*(a^2*b \\
& + b^3)*c*d*ef^2 - 12*I*(a^3 + a*b^2)*c^2*f^3 + 12*(a^2*b + b^3)*c^2*f^3)*c \\
& osh(d*x + c)*\sinh(d*x + c) + (6*I*(a^3 + a*b^2)*d^2*f^3*x^2 - 6*(a^2*b + b^ \\
& 3)*d^2*f^3*x^2 + 12*I*(a^3 + a*b^2)*d^2*ef^2*x - 12*(a^2*b + b^3)*d^2*ef^ \\
& 2*x + 12*I*(a^3 + a*b^2)*c*d*ef^2 - 12*(a^2*b + b^3)*c*d*ef^2 - 6*I*(a^3 \\
& + a*b^2)*c^2*f^3 + 6*(a^2*b + b^3)*c^2*f^3)*\sinh(d*x + c)^2*\log(-I*\cosh(d* \\
& x + c) - I*\sinh(d*x + c) + 1) - (-12*I*(a^3 + a*b^2)*f^3 + 12*(a^2*b + b^3) \\
& *f^3 + (-12*I*(a^3 + a*b^2)*f^3 + 12*(a^2*b + b^3)*f^3)*\cosh(d*x + c)^2 + ( \\
& -24*I*(a^3 + a*b^2)*f^3 + 24*(a^2*b + b^3)*f^3)*\cosh(d*x + c)*\sinh(d*x + c) \\
& + (-12*I*(a^3 + a*b^2)*f^3 + 12*(a^2*b + b^3)*f^3)*\sinh(d*x + c)^2)*\text{polylo} \\
& \text{g}(3, I*\cosh(d*x + c) + I*\sinh(d*x + c)) - (12*I*(a^3 + a*b^2)*f^3 + 12*(a^2 \\
& *b + b^3)*f^3 + (12*I*(a^3 + a*b^2)*f^3 + 12*(a^2*b + b^3)*f^3)*\cosh(d*x + \\
& c)^2 + (24*I*(a^3 + a*b^2)*f^3 + 24*(a^2*b + b^3)*f^3)*\cosh(d*x + c)*\sinh(d \\
& *x + c) + (12*I*(a^3 + a*b^2)*f^3 + 12*(a^2*b + b^3)*f^3)*\sinh(d*x + c)^2)* \\
& \text{polylog}(3, -I*\cosh(d*x + c) - I*\sinh(d*x + c)) + 4*((a^3 + a*b^2)*d^3*f^3*x \\
& ^3 + 3*(a^3 + a*b^2)*d^3*ef^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + (a^3 + a \\
& *b^2)*d^3*e^3 - 2*((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*d^3*ef^2*x^ \\
& 2 + 3*(a^2*b + b^3)*d^3*e^2*f*x + 3*(a^2*b + b^3)*c*d^2*e^2*f - 3*(a^2*b + \\
& b^3)*c^2*d*ef^2 + (a^2*b + b^3)*c^3*f^3)*\cosh(d*x + c))*\sinh(d*x + c))/((a \\
& ^4 + 2*a^2*b^2 + b^4)*d^4*\cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^4*c \\
& osh(d*x + c)*\sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*d^4*\sinh(d*x + c)^2 + \\
& (a^4 + 2*a^2*b^2 + b^4)*d^4)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.354 \quad \int \frac{(e+fx)^2 \operatorname{sech}(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=648

$$\frac{2abf(e+fx)\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)^{3/2}} + \frac{2abf(e+fx)\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2(a^2+b^2)^{3/2}} + \frac{a^2 f^2 \operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{bd^3(a^2+b^2)} - \frac{2iaf^2}{d}$$

[Out]  $(e + f*x)^2/(b*d) - (a^2*(e + f*x)^2)/(b*(a^2 + b^2)*d) + (4*a*f*(e + f*x)*\operatorname{ArcTan}[E^{(c + d*x)}])/((a^2 + b^2)*d^2) - (a*b*(e + f*x)^2*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])])/((a^2 + b^2)^{(3/2)*d}) + (a*b*(e + f*x)^2*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])/((a^2 + b^2)^{(3/2)*d}) - (2*f*(e + f*x)*\operatorname{Log}[1 + E^{2*(c + d*x)}])/((b*d^2) + (2*a^2*f*(e + f*x)*\operatorname{Log}[1 + E^{2*(c + d*x)}])/((b*(a^2 + b^2)*d^2) - ((2*I)*a*f^2*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x)}])/((a^2 + b^2)*d^3) + ((2*I)*a*f^2*\operatorname{PolyLog}[2, I*E^{(c + d*x)}])/((a^2 + b^2)*d^3) - (2*a*b*f*(e + f*x)*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2]))])/((a^2 + b^2)^{(3/2)*d^2}) + (2*a*b*f*(e + f*x)*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2]))])/((a^2 + b^2)^{(3/2)*d^2}) - (f^2*\operatorname{PolyLog}[2, -E^{2*(c + d*x)}])/((b*d^3) + (a^2*f^2*\operatorname{PolyLog}[2, -E^{2*(c + d*x)}])/((b*(a^2 + b^2)*d^3) + (2*a*b*f^2*\operatorname{PolyLog}[3, -((b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2]))])/((a^2 + b^2)^{(3/2)*d^3}) - (2*a*b*f^2*\operatorname{PolyLog}[3, -((b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2]))])/((a^2 + b^2)^{(3/2)*d^3}) - (a*(e + f*x)^2*\operatorname{Sech}[c + d*x])/((a^2 + b^2)*d) + ((e + f*x)^2*\operatorname{Tanh}[c + d*x])/((b*d) - (a^2*(e + f*x)^2*\operatorname{Tanh}[c + d*x])/((b*(a^2 + b^2)*d)$

**Rubi [A]** time = 1.32148, antiderivative size = 648, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 15, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$ , Rules used = {5583, 4184, 3718, 2190, 2279, 2391, 5573, 3322, 2264, 2531, 2282, 6589, 6742, 5451, 4180}

$$\frac{2abf(e+fx)\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)^{3/2}} + \frac{2abf(e+fx)\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2(a^2+b^2)^{3/2}} + \frac{a^2 f^2 \operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{bd^3(a^2+b^2)} - \frac{2iaf^2}{d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e + f*x)^2*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x]/(a + b*\operatorname{Sinh}[c + d*x]), x]$

[Out]  $(e + f*x)^2/(b*d) - (a^2*(e + f*x)^2)/(b*(a^2 + b^2)*d) + (4*a*f*(e + f*x)*\operatorname{ArcTan}[E^{(c + d*x)}])/((a^2 + b^2)*d^2) - (a*b*(e + f*x)^2*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])])/((a^2 + b^2)^{(3/2)*d}) + (a*b*(e + f*x)^2*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])/((a^2 + b^2)^{(3/2)*d}) - (2*f*(e + f*x)*\operatorname{Log}[1 + E^{2*(c + d*x)}])/((b*d^2) + (2*a^2*f*(e + f*x)*\operatorname{Log}[1 + E^{2*(c + d*x)}])/((b*(a^2 + b^2)*d^2) - ((2*I)*a*f^2*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x)}])/((a^2 + b^2)*d^3) + ((2*I)*a*f^2*\operatorname{PolyLog}[2, I*E^{(c + d*x)}])/((a^2 + b^2)*d^3) - (2*a*b*f*(e + f*x)*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2]))])/((a^2 + b^2)^{(3/2)*d^2}) + (2*a*b*f*(e + f*x)*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2]))])/((a^2 + b^2)^{(3/2)*d^2}) - (f^2*\operatorname{PolyLog}[2, -E^{2*(c + d*x)}])/((b*d^3) + (a^2*f^2*\operatorname{PolyLog}[2, -E^{2*(c + d*x)}])/((b*(a^2 + b^2)*d^3) + (2*a*b*f^2*\operatorname{PolyLog}[3, -((b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2]))])/((a^2 + b^2)^{(3/2)*d^3}) - (2*a*b*f^2*\operatorname{PolyLog}[3, -((b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2]))])/((a^2 + b^2)^{(3/2)*d^3}) - (a*(e + f*x)^2*\operatorname{Sech}[c + d*x])/((a^2 + b^2)*d) + ((e + f*x)^2*\operatorname{Tanh}[c + d*x])/((b*d) - (a^2*(e + f*x)^2*\operatorname{Tanh}[c + d*x])/((b*(a^2 + b^2)*d)$

$$\begin{aligned} & [1 + (bE^{(c+dx)})/(a + \sqrt{a^2 + b^2})]/((a^2 + b^2)^{3/2}d) - (2f(e + fx) \cdot \text{Log}[1 + E^{(2(c+dx))}]/(b^2d^2) + (2a^2f(e + fx) \cdot \text{Log}[1 + E^{(2(c+dx))}]/(b(a^2 + b^2)d^2) - ((2I)af^2 \cdot \text{PolyLog}[2, (-I)E^{(c+dx)}]/((a^2 + b^2)d^3) + ((2I)af^2 \cdot \text{PolyLog}[2, I E^{(c+dx)}]/((a^2 + b^2)d^3) - (2abf(e + fx) \cdot \text{PolyLog}[2, -(bE^{(c+dx)})/(a - \sqrt{a^2 + b^2})])]/((a^2 + b^2)^{3/2}d^2) + (2abf(e + fx) \cdot \text{PolyLog}[2, -(bE^{(c+dx)})/(a + \sqrt{a^2 + b^2})])]/((a^2 + b^2)^{3/2}d^2) - (f^2 \cdot \text{PolyLog}[2, -E^{(2(c+dx))}]/(b^2d^3) + (a^2f^2 \cdot \text{PolyLog}[2, -E^{(2(c+dx))}]/(b(a^2 + b^2)d^3) + (2abf^2 \cdot \text{PolyLog}[3, -(bE^{(c+dx)})/(a - \sqrt{a^2 + b^2})])]/((a^2 + b^2)^{3/2}d^3) - (2abf^2 \cdot \text{PolyLog}[3, -(bE^{(c+dx)})/(a + \sqrt{a^2 + b^2})])]/((a^2 + b^2)^{3/2}d^3) - (a(e + fx)^2 \cdot \text{Sech}[c + dx])/((a^2 + b^2)d) + ((e + fx)^2 \cdot \text{Tanh}[c + dx])/(bd) - (a^2(e + fx)^2 \cdot \text{Tanh}[c + dx])/(b(a^2 + b^2)d) \end{aligned}$$

### Rule 5583

```
Int[(((e_) + (f_)*(x_))^(m_)*Sech[(c_) + (d_)*(x_)]^(p_)*Tanh[(c_) + (d_)*(x_)]^(n_))/(a_ + (b_)*Sinh[(c_) + (d_)*(x_)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 4184

```
Int[csc[(e_) + (f_)*(x_)]^2*((c_) + (d_)*(x_))^(m_), x_Symbol] := -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 3718

```
Int[(((c_) + (d_)*(x_))^(m_)*tan[(e_) + (Complex[0, fz_])*(f_)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))]/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/(a_ + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e + f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[(((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])* (f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_.)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_.)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
```

```
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

### Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n), x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(e+fx)^2 \operatorname{sech}(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \operatorname{sech}^2(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{(e+fx)^2 \tanh(c+dx)}{bd} - \frac{a \int (e+fx)^2 \operatorname{sech}^2(c+dx)(a-b \sinh(c+dx)) dx}{b(a^2+b^2)} \\
&= \frac{(e+fx)^2}{bd} + \frac{(e+fx)^2 \tanh(c+dx)}{bd} - \frac{a \int (a(e+fx)^2 \operatorname{sech}^2(c+dx) - b(e+fx)^2 \sinh(c+dx)) dx}{b(a^2+b^2)} \\
&= \frac{(e+fx)^2}{bd} - \frac{2f(e+fx) \log(1+e^{2(c+dx)})}{bd^2} + \frac{(e+fx)^2 \tanh(c+dx)}{bd} - \frac{(2ab \int (e+fx)^2 \operatorname{sech}^2(c+dx) dx)}{b(a^2+b^2)} \\
&= \frac{(e+fx)^2}{bd} - \frac{ab(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} + \frac{ab(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} \\
&= \frac{(e+fx)^2}{bd} - \frac{a^2(e+fx)^2}{b(a^2+b^2)d} + \frac{4af(e+fx) \tan^{-1}(e^{c+dx})}{(a^2+b^2)d^2} - \frac{ab(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)} \\
&= \frac{(e+fx)^2}{bd} - \frac{a^2(e+fx)^2}{b(a^2+b^2)d} + \frac{4af(e+fx) \tan^{-1}(e^{c+dx})}{(a^2+b^2)d^2} - \frac{ab(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)} \\
&= \frac{(e+fx)^2}{bd} - \frac{a^2(e+fx)^2}{b(a^2+b^2)d} + \frac{4af(e+fx) \tan^{-1}(e^{c+dx})}{(a^2+b^2)d^2} - \frac{ab(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)} \\
&= \frac{(e+fx)^2}{bd} - \frac{a^2(e+fx)^2}{b(a^2+b^2)d} + \frac{4af(e+fx) \tan^{-1}(e^{c+dx})}{(a^2+b^2)d^2} - \frac{ab(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)}
\end{aligned}$$

**Mathematica [A]** time = 8.18767, size = 906, normalized size = 1.4

$$2a \left( \frac{2 \tan^{-1}\left(\frac{\sinh(c)+\cosh(c) \tanh\left(\frac{dx}{2}\right)}{\sqrt{\cosh^2(c)-\sinh^2(c)}}\right) \tanh^{-1}(\coth(c))}{\sqrt{\cosh^2(c)-\sinh^2(c)}} - \frac{\operatorname{icsch}(c)\left(i(dx+\tanh^{-1}(\coth(c)))\left(\log\left(1-e^{-dx-\tanh^{-1}(\coth(c))}\right)-\log\left(1+e^{-dx-\tanh^{-1}(\coth(c))}\right)\right)\right)}{\sqrt{1-\coth^2(c)}} \right)$$

$(a^2+b^2)d^3$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x
]
```

```
[Out] (a*b*(2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]) - d^2*f^2*x^2*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2]]) + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]]) + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^
2]]) - 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]]) +
2*d*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) + 2*f
^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]]) - 2*f^2*PolyLog[3, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])]))/((a^2 + b^2)^(3/2)*d^3) - (2*b*e*
f*Sech[c]*(Cosh[c]*Log[Cosh[c]*Cosh[d*x] + Sinh[c]*Sinh[d*x]] - d*x*Sinh[c]
))/((a^2 + b^2)*d^2*(Cosh[c]^2 - Sinh[c]^2)) + (4*a*e*f*ArcTan[(Sinh[c] + C
osh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]])/((a^2 + b^2)*d^2*Sqrt[C
osh[c]^2 - Sinh[c]^2]) + (b*f^2*Csch[c]*(-(d^2*x^2)/E^ArcTanh[Coth[c]])) +
(I*Coth[c]*(-(d*x*(-Pi + (2*I)*ArcTanh[Coth[c]])) - Pi*Log[1 + E^(2*d*x)] -
2*(I*d*x + I*ArcTanh[Coth[c]])*Log[1 - E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]
]])]) + Pi*Log[Cosh[d*x]] + (2*I)*ArcTanh[Coth[c]]*Log[I*Sinh[d*x + ArcTanh[
Coth[c]]]]) + I*PolyLog[2, E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]])])))/Sqrt[1
- Coth[c]^2]*Sech[c])/((a^2 + b^2)*d^3*Sqrt[Csch[c]^2*(-Cosh[c]^2 + Sinh[c]
^2)]) + (2*a*f^2*(((I)*Csch[c]*(I*(d*x + ArcTanh[Coth[c]]))*(Log[1 - E^(-
d*x) - ArcTanh[Coth[c]]]) - Log[1 + E^(-d*x) - ArcTanh[Coth[c]]]) + I*(Po
lyLog[2, -E^(-d*x) - ArcTanh[Coth[c]]]) - PolyLog[2, E^(-d*x) - ArcTanh[C
oth[c]]]))))/Sqrt[1 - Coth[c]^2] - (2*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)/
2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]]*ArcTanh[Coth[c]]/Sqrt[Cosh[c]^2 - Sinh[c]
^2]))/((a^2 + b^2)*d^3) + (Sech[c]*Sech[c + d*x]*(-(a*e^2*Cosh[c]) - 2*a*e*
f*x*Cosh[c] - a*f^2*x^2*Cosh[c] + b*e^2*Sinh[d*x] + 2*b*e*f*x*Sinh[d*x] + b
*f^2*x^2*Sinh[d*x]))/((a^2 + b^2)*d)
```

**Maple [F]** time = 0.931, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \operatorname{sech}(dx + c) \tanh(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [C]** time = 3.44481, size = 8627, normalized size = 13.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*(a^2*b + b^3)*d^2*e^2 - 8*(a^2*b + b^3)*c*d*e*f + 4*(a^2*b + b^3)*c^2*f^2 - 4*((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*d^2*e*f*x + 2*(a^2*b + b^3)*c*d*e*f - (a^2*b + b^3)*c^2*f^2)*cosh(d*x + c)^2 - 4*((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*d^2*e*f*x + 2*(a^2*b + b^3)*c*d*e*f - (a^2*b + b^3)*c^2*f^2)*sinh(d*x + c)^2 + 4*(a*b^2*d*f^2*x + a*b^2*d*e*f + (a*b^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d*f^2*x + a*b^2*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 4*(a*b^2*d*f^2*x + a*b^2*d*e*f + (a*b^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d*f^2*x + a*b^2*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(d*x + c)^2 + 2*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(d*x + c)^2 + 2*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)
```

$$\begin{aligned}
& 2) * \log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + \\
& 2*a) + 2*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2 + \\
& (a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*\cosh(d*x + c)^2 + 2*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2} * \log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c)))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 2*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2 + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*\cosh(d*x + c)^2 + 2*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2} * \log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c)))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 4*(a*b^2*f^2*\cosh(d*x + c)^2 + 2*a*b^2*f^2*\cosh(d*x + c)*\sinh(d*x + c) + a*b^2*f^2*\sinh(d*x + c)^2 + a*b^2*f^2)*\sqrt{(a^2 + b^2)/b^2} * \text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c)))*\sqrt{(a^2 + b^2)/b^2})/b) + 4*(a*b^2*f^2*\cosh(d*x + c)^2 + 2*a*b^2*f^2*\cosh(d*x + c)*\sinh(d*x + c) + a*b^2*f^2*\sinh(d*x + c)^2 + a*b^2*f^2)*\sqrt{(a^2 + b^2)/b^2} * \text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c)))*\sqrt{(a^2 + b^2)/b^2})/b) + 4*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + (a^3 + a*b^2)*d^2*e^2)*\cosh(d*x + c) - (4*I*(a^3 + a*b^2)*f^2 - 4*(a^2*b + b^3)*f^2 + (4*I*(a^3 + a*b^2)*f^2 - 4*(a^2*b + b^3)*f^2)*\cosh(d*x + c)^2 + (8*I*(a^3 + a*b^2)*f^2 - 8*(a^2*b + b^3)*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (4*I*(a^3 + a*b^2)*f^2 - 4*(a^2*b + b^3)*f^2)*\sinh(d*x + c)^2)*\text{dilog}(I*\cosh(d*x + c) + I*\sinh(d*x + c)) - (-4*I*(a^3 + a*b^2)*f^2 - 4*(a^2*b + b^3)*f^2 + (-4*I*(a^3 + a*b^2)*f^2 - 4*(a^2*b + b^3)*f^2)*\cosh(d*x + c)^2 + (-8*I*(a^3 + a*b^2)*f^2 - 8*(a^2*b + b^3)*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (-4*I*(a^3 + a*b^2)*f^2 - 4*(a^2*b + b^3)*f^2)*\sinh(d*x + c)^2)*\text{dilog}(-I*\cosh(d*x + c) - I*\sinh(d*x + c)) - (4*I*(a^3 + a*b^2)*d*e*f - 4*(a^2*b + b^3)*d*e*f - 4*I*(a^3 + a*b^2)*c*f^2 + 4*(a^2*b + b^3)*c*f^2 + (4*I*(a^3 + a*b^2)*d*e*f - 4*(a^2*b + b^3)*d*e*f - 4*I*(a^3 + a*b^2)*c*f^2 + 4*(a^2*b + b^3)*c*f^2)*\cosh(d*x + c)^2 + (8*I*(a^3 + a*b^2)*d*e*f - 8*(a^2*b + b^3)*d*e*f - 8*I*(a^3 + a*b^2)*c*f^2 + 8*(a^2*b + b^3)*c*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (4*I*(a^3 + a*b^2)*d*e*f - 4*(a^2*b + b^3)*d*e*f - 4*I*(a^3 + a*b^2)*c*f^2 + 4*(a^2*b + b^3)*c*f^2)*\sinh(d*x + c)^2)*\log(\cosh(d*x + c) + \sinh(d*x + c) + I) - (-4*I*(a^3 + a*b^2)*d*e*f - 4*(a^2*b + b^3)*d*e*f + 4*I*(a^3 + a*b^2)*c*f^2 + 4*(a^2*b + b^3)*c*f^2 + (-4*I*(a^3 + a*b^2)*d*e*f - 4*(a^2*b + b^3)*d*e*f + 4*I*(a^3 + a*b^2)*c*f^2 + 4*(a^2*b + b^3)*c*f^2)*\cosh(d*x + c)^2 + (-8*I*(a^3 + a*b^2)*d*e*f - 8*(a^2*b + b^3)*d*e*f + 8*I*(a^3 + a*b^2)*c*f^2 + 8*(a^2*b + b^3)*c*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (-4*I*(a^3 + a*b^2)*d*e*f - 4*(a^2*b + b^3)*d*e*f + 4*I*(a^3 + a*b^2)*c*f^2 + 4*(a^2*b + b^3)*c*f^2)*\sinh(d*x + c)^2)*\log(\cosh(d*x + c) + \sinh(d*x + c) - I) - (-4*I*(a^3 + a*b^2)*d*f^2*x - 4*(a^2*b + b^3)*d*f^2*x - 4*I*(a^3 + a*b^2)*c*f^2 - 4*(a^2*b + b^3)*
\end{aligned}$$

$$\begin{aligned}
& c*f^2 + (-4*I*(a^3 + a*b^2)*d*f^2*x - 4*(a^2*b + b^3)*d*f^2*x - 4*I*(a^3 + \\
& a*b^2)*c*f^2 - 4*(a^2*b + b^3)*c*f^2)*\cosh(d*x + c)^2 + (-8*I*(a^3 + a*b^2) \\
& *d*f^2*x - 8*(a^2*b + b^3)*d*f^2*x - 8*I*(a^3 + a*b^2)*c*f^2 - 8*(a^2*b + b \\
& ^3)*c*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (-4*I*(a^3 + a*b^2)*d*f^2*x - 4*(a \\
& ^2*b + b^3)*d*f^2*x - 4*I*(a^3 + a*b^2)*c*f^2 - 4*(a^2*b + b^3)*c*f^2)*\sinh \\
& (d*x + c)^2*\log(I*\cosh(d*x + c) + I*\sinh(d*x + c) + 1) - (4*I*(a^3 + a*b^2) \\
& )*d*f^2*x - 4*(a^2*b + b^3)*d*f^2*x + 4*I*(a^3 + a*b^2)*c*f^2 - 4*(a^2*b + \\
& b^3)*c*f^2 + (4*I*(a^3 + a*b^2)*d*f^2*x - 4*(a^2*b + b^3)*d*f^2*x + 4*I*(a^ \\
& 3 + a*b^2)*c*f^2 - 4*(a^2*b + b^3)*c*f^2)*\cosh(d*x + c)^2 + (8*I*(a^3 + a*b \\
& ^2)*d*f^2*x - 8*(a^2*b + b^3)*d*f^2*x + 8*I*(a^3 + a*b^2)*c*f^2 - 8*(a^2*b \\
& + b^3)*c*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (4*I*(a^3 + a*b^2)*d*f^2*x - 4* \\
& (a^2*b + b^3)*d*f^2*x + 4*I*(a^3 + a*b^2)*c*f^2 - 4*(a^2*b + b^3)*c*f^2)*\si \\
& nh(d*x + c)^2*\log(-I*\cosh(d*x + c) - I*\sinh(d*x + c) + 1) + 4*((a^3 + a*b^ \\
& 2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + (a^3 + a*b^2)*d^2*e^2 - 2*((a^ \\
& 2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*d^2*e*f*x + 2*(a^2*b + b^3)*c*d*e* \\
& f - (a^2*b + b^3)*c^2*f^2)*\cosh(d*x + c))*\sinh(d*x + c))/((a^4 + 2*a^2*b^2 \\
& + b^4)*d^3*\cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*\cosh(d*x + c)*\si \\
& nh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*d^3*\sinh(d*x + c)^2 + (a^4 + 2*a^2*b^ \\
& 2 + b^4)*d^3)
\end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^2 \tanh(c + dx) \operatorname{sech}(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*sech(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*\*2\*tanh(c + d\*x)\*sech(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sech(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.355 \quad \int \frac{(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=335

$$-\frac{abf\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)^{3/2}} + \frac{abf\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2+a}}\right)}{d^2(a^2+b^2)^{3/2}} + \frac{af\tan^{-1}(\sinh(c+dx))}{d^2(a^2+b^2)} + \frac{a^2f\log(\cosh(c+dx))}{bd^2(a^2+b^2)} - \frac{ab($$

[Out] (a\*f\*ArcTan[Sinh[c + d\*x]])/((a^2 + b^2)\*d^2) - (a\*b\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d) + (a\*b\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d) - (f\*Log[Cosh[c + d\*x]])/(b\*d^2) + (a^2\*f\*Log[Cosh[c + d\*x]])/(b\*(a^2 + b^2)\*d^2) - (a\*b\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)\*d^2) + (a\*b\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)\*d^2) - (a\*(e + f\*x)\*Sech[c + d\*x])/((a^2 + b^2)\*d) + ((e + f\*x)\*Tanh[c + d\*x])/(b\*d) - (a^2\*(e + f\*x)\*Tanh[c + d\*x])/(b\*(a^2 + b^2)\*d)

**Rubi [A]** time = 0.662796, antiderivative size = 335, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 12, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {5583, 4184, 3475, 5573, 3322, 2264, 2190, 2279, 2391, 6742, 5451, 3770}

$$-\frac{abf\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2(a^2+b^2)^{3/2}} + \frac{abf\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2+a}}\right)}{d^2(a^2+b^2)^{3/2}} + \frac{af\tan^{-1}(\sinh(c+dx))}{d^2(a^2+b^2)} + \frac{a^2f\log(\cosh(c+dx))}{bd^2(a^2+b^2)} - \frac{ab($$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Sech[c + d\*x]\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (a\*f\*ArcTan[Sinh[c + d\*x]])/((a^2 + b^2)\*d^2) - (a\*b\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d) + (a\*b\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^(3/2)\*d) - (f\*Log[Cosh[c + d\*x]])/(b\*d^2) + (a^2\*f\*Log[Cosh[c + d\*x]])/(b\*(a^2 + b^2)\*d^2) - (a\*b\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)\*d^2) + (a\*b\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)\*d^2) - (a\*(e + f\*x)\*Sech[c + d\*x])/((a^2 + b^2)\*d) + ((e + f\*x)\*Tanh[c + d\*x])/(b\*d) - (a^2\*(e + f\*x)\*Tanh[c + d\*x])/(b\*(a^2 + b^2)\*d)

**Rule 5583**

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
(d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

#### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[p((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

#### Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

#### Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

#### Rule 3322

```
Int[(((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

#### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

#### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
```



```

[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

### Rule 2279

```

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

```

### Rule 2391

```

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

```

### Rule 6742

```

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]

```

### Rule 5451

```

Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

```

### Rule 3770

```

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\operatorname{sech}^2(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx}{b} \\
&= \frac{(e+fx)\tanh(c+dx)}{bd} - \frac{a \int (e+fx)\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx)) dx}{b(a^2+b^2)} \\
&= -\frac{f \log(\cosh(c+dx))}{bd^2} + \frac{(e+fx)\tanh(c+dx)}{bd} - \frac{a \int (a(e+fx)\operatorname{sech}^2(c+dx) dx)}{b(a^2+b^2)} \\
&= -\frac{f \log(\cosh(c+dx))}{bd^2} + \frac{(e+fx)\tanh(c+dx)}{bd} - \frac{(2ab^2) \int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{(a^2+b^2)^{3/2}} \\
&= -\frac{ab(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} + \frac{ab(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} - \frac{f \log(\cosh(c+dx))}{bd} \\
&= \frac{af \tan^{-1}(\sinh(c+dx))}{(a^2+b^2)d^2} - \frac{ab(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} + \frac{ab(e+fx) \log\left(1 - \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} \\
&= \frac{af \tan^{-1}(\sinh(c+dx))}{(a^2+b^2)d^2} - \frac{ab(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} + \frac{ab(e+fx) \log\left(1 - \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d}
\end{aligned}$$

**Mathematica [A]** time = 3.10648, size = 285, normalized size = 0.85

$$\frac{ab\left(-f \operatorname{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) + f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) + 2de \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) - f(c+dx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right) + f(c+dx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right) - 2cf \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)\right)}{(a^2+b^2)^{3/2} d^2}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Sech[c + d\*x]\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]), x]

[Out] ((2\*a\*f\*ArcTan[Tanh[(c + d\*x)/2]])/(a^2 + b^2) - (b\*f\*Log[Cosh[c + d\*x]])/(a^2 + b^2) + (a\*b\*(2\*d\*e\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]] - 2\*c\*f\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]] - f\*(c + d\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]) + f\*(c + d\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]) - f\*PolyLog[2, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] + f\*PolyLog[2, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])]))/(a^2 + b^2)^(3/2)

$$) + (d*(e + f*x)*\text{Sech}[c + d*x]*(-a + b*\text{Sinh}[c + d*x]))/(a^2 + b^2))/d^2$$

**Maple [B]** time = 0.168, size = 1858, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)`

[Out] 
$$\begin{aligned} & -1/(a^2+b^2)^2/d^2*f*b*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)*a^2-2/(a^2+b^2) \\ & )/d^2*f*b^3/(2*a^2+2*b^2)*\ln(1+\exp(2*d*x+2*c))+1/(a^2+b^2)/d^2*f*b^3/(2*a^2 \\ & +2*b^2)*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)+4/(a^2+b^2)/d^2*a^3*f/(2*a^2+ \\ & 2*b^2)*\arctan(\exp(d*x+c))+2/(a^2+b^2)^{(5/2)}/d^2*f*b*\operatorname{arctanh}(1/2*(2*b*\exp(d* \\ & x+c)+2*a)/(a^2+b^2)^{(1/2}))*a^3+2/(a^2+b^2)^{(5/2)}/d^2*f*b^3*\operatorname{arctanh}(1/2*(2*b \\ & * \exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2}))*a+4/(a^2+b^2)/d^2*f*b^2/(2*a^2+2*b^2)*a*a \\ & \operatorname{rctan}(\exp(d*x+c))-2/(a^2+b^2)/d^2*a^2*b*f/(2*a^2+2*b^2)*\ln(1+\exp(2*d*x+2*c) \\ & )+2/(a^2+b^2)/d^2*a^2*b*f/(2*a^2+2*b^2)*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)- \\ & b)-2/(a^2+b^2)^{(3/2)}/d^2*a*b^3*f*c/(2*a^2+2*b^2)*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c) \\ & )+2*a)/(a^2+b^2)^{(1/2}))-2/(a^2+b^2)^{(3/2)}/d^2*a^3*b*f*c/(2*a^2+2*b^2)*\operatorname{arcta} \\ & \operatorname{nh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2}))-2/(a^2+b^2)^{(1/2)}/d^2*a*b*f*c/ \\ & (2*a^2+2*b^2)*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2}))+2/(a^2+b^2) \\ & ^{(3/2)}/d^2*a*b^3*f/(2*a^2+2*b^2)*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2}))+a)/(a+(a^ \\ & 2+b^2)^{(1/2}))*c-2/(a^2+b^2)^{(3/2)}/d^2*a^3*b*f/(2*a^2+2*b^2)*\ln((-b*\exp(d*x \\ & +c)+(a^2+b^2)^{(1/2}))-a)/(-a+(a^2+b^2)^{(1/2}))*c-2/(a^2+b^2)^{(3/2)}/d^2*a*b^3* \\ & f/(2*a^2+2*b^2)*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2}))-a)/(-a+(a^2+b^2)^{(1/2}))* \\ & c+2/(a^2+b^2)^{(3/2)}/d^2*a^3*b*f/(2*a^2+2*b^2)*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1 \\ & /2}))+a)/(a+(a^2+b^2)^{(1/2}))*c-2/(a^2+b^2)^{(3/2)}/d^2*a*b^3*f/(2*a^2+2*b^2)*\ln( \\ & (-b*\exp(d*x+c)+(a^2+b^2)^{(1/2}))-a)/(-a+(a^2+b^2)^{(1/2}))*x+2/(a^2+b^2)^{(3/2) \\ & }/d^2*a^3*b*f/(2*a^2+2*b^2)*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2}))+a)/(a+(a^2+b^2)^{( \\ & 1/2}))*x+2/(a^2+b^2)^{(3/2)}/d^2*a*b^3*f/(2*a^2+2*b^2)*\ln((b*\exp(d*x+c)+(a^2+b^ \\ & 2)^{(1/2}))+a)/(a+(a^2+b^2)^{(1/2}))*x-2/(a^2+b^2)^{(3/2)}/d^2*a^3*b*f/(2*a^2+2*b^2) \\ & )*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2}))-a)/(-a+(a^2+b^2)^{(1/2}))*x+2/(a^2+b^2)^{( \\ & 3/2)}/d^2*a*b^3*e/(2*a^2+2*b^2)*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1 \\ & /2}))+2/(a^2+b^2)^{(3/2)}/d^2*a^3*b*e/(2*a^2+2*b^2)*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+ \\ & 2*a)/(a^2+b^2)^{(1/2}))+2/(a^2+b^2)^{(1/2)}/d^2*a*b*e/(2*a^2+2*b^2)*\operatorname{arctanh}(1/2*( \\ & 2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2}))-2/(a^2+b^2)^{(3/2)}/d^2*a*b^3*f/(2*a^2+2 \\ & *b^2)*\operatorname{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2}))-a)/(-a+(a^2+b^2)^{(1/2}))+2/(a^2+ \\ & b^2)^{(3/2)}/d^2*a^3*b*f/(2*a^2+2*b^2)*\operatorname{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2}))+a) \\ & / (a+(a^2+b^2)^{(1/2}))+2/(a^2+b^2)^{(3/2)}/d^2*a*b^3*f/(2*a^2+2*b^2)*\operatorname{dilog}((b* \\ & \exp(d*x+c)+(a^2+b^2)^{(1/2}))+a)/(a+(a^2+b^2)^{(1/2}))-2/(a^2+b^2)^{(3/2)}/d^2*a^ \\ & 3*b*f/(2*a^2+2*b^2)*\operatorname{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2}))-a)/(-a+(a^2+b^2)^{($$

$$\begin{aligned} & 1/2)) - 2/(a^2+b^2)^{(1/2)}/d^2*a*b*f/(2*a^2+2*b^2)*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c) \\ & +2*a)/(a^2+b^2)^{(1/2)}) - 2/(a^2+b^2)^{(3/2)}/d^2*a*b^3*f/(2*a^2+2*b^2)*\operatorname{arctanh} \\ & (1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) - 2/(a^2+b^2)^{(3/2)}/d^2*a^3*b*f/(2 \\ & *a^2+2*b^2)*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) + 2/(a^2+b^2)/d \\ & ^2*b*f*\ln(\exp(d*x+c)) - 1/2/(a^2+b^2)^2/d^2*f*b^3*\ln(b*\exp(2*d*x+2*c)+2*a*\exp \\ & (d*x+c)-b) - 2*(f*x+e)*(a*\exp(d*x+c)+b)/d/(a^2+b^2)/(1+\exp(2*d*x+2*c)) \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.50921, size = 3272, normalized size = 9.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & (2*(a^2*b + b^3)*d*f*x*\cosh(d*x + c)^2 + 2*(a^2*b + b^3)*d*f*x*\sinh(d*x + c) \\ & )^2 - 2*(a^2*b + b^3)*d*e - (a*b^2*f*\cosh(d*x + c)^2 + 2*a*b^2*f*\cosh(d*x + \\ & c)*\sinh(d*x + c) + a*b^2*f*\sinh(d*x + c)^2 + a*b^2*f)*\sqrt{(a^2 + b^2)/b^2} \\ & )*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x \\ & + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + (a*b^2*f*\cosh(d*x + c)^2 + 2*a*b^2 \\ & *f*\cosh(d*x + c)*\sinh(d*x + c) + a*b^2*f*\sinh(d*x + c)^2 + a*b^2*f)*\sqrt{(a^2 + b^2)/b^2} \\ & )*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) \\ & + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + (a*b^2*d*e - a*b^2* \\ & c*f + (a*b^2*d*e - a*b^2*c*f)*\cosh(d*x + c)^2 + 2*(a*b^2*d*e - a*b^2*c*f)*\cosh \\ & (d*x + c)*\sinh(d*x + c) + (a*b^2*d*e - a*b^2*c*f)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2} \\ & )*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - (a*b^2*d*e - a*b^2*c*f + (a*b^2*d*e - a*b^2*c*f)*\cosh \\ & (d*x + c)^2 + 2*(a*b^2*d*e - a*b^2*c*f)*\cosh(d*x + c)*\sinh(d*x + c) + (a*b^2 \end{aligned}$$

```

*d*e - a*b^2*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x +
c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (a*b^2*d*f*x +
a*b^2*c*f + (a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*f*x + a
*b^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d*f*x + a*b^2*c*f)*sinh(d*x
+ c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (a*b^2*d*f
*x + a*b^2*c*f + (a*b^2*d*f*x + a*b^2*c*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*f*x
+ a*b^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a*b^2*d*f*x + a*b^2*c*f)*sinh(
d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) -
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*((a^
3 + a*b^2)*f*cosh(d*x + c)^2 + 2*(a^3 + a*b^2)*f*cosh(d*x + c)*sinh(d*x + c
) + (a^3 + a*b^2)*f*sinh(d*x + c)^2 + (a^3 + a*b^2)*f)*arctan(cosh(d*x + c)
+ sinh(d*x + c)) - 2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e)*cosh(d*x +
c) - ((a^2*b + b^3)*f*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f*cosh(d*x + c)*sin
h(d*x + c) + (a^2*b + b^3)*f*sinh(d*x + c)^2 + (a^2*b + b^3)*f)*log(2*cosh(
d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(2*(a^2*b + b^3)*d*f*x*cosh(d
*x + c) - (a^3 + a*b^2)*d*f*x - (a^3 + a*b^2)*d*e)*sinh(d*x + c))/((a^4 + 2
*a^2*b^2 + b^4)*d^2*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(d*
x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*d^2*sinh(d*x + c)^2 + (a^4 +
2*a^2*b^2 + b^4)*d^2)

```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \tanh(c + dx) \operatorname{sech}(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*tanh(c + d*x)*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

[Out] Timed out

$$3.356 \quad \int \frac{\mathbf{sech}(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=78

$$\frac{2ab \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}} - \frac{\mathbf{sech}(c+dx)(a-b \sinh(c+dx))}{d(a^2+b^2)}$$

[Out] (2\*a\*b\*ArcTanh[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[a^2 + b^2]]/((a^2 + b^2)^(3/2)\*d) - (Sech[c + d\*x]\*(a - b\*Sinh[c + d\*x]))/((a^2 + b^2)\*d)

**Rubi [A]** time = 0.112229, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$ , Rules used = {2866, 12, 2660, 618, 204}

$$\frac{2ab \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}} - \frac{\mathbf{sech}(c+dx)(a-b \sinh(c+dx))}{d(a^2+b^2)}$$

Antiderivative was successfully verified.

[In] Int[(Sech[c + d\*x]\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (2\*a\*b\*ArcTanh[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[a^2 + b^2]]/((a^2 + b^2)^(3/2)\*d) - (Sech[c + d\*x]\*(a - b\*Sinh[c + d\*x]))/((a^2 + b^2)\*d)

### Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_.)]*(g_.))^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sinh[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sinh[e + f*x])^m*Simp[c*(a^2*(p + 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*m]
```

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 2660

Int[((a\_) + (b\_)\*sin[(c\_) + (d\_)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + 2\*b\*e\*x + a\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

### Rule 618

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{\operatorname{sech}(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx &= -\frac{\operatorname{sech}(c+dx)(a-b \sinh(c+dx))}{(a^2+b^2)d} - \frac{\int \frac{ab}{a+b \sinh(c+dx)} dx}{a^2+b^2} \\
 &= -\frac{\operatorname{sech}(c+dx)(a-b \sinh(c+dx))}{(a^2+b^2)d} - \frac{(ab) \int \frac{1}{a+b \sinh(c+dx)} dx}{a^2+b^2} \\
 &= -\frac{\operatorname{sech}(c+dx)(a-b \sinh(c+dx))}{(a^2+b^2)d} + \frac{(2iab) \operatorname{Subst}\left(\int \frac{1}{a-2ibx+ax^2} dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{(a^2+b^2)d} \\
 &= -\frac{\operatorname{sech}(c+dx)(a-b \sinh(c+dx))}{(a^2+b^2)d} - \frac{(4iab) \operatorname{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2} dx, x, -2ib+2a \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{(a^2+b^2)d} \\
 &= \frac{2ab \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d} - \frac{\operatorname{sech}(c+dx)(a-b \sinh(c+dx))}{(a^2+b^2)d}
 \end{aligned}$$



**Mathematica [A]** time = 0.192775, size = 104, normalized size = 1.33

$$\frac{b\sqrt{-a^2 - b^2} \tanh(c + dx) - a\sqrt{-a^2 - b^2} \operatorname{sech}(c + dx) - 2ab \tan^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2 - b^2}}\right)}{d(-a^2 - b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sech[c + d\*x]\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]), x]

[Out] -((-2\*a\*b\*ArcTan[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[-a^2 - b^2]] - a\*Sqrt[-a^2 - b^2]\*Sech[c + d\*x] + b\*Sqrt[-a^2 - b^2]\*Tanh[c + d\*x])/((-a^2 - b^2)^(3/2)\*d))

**Maple [A]** time = 0.001, size = 100, normalized size = 1.3

$$\frac{1}{d} \left( -4 \frac{ab}{(2a^2 + 2b^2) \sqrt{a^2 + b^2}} \operatorname{Artanh} \left( \frac{1}{2} \frac{2a \tanh(1/2 dx + c/2) - 2b}{\sqrt{a^2 + b^2}} \right) - 2 \frac{-\tanh(1/2 dx + c/2) b + a}{(a^2 + b^2) ((\tanh(1/2 dx + c/2))^2 + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)), x)

[Out] 1/d\*(-4\*a\*b/(2\*a^2+2\*b^2)/(a^2+b^2)^(1/2)\*arctanh(1/2\*(2\*a\*tanh(1/2\*d\*x+1/2\*c)-2\*b)/(a^2+b^2)^(1/2))-2/(a^2+b^2)\*(-tanh(1/2\*d\*x+1/2\*c)\*b+a)/(tanh(1/2\*d\*x+1/2\*c)^2+1))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)), x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [B]** time = 2.15745, size = 884, normalized size = 11.33

$$\frac{2a^2b + 2b^3 - (ab \cosh(dx + c)^2 + 2ab \cosh(dx + c) \sinh(dx + c) + ab \sinh(dx + c)^2 + ab) \sqrt{a^2 + b^2} \log\left(\frac{b^2 \cosh(dx + c)^2 + a^2 \sinh(dx + c)^2 + 2ab \cosh(dx + c) \sinh(dx + c)}{(a^4 + 2a^2b^2 + b^4)d \cosh(dx + c)^2 + 2(a^4 + 2a^2b^2 + b^4)d \sinh(dx + c)^2 + (a^4 + 2a^2b^2 + b^4)d}\right)}{(a^4 + 2a^2b^2 + b^4)d \cosh(dx + c)^2 + 2(a^4 + 2a^2b^2 + b^4)d \sinh(dx + c)^2 + (a^4 + 2a^2b^2 + b^4)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-(2a^2b + 2b^3 - (ab \cosh(dx + c)^2 + 2ab \cosh(dx + c) \sinh(dx + c) + ab \sinh(dx + c)^2 + ab) \sqrt{a^2 + b^2} \log((b^2 \cosh(dx + c)^2 + a^2 \sinh(dx + c)^2 + 2ab \cosh(dx + c) \sinh(dx + c) + a^2 + b^2 + 2(b^2 \cosh(dx + c) + a \sinh(dx + c) + a) \sqrt{a^2 + b^2} (b \cosh(dx + c) + b \sinh(dx + c) + a)) / (b \cosh(dx + c)^2 + b \sinh(dx + c)^2 + 2a \cosh(dx + c) + 2(b \cosh(dx + c) + a) \sinh(dx + c) - b)) + 2(a^3 + ab^2) \cosh(dx + c) + 2(a^3 + ab^2) \sinh(dx + c)) / ((a^4 + 2a^2b^2 + b^4) d \cosh(dx + c)^2 + 2(a^4 + 2a^2b^2 + b^4) d \cosh(dx + c) \sinh(dx + c) + (a^4 + 2a^2b^2 + b^4) d \sinh(dx + c)^2 + (a^4 + 2a^2b^2 + b^4) d)$$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(c + dx) \operatorname{sech}(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(tanh(c + d\*x)\*sech(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

---

**Giac [A]** time = 1.27738, size = 158, normalized size = 2.03

$$\frac{ab \log\left(\frac{-2be^{dx+2c} - 2ae^c - 2\sqrt{a^2+b^2}e^c}{-2be^{dx+2c} - 2ae^c + 2\sqrt{a^2+b^2}e^c}\right)}{(a^2+b^2)^{\frac{3}{2}}} - \frac{2(ae^{dx+c}+b)}{(a^2+b^2)(e^{2dx+2c}+1)}$$


---

$d$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*b*log(abs(-2*b*e^(d*x + 2*c) - 2*a*e^c - 2*sqrt(a^2 + b^2)*e^c)/abs(-2*b
*e^(d*x + 2*c) - 2*a*e^c + 2*sqrt(a^2 + b^2)*e^c))/(a^2 + b^2)^(3/2) - 2*(a
*e^(d*x + c) + b)/((a^2 + b^2)*(e^(2*d*x + 2*c) + 1))/d
```

$$3.357 \quad \int \frac{\operatorname{sech}(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable} \left( \frac{\tanh(c+dx) \operatorname{sech}(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[(Sech[c + d\*x]\*Tanh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0586461, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{sech}(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Sech[c + d\*x]\*Tanh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int] [(Sech[c + d\*x]\*Tanh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\operatorname{sech}(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [A]** time = 60.4988, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sech[c + d\*x]\*Tanh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Integrate[(Sech[c + d\*x]\*Tanh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Maple [A]** time = 0.559, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(dx + c) \tanh(dx + c)}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d\*x+c)\*tanh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x)

[Out] int(sech(d\*x+c)\*tanh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-2ab \int -\frac{e^{(dx+c)}}{a^2be + b^3e + (a^2bf + b^3f)x - (a^2bee^{(2c)} + b^3ee^{(2c)} + (a^2bfe^{(2c)} + b^3fe^{(2c)})x)e^{2dx} - 2(a^3ee^c + ab^2ee^c + (a^3f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)\*tanh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x, algorithm="maxima")

[Out] -2\*a\*b\*integrate(-e^(d\*x + c)/(a^2\*b\*e + b^3\*e + (a^2\*b\*f + b^3\*f)\*x - (a^2\*b\*e\*e^(2\*c) + b^3\*e\*e^(2\*c) + (a^2\*b\*f\*e^(2\*c) + b^3\*f\*e^(2\*c))\*x)\*e^(2\*d\*x) - 2\*(a^3\*e\*e^c + a\*b^2\*e\*e^c + (a^3\*f\*e^c + a\*b^2\*f\*e^c)\*x)\*e^(d\*x)), x) - 2\*(a\*e^(d\*x + c) + b)/(a^2\*d\*e + b^2\*d\*e + (a^2\*d\*f + b^2\*d\*f)\*x + (a^2\*d\*e\*e^(2\*c) + b^2\*d\*e\*e^(2\*c) + (a^2\*d\*f\*e^(2\*c) + b^2\*d\*f\*e^(2\*c))\*x)\*e^(2\*d\*x)) - 2\*integrate((a\*f\*e^(d\*x + c) + b\*f)/(a^2\*d\*e^2 + b^2\*d\*e^2 + (a^2\*d\*f^2 + b^2\*d\*f^2)\*x^2 + 2\*(a^2\*d\*e\*f + b^2\*d\*e\*f)\*x + (a^2\*d\*e^2\*e^(2\*c) + b^2\*d\*e^2\*e^(2\*c) + (a^2\*d\*f^2\*e^(2\*c) + b^2\*d\*f^2\*e^(2\*c))\*x^2 + 2\*(a^2\*d\*e\*f\*e^(2\*c) + b^2\*d\*e\*f\*e^(2\*c))\*x)\*e^(2\*d\*x)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{sech}(dx + c) \tanh(dx + c)}{afx + ae + (bfx + be) \sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] integral(sech(d*x + c)*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
+ c)), x)
```

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(c + dx) \operatorname{sech}(c + dx)}{(a + b \sinh(c + dx))(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)*sech(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] Timed out
```

$$3.358 \quad \int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=1176

result too large to display

```
[Out] ((e + f*x)^2*ArcTan[E^(c + d*x)]/(b*d) - (2*a^2*b*(e + f*x)^2*ArcTan[E^(c + d*x)]/((a^2 + b^2)^2*d) - (a^2*(e + f*x)^2*ArcTan[E^(c + d*x)]/(b*(a^2 + b^2)*d) - (f^2*ArcTan[Sinh[c + d*x]]/(b*d^3) + (a^2*f^2*ArcTan[Sinh[c + d*x]]/(b*(a^2 + b^2)*d^3) - (a*b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/((a^2 + b^2)^2*d) - (a*b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/((a^2 + b^2)^2*d) + (a*b^2*(e + f*x)^2*Log[1 + E^(2*(c + d*x))]/((a^2 + b^2)^2*d) - (a*f^2*Log[Cosh[c + d*x]]/((a^2 + b^2)*d^3) - (I*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)]/(b*d^2) + ((2*I)*a^2*b*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)]/((a^2 + b^2)^2*d^2) + (I*a^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)]/(b*(a^2 + b^2)*d^2) + (I*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/(b*d^2) - ((2*I)*a^2*b*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/((a^2 + b^2)^2*d^2) - (I*a^2*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/(b*(a^2 + b^2)*d^2) - (2*a*b^2*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]/((a^2 + b^2)^2*d^2) - (2*a*b^2*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]/((a^2 + b^2)^2*d^2) + (a*b^2*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))]/((a^2 + b^2)^2*d^2) + (I*f^2*PolyLog[3, (-I)*E^(c + d*x)]/(b*d^3) - ((2*I)*a^2*b*f^2*PolyLog[3, (-I)*E^(c + d*x)]/((a^2 + b^2)^2*d^3) - (I*a^2*f^2*PolyLog[3, (-I)*E^(c + d*x)]/(b*(a^2 + b^2)*d^3) - (I*f^2*PolyLog[3, I*E^(c + d*x)]/(b*d^3) + ((2*I)*a^2*b*f^2*PolyLog[3, I*E^(c + d*x)]/((a^2 + b^2)^2*d^3) + (I*a^2*f^2*PolyLog[3, I*E^(c + d*x)]/(b*(a^2 + b^2)*d^3) + (2*a*b^2*f^2*PolyLog[3, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]/((a^2 + b^2)^2*d^3) + (2*a*b^2*f^2*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]/((a^2 + b^2)^2*d^3) - (a*b^2*f^2*PolyLog[3, -E^(2*(c + d*x))]/(2*(a^2 + b^2)^2*d^3) + (f*(e + f*x)*Sech[c + d*x]/(b*d^2) - (a^2*f*(e + f*x)*Sech[c + d*x]/(b*(a^2 + b^2)*d^2) - (a*(e + f*x)^2*Sech[c + d*x]^2)/(2*(a^2 + b^2)*d) + (a*f*(e + f*x)*Tanh[c + d*x]/((a^2 + b^2)*d^2) + ((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*b*d) - (a^2*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*b*(a^2 + b^2)*d)
```

---

**Rubi [A]** time = 1.69576, antiderivative size = 1176, normalized size of antiderivative = 1., number of steps used = 49, number of rules used = 15, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.441$ , Rules used = {5583, 4186, 3770, 4180, 2531, 2282, 6589, 5573, 5561, 2190, 6742, 3718,

5451, 4184, 3475}

$$\frac{(e+fx)^2 \tan^{-1}(e^{c+dx}) a^2}{b(a^2+b^2)d} - \frac{2b(e+fx)^2 \tan^{-1}(e^{c+dx}) a^2}{(a^2+b^2)^2 d} + \frac{f^2 \tan^{-1}(\sinh(c+dx)) a^2}{b(a^2+b^2)d^3} + \frac{if(e+fx) \text{PolyLog}(2, -ie^{c+dx})}{b(a^2+b^2)d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((e + f*x)^2*ArcTan[E^(c + d*x)]/(b*d) - (2*a^2*b*(e + f*x)^2*ArcTan[E^(c + d*x)]/((a^2 + b^2)^2*d) - (a^2*(e + f*x)^2*ArcTan[E^(c + d*x)]/(b*(a^2 + b^2)*d) - (f^2*ArcTan[Sinh[c + d*x]])/(b*d^3) + (a^2*f^2*ArcTan[Sinh[c + d*x]])/(b*(a^2 + b^2)*d^3) - (a*b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2 + b^2)^2*d) - (a*b^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2 + b^2)^2*d) + (a*b^2*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(a^2 + b^2)^2*d) - (a*f^2*Log[Cosh[c + d*x]])/(a^2 + b^2)*d^3) - (I*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)]/(b*d^2) + ((2*I)*a^2*b*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)]/((a^2 + b^2)^2*d^2) + (I*a^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)]/(b*(a^2 + b^2)*d^2) + (I*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/(b*d^2) - ((2*I)*a^2*b*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/((a^2 + b^2)^2*d^2) - (I*a^2*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/(b*(a^2 + b^2)*d^2) - (2*a*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2 + b^2)^2*d^2) - (2*a*b^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2 + b^2)^2*d^2) + (a*b^2*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(a^2 + b^2)^2*d^2) + (I*f^2*PolyLog[3, (-I)*E^(c + d*x)]/(b*d^3) - ((2*I)*a^2*b*f^2*PolyLog[3, (-I)*E^(c + d*x)]/((a^2 + b^2)^2*d^3) - (I*a^2*f^2*PolyLog[3, (-I)*E^(c + d*x)]/(b*(a^2 + b^2)*d^3) - (I*f^2*PolyLog[3, I*E^(c + d*x)]/(b*d^3) + ((2*I)*a^2*b*f^2*PolyLog[3, I*E^(c + d*x)]/((a^2 + b^2)^2*d^3) + (I*a^2*f^2*PolyLog[3, I*E^(c + d*x)]/(b*(a^2 + b^2)*d^3) + (2*a*b^2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2 + b^2)^2*d^3) + (2*a*b^2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2 + b^2)^2*d^3) - (a*b^2*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^3) + (f*(e + f*x)*Sech[c + d*x])/(b*d^2) - (a^2*f*(e + f*x)*Sech[c + d*x])/(b*(a^2 + b^2)*d^2) - (a*(e + f*x)^2*Sech[c + d*x]^2)/(2*(a^2 + b^2)*d) + (a*f*(e + f*x)*Tanh[c + d*x])/(a^2 + b^2)*d^2) + ((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*b*d) - (a^2*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*b*(a^2 + b^2)*d)
```

**Rule 5583**

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x
```



```
] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:= -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x]
+ (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*
(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*
(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/
(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*(c_.) + (d_.)*(x_)^(m_.), x_Symbol]
:= Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]
+ Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x])
/; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.))]*(f_.) + (g_.)*(x_)^(m_.), x_Symbol]
:= -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x]
+ Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x]
/; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol]
:> Dist[b^2/(a^2 + b^2), Int[((e + f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]
+ Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol]
:> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x]
+ (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x]
+ Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol]
:> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x]
&& IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rule 3718

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol]
:> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{c, d, e, f, fz}, x]
&& IGtQ[m, 0]
```

Rule 5451

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
```

```
(b_.)*(x_)^(p_), x_Symbol] := -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

#### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[
((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

#### Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \operatorname{sech}^3(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^2 \operatorname{sech}^3(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{f(e+fx) \operatorname{sech}(c+dx)}{bd^2} + \frac{(e+fx)^2 \operatorname{sech}(c+dx) \tanh(c+dx)}{2bd} + \frac{\int (e+fx)}{b} \\
&= \frac{(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{f^2 \tan^{-1}(\sinh(c+dx))}{bd^3} + \frac{f(e+fx) \operatorname{sech}(c+dx)}{bd^2} \\
&= \frac{ab^2(e+fx)^3}{3(a^2+b^2)^2 f} + \frac{(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{f^2 \tan^{-1}(\sinh(c+dx))}{bd^3} - \frac{if(e+fx)}{b} \\
&= \frac{ab^2(e+fx)^3}{3(a^2+b^2)^2 f} + \frac{(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{f^2 \tan^{-1}(\sinh(c+dx))}{bd^3} - \frac{ab^2(e+fx)}{b} \\
&= \frac{(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2b(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} - \frac{a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} \\
&= \frac{(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2b(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} - \frac{a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} \\
&= \frac{(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2b(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} - \frac{a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} \\
&= \frac{(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2b(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} - \frac{a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} \\
&= \frac{(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2b(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} - \frac{a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d} \\
&= \frac{(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2b(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} - \frac{a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d}
\end{aligned}$$

**Mathematica [B]** time = 31.4769, size = 3390, normalized size = 2.88

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Sech[c + d\*x]^2\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x])

,x]

```
[Out] (-12*a*b^2*d^3*e^2*E^(2*c)*x + 12*a^3*d*E^(2*c)*f^2*x + 12*a*b^2*d*E^(2*c)*
f^2*x - 12*a*b^2*d^3*e*E^(2*c)*f*x^2 - 4*a*b^2*d^3*E^(2*c)*f^2*x^3 - 6*a^2*
b*d^2*e^2*ArcTan[E^(c + d*x)] + 6*b^3*d^2*e^2*ArcTan[E^(c + d*x)] - 6*a^2*b
*d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] + 6*b^3*d^2*e^2*E^(2*c)*ArcTan[E^(c +
d*x)] - 12*a^2*b*f^2*ArcTan[E^(c + d*x)] - 12*b^3*f^2*ArcTan[E^(c + d*x)] -
12*a^2*b*E^(2*c)*f^2*ArcTan[E^(c + d*x)] - 12*b^3*E^(2*c)*f^2*ArcTan[E^(c
+ d*x)] - (6*I)*a^2*b*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*b^3*d^2*e*f*
x*Log[1 - I*E^(c + d*x)] - (6*I)*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d
*x)] + (6*I)*b^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] - (3*I)*a^2*b*d^2
*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*b^3*d^2*f^2*x^2*Log[1 - I*E^(c + d*
x)] - (3*I)*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*b^3*d^
2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (6*I)*a^2*b*d^2*e*f*x*Log[1 + I*
E^(c + d*x)] - (6*I)*b^3*d^2*e*f*x*Log[1 + I*E^(c + d*x)] + (6*I)*a^2*b*d^2
*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (6*I)*b^3*d^2*e*E^(2*c)*f*x*Log[1 +
I*E^(c + d*x)] + (3*I)*a^2*b*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (3*I)*b^
3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] + (3*I)*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[
1 + I*E^(c + d*x)] - (3*I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 + I*E^(c + d*x)] +
6*a*b^2*d^2*e^2*Log[1 + E^(2*(c + d*x))] + 6*a*b^2*d^2*e^2*E^(2*c)*Log[1 +
E^(2*(c + d*x))] - 6*a^3*f^2*Log[1 + E^(2*(c + d*x))] - 6*a*b^2*f^2*Log[1
+ E^(2*(c + d*x))] - 6*a^3*E^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] - 6*a*b^2*E
^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] + 12*a*b^2*d^2*e*f*x*Log[1 + E^(2*(c +
d*x))] + 12*a*b^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))] + 6*a*b^2*d^2*
f^2*x^2*Log[1 + E^(2*(c + d*x))] + 6*a*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(2
*(c + d*x))] + (6*I)*b*(a^2 - b^2)*d*(1 + E^(2*c))*f*(e + f*x)*PolyLog[2, (
-I)*E^(c + d*x)] + (6*I)*b*(-a^2 + b^2)*d*(1 + E^(2*c))*f*(e + f*x)*PolyLog
[2, I*E^(c + d*x)] + 6*a*b^2*d*e*f*PolyLog[2, -E^(2*(c + d*x))] + 6*a*b^2*d
*e*E^(2*c)*f*PolyLog[2, -E^(2*(c + d*x))] + 6*a*b^2*d*f^2*x*PolyLog[2, -E^(
2*(c + d*x))] + 6*a*b^2*d*E^(2*c)*f^2*x*PolyLog[2, -E^(2*(c + d*x))] - (6*I
)*a^2*b*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I)*b^3*f^2*PolyLog[3, (-I)*E^
(c + d*x)] - (6*I)*a^2*b*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I)*b
^3*E^(2*c)*f^2*PolyLog[3, (-I)*E^(c + d*x)] + (6*I)*a^2*b*f^2*PolyLog[3, I*
E^(c + d*x)] - (6*I)*b^3*f^2*PolyLog[3, I*E^(c + d*x)] + (6*I)*a^2*b*E^(2*c
)*f^2*PolyLog[3, I*E^(c + d*x)] - (6*I)*b^3*E^(2*c)*f^2*PolyLog[3, I*E^(c +
d*x)] - 3*a*b^2*f^2*PolyLog[3, -E^(2*(c + d*x))] - 3*a*b^2*E^(2*c)*f^2*Pol
yLog[3, -E^(2*(c + d*x))]/(6*(a^2 + b^2)^2*d^3*(1 + E^(2*c))) + (a*b^2*(6*
e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2
]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d
) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-
a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[
(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(
a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-
a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x))])
```

$$\begin{aligned}
& ])/d - (3e^{2c}E^{2c} \operatorname{Log}[2aE^c + b(-1 + E^{2(c+dx)})])/d + \\
& (6efx \operatorname{Log}[1 + (bE^{2c+dx})/(aE^c - \sqrt{(a^2 + b^2)E^{2c}})])/d \\
& - (6eE^{2c}fx \operatorname{Log}[1 + (bE^{2c+dx})/(aE^c - \sqrt{(a^2 + b^2)E^{2c}})])/d + (3f^2x^2 \operatorname{Log}[1 + (bE^{2c+dx})/(aE^c - \sqrt{(a^2 + b^2)E^{2c}})])/d - (3E^{2c}f^2x^2 \operatorname{Log}[1 + (bE^{2c+dx})/(aE^c - \sqrt{(a^2 + b^2)E^{2c}})])/d + (6efx \operatorname{Log}[1 + (bE^{2c+dx})/(aE^c + \sqrt{(a^2 + b^2)E^{2c}})])/d - (6eE^{2c}fx \operatorname{Log}[1 + (bE^{2c+dx})/(aE^c + \sqrt{(a^2 + b^2)E^{2c}})])/d + (3f^2x^2 \operatorname{Log}[1 + (bE^{2c+dx})/(aE^c + \sqrt{(a^2 + b^2)E^{2c}})])/d - (3E^{2c}f^2x^2 \operatorname{Log}[1 + (bE^{2c+dx})/(aE^c + \sqrt{(a^2 + b^2)E^{2c}})])/d - (6(-1 + E^{2c})ef \\
& (e + fx) \operatorname{PolyLog}[2, -(bE^{2c+dx})/(aE^c - \sqrt{(a^2 + b^2)E^{2c}})])/d^2 - (6(-1 + E^{2c})ef(e + fx) \operatorname{PolyLog}[2, -(bE^{2c+dx})/(aE^c + \sqrt{(a^2 + b^2)E^{2c}})])/d^2 - (6f^2 \operatorname{PolyLog}[3, -(bE^{2c+dx})/(aE^c - \sqrt{(a^2 + b^2)E^{2c}})])/d^3 + (6E^{2c}f^2 \operatorname{PolyLog}[3, -(bE^{2c+dx})/(aE^c - \sqrt{(a^2 + b^2)E^{2c}})])/d^3 - (6f^2 \operatorname{PolyLog}[3, -(bE^{2c+dx})/(aE^c + \sqrt{(a^2 + b^2)E^{2c}})])/d^3 + (6E^{2c}f^2 \operatorname{PolyLog}[3, -(bE^{2c+dx})/(aE^c + \sqrt{(a^2 + b^2)E^{2c}})])/d^3)/(3(a^2 + b^2)^2(-1 + E^{2c})) + (\operatorname{Csch}[c] \operatorname{Sech}[c] \operatorname{Sech}[c + dx]^2(6a^3ef + 6ab^2ef - 12ab^2d^2e^2x + 6a^3f^2x + 6ab^2f^2x - 12ab^2d^2efx^2 - 4ab^2d^2f^2x^3 - 6a^3ef \operatorname{Cosh}[2c] - 6ab^2ef \operatorname{Cosh}[2c] - 6a^3f^2x \operatorname{Cosh}[2c] - 6ab^2f^2x \operatorname{Cosh}[2c] - 6a^3ef \operatorname{Cosh}[2dx] - 6ab^2ef \operatorname{Cosh}[2dx] - 6a^3f^2x \operatorname{Cosh}[2dx] - 6ab^2f^2x \operatorname{Cosh}[2dx] - 3a^2bde^2 \operatorname{Cosh}[c - dx] - 3b^3de^2 \operatorname{Cosh}[c - dx] - 6a^2bdefx \operatorname{Cosh}[c - dx] - 6b^3defx \operatorname{Cosh}[c - dx] - 3a^2bdf^2x^2 \operatorname{Cosh}[c - dx] - 3b^3df^2x^2 \operatorname{Cosh}[c - dx] + 3a^2bde^2 \operatorname{Cosh}[3c + dx] + 3b^3de^2 \operatorname{Cosh}[3c + dx] + 6a^2bdefx \operatorname{Cosh}[3c + dx] + 6b^3defx \operatorname{Cosh}[3c + dx] + 3a^2bdf^2x^2 \operatorname{Cosh}[3c + dx] + 3b^3df^2x^2 \operatorname{Cosh}[3c + dx] + 6a^3ef \operatorname{Cosh}[2c + 2dx] + 6ab^2ef \operatorname{Cosh}[2c + 2dx] - 12ab^2d^2e^2x \operatorname{Cosh}[2c + 2dx] + 6a^3f^2x \operatorname{Cosh}[2c + 2dx] + 6ab^2f^2x \operatorname{Cosh}[2c + 2dx] - 12ab^2d^2efx^2 \operatorname{Cosh}[2c + 2dx] - 4ab^2d^2f^2x^3 \operatorname{Cosh}[2c + 2dx] - 6a^3de^2 \operatorname{Sinh}[2c] - 6ab^2de^2 \operatorname{Sinh}[2c] - 12a^3defx \operatorname{Sinh}[2c] - 12ab^2defx \operatorname{Sinh}[2c] - 6a^3df^2x^2 \operatorname{Sinh}[2c] - 6ab^2df^2x^2 \operatorname{Sinh}[2c] + 6a^2bde^2 \operatorname{Sinh}[c - dx] + 6b^3de^2 \operatorname{Sinh}[c - dx] + 6a^2bde^2 \operatorname{Sinh}[3c + dx] + 6b^3de^2 \operatorname{Sinh}[3c + dx] + 6a^2bdf^2x \operatorname{Sinh}[c - dx] + 6b^3df^2x \operatorname{Sinh}[c - dx] + 6a^2bde^2 \operatorname{Sinh}[3c + dx] + 6b^3de^2 \operatorname{Sinh}[3c + dx]))/(24(a^2 + b^2)^2d^2)
\end{aligned}$$

**Maple [F]** time = 0.454, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\operatorname{sech}(dx + c))^2 \tanh(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)^2*\text{sech}(d*x+c)^2*\text{tanh}(d*x+c)/(a+b*\sinh(d*x+c)),x)$

[Out]  $\text{int}((f*x+e)^2*\text{sech}(d*x+c)^2*\text{tanh}(d*x+c)/(a+b*\sinh(d*x+c)),x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x+e)^2*\text{sech}(d*x+c)^2*\text{tanh}(d*x+c)/(a+b*\sinh(d*x+c)),x, \text{algorithm}="maxima")$

[Out] 
$$\begin{aligned} & -a^2*b*d^2*f^2*\text{integrate}(x^2*e^{(d*x+c)}/(a^4*d^2*e^{(2*d*x+2*c)} + 2*a^2*b^2*d^2 \\ & + b^4*d^2), x) + b^3*d^2*f^2*\text{integrate}(x^2*e^{(d*x+c)}/(a^4*d^2*e^{(2*d*x+2*c)} + 2*a^2*b^2*d^2*e^{(2*d*x+2*c)} \\ & + b^4*d^2), x) - 2*a*b^2*d^2*f^2*\text{integrate}(x^2/(a^4*d^2*e^{(2*d*x+2*c)} + 2*a^2*b^2*d^2*e^{(2*d*x+2*c)} + b^4*d^2 \\ & + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a^2*b*d^2*e*f*\text{integrate}(x*e^{(d*x+c)}/(a^4*d^2*e^{(2*d*x+2*c)} + 2*a^2*b^2*d^2*e^{(2*d*x+2*c)} \\ & + b^4*d^2), x) + 2*b^3*d^2*e*f*\text{integrate}(x*e^{(d*x+c)}/(a^4*d^2*e^{(2*d*x+2*c)} + 2*a^2*b^2*d^2*e^{(2*d*x+2*c)} + b^4*d^2), x) \\ & - 4*a*b^2*d^2*e*f*\text{integrate}(x/(a^4*d^2*e^{(2*d*x+2*c)} + 2*a^2*b^2*d^2*e^{(2*d*x+2*c)} + b^4*d^2), x) + a^3*f^2*(2*(d*x+c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) \\ & - \log(e^{(2*d*x+2*c)} + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) + a*b^2*f^2*(2*(d*x+c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) \\ & - \log(e^{(2*d*x+2*c)} + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) - (a*b^2*\log(-2*a*e^{(-d*x-c)} + b*e^{(-2*d*x-2*c)} - b)/((a^4 + 2*a^2*b^2 + b^4)*d) \\ & - a*b^2*\log(e^{(-2*d*x-2*c)} + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^2*b - b^3)*\arctan(e^{(-d*x-c)})/((a^4 + 2*a^2*b^2 + b^4)*d) - (b*e^{(-d*x-c)} - 2*a*e^{(-2*d*x-2*c)} - b*e^{(-3*d*x-3*c)})/((a^2 + b^2 + 2*(a^2 + b^2)*e^{(-2*d*x-2*c)} + (a^2 + b^2)*e^{(-4*d*x-4*c)})*d)) * e^2 - 2*a^2*b*f^2*\arctan(e^{(d*x+c)})/((a^4 + 2*a^2*b^2 + b^4)*d^3) \\ & - 2*b^3*f^2*\arctan(e^{(d*x+c)})/((a^4 + 2*a^2*b^2 + b^4)*d^3) - (2*a*f^2*x + 2*a*e*f - (b*d*f^2*x^2*e^{(3*c)} + 2*b*e*f*e^{(3*c)} + 2*(d*e*f + f^2)*b*x*e^{(3*c)})) * e^{(3*d*x)} + 2*(a*d*f^2*x^2*e^{(2*c)} + a*e*f*e^{(2*c)} + (2*d*e*f + f^2)*a*x*e^{(2*c)}) * e^{(2*d*x)} + (b*d*f^2*x^2*e^c - 2*b*e*f*e^c + 2*(d*e*f - f^2)*b*x*e^c) * e^{(d*x)}/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^{(4*c)} + b^2*d^2*e^{(4*c)})) * e^{(4*d*x)} + 2*(a^2*d^2*e^{(2*c)} + \end{aligned}$$

```
b^2*d^2*e^(2*c))*e^(2*d*x)) + integrate(2*(a*b^3*f^2*x^2 + 2*a*b^3*e*f*x -
(a^2*b^2*f^2*x^2*e^c + 2*a^2*b^2*e*f*x*e^c))*e^(d*x))/(a^4*b + 2*a^2*b^3 + b
^5 - (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e
^c + 2*a^3*b^2*e^c + a*b^4*e^c))*e^(d*x)), x)
```

**Fricas [C]** time = 5.5478, size = 24520, normalized size = 20.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*cosh(d*x + c)^4 + 4*((
a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*sinh(d*x + c)^4 - 4*(a^3 + a*b^
2)*d*e*f + 4*(a^3 + a*b^2)*c*f^2 + 2*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b +
b^3)*d^2*e^2 + 2*(a^2*b + b^3)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (a^2*b +
b^3)*d*f^2)*x)*cosh(d*x + c)^3 + 2*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^
3)*d^2*e^2 + 2*(a^2*b + b^3)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (a^2*b + b^
3)*d*f^2)*x + 8*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*cosh(d*x + c)
)*sinh(d*x + c)^3 - 4*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a*b^2)*d^2*e^2 +
(a^3 + a*b^2)*d*e*f - 2*(a^3 + a*b^2)*c*f^2 + (2*(a^3 + a*b^2)*d^2*e*f - (a
^3 + a*b^2)*d*f^2)*x)*cosh(d*x + c)^2 - 2*(2*(a^3 + a*b^2)*d^2*f^2*x^2 + 2*
(a^3 + a*b^2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f - 4*(a^3 + a*b^2)*c*f^2 - 12*
((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*cosh(d*x + c)^2 + 2*(2*(a^3 +
a*b^2)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x - 3*((a^2*b + b^3)*d^2*f^2*x^2 + (
a^2*b + b^3)*d^2*e^2 + 2*(a^2*b + b^3)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (
a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*((a^2*b + b^3)*d^
2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f + 2*((a^2*b + b^3)
)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c) - 4*(a*b^2*d*f^2*x + a*b^
2*d*e*f + (a*b^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)^4 + 4*(a*b^2*d*f^2*x
+ a*b^2*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b^2*d*f^2*x + a*b^2*d*e*f)
)*sinh(d*x + c)^4 + 2*(a*b^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)^2 + 2*(a*
b^2*d*f^2*x + a*b^2*d*e*f + 3*(a*b^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)^2)
)*sinh(d*x + c)^2 + 4*((a*b^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)^3 + (a*b
^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x +
c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2) - b)/b + 1) - 4*(a*b^2*d*f^2*x + a*b^2*d*e*f + (a*b^2*d*f^2*x + a*b^
2*d*e*f)*cosh(d*x + c)^4 + 4*(a*b^2*d*f^2*x + a*b^2*d*e*f)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a*b^2*d*f^2*x + a*b^2*d*e*f)*sinh(d*x + c)^4 + 2*(a*b^2*d*
f^2*x + a*b^2*d*e*f)*cosh(d*x + c)^2 + 2*(a*b^2*d*f^2*x + a*b^2*d*e*f + 3*(
```



$$\begin{aligned}
& a*b^2*d*f^2*x + a*b^2*d*e*f)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*((a*b^2*d \\
& *f^2*x + a*b^2*d*e*f)*\cosh(d*x + c)^3 + (a*b^2*d*f^2*x + a*b^2*d*e*f)*\cosh( \\
& d*x + c))*\sinh(d*x + c))*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh \\
& (d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + (4*a*b^2*d \\
& *f^2*x + 4*a*b^2*d*e*f - 2*I*(a^2*b - b^3)*d*f^2*x + (4*a*b^2*d*f^2*x + 4*a \\
& *b^2*d*e*f - 2*I*(a^2*b - b^3)*d*f^2*x - 2*I*(a^2*b - b^3)*d*e*f)*\cosh(d*x \\
& + c)^4 + (16*a*b^2*d*f^2*x + 16*a*b^2*d*e*f - 8*I*(a^2*b - b^3)*d*f^2*x - 8 \\
& *I*(a^2*b - b^3)*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (4*a*b^2*d*f^2*x + \\
& 4*a*b^2*d*e*f - 2*I*(a^2*b - b^3)*d*f^2*x - 2*I*(a^2*b - b^3)*d*e*f)*\sinh(d \\
& *x + c)^4 - 2*I*(a^2*b - b^3)*d*e*f + (8*a*b^2*d*f^2*x + 8*a*b^2*d*e*f - 4* \\
& I*(a^2*b - b^3)*d*f^2*x - 4*I*(a^2*b - b^3)*d*e*f)*\cosh(d*x + c)^2 + (8*a*b \\
& ^2*d*f^2*x + 8*a*b^2*d*e*f - 4*I*(a^2*b - b^3)*d*f^2*x - 4*I*(a^2*b - b^3)* \\
& d*e*f + (24*a*b^2*d*f^2*x + 24*a*b^2*d*e*f - 12*I*(a^2*b - b^3)*d*f^2*x - 1 \\
& 2*I*(a^2*b - b^3)*d*e*f)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + ((16*a*b^2*d*f^ \\
& 2*x + 16*a*b^2*d*e*f - 8*I*(a^2*b - b^3)*d*f^2*x - 8*I*(a^2*b - b^3)*d*e*f) \\
& *\cosh(d*x + c)^3 + (16*a*b^2*d*f^2*x + 16*a*b^2*d*e*f - 8*I*(a^2*b - b^3)*d \\
& *f^2*x - 8*I*(a^2*b - b^3)*d*e*f)*\cosh(d*x + c))*\sinh(d*x + c))*\operatorname{dilog}(I*\cos \\
& h(d*x + c) + I*\sinh(d*x + c)) + (4*a*b^2*d*f^2*x + 4*a*b^2*d*e*f + 2*I*(a^2 \\
& *b - b^3)*d*f^2*x + (4*a*b^2*d*f^2*x + 4*a*b^2*d*e*f + 2*I*(a^2*b - b^3)*d* \\
& f^2*x + 2*I*(a^2*b - b^3)*d*e*f)*\cosh(d*x + c)^4 + (16*a*b^2*d*f^2*x + 16*a \\
& *b^2*d*e*f + 8*I*(a^2*b - b^3)*d*f^2*x + 8*I*(a^2*b - b^3)*d*e*f)*\cosh(d*x \\
& + c)*\sinh(d*x + c)^3 + (4*a*b^2*d*f^2*x + 4*a*b^2*d*e*f + 2*I*(a^2*b - b^3) \\
& *d*f^2*x + 2*I*(a^2*b - b^3)*d*e*f)*\sinh(d*x + c)^4 + 2*I*(a^2*b - b^3)*d*e \\
& *f + (8*a*b^2*d*f^2*x + 8*a*b^2*d*e*f + 4*I*(a^2*b - b^3)*d*f^2*x + 4*I*(a^ \\
& 2*b - b^3)*d*e*f)*\cosh(d*x + c)^2 + (8*a*b^2*d*f^2*x + 8*a*b^2*d*e*f + 4*I* \\
& (a^2*b - b^3)*d*f^2*x + 4*I*(a^2*b - b^3)*d*e*f + (24*a*b^2*d*f^2*x + 24*a* \\
& b^2*d*e*f + 12*I*(a^2*b - b^3)*d*f^2*x + 12*I*(a^2*b - b^3)*d*e*f)*\cosh(d*x \\
& + c)^2*\sinh(d*x + c)^2 + ((16*a*b^2*d*f^2*x + 16*a*b^2*d*e*f + 8*I*(a^2*b \\
& - b^3)*d*f^2*x + 8*I*(a^2*b - b^3)*d*e*f)*\cosh(d*x + c)^3 + (16*a*b^2*d*f^ \\
& 2*x + 16*a*b^2*d*e*f + 8*I*(a^2*b - b^3)*d*f^2*x + 8*I*(a^2*b - b^3)*d*e*f) \\
& *\cosh(d*x + c))*\sinh(d*x + c))*\operatorname{dilog}(-I*\cosh(d*x + c) - I*\sinh(d*x + c)) - \\
& 2*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2 + (a*b^2*d^2*e^2 - 2*a*b \\
& ^2*c*d*e*f + a*b^2*c^2*f^2)*\cosh(d*x + c)^4 + 4*(a*b^2*d^2*e^2 - 2*a*b^2*c* \\
& d*e*f + a*b^2*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a*b^2*d^2*e^2 - 2*a \\
& *b^2*c*d*e*f + a*b^2*c^2*f^2)*\sinh(d*x + c)^4 + 2*(a*b^2*d^2*e^2 - 2*a*b^2* \\
& c*d*e*f + a*b^2*c^2*f^2)*\cosh(d*x + c)^2 + 2*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e \\
& *f + a*b^2*c^2*f^2 + 3*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*\co \\
& sh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^ \\
& 2*c^2*f^2)*\cosh(d*x + c)^3 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f \\
& ^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) \\
& + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - 2*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + \\
& a*b^2*c^2*f^2 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*\cosh(d*x \\
& + c)^4 + 4*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*\cosh(d*x + c)* \\
& \sinh(d*x + c)^3 + (a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*\sinh(d* \\
& x + c)^4 + 2*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*\cosh(d*x + c
\end{aligned}$$

$$\begin{aligned}
&)^2 + 2*(a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2 + 3*(a*b^2*d^2*e^2 \\
&- 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*(( \\
&a*b^2*d^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*\cosh(d*x + c)^3 + (a*b^2*d \\
&^2*e^2 - 2*a*b^2*c*d*e*f + a*b^2*c^2*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log \\
&(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - \\
&2*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2 \\
&+ (a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2 \\
&))*\cosh(d*x + c)^4 + 4*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d* \\
&e*f - a*b^2*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a*b^2*d^2*f^2*x^2 + 2 \\
&*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*\sinh(d*x + c)^4 + 2*(a* \\
&b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*\cosh \\
&(d*x + c)^2 + 2*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - \\
&a*b^2*c^2*f^2 + 3*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f \\
&- a*b^2*c^2*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((a*b^2*d^2*f^2*x^2 + \\
&2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*\cosh(d*x + c)^3 + (a* \\
&b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*\cosh \\
&(d*x + c))*\sinh(d*x + c))*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh \\
&(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 2*(a*b^2*d^2*f \\
&^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2 + (a*b^2*d^2*f \\
&^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*\cosh(d*x + c) \\
&^4 + 4*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2 \\
&f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f* \\
&x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*\sinh(d*x + c)^4 + 2*(a*b^2*d^2*f^2*x^2 \\
&+ 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*\cosh(d*x + c)^2 + 2 \\
&*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2 + \\
&3*(a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2 \\
&))*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e* \\
&f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*\cosh(d*x + c)^3 + (a*b^2*d^2*f^2*x^2 \\
&+ 2*a*b^2*d^2*e*f*x + 2*a*b^2*c*d*e*f - a*b^2*c^2*f^2)*\cosh(d*x + c))*\sinh \\
&(d*x + c))*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*s \\
&inh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + (2*a*b^2*d^2*e^2 - 4*a*b^2*c* \\
&d*e*f - I*(a^2*b - b^3)*d^2*e^2 + 2*I*(a^2*b - b^3)*c*d*e*f + (2*a*b^2*d^2* \\
&e^2 - 4*a*b^2*c*d*e*f - I*(a^2*b - b^3)*d^2*e^2 + 2*I*(a^2*b - b^3)*c*d*e*f \\
&+ 2*(a*b^2*c^2 - a^3 - a*b^2)*f^2 - I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2 \\
&))*f^2)*\cosh(d*x + c)^4 + (8*a*b^2*d^2*e^2 - 16*a*b^2*c*d*e*f - 4*I*(a^2*b - \\
&b^3)*d^2*e^2 + 8*I*(a^2*b - b^3)*c*d*e*f + 8*(a*b^2*c^2 - a^3 - a*b^2)*f^2 \\
&- 4*I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2))*f^2)*\cosh(d*x + c)*\sinh(d*x + \\
&c)^3 + (2*a*b^2*d^2*e^2 - 4*a*b^2*c*d*e*f - I*(a^2*b - b^3)*d^2*e^2 + 2*I*( \\
&a^2*b - b^3)*c*d*e*f + 2*(a*b^2*c^2 - a^3 - a*b^2)*f^2 - I*(2*a^2*b + 2*b^3 \\
&+ (a^2*b - b^3)*c^2))*f^2)*\sinh(d*x + c)^4 + 2*(a*b^2*c^2 - a^3 - a*b^2)*f^ \\
&2 - I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2))*f^2 + (4*a*b^2*d^2*e^2 - 8*a*b^ \\
&2*c*d*e*f - 2*I*(a^2*b - b^3)*d^2*e^2 + 4*I*(a^2*b - b^3)*c*d*e*f + 4*(a*b^ \\
&2*c^2 - a^3 - a*b^2)*f^2 - 2*I*(2*a^2*b + 2*b^3 + (a^2*b - b^3)*c^2))*f^2)*c \\
&osh(d*x + c)^2 + (4*a*b^2*d^2*e^2 - 8*a*b^2*c*d*e*f - 2*I*(a^2*b - b^3)*d^2 \\
&*e^2 + 4*I*(a^2*b - b^3)*c*d*e*f + 4*(a*b^2*c^2 - a^3 - a*b^2)*f^2 - 2*I*(2
\end{aligned}$$

$$\begin{aligned}
& a^2b + 2b^3 + (a^2b - b^3)c^2)f^2 + (12ab^2d^2e^2 - 24ab^2cde^2 \\
& e^2f - 6I(a^2b - b^3)d^2e^2 + 12I(a^2b - b^3)cde^2f + 12(ab^2c^2 \\
& - a^3 - ab^2)f^2 - 6I(2a^2b + 2b^3 + (a^2b - b^3)c^2)f^2)cosh( \\
& dx + c)^2)sinh(dx + c)^2 + ((8ab^2d^2e^2 - 16ab^2cde^2f - 4I(a \\
& ^2b - b^3)d^2e^2 + 8I(a^2b - b^3)cde^2f + 8(ab^2c^2 - a^3 - ab^2) \\
& )f^2 - 4I(2a^2b + 2b^3 + (a^2b - b^3)c^2)f^2)cosh(dx + c)^3 + ( \\
& 8ab^2d^2e^2 - 16ab^2cde^2f - 4I(a^2b - b^3)d^2e^2 + 8I(a^2b \\
& - b^3)cde^2f + 8(ab^2c^2 - a^3 - ab^2)f^2 - 4I(2a^2b + 2b^3 + \\
& (a^2b - b^3)c^2)f^2)cosh(dx + c))sinh(dx + c))log(cosh(dx + c) + s \\
& inh(dx + c) + I) + (2ab^2d^2e^2 - 4ab^2cde^2f + I(a^2b - b^3)d^ \\
& 2e^2 - 2I(a^2b - b^3)cde^2f + (2ab^2d^2e^2 - 4ab^2cde^2f + I \\
& (a^2b - b^3)d^2e^2 - 2I(a^2b - b^3)cde^2f + 2(ab^2c^2 - a^3 - a \\
& b^2)f^2 + I(2a^2b + 2b^3 + (a^2b - b^3)c^2)f^2)cosh(dx + c)^4 + ( \\
& 8ab^2d^2e^2 - 16ab^2cde^2f + 4I(a^2b - b^3)d^2e^2 - 8I(a^2b \\
& - b^3)cde^2f + 8(ab^2c^2 - a^3 - ab^2)f^2 + 4I(2a^2b + 2b^3 + \\
& (a^2b - b^3)c^2)f^2)cosh(dx + c)sinh(dx + c)^3 + (2ab^2d^2e^2 - \\
& 4ab^2cde^2f + I(a^2b - b^3)d^2e^2 - 2I(a^2b - b^3)cde^2f + 2( \\
& ab^2c^2 - a^3 - ab^2)f^2 + I(2a^2b + 2b^3 + (a^2b - b^3)c^2)f^2) \\
& )sinh(dx + c)^4 + 2(ab^2c^2 - a^3 - ab^2)f^2 + I(2a^2b + 2b^3 + ( \\
& a^2b - b^3)c^2)f^2 + (4ab^2d^2e^2 - 8ab^2cde^2f + 2I(a^2b - b \\
& ^3)d^2e^2 - 4I(a^2b - b^3)cde^2f + 4(ab^2c^2 - a^3 - ab^2)f^2 + \\
& 2I(2a^2b + 2b^3 + (a^2b - b^3)c^2)f^2)cosh(dx + c)^2 + (4ab^2d^ \\
& 2e^2 - 8ab^2cde^2f + 2I(a^2b - b^3)d^2e^2 - 4I(a^2b - b^3)c \\
& )de^2f + 4(ab^2c^2 - a^3 - ab^2)f^2 + 2I(2a^2b + 2b^3 + (a^2b - \\
& b^3)c^2)f^2 + (12ab^2d^2e^2 - 24ab^2cde^2f + 6I(a^2b - b^3)d^ \\
& 2e^2 - 12I(a^2b - b^3)cde^2f + 12(ab^2c^2 - a^3 - ab^2)f^2 + 6I \\
& ) \\
& (2a^2b + 2b^3 + (a^2b - b^3)c^2)f^2)cosh(dx + c)^2)sinh(dx + c)^ \\
& 2 + ((8ab^2d^2e^2 - 16ab^2cde^2f + 4I(a^2b - b^3)d^2e^2 - 8I \\
& (a^2b - b^3)cde^2f + 8(ab^2c^2 - a^3 - ab^2)f^2 + 4I(2a^2b + 2 \\
& b^3 + (a^2b - b^3)c^2)f^2)cosh(dx + c)^3 + (8ab^2d^2e^2 - 16ab^2 \\
& )cde^2f + 4I(a^2b - b^3)d^2e^2 - 8I(a^2b - b^3)cde^2f + 8(ab^2 \\
& )c^2 - a^3 - ab^2)f^2 + 4I(2a^2b + 2b^3 + (a^2b - b^3)c^2)f^2)co \\
& sh(dx + c))sinh(dx + c))log(cosh(dx + c) + sinh(dx + c) - I) + (2ab \\
& ^2d^2f^2x^2 + 4ab^2d^2e^2fx + 4ab^2cde^2f - 2ab^2c^2f^2 + I \\
& (a^2b - b^3)d^2f^2x^2 + 2I(a^2b - b^3)d^2e^2fx + 2I(a^2b - b^3) \\
& )cde^2f - I(a^2b - b^3)c^2f^2 + (2ab^2d^2f^2x^2 + 4ab^2d^2e^2 \\
& )fx + 4ab^2cde^2f - 2ab^2c^2f^2 + I(a^2b - b^3)d^2f^2x^2 + 2I \\
& (a^2b - b^3)d^2e^2fx + 2I(a^2b - b^3)cde^2f - I(a^2b - b^3)c^2f \\
& ^2)cosh(dx + c)^4 + (8ab^2d^2f^2x^2 + 16ab^2d^2e^2fx + 16ab^2 \\
& )cde^2f - 8ab^2c^2f^2 + 4I(a^2b - b^3)d^2f^2x^2 + 8I(a^2b - b^ \\
& ^3)d^2e^2fx + 8I(a^2b - b^3)cde^2f - 4I(a^2b - b^3)c^2f^2)cosh( \\
& dx + c)sinh(dx + c)^3 + (2ab^2d^2f^2x^2 + 4ab^2d^2e^2fx + 4ab \\
& ^2)cde^2f - 2ab^2c^2f^2 + I(a^2b - b^3)d^2f^2x^2 + 2I(a^2b - b \\
& ^3)d^2e^2fx + 2I(a^2b - b^3)cde^2f - I(a^2b - b^3)c^2f^2)sinh(d \\
& x + c)^4 + (4ab^2d^2f^2x^2 + 8ab^2d^2e^2fx + 8ab^2cde^2f - 4
\end{aligned}$$

$$\begin{aligned}
& a^2 b^2 c^2 f^2 + 2 I (a^2 b - b^3) d^2 f^2 x^2 + 4 I (a^2 b - b^3) d^2 e f x \\
& + 4 I (a^2 b - b^3) c d e f - 2 I (a^2 b - b^3) c^2 f^2) \cosh(dx + c)^2 + \\
& (4 a^2 b^2 d^2 f^2 x^2 + 8 a^2 b^2 d^2 e f x + 8 a^2 b^2 c d e f - 4 a^2 b^2 c^2 f^2 \\
& ^2 + 2 I (a^2 b - b^3) d^2 f^2 x^2 + 4 I (a^2 b - b^3) d^2 e f x + 4 I (a^2 \\
& * b - b^3) c d e f - 2 I (a^2 b - b^3) c^2 f^2 + (12 a^2 b^2 d^2 f^2 x^2 + 24 a^2 \\
& a^2 b^2 d^2 e f x + 24 a^2 b^2 c d e f - 12 a^2 b^2 c^2 f^2 + 6 I (a^2 b - b^3) d^2 \\
& ^2 f^2 x^2 + 12 I (a^2 b - b^3) d^2 e f x + 12 I (a^2 b - b^3) c d e f - 6 I \\
& I (a^2 b - b^3) c^2 f^2) \cosh(dx + c)^2 \sinh(dx + c)^2 + ((8 a^2 b^2 d^2 f^2 \\
& ^2 x^2 + 16 a^2 b^2 d^2 e f x + 16 a^2 b^2 c d e f - 8 a^2 b^2 c^2 f^2 + 4 I (a^2 \\
& * b - b^3) d^2 f^2 x^2 + 8 I (a^2 b - b^3) d^2 e f x + 8 I (a^2 b - b^3) c d \\
& * e f - 4 I (a^2 b - b^3) c^2 f^2) \cosh(dx + c)^3 + (8 a^2 b^2 d^2 f^2 x^2 + \\
& 16 a^2 b^2 d^2 e f x + 16 a^2 b^2 c d e f - 8 a^2 b^2 c^2 f^2 + 4 I (a^2 b - b^3) \\
& * d^2 f^2 x^2 + 8 I (a^2 b - b^3) d^2 e f x + 8 I (a^2 b - b^3) c d e f - 4 I \\
& I (a^2 b - b^3) c^2 f^2) \cosh(dx + c) \sinh(dx + c) \log(I \cosh(dx + c) \\
& + I \sinh(dx + c) + 1) + (2 a^2 b^2 d^2 f^2 x^2 + 4 a^2 b^2 d^2 e f x + 4 a^2 b^2 \\
& * c d e f - 2 a^2 b^2 c^2 f^2 - I (a^2 b - b^3) d^2 f^2 x^2 - 2 I (a^2 b - b^3) \\
& ) d^2 e f x - 2 I (a^2 b - b^3) c d e f + I (a^2 b - b^3) c^2 f^2 + (2 a^2 b^2 \\
& 2 d^2 f^2 x^2 + 4 a^2 b^2 d^2 e f x + 4 a^2 b^2 c d e f - 2 a^2 b^2 c^2 f^2 - I ( \\
& a^2 b - b^3) d^2 f^2 x^2 - 2 I (a^2 b - b^3) d^2 e f x - 2 I (a^2 b - b^3) * \\
& c d e f + I (a^2 b - b^3) c^2 f^2) \cosh(dx + c)^4 + (8 a^2 b^2 d^2 f^2 x^2 + \\
& 16 a^2 b^2 d^2 e f x + 16 a^2 b^2 c d e f - 8 a^2 b^2 c^2 f^2 - 4 I (a^2 b - b^3) \\
& ) d^2 f^2 x^2 - 8 I (a^2 b - b^3) d^2 e f x - 8 I (a^2 b - b^3) c d e f + 4 \\
& * I (a^2 b - b^3) c^2 f^2) \cosh(dx + c) \sinh(dx + c)^3 + (2 a^2 b^2 d^2 f^2 * \\
& x^2 + 4 a^2 b^2 d^2 e f x + 4 a^2 b^2 c d e f - 2 a^2 b^2 c^2 f^2 - I (a^2 b - b^ \\
& 3) d^2 f^2 x^2 - 2 I (a^2 b - b^3) d^2 e f x - 2 I (a^2 b - b^3) c d e f + \\
& I (a^2 b - b^3) c^2 f^2) \sinh(dx + c)^4 + (4 a^2 b^2 d^2 f^2 x^2 + 8 a^2 b^2 d^ \\
& ^2 e f x + 8 a^2 b^2 c d e f - 4 a^2 b^2 c^2 f^2 - 2 I (a^2 b - b^3) d^2 f^2 x^ \\
& 2 - 4 I (a^2 b - b^3) d^2 e f x - 4 I (a^2 b - b^3) c d e f + 2 I (a^2 b - \\
& b^3) c^2 f^2) \cosh(dx + c)^2 + (4 a^2 b^2 d^2 f^2 x^2 + 8 a^2 b^2 d^2 e f x + \\
& 8 a^2 b^2 c d e f - 4 a^2 b^2 c^2 f^2 - 2 I (a^2 b - b^3) d^2 f^2 x^2 - 4 I (a^ \\
& 2 * b - b^3) d^2 e f x - 4 I (a^2 b - b^3) c d e f + 2 I (a^2 b - b^3) c^2 f^ \\
& 2 + (12 a^2 b^2 d^2 f^2 x^2 + 24 a^2 b^2 d^2 e f x + 24 a^2 b^2 c d e f - 12 a^2 b^ \\
& 2 * c^2 f^2 - 6 I (a^2 b - b^3) d^2 f^2 x^2 - 12 I (a^2 b - b^3) d^2 e f x - \\
& 12 I (a^2 b - b^3) c d e f + 6 I (a^2 b - b^3) c^2 f^2) \cosh(dx + c)^2 \sinh \\
& (dx + c)^2 + ((8 a^2 b^2 d^2 f^2 x^2 + 16 a^2 b^2 d^2 e f x + 16 a^2 b^2 c d e \\
& * f - 8 a^2 b^2 c^2 f^2 - 4 I (a^2 b - b^3) d^2 f^2 x^2 - 8 I (a^2 b - b^3) d^ \\
& 2 * e f x - 8 I (a^2 b - b^3) c d e f + 4 I (a^2 b - b^3) c^2 f^2) \cosh(dx + \\
& c)^3 + (8 a^2 b^2 d^2 f^2 x^2 + 16 a^2 b^2 d^2 e f x + 16 a^2 b^2 c d e f - 8 a^ \\
& b^2 * c^2 f^2 - 4 I (a^2 b - b^3) d^2 f^2 x^2 - 8 I (a^2 b - b^3) d^2 e f x - \\
& 8 I (a^2 b - b^3) c d e f + 4 I (a^2 b - b^3) c^2 f^2) \cosh(dx + c) \sinh \\
& (dx + c) \log(-I \cosh(dx + c) - I \sinh(dx + c) + 1) + 4 (a^2 b^2 f^2 \cosh \\
& (dx + c)^4 + 4 a^2 b^2 f^2 \cosh(dx + c) \sinh(dx + c)^3 + a^2 b^2 f^2 \sinh(dx \\
& + c)^4 + 2 a^2 b^2 f^2 \cosh(dx + c)^2 + a^2 b^2 f^2 + 2 (3 a^2 b^2 f^2 \cosh(dx \\
& + c)^2 + a^2 b^2 f^2) \sinh(dx + c)^2 + 4 (a^2 b^2 f^2 \cosh(dx + c)^3 + a^2 b^2 \\
& * f^2 \cosh(dx + c) \sinh(dx + c)) \sinh(dx + c) \operatorname{polylog}(3, (a \cosh(dx + c) + a \sinh(dx
\end{aligned}$$

$$\begin{aligned}
& + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2})/b + 4*( \\
& a*b^2*f^2*\cosh(dx + c)^4 + 4*a*b^2*f^2*\cosh(dx + c)*\sinh(dx + c)^3 + a*b \\
& ^2*f^2*\sinh(dx + c)^4 + 2*a*b^2*f^2*\cosh(dx + c)^2 + a*b^2*f^2 + 2*(3*a*b \\
& ^2*f^2*\cosh(dx + c)^2 + a*b^2*f^2)*\sinh(dx + c)^2 + 4*(a*b^2*f^2*\cosh(dx \\
& + c)^3 + a*b^2*f^2*\cosh(dx + c))*\sinh(dx + c))*\text{polylog}(3, (a*\cosh(dx + \\
& c) + a*\sinh(dx + c) - (b*\cosh(dx + c) + b*\sinh(dx + c))*\sqrt{(a^2 + b^2) \\
& /b^2}))/b - (4*a*b^2*f^2 + (4*a*b^2*f^2 - 2*I*(a^2*b - b^3)*f^2)*\cosh(dx + \\
& c)^4 + (16*a*b^2*f^2 - 8*I*(a^2*b - b^3)*f^2)*\cosh(dx + c)*\sinh(dx + c)^ \\
& 3 + (4*a*b^2*f^2 - 2*I*(a^2*b - b^3)*f^2)*\sinh(dx + c)^4 - 2*I*(a^2*b - b^ \\
& 3)*f^2 + (8*a*b^2*f^2 - 4*I*(a^2*b - b^3)*f^2)*\cosh(dx + c)^2 + (8*a*b^2*f \\
& ^2 - 4*I*(a^2*b - b^3)*f^2 + (24*a*b^2*f^2 - 12*I*(a^2*b - b^3)*f^2)*\cosh(d \\
& *x + c)^2)*\sinh(dx + c)^2 + ((16*a*b^2*f^2 - 8*I*(a^2*b - b^3)*f^2)*\cosh(d \\
& *x + c)^3 + (16*a*b^2*f^2 - 8*I*(a^2*b - b^3)*f^2)*\cosh(dx + c))*\sinh(dx \\
& + c))*\text{polylog}(3, I*\cosh(dx + c) + I*\sinh(dx + c)) - (4*a*b^2*f^2 + (4*a*b \\
& ^2*f^2 + 2*I*(a^2*b - b^3)*f^2)*\cosh(dx + c)^4 + (16*a*b^2*f^2 + 8*I*(a^2*b \\
& - b^3)*f^2)*\cosh(dx + c)*\sinh(dx + c)^3 + (4*a*b^2*f^2 + 2*I*(a^2*b - b \\
& ^3)*f^2)*\sinh(dx + c)^4 + 2*I*(a^2*b - b^3)*f^2 + (8*a*b^2*f^2 + 4*I*(a^2*b \\
& - b^3)*f^2)*\cosh(dx + c)^2 + (8*a*b^2*f^2 + 4*I*(a^2*b - b^3)*f^2 + (24* \\
& a*b^2*f^2 + 12*I*(a^2*b - b^3)*f^2)*\cosh(dx + c)^2)*\sinh(dx + c)^2 + ((16 \\
& *a*b^2*f^2 + 8*I*(a^2*b - b^3)*f^2)*\cosh(dx + c)^3 + (16*a*b^2*f^2 + 8*I*( \\
& a^2*b - b^3)*f^2)*\cosh(dx + c))*\sinh(dx + c))*\text{polylog}(3, -I*\cosh(dx + c) \\
& - I*\sinh(dx + c)) - 2*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 \\
& - 2*(a^2*b + b^3)*d*e*f - 8*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*c \\
& osh(dx + c)^3 - 3*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 + 2*( \\
& a^2*b + b^3)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (a^2*b + b^3)*d*f^2)*x)*cos \\
& h(dx + c)^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x + 4*((a^3 \\
& + a*b^2)*d^2*f^2*x^2 + (a^3 + a*b^2)*d^2*e^2 + (a^3 + a*b^2)*d*e*f - 2*(a^3 \\
& + a*b^2)*c*f^2 + (2*(a^3 + a*b^2)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x)*\cosh(d \\
& *x + c))*\sinh(dx + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3*\cosh(dx + c)^4 + 4*(a \\
& ^4 + 2*a^2*b^2 + b^4)*d^3*\cosh(dx + c)*\sinh(dx + c)^3 + (a^4 + 2*a^2*b^2 \\
& + b^4)*d^3*\sinh(dx + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*\cosh(dx + c)^2 \\
& + (a^4 + 2*a^2*b^2 + b^4)*d^3 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d^3*\cosh(dx + \\
& c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^3)*\sinh(dx + c)^2 + 4*((a^4 + 2*a^2*b^2 \\
& + b^4)*d^3*\cosh(dx + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d^3*\cosh(dx + c))*\sin \\
& h(dx + c))
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*sech(dx+c)\*\*2\*tanh(dx+c)/(a+b\*sinh(dx+c)),x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sech(d\*x+c)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.359 \quad \int \frac{(e+fx)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=711

$$\frac{ia^2 f \operatorname{PolyLog}(2, -ie^{c+dx})}{2bd^2(a^2 + b^2)} + \frac{ia^2 b f \operatorname{PolyLog}(2, -ie^{c+dx})}{d^2(a^2 + b^2)^2} - \frac{ia^2 f \operatorname{PolyLog}(2, ie^{c+dx})}{2bd^2(a^2 + b^2)} - \frac{ia^2 b f \operatorname{PolyLog}(2, ie^{c+dx})}{d^2(a^2 + b^2)^2} - \frac{ab^2 f}{d}$$

```
[Out] ((e + f*x)*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*b*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)^2*d) - (a^2*(e + f*x)*ArcTan[E^(c + d*x)])/(b*(a^2 + b^2)*d) - (a*b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (a*b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (a*b^2*(e + f*x)*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)^2*d) - ((I/2)*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + (I*a^2*b*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + ((I/2)*a^2*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) + ((I/2)*f*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - (I*a^2*b*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - ((I/2)*a^2*f*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (a*b^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (a*b^2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) + (a*b^2*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^2) + (f*Sech[c + d*x])/(2*b*d^2) - (a^2*f*Sech[c + d*x])/(2*b*(a^2 + b^2)*d^2) - (a*(e + f*x)*Sech[c + d*x]^2)/(2*(a^2 + b^2)*d) + (a*f*Tanh[c + d*x])/(2*(a^2 + b^2)*d^2) + ((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b*d) - (a^2*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b*(a^2 + b^2)*d)
```

**Rubi [A]** time = 0.993033, antiderivative size = 711, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 13, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$ , Rules used = {5583, 4185, 4180, 2279, 2391, 5573, 5561, 2190, 6742, 3718, 5451, 3767, 8}

$$\frac{ia^2 f \operatorname{PolyLog}(2, -ie^{c+dx})}{2bd^2(a^2 + b^2)} + \frac{ia^2 b f \operatorname{PolyLog}(2, -ie^{c+dx})}{d^2(a^2 + b^2)^2} - \frac{ia^2 f \operatorname{PolyLog}(2, ie^{c+dx})}{2bd^2(a^2 + b^2)} - \frac{ia^2 b f \operatorname{PolyLog}(2, ie^{c+dx})}{d^2(a^2 + b^2)^2} - \frac{ab^2 f}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] ((e + f*x)*ArcTan[E^(c + d*x)])/(b*d) - (2*a^2*b*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)^2*d) - (a^2*(e + f*x)*ArcTan[E^(c + d*x)])/(b*(a^2 + b^2)*d) - (a*b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (a*b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (a*b^2*(e + f*x)*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)^2*d) - ((I/2)*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + (I*a^2*b*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + ((I/2)*a^2*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) + ((I/2)*f*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - (I*a^2*b*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - ((I/2)*a^2*f*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)*d^2) - (a*b^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (a*b^2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) + (a*b^2*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^2) + (f*Sech[c + d*x])/(2*b*d^2) - (a^2*f*Sech[c + d*x])/(2*b*(a^2 + b^2)*d^2) - (a*(e + f*x)*Sech[c + d*x]^2)/(2*(a^2 + b^2)*d) + (a*f*Tanh[c + d*x])/(2*(a^2 + b^2)*d^2) + ((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b*d) - (a^2*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b*(a^2 + b^2)*d)
```

```

*d) - (a*b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]/((a^2 + b^2)^2*d) - (a*b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]/((a^2 + b^2)^2*d) + (a*b^2*(e + f*x)*Log[1 + E^(2*(c + d*x))]/((a^2 + b^2)^2*d) - ((I/2)*f*PolyLog[2, (-I)*E^(c + d*x)]/(b*d^2) + (I*a^2*b*f*PolyLog[2, (-I)*E^(c + d*x)]/((a^2 + b^2)^2*d^2) + ((I/2)*a^2*f*PolyLog[2, (-I)*E^(c + d*x)]/(b*(a^2 + b^2)*d^2) + ((I/2)*f*PolyLog[2, I*E^(c + d*x)]/(b*d^2) - (I*a^2*b*f*PolyLog[2, I*E^(c + d*x)]/((a^2 + b^2)^2*d^2) - ((I/2)*a^2*f*PolyLog[2, I*E^(c + d*x)]/(b*(a^2 + b^2)*d^2) - (a*b^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)^2*d^2) - (a*b^2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 + b^2)^2*d^2) + (a*b^2*f*PolyLog[2, -E^(2*(c + d*x))]/(2*(a^2 + b^2)^2*d^2) + (f*Sech[c + d*x])/(2*b*d^2) - (a^2*f*Sech[c + d*x])/(2*b*(a^2 + b^2)*d^2) - (a*(e + f*x)*Sech[c + d*x]^2)/(2*(a^2 + b^2)*d) + (a*f*Tanh[c + d*x])/(2*(a^2 + b^2)*d^2) + ((e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b*d) - (a^2*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b*(a^2 + b^2)*d)

```

### Rule 5583

```

Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

```

### Rule 4185

```

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_)), x_Symbol] := -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]

```

### Rule 4180

```

Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)]]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]

```

### Rule 2279

```

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]

```



```
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 5573

```
Int[(((e_.) + (f_.)*(x_)^(m_.))*Sech[(c_.) + (d_.)*(x_)^(n_.)]/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)])*((e_.) + (f_.)*(x_)^(m_.)))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_)^(m_.)))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 3718

```
Int[(((c_.) + (d_.)*(x_)^(m_.))*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] := -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e + fx)\operatorname{sech}^2(c + dx)\tanh(c + dx)}{a + b\sinh(c + dx)} dx &= \frac{\int (e + fx)\operatorname{sech}^3(c + dx) dx}{b} - \frac{a \int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx}{b} \\
&= \frac{f\operatorname{sech}(c + dx)}{2bd^2} + \frac{(e + fx)\operatorname{sech}(c + dx)\tanh(c + dx)}{2bd} + \frac{\int (e + fx)\operatorname{sech}(c + dx) dx}{2b} \\
&= \frac{(e + fx)\tan^{-1}(e^{c+dx})}{bd} + \frac{f\operatorname{sech}(c + dx)}{2bd^2} + \frac{(e + fx)\operatorname{sech}(c + dx)\tanh(c + dx)}{2bd} \\
&= \frac{ab^2(e + fx)^2}{2(a^2 + b^2)^2 f} + \frac{(e + fx)\tan^{-1}(e^{c+dx})}{bd} + \frac{f\operatorname{sech}(c + dx)}{2bd^2} + \frac{(e + fx)\operatorname{sech}(c + dx)\tanh(c + dx)}{2bd} \\
&= \frac{ab^2(e + fx)^2}{2(a^2 + b^2)^2 f} + \frac{(e + fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{ab^2(e + fx)\log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{(a^2 + b^2)^2 d} \\
&= \frac{(e + fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2b(e + fx)\tan^{-1}(e^{c+dx})}{(a^2 + b^2)^2 d} - \frac{a^2(e + fx)\tan^{-1}(e^{c+dx})}{b(a^2 + b^2)d} \\
&= \frac{(e + fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2b(e + fx)\tan^{-1}(e^{c+dx})}{(a^2 + b^2)^2 d} - \frac{a^2(e + fx)\tan^{-1}(e^{c+dx})}{b(a^2 + b^2)d} \\
&= \frac{(e + fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2b(e + fx)\tan^{-1}(e^{c+dx})}{(a^2 + b^2)^2 d} - \frac{a^2(e + fx)\tan^{-1}(e^{c+dx})}{b(a^2 + b^2)d} \\
&= \frac{(e + fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2b(e + fx)\tan^{-1}(e^{c+dx})}{(a^2 + b^2)^2 d} - \frac{a^2(e + fx)\tan^{-1}(e^{c+dx})}{b(a^2 + b^2)d}
\end{aligned}$$

**Mathematica [A]** time = 8.54467, size = 587, normalized size = 0.83

$$b \left( i f (a^2 - b^2) \operatorname{PolyLog} \left( 2, -i e^{c+dx} \right) - i f (a^2 - b^2) \operatorname{PolyLog} \left( 2, i e^{c+dx} \right) + a b f \operatorname{PolyLog} \left( 2, -e^{2(c+dx)} \right) - 2 a^2 d e \tan^{-1} \left( e^{c+dx} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sech[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
]
```

```
[Out] (-2*a*b^2*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]) + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]) + b*(-2*a*b*d*e*(c + d*x) + 2*a*b*c*f*(c + d*x) - a*b*f*(c + d*x)^2 - 2*a^2*d*e*ArcTan[E^(c + d*x)] + 2*b^2*d*e*ArcTan[E^(c + d*x)] + 2*a^2*c*f*ArcTan[E^(c + d*x)] - 2*b^2*c*f*ArcTan[E^(c + d*x)] - I*a^2*f*(c + d*x)*Log[1 - I*E^(c + d*x)] + I*b^2*f*(c + d*x)*Log[1 - I*E^(c + d*x)] + I*a^2*f*(c + d*x)*Log[1 + I*E^(c + d*x)] - I*b^2*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + 2*a*b*d*e*Log[1 + E^(2*(c + d*x))] - 2*a*b*c*f*Log[1 + E^(2*(c + d*x))] + 2*a*b*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] + I*(a^2 - b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] - I*(a^2 - b^2)*f*PolyLog[2, I*E^(c + d*x)] + a*b*f*PolyLog[2, -E^(2*(c + d*x))] + (a^2 + b^2)*f*Sech[c + d*x]*(b + a*Sinh[c + d*x]) + (a^2 + b^2)*d*(e + f*x)*Sech[c + d*x]^2*(-a + b*Sinh[c + d*x]))/(2*(a^2 + b^2)^2*d^2)
```

**Maple [B]** time = 0.19, size = 2074, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d^2/(a^2+b^2)^(3/2)*b^2*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^2-1/d/(a^2+b^2)^(3/2)*b^4*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/d/(a^2+b^2)^(1/2)*b^2*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+I*b/(a^2+b^2)/d^2*a^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c+I*b/(a^2+b^2)/d*a^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x-I*b/(a^2+b^2)/d*a^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x-I*b/(a^2+b^2)/d^2*a^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c+I*b/(a^2+b^2)/d^2*a^2*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))+2*b^2/(a^2+b^2)/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*x+2*b^2/(a^2+b^2)/d^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*c-2*b^2/(a^2+b^2)/d*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2))-a)/(-a+(a^2+b^2)^(1/2)))*a*x-2*b^2/(a^2+b^2)/d^2*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2))-a)/(-a+(a^2+b^2)^(1/2)))*a*c-2*b^2/(a^2+b^2)/d*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a*x-I*b^3/(a^2+b^2)/d*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x-I*b^3/(a^2+b^2)/d^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c-I*b/(a^2+b^2)/d^2*a^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))+2*b^2/(a^2+b^2)/d*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*a*x+2*b^2/(a^2+b^2)/d^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*a*c+I*b^3/(a^2+b^2)/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x+I*b^3/(a^2+b^2)/d^2*f/(2*a^2+2
```

$$\begin{aligned}
 & *b^2 * \ln(1 - I * \exp(dx+c)) * c + 2*b / (a^2+b^2) / d^2 * a^2 * f * c / (2*a^2+2*b^2) * \arctan(\exp(dx+c)) \\
 & - 2*b^2 / (a^2+b^2) / d^2 * f * c / (2*a^2+2*b^2) * a * \ln(1 + \exp(2*d*x+2*c)) + 2*b^2 / (a^2+b^2) / d^2 * f * c / (2*a^2+2*b^2) * a * \ln(b * \exp(2*d*x+2*c) + 2*a * \exp(dx+c) - b) \\
 & - 2*b^2 / (a^2+b^2) / d^2 * f / (2*a^2+2*b^2) * \ln((b * \exp(dx+c) + (a^2+b^2)^{(1/2)} + a) / (a + (a^2+b^2)^{(1/2)})) \\
 & * a * c - 2*b^2 / (a^2+b^2) / d * e / (2*a^2+2*b^2) * a * \ln(b * \exp(2*d*x+2*c) + 2*a * \exp(dx+c) - b) \\
 & + 2*b^2 / (a^2+b^2) / d * e / (2*a^2+2*b^2) * a * \ln(1 + \exp(2*d*x+2*c)) - 2*b^3 / (a^2+b^2) / d^2 * f * c / (2*a^2+2*b^2) * \arctan(\exp(dx+c)) + 2*b^2 / (a^2+b^2) / d^2 * f / (2*a^2+2*b^2) * \operatorname{dilog}(1 + I * \exp(dx+c)) \\
 & * a + 2*b^2 / (a^2+b^2) / d^2 * f / (2*a^2+2*b^2) * \operatorname{dilog}(1 - I * \exp(dx+c)) * a - 2*b^2 / (a^2+b^2) / d^2 * f / (2*a^2+2*b^2) * \operatorname{dilog}((-b * \exp(dx+c) + (a^2+b^2)^{(1/2)} - a) / (-a + (a^2+b^2)^{(1/2)})) \\
 & * a + I * b^3 / (a^2+b^2) / d^2 * f / (2*a^2+2*b^2) * \operatorname{dilog}(1 - I * \exp(dx+c)) - 2*b^2 / (a^2+b^2) / d^2 * f / (2*a^2+2*b^2) * \operatorname{dilog}((b * \exp(dx+c) + (a^2+b^2)^{(1/2)} + a) / (a + (a^2+b^2)^{(1/2)})) \\
 & * a - 2*b / (a^2+b^2) / d * a^2 * e / (2*a^2+2*b^2) * \arctan(\exp(dx+c)) - I * b^3 / (a^2+b^2) / d^2 * f / (2*a^2+2*b^2) * \operatorname{dilog}(1 + I * \exp(dx+c)) \\
 & + 1/d^2 / (a^2+b^2)^{(3/2)} * b^4 * f * c / (2*a^2+2*b^2) * \operatorname{arctanh}(1/2 * (2*b * \exp(dx+c) + 2*a) / (a^2+b^2)^{(1/2)}) - 1/d / (a^2+b^2)^{(3/2)} * b^2 * e / (2*a^2+2*b^2) * \operatorname{arctanh}(1/2 * (2*b * \exp(dx+c) + 2*a) / (a^2+b^2)^{(1/2)}) \\
 & * a^2 - 1/d^2 / (a^2+b^2)^{(1/2)} * b^2 * f * c / (2*a^2+2*b^2) * \operatorname{arctanh}(1/2 * (2*b * \exp(dx+c) + 2*a) / (a^2+b^2)^{(1/2)}) - (b * d * f * x * \exp(3*d*x+3*c) + 2*a * d * f * x * \exp(2*d*x+2*c) - b * d * e * \exp(3*d*x+3*c) + 2*a * d * e * \exp(2*d*x+2*c) + b * d * f * x * \exp(dx+c) - b * f * \exp(3*d*x+3*c) + a * f * \exp(2*d*x+2*c) + b * d * e * \exp(dx+c) - f * b * \exp(dx+c) + a * f) / d^2 / (a^2+b^2) / (1 + \exp(2*d*x+2*c))^2 + 2*b^3 / (a^2+b^2) / d * e / (2*a^2+2*b^2) * \arctan(\exp(dx+c))
 \end{aligned}$$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\left( \frac{ab^2 \log(-2ae^{-dx-c}) + be^{-2dx-2c} - b}{(a^4 + 2a^2b^2 + b^4)d} - \frac{ab^2 \log(e^{-2dx-2c} + 1)}{(a^4 + 2a^2b^2 + b^4)d} - \frac{(a^2b - b^3) \arctan(e^{-dx-c})}{(a^4 + 2a^2b^2 + b^4)d} - \frac{be^{-dx-c}}{(a^2 + b^2 + 2(a^2 + b^2))} \right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(d\*x+c)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned}
 & -(a*b^2 * \log(-2*a*e^{-d*x - c}) + b*e^{-2*d*x - 2*c} - b) / ((a^4 + 2*a^2*b^2 + b^4) * d) - a*b^2 * \log(e^{-2*d*x - 2*c} + 1) / ((a^4 + 2*a^2*b^2 + b^4) * d) - (a^2*b - b^3) * \arctan(e^{-d*x - c}) / ((a^4 + 2*a^2*b^2 + b^4) * d) - (b*e^{-d*x - c} - 2*a*e^{-2*d*x - 2*c} - b*e^{-3*d*x - 3*c}) / ((a^2 + b^2 + 2*(a^2 + b^2)) * e^{-2*d*x - 2*c} + (a^2 + b^2) * e^{-4*d*x - 4*c}) * d) * e + f * ((b*d*x * e^{3*c} + b * e^{3*c}) * e^{3*d*x} - (2*a*d*x * e^{2*c} + a * e^{2*c}) * e^{2*d*x} - (b*d*x * e^c - b * e^c) * e^{d*x} - a) / (a^2*d^2 + b^2*d^2 + (a^2*d^2 * e^{4*c} + b^2*d^2 * e^{4*c}) * e^{4*d*x} + 2*(a^2*d^2 * e^{2*c} + b^2*d^2 * e^{2*c}) * e^{2*d*x}) + 4 * \operatorname{integrate}(-1/2 * (a^2*b^2 * x * e^{d*x + c} - a*b^3 * x) / (a^4*b + 2*a^2*b^3 + b^5 - (a^4*b * e^{2*c} + 2*a^2*b^3 * e^{2*c} + b^5 * e^{2*c})) * e^{2*d*x} - 2*(a^5 * e^c +
 \end{aligned}$$

```
2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) - 4*integrate(1/4*(2*a*b^2*x + (a^
2*b*e^c - b^3*e^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c) + 2*a^2
*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x))
```

**Fricas [B]** time = 3.69688, size = 11628, normalized size = 16.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/2*(2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^2*b + b^3)*f)*cosh(d*x
+ c)^3 + 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^2*b + b^3)*f)*sin
h(d*x + c)^3 - 2*(2*(a^3 + a*b^2)*d*f*x + 2*(a^3 + a*b^2)*d*e + (a^3 + a*b^
2)*f)*cosh(d*x + c)^2 - 2*(2*(a^3 + a*b^2)*d*f*x + 2*(a^3 + a*b^2)*d*e + (a
^3 + a*b^2)*f - 3*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^2*b + b^3)*
f)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(a^3 + a*b^2)*f - 2*((a^2*b + b^3)*d*
f*x + (a^2*b + b^3)*d*e - (a^2*b + b^3)*f)*cosh(d*x + c) - 2*(a*b^2*f*cosh(
d*x + c)^4 + 4*a*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3 + a*b^2*f*sinh(d*x + c
)^4 + 2*a*b^2*f*cosh(d*x + c)^2 + a*b^2*f + 2*(3*a*b^2*f*cosh(d*x + c)^2 +
a*b^2*f)*sinh(d*x + c)^2 + 4*(a*b^2*f*cosh(d*x + c)^3 + a*b^2*f*cosh(d*x +
c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(a*b^2*f*cosh(
d*x + c)^4 + 4*a*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3 + a*b^2*f*sinh(d*x + c
)^4 + 2*a*b^2*f*cosh(d*x + c)^2 + a*b^2*f + 2*(3*a*b^2*f*cosh(d*x + c)^2 +
a*b^2*f)*sinh(d*x + c)^2 + 4*(a*b^2*f*cosh(d*x + c)^3 + a*b^2*f*cosh(d*x +
c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + ((2*a*b^2*f - I*
(a^2*b - b^3)*f)*cosh(d*x + c)^4 + (8*a*b^2*f - 4*I*(a^2*b - b^3)*f)*cosh(d
*x + c)*sinh(d*x + c)^3 + (2*a*b^2*f - I*(a^2*b - b^3)*f)*sinh(d*x + c)^4 +
2*a*b^2*f + (4*a*b^2*f - 2*I*(a^2*b - b^3)*f)*cosh(d*x + c)^2 + (4*a*b^2*f
+ (12*a*b^2*f - 6*I*(a^2*b - b^3)*f)*cosh(d*x + c)^2 - 2*I*(a^2*b - b^3)*f
)*sinh(d*x + c)^2 - I*(a^2*b - b^3)*f + ((8*a*b^2*f - 4*I*(a^2*b - b^3)*f)*
cosh(d*x + c)^3 + (8*a*b^2*f - 4*I*(a^2*b - b^3)*f)*cosh(d*x + c))*sinh(d*x
+ c))*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + ((2*a*b^2*f + I*(a^2*b -
b^3)*f)*cosh(d*x + c)^4 + (8*a*b^2*f + 4*I*(a^2*b - b^3)*f)*cosh(d*x + c)*s
inh(d*x + c)^3 + (2*a*b^2*f + I*(a^2*b - b^3)*f)*sinh(d*x + c)^4 + 2*a*b^2*
f + (4*a*b^2*f + 2*I*(a^2*b - b^3)*f)*cosh(d*x + c)^2 + (4*a*b^2*f + (12*a*
b^2*f + 6*I*(a^2*b - b^3)*f)*cosh(d*x + c)^2 + 2*I*(a^2*b - b^3)*f)*sinh(d*
x + c)^2 + I*(a^2*b - b^3)*f + ((8*a*b^2*f + 4*I*(a^2*b - b^3)*f)*cosh(d*x
```

$$\begin{aligned}
& + c)^3 + (8*a*b^2*f + 4*I*(a^2*b - b^3)*f)*\cosh(d*x + c))*\sinh(d*x + c))*di \\
& \log(-I*\cosh(d*x + c) - I*\sinh(d*x + c)) - 2*(a*b^2*d*e - a*b^2*c*f + (a*b^2 \\
& *d*e - a*b^2*c*f)*\cosh(d*x + c)^4 + 4*(a*b^2*d*e - a*b^2*c*f)*\cosh(d*x + c) \\
& *\sinh(d*x + c)^3 + (a*b^2*d*e - a*b^2*c*f)*\sinh(d*x + c)^4 + 2*(a*b^2*d*e - \\
& a*b^2*c*f)*\cosh(d*x + c)^2 + 2*(a*b^2*d*e - a*b^2*c*f + 3*(a*b^2*d*e - a*b \\
& ^2*c*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((a*b^2*d*e - a*b^2*c*f)*\cosh( \\
& d*x + c)^3 + (a*b^2*d*e - a*b^2*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2*b* \\
& \cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - 2*(a \\
& *b^2*d*e - a*b^2*c*f + (a*b^2*d*e - a*b^2*c*f)*\cosh(d*x + c)^4 + 4*(a*b^2*d \\
& *e - a*b^2*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a*b^2*d*e - a*b^2*c*f)*\sin \\
& h(d*x + c)^4 + 2*(a*b^2*d*e - a*b^2*c*f)*\cosh(d*x + c)^2 + 2*(a*b^2*d*e - a \\
& *b^2*c*f + 3*(a*b^2*d*e - a*b^2*c*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*( \\
& (a*b^2*d*e - a*b^2*c*f)*\cosh(d*x + c)^3 + (a*b^2*d*e - a*b^2*c*f)*\cosh(d*x \\
& + c))*\sinh(d*x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{( \\
& a^2 + b^2)/b^2} + 2*a) - 2*(a*b^2*d*f*x + a*b^2*c*f + (a*b^2*d*f*x + a*b^2* \\
& c*f)*\cosh(d*x + c)^4 + 4*(a*b^2*d*f*x + a*b^2*c*f)*\cosh(d*x + c)*\sinh(d*x + \\
& c)^3 + (a*b^2*d*f*x + a*b^2*c*f)*\sinh(d*x + c)^4 + 2*(a*b^2*d*f*x + a*b^2* \\
& c*f)*\cosh(d*x + c)^2 + 2*(a*b^2*d*f*x + a*b^2*c*f + 3*(a*b^2*d*f*x + a*b^2* \\
& c*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((a*b^2*d*f*x + a*b^2*c*f)*\cosh(d \\
& *x + c)^3 + (a*b^2*d*f*x + a*b^2*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(-(a \\
& *\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{ \\
& ((a^2 + b^2)/b^2) - b)/b) - 2*(a*b^2*d*f*x + a*b^2*c*f + (a*b^2*d*f*x + a*b \\
& ^2*c*f)*\cosh(d*x + c)^4 + 4*(a*b^2*d*f*x + a*b^2*c*f)*\cosh(d*x + c)*\sinh(d* \\
& x + c)^3 + (a*b^2*d*f*x + a*b^2*c*f)*\sinh(d*x + c)^4 + 2*(a*b^2*d*f*x + a*b \\
& ^2*c*f)*\cosh(d*x + c)^2 + 2*(a*b^2*d*f*x + a*b^2*c*f + 3*(a*b^2*d*f*x + a*b \\
& ^2*c*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((a*b^2*d*f*x + a*b^2*c*f)*\cos \\
& h(d*x + c)^3 + (a*b^2*d*f*x + a*b^2*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log( \\
& -(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{ \\
& ((a^2 + b^2)/b^2) - b)/b) + (2*a*b^2*d*e - 2*a*b^2*c*f + (2*a*b^2*d*e - \\
& 2*a*b^2*c*f - I*(a^2*b - b^3)*d*e + I*(a^2*b - b^3)*c*f)*\cosh(d*x + c)^4 + \\
& (8*a*b^2*d*e - 8*a*b^2*c*f - 4*I*(a^2*b - b^3)*d*e + 4*I*(a^2*b - b^3)*c*f) \\
& *\cosh(d*x + c)*\sinh(d*x + c)^3 + (2*a*b^2*d*e - 2*a*b^2*c*f - I*(a^2*b - b^ \\
& 3)*d*e + I*(a^2*b - b^3)*c*f)*\sinh(d*x + c)^4 - I*(a^2*b - b^3)*d*e + I*(a^ \\
& 2*b - b^3)*c*f + (4*a*b^2*d*e - 4*a*b^2*c*f - 2*I*(a^2*b - b^3)*d*e + 2*I*( \\
& a^2*b - b^3)*c*f)*\cosh(d*x + c)^2 + (4*a*b^2*d*e - 4*a*b^2*c*f - 2*I*(a^2*b \\
& - b^3)*d*e + 2*I*(a^2*b - b^3)*c*f + (12*a*b^2*d*e - 12*a*b^2*c*f - 6*I*(a \\
& ^2*b - b^3)*d*e + 6*I*(a^2*b - b^3)*c*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + \\
& ((8*a*b^2*d*e - 8*a*b^2*c*f - 4*I*(a^2*b - b^3)*d*e + 4*I*(a^2*b - b^3)*c* \\
& f)*\cosh(d*x + c)^3 + (8*a*b^2*d*e - 8*a*b^2*c*f - 4*I*(a^2*b - b^3)*d*e + 4 \\
& *I*(a^2*b - b^3)*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sin \\
& h(d*x + c) + I) + (2*a*b^2*d*e - 2*a*b^2*c*f + (2*a*b^2*d*e - 2*a*b^2*c*f + \\
& I*(a^2*b - b^3)*d*e - I*(a^2*b - b^3)*c*f)*\cosh(d*x + c)^4 + (8*a*b^2*d*e \\
& - 8*a*b^2*c*f + 4*I*(a^2*b - b^3)*d*e - 4*I*(a^2*b - b^3)*c*f)*\cosh(d*x + c \\
& )*\sinh(d*x + c)^3 + (2*a*b^2*d*e - 2*a*b^2*c*f + I*(a^2*b - b^3)*d*e - I*(a \\
& ^2*b - b^3)*c*f)*\sinh(d*x + c)^4 + I*(a^2*b - b^3)*d*e - I*(a^2*b - b^3)*c*
\end{aligned}$$

$$\begin{aligned}
& f + (4*a*b^2*d*e - 4*a*b^2*c*f + 2*I*(a^2*b - b^3)*d*e - 2*I*(a^2*b - b^3)* \\
& c*f)*\cosh(d*x + c)^2 + (4*a*b^2*d*e - 4*a*b^2*c*f + 2*I*(a^2*b - b^3)*d*e - \\
& 2*I*(a^2*b - b^3)*c*f + (12*a*b^2*d*e - 12*a*b^2*c*f + 6*I*(a^2*b - b^3)*d \\
& *e - 6*I*(a^2*b - b^3)*c*f)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + ((8*a*b^2*d* \\
& e - 8*a*b^2*c*f + 4*I*(a^2*b - b^3)*d*e - 4*I*(a^2*b - b^3)*c*f)*\cosh(d*x + \\
& c)^3 + (8*a*b^2*d*e - 8*a*b^2*c*f + 4*I*(a^2*b - b^3)*d*e - 4*I*(a^2*b - b \\
& ^3)*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - \\
& I) + (2*a*b^2*d*f*x + 2*a*b^2*c*f + (2*a*b^2*d*f*x + 2*a*b^2*c*f + I*(a^2*b \\
& - b^3)*d*f*x + I*(a^2*b - b^3)*c*f)*\cosh(d*x + c)^4 + (8*a*b^2*d*f*x + 8*a \\
& *b^2*c*f + 4*I*(a^2*b - b^3)*d*f*x + 4*I*(a^2*b - b^3)*c*f)*\cosh(d*x + c)*\s \\
& \sinh(d*x + c)^3 + (2*a*b^2*d*f*x + 2*a*b^2*c*f + I*(a^2*b - b^3)*d*f*x + I*( \\
& a^2*b - b^3)*c*f)*\sinh(d*x + c)^4 + I*(a^2*b - b^3)*d*f*x + I*(a^2*b - b^3) \\
& *c*f + (4*a*b^2*d*f*x + 4*a*b^2*c*f + 2*I*(a^2*b - b^3)*d*f*x + 2*I*(a^2*b \\
& - b^3)*c*f)*\cosh(d*x + c)^2 + (4*a*b^2*d*f*x + 4*a*b^2*c*f + 2*I*(a^2*b - b \\
& ^3)*d*f*x + 2*I*(a^2*b - b^3)*c*f + (12*a*b^2*d*f*x + 12*a*b^2*c*f + 6*I*(a \\
& ^2*b - b^3)*d*f*x + 6*I*(a^2*b - b^3)*c*f)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 \\
& + ((8*a*b^2*d*f*x + 8*a*b^2*c*f + 4*I*(a^2*b - b^3)*d*f*x + 4*I*(a^2*b - b \\
& ^3)*c*f)*\cosh(d*x + c)^3 + (8*a*b^2*d*f*x + 8*a*b^2*c*f + 4*I*(a^2*b - b^3) \\
& *d*f*x + 4*I*(a^2*b - b^3)*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(I*\cosh(d* \\
& x + c) + I*\sinh(d*x + c) + 1) + (2*a*b^2*d*f*x + 2*a*b^2*c*f + (2*a*b^2*d*f \\
& *x + 2*a*b^2*c*f - I*(a^2*b - b^3)*d*f*x - I*(a^2*b - b^3)*c*f)*\cosh(d*x + \\
& c)^4 + (8*a*b^2*d*f*x + 8*a*b^2*c*f - 4*I*(a^2*b - b^3)*d*f*x - 4*I*(a^2*b \\
& - b^3)*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (2*a*b^2*d*f*x + 2*a*b^2*c*f - \\
& I*(a^2*b - b^3)*d*f*x - I*(a^2*b - b^3)*c*f)*\sinh(d*x + c)^4 - I*(a^2*b - b \\
& ^3)*d*f*x - I*(a^2*b - b^3)*c*f + (4*a*b^2*d*f*x + 4*a*b^2*c*f - 2*I*(a^2*b \\
& - b^3)*d*f*x - 2*I*(a^2*b - b^3)*c*f)*\cosh(d*x + c)^2 + (4*a*b^2*d*f*x + 4 \\
& *a*b^2*c*f - 2*I*(a^2*b - b^3)*d*f*x - 2*I*(a^2*b - b^3)*c*f + (12*a*b^2*d* \\
& f*x + 12*a*b^2*c*f - 6*I*(a^2*b - b^3)*d*f*x - 6*I*(a^2*b - b^3)*c*f)*\cosh( \\
& d*x + c)^2*\sinh(d*x + c)^2 + ((8*a*b^2*d*f*x + 8*a*b^2*c*f - 4*I*(a^2*b - \\
& b^3)*d*f*x - 4*I*(a^2*b - b^3)*c*f)*\cosh(d*x + c)^3 + (8*a*b^2*d*f*x + 8*a* \\
& b^2*c*f - 4*I*(a^2*b - b^3)*d*f*x - 4*I*(a^2*b - b^3)*c*f)*\cosh(d*x + c))*\s \\
& \sinh(d*x + c))*\log(-I*\cosh(d*x + c) - I*\sinh(d*x + c) + 1) - 2*((a^2*b + b^3 \\
& )*d*f*x + (a^2*b + b^3)*d*e - 3*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + \\
& (a^2*b + b^3)*f)*\cosh(d*x + c)^2 - (a^2*b + b^3)*f + 2*(2*(a^3 + a*b^2)*d*f \\
& *x + 2*(a^3 + a*b^2)*d*e + (a^3 + a*b^2)*f)*\cosh(d*x + c))*\sinh(d*x + c))/ \\
& (a^4 + 2*a^2*b^2 + b^4)*d^2*\cosh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d^2 \\
& *\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d^2*\sinh(d*x + c)^ \\
& 4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*\cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4) \\
& *d^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d^2*\cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 \\
& + b^4)*d^2)*\sinh(d*x + c)^2 + 4*((a^4 + 2*a^2*b^2 + b^4)*d^2*\cosh(d*x + c)^ \\
& 3 + (a^4 + 2*a^2*b^2 + b^4)*d^2*\cosh(d*x + c))*\sinh(d*x + c))
\end{aligned}$$


---



**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \tanh(c + dx) \operatorname{sech}^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(d\*x+c)\*\*2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*tanh(c + d\*x)\*sech(c + d\*x)\*\*2/(a + b\*sinh(c + d\*x)), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \operatorname{sech}(dx + c)^2 \tanh(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(d\*x+c)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)\*sech(d\*x + c)^2\*tanh(d\*x + c)/(b\*sinh(d\*x + c) + a), x)

$$3.360 \quad \int \frac{\operatorname{sech}^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=122

$$-\frac{ab^2 \log(a+b \sinh(c+dx))}{d(a^2+b^2)^2} - \frac{b(a^2-b^2) \tan^{-1}(\sinh(c+dx))}{2d(a^2+b^2)^2} + \frac{ab^2 \log(\cosh(c+dx))}{d(a^2+b^2)^2} - \frac{\operatorname{sech}^2(c+dx)(a-b \sinh(c+dx))}{2d(a^2+b^2)}$$

[Out]  $-(b*(a^2 - b^2)*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(2*(a^2 + b^2)^2*d) + (a*b^2*\operatorname{Log}[\operatorname{Cos}h[c + d*x]])/((a^2 + b^2)^2*d) - (a*b^2*\operatorname{Log}[a + b*\operatorname{Sinh}[c + d*x]])/((a^2 + b^2)^2*d) - (\operatorname{Sech}[c + d*x]^2*(a - b*\operatorname{Sinh}[c + d*x]))/(2*(a^2 + b^2)*d)$

**Rubi [A]** time = 0.199132, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$ , Rules used = {2837, 12, 823, 801, 635, 203, 260}

$$-\frac{ab^2 \log(a+b \sinh(c+dx))}{d(a^2+b^2)^2} - \frac{b(a^2-b^2) \tan^{-1}(\sinh(c+dx))}{2d(a^2+b^2)^2} + \frac{ab^2 \log(\cosh(c+dx))}{d(a^2+b^2)^2} - \frac{\operatorname{sech}^2(c+dx)(a-b \sinh(c+dx))}{2d(a^2+b^2)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Sech}[c + d*x]^2*\operatorname{Tanh}[c + d*x])/(a + b*\operatorname{Sinh}[c + d*x]), x]$

[Out]  $-(b*(a^2 - b^2)*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(2*(a^2 + b^2)^2*d) + (a*b^2*\operatorname{Log}[\operatorname{Cos}h[c + d*x]])/((a^2 + b^2)^2*d) - (a*b^2*\operatorname{Log}[a + b*\operatorname{Sinh}[c + d*x]])/((a^2 + b^2)^2*d) - (\operatorname{Sech}[c + d*x]^2*(a - b*\operatorname{Sinh}[c + d*x]))/(2*(a^2 + b^2)*d)$

### Rule 2837

$\operatorname{Int}[\cos[(e_.) + (f_.)*(x_)]^{(p_)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^{(m_.)})*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]^{(n_.)}), x\_Symbol] \rightarrow \operatorname{Dist}[1/(b^p*f), \operatorname{Subst}[\operatorname{Int}[(a+x)^m*(c+(d*x)/b)^n*(b^2-x^2)^{(p-1)/2}], x], x, b*\sin[e+f*x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f, m, n\}, x \ \&\amp; \ \operatorname{IntegerQ}[(p-1)/2] \ \&\amp; \ \operatorname{NeQ}[a^2 - b^2, 0]$

### Rule 12

$\operatorname{Int}[(a_)*(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$   $\operatorname{FreeQ}[a, x] \ \&\amp; \ \operatorname{!Match}Q[u, (b_)*(v_)] /;$   $\operatorname{FreeQ}[b, x]$

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{b^3 \operatorname{Subst}\left(\int \frac{x}{b(a+x)(-b^2-x^2)^2} dx, x, b \sinh(c+dx)\right)}{d} \\
&= \frac{b^2 \operatorname{Subst}\left(\int \frac{x}{(a+x)(-b^2-x^2)^2} dx, x, b \sinh(c+dx)\right)}{d} \\
&= -\frac{\operatorname{sech}^2(c+dx)(a-b \sinh(c+dx))}{2(a^2+b^2)d} + \frac{\operatorname{Subst}\left(\int \frac{ab^2-b^2x}{(a+x)(-b^2-x^2)} dx, x, b \sinh(c+dx)\right)}{2(a^2+b^2)d} \\
&= -\frac{\operatorname{sech}^2(c+dx)(a-b \sinh(c+dx))}{2(a^2+b^2)d} + \frac{\operatorname{Subst}\left(\int \left(-\frac{2ab^2}{(a^2+b^2)(a+x)} + \frac{b^2(-a^2+b^2+2ax)}{(a^2+b^2)(b^2+x^2)}\right) dx, x, b \sinh(c+dx)\right)}{2(a^2+b^2)d} \\
&= -\frac{ab^2 \log(a+b \sinh(c+dx))}{(a^2+b^2)^2 d} - \frac{\operatorname{sech}^2(c+dx)(a-b \sinh(c+dx))}{2(a^2+b^2)d} + \frac{b^2 \operatorname{Subst}\left(\int \frac{-a^2}{b^2+x^2} dx, x, b \sinh(c+dx)\right)}{(a^2+b^2)d} \\
&= -\frac{ab^2 \log(a+b \sinh(c+dx))}{(a^2+b^2)^2 d} - \frac{\operatorname{sech}^2(c+dx)(a-b \sinh(c+dx))}{2(a^2+b^2)d} + \frac{(ab^2) \operatorname{Subst}\left(\int \frac{1}{b^2+x^2} dx, x, b \sinh(c+dx)\right)}{(a^2+b^2)d} \\
&= -\frac{b(a^2-b^2) \tan^{-1}(\sinh(c+dx))}{2(a^2+b^2)^2 d} + \frac{ab^2 \log(\cosh(c+dx))}{(a^2+b^2)^2 d} - \frac{ab^2 \log(a+b \sinh(c+dx))}{(a^2+b^2)^2 d}
\end{aligned}$$

**Mathematica [A]** time = 0.208201, size = 105, normalized size = 0.86

$$\frac{-a(a^2+b^2) \operatorname{sech}^2(c+dx) + b(a^2+b^2) \tanh(c+dx) \operatorname{sech}(c+dx) + 2b\left((b^2-a^2) \tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right) + ab(\log(\cosh(c+dx)))}{2d(a^2+b^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Sech[c + d\*x]^2\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (2\*b\*((-a^2 + b^2)\*ArcTan[Tanh[(c + d\*x)/2]] + a\*b\*(Log[Cosh[c + d\*x]] - Log[a + b\*Sinh[c + d\*x]])) - a\*(a^2 + b^2)\*Sech[c + d\*x]^2 + b\*(a^2 + b^2)\*Sech[c + d\*x]\*Tanh[c + d\*x])/(2\*(a^2 + b^2)^2\*d)

**Maple [B]** time = 0.003, size = 474, normalized size = 3.9

$$-2 \frac{ab^2 \ln\left(\left(\tanh\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2 a - 2 \tanh\left(\frac{1}{2}dx + \frac{c}{2}\right) b - a\right)}{d\left(2a^4 + 4a^2b^2 + 2b^4\right)} - \frac{a^2b}{d\left(a^4 + 2a^2b^2 + b^4\right)} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3 \left(\tanh\left(\frac{dx}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)`

[Out] 
$$\begin{aligned} & -2/d*a*b^2/(2*a^4+4*a^2*b^2+2*b^4)*\ln(\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)*b-a)-1/d/(a^4+2*a^2*b^2+b^4)/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)^3*a^2*b-1/d/(a^4+2*a^2*b^2+b^4)/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)^3*b^3+2/d/(a^4+2*a^2*b^2+b^4)/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)^2*a^3+2/d/(a^4+2*a^2*b^2+b^4)/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)^2*a*b^2+1/d/(a^4+2*a^2*b^2+b^4)/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)*a^2*b+1/d/(a^4+2*a^2*b^2+b^4)/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)*b^3+1/d/(a^4+2*a^2*b^2+b^4)*\ln(\tanh(1/2*d*x+1/2*c)^2+1)*a*b^2-1/d/(a^4+2*a^2*b^2+b^4)*\arctan(\tanh(1/2*d*x+1/2*c))*a^2*b+1/d/(a^4+2*a^2*b^2+b^4)*\arctan(\tanh(1/2*d*x+1/2*c))*b^3 \end{aligned}$$

**Maxima [A]** time = 1.58781, size = 294, normalized size = 2.41

$$-\frac{ab^2 \log\left(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b\right)}{\left(a^4 + 2a^2b^2 + b^4\right)d} + \frac{ab^2 \log\left(e^{(-2dx-2c)} + 1\right)}{\left(a^4 + 2a^2b^2 + b^4\right)d} + \frac{\left(a^2b - b^3\right) \arctan\left(e^{(-dx-c)}\right)}{\left(a^4 + 2a^2b^2 + b^4\right)d} + \frac{be^{(-dx-c)}}{\left(a^2 + b^2 + 2\left(a^2 + b^2\right)\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & -a*b^2*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/((a^4 + 2*a^2*b^2 + b^4)*d) + a*b^2*\log(e^{(-2*d*x - 2*c)} + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) + (a^2*b - b^3)*\arctan(e^{(-d*x - c)})/((a^4 + 2*a^2*b^2 + b^4)*d) + (b*e^{(-d*x - c)} - 2*a*e^{(-2*d*x - 2*c)} - b*e^{(-3*d*x - 3*c)})/((a^2 + b^2 + 2*(a^2 + b^2))*e^{(-2*d*x - 2*c)} + (a^2 + b^2)*e^{(-4*d*x - 4*c)})*d \end{aligned}$$

**Fricas [B]** time = 2.46903, size = 2240, normalized size = 18.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((a^2*b + b^3)*cosh(d*x + c)^3 + (a^2*b + b^3)*sinh(d*x + c)^3 - 2*(a^3 + a
*b^2)*cosh(d*x + c)^2 - (2*a^3 + 2*a*b^2 - 3*(a^2*b + b^3)*cosh(d*x + c))*s
inh(d*x + c)^2 - ((a^2*b - b^3)*cosh(d*x + c)^4 + 4*(a^2*b - b^3)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (a^2*b - b^3)*sinh(d*x + c)^4 + a^2*b - b^3 + 2*(a^2
*b - b^3)*cosh(d*x + c)^2 + 2*(a^2*b - b^3 + 3*(a^2*b - b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 4*((a^2*b - b^3)*cosh(d*x + c)^3 + (a^2*b - b^3)*cosh(
d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (a^2*b + b
^3)*cosh(d*x + c) - (a*b^2*cosh(d*x + c)^4 + 4*a*b^2*cosh(d*x + c)*sinh(d*x
+ c)^3 + a*b^2*sinh(d*x + c)^4 + 2*a*b^2*cosh(d*x + c)^2 + a*b^2 + 2*(3*a
*b^2*cosh(d*x + c)^2 + a*b^2)*sinh(d*x + c)^2 + 4*(a*b^2*cosh(d*x + c)^3 + a
*b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x +
c) - sinh(d*x + c))) + (a*b^2*cosh(d*x + c)^4 + 4*a*b^2*cosh(d*x + c)*sinh(
d*x + c)^3 + a*b^2*sinh(d*x + c)^4 + 2*a*b^2*cosh(d*x + c)^2 + a*b^2 + 2*(3
*a*b^2*cosh(d*x + c)^2 + a*b^2)*sinh(d*x + c)^2 + 4*(a*b^2*cosh(d*x + c)^3
+ a*b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) -
sinh(d*x + c))) - (a^2*b + b^3 - 3*(a^2*b + b^3)*cosh(d*x + c)^2 + 4*(a^3 +
a*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x +
c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 +
2*a^2*b^2 + b^4)*d*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x +
c)^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 +
b^4)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d + 4*((a^4 + 2*a^2*b^2
+ b^4)*d*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c))*sinh(d
*x + c))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(c + dx) \operatorname{sech}^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)*sech(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

---

**Giac [A]** time = 1.35872, size = 304, normalized size = 2.49

$$\frac{ab^2 \log(e^{(2dx+2c)+1})}{a^4+2a^2b^2+b^4} - \frac{ab^2 \log(|-be^{(2dx+2c)}-2ae^{(dx+c)}+b|)}{a^4+2a^2b^2+b^4} - \frac{(a^2be^c-b^3e^c) \arctan(e^{(dx+c)})e^{(-c)}}{a^4+2a^2b^2+b^4} + \frac{(a^2be^{(3c)}+b^3e^{(3c)})e^{(3dx)}-2(a^3e^{(2c)}+ab^2e^{(2c)})e^{(2dx)}-(a^2+b^2)^2(e^{(2dx+2c)+1})^2}{(a^2+b^2)^2(e^{(2dx+2c)+1})^2}$$


---

$d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] (a\*b^2\*log(e^(2\*d\*x + 2\*c) + 1)/(a^4 + 2\*a^2\*b^2 + b^4) - a\*b^2\*log(abs(-b\*e^(2\*d\*x + 2\*c) - 2\*a\*e^(d\*x + c) + b))/(a^4 + 2\*a^2\*b^2 + b^4) - (a^2\*b\*e^c - b^3\*e^c)\*arctan(e^(d\*x + c))\*e^(-c)/(a^4 + 2\*a^2\*b^2 + b^4) + ((a^2\*b\*e^(3\*c) + b^3\*e^(3\*c))\*e^(3\*d\*x) - 2\*(a^3\*e^(2\*c) + a\*b^2\*e^(2\*c))\*e^(2\*d\*x) - (a^2\*b\*e^c + b^3\*e^c)\*e^(d\*x))/((a^2 + b^2)^2\*(e^(2\*d\*x + 2\*c) + 1)^2)/d

$$3.361 \quad \int \frac{\operatorname{sech}^2(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable} \left( \frac{\tanh(c+dx) \operatorname{sech}^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[(Sech[c + d\*x]^2\*Tanh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x]))], x]

**Rubi [A]** time = 0.0856575, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{sech}^2(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Sech[c + d\*x]^2\*Tanh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int] [(Sech[c + d\*x]^2\*Tanh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x]))], x]

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\operatorname{sech}^2(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [A]** time = 142.055, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sech[c + d\*x]^2\*Tanh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]



[Out] Integrate[(Sech[c + d\*x]^2\*Tanh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Maple [A]** time = 1.159, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{sech}(dx + c))^2 \tanh(dx + c)}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d\*x+c)^2\*tanh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(sech(d\*x+c)^2\*tanh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)^2\*tanh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] (a\*f + (b\*d\*f\*x\*e^(3\*c) + (d\*e - f)\*b\*e^(3\*c))\*e^(3\*d\*x) - (2\*a\*d\*f\*x\*e^(2\*c) + (2\*d\*e - f)\*a\*e^(2\*c))\*e^(2\*d\*x) - (b\*d\*f\*x\*e^c + (d\*e + f)\*b\*e^c)\*e^(d\*x))/(a^2\*d^2\*e^2 + b^2\*d^2\*e^2 + (a^2\*d^2\*f^2 + b^2\*d^2\*f^2)\*x^2 + 2\*(a^2\*d^2\*e\*f + b^2\*d^2\*e\*f)\*x + (a^2\*d^2\*e^2\*e^(4\*c) + b^2\*d^2\*e^2\*e^(4\*c) + (a^2\*d^2\*f^2\*e^(4\*c) + b^2\*d^2\*f^2\*e^(4\*c))\*x^2 + 2\*(a^2\*d^2\*e\*f\*e^(4\*c) + b^2\*d^2\*e\*f\*e^(4\*c))\*x)\*e^(4\*d\*x) + 2\*(a^2\*d^2\*e^2\*e^(2\*c) + b^2\*d^2\*e^2\*e^(2\*c) + (a^2\*d^2\*f^2\*e^(2\*c) + b^2\*d^2\*f^2\*e^(2\*c))\*x^2 + 2\*(a^2\*d^2\*e\*f\*e^(2\*c) + b^2\*d^2\*e\*f\*e^(2\*c))\*x)\*e^(2\*d\*x)) - 4\*integrate(1/4\*(2\*a\*b^2\*d^2\*f^2\*x^2 + 4\*a\*b^2\*d^2\*e\*f\*x - 2\*a^3\*f^2 + 2\*(d^2\*e^2 - f^2)\*a\*b^2 + ((d^2\*e^2 + 2\*f^2)\*a^2\*b\*e^c - (d^2\*e^2 - 2\*f^2)\*b^3\*e^c + (a^2\*b\*d^2\*f^2\*e^c - b^3\*d^2\*f^2\*e^c)\*x^2 + 2\*(a^2\*b\*d^2\*e\*f\*e^c - b^3\*d^2\*e\*f\*e^c)\*x)\*e^(d\*x))/(a^4\*d^2\*e^3 + 2\*a^2\*b^2\*d^2\*e^3 + b^4\*d^2\*e^3 + (a^4\*d^2\*f^3 + 2\*a^2\*b^2\*d^2\*f^3 + b^4\*d^2\*f^3)\*x^3 + 3\*(a^4\*d^2\*e\*f^2 + 2\*a^2\*b^2\*d^2\*e\*f^2 + b^4\*d^2\*e\*f^2)\*x^2 + 3\*(a^4\*d^2\*e^2\*f + 2\*a^2\*b^2\*d^2\*e^2\*f + b^4\*d^2\*e^2\*f)\*x + (a^4\*d^2\*e^3\*e^(2\*c) + 2\*a^2\*b^2\*d^2\*e^3\*e^(2\*c) + b^4\*d^2\*e^3\*e^(2\*c) + (a^4\*d^2\*f^3\*e^(2\*c) + 2\*a^2\*b^2\*d^2\*f^3\*e^(2\*c) + b^4\*d^2\*f^3\*e^(2\*c))\*x^3 + 3\*(a

$$\begin{aligned} &^4*d^2*e*f^2*e^{(2*c)} + 2*a^2*b^2*d^2*e*f^2*e^{(2*c)} + b^4*d^2*e*f^2*e^{(2*c)} \\ &*x^2 + 3*(a^4*d^2*e^2*f*e^{(2*c)} + 2*a^2*b^2*d^2*e^2*f*e^{(2*c)} + b^4*d^2*e^2 \\ &*f*e^{(2*c)})*x)*e^{(2*d*x)}, x) + 4*integrate(-1/2*(a^2*b^2*e^{(d*x + c)} - a*b \\ &^3)/(a^4*b*e + 2*a^2*b^3*e + b^5*e + (a^4*b*f + 2*a^2*b^3*f + b^5*f)*x - (a \\ &^4*b*e*e^{(2*c)} + 2*a^2*b^3*e*e^{(2*c)} + b^5*e*e^{(2*c)} + (a^4*b*f*e^{(2*c)} + 2 \\ &*a^2*b^3*f*e^{(2*c)} + b^5*f*e^{(2*c)})*x)*e^{(2*d*x)} - 2*(a^5*e*e^c + 2*a^3*b^2 \\ &*e*e^c + a*b^4*e*e^c + (a^5*f*e^c + 2*a^3*b^2*f*e^c + a*b^4*f*e^c)*x)*e^{(d* \\ &x)}), x) \end{aligned}$$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{sech}(dx + c)^2 \tanh(dx + c)}{afx + ae + (bf x + be) \sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sech(d*x + c)^2*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(c + dx) \text{sech}^2(c + dx)}{(a + b \sinh(c + dx))(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)*sech(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.362 \quad \int \frac{(e+fx)^3 \cosh(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=606

$$\frac{6a^2 f^2 (e+fx) \text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^3} - \frac{6a^2 f^2 (e+fx) \text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3 d^3} + \frac{3a^2 f (e+fx)^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^2}$$

[Out]  $(3*f^3*x)/(8*b*d^3) + (e+fx)^3/(4*b*d) - (a^2*(e+fx)^4)/(4*b^3*f) + (6*a*f^3*\text{Cosh}[c+dx])/(b^2*d^4) + (3*a*f*(e+fx)^2*\text{Cosh}[c+dx])/(b^2*d^2) + (a^2*(e+fx)^3*\text{Log}[1+(b*E^{c+dx})/(a-\text{Sqrt}[a^2+b^2])])/(b^3*d) + (a^2*(e+fx)^3*\text{Log}[1+(b*E^{c+dx})/(a+\text{Sqrt}[a^2+b^2])])/(b^3*d) + (3*a^2*f*(e+fx)^2*\text{PolyLog}[2, -((b*E^{c+dx})/(a-\text{Sqrt}[a^2+b^2]))])/(b^3*d^2) + (3*a^2*f*(e+fx)^2*\text{PolyLog}[2, -((b*E^{c+dx})/(a+\text{Sqrt}[a^2+b^2]))])/(b^3*d^2) - (6*a^2*f^2*(e+fx)*\text{PolyLog}[3, -((b*E^{c+dx})/(a-\text{Sqrt}[a^2+b^2]))])/(b^3*d^3) - (6*a^2*f^2*(e+fx)*\text{PolyLog}[3, -((b*E^{c+dx})/(a+\text{Sqrt}[a^2+b^2]))])/(b^3*d^3) + (6*a^2*f^3*\text{PolyLog}[4, -((b*E^{c+dx})/(a-\text{Sqrt}[a^2+b^2]))])/(b^3*d^4) + (6*a^2*f^3*\text{PolyLog}[4, -((b*E^{c+dx})/(a+\text{Sqrt}[a^2+b^2]))])/(b^3*d^4) - (6*a*f^2*(e+fx)*\text{Sinh}[c+dx])/(b^2*d^3) - (a*(e+fx)^3*\text{Sinh}[c+dx])/(b^2*d) - (3*f^3*\text{Cosh}[c+dx]*\text{Sinh}[c+dx])/(8*b*d^4) - (3*f*(e+fx)^2*\text{Cosh}[c+dx]*\text{Sinh}[c+dx])/(4*b*d^2) + (3*f^2*(e+fx)*\text{Sinh}[c+dx]^2)/(4*b*d^3) + ((e+fx)^3*\text{Sinh}[c+dx]^2)/(2*b*d)$

**Rubi [A]** time = 0.879494, antiderivative size = 606, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 14, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {5579, 5446, 3311, 32, 2635, 8, 3296, 2638, 5561, 2190, 2531, 6609, 2282, 6589}

$$\frac{6a^2 f^2 (e+fx) \text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^3} - \frac{6a^2 f^2 (e+fx) \text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3 d^3} + \frac{3a^2 f (e+fx)^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(e+fx)^3*\text{Cosh}[c+dx]*\text{Sinh}[c+dx]^2}{(a+b*\text{Sinh}[c+dx])}, x]$

[Out]  $(3*f^3*x)/(8*b*d^3) + (e+fx)^3/(4*b*d) - (a^2*(e+fx)^4)/(4*b^3*f) + (6*a*f^3*\text{Cosh}[c+dx])/(b^2*d^4) + (3*a*f*(e+fx)^2*\text{Cosh}[c+dx])/(b^2*d^2) + (a^2*(e+fx)^3*\text{Log}[1+(b*E^{c+dx})/(a-\text{Sqrt}[a^2+b^2])])/(b^3*d) + (a^2*(e+fx)^3*\text{Log}[1+(b*E^{c+dx})/(a+\text{Sqrt}[a^2+b^2])])/(b^3*d) + (3*a^2*f*(e+fx)^2*\text{PolyLog}[2, -((b*E^{c+dx})/(a-\text{Sqrt}[a^2+b^2]))])/(b^3*d^2) + (3*a^2*f*(e+fx)^2*\text{PolyLog}[2, -((b*E^{c+dx})/(a+\text{Sqrt}[a^2+b^2]))])/(b^3*d^2) - (6*a^2*f^2*(e+fx)*\text{PolyLog}[3, -((b*E^{c+dx})/(a-\text{Sqrt}[a^2+b^2]))])/(b^3*d^3) - (6*a^2*f^2*(e+fx)*\text{PolyLog}[3, -((b*E^{c+dx})/(a+\text{Sqrt}[a^2+b^2]))])/(b^3*d^3) + (6*a^2*f^3*\text{PolyLog}[4, -((b*E^{c+dx})/(a-\text{Sqrt}[a^2+b^2]))])/(b^3*d^4) + (6*a^2*f^3*\text{PolyLog}[4, -((b*E^{c+dx})/(a+\text{Sqrt}[a^2+b^2]))])/(b^3*d^4) - (6*a*f^2*(e+fx)*\text{Sinh}[c+dx])/(b^2*d^3) - (a*(e+fx)^3*\text{Sinh}[c+dx])/(b^2*d) - (3*f^3*\text{Cosh}[c+dx]*\text{Sinh}[c+dx])/(8*b*d^4) - (3*f*(e+fx)^2*\text{Cosh}[c+dx]*\text{Sinh}[c+dx])/(4*b*d^2) + (3*f^2*(e+fx)*\text{Sinh}[c+dx]^2)/(4*b*d^3) + ((e+fx)^3*\text{Sinh}[c+dx]^2)/(2*b*d)$

$$\begin{aligned} & \text{Int}[\dots] / (b^3 d^2) + (3 a^2 f (e + f x)^2 \text{PolyLog}[2, -((b E^{(c + d x)}) / (a + \sqrt{a^2 + b^2}))]) / (b^3 d^2) \\ & - (6 a^2 f^2 (e + f x) \text{PolyLog}[3, -((b E^{(c + d x)}) / (a - \sqrt{a^2 + b^2}))]) / (b^3 d^3) - (6 a^2 f^2 (e + f x) \text{PolyLog}[3, - \\ & ((b E^{(c + d x)}) / (a + \sqrt{a^2 + b^2}))]) / (b^3 d^3) + (6 a^2 f^3 \text{PolyLog}[4, -((b E^{(c + d x)}) / (a - \sqrt{a^2 + b^2}))]) / (b^3 d^4) \\ & + (6 a^2 f^3 \text{PolyLog}[4, -((b E^{(c + d x)}) / (a + \sqrt{a^2 + b^2}))]) / (b^3 d^4) - (6 a f^2 (e + f x) \text{Sinh}[c + d x]) / (b^2 d^3) \\ & - (a (e + f x)^3 \text{Sinh}[c + d x]) / (b^2 d) - (3 f^3 \text{Cosh}[c + d x] \text{Sinh}[c + d x]) / (8 b d^4) - (3 f (e + f x)^2 \text{Cosh}[c + d x] \text{Sinh}[c + d x]) / (4 b d^2) \\ & + (3 f^2 (e + f x) \text{Sinh}[c + d x]^2) / (4 b d^3) + ((e + f x)^3 \text{Sinh}[c + d x]^2) / (2 b d) \end{aligned}$$
Rule 5579

$$\begin{aligned} & \text{Int}[(\text{Cosh}[(c_.) + (d_.) (x_)]^{(p_.)} ((e_.) + (f_.) (x_))^{(m_.)} \text{Sinh}[(c_.) + (d_.) (x_)]^{(n_.)}) / ((a_.) + (b_.) \text{Sinh}[(c_.) + (d_.) (x_)]), x\_Symbol] \text{ :> } D \\ & \text{ist}[1/b, \text{Int}[(e + f x)^m \text{Cosh}[c + d x]^p \text{Sinh}[c + d x]^{(n - 1)}, x], x] - \text{Dist}[a/b, \text{Int}[(e + f x)^m \text{Cosh}[c + d x]^p \text{Sinh}[c + d x]^{(n - 1)} / (a + b \text{Sinh}[c + d x]), x], x] /; \\ & \text{FreeQ}\{a, b, c, d, e, f\}, x\} \ \&\& \text{IGtQ}[m, 0] \ \&\& \text{IGtQ}[n, 0] \ \&\& \text{IGtQ}[p, 0] \end{aligned}$$
Rule 5446

$$\begin{aligned} & \text{Int}[\text{Cosh}[(a_.) + (b_.) (x_)]^{(m_.)} ((c_.) + (d_.) (x_))^{(n_.)} \text{Sinh}[(a_.) + (b_.) (x_)]^{(n_.)}, x\_Symbol] \text{ :> } \text{Simp}[(c + d x)^m \text{Sinh}[a + b x]^{(n + 1)} / (b (n + 1)), x] \\ & - \text{Dist}[(d m) / (b (n + 1)), \text{Int}[(c + d x)^{(m - 1)} \text{Sinh}[a + b x]^{(n + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x\} \ \&\& \text{IGtQ}[m, 0] \ \&\& \text{NeQ}[n, -1] \end{aligned}$$
Rule 3311

$$\begin{aligned} & \text{Int}[(c_.) + (d_.) (x_)]^{(m_.)} ((b_.) \sin[(e_.) + (f_.) (x_)]^{(n_.)}), x\_Symbol] \text{ :> } \text{Simp}[(d m (c + d x)^{(m - 1)} (b \sin[e + f x])^n) / (f^2 n^2), x] + (\text{Dist} \\ & [(b^2 (n - 1)) / n, \text{Int}[(c + d x)^m (b \sin[e + f x])^{(n - 2)}, x], x] - \text{Dist}[(d^2 m (m - 1)) / (f^2 n^2), \text{Int}[(c + d x)^{(m - 2)} (b \sin[e + f x])^n, x], x] \\ & - \text{Simp}[(b (c + d x)^m \text{Cos}[e + f x] (b \sin[e + f x])^{(n - 1)}) / (f n), x]) /; \text{FreeQ}\{b, c, d, e, f\}, x\} \ \&\& \text{GtQ}[n, 1] \ \&\& \text{GtQ}[m, 1] \end{aligned}$$
Rule 32

$$\text{Int}[(a_.) + (b_.) (x_)]^{(m_.)}, x\_Symbol] \text{ :> } \text{Simp}[(a + b x)^{(m + 1)} / (b (m + 1)), x] /; \text{FreeQ}\{a, b, m\}, x\} \ \&\& \text{NeQ}[m, -1]$$
Rule 2635

$$\text{Int}[(b_.) \sin[(c_.) + (d_.) (x_)]^{(n_.)}, x\_Symbol] \text{ :> } -\text{Simp}[(b \text{Cos}[c + d x]) (b \sin[c + d x])^{(n - 1)}] / (d n), x] + \text{Dist}[(b^2 (n - 1)) / n, \text{Int}[(b \sin[c$$

+ d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[  
((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[  
e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ  
[{c, d}, x]

### Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sin  
h[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)),  
x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x))  
, x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x))  
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/  
((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp  
[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Di  
st[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))  
)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)  
\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))  
)^n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m -  
1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))^n)], x], x] /; FreeQ[{F, a, b, c, e, f,  
g, n}, x] && GtQ[m, 0]

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*(a_.) + (b_.
)*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*(a_.) + (b_.)*(x_)]^p]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \cosh(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \cosh(c+dx) \sinh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^3 \cosh(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{(e+fx)^3 \sinh^2(c+dx)}{2bd} - \frac{a \int (e+fx)^3 \cosh(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)^3 \cosh(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
&= -\frac{a^2(e+fx)^4}{4b^3f} - \frac{a(e+fx)^3 \sinh(c+dx)}{b^2d} - \frac{3f(e+fx)^2 \cosh(c+dx) \sinh(c+dx)}{4bd^2} \\
&= \frac{(e+fx)^3}{4bd} - \frac{a^2(e+fx)^4}{4b^3f} + \frac{3af(e+fx)^2 \cosh(c+dx)}{b^2d^2} + \frac{a^2(e+fx)^3 \log\left(\frac{1+\sinh(c+dx)}{1-\sinh(c+dx)}\right)}{b^3d} \\
&= \frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{a^2(e+fx)^4}{4b^3f} + \frac{3af(e+fx)^2 \cosh(c+dx)}{b^2d^2} + \frac{a^2(e+fx)^3 \log\left(\frac{1+\sinh(c+dx)}{1-\sinh(c+dx)}\right)}{b^3d} \\
&= \frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{a^2(e+fx)^4}{4b^3f} + \frac{6af^3 \cosh(c+dx)}{b^2d^4} + \frac{3af(e+fx)^2 \cosh(c+dx)}{b^2d^2} \\
&= \frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{a^2(e+fx)^4}{4b^3f} + \frac{6af^3 \cosh(c+dx)}{b^2d^4} + \frac{3af(e+fx)^2 \cosh(c+dx)}{b^2d^2} \\
&= \frac{3f^3x}{8bd^3} + \frac{(e+fx)^3}{4bd} - \frac{a^2(e+fx)^4}{4b^3f} + \frac{6af^3 \cosh(c+dx)}{b^2d^4} + \frac{3af(e+fx)^2 \cosh(c+dx)}{b^2d^2}
\end{aligned}$$

**Mathematica [B]** time = 28.4807, size = 2872, normalized size = 4.74

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(e^3*Log[a + b*Sinh[c + d*x]])/(4*b*d) - (3*e^2*f*(-x^2/(2*b) + (x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]))/(b*d) + (x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]))/(b*d) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]])/(b*d^2) + PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(b*d^2))/4 - (3*e*f^2*(-x^3/(3*b) + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]))/(b*d) + (x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(b*d) + (2*x*PolyLog[2, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(b*d^2) +
```



$$\begin{aligned}
& (2*x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(b*d^2) - (2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]/(b*d^3) - (2*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))/(b*d^3)))/4 - (f^3*(-x^4/(4*b) + (x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]/(b*d) + (x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]/(b*d) + (3*x^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(b*d^2) + (3*x^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(b*d^2) - (6*x*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(b*d^3) - (6*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(b*d^3) + (6*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])]/(b*d^4) + (6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(b*d^4)))/4 + (f^3*((4*a^2 + b^2)*x^4*Cosh[c]*Csch[c/2]*Sech[c/2])/(8*b^3) - (4*a*Cosh[d*x]*(-6*Cosh[c] - 3*d^2*x^2*Cosh[c] + 6*d*x*Sinh[c] + d^3*x^3*Sinh[c]))/(b^2*d^4) + (Cosh[2*d*x]*(6*d*x*Cosh[2*c] + 4*d^3*x^3*Cosh[2*c] - 3*Sinh[2*c] - 6*d^2*x^2*Sinh[2*c]))/(4*b*d^4) - ((4*a^2 + b^2)*(x^4 - (2*b^2*(d^3*x^3*Log[1 + ((a - Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x])))/b] - 3*d^2*x^2*PolyLog[2, ((-a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 6*d*x*PolyLog[3, ((-a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 6*PolyLog[4, ((-a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b])*(-1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 + b^2]*(-a + Sqrt[a^2 + b^2])*d^4) - (2*b^2*(d^3*x^3*Log[1 + ((a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 3*d^2*x^2*PolyLog[2, -(((a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b)] - 6*d*x*PolyLog[3, -(((a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b)] - 6*PolyLog[4, -(((a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b)])))*(-1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 + b^2]*(a + Sqrt[a^2 + b^2])*d^4) + (2*a*(d^3*x^3*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])]) + 3*d^2*x^2*PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])]) - 6*d*x*PolyLog[3, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])]) + 6*PolyLog[4, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])])*(-1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 + b^2]*d^4) - (2*a*(d^3*x^3*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])]) + 3*d^2*x^2*PolyLog[2, -(b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])]) - 6*d*x*PolyLog[3, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))] + 6*PolyLog[4, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))])*(-1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2 + b^2]*d^4))/(2*b^3*(-1 + Cosh[2*c] + Sinh[2*c])) - (4*a*(6*d*x*Cosh[c] + d^3*x^3*Cosh[c] - 6*Sinh[c] - 3*d^2*x^2*Sinh[c])*Sinh[d*x])/(b^2*d^4) + ((-3*Cosh[2*c] - 6*d^2*x^2*Cosh[2*c] + 6*d*x*Sinh[2*c] + 4*d^3*x^3*Sinh[2*c])*Sinh[2*d*x])/(4*b*d^4))/4 + (e*f^2*(2*(4*a^2 + b^2)*x^3*Coth[c] - (24*a*b*Cosh[d*x]*(-2*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c]))/d^3 + (3*b^2*Cosh[2*d*x]*((1 + 2*d^2*x^2)*Cosh[2*c] - 2*d*x*Sinh[2*c]))/d^3 - (4*a^2 + b^2)*(-1 + Coth[c])*(2*x^3 + (6*a*(d^2*x^2*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])]) + 2*d*x*PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])]) - 2*PolyLog[3, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])])*Sinh[c]*(Cosh[c] + Sinh[c]))/(Sqrt[a^2 + b^2]*d^3) - (3*b^2*(d^2*x^2*Log[1 + ((a - Sqrt
\end{aligned}$$

$$\begin{aligned}
& [a^2 + b^2])*(\text{Cosh}[c + d*x] - \text{Sinh}[c + d*x])/b] - 2*d*x*\text{PolyLog}[2, ((-a + \\
& \text{Sqrt}[a^2 + b^2])*(\text{Cosh}[c + d*x] - \text{Sinh}[c + d*x])/b] - 2*\text{PolyLog}[3, ((-a + \\
& \text{Sqrt}[a^2 + b^2])*(\text{Cosh}[c + d*x] - \text{Sinh}[c + d*x])/b))*(-1 + \text{Cosh}[2*c] + \text{Sin} \\
& \text{h}[2*c]))/(\text{Sqrt}[a^2 + b^2]*(-a + \text{Sqrt}[a^2 + b^2])*d^3) - (3*b^2*(d^2*x^2*\text{Log} \\
& [1 + ((a + \text{Sqrt}[a^2 + b^2])*(\text{Cosh}[c + d*x] - \text{Sinh}[c + d*x])/b] - 2*d*x*\text{Pol} \\
& \text{yLog}[2, ((a + \text{Sqrt}[a^2 + b^2])*(-\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x])/b] - 2*\text{Pol} \\
& \text{yLog}[3, ((a + \text{Sqrt}[a^2 + b^2])*(-\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x])/b))*(-1 + \\
& \text{Cosh}[2*c] + \text{Sinh}[2*c]))/(\text{Sqrt}[a^2 + b^2]*(a + \text{Sqrt}[a^2 + b^2])*d^3) - (3*a* \\
& (d^2*x^2*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a + \text{Sqrt}[a^2 + b^2])] \\
& + 2*d*x*\text{PolyLog}[2, -((b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a + \text{Sqrt}[a^2 + b \\
& ^2]))] - 2*\text{PolyLog}[3, -((b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a + \text{Sqrt}[a^2 + \\
& b^2]))])*(-1 + \text{Cosh}[2*c] + \text{Sinh}[2*c]))/(\text{Sqrt}[a^2 + b^2]*d^3)) - (24*a*b*(( \\
& 2 + d^2*x^2)*\text{Cosh}[c] - 2*d*x*\text{Sinh}[c])*\text{Sinh}[d*x])/d^3 + (3*b^2*(-2*d*x*\text{Cosh}[ \\
& 2*c] + (1 + 2*d^2*x^2)*\text{Sinh}[2*c])*\text{Sinh}[2*d*x])/d^3)/(8*b^3) + (e^3*((4*a^2 \\
& + b^2)*\text{Log}[a + b*\text{Sinh}[c + d*x]] - 4*a*b*\text{Sinh}[c + d*x] + 2*b^2*\text{Sinh}[c + d*x \\
& ^2))/(4*b^3*d) + (3*e^2*f*(8*a*b*\text{Cosh}[c + d*x] + 2*b^2*d*x*\text{Cosh}[2*(c + d*x \\
& )] + 2*(4*a^2 + b^2)*(-(c + d*x)^2/2 + (c + d*x)*\text{Log}[1 + (b*E^(c + d*x))/(a \\
& - \text{Sqrt}[a^2 + b^2]]) + (c + d*x)*\text{Log}[1 + (b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b^ \\
& 2]]) - c*\text{Log}[a + b*\text{Sinh}[c + d*x]] + \text{PolyLog}[2, (b*E^(c + d*x))/(-a + \text{Sqrt}[a \\
& ^2 + b^2]]) + \text{PolyLog}[2, -((b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b^2]))]) - 8*a*b \\
& *d*x*\text{Sinh}[c + d*x] - b^2*\text{Sinh}[2*(c + d*x)])/(8*b^3*d^2)
\end{aligned}$$

**Maple [F]** time = 0.216, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c) (\sinh(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*cosh(d\*x+c)\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*cosh(d\*x+c)\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $\frac{1}{8}e^3(8(dx+c)a^2/(b^3d) - (4ae^{-(dx+c)} - b)e^{(2dx+2c)})/(b^2d) + 8a^2\log(-2ae^{-(dx+c)} + be^{-(2dx+2c)} - b)/(b^3d) + (4ae^{-(dx+c)} + be^{-(2dx+2c)})/(b^2d) + \frac{1}{32}(8a^2d^4f^3x^4e^{(2c)} + 32a^2d^4e^2f^2x^3e^{(2c)} + 48a^2d^4e^2f^2x^2e^{(2c)} + (4b^2d^3f^3x^3e^{(4c)} + 6(2d^3e^2f^2 - d^2f^3)b^2x^2e^{(4c)} + 6(2d^3e^2f - 2d^2e^2f + df^3)b^2xe^{(4c)} - 3(2d^2e^2f - 2de^2f + f^3)b^2e^{(4c)})e^{(2dx)} - 16(abd^3f^3x^3e^{(3c)} + 3(d^3e^2f - d^2f^3)abx^2e^{(3c)} + 3(d^3e^2f - 2d^2e^2f + 2df^3)abxe^{(3c)} - 3(d^2e^2f - 2de^2f + 2f^3)abbe^{(3c)})e^{(dx)} + 16(abd^3f^3x^3e^c + 3(d^3e^2f + d^2f^3)abx^2e^c + 3(d^3e^2f + 2d^2e^2f + 2df^3)abxe^c + 3(d^2e^2f + 2de^2f + 2f^3)abbe^c)e^{(-dx)} + (4b^2d^3f^3x^3 + 6(2d^3e^2f + d^2f^3)b^2x^2 + 6(2d^3e^2f + 2d^2e^2f + df^3)b^2x + 3(2d^2e^2f + 2de^2f + f^3)b^2)e^{(-2dx)})e^{(-2c)}/(b^3d^4) - \int (-2(a^2bf^3x^3 + 3a^2b^2e^2f^2x^2 + 3a^2b^2e^2f^2x - (a^3f^3x^3e^c + 3a^3e^2f^2x^2e^c + 3a^3e^2f^2x^2e^c)e^{(dx)})/(b^4e^{(2dx+2c)} + 2ab^3e^{(dx+c)} - b^4), x)$

---

**Fricas [C]** time = 2.986, size = 8556, normalized size = 14.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{32}(4b^2d^3f^3x^3 + 4b^2d^3e^3 + 6b^2d^2e^2f + 6b^2d^2e^2f^2 + 3b^2f^3 + (4b^2d^3f^3x^3 + 4b^2d^3e^3 - 6b^2d^2e^2f + 6b^2d^2e^2f^2 - 3b^2f^3 + 6(2b^2d^3e^2f - b^2d^2f^3)x^2 + 6(2b^2d^3e^2f - 2b^2d^2e^2f + b^2df^3)x)*\cosh(dx+c)^4 + (4b^2d^3f^3x^3 + 4b^2d^3e^3 - 6b^2d^2e^2f + 6b^2d^2e^2f^2 - 3b^2f^3 + 6(2b^2d^3e^2f - b^2d^2f^3)x^2 + 6(2b^2d^3e^2f - 2b^2d^2e^2f + b^2df^3)x)*\sinh(dx+c)^4 - 16(abd^3f^3x^3 + abd^3e^3 - 3abd^2e^2f + 6abd^2e^2f^2 - 6abf^3 + 3(abd^3e^2f - abd^2f^3)x^2 + 3(abd^3e^2f - 2abd^2e^2f + 2abd^2f^3)x)*\cosh(dx+c)^3 - 4(4abd^3f^3x^3 + 4abd^3e^3 - 12abd^2e^2f + 24abd^2e^2f^2 - 24abf^3 + 12(abd^3e^2f - abd^2f^3)x^2 + 12(abd^3e^2f - 2abd^2e^2f + 2abd^2f^3)x - (4b^2d^3f^3x^3 + 4b^2d^3e^3 - 6b^2d^2e^2f + 6b^2d^2e^2f^2 - 3b^2f^3 + 6(2b^2d^3e^2f - b^2d^2f^3)x^2 + 6($



```

*f*x + 3*a^2*c*d^2*e^2*f - 3*a^2*c^2*d*e*f^2 + a^2*c^3*f^3)*sinh(d*x + c)^2
)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 192*(a^2*f^3*cosh(d*x + c)^2 + 2*a^2*f
^3*cosh(d*x + c)*sinh(d*x + c) + a^2*f^3*sinh(d*x + c)^2)*polylog(4, (a*cos
h(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 + b^2)/b^2))/b) + 192*(a^2*f^3*cosh(d*x + c)^2 + 2*a^2*f^3*cosh(d*x + c)*
sinh(d*x + c) + a^2*f^3*sinh(d*x + c)^2)*polylog(4, (a*cosh(d*x + c) + a*si
nh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b)
- 192*((a^2*d*f^3*x + a^2*d*e*f^2)*cosh(d*x + c)^2 + 2*(a^2*d*f^3*x + a^2*
d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d*f^3*x + a^2*d*e*f^2)*sinh(d*x
+ c)^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 192*((a^2*d*f^3*x + a^2*d*e*f
^2)*cosh(d*x + c)^2 + 2*(a^2*d*f^3*x + a^2*d*e*f^2)*cosh(d*x + c)*sinh(d*x
+ c) + (a^2*d*f^3*x + a^2*d*e*f^2)*sinh(d*x + c)^2)*polylog(3, (a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2))/b) + 4*(4*a*b*d^3*f^3*x^3 + 4*a*b*d^3*e^3 + 12*a*b*d^2*e^2*f + 24*
a*b*d*e*f^2 + 24*a*b*f^3 + (4*b^2*d^3*f^3*x^3 + 4*b^2*d^3*e^3 - 6*b^2*d^2*e
^2*f + 6*b^2*d*e*f^2 - 3*b^2*f^3 + 6*(2*b^2*d^3*e*f^2 - b^2*d^2*f^3)*x^2 +
6*(2*b^2*d^3*e^2*f - 2*b^2*d^2*e*f^2 + b^2*d*f^3)*x)*cosh(d*x + c)^3 + 12*(
a*b*d^3*e*f^2 + a*b*d^2*f^3)*x^2 - 12*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 - 3*a*
b*d^2*e^2*f + 6*a*b*d*e*f^2 - 6*a*b*f^3 + 3*(a*b*d^3*e*f^2 - a*b*d^2*f^3)*x
^2 + 3*(a*b*d^3*e^2*f - 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x)*cosh(d*x + c)^2 +
12*(a*b*d^3*e^2*f + 2*a*b*d^2*e*f^2 + 2*a*b*d*f^3)*x - 4*(a^2*d^4*f^3*x^4
+ 4*a^2*d^4*e*f^2*x^3 + 6*a^2*d^4*e^2*f*x^2 + 4*a^2*d^4*e^3*x + 8*a^2*c*d^3
*e^3 - 12*a^2*c^2*d^2*e^2*f + 8*a^2*c^3*d*e*f^2 - 2*a^2*c^4*f^3)*cosh(d*x +
c))*sinh(d*x + c))/(b^3*d^4*cosh(d*x + c)^2 + 2*b^3*d^4*cosh(d*x + c)*sinh
(d*x + c) + b^3*d^4*sinh(d*x + c)^2)

```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cosh(d\*x+c)\*sinh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c) \sinh(dx + c)^2}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

$$3.363 \quad \int \frac{(e+fx)^2 \cosh(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=449

$$\frac{2a^2 f(e+fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^2} + \frac{2a^2 f(e+fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3 d^2} - \frac{2a^2 f^2 \text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^3} - \frac{2a^2 f^2 \text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3 d^3}$$

```
[Out] (e*f*x)/(2*b*d) + (f^2*x^2)/(4*b*d) - (a^2*(e + f*x)^3)/(3*b^3*f) + (2*a*f*(e + f*x)*Cosh[c + d*x])/(b^2*d^2) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*d) + (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^2) + (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) - (2*a^2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^3) - (2*a^2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^3) - (2*a*f^2*Sinh[c + d*x])/(b^2*d^3) - (a*(e + f*x)^2*Sinh[c + d*x])/(b^2*d) - (f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d^2) + (f^2*Sinh[c + d*x]^2)/(4*b*d^3) + ((e + f*x)^2*Sinh[c + d*x]^2)/(2*b*d)
```

**Rubi [A]** time = 0.716799, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {5579, 5446, 3310, 3296, 2637, 5561, 2190, 2531, 2282, 6589}

$$\frac{2a^2 f(e+fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^2} + \frac{2a^2 f(e+fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3 d^2} - \frac{2a^2 f^2 \text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^3} - \frac{2a^2 f^2 \text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3 d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e*f*x)/(2*b*d) + (f^2*x^2)/(4*b*d) - (a^2*(e + f*x)^3)/(3*b^3*f) + (2*a*f*(e + f*x)*Cosh[c + d*x])/(b^2*d^2) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*d) + (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^2) + (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) - (2*a^2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^3) - (2*a^2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^3) - (2*a*f^2*Sinh[c + d*x])/(b^2*d^3) - (a*(e + f*x)^2*Sinh[c + d*x])/(b^2*d) - (f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d^2) + (f^2*Sinh[c + d*x]^2)/(4*b*d^3) +
```

$((e + f*x)^2*\text{Sinh}[c + d*x]^2)/(2*b*d)$

### Rule 5579

$\text{Int}[(\text{Cosh}[(c_.) + (d_.)*(x_.)]^{(p_.)}*((e_.) + (f_.)*(x_.))^{(m_.)}*\text{Sinh}[(c_.) + (d_.)*(x_.)]^{(n_.)})/((a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_.)]), x\_Symbol] \rightarrow \text{Dist}[1/b, \text{Int}[(e + f*x)^m*\text{Cosh}[c + d*x]^p*\text{Sinh}[c + d*x]^{(n - 1)}, x], x] - \text{Dist}[a/b, \text{Int}[(e + f*x)^m*\text{Cosh}[c + d*x]^p*\text{Sinh}[c + d*x]^{(n - 1)}]/(a + b*\text{Sinh}[c + d*x]), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5446

$\text{Int}[\text{Cosh}[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sinh}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Sinh}[a + b*x]^{(n + 1)}]/(b*(n + 1)), x] - \text{Dist}[(d*m)/(b*(n + 1)), \text{Int}[(c + d*x)^{(m - 1)}*\text{Sinh}[a + b*x]^{(n + 1)}, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

### Rule 3310

$\text{Int}[(c_.) + (d_.)*(x_.)]*((b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(d*(b*\sin[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n - 1))/n, \text{Int}[(c + d*x)*(b*\sin[e + f*x])^{(n - 2)}, x], x] - \text{Simp}[(b*(c + d*x)*\cos[e + f*x]*(b*\sin[e + f*x])^{(n - 1)}]/(f*n), x)) /;$  FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

### Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\cos[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m - 1)}*\cos[e + f*x], x], x] /;$  FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2637

$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Simp}[\sin[c + d*x]/d, x] /;$  FreeQ[{c, d}, x]

### Rule 5561

$\text{Int}[(\text{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))^{(m_.)})/((a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_.)]), x\_Symbol] \rightarrow -\text{Simp}[(e + f*x)^{(m + 1)}]/(b*f*(m + 1)), x] + (\text{Int}[(e + f*x)^m*\text{E}^{(c + d*x)}]/(a - \text{Rt}[a^2 + b^2, 2] + b*\text{E}^{(c + d*x)}), x] + \text{Int}[(e + f*x)^m*\text{E}^{(c + d*x)}]/(a + \text{Rt}[a^2 + b^2, 2] + b*\text{E}^{(c + d*x)}), x)) /;$  FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]



Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n])]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \cosh(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \cosh(c+dx) \sinh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^2 \cosh(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{(e+fx)^2 \sinh^2(c+dx)}{2bd} - \frac{a \int (e+fx)^2 \cosh(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)^2 \cosh(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
&= -\frac{a^2(e+fx)^3}{3b^3f} - \frac{a(e+fx)^2 \sinh(c+dx)}{b^2d} - \frac{f(e+fx) \cosh(c+dx) \sinh(c+dx)}{2bd^2} \\
&= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} - \frac{a^2(e+fx)^3}{3b^3f} + \frac{2af(e+fx) \cosh(c+dx)}{b^2d^2} + \frac{a^2(e+fx)^2 \log\left(\frac{a+b \sinh(c+dx)}{a-\sqrt{a^2+b^2}}\right)}{b^3d} \\
&= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} - \frac{a^2(e+fx)^3}{3b^3f} + \frac{2af(e+fx) \cosh(c+dx)}{b^2d^2} + \frac{a^2(e+fx)^2 \log\left(\frac{a+b \sinh(c+dx)}{a-\sqrt{a^2+b^2}}\right)}{b^3d} \\
&= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} - \frac{a^2(e+fx)^3}{3b^3f} + \frac{2af(e+fx) \cosh(c+dx)}{b^2d^2} + \frac{a^2(e+fx)^2 \log\left(\frac{a+b \sinh(c+dx)}{a-\sqrt{a^2+b^2}}\right)}{b^3d} \\
&= \frac{efx}{2bd} + \frac{f^2x^2}{4bd} - \frac{a^2(e+fx)^3}{3b^3f} + \frac{2af(e+fx) \cosh(c+dx)}{b^2d^2} + \frac{a^2(e+fx)^2 \log\left(\frac{a+b \sinh(c+dx)}{a-\sqrt{a^2+b^2}}\right)}{b^3d}
\end{aligned}$$

**Mathematica [B]** time = 11.766, size = 1496, normalized size = 3.33

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(e^2*Log[a + b*Sinh[c + d*x]])/(4*b*d) - (e*f*(-x^2/(2*b) + (x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]))/(b*d) + (x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]))/(b*d) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]])/(b*d^2) + PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(b*d^2))/2 - (f^2*(-x^3/(3*b) + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]))/(b*d) + (x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]))/(b*d) + (2*x*PolyLog[2, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(b*d^2) + (2*x*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(b*d^2) - (2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]])/(b*d^3) - (2*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(b*d^3)))/4 + (f^2*(2*(4*a^2 + b^2)*x^3*Co
```

```

th[c] - (24*a*b*Cosh[d*x]*(-2*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c]))/d^3 + (
3*b^2*Cosh[2*d*x]*((1 + 2*d^2*x^2)*Cosh[2*c] - 2*d*x*Sinh[2*c]))/d^3 - (4*a
^2 + b^2)*(-1 + Coth[c])*(2*x^3 + (6*a*(d^2*x^2*Log[1 + (b*(Cosh[c + d*x] +
Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])]) + 2*d*x*PolyLog[2, (b*(Cosh[c + d*x]
+ Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])]) - 2*PolyLog[3, (b*(Cosh[c + d*x]
+ Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])])*(Sinh[c]*(Cosh[c] + Sinh[c]))/(
Sqrt[a^2 + b^2]*d^3) - (3*b^2*(d^2*x^2*Log[1 + ((a - Sqrt[a^2 + b^2])*(Cosh
[c + d*x] - Sinh[c + d*x]))/b] - 2*d*x*PolyLog[2, ((-a + Sqrt[a^2 + b^2])*(
Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*PolyLog[3, ((-a + Sqrt[a^2 + b^2])*(
Cosh[c + d*x] - Sinh[c + d*x]))/b])*(-1 + Cosh[2*c] + Sinh[2*c]))/(Sqrt[a^2
+ b^2]*(-a + Sqrt[a^2 + b^2])*d^3) - (3*b^2*(d^2*x^2*Log[1 + ((a + Sqrt[a^
2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*d*x*PolyLog[2, ((a + Sqrt
[a^2 + b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b] - 2*PolyLog[3, ((a + Sqrt
[a^2 + b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b])*(-1 + Cosh[2*c] + Sinh[2
*c]))/(Sqrt[a^2 + b^2]*(a + Sqrt[a^2 + b^2])*d^3) - (3*a*(d^2*x^2*Log[1 + (
b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])]) + 2*d*x*PolyLog[2
, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))]) - 2*PolyLog
[3, -((b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2]))])*(-1 + Co
sh[2*c] + Sinh[2*c]))/(Sqrt[a^2 + b^2]*d^3) - (24*a*b*((2 + d^2*x^2)*Cosh[
c] - 2*d*x*Sinh[c])*Sinh[d*x])/d^3 + (3*b^2*(-2*d*x*Cosh[2*c] + (1 + 2*d^2*
x^2)*Sinh[2*c])*Sinh[2*d*x])/d^3)/(24*b^3) + (e^2*((4*a^2 + b^2)*Log[a + b
*Sinh[c + d*x]] - 4*a*b*Sinh[c + d*x] + 2*b^2*Sinh[c + d*x]^2))/(4*b^3*d) +
(e*f*(8*a*b*Cosh[c + d*x] + 2*b^2*d*x*Cosh[2*(c + d*x)] + 2*(4*a^2 + b^2)*
(-c + d*x)^2/2 + (c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])
+ (c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) - c*Log[a + b*Si
nh[c + d*x]] + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]]) + PolyLog
[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) - 8*a*b*d*x*Sinh[c + d*x] -
b^2*Sinh[2*(c + d*x)))/(4*b^3*d^2)

```

---

**Maple [F]** time = 0.302, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c) (\sinh(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*cosh(d\*x+c)\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*cosh(d\*x+c)\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{8} e^2 \left( \frac{8(dx+c)a^2}{b^3d} - \frac{(4ae^{(-dx-c)} - b)e^{(2dx+2c)}}{b^2d} + \frac{8a^2 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{b^3d} + \frac{4ae^{(-dx-c)} + be^{(-2dx-2c)}}{b^2d} \right) + \frac{(16}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*e^2*(8*(d*x + c)*a^2/(b^3*d) - (4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) + 8*a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d) + (4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d)) + 1/48*(16*a^2*d^3*f^2*x^3*e^(2*c) + 48*a^2*d^3*e*f*x^2*e^(2*c) + 3*(2*b^2*d^2*f^2*x^2*e^(4*c) + 2*(2*d^2*e*f - d*f^2)*b^2*x*e^(4*c) - (2*d*e*f - f^2)*b^2*e^(4*c))*e^(2*d*x) - 24*(a*b*d^2*f^2*x^2*e^(3*c) + 2*(d^2*e*f - d*f^2)*a*b*x*e^(3*c) - 2*(d*e*f - f^2)*a*b*e^(3*c))*e^(d*x) + 24*(a*b*d^2*f^2*x^2*e^c + 2*(d^2*e*f + d*f^2)*a*b*x*e^c + 2*(d*e*f + f^2)*a*b*e^c)*e^(-d*x) + 3*(2*b^2*d^2*f^2*x^2 + 2*(2*d^2*e*f + d*f^2)*b^2*x + (2*d*e*f + f^2)*b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^3) - integrate(-2*(a^2*b*f^2*x^2 + 2*a^2*b*e*f*x - (a^3*f^2*x^2*e^c + 2*a^3*e*f*x*e^c)*e^(d*x))/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x)
```

**Fricas [C]** time = 2.6464, size = 5513, normalized size = 12.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(6*b^2*d^2*f^2*x^2 + 6*b^2*d^2*e^2 + 6*b^2*d*e*f + 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*cosh(d*x + c)^4 + 3*(2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*sinh(d*x + c)^4 + 3*b^2*f^2 - 24*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*cosh(d*x + c)^3 - 12*(2*a*b*d^2*f^2*x^2 + 2*a*b*d^2*e^2 - 4*a*b*d*e*f + 4*a*b*f^2 + 4*(a*b*d^2*e*f - a*b*d*f^2)*x - (2*b^2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 16*(a^2*d^3*f^2*x^3 + 3*a^2*d^3*e*f*x^2 + 3*a^2*d^3*e^2*x + 6*a^2*c*d^2*e^2 - 6*a^2*c^2*d*e*f + 2*a^2*c^3*f^2)*cosh(d*
```

$$\begin{aligned}
& x + c)^2 - 2*(8*a^2*d^3*f^2*x^3 + 24*a^2*d^3*e*f*x^2 + 24*a^2*d^3*e^2*x + 4 \\
& 8*a^2*c*d^2*e^2 - 48*a^2*c^2*d*e*f + 16*a^2*c^3*f^2 - 9*(2*b^2*d^2*f^2*x^2 \\
& + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - b^2*d*f^2)*x)* \\
& \cosh(d*x + c)^2 + 36*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f \\
& ^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 6*(2*b \\
& ^2*d^2*e*f + b^2*d*f^2)*x + 24*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 + 2*a*b*d*e*f \\
& + 2*a*b*f^2 + 2*(a*b*d^2*e*f + a*b*d*f^2)*x)*\cosh(d*x + c) + 96*((a^2*d*f^ \\
& 2*x + a^2*d*e*f)*\cosh(d*x + c)^2 + 2*(a^2*d*f^2*x + a^2*d*e*f)*\cosh(d*x + c \\
& )*\sinh(d*x + c) + (a^2*d*f^2*x + a^2*d*e*f)*\sinh(d*x + c)^2)*\operatorname{dilog}((a*\cosh( \\
& d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 \\
& + b^2)/b^2} - b)/b + 1) + 96*((a^2*d*f^2*x + a^2*d*e*f)*\cosh(d*x + c)^2 + 2 \\
& *(a^2*d*f^2*x + a^2*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2*d*f^2*x + a^2 \\
& *d*e*f)*\sinh(d*x + c)^2)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh \\
& (d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 48*((a^2*d \\
& ^2*e^2 - 2*a^2*c*d*e*f + a^2*c^2*f^2)*\cosh(d*x + c)^2 + 2*(a^2*d^2*e^2 - 2* \\
& a^2*c*d*e*f + a^2*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2*d^2*e^2 - 2*a \\
& ^2*c*d*e*f + a^2*c^2*f^2)*\sinh(d*x + c)^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh \\
& (d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 48*((a^2*d^2*e^2 - 2*a^2*c*d \\
& *e*f + a^2*c^2*f^2)*\cosh(d*x + c)^2 + 2*(a^2*d^2*e^2 - 2*a^2*c*d*e*f + a^2* \\
& c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2*d^2*e^2 - 2*a^2*c*d*e*f + a^2*c \\
& ^2*f^2)*\sinh(d*x + c)^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*sq \\
& rt((a^2 + b^2)/b^2) + 2*a) + 48*((a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + 2*a^2 \\
& *c*d*e*f - a^2*c^2*f^2)*\cosh(d*x + c)^2 + 2*(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e* \\
& f*x + 2*a^2*c*d*e*f - a^2*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2*d^2*f \\
& ^2*x^2 + 2*a^2*d^2*e*f*x + 2*a^2*c*d*e*f - a^2*c^2*f^2)*\sinh(d*x + c)^2)*lo \\
& g(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c)) \\
& *\sqrt{(a^2 + b^2)/b^2} - b)/b) + 48*((a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + 2 \\
& *a^2*c*d*e*f - a^2*c^2*f^2)*\cosh(d*x + c)^2 + 2*(a^2*d^2*f^2*x^2 + 2*a^2*d^2 \\
& *e*f*x + 2*a^2*c*d*e*f - a^2*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2*d \\
& ^2*f^2*x^2 + 2*a^2*d^2*e*f*x + 2*a^2*c*d*e*f - a^2*c^2*f^2)*\sinh(d*x + c)^2 \\
& )*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + \\
& c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 96*(a^2*f^2*\cosh(d*x + c)^2 + 2*a^2*f^ \\
& 2*\cosh(d*x + c)*\sinh(d*x + c) + a^2*f^2*\sinh(d*x + c)^2)*\operatorname{polylog}(3, (a*\cosh \\
& (d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 \\
& + b^2)/b^2}))/b) - 96*(a^2*f^2*\cosh(d*x + c)^2 + 2*a^2*f^2*\cosh(d*x + c)*si \\
& nh(d*x + c) + a^2*f^2*\sinh(d*x + c)^2)*\operatorname{polylog}(3, (a*\cosh(d*x + c) + a*\sinh \\
& (d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) + \\
& 4*(6*a*b*d^2*f^2*x^2 + 6*a*b*d^2*e^2 + 12*a*b*d*e*f + 12*a*b*f^2 + 3*(2*b^ \\
& 2*d^2*f^2*x^2 + 2*b^2*d^2*e^2 - 2*b^2*d*e*f + b^2*f^2 + 2*(2*b^2*d^2*e*f - \\
& b^2*d*f^2)*x)*\cosh(d*x + c)^3 - 18*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d \\
& *e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*\cosh(d*x + c)^2 + 12*(a*b \\
& *d^2*e*f + a*b*d*f^2)*x - 8*(a^2*d^3*f^2*x^3 + 3*a^2*d^3*e*f*x^2 + 3*a^2*d^ \\
& 3*e^2*x + 6*a^2*c*d^2*e^2 - 6*a^2*c^2*d*e*f + 2*a^2*c^3*f^2)*\cosh(d*x + c)) \\
& *\sinh(d*x + c))/(b^3*d^3*\cosh(d*x + c)^2 + 2*b^3*d^3*\cosh(d*x + c)*\sinh(d*x \\
& + c) + b^3*d^3*\sinh(d*x + c)^2)
\end{aligned}$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cosh(d\*x+c)\*sinh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c) \sinh(dx + c)^2}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*cosh(d\*x + c)\*sinh(d\*x + c)^2/(b\*sinh(d\*x + c) + a), x)

$$3.364 \quad \int \frac{(e+fx) \cosh(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=278

$$\frac{a^2 f \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^2} + \frac{a^2 f \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3 d^2} + \frac{a^2(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{b^3 d} + \frac{a^2(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3 d}$$

```
[Out] (f*x)/(4*b*d) - (a^2*(e + f*x)^2)/(2*b^3*f) + (a*f*Cosh[c + d*x])/(b^2*d^2)
+ (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) +
(a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*d) +
(a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^2) + (a^
2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) - (a*(e
+ f*x)*Sinh[c + d*x])/(b^2*d) - (f*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^2)
+ ((e + f*x)*Sinh[c + d*x]^2)/(2*b*d)
```

**Rubi [A]** time = 0.418473, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {5579, 5446, 2635, 8, 3296, 2638, 5561, 2190, 2279, 2391}

$$\frac{a^2 f \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^3 d^2} + \frac{a^2 f \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3 d^2} + \frac{a^2(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{b^3 d} + \frac{a^2(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^3 d}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (f*x)/(4*b*d) - (a^2*(e + f*x)^2)/(2*b^3*f) + (a*f*Cosh[c + d*x])/(b^2*d^2)
+ (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^3*d) +
(a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^3*d) +
(a^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^3*d^2) + (a^
2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^3*d^2) - (a*(e
+ f*x)*Sinh[c + d*x])/(b^2*d) - (f*Cosh[c + d*x]*Sinh[c + d*x])/(4*b*d^2)
+ ((e + f*x)*Sinh[c + d*x]^2)/(2*b*d)
```

### Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
(d_.)*(x_)]^(n_.)]/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
```

$[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$

### Rule 5446

$\text{Int}[\text{Cosh}[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sinh}[(a_.) + (b_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Sinh}[a + b*x]^{(n + 1)})/(b*(n + 1)), x] - \text{Dist}[(d*m)/(b*(n + 1)), \text{Int}[(c + d*x)^{(m - 1)}*\text{Sinh}[a + b*x]^{(n + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x\} \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[n, -1]$

### Rule 2635

$\text{Int}[(b_.)*\sin[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n - 1)})/(d*n), x] + \text{Dist}[(b^2*(n - 1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n - 2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x\} \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

### Rule 8

$\text{Int}[a_, x\_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

### Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x])/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m - 1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{GtQ}[m, 0]$

### Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}\{c, d\}, x]$

### Rule 5561

$\text{Int}[(\text{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))^{(m_.)})/((a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_.)]), x\_Symbol] \rightarrow -\text{Simp}[(e + f*x)^{(m + 1)}/(b*f*(m + 1)), x] + (\text{Int}[(e + f*x)^m*\text{E}^{(c + d*x)}]/(a - \text{Rt}[a^2 + b^2, 2] + b*\text{E}^{(c + d*x)}), x] + \text{Int}[(e + f*x)^m*\text{E}^{(c + d*x)}]/(a + \text{Rt}[a^2 + b^2, 2] + b*\text{E}^{(c + d*x)}), x)) /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[a^2 + b^2, 0]$

### Rule 2190

$\text{Int}[(F_.)^{((g_.)*((e_.) + (f_.)*(x_.)))^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}}/((a_.) + (b_.)*((F_.)^{((g_.)*((e_.) + (f_.)*(x_.)))^{(n_.)}}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]]/(b*f*g*n*\text{Log}[F]), x] - \text{Di}$



```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{(e + fx) \cosh(c + dx) \sinh^2(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\int (e + fx) \cosh(c + dx) \sinh(c + dx) dx}{b} - \frac{a \int \frac{(e + fx) \cosh(c + dx) \sinh(c + dx)}{a + b \sinh(c + dx)} dx}{b} \\
 &= \frac{(e + fx) \sinh^2(c + dx)}{2bd} - \frac{a \int (e + fx) \cosh(c + dx) dx}{b^2} + \frac{a^2 \int \frac{(e + fx) \cosh(c + dx)}{a + b \sinh(c + dx)} dx}{b^2} \\
 &= -\frac{a^2(e + fx)^2}{2b^3f} - \frac{a(e + fx) \sinh(c + dx)}{b^2d} - \frac{f \cosh(c + dx) \sinh(c + dx)}{4bd^2} + \frac{e \cosh(c + dx) \sinh(c + dx)}{4bd^2} \\
 &= \frac{fx}{4bd} - \frac{a^2(e + fx)^2}{2b^3f} + \frac{af \cosh(c + dx)}{b^2d^2} + \frac{a^2(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^3d} + \frac{e \cosh(c + dx) \sinh(c + dx)}{4bd^2} \\
 &= \frac{fx}{4bd} - \frac{a^2(e + fx)^2}{2b^3f} + \frac{af \cosh(c + dx)}{b^2d^2} + \frac{a^2(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^3d} + \frac{e \cosh(c + dx) \sinh(c + dx)}{4bd^2} \\
 &= \frac{fx}{4bd} - \frac{a^2(e + fx)^2}{2b^3f} + \frac{af \cosh(c + dx)}{b^2d^2} + \frac{a^2(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^3d} + \frac{e \cosh(c + dx) \sinh(c + dx)}{4bd^2}
 \end{aligned}$$

**Mathematica [A]** time = 1.08718, size = 423, normalized size = 1.52

$$\frac{b^2 f \left( -2 \operatorname{PolyLog} \left( 2, \frac{be^{c+dx}}{\sqrt{a^2 + b^2} - a} \right) - 2 \operatorname{PolyLog} \left( 2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a} \right) + dx \left( -2 \log \left( \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}} + 1 \right) - 2 \log \left( \frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a} + 1 \right) + dx \right) \right)}{b^3 d}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Cosh[c + d\*x]\*Sinh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x  
]

[Out] (-2\*b^2\*d\*e\*Log[a + b\*Sinh[c + d\*x]] + b^2\*f\*(d\*x\*(d\*x - 2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]) - 2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]) - 2\*PolyLog[2, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2]]) - 2\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))]) + 2\*d\*e\*((4\*a^2 + b^2)\*Log[a + b\*Sinh[c + d\*x]] - 4\*a\*b\*Sinh[c + d\*x] + 2\*b^2\*Sinh[c + d\*x]^2) + f\*(8\*a\*b\*Cosh[c + d\*x] + 2\*b^2\*d\*x\*Cosh[2\*(c + d\*x)] + 2\*(4\*a^2 + b^2)\*(-(c + d\*x)^2/2 + (c + d\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]) + (c + d\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]) - c\*Log[a + b\*Sinh[c + d\*x]] + PolyLog[2, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2]]) + PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))]) - 8\*a\*b\*d\*x\*Sinh[c + d\*x] - b^2\*Sinh[2\*(c + d\*x)]))/(8\*b^3\*d^2)

**Maple [B]** time = 0.116, size = 565, normalized size = 2.

$$-\frac{a^2 f x^2}{2 b^3} + \frac{a^2 e x}{b^3} + \frac{(2 d f x + 2 d e - f) e^{2 d x + 2 c}}{16 d^2 b} - \frac{a (d f x + d e - f) e^{d x + c}}{2 b^2 d^2} + \frac{a (d f x + d e + f) e^{-d x - c}}{2 b^2 d^2} + \frac{(2 d f x + 2 d e + f) e^{-d x - c}}{16 d^2 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)\*cosh(d\*x+c)\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] -1/2\*a^2\*f\*x^2/b^3+a^2\*e\*x/b^3+1/16\*(2\*d\*f\*x+2\*d\*e-f)/d^2/b\*exp(2\*d\*x+2\*c)-1/2\*a\*(d\*f\*x+d\*e-f)/b^2/d^2\*exp(d\*x+c)+1/2\*a\*(d\*f\*x+d\*e+f)/b^2/d^2\*exp(-d\*x-c)+1/16\*(2\*d\*f\*x+2\*d\*e+f)/d^2/b\*exp(-2\*d\*x-2\*c)-1/b^3/d^2\*a^2\*f\*c\*ln(b\*exp(2\*d\*x+2\*c)+2\*a\*exp(d\*x+c)-b)+2/b^3/d^2\*a^2\*f\*c\*ln(exp(d\*x+c))+1/b^3/d\*a^2\*e\*ln(b\*exp(2\*d\*x+2\*c)+2\*a\*exp(d\*x+c)-b)-2/b^3/d\*a^2\*e\*ln(exp(d\*x+c))+1/b^3/d\*a^2\*f\*ln((-b\*exp(d\*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))))\*x+1/b^3/d^2\*a^2\*f\*ln((-b\*exp(d\*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))))\*c+1/b^3/d^2\*a^2\*f\*ln((b\*exp(d\*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))))\*x+1/b^3/d^2\*a^2\*f\*ln((b\*exp(d\*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))))\*c+1/b^3/d^2\*a^2\*f\*dilog((b\*exp(d\*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))))+1/b^3/d^2\*a^2\*f\*dilog((-b\*exp(d\*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))))-2/b^3/d\*a^2\*f\*c\*x-1/b^3/d^2\*a^2\*f\*c^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{8} e^{\left( \frac{8(dx+c)a^2}{b^3d} - \frac{(4ae^{-dx-c} - b)e^{(2dx+2c)}}{b^2d} + \frac{8a^2 \log(-2ae^{-dx-c} + be^{(-2dx-2c)} - b)}{b^3d} + \frac{4ae^{-dx-c} + be^{(-2dx-2c)}}{b^2d} \right) + \frac{1}{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] 1/8*e*(8*(d*x + c)*a^2/(b^3*d) - (4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^
2*d) + 8*a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d) + (4*a
*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d)) + 1/16*f*((8*a^2*d^2*x^2*e^(2*
c) + (2*b^2*d*x*e^(4*c) - b^2*e^(4*c))*e^(2*d*x) - 8*(a*b*d*x*e^(3*c) - a*b
*e^(3*c))*e^(d*x) + 8*(a*b*d*x*e^c + a*b*e^c)*e^(-d*x) + (2*b^2*d*x + b^2)*
e^(-2*d*x))*e^(-2*c)/(b^3*d^2) - 2*integrate(16*(a^3*x*e^(d*x + c) - a^2*b*
x)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) - b^4), x))
```

**Fricas [B]** time = 2.38726, size = 3054, normalized size = 10.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/16*(2*b^2*d*f*x + (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^4 + (2*
b^2*d*f*x + 2*b^2*d*e - b^2*f)*sinh(d*x + c)^4 + 2*b^2*d*e - 8*(a*b*d*f*x +
a*b*d*e - a*b*f)*cosh(d*x + c)^3 - 4*(2*a*b*d*f*x + 2*a*b*d*e - 2*a*b*f -
(2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c))*sinh(d*x + c)^3 + b^2*f -
8*(a^2*d^2*f*x^2 + 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f)*cosh(d*x + c)
^2 - 2*(4*a^2*d^2*f*x^2 + 8*a^2*d^2*e*x + 16*a^2*c*d*e - 8*a^2*c^2*f - 3*(2
*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^2 + 12*(a*b*d*f*x + a*b*d*e -
a*b*f)*cosh(d*x + c))*sinh(d*x + c)^2 + 8*(a*b*d*f*x + a*b*d*e + a*b*f)*co
sh(d*x + c) + 16*(a^2*f*cosh(d*x + c)^2 + 2*a^2*f*cosh(d*x + c)*sinh(d*x +
c) + a^2*f*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 16*(a^2
*f*cosh(d*x + c)^2 + 2*a^2*f*cosh(d*x + c)*sinh(d*x + c) + a^2*f*sinh(d*x +
c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 16*((a^2*d*e - a^2*c*f)*cosh
```

```
(d*x + c)^2 + 2*(a^2*d*e - a^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d*e
- a^2*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b
*sqrt((a^2 + b^2)/b^2) + 2*a) + 16*((a^2*d*e - a^2*c*f)*cosh(d*x + c)^2 + 2
*(a^2*d*e - a^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d*e - a^2*c*f)*sinh
(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^
2)/b^2) + 2*a) + 16*((a^2*d*f*x + a^2*c*f)*cosh(d*x + c)^2 + 2*(a^2*d*f*x +
a^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d*f*x + a^2*c*f)*sinh(d*x + c)
^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 16*((a^2*d*f*x + a^2*c*f)*cosh(d*x +
c)^2 + 2*(a^2*d*f*x + a^2*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*d*f*x +
a^2*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(2*a*b*d*f*x
+ 2*a*b*d*e + (2*b^2*d*f*x + 2*b^2*d*e - b^2*f)*cosh(d*x + c)^3 + 2*a*b*f
- 6*(a*b*d*f*x + a*b*d*e - a*b*f)*cosh(d*x + c)^2 - 4*(a^2*d^2*f*x^2 + 2*a^
2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d
^2*cosh(d*x + c)^2 + 2*b^3*d^2*cosh(d*x + c)*sinh(d*x + c) + b^3*d^2*sinh(d
*x + c)^2)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \cosh(dx + c) \sinh(dx + c)^2}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

$$3.365 \quad \int \frac{\cosh(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=55

$$\frac{a^2 \log(a + b \sinh(c + dx))}{b^3 d} - \frac{a \sinh(c + dx)}{b^2 d} + \frac{\sinh^2(c + dx)}{2bd}$$

[Out] (a^2\*Log[a + b\*Sinh[c + d\*x]])/(b^3\*d) - (a\*Sinh[c + d\*x])/(b^2\*d) + Sinh[c + d\*x]^2/(2\*b\*d)

**Rubi [A]** time = 0.0808121, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2833, 12, 43}

$$\frac{a^2 \log(a + b \sinh(c + dx))}{b^3 d} - \frac{a \sinh(c + dx)}{b^2 d} + \frac{\sinh^2(c + dx)}{2bd}$$

Antiderivative was successfully verified.

[In] Int[(Cosh[c + d\*x]\*Sinh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (a^2\*Log[a + b\*Sinh[c + d\*x]])/(b^3\*d) - (a\*Sinh[c + d\*x])/(b^2\*d) + Sinh[c + d\*x]^2/(2\*b\*d)

### Rule 2833

Int[cos[(e\_.) + (f\_.)\*(x\_.)]\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Dist[1/(b\*f), Subst[Int[(a + x)^m\*(c + (d\*x)/b)^n, x], x, b\*Sin[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]

### Rule 12

Int[(a\_.)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_.)\*(v\_) /; FreeQ[b, x]]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le

Q[7\*m + 4\*n + 4, 0] || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\cosh(c + dx) \sinh^2(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\text{Subst}\left(\int \frac{x^2}{b^2(a+x)} dx, x, b \sinh(c + dx)\right)}{bd} \\ &= \frac{\text{Subst}\left(\int \frac{x^2}{a+x} dx, x, b \sinh(c + dx)\right)}{b^3d} \\ &= \frac{\text{Subst}\left(\int \left(-a + x + \frac{a^2}{a+x}\right) dx, x, b \sinh(c + dx)\right)}{b^3d} \\ &= \frac{a^2 \log(a + b \sinh(c + dx))}{b^3d} - \frac{a \sinh(c + dx)}{b^2d} + \frac{\sinh^2(c + dx)}{2bd} \end{aligned}$$

**Mathematica [A]** time = 0.0711854, size = 49, normalized size = 0.89

$$\frac{2a^2 \log(a + b \sinh(c + dx)) - 2ab \sinh(c + dx) + b^2 \sinh^2(c + dx)}{2b^3d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cosh[c + d\*x]\*Sinh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (2\*a^2\*Log[a + b\*Sinh[c + d\*x]] - 2\*a\*b\*Sinh[c + d\*x] + b^2\*Sinh[c + d\*x]^2)/(2\*b^3\*d)

**Maple [A]** time = 0.017, size = 54, normalized size = 1.

$$\frac{a^2 \ln(a + b \sinh(dx + c))}{b^3d} - \frac{a \sinh(dx + c)}{b^2d} + \frac{(\sinh(dx + c))^2}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] a^2\*ln(a+b\*sinh(d\*x+c))/b^3/d-a\*sinh(d\*x+c)/b^2/d+1/2\*sinh(d\*x+c)^2/b/d

---

**Maxima [B]** time = 1.05146, size = 161, normalized size = 2.93

$$\frac{(dx+c)a^2}{b^3d} - \frac{(4ae^{(-dx-c)} - b)e^{(2dx+2c)}}{8b^2d} + \frac{a^2 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{b^3d} + \frac{4ae^{(-dx-c)} + be^{(-2dx-2c)}}{8b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] (d*x + c)*a^2/(b^3*d) - 1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d)
+ a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^3*d) + 1/8*(4*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c))/(b^2*d)
```

---

**Fricas [B]** time = 2.21071, size = 784, normalized size = 14.25

$$8a^2dx \cosh(dx+c)^2 - b^2 \cosh(dx+c)^4 - b^2 \sinh(dx+c)^4 + 4ab \cosh(dx+c)^3 - 4(b^2 \cosh(dx+c) - ab) \sinh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8*(8*a^2*d*x*cosh(d*x + c)^2 - b^2*cosh(d*x + c)^4 - b^2*sinh(d*x + c)^4
+ 4*a*b*cosh(d*x + c)^3 - 4*(b^2*cosh(d*x + c) - a*b)*sinh(d*x + c)^3 - 4*
a*b*cosh(d*x + c) + 2*(4*a^2*d*x - 3*b^2*cosh(d*x + c)^2 + 6*a*b*cosh(d*x +
c))*sinh(d*x + c)^2 - b^2 - 8*(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*s
inh(d*x + c) + a^2*sinh(d*x + c)^2)*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x +
c) - sinh(d*x + c))) + 4*(4*a^2*d*x*cosh(d*x + c) - b^2*cosh(d*x + c)^3 +
3*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^2 + 2*b^3*
d*cosh(d*x + c)*sinh(d*x + c) + b^3*d*sinh(d*x + c)^2)
```

---

**Sympy [A]** time = 3.59674, size = 87, normalized size = 1.58

$$\begin{cases} \frac{x \sinh^2(c) \cosh(c)}{a} & \text{for } b = 0 \wedge d = 0 \\ \frac{\sinh^3(c+dx)}{3ad} & \text{for } b = 0 \\ \frac{x \sinh^2(c) \cosh(c)}{a+b \sinh(c)} & \text{for } d = 0 \\ \frac{a^2 \log\left(\frac{a}{b} + \sinh(c+dx)\right)}{b^3 d} - \frac{a \sinh(c+dx)}{b^2 d} + \frac{\sinh^2(c+dx)}{2bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*sinh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Piecewise((x\*sinh(c)\*\*2\*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (sinh(c + d\*x)\*\*3/(3\*a\*d), Eq(b, 0)), (x\*sinh(c)\*\*2\*cosh(c)/(a + b\*sinh(c)), Eq(d, 0)), (a\*\*2\*log(a/b + sinh(c + d\*x))/(b\*\*3\*d) - a\*sinh(c + d\*x)/(b\*\*2\*d) + sinh(c + d\*x)\*\*2/(2\*b\*d), True))

**Giac [A]** time = 1.29886, size = 123, normalized size = 2.24

$$\frac{a^2 \log\left(\left|b(e^{(dx+c)} - e^{(-dx-c)}) + 2a\right|\right)}{b^3 d} + \frac{bd(e^{(dx+c)} - e^{(-dx-c)})^2 - 4ad(e^{(dx+c)} - e^{(-dx-c)})}{8b^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] a^2\*log(abs(b\*(e^(d\*x + c) - e^(-d\*x - c)) + 2\*a))/(b^3\*d) + 1/8\*(b\*d\*(e^(d\*x + c) - e^(-d\*x - c))^2 - 4\*a\*d\*(e^(d\*x + c) - e^(-d\*x - c)))/(b^2\*d^2)



$$3.366 \quad \int \frac{\cosh(c+dx) \sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable} \left( \frac{\sinh^2(c+dx) \cosh(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[(Cosh[c + d\*x]\*Sinh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0869597, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cosh(c+dx) \sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Cosh[c + d\*x]\*Sinh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] Defer[Int] [(Cosh[c + d\*x]\*Sinh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\cosh(c+dx) \sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\cosh(c+dx) \sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.002, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Cosh[c + d\*x]\*Sinh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] \$Aborted

**Maple [A]** time = 0.26, size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c) (\sinh(dx+c))^2}{(fx+e)(a+b \sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)\*sinh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(cosh(d\*x+c)\*sinh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{e^{(-2c+\frac{2de}{f})} E_1\left(\frac{2(fx+e)d}{f}\right)}{4bf} + \frac{ae^{(-c+\frac{de}{f})} E_1\left(\frac{(fx+e)d}{f}\right)}{2b^2f} + \frac{ae^{(c-\frac{de}{f})} E_1\left(-\frac{(fx+e)d}{f}\right)}{2b^2f} - \frac{e^{(2c-\frac{2de}{f})} E_1\left(-\frac{2(fx+e)d}{f}\right)}{4bf} + \frac{a^2 \log(fx+e)}{b^3f} - \frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*sinh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 1/4\*e^(-2\*c + 2\*d\*e/f)\*exp\_integral\_e(1, 2\*(f\*x + e)\*d/f)/(b\*f) + 1/2\*a\*e^(-c + d\*e/f)\*exp\_integral\_e(1, (f\*x + e)\*d/f)/(b^2\*f) + 1/2\*a\*e^(c - d\*e/f)\*exp\_integral\_e(1, -(f\*x + e)\*d/f)/(b^2\*f) - 1/4\*e^(2\*c - 2\*d\*e/f)\*exp\_integral\_e(1, -2\*(f\*x + e)\*d/f)/(b\*f) + a^2\*log(f\*x + e)/(b^3\*f) - 1/8\*integrate(-16\*(a^3\*e^(d\*x + c) - a^2\*b)/(b^4\*f\*x + b^4\*e - (b^4\*f\*x\*e^(2\*c) + b^4\*e\*e^(2\*c)))\*e^(2\*d\*x) - 2\*(a\*b^3\*f\*x\*e^c + a\*b^3\*e\*e^c)\*e^(d\*x), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx+c) \sinh(dx+c)^2}{afx+ae+(bfx+be) \sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)*sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c) \sinh(dx + c)^2}{(fx + e)(b \sinh(dx + c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)*sinh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

$$3.367 \quad \int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=897

$$\frac{a(e+fx)^4}{8b^2f} - \frac{a^3(e+fx)^4}{4b^4f} + \frac{\cosh^3(c+dx)(e+fx)^3}{3bd} + \frac{a^2 \cosh(c+dx)(e+fx)^3}{b^3d} + \frac{a^2 \sqrt{a^2+b^2} \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}} + 1\right)(e+fx)}{b^4d}$$

[Out]  $(-3*a*e*f^2*x)/(4*b^2*d^2) - (3*a*f^3*x^2)/(8*b^2*d^2) - (a^3*(e+f*x)^4)/(4*b^4*f) - (a*(e+f*x)^4)/(8*b^2*f) + (6*a^2*f^2*(e+f*x)*\text{Cosh}[c+dx])/ (b^3*d^3) + (4*f^2*(e+f*x)*\text{Cosh}[c+dx])/ (3*b*d^3) + (a^2*(e+f*x)^3*\text{Cosh}[c+dx])/ (b^3*d) + (3*a*f^3*\text{Cosh}[c+dx]^2)/(8*b^2*d^4) + (3*a*f*(e+f*x)^2*\text{Cosh}[c+dx]^2)/(4*b^2*d^2) + (2*f^2*(e+f*x)*\text{Cosh}[c+dx]^3)/(9*b*d^3) + ((e+f*x)^3*\text{Cosh}[c+dx]^3)/(3*b*d) + (a^2*\text{Sqrt}[a^2+b^2]*(e+f*x)^3*\text{Log}[1+(b*E^(c+dx))/(a-\text{Sqrt}[a^2+b^2])])/ (b^4*d) - (a^2*\text{Sqrt}[a^2+b^2]*(e+f*x)^3*\text{Log}[1+(b*E^(c+dx))/(a+\text{Sqrt}[a^2+b^2])])/ (b^4*d) + (3*a^2*\text{Sqrt}[a^2+b^2]*f*(e+f*x)^2*\text{PolyLog}[2,-((b*E^(c+dx))/(a-\text{Sqrt}[a^2+b^2]))])/ (b^4*d^2) - (3*a^2*\text{Sqrt}[a^2+b^2]*f*(e+f*x)^2*\text{PolyLog}[2,-((b*E^(c+dx))/(a+\text{Sqrt}[a^2+b^2]))])/ (b^4*d^2) - (6*a^2*\text{Sqrt}[a^2+b^2]*f^2*(e+f*x)*\text{PolyLog}[3,-((b*E^(c+dx))/(a-\text{Sqrt}[a^2+b^2]))])/ (b^4*d^3) + (6*a^2*\text{Sqrt}[a^2+b^2]*f^2*(e+f*x)*\text{PolyLog}[3,-((b*E^(c+dx))/(a+\text{Sqrt}[a^2+b^2]))])/ (b^4*d^3) + (6*a^2*\text{Sqrt}[a^2+b^2]*f^3*\text{PolyLog}[4,-((b*E^(c+dx))/(a-\text{Sqrt}[a^2+b^2]))])/ (b^4*d^4) - (6*a^2*\text{Sqrt}[a^2+b^2]*f^3*\text{PolyLog}[4,-((b*E^(c+dx))/(a+\text{Sqrt}[a^2+b^2]))])/ (b^4*d^4) - (6*a^2*f^3*\text{Sinh}[c+dx])/ (b^3*d^4) - (14*f^3*\text{Sinh}[c+dx])/ (9*b*d^4) - (3*a^2*f*(e+f*x)^2*\text{Sinh}[c+dx])/ (b^3*d^2) - (2*f*(e+f*x)^2*\text{Sinh}[c+dx])/ (3*b*d^2) - (3*a*f^2*(e+f*x)*\text{Cosh}[c+dx]*\text{Sinh}[c+dx])/ (4*b^2*d^3) - (a*(e+f*x)^3*\text{Cosh}[c+dx]*\text{Sinh}[c+dx])/ (2*b^2*d) - (f*(e+f*x)^2*\text{Cosh}[c+dx]^2*\text{Sinh}[c+dx])/ (3*b*d^2) - (2*f^3*\text{Sinh}[c+dx]^3)/(27*b*d^4)$

**Rubi [A]** time = 1.47301, antiderivative size = 897, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 16, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {5579, 5447, 3311, 3296, 2637, 2633, 32, 3310, 5565, 3322, 2264, 2190, 2531, 6609, 2282, 6589}

$$\frac{a(e+fx)^4}{8b^2f} - \frac{a^3(e+fx)^4}{4b^4f} + \frac{\cosh^3(c+dx)(e+fx)^3}{3bd} + \frac{a^2 \cosh(c+dx)(e+fx)^3}{b^3d} + \frac{a^2 \sqrt{a^2+b^2} \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}} + 1\right)(e+fx)}{b^4d}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Cosh[c + d\*x]^2\*Sinh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] 
$$\begin{aligned} & (-3*a*e*f^2*x)/(4*b^2*d^2) - (3*a*f^3*x^2)/(8*b^2*d^2) - (a^3*(e + f*x)^4)/ \\ & (4*b^4*f) - (a*(e + f*x)^4)/(8*b^2*f) + (6*a^2*f^2*(e + f*x)*Cosh[c + d*x]) \\ & / (b^3*d^3) + (4*f^2*(e + f*x)*Cosh[c + d*x])/(3*b*d^3) + (a^2*(e + f*x)^3*C \\ & osh[c + d*x])/(b^3*d) + (3*a*f^3*Cosh[c + d*x]^2)/(8*b^2*d^4) + (3*a*f*(e + \\ & f*x)^2*Cosh[c + d*x]^2)/(4*b^2*d^2) + (2*f^2*(e + f*x)*Cosh[c + d*x]^3)/(9 \\ & *b*d^3) + ((e + f*x)^3*Cosh[c + d*x]^3)/(3*b*d) + (a^2*sqrt[a^2 + b^2]*(e + \\ & f*x)^3*Log[1 + (b*E^(c + d*x))/(a - sqrt[a^2 + b^2])])/(b^4*d) - (a^2*sqrt \\ & [a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + sqrt[a^2 + b^2])])/(b^ \\ & 4*d) + (3*a^2*sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -(b*E^(c + d*x))/(a \\ & - sqrt[a^2 + b^2])])/(b^4*d^2) - (3*a^2*sqrt[a^2 + b^2]*f*(e + f*x)^2*Pol \\ & yLog[2, -(b*E^(c + d*x))/(a + sqrt[a^2 + b^2])])/(b^4*d^2) - (6*a^2*sqrt[ \\ & a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -(b*E^(c + d*x))/(a - sqrt[a^2 + b^2]) \\ & ])/(b^4*d^3) + (6*a^2*sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -(b*E^(c + \\ & d*x))/(a + sqrt[a^2 + b^2])])/(b^4*d^3) + (6*a^2*sqrt[a^2 + b^2]*f^3*Poly \\ & Log[4, -(b*E^(c + d*x))/(a - sqrt[a^2 + b^2])])/(b^4*d^4) - (6*a^2*sqrt[ \\ & a^2 + b^2]*f^3*PolyLog[4, -(b*E^(c + d*x))/(a + sqrt[a^2 + b^2])])/(b^4*d^ \\ & 4) - (6*a^2*f^3*Sinh[c + d*x])/(b^3*d^4) - (14*f^3*Sinh[c + d*x])/(9*b*d^4) \\ & - (3*a^2*f*(e + f*x)^2*Sinh[c + d*x])/(b^3*d^2) - (2*f*(e + f*x)^2*Sinh[ \\ & c + d*x])/(3*b*d^2) - (3*a*f^2*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^2* \\ & d^3) - (a*(e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d) - (f*(e + f*x) \\ & ^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b*d^2) - (2*f^3*Sinh[c + d*x]^3)/(27*b \\ & *d^4) \end{aligned}$$

### Rule 5579

Int[(Cosh[(c\_.) + (d\_.)\*(x\_.)]^(p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^p\*Sinh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Cosh[c + d\*x]^p\*Sinh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5447

Int[Cosh[(a\_.) + (b\_.)\*(x\_.)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*Sinh[(a\_.) + (b\_.)\*(x\_.)], x\_Symbol] := Simp[((c + d\*x)^m\*Cosh[a + b\*x]^(n + 1))/(b\*(n + 1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*Cosh[a + b\*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]^(n\_.), x\_Symbol]

```

1] := Simp[(d*m*(c + d*x)^(m - 1)*(b*SIN[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*SIN[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*SIN[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*SIN[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

```

### Rule 3296

```

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

```

### Rule 2637

```

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[SIN[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

```

### Rule 2633

```

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]

```

### Rule 32

```

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

```

### Rule 3310

```

Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
Simp[(d*(b*SIN[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*SIN[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*SIN[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]

```

### Rule 5565

```

Int[(Cosh[(c_.) + (d_.)*(x_)])^(n_)*((e_.) + (f_.)*(x_))^(m_.)/((a_.) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_] *
(f_.)*(x_))], x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :=> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :=> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
)))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :=> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] :=> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_.)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```





**Mathematica [A]** time = 7.79542, size = 1667, normalized size = 1.86

result too large to display

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^3\*Cosh[c + d\*x]^2\*Sinh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] 
$$-(432*a^3*d^4*e^3*x + 216*a*b^2*d^4*e^3*x + 648*a^3*d^4*e^2*f*x^2 + 324*a*b^2*d^4*e^2*f*x^2 + 432*a^3*d^4*e*f^2*x^3 + 216*a*b^2*d^4*e*f^2*x^3 + 108*a^3*d^4*f^3*x^4 + 54*a*b^2*d^4*f^3*x^4 + 864*a^2*\text{Sqrt}[a^2 + b^2]*d^3*e^3*\text{ArcTanh}[(a + b*E^{(c + d*x)})/\text{Sqrt}[a^2 + b^2]] - 432*a^2*b*d^3*e^3*\text{Cosh}[c + d*x] - 108*b^3*d^3*e^3*\text{Cosh}[c + d*x] - 2592*a^2*b*d*e*f^2*\text{Cosh}[c + d*x] - 648*b^3*d*e*f^2*\text{Cosh}[c + d*x] - 1296*a^2*b*d^3*e^2*f*x*\text{Cosh}[c + d*x] - 324*b^3*d^3*e^2*f*x*\text{Cosh}[c + d*x] - 2592*a^2*b*d*f^3*x*\text{Cosh}[c + d*x] - 648*b^3*d*f^3*x*\text{Cosh}[c + d*x] - 1296*a^2*b*d^3*e*f^2*x^2*\text{Cosh}[c + d*x] - 324*b^3*d^3*e*f^2*x^2*\text{Cosh}[c + d*x] - 432*a^2*b*d^3*f^3*x^3*\text{Cosh}[c + d*x] - 108*b^3*d^3*f^3*x^3*\text{Cosh}[c + d*x] - 162*a*b^2*d^2*e^2*f*\text{Cosh}[2*(c + d*x)] - 81*a*b^2*f^3*\text{Cosh}[2*(c + d*x)] - 324*a*b^2*d^2*e*f^2*x*\text{Cosh}[2*(c + d*x)] - 162*a*b^2*d^2*f^3*x^2*\text{Cosh}[2*(c + d*x)] - 36*b^3*d^3*e^3*\text{Cosh}[3*(c + d*x)] - 24*b^3*d*e*f^2*\text{Cosh}[3*(c + d*x)] - 108*b^3*d^3*e^2*f*x*\text{Cosh}[3*(c + d*x)] - 24*b^3*d*f^3*x*\text{Cosh}[3*(c + d*x)] - 108*b^3*d^3*e*f^2*x^2*\text{Cosh}[3*(c + d*x)] - 36*b^3*d^3*f^3*x^3*\text{Cosh}[3*(c + d*x)] - 1296*a^2*\text{Sqrt}[a^2 + b^2]*d^3*e^2*f*x*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])] - 1296*a^2*\text{Sqrt}[a^2 + b^2]*d^3*e*f^2*x^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])] - 432*a^2*\text{Sqrt}[a^2 + b^2]*d^3*f^3*x^3*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])] + 1296*a^2*\text{Sqrt}[a^2 + b^2]*d^3*e^2*f*x*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])] + 1296*a^2*\text{Sqrt}[a^2 + b^2]*d^3*e*f^2*x^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])] + 432*a^2*\text{Sqrt}[a^2 + b^2]*d^3*f^3*x^3*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])] - 1296*a^2*\text{Sqrt}[a^2 + b^2]*d^2*f*(e + f*x)^2*\text{PolyLog}[2, (b*E^{(c + d*x)})/(-a + \text{Sqrt}[a^2 + b^2])] + 1296*a^2*\text{Sqrt}[a^2 + b^2]*d^2*f*(e + f*x)^2*\text{PolyLog}[2, -(b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])] + 2592*a^2*\text{Sqrt}[a^2 + b^2]*d*e*f^2*\text{PolyLog}[3, (b*E^{(c + d*x)})/(-a + \text{Sqrt}[a^2 + b^2])] + 2592*a^2*\text{Sqrt}[a^2 + b^2]*d*f^3*x*\text{PolyLog}[3, (b*E^{(c + d*x)})/(-a + \text{Sqrt}[a^2 + b^2])] - 2592*a^2*\text{Sqrt}[a^2 + b^2]*d*e*f^2*\text{PolyLog}[3, -(b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])] - 2592*a^2*\text{Sqrt}[a^2 + b^2]*d*f^3*x*\text{PolyLog}[3, -(b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])] - 2592*a^2*\text{Sqrt}[a^2 + b^2]*f^3*\text{PolyLog}[4, (b*E^{(c + d*x)})/(-a + \text{Sqrt}[a^2 + b^2])] + 2592*a^2*\text{Sqrt}[a^2 + b^2]*f^3*\text{PolyLog}[4, -(b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])] + 1296*a^2*b*d^2*e^2*f*\text{Sinh}[c + d*x] + 324*b^3*d^2*e^2*f*\text{Sinh}[c + d*x] + 2592*a^2*b*f^3*\text{Sinh}[c + d*x] + 648*b^3*f^3*\text{Sinh}[c + d*x] + 2592*a^2*b*d^2*e*f^2*x*\text{Sinh}[c + d*x] + 648*b^3*d^2*e*f^2*x*\text{Sinh}[c + d*x] + 1296*a^2*b*d^2*f^3*x^2*\text{Sinh}[c + d*x] + 324*b^3*d^2*f^3*x^2*\text{Sinh}[c + d*x] + 108*a*b^2*d^3*e^3*\text{Sinh}[2*(c + d*x)] + 16$$

$$\frac{2ab^2de^2\sinh[2(c+dx)] + 324ab^2d^3e^2fx\sinh[2(c+dx)] + 162ab^2d^3f^3x\sinh[2(c+dx)] + 324ab^2d^3e^2fx^2\sinh[2(c+dx)] + 108ab^2d^3f^3x^3\sinh[2(c+dx)] + 36b^3d^2e^2f\sinh[3(c+dx)] + 8b^3f^3\sinh[3(c+dx)] + 72b^3d^2e^2fx\sinh[3(c+dx)] + 36b^3d^2f^3x^2\sinh[3(c+dx)]}{(432b^4d^4)}$$

**Maple [F]** time = 0.217, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 (\cosh(dx+c))^2 (\sinh(dx+c))^2}{a+b\sinh(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*cosh(d\*x+c)^2\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*cosh(d\*x+c)^2\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)^2\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 3.59105, size = 15351, normalized size = 17.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)^2\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

```
[Out] 1/864*(36*b^3*d^3*f^3*x^3 + 36*b^3*d^3*e^3 + 36*b^3*d^2*e^2*f + 24*b^3*d*e*f^2 + 4*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*cosh(d*x + c)^6 + 4*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*sinh(d*x + c)^6 + 8*b^3*f^3 - 27*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x)*cosh(d*x + c)^5 - 3*(36*a*b^2*d^3*f^3*x^3 + 36*a*b^2*d^3*e^3 - 54*a*b^2*d^2*e^2*f + 54*a*b^2*d*e*f^2 - 27*a*b^2*f^3 + 54*(2*a*b^2*d^3*e*f^2 - a*b^2*d^2*f^3)*x^2 + 54*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x - 8*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 108*((4*a^2*b + b^3)*d^3*f^3*x^3 + (4*a^2*b + b^3)*d^3*e^3 - 3*(4*a^2*b + b^3)*d^2*e^2*f + 6*(4*a^2*b + b^3)*d*e*f^2 - 6*(4*a^2*b + b^3)*f^3 + 3*((4*a^2*b + b^3)*d^3*e*f^2 - (4*a^2*b + b^3)*d^2*f^3)*x^2 + 3*((4*a^2*b + b^3)*d^3*e^2*f - 2*(4*a^2*b + b^3)*d^2*e*f^2 + 2*(4*a^2*b + b^3)*d*f^3)*x)*cosh(d*x + c)^4 + 3*(36*(4*a^2*b + b^3)*d^3*f^3*x^3 + 36*(4*a^2*b + b^3)*d^3*e^3 - 108*(4*a^2*b + b^3)*d^2*e^2*f + 216*(4*a^2*b + b^3)*d*e*f^2 - 216*(4*a^2*b + b^3)*f^3 + 108*((4*a^2*b + b^3)*d^3*e*f^2 - (4*a^2*b + b^3)*d^2*f^3)*x^2 + 20*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*cosh(d*x + c)^2 + 108*((4*a^2*b + b^3)*d^3*e^2*f - 2*(4*a^2*b + b^3)*d^2*e*f^2 + 2*(4*a^2*b + b^3)*d*f^3)*x - 45*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^4 - 108*((2*a^3 + a*b^2)*d^4*f^3*x^4 + 4*(2*a^3 + a*b^2)*d^4*e*f^2*x^3 + 6*(2*a^3 + a*b^2)*d^4*e^2*f*x^2 + 4*(2*a^3 + a*b^2)*d^4*e^3*x)*cosh(d*x + c)^3 - 2*(54*(2*a^3 + a*b^2)*d^4*f^3*x^4 + 216*(2*a^3 + a*b^2)*d^4*e*f^2*x^3 + 324*(2*a^3 + a*b^2)*d^4*e^2*f*x^2 + 216*(2*a^3 + a*b^2)*d^4*e^3*x - 40*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*cosh(d*x + c)^3 + 135*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x)*cosh(d*x + c)^2 - 216*((4*a^2*b + b^3)*d^3*f^3*x^3 + (4*a^2*b + b^3)*d^3*e^3 - 3*(4*a^2*b + b^3)*d^2*e^2*f + 6*(4*a^2*b + b^3)*d*e*f^2 - 6*(4*a^2*b + b^3)*f^3 + 3*((4*a^2*b + b^3)*d^3*e*f^2 - (4*a^2*b + b^3)*d^2*f^3)*x^2 + 3*((4*a^2*b + b^3)*d^3*e^2*f - 2*(4*a^2*b + b^3)*d^2*e*f^2 + 2*(4*a^2*b + b^3)*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 36*(3*b^3*d^3*e*f^2 + b^3*d^2*f^3)*x^2 + 108*((4*a^2*b + b^3)*d^3*f^3*x^3 + (4*a^2*b + b^3)*d^3*e^3 + 3*(4*a^2*b + b^3)*d^2*e^2*f + 6*(4*a^2*b + b^3)*d*e*f^2 + 6*(4
```

$$\begin{aligned}
& *a^2*b + b^3)*f^3 + 3*((4*a^2*b + b^3)*d^3*e*f^2 + (4*a^2*b + b^3)*d^2*f^3) \\
& *x^2 + 3*((4*a^2*b + b^3)*d^3*e^2*f + 2*(4*a^2*b + b^3)*d^2*e*f^2 + 2*(4*a^ \\
& 2*b + b^3)*d*f^3)*x)*\cosh(d*x + c)^2 + 6*(18*(4*a^2*b + b^3)*d^3*f^3*x^3 + \\
& 18*(4*a^2*b + b^3)*d^3*e^3 + 54*(4*a^2*b + b^3)*d^2*e^2*f + 108*(4*a^2*b + \\
& b^3)*d*e*f^2 + 10*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6* \\
& b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3* \\
& d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*\cosh(d*x + c)^4 + 108*(4*a^2* \\
& b + b^3)*f^3 - 45*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 - 6*a*b^2*d^2*e^2* \\
& f + 6*a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 - a*b^2*d^2*f^3)*x \\
& ^2 + 6*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x)*\cosh(d*x + \\
& c)^3 + 54*((4*a^2*b + b^3)*d^3*e*f^2 + (4*a^2*b + b^3)*d^2*f^3)*x^2 + 108*( \\
& (4*a^2*b + b^3)*d^3*f^3*x^3 + (4*a^2*b + b^3)*d^3*e^3 - 3*(4*a^2*b + b^3)*d \\
& ^2*e^2*f + 6*(4*a^2*b + b^3)*d*e*f^2 - 6*(4*a^2*b + b^3)*f^3 + 3*((4*a^2*b \\
& + b^3)*d^3*e*f^2 - (4*a^2*b + b^3)*d^2*f^3)*x^2 + 3*((4*a^2*b + b^3)*d^3*e^ \\
& 2*f - 2*(4*a^2*b + b^3)*d^2*e*f^2 + 2*(4*a^2*b + b^3)*d*f^3)*x)*\cosh(d*x + \\
& c)^2 + 54*((4*a^2*b + b^3)*d^3*e^2*f + 2*(4*a^2*b + b^3)*d^2*e*f^2 + 2*(4*a \\
& ^2*b + b^3)*d*f^3)*x - 54*((2*a^3 + a*b^2)*d^4*f^3*x^4 + 4*(2*a^3 + a*b^2)* \\
& d^4*e*f^2*x^3 + 6*(2*a^3 + a*b^2)*d^4*e^2*f*x^2 + 4*(2*a^3 + a*b^2)*d^4*e^3 \\
& *x)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 2592*((a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2 \\
& *e*f^2*x + a^2*b*d^2*e^2*f)*\cosh(d*x + c)^3 + 3*(a^2*b*d^2*f^3*x^2 + 2*a^2* \\
& b*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*(a^2*b*d \\
& ^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*\cosh(d*x + c)*\sinh(d*x \\
& + c)^2 + (a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*\sinh(d \\
& *x + c)^3)*\sqrt{((a^2 + b^2)/b^2)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + \\
& (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 25 \\
& 92*((a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*\cosh(d*x + \\
& c)^3 + 3*(a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*\cosh(d \\
& *x + c)^2*\sinh(d*x + c) + 3*(a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2* \\
& b*d^2*e^2*f)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a^2*b*d^2*f^3*x^2 + 2*a^2*b*d \\
& ^2*e*f^2*x + a^2*b*d^2*e^2*f)*\sinh(d*x + c)^3)*\sqrt{(a^2 + b^2)/b^2)*\operatorname{dilog} \\
& (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{ \\
& (a^2 + b^2)/b^2} - b)/b + 1) - 864*((a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f \\
& + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*\cosh(d*x + c)^3 + 3*(a^2*b*d^3*e^3 \\
& - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*\cosh(d*x + c)^ \\
& 2*\sinh(d*x + c) + 3*(a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e* \\
& f^2 - a^2*b*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a^2*b*d^3*e^3 - 3*a^2 \\
& *b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*\sinh(d*x + c)^3)*\sqrt{ \\
& ((a^2 + b^2)/b^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 \\
& + b^2)/b^2} + 2*a) + 864*((a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c \\
& ^2*d*e*f^2 - a^2*b*c^3*f^3)*\cosh(d*x + c)^3 + 3*(a^2*b*d^3*e^3 - 3*a^2*b*c* \\
& d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*\cosh(d*x + c)^2*\sinh(d*x + \\
& c) + 3*(a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b* \\
& c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2 \\
& *f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*\sinh(d*x + c)^3)*\sqrt{(a^2 + b^2) \\
& /b^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2})
\end{aligned}$$

$$\begin{aligned}
& + 2*a) + 864*((a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*\cosh(d*x + c)^3 + 3*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*\sinh(d*x + c)^3)*\sqrt{(a^2 + b^2)/b^2}*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c)))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 864*((a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*\cosh(d*x + c)^3 + 3*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*\sinh(d*x + c)^3)*\sqrt{(a^2 + b^2)/b^2}*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c)))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + 5184*(a^2*b*f^3*\cosh(d*x + c)^3 + 3*a^2*b*f^3*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*a^2*b*f^3*\cosh(d*x + c)*\sinh(d*x + c)^2 + a^2*b*f^3*\sinh(d*x + c)^3)*\sqrt{(a^2 + b^2)/b^2}*polylog(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) - 5184*(a^2*b*f^3*\cosh(d*x + c)^3 + 3*a^2*b*f^3*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*a^2*b*f^3*\cosh(d*x + c)*\sinh(d*x + c)^2 + a^2*b*f^3*\sinh(d*x + c)^3)*\sqrt{(a^2 + b^2)/b^2}*polylog(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) - 5184*((a^2*b*d*f^3*x + a^2*b*d*e*f^2)*\cosh(d*x + c)^3 + 3*(a^2*b*d*f^3*x + a^2*b*d*e*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*(a^2*b*d*f^3*x + a^2*b*d*e*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a^2*b*d*f^3*x + a^2*b*d*e*f^2)*\sinh(d*x + c)^3)*\sqrt{(a^2 + b^2)/b^2}*polylog(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) + 5184*((a^2*b*d*f^3*x + a^2*b*d*e*f^2)*\cosh(d*x + c)^3 + 3*(a^2*b*d*f^3*x + a^2*b*d*e*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*(a^2*b*d*f^3*x + a^2*b*d*e*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a^2*b*d*f^3*x + a^2*b*d*e*f^2)*\sinh(d*x + c)^3)*\sqrt{(a^2 + b^2)/b^2}*polylog(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) + 12*(9*b^3*d^3*e^2*f + 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x + 27*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 + 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 + 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 + a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f + 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x)*\cosh(d*x + c) + 3*(36*a*b^2*d^3*f^3*x^3 + 36*a*b^2*d^3*e^3 + 54*a*b^2*d^2*e^2*f + 54*a*b^2*d*e*f^2 + 27*a*b^2*f^3 + 8*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*\cosh(d*x + c)^5 -
\end{aligned}$$

$$45*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d^3*e^3 - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3*e*f^2 - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x)*\cosh(d*x + c)^4 + 144*((4*a^2*b + b^3)*d^3*f^3*x^3 + (4*a^2*b + b^3)*d^3*e^3 - 3*(4*a^2*b + b^3)*d^2*e^2*f + 6*(4*a^2*b + b^3)*d*e*f^2 - 6*(4*a^2*b + b^3)*f^3 + 3*((4*a^2*b + b^3)*d^3*e*f^2 - (4*a^2*b + b^3)*d^2*f^3)*x^2 + 3*((4*a^2*b + b^3)*d^3*e^2*f - 2*(4*a^2*b + b^3)*d^2*e*f^2 + 2*(4*a^2*b + b^3)*d*f^3)*x)*\cosh(d*x + c)^3 + 54*(2*a*b^2*d^3*e*f^2 + a*b^2*d^2*f^3)*x^2 - 108*((2*a^3 + a*b^2)*d^4*f^3*x^4 + 4*(2*a^3 + a*b^2)*d^4*e*f^2*x^3 + 6*(2*a^3 + a*b^2)*d^4*e^2*f*x^2 + 4*(2*a^3 + a*b^2)*d^4*e^3*x)*\cosh(d*x + c)^2 + 54*(2*a*b^2*d^3*e^2*f + 2*a*b^2*d^2*e*f^2 + a*b^2*d*f^3)*x + 72*((4*a^2*b + b^3)*d^3*f^3*x^3 + (4*a^2*b + b^3)*d^3*e^3 + 3*(4*a^2*b + b^3)*d^2*e^2*f + 6*(4*a^2*b + b^3)*d*e*f^2 + 6*(4*a^2*b + b^3)*f^3 + 3*((4*a^2*b + b^3)*d^3*e*f^2 + (4*a^2*b + b^3)*d^2*f^3)*x^2 + 3*((4*a^2*b + b^3)*d^3*e^2*f + 2*(4*a^2*b + b^3)*d^2*e*f^2 + 2*(4*a^2*b + b^3)*d*f^3)*x)*\cosh(d*x + c))*\sinh(d*x + c)/(b^4*d^4*\cosh(d*x + c)^3 + 3*b^4*d^4*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*b^4*d^4*\cosh(d*x + c)*\sinh(d*x + c)^2 + b^4*d^4*\sinh(d*x + c)^3)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cosh(d\*x+c)\*\*2\*sinh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c)^2 \sinh(dx + c)^2}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)^2\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*cosh(d\*x + c)^2\*sinh(d\*x + c)^2/(b\*sinh(d\*x + c) + a), x)

$$3.368 \quad \int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=649

$$\frac{2a^2 f \sqrt{a^2 + b^2} (e + fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^4 d^2} - \frac{2a^2 f \sqrt{a^2 + b^2} (e + fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right)}{b^4 d^2} - \frac{2a^2 f^2 \sqrt{a^2 + b^2} \text{PolyLog}\left(3, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^4 d^2} - \frac{2a^2 f^2 \sqrt{a^2 + b^2} \text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right)}{b^4 d^2}$$

```
[Out] -(a*f^2*x)/(4*b^2*d^2) - (a^3*(e + f*x)^3)/(3*b^4*f) - (a*(e + f*x)^3)/(6*b^2*f) + (2*a^2*f^2*Cosh[c + d*x])/(b^3*d^3) + (4*f^2*Cosh[c + d*x])/(9*b*d^3) + (a^2*(e + f*x)^2*Cosh[c + d*x])/(b^3*d) + (a*f*(e + f*x)*Cosh[c + d*x]^2)/(2*b^2*d^2) + (2*f^2*Cosh[c + d*x]^3)/(27*b*d^3) + ((e + f*x)^2*Cosh[c + d*x]^3)/(3*b*d) + (a^2*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]))/(b^4*d) - (a^2*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]))/(b^4*d) + (2*a^2*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^2) - (2*a^2*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^2) - (2*a^2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^3) + (2*a^2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^3) - (2*a^2*f*(e + f*x)*Sinh[c + d*x])/(b^3*d^2) - (4*f*(e + f*x)*Sinh[c + d*x])/(9*b*d^2) - (a*f^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^2*d^3) - (a*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d) - (2*f*(e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(9*b*d^2)
```

**Rubi [A]** time = 1.20186, antiderivative size = 649, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 16, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {5579, 5447, 3310, 3296, 2638, 3311, 32, 2635, 8, 5565, 3322, 2264, 2190, 2531, 2282, 6589}

$$\frac{2a^2 f \sqrt{a^2 + b^2} (e + fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^4 d^2} - \frac{2a^2 f \sqrt{a^2 + b^2} (e + fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right)}{b^4 d^2} - \frac{2a^2 f^2 \sqrt{a^2 + b^2} \text{PolyLog}\left(3, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^4 d^2} - \frac{2a^2 f^2 \sqrt{a^2 + b^2} \text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right)}{b^4 d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(a*f^2*x)/(4*b^2*d^2) - (a^3*(e + f*x)^3)/(3*b^4*f) - (a*(e + f*x)^3)/(6*b^2*f) + (2*a^2*f^2*Cosh[c + d*x])/(b^3*d^3) + (4*f^2*Cosh[c + d*x])/(9*b*d^3) + (a^2*(e + f*x)^2*Cosh[c + d*x])/(b^3*d) + (a*f*(e + f*x)*Cosh[c + d*x]^2)/(2*b^2*d^2) + (2*f^2*Cosh[c + d*x]^3)/(27*b*d^3) + ((e + f*x)^2*Cosh[c + d*x]^3)/(3*b*d)
```

$$\begin{aligned}
& + d*x]^3)/(3*b*d) + (a^2*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x) \\
& )/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a^2*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 \\
& + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) + (2*a^2*Sqrt[a^2 + b^2]* \\
& f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^2) \\
& - (2*a^2*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqr \\
& t[a^2 + b^2]))])/(b^4*d^2) - (2*a^2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^( \\
& c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^3) + (2*a^2*Sqrt[a^2 + b^2]*f^2*P \\
& olyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^4*d^3) - (2*a^2*f*( \\
& e + f*x)*Sinh[c + d*x])/(b^3*d^2) - (4*f*(e + f*x)*Sinh[c + d*x])/(9*b*d^2) \\
& - (a*f^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^2*d^3) - (a*(e + f*x)^2*Cosh[c \\
& + d*x]*Sinh[c + d*x])/(2*b^2*d) - (2*f*(e + f*x)*Cosh[c + d*x]^2*Sinh[c + d \\
& *x])/(9*b*d^2)
\end{aligned}$$
Rule 5579

```

Int[(Cosh[(c_.) + (d_.)*(x_.)]^(p_.)*((e_.) + (f_.)*(x_.))^(m_.)*Sinh[(c_.) +
(d_.)*(x_.)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
0] && IGtQ[p, 0]

```

Rule 5447

```

Int[Cosh[(a_.) + (b_.)*(x_.)]^(n_.)*((c_.) + (d_.)*(x_.))^(m_.)*Sinh[(a_.) +
(b_.)*(x_.)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

```

Rule 3310

```

Int[((c_.) + (d_.)*(x_.))*((b_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :>
Simp[(d*(b*Ssin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Ssin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Ssin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]

```

Rule 3296

```

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

```

Rule 2638



Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*Ssin[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*Ssin[e + f\*x])^(n - 2), x], x] - Dist[(d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*Ssin[e + f\*x])^n, x], x] - Simp[(b\*(c + d\*x)^m\*Cos[e + f\*x]\*(b\*Ssin[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x]\*(b\*Ssin[c + d\*x])^(n - 1))/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Ssin[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rule 5565

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)])^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(m\_.)/((a\_) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Dist[a/b^2, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

### Rule 3322

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_) + (b\_.)\*sin[(e\_.) + (Complex[0, fz\_])\* (f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(-(I\*b) + 2\*a\*E^(-(I\*e) + f\*fz\*x) + I\*b\*E^(2\*(-(I\*e) + f\*fz\*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \cosh^2(c+dx) \sinh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{(e+fx)^2 \cosh^3(c+dx)}{3bd} - \frac{a \int (e+fx)^2 \cosh^2(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)^2 \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
&= \frac{af(e+fx) \cosh^2(c+dx)}{2b^2d^2} + \frac{2f^2 \cosh^3(c+dx)}{27bd^3} + \frac{(e+fx)^2 \cosh^3(c+dx)}{3bd} \\
&= -\frac{a^3(e+fx)^3}{3b^4f} - \frac{a(e+fx)^3}{6b^2f} + \frac{a^2(e+fx)^2 \cosh(c+dx)}{b^3d} + \frac{af(e+fx) \cosh(c+dx)}{2b^2d^2} \\
&= -\frac{af^2x}{4b^2d^2} - \frac{a^3(e+fx)^3}{3b^4f} - \frac{a(e+fx)^3}{6b^2f} + \frac{4f^2 \cosh(c+dx)}{9bd^3} + \frac{a^2(e+fx)^2 \cosh(c+dx)}{b^3d} \\
&= -\frac{af^2x}{4b^2d^2} - \frac{a^3(e+fx)^3}{3b^4f} - \frac{a(e+fx)^3}{6b^2f} + \frac{2a^2f^2 \cosh(c+dx)}{b^3d^3} + \frac{4f^2 \cosh(c+dx)}{9bd^3} \\
&= -\frac{af^2x}{4b^2d^2} - \frac{a^3(e+fx)^3}{3b^4f} - \frac{a(e+fx)^3}{6b^2f} + \frac{2a^2f^2 \cosh(c+dx)}{b^3d^3} + \frac{4f^2 \cosh(c+dx)}{9bd^3} \\
&= -\frac{af^2x}{4b^2d^2} - \frac{a^3(e+fx)^3}{3b^4f} - \frac{a(e+fx)^3}{6b^2f} + \frac{2a^2f^2 \cosh(c+dx)}{b^3d^3} + \frac{4f^2 \cosh(c+dx)}{9bd^3} \\
&= -\frac{af^2x}{4b^2d^2} - \frac{a^3(e+fx)^3}{3b^4f} - \frac{a(e+fx)^3}{6b^2f} + \frac{2a^2f^2 \cosh(c+dx)}{b^3d^3} + \frac{4f^2 \cosh(c+dx)}{9bd^3}
\end{aligned}$$

**Mathematica [A]** time = 4.98927, size = 966, normalized size = 1.49

$$-54d^2e^2 \cosh(c+dx)b^3 - 108f^2 \cosh(c+dx)b^3 - 54d^2f^2x^2 \cosh(c+dx)b^3 - 108d^2efx \cosh(c+dx)b^3 - 18d^2e^2 \cosh(c+dx)b^3$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Cosh[c + d\*x]^2\*Sinh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out]  $-(216*a^3*d^3*e^2*x + 108*a*b^2*d^3*e^2*x + 216*a^3*d^3*e*f*x^2 + 108*a*b^2*d^3*e*f*x^2 + 72*a^3*d^3*f^2*x^3 + 36*a*b^2*d^3*f^2*x^3 + 432*a^2*\text{Sqrt}[a^2 + b^2]*d^2*e^2*\text{ArcTanh}[(a + b*E^{(c + d*x)})/\text{Sqrt}[a^2 + b^2]] - 216*a^2*b*d^2$

$$\begin{aligned}
& 2e^{2c} \cosh[c + dx] - 54b^3 d^2 e^{2c} \cosh[c + dx] - 432a^2 b f^2 \cosh[c + dx] - 108b^3 f^2 \cosh[c + dx] - 432a^2 b d^2 e f x \cosh[c + dx] - 108b^3 d^2 e f x \cosh[c + dx] - 216a^2 b d^2 f^2 x^2 \cosh[c + dx] - 54b^3 d^2 f^2 x^2 \cosh[c + dx] - 54a^2 b^2 d e f \cosh[2(c + dx)] - 54a^2 b^2 d f^2 x \cosh[2(c + dx)] - 18b^3 d^2 e^{2c} \cosh[3(c + dx)] - 4b^3 f^2 \cosh[3(c + dx)] - 36b^3 d^2 e f x \cosh[3(c + dx)] - 18b^3 d^2 f^2 x^2 \cosh[3(c + dx)] - 432a^2 \sqrt{a^2 + b^2} d^2 e f x \operatorname{Log}\left[1 + \frac{bE^{c+dx}}{a - \sqrt{a^2 + b^2}}\right] - 216a^2 \sqrt{a^2 + b^2} d^2 f^2 x^2 \operatorname{Log}\left[1 + \frac{bE^{c+dx}}{a - \sqrt{a^2 + b^2}}\right] + 432a^2 \sqrt{a^2 + b^2} d^2 e f x \operatorname{Log}\left[1 + \frac{bE^{c+dx}}{a + \sqrt{a^2 + b^2}}\right] + 216a^2 \sqrt{a^2 + b^2} d^2 f^2 x^2 \operatorname{Log}\left[1 + \frac{bE^{c+dx}}{a + \sqrt{a^2 + b^2}}\right] - 432a^2 \sqrt{a^2 + b^2} d f (e + f x) \operatorname{PolyLog}\left[2, \frac{bE^{c+dx}}{-a + \sqrt{a^2 + b^2}}\right] + 432a^2 \sqrt{a^2 + b^2} d f (e + f x) \operatorname{PolyLog}\left[2, -\left(\frac{bE^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)\right] + 432a^2 \sqrt{a^2 + b^2} f^2 \operatorname{PolyLog}\left[3, \frac{bE^{c+dx}}{-a + \sqrt{a^2 + b^2}}\right] - 432a^2 \sqrt{a^2 + b^2} f^2 \operatorname{PolyLog}\left[3, -\left(\frac{bE^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)\right] + 432a^2 b d e f \sinh[c + dx] + 108b^3 d e f \sinh[c + dx] + 432a^2 b d f^2 x \sinh[c + dx] + 108b^3 d f^2 x \sinh[c + dx] + 54a^2 b^2 d^2 e^2 \sinh[2(c + dx)] + 27a^2 b^2 f^2 \sinh[2(c + dx)] + 108a^2 b^2 d^2 e f x \sinh[2(c + dx)] + 54a^2 b^2 d^2 f^2 x^2 \sinh[2(c + dx)] + 12b^3 d e f \sinh[3(c + dx)] + 12b^3 d f^2 x \sinh[3(c + dx)] \Big/ (216b^4 d^3)
\end{aligned}$$

**Maple [F]** time = 0.228, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\cosh(dx + c))^2 (\sinh(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*cosh(d\*x+c)^2\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*cosh(d\*x+c)^2\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [C]** time = 3.49251, size = 9708, normalized size = 14.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/432*(18*b^3*d^2*f^2*x^2 + 18*b^3*d^2*e^2 + 2*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x + c)^6 + 2*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*sinh(d*x + c)^6 + 12*b^3*d*e*f - 27*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*cosh(d*x + c)^5 - 3*(18*a*b^2*d^2*f^2*x^2 + 18*a*b^2*d^2*e^2 - 18*a*b^2*d*e*f + 9*a*b^2*f^2 + 18*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x - 4*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*b^3*f^2 + 54*((4*a^2*b + b^3)*d^2*f^2*x^2 + (4*a^2*b + b^3)*d^2*e^2 - 2*(4*a^2*b + b^3)*d*e*f + 2*(4*a^2*b + b^3)*f^2 + 2*((4*a^2*b + b^3)*d^2*e*f - (4*a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c)^4 + 3*(18*(4*a^2*b + b^3)*d^2*f^2*x^2 + 18*(4*a^2*b + b^3)*d^2*e^2 - 36*(4*a^2*b + b^3)*d*e*f + 36*(4*a^2*b + b^3)*f^2 + 10*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x + c)^2 + 36*((4*a^2*b + b^3)*d^2*e*f - (4*a^2*b + b^3)*d*f^2)*x - 45*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^4 - 72*((2*a^3 + a*b^2)*d^3*f^2*x^3 + 3*(2*a^3 + a*b^2)*d^3*e*f*x^2 + 3*(2*a^3 + a*b^2)*d^3*e^2*x)*cosh(d*x + c)^3 - 2*(36*(2*a^3 + a*b^2)*d^3*f^2*x^3 + 108*(2*a^3 + a*b^2)*d^3*e*f*x^2 + 108*(2*a^3 + a*b^2)*d^3*e^2*x - 20*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*cosh(d*x + c)^3 + 135*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*cosh(d*x + c)^2 - 108*((4*a^2*b + b^3)*d^2*f^2*x^2 + (4*a^2*b + b^3)*d^2*e^2 - 2*(4*a^2*b + b^3)*d*e*f + 2*(4*a^2*b + b^3)*f^2 + 2*((4*a^2*b + b^3)*d^2*e*f - (4*a^2*b + b^3)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 54*((4*a^2*b + b^3)*d^2*f^2*x^2 + (4*a^2*b + b^3)*d^2*e^2 + 2*(4*a^2*b + b^3)*d*e*f + 2*(4*a^2*b + b^3)*f^2 + 2*((4*a^2*b + b^3)*d^2*e*f + (4*a^2*b
```

$$\begin{aligned}
& + b^3) * d * f^2) * x) * \cosh(d * x + c)^2 + 6 * (9 * (4 * a^2 * b + b^3) * d^2 * f^2 * x^2 + 9 * (4 * a^2 * b + b^3) * d^2 * e^2 + 5 * (9 * b^3 * d^2 * f^2 * x^2 + 9 * b^3 * d^2 * e^2 - 6 * b^3 * d * e * f + 2 * b^3 * f^2 + 6 * (3 * b^3 * d^2 * e * f - b^3 * d * f^2) * x) * \cosh(d * x + c)^4 + 18 * (4 * a^2 * b + b^3) * d * e * f - 45 * (2 * a * b^2 * d^2 * f^2 * x^2 + 2 * a * b^2 * d^2 * e^2 - 2 * a * b^2 * d * e * f + a * b^2 * f^2 + 2 * (2 * a * b^2 * d^2 * e * f - a * b^2 * d * f^2) * x) * \cosh(d * x + c)^3 + 18 * (4 * a^2 * b + b^3) * f^2 + 54 * ((4 * a^2 * b + b^3) * d^2 * f^2 * x^2 + (4 * a^2 * b + b^3) * d^2 * e^2 - 2 * (4 * a^2 * b + b^3) * d * e * f + 2 * (4 * a^2 * b + b^3) * f^2 + 2 * ((4 * a^2 * b + b^3) * d^2 * e * f - (4 * a^2 * b + b^3) * d * f^2) * x) * \cosh(d * x + c)^2 + 18 * ((4 * a^2 * b + b^3) * d^2 * e * f + (4 * a^2 * b + b^3) * d * f^2) * x - 36 * ((2 * a^3 + a * b^2) * d^3 * f^2 * x^3 + 3 * (2 * a^3 + a * b^2) * d^3 * e * f * x^2 + 3 * (2 * a^3 + a * b^2) * d^3 * e^2 * x) * \cosh(d * x + c)) * \sinh(d * x + c)^2 + 864 * ((a^2 * b * d * f^2 * x + a^2 * b * d * e * f) * \cosh(d * x + c)^3 + 3 * (a^2 * b * d * f^2 * x + a^2 * b * d * e * f) * \cosh(d * x + c)^2 * \sinh(d * x + c) + 3 * (a^2 * b * d * f^2 * x + a^2 * b * d * e * f) * \cosh(d * x + c) * \sinh(d * x + c)^2 + (a^2 * b * d * f^2 * x + a^2 * b * d * e * f) * \sinh(d * x + c)^3) * \sqrt{(a^2 + b^2) / b^2} * \operatorname{dilog}((a * \cosh(d * x + c) + a * \sinh(d * x + c) + (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b + 1) - 864 * ((a^2 * b * d * f^2 * x + a^2 * b * d * e * f) * \cosh(d * x + c)^3 + 3 * (a^2 * b * d * f^2 * x + a^2 * b * d * e * f) * \cosh(d * x + c)^2 * \sinh(d * x + c) + 3 * (a^2 * b * d * f^2 * x + a^2 * b * d * e * f) * \cosh(d * x + c) * \sinh(d * x + c)^2 + (a^2 * b * d * f^2 * x + a^2 * b * d * e * f) * \sinh(d * x + c)^3) * \sqrt{(a^2 + b^2) / b^2} * \operatorname{dilog}((a * \cosh(d * x + c) + a * \sinh(d * x + c) - (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b + 1) - 432 * ((a^2 * b * d^2 * e^2 - 2 * a^2 * b * c * d * e * f + a^2 * b * c^2 * f^2) * \cosh(d * x + c)^3 + 3 * (a^2 * b * d^2 * e^2 - 2 * a^2 * b * c * d * e * f + a^2 * b * c^2 * f^2) * \cosh(d * x + c)^2 * \sinh(d * x + c) + 3 * (a^2 * b * d^2 * e^2 - 2 * a^2 * b * c * d * e * f + a^2 * b * c^2 * f^2) * \cosh(d * x + c) * \sinh(d * x + c)^2 + (a^2 * b * d^2 * e^2 - 2 * a^2 * b * c * d * e * f + a^2 * b * c^2 * f^2) * \sinh(d * x + c)^3) * \sqrt{(a^2 + b^2) / b^2} * \log(2 * b * \cosh(d * x + c) + 2 * b * \sinh(d * x + c) + 2 * b * \sqrt{(a^2 + b^2) / b^2} + 2 * a) + 432 * ((a^2 * b * d^2 * e^2 - 2 * a^2 * b * c * d * e * f + a^2 * b * c^2 * f^2) * \cosh(d * x + c)^3 + 3 * (a^2 * b * d^2 * e^2 - 2 * a^2 * b * c * d * e * f + a^2 * b * c^2 * f^2) * \cosh(d * x + c)^2 * \sinh(d * x + c) + 3 * (a^2 * b * d^2 * e^2 - 2 * a^2 * b * c * d * e * f + a^2 * b * c^2 * f^2) * \cosh(d * x + c) * \sinh(d * x + c)^2 + (a^2 * b * d^2 * e^2 - 2 * a^2 * b * c * d * e * f + a^2 * b * c^2 * f^2) * \sinh(d * x + c)^3) * \sqrt{(a^2 + b^2) / b^2} * \log(2 * b * \cosh(d * x + c) + 2 * b * \sinh(d * x + c) - 2 * b * \sqrt{(a^2 + b^2) / b^2} + 2 * a) + 432 * ((a^2 * b * d^2 * f^2 * x^2 + 2 * a^2 * b * d^2 * e * f * x + 2 * a^2 * b * c * d * e * f - a^2 * b * c^2 * f^2) * \cosh(d * x + c)^3 + 3 * (a^2 * b * d^2 * f^2 * x^2 + 2 * a^2 * b * d^2 * e * f * x + 2 * a^2 * b * c * d * e * f - a^2 * b * c^2 * f^2) * \cosh(d * x + c)^2 * \sinh(d * x + c) + 3 * (a^2 * b * d^2 * f^2 * x^2 + 2 * a^2 * b * d^2 * e * f * x + 2 * a^2 * b * c * d * e * f - a^2 * b * c^2 * f^2) * \cosh(d * x + c) * \sinh(d * x + c)^2 + (a^2 * b * d^2 * f^2 * x^2 + 2 * a^2 * b * d^2 * e * f * x + 2 * a^2 * b * c * d * e * f - a^2 * b * c^2 * f^2) * \sinh(d * x + c)^3) * \sqrt{(a^2 + b^2) / b^2} * \log(-(a * \cosh(d * x + c) + a * \sinh(d * x + c) + (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b) - 432 * ((a^2 * b * d^2 * f^2 * x^2 + 2 * a^2 * b * d^2 * e * f * x + 2 * a^2 * b * c * d * e * f - a^2 * b * c^2 * f^2) * \cosh(d * x + c)^3 + 3 * (a^2 * b * d^2 * f^2 * x^2 + 2 * a^2 * b * d^2 * e * f * x + 2 * a^2 * b * c * d * e * f - a^2 * b * c^2 * f^2) * \cosh(d * x + c)^2 * \sinh(d * x + c) + 3 * (a^2 * b * d^2 * f^2 * x^2 + 2 * a^2 * b * d^2 * e * f * x + 2 * a^2 * b * c * d * e * f - a^2 * b * c^2 * f^2) * \cosh(d * x + c) * \sinh(d * x + c)^2 + (a^2 * b * d^2 * f^2 * x^2 + 2 * a^2 * b * d^2 * e * f * x + 2 * a^2 * b * c * d * e * f - a^2 * b * c^2 * f^2) * \sinh(d * x + c)^3) * \sqrt{(a^2 + b^2) / b^2} * \log(-(a * \cosh(d * x + c) + a * \sinh(d * x + c) - (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b)
\end{aligned}$$

$$\begin{aligned}
&/b) - 864*(a^2*b*f^2*\cosh(d*x + c)^3 + 3*a^2*b*f^2*\cosh(d*x + c)^2*\sinh(d*x \\
&+ c) + 3*a^2*b*f^2*\cosh(d*x + c)*\sinh(d*x + c)^2 + a^2*b*f^2*\sinh(d*x + c) \\
&^3)*\sqrt{((a^2 + b^2)/b^2)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + ( \\
&b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2)})/b) + 864*(a^2*b*f \\
&^2*\cosh(d*x + c)^3 + 3*a^2*b*f^2*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*a^2*b*f^ \\
&2*\cosh(d*x + c)*\sinh(d*x + c)^2 + a^2*b*f^2*\sinh(d*x + c)^3)*\sqrt{((a^2 + b^ \\
&2)/b^2)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + \\
&b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2)})/b) + 12*(3*b^3*d^2*e*f + b^3*d*f^2) \\
&*x + 27*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 + 2*a*b^2*d*e*f + a*b^2*f^2 \\
&+ 2*(2*a*b^2*d^2*e*f + a*b^2*d*f^2)*x)*\cosh(d*x + c) + 3*(18*a*b^2*d^2*f^2*x \\
&x^2 + 18*a*b^2*d^2*e^2 + 18*a*b^2*d*e*f + 4*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e \\
&e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*\cosh(d*x + \\
&c)^5 + 9*a*b^2*f^2 - 45*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d \\
&*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*\cosh(d*x + c)^4 + 7 \\
&2*((4*a^2*b + b^3)*d^2*f^2*x^2 + (4*a^2*b + b^3)*d^2*e^2 - 2*(4*a^2*b + b^3) \\
&)*d*e*f + 2*(4*a^2*b + b^3)*f^2 + 2*((4*a^2*b + b^3)*d^2*e*f - (4*a^2*b + b \\
&^3)*d*f^2)*x)*\cosh(d*x + c)^3 - 72*((2*a^3 + a*b^2)*d^3*f^2*x^3 + 3*(2*a^3 \\
&+ a*b^2)*d^3*e*f*x^2 + 3*(2*a^3 + a*b^2)*d^3*e^2*x)*\cosh(d*x + c)^2 + 18*(2 \\
&)*a*b^2*d^2*e*f + a*b^2*d*f^2)*x + 36*((4*a^2*b + b^3)*d^2*f^2*x^2 + (4*a^2* \\
&b + b^3)*d^2*e^2 + 2*(4*a^2*b + b^3)*d*e*f + 2*(4*a^2*b + b^3)*f^2 + 2*((4* \\
&a^2*b + b^3)*d^2*e*f + (4*a^2*b + b^3)*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + \\
&c))/(b^4*d^3*\cosh(d*x + c)^3 + 3*b^4*d^3*\cosh(d*x + c)^2*\sinh(d*x + c) + 3* \\
&b^4*d^3*\cosh(d*x + c)*\sinh(d*x + c)^2 + b^4*d^3*\sinh(d*x + c)^3)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cosh(d\*x+c)\*\*2\*sinh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c)^2 \sinh(dx + c)^2}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)^2*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```



$$3.369 \quad \int \frac{(e+fx) \cosh^2(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=403

$$\frac{a^2 f \sqrt{a^2 + b^2} \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^4 d^2} - \frac{a^2 f \sqrt{a^2 + b^2} \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right)}{b^4 d^2} - \frac{a^2 f \sinh(c + dx)}{b^3 d^2} + \frac{a^2 \sqrt{a^2 + b^2} (e + fx)}{b^4}$$

```
[Out] -((a^3*e*x)/b^4) - (a*e*x)/(2*b^2) - (a^3*f*x^2)/(2*b^4) - (a*f*x^2)/(4*b^2)
) + (a^2*(e + f*x)*Cosh[c + d*x])/(b^3*d) + (a*f*Cosh[c + d*x]^2)/(4*b^2*d^2)
+ ((e + f*x)*Cosh[c + d*x]^3)/(3*b*d) + (a^2*Sqrt[a^2 + b^2]*(e + f*x)*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(b^4*d) - (a^2*Sqrt[a^2 + b^2]
*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(b^4*d) + (a^2
*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(b
^4*d^2) - (a^2*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))]/(b^4*d^2) - (a^2*f*Sinh[c + d*x])/(b^3*d^2) - (f*Sinh[c + d*x])
/(3*b*d^2) - (a*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d) - (f*Sinh[
c + d*x]^3)/(9*b*d^2)
```

---

**Rubi [A]** time = 0.690013, antiderivative size = 403, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 12, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$ , Rules used = {5579, 5447, 2633, 3310, 5565, 3296, 2637, 3322, 2264, 2190, 2279, 2391}

$$\frac{a^2 f \sqrt{a^2 + b^2} \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^4 d^2} - \frac{a^2 f \sqrt{a^2 + b^2} \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right)}{b^4 d^2} - \frac{a^2 f \sinh(c + dx)}{b^3 d^2} + \frac{a^2 \sqrt{a^2 + b^2} (e + fx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a^3*e*x)/b^4) - (a*e*x)/(2*b^2) - (a^3*f*x^2)/(2*b^4) - (a*f*x^2)/(4*b^2)
) + (a^2*(e + f*x)*Cosh[c + d*x])/(b^3*d) + (a*f*Cosh[c + d*x]^2)/(4*b^2*d^2)
+ ((e + f*x)*Cosh[c + d*x]^3)/(3*b*d) + (a^2*Sqrt[a^2 + b^2]*(e + f*x)*L
og[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(b^4*d) - (a^2*Sqrt[a^2 + b^2]
*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(b^4*d) + (a^2
*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(b
^4*d^2) - (a^2*Sqrt[a^2 + b^2]*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2]))]/(b^4*d^2) - (a^2*f*Sinh[c + d*x])/(b^3*d^2) - (f*Sinh[c + d*x])
/(3*b*d^2) - (a*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^2*d) - (f*Sinh[
c + d*x]^3)/(9*b*d^2)
```

Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
(d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^(n - 1)/2], x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Simp[(d*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*Sine[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*Sine[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_]*)
(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx) \cosh^2(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx) \cosh^2(c+dx) \sinh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx) \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{(e+fx) \cosh^3(c+dx)}{3bd} - \frac{a \int (e+fx) \cosh^2(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx) \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
&= \frac{af \cosh^2(c+dx)}{4b^2d^2} + \frac{(e+fx) \cosh^3(c+dx)}{3bd} - \frac{a(e+fx) \cosh(c+dx) \sinh(c+dx)}{2b^2d} \\
&= -\frac{a^3ex}{b^4} - \frac{aex}{2b^2} - \frac{a^3fx^2}{2b^4} - \frac{afx^2}{4b^2} + \frac{a^2(e+fx) \cosh(c+dx)}{b^3d} + \frac{af \cosh^2(c+dx)}{4b^2d^2} \\
&= -\frac{a^3ex}{b^4} - \frac{aex}{2b^2} - \frac{a^3fx^2}{2b^4} - \frac{afx^2}{4b^2} + \frac{a^2(e+fx) \cosh(c+dx)}{b^3d} + \frac{af \cosh^2(c+dx)}{4b^2d^2} \\
&= -\frac{a^3ex}{b^4} - \frac{aex}{2b^2} - \frac{a^3fx^2}{2b^4} - \frac{afx^2}{4b^2} + \frac{a^2(e+fx) \cosh(c+dx)}{b^3d} + \frac{af \cosh^2(c+dx)}{4b^2d^2} \\
&= -\frac{a^3ex}{b^4} - \frac{aex}{2b^2} - \frac{a^3fx^2}{2b^4} - \frac{afx^2}{4b^2} + \frac{a^2(e+fx) \cosh(c+dx)}{b^3d} + \frac{af \cosh^2(c+dx)}{4b^2d^2} \\
&= -\frac{a^3ex}{b^4} - \frac{aex}{2b^2} - \frac{a^3fx^2}{2b^4} - \frac{afx^2}{4b^2} + \frac{a^2(e+fx) \cosh(c+dx)}{b^3d} + \frac{af \cosh^2(c+dx)}{4b^2d^2}
\end{aligned}$$

**Mathematica [A]** time = 3.05828, size = 676, normalized size = 1.68

$$-\frac{72a^2f\sqrt{a^2+b^2}\text{PolyLog}\left(2, \frac{b(\sinh(c+dx)+\cosh(c+dx))}{\sqrt{a^2+b^2}-a}\right) + 72a^2f\sqrt{a^2+b^2}\text{PolyLog}\left(2, -\frac{b(\sinh(c+dx)+\cosh(c+dx))}{\sqrt{a^2+b^2}+a}\right) + 144a^2de\sqrt{a^2+b^2}}{\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]), x]
```

```
[Out] -(72*a^3*c*d*e + 36*a*b^2*c*d*e - 36*a^3*c^2*f - 18*a*b^2*c^2*f + 72*a^3*d^2*e*x + 36*a*b^2*d^2*e*x + 36*a^3*d^2*f*x^2 + 18*a*b^2*d^2*f*x^2 + 144*a^2*Sqrt[a^2 + b^2]*d*e*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] - 144*a^2*Sqrt[a^2 + b^2]*c*f*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] - 72*a^2*b*d*e*Cosh[c + d*x] - 18*b^3*d*e*Cosh[c + d*x] - 72*a^2*b*d*f*x*Cosh[c + d*x] - 18*b^3*d*f*x*Cosh[c + d*x] - 9*a*b^2*f*Cosh[2*(c + d*x)] - 6*b^3*d*e*Cosh[3*(c + d*x)] - 6*b^3*d*f*x*Cosh[3*(c + d*x)])
```

$$\begin{aligned}
& 3*(c + d*x)] - 72*a^2*\text{Sqrt}[a^2 + b^2]*c*f*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a - \text{Sqrt}[a^2 + b^2])] - 72*a^2*\text{Sqrt}[a^2 + b^2]*d*f*x*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a - \text{Sqrt}[a^2 + b^2])] + 72*a^2*\text{Sqrt}[a^2 + b^2]*c*f*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a + \text{Sqrt}[a^2 + b^2])] + 72*a^2*\text{Sqrt}[a^2 + b^2]*d*f*x*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a + \text{Sqrt}[a^2 + b^2])] - 72*a^2*\text{Sqrt}[a^2 + b^2]*f*\text{PolyLog}[2, (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(-a + \text{Sqrt}[a^2 + b^2])] + 72*a^2*\text{Sqrt}[a^2 + b^2]*f*\text{PolyLog}[2, -(b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a + \text{Sqrt}[a^2 + b^2])] + 72*a^2*b*f*\text{Sinh}[c + d*x] + 18*b^3*f*\text{Sinh}[c + d*x] + 18*a*b^2*d*e*\text{Sinh}[2*(c + d*x)] + 18*a*b^2*d*f*x*\text{Sinh}[2*(c + d*x)] + 2*b^3*f*\text{Sinh}[3*(c + d*x)]/(72*b^4*d^2)
\end{aligned}$$

**Maple [B]** time = 0.106, size = 1128, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)*\cosh(d*x+c)^2*\sinh(d*x+c)^2/(a+b*\sinh(d*x+c)), x)$

[Out] 
$$\begin{aligned}
& -1/16*a*(2*d*f*x+2*d*e-f)/b^2/d^2*\exp(2*d*x+2*c)+1/16*a*(2*d*f*x+2*d*e+f)/b^2/d^2*\exp(-2*d*x-2*c)-1/2*a^3*f*x^2/b^4-1/4*a*f*x^2/b^2-a^4/b^4/d^2*f/(a^2+b^2)^{(1/2)}*\text{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))-2*a^4/b^4/d^2*f/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+a^4/b^4/d^2*f/(a^2+b^2)^{(1/2)}*\text{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))+1/8*(4*a^2+b^2)*(d*f*x+d*e+f)/b^3/d^2*\exp(-d*x-c)-a^3*e*x/b^4-1/2*a*e*x/b^2-2*a^2/b^2/d^2*f/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+a^2/b^2/d^2*f/(a^2+b^2)^{(1/2)}*\text{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))-a^2/b^2/d^2*f/(a^2+b^2)^{(1/2)}*\text{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))+a^4/b^4/d^2*f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*x+a^4/b^4/d^2*f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*c-a^4/b^4/d^2*f/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*x-a^4/b^4/d^2*f/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*c+2*a^4/b^4/d^2*f*c/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+a^2/b^2/d^2*f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*x+a^2/b^2/d^2*f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*c-a^2/b^2/d^2*f/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*x-a^2/b^2/d^2*f/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*c+2*a^2/b^2/d^2*f*c/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+1/72*(3*d*f*x+3*d*e+f)/d^2/b*\exp(-3*d*x)
\end{aligned}$$

$$x-3*c)+1/72*(3*d*f*x+3*d*e-f)/d^2/b*\exp(3*d*x+3*c)+1/8*(4*a^2*d*f*x+b^2*d*f*x+4*a^2*d*e+b^2*d*e-4*a^2*f-b^2*f)/b^3/d^2*\exp(d*x+c)$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^2\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.80926, size = 5268, normalized size = 13.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^2\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 1/144*(2*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*\cosh(d*x + c)^6 + 2*(3*b^3*d*f*x \\ & + 3*b^3*d*e - b^3*f)*\sinh(d*x + c)^6 + 6*b^3*d*f*x - 9*(2*a*b^2*d*f*x + 2* \\ & a*b^2*d*e - a*b^2*f)*\cosh(d*x + c)^5 - 3*(6*a*b^2*d*f*x + 6*a*b^2*d*e - 3*a \\ & *b^2*f - 4*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*\cosh(d*x + c))*\sinh(d*x + c)^5 \\ & + 6*b^3*d*e + 18*((4*a^2*b + b^3)*d*f*x + (4*a^2*b + b^3)*d*e - (4*a^2*b + \\ & b^3)*f)*\cosh(d*x + c)^4 + 3*(6*(4*a^2*b + b^3)*d*f*x + 6*(4*a^2*b + b^3)*d \\ & *e + 10*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*\cosh(d*x + c)^2 - 6*(4*a^2*b + b^ \\ & 3)*f - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*\cosh(d*x + c))*\sinh(d*x + \\ & c)^4 + 2*b^3*f - 36*((2*a^3 + a*b^2)*d^2*f*x^2 + 2*(2*a^3 + a*b^2)*d^2*e*x \\ & )*\cosh(d*x + c)^3 - 2*(18*(2*a^3 + a*b^2)*d^2*f*x^2 + 36*(2*a^3 + a*b^2)*d^ \\ & 2*e*x - 20*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*\cosh(d*x + c)^3 + 45*(2*a*b^2* \\ & d*f*x + 2*a*b^2*d*e - a*b^2*f)*\cosh(d*x + c)^2 - 36*((4*a^2*b + b^3)*d*f*x \\ & + (4*a^2*b + b^3)*d*e - (4*a^2*b + b^3)*f)*\cosh(d*x + c))*\sinh(d*x + c)^3 + \\ & 18*((4*a^2*b + b^3)*d*f*x + (4*a^2*b + b^3)*d*e + (4*a^2*b + b^3)*f)*\cosh( \\ & d*x + c)^2 + 6*(5*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*\cosh(d*x + c)^4 + 3*(4* \\ & a^2*b + b^3)*d*f*x - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*\cosh(d*x + \\ & c)^3 + 3*(4*a^2*b + b^3)*d*e + 18*((4*a^2*b + b^3)*d*f*x + (4*a^2*b + b^3)* \end{aligned}$$

$$\begin{aligned}
& d * e - (4 * a^2 * b + b^3) * f * \cosh(d * x + c)^2 + 3 * (4 * a^2 * b + b^3) * f - 18 * ((2 * a^3 \\
& + a * b^2) * d^2 * f * x^2 + 2 * (2 * a^3 + a * b^2) * d^2 * e * x) * \cosh(d * x + c) * \sinh(d * x + \\
& c)^2 + 144 * (a^2 * b * f * \cosh(d * x + c)^3 + 3 * a^2 * b * f * \cosh(d * x + c)^2 * \sinh(d * x + \\
& c) + 3 * a^2 * b * f * \cosh(d * x + c) * \sinh(d * x + c)^2 + a^2 * b * f * \sinh(d * x + c)^3) * \text{sqrt} \\
& \text{t}((a^2 + b^2) / b^2) * \text{dilog}((a * \cosh(d * x + c) + a * \sinh(d * x + c) + (b * \cosh(d * x + \\
& c) + b * \sinh(d * x + c)) * \text{sqrt}((a^2 + b^2) / b^2) - b) / b + 1) - 144 * (a^2 * b * f * \cosh \\
& (d * x + c)^3 + 3 * a^2 * b * f * \cosh(d * x + c)^2 * \sinh(d * x + c) + 3 * a^2 * b * f * \cosh(d * x \\
& + c) * \sinh(d * x + c)^2 + a^2 * b * f * \sinh(d * x + c)^3) * \text{sqrt}((a^2 + b^2) / b^2) * \text{dilo} \\
& \text{g}((a * \cosh(d * x + c) + a * \sinh(d * x + c) - (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \\
& \text{sqrt}((a^2 + b^2) / b^2) - b) / b + 1) - 144 * ((a^2 * b * d * e - a^2 * b * c * f) * \cosh(d * x + \\
& c)^3 + 3 * (a^2 * b * d * e - a^2 * b * c * f) * \cosh(d * x + c)^2 * \sinh(d * x + c) + 3 * (a^2 * b * \\
& d * e - a^2 * b * c * f) * \cosh(d * x + c) * \sinh(d * x + c)^2 + (a^2 * b * d * e - a^2 * b * c * f) * \text{si} \\
& \text{nh}(d * x + c)^3) * \text{sqrt}((a^2 + b^2) / b^2) * \log(2 * b * \cosh(d * x + c) + 2 * b * \sinh(d * x + \\
& c) + 2 * b * \text{sqrt}((a^2 + b^2) / b^2) + 2 * a) + 144 * ((a^2 * b * d * e - a^2 * b * c * f) * \cosh( \\
& d * x + c)^3 + 3 * (a^2 * b * d * e - a^2 * b * c * f) * \cosh(d * x + c)^2 * \sinh(d * x + c) + 3 * (a \\
& ^2 * b * d * e - a^2 * b * c * f) * \cosh(d * x + c) * \sinh(d * x + c)^2 + (a^2 * b * d * e - a^2 * b * c * \\
& f) * \sinh(d * x + c)^3) * \text{sqrt}((a^2 + b^2) / b^2) * \log(2 * b * \cosh(d * x + c) + 2 * b * \sinh( \\
& d * x + c) - 2 * b * \text{sqrt}((a^2 + b^2) / b^2) + 2 * a) + 144 * ((a^2 * b * d * f * x + a^2 * b * c * f \\
& ) * \cosh(d * x + c)^3 + 3 * (a^2 * b * d * f * x + a^2 * b * c * f) * \cosh(d * x + c)^2 * \sinh(d * x + \\
& c) + 3 * (a^2 * b * d * f * x + a^2 * b * c * f) * \cosh(d * x + c) * \sinh(d * x + c)^2 + (a^2 * b * d * f \\
& * x + a^2 * b * c * f) * \sinh(d * x + c)^3) * \text{sqrt}((a^2 + b^2) / b^2) * \log(-(a * \cosh(d * x + c \\
& ) + a * \sinh(d * x + c) + (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \text{sqrt}((a^2 + b^2) / \\
& b^2) - b) / b) - 144 * ((a^2 * b * d * f * x + a^2 * b * c * f) * \cosh(d * x + c)^3 + 3 * (a^2 * b * d * \\
& f * x + a^2 * b * c * f) * \cosh(d * x + c)^2 * \sinh(d * x + c) + 3 * (a^2 * b * d * f * x + a^2 * b * c * f \\
& ) * \cosh(d * x + c) * \sinh(d * x + c)^2 + (a^2 * b * d * f * x + a^2 * b * c * f) * \sinh(d * x + c)^3 \\
& ) * \text{sqrt}((a^2 + b^2) / b^2) * \log(-(a * \cosh(d * x + c) + a * \sinh(d * x + c) - (b * \cosh(d \\
& * x + c) + b * \sinh(d * x + c)) * \text{sqrt}((a^2 + b^2) / b^2) - b) / b) + 9 * (2 * a * b^2 * d * f * x \\
& + 2 * a * b^2 * d * e + a * b^2 * f) * \cosh(d * x + c) + 3 * (6 * a * b^2 * d * f * x + 4 * (3 * b^3 * d * f * x \\
& + 3 * b^3 * d * e - b^3 * f) * \cosh(d * x + c)^5 + 6 * a * b^2 * d * e - 15 * (2 * a * b^2 * d * f * x + 2 \\
& * a * b^2 * d * e - a * b^2 * f) * \cosh(d * x + c)^4 + 3 * a * b^2 * f + 24 * ((4 * a^2 * b + b^3) * d * f \\
& * x + (4 * a^2 * b + b^3) * d * e - (4 * a^2 * b + b^3) * f) * \cosh(d * x + c)^3 - 36 * ((2 * a^3 \\
& + a * b^2) * d^2 * f * x^2 + 2 * (2 * a^3 + a * b^2) * d^2 * e * x) * \cosh(d * x + c)^2 + 12 * ((4 * a^2 * b \\
& + b^3) * d * f * x + (4 * a^2 * b + b^3) * d * e + (4 * a^2 * b + b^3) * f) * \cosh(d * x + c) * \\
& \sinh(d * x + c)) / (b^4 * d^2 * \cosh(d * x + c)^3 + 3 * b^4 * d^2 * \cosh(d * x + c)^2 * \sinh(d * x \\
& + c) + 3 * b^4 * d^2 * \cosh(d * x + c) * \sinh(d * x + c)^2 + b^4 * d^2 * \sinh(d * x + c)^3)
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)\*\*2\*sinh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \cosh(dx + c)^2 \sinh(dx + c)^2}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^2\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)\*cosh(d\*x + c)^2\*sinh(d\*x + c)^2/(b\*sinh(d\*x + c) + a), x)



$$3.370 \quad \int \frac{\cosh^2(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=141

$$\frac{(3a^2 + b^2) \cosh(c + dx)}{3b^3d} - \frac{2a^2\sqrt{a^2 + b^2} \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^4d} - \frac{ax(2a^2 + b^2)}{2b^4} - \frac{a \sinh(c + dx) \cosh(c + dx)}{2b^2d} + \frac{\sinh(c + dx)^2}{3b^2d}$$

[Out]  $-(a*(2*a^2 + b^2)*x)/(2*b^4) - (2*a^2*\text{Sqrt}[a^2 + b^2]*\text{ArcTanh}[(b - a*\text{Tanh}[(c + d*x)/2]]/\text{Sqrt}[a^2 + b^2]))/(b^4*d) + ((3*a^2 + b^2)*\text{Cosh}[c + d*x])/(3*b^3*d) - (a*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(2*b^2*d) + (\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x]^2)/(3*b*d)$

**Rubi [A]** time = 0.498199, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$ , Rules used = {2889, 3050, 3049, 3023, 2735, 2660, 618, 204}

$$\frac{(3a^2 + b^2) \cosh(c + dx)}{3b^3d} - \frac{2a^2\sqrt{a^2 + b^2} \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^4d} - \frac{ax(2a^2 + b^2)}{2b^4} - \frac{a \sinh(c + dx) \cosh(c + dx)}{2b^2d} + \frac{\sinh(c + dx)^2}{3b^2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Cosh}[c + d*x]^2*\text{Sinh}[c + d*x]^2)/(a + b*\text{Sinh}[c + d*x]), x]$

[Out]  $-(a*(2*a^2 + b^2)*x)/(2*b^4) - (2*a^2*\text{Sqrt}[a^2 + b^2]*\text{ArcTanh}[(b - a*\text{Tanh}[(c + d*x)/2]]/\text{Sqrt}[a^2 + b^2]))/(b^4*d) + ((3*a^2 + b^2)*\text{Cosh}[c + d*x])/(3*b^3*d) - (a*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(2*b^2*d) + (\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x]^2)/(3*b*d)$

### Rule 2889

$\text{Int}[\cos[(e_.) + (f_.)*(x_.)]^2*((d_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}, x\_Symbol] \rightarrow \text{Int}[(d*\text{Sin}[e + f*x])^n*(a + b*\text{Sin}[e + f*x])^m*(1 - \text{Sin}[e + f*x]^2), x] /;$  FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2\*m, 2\*n])

### Rule 3050

$\text{Int}[(a_. + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}*((A_.) + (C_.)*\sin[(e_.) + (f_.)*(x_.)]^2), x\_Symbol] :$

```
> -Simp[(C*cos[e + f*x]*(a + b*sin[e + f*x])^m*(c + d*sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*sin[e + f*x])^
(m - 1)*(c + d*sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0]
)))
```

### Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*cos[e + f*x]*(a + b*sin[e + f*x]
)^m*(c + d*sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*sin[e + f*x])^(m - 1)*(c + d*sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

### Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*cos
[e + f*x]*(a + b*sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

### Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_
.)*(x_)]), x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

### Rule 2660

```
Int[((a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 204

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^2(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx &= \int \frac{\sinh^2(c+dx) (1 + \sinh^2(c+dx))}{a+b \sinh(c+dx)} dx \\
 &= \frac{\cosh(c+dx) \sinh^2(c+dx)}{3bd} + \frac{\int \frac{\sinh(c+dx)(-2a+b \sinh(c+dx)-3a \sinh^2(c+dx))}{a+b \sinh(c+dx)} dx}{3b} \\
 &= -\frac{a \cosh(c+dx) \sinh(c+dx)}{2b^2d} + \frac{\cosh(c+dx) \sinh^2(c+dx)}{3bd} + \frac{\int \frac{3a^2-ab \sinh(c+dx)+2}{a+b \sinh(c+dx)} dx}{6} \\
 &= \frac{(3a^2+b^2) \cosh(c+dx)}{3b^3d} - \frac{a \cosh(c+dx) \sinh(c+dx)}{2b^2d} + \frac{\cosh(c+dx) \sinh^2(c+dx)}{3bd} \\
 &= -\frac{a(2a^2+b^2)x}{2b^4} + \frac{(3a^2+b^2) \cosh(c+dx)}{3b^3d} - \frac{a \cosh(c+dx) \sinh(c+dx)}{2b^2d} + \frac{\cosh(c+dx) \sinh^2(c+dx)}{3bd} \\
 &= -\frac{a(2a^2+b^2)x}{2b^4} + \frac{(3a^2+b^2) \cosh(c+dx)}{3b^3d} - \frac{a \cosh(c+dx) \sinh(c+dx)}{2b^2d} + \frac{\cosh(c+dx) \sinh^2(c+dx)}{3bd} \\
 &= -\frac{a(2a^2+b^2)x}{2b^4} + \frac{(3a^2+b^2) \cosh(c+dx)}{3b^3d} - \frac{a \cosh(c+dx) \sinh(c+dx)}{2b^2d} + \frac{\cosh(c+dx) \sinh^2(c+dx)}{3bd} \\
 &= -\frac{a(2a^2+b^2)x}{2b^4} - \frac{2a^2\sqrt{a^2+b^2} \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^4d} + \frac{(3a^2+b^2) \cosh(c+dx)}{3b^3d}
 \end{aligned}$$

**Mathematica [A]** time = 0.391655, size = 123, normalized size = 0.87

$$\frac{3b(4a^2+b^2) \cosh(c+dx) - 3a \left( 2(2a^2+b^2)(c+dx) + 8a\sqrt{-a^2-b^2} \tan^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right) + b^2 \sinh(2(c+dx)) \right) + b^2 \sinh^2(c+dx)}{12b^4d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cosh[c + d\*x]^2\*Sinh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (3\*b\*(4\*a^2 + b^2)\*Cosh[c + d\*x] + b^3\*Cosh[3\*(c + d\*x)] - 3\*a\*(2\*(2\*a^2 + b^2)\*(c + d\*x) + 8\*a\*Sqrt[-a^2 - b^2]\*ArcTan[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[-a^2 - b^2]] + b^2\*Sinh[2\*(c + d\*x)])/(12\*b^4\*d)

**Maple [B]** time = 0.037, size = 398, normalized size = 2.8

$$\frac{1}{3bd} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-3} - \frac{1}{2bd} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-2} + \frac{a}{2db^2} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-2} + \frac{a^2}{db^3} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)^2\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] 1/3/d/b/(tanh(1/2\*d\*x+1/2\*c)+1)^3-1/2/d/b/(tanh(1/2\*d\*x+1/2\*c)+1)^2+1/2/d/b^2/(tanh(1/2\*d\*x+1/2\*c)+1)^2\*a+1/d/b^3/(tanh(1/2\*d\*x+1/2\*c)+1)\*a^2-1/2/d/b^2/(tanh(1/2\*d\*x+1/2\*c)+1)\*a+1/2/d/b/(tanh(1/2\*d\*x+1/2\*c)+1)-1/d\*a^3/b^4\*ln(tanh(1/2\*d\*x+1/2\*c)+1)-1/2/d\*a/b^2\*ln(tanh(1/2\*d\*x+1/2\*c)+1)-1/3/d/b/(tanh(1/2\*d\*x+1/2\*c)-1)^3-1/2/d/b^2/(tanh(1/2\*d\*x+1/2\*c)-1)^2\*a-1/2/d/b/(tanh(1/2\*d\*x+1/2\*c)-1)^2-1/d/b^3/(tanh(1/2\*d\*x+1/2\*c)-1)\*a^2-1/2/d/b^2/(tanh(1/2\*d\*x+1/2\*c)-1)\*a-1/2/d/b/(tanh(1/2\*d\*x+1/2\*c)-1)+1/d\*a^3/b^4\*ln(tanh(1/2\*d\*x+1/2\*c)-1)+1/2/d\*a/b^2\*ln(tanh(1/2\*d\*x+1/2\*c)-1)+2/d\*a^2\*(a^2+b^2)^(1/2)/b^4\*arctanh(1/2\*(2\*a\*tanh(1/2\*d\*x+1/2\*c)-2\*b)/(a^2+b^2)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.13802, size = 1882, normalized size = 13.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\frac{1}{24} \cdot (b^3 \cosh(dx+c)^6 + b^3 \sinh(dx+c)^6 - 3ab^2 \cosh(dx+c)^5 - 12(2a^3 + ab^2)dx \cosh(dx+c)^3 + 3(2b^3 \cosh(dx+c) - ab^2) \sinh(dx+c)^5 + 3(4a^2b + b^3) \cosh(dx+c)^4 + 3(5b^3 \cosh(dx+c)^2 - 5ab^2 \cosh(dx+c) + 4a^2b + b^3) \sinh(dx+c)^4 + 3ab^2 \cosh(dx+c) + 2(10b^3 \cosh(dx+c)^3 - 15ab^2 \cosh(dx+c)^2 - 6(2a^3 + ab^2)dx + 6(4a^2b + b^3) \cosh(dx+c)) \sinh(dx+c)^3 + b^3 + 3(4a^2b + b^3) \cosh(dx+c)^2 + 3(5b^3 \cosh(dx+c)^4 - 10ab^2 \cosh(dx+c)^3 - 12(2a^3 + ab^2)dx \cosh(dx+c) + 4a^2b + b^3 + 6(4a^2b + b^3) \cosh(dx+c)^2) \sinh(dx+c)^2 + 24(a^2 \cosh(dx+c)^3 + 3a^2 \cosh(dx+c)^2 \sinh(dx+c) + 3a^2 \cosh(dx+c) \sinh(dx+c)^2 + a^2 \sinh(dx+c)^3) \sqrt{a^2 + b^2} \log\left(\frac{(b^2 \cosh(dx+c)^2 + b^2 \sinh(dx+c)^2 + 2ab \cosh(dx+c) + 2a^2 + b^2 + 2(b^2 \cosh(dx+c) + ab) \sinh(dx+c) - 2\sqrt{a^2 + b^2}(b \cosh(dx+c) + b \sinh(dx+c) + a))}{(b \cosh(dx+c)^2 + b \sinh(dx+c)^2 + 2a \cosh(dx+c) + 2(b \cosh(dx+c) + a) \sinh(dx+c) - b)}\right) + 3(2b^3 \cosh(dx+c)^5 - 5ab^2 \cosh(dx+c)^4 - 12(2a^3 + ab^2)dx \cosh(dx+c)^2 + 4(4a^2b + b^3) \cosh(dx+c)^3 + ab^2 + 2(4a^2b + b^3) \cosh(dx+c)) \sinh(dx+c) / (b^4 d \cosh(dx+c)^3 + 3b^4 d \cosh(dx+c)^2 \sinh(dx+c) + 3b^4 d \cosh(dx+c) \sinh(dx+c)^2 + b^4 d \sinh(dx+c)^3)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*\*2\*sinh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [A]** time = 1.16917, size = 309, normalized size = 2.19

$$-\frac{(2a^3 + ab^2)(dx + c)}{2b^4d} + \frac{(3ab^2e^{(dx+c)} + b^3 + 3(4a^2b + b^3)e^{(2dx+2c)})e^{(-3dx-3c)}}{24b^4d} + \frac{(a^4 + a^2b^2) \log\left(\frac{|2be^{(dx+c)} + 2a - 2\sqrt{a^2+b^2}|}{|2be^{(dx+c)} + 2a + 2\sqrt{a^2+b^2}|}\right)}{\sqrt{a^2 + b^2}b^4d} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^3 + a*b^2)*(d*x + c)/(b^4*d) + 1/24*(3*a*b^2*e^(d*x + c) + b^3 + 3*(4*a^2*b + b^3)*e^(2*d*x + 2*c))*e^(-3*d*x - 3*c)/(b^4*d) + (a^4 + a^2*b^2)*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4*d) + 1/24*(b^2*d^2*e^(3*d*x + 3*c) - 3*a*b*d^2*e^(2*d*x + 2*c) + 12*a^2*d^2*e^(d*x + c) + 3*b^2*d^2*e^(d*x + c))/(b^3*d^3)
```

$$3.371 \quad \int \frac{\cosh^2(c+dx) \sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=38

$$\text{Unintegrable} \left( \frac{\sinh^2(c+dx) \cosh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[(Cosh[c + d\*x]^2\*Sinh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.125769, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cosh^2(c+dx) \sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Cosh[c + d\*x]^2\*Sinh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] Defer[Int] [(Cosh[c + d\*x]^2\*Sinh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\cosh^2(c+dx) \sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\cosh^2(c+dx) \sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.001, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Cosh[c + d\*x]^2\*Sinh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] \$Aborted

**Maple [A]** time = 0.135, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx+c))^2 (\sinh(dx+c))^2}{(fx+e)(a+b\sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)^2\*sinh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(cosh(d\*x+c)^2\*sinh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$2(a^4e^c + a^2b^2e^c) \int -\frac{e^{(dx)}}{b^5fx + b^5e - (b^5fxe^{2c} + b^5ee^{2c})e^{2dx}} - 2(ab^4fxe^c + ab^4ee^c)e^{(dx)} dx + \frac{e^{(-3c+\frac{3de}{f})} E_1\left(\frac{3(fx+e)d}{f}\right)}{8bf} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2\*sinh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 2\*(a^4\*e^c + a^2\*b^2\*e^c)\*integrate(-e^(d\*x)/(b^5\*f\*x + b^5\*e - (b^5\*f\*x\*e^(2\*c) + b^5\*e\*e^(2\*c)))\*e^(2\*d\*x) - 2\*(a\*b^4\*f\*x\*e^c + a\*b^4\*e\*e^c)\*e^(d\*x), x) + 1/8\*e^(-3\*c + 3\*d\*e/f)\*exp\_integral\_e(1, 3\*(f\*x + e)\*d/f)/(b\*f) + 1/4\*a\*e^(-2\*c + 2\*d\*e/f)\*exp\_integral\_e(1, 2\*(f\*x + e)\*d/f)/(b^2\*f) + 1/4\*a\*e^(2\*c - 2\*d\*e/f)\*exp\_integral\_e(1, -2\*(f\*x + e)\*d/f)/(b^2\*f) - 1/8\*e^(3\*c - 3\*d\*e/f)\*exp\_integral\_e(1, -3\*(f\*x + e)\*d/f)/(b\*f) + 1/8\*(4\*a^2 + b^2)\*e^(-c + d\*e/f)\*exp\_integral\_e(1, (f\*x + e)\*d/f)/(b^3\*f) - 1/8\*(4\*a^2\*e^c + b^2\*e^c)\*e^(-d\*e/f)\*exp\_integral\_e(1, -(f\*x + e)\*d/f)/(b^3\*f) - 1/2\*(2\*a^3 + a\*b^2)\*log(f\*x + e)/(b^4\*f)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx+c)^2 \sinh(dx+c)^2}{afx+ae+(bfx+be)\sinh(dx+c)}, x\right)$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
m="fricas")
```

```
[Out] integral(cosh(d*x + c)^2*sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(
d*x + c)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

sage<sub>0</sub>x

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm
m="giac")
```

```
[Out] sage0*x
```

$$3.372 \quad \int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=1123

result too large to display

```
[Out] (3*a^2*f^3*x)/(8*b^3*d^3) - (45*f^3*x)/(256*b*d^3) + (a^2*(e + f*x)^3)/(4*b^3*d) - (3*(e + f*x)^3)/(32*b*d) - (a^2*(a^2 + b^2)*(e + f*x)^4)/(4*b^5*f) + (6*a^3*f^3*Cosh[c + d*x])/(b^4*d^4) + (40*a*f^3*Cosh[c + d*x])/(9*b^2*d^4) + (3*a^3*f*(e + f*x)^2*Cosh[c + d*x])/(b^4*d^2) + (2*a*f*(e + f*x)^2*Cosh[c + d*x])/(b^2*d^2) + (9*f^2*(e + f*x)*Cosh[c + d*x]^2)/(32*b*d^3) + (2*a*f^3*Cosh[c + d*x]^3)/(27*b^2*d^4) + (a*f*(e + f*x)^2*Cosh[c + d*x]^3)/(3*b^2*d^2) + (3*f^2*(e + f*x)*Cosh[c + d*x]^4)/(32*b*d^3) + ((e + f*x)^3*Cosh[c + d*x]^4)/(4*b*d) + (a^2*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^5*d) + (a^2*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^5*d) + (3*a^2*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^2) + (3*a^2*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^2) - (6*a^2*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^3) - (6*a^2*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^3) + (6*a^2*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^4) + (6*a^2*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^4) - (6*a^3*f^2*(e + f*x)*Sinh[c + d*x])/(b^4*d^3) - (40*a*f^2*(e + f*x)*Sinh[c + d*x])/(9*b^2*d^3) - (a^3*(e + f*x)^3*Sinh[c + d*x])/(b^4*d) - (2*a*(e + f*x)^3*Sinh[c + d*x])/(3*b^2*d) - (3*a^2*f^3*Cosh[c + d*x]*Sinh[c + d*x])/(8*b^3*d^4) - (45*f^3*Cosh[c + d*x]*Sinh[c + d*x])/(256*b*d^4) - (3*a^2*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^3*d^2) - (9*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(32*b*d^2) - (2*a*f^2*(e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(9*b^2*d^3) - (a*(e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b^2*d) - (3*f^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(128*b*d^4) - (3*f*(e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(16*b*d^2) + (3*a^2*f^2*(e + f*x)*Sinh[c + d*x]^2)/(4*b^3*d^3) + (a^2*(e + f*x)^3*Sinh[c + d*x]^2)/(2*b^3*d)
```

---

**Rubi [A]** time = 1.52371, antiderivative size = 1123, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 17, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.472$ , Rules used = {5579, 5447, 3311, 32, 2635, 8, 3296, 2638, 3310, 5565, 5446, 5561,

2190, 2531, 6609, 2282, 6589}

$$-\frac{a^2(a^2+b^2)(e+fx)^4}{4b^5f} + \frac{\cosh^4(c+dx)(e+fx)^3}{4bd} + \frac{a^2\sinh^2(c+dx)(e+fx)^3}{2b^3d} + \frac{a^2(a^2+b^2)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)(e+fx)}{b^5d}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Cosh[c + d\*x]^3\*Sinh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (3\*a^2\*f^3\*x)/(8\*b^3\*d^3) - (45\*f^3\*x)/(256\*b\*d^3) + (a^2\*(e + f\*x)^3)/(4\*b^3\*d) - (3\*(e + f\*x)^3)/(32\*b\*d) - (a^2\*(a^2 + b^2)\*(e + f\*x)^4)/(4\*b^5\*f) + (6\*a^3\*f^3\*Cosh[c + d\*x])/(b^4\*d^4) + (40\*a\*f^3\*Cosh[c + d\*x])/(9\*b^2\*d^4) + (3\*a^3\*f\*(e + f\*x)^2\*Cosh[c + d\*x])/(b^4\*d^2) + (2\*a\*f\*(e + f\*x)^2\*Cosh[c + d\*x])/(b^2\*d^2) + (9\*f^2\*(e + f\*x)\*Cosh[c + d\*x]^2)/(32\*b\*d^3) + (2\*a\*f^3\*Cosh[c + d\*x]^3)/(27\*b^2\*d^4) + (a\*f\*(e + f\*x)^2\*Cosh[c + d\*x]^3)/(3\*b^2\*d^2) + (3\*f^2\*(e + f\*x)\*Cosh[c + d\*x]^4)/(32\*b\*d^3) + ((e + f\*x)^3\*Cosh[c + d\*x]^4)/(4\*b\*d) + (a^2\*(a^2 + b^2)\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b^5\*d) + (a^2\*(a^2 + b^2)\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b^5\*d) + (3\*a^2\*(a^2 + b^2)\*f\*(e + f\*x)^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^5\*d^2) + (3\*a^2\*(a^2 + b^2)\*f\*(e + f\*x)^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^5\*d^2) - (6\*a^2\*(a^2 + b^2)\*f^2\*(e + f\*x)\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^5\*d^3) - (6\*a^2\*(a^2 + b^2)\*f^2\*(e + f\*x)\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^5\*d^3) + (6\*a^2\*(a^2 + b^2)\*f^3\*PolyLog[4, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^5\*d^4) + (6\*a^2\*(a^2 + b^2)\*f^3\*PolyLog[4, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^5\*d^4) - (6\*a^3\*f^2\*(e + f\*x)\*Sinh[c + d\*x])/(b^4\*d^3) - (40\*a\*f^2\*(e + f\*x)\*Sinh[c + d\*x])/(9\*b^2\*d^3) - (a^3\*(e + f\*x)^3\*Sinh[c + d\*x])/(b^4\*d) - (2\*a\*(e + f\*x)^3\*Sinh[c + d\*x])/(3\*b^2\*d) - (3\*a^2\*f^3\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(8\*b^3\*d^4) - (45\*f^3\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(256\*b\*d^4) - (3\*a^2\*f\*(e + f\*x)^2\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(4\*b^3\*d^2) - (9\*f\*(e + f\*x)^2\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(32\*b\*d^2) - (2\*a\*f^2\*(e + f\*x)\*Cosh[c + d\*x]^2\*Sinh[c + d\*x])/(9\*b^2\*d^3) - (a\*(e + f\*x)^3\*Cosh[c + d\*x]^2\*Sinh[c + d\*x])/(3\*b^2\*d) - (3\*f^3\*Cosh[c + d\*x]^3\*Sinh[c + d\*x])/(128\*b\*d^4) - (3\*f\*(e + f\*x)^2\*Cosh[c + d\*x]^3\*Sinh[c + d\*x])/(16\*b\*d^2) + (3\*a^2\*f^2\*(e + f\*x)\*Sinh[c + d\*x]^2)/(4\*b^3\*d^3) + (a^2\*(e + f\*x)^3\*Sinh[c + d\*x]^2)/(2\*b^3\*d)

### Rule 5579

Int[(Cosh[(c\_.) + (d\_.)\*(x\_.)]^(p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]^(n\_.)]/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^p\*Sinh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Cosh[c + d\*x]^p\*Sinh[c + d\*x]^(n - 1))/(a + b\*Sinh

$[c + d*x]), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5447

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sinh[(a\_.) + (b\_.)\*(x\_)], x\_Symbol] := Simp[((c + d\*x)^m\*Cosh[a + b\*x]^(n + 1))/(b\*(n + 1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*Cosh[a + b\*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

### Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*Sine[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*Sine[e + f\*x])^(n - 2), x], x] - Dist[(d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*Sine[e + f\*x])^n, x], x] - Simp[(b\*(c + d\*x)^m\*Cos[e + f\*x]\*(b\*Sine[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_.), x\_Symbol] := -Simp[(b\*Cos[c + d\*x]\*(b\*Sine[c + d\*x])^(n - 1))/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Sine[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
  Simp[(d*(b*SIN[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*SIN[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*SIN[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_))*((e_.) + (f_.)*(x_))^(m_.)]/((a_.) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.)]/((a_.) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))*((c_.) + (d_.)*(x_))^(m_.)]/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x
)))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
```

```

)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]

```

### Rule 6609

```

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_)))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]

```

### Rule 2282

```

Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

### Rule 6589

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \cosh^3(c+dx) \sinh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{(e+fx)^3 \cosh^4(c+dx)}{4bd} - \frac{a \int (e+fx)^3 \cosh^3(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)^3 \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
&= \frac{af(e+fx)^2 \cosh^3(c+dx)}{3b^2d^2} + \frac{3f^2(e+fx) \cosh^4(c+dx)}{32bd^3} + \frac{(e+fx)^3 \cosh^3(c+dx)}{4bd} \\
&= -\frac{a^2(a^2+b^2)(e+fx)^4}{4b^5f} + \frac{9f^2(e+fx) \cosh^2(c+dx)}{32bd^3} + \frac{2af^3 \cosh^3(c+dx)}{27b^2d^4} \\
&= -\frac{3(e+fx)^3}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^4}{4b^5f} + \frac{3a^3f(e+fx)^2 \cosh(c+dx)}{b^4d^2} + \frac{2af^3 \cosh^3(c+dx)}{27b^2d^4} \\
&= -\frac{45f^3x}{256bd^3} + \frac{a^2(e+fx)^3}{4b^3d} - \frac{3(e+fx)^3}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^4}{4b^5f} + \frac{4af^3 \cosh^3(c+dx)}{9b^2d^4} \\
&= \frac{3a^2f^3x}{8b^3d^3} - \frac{45f^3x}{256bd^3} + \frac{a^2(e+fx)^3}{4b^3d} - \frac{3(e+fx)^3}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^4}{4b^5f} + \frac{4af^3 \cosh^3(c+dx)}{9b^2d^4} \\
&= \frac{3a^2f^3x}{8b^3d^3} - \frac{45f^3x}{256bd^3} + \frac{a^2(e+fx)^3}{4b^3d} - \frac{3(e+fx)^3}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^4}{4b^5f} + \frac{4af^3 \cosh^3(c+dx)}{9b^2d^4} \\
&= \frac{3a^2f^3x}{8b^3d^3} - \frac{45f^3x}{256bd^3} + \frac{a^2(e+fx)^3}{4b^3d} - \frac{3(e+fx)^3}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^4}{4b^5f} + \frac{4af^3 \cosh^3(c+dx)}{9b^2d^4}
\end{aligned}$$

**Mathematica [B]** time = 42.1848, size = 8734, normalized size = 7.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Result too large to show
```

**Maple [F]** time = 0.273, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 (\cosh(dx + c))^3 (\sinh(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*cosh(d\*x+c)^3\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*cosh(d\*x+c)^3\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)^3\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/192*e^3*((8*a*b^2*e^{(-d*x - c)} - 3*b^3 - 12*(2*a^2*b + b^3))*e^{(-2*d*x - 2*c)} + 24*(4*a^3 + 3*a*b^2)*e^{(-3*d*x - 3*c)})*e^{(4*d*x + 4*c)}/(b^4*d) - 192 \\ & *(a^4 + a^2*b^2)*(d*x + c)/(b^5*d) - (8*a*b^2*e^{(-3*d*x - 3*c)} + 3*b^3*e^{(-4*d*x - 4*c)} + 24*(4*a^3 + 3*a*b^2)*e^{(-d*x - c)} + 12*(2*a^2*b + b^3)*e^{(-2*d*x - 2*c)})/(b^4*d) - 192*(a^4 + a^2*b^2)*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/(b^5*d) + 1/55296*(13824*(a^4*d^4*f^3*e^{(4*c)} + a^2*b^2*d^4*f^3*e^{(4*c)})*x^4 + 55296*(a^4*d^4*e*f^2*e^{(4*c)} + a^2*b^2*d^4*e*f^2*e^{(4*c)})*x^3 + 82944*(a^4*d^4*e^2*f*e^{(4*c)} + a^2*b^2*d^4*e^2*f*e^{(4*c)})*x^2 + 27*(32*b^4*d^3*f^3*x^3*e^{(8*c)} + 24*(4*d^3*e*f^2 - d^2*f^3)*b^4*x^2*e^{(8*c)} + 12*(8*d^3*e^2*f - 4*d^2*e*f^2 + d*f^3)*b^4*x*e^{(8*c)} - 3*(8*d^2*e^2*f - 4*d*e*f^2 + f^3)*b^4*e^{(8*c)})*e^{(4*d*x)} - 256*(9*a*b^3*d^3*f^3*x^3*e^{(7*c)} + 9*(3*d^3*e*f^2 - d^2*f^3)*a*b^3*x^2*e^{(7*c)} + 3*(9*d^3*e^2*f - 6*d^2*e*f^2 + 2*d*f^3)*a*b^3*x*e^{(7*c)} - (9*d^2*e^2*f - 6*d*e*f^2 + 2*f^3)*a*b^3*e^{(7*c)})*e^{(3*d*x)} - 864*(6*(2*d^2*e^2*f - 2*d*e*f^2 + f^3)*a^2*b^2*e^{(6*c)} + 3*(2*d^2*e^2*f - 2*d*e*f^2 + f^3)*b^4*e^{(6*c)} - 4*(2*a^2*b^2*d^3*f^3*e^{(6*c)} + b^4*d^3*f^3*e^{(6*c)})*x^3 - 6*(2*(2*d^3*e*f^2 - d^2*f^3)*a^2*b^2*e^{(6*c)} + (2*d^3*e*f^2 - d^2*f^3)*b^4*e^{(6*c)})*x^2 - 6*(2*(2*d^3*e^2*f - 2*d^2*e*f^2 + d*f^3)*a^2*b^2*e^{(6*c)} + (2*d^3*e^2*f - 2*d^2*e*f^2 + d*f^3)*b^4*e^{(6*c)})*x)*e^{(2*d*x)} + 6912*(12*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a^3*b*e^{(5*c)} + 9*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a*b^3*e^{(5*c)} - (4*a^3*b*d^3*f^3*e^{(5*c)} + 3*a*b^3*d^3*f^3*e^{(5*c)})*x^3 - 3*(4*(d^3*e*f^2 - d^2*f^3)*a^3*b*e^{(5*c)} + \end{aligned}$$



$$\begin{aligned}
& 3*(d^3*e*f^2 - d^2*f^3)*a*b^3*e^{(5*c)}*x^2 - 3*(4*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*a^3*b*e^{(5*c)} + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*a*b^3*e^{(5*c)})*x)*e^{(d*x)} + 6912*(12*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*a^3*b*e^{(3*c)} + 9*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*a*b^3*e^{(3*c)} + (4*a^3*b*d^3*f^3*e^{(3*c)} + 3*a*b^3*d^3*f^3*e^{(3*c)})*x^3 + 3*(4*(d^3*e*f^2 + d^2*f^3)*a^3*b*e^{(3*c)} + 3*(d^3*e*f^2 + d^2*f^3)*a*b^3*e^{(3*c)})*x^2 + 3*(4*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*a^3*b*e^{(3*c)} + 3*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*a*b^3*e^{(3*c)})*x)*e^{(-d*x)} + 864*(6*(2*d^2*e^2*f + 2*d*e*f^2 + f^3)*a^2*b^2*e^{(2*c)} + 3*(2*d^2*e^2*f + 2*d*e*f^2 + f^3)*b^4*e^{(2*c)} + 4*(2*a^2*b^2*d^3*f^3*e^{(2*c)} + b^4*d^3*f^3*e^{(2*c)})*x^3 + 6*(2*(2*d^3*e*f^2 + d^2*f^3)*a^2*b^2*e^{(2*c)} + (2*d^3*e*f^2 + d^2*f^3)*b^4*e^{(2*c)})*x^2 + 6*(2*(2*d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*a^2*b^2*e^{(2*c)} + (2*d^3*e^2*f + 2*d^2*e*f^2 + d*f^3)*b^4*e^{(2*c)})*x)*e^{(-2*d*x)} + 256*(9*a*b^3*d^3*f^3*x^3*e^c + 9*(3*d^3*e*f^2 + d^2*f^3)*a*b^3*x^2*e^c + 3*(9*d^3*e^2*f + 6*d^2*e*f^2 + 2*d*f^3)*a*b^3*x*e^c + (9*d^2*e^2*f + 6*d*e*f^2 + 2*f^3)*a*b^3*e^c)*e^{(-3*d*x)} + 27*(32*b^4*d^3*f^3*x^3 + 24*(4*d^3*e*f^2 + d^2*f^3)*b^4*x^2 + 12*(8*d^3*e^2*f + 4*d^2*e*f^2 + d*f^3)*b^4*x + 3*(8*d^2*e^2*f + 4*d*e*f^2 + f^3)*b^4)*e^{(-4*d*x)}*e^{(-4*c)}/(b^5*d^4) - \text{integrate}(-2*((a^4*b*f^3 + a^2*b^3*f^3)*x^3 + 3*(a^4*b*e*f^2 + a^2*b^3*e*f^2)*x^2 + 3*(a^4*b*e^2*f + a^2*b^3*e^2*f)*x - ((a^5*f^3*e^c + a^3*b^2*f^3*e^c)*x^3 + 3*(a^5*e*f^2*e^c + a^3*b^2*e*f^2*e^c)*x^2 + 3*(a^5*e^2*f*e^c + a^3*b^2*e^2*f*e^c)*x)*e^{(d*x)})/(b^6*e^{(2*d*x + 2*c)} + 2*a*b^5*e^{(d*x + c)} - b^6), x)
\end{aligned}$$


---

**Fricas [C]** time = 4.35762, size = 26953, normalized size = 24.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)^3\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 1/55296\*(864\*b^4\*d^3\*f^3\*x^3 + 864\*b^4\*d^3\*e^3 + 648\*b^4\*d^2\*e^2\*f + 27\*(32\*b^4\*d^3\*f^3\*x^3 + 32\*b^4\*d^3\*e^3 - 24\*b^4\*d^2\*e^2\*f + 12\*b^4\*d\*e\*f^2 - 3\*b^4\*f^3 + 24\*(4\*b^4\*d^3\*e\*f^2 - b^4\*d^2\*f^3)\*x^2 + 12\*(8\*b^4\*d^3\*e^2\*f - 4\*b^4\*d^2\*e\*f^2 + b^4\*d\*f^3)\*x)\*cosh(d\*x + c)^8 + 27\*(32\*b^4\*d^3\*f^3\*x^3 + 32\*b^4\*d^3\*e^3 - 24\*b^4\*d^2\*e^2\*f + 12\*b^4\*d\*e\*f^2 - 3\*b^4\*f^3 + 24\*(4\*b^4\*d^3\*e\*f^2 - b^4\*d^2\*f^3)\*x^2 + 12\*(8\*b^4\*d^3\*e^2\*f - 4\*b^4\*d^2\*e\*f^2 + b^4\*d\*f^3)\*x)\*sinh(d\*x + c)^8 + 324\*b^4\*d\*e\*f^2 - 256\*(9\*a\*b^3\*d^3\*f^3\*x^3 + 9\*a\*b^3\*d^3\*e^3 - 9\*a\*b^3\*d^2\*e^2\*f + 6\*a\*b^3\*d\*e\*f^2 - 2\*a\*b^3\*f^3 + 9\*(3\*a\*b^3\*d^3\*e\*f^2 - a\*b^3\*d^2\*f^3)\*x^2 + 3\*(9\*a\*b^3\*d^3\*e^2\*f - 6\*a\*b^3\*d^2\*e\*f^2 + 2\*a\*b^3\*d\*f^3)\*x)\*cosh(d\*x + c)^7 - 8\*(288\*a\*b^3\*d^3\*f^3\*x^3 + 288\*a\*b^3\*

$$\begin{aligned}
& d^3e^3 - 288ab^3d^2e^2f + 192ab^3d^2e^2f^2 - 64ab^3d^2e^2f^3 + 288(3a \\
& * b^3d^3e^2f^2 - ab^3d^2e^2f^3)x^2 + 96(9ab^3d^3e^2f - 6ab^3d^2e^2 \\
& * f^2 + 2ab^3d^2e^2f^3)x - 27(32b^4d^3f^3x^3 + 32b^4d^3e^3 - 24b^4d^3 \\
& d^2e^2f + 12b^4d^2e^2f^2 - 3b^4d^2e^2f^3 + 24(4b^4d^3e^2f^2 - b^4d^2e^2f^3) \\
& * x^2 + 12(8b^4d^3e^2f - 4b^4d^2e^2f^2 + b^4d^2e^2f^3)x) * \cosh(dx + c) \\
& * \sinh(dx + c)^7 + 81b^4d^3f^3 + 864(4(2a^2b^2 + b^4)d^3f^3x^3 + 4(2 \\
& * a^2b^2 + b^4)d^3e^3 - 6(2a^2b^2 + b^4)d^2e^2f + 6(2a^2b^2 + b^4) \\
& * d^2e^2f^2 - 3(2a^2b^2 + b^4)f^3 + 6(2(2a^2b^2 + b^4)d^3e^2f^2 - ( \\
& 2a^2b^2 + b^4)d^2e^2f^3)x^2 + 6(2(2a^2b^2 + b^4)d^3e^2f - 2(2a^2 \\
& * b^2 + b^4)d^2e^2f^2 + (2a^2b^2 + b^4)d^2e^2f^3)x) * \cosh(dx + c)^6 + 4(86 \\
& 4(2a^2b^2 + b^4)d^3f^3x^3 + 864(2a^2b^2 + b^4)d^3e^3 - 1296(2a^2 \\
& * b^2 + b^4)d^2e^2f + 1296(2a^2b^2 + b^4)d^2e^2f^2 - 648(2a^2b^2 + \\
& b^4)f^3 + 1296(2(2a^2b^2 + b^4)d^3e^2f^2 - (2a^2b^2 + b^4)d^2e^2f^3) \\
& )x^2 + 189(32b^4d^3f^3x^3 + 32b^4d^3e^3 - 24b^4d^2e^2f + 12b^4 \\
& d^2e^2f^2 - 3b^4d^2e^2f^3 + 24(4b^4d^3e^2f^2 - b^4d^2e^2f^3)x^2 + 12(8b^4d^3 \\
& d^3e^2f - 4b^4d^2e^2f^2 + b^4d^2e^2f^3)x) * \cosh(dx + c)^2 + 1296(2(2a^2 \\
& * b^2 + b^4)d^3e^2f - 2(2a^2b^2 + b^4)d^2e^2f^2 + (2a^2b^2 + b^4) \\
& * d^2e^2f^3)x - 448(9ab^3d^3f^3x^3 + 9ab^3d^3e^3 - 9ab^3d^2e^2f + \\
& 6ab^3d^2e^2f^2 - 2ab^3d^2e^2f^3 + 9(3ab^3d^3e^2f^2 - ab^3d^2e^2f^3)x^2 \\
& + 3(9ab^3d^3e^2f - 6ab^3d^2e^2f^2 + 2ab^3d^2e^2f^3)x) * \cosh(dx + c) \\
& ) * \sinh(dx + c)^6 - 6912((4a^3b + 3ab^3)d^3f^3x^3 + (4a^3b + 3ab^3) \\
& * d^3e^3 - 3(4a^3b + 3ab^3)d^2e^2f + 6(4a^3b + 3ab^3)d^2e^2 \\
& * f^2 - 6(4a^3b + 3ab^3)f^3 + 3((4a^3b + 3ab^3)d^3e^2f^2 - (4a^3 \\
& * b + 3ab^3)d^2e^2f^3)x^2 + 3((4a^3b + 3ab^3)d^3e^2f - 2(4a^3b \\
& + 3ab^3)d^2e^2f^2 + 2(4a^3b + 3ab^3)d^2e^2f^3)x) * \cosh(dx + c)^5 - 2 \\
& 4(288(4a^3b + 3ab^3)d^3f^3x^3 + 288(4a^3b + 3ab^3)d^3e^3 - \\
& 864(4a^3b + 3ab^3)d^2e^2f + 1728(4a^3b + 3ab^3)d^2e^2f^2 - 1728 \\
& * (4a^3b + 3ab^3)f^3 - 63(32b^4d^3f^3x^3 + 32b^4d^3e^3 - 24b^4d^3 \\
& d^2e^2f + 12b^4d^2e^2f^2 - 3b^4d^2e^2f^3 + 24(4b^4d^3e^2f^2 - b^4d^2e^2f^3) \\
& )x^2 + 12(8b^4d^3e^2f - 4b^4d^2e^2f^2 + b^4d^2e^2f^3)x) * \cosh(dx + c) \\
& ^3 + 864((4a^3b + 3ab^3)d^3e^2f^2 - (4a^3b + 3ab^3)d^2e^2f^3)x^2 \\
& + 224(9ab^3d^3f^3x^3 + 9ab^3d^3e^3 - 9ab^3d^2e^2f + 6ab^3d^2 \\
& * e^2f^2 - 2ab^3d^2e^2f^3 + 9(3ab^3d^3e^2f^2 - ab^3d^2e^2f^3)x^2 + 3(9ab^3 \\
& * d^3e^2f - 6ab^3d^2e^2f^2 + 2ab^3d^2e^2f^3)x) * \cosh(dx + c)^2 + 864 \\
& * ((4a^3b + 3ab^3)d^3e^2f - 2(4a^3b + 3ab^3)d^2e^2f^2 + 2(4a^3 \\
& * b + 3ab^3)d^2e^2f^3)x - 216(4(2a^2b^2 + b^4)d^3f^3x^3 + 4(2a^2b^2 \\
& + b^4)d^3e^3 - 6(2a^2b^2 + b^4)d^2e^2f + 6(2a^2b^2 + b^4)d^2e^2 \\
& * f^2 - 3(2a^2b^2 + b^4)f^3 + 6(2(2a^2b^2 + b^4)d^3e^2f^2 - (2a^2b^2 \\
& + b^4)d^2e^2f^3)x^2 + 6(2(2a^2b^2 + b^4)d^3e^2f - 2(2a^2b^2 + \\
& b^4)d^2e^2f^2 + (2a^2b^2 + b^4)d^2e^2f^3)x) * \cosh(dx + c) * \sinh(dx + c)^5 \\
& - 13824((a^4 + a^2b^2)d^4f^3x^4 + 4(a^4 + a^2b^2)d^4e^2f^2x^3 + \\
& 6(a^4 + a^2b^2)d^4e^2f^2x^2 + 4(a^4 + a^2b^2)d^4e^3x + 8(a^4 + a^2 \\
& * b^2) * c * d^3e^3 - 12(a^4 + a^2b^2) * c^2 * d^2e^2f + 8(a^4 + a^2b^2) * c^3 \\
& * d^2e^2f^2 - 2(a^4 + a^2b^2) * c^4 * f^3) * \cosh(dx + c)^4 - 2(6912(a^4 + a^2b^2) \\
& * d^4f^3x^4 + 27648(a^4 + a^2b^2) * d^4e^2f^2x^3 + 41472(a^4 + a^2b^2)
\end{aligned}$$

$$\begin{aligned}
& ^2)*d^4*e^2*f*x^2 + 27648*(a^4 + a^2*b^2)*d^4*e^3*x + 55296*(a^4 + a^2*b^2) \\
& *c*d^3*e^3 - 82944*(a^4 + a^2*b^2)*c^2*d^2*e^2*f + 55296*(a^4 + a^2*b^2)*c^ \\
& 3*d*e*f^2 - 13824*(a^4 + a^2*b^2)*c^4*f^3 - 945*(32*b^4*d^3*f^3*x^3 + 32*b^ \\
& 4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3*e \\
& *f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f^3 \\
& )*x)*\cosh(d*x + c)^4 + 4480*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^ \\
& 3*d^2*e^2*f + 6*a*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*d^ \\
& 2*f^3)*x^2 + 3*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x) \\
& *\cosh(d*x + c)^3 - 6480*(4*(2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*(2*a^2*b^2 + b \\
& ^4)*d^3*e^3 - 6*(2*a^2*b^2 + b^4)*d^2*e^2*f + 6*(2*a^2*b^2 + b^4)*d*e*f^2 - \\
& 3*(2*a^2*b^2 + b^4)*f^3 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e*f^2 - (2*a^2*b^2 + \\
& b^4)*d^2*f^3)*x^2 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e^2*f - 2*(2*a^2*b^2 + b^4)* \\
& d^2*e*f^2 + (2*a^2*b^2 + b^4)*d*f^3)*x)*\cosh(d*x + c)^2 + 17280*((4*a^3*b + \\
& 3*a*b^3)*d^3*f^3*x^3 + (4*a^3*b + 3*a*b^3)*d^3*e^3 - 3*(4*a^3*b + 3*a*b^3) \\
& *d^2*e^2*f + 6*(4*a^3*b + 3*a*b^3)*d*e*f^2 - 6*(4*a^3*b + 3*a*b^3)*f^3 + 3* \\
& ((4*a^3*b + 3*a*b^3)*d^3*e*f^2 - (4*a^3*b + 3*a*b^3)*d^2*f^3)*x^2 + 3*((4*a \\
& ^3*b + 3*a*b^3)*d^3*e^2*f - 2*(4*a^3*b + 3*a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + \\
& 3*a*b^3)*d*f^3)*x)*\cosh(d*x + c)*\sinh(d*x + c)^4 + 6912*((4*a^3*b + 3*a*b^ \\
& 3)*d^3*f^3*x^3 + (4*a^3*b + 3*a*b^3)*d^3*e^3 + 3*(4*a^3*b + 3*a*b^3)*d^2*e^ \\
& 2*f + 6*(4*a^3*b + 3*a*b^3)*d*e*f^2 + 6*(4*a^3*b + 3*a*b^3)*f^3 + 3*((4*a^3 \\
& *b + 3*a*b^3)*d^3*e*f^2 + (4*a^3*b + 3*a*b^3)*d^2*f^3)*x^2 + 3*((4*a^3*b + \\
& 3*a*b^3)*d^3*e^2*f + 2*(4*a^3*b + 3*a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + 3*a*b^3 \\
& )*d*f^3)*x)*\cosh(d*x + c)^3 + 8*(864*(4*a^3*b + 3*a*b^3)*d^3*f^3*x^3 + 864* \\
& (4*a^3*b + 3*a*b^3)*d^3*e^3 + 2592*(4*a^3*b + 3*a*b^3)*d^2*e^2*f + 189*(32* \\
& b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b^ \\
& 4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f - 4*b^ \\
& 4*d^2*e*f^2 + b^4*d*f^3)*x)*\cosh(d*x + c)^5 + 5184*(4*a^3*b + 3*a*b^3)*d*e* \\
& f^2 - 1120*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^3*d^2*e^2*f + 6*a \\
& *b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + 3* \\
& (9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)*\cosh(d*x + c)^4 \\
& + 5184*(4*a^3*b + 3*a*b^3)*f^3 + 2160*(4*(2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 4* \\
& (2*a^2*b^2 + b^4)*d^3*e^3 - 6*(2*a^2*b^2 + b^4)*d^2*e^2*f + 6*(2*a^2*b^2 + \\
& b^4)*d*e*f^2 - 3*(2*a^2*b^2 + b^4)*f^3 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e*f^2 - \\
& (2*a^2*b^2 + b^4)*d^2*f^3)*x^2 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e^2*f - 2*(2*a \\
& ^2*b^2 + b^4)*d^2*e*f^2 + (2*a^2*b^2 + b^4)*d*f^3)*x)*\cosh(d*x + c)^3 + 259 \\
& 2*((4*a^3*b + 3*a*b^3)*d^3*e*f^2 + (4*a^3*b + 3*a*b^3)*d^2*f^3)*x^2 - 8640* \\
& ((4*a^3*b + 3*a*b^3)*d^3*f^3*x^3 + (4*a^3*b + 3*a*b^3)*d^3*e^3 - 3*(4*a^3*b \\
& + 3*a*b^3)*d^2*e^2*f + 6*(4*a^3*b + 3*a*b^3)*d*e*f^2 - 6*(4*a^3*b + 3*a*b^ \\
& 3)*f^3 + 3*((4*a^3*b + 3*a*b^3)*d^3*e*f^2 - (4*a^3*b + 3*a*b^3)*d^2*f^3)*x^ \\
& 2 + 3*((4*a^3*b + 3*a*b^3)*d^3*e^2*f - 2*(4*a^3*b + 3*a*b^3)*d^2*e*f^2 + 2* \\
& (4*a^3*b + 3*a*b^3)*d*f^3)*x)*\cosh(d*x + c)^2 + 2592*((4*a^3*b + 3*a*b^3)*d \\
& ^3*e^2*f + 2*(4*a^3*b + 3*a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + 3*a*b^3)*d*f^3)*x \\
& - 6912*((a^4 + a^2*b^2)*d^4*f^3*x^4 + 4*(a^4 + a^2*b^2)*d^4*e*f^2*x^3 + 6* \\
& (a^4 + a^2*b^2)*d^4*e^2*f*x^2 + 4*(a^4 + a^2*b^2)*d^4*e^3*x + 8*(a^4 + a^2* \\
& b^2)*c*d^3*e^3 - 12*(a^4 + a^2*b^2)*c^2*d^2*e^2*f + 8*(a^4 + a^2*b^2)*c^3*d
\end{aligned}$$

$$\begin{aligned}
& *e*f^2 - 2*(a^4 + a^2*b^2)*c^4*f^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 648*(4 \\
& *b^4*d^3*e*f^2 + b^4*d^2*f^3)*x^2 + 864*(4*(2*a^2*b^2 + b^4)*d^3*f^3*x^3 + \\
& 4*(2*a^2*b^2 + b^4)*d^3*e^3 + 6*(2*a^2*b^2 + b^4)*d^2*e^2*f + 6*(2*a^2*b^2 \\
& + b^4)*d*e*f^2 + 3*(2*a^2*b^2 + b^4)*f^3 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e*f^2 \\
& + (2*a^2*b^2 + b^4)*d^2*f^3)*x^2 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e^2*f + 2*(2 \\
& *a^2*b^2 + b^4)*d^2*e*f^2 + (2*a^2*b^2 + b^4)*d*f^3)*x)*\cosh(d*x + c)^2 + 1 \\
& 2*(288*(2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 288*(2*a^2*b^2 + b^4)*d^3*e^3 + 63*( \\
& 32*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 - 3 \\
& *b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f - 4 \\
& *b^4*d^2*e*f^2 + b^4*d*f^3)*x)*\cosh(d*x + c)^6 + 432*(2*a^2*b^2 + b^4)*d^2* \\
& e^2*f - 448*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^3*d^2*e^2*f + 6* \\
& a*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + 3 \\
& *(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)*\cosh(d*x + c)^5 \\
& + 432*(2*a^2*b^2 + b^4)*d*e*f^2 + 1080*(4*(2*a^2*b^2 + b^4)*d^3*f^3*x^3 + \\
& 4*(2*a^2*b^2 + b^4)*d^3*e^3 - 6*(2*a^2*b^2 + b^4)*d^2*e^2*f + 6*(2*a^2*b^2 \\
& + b^4)*d*e*f^2 - 3*(2*a^2*b^2 + b^4)*f^3 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e*f^2 \\
& - (2*a^2*b^2 + b^4)*d^2*f^3)*x^2 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e^2*f - 2*(2 \\
& *a^2*b^2 + b^4)*d^2*e*f^2 + (2*a^2*b^2 + b^4)*d*f^3)*x)*\cosh(d*x + c)^4 + 2 \\
& 16*(2*a^2*b^2 + b^4)*f^3 - 5760*((4*a^3*b + 3*a*b^3)*d^3*f^3*x^3 + (4*a^3*b \\
& + 3*a*b^3)*d^3*e^3 - 3*(4*a^3*b + 3*a*b^3)*d^2*e^2*f + 6*(4*a^3*b + 3*a*b^ \\
& 3)*d*e*f^2 - 6*(4*a^3*b + 3*a*b^3)*f^3 + 3*((4*a^3*b + 3*a*b^3)*d^3*e*f^2 - \\
& (4*a^3*b + 3*a*b^3)*d^2*f^3)*x^2 + 3*((4*a^3*b + 3*a*b^3)*d^3*e^2*f - 2*(4 \\
& *a^3*b + 3*a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + 3*a*b^3)*d*f^3)*x)*\cosh(d*x + c) \\
& ^3 + 432*(2*(2*a^2*b^2 + b^4)*d^3*e*f^2 + (2*a^2*b^2 + b^4)*d^2*f^3)*x^2 - \\
& 6912*((a^4 + a^2*b^2)*d^4*f^3*x^4 + 4*(a^4 + a^2*b^2)*d^4*e*f^2*x^3 + 6*(a^ \\
& 4 + a^2*b^2)*d^4*e^2*f*x^2 + 4*(a^4 + a^2*b^2)*d^4*e^3*x + 8*(a^4 + a^2*b^2 \\
& )*c*d^3*e^3 - 12*(a^4 + a^2*b^2)*c^2*d^2*e^2*f + 8*(a^4 + a^2*b^2)*c^3*d*e* \\
& f^2 - 2*(a^4 + a^2*b^2)*c^4*f^3)*\cosh(d*x + c)^2 + 432*(2*(2*a^2*b^2 + b^4) \\
& *d^3*e^2*f + 2*(2*a^2*b^2 + b^4)*d^2*e*f^2 + (2*a^2*b^2 + b^4)*d*f^3)*x + 1 \\
& 728*((4*a^3*b + 3*a*b^3)*d^3*f^3*x^3 + (4*a^3*b + 3*a*b^3)*d^3*e^3 + 3*(4*a \\
& ^3*b + 3*a*b^3)*d^2*e^2*f + 6*(4*a^3*b + 3*a*b^3)*d*e*f^2 + 6*(4*a^3*b + 3* \\
& a*b^3)*f^3 + 3*((4*a^3*b + 3*a*b^3)*d^3*e*f^2 + (4*a^3*b + 3*a*b^3)*d^2*f^3 \\
& )*x^2 + 3*((4*a^3*b + 3*a*b^3)*d^3*e^2*f + 2*(4*a^3*b + 3*a*b^3)*d^2*e*f^2 \\
& + 2*(4*a^3*b + 3*a*b^3)*d*f^3)*x)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 324*(8*b \\
& ^4*d^3*e^2*f + 4*b^4*d^2*e*f^2 + b^4*d*f^3)*x + 256*(9*a*b^3*d^3*f^3*x^3 + \\
& 9*a*b^3*d^3*e^3 + 9*a*b^3*d^2*e^2*f + 6*a*b^3*d*e*f^2 + 2*a*b^3*f^3 + 9*(3* \\
& a*b^3*d^3*e*f^2 + a*b^3*d^2*f^3)*x^2 + 3*(9*a*b^3*d^3*e^2*f + 6*a*b^3*d^2*e \\
& *f^2 + 2*a*b^3*d*f^3)*x)*\cosh(d*x + c) + 165888*((a^4 + a^2*b^2)*d^2*f^3*x \\
& ^2 + 2*(a^4 + a^2*b^2)*d^2*e*f^2*x + (a^4 + a^2*b^2)*d^2*e^2*f)*\cosh(d*x + \\
& c)^4 + 4*((a^4 + a^2*b^2)*d^2*f^3*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f^2*x + (a^ \\
& 4 + a^2*b^2)*d^2*e^2*f)*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*((a^4 + a^2*b^2)* \\
& d^2*f^3*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f^2*x + (a^4 + a^2*b^2)*d^2*e^2*f)*co \\
& sh(d*x + c)^2*\sinh(d*x + c)^2 + 4*((a^4 + a^2*b^2)*d^2*f^3*x^2 + 2*(a^4 + a \\
& ^2*b^2)*d^2*e*f^2*x + (a^4 + a^2*b^2)*d^2*e^2*f)*\cosh(d*x + c)*\sinh(d*x + c \\
& )^3 + ((a^4 + a^2*b^2)*d^2*f^3*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f^2*x + (a^4 +
\end{aligned}$$



$$\begin{aligned}
& 3 + 3*(a^4 + a^2*b^2)*d^3*e*f^2*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*f*x + 3*(a^4 + a^2*b^2)*c*d^2*e^2*f - 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 + (a^4 + a^2*b^2)*c^3*f^3)*\sinh(d*x + c)^4)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2) - b}/b) + 55296*(((a^4 + a^2*b^2)*d^3*f^3*x^3 + 3*(a^4 + a^2*b^2)*d^3*e*f^2*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*f*x + 3*(a^4 + a^2*b^2)*c*d^2*e^2*f - 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 + (a^4 + a^2*b^2)*c^3*f^3)*\cosh(d*x + c)^4 + 4*((a^4 + a^2*b^2)*d^3*f^3*x^3 + 3*(a^4 + a^2*b^2)*d^3*e*f^2*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*f*x + 3*(a^4 + a^2*b^2)*c*d^2*e^2*f - 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 + (a^4 + a^2*b^2)*c^3*f^3)*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*((a^4 + a^2*b^2)*d^3*f^3*x^3 + 3*(a^4 + a^2*b^2)*d^3*e*f^2*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*f*x + 3*(a^4 + a^2*b^2)*c*d^2*e^2*f - 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 + (a^4 + a^2*b^2)*c^3*f^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*((a^4 + a^2*b^2)*d^3*f^3*x^3 + 3*(a^4 + a^2*b^2)*d^3*e*f^2*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*f*x + 3*(a^4 + a^2*b^2)*c*d^2*e^2*f - 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 + (a^4 + a^2*b^2)*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^4 + a^2*b^2)*d^3*f^3*x^3 + 3*(a^4 + a^2*b^2)*d^3*e*f^2*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*f*x + 3*(a^4 + a^2*b^2)*c*d^2*e^2*f - 3*(a^4 + a^2*b^2)*c^2*d*e*f^2 + (a^4 + a^2*b^2)*c^3*f^3)*\sinh(d*x + c)^4)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2) - b}/b) + 331776*(((a^4 + a^2*b^2)*f^3*\cosh(d*x + c)^4 + 4*(a^4 + a^2*b^2)*f^3*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*(a^4 + a^2*b^2)*f^3*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*(a^4 + a^2*b^2)*f^3*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^4 + a^2*b^2)*f^3*\sinh(d*x + c)^4)*\text{polylog}(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2)}/b) + 331776*(((a^4 + a^2*b^2)*f^3*\cosh(d*x + c)^4 + 4*(a^4 + a^2*b^2)*f^3*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*(a^4 + a^2*b^2)*f^3*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*(a^4 + a^2*b^2)*f^3*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^4 + a^2*b^2)*f^3*\sinh(d*x + c)^4)*\text{polylog}(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2)}/b) - 331776*(((a^4 + a^2*b^2)*d*f^3*x + (a^4 + a^2*b^2)*d*e*f^2)*\cosh(d*x + c)^4 + 4*((a^4 + a^2*b^2)*d*f^3*x + (a^4 + a^2*b^2)*d*e*f^2)*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*((a^4 + a^2*b^2)*d*f^3*x + (a^4 + a^2*b^2)*d*e*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*((a^4 + a^2*b^2)*d*f^3*x + (a^4 + a^2*b^2)*d*e*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^4 + a^2*b^2)*d*f^3*x + (a^4 + a^2*b^2)*d*e*f^2)*\sinh(d*x + c)^4)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2)}/b) - 331776*(((a^4 + a^2*b^2)*d*f^3*x + (a^4 + a^2*b^2)*d*e*f^2)*\cosh(d*x + c)^4 + 4*((a^4 + a^2*b^2)*d*f^3*x + (a^4 + a^2*b^2)*d*e*f^2)*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*((a^4 + a^2*b^2)*d*f^3*x + (a^4 + a^2*b^2)*d*e*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*((a^4 + a^2*b^2)*d*f^3*x + (a^4 + a^2*b^2)*d*e*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^4 + a^2*b^2)*d*f^3*x + (a^4 + a^2*b^2)*d*e*f^2)*\sinh(d*x + c)^4)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2)}/b) + 8*(288*a*b^3*d^3*f^3*x^3 + 288*a*b^3*d^3*e^3 + 288*a*b^3*d^2*e^2*f + 192*a*b^3*d*e*f^2 + 27*(32*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*d^2*e^2*f
\end{aligned}$$

$$\begin{aligned}
& + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)*x^2 + 12 \\
& *(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f^3)*x)*\cosh(d*x + c)^7 + 64*a* \\
& b^3*f^3 - 224*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^3*d^2*e^2*f + \\
& 6*a*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + \\
& 3*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)*\cosh(d*x + c) \\
& ^6 + 648*(4*(2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*(2*a^2*b^2 + b^4)*d^3*e^3 - 6 \\
& *(2*a^2*b^2 + b^4)*d^2*e^2*f + 6*(2*a^2*b^2 + b^4)*d*e*f^2 - 3*(2*a^2*b^2 + \\
& b^4)*f^3 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e*f^2 - (2*a^2*b^2 + b^4)*d^2*f^3)*x \\
& ^2 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e^2*f - 2*(2*a^2*b^2 + b^4)*d^2*e*f^2 + (2* \\
& a^2*b^2 + b^4)*d*f^3)*x)*\cosh(d*x + c)^5 - 4320*((4*a^3*b + 3*a*b^3)*d^3*f^ \\
& 3*x^3 + (4*a^3*b + 3*a*b^3)*d^3*e^3 - 3*(4*a^3*b + 3*a*b^3)*d^2*e^2*f + 6*( \\
& 4*a^3*b + 3*a*b^3)*d*e*f^2 - 6*(4*a^3*b + 3*a*b^3)*f^3 + 3*((4*a^3*b + 3*a* \\
& b^3)*d^3*e*f^2 - (4*a^3*b + 3*a*b^3)*d^2*f^3)*x^2 + 3*((4*a^3*b + 3*a*b^3)* \\
& d^3*e^2*f - 2*(4*a^3*b + 3*a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + 3*a*b^3)*d*f^3)* \\
& x)*\cosh(d*x + c)^4 - 6912*((a^4 + a^2*b^2)*d^4*f^3*x^4 + 4*(a^4 + a^2*b^2)* \\
& d^4*e*f^2*x^3 + 6*(a^4 + a^2*b^2)*d^4*e^2*f*x^2 + 4*(a^4 + a^2*b^2)*d^4*e^3 \\
& *x + 8*(a^4 + a^2*b^2)*c*d^3*e^3 - 12*(a^4 + a^2*b^2)*c^2*d^2*e^2*f + 8*(a^ \\
& 4 + a^2*b^2)*c^3*d*e*f^2 - 2*(a^4 + a^2*b^2)*c^4*f^3)*\cosh(d*x + c)^3 + 288 \\
& *(3*a*b^3*d^3*e*f^2 + a*b^3*d^2*f^3)*x^2 + 2592*((4*a^3*b + 3*a*b^3)*d^3*f^ \\
& 3*x^3 + (4*a^3*b + 3*a*b^3)*d^3*e^3 + 3*(4*a^3*b + 3*a*b^3)*d^2*e^2*f + 6*( \\
& 4*a^3*b + 3*a*b^3)*d*e*f^2 + 6*(4*a^3*b + 3*a*b^3)*f^3 + 3*((4*a^3*b + 3*a* \\
& b^3)*d^3*e*f^2 + (4*a^3*b + 3*a*b^3)*d^2*f^3)*x^2 + 3*((4*a^3*b + 3*a*b^3)* \\
& d^3*e^2*f + 2*(4*a^3*b + 3*a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + 3*a*b^3)*d*f^3)* \\
& x)*\cosh(d*x + c)^2 + 96*(9*a*b^3*d^3*e^2*f + 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d* \\
& f^3)*x + 216*(4*(2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 4*(2*a^2*b^2 + b^4)*d^3*e^3 \\
& + 6*(2*a^2*b^2 + b^4)*d^2*e^2*f + 6*(2*a^2*b^2 + b^4)*d*e*f^2 + 3*(2*a^2*b \\
& ^2 + b^4)*f^3 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e*f^2 + (2*a^2*b^2 + b^4)*d^2*f^ \\
& 3)*x^2 + 6*(2*(2*a^2*b^2 + b^4)*d^3*e^2*f + 2*(2*a^2*b^2 + b^4)*d^2*e*f^2 + \\
& (2*a^2*b^2 + b^4)*d*f^3)*x)*\cosh(d*x + c))*\sinh(d*x + c))/(b^5*d^4*\cosh(d* \\
& x + c)^4 + 4*b^5*d^4*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*b^5*d^4*\cosh(d*x + c \\
& )^2*\sinh(d*x + c)^2 + 4*b^5*d^4*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^5*d^4*\sin \\
& h(d*x + c)^4)
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cosh(d\*x+c)\*\*3\*sinh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c)^3 \sinh(dx + c)^2}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)^3*sinh(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```



$$3.373 \quad \int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=819

$$\frac{f^2 \cosh^4(c+dx)}{32bd^3} + \frac{(e+fx)^2 \cosh^4(c+dx)}{4bd} + \frac{2af(e+fx) \cosh^3(c+dx)}{9b^2d^2} - \frac{f(e+fx) \sinh(c+dx) \cosh^3(c+dx)}{8bd^2} + \frac{3f}{32bd^3}$$

```
[Out] (a^2*e*f*x)/(2*b^3*d) - (3*e*f*x)/(16*b*d) + (a^2*f^2*x^2)/(4*b^3*d) - (3*f^2*x^2)/(32*b*d) - (a^2*(a^2 + b^2)*(e + f*x)^3)/(3*b^5*f) + (2*a^3*f*(e + f*x)*Cosh[c + d*x])/(b^4*d^2) + (4*a*f*(e + f*x)*Cosh[c + d*x])/(3*b^2*d^2) + (3*f^2*Cosh[c + d*x]^2)/(32*b*d^3) + (2*a*f*(e + f*x)*Cosh[c + d*x]^3)/(9*b^2*d^2) + (f^2*Cosh[c + d*x]^4)/(32*b*d^3) + ((e + f*x)^2*Cosh[c + d*x]^4)/(4*b*d) + (a^2*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(b^5*d) + (a^2*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(b^5*d) + (2*a^2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(b^5*d^2) + (2*a^2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(b^5*d^2) - (2*a^2*(a^2 + b^2)*f^2*PolyLog[3, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(b^5*d^3) - (2*a^2*(a^2 + b^2)*f^2*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(b^5*d^3) - (2*a^3*f^2*Sinh[c + d*x])/(b^4*d^3) - (14*a*f^2*Sinh[c + d*x])/(9*b^2*d^3) - (a^3*(e + f*x)^2*Sinh[c + d*x])/(b^4*d) - (2*a*(e + f*x)^2*Sinh[c + d*x])/(3*b^2*d) - (a^2*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^3*d^2) - (3*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(16*b*d^2) - (a*(e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b^2*d) - (f*(e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x])/(8*b*d^2) + (a^2*f^2*Sinh[c + d*x]^2)/(4*b^3*d^3) + (a^2*(e + f*x)^2*Sinh[c + d*x]^2)/(2*b^3*d) - (2*a*f^2*Sinh[c + d*x]^3)/(27*b^2*d^3)
```

**Rubi [A]** time = 1.17102, antiderivative size = 819, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 14, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$ , Rules used = {5579, 5447, 3310, 3311, 3296, 2637, 2633, 5565, 5446, 5561, 2190, 2531, 2282, 6589}

$$\frac{f^2 \cosh^4(c+dx)}{32bd^3} + \frac{(e+fx)^2 \cosh^4(c+dx)}{4bd} + \frac{2af(e+fx) \cosh^3(c+dx)}{9b^2d^2} - \frac{f(e+fx) \sinh(c+dx) \cosh^3(c+dx)}{8bd^2} + \frac{3f}{32bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*e*f*x)/(2*b^3*d) - (3*e*f*x)/(16*b*d) + (a^2*f^2*x^2)/(4*b^3*d) - (3*f^2*x^2)/(32*b*d) - (a^2*(a^2 + b^2)*(e + f*x)^3)/(3*b^5*f) + (2*a^3*f*(e + f*x)*Cosh[c + d*x])/(b^4*d^2) + (4*a*f*(e + f*x)*Cosh[c + d*x])/(3*b^2*d^2) + (3*f^2*Cosh[c + d*x]^2)/(32*b*d^3) + (2*a*f*(e + f*x)*Cosh[c + d*x]^3)/(9*b^2*d^2) + (f^2*Cosh[c + d*x]^4)/(32*b*d^3) + ((e + f*x)^2*Cosh[c + d*x]^4)/(4*b*d) + (a^2*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^5*d) + (a^2*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^5*d) + (2*a^2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^2) + (2*a^2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^2) - (2*a^2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^3) - (2*a^2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^3) - (2*a^3*f^2*Sinh[c + d*x])/(b^4*d^3) - (14*a*f^2*Sinh[c + d*x])/(9*b^2*d^3) - (a^3*(e + f*x)^2*Sinh[c + d*x])/(b^4*d) - (2*a*(e + f*x)^2*Sinh[c + d*x])/(3*b^2*d) - (a^2*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^3*d^2) - (3*f*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(16*b*d^2) - (a*(e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b^2*d) - (f*(e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x])/(8*b*d^2) + (a^2*f^2*Sinh[c + d*x]^2)/(4*b^3*d^3) + (a^2*(e + f*x)^2*Sinh[c + d*x]^2)/(2*b^3*d) - (2*a*f^2*Sinh[c + d*x]^3)/(27*b^2*d^3)
```

### Rule 5579

```
Int[(Cosh[(c_) + (d_)*(x_)]^(p_)*((e_) + (f_)*(x_))^(m_)*Sinh[(c_) + (d_)*(x_)]^(n_)]/((a_) + (b_)*Sinh[(c_) + (d_)*(x_)]), x_Symbol] :> Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 5447

```
Int[Cosh[(a_) + (b_)*(x_)]^(n_)*((c_) + (d_)*(x_))^(m_)*Sinh[(a_) + (b_)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### Rule 3310

```
Int[((c_) + (d_)*(x_))*((b_)*sin[(e_) + (f_)*(x_)]^(n_)), x_Symbol] :> Simp[(d*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*Sine[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*Sine[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]
```

Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*SIN[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*SIN[e + f\*x])^(n - 2), x], x] - Dist[(d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*SIN[e + f\*x])^n, x], x] - Simp[(b\*(c + d\*x)^m\*cos[e + f\*x]\*(b\*SIN[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[(c + d\*x)^m\*cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[SIN[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2633

Int[sin[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 5565

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(m\_))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Dist[a/b^2, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

Rule 5446

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_)\*Sinh[(a\_.) + (b\_.)\*(x\_)]^(n\_), x\_Symbol] := Simp[(c + d\*x)^m\*Sinh[a + b\*x]^(n + 1)/(b\*(n + 1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*Sinh[a + b\*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_))/((a\_.) + (b\_.)\*Sin

```
h[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_.)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \cosh^3(c+dx) \sinh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{(e+fx)^2 \cosh^4(c+dx)}{4bd} - \frac{a \int (e+fx)^2 \cosh^3(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)^2 \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
&= \frac{2af(e+fx) \cosh^3(c+dx)}{9b^2d^2} + \frac{f^2 \cosh^4(c+dx)}{32bd^3} + \frac{(e+fx)^2 \cosh^4(c+dx)}{4bd} \\
&= -\frac{a^2(a^2+b^2)(e+fx)^3}{3b^5f} + \frac{3f^2 \cosh^2(c+dx)}{32bd^3} + \frac{2af(e+fx) \cosh^3(c+dx)}{9b^2d^2} \\
&= -\frac{3efx}{16bd} - \frac{3f^2x^2}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^3}{3b^5f} + \frac{2a^3f(e+fx) \cosh(c+dx)}{b^4d^2} + \dots \\
&= \frac{a^2efx}{2b^3d} - \frac{3efx}{16bd} + \frac{a^2f^2x^2}{4b^3d} - \frac{3f^2x^2}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^3}{3b^5f} + \frac{2a^3f(e+fx)}{b^4d^2} + \dots \\
&= \frac{a^2efx}{2b^3d} - \frac{3efx}{16bd} + \frac{a^2f^2x^2}{4b^3d} - \frac{3f^2x^2}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^3}{3b^5f} + \frac{2a^3f(e+fx)}{b^4d^2} + \dots \\
&= \frac{a^2efx}{2b^3d} - \frac{3efx}{16bd} + \frac{a^2f^2x^2}{4b^3d} - \frac{3f^2x^2}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^3}{3b^5f} + \frac{2a^3f(e+fx)}{b^4d^2} + \dots
\end{aligned}$$

**Mathematica [B]** time = 19.3254, size = 5198, normalized size = 6.35

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Cosh[c + d\*x]^3\*Sinh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] Result too large to show

**Maple [F]** time = 0.243, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^2 (\cosh(dx+c))^3 (\sinh(dx+c))^2}{a+b \sinh(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/192*e^2*((8*a*b^2*e^(-d*x - c) - 3*b^3 - 12*(2*a^2*b + b^3)*e^(-2*d*x - 2*c) + 24*(4*a^3 + 3*a*b^2)*e^(-3*d*x - 3*c))*e^(4*d*x + 4*c)/(b^4*d) - 192*(a^4 + a^2*b^2)*(d*x + c)/(b^5*d) - (8*a*b^2*e^(-3*d*x - 3*c) + 3*b^3*e^(-4*d*x - 4*c) + 24*(4*a^3 + 3*a*b^2)*e^(-d*x - c) + 12*(2*a^2*b + b^3)*e^(-2*d*x - 2*c))/(b^4*d) - 192*(a^4 + a^2*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^5*d) + 1/13824*(4608*(a^4*d^3*f^2*e^(4*c) + a^2*b^2*d^3*f^2*e^(4*c))*x^3 + 13824*(a^4*d^3*e*f*e^(4*c) + a^2*b^2*d^3*e*f*e^(4*c))*x^2 + 27*(8*b^4*d^2*f^2*x^2*e^(8*c) + 4*(4*d^2*e*f - d*f^2)*b^4*x*e^(8*c) - (4*d*e*f - f^2)*b^4*e^(8*c))*e^(4*d*x) - 64*(9*a*b^3*d^2*f^2*x^2*e^(7*c) + 6*(3*d^2*e*f - d*f^2)*a*b^3*x*e^(7*c) - 2*(3*d*e*f - f^2)*a*b^3*e^(7*c))*e^(3*d*x) - 432*(2*(2*d*e*f - f^2)*a^2*b^2*e^(6*c) + (2*d*e*f - f^2)*b^4*e^(6*c) - 2*(2*a^2*b^2*d^2*f^2*e^(6*c) + b^4*d^2*f^2*e^(6*c))*x^2 - 2*(2*(2*d^2*e*f - d*f^2)*a^2*b^2*e^(6*c) + (2*d^2*e*f - d*f^2)*b^4*e^(6*c))*x)*e^(2*d*x) + 1728*(8*(d*e*f - f^2)*a^3*b*e^(5*c) + 6*(d*e*f - f^2)*a*b^3*e^(5*c) - (4*a^3*b*d^2*f^2*e^(5*c) + 3*a*b^3*d^2*f^2*e^(5*c))*x^2 - 2*(4*(d^2*e*f - d*f^2)*a^3*b*e^(5*c) + 3*(d^2*e*f - d*f^2)*a*b^3*e^(5*c))*x)*e^(d*x) + 1728*(8*(d*e*f + f^2)*a^3*b*e^(3*c) + 6*(d*e*f + f^2)*a*b^3*e^(3*c) + (4*a^3*b*d^2*f^2*e^(3*c) + 3*a*b^3*d^2*f^2*e^(3*c))*x^2 + 2*(4*(d^2*e*f + d*f^2)*a^3*b*e^(3*c) + 3*(d^2*e*f + d*f^2)*a*b^3*e^(3*c))*x)*e^(-d*x) + 432*(2*(2*d*e*f + f^2)*a^2*b^2*e^(2*c) + (2*d*e*f + f^2)*b^4*e^(2*c) + 2*(2*a^2*b^2*d^2*f^2*e^(2*c) + b^4*d^2*f^2*e^(2*c))*x^2 + 2*(2*(2*d^2*e*f + d*f^2)*a^2*b^2*e^(2*c) + (2*d^2*e*f + d*f^2)*b^4*e^(2*c))*x)*e^(-2*d*x) + 64*(9*a*b^3*d^2*f^2*x^2*e^c + 6*(3*d^2*e*f + d*f^2)*a*b^3*x*e^c + 2*(3*d*e*f + f^2)*a*b^3*e^c)*e^(-3*d*x) + 27*(8*b^4*d^2*f^2*x^2 + 4*(4*d^2*e*f + d*f^2)*b^4*x + (4*d*e*f + f^2)*b^4)*e^(-4*d*x)*e^(-4*c)/(b^5*d^3) - integrate(-2*((a^4*b*f^2 + a^2*b^3*f^2)*x^2 + 2*(a^4*b*e*f + a^2*b^3*e*f)*x - ((a^5*f^2*e^c + a^3*b^2*f^2*e^c)*x^2 + 2*(a^5*e*f*e^c + a^3*b^2*e*f*e^c)*x)*e^(d*x))/(b^6*e^(2*d*x
```

+ 2\*c) + 2\*a\*b^5\*e^(d\*x + c) - b^6), x)

---

**Fricas [C]** time = 3.44704, size = 16764, normalized size = 20.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)^3\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 1/13824\*(216\*b^4\*d^2\*f^2\*x^2 + 27\*(8\*b^4\*d^2\*f^2\*x^2 + 8\*b^4\*d^2\*e^2 - 4\*b^4\*d\*e\*f + b^4\*f^2 + 4\*(4\*b^4\*d^2\*e\*f - b^4\*d\*f^2)\*x)\*cosh(d\*x + c)^8 + 27\*(8\*b^4\*d^2\*f^2\*x^2 + 8\*b^4\*d^2\*e^2 - 4\*b^4\*d\*e\*f + b^4\*f^2 + 4\*(4\*b^4\*d^2\*e\*f - b^4\*d\*f^2)\*x)\*sinh(d\*x + c)^8 + 216\*b^4\*d^2\*e^2 - 64\*(9\*a\*b^3\*d^2\*f^2\*x^2 + 9\*a\*b^3\*d^2\*e^2 - 6\*a\*b^3\*d\*e\*f + 2\*a\*b^3\*f^2 + 6\*(3\*a\*b^3\*d^2\*e\*f - a\*b^3\*d\*f^2)\*x)\*cosh(d\*x + c)^7 - 8\*(72\*a\*b^3\*d^2\*f^2\*x^2 + 72\*a\*b^3\*d^2\*e^2 - 48\*a\*b^3\*d\*e\*f + 16\*a\*b^3\*f^2 + 48\*(3\*a\*b^3\*d^2\*e\*f - a\*b^3\*d\*f^2)\*x - 27\*(8\*b^4\*d^2\*f^2\*x^2 + 8\*b^4\*d^2\*e^2 - 4\*b^4\*d\*e\*f + b^4\*f^2 + 4\*(4\*b^4\*d^2\*e\*f - b^4\*d\*f^2)\*x)\*cosh(d\*x + c))\*sinh(d\*x + c)^7 + 108\*b^4\*d\*e\*f + 432\*(2\*(2\*a^2\*b^2 + b^4)\*d^2\*f^2\*x^2 + 2\*(2\*a^2\*b^2 + b^4)\*d^2\*e^2 - 2\*(2\*a^2\*b^2 + b^4)\*d\*e\*f + (2\*a^2\*b^2 + b^4)\*f^2 + 2\*(2\*(2\*a^2\*b^2 + b^4)\*d^2\*e\*f - (2\*a^2\*b^2 + b^4)\*d\*f^2)\*x)\*cosh(d\*x + c)^6 + 4\*(216\*(2\*a^2\*b^2 + b^4)\*d^2\*f^2\*x^2 + 216\*(2\*a^2\*b^2 + b^4)\*d^2\*e^2 - 216\*(2\*a^2\*b^2 + b^4)\*d\*e\*f + 108\*(2\*a^2\*b^2 + b^4)\*f^2 + 189\*(8\*b^4\*d^2\*f^2\*x^2 + 8\*b^4\*d^2\*e^2 - 4\*b^4\*d\*e\*f + b^4\*f^2 + 4\*(4\*b^4\*d^2\*e\*f - b^4\*d\*f^2)\*x)\*cosh(d\*x + c)^2 + 216\*(2\*(2\*a^2\*b^2 + b^4)\*d^2\*e\*f - (2\*a^2\*b^2 + b^4)\*d\*f^2)\*x - 112\*(9\*a\*b^3\*d^2\*f^2\*x^2 + 9\*a\*b^3\*d^2\*e^2 - 6\*a\*b^3\*d\*e\*f + 2\*a\*b^3\*f^2 + 6\*(3\*a\*b^3\*d^2\*e\*f - a\*b^3\*d\*f^2)\*x)\*cosh(d\*x + c))\*sinh(d\*x + c)^6 + 27\*b^4\*f^2 - 1728\*((4\*a^3\*b + 3\*a\*b^3)\*d^2\*f^2\*x^2 + (4\*a^3\*b + 3\*a\*b^3)\*d^2\*e^2 - 2\*(4\*a^3\*b + 3\*a\*b^3)\*d\*e\*f + 2\*(4\*a^3\*b + 3\*a\*b^3)\*f^2 + 2\*((4\*a^3\*b + 3\*a\*b^3)\*d^2\*e\*f - (4\*a^3\*b + 3\*a\*b^3)\*d\*f^2)\*x)\*cosh(d\*x + c)^5 - 24\*(72\*(4\*a^3\*b + 3\*a\*b^3)\*d^2\*f^2\*x^2 + 72\*(4\*a^3\*b + 3\*a\*b^3)\*d^2\*e^2 - 144\*(4\*a^3\*b + 3\*a\*b^3)\*d\*e\*f - 63\*(8\*b^4\*d^2\*f^2\*x^2 + 8\*b^4\*d^2\*e^2 - 4\*b^4\*d\*e\*f + b^4\*f^2 + 4\*(4\*b^4\*d^2\*e\*f - b^4\*d\*f^2)\*x)\*cosh(d\*x + c)^3 + 144\*(4\*a^3\*b + 3\*a\*b^3)\*f^2 + 56\*(9\*a\*b^3\*d^2\*f^2\*x^2 + 9\*a\*b^3\*d^2\*e^2 - 6\*a\*b^3\*d\*e\*f + 2\*a\*b^3\*f^2 + 6\*(3\*a\*b^3\*d^2\*e\*f - a\*b^3\*d\*f^2)\*x)\*cosh(d\*x + c)^2 + 144\*((4\*a^3\*b + 3\*a\*b^3)\*d^2\*e\*f - (4\*a^3\*b + 3\*a\*b^3)\*d\*f^2)\*x - 108\*(2\*(2\*a^2\*b^2 + b^4)\*d^2\*f^2\*x^2 + 2\*(2\*a^2\*b^2 + b^4)\*d^2\*e^2 - 2\*(2\*a^2\*b^2 + b^4)\*d\*e\*f + (2\*a^2\*b^2 + b^4)\*f^2 + 2\*(2\*(2\*a^2\*b^2 + b^4)\*d^2\*e\*f - (2\*a^2\*b^2 + b^4)\*d\*f^2)\*x)\*cosh(d\*x + c))\*sinh(d\*x + c)^5 - 4608\*((a^4 + a^2\*b^2)\*d^3\*f^2\*x^3 + 3\*(a^4 + a^2\*b^2)\*d^3\*e\*f\*x^2 + 3\*(a^4 + a^2\*b^2)\*d^3\*e^2\*x + 6\*(a^4 + a^2\*b^2)\*c\*

$$\begin{aligned}
& d^2e^2 - 6*(a^4 + a^2*b^2)*c^2*d*e*f + 2*(a^4 + a^2*b^2)*c^3*f^2)*\cosh(d*x \\
& + c)^4 - 2*(2304*(a^4 + a^2*b^2)*d^3*f^2*x^3 + 6912*(a^4 + a^2*b^2)*d^3*e \\
& f*x^2 + 6912*(a^4 + a^2*b^2)*d^3*e^2*x + 13824*(a^4 + a^2*b^2)*c*d^2*e^2 - \\
& 13824*(a^4 + a^2*b^2)*c^2*d*e*f + 4608*(a^4 + a^2*b^2)*c^3*f^2 - 945*(8*b^4 \\
& *d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*f + b^4*f^2 + 4*(4*b^4*d^2*e*f - b \\
& ^4*d*f^2)*x)*\cosh(d*x + c)^4 + 1120*(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*d^2*e^2 \\
& - 6*a*b^3*d*e*f + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*e*f - a*b^3*d*f^2)*x)*\cosh(d \\
& *x + c)^3 - 3240*(2*(2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(2*a^2*b^2 + b^4)*d^2 \\
& *e^2 - 2*(2*a^2*b^2 + b^4)*d*e*f + (2*a^2*b^2 + b^4)*f^2 + 2*(2*(2*a^2*b^2 \\
& + b^4)*d^2*e*f - (2*a^2*b^2 + b^4)*d*f^2)*x)*\cosh(d*x + c)^2 + 4320*((4*a^3 \\
& *b + 3*a*b^3)*d^2*f^2*x^2 + (4*a^3*b + 3*a*b^3)*d^2*e^2 - 2*(4*a^3*b + 3*a* \\
& b^3)*d*e*f + 2*(4*a^3*b + 3*a*b^3)*f^2 + 2*((4*a^3*b + 3*a*b^3)*d^2*e*f - ( \\
& 4*a^3*b + 3*a*b^3)*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^4 + 1728*((4*a^3*b \\
& + 3*a*b^3)*d^2*f^2*x^2 + (4*a^3*b + 3*a*b^3)*d^2*e^2 + 2*(4*a^3*b + 3*a*b \\
& ^3)*d*e*f + 2*(4*a^3*b + 3*a*b^3)*f^2 + 2*((4*a^3*b + 3*a*b^3)*d^2*e*f + (4 \\
& *a^3*b + 3*a*b^3)*d*f^2)*x)*\cosh(d*x + c)^3 + 8*(216*(4*a^3*b + 3*a*b^3)*d^ \\
& 2*f^2*x^2 + 189*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*f + b^4*f^2 \\
& + 4*(4*b^4*d^2*e*f - b^4*d*f^2)*x)*\cosh(d*x + c)^5 + 216*(4*a^3*b + 3*a*b^3 \\
& )*d^2*e^2 - 280*(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*d^2*e^2 - 6*a*b^3*d*e*f + 2* \\
& a*b^3*f^2 + 6*(3*a*b^3*d^2*e*f - a*b^3*d*f^2)*x)*\cosh(d*x + c)^4 + 432*(4*a \\
& ^3*b + 3*a*b^3)*d*e*f + 1080*(2*(2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(2*a^2*b^ \\
& 2 + b^4)*d^2*e^2 - 2*(2*a^2*b^2 + b^4)*d*e*f + (2*a^2*b^2 + b^4)*f^2 + 2*(2 \\
& *(2*a^2*b^2 + b^4)*d^2*e*f - (2*a^2*b^2 + b^4)*d*f^2)*x)*\cosh(d*x + c)^3 + \\
& 432*(4*a^3*b + 3*a*b^3)*f^2 - 2160*((4*a^3*b + 3*a*b^3)*d^2*f^2*x^2 + (4*a^ \\
& 3*b + 3*a*b^3)*d^2*e^2 - 2*(4*a^3*b + 3*a*b^3)*d*e*f + 2*(4*a^3*b + 3*a*b^3 \\
& )*f^2 + 2*((4*a^3*b + 3*a*b^3)*d^2*e*f - (4*a^3*b + 3*a*b^3)*d*f^2)*x)*\cosh \\
& (d*x + c)^2 + 432*((4*a^3*b + 3*a*b^3)*d^2*e*f + (4*a^3*b + 3*a*b^3)*d*f^2) \\
& *x - 2304*((a^4 + a^2*b^2)*d^3*f^2*x^3 + 3*(a^4 + a^2*b^2)*d^3*e*f*x^2 + 3* \\
& (a^4 + a^2*b^2)*d^3*e^2*x + 6*(a^4 + a^2*b^2)*c*d^2*e^2 - 6*(a^4 + a^2*b^2) \\
& *c^2*d*e*f + 2*(a^4 + a^2*b^2)*c^3*f^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 43 \\
& 2*(2*(2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(2*a^2*b^2 + b^4)*d^2*e^2 + 2*(2*a^2 \\
& *b^2 + b^4)*d*e*f + (2*a^2*b^2 + b^4)*f^2 + 2*(2*(2*a^2*b^2 + b^4)*d^2*e*f \\
& + (2*a^2*b^2 + b^4)*d*f^2)*x)*\cosh(d*x + c)^2 + 12*(72*(2*a^2*b^2 + b^4)*d^ \\
& 2*f^2*x^2 + 63*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*f + b^4*f^2 + \\
& 4*(4*b^4*d^2*e*f - b^4*d*f^2)*x)*\cosh(d*x + c)^6 - 112*(9*a*b^3*d^2*f^2*x^ \\
& 2 + 9*a*b^3*d^2*e^2 - 6*a*b^3*d*e*f + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*e*f - a* \\
& b^3*d*f^2)*x)*\cosh(d*x + c)^5 + 72*(2*a^2*b^2 + b^4)*d^2*e^2 + 540*(2*(2*a^ \\
& 2*b^2 + b^4)*d^2*f^2*x^2 + 2*(2*a^2*b^2 + b^4)*d^2*e^2 - 2*(2*a^2*b^2 + b^4 \\
& )*d*e*f + (2*a^2*b^2 + b^4)*f^2 + 2*(2*(2*a^2*b^2 + b^4)*d^2*e*f - (2*a^2*b \\
& ^2 + b^4)*d*f^2)*x)*\cosh(d*x + c)^4 + 72*(2*a^2*b^2 + b^4)*d*e*f - 1440*((4 \\
& *a^3*b + 3*a*b^3)*d^2*f^2*x^2 + (4*a^3*b + 3*a*b^3)*d^2*e^2 - 2*(4*a^3*b + \\
& 3*a*b^3)*d*e*f + 2*(4*a^3*b + 3*a*b^3)*f^2 + 2*((4*a^3*b + 3*a*b^3)*d^2*e*f \\
& - (4*a^3*b + 3*a*b^3)*d*f^2)*x)*\cosh(d*x + c)^3 + 36*(2*a^2*b^2 + b^4)*f^2 \\
& - 2304*((a^4 + a^2*b^2)*d^3*f^2*x^3 + 3*(a^4 + a^2*b^2)*d^3*e*f*x^2 + 3*(a \\
& ^4 + a^2*b^2)*d^3*e^2*x + 6*(a^4 + a^2*b^2)*c*d^2*e^2 - 6*(a^4 + a^2*b^2)*c
\end{aligned}$$



$$\begin{aligned}
&^2*d*e*f + 2*(a^4 + a^2*b^2)*c^3*f^2)*\cosh(d*x + c)^2 + 72*(2*(2*a^2*b^2 + \\
&b^4)*d^2*e*f + (2*a^2*b^2 + b^4)*d*f^2)*x + 432*((4*a^3*b + 3*a*b^3)*d^2*f^2 \\
&2*x^2 + (4*a^3*b + 3*a*b^3)*d^2*e^2 + 2*(4*a^3*b + 3*a*b^3)*d*e*f + 2*(4*a^ \\
&3*b + 3*a*b^3)*f^2 + 2*((4*a^3*b + 3*a*b^3)*d^2*e*f + (4*a^3*b + 3*a*b^3)*d \\
&*f^2)*x)*\cosh(d*x + c)*\sinh(d*x + c)^2 + 108*(4*b^4*d^2*e*f + b^4*d*f^2)*x \\
&+ 64*(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*d^2*e^2 + 6*a*b^3*d*e*f + 2*a*b^3*f^2 \\
&+ 6*(3*a*b^3*d^2*e*f + a*b^3*d*f^2)*x)*\cosh(d*x + c) + 27648*(((a^4 + a^2*b \\
&^2)*d*f^2*x + (a^4 + a^2*b^2)*d*e*f)*\cosh(d*x + c)^4 + 4*((a^4 + a^2*b^2)*d \\
&*f^2*x + (a^4 + a^2*b^2)*d*e*f)*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*((a^4 + a \\
&^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*d*e*f)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + \\
&4*((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*d*e*f)*\cosh(d*x + c)*\sinh(d*x \\
&+ c)^3 + ((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*d*e*f)*\sinh(d*x + c)^4) \\
&*dilog((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + \\
&c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 27648*(((a^4 + a^2*b^2)*d*f^2*x + \\
&(a^4 + a^2*b^2)*d*e*f)*\cosh(d*x + c)^4 + 4*((a^4 + a^2*b^2)*d*f^2*x + (a^4 \\
&+ a^2*b^2)*d*e*f)*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*((a^4 + a^2*b^2)*d*f^2* \\
&x + (a^4 + a^2*b^2)*d*e*f)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*((a^4 + a^2* \\
&b^2)*d*f^2*x + (a^4 + a^2*b^2)*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^4 \\
&+ a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*d*e*f)*\sinh(d*x + c)^4)*dilog((a*\cosh \\
&(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 \\
&+ b^2)/b^2} - b)/b + 1) + 13824*(((a^4 + a^2*b^2)*d^2*e^2 - 2*(a^4 + a^2*b \\
&^2)*c*d*e*f + (a^4 + a^2*b^2)*c^2*f^2)*\cosh(d*x + c)^4 + 4*((a^4 + a^2*b^2) \\
&*d^2*e^2 - 2*(a^4 + a^2*b^2)*c*d*e*f + (a^4 + a^2*b^2)*c^2*f^2)*\cosh(d*x + \\
&c)^3*\sinh(d*x + c) + 6*((a^4 + a^2*b^2)*d^2*e^2 - 2*(a^4 + a^2*b^2)*c*d*e*f \\
&+ (a^4 + a^2*b^2)*c^2*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*((a^4 + a^2 \\
&*b^2)*d^2*e^2 - 2*(a^4 + a^2*b^2)*c*d*e*f + (a^4 + a^2*b^2)*c^2*f^2)*\cosh(d \\
&*x + c)*\sinh(d*x + c)^3 + ((a^4 + a^2*b^2)*d^2*e^2 - 2*(a^4 + a^2*b^2)*c*d* \\
&e*f + (a^4 + a^2*b^2)*c^2*f^2)*\sinh(d*x + c)^4)*\log(2*b*\cosh(d*x + c) + 2*b \\
&*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 13824*(((a^4 + a^2*b^2) \\
&*d^2*e^2 - 2*(a^4 + a^2*b^2)*c*d*e*f + (a^4 + a^2*b^2)*c^2*f^2)*\cosh(d*x + \\
&c)^4 + 4*((a^4 + a^2*b^2)*d^2*e^2 - 2*(a^4 + a^2*b^2)*c*d*e*f + (a^4 + a^2* \\
&b^2)*c^2*f^2)*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*((a^4 + a^2*b^2)*d^2*e^2 - \\
&2*(a^4 + a^2*b^2)*c*d*e*f + (a^4 + a^2*b^2)*c^2*f^2)*\cosh(d*x + c)^2*\sinh(d \\
&*x + c)^2 + 4*((a^4 + a^2*b^2)*d^2*e^2 - 2*(a^4 + a^2*b^2)*c*d*e*f + (a^4 + \\
&a^2*b^2)*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^4 + a^2*b^2)*d^2*e^2 \\
&- 2*(a^4 + a^2*b^2)*c*d*e*f + (a^4 + a^2*b^2)*c^2*f^2)*\sinh(d*x + c)^4)*\lo \\
&g(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) \\
&+ 13824*(((a^4 + a^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a^ \\
&4 + a^2*b^2)*c*d*e*f - (a^4 + a^2*b^2)*c^2*f^2)*\cosh(d*x + c)^4 + 4*((a^4 + \\
&a^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a^4 + a^2*b^2)*c*d \\
&*e*f - (a^4 + a^2*b^2)*c^2*f^2)*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*((a^4 + a \\
&^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a^4 + a^2*b^2)*c*d*e \\
&*f - (a^4 + a^2*b^2)*c^2*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*((a^4 + a \\
&^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a^4 + a^2*b^2)*c*d*e \\
&*f - (a^4 + a^2*b^2)*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^4 + a^2*b
\end{aligned}$$

$$\begin{aligned}
& \text{^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a^4 + a^2*b^2)*c*d*e*f -} \\
& (a^4 + a^2*b^2)*c^2*f^2)*\sinh(d*x + c)^4)*\log(-(a*\cosh(d*x + c) + a*\sinh(d \\
& *x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2) - b)/b} \\
& + 13824*(((a^4 + a^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a \\
& ^4 + a^2*b^2)*c*d*e*f - (a^4 + a^2*b^2)*c^2*f^2)*\cosh(d*x + c)^4 + 4*((a^4 \\
& + a^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a^4 + a^2*b^2)*c* \\
& d*e*f - (a^4 + a^2*b^2)*c^2*f^2)*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*((a^4 + \\
& a^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a^4 + a^2*b^2)*c*d* \\
& e*f - (a^4 + a^2*b^2)*c^2*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*((a^4 + \\
& a^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a^4 + a^2*b^2)*c*d* \\
& e*f - (a^4 + a^2*b^2)*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^4 + a^2* \\
& b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + 2*(a^4 + a^2*b^2)*c*d*e*f \\
& - (a^4 + a^2*b^2)*c^2*f^2)*\sinh(d*x + c)^4)*\log(-(a*\cosh(d*x + c) + a*\sinh( \\
& d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2) - b)/b} \\
& ) - 27648*((a^4 + a^2*b^2)*f^2*\cosh(d*x + c)^4 + 4*(a^4 + a^2*b^2)*f^2*\cosh \\
& (d*x + c)^3*\sinh(d*x + c) + 6*(a^4 + a^2*b^2)*f^2*\cosh(d*x + c)^2*\sinh(d*x \\
& + c)^2 + 4*(a^4 + a^2*b^2)*f^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^4 + a^2*b \\
& ^2)*f^2*\sinh(d*x + c)^4)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b \\
& *\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2)}/b) - 27648*((a^4 + \\
& a^2*b^2)*f^2*\cosh(d*x + c)^4 + 4*(a^4 + a^2*b^2)*f^2*\cosh(d*x + c)^3*\sinh( \\
& d*x + c) + 6*(a^4 + a^2*b^2)*f^2*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*(a^4 + \\
& a^2*b^2)*f^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^4 + a^2*b^2)*f^2*\sinh(d*x \\
& + c)^4)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + \\
& b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2)}/b) + 8*(72*a*b^3*d^2*f^2*x^2 + 72*a \\
& *b^3*d^2*e^2 + 27*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*f + b^4*f^ \\
& 2 + 4*(4*b^4*d^2*e*f - b^4*d*f^2)*x)*\cosh(d*x + c)^7 + 48*a*b^3*d*e*f - 56* \\
& (9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*d^2*e^2 - 6*a*b^3*d*e*f + 2*a*b^3*f^2 + 6*(3 \\
& *a*b^3*d^2*e*f - a*b^3*d*f^2)*x)*\cosh(d*x + c)^6 + 16*a*b^3*f^2 + 324*(2*(2 \\
& *a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(2*a^2*b^2 + b^4)*d^2*e^2 - 2*(2*a^2*b^2 + \\
& b^4)*d*e*f + (2*a^2*b^2 + b^4)*f^2 + 2*(2*(2*a^2*b^2 + b^4)*d^2*e*f - (2*a^ \\
& 2*b^2 + b^4)*d*f^2)*x)*\cosh(d*x + c)^5 - 1080*((4*a^3*b + 3*a*b^3)*d^2*f^2* \\
& x^2 + (4*a^3*b + 3*a*b^3)*d^2*e^2 - 2*(4*a^3*b + 3*a*b^3)*d*e*f + 2*(4*a^3* \\
& b + 3*a*b^3)*f^2 + 2*((4*a^3*b + 3*a*b^3)*d^2*e*f - (4*a^3*b + 3*a*b^3)*d*f \\
& ^2)*x)*\cosh(d*x + c)^4 - 2304*((a^4 + a^2*b^2)*d^3*f^2*x^3 + 3*(a^4 + a^2*b \\
& ^2)*d^3*e*f*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*x + 6*(a^4 + a^2*b^2)*c*d^2*e^2 \\
& - 6*(a^4 + a^2*b^2)*c^2*d*e*f + 2*(a^4 + a^2*b^2)*c^3*f^2)*\cosh(d*x + c)^3 \\
& + 648*((4*a^3*b + 3*a*b^3)*d^2*f^2*x^2 + (4*a^3*b + 3*a*b^3)*d^2*e^2 + 2*( \\
& 4*a^3*b + 3*a*b^3)*d*e*f + 2*(4*a^3*b + 3*a*b^3)*f^2 + 2*((4*a^3*b + 3*a*b^ \\
& 3)*d^2*e*f + (4*a^3*b + 3*a*b^3)*d*f^2)*x)*\cosh(d*x + c)^2 + 48*(3*a*b^3*d^ \\
& 2*e*f + a*b^3*d*f^2)*x + 108*(2*(2*a^2*b^2 + b^4)*d^2*f^2*x^2 + 2*(2*a^2*b^ \\
& 2 + b^4)*d^2*e^2 + 2*(2*a^2*b^2 + b^4)*d*e*f + (2*a^2*b^2 + b^4)*f^2 + 2*( \\
& 2*(2*a^2*b^2 + b^4)*d^2*e*f + (2*a^2*b^2 + b^4)*d*f^2)*x)*\cosh(d*x + c))*\sin \\
& h(d*x + c))/(b^5*d^3*\cosh(d*x + c)^4 + 4*b^5*d^3*\cosh(d*x + c)^3*\sinh(d*x + \\
& c) + 6*b^5*d^3*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*b^5*d^3*\cosh(d*x + c)*\sin \\
& h(d*x + c)^3 + b^5*d^3*\sinh(d*x + c)^4)
\end{aligned}$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cosh(d\*x+c)\*\*3\*sinh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c)^3 \sinh(dx + c)^2}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)^3\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*cosh(d\*x + c)^3\*sinh(d\*x + c)^2/(b\*sinh(d\*x + c) + a), x)

$$3.374 \quad \int \frac{(e+fx) \cosh^3(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=499

$$\frac{a^2 f (a^2 + b^2) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5 d^2} + \frac{a^2 f (a^2 + b^2) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^5 d^2} + \frac{a^3 f \cosh(c+dx)}{b^4 d^2} - \frac{a^2 f \sinh(c+dx) \cosh(c+dx)}{4b^3 d^2}$$

[Out] (a^2\*f\*x)/(4\*b^3\*d) - (3\*f\*x)/(32\*b\*d) - (a^2\*(a^2 + b^2)\*(e + f\*x)^2)/(2\*b^5\*f) + (a^3\*f\*Cosh[c + d\*x])/(b^4\*d^2) + (2\*a\*f\*Cosh[c + d\*x])/(3\*b^2\*d^2) + (a\*f\*Cosh[c + d\*x]^3)/(9\*b^2\*d^2) + ((e + f\*x)\*Cosh[c + d\*x]^4)/(4\*b\*d) + (a^2\*(a^2 + b^2)\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/(b^5\*d) + (a^2\*(a^2 + b^2)\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/(b^5\*d) + (a^2\*(a^2 + b^2)\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))]/(b^5\*d^2) + (a^2\*(a^2 + b^2)\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))]/(b^5\*d^2) - (a^3\*(e + f\*x)\*Sinh[c + d\*x])/(b^4\*d) - (2\*a\*(e + f\*x)\*Sinh[c + d\*x])/(3\*b^2\*d) - (a^2\*f\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(4\*b^3\*d^2) - (3\*f\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(32\*b\*d^2) - (a\*(e + f\*x)\*Cosh[c + d\*x]^2\*Sinh[c + d\*x])/(3\*b^2\*d) - (f\*Cosh[c + d\*x]^3\*Sinh[c + d\*x])/(16\*b\*d^2) + (a^2\*(e + f\*x)\*Sinh[c + d\*x]^2)/(2\*b^3\*d)

**Rubi [A]** time = 0.675401, antiderivative size = 499, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 13, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}}$  = 0.382, Rules used = {5579, 5447, 2635, 8, 3310, 3296, 2638, 5565, 5446, 5561, 2190, 2279, 2391}

$$\frac{a^2 f (a^2 + b^2) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5 d^2} + \frac{a^2 f (a^2 + b^2) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^5 d^2} + \frac{a^3 f \cosh(c+dx)}{b^4 d^2} - \frac{a^2 f \sinh(c+dx) \cosh(c+dx)}{4b^3 d^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Cosh[c + d\*x]^3\*Sinh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (a^2\*f\*x)/(4\*b^3\*d) - (3\*f\*x)/(32\*b\*d) - (a^2\*(a^2 + b^2)\*(e + f\*x)^2)/(2\*b^5\*f) + (a^3\*f\*Cosh[c + d\*x])/(b^4\*d^2) + (2\*a\*f\*Cosh[c + d\*x])/(3\*b^2\*d^2) + (a\*f\*Cosh[c + d\*x]^3)/(9\*b^2\*d^2) + ((e + f\*x)\*Cosh[c + d\*x]^4)/(4\*b\*d) + (a^2\*(a^2 + b^2)\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/(b^5\*d) + (a^2\*(a^2 + b^2)\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/(b^5\*d) + (a^2\*(a^2 + b^2)\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))]/(b^5\*d^2) + (a^2\*(a^2 + b^2)\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))]/(b^5\*d^2) - (a^3\*(e + f\*x)\*Sinh[c + d\*x])/(b^4\*d)

$$4*d) - (2*a*(e + f*x)*\text{Sinh}[c + d*x])/(3*b^2*d) - (a^2*f*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(4*b^3*d^2) - (3*f*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(32*b*d^2) - (a*(e + f*x)*\text{Cosh}[c + d*x]^2*\text{Sinh}[c + d*x])/(3*b^2*d) - (f*\text{Cosh}[c + d*x]^3*\text{Sinh}[c + d*x])/(16*b*d^2) + (a^2*(e + f*x)*\text{Sinh}[c + d*x]^2)/(2*b^3*d)$$

### Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.)]/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] := Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Sinh[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sinh[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

### Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

### Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(d*(b*Sinh[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*Sinh[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*Sinh[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]
```

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

$e + f*x$ ,  $x$ ],  $x$ ] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rule 5565

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)])^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(m\_)]/((a\_) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Dist[a/b^2, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

### Rule 5446

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_)\*Sinh[(a\_.) + (b\_.)\*(x\_)]^(n\_), x\_Symbol] := Simp[((c + d\*x)^m\*Sinh[a + b\*x]^(n + 1))/(b\*(n + 1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*Sinh[a + b\*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

### Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_)]/((a\_) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_)\*((c\_.) + (d\_.)\*(x\_))^(m\_)]/((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned}
 \int \frac{(e+fx) \cosh^3(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx) \cosh^3(c+dx) \sinh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx) \cosh^3(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
 &= \frac{(e+fx) \cosh^4(c+dx)}{4bd} - \frac{a \int (e+fx) \cosh^3(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx) \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
 &= \frac{af \cosh^3(c+dx)}{9b^2d^2} + \frac{(e+fx) \cosh^4(c+dx)}{4bd} - \frac{a(e+fx) \cosh^2(c+dx) \sinh(c+dx)}{3b^2d} \\
 &= -\frac{a^2(a^2+b^2)(e+fx)^2}{2b^5f} + \frac{af \cosh^3(c+dx)}{9b^2d^2} + \frac{(e+fx) \cosh^4(c+dx)}{4bd} - \frac{a^2 \int \frac{(e+fx) \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
 &= -\frac{3fx}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^2}{2b^5f} + \frac{a^3f \cosh(c+dx)}{b^4d^2} + \frac{2af \cosh(c+dx)}{3b^2d^2} + \frac{a^2 \int \frac{(e+fx) \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
 &= \frac{a^2fx}{4b^3d} - \frac{3fx}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^2}{2b^5f} + \frac{a^3f \cosh(c+dx)}{b^4d^2} + \frac{2af \cosh(c+dx)}{3b^2d^2} + \frac{a^2 \int \frac{(e+fx) \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
 &= \frac{a^2fx}{4b^3d} - \frac{3fx}{32bd} - \frac{a^2(a^2+b^2)(e+fx)^2}{2b^5f} + \frac{a^3f \cosh(c+dx)}{b^4d^2} + \frac{2af \cosh(c+dx)}{3b^2d^2} + \frac{a^2 \int \frac{(e+fx) \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx}{b^2}
 \end{aligned}$$

**Mathematica [A]** time = 3.20973, size = 853, normalized size = 1.71

$$-144de \log(a+b \sinh(c+dx))b^4 + 72f \left( dx \left( dx - 2 \log \left( \frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}} + 1 \right) - 2 \log \left( \frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}} + 1 \right) \right) - 2 \text{PolyLog} \left( 2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((e+f\*x)\*Cosh[c+d\*x]^3\*Sinh[c+d\*x]^2)/(a+b\*Sinh[c+d\*x]),x]

[Out] (-144\*b^4\*d\*e\*Log[a+b\*Sinh[c+d\*x]] + 72\*b^4\*f\*(d\*x\*(d\*x - 2\*Log[1 + (b\*E^(c+d\*x))/(a - Sqrt[a^2 + b^2]]) - 2\*Log[1 + (b\*E^(c+d\*x))/(a + Sqrt[a

$$\begin{aligned} &^2 + b^2])) - 2*\text{PolyLog}[2, (b*E^{(c + d*x)})/(-a + \text{Sqrt}[a^2 + b^2])] - 2*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))] + 72*b^2*d*e*((4*a^2 + b^2)*\text{Log}[a + b*\text{Sinh}[c + d*x]] - 4*a*b*\text{Sinh}[c + d*x] + 2*b^2*\text{Sinh}[c + d*x]^2) \\ &+ 24*d*e*(3*(16*a^4 + 12*a^2*b^2 + b^4)*\text{Log}[a + b*\text{Sinh}[c + d*x]] - 12*a*b*(4*a^2 + 3*b^2)*\text{Sinh}[c + d*x] + 6*b^2*(4*a^2 + 3*b^2)*\text{Sinh}[c + d*x]^2 - 16*a*b^3*\text{Sinh}[c + d*x]^3 + 12*b^4*\text{Sinh}[c + d*x]^4) + 36*b^2*f*(8*a*b*\text{Cosh}[c + d*x] + 2*b^2*d*x*\text{Cosh}[2*(c + d*x)] + 2*(4*a^2 + b^2)*(-(c + d*x)^2/2 + (c + d*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])] + (c + d*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])] - c*\text{Log}[a + b*\text{Sinh}[c + d*x]] + \text{PolyLog}[2, (b*E^{(c + d*x)})/(-a + \text{Sqrt}[a^2 + b^2])] + \text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))] - 8*a*b*d*x*\text{Sinh}[c + d*x] - b^2*\text{Sinh}[2*(c + d*x)]) + f*(576*a*b*(2*a^2 + b^2)*\text{Cosh}[c + d*x] + 72*b^2*(4*a^2 + b^2)*d*x*\text{Cosh}[2*(c + d*x)] + 32*a*b^3*\text{Cosh}[3*(c + d*x)] + 36*b^4*d*x*\text{Cosh}[4*(c + d*x)] + 72*(16*a^4 + 12*a^2*b^2 + b^4)*(-(c + d*x)^2/2 + (c + d*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])] + (c + d*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])] - c*\text{Log}[a + b*\text{Sinh}[c + d*x]] + \text{PolyLog}[2, (b*E^{(c + d*x)})/(-a + \text{Sqrt}[a^2 + b^2])] + \text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))] - 576*a*b*(2*a^2 + b^2)*d*x*\text{Sinh}[c + d*x] - 36*b^2*(4*a^2 + b^2)*\text{Sinh}[2*(c + d*x)] - 96*a*b^3*d*x*\text{Sinh}[3*(c + d*x)] - 9*b^4*\text{Sinh}[4*(c + d*x)])))/(1152*b^5*d^2) \end{aligned}$$

**Maple [B]** time = 0.118, size = 1217, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)*\cosh(d*x+c)^3*\sinh(d*x+c)^2/(a+b*\sinh(d*x+c)),x)$

[Out]  $a^4/b^5/d^2*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) * c + a^4/b^5/d*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) * x + a^4/b^5/d^2*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) * c + a^4/b^5/d*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) * x - a^4/b^5/d^2*f*c*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)+2*a^4/b^5/d^2*f*c*\ln(\exp(d*x+c))-2*a^4/b^5/d*f*c*x-1/2*a^4*f*x^2/b^5-1/2*a^2*f*x^2/b^3+1/8*a*(4*a^2+3*b^2)*(d*f*x+d*e+f)/b^4/d^2*\exp(-d*x-c)-1/b^3/d^2*a^2*f*c*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)+2/b^3/d^2*a^2*f*c*\ln(\exp(d*x+c))+1/b^3/d*a^2*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) * x + 1/b^3/d^2*a^2*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) * c + 1/b^3/d*a^2*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) * x + 1/b^3/d^2*a^2*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) * c - 2/b^3/d*a^2*f*c*x + a^4*e*x/b^5+a^2*e*x/b^3+1/256*(4*d*f*x+4*d*e-f)/d^2/b*\exp(4*d*x+4*c)+1/32*(4*a^2*$



$$\begin{aligned}
& d*f*x+2*b^2*d*f*x+4*a^2*d*e+2*b^2*d*e-2*a^2*f-b^2*f)/b^3/d^2*\exp(2*d*x+2*c) \\
& -1/72*a*(3*d*f*x+3*d*e-f)/b^2/d^2*\exp(3*d*x+3*c)-1/b^3/d^2*a^2*f*c^2+1/b^3/ \\
& d^2*a^2*f*dilog((b*\exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/b^3 \\
& /d^2*a^2*f*dilog((-b*\exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/ \\
& b^3/d*a^2*e*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)-2/b^3/d*a^2*e*\ln(\exp(d*x+ \\
& c))+1/256*(4*d*f*x+4*d*e+f)/d^2/b*\exp(-4*d*x-4*c)+1/32*(2*a^2+b^2)*(2*d*f*x \\
& +2*d*e+f)/b^3/d^2*\exp(-2*d*x-2*c)+1/72*a*(3*d*f*x+3*d*e+f)/b^2/d^2*\exp(-3*d \\
& *x-3*c)-1/8*a*(4*a^2*d*f*x+3*b^2*d*f*x+4*a^2*d*e+3*b^2*d*e-4*a^2*f-3*b^2*f) \\
& /b^4/d^2*\exp(d*x+c)-a^4/b^5/d^2*f*c^2+a^4/b^5/d*e*\ln(b*\exp(2*d*x+2*c)+2*a*e \\
& \exp(d*x+c)-b)-2*a^4/b^5/d*e*\ln(\exp(d*x+c))+a^4/b^5/d^2*f*dilog((b*\exp(d*x+c) \\
& +(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+a^4/b^5/d^2*f*dilog((-b*\exp(d*x+c) \\
& +(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^3\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned}
& -1/192*e*((8*a*b^2*e^{(-d*x - c)} - 3*b^3 - 12*(2*a^2*b + b^3)*e^{(-2*d*x - 2*c)} + 24*(4*a^3 + 3*a*b^2)*e^{(-3*d*x - 3*c)})*e^{(4*d*x + 4*c)})/(b^4*d) - 192*( \\
& a^4 + a^2*b^2)*(d*x + c)/(b^5*d) - (8*a*b^2*e^{(-3*d*x - 3*c)} + 3*b^3*e^{(-4*d*x - 4*c)} + 24*(4*a^3 + 3*a*b^2)*e^{(-d*x - c)} + 12*(2*a^2*b + b^3)*e^{(-2*d*x - 2*c)})/(b^4*d) - 192*(a^4 + a^2*b^2)*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/(b^5*d) + 1/2304*f*((1152*(a^4*d^2*e^{(4*c)} + a^2*b^2*d^2*e^{(4*c)})*x^2 + 9*(4*b^4*d*x*e^{(8*c)} - b^4*e^{(8*c)})*e^{(4*d*x)} - 32*(3*a*b^3*d*x*e^{(7*c)} - a*b^3*e^{(7*c)})*e^{(3*d*x)} - 72*(2*a^2*b^2*e^{(6*c)} + b^4*e^{(6*c)} - 2*(2*a^2*b^2*d*e^{(6*c)} + b^4*d*e^{(6*c)})*x)*e^{(2*d*x)} + 288*(4*a^3*b*e^{(5*c)} + 3*a*b^3*e^{(5*c)} - (4*a^3*b*d*e^{(5*c)} + 3*a*b^3*d*e^{(5*c)})*x)*e^{(d*x)} + 288*(4*a^3*b*e^{(3*c)} + 3*a*b^3*e^{(3*c)} + (4*a^3*b*d*e^{(3*c)} + 3*a*b^3*d*e^{(3*c)})*x)*e^{(-d*x)} + 72*(2*a^2*b^2*e^{(2*c)} + b^4*e^{(2*c)} + 2*(2*a^2*b^2*d*e^{(2*c)} + b^4*d*e^{(2*c)})*x)*e^{(-2*d*x)} + 32*(3*a*b^3*d*x*e^c + a*b^3*e^c)*e^{(-3*d*x)} + 9*(4*b^4*d*x + b^4)*e^{(-4*d*x)})*e^{(-4*c)})/(b^5*d^2) - 72*integrate(64*((a^5*e^c + a^3*b^2*e^c)*x*e^{(d*x)} - (a^4*b + a^2*b^3)*x)/(b^6*e^{(2*d*x + 2*c)} + 2*a*b^5*e^{(d*x + c)} - b^6), x)
\end{aligned}$$



$$\begin{aligned}
& + b^4) * d * f * x - 240 * ((4 * a^3 * b + 3 * a * b^3) * d * f * x + (4 * a^3 * b + 3 * a * b^3) * d * e - \\
& (4 * a^3 * b + 3 * a * b^3) * f) * \cosh(d * x + c)^3 + 12 * (2 * a^2 * b^2 + b^4) * d * e - 576 * ((a \\
& ^4 + a^2 * b^2) * d^2 * f * x^2 + 2 * (a^4 + a^2 * b^2) * d^2 * e * x + 4 * (a^4 + a^2 * b^2) * c * d \\
& * e - 2 * (a^4 + a^2 * b^2) * c^2 * f) * \cosh(d * x + c)^2 + 6 * (2 * a^2 * b^2 + b^4) * f + 72 * \\
& ((4 * a^3 * b + 3 * a * b^3) * d * f * x + (4 * a^3 * b + 3 * a * b^3) * d * e + (4 * a^3 * b + 3 * a * b^3) * \\
& f) * \cosh(d * x + c) * \sinh(d * x + c)^2 + 32 * (3 * a * b^3 * d * f * x + 3 * a * b^3 * d * e + a * b^3 \\
& * f) * \cosh(d * x + c) + 2304 * ((a^4 + a^2 * b^2) * f * \cosh(d * x + c)^4 + 4 * (a^4 + a^2 * \\
& b^2) * f * \cosh(d * x + c)^3 * \sinh(d * x + c) + 6 * (a^4 + a^2 * b^2) * f * \cosh(d * x + c)^2 * \\
& \sinh(d * x + c)^2 + 4 * (a^4 + a^2 * b^2) * f * \cosh(d * x + c) * \sinh(d * x + c)^3 + (a^4 \\
& + a^2 * b^2) * f * \sinh(d * x + c)^4) * \operatorname{dilog}((a * \cosh(d * x + c) + a * \sinh(d * x + c) + (b \\
& * \cosh(d * x + c) + b * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b + 1) + 2304 * \\
& ((a^4 + a^2 * b^2) * f * \cosh(d * x + c)^4 + 4 * (a^4 + a^2 * b^2) * f * \cosh(d * x + c)^3 * \operatorname{si} \\
& \sinh(d * x + c) + 6 * (a^4 + a^2 * b^2) * f * \cosh(d * x + c)^2 * \sinh(d * x + c)^2 + 4 * (a^4 \\
& + a^2 * b^2) * f * \cosh(d * x + c) * \sinh(d * x + c)^3 + (a^4 + a^2 * b^2) * f * \sinh(d * x + c \\
& )^4) * \operatorname{dilog}((a * \cosh(d * x + c) + a * \sinh(d * x + c) - (b * \cosh(d * x + c) + b * \sinh(d \\
& * x + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b + 1) + 2304 * (((a^4 + a^2 * b^2) * d * e - ( \\
& a^4 + a^2 * b^2) * c * f) * \cosh(d * x + c)^4 + 4 * ((a^4 + a^2 * b^2) * d * e - (a^4 + a^2 * b \\
& ^2) * c * f) * \cosh(d * x + c)^3 * \sinh(d * x + c) + 6 * ((a^4 + a^2 * b^2) * d * e - (a^4 + a^ \\
& 2 * b^2) * c * f) * \cosh(d * x + c)^2 * \sinh(d * x + c)^2 + 4 * ((a^4 + a^2 * b^2) * d * e - (a^4 \\
& + a^2 * b^2) * c * f) * \cosh(d * x + c) * \sinh(d * x + c)^3 + ((a^4 + a^2 * b^2) * d * e - (a^ \\
& 4 + a^2 * b^2) * c * f) * \sinh(d * x + c)^4) * \log(2 * b * \cosh(d * x + c) + 2 * b * \sinh(d * x + c \\
& ) + 2 * b * \sqrt{(a^2 + b^2) / b^2} + 2 * a) + 2304 * (((a^4 + a^2 * b^2) * d * e - (a^4 + \\
& a^2 * b^2) * c * f) * \cosh(d * x + c)^4 + 4 * ((a^4 + a^2 * b^2) * d * e - (a^4 + a^2 * b^2) * c * \\
& f) * \cosh(d * x + c)^3 * \sinh(d * x + c) + 6 * ((a^4 + a^2 * b^2) * d * e - (a^4 + a^2 * b^2) \\
& * c * f) * \cosh(d * x + c)^2 * \sinh(d * x + c)^2 + 4 * ((a^4 + a^2 * b^2) * d * e - (a^4 + a^2 \\
& * b^2) * c * f) * \cosh(d * x + c) * \sinh(d * x + c)^3 + ((a^4 + a^2 * b^2) * d * e - (a^4 + a^ \\
& 2 * b^2) * c * f) * \sinh(d * x + c)^4) * \log(2 * b * \cosh(d * x + c) + 2 * b * \sinh(d * x + c) - 2 * \\
& b * \sqrt{(a^2 + b^2) / b^2} + 2 * a) + 2304 * (((a^4 + a^2 * b^2) * d * f * x + (a^4 + a^2 * \\
& b^2) * c * f) * \cosh(d * x + c)^4 + 4 * ((a^4 + a^2 * b^2) * d * f * x + (a^4 + a^2 * b^2) * c * f) \\
& * \cosh(d * x + c)^3 * \sinh(d * x + c) + 6 * ((a^4 + a^2 * b^2) * d * f * x + (a^4 + a^2 * b^2) \\
& * c * f) * \cosh(d * x + c)^2 * \sinh(d * x + c)^2 + 4 * ((a^4 + a^2 * b^2) * d * f * x + (a^4 + a \\
& ^2 * b^2) * c * f) * \cosh(d * x + c) * \sinh(d * x + c)^3 + ((a^4 + a^2 * b^2) * d * f * x + (a^4 \\
& + a^2 * b^2) * c * f) * \sinh(d * x + c)^4) * \log(-(a * \cosh(d * x + c) + a * \sinh(d * x + c) + \\
& (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b) + 2304 * (( \\
& (a^4 + a^2 * b^2) * d * f * x + (a^4 + a^2 * b^2) * c * f) * \cosh(d * x + c)^4 + 4 * ((a^4 + a^ \\
& 2 * b^2) * d * f * x + (a^4 + a^2 * b^2) * c * f) * \cosh(d * x + c)^3 * \sinh(d * x + c) + 6 * ((a^4 \\
& + a^2 * b^2) * d * f * x + (a^4 + a^2 * b^2) * c * f) * \cosh(d * x + c)^2 * \sinh(d * x + c)^2 + \\
& 4 * ((a^4 + a^2 * b^2) * d * f * x + (a^4 + a^2 * b^2) * c * f) * \cosh(d * x + c) * \sinh(d * x + c) \\
& ^3 + ((a^4 + a^2 * b^2) * d * f * x + (a^4 + a^2 * b^2) * c * f) * \sinh(d * x + c)^4) * \log(-(a \\
& * \cosh(d * x + c) + a * \sinh(d * x + c) - (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \sqrt{ \\
& (a^2 + b^2) / b^2} - b) / b) + 8 * (9 * (4 * b^4 * d * f * x + 4 * b^4 * d * e - b^4 * f) * \cosh(d * x \\
& + c)^7 + 12 * a * b^3 * d * f * x - 28 * (3 * a * b^3 * d * f * x + 3 * a * b^3 * d * e - a * b^3 * f) * \cosh( \\
& d * x + c)^6 + 12 * a * b^3 * d * e + 54 * (2 * (2 * a^2 * b^2 + b^4) * d * f * x + 2 * (2 * a^2 * b^2 + \\
& b^4) * d * e - (2 * a^2 * b^2 + b^4) * f) * \cosh(d * x + c)^5 + 4 * a * b^3 * f - 180 * ((4 * a^3 * b \\
& + 3 * a * b^3) * d * f * x + (4 * a^3 * b + 3 * a * b^3) * d * e - (4 * a^3 * b + 3 * a * b^3) * f) * \cosh(d
\end{aligned}$$

```
*x + c)^4 - 576*((a^4 + a^2*b^2)*d^2*f*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*x + 4*
(a^4 + a^2*b^2)*c*d*e - 2*(a^4 + a^2*b^2)*c^2*f)*cosh(d*x + c)^3 + 108*((4*
a^3*b + 3*a*b^3)*d*f*x + (4*a^3*b + 3*a*b^3)*d*e + (4*a^3*b + 3*a*b^3)*f)*c
osh(d*x + c)^2 + 18*(2*(2*a^2*b^2 + b^4)*d*f*x + 2*(2*a^2*b^2 + b^4)*d*e +
(2*a^2*b^2 + b^4)*f)*cosh(d*x + c))*sinh(d*x + c))/(b^5*d^2*cosh(d*x + c)^4
+ 4*b^5*d^2*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^5*d^2*cosh(d*x + c)^2*sinh
(d*x + c)^2 + 4*b^5*d^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^5*d^2*sinh(d*x +
c)^4)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)**3*sinh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \cosh(dx + c)^3 \sinh(dx + c)^2}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)^3*sinh(d*x + c)^2/(b*sinh(d*x + c) + a),
x)
```

$$3.375 \quad \int \frac{\cosh^3(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=113

$$\frac{(a^2 + b^2) \sinh^2(c + dx)}{2b^3d} - \frac{a(a^2 + b^2) \sinh(c + dx)}{b^4d} + \frac{a^2(a^2 + b^2) \log(a + b \sinh(c + dx))}{b^5d} - \frac{a \sinh^3(c + dx)}{3b^2d} + \frac{\sinh^4(c + dx)}{4bd}$$

[Out] (a^2\*(a^2 + b^2)\*Log[a + b\*Sinh[c + d\*x]])/(b^5\*d) - (a\*(a^2 + b^2)\*Sinh[c + d\*x])/(b^4\*d) + ((a^2 + b^2)\*Sinh[c + d\*x]^2)/(2\*b^3\*d) - (a\*Sinh[c + d\*x]^3)/(3\*b^2\*d) + Sinh[c + d\*x]^4/(4\*b\*d)

**Rubi [A]** time = 0.162831, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {2837, 12, 894}

$$\frac{(a^2 + b^2) \sinh^2(c + dx)}{2b^3d} - \frac{a(a^2 + b^2) \sinh(c + dx)}{b^4d} + \frac{a^2(a^2 + b^2) \log(a + b \sinh(c + dx))}{b^5d} - \frac{a \sinh^3(c + dx)}{3b^2d} + \frac{\sinh^4(c + dx)}{4bd}$$

Antiderivative was successfully verified.

[In] Int[(Cosh[c + d\*x]^3\*Sinh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (a^2\*(a^2 + b^2)\*Log[a + b\*Sinh[c + d\*x]])/(b^5\*d) - (a\*(a^2 + b^2)\*Sinh[c + d\*x])/(b^4\*d) + ((a^2 + b^2)\*Sinh[c + d\*x]^2)/(2\*b^3\*d) - (a\*Sinh[c + d\*x]^3)/(3\*b^2\*d) + Sinh[c + d\*x]^4/(4\*b\*d)

### Rule 2837

Int[cos[(e\_.) + (f\_.)\*(x\_.)]^(p\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Dist[1/(b^p\*f), Subst[Int[(a + x)^m\*(c + (d\*x)/b)^n\*(b^2 - x^2)^((p - 1)/2), x], x, b\*Sinh[e + f\*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

### Rule 12

Int[(a\_.)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_.)\*(v\_) /; FreeQ[b, x]]

### Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

### Rubi steps

$$\begin{aligned} \int \frac{\cosh^3(c + dx) \sinh^2(c + dx)}{a + b \sinh(c + dx)} dx &= -\frac{\text{Subst}\left(\int \frac{x^2(-b^2-x^2)}{b^2(a+x)} dx, x, b \sinh(c + dx)\right)}{b^3 d} \\ &= -\frac{\text{Subst}\left(\int \frac{x^2(-b^2-x^2)}{a+x} dx, x, b \sinh(c + dx)\right)}{b^5 d} \\ &= -\frac{\text{Subst}\left(\int \left(a(a^2 + b^2) - (a^2 + b^2)x + ax^2 - x^3 - \frac{a^2(a^2+b^2)}{a+x}\right) dx, x, b \sinh(c + dx)\right)}{b^5 d} \\ &= \frac{a^2(a^2 + b^2) \log(a + b \sinh(c + dx))}{b^5 d} - \frac{a(a^2 + b^2) \sinh(c + dx)}{b^4 d} + \frac{(a^2 + b^2) \sinh^2(c + dx)}{2b^3 d} \end{aligned}$$

**Mathematica [A]** time = 0.302434, size = 98, normalized size = 0.87

$$\frac{6b^2(a^2 + b^2) \sinh^2(c + dx) - 12ab(a^2 + b^2) \sinh(c + dx) + 12a^2(a^2 + b^2) \log(a + b \sinh(c + dx)) - 4ab^3 \sinh^3(c + dx) + 3b^4 \sinh^4(c + dx)}{12b^5 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[c + d*x]^3*Sinh[c + d*x]^2)/(a + b*Sinh[c + d*x]), x]
```

```
[Out] (12*a^2*(a^2 + b^2)*Log[a + b*Sinh[c + d*x]] - 12*a*b*(a^2 + b^2)*Sinh[c + d*x] + 6*b^2*(a^2 + b^2)*Sinh[c + d*x]^2 - 4*a*b^3*Sinh[c + d*x]^3 + 3*b^4*Sinh[c + d*x]^4)/(12*b^5*d)
```

**Maple [B]** time = 0.046, size = 614, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(\cosh(dx+c)^3 \sinh(dx+c)^2 / (a+b \sinh(dx+c)), x)$

[Out]  $\frac{5}{8} \frac{d}{b} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1)^2} + \frac{5}{8} \frac{d}{b} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1)^2} - \frac{1}{d} \frac{a^4}{b^5} \ln(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1) + \frac{1}{d} \frac{a^4}{b^5} \ln(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1) - \frac{2}{d} \frac{a^2}{b^4} \tanh(\frac{1}{2}dx + \frac{1}{2}c) + \frac{1}{2} \frac{d}{b^3} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1)^2} \frac{a^2}{b^4} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1)} \frac{a^3}{b^5} \ln(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1) + \frac{1}{3} \frac{d}{b^2} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1)^3} \frac{a}{b^2} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1)^3} \frac{a}{b^3} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1)^2} \frac{a^2}{b^4} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1)} \frac{a^3}{b^5} \frac{1}{2} \frac{d}{b^2} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1)^2} \frac{a}{b^3} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1)} \frac{a^2}{b^4} \frac{1}{2} \frac{d}{b^2} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1)^2} \frac{a}{b^3} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1)} \frac{a^2}{b^4} \frac{1}{b^3} \ln(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1) \frac{a}{b^3} \ln(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1) \frac{a}{b^4} \frac{1}{d} \frac{1}{b^3} \ln(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1) \frac{a}{b^2} \frac{1}{d} \frac{1}{b^2} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1)} \frac{a}{b^3} \ln(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1) \frac{a}{b^4} \frac{1}{4} \frac{d}{b} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1)^4} + \frac{1}{4} \frac{d}{b} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1)^4} - \frac{1}{2} \frac{d}{b} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1)^3} + \frac{1}{2} \frac{d}{b} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1)^3} - \frac{3}{8} \frac{d}{b} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1)} + \frac{3}{8} \frac{d}{b} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1)}$

**Maxima [B]** time = 1.04974, size = 316, normalized size = 2.8

$$\frac{(8ab^2e^{-dx-c} - 3b^3 - 12(2a^2b + b^3)e^{-2dx-2c}) + 24(4a^3 + 3ab^2)e^{-3dx-3c})e^{4dx+4c}}{192b^4d} + \frac{(a^4 + a^2b^2)(dx+c)}{b^5d} + \frac{8ab^2e^{-dx-c}}{b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\cosh(dx+c)^3 \sinh(dx+c)^2 / (a+b \sinh(dx+c)), x, \text{algorithm}="maxima")$

[Out]  $-\frac{1}{192} (8a^2b^2e^{-dx-c} - 3b^3 - 12(2a^2b + b^3)e^{-2dx-2c}) + 24(4a^3 + 3a^2b^2)e^{-3dx-3c})e^{4dx+4c} / (b^4d) + (a^4 + a^2b^2)(dx+c) / (b^5d) + \frac{1}{192} (8a^2b^2e^{-3dx-3c} + 3b^3e^{-4dx-4c} + 24(4a^3 + 3a^2b^2)e^{-dx-c} + 12(2a^2b + b^3)e^{-2dx-2c}) / (b^4d) + (a^4 + a^2b^2) \log(-2ae^{-dx-c} + be^{-2dx-2c} - b) / (b^5d)$

**Fricas [B]** time = 2.23817, size = 2631, normalized size = 23.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^3\*sinh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\frac{1}{192} (3b^4 \cosh(dx+c)^8 + 3b^4 \sinh(dx+c)^8 - 8ab^3 \cosh(dx+c)^7 + 8(3b^4 \cosh(dx+c) - ab^3) \sinh(dx+c)^7 - 192(a^4 + a^2b^2) dx \cosh(dx+c)^4 + 12(2a^2b^2 + b^4) \cosh(dx+c)^6 + 4(21b^4 \cosh(dx+c)^2 - 14ab^3 \cosh(dx+c) + 6a^2b^2 + 3b^4) \sinh(dx+c)^6 - 24(4a^3b + 3ab^3) \cosh(dx+c)^5 + 24(7b^4 \cosh(dx+c)^3 - 7ab^3 \cosh(dx+c)^2 - 4a^3b - 3ab^3 + 3(2a^2b^2 + b^4) \cosh(dx+c)) \sinh(dx+c)^5 + 8ab^3 \cosh(dx+c) + 2(105b^4 \cosh(dx+c)^4 - 140ab^3 \cosh(dx+c)^3 - 96(a^4 + a^2b^2) dx + 90(2a^2b^2 + b^4) \cosh(dx+c)^2 - 60(4a^3b + 3ab^3) \cosh(dx+c)) \sinh(dx+c)^4 + 3b^4 + 24(4a^3b + 3ab^3) \cosh(dx+c)^3 + 8(21b^4 \cosh(dx+c)^5 - 35ab^3 \cosh(dx+c)^4 + 12a^3b + 9ab^3 - 96(a^4 + a^2b^2) dx \cosh(dx+c) + 30(2a^2b^2 + b^4) \cosh(dx+c)^3 - 30(4a^3b + 3ab^3) \cosh(dx+c)^2) \sinh(dx+c)^3 + 12(2a^2b^2 + b^4) \cosh(dx+c)^2 + 12(7b^4 \cosh(dx+c)^6 - 14ab^3 \cosh(dx+c)^5 - 96(a^4 + a^2b^2) dx \cosh(dx+c)^2 + 15(2a^2b^2 + b^4) \cosh(dx+c)^4 + 2a^2b^2 + b^4 - 20(4a^3b + 3ab^3) \cosh(dx+c)^3 + 6(4a^3b + 3ab^3) \cosh(dx+c)) \sinh(dx+c)^2 + 192((a^4 + a^2b^2) \cosh(dx+c)^4 + 4(a^4 + a^2b^2) \cosh(dx+c)^3 \sinh(dx+c) + 6(a^4 + a^2b^2) \cosh(dx+c)^2 \sinh(dx+c)^2 + 4(a^4 + a^2b^2) \cosh(dx+c) \sinh(dx+c)^3 + (a^4 + a^2b^2) \sinh(dx+c)^4) \log(2(b \sinh(dx+c) + a) / (\cosh(dx+c) - \sinh(dx+c))) + 8(3b^4 \cosh(dx+c)^7 - 7ab^3 \cosh(dx+c)^6 - 96(a^4 + a^2b^2) dx \cosh(dx+c)^3 + 9(2a^2b^2 + b^4) \cosh(dx+c)^5 - 15(4a^3b + 3ab^3) \cosh(dx+c)^4 + ab^3 + 9(4a^3b + 3ab^3) \cosh(dx+c)^2 + 3(2a^2b^2 + b^4) \cosh(dx+c)) \sinh(dx+c) / (b^5 dx \cosh(dx+c)^4 + 4b^5 dx \cosh(dx+c)^3 \sinh(dx+c) + 6b^5 dx \cosh(dx+c)^2 \sinh(dx+c)^2 + 4b^5 dx \cosh(dx+c) \sinh(dx+c)^3 + b^5 dx \sinh(dx+c)^4)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*\*3\*sinh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out



**Giac [B]** time = 1.20187, size = 298, normalized size = 2.64

$$\frac{(a^4 + a^2 b^2) \log\left(\left|b(e^{(dx+c)} - e^{(-dx-c)}) + 2a\right|\right)}{b^5 d} + \frac{3b^3 d^3 (e^{(dx+c)} - e^{(-dx-c)})^4 - 8ab^2 d^3 (e^{(dx+c)} - e^{(-dx-c)})^3 + 24a^2 b d^3 (e^{(dx+c)} - e^{(-dx-c)})^2 - 96a^3 d^3 (e^{(dx+c)} - e^{(-dx-c)}) - 96a^2 b^2 d^3 (e^{(dx+c)} - e^{(-dx-c)})}{b^4 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (a^4 + a^2*b^2)*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(b^5*d) + 1/192*(3*b^3*d^3*(e^(d*x + c) - e^(-d*x - c))^4 - 8*a*b^2*d^3*(e^(d*x + c) - e^(-d*x - c))^3 + 24*a^2*b*d^3*(e^(d*x + c) - e^(-d*x - c))^2 + 24*b^3*d^3*(e^(d*x + c) - e^(-d*x - c))^2 - 96*a^3*d^3*(e^(d*x + c) - e^(-d*x - c)) - 96*a*b^2*d^3*(e^(d*x + c) - e^(-d*x - c)))/(b^4*d^4)
```

$$3.376 \quad \int \frac{\cosh^3(c+dx) \sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=38

$$\text{Unintegrable} \left( \frac{\sinh^2(c+dx) \cosh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[(Cosh[c + d\*x]^3\*Sinh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.13164, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cosh^3(c+dx) \sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Cosh[c + d\*x]^3\*Sinh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] Defer[Int] [(Cosh[c + d\*x]^3\*Sinh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\cosh^3(c+dx) \sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\cosh^3(c+dx) \sinh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.001, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Cosh[c + d\*x]^3\*Sinh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] \$Aborted

**Maple [A]** time = 0.194, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx + c))^3 (\sinh(dx + c))^2}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)^3\*sinh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(cosh(d\*x+c)^3\*sinh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{e^{\left(-4c + \frac{4de}{f}\right)} E_1\left(\frac{4(fx+e)d}{f}\right)}{16bf} + \frac{ae^{\left(-3c + \frac{3de}{f}\right)} E_1\left(\frac{3(fx+e)d}{f}\right)}{8b^2f} + \frac{ae^{\left(3c - \frac{3de}{f}\right)} E_1\left(-\frac{3(fx+e)d}{f}\right)}{8b^2f} - \frac{e^{\left(4c - \frac{4de}{f}\right)} E_1\left(-\frac{4(fx+e)d}{f}\right)}{16bf} + \frac{(2a^2 + b^2)e^{\left(-2c + \frac{2de}{f}\right)} E_1\left(\frac{2(fx+e)d}{f}\right)}{8b^3f} - \frac{1}{8} \frac{(2a^2 + b^2)e^{\left(-2c + \frac{2de}{f}\right)}}{b^3f} + \frac{1}{8} \frac{(4a^3 + 3ab^2)e^{\left(-c + \frac{de}{f}\right)} E_1\left(\frac{(fx+e)d}{f}\right)}{b^4f} + \frac{1}{8} \frac{(4a^3e^c + 3ab^2e^c)e^{\left(-\frac{de}{f}\right)} E_1\left(-\frac{(fx+e)d}{f}\right)}{b^4f} + \frac{(a^4 + a^2b^2)\log(fx + e)}{b^5f} - \frac{1}{32} \frac{\int (64(a^4b + a^2b^3 - (a^5e^c + a^3b^2e^c))e^{dx})}{(b^6fx + b^6e - (b^6fx * e^{2c} + b^6e * e^{2c}))e^{2dx}} - 2 \frac{(ab^5fxe^c + ab^5e * e^c)e^{dx}}{(b^6fx + b^6e - (b^6fx * e^{2c} + b^6e * e^{2c}))e^{2dx}}, x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^3\*sinh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm m="maxima")

[Out] 1/16\*e^(-4\*c + 4\*d\*e/f)\*exp\_integral\_e(1, 4\*(f\*x + e)\*d/f)/(b\*f) + 1/8\*a\*e^(-3\*c + 3\*d\*e/f)\*exp\_integral\_e(1, 3\*(f\*x + e)\*d/f)/(b^2\*f) + 1/8\*a\*e^(3\*c - 3\*d\*e/f)\*exp\_integral\_e(1, -3\*(f\*x + e)\*d/f)/(b^2\*f) - 1/16\*e^(4\*c - 4\*d\*e/f)\*exp\_integral\_e(1, -4\*(f\*x + e)\*d/f)/(b\*f) + 1/8\*(2\*a^2 + b^2)\*e^(-2\*c + 2\*d\*e/f)\*exp\_integral\_e(1, 2\*(f\*x + e)\*d/f)/(b^3\*f) - 1/8\*(2\*a^2\*e^(2\*c) + b^2\*e^(2\*c))\*e^(-2\*d\*e/f)\*exp\_integral\_e(1, -2\*(f\*x + e)\*d/f)/(b^3\*f) + 1/8\*(4\*a^3 + 3\*a\*b^2)\*e^(-c + d\*e/f)\*exp\_integral\_e(1, (f\*x + e)\*d/f)/(b^4\*f) + 1/8\*(4\*a^3\*e^c + 3\*a\*b^2\*e^c)\*e^(-d\*e/f)\*exp\_integral\_e(1, -(f\*x + e)\*d/f)/(b^4\*f) + (a^4 + a^2\*b^2)\*log(f\*x + e)/(b^5\*f) - 1/32\*integrate(64\*(a^4\*b + a^2\*b^3 - (a^5\*e^c + a^3\*b^2\*e^c))\*e^(d\*x))/(b^6\*f\*x + b^6\*e - (b^6\*f\*x \* e^(2\*c) + b^6\*e \* e^(2\*c))\*e^(2\*d\*x) - 2\*(a\*b^5\*f\*x\*e^c + a\*b^5\*e \* e^c)\*e^(d\*x)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\cosh(dx+c)^3 \sinh(dx+c)^2}{afx+ae+(bfx+be)\sinh(dx+c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^3*sinh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*sinh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^3 \sinh(dx+c)^2}{(fx+e)(b \sinh(dx+c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*sinh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^3*sinh(d*x + c)^2/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

$$3.377 \quad \int \frac{(e+fx)^3 \sinh(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=1218

result too large to display

```
[Out] -(e + f*x)^4/(4*b*f) - (2*a*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b^2*d) + (2*a
^3*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d) + (a^2*(e + f*x)^3*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d) + (a^2*(e
+ f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d)
+ ((e + f*x)^3*Log[1 + E^(2*(c + d*x))])/(b*d) - (a^2*(e + f*x)^3*Log[1 +
E^(2*(c + d*x))])/(b*(a^2 + b^2)*d) + ((3*I)*a*f*(e + f*x)^2*PolyLog[2, (-I
)*E^(c + d*x)])/(b^2*d^2) - ((3*I)*a^3*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c +
d*x)])/(b^2*(a^2 + b^2)*d^2) - ((3*I)*a*f*(e + f*x)^2*PolyLog[2, I*E^(c +
d*x)])/(b^2*d^2) + ((3*I)*a^3*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b^2
*(a^2 + b^2)*d^2) + (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^2) + (3*a^2*f*(e + f*x)^2*PolyLog[2, -
((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^2) + (3*f*(e + f
*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*b*d^2) - (3*a^2*f*(e + f*x)^2*PolyLo
g[2, -E^(2*(c + d*x))])/(2*b*(a^2 + b^2)*d^2) - ((6*I)*a*f^2*(e + f*x)*Poly
Log[3, (-I)*E^(c + d*x)])/(b^2*d^3) + ((6*I)*a^3*f^2*(e + f*x)*PolyLog[3, (
-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) + ((6*I)*a*f^2*(e + f*x)*PolyLog[3,
I*E^(c + d*x)])/(b^2*d^3) - ((6*I)*a^3*f^2*(e + f*x)*PolyLog[3, I*E^(c + d
*x)])/(b^2*(a^2 + b^2)*d^3) - (6*a^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^3) - (6*a^2*f^2*(e + f*x)*Po
lyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)*d^3) - (
3*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*b*d^3) + (3*a^2*f^2*(e + f
*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*b*(a^2 + b^2)*d^3) + ((6*I)*a*f^3*Poly
Log[4, (-I)*E^(c + d*x)])/(b^2*d^4) - ((6*I)*a^3*f^3*PolyLog[4, (-I)*E^(c +
d*x)])/(b^2*(a^2 + b^2)*d^4) - ((6*I)*a*f^3*PolyLog[4, I*E^(c + d*x)])/(b^
2*d^4) + ((6*I)*a^3*f^3*PolyLog[4, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^4) +
(6*a^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 +
b^2)*d^4) + (6*a^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]
)/(b*(a^2 + b^2)*d^4) + (3*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*b*d^4) - (3
*a^2*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*b*(a^2 + b^2)*d^4)
```

---

**Rubi [A]** time = 1.69431, antiderivative size = 1218, normalized size of antiderivative = 1., number of steps used = 46, number of rules used = 12, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} =$

0.375, Rules used = {5581, 3718, 2190, 2531, 6609, 2282, 6589, 5567, 4180, 5573, 5561, 6742}

$$-\frac{(e+fx)^4}{4bf} + \frac{2a^3 \tan^{-1}\left(\frac{e^{c+dx}}{a-\sqrt{a^2+b^2}}\right)(e+fx)^3}{b^2(a^2+b^2)d} - \frac{2a \tan^{-1}\left(\frac{e^{c+dx}}{a-\sqrt{a^2+b^2}}\right)(e+fx)^3}{b^2d} + \frac{a^2 \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}} + 1\right)(e+fx)^3}{b(a^2+b^2)d} + \frac{a^2 \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}} + 1\right)(e+fx)^3}{b(a^2+b^2)d}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Sinh[c + d\*x]\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out]  $-(e+fx)^4/(4*b*f) - (2*a*(e+fx)^3*ArcTan[E^{(c+d*x)}])/(b^2*d) + (2*a^3*(e+fx)^3*ArcTan[E^{(c+d*x)}])/(b^2*(a^2+b^2)*d) + (a^2*(e+fx)^3*Log[1+(b*E^{(c+d*x)})/(a-Sqrt[a^2+b^2])])/(b*(a^2+b^2)*d) + (a^2*(e+fx)^3*Log[1+(b*E^{(c+d*x)})/(a+Sqrt[a^2+b^2])])/(b*(a^2+b^2)*d) + ((e+fx)^3*Log[1+E^{(2*(c+d*x))}])/(b*d) - (a^2*(e+fx)^3*Log[1+E^{(2*(c+d*x))}])/(b*(a^2+b^2)*d) + ((3*I)*a*f*(e+fx)^2*PolyLog[2,(-I)*E^{(c+d*x)}])/(b^2*d^2) - ((3*I)*a^3*f*(e+fx)^2*PolyLog[2,(-I)*E^{(c+d*x)}])/(b^2*(a^2+b^2)*d^2) - ((3*I)*a*f*(e+fx)^2*PolyLog[2,I*E^{(c+d*x)}])/(b^2*d^2) + ((3*I)*a^3*f*(e+fx)^2*PolyLog[2,I*E^{(c+d*x)}])/(b^2*(a^2+b^2)*d^2) + (3*a^2*f*(e+fx)^2*PolyLog[2,-(b*E^{(c+d*x)})/(a-Sqrt[a^2+b^2])])/(b*(a^2+b^2)*d^2) + (3*a^2*f*(e+fx)^2*PolyLog[2,-(b*E^{(c+d*x)})/(a+Sqrt[a^2+b^2])])/(b*(a^2+b^2)*d^2) + (3*f*(e+fx)^2*PolyLog[2,-E^{(2*(c+d*x))}])/(2*b*d^2) - (3*a^2*f*(e+fx)^2*PolyLog[2,-E^{(2*(c+d*x))}])/(2*b*(a^2+b^2)*d^2) - ((6*I)*a*f^2*(e+fx)*PolyLog[3,(-I)*E^{(c+d*x)}])/(b^2*d^3) + ((6*I)*a^3*f^2*(e+fx)*PolyLog[3,(-I)*E^{(c+d*x)}])/(b^2*(a^2+b^2)*d^3) + ((6*I)*a*f^2*(e+fx)*PolyLog[3,I*E^{(c+d*x)}])/(b^2*d^3) - ((6*I)*a^3*f^2*(e+fx)*PolyLog[3,I*E^{(c+d*x)}])/(b^2*(a^2+b^2)*d^3) - (6*a^2*f^2*(e+fx)*PolyLog[3,-(b*E^{(c+d*x)})/(a-Sqrt[a^2+b^2])])/(b*(a^2+b^2)*d^3) - (6*a^2*f^2*(e+fx)*PolyLog[3,-(b*E^{(c+d*x)})/(a+Sqrt[a^2+b^2])])/(b*(a^2+b^2)*d^3) - (3*f^2*(e+fx)*PolyLog[3,-E^{(2*(c+d*x))}])/(2*b*d^3) + (3*a^2*f^2*(e+fx)*PolyLog[3,-E^{(2*(c+d*x))}])/(2*b*(a^2+b^2)*d^3) + ((6*I)*a*f^3*PolyLog[4,(-I)*E^{(c+d*x)}])/(b^2*d^4) - ((6*I)*a^3*f^3*PolyLog[4,(-I)*E^{(c+d*x)}])/(b^2*(a^2+b^2)*d^4) - ((6*I)*a*f^3*PolyLog[4,I*E^{(c+d*x)}])/(b^2*d^4) + ((6*I)*a^3*f^3*PolyLog[4,I*E^{(c+d*x)}])/(b^2*(a^2+b^2)*d^4) + (6*a^2*f^3*PolyLog[4,-(b*E^{(c+d*x)})/(a-Sqrt[a^2+b^2])])/(b*(a^2+b^2)*d^4) + (6*a^2*f^3*PolyLog[4,-(b*E^{(c+d*x)})/(a+Sqrt[a^2+b^2])])/(b*(a^2+b^2)*d^4) + (3*f^3*PolyLog[4,-E^{(2*(c+d*x))}])/(4*b*d^4) - (3*a^2*f^3*PolyLog[4,-E^{(2*(c+d*x))}])/(4*b*(a^2+b^2)*d^4)$

### Rule 5581

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]^(p\_.)\*Tanh[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] :> Dist[1/b, Int[(e + f\*x)^m\*Sinh[c + d\*x]^(p-1)\*Tanh[c + d\*x]^n, x] - Di

```
st[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n)/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
0] && IGtQ[p, 0]
```

### Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

### Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol]
:> Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[m, 0] && IGtQ[n, 0]
```

### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x]
&& IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol]
:> Dist[b^2/(a^2 + b^2), Int[((e + f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol]
:> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rule 6742

```
Int[u_, x_Symbol]
:> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(e+fx)^3 \sinh(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \tanh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^3 \tanh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= -\frac{(e+fx)^4}{4bf} - \frac{a \int (e+fx)^3 \operatorname{sech}(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)^3 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} + \dots \\
&= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{(e+fx)^3 \log(1+e^{2(c+dx)})}{bd} + \dots \\
&= -\frac{(e+fx)^4}{4bf} - \frac{a^2(e+fx)^4}{4b(a^2+b^2)f} - \frac{2a(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{(e+fx)^3 \log(1+e^{2(c+dx)})}{bd} + \dots \\
&= -\frac{(e+fx)^4}{4bf} - \frac{a^2(e+fx)^4}{4b(a^2+b^2)f} - \frac{2a(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{a^2(e+fx)^3 \log(1+e^{2(c+dx)})}{b(a^2+b^2)} + \dots \\
&= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d} + \dots \\
&= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d} + \dots \\
&= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d} + \dots \\
&= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d} + \dots \\
&= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d} + \dots \\
&= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d} + \dots \\
&= -\frac{(e+fx)^4}{4bf} - \frac{2a(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d} + \dots
\end{aligned}$$

**Mathematica [B]** time = 26.8379, size = 3387, normalized size = 2.78

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^3\*Sinh[c + d\*x]\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x  
]

[Out] 
$$\begin{aligned} & (-8*b*d^4*e^3*E^{(2*c)}*x - 12*b*d^4*e^2*E^{(2*c)}*f*x^2 - 8*b*d^4*e*E^{(2*c)}*f^2*x^3 - 2*b*d^4*E^{(2*c)}*f^3*x^4 - 8*a*d^3*e^3*ArcTan[E^{(c + d*x)}] - 8*a*d^3*e^3*E^{(2*c)}*ArcTan[E^{(c + d*x)}] - (12*I)*a*d^3*e^2*f*x*Log[1 - I*E^{(c + d*x)}] - (12*I)*a*d^3*e^2*E^{(2*c)}*f*x*Log[1 - I*E^{(c + d*x)}] - (12*I)*a*d^3*e*f^2*x^2*Log[1 - I*E^{(c + d*x)}] - (4*I)*a*d^3*f^3*x^3*Log[1 - I*E^{(c + d*x)}] - (4*I)*a*d^3*E^{(2*c)}*f^3*x^3*Log[1 - I*E^{(c + d*x)}] + (12*I)*a*d^3*e^2*f*x*Log[1 + I*E^{(c + d*x)}] + (12*I)*a*d^3*e*f^2*x^2*Log[1 + I*E^{(c + d*x)}] + (12*I)*a*d^3*e*E^{(2*c)}*f^2*x^2*Log[1 + I*E^{(c + d*x)}] + (4*I)*a*d^3*f^3*x^3*Log[1 + I*E^{(c + d*x)}] + (4*I)*a*d^3*E^{(2*c)}*f^3*x^3*Log[1 + I*E^{(c + d*x)}] + 4*b*d^3*e^3*E^{(2*c)}*Log[1 + E^{(2*(c + d*x))}] + 4*b*d^3*e^3*E^{(2*c)}*Log[1 + E^{(2*(c + d*x))}] + 12*b*d^3*e^2*f*x*Log[1 + E^{(2*(c + d*x))}] + 12*b*d^3*e^2*E^{(2*c)}*f*x*Log[1 + E^{(2*(c + d*x))}] + 12*b*d^3*e*f^2*x^2*Log[1 + E^{(2*(c + d*x))}] + 12*b*d^3*e*E^{(2*c)}*f^2*x^2*Log[1 + E^{(2*(c + d*x))}] + 4*b*d^3*f^3*x^3*Log[1 + E^{(2*(c + d*x))}] + 4*b*d^3*E^{(2*c)}*f^3*x^3*Log[1 + E^{(2*(c + d*x))}] + (12*I)*a*d^2*(1 + E^{(2*c)})*f*(e + f*x)^2*PolyLog[2, (-I)*E^{(c + d*x)}] - (12*I)*a*d^2*(1 + E^{(2*c)})*f*(e + f*x)^2*PolyLog[2, I*E^{(c + d*x)}] + 6*b*d^2*e^2*f*PolyLog[2, -E^{(2*(c + d*x))}] + 6*b*d^2*e^2*E^{(2*c)}*f*PolyLog[2, -E^{(2*(c + d*x))}] + 12*b*d^2*e*f^2*x*PolyLog[2, -E^{(2*(c + d*x))}] + 12*b*d^2*e*E^{(2*c)}*f^2*x*PolyLog[2, -E^{(2*(c + d*x))}] + 6*b*d^2*f^3*x^2*PolyLog[2, -E^{(2*(c + d*x))}] + 6*b*d^2*E^{(2*c)}*f^3*x^2*PolyLog[2, -E^{(2*(c + d*x))}] - (24*I)*a*d*e*f^2*PolyLog[3, (-I)*E^{(c + d*x)}] - (24*I)*a*d*e*E^{(2*c)}*f^2*PolyLog[3, (-I)*E^{(c + d*x)}] - (24*I)*a*d*f^3*x*PolyLog[3, (-I)*E^{(c + d*x)}] + (24*I)*a*d*e*f^2*PolyLog[3, I*E^{(c + d*x)}] + (24*I)*a*d*e*E^{(2*c)}*f^2*PolyLog[3, I*E^{(c + d*x)}] + (24*I)*a*d*f^3*x*PolyLog[3, I*E^{(c + d*x)}] + (24*I)*a*d*E^{(2*c)}*f^3*x*PolyLog[3, I*E^{(c + d*x)}] - 6*b*d*e*f^2*PolyLog[3, -E^{(2*(c + d*x))}] - 6*b*d*e*E^{(2*c)}*f^2*PolyLog[3, -E^{(2*(c + d*x))}] - 6*b*d*f^3*x*PolyLog[3, -E^{(2*(c + d*x))}] - 6*b*d*E^{(2*c)}*f^3*x*PolyLog[3, -E^{(2*(c + d*x))}] + (24*I)*a*f^3*PolyLog[4, (-I)*E^{(c + d*x)}] + (24*I)*a*E^{(2*c)}*f^3*PolyLog[4, (-I)*E^{(c + d*x)}] - (24*I)*a*f^3*PolyLog[4, I*E^{(c + d*x)}] - (24*I)*a*E^{(2*c)}*f^3*PolyLog[4, I*E^{(c + d*x)}] + 3*b*f^3*PolyLog[4, -E^{(2*(c + d*x))}] + 3*b*E^{(2*c)}*f^3*PolyLog[4, -E^{(2*(c + d*x))}])/(4*(a^2 + b^2)*d^4*(1 + E^{(2*c)})) - (a^2*(4*e^3*E^{(2*c)}*x + 6*e^2*E^{(2*c)}*f*x^2 + 4*e*E^{(2*c)}*f^2*x^3 + E^{(2*c)}*f^3*x^4 + (4*a*Sqrt[a^2 + b^2]*e^3*ArcTan[(a + b*E^{(c + d*x)})/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*E^{(2*c)}*ArcTan[(a + b*E^{(c + d*x)})/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*ArcTanh[(a + b*E^{(c + d*x)})/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (4*a*Sqrt[-(a^2 + b^2)^2]*e^3*E^{(2*c)}*ArcTanh[(a + b*E^{(c + d*x)})/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (2*e^3*Log[2*a*E^{(c + d*x)} + b*(-1 + E^{(2*(c + d*x))})])/d - (2*e$$

$$\begin{aligned}
&^3E^{(2c)} \cdot \text{Log}[2aE^{(c+d*x)} + b(-1 + E^{(2(c+d*x))})] / d + (6e^{2f*x} \cdot \text{Log}[1 + (bE^{(2c+d*x)}) / (aE^c - \text{Sqrt}[(a^2 + b^2)E^{(2c)}])] / d - (6e^{2c} \cdot E^{(2c)} \cdot f*x \cdot \text{Log}[1 + (bE^{(2c+d*x)}) / (aE^c - \text{Sqrt}[(a^2 + b^2)E^{(2c)}])] / d + (6e \cdot f^2 \cdot x^2 \cdot \text{Log}[1 + (bE^{(2c+d*x)}) / (aE^c - \text{Sqrt}[(a^2 + b^2)E^{(2c)}])] / d - (6e \cdot E^{(2c)} \cdot f^2 \cdot x^2 \cdot \text{Log}[1 + (bE^{(2c+d*x)}) / (aE^c - \text{Sqrt}[(a^2 + b^2)E^{(2c)}])] / d + (2f^3 \cdot x^3 \cdot \text{Log}[1 + (bE^{(2c+d*x)}) / (aE^c - \text{Sqrt}[(a^2 + b^2)E^{(2c)}])] / d - (2E^{(2c)} \cdot f^3 \cdot x^3 \cdot \text{Log}[1 + (bE^{(2c+d*x)}) / (aE^c - \text{Sqrt}[(a^2 + b^2)E^{(2c)}])] / d + (6e^{2f*x} \cdot \text{Log}[1 + (bE^{(2c+d*x)}) / (aE^c + \text{Sqrt}[(a^2 + b^2)E^{(2c)}])] / d - (6e^{2c} \cdot E^{(2c)} \cdot f*x \cdot \text{Log}[1 + (bE^{(2c+d*x)}) / (aE^c + \text{Sqrt}[(a^2 + b^2)E^{(2c)}])] / d + (6e \cdot f^2 \cdot x^2 \cdot \text{Log}[1 + (bE^{(2c+d*x)}) / (aE^c + \text{Sqrt}[(a^2 + b^2)E^{(2c)}])] / d - (6e \cdot E^{(2c)} \cdot f^2 \cdot x^2 \cdot \text{Log}[1 + (bE^{(2c+d*x)}) / (aE^c + \text{Sqrt}[(a^2 + b^2)E^{(2c)}])] / d + (2f^3 \cdot x^3 \cdot \text{Log}[1 + (bE^{(2c+d*x)}) / (aE^c + \text{Sqrt}[(a^2 + b^2)E^{(2c)}])] / d - (2E^{(2c)} \cdot f^3 \cdot x^3 \cdot \text{Log}[1 + (bE^{(2c+d*x)}) / (aE^c + \text{Sqrt}[(a^2 + b^2)E^{(2c)}])] / d - (6(-1 + E^{(2c)}) \cdot f \cdot (e + f*x)^2 \cdot \text{PolyLog}[2, -((bE^{(2c+d*x)}) / (aE^c - \text{Sqrt}[(a^2 + b^2)E^{(2c)}])]) / d^2 - (6(-1 + E^{(2c)}) \cdot f \cdot (e + f*x)^2 \cdot \text{PolyLog}[2, -((bE^{(2c+d*x)}) / (aE^c + \text{Sqrt}[(a^2 + b^2)E^{(2c)}])]) / d^2 - (12e \cdot f^2 \cdot \text{PolyLog}[3, -((bE^{(2c+d*x)}) / (aE^c - \text{Sqrt}[(a^2 + b^2)E^{(2c)}])]) / d^3 + (12e \cdot E^{(2c)} \cdot f^2 \cdot \text{PolyLog}[3, -((bE^{(2c+d*x)}) / (aE^c - \text{Sqrt}[(a^2 + b^2)E^{(2c)}])]) / d^3 - (12f^3 \cdot x \cdot \text{PolyLog}[3, -((bE^{(2c+d*x)}) / (aE^c - \text{Sqrt}[(a^2 + b^2)E^{(2c)}])]) / d^3 + (12E^{(2c)} \cdot f^3 \cdot x \cdot \text{PolyLog}[3, -((bE^{(2c+d*x)}) / (aE^c - \text{Sqrt}[(a^2 + b^2)E^{(2c)}])]) / d^3 - (12e \cdot f^2 \cdot \text{PolyLog}[3, -((bE^{(2c+d*x)}) / (aE^c + \text{Sqrt}[(a^2 + b^2)E^{(2c)}])]) / d^3 + (12e \cdot E^{(2c)} \cdot f^2 \cdot \text{PolyLog}[3, -((bE^{(2c+d*x)}) / (aE^c + \text{Sqrt}[(a^2 + b^2)E^{(2c)}])]) / d^3 - (12f^3 \cdot x \cdot \text{PolyLog}[3, -((bE^{(2c+d*x)}) / (aE^c + \text{Sqrt}[(a^2 + b^2)E^{(2c)}])]) / d^3 + (12E^{(2c)} \cdot f^3 \cdot x \cdot \text{PolyLog}[3, -((bE^{(2c+d*x)}) / (aE^c + \text{Sqrt}[(a^2 + b^2)E^{(2c)}])]) / d^3 + (12f^3 \cdot \text{PolyLog}[4, -((bE^{(2c+d*x)}) / (aE^c - \text{Sqrt}[(a^2 + b^2)E^{(2c)}])]) / d^4 - (12E^{(2c)} \cdot f^3 \cdot \text{PolyLog}[4, -((bE^{(2c+d*x)}) / (aE^c - \text{Sqrt}[(a^2 + b^2)E^{(2c)}])]) / d^4 + (12f^3 \cdot \text{PolyLog}[4, -((bE^{(2c+d*x)}) / (aE^c + \text{Sqrt}[(a^2 + b^2)E^{(2c)}])]) / d^4 - (12E^{(2c)} \cdot f^3 \cdot \text{PolyLog}[4, -((bE^{(2c+d*x)}) / (aE^c + \text{Sqrt}[(a^2 + b^2)E^{(2c)}])]) / d^4) / (2b \cdot (a^2 + b^2) \cdot (-1 + E^{(2c)})) + ((4a^2 \cdot e^3 \cdot x - 4b^2 \cdot e^3 \cdot x + 6a^2 \cdot e^2 \cdot f \cdot x^2 - 6b^2 \cdot e^2 \cdot f \cdot x^2 + 4a^2 \cdot e \cdot f^2 \cdot x^3 - 4b^2 \cdot e \cdot f^2 \cdot x^3 + a^2 \cdot f^3 \cdot x^4 - b^2 \cdot f^3 \cdot x^4 + 4a^2 \cdot e^3 \cdot x \cdot \text{Cosh}[2c] + 4b^2 \cdot e^3 \cdot x \cdot \text{Cosh}[2c] + 6a^2 \cdot e^2 \cdot f \cdot x^2 \cdot \text{Cosh}[2c] + 6b^2 \cdot e^2 \cdot f \cdot x^2 \cdot \text{Cosh}[2c] + 4a^2 \cdot e \cdot f^2 \cdot x^3 \cdot \text{Cosh}[2c] + 4b^2 \cdot e \cdot f^2 \cdot x^3 \cdot \text{Cosh}[2c] + a^2 \cdot f^3 \cdot x^4 \cdot \text{Cosh}[2c] + b^2 \cdot f^3 \cdot x^4 \cdot \text{Cosh}[2c]) \cdot \text{Csch}[c] \cdot \text{Sech}[c]) / (8b \cdot (a^2 + b^2))
\end{aligned}$$

**Maple [F]** time = 0.652, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \sinh(dx + c) \tanh(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)`

[Out] `int((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$e^3 \left( \frac{a^2 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{(a^2b + b^3)d} + \frac{2a \arctan(e^{(-dx-c)})}{(a^2 + b^2)d} + \frac{b \log(e^{(-2dx-2c)} + 1)}{(a^2 + b^2)d} + \frac{dx + c}{bd} \right) + \frac{f^3x^4 + 4ef^2x^3 + 6efx^2 + 6efx + 6e}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `e^3*(a^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b + b^3)*d) + 2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^2 + b^2)*d) + (d*x + c)/(b*d)) + 1/4*(f^3*x^4 + 4*e*f^2*x^3 + 6*e^2*f*x^2)/b - integrate(2*(a^2*b*f^3*x^3 + 3*a^2*b*e*f^2*x^2 + 3*a^2*b*e^2*f*x - (a^3*f^3*x^3*e^c + 3*a^3*e*f^2*x^2*e^c + 3*a^3*e^2*f*x*e^c)*e^(d*x))/(a^2*b^2 + b^4 - (a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x) - 2*(a^3*b*e^c + a*b^3*e^c)*e^(d*x)), x) - integrate(2*(b*f^3*x^3 + 3*b*e*f^2*x^2 + 3*b*e^2*f*x + (a*f^3*x^3*e^c + 3*a*e*f^2*x^2*e^c + 3*a*e^2*f*x*e^c)*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)`

**Fricas [C]** time = 3.27461, size = 4721, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] `-1/4*((a^2 + b^2)*d^4*f^3*x^4 + 4*(a^2 + b^2)*d^4*e*f^2*x^3 + 6*(a^2 + b^2)*d^4*e^2*f*x^2 + 4*(a^2 + b^2)*d^4*e^3*x - 24*a^2*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 24*a^2*f^3*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) -`

$$\begin{aligned}
& (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} / b - 12(a^2 d^2 f^3 x^2 + 2a^2 d^2 e f^2 x + a^2 d^2 e^2 f) \operatorname{dilog}((a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b) / b + 1) - 12(a^2 d^2 f^3 x^2 + 2a^2 d^2 e f^2 x + a^2 d^2 e^2 f) \operatorname{dilog}((a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b) / b + 1) - (-12I a b d^2 f^3 x^2 + 12b^2 d^2 f^3 x^2 - 24I a b d^2 e f^2 x + 24b^2 d^2 e f^2 x - 12I a b d^2 e^2 f + 12b^2 d^2 e^2 f) \operatorname{dilog}(I \cosh(dx + c) + I \sinh(dx + c)) - (12I a b d^2 f^3 x^2 + 12b^2 d^2 f^3 x^2 + 24I a b d^2 e f^2 x + 24b^2 d^2 e f^2 x + 12I a b d^2 e^2 f + 12b^2 d^2 e^2 f) \operatorname{dilog}(-I \cosh(dx + c) - I \sinh(dx + c)) \\
& - 4(a^2 d^3 e^3 - 3a^2 c d^2 e^2 f + 3a^2 c^2 d e f^2 - a^2 c^3 f^3) \log(2b \cosh(dx + c) + 2b \sinh(dx + c) + 2b \sqrt{(a^2 + b^2)/b^2} + 2a) - 4(a^2 d^3 e^3 - 3a^2 c d^2 e^2 f + 3a^2 c^2 d e f^2 - a^2 c^3 f^3) \log(2b \cosh(dx + c) + 2b \sinh(dx + c) - 2b \sqrt{(a^2 + b^2)/b^2} + 2a) - 4(a^2 d^3 f^3 x^3 + 3a^2 d^3 e f^2 x^2 + 3a^2 d^3 e^2 f x + 3a^2 c d^2 e^2 f - 3a^2 c^2 d e f^2 + a^2 c^3 f^3) \log(-(a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b) / b) - 4(a^2 d^3 f^3 x^3 + 3a^2 d^3 e f^2 x^2 + 3a^2 d^3 e^2 f x + 3a^2 c d^2 e^2 f - 3a^2 c^2 d e f^2 + a^2 c^3 f^3) \log(-(a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b) / b) \\
& - (-4I a b d^3 e^3 + 4b^2 d^3 e^3 + 12I a b c d^2 e^2 f - 12b^2 c d^2 e^2 f - 12I a b c^2 d e f^2 + 12b^2 c^2 d e f^2 + 4I a b c^3 f^3 - 4b^2 c^3 f^3) \log(\cosh(dx + c) + \sinh(dx + c) + I) - (4I a b d^3 e^3 + 4b^2 d^3 e^3 - 12I a b c d^2 e^2 f - 12b^2 c d^2 e^2 f + 12I a b c^2 d e f^2 - 12b^2 c^2 d e f^2 - 4I a b c^3 f^3 - 4b^2 c^3 f^3) \log(\cosh(dx + c) + \sinh(dx + c) - I) - (4I a b d^3 f^3 x^3 + 4b^2 d^3 f^3 x^3 + 12I a b d^3 e f^2 x^2 + 12b^2 d^3 e f^2 x^2 + 12I a b d^3 e^2 f x + 12b^2 d^3 e^2 f x + 12I a b c d^2 e^2 f + 12b^2 c d^2 e^2 f - 12I a b c^2 d e f^2 - 12b^2 c^2 d e f^2 + 4I a b c^3 f^3 + 4b^2 c^3 f^3) \log(I \cosh(dx + c) + I \sinh(dx + c) + 1) - (-4I a b d^3 f^3 x^3 + 4b^2 d^3 f^3 x^3 - 12I a b d^3 e f^2 x^2 + 12b^2 d^3 e f^2 x^2 - 12I a b d^3 e^2 f x + 12b^2 d^3 e^2 f x - 12I a b c d^2 e^2 f + 12b^2 c d^2 e^2 f + 12I a b c^2 d e f^2 - 12b^2 c^2 d e f^2 - 4I a b c^3 f^3 + 4b^2 c^3 f^3) \log(-I \cosh(dx + c) - I \sinh(dx + c) + 1) + 24(I a b f^3 - b^2 f^3) \operatorname{polylog}(4, I \cosh(dx + c) + I \sinh(dx + c)) + 24(-I a b f^3 - b^2 f^3) \operatorname{polylog}(4, -I \cosh(dx + c) - I \sinh(dx + c)) + 24(a^2 d f^3 x + a^2 d e f^2) \operatorname{polylog}(3, (a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2}) / b) + 24(a^2 d f^3 x + a^2 d e f^2) \operatorname{polylog}(3, (a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2}) / b) - (24I a b d f^3 x - 24b^2 d f^3 x + 24I a b d e f^2 - 24b^2 d e f^2) \operatorname{polylog}(3, I \cosh(dx + c) + I \sinh(dx + c)) - (-24I a b d f^3 x - 24b^2 d f^3 x - 24I a b d e f^2 - 24b^2 d e f^2) \operatorname{polylog}(3, -I \cosh(dx + c) - I \sinh(dx + c)) / ((a^2 b + b^3) d^4)
\end{aligned}$$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^3 \sinh(c + dx) \tanh(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*sinh(c + d*x)*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.378 \quad \int \frac{(e+fx)^2 \sinh(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=861

$$\frac{2(e+fx)^2 \tan^{-1}(e^{c+dx}) a^3}{b^2(a^2+b^2)d} - \frac{2if(e+fx)\text{PolyLog}(2, -ie^{c+dx}) a^3}{b^2(a^2+b^2)d^2} + \frac{2if(e+fx)\text{PolyLog}(2, ie^{c+dx}) a^3}{b^2(a^2+b^2)d^2} + \frac{2if^2\text{PolyLog}}{b^2(a^2+b^2)d^2}$$

```
[Out] -(e + f*x)^3/(3*b*f) - (2*a*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^2*d) + (2*a^3*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d) + ((e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b*d) - (a^2*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b*(a^2 + b^2)*d) + ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*d^2) - ((2*I)*a^3*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) - ((2*I)*a*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b^2*d^2) + ((2*I)*a^3*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) + (2*a^2*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d^2) + (2*a^2*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d^2) + (f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(b*d^2) - (a^2*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(b*(a^2 + b^2)*d^2) - ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b^2*d^3) + ((2*I)*a^3*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) + ((2*I)*a*f^2*PolyLog[3, I*E^(c + d*x)])/(b^2*d^3) - ((2*I)*a^3*f^2*PolyLog[3, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) - (2*a^2*f^2*PolyLog[3, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d^3) - (2*a^2*f^2*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)*d^3) - (f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*b*d^3) + (a^2*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*b*(a^2 + b^2)*d^3)
```

**Rubi [A]** time = 1.36649, antiderivative size = 861, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 11, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$ , Rules used = {5581, 3718, 2190, 2531, 2282, 6589, 5567, 4180, 5573, 5561, 6742}

$$\frac{2(e+fx)^2 \tan^{-1}(e^{c+dx}) a^3}{b^2(a^2+b^2)d} - \frac{2if(e+fx)\text{PolyLog}(2, -ie^{c+dx}) a^3}{b^2(a^2+b^2)d^2} + \frac{2if(e+fx)\text{PolyLog}(2, ie^{c+dx}) a^3}{b^2(a^2+b^2)d^2} + \frac{2if^2\text{PolyLog}}{b^2(a^2+b^2)d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(e + f*x)^3/(3*b*f) - (2*a*(e + f*x)^2*ArcTan[E^(c + d*x)]/(b^2*d) + (2*a^3*(e + f*x)^2*ArcTan[E^(c + d*x)]/(b^2*(a^2 + b^2)*d) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(b*(a^2 + b^2)*d) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(b*(a^2 + b^2)*d) + ((e + f*x)^2*Log[1 + E^(2*(c + d*x))]/(b*d) - (a^2*(e + f*x)^2*Log[1 + E^(2*(c + d*x))]/(b*(a^2 + b^2)*d) + ((2*I)*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)]/(b^2*d^2) - ((2*I)*a^3*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)]/(b^2*(a^2 + b^2)*d^2) - ((2*I)*a*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/(b^2*d^2) + ((2*I)*a^3*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/(b^2*(a^2 + b^2)*d^2) + (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(b*(a^2 + b^2)*d^2) + (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(b*(a^2 + b^2)*d^2) + (f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))]/(b*d^2) - (a^2*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))]/(b*(a^2 + b^2)*d^2) - ((2*I)*a*f^2*PolyLog[3, (-I)*E^(c + d*x)]/(b^2*d^3) + ((2*I)*a^3*f^2*PolyLog[3, (-I)*E^(c + d*x)]/(b^2*(a^2 + b^2)*d^3) + ((2*I)*a*f^2*PolyLog[3, I*E^(c + d*x)]/(b^2*d^3) - ((2*I)*a^3*f^2*PolyLog[3, I*E^(c + d*x)]/(b^2*(a^2 + b^2)*d^3) - (2*a^2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(b*(a^2 + b^2)*d^3) - (2*a^2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(b*(a^2 + b^2)*d^3) - (f^2*PolyLog[3, -E^(2*(c + d*x))]/(2*b*d^3) + (a^2*f^2*PolyLog[3, -E^(2*(c + d*x))]/(2*b*(a^2 + b^2)*d^3)
```

### Rule 5581

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Dist[a/b, Int[(((e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 3718

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[(((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(1 + E^(2*(-I*e) + f*fz*x))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n]/a)]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```



Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*(a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*(a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sech[c
+ d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)]]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
```

$b^2$ ), Int[(e + f\*x)^m\*Sech[c + d\*x]^n\*(a - b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]

### Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_.)]\*((e\_.) + (f\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \sinh(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \tanh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^2 \tanh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= -\frac{(e+fx)^3}{3bf} - \frac{a \int (e+fx)^2 \operatorname{sech}(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} + \frac{2 \int (e+fx)^2 \operatorname{sech}(c+dx) dx}{b} \\
&= -\frac{(e+fx)^3}{3bf} - \frac{2a(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{(e+fx)^2 \log(1+e^{2(c+dx)})}{bd} + \frac{a^2 \int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
&= -\frac{(e+fx)^3}{3bf} - \frac{a^2(e+fx)^3}{3b(a^2+b^2)f} - \frac{2a(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{(e+fx)^2 \log(1+e^{2(c+dx)})}{bd} \\
&= -\frac{(e+fx)^3}{3bf} - \frac{a^2(e+fx)^3}{3b(a^2+b^2)f} - \frac{2a(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{a^2(e+fx)^2 \log(1+e^{2(c+dx)})}{b(a^2+b^2)} \\
&= -\frac{(e+fx)^3}{3bf} - \frac{2a(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d} + \frac{a^2 \int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
&= -\frac{(e+fx)^3}{3bf} - \frac{2a(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d} + \frac{a^2 \int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
&= -\frac{(e+fx)^3}{3bf} - \frac{2a(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d} + \frac{a^2 \int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
&= -\frac{(e+fx)^3}{3bf} - \frac{2a(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d} + \frac{a^2 \int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
&= -\frac{(e+fx)^3}{3bf} - \frac{2a(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d} + \frac{a^2 \int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx}{b^2}
\end{aligned}$$

**Mathematica [B]** time = 19.7431, size = 1759, normalized size = 2.04

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x]*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
]
```

```

[Out] -(12*b*d^3*e^2*E^(2*c)*x - 12*b*d^3*e^2*(1 + E^(2*c))*x - 12*b*d^3*e*f*x^2
- 4*b*d^3*f^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] + 6*b*d^
2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*a*d*e*(1 +
E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - PolyLog
[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + 6*b*d*e*(1 + E^(2*c))*
f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]) +
(6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log[1
+ I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2, I*
E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*x)]
) + b*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))]) - 6
*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))])/(6*(a^
2 + b^2)*d^3*(1 + E^(2*c))) - (a^2*(6*e^2*E^(2*c)*x + 6*e*E^(2*c))*f*x^2 + 2
*e^2*c*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt
[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(
2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) -
(6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])
/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a
+ b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*
E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))]/d - (3*e^2*E^(2*c)*Log[2*a*E^(c +
d*x) + b*(-1 + E^(2*(c + d*x)))]/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))]/(a
*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))/d - (6*e*E^(2*c))*f*x*Log[1 + (b*E^(2*c
+ d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (3*f^2*x^2*Log[1 + (b*E^(
2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (3*E^(2*c))*f^2*x^2*Lo
g[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])])/d + (6*e*f*x*
Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d - (6*e*E^(
2*c))*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])])/d
+ (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])
])/d - (3*E^(2*c))*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^
2)*E^(2*c)])])/d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d
*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^2 - (6*(-1 + E^(2*c))*f*(e +
f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/
d^2 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*
c)]))])/d^3 + (6*E^(2*c))*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(
a^2 + b^2)*E^(2*c)]))])/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c
+ Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3 + (6*E^(2*c))*f^2*PolyLog[3, -((b*E^(2*c
+ d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))])/d^3)/(3*b*(a^2 + b^2)*(-1
+ E^(2*c))) + ((3*a^2*e^2*x - 3*b^2*e^2*x + 3*a^2*e*f*x^2 - 3*b^2*e*f*x^2 +
a^2*f^2*x^3 - b^2*f^2*x^3 + 3*a^2*e^2*x*Cosh[2*c] + 3*b^2*e^2*x*Cosh[2*c]
+ 3*a^2*e*f*x^2*Cosh[2*c] + 3*b^2*e*f*x^2*Cosh[2*c] + a^2*f^2*x^3*Cosh[2*c]
+ b^2*f^2*x^3*Cosh[2*c])*Csch[c]*Sech[c])/(6*b*(a^2 + b^2))

```

**Maple [F]** time = 0.49, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \sinh(dx + c) \tanh(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*sinh(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*sinh(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$e^2 \left( \frac{a^2 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{(a^2b + b^3)d} + \frac{2a \arctan(e^{(-dx-c)})}{(a^2 + b^2)d} + \frac{b \log(e^{(-2dx-2c)} + 1)}{(a^2 + b^2)d} + \frac{dx + c}{bd} \right) + \frac{f^2x^3 + 3efx^2}{3b} - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sinh(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] e^2\*(a^2\*log(-2\*a\*e^(-d\*x - c) + b\*e^(-2\*d\*x - 2\*c) - b)/((a^2\*b + b^3)\*d) + 2\*a\*arctan(e^(-d\*x - c))/((a^2 + b^2)\*d) + b\*log(e^(-2\*d\*x - 2\*c) + 1)/((a^2 + b^2)\*d) + (d\*x + c)/(b\*d)) + 1/3\*(f^2\*x^3 + 3\*e\*f\*x^2)/b - integrate(2\*(a^2\*b\*f^2\*x^2 + 2\*a^2\*b\*e\*f\*x - (a^3\*f^2\*x^2\*e^c + 2\*a^3\*e\*f\*x\*e^c)\*e^(d\*x))/(a^2\*b^2 + b^4 - (a^2\*b^2\*e^(2\*c) + b^4\*e^(2\*c))\*e^(2\*d\*x) - 2\*(a^3\*b\*e^c + a\*b^3\*e^c)\*e^(d\*x)), x) - integrate(2\*(b\*f^2\*x^2 + 2\*b\*e\*f\*x + (a\*f^2\*x^2\*e^c + 2\*a\*e\*f\*x\*e^c)\*e^(d\*x))/(a^2 + b^2 + (a^2\*e^(2\*c) + b^2\*e^(2\*c))\*e^(2\*d\*x)), x)

**Fricas [C]** time = 2.84092, size = 3077, normalized size = 3.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sinh(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

```
[Out] -1/3*((a^2 + b^2)*d^3*f^2*x^3 + 3*(a^2 + b^2)*d^3*e*f*x^2 + 3*(a^2 + b^2)*d^3*e^2*x + 6*a^2*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 6*a^2*f^2*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*(a^2*d*f^2*x + a^2*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 6*(a^2*d*f^2*x + a^2*d*e*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (-6*I*a*b*d*f^2*x + 6*b^2*d*f^2*x - 6*I*a*b*d*e*f + 6*b^2*d*e*f)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - (6*I*a*b*d*f^2*x + 6*b^2*d*f^2*x + 6*I*a*b*d*e*f + 6*b^2*d*e*f)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) - 3*(a^2*d^2*e^2 - 2*a^2*c*d*e*f + a^2*c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(a^2*d^2*e^2 - 2*a^2*c*d*e*f + a^2*c^2*f^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 3*(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + 2*a^2*c*d*e*f - a^2*c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3*(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + 2*a^2*c*d*e*f - a^2*c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (-3*I*a*b*d^2*e^2 + 3*b^2*d^2*e^2 + 6*I*a*b*c*d*e*f - 6*b^2*c*d*e*f - 3*I*a*b*c^2*f^2 + 3*b^2*c^2*f^2)*log(cosh(d*x + c) + sinh(d*x + c) + I) - (3*I*a*b*d^2*e^2 + 3*b^2*d^2*e^2 - 6*I*a*b*c*d*e*f - 6*b^2*c*d*e*f + 3*I*a*b*c^2*f^2 + 3*b^2*c^2*f^2)*log(cosh(d*x + c) + sinh(d*x + c) - I) - (3*I*a*b*d^2*f^2*x^2 + 3*b^2*d^2*f^2*x^2 + 6*I*a*b*d^2*e*f*x + 6*b^2*d^2*e*f*x + 6*I*a*b*c*d*e*f + 6*b^2*c*d*e*f - 3*I*a*b*c^2*f^2 - 3*b^2*c^2*f^2)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) - (-3*I*a*b*d^2*f^2*x^2 + 3*b^2*d^2*f^2*x^2 - 6*I*a*b*d^2*e*f*x + 6*b^2*d^2*e*f*x - 6*I*a*b*c*d*e*f + 6*b^2*c*d*e*f + 3*I*a*b*c^2*f^2 - 3*b^2*c^2*f^2)*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) + 6*(-I*a*b*f^2 + b^2*f^2)*polylog(3, I*cosh(d*x + c) + I*sinh(d*x + c)) + 6*(I*a*b*f^2 + b^2*f^2)*polylog(3, -I*cosh(d*x + c) - I*sinh(d*x + c)))/((a^2*b + b^3)*d^3)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^2 \sinh(c + dx) \tanh(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

[Out]  $\text{Integral}((e + f*x)**2*\sinh(c + d*x)*\tanh(c + d*x)/(a + b*\sinh(c + d*x)), x)$

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x+e)^2*\sinh(d*x+c)*\tanh(d*x+c)/(a+b*\sinh(d*x+c)),x, \text{algorithm}="giac")$

[Out] Timed out

$$3.379 \quad \int \frac{(e+fx) \sinh(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=516

$$-\frac{ia^3 f \text{PolyLog}\left(2, -ie^{c+dx}\right)}{b^2 d^2 (a^2 + b^2)} + \frac{ia^3 f \text{PolyLog}\left(2, ie^{c+dx}\right)}{b^2 d^2 (a^2 + b^2)} + \frac{a^2 f \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2 (a^2 + b^2)} + \frac{a^2 f \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2 (a^2 + b^2)} - a$$

[Out]  $-(e + f*x)^2/(2*b*f) - (2*a*(e + f*x)*\text{ArcTan}[E^{(c + d*x)}])/(b^2*d) + (2*a^3*(e + f*x)*\text{ArcTan}[E^{(c + d*x)}])/(b^2*(a^2 + b^2)*d) + (a^2*(e + f*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])])/(b*(a^2 + b^2)*d) + (a^2*(e + f*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(b*(a^2 + b^2)*d) + ((e + f*x)*\text{Log}[1 + E^{(2*(c + d*x))}])/(b*d) - (a^2*(e + f*x)*\text{Log}[1 + E^{(2*(c + d*x))}])/(b*(a^2 + b^2)*d) + (I*a*f*\text{PolyLog}[2, (-I)*E^{(c + d*x)}])/(b^2*d^2) - (I*a^3*f*\text{PolyLog}[2, (-I)*E^{(c + d*x)}])/(b^2*(a^2 + b^2)*d^2) - (I*a*f*\text{PolyLog}[2, I*E^{(c + d*x)}])/(b^2*d^2) + (I*a^3*f*\text{PolyLog}[2, I*E^{(c + d*x)}])/(b^2*(a^2 + b^2)*d^2) + (a^2*f*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)*d^2) + (a^2*f*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(b*(a^2 + b^2)*d^2) + (f*\text{PolyLog}[2, -E^{(2*(c + d*x))}])/(2*b*d^2) - (a^2*f*\text{PolyLog}[2, -E^{(2*(c + d*x))}])/(2*b*(a^2 + b^2)*d^2)$

**Rubi [A]** time = 0.775852, antiderivative size = 516, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 10, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {5581, 3718, 2190, 2279, 2391, 5567, 4180, 5573, 5561, 6742}

$$-\frac{ia^3 f \text{PolyLog}\left(2, -ie^{c+dx}\right)}{b^2 d^2 (a^2 + b^2)} + \frac{ia^3 f \text{PolyLog}\left(2, ie^{c+dx}\right)}{b^2 d^2 (a^2 + b^2)} + \frac{a^2 f \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2 (a^2 + b^2)} + \frac{a^2 f \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2 (a^2 + b^2)} - a$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Sinh[c + d\*x]\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out]  $-(e + f*x)^2/(2*b*f) - (2*a*(e + f*x)*\text{ArcTan}[E^{(c + d*x)}])/(b^2*d) + (2*a^3*(e + f*x)*\text{ArcTan}[E^{(c + d*x)}])/(b^2*(a^2 + b^2)*d) + (a^2*(e + f*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])])/(b*(a^2 + b^2)*d) + (a^2*(e + f*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(b*(a^2 + b^2)*d) + ((e + f*x)*\text{Log}[1 + E^{(2*(c + d*x))}])/(b*d) - (a^2*(e + f*x)*\text{Log}[1 + E^{(2*(c + d*x))}])/(b*(a^2 + b^2)*d) + (I*a*f*\text{PolyLog}[2, (-I)*E^{(c + d*x)}])/(b^2*d^2) - (I*a^3*f*\text{PolyLog}[2, (-I)*E^{(c + d*x)}])/(b^2*(a^2 + b^2)*d^2) - (I*a*f*\text{PolyLog}[2, I*E^{(c + d*x)}])/(b^2*d^2) + (I*a^3*f*\text{PolyLog}[2, I*E^{(c + d*x)}])/(b^2*(a^2 + b^2)*d^2)$



$$\frac{(a^2 + b^2)d^2 + (a^2 f \text{PolyLog}[2, -((bE^{(c+dx)})/(a - \text{Sqrt}[a^2 + b^2]))])/(b(a^2 + b^2)d^2) + (a^2 f \text{PolyLog}[2, -((bE^{(c+dx)})/(a + \text{Sqrt}[a^2 + b^2]))])/(b(a^2 + b^2)d^2) + (f \text{PolyLog}[2, -E^{(2(c+dx))}])/(2b d^2) - (a^2 f \text{PolyLog}[2, -E^{(2(c+dx))}])/(2b(a^2 + b^2)d^2)}$$
Rule 5581

$$\text{Int}[(((e_{.}) + (f_{.})(x_{.}))^{(m_{.})} \text{Sinh}[(c_{.}) + (d_{.})(x_{.})]^{(p_{.})} \text{Tanh}[(c_{.}) + (d_{.})(x_{.})]^{(n_{.})})/((a_{.}) + (b_{.}) \text{Sinh}[(c_{.}) + (d_{.})(x_{.})]), x_{\text{Symbol}}] \text{ :> Dist}[1/b, \text{Int}[(e + f x)^m \text{Sinh}[c + d x]^{(p-1)} \text{Tanh}[c + d x]^n, x] - \text{Dist}[a/b, \text{Int}[(e + f x)^m \text{Sinh}[c + d x]^{(p-1)} \text{Tanh}[c + d x]^n / (a + b \text{Sinh}[c + d x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$$
Rule 3718

$$\text{Int}[(((c_{.}) + (d_{.})(x_{.}))^{(m_{.})} \tan[(e_{.}) + (\text{Complex}[0, fz_{.}]) (f_{.})(x_{.})]), x_{\text{Symbol}}] \text{ :> -Simp}[(I(c + dx)^{(m+1)})/(d(m+1)), x] + \text{Dist}[2I, \text{Int}[(c + dx)^m E^{(2(-Ie + f fz x))} / (1 + E^{(2(-Ie + f fz x))}), x], x] /; \text{FreeQ}[\{c, d, e, f, fz\}, x] \&\& \text{IGtQ}[m, 0]$$
Rule 2190

$$\text{Int}[(((F_{.})^{((g_{.}) * ((e_{.}) + (f_{.})(x_{.})))})^{(n_{.})} * ((c_{.}) + (d_{.})(x_{.}))^{(m_{.})}) / ((a_{.}) + (b_{.}) * ((F_{.})^{((g_{.}) * ((e_{.}) + (f_{.})(x_{.})))})^{(n_{.})}), x_{\text{Symbol}}] \text{ :> Simp}[(c + dx)^m \text{Log}[1 + (b(F^{(g(e + f x)))})^n / a] / (b f g^n \text{Log}[F]), x] - \text{Dist}[(d m) / (b f g^n \text{Log}[F]), \text{Int}[(c + dx)^{(m-1)} \text{Log}[1 + (b(F^{(g(e + f x)))})^n / a], x], x] /; \text{FreeQ}[\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$$
Rule 2279

$$\text{Int}[\text{Log}[(a_{.}) + (b_{.}) * ((F_{.})^{((e_{.}) * ((c_{.}) + (d_{.})(x_{.})))})^{(n_{.})}], x_{\text{Symbol}}] \text{ :> Dist}[1/(d e^n \text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b x] / x, x], x, (F^{(e(c + dx))})^n], x] /; \text{FreeQ}[\{F, a, b, c, d, e, n\}, x] \&\& \text{GtQ}[a, 0]$$
Rule 2391

$$\text{Int}[\text{Log}[(c_{.}) * ((d_{.}) + (e_{.})(x_{.})^{(n_{.})})] / (x_{.}), x_{\text{Symbol}}] \text{ :> -Simp}[\text{PolyLog}[2, -(c e x^n)] / n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \&\& \text{EqQ}[c d, 1]$$
Rule 5567

$$\text{Int}[(((e_{.}) + (f_{.})(x_{.}))^{(m_{.})} \text{Tanh}[(c_{.}) + (d_{.})(x_{.})]^{(n_{.})}) / ((a_{.}) + (b_{.}) \text{Sinh}[(c_{.}) + (d_{.})(x_{.})]), x_{\text{Symbol}}] \text{ :> Dist}[1/b, \text{Int}[(e + f x)^m \text{Sech}[c + d x] \text{Tanh}[c + d x]^{(n-1)}, x], x] - \text{Dist}[a/b, \text{Int}[(e + f x)^m \text{Sech}[c + d x] \text{Tanh}[c + d x]^{(n-1)}, x], x]$$

$d*x]*\text{Tanh}[c + d*x]^{(n - 1)}/(a + b*\text{Sinh}[c + d*x]), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 4180

$\text{Int}[\text{csc}[(e_.) + \text{Pi}*(k_.) + (\text{Complex}[0, \text{fz}_])*(f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] := \text{Simp}[(-2*(c + d*x)^m*\text{ArcTanh}[E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}])/(f*fz*I), x] + (-\text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 - E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}], x], x] + \text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}], x], x)] /;$  FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 5573

$\text{Int}[(((e_.) + (f_.)*(x_.))^{(m_.)}*\text{Sech}[(c_.) + (d_.)*(x_.)]^{(n_.)})/((a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_.)]), x\_Symbol] := \text{Dist}[b^2/(a^2 + b^2), \text{Int}[(e + f*x)^m*\text{Sech}[c + d*x]^{(n - 2)}]/(a + b*\text{Sinh}[c + d*x]), x], x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(e + f*x)^m*\text{Sech}[c + d*x]^n*(a - b*\text{Sinh}[c + d*x]), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]

### Rule 5561

$\text{Int}[(\text{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))^{(m_.)})/((a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_.)]), x\_Symbol] := -\text{Simp}[(e + f*x)^{(m + 1)}/(b*f*(m + 1)), x] + (\text{Int}[(e + f*x)^m*E^{(c + d*x)}/(a - \text{Rt}[a^2 + b^2, 2] + b*E^{(c + d*x)}), x] + \text{Int}[(e + f*x)^m*E^{(c + d*x)}/(a + \text{Rt}[a^2 + b^2, 2] + b*E^{(c + d*x)}), x]) /;$  FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

### Rule 6742

$\text{Int}[u_, x\_Symbol] := \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /;$  SumQ[v]

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx) \sinh(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx) \tanh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= -\frac{(e+fx)^2}{2bf} - \frac{a \int (e+fx) \operatorname{sech}(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} + \frac{2 \int \frac{e^2}{a+b \sinh(c+dx)} dx}{b^2} \\
&= -\frac{(e+fx)^2}{2bf} - \frac{2a(e+fx) \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{(e+fx) \log(1+e^{2(c+dx)})}{bd} + \frac{a^2 \int \frac{e^2}{a+b \sinh(c+dx)} dx}{b^2} \\
&= -\frac{(e+fx)^2}{2bf} - \frac{a^2(e+fx)^2}{2b(a^2+b^2)f} - \frac{2a(e+fx) \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{(e+fx) \log(1+e^{2(c+dx)})}{bd} \\
&= -\frac{(e+fx)^2}{2bf} - \frac{a^2(e+fx)^2}{2b(a^2+b^2)f} - \frac{2a(e+fx) \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{a^2(e+fx) \log(1+e^{2(c+dx)})}{b(a^2+b^2)} \\
&= -\frac{(e+fx)^2}{2bf} - \frac{2a(e+fx) \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx) \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d} + \frac{a^2(e+fx) \log(1+e^{2(c+dx)})}{b(a^2+b^2)} \\
&= -\frac{(e+fx)^2}{2bf} - \frac{2a(e+fx) \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx) \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d} + \frac{a^2(e+fx) \log(1+e^{2(c+dx)})}{b(a^2+b^2)} \\
&= -\frac{(e+fx)^2}{2bf} - \frac{2a(e+fx) \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx) \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d} + \frac{a^2(e+fx) \log(1+e^{2(c+dx)})}{b(a^2+b^2)} \\
&= -\frac{(e+fx)^2}{2bf} - \frac{2a(e+fx) \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx) \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d} + \frac{a^2(e+fx) \log(1+e^{2(c+dx)})}{b(a^2+b^2)}
\end{aligned}$$

**Mathematica [A]** time = 2.87536, size = 438, normalized size = 0.85

$$\frac{a^2 \left( f \operatorname{PolyLog} \left( 2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a} \right) + f \operatorname{PolyLog} \left( 2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} \right) + f(c+dx) \log \left( \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1 \right) + f(c+dx) \log \left( \frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} + 1 \right) + de \log(a+b \sinh(c+dx)) - cf \log(a+b \sinh(c+dx)) \right)}{b}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)\*Sinh[c + d\*x]\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out]  $(-(b*d*e*(c + d*x)) + b*c*f*(c + d*x) - (b*f*(c + d*x)^2)/2 - 2*a*d*e*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] + 2*a*c*f*ArcTan[Cosh[c + d*x] + Sinh[c +$

$$\begin{aligned}
& d*x]] - 2*a*f*(c + d*x)*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] + b*d*e*Log[1 \\
& + Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] - b*c*f*Log[1 + Cosh[2*(c + d*x)] \\
& + Sinh[2*(c + d*x)]] + b*f*(c + d*x)*Log[1 + Cosh[2*(c + d*x)] + Sinh[2*(c \\
& + d*x)]] + (a^2*(-f*(c + d*x)^2/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/ \\
& (a - Sqrt[a^2 + b^2])]) + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 \\
& + b^2])] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f* \\
& PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -(b*E^(c \\
& + d*x))/(a + Sqrt[a^2 + b^2])])]/b + I*a*f*PolyLog[2, (-I)*(Cosh[c + d*x] \\
& + Sinh[c + d*x])] - I*a*f*PolyLog[2, I*(Cosh[c + d*x] + Sinh[c + d*x])] + \\
& (b*f*PolyLog[2, -Cosh[2*(c + d*x)] - Sinh[2*(c + d*x)]]/2)/((a^2 + b^2)*d^ \\
& 2)
\end{aligned}$$

**Maple [B]** time = 0.225, size = 3882, normalized size = 7.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)\*sinh(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] 
$$\begin{aligned}
& -1/2*f*x^2/b-1/d^2/b*f*c^2-2/d/b*e*\ln(\exp(d*x+c))+2/d^2*f/(2*a^2+2*b^2)*\ln( \\
& 1+I*\exp(d*x+c))*b*c+2/d*f/(2*a^2+2*b^2)*\ln(1-I*\exp(d*x+c))*b*x+2/d^2*f/(2*a \\
& ^2+2*b^2)*\ln(1-I*\exp(d*x+c))*b*c-1/d*f*b/(2*a^2+2*b^2)*\ln((-b*\exp(d*x+c)+(a \\
& ^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*x-1/d^2*f*b/(2*a^2+2*b^2)*\ln((-b*\exp \\
& (d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*c-1/d*f*b/(2*a^2+2*b^2)*\ln \\
& ((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*x-1/d^2*f*b/(2*a^2+2 \\
& *b^2)*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*c-2/d^2*f*c/ \\
& (2*a^2+2*b^2)*b*\ln(1+\exp(2*d*x+2*c))+4/d^2*f*c/(2*a^2+2*b^2)*a*\arctan(\exp(d \\
& *x+c))+1/d^2*f*c*b/(2*a^2+2*b^2)*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)+2/d* \\
& f/(2*a^2+2*b^2)*\ln(1+I*\exp(d*x+c))*b*x+e*x/b-2/(a^2+b^2)^{(1/2)}/d^2*a*b*f*c/ \\
& (2*a^2+2*b^2)*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}+2*b/d^2*a*f \\
& /(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+ \\
& b^2)^{(1/2)}))*c-2/b/d^2*a^3*f*c/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2 \\
& *b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}-2/b/d^2*a*f*c/(2*a^2+2*b^2)*(a^2+b^2)^{( \\
& 1/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}+2/b/d*a^3*f/(2*a^2+2 \\
& *b^2)/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2 \\
& )))*x+2/b/d^2*a^3*f/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2 \\
& )^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*c-2/b/d*a^3*f/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2) \\
& }*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*x-2/b/d^2*a^3*f \\
& /(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^ \\
& 2+b^2)^{(1/2)}))*c-2*b/d*a*f/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+ \\
& (a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*x-2*b/d^2*a*f/(2*a^2+2*b^2)/(a^2+b
\end{aligned}$$

$$\begin{aligned}
& ^2)^{(1/2)} * \ln((-b * \exp(dx+c) + (a^2+b^2)^{(1/2)} - a) / (-a + (a^2+b^2)^{(1/2)})) * c + 2*b / \\
& d * a * f / (2 * a^2 + 2 * b^2) / (a^2+b^2)^{(1/2)} * \ln((b * \exp(dx+c) + (a^2+b^2)^{(1/2)} + a) / (a + \\
& (a^2+b^2)^{(1/2)})) * x + 2 / (a^2+b^2)^{(1/2)} / d * a * b * e / (2 * a^2 + 2 * b^2) * \operatorname{arctanh}(1/2 * (2 * \\
& b * \exp(dx+c) + 2 * a) / (a^2+b^2)^{(1/2)}) - 2 / d / b * f * c * x + 2 / d^2 / b * f * c * \ln(\exp(dx+c)) + 1 \\
& / 2 * b / d^2 * f / (a^2+b^2) * \ln((b * \exp(dx+c) + (a^2+b^2)^{(1/2)} + a) / (a + (a^2+b^2)^{(1/2)} \\
& )) * c - b / d^2 * f / (a^2+b^2)^{(3/2)} * \operatorname{dilog}((b * \exp(dx+c) + (a^2+b^2)^{(1/2)} + a) / (a + (a^2 \\
& + b^2)^{(1/2)})) * a + b / d^2 * f / (a^2+b^2)^{(3/2)} * \operatorname{dilog}((-b * \exp(dx+c) + (a^2+b^2)^{(1/2)} \\
& ) - a) / (-a + (a^2+b^2)^{(1/2)})) * a - 1/2 * b / d^2 * f * c / (a^2+b^2) * \ln(b * \exp(2 * dx + 2 * c) + 2 * \\
& a * \exp(dx+c) - b) + 1/2 * b / d * f / (a^2+b^2) * \ln((-b * \exp(dx+c) + (a^2+b^2)^{(1/2)} - a) / (- \\
& a + (a^2+b^2)^{(1/2)})) * x + 1/2 * b / d^2 * f / (a^2+b^2) * \ln((-b * \exp(dx+c) + (a^2+b^2)^{(1/2)} \\
& ) - a) / (-a + (a^2+b^2)^{(1/2)})) * c + 1/b / d * e / (a^2+b^2) * \ln(b * \exp(2 * dx + 2 * c) + 2 * a * \exp \\
& (dx+c) - b) * a^2 - 2 * I / d^2 * a * f / (2 * a^2 + 2 * b^2) * \operatorname{dilog}(1 - I * \exp(dx+c)) + 2 * I / d^2 * a * f / \\
& (2 * a^2 + 2 * b^2) * \operatorname{dilog}(1 + I * \exp(dx+c)) - 2 / b / d * e / (a^2+b^2)^{(3/2)} * \operatorname{arctanh}(1/2 * (2 * \\
& b * \exp(dx+c) + 2 * a) / (a^2+b^2)^{(1/2)}) * a^3 - 2 * b / d * e / (a^2+b^2)^{(3/2)} * \operatorname{arctanh}(1/2 * \\
& (2 * b * \exp(dx+c) + 2 * a) / (a^2+b^2)^{(1/2)}) * a + 1/b / d^2 * f / (a^2+b^2) * \operatorname{dilog}((b * \exp(dx \\
& + c) + (a^2+b^2)^{(1/2)} + a) / (a + (a^2+b^2)^{(1/2)})) * a^2 + 1/b / d^2 * f / (a^2+b^2) * \operatorname{dilog} \\
& ((-b * \exp(dx+c) + (a^2+b^2)^{(1/2)} - a) / (-a + (a^2+b^2)^{(1/2)})) * a^2 - 1/b / d^2 * f / (a^2 + \\
& b^2)^{(3/2)} * \operatorname{dilog}((b * \exp(dx+c) + (a^2+b^2)^{(1/2)} + a) / (a + (a^2+b^2)^{(1/2)})) * a^3 + \\
& 1/b / d^2 * f / (a^2+b^2)^{(3/2)} * \operatorname{dilog}((-b * \exp(dx+c) + (a^2+b^2)^{(1/2)} - a) / (-a + (a^2 + \\
& b^2)^{(1/2)})) * a^3 + 1/2 * b / d * f / (a^2+b^2) * \ln((b * \exp(dx+c) + (a^2+b^2)^{(1/2)} + a) / (a \\
& + (a^2+b^2)^{(1/2)})) * x + 2 / d * e / (2 * a^2 + 2 * b^2) * b * \ln(1 + \exp(2 * dx + 2 * c)) - 4 / d * e / (2 * a^ \\
& 2 + 2 * b^2) * a * \operatorname{arctan}(\exp(dx+c)) - 1 / d * e * b / (2 * a^2 + 2 * b^2) * \ln(b * \exp(2 * dx + 2 * c) + 2 * a \\
& * \exp(dx+c) - b) + 2 / d^2 * f / (2 * a^2 + 2 * b^2) * \operatorname{dilog}(1 + I * \exp(dx+c)) * b + 2 / d^2 * f / (2 * a^2 \\
& + 2 * b^2) * \operatorname{dilog}(1 - I * \exp(dx+c)) * b - 1 / d^2 * f * b / (2 * a^2 + 2 * b^2) * \operatorname{dilog}((-b * \exp(dx+c \\
& ) + (a^2+b^2)^{(1/2)} - a) / (-a + (a^2+b^2)^{(1/2)})) - 1 / d^2 * f * b / (2 * a^2 + 2 * b^2) * \operatorname{dilog}((b \\
& * \exp(dx+c) + (a^2+b^2)^{(1/2)} + a) / (a + (a^2+b^2)^{(1/2)})) + 1/2 * b / d * e / (a^2+b^2) * \ln \\
& (b * \exp(2 * dx + 2 * c) + 2 * a * \exp(dx+c) - b) + 1/2 * b / d^2 * f / (a^2+b^2) * \operatorname{dilog}((b * \exp(dx+c \\
& ) + (a^2+b^2)^{(1/2)} + a) / (a + (a^2+b^2)^{(1/2)})) + 1/2 * b / d^2 * f / (a^2+b^2) * \operatorname{dilog}((-b * e \\
& xp(dx+c) + (a^2+b^2)^{(1/2)} - a) / (-a + (a^2+b^2)^{(1/2)})) + 2 * b / d^2 * a * f / (2 * a^2 + 2 * b^2 \\
& ) / (a^2+b^2)^{(1/2)} * \operatorname{dilog}((b * \exp(dx+c) + (a^2+b^2)^{(1/2)} + a) / (a + (a^2+b^2)^{(1/2)} \\
& )) - 2 * b / d^2 * a * f / (2 * a^2 + 2 * b^2) / (a^2+b^2)^{(1/2)} * \operatorname{dilog}((-b * \exp(dx+c) + (a^2+b^2) \\
& ^{(1/2)} - a) / (-a + (a^2+b^2)^{(1/2)})) + 2 * b / d^2 * f * c / (a^2+b^2)^{(3/2)} * \operatorname{arctanh}(1/2 * (2 * \\
& b * \exp(dx+c) + 2 * a) / (a^2+b^2)^{(1/2)}) * a - 2 / b / d^2 * a^3 * f / (2 * a^2 + 2 * b^2) / (a^2+b^2)^ \\
& ^{(1/2)} * \operatorname{dilog}((-b * \exp(dx+c) + (a^2+b^2)^{(1/2)} - a) / (-a + (a^2+b^2)^{(1/2)})) + 2 / b / d^2 \\
& * f * c / (a^2+b^2)^{(3/2)} * \operatorname{arctanh}(1/2 * (2 * b * \exp(dx+c) + 2 * a) / (a^2+b^2)^{(1/2)}) * a^3 - \\
& 1/b / d^2 * f * c / (a^2+b^2) * \ln(b * \exp(2 * dx + 2 * c) + 2 * a * \exp(dx+c) - b) * a^2 + 2 / b / d^2 * a^3 \\
& * f / (2 * a^2 + 2 * b^2) / (a^2+b^2)^{(1/2)} * \operatorname{dilog}((b * \exp(dx+c) + (a^2+b^2)^{(1/2)} + a) / (a + \\
& (a^2+b^2)^{(1/2)})) + 1/b / d^2 * f / (a^2+b^2)^{(3/2)} * \ln((-b * \exp(dx+c) + (a^2+b^2)^{(1/2)} \\
& ) - a) / (-a + (a^2+b^2)^{(1/2)})) * a^3 * c + 1/b / d * f / (a^2+b^2) * \ln((-b * \exp(dx+c) + (a^2 + \\
& b^2)^{(1/2)} - a) / (-a + (a^2+b^2)^{(1/2)})) * a^2 * x + 1/b / d^2 * f / (a^2+b^2) * \ln((-b * \exp(dx \\
& + c) + (a^2+b^2)^{(1/2)} - a) / (-a + (a^2+b^2)^{(1/2)})) * a^2 * c + 1/b / d * f / (a^2+b^2) * \ln((b \\
& * \exp(dx+c) + (a^2+b^2)^{(1/2)} + a) / (a + (a^2+b^2)^{(1/2)})) * a^2 * x + 1/b / d^2 * f / (a^2+b^ \\
& 2) * \ln((b * \exp(dx+c) + (a^2+b^2)^{(1/2)} + a) / (a + (a^2+b^2)^{(1/2)})) * a^2 * c - 1/b / d * f / ( \\
& a^2+b^2)^{(3/2)} * \ln((b * \exp(dx+c) + (a^2+b^2)^{(1/2)} + a) / (a + (a^2+b^2)^{(1/2)})) * a^3 \\
& * x - 1/b / d^2 * f / (a^2+b^2)^{(3/2)} * \ln((b * \exp(dx+c) + (a^2+b^2)^{(1/2)} + a) / (a + (a^2+b^
\end{aligned}$$

$$2)^{(1/2)}) * a^3 * c + b/d * f / (a^2 + b^2)^{(3/2)} * \ln((-b * \exp(dx+c) + (a^2 + b^2)^{(1/2)} - a) / (-a + (a^2 + b^2)^{(1/2)})) * a * x + b/d^2 * f / (a^2 + b^2)^{(3/2)} * \ln((-b * \exp(dx+c) + (a^2 + b^2)^{(1/2)} - a) / (-a + (a^2 + b^2)^{(1/2)})) * a * c - b/d * f / (a^2 + b^2)^{(3/2)} * \ln((b * \exp(dx+c) + (a^2 + b^2)^{(1/2)} + a) / (a + (a^2 + b^2)^{(1/2)})) * a * x - b/d^2 * f / (a^2 + b^2)^{(3/2)} * \ln((b * \exp(dx+c) + (a^2 + b^2)^{(1/2)} + a) / (a + (a^2 + b^2)^{(1/2)})) * a * c + 2/b/d * a * e / (2 * a^2 + 2 * b^2) * (a^2 + b^2)^{(1/2)} * \operatorname{arctanh}(1/2 * (2 * b * \exp(dx+c) + 2 * a) / (a^2 + b^2)^{(1/2)}) + 2/b/d * a^3 * e / (2 * a^2 + 2 * b^2) / (a^2 + b^2)^{(1/2)} * \operatorname{arctanh}(1/2 * (2 * b * \exp(dx+c) + 2 * a) / (a^2 + b^2)^{(1/2)}) + 1/b/d * f / (a^2 + b^2)^{(3/2)} * \ln((-b * \exp(dx+c) + (a^2 + b^2)^{(1/2)} - a) / (-a + (a^2 + b^2)^{(1/2)})) * a^3 * x - 2 * I/d^2 * a * f / (2 * a^2 + 2 * b^2) * \ln(1 - I * \exp(dx+c)) * c + 2 * I/d^2 * a * f / (2 * a^2 + 2 * b^2) * \ln(1 + I * \exp(dx+c)) * c - 2 * I/d * a * f / (2 * a^2 + 2 * b^2) * \ln(1 - I * \exp(dx+c)) * x + 2 * I/d * a * f / (2 * a^2 + 2 * b^2) * \ln(1 + I * \exp(dx+c)) * x$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} f \left( \frac{x^2}{b} - \int - \frac{4(a^3 x e^{dx+c} - a^2 b x)}{a^2 b^2 + b^4 - (a^2 b^2 e^{2c} + b^4 e^{2c}) e^{2dx} - 2(a^3 b e^c + a b^3 e^c) e^{dx}} dx - \int \frac{4(ax e^{dx+c} + bx)}{a^2 + b^2 + (a^2 e^{2c} + b^2 e^{2c}) e^{2dx}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 1/2\*f\*(x^2/b - integrate(-4\*(a^3\*x\*e^(d\*x + c) - a^2\*b\*x)/(a^2\*b^2 + b^4 - (a^2\*b^2\*e^(2\*c) + b^4\*e^(2\*c))\*e^(2\*d\*x) - 2\*(a^3\*b\*e^c + a\*b^3\*e^c)\*e^(d\*x)), x) - integrate(4\*(a\*x\*e^(d\*x + c) + b\*x)/(a^2 + b^2 + (a^2\*e^(2\*c) + b^2\*e^(2\*c))\*e^(2\*d\*x)), x) + e\*(a^2\*log(-2\*a\*e^(-d\*x - c) + b\*e^(-2\*d\*x - 2\*c) - b)/((a^2\*b + b^3)\*d) + 2\*a\*arctan(e^(-d\*x - c))/((a^2 + b^2)\*d) + b\*log(e^(-2\*d\*x - 2\*c) + 1)/((a^2 + b^2)\*d) + (d\*x + c)/(b\*d))

**Fricas [A]** time = 2.5035, size = 1767, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] -1/2\*((a^2 + b^2)\*d^2\*f\*x^2 + 2\*(a^2 + b^2)\*d^2\*e\*x - 2\*a^2\*f\*dilog((a\*cosh(d\*x + c) + a\*sinh(d\*x + c) + (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2

$$\begin{aligned}
& + b^2/b^2) - b)/b + 1) - 2a^2f \operatorname{dilog}((a \cosh(dx + c) + a \sinh(dx + c) \\
& - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + \\
& 2(Iabf - b^2f) \operatorname{dilog}(I \cosh(dx + c) + I \sinh(dx + c)) + 2(-Iabf \\
& - b^2f) \operatorname{dilog}(-I \cosh(dx + c) - I \sinh(dx + c)) - 2(a^2de - a^2cf) \log \\
& \log(2b \cosh(dx + c) + 2b \sinh(dx + c) + 2b \sqrt{(a^2 + b^2)/b^2} + 2a \\
& ) - 2(a^2de - a^2cf) \log(2b \cosh(dx + c) + 2b \sinh(dx + c) - 2b \sqrt{(a^2 + b^2)/b^2} + 2a) \\
& - 2(a^2dfx + a^2cf) \log(-(a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b) - 2(a^2dfx + a^2cf) \log(-(a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b) - (-2Iabd + 2b^2de + 2Iabc - 2b^2cf) \log(\cosh(dx + c) + \sinh(dx + c) + I) - (2Iabd + 2b^2de - 2Iabc - 2b^2cf) \log(\cosh(dx + c) + \sinh(dx + c) - I) - (2Iabd + 2b^2de + 2Iabc + 2b^2cf) \log(I \cosh(dx + c) + I \sinh(dx + c) + 1) - (-2Iabd + 2b^2de - 2Iabc + 2b^2cf) \log(-I \cosh(dx + c) - I \sinh(dx + c) + 1)) / ((a^2b + b^3)d^2)
\end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \sinh(c + dx) \tanh(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*sinh(c + d\*x)\*tanh(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.380 \quad \int \frac{\sinh(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=74

$$\frac{a^2 \log(a + b \sinh(c + dx))}{bd(a^2 + b^2)} - \frac{a \tan^{-1}(\sinh(c + dx))}{d(a^2 + b^2)} + \frac{b \log(\cosh(c + dx))}{d(a^2 + b^2)}$$

[Out]  $-\left(\frac{a \operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]]}{(a^2 + b^2)*d}\right) + \left(\frac{b \operatorname{Log}[\operatorname{Cosh}[c + d*x]]}{(a^2 + b^2)*d}\right) + \left(\frac{a^2 \operatorname{Log}[a + b \operatorname{Sinh}[c + d*x]]}{b*(a^2 + b^2)*d}\right)$

**Rubi [A]** time = 0.159761, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$ , Rules used = {2837, 12, 1629, 635, 203, 260}

$$\frac{a^2 \log(a + b \sinh(c + dx))}{bd(a^2 + b^2)} - \frac{a \tan^{-1}(\sinh(c + dx))}{d(a^2 + b^2)} + \frac{b \log(\cosh(c + dx))}{d(a^2 + b^2)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Sinh}[c + d*x]*\operatorname{Tanh}[c + d*x])/(a + b*\operatorname{Sinh}[c + d*x]), x]$

[Out]  $-\left(\frac{a \operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]]}{(a^2 + b^2)*d}\right) + \left(\frac{b \operatorname{Log}[\operatorname{Cosh}[c + d*x]]}{(a^2 + b^2)*d}\right) + \left(\frac{a^2 \operatorname{Log}[a + b \operatorname{Sinh}[c + d*x]]}{b*(a^2 + b^2)*d}\right)$

### Rule 2837

$\operatorname{Int}[\cos[(e_.) + (f_.)*(x_.)]^{(p_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(m_.)})^{((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)})}, x\_Symbol] \rightarrow \operatorname{Dist}[1/(b^p f), \operatorname{Subst}[\operatorname{Int}[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^{(p-1)/2}], x], x, b*\operatorname{Sin}[e + f*x], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f, m, n\}, x \&\& \operatorname{IntegerQ}[(p-1)/2] \&\& \operatorname{NeQ}[a^2 - b^2, 0]$

### Rule 12

$\operatorname{Int}[(a_)*(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$   $\operatorname{FreeQ}[a, x] \&\& \operatorname{!MatchQ}[u, (b_)*(v_)] /;$   $\operatorname{FreeQ}[b, x]$

### Rule 1629

$\operatorname{Int}[(Pq_)*((d_) + (e_.)*(x_.))^{(m_.)}*((a_.) + (c_.)*(x_.)^2)^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /;$   $\operatorname{FreeQ}\{a, c,$



d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

### Rule 635

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

### Rule 203

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[Rt[b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 260

Int[(x\_)^(m\_)/((a\_) + (b\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{\sinh(c + dx) \tanh(c + dx)}{a + b \sinh(c + dx)} dx &= -\frac{b \operatorname{Subst}\left(\int \frac{x^2}{b^2(a+x)(-b^2-x^2)} dx, x, b \sinh(c + dx)\right)}{d} \\
 &= -\frac{\operatorname{Subst}\left(\int \frac{x^2}{(a+x)(-b^2-x^2)} dx, x, b \sinh(c + dx)\right)}{bd} \\
 &= -\frac{\operatorname{Subst}\left(\int \left(-\frac{a^2}{(a^2+b^2)(a+x)} + \frac{b^2(a-x)}{(a^2+b^2)(b^2+x^2)}\right) dx, x, b \sinh(c + dx)\right)}{bd} \\
 &= \frac{a^2 \log(a + b \sinh(c + dx))}{b(a^2 + b^2)d} - \frac{b \operatorname{Subst}\left(\int \frac{a-x}{b^2+x^2} dx, x, b \sinh(c + dx)\right)}{(a^2 + b^2)d} \\
 &= \frac{a^2 \log(a + b \sinh(c + dx))}{b(a^2 + b^2)d} + \frac{b \operatorname{Subst}\left(\int \frac{x}{b^2+x^2} dx, x, b \sinh(c + dx)\right)}{(a^2 + b^2)d} - \frac{(ab) \operatorname{Subst}\left(\int \frac{1}{b^2+x^2} dx, x, b \sinh(c + dx)\right)}{(a^2 + b^2)d} \\
 &= -\frac{a \tan^{-1}(\sinh(c + dx))}{(a^2 + b^2)d} + \frac{b \log(\cosh(c + dx))}{(a^2 + b^2)d} + \frac{a^2 \log(a + b \sinh(c + dx))}{b(a^2 + b^2)d}
 \end{aligned}$$

**Mathematica [C]** time = 0.085892, size = 78, normalized size = 1.05

$$\frac{2a^2 \log(a + b \sinh(c + dx)) + b(b + ia) \log(-\sinh(c + dx) + i) + b(b - ia) \log(\sinh(c + dx) + i)}{2bd(a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sinh[c + d\*x]\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (b\*(I\*a + b)\*Log[I - Sinh[c + d\*x]] + b\*((-I)\*a + b)\*Log[I + Sinh[c + d\*x]] + 2\*a^2\*Log[a + b\*Sinh[c + d\*x]])/(2\*b\*(a^2 + b^2)\*d)

**Maple [B]** time = 0.001, size = 153, normalized size = 2.1

$$-\frac{1}{bd} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) - \frac{1}{bd} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + \frac{a^2}{bd(a^2 + b^2)} \ln\left(\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2 a - 2 \tanh\left(\frac{1}{2} dx + \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] -1/d/b\*ln(tanh(1/2\*d\*x+1/2\*c)+1)-1/d/b\*ln(tanh(1/2\*d\*x+1/2\*c)-1)+1/d\*a^2/b/(a^2+b^2)\*ln(tanh(1/2\*d\*x+1/2\*c)^2\*a-2\*tanh(1/2\*d\*x+1/2\*c)\*b-a)+4/d/(4\*a^2+4\*b^2)\*b\*ln(tanh(1/2\*d\*x+1/2\*c)^2+1)-8/d/(4\*a^2+4\*b^2)\*a\*arctan(tanh(1/2\*d\*x+1/2\*c))

**Maxima [A]** time = 1.62767, size = 149, normalized size = 2.01

$$\frac{a^2 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{(a^2b + b^3)d} + \frac{2a \arctan(e^{(-dx-c)})}{(a^2 + b^2)d} + \frac{b \log(e^{(-2dx-2c)} + 1)}{(a^2 + b^2)d} + \frac{dx + c}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] a^2\*log(-2\*a\*e^(-d\*x - c) + b\*e^(-2\*d\*x - 2\*c) - b)/((a^2\*b + b^3)\*d) + 2\*a\*arctan(e^(-d\*x - c))/((a^2 + b^2)\*d) + b\*log(e^(-2\*d\*x - 2\*c) + 1)/((a^2 +

$$b^2*d) + (d*x + c)/(b*d)$$

**Fricas [A]** time = 2.30851, size = 284, normalized size = 3.84

$$\frac{(a^2 + b^2)dx + 2ab \arctan(\cosh(dx + c) + \sinh(dx + c)) - a^2 \log\left(\frac{2(b \sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right) - b^2 \log\left(\frac{2 \cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{(a^2b + b^3)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] -((a^2 + b^2)\*d\*x + 2\*a\*b\*arctan(cosh(d\*x + c) + sinh(d\*x + c)) - a^2\*log(2\*(b\*sinh(d\*x + c) + a)/(cosh(d\*x + c) - sinh(d\*x + c))) - b^2\*log(2\*cosh(d\*x + c)/(cosh(d\*x + c) - sinh(d\*x + c))))/((a^2\*b + b^3)\*d)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(c + dx) \tanh(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(sinh(c + d\*x)\*tanh(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

**Giac [A]** time = 1.52435, size = 128, normalized size = 1.73

$$\frac{\frac{a^2 \log(|be^{(2dx+2c)}+2ae^{(dx+c)}-b|)}{a^2b+b^3} - \frac{dx}{b} - \frac{2a \arctan(e^{(dx+c)})}{a^2+b^2} + \frac{b \log(e^{(2dx+2c)}+1)}{a^2+b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

```
[Out] (a^2*log(abs(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - b))/(a^2*b + b^3) - d*x/  
b - 2*a*arctan(e^(d*x + c))/(a^2 + b^2) + b*log(e^(2*d*x + 2*c) + 1)/(a^2 +  
b^2))/d
```

$$3.381 \quad \int \frac{\sinh(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable} \left( \frac{\sinh(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[(Sinh[c + d\*x]\*Tanh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0598635, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sinh(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Sinh[c + d\*x]\*Tanh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int] [(Sinh[c + d\*x]\*Tanh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\sinh(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\sinh(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.001, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Sinh[c + d\*x]\*Tanh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] \$Aborted

**Maple [A]** time = 0.421, size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)\tanh(dx+c)}{(fx+e)(a+b\sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d\*x+c)\*tanh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(sinh(d\*x+c)\*tanh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{\log(fx+e)}{bf} - \frac{1}{2} \int -\frac{4(a^3e^{dx+c} - a^2b)}{a^2b^2e + b^4e + (a^2b^2f + b^4f)x - (a^2b^2ee^{2c} + b^4ee^{2c}) + (a^2b^2fe^{2c} + b^4fe^{2c})x} e^{2dx} - 2(a^3bee^c -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)\*tanh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] log(f\*x + e)/(b\*f) - 1/2\*integrate(-4\*(a^3\*e^(d\*x + c) - a^2\*b)/(a^2\*b^2\*e + b^4\*e + (a^2\*b^2\*f + b^4\*f)\*x - (a^2\*b^2\*e\*e^(2\*c) + b^4\*e\*e^(2\*c) + (a^2\*b^2\*f\*e^(2\*c) + b^4\*f\*e^(2\*c))\*x)\*e^(2\*d\*x) - 2\*(a^3\*b\*e\*e^c + a\*b^3\*e\*e^c + (a^3\*b\*f\*e^c + a\*b^3\*f\*e^c)\*x)\*e^(d\*x)), x) - 1/2\*integrate(4\*(a\*e^(d\*x + c) + b)/(a^2\*e + b^2\*e + (a^2\*f + b^2\*f)\*x + (a^2\*e\*e^(2\*c) + b^2\*e\*e^(2\*c) + (a^2\*f\*e^(2\*c) + b^2\*f\*e^(2\*c))\*x)\*e^(2\*d\*x)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh(dx+c)\tanh(dx+c)}{afx+ae+(bfx+be)\sinh(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sinh(d*x + c)*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(c + dx) \tanh(c + dx)}{(a + b \sinh(c + dx))(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)*tanh(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.382 \quad \int \frac{(e+fx)^3 \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=1118

result too large to display

```
[Out] -((a*(e + f*x)^3)/(b^2*d)) + (a^3*(e + f*x)^3)/(b^2*(a^2 + b^2)*d) + (6*f*(
e + f*x)^2*ArcTan[E^(c + d*x)]/(b*d^2) - (6*a^2*f*(e + f*x)^2*ArcTan[E^(c
+ d*x)]/(b*(a^2 + b^2)*d^2) + (a^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*d) - (a^2*(e + f*x)^3*Log[1 + (b*E^
(c + d*x))/(a + Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*d) + (3*a*f*(e + f*x)
^2*Log[1 + E^(2*(c + d*x))]/(b^2*d^2) - (3*a^3*f*(e + f*x)^2*Log[1 + E^(2*
(c + d*x))]/(b^2*(a^2 + b^2)*d^2) - ((6*I)*f^2*(e + f*x)*PolyLog[2, (-I)*E
^(c + d*x)]/(b*d^3) + ((6*I)*a^2*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)
]/(b*(a^2 + b^2)*d^3) + ((6*I)*f^2*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/(b
*d^3) - ((6*I)*a^2*f^2*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/(b*(a^2 + b^2)*
d^3) + (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2]))]/((a^2 + b^2)^(3/2)*d^2) - (3*a^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 + b^2)^(3/2)*d^2) + (3*a*f^2*(e + f*
x)*PolyLog[2, -E^(2*(c + d*x))]/(b^2*d^3) - (3*a^3*f^2*(e + f*x)*PolyLog[2
, -E^(2*(c + d*x))]/(b^2*(a^2 + b^2)*d^3) + ((6*I)*f^3*PolyLog[3, (-I)*E^(
c + d*x)]/(b*d^4) - ((6*I)*a^2*f^3*PolyLog[3, (-I)*E^(c + d*x)]/(b*(a^2 +
b^2)*d^4) - ((6*I)*f^3*PolyLog[3, I*E^(c + d*x)]/(b*d^4) + ((6*I)*a^2*f^3
*PolyLog[3, I*E^(c + d*x)]/(b*(a^2 + b^2)*d^4) - (6*a^2*f^2*(e + f*x)*Poly
Log[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)^(3/2)*d^3) +
(6*a^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/
((a^2 + b^2)^(3/2)*d^3) - (3*a*f^3*PolyLog[3, -E^(2*(c + d*x))]/(2*b^2*d^
4) + (3*a^3*f^3*PolyLog[3, -E^(2*(c + d*x))]/(2*b^2*(a^2 + b^2)*d^4) + (6*
a^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)^
(3/2)*d^4) - (6*a^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])
)]/((a^2 + b^2)^(3/2)*d^4) - ((e + f*x)^3*Sech[c + d*x])/(b*d) + (a^2*(e +
f*x)^3*Sech[c + d*x])/(b*(a^2 + b^2)*d) - (a*(e + f*x)^3*Tanh[c + d*x])/(b^
2*d) + (a^3*(e + f*x)^3*Tanh[c + d*x])/(b^2*(a^2 + b^2)*d)
```

**Rubi [A]** time = 1.98044, antiderivative size = 1118, normalized size of antiderivative = 1., number of steps used = 45, number of rules used = 15, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.536$ , Rules used = {5567, 5451, 4180, 2531, 2282, 6589, 5583, 4184, 3718, 2190, 5573, 3322, 2264, 6609, 6742}

$$\frac{(e+fx)^3 a^3}{b^2(a^2+b^2)d} - \frac{3f(e+fx)^2 \log(1+e^{2(c+dx)}) a^3}{b^2(a^2+b^2)d^2} - \frac{3f^2(e+fx) \text{PolyLog}(2, -e^{2(c+dx)}) a^3}{b^2(a^2+b^2)d^3} + \frac{3f^3 \text{PolyLog}(3, -e^{2(c+dx)}) a^3}{2b^2(a^2+b^2)d^4}$$



Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Tanh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] 
$$-\frac{(a(e+fx)^3)}{b^2d} + \frac{a^3(e+fx)^3}{b^2(a^2+b^2)d} + (6f(e+fx)^2 \operatorname{ArcTan}[E^{c+dx}] / (bd^2) - (6a^2f(e+fx)^2 \operatorname{ArcTan}[E^{c+dx}]] / (b(a^2+b^2)d^2) + (a^2(e+fx)^3 \operatorname{Log}[1 + (bE^{c+dx}) / (a - \sqrt{a^2+b^2})]) / ((a^2+b^2)^{3/2}d) - (a^2(e+fx)^3 \operatorname{Log}[1 + (bE^{c+dx}) / (a + \sqrt{a^2+b^2})]) / ((a^2+b^2)^{3/2}d) + (3af(e+fx)^2 \operatorname{Log}[1 + E^{2(c+dx)}]) / (b^2d^2) - (3a^3f(e+fx)^2 \operatorname{Log}[1 + E^{2(c+dx)}]) / (b^2(a^2+b^2)d^2) - ((6I)f^2(e+fx) \operatorname{PolyLog}[2, (-I)E^{c+dx}]) / (bd^3) + ((6I)a^2f^2(e+fx) \operatorname{PolyLog}[2, (-I)E^{c+dx}]) / (b(a^2+b^2)d^3) + ((6I)f^2(e+fx) \operatorname{PolyLog}[2, IE^{c+dx}]) / (bd^3) - ((6I)a^2f^2(e+fx) \operatorname{PolyLog}[2, IE^{c+dx}]) / (b(a^2+b^2)d^3) + (3a^2f(e+fx)^2 \operatorname{PolyLog}[2, -(bE^{c+dx}) / (a - \sqrt{a^2+b^2})]) / ((a^2+b^2)^{3/2}d^2) - (3a^2f(e+fx)^2 \operatorname{PolyLog}[2, -(bE^{c+dx}) / (a + \sqrt{a^2+b^2})]) / ((a^2+b^2)^{3/2}d^2) + (3af^2(e+fx) \operatorname{PolyLog}[2, -E^{2(c+dx)}]) / (b^2d^3) - (3a^3f^2(e+fx) \operatorname{PolyLog}[2, -E^{2(c+dx)}]) / (b^2(a^2+b^2)d^3) + ((6I)f^3 \operatorname{PolyLog}[3, (-I)E^{c+dx}]) / (bd^4) - ((6I)a^2f^3 \operatorname{PolyLog}[3, (-I)E^{c+dx}]) / (b(a^2+b^2)d^4) - ((6I)f^3 \operatorname{PolyLog}[3, IE^{c+dx}]) / (bd^4) + ((6I)a^2f^3 \operatorname{PolyLog}[3, IE^{c+dx}]) / (b(a^2+b^2)d^4) - (6a^2f^2(e+fx) \operatorname{PolyLog}[3, -(bE^{c+dx}) / (a - \sqrt{a^2+b^2})]) / ((a^2+b^2)^{3/2}d^3) + (6a^2f^2(e+fx) \operatorname{PolyLog}[3, -(bE^{c+dx}) / (a + \sqrt{a^2+b^2})]) / ((a^2+b^2)^{3/2}d^3) - (3af^3 \operatorname{PolyLog}[3, -E^{2(c+dx)}]) / (2b^2d^4) + (3a^3f^3 \operatorname{PolyLog}[3, -E^{2(c+dx)}]) / (2b^2(a^2+b^2)d^4) + (6a^2f^3 \operatorname{PolyLog}[4, -(bE^{c+dx}) / (a - \sqrt{a^2+b^2})]) / ((a^2+b^2)^{3/2}d^4) - (6a^2f^3 \operatorname{PolyLog}[4, -(bE^{c+dx}) / (a + \sqrt{a^2+b^2})]) / ((a^2+b^2)^{3/2}d^4) - ((e+fx)^3 \operatorname{Sech}[c+dx]) / (bd) + (a^2(e+fx)^3 \operatorname{Sech}[c+dx]) / (b(a^2+b^2)d) - (a(e+fx)^3 \operatorname{Tanh}[c+dx]) / (b^2d) + (a^3(e+fx)^3 \operatorname{Tanh}[c+dx]) / (b^2(a^2+b^2)d)$$

### Rule 5567

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Tanh[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Sech[c + d\*x]\*Tanh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sech[c + d\*x]\*Tanh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 5451

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sech[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*Tanh[(a\_.) + (b\_.)\*(x\_)]^(p\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Sech[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Sech[a + b\*x]^n, x], x] /; Fre

$eQ[\{a, b, c, d, n\}, x] \ \&\& \ EqQ[p, 1] \ \&\& \ GtQ[m, 0]$

### Rule 4180

$\text{Int}[\text{csc}[(e_.) + \text{Pi}*(k_.) + (\text{Complex}[0, \text{fz}_.])*(f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{\text{m}_.}, x\_Symbol] \ :> \text{Simp}[(-2*(c + d*x)^{\text{m}}*\text{ArcTanh}[E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}])/(f*fz*I), x] + (-\text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{\text{m} - 1}*\text{Log}[1 - E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}], x], x] + \text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{\text{m} - 1}*\text{Log}[1 + E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}], x], x]) \ /; \text{FreeQ}[\{c, d, e, f, \text{fz}\}, x] \ \&\& \ \text{IntegerQ}[2*k] \ \&\& \ \text{IGtQ}[m, 0]$

### Rule 2531

$\text{Int}[\text{Log}[1 + (e_.)*((F_)^{((c_.)*((a_.) + (b_.)*(x_.)))^{\text{n}_.})}]*((f_.) + (g_.)*(x_.))^{\text{m}_.}, x\_Symbol] \ :> -\text{Simp}[(f + g*x)^{\text{m}}*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x))^{\text{n}})})^{\text{n}}])/(b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{\text{m} - 1}*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x))^{\text{n}})})^{\text{n}}], x], x] \ /; \text{FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \ \&\& \ \text{GtQ}[m, 0]$

### Rule 2282

$\text{Int}[u_, x\_Symbol] \ :> \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] \ /; \text{FunctionOfExponentialQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (w_)*((a_.)*(v_)^{\text{n}_.})^{\text{m}_.}] \ /; \text{FreeQ}[\{a, m, n\}, x] \ \&\& \ \text{IntegerQ}[m*n] \ \&\& \ !\text{MatchQ}[u, E^{((c_.)*((a_.) + (b_.)*x))* (F_)[v_]}] \ /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{InverseFunctionQ}[F[x]]]$

### Rule 6589

$\text{Int}[\text{PolyLog}[n_, (c_.)*((a_.) + (b_.)*(x_.))^{\text{p}_.}]/((d_.) + (e_.)*(x_.)), x\_Symbol] \ :> \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^{\text{p}}]/(e*p), x] \ /; \text{FreeQ}[\{a, b, c, d, e, n, p\}, x] \ \&\& \ \text{EqQ}[b*d, a*e]$

### Rule 5583

$\text{Int}[(((e_.) + (f_.)*(x_.))^{\text{m}_.})*\text{Sech}[(c_.) + (d_.)*(x_.)]^{\text{p}_.}*\text{Tanh}[(c_.) + (d_.)*(x_.)]^{\text{n}_.})/((a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_.)]), x\_Symbol] \ :> \text{Dist}[1/b, \text{Int}[(e + f*x)^{\text{m}}*\text{Sech}[c + d*x]^{\text{p} + 1}*\text{Tanh}[c + d*x]^{\text{n} - 1}, x], x] - \text{Dist}[a/b, \text{Int}[((e + f*x)^{\text{m}}*\text{Sech}[c + d*x]^{\text{p} + 1}*\text{Tanh}[c + d*x]^{\text{n} - 1})/(a + b*\text{Sinh}[c + d*x]), x], x] \ /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$

### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-I*e) + f*fz*x))]/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

### Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 6609

```

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]

```

### Rule 6742

```

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \operatorname{sech}(c+dx) \tanh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^3 \operatorname{sech}(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= -\frac{(e+fx)^3 \operatorname{sech}(c+dx)}{bd} - \frac{a \int (e+fx)^3 \operatorname{sech}^2(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)^3 \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} + \\
&= \frac{6f(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd^2} - \frac{(e+fx)^3 \operatorname{sech}(c+dx)}{bd} - \frac{a(e+fx)^3 \tanh(c+dx)}{b^2 d} + \frac{a^2 \int}{b^2 d} \\
&= -\frac{a(e+fx)^3}{b^2 d} + \frac{6f(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd^2} - \frac{6if^2(e+fx) \operatorname{Li}_2(-ie^{c+dx})}{bd^3} + \frac{6if^2(e+fx) \operatorname{Li}_2(-ie^{c+dx})}{bd^3} \\
&= -\frac{a(e+fx)^3}{b^2 d} + \frac{6f(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd^2} + \frac{3af(e+fx)^2 \log(1+e^{2(c+dx)})}{b^2 d^2} - \frac{6if^2(e+fx) \operatorname{Li}_2(-ie^{c+dx})}{bd^3} \\
&= -\frac{a(e+fx)^3}{b^2 d} + \frac{6f(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd^2} + \frac{a^2(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} - \frac{a^2(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} \\
&= -\frac{a(e+fx)^3}{b^2 d} + \frac{a^3(e+fx)^3}{b^2(a^2+b^2)d} + \frac{6f(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd^2} - \frac{6a^2 f(e+fx)^2 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d^2} \\
&= -\frac{a(e+fx)^3}{b^2 d} + \frac{a^3(e+fx)^3}{b^2(a^2+b^2)d} + \frac{6f(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd^2} - \frac{6a^2 f(e+fx)^2 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d^2} \\
&= -\frac{a(e+fx)^3}{b^2 d} + \frac{a^3(e+fx)^3}{b^2(a^2+b^2)d} + \frac{6f(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd^2} - \frac{6a^2 f(e+fx)^2 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d^2} \\
&= -\frac{a(e+fx)^3}{b^2 d} + \frac{a^3(e+fx)^3}{b^2(a^2+b^2)d} + \frac{6f(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd^2} - \frac{6a^2 f(e+fx)^2 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d^2} \\
&= -\frac{a(e+fx)^3}{b^2 d} + \frac{a^3(e+fx)^3}{b^2(a^2+b^2)d} + \frac{6f(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd^2} - \frac{6a^2 f(e+fx)^2 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d^2} \\
&= -\frac{a(e+fx)^3}{b^2 d} + \frac{a^3(e+fx)^3}{b^2(a^2+b^2)d} + \frac{6f(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd^2} - \frac{6a^2 f(e+fx)^2 \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d^2}
\end{aligned}$$

**Mathematica [A]** time = 13.669, size = 1147, normalized size = 1.03

$$\left(-2e^3 \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) d^3 + f^3 x^3 \log\left(\frac{e^{c+dx} b}{a-\sqrt{a^2+b^2}} + 1\right) d^3 + 3ef^2 x^2 \log\left(\frac{e^{c+dx} b}{a-\sqrt{a^2+b^2}} + 1\right) d^3 + 3e^2 f x \log\left(\frac{e^{c+dx} b}{a-\sqrt{a^2+b^2}} + 1\right) d^3 - \dots\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^3*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (f*(-12*a*d^3*e^2*E^(2*c)*x + 12*a*d^3*e^2*(1 + E^(2*c))*x + 12*a*d^3*e*f*x^2 + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^(2*c))*ArcTan[E^(c + d*x)] - 6*a*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))]) + (12*I)*b*d*e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(c + d*x)]) - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) - 6*a*d*e*(1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2, -E^(2*(c + d*x))]) + (6*I)*b*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*x)] - d^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] + 2*d*x*PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyLog[3, I*E^(c + d*x)]) - a*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^(2*(c + d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))] + 3*PolyLog[3, -E^(2*(c + d*x))]))/(2*(a^2 + b^2)*d^4*(1 + E^(2*c))) + (a^2*(-2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 3*d^2*f*(e + f*x)^2*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 6*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*e*f^2*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 6*d*f^3*x*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 6*f^3*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*f^3*PolyLog[4, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]))/(a^2 + b^2)^(3/2)*d^4) + (Sech[c]*Sech[c + d*x]*(-b*e^3*Cosh[c] - 3*b*e^2*f*x*Cosh[c] - 3*b*e*f^2*x^2*Cosh[c] - b*f^3*x^3*Cosh[c] - a*e^3*Sinh[d*x] - 3*a*e^2*f*x*Sinh[d*x] - 3*a*e*f^2*x^2*Sinh[d*x] - a*f^3*x^3*Sinh[d*x]))/((a^2 + b^2)*d)
```

---

**Maple [F]** time = 0.732, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 (\tanh(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [C]** time = 4.22311, size = 14864, normalized size = 13.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*(a^3 + a*b^2)*d^3*e^3 - 12*(a^3 + a*b^2)*c*d^2*e^2*f + 12*(a^3 + a*b^2)*c^2*d*e*f^2 - 4*(a^3 + a*b^2)*c^3*f^3 - 4*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*cosh(d*x + c)^2 - 4*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*sinh(d*x + c)^2 + 6*(a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f + (a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*cosh(d*x + c)^2 + 2*(a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*sinh(d*x + c) + (a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 6*(a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f + (a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*cosh(d*x + c)^2 + 2*(a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*sinh(d*x + c) + (a^2*b*d^2*f^3*x^2 + 2*a^2*b*d^2*e*f^2*x + a^2*b*d^2*e^2*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3 + (a^2*b*d^3*e^3 - 3*a
```

$$\begin{aligned}
& ^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*\cosh(d*x + c)^2 + 2 \\
& *(a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3 \\
& )*\cosh(d*x + c)*\sinh(d*x + c) + (a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^ \\
& 2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2)*\log \\
& (2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + \\
& 2*(a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f \\
& ^3 + (a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f \\
& ^3)*\cosh(d*x + c)^2 + 2*(a^2*b*d^3*e^3 - 3*a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2 \\
& *d*e*f^2 - a^2*b*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2*b*d^3*e^3 - 3 \\
& *a^2*b*c*d^2*e^2*f + 3*a^2*b*c^2*d*e*f^2 - a^2*b*c^3*f^3)*\sinh(d*x + c)^2)* \\
& \sqrt{(a^2 + b^2)/b^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{( \\
& a^2 + b^2)/b^2} + 2*a) + 2*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3* \\
& a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f \\
& ^3 + (a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a \\
& ^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*\cosh(d*x + c)^2 + 2 \\
& *(a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b \\
& *c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x \\
& + c) + (a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3 \\
& *a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3)*\sinh(d*x + c)^2)* \\
& \sqrt{(a^2 + b^2)/b^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x \\
& + c) + b*\sinh(d*x + c)))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 2*(a^2*b*d^3*f^3*x \\
& ^3 + 3*a^2*b*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3* \\
& a^2*b*c^2*d*e*f^2 + a^2*b*c^3*f^3 + (a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2* \\
& x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2 \\
& *b*c^3*f^3)*\cosh(d*x + c)^2 + 2*(a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^2*x^2 \\
& + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a^2*b*c \\
& ^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2*b*d^3*f^3*x^3 + 3*a^2*b*d^3*e*f^ \\
& 2*x^2 + 3*a^2*b*d^3*e^2*f*x + 3*a^2*b*c*d^2*e^2*f - 3*a^2*b*c^2*d*e*f^2 + a \\
& ^2*b*c^3*f^3)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2)*\log(-(a*\cosh(d*x + c) \\
& + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c)))*\sqrt{(a^2 + b^2)/b^ \\
& 2) - b)/b) + 12*(a^2*b*f^3*\cosh(d*x + c)^2 + 2*a^2*b*f^3*\cosh(d*x + c)*\sinh \\
& (d*x + c) + a^2*b*f^3*\sinh(d*x + c)^2 + a^2*b*f^3)*\sqrt{(a^2 + b^2)/b^2)*\text{poly} \\
& \text{log}(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x \\
& + c)))*\sqrt{(a^2 + b^2)/b^2))/b) - 12*(a^2*b*f^3*\cosh(d*x + c)^2 + 2*a^2*b* \\
& f^3*\cosh(d*x + c)*\sinh(d*x + c) + a^2*b*f^3*\sinh(d*x + c)^2 + a^2*b*f^3)*\sqrt{ \\
& (a^2 + b^2)/b^2)*\text{polylog}(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh \\
& (d*x + c) + b*\sinh(d*x + c)))*\sqrt{(a^2 + b^2)/b^2))/b) - 12*(a^2*b*d*f^3*x \\
& + a^2*b*d*e*f^2 + (a^2*b*d*f^3*x + a^2*b*d*e*f^2)*\cosh(d*x + c)^2 + 2*(a^2* \\
& b*d*f^3*x + a^2*b*d*e*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2*b*d*f^3*x + a \\
& ^2*b*d*e*f^2)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2)*\text{polylog}(3, (a*\cosh(d*x \\
& + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c)))*\sqrt{(a^2 + b \\
& ^2)/b^2))/b) + 12*(a^2*b*d*f^3*x + a^2*b*d*e*f^2 + (a^2*b*d*f^3*x + a^2*b*d \\
& *e*f^2)*\cosh(d*x + c)^2 + 2*(a^2*b*d*f^3*x + a^2*b*d*e*f^2)*\cosh(d*x + c)*\text{sinh} \\
& (d*x + c) + (a^2*b*d*f^3*x + a^2*b*d*e*f^2)*\sinh(d*x + c)^2)*\sqrt{(a^2 + \\
& b^2)/b^2)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c)
\end{aligned}$$



$$\begin{aligned}
 &+ b \sinh(dx + c) \sqrt{(a^2 + b^2)/b^2})/b - 4*((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*d^3*e*f^2*x^2 + 3*(a^2*b + b^3)*d^3*e^2*f*x + (a^2*b + b^3)*d^3*e^3)*\cosh(dx + c) + (12*(a^3 + a*b^2)*d*f^3*x + 12*I*(a^2*b + b^3)*d*f^3*x + 12*(a^3 + a*b^2)*d*e*f^2 + 12*I*(a^2*b + b^3)*d*e*f^2 + (12*(a^3 + a*b^2)*d*f^3*x + 12*I*(a^2*b + b^3)*d*f^3*x + 12*(a^3 + a*b^2)*d*e*f^2 + 12*I*(a^2*b + b^3)*d*e*f^2)*\cosh(dx + c)^2 + (24*(a^3 + a*b^2)*d*f^3*x + 24*I*(a^2*b + b^3)*d*f^3*x + 24*(a^3 + a*b^2)*d*e*f^2 + 24*I*(a^2*b + b^3)*d*e*f^2)*\cosh(dx + c)*\sinh(dx + c) + (12*(a^3 + a*b^2)*d*f^3*x + 12*I*(a^2*b + b^3)*d*f^3*x + 12*(a^3 + a*b^2)*d*e*f^2 + 12*I*(a^2*b + b^3)*d*e*f^2)*\sinh(dx + c)^2*\operatorname{dilog}(I*\cosh(dx + c) + I*\sinh(dx + c)) + (12*(a^3 + a*b^2)*d*f^3*x - 12*I*(a^2*b + b^3)*d*f^3*x + 12*(a^3 + a*b^2)*d*e*f^2 - 12*I*(a^2*b + b^3)*d*e*f^2 + (12*(a^3 + a*b^2)*d*f^3*x - 12*I*(a^2*b + b^3)*d*f^3*x + 12*(a^3 + a*b^2)*d*e*f^2 - 12*I*(a^2*b + b^3)*d*e*f^2)*\cosh(dx + c)^2 + (24*(a^3 + a*b^2)*d*f^3*x - 24*I*(a^2*b + b^3)*d*f^3*x + 24*(a^3 + a*b^2)*d*e*f^2 - 24*I*(a^2*b + b^3)*d*e*f^2)*\cosh(dx + c)*\sinh(dx + c) + (12*(a^3 + a*b^2)*d*f^3*x - 12*I*(a^2*b + b^3)*d*f^3*x + 12*(a^3 + a*b^2)*d*e*f^2 - 12*I*(a^2*b + b^3)*d*e*f^2)*\sinh(dx + c)^2*\operatorname{dilog}(-I*\cosh(dx + c) - I*\sinh(dx + c)) + (6*(a^3 + a*b^2)*d^2*e^2*f + 6*I*(a^2*b + b^3)*d^2*e^2*f - 12*(a^3 + a*b^2)*c*d*e*f^2 - 12*I*(a^2*b + b^3)*c*d*e*f^2 + 6*(a^3 + a*b^2)*c^2*f^3 + 6*I*(a^2*b + b^3)*c^2*f^3 + (6*(a^3 + a*b^2)*d^2*e^2*f + 6*I*(a^2*b + b^3)*d^2*e^2*f - 12*(a^3 + a*b^2)*c*d*e*f^2 - 12*I*(a^2*b + b^3)*c*d*e*f^2 + 6*(a^3 + a*b^2)*c^2*f^3 + 6*I*(a^2*b + b^3)*c^2*f^3)*\cosh(dx + c)^2 + (12*(a^3 + a*b^2)*d^2*e^2*f + 12*I*(a^2*b + b^3)*d^2*e^2*f - 24*(a^3 + a*b^2)*c*d*e*f^2 - 24*I*(a^2*b + b^3)*c*d*e*f^2 + 12*(a^3 + a*b^2)*c^2*f^3 + 12*I*(a^2*b + b^3)*c^2*f^3)*\cosh(dx + c)*\sinh(dx + c) + (6*(a^3 + a*b^2)*d^2*e^2*f + 6*I*(a^2*b + b^3)*d^2*e^2*f - 12*(a^3 + a*b^2)*c*d*e*f^2 - 12*I*(a^2*b + b^3)*c*d*e*f^2 + 6*(a^3 + a*b^2)*c^2*f^3 + 6*I*(a^2*b + b^3)*c^2*f^3)*\sinh(dx + c)^2*\log(\cosh(dx + c) + \sinh(dx + c) + I) + (6*(a^3 + a*b^2)*d^2*e^2*f - 6*I*(a^2*b + b^3)*d^2*e^2*f - 12*(a^3 + a*b^2)*c*d*e*f^2 + 12*I*(a^2*b + b^3)*c*d*e*f^2 + 6*(a^3 + a*b^2)*c^2*f^3 - 6*I*(a^2*b + b^3)*c^2*f^3 + (6*(a^3 + a*b^2)*d^2*e^2*f - 6*I*(a^2*b + b^3)*d^2*e^2*f - 12*(a^3 + a*b^2)*c*d*e*f^2 + 12*I*(a^2*b + b^3)*c*d*e*f^2 + 6*(a^3 + a*b^2)*c^2*f^3 - 6*I*(a^2*b + b^3)*c^2*f^3)*\cosh(dx + c)^2 + (12*(a^3 + a*b^2)*d^2*e^2*f - 12*I*(a^2*b + b^3)*d^2*e^2*f - 24*(a^3 + a*b^2)*c*d*e*f^2 + 24*I*(a^2*b + b^3)*c*d*e*f^2 + 12*(a^3 + a*b^2)*c^2*f^3 - 12*I*(a^2*b + b^3)*c^2*f^3)*\cosh(dx + c)*\sinh(dx + c) + (6*(a^3 + a*b^2)*d^2*e^2*f - 6*I*(a^2*b + b^3)*d^2*e^2*f - 12*(a^3 + a*b^2)*c*d*e*f^2 + 12*I*(a^2*b + b^3)*c*d*e*f^2 + 6*(a^3 + a*b^2)*c^2*f^3 - 6*I*(a^2*b + b^3)*c^2*f^3)*\sinh(dx + c)^2*\log(\cosh(dx + c) + \sinh(dx + c) - I) + (6*(a^3 + a*b^2)*d^2*f^3*x^2 - 6*I*(a^2*b + b^3)*d^2*f^3*x^2 + 12*(a^3 + a*b^2)*d^2*e*f^2*x - 12*I*(a^2*b + b^3)*d^2*e*f^2*x + 12*(a^3 + a*b^2)*c*d*e*f^2 - 12*I*(a^2*b + b^3)*c*d*e*f^2 - 6*(a^3 + a*b^2)*c^2*f^3 + 6*I*(a^2*b + b^3)*c^2*f^3 + (6*(a^3 + a*b^2)*d^2*f^3*x^2 - 6*I*(a^2*b + b^3)*d^2*f^3*x^2 + 12*(a^3 + a*b^2)*d^2*e*f^2*x - 12*I*(a^2*b + b^3)*d^2*e*f^2*x + 12*(a^3 + a*b^2)*c*d*e*f^2 - 12*I*(a^2*b + b^3)*c*d*e*f^2 - 6*(a^3 + a*b^2)*c^2*f^3 + 6*I*(a^2*b + b^3)*c^2*f^3)*c
 \end{aligned}$$

```

osh(d*x + c)^2 + (12*(a^3 + a*b^2)*d^2*f^3*x^2 - 12*I*(a^2*b + b^3)*d^2*f^3
*x^2 + 24*(a^3 + a*b^2)*d^2*e*f^2*x - 24*I*(a^2*b + b^3)*d^2*e*f^2*x + 24*(
a^3 + a*b^2)*c*d*e*f^2 - 24*I*(a^2*b + b^3)*c*d*e*f^2 - 12*(a^3 + a*b^2)*c^
2*f^3 + 12*I*(a^2*b + b^3)*c^2*f^3)*cosh(d*x + c)*sinh(d*x + c) + (6*(a^3 +
a*b^2)*d^2*f^3*x^2 - 6*I*(a^2*b + b^3)*d^2*f^3*x^2 + 12*(a^3 + a*b^2)*d^2*
e*f^2*x - 12*I*(a^2*b + b^3)*d^2*e*f^2*x + 12*(a^3 + a*b^2)*c*d*e*f^2 - 12*
I*(a^2*b + b^3)*c*d*e*f^2 - 6*(a^3 + a*b^2)*c^2*f^3 + 6*I*(a^2*b + b^3)*c^2
*f^3)*sinh(d*x + c)^2*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1) + (6*(a^3
+ a*b^2)*d^2*f^3*x^2 + 6*I*(a^2*b + b^3)*d^2*f^3*x^2 + 12*(a^3 + a*b^2)*d^
2*e*f^2*x + 12*I*(a^2*b + b^3)*d^2*e*f^2*x + 12*(a^3 + a*b^2)*c*d*e*f^2 + 1
2*I*(a^2*b + b^3)*c*d*e*f^2 - 6*(a^3 + a*b^2)*c^2*f^3 - 6*I*(a^2*b + b^3)*c
^2*f^3 + (6*(a^3 + a*b^2)*d^2*f^3*x^2 + 6*I*(a^2*b + b^3)*d^2*f^3*x^2 + 12*
(a^3 + a*b^2)*d^2*e*f^2*x + 12*I*(a^2*b + b^3)*d^2*e*f^2*x + 12*(a^3 + a*b^
2)*c*d*e*f^2 + 12*I*(a^2*b + b^3)*c*d*e*f^2 - 6*(a^3 + a*b^2)*c^2*f^3 - 6*I
*(a^2*b + b^3)*c^2*f^3)*cosh(d*x + c)^2 + (12*(a^3 + a*b^2)*d^2*f^3*x^2 + 1
2*I*(a^2*b + b^3)*d^2*f^3*x^2 + 24*(a^3 + a*b^2)*d^2*e*f^2*x + 24*I*(a^2*b
+ b^3)*d^2*e*f^2*x + 24*(a^3 + a*b^2)*c*d*e*f^2 + 24*I*(a^2*b + b^3)*c*d*e*
f^2 - 12*(a^3 + a*b^2)*c^2*f^3 - 12*I*(a^2*b + b^3)*c^2*f^3)*cosh(d*x + c)*
sinh(d*x + c) + (6*(a^3 + a*b^2)*d^2*f^3*x^2 + 6*I*(a^2*b + b^3)*d^2*f^3*x^
2 + 12*(a^3 + a*b^2)*d^2*e*f^2*x + 12*I*(a^2*b + b^3)*d^2*e*f^2*x + 12*(a^3
+ a*b^2)*c*d*e*f^2 + 12*I*(a^2*b + b^3)*c*d*e*f^2 - 6*(a^3 + a*b^2)*c^2*f^
3 - 6*I*(a^2*b + b^3)*c^2*f^3)*sinh(d*x + c)^2*log(-I*cosh(d*x + c) - I*si
nh(d*x + c) + 1) - 12*((a^3 + a*b^2)*f^3 + I*(a^2*b + b^3)*f^3 + ((a^3 + a*
b^2)*f^3 + I*(a^2*b + b^3)*f^3)*cosh(d*x + c)^2 + 2*((a^3 + a*b^2)*f^3 + I*
(a^2*b + b^3)*f^3)*cosh(d*x + c)*sinh(d*x + c) + ((a^3 + a*b^2)*f^3 + I*(a^
2*b + b^3)*f^3)*sinh(d*x + c)^2)*polylog(3, I*cosh(d*x + c) + I*sinh(d*x +
c)) - (12*(a^3 + a*b^2)*f^3 - 12*I*(a^2*b + b^3)*f^3 + 12*((a^3 + a*b^2)*f^
3 - I*(a^2*b + b^3)*f^3)*cosh(d*x + c)^2 + 24*((a^3 + a*b^2)*f^3 - I*(a^2*b
+ b^3)*f^3)*cosh(d*x + c)*sinh(d*x + c) + 12*((a^3 + a*b^2)*f^3 - I*(a^2*b
+ b^3)*f^3)*sinh(d*x + c)^2)*polylog(3, -I*cosh(d*x + c) - I*sinh(d*x + c)
) - 4*((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*d^3*e*f^2*x^2 + 3*(a^2*b
+ b^3)*d^3*e^2*f*x + (a^2*b + b^3)*d^3*e^3 + 2*((a^3 + a*b^2)*d^3*f^3*x^3
+ 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + 3*(a^3 + a*
b^2)*c*d^2*e^2*f - 3*(a^3 + a*b^2)*c^2*d*e*f^2 + (a^3 + a*b^2)*c^3*f^3)*cos
h(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^4*cosh(d*x + c)^2 + 2
*(a^4 + 2*a^2*b^2 + b^4)*d^4*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2
+ b^4)*d^4*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^4)

```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^3 \tanh^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*tanh(c + d*x)**2/(a + b*sinh(c + d*x)), x)
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.383 \quad \int \frac{(e+fx)^2 \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=772

$$\frac{2a^2 f(e+fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2 (a^2+b^2)^{3/2}} - \frac{2a^2 f(e+fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2 (a^2+b^2)^{3/2}} - \frac{a^3 f^2 \text{PolyLog}\left(2, -e^{2(c+dx)}\right)}{b^2 d^3 (a^2+b^2)} + \frac{2ia^2 f^2 \text{Po}}{bd}$$

[Out]  $-\left(\frac{a(e+fx)^2}{b^2 d}\right) + \frac{a^3(e+fx)^2}{b^2(a^2+b^2)d} + (4f(e+fx) \text{ArcTan}[E^{(c+dx)}]) / (b d^2) - (4a^2 f(e+fx) \text{ArcTan}[E^{(c+dx)}]) / (b(a^2+b^2)d^2) + \frac{a^2(e+fx)^2 \text{Log}[1+(bE^{(c+dx)}) / (a-\sqrt{a^2+b^2})]}{(a^2+b^2)^{3/2} d} - \frac{a^2(e+fx)^2 \text{Log}[1+(bE^{(c+dx)}) / (a+\sqrt{a^2+b^2})]}{(a^2+b^2)^{3/2} d} + \frac{2a f(e+fx) \text{Log}[1+E^{2(c+dx)}]}{b^2 d^2} - \frac{2a^3 f(e+fx) \text{Log}[1+E^{2(c+dx)}]}{b^2(a^2+b^2)d^2} - \frac{(2I) f^2 \text{PolyLog}[2, (-I)E^{(c+dx)}]}{b d^3} + \frac{(2I) a^2 f^2 \text{PolyLog}[2, (-I)E^{(c+dx)}]}{b(a^2+b^2)d^3} + \frac{(2I) f^2 \text{PolyLog}[2, I E^{(c+dx)}]}{b d^3} - \frac{(2I) a^2 f^2 \text{PolyLog}[2, I E^{(c+dx)}]}{b(a^2+b^2)d^3} + \frac{2a^2 f(e+fx) \text{PolyLog}[2, -(bE^{(c+dx)}) / (a-\sqrt{a^2+b^2})]}{(a^2+b^2)^{3/2} d^2} - \frac{2a^2 f(e+fx) \text{PolyLog}[2, -(bE^{(c+dx)}) / (a+\sqrt{a^2+b^2})]}{(a^2+b^2)^{3/2} d^2} + \frac{a f^2 \text{PolyLog}[2, -E^{2(c+dx)}]}{b^2 d^3} - \frac{a^3 f^2 \text{PolyLog}[2, -E^{2(c+dx)}]}{b^2(a^2+b^2)d^3} - \frac{2a^2 f^2 \text{PolyLog}[3, -(bE^{(c+dx)}) / (a-\sqrt{a^2+b^2})]}{(a^2+b^2)^{3/2} d^3} + \frac{2a^2 f^2 \text{PolyLog}[3, -(bE^{(c+dx)}) / (a+\sqrt{a^2+b^2})]}{(a^2+b^2)^{3/2} d^3} - \frac{(e+fx)^2 \text{Sech}[c+dx]}{b d} + \frac{a^2(e+fx)^2 \text{Sech}[c+dx]}{b(a^2+b^2)d} - \frac{a(e+fx)^2 \text{Tanh}[c+dx]}{b^2 d} + \frac{a^3(e+fx)^2 \text{Tanh}[c+dx]}{b^2(a^2+b^2)d}$

**Rubi [A]** time = 1.53284, antiderivative size = 772, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 16, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$ , Rules used = {5567, 5451, 4180, 2279, 2391, 5583, 4184, 3718, 2190, 5573, 3322, 2264, 2531, 2282, 6589, 6742}

$$\frac{2a^2 f(e+fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2 (a^2+b^2)^{3/2}} - \frac{2a^2 f(e+fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2 (a^2+b^2)^{3/2}} - \frac{a^3 f^2 \text{PolyLog}\left(2, -e^{2(c+dx)}\right)}{b^2 d^3 (a^2+b^2)} + \frac{2ia^2 f^2 \text{Po}}{bd}$$

Antiderivative was successfully verified.

[In] Int[((e+fx)^2\*Tanh[c+dx]^2)/(a+b\*Sinh[c+dx]),x]

```
[Out] -((a*(e + f*x)^2)/(b^2*d)) + (a^3*(e + f*x)^2)/(b^2*(a^2 + b^2)*d) + (4*f*(
e + f*x)*ArcTan[E^(c + d*x)]/(b*d^2) - (4*a^2*f*(e + f*x)*ArcTan[E^(c + d*
x)]/(b*(a^2 + b^2)*d^2) + (a^2*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2]]))/((a^2 + b^2)^(3/2)*d) - (a^2*(e + f*x)^2*Log[1 + (b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]]))/((a^2 + b^2)^(3/2)*d) + (2*a*f*(e + f*x)*Log
[1 + E^(2*(c + d*x))]/(b^2*d^2) - (2*a^3*f*(e + f*x)*Log[1 + E^(2*(c + d*x
))]/(b^2*(a^2 + b^2)*d^2) - ((2*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)]/(b*d^
3) + ((2*I)*a^2*f^2*PolyLog[2, (-I)*E^(c + d*x)]/(b*(a^2 + b^2)*d^3) + ((2
*I)*f^2*PolyLog[2, I*E^(c + d*x)]/(b*d^3) - ((2*I)*a^2*f^2*PolyLog[2, I*E^
(c + d*x)]/(b*(a^2 + b^2)*d^3) + (2*a^2*f*(e + f*x)*PolyLog[2, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (2*a^2*f*(e + f*x
)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*
d^2) + (a*f^2*PolyLog[2, -E^(2*(c + d*x))]/(b^2*d^3) - (a^3*f^2*PolyLog[2,
-E^(2*(c + d*x))]/(b^2*(a^2 + b^2)*d^3) - (2*a^2*f^2*PolyLog[3, -((b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) + (2*a^2*f^2*Poly
Log[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^3) -
((e + f*x)^2*Sech[c + d*x]/(b*d) + (a^2*(e + f*x)^2*Sech[c + d*x]/(b*(a^
2 + b^2)*d) - (a*(e + f*x)^2*Tanh[c + d*x]/(b^2*d) + (a^3*(e + f*x)^2*Tanh
[c + d*x]/(b^2*(a^2 + b^2)*d)
```

### Rule 5567

```
Int[(((e_.) + (f_.)*(x_.))^(m_.)*Tanh[(c_.) + (d_.)*(x_.)]^(n_.))/((a_.) + (b_
_.)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sech[c
+ d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

### Rule 5451

```
Int[(((c_.) + (d_.)*(x_.))^(m_.)*Sech[(a_.) + (b_.)*(x_.)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_.)]^(p_.), x_Symbol] := -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b^n),
x] + Dist[(d*m)/(b^n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_.)]*((c_.) + (d_.)*(x_
.))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)]]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5583

```
Int[(((e_) + (f_)*(x_))^(m_)*Sech[(c_) + (d_)*(x_)]^(p_)*Tanh[(c_) +
(d_)*(x_)]^(n_))/((a_) + (b_)*Sinh[(c_) + (d_)*(x_)]), x_Symbol] := D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x]
] - Dist[a/b, Int[(((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))
)/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0
] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4184

```
Int[csc[(e_) + (f_)*(x_)]^2*((c_) + (d_)*(x_))^(m_), x_Symbol] := -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[(((c_) + (d_)*(x_))^(m_)*tan[(e_) + (Complex[0, fz_])*(f_)*(x_)], x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[(((c
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 5573

```
Int[(((e_) + (f_)*(x_))^(m_)*Sech[(c_) + (d_)*(x_)]^(n_))/((a_) + (b_
)*Sinh[(c_) + (d_)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[(e +
```

```
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

### Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \operatorname{sech}(c+dx) \tanh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^2 \operatorname{sech}(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= -\frac{(e+fx)^2 \operatorname{sech}(c+dx)}{bd} - \frac{a \int (e+fx)^2 \operatorname{sech}^2(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} + \dots \\
&= \frac{4f(e+fx) \tan^{-1}(e^{c+dx})}{bd^2} - \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{bd} - \frac{a(e+fx)^2 \tanh(c+dx)}{b^2 d} + \frac{a^2 \int \frac{1}{a+b \sinh(c+dx)} dx}{a} \\
&= -\frac{a(e+fx)^2}{b^2 d} + \frac{4f(e+fx) \tan^{-1}(e^{c+dx})}{bd^2} - \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{bd} - \frac{a(e+fx)^2 \tanh(c+dx)}{b^2 d} \\
&= -\frac{a(e+fx)^2}{b^2 d} + \frac{4f(e+fx) \tan^{-1}(e^{c+dx})}{bd^2} + \frac{2af(e+fx) \log(1+e^{2(c+dx)})}{b^2 d^2} - \frac{2if^2 \operatorname{Li}_2(-ie^{c+dx})}{bd^3} \\
&= -\frac{a(e+fx)^2}{b^2 d} + \frac{4f(e+fx) \tan^{-1}(e^{c+dx})}{bd^2} + \frac{a^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2} d} - \frac{a^2(e+fx)^2}{(a^2+b^2)d} \\
&= -\frac{a(e+fx)^2}{b^2 d} + \frac{a^3(e+fx)^2}{b^2(a^2+b^2)d} + \frac{4f(e+fx) \tan^{-1}(e^{c+dx})}{bd^2} - \frac{4a^2 f(e+fx) \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d^2} \\
&= -\frac{a(e+fx)^2}{b^2 d} + \frac{a^3(e+fx)^2}{b^2(a^2+b^2)d} + \frac{4f(e+fx) \tan^{-1}(e^{c+dx})}{bd^2} - \frac{4a^2 f(e+fx) \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d^2} \\
&= -\frac{a(e+fx)^2}{b^2 d} + \frac{a^3(e+fx)^2}{b^2(a^2+b^2)d} + \frac{4f(e+fx) \tan^{-1}(e^{c+dx})}{bd^2} - \frac{4a^2 f(e+fx) \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d^2} \\
&= -\frac{a(e+fx)^2}{b^2 d} + \frac{a^3(e+fx)^2}{b^2(a^2+b^2)d} + \frac{4f(e+fx) \tan^{-1}(e^{c+dx})}{bd^2} - \frac{4a^2 f(e+fx) \tan^{-1}(e^{c+dx})}{b(a^2+b^2)d^2}
\end{aligned}$$



**Mathematica [A]** time = 8.48947, size = 910, normalized size = 1.18

$$\left(-2e^2 \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)d^2 + f^2x^2 \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}} + 1\right)d^2 + 2efx \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}} + 1\right)d^2 - f^2x^2 \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}} + 1\right)d^2 - 2ef\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Tanh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (a^2\*(-2\*d^2\*e^2\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]] + 2\*d^2\*e\*f\*x\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])] + d^2\*f^2\*x^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])] - 2\*d^2\*e\*f\*x\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] - d^2\*f^2\*x^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])]) + 2\*d\*f\*(e + f\*x)\*PolyLog[2, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] - 2\*d\*f\*(e + f\*x)\*PolyLog[2, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] - 2\*f^2\*PolyLog[3, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] + 2\*f^2\*PolyLog[3, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^2 + b^2)^(3/2)\*d^3 + (2\*a\*e\*f\*Sech[c]\*(Cosh[c]\*Log[Cosh[c]\*Cosh[d\*x] + Sinh[c]\*Sinh[d\*x]] - d\*x\*Sinh[c]))/(a^2 + b^2)\*d^2\*(Cosh[c]^2 - Sinh[c]^2) + (4\*b\*e\*f\*ArcTan[(Sinh[c] + Cosh[c]\*Tanh[(d\*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]])/(a^2 + b^2)\*d^2\*Sqrt[Cosh[c]^2 - Sinh[c]^2] - (a\*f^2\*Csch[c]\*(-(d^2\*x^2)/E^ArcTanh[Coth[c]]) + (I\*Coth[c]\*(-(d\*x\*(-Pi + (2\*I)\*ArcTanh[Coth[c]])) - Pi\*Log[1 + E^(2\*d\*x)] - 2\*(I\*d\*x + I\*ArcTanh[Coth[c]])\*Log[1 - E^((2\*I)\*(I\*d\*x + I\*ArcTanh[Coth[c]])])]) + Pi\*Log[Cosh[d\*x]] + (2\*I)\*ArcTanh[Coth[c]]\*Log[I\*Sinh[d\*x] + ArcTanh[Coth[c]]]) + I\*PolyLog[2, E^((2\*I)\*(I\*d\*x + I\*ArcTanh[Coth[c]])]))/Sqrt[1 - Coth[c]^2]\*Sech[c])/((a^2 + b^2)\*d^3\*Sqrt[Csch[c]^2\*(-Cosh[c]^2 + Sinh[c]^2)]) + (2\*b\*f^2\*(((I)\*Csch[c]\*(I\*(d\*x + ArcTanh[Coth[c]]))\*(Log[1 - E^(-(d\*x) - ArcTanh[Coth[c]])] - Log[1 + E^(-(d\*x) - ArcTanh[Coth[c]])]) + I\*(PolyLog[2, -E^(-(d\*x) - ArcTanh[Coth[c]])] - PolyLog[2, E^(-(d\*x) - ArcTanh[Coth[c]])])))/Sqrt[1 - Coth[c]^2] - (2\*ArcTan[(Sinh[c] + Cosh[c]\*Tanh[(d\*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]]\*ArcTanh[Coth[c]])/Sqrt[Cosh[c]^2 - Sinh[c]^2])/(a^2 + b^2)\*d^3 + (Sech[c]\*Sech[c + d\*x]\*(-(b\*e^2\*Cosh[c]) - 2\*b\*e\*f\*x\*Cosh[c] - b\*f^2\*x^2\*Cosh[c] - a\*e^2\*Sinh[d\*x] - 2\*a\*e\*f\*x\*Sinh[d\*x] - a\*f^2\*x^2\*Sinh[d\*x]))/(a^2 + b^2)\*d)

**Maple [F]** time = 0.576, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\tanh(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [C]** time = 3.41043, size = 8585, normalized size = 11.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(4*(a^3 + a*b^2)*d^2*e^2 - 8*(a^3 + a*b^2)*c*d*e*f + 4*(a^3 + a*b^2)*c^2*f^2 - 4*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*cosh(d*x + c)^2 - 4*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*sinh(d*x + c)^2 + 4*(a^2*b*d*f^2*x + a^2*b*d*e*f + (a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d*f^2*x + a^2*b*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 4*(a^2*b*d*f^2*x + a^2*b*d*e*f + (a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d*f^2*x + a^2*b*d*e*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2 + (a^2*b*d^2*e^2 - 2*a^2*b*c*d*e*f + a^2*b*c^2*f^2)*cosh(d*x + c)^2 + 2*(a^2*b*d^2*e^2
```



```

3 + a*b^2)*d*e*f - 4*I*(a^2*b + b^3)*d*e*f - 4*(a^3 + a*b^2)*c*f^2 + 4*I*(a
^2*b + b^3)*c*f^2)*cosh(d*x + c)^2 + (8*(a^3 + a*b^2)*d*e*f - 8*I*(a^2*b +
b^3)*d*e*f - 8*(a^3 + a*b^2)*c*f^2 + 8*I*(a^2*b + b^3)*c*f^2)*cosh(d*x + c)
*sinh(d*x + c) + (4*(a^3 + a*b^2)*d*e*f - 4*I*(a^2*b + b^3)*d*e*f - 4*(a^3
+ a*b^2)*c*f^2 + 4*I*(a^2*b + b^3)*c*f^2)*sinh(d*x + c)^2*log(cosh(d*x + c
) + sinh(d*x + c) - I) + (4*(a^3 + a*b^2)*d*f^2*x - 4*I*(a^2*b + b^3)*d*f^2
*x + 4*(a^3 + a*b^2)*c*f^2 - 4*I*(a^2*b + b^3)*c*f^2 + (4*(a^3 + a*b^2)*d*f
^2*x - 4*I*(a^2*b + b^3)*d*f^2*x + 4*(a^3 + a*b^2)*c*f^2 - 4*I*(a^2*b + b^3
)*c*f^2)*cosh(d*x + c)^2 + (8*(a^3 + a*b^2)*d*f^2*x - 8*I*(a^2*b + b^3)*d*f
^2*x + 8*(a^3 + a*b^2)*c*f^2 - 8*I*(a^2*b + b^3)*c*f^2)*cosh(d*x + c)*sinh(
d*x + c) + (4*(a^3 + a*b^2)*d*f^2*x - 4*I*(a^2*b + b^3)*d*f^2*x + 4*(a^3 +
a*b^2)*c*f^2 - 4*I*(a^2*b + b^3)*c*f^2)*sinh(d*x + c)^2*log(I*cosh(d*x + c
) + I*sinh(d*x + c) + 1) + (4*(a^3 + a*b^2)*d*f^2*x + 4*I*(a^2*b + b^3)*d*f
^2*x + 4*(a^3 + a*b^2)*c*f^2 + 4*I*(a^2*b + b^3)*c*f^2 + (4*(a^3 + a*b^2)*d
*f^2*x + 4*I*(a^2*b + b^3)*d*f^2*x + 4*(a^3 + a*b^2)*c*f^2 + 4*I*(a^2*b + b
^3)*c*f^2)*cosh(d*x + c)^2 + (8*(a^3 + a*b^2)*d*f^2*x + 8*I*(a^2*b + b^3)*d
*f^2*x + 8*(a^3 + a*b^2)*c*f^2 + 8*I*(a^2*b + b^3)*c*f^2)*cosh(d*x + c)*sin
h(d*x + c) + (4*(a^3 + a*b^2)*d*f^2*x + 4*I*(a^2*b + b^3)*d*f^2*x + 4*(a^3
+ a*b^2)*c*f^2 + 4*I*(a^2*b + b^3)*c*f^2)*sinh(d*x + c)^2*log(-I*cosh(d*x
+ c) - I*sinh(d*x + c) + 1) - 4*((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3
)*d^2*e*f*x + (a^2*b + b^3)*d^2*e^2 + 2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3
+ a*b^2)*d^2*e*f*x + 2*(a^3 + a*b^2)*c*d*e*f - (a^3 + a*b^2)*c^2*f^2)*cosh
(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)^2 + 2*
(a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2
+ b^4)*d^3*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^3)

```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^2 \tanh^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*tanh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*\*2\*tanh(c + d\*x)\*\*2/(a + b\*sinh(c + d\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

sage<sub>0</sub>x

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```

$$3.384 \quad \int \frac{(e+fx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=385

$$\frac{a^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2 (a^2+b^2)^{3/2}} - \frac{a^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2 (a^2+b^2)^{3/2}} - \frac{a^2 f \tan^{-1}(\sinh(c+dx))}{bd^2 (a^2+b^2)} - \frac{a^3 f \log(\cosh(c+dx))}{b^2 d^2 (a^2+b^2)} + \frac{a^2(e+fx)}{d^2 (a^2+b^2)^{3/2}}$$

```
[Out] (f*ArcTan[Sinh[c + d*x]])/(b*d^2) - (a^2*f*ArcTan[Sinh[c + d*x]])/(b*(a^2 +
b^2)*d^2) + (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])
/((a^2 + b^2)^(3/2)*d) - (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (a*f*Log[Cosh[c + d*x]])/(b^2*d^2) - (
a^3*f*Log[Cosh[c + d*x]])/(b^2*(a^2 + b^2)*d^2) + (a^2*f*PolyLog[2, -((b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (a^2*f*PolyLo
g[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (
(e + f*x)*Sech[c + d*x])/(b*d) + (a^2*(e + f*x)*Sech[c + d*x])/(b*(a^2 + b^
2)*d) - (a*(e + f*x)*Tanh[c + d*x])/(b^2*d) + (a^3*(e + f*x)*Tanh[c + d*x]
)/(b^2*(a^2 + b^2)*d)
```

**Rubi [A]** time = 0.758024, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 13, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5567, 5451, 3770, 5583, 4184, 3475, 5573, 3322, 2264, 2190, 2279, 2391, 6742}

$$\frac{a^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{d^2 (a^2+b^2)^{3/2}} - \frac{a^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{d^2 (a^2+b^2)^{3/2}} - \frac{a^2 f \tan^{-1}(\sinh(c+dx))}{bd^2 (a^2+b^2)} - \frac{a^3 f \log(\cosh(c+dx))}{b^2 d^2 (a^2+b^2)} + \frac{a^2(e+fx)}{d^2 (a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (f*ArcTan[Sinh[c + d*x]])/(b*d^2) - (a^2*f*ArcTan[Sinh[c + d*x]])/(b*(a^2 +
b^2)*d^2) + (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])
/((a^2 + b^2)^(3/2)*d) - (a^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a
^2 + b^2])])/((a^2 + b^2)^(3/2)*d) + (a*f*Log[Cosh[c + d*x]])/(b^2*d^2) - (
a^3*f*Log[Cosh[c + d*x]])/(b^2*(a^2 + b^2)*d^2) + (a^2*f*PolyLog[2, -((b*E^
(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (a^2*f*PolyLo
g[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^(3/2)*d^2) - (
(e + f*x)*Sech[c + d*x])/(b*d) + (a^2*(e + f*x)*Sech[c + d*x])/(b*(a^2 + b^
2)*d) - (a*(e + f*x)*Tanh[c + d*x])/(b^2*d) + (a^3*(e + f*x)*Tanh[c + d*x]
)/(b^2*(a^2 + b^2)*d)
```

$/(b^2*(a^2 + b^2)*d)$

### Rule 5567

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Tanh[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Sech[c + d\*x]\*Tanh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sech[c + d\*x]\*Tanh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 5451

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sech[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*Tanh[(a\_.) + (b\_.)\*(x\_)]^(p\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Sech[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Sech[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

### Rule 5583

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(p\_.)\*Tanh[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Sech[c + d\*x]^(p + 1)\*Tanh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sech[c + d\*x]^(p + 1)\*Tanh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

### Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

### Rule 3322

```
Int[(((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```



Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\operatorname{sech}(c+dx)\tanh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)} dx}{b} \\
&= -\frac{(e+fx)\operatorname{sech}(c+dx)}{bd} - \frac{a \int (e+fx)\operatorname{sech}^2(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx}{b^2} + \frac{f \int}{a^2 +} \\
&= \frac{f \tan^{-1}(\sinh(c+dx))}{bd^2} - \frac{(e+fx)\operatorname{sech}(c+dx)}{bd} - \frac{a(e+fx)\tanh(c+dx)}{b^2d} + \frac{a^2 \int \frac{e+f}{a+b\sinh}}{a^2 +} \\
&= \frac{f \tan^{-1}(\sinh(c+dx))}{bd^2} + \frac{af \log(\cosh(c+dx))}{b^2d^2} - \frac{(e+fx)\operatorname{sech}(c+dx)}{bd} - \frac{a(e+fx)\tanh}{b^2d} \\
&= \frac{f \tan^{-1}(\sinh(c+dx))}{bd^2} + \frac{af \log(\cosh(c+dx))}{b^2d^2} - \frac{(e+fx)\operatorname{sech}(c+dx)}{bd} - \frac{a(e+fx)\tanh}{b^2d} \\
&= \frac{f \tan^{-1}(\sinh(c+dx))}{bd^2} + \frac{a^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d} - \frac{a^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d} \\
&= \frac{f \tan^{-1}(\sinh(c+dx))}{bd^2} - \frac{a^2 f \tan^{-1}(\sinh(c+dx))}{b(a^2+b^2)d^2} + \frac{a^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d} - \frac{a^2}{(a^2+b^2)^{3/2}d} \\
&= \frac{f \tan^{-1}(\sinh(c+dx))}{bd^2} - \frac{a^2 f \tan^{-1}(\sinh(c+dx))}{b(a^2+b^2)d^2} + \frac{a^2(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d} - \frac{a^2}{(a^2+b^2)^{3/2}d}
\end{aligned}$$

**Mathematica [A]** time = 3.05218, size = 284, normalized size = 0.74

$$\frac{a^2 \left( f \operatorname{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) - f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) - 2de \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) + f(c+dx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right) - f(c+dx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right) + 2cf \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) \right)}{(a^2+b^2)^{3/2}d^2}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Tanh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]), x]

[Out] ((2\*b\*f\*ArcTan[Tanh[(c + d\*x)/2]])/(a^2 + b^2) + (a\*f\*Log[Cosh[c + d\*x]])/(a^2 + b^2) + (a^2\*(-2\*d\*e\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]] + 2\*

$$c*f*\text{ArcTanh}[(a + b*E^{(c + d*x)})/\text{Sqrt}[a^2 + b^2]] + f*(c + d*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])] - f*(c + d*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])] + f*\text{PolyLog}[2, (b*E^{(c + d*x)})/(-a + \text{Sqrt}[a^2 + b^2])] - f*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))]/(a^2 + b^2)^{(3/2)} - (d*(e + f*x)*\text{Sech}[c + d*x]*(b + a*\text{Sinh}[c + d*x]))/(a^2 + b^2)/d^2$$

**Maple [B]** time = 0.162, size = 1928, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)*\tanh(d*x+c)^2/(a+b*\sinh(d*x+c)),x)$

[Out] 
$$\begin{aligned} & -2/d^2/(a^2+b^2)^{(3/2)}*b^2*f/(2*a^2+2*b^2)*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) * a^2*c+2/d^2/(a^2+b^2)^{(3/2)}*b^2*f*c/(2*a^2+2*b^2)* \\ & \text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) * a^2+2/d/(a^2+b^2)^{(3/2)}*b^2*f/(2*a^2+2*b^2)* \\ & \ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) * a^2*x-2/d/(a^2+b^2)^{(3/2)}*b^2*f/(2*a^2+2*b^2)* \\ & \ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) * a^2*x+2/d^2/(a^2+b^2)^{(3/2)}*b^2*f/(2*a^2+2*b^2)* \\ & \ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) * a^2*c-2/d*e/(2*a^2+2*b^2)/ \\ & (a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) * a^2-2/(a^2+b^2)^{(3/2)}/d^2*a^4*f/(2*a^2+2*b^2)* \\ & \text{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))+2/(a^2+b^2)^{(3/2)}/d^2*a^4*f/(2*a^2+2*b^2)* \\ & \text{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))-2/(a^2+b^2)^{(3/2)}/d*a^4*e/(2*a^2+2*b^2)* \\ & \text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-2/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)* \\ & \text{arctan}(\exp(d*x+c))-2/d^2/(a^2+b^2)^{(5/2)}*a^4*f*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+1/2/d^2/(a^2+b^2)^2*a*b^2*f* \\ & \ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)+2/d^2/(a^2+b^2)*a^3*f/(2*a^2+2*b^2)* \\ & \ln(1+\exp(2*d*x+2*c))+2/d^2*f*c/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) * a^2-2/d^2/(a^2+b^2)^{(3/2)}*b^4*f/(2*a^2+2*b^2)* \\ & \text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+4/d^2/(a^2+b^2)*a^2*b*f/(2*a^2+2*b^2)* \\ & \text{arctan}(\exp(d*x+c))-2/d^2/(a^2+b^2)^{(5/2)}*a^2*b^2*f*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+4/d^2/(a^2+b^2)^{(1/2)}*a^2*f/(2*a^2+2*b^2)* \\ & \text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+2/d^2/(a^2+b^2)*b^2*f/(2*a^2+2*b^2)*a*\ln(1+\exp(2*d*x+2*c))-1/d^2/(a^2+b^2)*b^2*f/(2*a^2+2*b^2)*a*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)+2/d^2/(a^2+b^2)^{(1/2)}*b^2*f/(2*a^2+2*b^2)* \\ & \text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+2/d^2/(a^2+b^2)^{(3/2)}*b^2*f/(2*a^2+2*b^2)* \\ & \text{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) * a^2-2/d/(a^2+b^2)^{(3/2)}*b^2*e/(2*a^2+2*b^2)* \\ & \text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) * a^2-2/(a^2+b^2)^{(3/2)}/d \end{aligned}$$

```

*a^4*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)
)))*x-2/d^2/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+
2*a)/(a^2+b^2)^(1/2))*a^2-2/d^2/(a^2+b^2)^(3/2)*b^2*f/(2*a^2+2*b^2)*dilog((
b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a^2+2/(a^2+b^2)^(3/2)/
d^2*a^4*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))
+2/(a^2+b^2)^(3/2)/d*a^4*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-
a)/(-a+(a^2+b^2)^(1/2)))*x-2/(a^2+b^2)^(3/2)/d^2*a^4*f/(2*a^2+2*b^2)*ln((b*
exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/(a^2+b^2)^(3/2)/d^2*
a^4*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2
)))*c+1/d^2/(a^2+b^2)^2*a^3*f*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d^2/(
a^2+b^2)*a*f*ln(exp(d*x+c))+2*(f*x+e)*(-b*exp(d*x+c)+a)/d/(a^2+b^2)/(1+exp(
2*d*x+2*c))

```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*tanh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.58481, size = 3274, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*tanh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

```

[Out] -(2*(a^3 + a*b^2)*d*f*x*cosh(d*x + c)^2 + 2*(a^3 + a*b^2)*d*f*x*sinh(d*x +
c)^2 - 2*(a^3 + a*b^2)*d*e - (a^2*b*f*cosh(d*x + c)^2 + 2*a^2*b*f*cosh(d*x
+ c)*sinh(d*x + c) + a^2*b*f*sinh(d*x + c)^2 + a^2*b*f)*sqrt((a^2 + b^2)/b^
2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a^2*b*f*cosh(d*x + c)^2 + 2*a^2
*b*f*cosh(d*x + c)*sinh(d*x + c) + a^2*b*f*sinh(d*x + c)^2 + a^2*b*f)*sqrt(
(a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c
) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (a^2*b*d*e - a^2*b
*c*f + (a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*e - a^2*b*c*f)*

```

```

cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d*e - a^2*b*c*f)*sinh(d*x + c)^2)*sqrt
((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2
+ b^2)/b^2) + 2*a) - (a^2*b*d*e - a^2*b*c*f + (a^2*b*d*e - a^2*b*c*f)*cosh
(d*x + c)^2 + 2*(a^2*b*d*e - a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*
b*d*e - a^2*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x
+ c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - (a^2*b*d*f*x
+ a^2*b*c*f + (a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*f*x +
a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d*f*x + a^2*b*c*f)*sinh(d*x
+ c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (a^2*b*d*
f*x + a^2*b*c*f + (a^2*b*d*f*x + a^2*b*c*f)*cosh(d*x + c)^2 + 2*(a^2*b*d*f*
x + a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^2*b*d*f*x + a^2*b*c*f)*sinh
(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
- (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*((a
^2*b + b^3)*f*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f*cosh(d*x + c)*sinh(d*x +
c) + (a^2*b + b^3)*f*sinh(d*x + c)^2 + (a^2*b + b^3)*f)*arctan(cosh(d*x + c
) + sinh(d*x + c)) + 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e)*cosh(d*x +
c) - ((a^3 + a*b^2)*f*cosh(d*x + c)^2 + 2*(a^3 + a*b^2)*f*cosh(d*x + c)*si
nh(d*x + c) + (a^3 + a*b^2)*f*sinh(d*x + c)^2 + (a^3 + a*b^2)*f)*log(2*cosh
(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(2*(a^3 + a*b^2)*d*f*x*cosh(
d*x + c) + (a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e)*sinh(d*x + c))/((a^4 +
2*a^2*b^2 + b^4)*d^2*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*cosh(
d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*d^2*sinh(d*x + c)^2 + (a^4
+ 2*a^2*b^2 + b^4)*d^2)

```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \tanh^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*tanh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*tanh(c + d\*x)\*\*2/(a + b\*sinh(c + d\*x)), x)

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.385 \quad \int \frac{\tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=90

$$-\frac{2a^2 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}} - \frac{a \tanh(c+dx)}{d(a^2+b^2)} - \frac{b \operatorname{sech}(c+dx)}{d(a^2+b^2)}$$

[Out]  $(-2*a^2*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*d) - (b*Sech[c + d*x])/((a^2 + b^2)*d) - (a*Tanh[c + d*x])/((a^2 + b^2)*d)$

**Rubi [A]** time = 0.105545, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2727, 3767, 8, 2606, 2660, 618, 204}

$$-\frac{2a^2 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d(a^2+b^2)^{3/2}} - \frac{a \tanh(c+dx)}{d(a^2+b^2)} - \frac{b \operatorname{sech}(c+dx)}{d(a^2+b^2)}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d\*x]^2/(a + b\*Sinh[c + d\*x]),x]

[Out]  $(-2*a^2*ArcTanh[(b - a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 + b^2)^(3/2)*d) - (b*Sech[c + d*x])/((a^2 + b^2)*d) - (a*Tanh[c + d*x])/((a^2 + b^2)*d)$

#### Rule 2727

Int[((g\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(p\_)/((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]), x\_Symbol] :> Dist[a/(a^2 - b^2), Int[(g\*Tan[e + f\*x])^p/Sin[e + f\*x]^2, x], x] + (-Dist[(b\*g)/(a^2 - b^2), Int[(g\*Tan[e + f\*x])^(p - 1)/Cos[e + f\*x], x], x] - Dist[(a^2\*g^2)/(a^2 - b^2), Int[(g\*Tan[e + f\*x])^(p - 2)/(a + b\*Sin[e + f\*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2\*p] && GtQ[p, 1]

#### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rule 2606

Int[((a\_.)\*sec[(e\_.) + (f\_.)\*(x\_)]^(m\_.))\*((b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^(n\_.), x\_Symbol] := Dist[a/f, Subst[Int[(a\*x)^(m - 1)\*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f\*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

### Rule 2660

Int[((a\_) + (b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)]^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + 2\*b\*e\*x + a\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(c+dx)}{a+b\sinh(c+dx)} dx &= -\frac{a \int \operatorname{sech}^2(c+dx) dx}{a^2+b^2} + \frac{a^2 \int \frac{1}{a+b\sinh(c+dx)} dx}{a^2+b^2} + \frac{b \int \operatorname{sech}(c+dx) \tanh(c+dx) dx}{a^2+b^2} \\
&= -\frac{(ia) \operatorname{Subst}\left(\int 1 dx, x, -i \tanh(c+dx)\right)}{(a^2+b^2)d} - \frac{(2ia^2) \operatorname{Subst}\left(\int \frac{1}{a-2ibx+ax^2} dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{(a^2+b^2)d} \\
&= -\frac{b \operatorname{sech}(c+dx)}{(a^2+b^2)d} - \frac{a \tanh(c+dx)}{(a^2+b^2)d} + \frac{(4ia^2) \operatorname{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2} dx, x, -2ib+2a \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{(a^2+b^2)d} \\
&= -\frac{2a^2 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}d} - \frac{b \operatorname{sech}(c+dx)}{(a^2+b^2)d} - \frac{a \tanh(c+dx)}{(a^2+b^2)d}
\end{aligned}$$

**Mathematica [A]** time = 0.195656, size = 106, normalized size = 1.18

$$\frac{a \left( 2a \tan^{-1} \left( \frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}} \right) - \sqrt{-a^2-b^2} \tanh(c+dx) \right) - b \sqrt{-a^2-b^2} \operatorname{sech}(c+dx)}{d (-a^2-b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d\*x]^2/(a + b\*Sinh[c + d\*x]),x]

[Out] -((- (b\*Sqrt[-a^2 - b^2]\*Sech[c + d\*x]) + a\*(2\*a\*ArcTan[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[-a^2 - b^2]] - Sqrt[-a^2 - b^2]\*Tanh[c + d\*x]))/((-a^2 - b^2)^(3/2)\*d))

**Maple [A]** time = 0.001, size = 103, normalized size = 1.1

$$\frac{1}{d} \left( 8 \frac{a^2}{(4a^2 + 4b^2) \sqrt{a^2 + b^2}} \operatorname{Arctanh} \left( \frac{1}{2} \frac{2a \tanh(1/2 dx + c/2) - 2b}{\sqrt{a^2 + b^2}} \right) + 2 \frac{-a \tanh(1/2 dx + c/2) - b}{(a^2 + b^2) \left( (\tanh(1/2 dx + c/2))^2 + 1 \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)



```
[Out] 1/d*(8*a^2/(4*a^2+4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+2/(a^2+b^2)*(-a*tanh(1/2*d*x+1/2*c)-b)/(tanh(1/2*d*x+1/2*c)^2+1))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [B]** time = 2.12362, size = 883, normalized size = 9.81

$$\frac{2a^3 + 2ab^2 + (a^2 \cosh(dx+c)^2 + 2a^2 \cosh(dx+c) \sinh(dx+c) + a^2 \sinh(dx+c)^2 + a^2) \sqrt{a^2 + b^2} \log\left(\frac{b^2 \cosh(dx+c)^2 + a^2 \sinh(dx+c)^2 + 2ab \cosh(dx+c) \sinh(dx+c) + a^2}{(a^4 + 2a^2b^2 + b^4)d \cosh(dx+c)^2 + 2(a^4 + 2a^2b^2 + b^4)a \sinh(dx+c) + a^4}\right)}{(a^4 + 2a^2b^2 + b^4)d \cosh(dx+c)^2 + 2(a^4 + 2a^2b^2 + b^4)a \sinh(dx+c) + a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*a^3 + 2*a*b^2 + (a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 + a^2)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) - 2*(a^2*b + b^3)*cosh(d*x + c) - 2*(a^2*b + b^3)*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*d*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(tanh(c + d\*x)\*\*2/(a + b\*sinh(c + d\*x)), x)

**Giac [A]** time = 1.37836, size = 163, normalized size = 1.81

$$\frac{a^2 \log\left(\frac{-2be^{(dx+2c)} - 2ae^c - 2\sqrt{a^2+b^2}e^c}{-2be^{(dx+2c)} - 2ae^c + 2\sqrt{a^2+b^2}e^c}\right)}{(a^2+b^2)^{\frac{3}{2}}} + \frac{2(b e^{(dx+c)} - a)}{(a^2+b^2)(e^{(2dx+2c)}+1)}$$


---


$$d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out]  $-(a^2 \log(\text{abs}(-2*b*e^{(d*x + 2*c)} - 2*a*e^c - 2*\text{sqrt}(a^2 + b^2)*e^c)/\text{abs}(-2*b*e^{(d*x + 2*c)} - 2*a*e^c + 2*\text{sqrt}(a^2 + b^2)*e^c)))/(a^2 + b^2)^{(3/2)} + 2*(b*e^{(d*x + c)} - a)/((a^2 + b^2)*(e^{(2*d*x + 2*c)} + 1))/d$

$$3.386 \quad \int \frac{\tanh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Optimal. Leaf size=30

$$\text{Unintegrable} \left( \frac{\tanh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[Tanh[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0729658, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Tanh[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] Defer[Int][Tanh[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\tanh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.009, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[Tanh[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] \$Aborted

---

**Maple [A]** time = 0.487, size = 0, normalized size = 0.

$$\int \frac{(\tanh(dx+c))^2}{(fx+e)(a+b\sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(tanh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$2a^2 \int \frac{e^{(dx+c)}}{a^2be + b^3e + (a^2bf + b^3f)x - (a^2bee^{(2c)} + b^3ee^{(2c)} + (a^2bfe^{(2c)} + b^3fe^{(2c)})x} e^{(2dx)} - 2(a^3ee^c + ab^2ee^c + (a^3fe^c +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 2\*a^2\*integrate(-e^(d\*x + c)/(a^2\*b\*e + b^3\*e + (a^2\*b\*f + b^3\*f)\*x - (a^2\*b\*e\*e^(2\*c) + b^3\*e\*e^(2\*c) + (a^2\*b\*f\*e^(2\*c) + b^3\*f\*e^(2\*c))\*x)\*e^(2\*d\*x) - 2\*(a^3\*e\*e^c + a\*b^2\*e\*e^c + (a^3\*f\*e^c + a\*b^2\*f\*e^c)\*x)\*e^(d\*x)), x) - 2\*(b\*e^(d\*x + c) - a)/(a^2\*d\*e + b^2\*d\*e + (a^2\*d\*f + b^2\*d\*f)\*x + (a^2\*d\*e\*e^(2\*c) + b^2\*d\*e\*e^(2\*c) + (a^2\*d\*f\*e^(2\*c) + b^2\*d\*f\*e^(2\*c))\*x)\*e^(2\*d\*x) - integrate(2\*(b\*f\*e^(d\*x + c) - a\*f)/(a^2\*d\*e^2 + b^2\*d\*e^2 + (a^2\*d\*f^2 + b^2\*d\*f^2)\*x^2 + 2\*(a^2\*d\*e\*f + b^2\*d\*e\*f)\*x + (a^2\*d\*e^2\*e^(2\*c) + b^2\*d\*e^2\*e^(2\*c) + (a^2\*d\*f^2\*e^(2\*c) + b^2\*d\*f^2\*e^(2\*c))\*x^2 + 2\*(a^2\*d\*e\*f\*e^(2\*c) + b^2\*d\*e\*f\*e^(2\*c))\*x)\*e^(2\*d\*x)), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tanh(dx+c)^2}{afx+ae+(bfx+be)\sinh(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(tanh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(c + dx)}{(a + b \sinh(c + dx))(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.387 \quad \int \frac{(e+fx)^2 \operatorname{sech}(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=1256

result too large to display

```
[Out] -((a*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^2*d)) + (2*a^3*(e + f*x)^2*ArcTan[E^(c + d*x)])/((a^2 + b^2)^2*d) + (a^3*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d) + (a*f^2*ArcTan[Sinh[c + d*x]])/(b^2*d^3) - (a^3*f^2*ArcTan[Sinh[c + d*x]])/(b^2*(a^2 + b^2)*d^3) + (a^2*b*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2 + b^2)^2*d) + (a^2*b*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2 + b^2)^2*d) - (a^2*b*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(a^2 + b^2)^2*d) - (f^2*Log[Cosh[c + d*x]])/(b*d^3) + (a^2*f^2*Log[Cosh[c + d*x]])/(b*(a^2 + b^2)*d^3) + (I*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*d^2) - ((2*I)*a^3*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - (I*a^3*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) - (I*a*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + ((2*I)*a^3*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) + (2*a^2*b*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2 + b^2)^2*d^2) + (2*a^2*b*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2 + b^2)^2*d^2) - (a^2*b*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(a^2 + b^2)^2*d^2) - (I*a*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b^2*d^3) + ((2*I)*a^3*f^2*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^3) + (I*a^3*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) + (I*a*f^2*PolyLog[3, I*E^(c + d*x)])/(b^2*d^3) - ((2*I)*a^3*f^2*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)^2*d^3) - (I*a^3*f^2*PolyLog[3, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) - (2*a^2*b*f^2*PolyLog[3, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2 + b^2)^2*d^3) - (2*a^2*b*f^2*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2 + b^2)^2*d^3) + (a^2*b*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^3) - (a*f*(e + f*x)*Sech[c + d*x])/(b^2*d^2) + (a^3*f*(e + f*x)*Sech[c + d*x])/(b^2*(a^2 + b^2)*d^2) - ((e + f*x)^2*Sech[c + d*x]^2)/(2*b*d) + (a^2*(e + f*x)^2*Sech[c + d*x]^2)/(2*b*(a^2 + b^2)*d) + (f*(e + f*x)*Tanh[c + d*x])/(b*d^2) - (a^2*f*(e + f*x)*Tanh[c + d*x])/(b*(a^2 + b^2)*d^2) - (a*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*b^2*d) + (a^3*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*b^2*(a^2 + b^2)*d)
```

---

**Rubi [A]** time = 1.96754, antiderivative size = 1256, normalized size of antiderivative = 1., number of steps used = 53, number of rules used = 15, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.441$ , Rules used = {5583, 5451, 4184, 3475, 4186, 3770, 4180, 2531, 2282, 6589, 5573, 5561,

2190, 6742, 3718}

$$\frac{(e+fx)^2 \tan^{-1}(e^{c+dx}) a^3}{b^2(a^2+b^2)d} + \frac{2(e+fx)^2 \tan^{-1}(e^{c+dx}) a^3}{(a^2+b^2)^2 d} - \frac{f^2 \tan^{-1}(\sinh(c+dx)) a^3}{b^2(a^2+b^2)d^3} - \frac{if(e+fx) \text{PolyLog}(2, -ie^{c+dx})}{b^2(a^2+b^2)d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^2*d)) + (2*a^3*(e + f*x)^2*ArcTan[E^(c + d*x)]/((a^2 + b^2)^2*d) + (a^3*(e + f*x)^2*ArcTan[E^(c + d*x)]/(b^2*(a^2 + b^2)*d) + (a*f^2*ArcTan[Sinh[c + d*x]]/(b^2*d^3) - (a^3*f^2*ArcTan[Sinh[c + d*x]]/(b^2*(a^2 + b^2)*d^3) + (a^2*b*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/((a^2 + b^2)^2*d) + (a^2*b*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/((a^2 + b^2)^2*d) - (a^2*b*(e + f*x)^2*Log[1 + E^(2*(c + d*x))]/((a^2 + b^2)^2*d) - (f^2*Log[Cosh[c + d*x]]/(b*d^3) + (a^2*f^2*Log[Cosh[c + d*x]]/(b*(a^2 + b^2)*d^3) + (I*a*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)]/(b^2*d^2) - ((2*I)*a^3*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)]/((a^2 + b^2)^2*d^2) - (I*a^3*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/(b^2*d^2) + ((2*I)*a^3*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/((a^2 + b^2)^2*d^2) + (I*a^3*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/(b^2*(a^2 + b^2)*d^2) + (2*a^2*b*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)^2*d^2) + (2*a^2*b*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 + b^2)^2*d^2) - (a^2*b*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))]/((a^2 + b^2)^2*d^2) - (I*a*f^2*PolyLog[3, (-I)*E^(c + d*x)]/(b^2*d^3) + ((2*I)*a^3*f^2*PolyLog[3, (-I)*E^(c + d*x)]/((a^2 + b^2)^2*d^3) + (I*a^3*f^2*PolyLog[3, (-I)*E^(c + d*x)]/(b^2*(a^2 + b^2)*d^3) + (I*a*f^2*PolyLog[3, I*E^(c + d*x)]/(b^2*d^3) - ((2*I)*a^3*f^2*PolyLog[3, I*E^(c + d*x)]/((a^2 + b^2)^2*d^3) - (I*a^3*f^2*PolyLog[3, I*E^(c + d*x)]/(b^2*(a^2 + b^2)*d^3) - (2*a^2*b*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/((a^2 + b^2)^2*d^3) - (2*a^2*b*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/((a^2 + b^2)^2*d^3) + (a^2*b*f^2*PolyLog[3, -E^(2*(c + d*x))]/(2*(a^2 + b^2)^2*d^3) - (a*f*(e + f*x)*Sech[c + d*x]/(b^2*d^2) + (a^3*f*(e + f*x)*Sech[c + d*x]/(b^2*(a^2 + b^2)*d^2) - ((e + f*x)^2*Sech[c + d*x]^2)/(2*b*d) + (a^2*(e + f*x)^2*Sech[c + d*x]^2)/(2*b*(a^2 + b^2)*d) + (f*(e + f*x)*Tanh[c + d*x]/(b*d^2) - (a^2*f*(e + f*x)*Tanh[c + d*x]/(b*(a^2 + b^2)*d^2) - (a*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x]/(2*b^2*d) + (a^3*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x]/(2*b^2*(a^2 + b^2)*d)
```

**Rule 5583**

```
Int[((e_.) + (f_.)*(x_.))^(m_.)*Sech[(c_.) + (d_.)*(x_.)]^(p_.)*Tanh[(c_.) +
```

```
(d_.)*(x_)^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

#### Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n), x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

#### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

#### Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

#### Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

#### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

#### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-I*e) + f*fz*x)/E^(I*k*Pi)]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
```



$d*x)^{(m-1)}*\text{Log}[1 + E^{-(I*e) + f*fz*x}/E^{(I*k*Pi)}], x], x] /; \text{FreeQ}\{c, d, e, f, fz\}, x] \&\& \text{IntegerQ}[2*k] \&\& \text{IGtQ}[m, 0]$

### Rule 2531

$\text{Int}[\text{Log}[1 + (e\_)*((F\_)^{(c\_)*(a\_)} + (b\_)*(x\_))^{(n\_)}]*((f\_)+ (g\_)* (x\_))^{(m\_)}, x\_Symbol] := -\text{Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F^{c*(a + b*x)})^n)]/(b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{(m-1)}*\text{PolyLog}[2, -(e*(F^{c*(a + b*x)})^n)], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$

### Rule 2282

$\text{Int}[u_, x\_Symbol] := \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \&\& !\text{MatchQ}[u, (w\_)*((a\_)*(v\_)^{(n\_)})^{(m\_)} /; \text{FreeQ}\{a, m, n\}, x] \&\& \text{IntegerQ}[m*n] \&\& !\text{MatchQ}[u, E^{((c\_)*(a\_)} + (b\_)*x)* (F\_)[v\_]} /; \text{FreeQ}\{a, b, c\}, x] \&\& \text{InverseFunctionQ}[F[x]]]$

### Rule 6589

$\text{Int}[\text{PolyLog}[n_, (c\_)*((a\_)+ (b\_)*(x\_))^{(p\_)}]/((d\_)+ (e\_)*(x\_)), x\_Symbol] := \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x] \&\& \text{EqQ}[b*d, a*e]$

### Rule 5573

$\text{Int}[(((e\_)+ (f\_)*(x\_))^{(m\_)}*\text{Sech}[(c\_)+ (d\_)*(x\_)]^{(n\_)})/((a\_)+ (b\_)*\text{Sinh}[(c\_)+ (d\_)*(x\_)]), x\_Symbol] := \text{Dist}[b^2/(a^2 + b^2), \text{Int}[(e + f*x)^m*\text{Sech}[c + d*x]^{(n-2)}]/(a + b*\text{Sinh}[c + d*x]), x], x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(e + f*x)^m*\text{Sech}[c + d*x]^{(n-1)}*(a - b*\text{Sinh}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{IGtQ}[n, 0]$

### Rule 5561

$\text{Int}[(\text{Cosh}[(c\_)+ (d\_)*(x\_)]*((e\_)+ (f\_)*(x\_))^{(m\_)})/((a\_)+ (b\_)*\text{Sinh}[(c\_)+ (d\_)*(x\_)]), x\_Symbol] := -\text{Simp}[(e + f*x)^{(m+1)}/(b*f*(m+1)), x] + (\text{Int}[(e + f*x)^m*\text{E}^{(c + d*x)}/(a - \text{Rt}[a^2 + b^2, 2] + b*\text{E}^{(c + d*x)}), x] + \text{Int}[(e + f*x)^m*\text{E}^{(c + d*x)}/(a + \text{Rt}[a^2 + b^2, 2] + b*\text{E}^{(c + d*x)}), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[a^2 + b^2, 0]$

### Rule 2190

$\text{Int}[(((F\_)^{(g\_)*((e\_)+ (f\_)*(x\_))^{(n\_)})*((c\_)+ (d\_)*(x\_))^{(m\_)})/((a\_)+ (b\_)*((F\_)^{(g\_)*((e\_)+ (f\_)*(x\_))^{(n\_)}))^{(n\_)}), x\_Symbol] := \text{Simp}$

```

[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

### Rule 6742

```

Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]

```

### Rule 3718

```

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :=> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \operatorname{sech}(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \operatorname{sech}^2(c+dx) \tanh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= -\frac{(e+fx)^2 \operatorname{sech}^2(c+dx)}{2bd} - \frac{a \int (e+fx)^2 \operatorname{sech}^3(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)^2 \operatorname{sech}^3(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
&= -\frac{af(e+fx) \operatorname{sech}(c+dx)}{b^2 d^2} - \frac{(e+fx)^2 \operatorname{sech}^2(c+dx)}{2bd} + \frac{f(e+fx) \tanh(c+dx)}{bd^2} \\
&= -\frac{a(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{af^2 \tan^{-1}(\sinh(c+dx))}{b^2 d^3} - \frac{f^2 \log(\cosh(c+dx))}{bd^3} \\
&= -\frac{a^2 b (e+fx)^3}{3(a^2+b^2)^2 f} - \frac{a(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{af^2 \tan^{-1}(\sinh(c+dx))}{b^2 d^3} \\
&= -\frac{a^2 b (e+fx)^3}{3(a^2+b^2)^2 f} - \frac{a(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{af^2 \tan^{-1}(\sinh(c+dx))}{b^2 d^3} \\
&= -\frac{a(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} + \frac{a^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)} \\
&= -\frac{a(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} + \frac{a^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)} \\
&= -\frac{a(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} + \frac{a^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)} \\
&= -\frac{a(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} + \frac{a^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)} \\
&= -\frac{a(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2 d} + \frac{2a^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} + \frac{a^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)}
\end{aligned}$$

**Mathematica [B]** time = 31.3531, size = 3390, normalized size = 2.7

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Sech[c + d\*x]\*Tanh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] 
$$\begin{aligned} & -(-12a^2bd^3e^2E^{(2c)}x - 12a^2bd^3e^2E^{(2c)}f^2x - 12b^3d^3e^2E^{(2c)} \\ & f^2x - 12a^2bd^3e^2E^{(2c)}f^2x^2 - 4a^2bd^3E^{(2c)}f^2x^3 - 6a^3 \\ & d^2e^2\text{ArcTan}[E^{(c+d*x)}] + 6ab^2d^2e^2\text{ArcTan}[E^{(c+d*x)}] - 6a^3 \\ & d^2e^2E^{(2c)}\text{ArcTan}[E^{(c+d*x)}] + 6ab^2d^2e^2E^{(2c)}\text{ArcTan}[E^{(c+d*x)}] \\ & - 12a^3f^2\text{ArcTan}[E^{(c+d*x)}] - 12ab^2f^2\text{ArcTan}[E^{(c+d*x)}] \\ & - 12a^3E^{(2c)}f^2\text{ArcTan}[E^{(c+d*x)}] - 12ab^2E^{(2c)}f^2\text{ArcTan}[E^{(c+d*x)}] \\ & - (6I)a^3d^2efxx\text{Log}[1 - IE^{(c+d*x)}] + (6I)ab^2d^2ef \\ & xx\text{Log}[1 - IE^{(c+d*x)}] - (6I)a^3d^2e^2E^{(2c)}fxx\text{Log}[1 - IE^{(c+d*x)}] \\ & + (6I)ab^2d^2e^2E^{(2c)}fxx\text{Log}[1 - IE^{(c+d*x)}] - (3I)a^3d^2 \\ & f^2x^2\text{Log}[1 - IE^{(c+d*x)}] + (3I)ab^2d^2f^2x^2\text{Log}[1 - IE^{(c+d*x)}] \\ & - (3I)a^3d^2E^{(2c)}f^2x^2\text{Log}[1 - IE^{(c+d*x)}] + (3I)ab^2d^2 \\ & E^{(2c)}f^2x^2\text{Log}[1 - IE^{(c+d*x)}] + (6I)a^3d^2efxx\text{Log}[1 + IE^{(c+d*x)}] \\ & - (6I)ab^2d^2efxx\text{Log}[1 + IE^{(c+d*x)}] + (6I)a^3d^2 \\ & e^2E^{(2c)}fxx\text{Log}[1 + IE^{(c+d*x)}] - (6I)ab^2d^2e^2E^{(2c)}fxx\text{Log}[1 \\ & + IE^{(c+d*x)}] + (3I)a^3d^2f^2x^2\text{Log}[1 + IE^{(c+d*x)}] - (3I)ab^2 \\ & d^2f^2x^2\text{Log}[1 + IE^{(c+d*x)}] + (3I)a^3d^2E^{(2c)}f^2x^2\text{Log}[1 \\ & + IE^{(c+d*x)}] - (3I)ab^2d^2E^{(2c)}f^2x^2\text{Log}[1 + IE^{(c+d*x)}] \\ & + 6a^2bd^2e^2\text{Log}[1 + E^{(2(c+d*x))}] + 6a^2bd^2e^2E^{(2c)}\text{Log}[1 \\ & + E^{(2(c+d*x))}] + 6a^2bf^2\text{Log}[1 + E^{(2(c+d*x))}] + 6b^3f^2\text{Log}[1 \\ & + E^{(2(c+d*x))}] + 6a^2bE^{(2c)}f^2\text{Log}[1 + E^{(2(c+d*x))}] + 6b^3 \\ & E^{(2c)}f^2\text{Log}[1 + E^{(2(c+d*x))}] + 12a^2bd^2efxx\text{Log}[1 + E^{(2(c+d*x))}] \\ & + 12a^2bd^2e^2E^{(2c)}fxx\text{Log}[1 + E^{(2(c+d*x))}] + 6a^2bd^2 \\ & f^2x^2\text{Log}[1 + E^{(2(c+d*x))}] + 6a^2bd^2E^{(2c)}f^2x^2\text{Log}[1 + E^{(2(c+d*x))}] \\ & + (6I)a*(a^2 - b^2)*d*(1 + E^{(2c)})f*(e + f*x)*\text{PolyLog}[2, \\ & (-I)E^{(c+d*x)}] - (6I)a*(a^2 - b^2)*d*(1 + E^{(2c)})f*(e + f*x)*\text{PolyLog} \\ & [2, IE^{(c+d*x)}] + 6a^2bd^2efxx\text{PolyLog}[2, -E^{(2(c+d*x))}] + 6a^2bd \\ & e^2E^{(2c)}fxx\text{PolyLog}[2, -E^{(2(c+d*x))}] + 6a^2bd^2f^2xx\text{PolyLog}[2, -E^{(2(c+d*x))}] \\ & + 6a^2bd^2E^{(2c)}f^2xx\text{PolyLog}[2, -E^{(2(c+d*x))}] - (6I) \\ & a^3f^2\text{PolyLog}[3, (-I)E^{(c+d*x)}] + (6I)ab^2f^2\text{PolyLog}[3, (-I)E^{(c+d*x)}] \\ & - (6I)a^3E^{(2c)}f^2\text{PolyLog}[3, (-I)E^{(c+d*x)}] + (6I)ab^2 \\ & E^{(2c)}f^2\text{PolyLog}[3, (-I)E^{(c+d*x)}] + (6I)a^3f^2\text{PolyLog}[3, IE^{(c+d*x)}] \\ & - (6I)ab^2f^2\text{PolyLog}[3, IE^{(c+d*x)}] + (6I)a^3E^{(2c)} \\ & f^2\text{PolyLog}[3, IE^{(c+d*x)}] - (6I)ab^2E^{(2c)}f^2\text{PolyLog}[3, IE^{(c+d*x)}] \\ & - 3a^2bf^2\text{PolyLog}[3, -E^{(2(c+d*x))}] - 3a^2bE^{(2c)}f^2\text{Poly} \\ & \text{Log}[3, -E^{(2(c+d*x))}]/(6(a^2 + b^2)^2d^3(1 + E^{(2c)})) - (a^2b(6 \\ & e^2E^{(2c)}x + 6e^2E^{(2c)}f^2x^2 + 2E^{(2c)}f^2x^3 + (6a*\text{Sqrt}[a^2 + b^2] \\ & )*e^2\text{ArcTan}[(a + bE^{(c+d*x)})/\text{Sqrt}[-a^2 - b^2]])/(\text{Sqrt}[-(a^2 + b^2)^2]*d \\ & ) + (6a*\text{Sqrt}[-(a^2 + b^2)^2]*e^2E^{(2c)}\text{ArcTan}[(a + bE^{(c+d*x)})/\text{Sqrt}[- \\ & a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6a*\text{Sqrt}[-(a^2 + b^2)^2]*e^2\text{ArcTanh} \\ & [(a + bE^{(c+d*x)})/\text{Sqrt}[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6a*\text{Sqrt}[-( \\ & a^2 + b^2)^2]*e^2E^{(2c)}\text{ArcTanh}[(a + bE^{(c+d*x)})/\text{Sqrt}[a^2 + b^2]])/((- \end{aligned}$$

$$\begin{aligned}
& a^2 - b^2)^{(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))]/d - (3*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))]/d + \\
& (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]]])/d - \\
& (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]]])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)* \\
& E^(2*c)]]])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[ \\
& (a^2 + b^2)*E^(2*c)]]])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqr \\
& t[(a^2 + b^2)*E^(2*c)]]])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a \\
& *E^c + Sqrt[(a^2 + b^2)*E^(2*c)]]])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x) \\
& )/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]]])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E \\
& ^{(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]]])/d - (6*(-1 + E^(2*c))*f \\
& *(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c) \\
& ]))])/d^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a \\
& *E^c + Sqrt[(a^2 + b^2)*E^(2*c)]]))])/d^2 - (6*f^2*PolyLog[3, -((b*E^(2*c + \\
& d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]]))])/d^3 + (6*E^(2*c)*f^2*PolyLog[3 \\
& , -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]]))])/d^3 - (6*f^2*P \\
& olyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]]))])/d^3 + \\
& (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^( \\
& 2*c)]]))])/d^3)/(3*(a^2 + b^2)^2*(-1 + E^(2*c))) + (Csch[c]*Sech[c]*Sech[c \\
& + d*x]^2*(6*a^2*b*e*f + 6*b^3*e*f + 12*a^2*b*d^2*e^2*x + 6*a^2*b*f^2*x + 6* \\
& b^3*f^2*x + 12*a^2*b*d^2*e*f*x^2 + 4*a^2*b*d^2*f^2*x^3 - 6*a^2*b*e*f*Cosh[2 \\
& *c] - 6*b^3*e*f*Cosh[2*c] - 6*a^2*b*f^2*x*Cosh[2*c] - 6*b^3*f^2*x*Cosh[2*c] \\
& - 6*a^2*b*e*f*Cosh[2*d*x] - 6*b^3*e*f*Cosh[2*d*x] - 6*a^2*b*f^2*x*Cosh[2*d \\
& *x] - 6*b^3*f^2*x*Cosh[2*d*x] + 3*a^3*d*e^2*Cosh[c - d*x] + 3*a*b^2*d*e^2*C \\
& osh[c - d*x] + 6*a^3*d*e*f*x*Cosh[c - d*x] + 6*a*b^2*d*e*f*x*Cosh[c - d*x] \\
& + 3*a^3*d*f^2*x^2*Cosh[c - d*x] + 3*a*b^2*d*f^2*x^2*Cosh[c - d*x] - 3*a^3*d \\
& *e^2*Cosh[3*c + d*x] - 3*a*b^2*d*e^2*Cosh[3*c + d*x] - 6*a^3*d*e*f*x*Cosh[3 \\
& *c + d*x] - 6*a*b^2*d*e*f*x*Cosh[3*c + d*x] - 3*a^3*d*f^2*x^2*Cosh[3*c + d \\
& x] - 3*a*b^2*d*f^2*x^2*Cosh[3*c + d*x] + 6*a^2*b*e*f*Cosh[2*c + 2*d*x] + 6* \\
& b^3*e*f*Cosh[2*c + 2*d*x] + 12*a^2*b*d^2*e^2*x*Cosh[2*c + 2*d*x] + 6*a^2*b* \\
& f^2*x*Cosh[2*c + 2*d*x] + 6*b^3*f^2*x*Cosh[2*c + 2*d*x] + 12*a^2*b*d^2*e*f* \\
& x^2*Cosh[2*c + 2*d*x] + 4*a^2*b*d^2*f^2*x^3*Cosh[2*c + 2*d*x] - 6*a^2*b*d*e \\
& ^2*Sinh[2*c] - 6*b^3*d*e^2*Sinh[2*c] - 12*a^2*b*d*e*f*x*Sinh[2*c] - 12*b^3* \\
& d*e*f*x*Sinh[2*c] - 6*a^2*b*d*f^2*x^2*Sinh[2*c] - 6*b^3*d*f^2*x^2*Sinh[2*c] \\
& - 6*a^3*e*f*Sinh[c - d*x] - 6*a*b^2*e*f*Sinh[c - d*x] - 6*a^3*f^2*x*Sinh[c \\
& - d*x] - 6*a*b^2*f^2*x*Sinh[c - d*x] - 6*a^3*e*f*Sinh[3*c + d*x] - 6*a*b^2 \\
& *e*f*Sinh[3*c + d*x] - 6*a^3*f^2*x*Sinh[3*c + d*x] - 6*a*b^2*f^2*x*Sinh[3*c \\
& + d*x]))/(24*(a^2 + b^2)^2*d^2)
\end{aligned}$$

**Maple [F]** time = 0.448, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \operatorname{sech}(dx + c) (\tanh(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*sech(d\*x+c)\*tanh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*sech(d\*x+c)\*tanh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sech(d\*x+c)\*tanh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $a^3 d^2 f^2 \operatorname{integrate}(x^2 e^{(d x + c)} / (a^4 d^2 e^{(2 d x + 2 c)} + 2 a^2 b^2 d^2 e^{(2 d x + 2 c)} + b^4 d^2 e^{(2 d x + 2 c)} + a^4 d^2 + 2 a^2 b^2 d^2 + b^4 d^2), x) - a b^2 d^2 f^2 \operatorname{integrate}(x^2 e^{(d x + c)} / (a^4 d^2 e^{(2 d x + 2 c)} + 2 a^2 b^2 d^2 e^{(2 d x + 2 c)} + b^4 d^2 e^{(2 d x + 2 c)} + a^4 d^2 + 2 a^2 b^2 d^2 + b^4 d^2), x) + 2 a^2 b d^2 f^2 \operatorname{integrate}(x^2 / (a^4 d^2 e^{(2 d x + 2 c)} + 2 a^2 b^2 d^2 e^{(2 d x + 2 c)} + b^4 d^2 e^{(2 d x + 2 c)} + a^4 d^2 + 2 a^2 b^2 d^2 + b^4 d^2), x) + 2 a^3 d^2 e f \operatorname{integrate}(x e^{(d x + c)} / (a^4 d^2 e^{(2 d x + 2 c)} + 2 a^2 b^2 d^2 e^{(2 d x + 2 c)} + b^4 d^2 e^{(2 d x + 2 c)} + a^4 d^2 + 2 a^2 b^2 d^2 + b^4 d^2), x) - 2 a b^2 d^2 e f \operatorname{integrate}(x e^{(d x + c)} / (a^4 d^2 e^{(2 d x + 2 c)} + 2 a^2 b^2 d^2 e^{(2 d x + 2 c)} + b^4 d^2 e^{(2 d x + 2 c)} + a^4 d^2 + 2 a^2 b^2 d^2 + b^4 d^2), x) + 4 a^2 b d^2 e f \operatorname{integrate}(x / (a^4 d^2 e^{(2 d x + 2 c)} + 2 a^2 b^2 d^2 e^{(2 d x + 2 c)} + b^4 d^2 e^{(2 d x + 2 c)} + a^4 d^2 + 2 a^2 b^2 d^2 + b^4 d^2), x) + a^2 b f^2 (2 (d x + c) / ((a^4 + 2 a^2 b^2 + b^4) d^3) - \log(e^{(2 d x + 2 c)} + 1) / ((a^4 + 2 a^2 b^2 + b^4) d^3)) + b^3 f^2 (2 (d x + c) / ((a^4 + 2 a^2 b^2 + b^4) d^3) - \log(e^{(2 d x + 2 c)} + 1) / ((a^4 + 2 a^2 b^2 + b^4) d^3)) + (a^2 b \log(-2 a e^{(-d x - c)} + b e^{(-2 d x - 2 c)} - b) / ((a^4 + 2 a^2 b^2 + b^4) d) - a^2 b \log(e^{(-2 d x - 2 c)} + 1) / ((a^4 + 2 a^2 b^2 + b^4) d) - (a^3 - a b^2) \operatorname{arctan}(e^{(-d x - c)}) / ((a^4 + 2 a^2 b^2 + b^4) d) - (a e^{(-d x - c)} + 2 b e^{(-2 d x - 2 c)} - a e^{(-3 d x - 3 c)}) / ((a^2 + b^2 + 2 (a^2 + b^2) e^{(-2 d x - 2 c)} + (a^2 + b^2) e^{(-4 d x - 4 c)}) d)) e^2 + 2 a^3 f^2 \operatorname{arctan}(e^{(d$

```

*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3) + 2*a*b^2*f^2*arctan(e^(d*x + c))/((
a^4 + 2*a^2*b^2 + b^4)*d^3) - (2*b*f^2*x + 2*b*e*f + (a*d*f^2*x^2*e^(3*c) +
2*a*e*f*e^(3*c) + 2*(d*e*f + f^2)*a*x*e^(3*c))*e^(3*d*x) + 2*(b*d*f^2*x^2*
e^(2*c) + b*e*f*e^(2*c) + (2*d*e*f + f^2)*b*x*e^(2*c))*e^(2*d*x) - (a*d*f^2
*x^2*e^c - 2*a*e*f*e^c + 2*(d*e*f - f^2)*a*x*e^c)*e^(d*x))/(a^2*d^2 + b^2*d
^2 + (a^2*d^2*e^(4*c) + b^2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b
^2*d^2*e^(2*c))*e^(2*d*x)) - integrate(2*(a^2*b^2*f^2*x^2 + 2*a^2*b^2*e*f*x
- (a^3*b*f^2*x^2*e^c + 2*a^3*b*e*f*x*e^c)*e^(d*x))/(a^4*b + 2*a^2*b^3 + b^
5 - (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e^
c + 2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x)

```

---

**Fricas [C]** time = 5.42971, size = 24496, normalized size = 19.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm
m="fricas")

```

```

[Out] 1/2*(4*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*cosh(d*x + c)^4 + 4*((
a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*sinh(d*x + c)^4 - 4*(a^2*b + b^
3)*d*e*f + 4*(a^2*b + b^3)*c*f^2 - 2*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a*
b^2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^3 + a*b^2)*d^2*e*f + (a^3 + a*
b^2)*d*f^2)*x)*cosh(d*x + c)^3 - 2*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a*b^
2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^3 + a*b^2)*d^2*e*f + (a^3 + a*b^
2)*d*f^2)*x - 8*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*cosh(d*x + c)
)*sinh(d*x + c)^3 - 4*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 +
(a^2*b + b^3)*d*e*f - 2*(a^2*b + b^3)*c*f^2 + (2*(a^2*b + b^3)*d^2*e*f - (a
^2*b + b^3)*d*f^2)*x)*cosh(d*x + c)^2 - 2*(2*(a^2*b + b^3)*d^2*f^2*x^2 + 2*
(a^2*b + b^3)*d^2*e^2 + 2*(a^2*b + b^3)*d*e*f - 4*(a^2*b + b^3)*c*f^2 - 12*
((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*cosh(d*x + c)^2 + 2*(2*(a^2*b
+ b^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x + 3*((a^3 + a*b^2)*d^2*f^2*x^2 + (
a^3 + a*b^2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^3 + a*b^2)*d^2*e*f + (
a^3 + a*b^2)*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*((a^3 + a*b^2)*d^
2*f^2*x^2 + (a^3 + a*b^2)*d^2*e^2 - 2*(a^3 + a*b^2)*d*e*f + 2*((a^3 + a*b^2)
)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x)*cosh(d*x + c) + 4*(a^2*b*d*f^2*x + a^2*
b*d*e*f + (a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^4 + 4*(a^2*b*d*f^2*x
+ a^2*b*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b*d*f^2*x + a^2*b*d*e*f)
)*sinh(d*x + c)^4 + 2*(a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^2 + 2*(a^
2*b*d*f^2*x + a^2*b*d*e*f + 3*(a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 4*((a^2*b*d*f^2*x + a^2*b*d*e*f)*cosh(d*x + c)^3 + (a^2

```





$$\begin{aligned}
& ^2) \cosh(dx + c) \sinh(dx + c) \log(2b \cosh(dx + c) + 2b \sinh(dx + c) \\
& + 2b \sqrt{(a^2 + b^2)/b^2} + 2a) + 2(a^2 b d^2 e^2 - 2a^2 b c d e f + \\
& a^2 b c^2 f^2 + (a^2 b d^2 e^2 - 2a^2 b c d e f + a^2 b c^2 f^2) \cosh(dx \\
& + c)^4 + 4(a^2 b d^2 e^2 - 2a^2 b c d e f + a^2 b c^2 f^2) \cosh(dx + c) * \\
& \sinh(dx + c)^3 + (a^2 b d^2 e^2 - 2a^2 b c d e f + a^2 b c^2 f^2) \sinh(dx \\
& x + c)^4 + 2(a^2 b d^2 e^2 - 2a^2 b c d e f + a^2 b c^2 f^2) \cosh(dx + c \\
& )^2 + 2(a^2 b d^2 e^2 - 2a^2 b c d e f + a^2 b c^2 f^2 + 3(a^2 b d^2 e^2 \\
& - 2a^2 b c d e f + a^2 b c^2 f^2) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4(( \\
& a^2 b d^2 e^2 - 2a^2 b c d e f + a^2 b c^2 f^2) \cosh(dx + c)^3 + (a^2 b d \\
& ^2 e^2 - 2a^2 b c d e f + a^2 b c^2 f^2) \cosh(dx + c)) \sinh(dx + c) \log \\
& (2b \cosh(dx + c) + 2b \sinh(dx + c) - 2b \sqrt{(a^2 + b^2)/b^2} + 2a) + \\
& 2(a^2 b d^2 f^2 x^2 + 2a^2 b d^2 e f x + 2a^2 b c d e f - a^2 b c^2 f^2 \\
& + (a^2 b d^2 f^2 x^2 + 2a^2 b d^2 e f x + 2a^2 b c d e f - a^2 b c^2 f^2 \\
& ) \cosh(dx + c)^4 + 4(a^2 b d^2 f^2 x^2 + 2a^2 b d^2 e f x + 2a^2 b c d e \\
& e f - a^2 b c^2 f^2) \cosh(dx + c) \sinh(dx + c)^3 + (a^2 b d^2 f^2 x^2 + 2 \\
& * a^2 b d^2 e f x + 2a^2 b c d e f - a^2 b c^2 f^2) \sinh(dx + c)^4 + 2(a^ \\
& 2 b d^2 f^2 x^2 + 2a^2 b d^2 e f x + 2a^2 b c d e f - a^2 b c^2 f^2) \cosh \\
& (dx + c)^2 + 2(a^2 b d^2 f^2 x^2 + 2a^2 b d^2 e f x + 2a^2 b c d e f - \\
& a^2 b c^2 f^2 + 3(a^2 b d^2 f^2 x^2 + 2a^2 b d^2 e f x + 2a^2 b c d e f \\
& - a^2 b c^2 f^2) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4((a^2 b d^2 f^2 x^2 + \\
& 2a^2 b d^2 e f x + 2a^2 b c d e f - a^2 b c^2 f^2) \cosh(dx + c)^3 + (a^ \\
& 2 b d^2 f^2 x^2 + 2a^2 b d^2 e f x + 2a^2 b c d e f - a^2 b c^2 f^2) \cosh \\
& (dx + c)) \sinh(dx + c) \log(-(a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh \\
& (dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b) + 2(a^2 b d^2 f \\
& ^2 x^2 + 2a^2 b d^2 e f x + 2a^2 b c d e f - a^2 b c^2 f^2 + (a^2 b d^2 f \\
& ^2 x^2 + 2a^2 b d^2 e f x + 2a^2 b c d e f - a^2 b c^2 f^2) \cosh(dx + c) \\
& ^4 + 4(a^2 b d^2 f^2 x^2 + 2a^2 b d^2 e f x + 2a^2 b c d e f - a^2 b c^2 \\
& * f^2) \cosh(dx + c) \sinh(dx + c)^3 + (a^2 b d^2 f^2 x^2 + 2a^2 b d^2 e f x \\
& x + 2a^2 b c d e f - a^2 b c^2 f^2) \sinh(dx + c)^4 + 2(a^2 b d^2 f^2 x^2 \\
& + 2a^2 b d^2 e f x + 2a^2 b c d e f - a^2 b c^2 f^2) \cosh(dx + c)^2 + 2 \\
& * (a^2 b d^2 f^2 x^2 + 2a^2 b d^2 e f x + 2a^2 b c d e f - a^2 b c^2 f^2 + \\
& 3(a^2 b d^2 f^2 x^2 + 2a^2 b d^2 e f x + 2a^2 b c d e f - a^2 b c^2 f^2 \\
& ) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4((a^2 b d^2 f^2 x^2 + 2a^2 b d^2 e e \\
& f x + 2a^2 b c d e f - a^2 b c^2 f^2) \cosh(dx + c)^3 + (a^2 b d^2 f^2 x^2 \\
& + 2a^2 b d^2 e f x + 2a^2 b c d e f - a^2 b c^2 f^2) \cosh(dx + c)) \sinh \\
& (dx + c) \log(-(a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \\
& \sqrt{(a^2 + b^2)/b^2} - b)/b) - (2a^2 b d^2 e^2 - 4a^2 b c d e f - I(a^3 - a b^2) d^2 e^2 + 2I(a^3 - a b^2) c d e f + (2a^2 b d^2 e \\
& e^2 - 4a^2 b c d e f - I(a^3 - a b^2) d^2 e^2 + 2I(a^3 - a b^2) c d e f \\
& + 2(a^2 b c^2 + a^2 b + b^3) f^2 - I(2a^3 + 2a b^2 + (a^3 - a b^2) c^2 \\
& ) f^2) \cosh(dx + c)^4 + (8a^2 b d^2 e^2 - 16a^2 b c d e f - 4I(a^3 - a \\
& * b^2) d^2 e^2 + 8I(a^3 - a b^2) c d e f + 8(a^2 b c^2 + a^2 b + b^3) f^2 \\
& - 4I(2a^3 + 2a b^2 + (a^3 - a b^2) c^2) f^2) \cosh(dx + c) \sinh(dx + \\
& c)^3 + (2a^2 b d^2 e^2 - 4a^2 b c d e f - I(a^3 - a b^2) d^2 e^2 + 2I(a^ \\
& a^3 - a b^2) c d e f + 2(a^2 b c^2 + a^2 b + b^3) f^2 - I(2a^3 + 2a b^2
\end{aligned}$$

$$\begin{aligned}
& + (a^3 - a*b^2)*c^2)*f^2)*\sinh(d*x + c)^4 + 2*(a^2*b*c^2 + a^2*b + b^3)*f^2 \\
& - I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2 + (4*a^2*b*d^2*e^2 - 8*a^2*b*c*d*e*f - 2*I*(a^3 - a*b^2)*d^2*e^2 + 4*I*(a^3 - a*b^2)*c*d*e*f + 4*(a^2*b*c^2 + a^2*b + b^3)*f^2 - 2*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*\cosh(d*x + c)^2 + (4*a^2*b*d^2*e^2 - 8*a^2*b*c*d*e*f - 2*I*(a^3 - a*b^2)*d^2*e^2 + 4*I*(a^3 - a*b^2)*c*d*e*f + 4*(a^2*b*c^2 + a^2*b + b^3)*f^2 - 2*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2 + (12*a^2*b*d^2*e^2 - 24*a^2*b*c*d*e*f - 6*I*(a^3 - a*b^2)*d^2*e^2 + 12*I*(a^3 - a*b^2)*c*d*e*f + 12*(a^2*b*c^2 + a^2*b + b^3)*f^2 - 6*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + ((8*a^2*b*d^2*e^2 - 16*a^2*b*c*d*e*f - 4*I*(a^3 - a*b^2)*d^2*e^2 + 8*I*(a^3 - a*b^2)*c*d*e*f + 8*(a^2*b*c^2 + a^2*b + b^3)*f^2 - 4*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*\cosh(d*x + c)^3 + (8*a^2*b*d^2*e^2 - 16*a^2*b*c*d*e*f - 4*I*(a^3 - a*b^2)*d^2*e^2 + 8*I*(a^3 - a*b^2)*c*d*e*f + 8*(a^2*b*c^2 + a^2*b + b^3)*f^2 - 4*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + I) - (2*a^2*b*d^2*e^2 - 4*a^2*b*c*d*e*f + I*(a^3 - a*b^2)*d^2*e^2 - 2*I*(a^3 - a*b^2)*c*d*e*f + (2*a^2*b*d^2*e^2 - 4*a^2*b*c*d*e*f + I*(a^3 - a*b^2)*d^2*e^2 - 2*I*(a^3 - a*b^2)*c*d*e*f + 2*(a^2*b*c^2 + a^2*b + b^3)*f^2 + I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*\cosh(d*x + c)^4 + (8*a^2*b*d^2*e^2 - 16*a^2*b*c*d*e*f + 4*I*(a^3 - a*b^2)*d^2*e^2 - 8*I*(a^3 - a*b^2)*c*d*e*f + 8*(a^2*b*c^2 + a^2*b + b^3)*f^2 + 4*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (2*a^2*b*d^2*e^2 - 4*a^2*b*c*d*e*f + I*(a^3 - a*b^2)*d^2*e^2 - 2*I*(a^3 - a*b^2)*c*d*e*f + 2*(a^2*b*c^2 + a^2*b + b^3)*f^2 + I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*\sinh(d*x + c)^4 + 2*(a^2*b*c^2 + a^2*b + b^3)*f^2 + I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2 + (4*a^2*b*d^2*e^2 - 8*a^2*b*c*d*e*f + 2*I*(a^3 - a*b^2)*d^2*e^2 - 4*I*(a^3 - a*b^2)*c*d*e*f + 4*(a^2*b*c^2 + a^2*b + b^3)*f^2 + 2*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*\cosh(d*x + c)^2 + (4*a^2*b*d^2*e^2 - 8*a^2*b*c*d*e*f + 2*I*(a^3 - a*b^2)*d^2*e^2 - 4*I*(a^3 - a*b^2)*c*d*e*f + 4*(a^2*b*c^2 + a^2*b + b^3)*f^2 + 2*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2 + (12*a^2*b*d^2*e^2 - 24*a^2*b*c*d*e*f + 6*I*(a^3 - a*b^2)*d^2*e^2 - 12*I*(a^3 - a*b^2)*c*d*e*f + 12*(a^2*b*c^2 + a^2*b + b^3)*f^2 + 6*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + ((8*a^2*b*d^2*e^2 - 16*a^2*b*c*d*e*f + 4*I*(a^3 - a*b^2)*d^2*e^2 - 8*I*(a^3 - a*b^2)*c*d*e*f + 8*(a^2*b*c^2 + a^2*b + b^3)*f^2 + 4*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*\cosh(d*x + c)^3 + (8*a^2*b*d^2*e^2 - 16*a^2*b*c*d*e*f + 4*I*(a^3 - a*b^2)*d^2*e^2 - 8*I*(a^3 - a*b^2)*c*d*e*f + 8*(a^2*b*c^2 + a^2*b + b^3)*f^2 + 4*I*(2*a^3 + 2*a*b^2 + (a^3 - a*b^2)*c^2)*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - I) - (2*a^2*b*d^2*f^2*x^2 + 4*a^2*b*d^2*e*f*x + 4*a^2*b*c*d*e*f - 2*a^2*b*c^2*f^2 + I*(a^3 - a*b^2)*d^2*f^2*x^2 + 2*I*(a^3 - a*b^2)*d^2*e*f*x + 2*I*(a^3 - a*b^2)*c*d*e*f - I*(a^3 - a*b^2)*c^2*f^2 + (2*a^2*b*d^2*f^2*x^2 + 4*a^2*b*d^2*e*f*x + 4*a^2*b*c*d*e*f - 2*a^2*b*c^2*f^2 + I*(a^3 - a*b^2)*d^2*f^2*x^2 + 2*I*(a^3 - a*b^2)*d^2*e*f*x + 2*I*(a^3 - a*b^2)*c*d*e*f - I*(a^3 - a*b^2)*c^2*f^2)*\cosh(d*x + c)^4 + (8*a^2*b*d^2*f^2*x^2 + 16*a^2*b*d^2*e*f*x + 16*a^2*b*
\end{aligned}$$

$$\begin{aligned}
& c*d*e*f - 8*a^2*b*c^2*f^2 + 4*I*(a^3 - a*b^2)*d^2*f^2*x^2 + 8*I*(a^3 - a*b^2) \\
& *d^2*e*f*x + 8*I*(a^3 - a*b^2)*c*d*e*f - 4*I*(a^3 - a*b^2)*c^2*f^2)*\cosh( \\
& d*x + c)*\sinh(d*x + c)^3 + (2*a^2*b*d^2*f^2*x^2 + 4*a^2*b*d^2*e*f*x + 4*a^2 \\
& *b*c*d*e*f - 2*a^2*b*c^2*f^2 + I*(a^3 - a*b^2)*d^2*f^2*x^2 + 2*I*(a^3 - a*b \\
& ^2)*d^2*e*f*x + 2*I*(a^3 - a*b^2)*c*d*e*f - I*(a^3 - a*b^2)*c^2*f^2)*\sinh(d \\
& *x + c)^4 + (4*a^2*b*d^2*f^2*x^2 + 8*a^2*b*d^2*e*f*x + 8*a^2*b*c*d*e*f - 4* \\
& a^2*b*c^2*f^2 + 2*I*(a^3 - a*b^2)*d^2*f^2*x^2 + 4*I*(a^3 - a*b^2)*d^2*e*f*x \\
& + 4*I*(a^3 - a*b^2)*c*d*e*f - 2*I*(a^3 - a*b^2)*c^2*f^2)*\cosh(d*x + c)^2 + \\
& (4*a^2*b*d^2*f^2*x^2 + 8*a^2*b*d^2*e*f*x + 8*a^2*b*c*d*e*f - 4*a^2*b*c^2*f \\
& ^2 + 2*I*(a^3 - a*b^2)*d^2*f^2*x^2 + 4*I*(a^3 - a*b^2)*d^2*e*f*x + 4*I*(a^3 \\
& - a*b^2)*c*d*e*f - 2*I*(a^3 - a*b^2)*c^2*f^2 + (12*a^2*b*d^2*f^2*x^2 + 24* \\
& a^2*b*d^2*e*f*x + 24*a^2*b*c*d*e*f - 12*a^2*b*c^2*f^2 + 6*I*(a^3 - a*b^2)*d \\
& ^2*f^2*x^2 + 12*I*(a^3 - a*b^2)*d^2*e*f*x + 12*I*(a^3 - a*b^2)*c*d*e*f - 6* \\
& I*(a^3 - a*b^2)*c^2*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + ((8*a^2*b*d^2*f \\
& ^2*x^2 + 16*a^2*b*d^2*e*f*x + 16*a^2*b*c*d*e*f - 8*a^2*b*c^2*f^2 + 4*I*(a^3 \\
& - a*b^2)*d^2*f^2*x^2 + 8*I*(a^3 - a*b^2)*d^2*e*f*x + 8*I*(a^3 - a*b^2)*c*d \\
& *e*f - 4*I*(a^3 - a*b^2)*c^2*f^2)*\cosh(d*x + c)^3 + (8*a^2*b*d^2*f^2*x^2 + \\
& 16*a^2*b*d^2*e*f*x + 16*a^2*b*c*d*e*f - 8*a^2*b*c^2*f^2 + 4*I*(a^3 - a*b^2) \\
& *d^2*f^2*x^2 + 8*I*(a^3 - a*b^2)*d^2*e*f*x + 8*I*(a^3 - a*b^2)*c*d*e*f - 4* \\
& I*(a^3 - a*b^2)*c^2*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(I*\cosh(d*x + c) \\
& + I*\sinh(d*x + c) + 1) - (2*a^2*b*d^2*f^2*x^2 + 4*a^2*b*d^2*e*f*x + 4*a^2*b \\
& *c*d*e*f - 2*a^2*b*c^2*f^2 - I*(a^3 - a*b^2)*d^2*f^2*x^2 - 2*I*(a^3 - a*b^2) \\
& )*d^2*e*f*x - 2*I*(a^3 - a*b^2)*c*d*e*f + I*(a^3 - a*b^2)*c^2*f^2 + (2*a^2* \\
& b*d^2*f^2*x^2 + 4*a^2*b*d^2*e*f*x + 4*a^2*b*c*d*e*f - 2*a^2*b*c^2*f^2 - I*( \\
& a^3 - a*b^2)*d^2*f^2*x^2 - 2*I*(a^3 - a*b^2)*d^2*e*f*x - 2*I*(a^3 - a*b^2)* \\
& c*d*e*f + I*(a^3 - a*b^2)*c^2*f^2)*\cosh(d*x + c)^4 + (8*a^2*b*d^2*f^2*x^2 + \\
& 16*a^2*b*d^2*e*f*x + 16*a^2*b*c*d*e*f - 8*a^2*b*c^2*f^2 - 4*I*(a^3 - a*b^2) \\
& )*d^2*f^2*x^2 - 8*I*(a^3 - a*b^2)*d^2*e*f*x - 8*I*(a^3 - a*b^2)*c*d*e*f + 4 \\
& *I*(a^3 - a*b^2)*c^2*f^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (2*a^2*b*d^2*f^2* \\
& x^2 + 4*a^2*b*d^2*e*f*x + 4*a^2*b*c*d*e*f - 2*a^2*b*c^2*f^2 - I*(a^3 - a*b^ \\
& 2)*d^2*f^2*x^2 - 2*I*(a^3 - a*b^2)*d^2*e*f*x - 2*I*(a^3 - a*b^2)*c*d*e*f + \\
& I*(a^3 - a*b^2)*c^2*f^2)*\sinh(d*x + c)^4 + (4*a^2*b*d^2*f^2*x^2 + 8*a^2*b*d \\
& ^2*e*f*x + 8*a^2*b*c*d*e*f - 4*a^2*b*c^2*f^2 - 2*I*(a^3 - a*b^2)*d^2*f^2*x^ \\
& 2 - 4*I*(a^3 - a*b^2)*d^2*e*f*x - 4*I*(a^3 - a*b^2)*c*d*e*f + 2*I*(a^3 - a* \\
& b^2)*c^2*f^2)*\cosh(d*x + c)^2 + (4*a^2*b*d^2*f^2*x^2 + 8*a^2*b*d^2*e*f*x + \\
& 8*a^2*b*c*d*e*f - 4*a^2*b*c^2*f^2 - 2*I*(a^3 - a*b^2)*d^2*f^2*x^2 - 4*I*(a^ \\
& 3 - a*b^2)*d^2*e*f*x - 4*I*(a^3 - a*b^2)*c*d*e*f + 2*I*(a^3 - a*b^2)*c^2*f^ \\
& 2 + (12*a^2*b*d^2*f^2*x^2 + 24*a^2*b*d^2*e*f*x + 24*a^2*b*c*d*e*f - 12*a^2* \\
& b*c^2*f^2 - 6*I*(a^3 - a*b^2)*d^2*f^2*x^2 - 12*I*(a^3 - a*b^2)*d^2*e*f*x - \\
& 12*I*(a^3 - a*b^2)*c*d*e*f + 6*I*(a^3 - a*b^2)*c^2*f^2)*\cosh(d*x + c)^2)*\si \\
& nh(d*x + c)^2 + ((8*a^2*b*d^2*f^2*x^2 + 16*a^2*b*d^2*e*f*x + 16*a^2*b*c*d*e \\
& *f - 8*a^2*b*c^2*f^2 - 4*I*(a^3 - a*b^2)*d^2*f^2*x^2 - 8*I*(a^3 - a*b^2)*d^ \\
& 2*e*f*x - 8*I*(a^3 - a*b^2)*c*d*e*f + 4*I*(a^3 - a*b^2)*c^2*f^2)*\cosh(d*x + \\
& c)^3 + (8*a^2*b*d^2*f^2*x^2 + 16*a^2*b*d^2*e*f*x + 16*a^2*b*c*d*e*f - 8*a^ \\
& 2*b*c^2*f^2 - 4*I*(a^3 - a*b^2)*d^2*f^2*x^2 - 8*I*(a^3 - a*b^2)*d^2*e*f*x -
\end{aligned}$$

$$\begin{aligned}
& 8*I*(a^3 - a*b^2)*c*d*e*f + 4*I*(a^3 - a*b^2)*c^2*f^2)*\cosh(d*x + c))*\sinh \\
& (d*x + c))*\log(-I*\cosh(d*x + c) - I*\sinh(d*x + c) + 1) - 4*(a^2*b*f^2*\cosh( \\
& d*x + c)^4 + 4*a^2*b*f^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + a^2*b*f^2*\sinh(d*x \\
& + c)^4 + 2*a^2*b*f^2*\cosh(d*x + c)^2 + a^2*b*f^2 + 2*(3*a^2*b*f^2*\cosh(d*x \\
& + c)^2 + a^2*b*f^2)*\sinh(d*x + c)^2 + 4*(a^2*b*f^2*\cosh(d*x + c)^3 + a^2*b \\
& *f^2*\cosh(d*x + c))*\sinh(d*x + c))*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x \\
& + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\text{sqrt}((a^2 + b^2)/b^2))/b) - 4*( \\
& a^2*b*f^2*\cosh(d*x + c)^4 + 4*a^2*b*f^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + a^2 \\
& *b*f^2*\sinh(d*x + c)^4 + 2*a^2*b*f^2*\cosh(d*x + c)^2 + a^2*b*f^2 + 2*(3*a^2 \\
& *b*f^2*\cosh(d*x + c)^2 + a^2*b*f^2)*\sinh(d*x + c)^2 + 4*(a^2*b*f^2*\cosh(d*x \\
& + c)^3 + a^2*b*f^2*\cosh(d*x + c))*\sinh(d*x + c))*\text{polylog}(3, (a*\cosh(d*x + \\
& c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\text{sqrt}((a^2 + b^2) \\
& /b^2))/b) + (4*a^2*b*f^2 + 2*(2*a^2*b*f^2 - I*(a^3 - a*b^2)*f^2)*\cosh(d*x + \\
& c)^4 + 8*(2*a^2*b*f^2 - I*(a^3 - a*b^2)*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 \\
& + 2*(2*a^2*b*f^2 - I*(a^3 - a*b^2)*f^2)*\sinh(d*x + c)^4 - 2*I*(a^3 - a*b^2 \\
& )*f^2 + 4*(2*a^2*b*f^2 - I*(a^3 - a*b^2)*f^2)*\cosh(d*x + c)^2 + 4*(2*a^2*b* \\
& f^2 - I*(a^3 - a*b^2)*f^2 + 3*(2*a^2*b*f^2 - I*(a^3 - a*b^2)*f^2)*\cosh(d*x \\
& + c)^2)*\sinh(d*x + c)^2 + 8*((2*a^2*b*f^2 - I*(a^3 - a*b^2)*f^2)*\cosh(d*x + \\
& c)^3 + (2*a^2*b*f^2 - I*(a^3 - a*b^2)*f^2)*\cosh(d*x + c))*\sinh(d*x + c))* \\
& \text{polylog}(3, I*\cosh(d*x + c) + I*\sinh(d*x + c)) + 2*(2*a^2*b*f^2 + (2*a^2*b*f^ \\
& 2 + I*(a^3 - a*b^2)*f^2)*\cosh(d*x + c)^4 + 4*(2*a^2*b*f^2 + I*(a^3 - a*b^2) \\
& *f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (2*a^2*b*f^2 + I*(a^3 - a*b^2)*f^2)* \\
& \sinh(d*x + c)^4 + I*(a^3 - a*b^2)*f^2 + 2*(2*a^2*b*f^2 + I*(a^3 - a*b^2)*f^2) \\
& )*\cosh(d*x + c)^2 + 2*(2*a^2*b*f^2 + I*(a^3 - a*b^2)*f^2 + 3*(2*a^2*b*f^2 + \\
& I*(a^3 - a*b^2)*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((2*a^2*b*f^2 + \\
& I*(a^3 - a*b^2)*f^2)*\cosh(d*x + c)^3 + (2*a^2*b*f^2 + I*(a^3 - a*b^2)*f^2)* \\
& \cosh(d*x + c))*\sinh(d*x + c))*\text{polylog}(3, -I*\cosh(d*x + c) - I*\sinh(d*x + c) \\
& ) + 2*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a*b^2)*d^2*e^2 - 2*(a^3 + a*b^2)* \\
& d*e*f + 8*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*c*f^2)*\cosh(d*x + c)^3 - 3 \\
& *((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a*b^2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f \\
& + 2*((a^3 + a*b^2)*d^2*e*f + (a^3 + a*b^2)*d*f^2)*x)*\cosh(d*x + c)^2 + 2*( \\
& (a^3 + a*b^2)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x - 4*((a^2*b + b^3)*d^2*f^2*x \\
& ^2 + (a^2*b + b^3)*d^2*e^2 + (a^2*b + b^3)*d*e*f - 2*(a^2*b + b^3)*c*f^2 + \\
& (2*(a^2*b + b^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x \\
& + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3*\cosh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b \\
& ^4)*d^3*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d^3*\sinh(d* \\
& x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*\cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 \\
& + b^4)*d^3 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d^3*\cosh(d*x + c)^2 + (a^4 + 2*a \\
& ^2*b^2 + b^4)*d^3)*\sinh(d*x + c)^2 + 4*((a^4 + 2*a^2*b^2 + b^4)*d^3*\cosh(d* \\
& x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d^3*\cosh(d*x + c))*\sinh(d*x + c))
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sech(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.388 \quad \int \frac{(e+fx)\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=760

$$\frac{ia^3 f \operatorname{PolyLog}(2, -ie^{c+dx})}{2b^2 d^2 (a^2 + b^2)} - \frac{ia^3 f \operatorname{PolyLog}(2, -ie^{c+dx})}{d^2 (a^2 + b^2)^2} + \frac{ia^3 f \operatorname{PolyLog}(2, ie^{c+dx})}{2b^2 d^2 (a^2 + b^2)} + \frac{ia^3 f \operatorname{PolyLog}(2, ie^{c+dx})}{d^2 (a^2 + b^2)^2} + \frac{a^2 b f \operatorname{PolyLog}(2, -ie^{c+dx})}{d^2 (a^2 + b^2)^2}$$

```
[Out] -((a*(e + f*x)*ArcTan[E^(c + d*x)])/(b^2*d)) + (2*a^3*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)^2*d) + (a^3*(e + f*x)*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d) + (a^2*b*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (a^2*b*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (a^2*b*(e + f*x)*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)^2*d) + ((I/2)*a*f*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*d^2) - (I*a^3*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - ((I/2)*a^3*f*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) - ((I/2)*a*f*PolyLog[2, I*E^(c + d*x)])/(b^2*d^2) + (I*a^3*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) + ((I/2)*a^3*f*PolyLog[2, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) + (a^2*b*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) + (a^2*b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (a^2*b*f*PolyLog[2, -E^(2*(c + d*x))])/((2*(a^2 + b^2)^2*d^2) - (a*f*Sech[c + d*x])/(2*b^2*d^2) + (a^3*f*Sech[c + d*x])/(2*b^2*(a^2 + b^2)*d^2) - ((e + f*x)*Sech[c + d*x]^2)/(2*b*d) + (a^2*(e + f*x)*Sech[c + d*x]^2)/(2*b*(a^2 + b^2)*d) + (f*Tanh[c + d*x])/(2*b*d^2) - (a^2*f*Tanh[c + d*x])/(2*b*(a^2 + b^2)*d^2) - (a*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b^2*d) + (a^3*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b^2*(a^2 + b^2)*d)
```

**Rubi [A]** time = 1.14808, antiderivative size = 760, normalized size of antiderivative = 1., number of steps used = 42, number of rules used = 13, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$ , Rules used = {5583, 5451, 3767, 8, 4185, 4180, 2279, 2391, 5573, 5561, 2190, 6742, 3718}

$$\frac{ia^3 f \operatorname{PolyLog}(2, -ie^{c+dx})}{2b^2 d^2 (a^2 + b^2)} - \frac{ia^3 f \operatorname{PolyLog}(2, -ie^{c+dx})}{d^2 (a^2 + b^2)^2} + \frac{ia^3 f \operatorname{PolyLog}(2, ie^{c+dx})}{2b^2 d^2 (a^2 + b^2)} + \frac{ia^3 f \operatorname{PolyLog}(2, ie^{c+dx})}{d^2 (a^2 + b^2)^2} + \frac{a^2 b f \operatorname{PolyLog}(2, -ie^{c+dx})}{d^2 (a^2 + b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]), x]
```

```
[Out] -((a*(e + f*x)*ArcTan[E^(c + d*x)])/(b^2*d)) + (2*a^3*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)^2*d) + (a^3*(e + f*x)*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d) + (a^2*b*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) + (a^2*b*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/((a^2 + b^2)^2*d) - (a^2*b*(e + f*x)*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)^2*d) + ((I/2)*a*f*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*d^2) - (I*a^3*f*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) - ((I/2)*a^3*f*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) - ((I/2)*a*f*PolyLog[2, I*E^(c + d*x)])/(b^2*d^2) + (I*a^3*f*PolyLog[2, I*E^(c + d*x)])/(b^2*(a^2 + b^2)^2*d^2) + ((I/2)*a^3*f*PolyLog[2, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2) + (a^2*b*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) + (a^2*b*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (a^2*b*f*PolyLog[2, -E^(2*(c + d*x))])/((2*(a^2 + b^2)^2*d^2) - (a*f*Sech[c + d*x])/(2*b^2*d^2) + (a^3*f*Sech[c + d*x])/(2*b^2*(a^2 + b^2)*d^2) - ((e + f*x)*Sech[c + d*x]^2)/(2*b*d) + (a^2*(e + f*x)*Sech[c + d*x]^2)/(2*b*(a^2 + b^2)*d) + (f*Tanh[c + d*x])/(2*b*d^2) - (a^2*f*Tanh[c + d*x])/(2*b*(a^2 + b^2)*d^2) - (a*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b^2*d) + (a^3*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b^2*(a^2 + b^2)*d)
```

### Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 5451

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := -Simp[(((c + d*x)^m*Sech[a + b*x]^n)/(b*n), x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

### Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
-Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^m_], x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi))]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```



Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3718

```
Int[((c_) + (d_)*(x_))^(m_)*tan[(e_) + (Complex[0, fz_])*(f_)*(x_)], x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\operatorname{sech}^2(c+dx)\tanh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)} dx}{b} \\
&= -\frac{(e+fx)\operatorname{sech}^2(c+dx)}{2bd} - \frac{a \int (e+fx)\operatorname{sech}^3(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx}{b^2} \\
&= -\frac{af\operatorname{sech}(c+dx)}{2b^2d^2} - \frac{(e+fx)\operatorname{sech}^2(c+dx)}{2bd} - \frac{a(e+fx)\operatorname{sech}(c+dx)\tanh(c+dx)}{2b^2d} \\
&= -\frac{a(e+fx)\tan^{-1}(e^{c+dx})}{b^2d} - \frac{af\operatorname{sech}(c+dx)}{2b^2d^2} - \frac{(e+fx)\operatorname{sech}^2(c+dx)}{2bd} + \frac{f \tan^{-1}(e^{c+dx})}{b^2d} \\
&= -\frac{a^2b(e+fx)^2}{2(a^2+b^2)^2f} - \frac{a(e+fx)\tan^{-1}(e^{c+dx})}{b^2d} - \frac{af\operatorname{sech}(c+dx)}{2b^2d^2} - \frac{(e+fx)\operatorname{sech}^2(c+dx)}{2bd} \\
&= -\frac{a^2b(e+fx)^2}{2(a^2+b^2)^2f} - \frac{a(e+fx)\tan^{-1}(e^{c+dx})}{b^2d} + \frac{a^2b(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2d} \\
&= -\frac{a(e+fx)\tan^{-1}(e^{c+dx})}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}(e^{c+dx})}{(a^2+b^2)^2d} + \frac{a^3(e+fx)\tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d} \\
&= -\frac{a(e+fx)\tan^{-1}(e^{c+dx})}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}(e^{c+dx})}{(a^2+b^2)^2d} + \frac{a^3(e+fx)\tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d} \\
&= -\frac{a(e+fx)\tan^{-1}(e^{c+dx})}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}(e^{c+dx})}{(a^2+b^2)^2d} + \frac{a^3(e+fx)\tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d} \\
&= -\frac{a(e+fx)\tan^{-1}(e^{c+dx})}{b^2d} + \frac{2a^3(e+fx)\tan^{-1}(e^{c+dx})}{(a^2+b^2)^2d} + \frac{a^3(e+fx)\tan^{-1}(e^{c+dx})}{b^2(a^2+b^2)d}
\end{aligned}$$

**Mathematica [A]** time = 9.50031, size = 588, normalized size = 0.77

$$\frac{a(-if(a^2-b^2)\operatorname{PolyLog}(2,-ie^{c+dx})+if(a^2-b^2)\operatorname{PolyLog}(2,ie^{c+dx})-abf\operatorname{PolyLog}(2,-e^{2(c+dx)})+2a^2de\tan^{-1}(e^{c+dx}))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sech[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a^2*b*(-f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + a*(2*a*b*d*e*(c + d*x) - 2*a*b*c*f*(c + d*x) + a*b*f*(c + d*x)^2 + 2*a^2*d*e*ArcTan[E^(c + d*x)] - 2*b^2*d*e*ArcTan[E^(c + d*x)] - 2*a^2*c*f*ArcTan[E^(c + d*x)] + 2*b^2*c*f*ArcTan[E^(c + d*x)] + I*a^2*f*(c + d*x)*Log[1 - I*E^(c + d*x)] - I*b^2*f*(c + d*x)*Log[1 - I*E^(c + d*x)] - I*a^2*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + I*b^2*f*(c + d*x)*Log[1 + I*E^(c + d*x)] - 2*a*b*d*e*Log[1 + E^(2*(c + d*x))] + 2*a*b*c*f*Log[1 + E^(2*(c + d*x))] - 2*a*b*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] - I*(a^2 - b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] + I*(a^2 - b^2)*f*PolyLog[2, I*E^(c + d*x)] - a*b*f*PolyLog[2, -E^(2*(c + d*x))] - (a^2 + b^2)*d*(e + f*x)*Sech[c + d*x]^2*(b + a*Sinh[c + d*x]) + (a^2 + b^2)*f*Sech[c + d*x]*(-a + b*Sinh[c + d*x]))/(2*(a^2 + b^2)^2*d^2)
```

**Maple [B]** time = 0.174, size = 2068, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] 2/(a^2+b^2)/d*a^3*e/(2*a^2+2*b^2)*arctan(exp(d*x+c))+I/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))-2/(a^2+b^2)/d^2*a^3*f*c/(2*a^2+2*b^2)*arctan(exp(d*x+c))-I/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*dilog(1+I*exp(d*x+c))+I/(a^2+b^2)*a/d^2*b^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*c+I/(a^2+b^2)*a/d*b^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*x-I/(a^2+b^2)*a/d*b^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*x-1/(a^2+b^2)^(3/2)/d^2*a*b^3*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^(3/2)/d^2*a^3*b*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(1/2)/d^2*a*b*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-I/(a^2+b^2)*a/d^2*b^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*c-I/(a^2+b^2)*a/d^2*b^2*f/(2*a^2+2*b^2)*dilog(1-I*exp(d*x+c))+2/(a^2+b^2)*a^2/d*b*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2))-a)/(-a+(a^2+b^2)^(1/2))*x+2/(a^2+b^2)*a^2/d^2*b*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2))-a)/(-a+(a^2+b^2)^(1/2))*c-2/(a^2+b^2)*a^2/d^2*b*f*c/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/(a^2+b^2)*a^2/d^2*b*f*c/(2*a^2+2*b^2)*ln(1+ex
```

$$\begin{aligned}
& p(2dx+2c)) + I/(a^2+b^2) * a/d^2 * b^2 * f / (2a^2+2b^2) * \text{dilog}(1+I*\exp(dx+c)) - 2 \\
& / (a^2+b^2) * a^2/d * b * f / (2a^2+2b^2) * \ln(1-I*\exp(dx+c)) * x - 2 / (a^2+b^2) * a^2/d^2 \\
& * b * f / (2a^2+2b^2) * \ln(1-I*\exp(dx+c)) * c - 2 / (a^2+b^2) * a^2/d^2 * b * f / (2a^2+2b^2) \\
& * \ln(1+I*\exp(dx+c)) * c - I / (a^2+b^2) / d * a^3 * f / (2a^2+2b^2) * \ln(1+I*\exp(dx+c)) \\
& * x - I / (a^2+b^2) / d^2 * a^3 * f / (2a^2+2b^2) * \ln(1+I*\exp(dx+c)) * c + 2 / (a^2+b^2) * a^2 \\
& / d * b * f / (2a^2+2b^2) * \ln((b*\exp(dx+c) + (a^2+b^2)^{1/2} + a) / (a + (a^2+b^2)^{1/2})) \\
& * x + 1 / (a^2+b^2)^{3/2} / d * a * b^3 * e / (2a^2+2b^2) * \text{arctanh}(1/2 * (2 * b * \exp(dx+c) \\
& + 2 * a) / (a^2+b^2)^{1/2}) + 1 / (a^2+b^2)^{3/2} / d * a^3 * b * e / (2a^2+2b^2) * \text{arctanh}(1/ \\
& 2 * (2 * b * \exp(dx+c) + 2 * a) / (a^2+b^2)^{1/2}) + 2 / (a^2+b^2) * a^2 / d^2 * b * f / (2a^2+2b^2) \\
& * \ln((b*\exp(dx+c) + (a^2+b^2)^{1/2} + a) / (a + (a^2+b^2)^{1/2})) * c + 2 / (a^2+b^2) / d \\
& ^2 * a * b^2 * f * c / (2a^2+2b^2) * \text{arctan}(\exp(dx+c)) - 2 / (a^2+b^2) * a^2 / d * b * f / (2a^2+ \\
& 2b^2) * \ln(1+I*\exp(dx+c)) * x + I / (a^2+b^2) / d * a^3 * f / (2a^2+2b^2) * \ln(1-I*\exp(dx \\
& c)) * x + I / (a^2+b^2) / d^2 * a^3 * f / (2a^2+2b^2) * \ln(1-I*\exp(dx+c)) * c - 1 / (a^2+b^2) \\
& ^{1/2} / d * a * b * e / (2a^2+2b^2) * \text{arctanh}(1/2 * (2 * b * \exp(dx+c) + 2 * a) / (a^2+b^2)^{1/2}) - 2 / (a^2+b^2) / d * a * b^2 * e / (2a^2+2b^2) \\
& * \text{arctan}(\exp(dx+c)) + 2 / (a^2+b^2) * a^2 / d^2 * b * f / (2a^2+2b^2) * \text{dilog}((-b*\exp(dx+c) + (a^2+b^2)^{1/2} - a) / (-a + (a^2+b^2) \\
& ^{1/2})) - 2 / (a^2+b^2) * a^2 / d^2 * b * f / (2a^2+2b^2) * \text{dilog}(1-I*\exp(dx+c)) - 2 / (a^2+b^2) * a^2 / d^2 * b * f / (2a^2+2b^2) \\
& * \text{dilog}((b*\exp(dx+c) + (a^2+b^2)^{1/2} + a) / (a + (a^2+b^2)^{1/2})) - 2 / (a^2+b^2) * a^2 / d * b * e / (2a^2+2b^2) * \ln(1+\exp(2 * dx + 2 * c)) \\
& + 2 / (a^2+b^2) * a^2 / d * b * e / (2a^2+2b^2) * \ln(b*\exp(2 * dx + 2 * c) + 2 * a * \exp(dx+c) - b) - (a * d * f * x * \exp(3 * d \\
& * x + 3 * c) + a * d * e * \exp(3 * dx + 3 * c) + 2 * b * d * f * x * \exp(2 * dx + 2 * c) - a * d * f * x * \exp(dx+c) + a * \\
& f * \exp(3 * dx + 3 * c) + 2 * b * d * e * \exp(2 * dx + 2 * c) - a * d * e * \exp(dx+c) + b * f * \exp(2 * dx + 2 * c) \\
& + a * f * \exp(dx+c) + b * f) / d^2 / (a^2+b^2) / (1+\exp(2 * dx + 2 * c))^2
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left( \frac{a^2 b \log(-2ae^{-dx-c}) + be^{(-2dx-2c)} - b}{(a^4 + 2a^2b^2 + b^4)d} - \frac{a^2 b \log(e^{(-2dx-2c)} + 1)}{(a^4 + 2a^2b^2 + b^4)d} - \frac{(a^3 - ab^2) \arctan(e^{(-dx-c)})}{(a^4 + 2a^2b^2 + b^4)d} - \frac{ae^{(-dx-c)} + 2}{(a^2 + b^2 + 2(a^2 + b^2))} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sech(dx+c)\*tanh(dx+c)^2/(a+b\*sinh(dx+c)),x, algorithm="maxima")

[Out] (a^2\*b\*log(-2\*a\*e^(-d\*x - c) + b\*e^(-2\*d\*x - 2\*c) - b)/((a^4 + 2\*a^2\*b^2 + b^4)\*d) - a^2\*b\*log(e^(-2\*d\*x - 2\*c) + 1)/((a^4 + 2\*a^2\*b^2 + b^4)\*d) - (a^3 - a\*b^2)\*arctan(e^(-d\*x - c))/((a^4 + 2\*a^2\*b^2 + b^4)\*d) - (a\*e^(-d\*x - c) + 2\*b\*e^(-2\*d\*x - 2\*c) - a\*e^(-3\*d\*x - 3\*c))/((a^2 + b^2 + 2\*(a^2 + b^2))\*e^(-2\*d\*x - 2\*c) + (a^2 + b^2)\*e^(-4\*d\*x - 4\*c))\*d)) \* e - f \* (((a\*d\*x\*e^(3\*c) + a\*e^(3\*c))\*e^(3\*d\*x) + (2\*b\*d\*x\*e^(2\*c) + b\*e^(2\*c))\*e^(2\*d\*x) - (a\*d\*x\*e^c - a\*e^c)\*e^(d\*x) + b)/(a^2\*d^2 + b^2\*d^2 + (a^2\*d^2\*e^(4\*c) + b^2\*d^2\*

```
e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x)) + 2*integrate(-(a^3*b*x*e^(d*x + c) - a^2*b^2*x)/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b*e^(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c + 2*a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) - 2*integrate(1/2*(2*a^2*b*x + (a^3*e^c - a*b^2*e^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c) + 2*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x))
```

**Fricas [B]** time = 3.666, size = 11621, normalized size = 15.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a*b^2)*f)*cosh(d*x + c)^3 + 2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a*b^2)*f)*sinh(d*x + c)^3 + 2*(2*(a^2*b + b^3)*d*f*x + 2*(a^2*b + b^3)*d*e + (a^2*b + b^3)*f)*cosh(d*x + c)^2 + 2*(2*(a^2*b + b^3)*d*f*x + 2*(a^2*b + b^3)*d*e + (a^2*b + b^3)*f + 3*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 + a*b^2)*f)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a^2*b + b^3)*f - 2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e - (a^3 + a*b^2)*f)*cosh(d*x + c) - 2*(a^2*b*f*cosh(d*x + c)^4 + 4*a^2*b*f*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*b*f*sinh(d*x + c)^4 + 2*a^2*b*f*cosh(d*x + c)^2 + a^2*b*f + 2*(3*a^2*b*f*cosh(d*x + c)^2 + a^2*b*f)*sinh(d*x + c)^2 + 4*(a^2*b*f*cosh(d*x + c)^3 + a^2*b*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*(a^2*b*f*cosh(d*x + c)^4 + 4*a^2*b*f*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*b*f*sinh(d*x + c)^4 + 2*a^2*b*f*cosh(d*x + c)^2 + a^2*b*f + 2*(3*a^2*b*f*cosh(d*x + c)^2 + a^2*b*f)*sinh(d*x + c)^2 + 4*(a^2*b*f*cosh(d*x + c)^3 + a^2*b*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + ((2*a^2*b*f - I*(a^3 - a*b^2)*f)*cosh(d*x + c)^4 + 4*(2*a^2*b*f - I*(a^3 - a*b^2)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2*b*f - I*(a^3 - a*b^2)*f)*sinh(d*x + c)^4 + 2*a^2*b*f + 2*(2*a^2*b*f - I*(a^3 - a*b^2)*f)*cosh(d*x + c)^2 + 2*(2*a^2*b*f + 3*(2*a^2*b*f - I*(a^3 - a*b^2)*f)*cosh(d*x + c)^2 - I*(a^3 - a*b^2)*f)*sinh(d*x + c)^2 - I*(a^3 - a*b^2)*f + 4*((2*a^2*b*f - I*(a^3 - a*b^2)*f)*cosh(d*x + c)^3 + (2*a^2*b*f - I*(a^3 - a*b^2)*f)*cosh(d*x + c))*sinh(d*x + c))*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + ((2*a^2*b*f + I*(a^3 - a*b^2)*f)*cosh(d*x + c)^4 + 4*(2*a^2*b*f + I*(a^3 - a*b^2)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2*b*f + I*(a^3 - a*b^2)*f)*sinh(d*x + c)^4 + 2*a^2*b*f
```

$$\begin{aligned}
& + 2*(2*a^2*b*f + I*(a^3 - a*b^2)*f)*\cosh(d*x + c)^2 + 2*(2*a^2*b*f + 3*(2*a^2*b*f + I*(a^3 - a*b^2)*f)*\cosh(d*x + c)^2 + I*(a^3 - a*b^2)*f)*\sinh(d*x + c)^2 + I*(a^3 - a*b^2)*f + 4*((2*a^2*b*f + I*(a^3 - a*b^2)*f)*\cosh(d*x + c))^3 + (2*a^2*b*f + I*(a^3 - a*b^2)*f)*\cosh(d*x + c))*\sinh(d*x + c))*\operatorname{dilog}(-I*\cosh(d*x + c) - I*\sinh(d*x + c)) - 2*(a^2*b*d*e - a^2*b*c*f + (a^2*b*d*e - a^2*b*c*f)*\cosh(d*x + c))^4 + 4*(a^2*b*d*e - a^2*b*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^2*b*d*e - a^2*b*c*f)*\sinh(d*x + c)^4 + 2*(a^2*b*d*e - a^2*b*c*f)*\cosh(d*x + c)^2 + 2*(a^2*b*d*e - a^2*b*c*f + 3*(a^2*b*d*e - a^2*b*c*f)*\cosh(d*x + c))^2*\sinh(d*x + c)^2 + 4*((a^2*b*d*e - a^2*b*c*f)*\cosh(d*x + c))^3 + (a^2*b*d*e - a^2*b*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - 2*(a^2*b*d*e - a^2*b*c*f + (a^2*b*d*e - a^2*b*c*f)*\cosh(d*x + c))^4 + 4*(a^2*b*d*e - a^2*b*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^2*b*d*e - a^2*b*c*f)*\sinh(d*x + c)^4 + 2*(a^2*b*d*e - a^2*b*c*f)*\cosh(d*x + c)^2 + 2*(a^2*b*d*e - a^2*b*c*f + 3*(a^2*b*d*e - a^2*b*c*f)*\cosh(d*x + c))^2*\sinh(d*x + c)^2 + 4*((a^2*b*d*e - a^2*b*c*f)*\cosh(d*x + c))^3 + (a^2*b*d*e - a^2*b*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - 2*(a^2*b*d*f*x + a^2*b*c*f + (a^2*b*d*f*x + a^2*b*c*f)*\cosh(d*x + c))^4 + 4*(a^2*b*d*f*x + a^2*b*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^2*b*d*f*x + a^2*b*c*f)*\sinh(d*x + c)^4 + 2*(a^2*b*d*f*x + a^2*b*c*f)*\cosh(d*x + c)^2 + 2*(a^2*b*d*f*x + a^2*b*c*f + 3*(a^2*b*d*f*x + a^2*b*c*f)*\cosh(d*x + c))^2*\sinh(d*x + c)^2 + 4*((a^2*b*d*f*x + a^2*b*c*f)*\cosh(d*x + c))^3 + (a^2*b*d*f*x + a^2*b*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 2*(a^2*b*d*f*x + a^2*b*c*f + (a^2*b*d*f*x + a^2*b*c*f)*\cosh(d*x + c))^4 + 4*(a^2*b*d*f*x + a^2*b*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^2*b*d*f*x + a^2*b*c*f)*\sinh(d*x + c)^4 + 2*(a^2*b*d*f*x + a^2*b*c*f)*\cosh(d*x + c)^2 + 2*(a^2*b*d*f*x + a^2*b*c*f + 3*(a^2*b*d*f*x + a^2*b*c*f)*\cosh(d*x + c))^2*\sinh(d*x + c)^2 + 4*((a^2*b*d*f*x + a^2*b*c*f)*\cosh(d*x + c))^3 + (a^2*b*d*f*x + a^2*b*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + (2*a^2*b*d*e - 2*a^2*b*c*f + (2*a^2*b*d*e - 2*a^2*b*c*f - I*(a^3 - a*b^2)*d*e + I*(a^3 - a*b^2)*c*f)*\cosh(d*x + c))^4 + (8*a^2*b*d*e - 8*a^2*b*c*f - 4*I*(a^3 - a*b^2)*d*e + 4*I*(a^3 - a*b^2)*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (2*a^2*b*d*e - 2*a^2*b*c*f - I*(a^3 - a*b^2)*d*e + I*(a^3 - a*b^2)*c*f)*\sinh(d*x + c)^4 - I*(a^3 - a*b^2)*d*e + I*(a^3 - a*b^2)*c*f + (4*a^2*b*d*e - 4*a^2*b*c*f - 2*I*(a^3 - a*b^2)*d*e + 2*I*(a^3 - a*b^2)*c*f)*\cosh(d*x + c)^2 + (4*a^2*b*d*e - 4*a^2*b*c*f - 2*I*(a^3 - a*b^2)*d*e + 2*I*(a^3 - a*b^2)*c*f + (12*a^2*b*d*e - 12*a^2*b*c*f - 6*I*(a^3 - a*b^2)*d*e + 6*I*(a^3 - a*b^2)*c*f)*\cosh(d*x + c))^2*\sinh(d*x + c)^2 + ((8*a^2*b*d*e - 8*a^2*b*c*f - 4*I*(a^3 - a*b^2)*d*e + 4*I*(a^3 - a*b^2)*c*f)*\cosh(d*x + c))^3 + (8*a^2*b*d*e - 8*a^2*b*c*f - 4*I*(a^3 - a*b^2)*d*e + 4*I*(a^3 - a*b^2)*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + I) + (2*a^2*b*d*e - 2*a^2*b*c*f + (2*a^2*b*d*e - 2*a^2*b*c*f + I*(a^3 - a*b^2)*d*e - I*(a^3 - a*b^2)*c*f)*\cosh(d*x + c))^4 + (8*a^2*b*d*e - 8*a
\end{aligned}$$

$$\begin{aligned}
&^2*b*c*f + 4*I*(a^3 - a*b^2)*d*e - 4*I*(a^3 - a*b^2)*c*f)*\cosh(d*x + c)*\sin \\
&h(d*x + c)^3 + (2*a^2*b*d*e - 2*a^2*b*c*f + I*(a^3 - a*b^2)*d*e - I*(a^3 - \\
&a*b^2)*c*f)*\sinh(d*x + c)^4 + I*(a^3 - a*b^2)*d*e - I*(a^3 - a*b^2)*c*f + ( \\
&4*a^2*b*d*e - 4*a^2*b*c*f + 2*I*(a^3 - a*b^2)*d*e - 2*I*(a^3 - a*b^2)*c*f)* \\
&\cosh(d*x + c)^2 + (4*a^2*b*d*e - 4*a^2*b*c*f + 2*I*(a^3 - a*b^2)*d*e - 2*I* \\
&(a^3 - a*b^2)*c*f + (12*a^2*b*d*e - 12*a^2*b*c*f + 6*I*(a^3 - a*b^2)*d*e - \\
&6*I*(a^3 - a*b^2)*c*f)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + ((8*a^2*b*d*e - 8 \\
&a^2*b*c*f + 4*I*(a^3 - a*b^2)*d*e - 4*I*(a^3 - a*b^2)*c*f)*\cosh(d*x + c)^3 \\
&+ (8*a^2*b*d*e - 8*a^2*b*c*f + 4*I*(a^3 - a*b^2)*d*e - 4*I*(a^3 - a*b^2)*c \\
&*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - I) + \\
&(2*a^2*b*d*f*x + 2*a^2*b*c*f + (2*a^2*b*d*f*x + 2*a^2*b*c*f + I*(a^3 - a*b^ \\
&2)*d*f*x + I*(a^3 - a*b^2)*c*f)*\cosh(d*x + c)^4 + (8*a^2*b*d*f*x + 8*a^2*b* \\
&c*f + 4*I*(a^3 - a*b^2)*d*f*x + 4*I*(a^3 - a*b^2)*c*f)*\cosh(d*x + c)*\sinh(d \\
&x + c)^3 + (2*a^2*b*d*f*x + 2*a^2*b*c*f + I*(a^3 - a*b^2)*d*f*x + I*(a^3 - \\
&a*b^2)*c*f)*\sinh(d*x + c)^4 + I*(a^3 - a*b^2)*d*f*x + I*(a^3 - a*b^2)*c*f \\
&+ (4*a^2*b*d*f*x + 4*a^2*b*c*f + 2*I*(a^3 - a*b^2)*d*f*x + 2*I*(a^3 - a*b^2 \\
&))*c*f)*\cosh(d*x + c)^2 + (4*a^2*b*d*f*x + 4*a^2*b*c*f + 2*I*(a^3 - a*b^2)*d \\
&>*f*x + 2*I*(a^3 - a*b^2)*c*f + (12*a^2*b*d*f*x + 12*a^2*b*c*f + 6*I*(a^3 - \\
&a*b^2)*d*f*x + 6*I*(a^3 - a*b^2)*c*f)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + (( \\
&8*a^2*b*d*f*x + 8*a^2*b*c*f + 4*I*(a^3 - a*b^2)*d*f*x + 4*I*(a^3 - a*b^2)*c \\
&>f)*\cosh(d*x + c)^3 + (8*a^2*b*d*f*x + 8*a^2*b*c*f + 4*I*(a^3 - a*b^2)*d*f* \\
&x + 4*I*(a^3 - a*b^2)*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(I*\cosh(d*x + c \\
&) + I*\sinh(d*x + c) + 1) + (2*a^2*b*d*f*x + 2*a^2*b*c*f + (2*a^2*b*d*f*x + \\
&2*a^2*b*c*f - I*(a^3 - a*b^2)*d*f*x - I*(a^3 - a*b^2)*c*f)*\cosh(d*x + c)^4 \\
&+ (8*a^2*b*d*f*x + 8*a^2*b*c*f - 4*I*(a^3 - a*b^2)*d*f*x - 4*I*(a^3 - a*b^2 \\
&))*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (2*a^2*b*d*f*x + 2*a^2*b*c*f - I*(a^ \\
&3 - a*b^2)*d*f*x - I*(a^3 - a*b^2)*c*f)*\sinh(d*x + c)^4 - I*(a^3 - a*b^2)*d \\
&>*f*x - I*(a^3 - a*b^2)*c*f + (4*a^2*b*d*f*x + 4*a^2*b*c*f - 2*I*(a^3 - a*b^ \\
&2)*d*f*x - 2*I*(a^3 - a*b^2)*c*f)*\cosh(d*x + c)^2 + (4*a^2*b*d*f*x + 4*a^2* \\
&b*c*f - 2*I*(a^3 - a*b^2)*d*f*x - 2*I*(a^3 - a*b^2)*c*f + (12*a^2*b*d*f*x + \\
&12*a^2*b*c*f - 6*I*(a^3 - a*b^2)*d*f*x - 6*I*(a^3 - a*b^2)*c*f)*\cosh(d*x + \\
&c)^2*\sinh(d*x + c)^2 + ((8*a^2*b*d*f*x + 8*a^2*b*c*f - 4*I*(a^3 - a*b^2)* \\
&d*f*x - 4*I*(a^3 - a*b^2)*c*f)*\cosh(d*x + c)^3 + (8*a^2*b*d*f*x + 8*a^2*b*c \\
&>*f - 4*I*(a^3 - a*b^2)*d*f*x - 4*I*(a^3 - a*b^2)*c*f)*\cosh(d*x + c))*\sinh(d \\
&>*x + c))*\log(-I*\cosh(d*x + c) - I*\sinh(d*x + c) + 1) - 2*((a^3 + a*b^2)*d*f \\
&>*x + (a^3 + a*b^2)*d*e - 3*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e + (a^3 \\
&+ a*b^2)*f)*\cosh(d*x + c)^2 - (a^3 + a*b^2)*f - 2*(2*(a^2*b + b^3)*d*f*x + \\
&2*(a^2*b + b^3)*d*e + (a^2*b + b^3)*f)*\cosh(d*x + c))*\sinh(d*x + c))/((a^4 \\
&+ 2*a^2*b^2 + b^4)*d^2*\cosh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d^2*\cosh \\
&(d*x + c)*\sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d^2*\sinh(d*x + c)^4 + 2 \\
&)*(a^4 + 2*a^2*b^2 + b^4)*d^2*\cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^2 \\
&+ 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d^2*\cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4 \\
&))*d^2)*\sinh(d*x + c)^2 + 4*((a^4 + 2*a^2*b^2 + b^4)*d^2*\cosh(d*x + c)^3 + ( \\
&a^4 + 2*a^2*b^2 + b^4)*d^2*\cosh(d*x + c))*\sinh(d*x + c))
\end{aligned}$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```



$$3.389 \quad \int \frac{\operatorname{sech}(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=121

$$\frac{a^2 b \log(a + b \sinh(c + dx))}{d(a^2 + b^2)^2} + \frac{a(a^2 - b^2) \tan^{-1}(\sinh(c + dx))}{2d(a^2 + b^2)^2} - \frac{a^2 b \log(\cosh(c + dx))}{d(a^2 + b^2)^2} - \frac{\operatorname{sech}^2(c + dx)(a \sinh(c + dx))}{2d(a^2 + b^2)}$$

[Out] (a\*(a^2 - b^2)\*ArcTan[Sinh[c + d\*x]])/(2\*(a^2 + b^2)^2\*d) - (a^2\*b\*Log[Cosh[c + d\*x]])/((a^2 + b^2)^2\*d) + (a^2\*b\*Log[a + b\*Sinh[c + d\*x]])/((a^2 + b^2)^2\*d) - (Sech[c + d\*x]^2\*(b + a\*Sinh[c + d\*x]))/(2\*(a^2 + b^2)\*d)

**Rubi [A]** time = 0.229112, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$ , Rules used = {2837, 12, 1647, 801, 635, 203, 260}

$$\frac{a^2 b \log(a + b \sinh(c + dx))}{d(a^2 + b^2)^2} + \frac{a(a^2 - b^2) \tan^{-1}(\sinh(c + dx))}{2d(a^2 + b^2)^2} - \frac{a^2 b \log(\cosh(c + dx))}{d(a^2 + b^2)^2} - \frac{\operatorname{sech}^2(c + dx)(a \sinh(c + dx))}{2d(a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] Int[(Sech[c + d\*x]\*Tanh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (a\*(a^2 - b^2)\*ArcTan[Sinh[c + d\*x]])/(2\*(a^2 + b^2)^2\*d) - (a^2\*b\*Log[Cosh[c + d\*x]])/((a^2 + b^2)^2\*d) + (a^2\*b\*Log[a + b\*Sinh[c + d\*x]])/((a^2 + b^2)^2\*d) - (Sech[c + d\*x]^2\*(b + a\*Sinh[c + d\*x]))/(2\*(a^2 + b^2)\*d)

### Rule 2837

Int[cos[(e\_.) + (f\_.)\*(x\_.)]^(p\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Dist[1/(b^p\*f), Subst[Int[(a + x)^m\*(c + (d\*x)/b)^n\*(b^2 - x^2)^((p - 1)/2), x], x, b\*S in[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

### Rule 12

Int[(a\_.)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match Q[u, (b\_.)\*(v\_) /; FreeQ[b, x]]

Rule 1647

```
Int[(Pq_)*((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_)))/((a_) + (c_)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{b^3 \operatorname{Subst}\left(\int \frac{x^2}{b^2(a+x)(-b^2-x^2)^2} dx, x, b \sinh(c+dx)\right)}{d} \\
&= \frac{b \operatorname{Subst}\left(\int \frac{x^2}{(a+x)(-b^2-x^2)^2} dx, x, b \sinh(c+dx)\right)}{d} \\
&= -\frac{\operatorname{sech}^2(c+dx)(b+a \sinh(c+dx))}{2(a^2+b^2)d} - \frac{\operatorname{Subst}\left(\int \frac{\frac{a^2b^2}{a^2+b^2} - \frac{ab^2x}{a^2+b^2}}{(a+x)(-b^2-x^2)} dx, x, b \sinh(c+dx)\right)}{2bd} \\
&= -\frac{\operatorname{sech}^2(c+dx)(b+a \sinh(c+dx))}{2(a^2+b^2)d} - \frac{\operatorname{Subst}\left(\int \left(-\frac{2a^2b^2}{(a^2+b^2)^2(a+x)} - \frac{ab^2(a^2-b^2-2ax)}{(a^2+b^2)^2(b^2+x^2)}\right) dx, x, b \sinh(c+dx)\right)}{2bd} \\
&= \frac{a^2b \log(a+b \sinh(c+dx))}{(a^2+b^2)^2 d} - \frac{\operatorname{sech}^2(c+dx)(b+a \sinh(c+dx))}{2(a^2+b^2)d} + \frac{(ab) \operatorname{Subst}\left(\int \frac{1}{a+x} dx, x, b \sinh(c+dx)\right)}{2bd} \\
&= \frac{a^2b \log(a+b \sinh(c+dx))}{(a^2+b^2)^2 d} - \frac{\operatorname{sech}^2(c+dx)(b+a \sinh(c+dx))}{2(a^2+b^2)d} - \frac{(a^2b) \operatorname{Subst}\left(\int \frac{1}{a+x} dx, x, b \sinh(c+dx)\right)}{2bd} \\
&= \frac{a(a^2-b^2) \tan^{-1}(\sinh(c+dx))}{2(a^2+b^2)^2 d} - \frac{a^2b \log(\cosh(c+dx))}{(a^2+b^2)^2 d} + \frac{a^2b \log(a+b \sinh(c+dx))}{(a^2+b^2)^2 d}
\end{aligned}$$

**Mathematica [C]** time = 0.327329, size = 130, normalized size = 1.07

$$\frac{b(a^2+b^2) \operatorname{sech}^2(c+dx) + a(a^2+b^2) \tanh(c+dx) \operatorname{sech}(c+dx) + a((a^2+b^2) \tan^{-1}(\sinh(c+dx)) + a((b+ia) \log(a+b \sinh(c+dx))))}{2d(a^2+b^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Sech[c + d\*x]\*Tanh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] -(a\*((a^2 + b^2)\*ArcTan[Sinh[c + d\*x]] + a\*((I\*a + b)\*Log[I - Sinh[c + d\*x]] + ((-I)\*a + b)\*Log[I + Sinh[c + d\*x]] - 2\*b\*Log[a + b\*Sinh[c + d\*x]])) + b\*(a^2 + b^2)\*Sech[c + d\*x]^2 + a\*(a^2 + b^2)\*Sech[c + d\*x]\*Tanh[c + d\*x])/(2\*(a^2 + b^2)^2\*d)

**Maple [B]** time = 0.003, size = 475, normalized size = 3.9

$$4 \frac{a^2 b \ln\left(\left(\tanh\left(\frac{1}{2} dx + \frac{c}{2}\right)\right)^2 a - 2 \tanh\left(\frac{1}{2} dx + \frac{c}{2}\right) b - a\right)}{d\left(4a^4 + 8a^2b^2 + 4b^4\right)} + \frac{a^3}{d\left(a^4 + 2a^2b^2 + b^4\right)} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3 \left(\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2 + \frac{a}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)`

[Out]  $4/d*a^2*b/(4*a^4+8*a^2*b^2+4*b^4)*\ln(\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)*b-a)+1/d/(a^4+2*a^2*b^2+b^4)/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)^3*a^3+1/d/(a^4+2*a^2*b^2+b^4)/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)^3*a*b^2+2/d/(a^4+2*a^2*b^2+b^4)/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)^2*a^2*b+2/d/(a^4+2*a^2*b^2+b^4)/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)^2*b^3-1/d/(a^4+2*a^2*b^2+b^4)/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)*a^3-1/d/(a^4+2*a^2*b^2+b^4)/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)*a*b^2+1/d/(a^4+2*a^2*b^2+b^4)*\arctan(\tanh(1/2*d*x+1/2*c))*a^3-1/d/(a^4+2*a^2*b^2+b^4)*\arctan(\tanh(1/2*d*x+1/2*c))*a*b^2-1/d/(a^4+2*a^2*b^2+b^4)*\ln(\tanh(1/2*d*x+1/2*c)^2+1)*a^2*b$

**Maxima [A]** time = 1.77111, size = 296, normalized size = 2.45

$$\frac{a^2 b \log\left(-2 a e^{-dx-c} + b e^{-2 dx-2c} - b\right)}{\left(a^4 + 2 a^2 b^2 + b^4\right) d} - \frac{a^2 b \log\left(e^{-2 dx-2c} + 1\right)}{\left(a^4 + 2 a^2 b^2 + b^4\right) d} - \frac{\left(a^3 - a b^2\right) \arctan\left(e^{-dx-c}\right)}{\left(a^4 + 2 a^2 b^2 + b^4\right) d} - \frac{a e^{-dx-c} + 2 b e^{-2 dx-2c}}{\left(a^2 + b^2 + 2\left(a^2 + b^2\right) e^{-2 dx-2c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out]  $a^2*b*\log(-2*a*e^{-d*x-c} + b*e^{-2*d*x-2*c} - b)/((a^4 + 2*a^2*b^2 + b^4)*d) - a^2*b*\log(e^{-2*d*x-2*c} + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (a^3 - a*b^2)*\arctan(e^{-d*x-c})/((a^4 + 2*a^2*b^2 + b^4)*d) - (a*e^{-d*x-c} + 2*b*e^{-2*d*x-2*c} - a*e^{-3*d*x-3*c})/((a^2 + b^2 + 2*(a^2 + b^2)*e^{-2*d*x-2*c} + (a^2 + b^2)*e^{-4*d*x-4*c})*d)$

**Fricas [B]** time = 2.41718, size = 2241, normalized size = 18.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((a^3 + a*b^2)*cosh(d*x + c)^3 + (a^3 + a*b^2)*sinh(d*x + c)^3 + 2*(a^2*b + b^3)*cosh(d*x + c)^2 + (2*a^2*b + 2*b^3 + 3*(a^3 + a*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - ((a^3 - a*b^2)*cosh(d*x + c)^4 + 4*(a^3 - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 - a*b^2)*sinh(d*x + c)^4 + a^3 - a*b^2 + 2*(a^3 - a*b^2)*cosh(d*x + c)^2 + 2*(a^3 - a*b^2 + 3*(a^3 - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3 - a*b^2)*cosh(d*x + c)^3 + (a^3 - a*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (a^3 + a*b^2)*cosh(d*x + c) - (a^2*b*cosh(d*x + c)^4 + 4*a^2*b*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*b*sinh(d*x + c)^4 + 2*a^2*b*cosh(d*x + c)^2 + a^2*b + 2*(3*a^2*b*cosh(d*x + c)^2 + a^2*b)*sinh(d*x + c)^2 + 4*(a^2*b*cosh(d*x + c)^3 + a^2*b*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + (a^2*b*cosh(d*x + c)^4 + 4*a^2*b*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*b*sinh(d*x + c)^4 + 2*a^2*b*cosh(d*x + c)^2 + a^2*b + 2*(3*a^2*b*cosh(d*x + c)^2 + a^2*b)*sinh(d*x + c)^2 + 4*(a^2*b*cosh(d*x + c)^3 + a^2*b*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - (a^3 + a*b^2 - 3*(a^3 + a*b^2)*cosh(d*x + c)^2 - 4*(a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d + 4*((a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d*cosh(d*x + c))*sinh(d*x + c))
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(c + dx) \operatorname{sech}(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)**2*sech(c + d*x)/(a + b*sinh(c + d*x)), x)
```

**Giac [A]** time = 1.26063, size = 305, normalized size = 2.52

$$\frac{\frac{a^2 b \log(e^{(2dx+2c)+1})}{a^4+2a^2b^2+b^4} - \frac{a^2 b \log(|-be^{(2dx+2c)}-2ae^{(dx+c)}+b|)}{a^4+2a^2b^2+b^4} - \frac{(a^3e^c-ab^2e^c) \arctan(e^{(dx+c)})e^{(-c)}}{a^4+2a^2b^2+b^4} + \frac{(a^3e^{(3c)}+ab^2e^{(3c)})e^{(3dx)}+2(a^2be^{(2c)}+b^3e^{(2c)})e^{(2dx)}-(a^3e^{(3c)}-ab^2e^{(3c)})e^{(2dx)}}{(a^2+b^2)^2(e^{(2dx+2c)+1})^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d\*x+c)\*tanh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out]  $-(a^2*b*\log(e^{(2*d*x + 2*c)} + 1))/(a^4 + 2*a^2*b^2 + b^4) - a^2*b*\log(\text{abs}(-b*e^{(2*d*x + 2*c)} - 2*a*e^{(d*x + c)} + b))/(a^4 + 2*a^2*b^2 + b^4) - (a^3*e^c - a*b^2*e^c)*\arctan(e^{(d*x + c)})*e^{(-c)}/(a^4 + 2*a^2*b^2 + b^4) + ((a^3*e^{(3*c)} + a*b^2*e^{(3*c)})*e^{(3*d*x)} + 2*(a^2*b*e^{(2*c)} + b^3*e^{(2*c)})*e^{(2*d*x)} - (a^3*e^c + a*b^2*e^c)*e^{(d*x)})/((a^2 + b^2)^2*(e^{(2*d*x + 2*c)} + 1)^2)/d$

$$3.390 \quad \int \frac{\operatorname{sech}(c+dx) \tanh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Optimal. Leaf size=36

$$\text{Unintegrable} \left( \frac{\tanh^2(c+dx) \operatorname{sech}(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[(Sech[c + d\*x]\*Tanh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0818747, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{sech}(c+dx) \tanh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Sech[c + d\*x]\*Tanh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int] [(Sech[c + d\*x]\*Tanh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx) \tanh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\operatorname{sech}(c+dx) \tanh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.001, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Sech[c + d\*x]\*Tanh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] \$Aborted

**Maple [A]** time = 1.098, size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(dx+c) (\tanh(dx+c))^2}{(fx+e)(a+b \sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)`

[Out] `int(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out]  $(b*f - (a*d*f*x*e^{(3*c)} + (d*e - f)*a*e^{(3*c)})e^{(3*d*x)} - (2*b*d*f*x*e^{(2*c)} + (2*d*e - f)*b*e^{(2*c)})e^{(2*d*x)} + (a*d*f*x*e^c + (d*e + f)*a*e^c)e^{(d*x)}) / (a^2*d^2*e^2 + b^2*d^2*e^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*(a^2*d^2*e*f + b^2*d^2*e*f)*x + (a^2*d^2*e^2*e^{(4*c)} + b^2*d^2*e^2*e^{(4*c)} + (a^2*d^2*f^2*e^{(4*c)} + b^2*d^2*f^2*e^{(4*c)})x^2 + 2*(a^2*d^2*e*f*e^{(4*c)} + b^2*d^2*e*f*e^{(4*c)})x)e^{(4*d*x)} + 2*(a^2*d^2*e^2*e^{(2*c)} + b^2*d^2*e^2*e^{(2*c)} + (a^2*d^2*f^2*e^{(2*c)} + b^2*d^2*f^2*e^{(2*c)})x^2 + 2*(a^2*d^2*e*f*e^{(2*c)} + b^2*d^2*e*f*e^{(2*c)})x)e^{(2*d*x)}) + 2*\integrate(1/2*(2*a^2*b*d^2*f^2*x^2 + 4*a^2*b*d^2*e*f*x + 2*b^3*f^2 + 2*(d^2*e^2 + f^2)*a^2*b + ((d^2*e^2 + 2*f^2)*a^3*e^c - (d^2*e^2 - 2*f^2)*a*b^2*e^c + (a^3*d^2*f^2*e^c - a*b^2*d^2*f^2*e^c)*x^2 + 2*(a^3*d^2*e*f*e^c - a*b^2*d^2*e*f*e^c)*x)e^{(d*x)}) / (a^4*d^2*e^3 + 2*a^2*b^2*d^2*e^3 + b^4*d^2*e^3 + (a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*e*f^2 + 2*a^2*b^2*d^2*e*f^2 + b^4*d^2*e*f^2)*x^2 + 3*(a^4*d^2*e^2*f + 2*a^2*b^2*d^2*e^2*f + b^4*d^2*e^2*f)*x + (a^4*d^2*e^3*e^{(2*c)} + 2*a^2*b^2*d^2*e^3*e^{(2*c)} + b^4*d^2*e^3*e^{(2*c)} + (a^4*d^2*f^3*e^{(2*c)} + 2*a^2*b^2*d^2*f^3*e^{(2*c)} + b^4*d^2*f^3*e^{(2*c)})x^3 + 3*(a^4*d^2*e*f^2*e^{(2*c)} + 2*a^2*b^2*d^2*e*f^2*e^{(2*c)} + b^4*d^2*e*f^2*e^{(2*c)})$



```
*x^2 + 3*(a^4*d^2*e^2*f*e^(2*c) + 2*a^2*b^2*d^2*e^2*f*e^(2*c) + b^4*d^2*e^2
*f*e^(2*c))*x)*e^(2*d*x)), x) - 2*integrate(-(a^3*b*e^(d*x + c) - a^2*b^2)/
(a^4*b*e + 2*a^2*b^3*e + b^5*e + (a^4*b*f + 2*a^2*b^3*f + b^5*f)*x - (a^4*b
*e*e^(2*c) + 2*a^2*b^3*e*e^(2*c) + b^5*e*e^(2*c) + (a^4*b*f*e^(2*c) + 2*a^2
*b^3*f*e^(2*c) + b^5*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^5*e*e^c + 2*a^3*b^2*e*
e^c + a*b^4*e*e^c + (a^5*f*e^c + 2*a^3*b^2*f*e^c + a*b^4*f*e^c)*x)*e^(d*x)),
x)
```

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{sech}(dx+c)\tanh(dx+c)^2}{afx+ae+(bfx+be)\sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] integral(sech(d*x + c)*tanh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*
x + c)), x)
```

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(c+dx)\text{sech}(c+dx)}{(a+b\sinh(c+dx))(e+fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(tanh(c + d*x)**2*sech(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)),
x)
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.391 \quad \int \frac{(e+fx)^3 \cosh(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=792

$$\frac{6a^3 f^2(e+fx) \text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4 d^3} + \frac{6a^3 f^2(e+fx) \text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4 d^3} - \frac{3a^3 f(e+fx)^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4 d^2}$$

[Out]  $(-3*a*f^3*x)/(8*b^2*d^3) - (a*(e+f*x)^3)/(4*b^2*d) + (a^3*(e+f*x)^4)/(4*b^4*f) - (6*a^2*f^3*\text{Cosh}[c+d*x])/(b^3*d^4) + (14*f^3*\text{Cosh}[c+d*x])/(9*b*d^4) - (3*a^2*f*(e+f*x)^2*\text{Cosh}[c+d*x])/(b^3*d^2) + (2*f*(e+f*x)^2*\text{Cosh}[c+d*x])/(3*b*d^2) - (2*f^3*\text{Cosh}[c+d*x]^3)/(27*b*d^4) - (a^3*(e+f*x)^3*\text{Log}[1+(b*E^{(c+d*x)})/(a-\text{Sqrt}[a^2+b^2])])/(b^4*d) - (a^3*(e+f*x)^3*\text{Log}[1+(b*E^{(c+d*x)})/(a+\text{Sqrt}[a^2+b^2])])/(b^4*d) - (3*a^3*f*(e+f*x)^2*\text{PolyLog}[2, -((b*E^{(c+d*x)})/(a-\text{Sqrt}[a^2+b^2]))])/(b^4*d^2) - (3*a^3*f*(e+f*x)^2*\text{PolyLog}[2, -((b*E^{(c+d*x)})/(a+\text{Sqrt}[a^2+b^2]))])/(b^4*d^2) + (6*a^3*f^2*(e+f*x)*\text{PolyLog}[3, -((b*E^{(c+d*x)})/(a-\text{Sqrt}[a^2+b^2]))])/(b^4*d^3) + (6*a^3*f^2*(e+f*x)*\text{PolyLog}[3, -((b*E^{(c+d*x)})/(a+\text{Sqrt}[a^2+b^2]))])/(b^4*d^3) - (6*a^3*f^3*\text{PolyLog}[4, -((b*E^{(c+d*x)})/(a-\text{Sqrt}[a^2+b^2]))])/(b^4*d^4) - (6*a^3*f^3*\text{PolyLog}[4, -((b*E^{(c+d*x)})/(a+\text{Sqrt}[a^2+b^2]))])/(b^4*d^4) + (6*a^2*f^2*(e+f*x)*\text{Sinh}[c+d*x])/(b^3*d^3) - (4*f^2*(e+f*x)*\text{Sinh}[c+d*x])/(3*b*d^3) + (a^2*(e+f*x)^3*\text{Sinh}[c+d*x])/(b^3*d) + (3*a*f^3*\text{Cosh}[c+d*x]*\text{Sinh}[c+d*x])/(8*b^2*d^4) + (3*a*f*(e+f*x)^2*\text{Cosh}[c+d*x]*\text{Sinh}[c+d*x])/(4*b^2*d^2) - (3*a*f^2*(e+f*x)*\text{Sinh}[c+d*x]^2)/(4*b^2*d^3) - (a*(e+f*x)^3*\text{Sinh}[c+d*x]^2)/(2*b^2*d) - (f*(e+f*x)^2*\text{Cosh}[c+d*x]*\text{Sinh}[c+d*x]^2)/(3*b*d^2) + (2*f^2*(e+f*x)*\text{Sinh}[c+d*x]^3)/(9*b*d^3) + ((e+f*x)^3*\text{Sinh}[c+d*x]^3)/(3*b*d)$

**Rubi [A]** time = 1.19673, antiderivative size = 792, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 15, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.441$ , Rules used = {5579, 5446, 3311, 3296, 2638, 2633, 32, 2635, 8, 5561, 2190, 2531, 6609, 2282, 6589}

$$\frac{6a^3 f^2(e+fx) \text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4 d^3} + \frac{6a^3 f^2(e+fx) \text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4 d^3} - \frac{3a^3 f(e+fx)^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4 d^2}$$

Antiderivative was successfully verified.

[In] Int[((e+f\*x)^3\*Cosh[c+d\*x]\*Sinh[c+d\*x]^3)/(a+b\*Sinh[c+d\*x]),x]

```
[Out] (-3*a*f^3*x)/(8*b^2*d^3) - (a*(e + f*x)^3)/(4*b^2*d) + (a^3*(e + f*x)^4)/(4
*b^4*f) - (6*a^2*f^3*Cosh[c + d*x])/(b^3*d^4) + (14*f^3*Cosh[c + d*x])/(9*b
*d^4) - (3*a^2*f*(e + f*x)^2*Cosh[c + d*x])/(b^3*d^2) + (2*f*(e + f*x)^2*Co
sh[c + d*x])/(3*b*d^2) - (2*f^3*Cosh[c + d*x]^3)/(27*b*d^4) - (a^3*(e + f*x
)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^4*d) - (a^3*(e + f*x
)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^4*d) - (3*a^3*f*(e +
f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^4*d^2) - (
3*a^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(
b^4*d^2) + (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2]))])/(b^4*d^3) + (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))])/(b^4*d^3) - (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/(b^4*d^4) - (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]))])/(b^4*d^4) + (6*a^2*f^2*(e + f*x)*Sinh[c + d*x])/
(b^3*d^3) - (4*f^2*(e + f*x)*Sinh[c + d*x])/(3*b*d^3) + (a^2*(e + f*x)^3*Si
nh[c + d*x])/(b^3*d) + (3*a*f^3*Cosh[c + d*x]*Sinh[c + d*x])/(8*b^2*d^4) +
(3*a*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^2*d^2) - (3*a*f^2*(e +
f*x)*Sinh[c + d*x]^2)/(4*b^2*d^3) - (a*(e + f*x)^3*Sinh[c + d*x]^2)/(2*b^2
*d) - (f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^2)/(3*b*d^2) + (2*f^2*(e +
f*x)*Sinh[c + d*x]^3)/(9*b*d^3) + ((e + f*x)^3*Sinh[c + d*x]^3)/(3*b*d)
```

### Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
(d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
0] && IGtQ[p, 0]
```

### Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sinh[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sinh[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sinh[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cosh[e + f*x]*(b*Sinh[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*cos[c + d*x
]*sin[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)])*((e_.) + (f_.)*(x_))^(m_.)/((a_.) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[(e + f*x)^m*E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[(e + f*x)^m*E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
```

```

[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

### Rule 2531

```

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]

```

### Rule 6609

```

Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]

```

### Rule 2282

```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

### Rule 6589

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \cosh(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \cosh(c+dx) \sinh^2(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^3 \cosh(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{(e+fx)^3 \sinh^3(c+dx)}{3bd} - \frac{a \int (e+fx)^3 \cosh(c+dx) \sinh(c+dx) dx}{b^2} + \frac{a^2}{b^2} \\
&= -\frac{a(e+fx)^3 \sinh^2(c+dx)}{2b^2d} - \frac{f(e+fx)^2 \cosh(c+dx) \sinh^2(c+dx)}{3bd^2} + \frac{2f^2}{3bd^2} \\
&= \frac{a^3(e+fx)^4}{4b^4f} + \frac{2f(e+fx)^2 \cosh(c+dx)}{3bd^2} + \frac{a^2(e+fx)^3 \sinh(c+dx)}{b^3d} + \frac{3a^2}{b^3d} \\
&= -\frac{a(e+fx)^3}{4b^2d} + \frac{a^3(e+fx)^4}{4b^4f} + \frac{2f^3 \cosh(c+dx)}{9bd^4} - \frac{3a^2f(e+fx)^2 \cosh(c+dx)}{b^3d^2} \\
&= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a^3(e+fx)^4}{4b^4f} + \frac{14f^3 \cosh(c+dx)}{9bd^4} - \frac{3a^2f(e+fx)^2 \cosh(c+dx)}{b^3d^2} \\
&= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a^3(e+fx)^4}{4b^4f} - \frac{6a^2f^3 \cosh(c+dx)}{b^3d^4} + \frac{14f^3 \cosh(c+dx)}{9bd^4} \\
&= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a^3(e+fx)^4}{4b^4f} - \frac{6a^2f^3 \cosh(c+dx)}{b^3d^4} + \frac{14f^3 \cosh(c+dx)}{9bd^4} \\
&= -\frac{3af^3x}{8b^2d^3} - \frac{a(e+fx)^3}{4b^2d} + \frac{a^3(e+fx)^4}{4b^4f} - \frac{6a^2f^3 \cosh(c+dx)}{b^3d^4} + \frac{14f^3 \cosh(c+dx)}{9bd^4}
\end{aligned}$$

**Mathematica [B]** time = 29.333, size = 7460, normalized size = 9.42

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^3\*Cosh[c + d\*x]\*Sinh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] Result too large to show

**Maple [F]** time = 0.268, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c) (\sinh(dx + c))^3}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*cosh(d\*x+c)\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*cosh(d\*x+c)\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/24*e^3*(24*(d*x + c)*a^3/(b^4*d) + 24*a^3*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/(b^4*d) + (3*a*b*e^{(-d*x - c)} - b^2 - 3*(4*a^2 - b^2)*e^{(-2*d*x - 2*c)})*e^{(3*d*x + 3*c)}/(b^3*d) + (3*a*b*e^{(-2*d*x - 2*c)} + b^2*e^{(-3*d*x - 3*c)} + 3*(4*a^2 - b^2)*e^{(-d*x - c)})/(b^3*d)) - 1/864*(216*a^3*d^4*f^3*x^4*e^{(3*c)} + 864*a^3*d^4*e*f^2*x^3*e^{(3*c)} + 1296*a^3*d^4*e^2*f*x^2*e^{(3*c)} - 4*(9*b^3*d^3*f^3*x^3*e^{(6*c)} + 9*(3*d^3*e*f^2 - d^2*f^3)*b^3*x^2*e^{(6*c)} + 3*(9*d^3*e^2*f - 6*d^2*e*f^2 + 2*d*f^3)*b^3*x*e^{(6*c)} - (9*d^2*e^2*f - 6*d*e*f^2 + 2*f^3)*b^3*e^{(6*c)})*e^{(3*d*x)} + 27*(4*a*b^2*d^3*f^3*x^3*e^{(5*c)} + 6*(2*d^3*e*f^2 - d^2*f^3)*a*b^2*x^2*e^{(5*c)} + 6*(2*d^3*e^2*f - 2*d^2*e*f^2 + d*f^3)*a*b^2*x*e^{(5*c)} - 3*(2*d^2*e^2*f - 2*d*e*f^2 + f^3)*a*b^2*e^{(5*c)})*e^{(2*d*x)} + 108*(12*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*a^2*b*e^{(4*c)} - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*b^3*e^{(4*c)} - (4*a^2*b*d^3*f^3*e^{(4*c)} - b^3*d^3*f^3*e^{(4*c)})*x^3 - 3*(4*(d^3*e*f^2 - d^2*f^3)*a^2*b*e^{(4*c)} - (d^3*e*f^2 - d^2*f^3)*b^3*e^{(4*c)})*x^2 - 3*(4*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*a^2*b*e^{(4*c)} - (d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b^3*e^{(4*c)})*x)*e^{(d*x)} + 108*(12*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*a^2*b*e^{(2*c)} - 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*b^3*e^{(2*c)} + (4*a^2*b*d^3*f^3*e^{(2*c)} - b^3*d^3*f^3*e^{(2*c)})*x^3 + 3*(4*(d^3*e*f^2 + d^2*f^3)*a^2*b*e^{(2*c)} - (d^3*e*f^2 + d^2*f^3)*b^3*e^{(2*c)})*x^2 + 3*(4*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*a^2*b*e^{(2*c)} - (d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*b^3*e^{(2*c)})*x)*e^{(-d*x)} + 27*(4*a*b^2*d^3*f^3*x^3*e^c + 6*(2*d^3*e*f^2 + d^2*f^3)*a*b^2*x^2*e^c + 6*(2*d^3*e \end{aligned}$$



$$\begin{aligned} &^2*f + 2*d^2*e*f^2 + d*f^3)*a*b^2*x*e^c + 3*(2*d^2*e^2*f + 2*d*e*f^2 + f^3) \\ &*a*b^2*e^c)*e^{(-2*d*x)} + 4*(9*b^3*d^3*f^3*x^3 + 9*(3*d^3*e*f^2 + d^2*f^3)*b \\ &^3*x^2 + 3*(9*d^3*e^2*f + 6*d^2*e*f^2 + 2*d*f^3)*b^3*x + (9*d^2*e^2*f + 6*d \\ &*e*f^2 + 2*f^3)*b^3)*e^{(-3*d*x)}*e^{(-3*c)}/(b^4*d^4) + \text{integrate}(-2*(a^3*b*f \\ &^3*x^3 + 3*a^3*b*e*f^2*x^2 + 3*a^3*b*e^2*f*x - (a^4*f^3*x^3*e^c + 3*a^4*e*f \\ &^2*x^2*e^c + 3*a^4*e^2*f*x*e^c)*e^{(d*x)})/(b^5*e^{(2*d*x + 2*c)} + 2*a*b^4*e^{( \\ &d*x + c)} - b^5), x) \end{aligned}$$

**Fricas [C]** time = 3.38887, size = 14880, normalized size = 18.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] 
$$\begin{aligned} &-1/864*(36*b^3*d^3*f^3*x^3 + 36*b^3*d^3*e^3 + 36*b^3*d^2*e^2*f + 24*b^3*d*e \\ &*f^2 - 4*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f \\ &^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f \\ &- 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*\cosh(d*x + c)^6 - 4*(9*b^3*d^3*f^3*x^3 \\ &+ 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d \\ &^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d \\ &*f^3)*x)*\sinh(d*x + c)^6 + 8*b^3*f^3 + 27*(4*a*b^2*d^3*f^3*x^3 + 4*a*b^2*d \\ &^3*e^3 - 6*a*b^2*d^2*e^2*f + 6*a*b^2*d*e*f^2 - 3*a*b^2*f^3 + 6*(2*a*b^2*d^3 \\ &*e*f^2 - a*b^2*d^2*f^3)*x^2 + 6*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a \\ &b^2*d*f^3)*x)*\cosh(d*x + c)^5 + 3*(36*a*b^2*d^3*f^3*x^3 + 36*a*b^2*d^3*e^3 \\ &- 54*a*b^2*d^2*e^2*f + 54*a*b^2*d*e*f^2 - 27*a*b^2*f^3 + 54*(2*a*b^2*d^3*e \\ &f^2 - a*b^2*d^2*f^3)*x^2 + 54*(2*a*b^2*d^3*e^2*f - 2*a*b^2*d^2*e*f^2 + a*b^ \\ &2*d*f^3)*x - 8*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3 \\ &*d*e*f^2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3 \\ &*e^2*f - 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 - \\ &108*((4*a^2*b - b^3)*d^3*f^3*x^3 + (4*a^2*b - b^3)*d^3*e^3 - 3*(4*a^2*b - \\ &b^3)*d^2*e^2*f + 6*(4*a^2*b - b^3)*d*e*f^2 - 6*(4*a^2*b - b^3)*f^3 + 3*((4 \\ &a^2*b - b^3)*d^3*e*f^2 - (4*a^2*b - b^3)*d^2*f^3)*x^2 + 3*((4*a^2*b - b^3)* \\ &d^3*e^2*f - 2*(4*a^2*b - b^3)*d^2*e*f^2 + 2*(4*a^2*b - b^3)*d*f^3)*x)*\cosh( \\ &d*x + c)^4 - 3*(36*(4*a^2*b - b^3)*d^3*f^3*x^3 + 36*(4*a^2*b - b^3)*d^3*e^3 \\ &- 108*(4*a^2*b - b^3)*d^2*e^2*f + 216*(4*a^2*b - b^3)*d*e*f^2 - 216*(4*a^2 \\ &*b - b^3)*f^3 + 108*((4*a^2*b - b^3)*d^3*e*f^2 - (4*a^2*b - b^3)*d^2*f^3)*x \\ &^2 + 20*(9*b^3*d^3*f^3*x^3 + 9*b^3*d^3*e^3 - 9*b^3*d^2*e^2*f + 6*b^3*d*e*f^ \\ &2 - 2*b^3*f^3 + 9*(3*b^3*d^3*e*f^2 - b^3*d^2*f^3)*x^2 + 3*(9*b^3*d^3*e^2*f \\ &- 6*b^3*d^2*e*f^2 + 2*b^3*d*f^3)*x)*\cosh(d*x + c)^2 + 108*((4*a^2*b - b^3)* \end{aligned}$$

$$\begin{aligned}
& d^3 e^{2f} - 2(4a^2 b - b^3) d^2 e f^2 + 2(4a^2 b - b^3) d f^3 x - 45(4a^2 b^2 d^3 f^3 x^3 + 4a^2 b^2 d^3 e^3 - 6a^2 b^2 d^2 e^2 f + 6a^2 b^2 d e f^2 \\
& - 3a^2 b^2 f^3 + 6(2a^2 b^2 d^3 e f^2 - a^2 b^2 d^2 f^3) x^2 + 6(2a^2 b^2 d^3 e^2 f - 2a^2 b^2 d^2 e f^2 + a^2 b^2 d f^3) x) \cosh(dx + c) \sinh(dx + c)^4 \\
& - 216(a^3 d^4 f^3 x^4 + 4a^3 d^4 e f^2 x^3 + 6a^3 d^4 e^2 f x^2 + 4a^3 d^4 e^3 x + 8a^3 c d^3 e^3 - 12a^3 c^2 d^2 e^2 f + 8a^3 c^3 d e f^2 - 2 \\
& a^3 c^4 f^3) \cosh(dx + c)^3 - 2(108a^3 d^4 f^3 x^4 + 432a^3 d^4 e f^2 x^3 + 648a^3 d^4 e^2 f x^2 + 432a^3 d^4 e^3 x + 864a^3 c d^3 e^3 - 1296a^3 c^2 d^2 e^2 f \\
& + 864a^3 c^3 d e f^2 - 216a^3 c^4 f^3 + 40(9b^3 d^3 f^3 x^3 + 9b^3 d^3 e^3 - 9b^3 d^2 e^2 f + 6b^3 d e f^2 - 2b^3 f^3 + 9(3 \\
& b^3 d^3 e f^2 - b^3 d^2 f^3) x^2 + 3(9b^3 d^3 e^2 f - 6b^3 d^2 e f^2 + 2b^3 d f^3) x) \cosh(dx + c)^3 - 135(4a^2 b^2 d^3 f^3 x^3 + 4a^2 b^2 d^3 e^3 \\
& - 6a^2 b^2 d^2 e^2 f + 6a^2 b^2 d e f^2 - 3a^2 b^2 f^3 + 6(2a^2 b^2 d^3 e f^2 - a^2 b^2 d^2 e f^2 + a^2 b^2 d f^3) x) \cosh(dx + c)^2 + 216((4a^2 b - b^3) d^3 f^3 x^3 + (4a^2 b - b^3) \\
& d^3 e^3 - 3(4a^2 b - b^3) d^2 e^2 f + 6(4a^2 b - b^3) d e f^2 - 6(4a^2 b - b^3) f^3 + 3((4a^2 b - b^3) d^3 e f^2 - (4a^2 b - b^3) d^2 f^3) \\
& x^2 + 3((4a^2 b - b^3) d^3 e^2 f - 2(4a^2 b - b^3) d^2 e f^2 + 2(4a^2 b - b^3) d f^3) x) \cosh(dx + c) \sinh(dx + c)^3 + 36(3b^3 d^3 e f^2 + \\
& b^3 d^2 f^3) x^2 + 108((4a^2 b - b^3) d^3 f^3 x^3 + (4a^2 b - b^3) d^3 e^3 + 3(4a^2 b - b^3) d^2 e^2 f + 6(4a^2 b - b^3) d e f^2 + 6(4a^2 b \\
& - b^3) f^3 + 3((4a^2 b - b^3) d^3 e f^2 + (4a^2 b - b^3) d^2 f^3) x^2 + 3((4a^2 b - b^3) d^3 e^2 f + 2(4a^2 b - b^3) d^2 e f^2 + 2(4a^2 b - b^3) \\
& d f^3) x) \cosh(dx + c)^2 + 6(18(4a^2 b - b^3) d^3 f^3 x^3 + 18(4a^2 b - b^3) d^3 e^3 + 54(4a^2 b - b^3) d^2 e^2 f + 108(4a^2 b - b^3) d e f^2 \\
& - 10(9b^3 d^3 f^3 x^3 + 9b^3 d^3 e^3 - 9b^3 d^2 e^2 f + 6b^3 d e f^2 - 2b^3 f^3 + 9(3b^3 d^3 e f^2 - b^3 d^2 f^3) x^2 + 3(9b^3 d^3 e^2 f \\
& f - 6b^3 d^2 e f^2 + 2b^3 d f^3) x) \cosh(dx + c)^4 + 108(4a^2 b - b^3) f^3 + 45(4a^2 b^2 d^3 f^3 x^3 + 4a^2 b^2 d^3 e^3 - 6a^2 b^2 d^2 e^2 f + 6a^2 b^2 d e f^2 \\
& - 3a^2 b^2 f^3 + 6(2a^2 b^2 d^3 e f^2 - a^2 b^2 d^2 f^3) x^2 + 6(2a^2 b^2 d^3 e^2 f - 2a^2 b^2 d^2 e f^2 + a^2 b^2 d f^3) x) \cosh(dx + c)^3 + \\
& 54((4a^2 b - b^3) d^3 e f^2 + (4a^2 b - b^3) d^2 f^3) x^2 - 108((4a^2 b - b^3) d^3 f^3 x^3 + (4a^2 b - b^3) d^3 e^3 - 3(4a^2 b - b^3) d^2 e^2 f \\
& + 6(4a^2 b - b^3) d e f^2 - 6(4a^2 b - b^3) f^3 + 3((4a^2 b - b^3) d^3 e f^2 - (4a^2 b - b^3) d^2 f^3) x^2 + 3((4a^2 b - b^3) d^3 e^2 f - 2 \\
& (4a^2 b - b^3) d^2 e f^2 + 2(4a^2 b - b^3) d f^3) x) \cosh(dx + c)^2 + 54((4a^2 b - b^3) d^3 e^2 f + 2(4a^2 b - b^3) d^2 e f^2 + 2(4a^2 b - b^3) \\
& d f^3) x - 108(a^3 d^4 f^3 x^4 + 4a^3 d^4 e f^2 x^3 + 6a^3 d^4 e^2 f x^2 + 4a^3 d^4 e^3 x + 8a^3 c d^3 e^3 - 12a^3 c^2 d^2 e^2 f + 8a^3 c^3 d e f^2 - 2a^3 c^4 f^3) \cosh(dx + c) \sinh(dx + c)^2 + 12(9b^3 d^3 e^2 f \\
& + 6b^3 d^2 e f^2 + 2b^3 d f^3) x + 27(4a^2 b^2 d^3 f^3 x^3 + 4a^2 b^2 d^3 e^3 + 6a^2 b^2 d^2 e^2 f + 6a^2 b^2 d e f^2 + 3a^2 b^2 f^3 + 6(2a^2 b^2 d^3 e f^2 + a^2 b^2 d^2 e f^2 + \\
& a^2 b^2 d f^3) x) \cosh(dx + c) + 2592((a^3 d^2 f^3 x^2 + 2a^3 d^2 e f^2 x + a^3 d^2 e^2 f) \cosh(dx + c)^3 + 3(a^3 d^2 f^3 x^2 + 2a^3 d^2 e f^2 x +
\end{aligned}$$

$$\begin{aligned}
& a^3 d^2 e^{2f} \cosh(dx + c)^2 \sinh(dx + c) + 3(a^3 d^2 f^3 x^2 + 2a^3 d^2 e f^2 x + a^3 d^2 e^2 f) \cosh(dx + c) \sinh(dx + c)^2 + (a^3 d^2 f^3 x^2 + 2a^3 d^2 e f^2 x + a^3 d^2 e^2 f) \sinh(dx + c)^3 * \operatorname{dilog}((a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 2592((a^3 d^2 f^3 x^2 + 2a^3 d^2 e f^2 x + a^3 d^2 e^2 f) \cosh(dx + c)^3 + 3(a^3 d^2 f^3 x^2 + 2a^3 d^2 e f^2 x + a^3 d^2 e^2 f) \cosh(dx + c)^2 \sinh(dx + c) + 3(a^3 d^2 f^3 x^2 + 2a^3 d^2 e f^2 x + a^3 d^2 e^2 f) \cosh(dx + c) \sinh(dx + c)^2 + (a^3 d^2 f^3 x^2 + 2a^3 d^2 e f^2 x + a^3 d^2 e^2 f) \sinh(dx + c)^3) * \operatorname{dilog}((a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 864((a^3 d^3 e^3 - 3a^3 c d^2 e^2 f + 3a^3 c^2 d e f^2 - a^3 c^3 f^3) \cosh(dx + c)^3 + 3(a^3 d^3 e^3 - 3a^3 c d^2 e^2 f + 3a^3 c^2 d e f^2 - a^3 c^3 f^3) \cosh(dx + c)^2 \sinh(dx + c) + 3(a^3 d^3 e^3 - 3a^3 c d^2 e^2 f + 3a^3 c^2 d e f^2 - a^3 c^3 f^3) \cosh(dx + c) \sinh(dx + c)^2 + (a^3 d^3 e^3 - 3a^3 c d^2 e^2 f + 3a^3 c^2 d e f^2 - a^3 c^3 f^3) \sinh(dx + c)^3) * \log(2b \cosh(dx + c) + 2b \sinh(dx + c) + 2b \sqrt{(a^2 + b^2)/b^2} + 2a) + 864((a^3 d^3 e^3 - 3a^3 c d^2 e^2 f + 3a^3 c^2 d e f^2 - a^3 c^3 f^3) \cosh(dx + c)^3 + 3(a^3 d^3 e^3 - 3a^3 c d^2 e^2 f + 3a^3 c^2 d e f^2 - a^3 c^3 f^3) \cosh(dx + c)^2 \sinh(dx + c) + 3(a^3 d^3 e^3 - 3a^3 c d^2 e^2 f + 3a^3 c^2 d e f^2 - a^3 c^3 f^3) \cosh(dx + c) \sinh(dx + c)^2 + (a^3 d^3 e^3 - 3a^3 c d^2 e^2 f + 3a^3 c^2 d e f^2 - a^3 c^3 f^3) \sinh(dx + c)^3) * \log(2b \cosh(dx + c) + 2b \sinh(dx + c) - 2b \sqrt{(a^2 + b^2)/b^2} + 2a) + 864((a^3 d^3 f^3 x^3 + 3a^3 d^3 e f^2 x^2 + 3a^3 d^3 e^2 f x + 3a^3 c d^2 e^2 f - 3a^3 c^2 d e f^2 + a^3 c^3 f^3) \cosh(dx + c)^3 + 3(a^3 d^3 f^3 x^3 + 3a^3 d^3 e f^2 x^2 + 3a^3 d^3 e^2 f x + 3a^3 c d^2 e^2 f - 3a^3 c^2 d e f^2 + a^3 c^3 f^3) \cosh(dx + c)^2 \sinh(dx + c) + 3(a^3 d^3 f^3 x^3 + 3a^3 d^3 e f^2 x^2 + 3a^3 d^3 e^2 f x + 3a^3 c d^2 e^2 f - 3a^3 c^2 d e f^2 + a^3 c^3 f^3) \cosh(dx + c) \sinh(dx + c)^2 + (a^3 d^3 f^3 x^3 + 3a^3 d^3 e f^2 x^2 + 3a^3 d^3 e^2 f x + 3a^3 c d^2 e^2 f - 3a^3 c^2 d e f^2 + a^3 c^3 f^3) \sinh(dx + c)^3) * \log(-(a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b) + 864((a^3 d^3 f^3 x^3 + 3a^3 d^3 e f^2 x^2 + 3a^3 d^3 e^2 f x + 3a^3 c d^2 e^2 f - 3a^3 c^2 d e f^2 + a^3 c^3 f^3) \cosh(dx + c)^3 + 3(a^3 d^3 f^3 x^3 + 3a^3 d^3 e f^2 x^2 + 3a^3 d^3 e^2 f x + 3a^3 c d^2 e^2 f - 3a^3 c^2 d e f^2 + a^3 c^3 f^3) \cosh(dx + c)^2 \sinh(dx + c) + 3(a^3 d^3 f^3 x^3 + 3a^3 d^3 e f^2 x^2 + 3a^3 d^3 e^2 f x + 3a^3 c d^2 e^2 f - 3a^3 c^2 d e f^2 + a^3 c^3 f^3) \cosh(dx + c) \sinh(dx + c)^2 + (a^3 d^3 f^3 x^3 + 3a^3 d^3 e f^2 x^2 + 3a^3 d^3 e^2 f x + 3a^3 c d^2 e^2 f - 3a^3 c^2 d e f^2 + a^3 c^3 f^3) \sinh(dx + c)^3) * \log(-(a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b) + 5184(a^3 f^3 \cosh(dx + c)^3 + 3a^3 f^3 \cosh(dx + c)^2 \sinh(dx + c) + 3a^3 f^3 \cosh(dx + c) \sinh(dx + c)^2 + a^3 f^3 \sinh(dx + c)^3) * \operatorname{polylog}(4, (a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2})/b) + 5184(a^3 f^3 \cosh(dx + c)^3 + 3a^3 f^3 \cosh(dx + c)^2 \sinh(dx + c) + 3a^3 f^3 \cosh(dx + c) \sinh(dx + c)^2 + a^3 f^3 \sinh(dx + c)^3)
\end{aligned}$$

$$\begin{aligned}
& c) \sinh(dx + c)^2 + a^3 f^3 \sinh(dx + c)^3 \text{polylog}(4, (a \cosh(dx + c) \\
& + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2})/b) - 5184 * ((a^3 d^3 f^3 x + a^3 d^2 e f^2) \cosh(dx + c)^3 + 3 * (a^3 d^3 f^3 x \\
& + a^3 d^2 e f^2) \cosh(dx + c)^2 \sinh(dx + c) + 3 * (a^3 d^3 f^3 x + a^3 d^2 e f^2) \cosh(dx + c) \sinh(dx + c)^2 + (a^3 d^3 f^3 x + a^3 d^2 e f^2) \sinh(dx + c) \\
& )^3) \text{polylog}(3, (a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2})/b) - 5184 * ((a^3 d^3 f^3 x + a^3 d^2 e f^2) \\
& * \cosh(dx + c)^3 + 3 * (a^3 d^3 f^3 x + a^3 d^2 e f^2) \cosh(dx + c)^2 \sinh(dx + c) + 3 * (a^3 d^3 f^3 x + a^3 d^2 e f^2) \cosh(dx + c) \sinh(dx + c)^2 + (a^3 d^3 f^3 x + a^3 d^2 e f^2) \sinh(dx + c) \\
& )^3) \text{polylog}(3, (a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2})/b) + \\
& 3 * (36 * a^2 b^2 d^3 f^3 x^3 + 36 * a^2 b^2 d^3 e^3 + 54 * a^2 b^2 d^2 e^2 f + 54 * a^2 b^2 d^2 e f^2 + 27 * a^2 b^2 f^3 - 8 * (9 * b^3 d^3 f^3 x^3 + 9 * b^3 d^3 e^3 - 9 * b^3 d^2 e^2 f \\
& + 6 * b^3 d^2 e f^2 - 2 * b^3 f^3 + 9 * (3 * b^3 d^3 e f^2 - b^3 d^2 f^3) * x^2 + 3 * (9 * b^3 d^3 e^2 f - 6 * b^3 d^2 e f^2 + 2 * b^3 d^2 f^3) * x) * \cosh(dx + c)^5 + 45 \\
& * (4 * a^2 b^2 d^3 f^3 x^3 + 4 * a^2 b^2 d^3 e^3 - 6 * a^2 b^2 d^2 e^2 f + 6 * a^2 b^2 d^2 e f^2 - 3 * a^2 b^2 f^3 + 6 * (2 * a^2 b^2 d^3 e f^2 - a^2 b^2 d^2 f^3) * x^2 + 6 * (2 * a^2 b^2 d^3 e^2 f \\
& - 2 * a^2 b^2 d^2 e f^2 + a^2 b^2 d^2 f^3) * x) * \cosh(dx + c)^4 - 144 * ((4 * a^2 b - b^3) * d^3 f^3 x^3 + (4 * a^2 b - b^3) * d^3 e^3 - 3 * (4 * a^2 b - b^3) * d^2 e^2 f \\
& + 6 * (4 * a^2 b - b^3) * d^2 e f^2 - 6 * (4 * a^2 b - b^3) * f^3 + 3 * ((4 * a^2 b - b^3) * d^3 e f^2 - (4 * a^2 b - b^3) * d^2 f^3) * x^2 + 3 * ((4 * a^2 b - b^3) * d^3 e^2 f - \\
& 2 * (4 * a^2 b - b^3) * d^2 e f^2 + 2 * (4 * a^2 b - b^3) * d^2 f^3) * x) * \cosh(dx + c)^3 \\
& + 54 * (2 * a^2 b^2 d^3 e f^2 + a^2 b^2 d^2 f^3) * x^2 - 216 * (a^3 d^4 f^3 x^4 + 4 * a^3 d^4 e f^2 x^3 + 6 * a^3 d^4 e^2 f x^2 + 4 * a^3 d^4 e^3 x + 8 * a^3 c d^3 e^3 - \\
& 12 * a^3 c^2 d^2 e^2 f + 8 * a^3 c^3 d e f^2 - 2 * a^3 c^4 f^3) * \cosh(dx + c)^2 + \\
& 54 * (2 * a^2 b^2 d^3 e^2 f + 2 * a^2 b^2 d^2 e f^2 + a^2 b^2 d^2 f^3) * x + 72 * ((4 * a^2 b - b^3) * d^3 f^3 x^3 + (4 * a^2 b - b^3) * d^3 e^3 + 3 * (4 * a^2 b - b^3) * d^2 e^2 f \\
& + 6 * (4 * a^2 b - b^3) * d^2 e f^2 + 6 * (4 * a^2 b - b^3) * f^3 + 3 * ((4 * a^2 b - b^3) * d^3 e f^2 + (4 * a^2 b - b^3) * d^2 f^3) * x^2 + 3 * ((4 * a^2 b - b^3) * d^3 e^2 f + 2 * (4 * a^2 b - b^3) * d^2 e f^2 \\
& + 2 * (4 * a^2 b - b^3) * d^2 f^3) * x) * \cosh(dx + c) * \sinh(dx + c) / (b^4 d^4 \cosh(dx + c)^3 + 3 b^4 d^4 \cosh(dx + c)^2 \sinh(dx + c) \\
& ) + 3 b^4 d^4 \cosh(dx + c) \sinh(dx + c)^2 + b^4 d^4 \sinh(dx + c)^3
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cosh(dx+c)\*sinh(dx+c)\*\*3/(a+b\*sinh(dx+c)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c) \sinh(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^3*cosh(d*x + c)*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

$$3.392 \quad \int \frac{(e+fx)^2 \cosh(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=578

$$-\frac{2a^3 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4 d^2} - \frac{2a^3 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4 d^2} + \frac{2a^3 f^2 \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4 d^3} + \frac{2a^3 f^2 \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4 d^3}$$

[Out]  $-(a e f x)/(2 b^2 d) - (a f^2 x^2)/(4 b^2 d) + (a^3 (e + f x)^3)/(3 b^4 f) - (2 a^2 f (e + f x) \operatorname{Cosh}[c + d x])/(b^3 d^2) + (4 f (e + f x) \operatorname{Cosh}[c + d x])/(9 b d^2) - (a^3 (e + f x)^2 \operatorname{Log}[1 + (b E^{c+dx})/(a - \sqrt{a^2 + b^2})])/(b^4 d) - (a^3 (e + f x)^2 \operatorname{Log}[1 + (b E^{c+dx})/(a + \sqrt{a^2 + b^2})])/(b^4 d) - (2 a^3 f (e + f x) \operatorname{PolyLog}[2, -((b E^{c+dx})/(a - \sqrt{a^2 + b^2}))])/(b^4 d^2) - (2 a^3 f (e + f x) \operatorname{PolyLog}[2, -((b E^{c+dx})/(a + \sqrt{a^2 + b^2}))])/(b^4 d^2) + (2 a^3 f^2 \operatorname{PolyLog}[3, -((b E^{c+dx})/(a - \sqrt{a^2 + b^2}))])/(b^4 d^3) + (2 a^3 f^2 \operatorname{PolyLog}[3, -((b E^{c+dx})/(a + \sqrt{a^2 + b^2}))])/(b^4 d^3) + (2 a^2 f^2 \operatorname{Sinh}[c + d x])/(b^3 d^3) - (4 f^2 \operatorname{Sinh}[c + d x])/(9 b d^3) + (a^2 (e + f x)^2 \operatorname{Sinh}[c + d x])/(b^3 d) + (a f (e + f x) \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x])/(2 b^2 d^2) - (a f^2 \operatorname{Sinh}[c + d x]^2)/(4 b^2 d^3) - (a (e + f x)^2 \operatorname{Sinh}[c + d x]^2)/(2 b^2 d) - (2 f (e + f x) \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]^2)/(9 b d^2) + (2 f^2 \operatorname{Sinh}[c + d x]^3)/(27 b d^3) + ((e + f x)^2 \operatorname{Sinh}[c + d x]^3)/(3 b d)$

**Rubi [A]** time = 0.928188, antiderivative size = 578, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 10, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$ , Rules used = {5579, 5446, 3310, 3296, 2637, 5561, 2190, 2531, 2282, 6589}

$$-\frac{2a^3 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4 d^2} - \frac{2a^3 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4 d^2} + \frac{2a^3 f^2 \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4 d^3} + \frac{2a^3 f^2 \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4 d^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e + f x)^2 \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x]^3 / (a + b \operatorname{Sinh}[c + d x]), x]$

[Out]  $-(a e f x)/(2 b^2 d) - (a f^2 x^2)/(4 b^2 d) + (a^3 (e + f x)^3)/(3 b^4 f) - (2 a^2 f (e + f x) \operatorname{Cosh}[c + d x])/(b^3 d^2) + (4 f (e + f x) \operatorname{Cosh}[c + d x])/(9 b d^2) - (a^3 (e + f x)^2 \operatorname{Log}[1 + (b E^{c+dx})/(a - \sqrt{a^2 + b^2})])/(b^4 d) - (a^3 (e + f x)^2 \operatorname{Log}[1 + (b E^{c+dx})/(a + \sqrt{a^2 + b^2})])/(b^4 d) - (2 a^3 f (e + f x) \operatorname{PolyLog}[2, -((b E^{c+dx})/(a - \sqrt{a^2 + b^2}))])/(b^4 d^2) - (2 a^3 f (e + f x) \operatorname{PolyLog}[2, -((b E^{c+dx})/(a + \sqrt{a^2 + b^2}))])/(b^4 d^2) + (2 a^3 f^2 \operatorname{PolyLog}[3, -((b E^{c+dx})/(a - \sqrt{a^2 + b^2}))])/(b^4 d^3) + (2 a^3 f^2 \operatorname{PolyLog}[3, -((b E^{c+dx})/(a + \sqrt{a^2 + b^2}))])/(b^4 d^3)$

$$\begin{aligned} & (a - \sqrt{a^2 + b^2})]/(b^4 d^3) + (2a^3 f^2 \text{PolyLog}[3, -(bE^{(c+dx)}) \\ & )/(a + \sqrt{a^2 + b^2})]/(b^4 d^3) + (2a^2 f^2 \text{Sinh}[c + dx])/(b^3 d^3) \\ & - (4f^2 \text{Sinh}[c + dx])/(9b d^3) + (a^2 (e + fx)^2 \text{Sinh}[c + dx])/(b^3 d) \\ & + (a f (e + fx) \text{Cosh}[c + dx] \text{Sinh}[c + dx])/(2b^2 d^2) - (a f^2 \text{Sinh}[c \\ & + dx]^2)/(4b^2 d^3) - (a (e + fx)^2 \text{Sinh}[c + dx]^2)/(2b^2 d) - (2f (e \\ & + fx) \text{Cosh}[c + dx] \text{Sinh}[c + dx]^2)/(9b d^2) + (2f^2 \text{Sinh}[c + dx]^3)/ \\ & (27b d^3) + ((e + fx)^2 \text{Sinh}[c + dx]^3)/(3b d) \end{aligned}$$
Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_.)]^(p_.)*((e_.) + (f_.)*(x_.))^(m_.)*Sinh[(c_.) +
(d_.)*(x_.)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
0] && IGtQ[p, 0]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.)*Sinh[(a_.) + (b_.)*
(x_.)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_.))*((b_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :=
Simp[(d*(b*Sinh[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sinh[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sinh[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := -Simp[
((c + d*x)^m*Cosh[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(e+fx)^2 \cosh(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \cosh(c+dx) \sinh^2(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^2 \cosh(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{(e+fx)^2 \sinh^3(c+dx)}{3bd} - \frac{a \int (e+fx)^2 \cosh(c+dx) \sinh(c+dx) dx}{b^2} + \frac{a^2}{b^2} \\
&= -\frac{a(e+fx)^2 \sinh^2(c+dx)}{2b^2d} - \frac{2f(e+fx) \cosh(c+dx) \sinh^2(c+dx)}{9bd^2} + \frac{2f}{9bd^2} \\
&= \frac{a^3(e+fx)^3}{3b^4f} + \frac{4f(e+fx) \cosh(c+dx)}{9bd^2} + \frac{a^2(e+fx)^2 \sinh(c+dx)}{b^3d} + \frac{af}{9bd^2} \\
&= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a^3(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx) \cosh(c+dx)}{b^3d^2} + \frac{4f(e+fx)}{9bd^2} \\
&= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a^3(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx) \cosh(c+dx)}{b^3d^2} + \frac{4f(e+fx)}{9bd^2} \\
&= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a^3(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx) \cosh(c+dx)}{b^3d^2} + \frac{4f(e+fx)}{9bd^2} \\
&= -\frac{aefx}{2b^2d} - \frac{af^2x^2}{4b^2d} + \frac{a^3(e+fx)^3}{3b^4f} - \frac{2a^2f(e+fx) \cosh(c+dx)}{b^3d^2} + \frac{4f(e+fx)}{9bd^2}
\end{aligned}$$

**Mathematica [B]** time = 14.9508, size = 3510, normalized size = 6.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(f^2*(2*a*x^3*(-1 + Coth[c]) - 2*a*x^3*Coth[c] - (6*a*b^2*(d^2*x^2*Log[1 + ((a - Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*d*x*PolyLog[2, ((-a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*PolyLog[3, ((-a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b]))/(Sqrt[a^2 + b^2]*(-a + Sqrt[a^2 + b^2])*d^3) - (6*a*b^2*(d^2*x^2*Log[1 + ((a + Sqrt[a^2 + b^2])*(Cosh[c + d*x] - Sinh[c + d*x]))/b] - 2*d*x*PolyLog[2, ((a + Sqrt[a^2 + b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b] - 2*PolyLog[3, ((a + Sqrt[a^2 + b^2])*(-Cosh[c + d*x] + Sinh[c + d*x]))/b]))/(Sqrt[a^2 + b^2]*(a + Sqrt[a^2 + b^2])*d^3)
```

$$\begin{aligned}
& \text{rt}[a^2 + b^2]) * (-\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]) / b) / (\text{Sqrt}[a^2 + b^2] * (a + \\
& \text{Sqrt}[a^2 + b^2]) * d^3) + (6*a^2*(d^2*x^2*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c \\
& + d*x])) / (a - \text{Sqrt}[a^2 + b^2])] + 2*d*x*\text{PolyLog}[2, (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c \\
& + d*x])) / (-a + \text{Sqrt}[a^2 + b^2])] - 2*\text{PolyLog}[3, (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x])) / (-a + \text{Sqrt}[a^2 + b^2])]) / (\text{Sqrt}[a^2 + b^2]*d^3) - (6*a^2*(d^2*x^2*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x])) / (a + \text{Sqrt}[a^2 + b^2])] + 2*d*x*\text{PolyLog}[2, -((b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x])) / (a + \text{Sqrt}[a^2 + b^2]))] - 2*\text{PolyLog}[3, -((b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x])) / (a + \text{Sqrt}[a^2 + b^2]))]) / (\text{Sqrt}[a^2 + b^2]*d^3) + (6*b*\text{Cosh}[d*x]*(-2*d*x*\text{Cosh}[c] + (2 + d^2*x^2)*\text{Sinh}[c])) / d^3 + (6*b*((2 + d^2*x^2)*\text{Cosh}[c] - 2*d*x*\text{Sinh}[c])*\text{Sinh}[d*x]) / d^3)) / (12*b^2) + (e^2*((a*\text{Log}[a + b*\text{Sinh}[c + d*x]]) / b^2 - \text{Sinh}[c + d*x] / b)) / (2*d) - (e*f*(-(b*\text{Cosh}[c + d*x]) - a*(-(c + d*x)^2/2 + (c + d*x)*\text{Log}[1 + (b*E^(c + d*x)) / (a - \text{Sqrt}[a^2 + b^2])] + (c + d*x)*\text{Log}[1 + (b*E^(c + d*x)) / (a + \text{Sqrt}[a^2 + b^2])]) - c*\text{Log}[a + b*\text{Sinh}[c + d*x]] + \text{PolyLog}[2, (b*E^(c + d*x)) / (-a + \text{Sqrt}[a^2 + b^2])] + \text{PolyLog}[2, -((b*E^(c + d*x)) / (a + \text{Sqrt}[a^2 + b^2]))]) + b*d*x*\text{Sinh}[c + d*x]) / (b^2*d^2) + (e^2*(-3*a*(2*a^2 + b^2)*\text{Log}[a + b*\text{Sinh}[c + d*x]] + 3*b*(2*a^2 + b^2)*\text{Sinh}[c + d*x] - 3*a*b^2*\text{Sinh}[c + d*x]^2 + 2*b^3*\text{Sinh}[c + d*x]^3)) / (6*b^4*d) + (e*f*(-18*b*(4*a^2 + b^2)*\text{Cosh}[c + d*x] - 18*a*b^2*d*x*\text{Cosh}[2*(c + d*x)] - 2*b^3*\text{Cosh}[3*(c + d*x)] - 36*a*(2*a^2 + b^2)*(-(c + d*x)^2/2 + (c + d*x)*\text{Log}[1 + (b*E^(c + d*x)) / (a - \text{Sqrt}[a^2 + b^2])] + (c + d*x)*\text{Log}[1 + (b*E^(c + d*x)) / (a + \text{Sqrt}[a^2 + b^2])]) - c*\text{Log}[a + b*\text{Sinh}[c + d*x]] + \text{PolyLog}[2, (b*E^(c + d*x)) / (-a + \text{Sqrt}[a^2 + b^2])] + \text{PolyLog}[2, -((b*E^(c + d*x)) / (a + \text{Sqrt}[a^2 + b^2]))]) + 18*b*(4*a^2 + b^2)*d*x*\text{Sinh}[c + d*x] + 9*a*b^2*\text{Sinh}[2*(c + d*x)] + 6*b^3*d*x*\text{Sinh}[3*(c + d*x)])) / (36*b^4*d^2) + (f^2*((2*a*(2*a^2 + b^2)*(-1 + \text{Coth}[c])*(2*x^3 + (6*a*(d^2*x^2*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x])) / (a - \text{Sqrt}[a^2 + b^2])]) + 2*d*x*\text{PolyLog}[2, (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x])) / (-a + \text{Sqrt}[a^2 + b^2])]) - 2*\text{PolyLog}[3, (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x])) / (-a + \text{Sqrt}[a^2 + b^2])]) * \text{Sinh}[c] * (\text{Cosh}[c] + \text{Sinh}[c])) / (\text{Sqrt}[a^2 + b^2]*d^3) - (3*b^2*(d^2*x^2*\text{Log}[1 + ((a - \text{Sqrt}[a^2 + b^2])*(\text{Cosh}[c + d*x] - \text{Sinh}[c + d*x])) / b] - 2*d*x*\text{PolyLog}[2, ((-a + \text{Sqrt}[a^2 + b^2])*(\text{Cosh}[c + d*x] - \text{Sinh}[c + d*x])) / b] - 2*\text{PolyLog}[3, ((-a + \text{Sqrt}[a^2 + b^2])*(\text{Cosh}[c + d*x] - \text{Sinh}[c + d*x])) / b]) * (-1 + \text{Cosh}[2*c] + \text{Sinh}[2*c])) / (\text{Sqrt}[a^2 + b^2]*(a + \text{Sqrt}[a^2 + b^2])*d^3) - (3*b^2*(d^2*x^2*\text{Log}[1 + ((a + \text{Sqrt}[a^2 + b^2])*(\text{Cosh}[c + d*x] - \text{Sinh}[c + d*x])) / b] - 2*d*x*\text{PolyLog}[2, ((a + \text{Sqrt}[a^2 + b^2])*(-\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x])) / b] - 2*\text{PolyLog}[3, ((a + \text{Sqrt}[a^2 + b^2])*(-\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x])) / b]) * (-1 + \text{Cosh}[2*c] + \text{Sinh}[2*c])) / (\text{Sqrt}[a^2 + b^2]*(a + \text{Sqrt}[a^2 + b^2])*d^3) - (3*a*(d^2*x^2*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x])) / (a + \text{Sqrt}[a^2 + b^2])] + 2*d*x*\text{PolyLog}[2, -((b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x])) / (a + \text{Sqrt}[a^2 + b^2]))] - 2*\text{PolyLog}[3, -((b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x])) / (a + \text{Sqrt}[a^2 + b^2]))]) * (-1 + \text{Cosh}[2*c] + \text{Sinh}[2*c])) / (\text{Sqrt}[a^2 + b^2]*d^3)) / (3*b^4) + \text{Csch}[c] * (\text{Cosh}[3*c + 3*d*x] / (108*b^4*d^3) - \text{Sinh}[3*c + 3*d*x] / (108*b^4*d^3)) * (27*a*b^2*\text{Cosh}[d*x] + 54*a*b^2*d*x*\text{Cosh}[d*x] + 54*a*b^2*d^2*x^2*\text{Cosh}[d*x] - 27*a*b^2*\text{Cosh}[2*c + d*x] - 54*a*b^2*d*x*\text{Cosh}[2*c + d*x] - 54*a*b^2*d^2*x^2*\text{Cosh}[2*c + d*x] + 432*a^2*b*\text{Cosh}[c + 2*d*x] + 108*b^3
\end{aligned}$$

$$\begin{aligned}
& *Cosh[c + 2*d*x] + 432*a^2*b*d*x*Cosh[c + 2*d*x] + 108*b^3*d*x*Cosh[c + 2*d*x] \\
& *x] + 216*a^2*b*d^2*x^2*Cosh[c + 2*d*x] + 54*b^3*d^2*x^2*Cosh[c + 2*d*x] - \\
& 432*a^2*b*Cosh[3*c + 2*d*x] - 108*b^3*Cosh[3*c + 2*d*x] - 432*a^2*b*d*x*Cos \\
& h[3*c + 2*d*x] - 108*b^3*d*x*Cosh[3*c + 2*d*x] - 216*a^2*b*d^2*x^2*Cosh[3*c \\
& + 2*d*x] - 54*b^3*d^2*x^2*Cosh[3*c + 2*d*x] - 144*a^3*d^3*x^3*Cosh[2*c + 3 \\
& *d*x] - 72*a*b^2*d^3*x^3*Cosh[2*c + 3*d*x] - 144*a^3*d^3*x^3*Cosh[4*c + 3*d \\
& *x] - 72*a*b^2*d^3*x^3*Cosh[4*c + 3*d*x] - 432*a^2*b*Cosh[3*c + 4*d*x] - 10 \\
& 8*b^3*Cosh[3*c + 4*d*x] + 432*a^2*b*d*x*Cosh[3*c + 4*d*x] + 108*b^3*d*x*Cos \\
& h[3*c + 4*d*x] - 216*a^2*b*d^2*x^2*Cosh[3*c + 4*d*x] - 54*b^3*d^2*x^2*Cosh[ \\
& 3*c + 4*d*x] + 432*a^2*b*Cosh[5*c + 4*d*x] + 108*b^3*Cosh[5*c + 4*d*x] - 43 \\
& 2*a^2*b*d*x*Cosh[5*c + 4*d*x] - 108*b^3*d*x*Cosh[5*c + 4*d*x] + 216*a^2*b*d \\
& ^2*x^2*Cosh[5*c + 4*d*x] + 54*b^3*d^2*x^2*Cosh[5*c + 4*d*x] + 27*a*b^2*Cosh \\
& [4*c + 5*d*x] - 54*a*b^2*d*x*Cosh[4*c + 5*d*x] + 54*a*b^2*d^2*x^2*Cosh[4*c \\
& + 5*d*x] - 27*a*b^2*Cosh[6*c + 5*d*x] + 54*a*b^2*d*x*Cosh[6*c + 5*d*x] - 54 \\
& *a*b^2*d^2*x^2*Cosh[6*c + 5*d*x] - 4*b^3*Cosh[5*c + 6*d*x] + 12*b^3*d*x*Cos \\
& h[5*c + 6*d*x] - 18*b^3*d^2*x^2*Cosh[5*c + 6*d*x] + 4*b^3*Cosh[7*c + 6*d*x] \\
& - 12*b^3*d*x*Cosh[7*c + 6*d*x] + 18*b^3*d^2*x^2*Cosh[7*c + 6*d*x] - 8*b^3* \\
& Sinh[c] - 24*b^3*d*x*Sinh[c] - 36*b^3*d^2*x^2*Sinh[c] + 27*a*b^2*Sinh[d*x] \\
& + 54*a*b^2*d*x*Sinh[d*x] + 54*a*b^2*d^2*x^2*Sinh[d*x] - 27*a*b^2*Sinh[2*c + \\
& d*x] - 54*a*b^2*d*x*Sinh[2*c + d*x] - 54*a*b^2*d^2*x^2*Sinh[2*c + d*x] + 4 \\
& 32*a^2*b*Sinh[c + 2*d*x] + 108*b^3*Sinh[c + 2*d*x] + 432*a^2*b*d*x*Sinh[c + \\
& 2*d*x] + 108*b^3*d*x*Sinh[c + 2*d*x] + 216*a^2*b*d^2*x^2*Sinh[c + 2*d*x] + \\
& 54*b^3*d^2*x^2*Sinh[c + 2*d*x] - 432*a^2*b*Sinh[3*c + 2*d*x] - 108*b^3*Sin \\
& h[3*c + 2*d*x] - 432*a^2*b*d*x*Sinh[3*c + 2*d*x] - 108*b^3*d*x*Sinh[3*c + 2 \\
& *d*x] - 216*a^2*b*d^2*x^2*Sinh[3*c + 2*d*x] - 54*b^3*d^2*x^2*Sinh[3*c + 2*d \\
& *x] - 144*a^3*d^3*x^3*Sinh[2*c + 3*d*x] - 72*a*b^2*d^3*x^3*Sinh[2*c + 3*d*x] \\
& ] - 144*a^3*d^3*x^3*Sinh[4*c + 3*d*x] - 72*a*b^2*d^3*x^3*Sinh[4*c + 3*d*x] \\
& - 432*a^2*b*Sinh[3*c + 4*d*x] - 108*b^3*Sinh[3*c + 4*d*x] + 432*a^2*b*d*x*S \\
& inh[3*c + 4*d*x] + 108*b^3*d*x*Sinh[3*c + 4*d*x] - 216*a^2*b*d^2*x^2*Sinh[3 \\
& *c + 4*d*x] - 54*b^3*d^2*x^2*Sinh[3*c + 4*d*x] + 432*a^2*b*Sinh[5*c + 4*d*x] \\
& ] + 108*b^3*Sinh[5*c + 4*d*x] - 432*a^2*b*d*x*Sinh[5*c + 4*d*x] - 108*b^3*d \\
& *x*Sinh[5*c + 4*d*x] + 216*a^2*b*d^2*x^2*Sinh[5*c + 4*d*x] + 54*b^3*d^2*x^2 \\
& *Sinh[5*c + 4*d*x] + 27*a*b^2*Sinh[4*c + 5*d*x] - 54*a*b^2*d*x*Sinh[4*c + 5 \\
& *d*x] + 54*a*b^2*d^2*x^2*Sinh[4*c + 5*d*x] - 27*a*b^2*Sinh[6*c + 5*d*x] + 5 \\
& 4*a*b^2*d*x*Sinh[6*c + 5*d*x] - 54*a*b^2*d^2*x^2*Sinh[6*c + 5*d*x] - 4*b^3* \\
& Sinh[5*c + 6*d*x] + 12*b^3*d*x*Sinh[5*c + 6*d*x] - 18*b^3*d^2*x^2*Sinh[5*c \\
& + 6*d*x] + 4*b^3*Sinh[7*c + 6*d*x] - 12*b^3*d*x*Sinh[7*c + 6*d*x] + 18*b^3* \\
& d^2*x^2*Sinh[7*c + 6*d*x]))/8
\end{aligned}$$

**Maple [F]** time = 0.208, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c) (\sinh(dx + c))^3}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*cosh(d\*x+c)\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*cosh(d\*x+c)\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/24*e^2*(24*(d*x + c)*a^3/(b^4*d) + 24*a^3*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/(b^4*d) + (3*a*b*e^{(-d*x - c)} - b^2 - 3*(4*a^2 - b^2)*e^{(-2*d*x - 2*c)})*e^{(3*d*x + 3*c)}/(b^3*d) + (3*a*b*e^{(-2*d*x - 2*c)} + b^2*e^{(-3*d*x - 3*c)} + 3*(4*a^2 - b^2)*e^{(-d*x - c)})/(b^3*d)) - 1/432*(144*a^3*d^3*f^2*x^3*e^{(3*c)} + 432*a^3*d^3*e*f*x^2*e^{(3*c)} - 2*(9*b^3*d^2*f^2*x^2*e^{(6*c)} + 6*(3*d^2*e*f - d*f^2)*b^3*x*e^{(6*c)} - 2*(3*d*e*f - f^2)*b^3*e^{(6*c)})*e^{(3*d*x)} + 27*(2*a*b^2*d^2*f^2*x^2*e^{(5*c)} + 2*(2*d^2*e*f - d*f^2)*a*b^2*x*e^{(5*c)} - (2*d*e*f - f^2)*a*b^2*e^{(5*c)})*e^{(2*d*x)} + 54*(8*(d*e*f - f^2)*a^2*b*e^{(4*c)} - 2*(d*e*f - f^2)*b^3*e^{(4*c)} - (4*a^2*b*d^2*f^2*e^{(4*c)} - b^3*d^2*f^2*e^{(4*c)})*x^2 - 2*(4*(d^2*e*f - d*f^2)*a^2*b*e^{(4*c)} - (d^2*e*f - d*f^2)*b^3*e^{(4*c)})*x)*e^{(d*x)} + 54*(8*(d*e*f + f^2)*a^2*b*e^{(2*c)} - 2*(d*e*f + f^2)*b^3*e^{(2*c)} + (4*a^2*b*d^2*f^2*e^{(2*c)} - b^3*d^2*f^2*e^{(2*c)})*x^2 + 2*(4*(d^2*e*f + d*f^2)*a^2*b*e^{(2*c)} - (d^2*e*f + d*f^2)*b^3*e^{(2*c)})*x)*e^{(-d*x)} + 27*(2*a*b^2*d^2*f^2*x^2*e^c + 2*(2*d^2*e*f + d*f^2)*a*b^2*x*e^c + (2*d*e*f + f^2)*a*b^2*e^c)*e^{(-2*d*x)} + 2*(9*b^3*d^2*f^2*x^2 + 6*(3*d^2*e*f + d*f^2)*b^3*x + 2*(3*d*e*f + f^2)*b^3)*e^{(-3*d*x)})*e^{(-3*c)}/(b^4*d^3) + integrate(-2*(a^3*b*f^2*x^2 + 2*a^3*b*e*f*x - (a^4*f^2*x^2*e^c + 2*a^4*e*f*x*e^c)*e^{(d*x)})/(b^5*e^{(2*d*x + 2*c)} + 2*a*b^4*e^{(d*x + c)} - b^5), x) \end{aligned}$$

**Fricas [C]** time = 2.85816, size = 9353, normalized size = 16.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-1/432*(18*b^3*d^2*f^2*x^2 + 18*b^3*d^2*e^2 - 2*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*\cosh(d*x + c)^6 - 2*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*\sinh(d*x + c)^6 + 12*b^3*d*e*f + 27*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*\cosh(d*x + c)^5 + 3*(18*a*b^2*d^2*f^2*x^2 + 18*a*b^2*d^2*e^2 - 18*a*b^2*d*e*f + 9*a*b^2*f^2 + 18*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x - 4*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 4*b^3*f^2 - 54*((4*a^2*b - b^3)*d^2*f^2*x^2 + (4*a^2*b - b^3)*d^2*e^2 - 2*(4*a^2*b - b^3)*d*e*f + 2*(4*a^2*b - b^3)*f^2 + 2*((4*a^2*b - b^3)*d^2*e*f - (4*a^2*b - b^3)*d*f^2)*x)*\cosh(d*x + c)^4 - 3*(18*(4*a^2*b - b^3)*d^2*f^2*x^2 + 18*(4*a^2*b - b^3)*d^2*e^2 - 36*(4*a^2*b - b^3)*d*e*f + 36*(4*a^2*b - b^3)*f^2 + 10*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*\cosh(d*x + c)^2 + 36*((4*a^2*b - b^3)*d^2*e*f - (4*a^2*b - b^3)*d*f^2)*x - 45*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^4 - 144*(a^3*d^3*f^2*x^3 + 3*a^3*d^3*e*f*x^2 + 3*a^3*d^3*e^2*x + 6*a^3*c*d^2*e^2 - 6*a^3*c^2*d*e*f + 2*a^3*c^3*f^2)*\cosh(d*x + c)^3 - 2*(72*a^3*d^3*f^2*x^3 + 216*a^3*d^3*e*f*x^2 + 216*a^3*d^3*e^2*x + 432*a^3*c*d^2*e^2 - 432*a^3*c^2*d*e*f + 144*a^3*c^3*f^2 + 20*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*\cosh(d*x + c)^3 - 135*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*\cosh(d*x + c)^2 + 108*((4*a^2*b - b^3)*d^2*f^2*x^2 + (4*a^2*b - b^3)*d^2*e^2 - 2*(4*a^2*b - b^3)*d*e*f + 2*(4*a^2*b - b^3)*f^2 + 2*((4*a^2*b - b^3)*d^2*e*f - (4*a^2*b - b^3)*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 54*((4*a^2*b - b^3)*d^2*f^2*x^2 + (4*a^2*b - b^3)*d^2*e^2 + 2*(4*a^2*b - b^3)*d*e*f + 2*(4*a^2*b - b^3)*f^2 + 2*((4*a^2*b - b^3)*d^2*e*f + (4*a^2*b - b^3)*d*f^2)*x)*\cosh(d*x + c)^2 + 6*(9*(4*a^2*b - b^3)*d^2*f^2*x^2 + 9*(4*a^2*b - b^3)*d^2*e^2 - 5*(9*b^3*d^2*f^2*x^2 + 9*b^3*d^2*e^2 - 6*b^3*d*e*f + 2*b^3*f^2 + 6*(3*b^3*d^2*e*f - b^3*d*f^2)*x)*\cosh(d*x + c)^4 + 18*(4*a^2*b - b^3)*d*e*f + 45*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 - 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f - a*b^2*d*f^2)*x)*\cosh(d*x + c)^3 + 18*(4*a^2*b - b^3)*f^2 - 54*((4*a^2*b - b^3)*d^2*f^2*x^2 + (4*a^2*b - b^3)*d^2*e^2 - 2*(4*a^2*b - b^3)*d*e*f + 2*(4*a$$

$$\begin{aligned}
& ^2*b - b^3)*f^2 + 2*((4*a^2*b - b^3)*d^2*e*f - (4*a^2*b - b^3)*d*f^2)*x)*\text{cosh}(d*x + c)^2 + 18*((4*a^2*b - b^3)*d^2*e*f + (4*a^2*b - b^3)*d*f^2)*x - 72 \\
& *(a^3*d^3*f^2*x^3 + 3*a^3*d^3*e*f*x^2 + 3*a^3*d^3*e^2*x + 6*a^3*c*d^2*e^2 - 6*a^3*c^2*d*e*f + 2*a^3*c^3*f^2)*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^2 + 12*(3*b^3*d^2*e*f + b^3*d*f^2)*x + 27*(2*a*b^2*d^2*f^2*x^2 + 2*a*b^2*d^2*e^2 + 2*a*b^2*d*e*f + a*b^2*f^2 + 2*(2*a*b^2*d^2*e*f + a*b^2*d*f^2)*x)*\text{cosh}(d*x + c) \\
& + 864*((a^3*d*f^2*x + a^3*d*e*f)*\text{cosh}(d*x + c)^3 + 3*(a^3*d*f^2*x + a^3*d*e*f)*\text{cosh}(d*x + c)^2*\text{sinh}(d*x + c) + 3*(a^3*d*f^2*x + a^3*d*e*f)*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^2 + (a^3*d*f^2*x + a^3*d*e*f)*\text{sinh}(d*x + c)^3)*\text{dilog}((a*\text{cosh}(d*x + c) + a*\text{sinh}(d*x + c) + (b*\text{cosh}(d*x + c) + b*\text{sinh}(d*x + c)))*\text{sqrt}((a^2 + b^2)/b^2) - b)/b + 1) + 864*((a^3*d*f^2*x + a^3*d*e*f)*\text{cosh}(d*x + c)^3 + 3*(a^3*d*f^2*x + a^3*d*e*f)*\text{cosh}(d*x + c)^2*\text{sinh}(d*x + c) + 3*(a^3*d*f^2*x + a^3*d*e*f)*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^2 + (a^3*d*f^2*x + a^3*d*e*f)*\text{sinh}(d*x + c)^3)*\text{dilog}((a*\text{cosh}(d*x + c) + a*\text{sinh}(d*x + c) - (b*\text{cosh}(d*x + c) + b*\text{sinh}(d*x + c)))*\text{sqrt}((a^2 + b^2)/b^2) - b)/b + 1) + 432*((a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*\text{cosh}(d*x + c)^3 + 3*(a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*\text{cosh}(d*x + c)^2*\text{sinh}(d*x + c) + 3*(a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^2 + (a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*\text{sinh}(d*x + c)^3)*\log(2*b*\text{cosh}(d*x + c) + 2*b*\text{sinh}(d*x + c) + 2*b*\text{sqrt}((a^2 + b^2)/b^2) + 2*a) + 432*((a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*\text{cosh}(d*x + c)^3 + 3*(a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*\text{cosh}(d*x + c)^2*\text{sinh}(d*x + c) + 3*(a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^2 + (a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*\text{sinh}(d*x + c)^3)*\log(2*b*\text{cosh}(d*x + c) + 2*b*\text{sinh}(d*x + c) - 2*b*\text{sqrt}((a^2 + b^2)/b^2) + 2*a) + 432*((a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*\text{cosh}(d*x + c)^3 + 3*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*\text{cosh}(d*x + c)^2*\text{sinh}(d*x + c) + 3*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^2 + (a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*\text{sinh}(d*x + c)^3)*\log(-(a*\text{cosh}(d*x + c) + a*\text{sinh}(d*x + c) + (b*\text{cosh}(d*x + c) + b*\text{sinh}(d*x + c)))*\text{sqrt}((a^2 + b^2)/b^2) - b)/b) + 432*((a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*\text{cosh}(d*x + c)^3 + 3*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*\text{cosh}(d*x + c)^2*\text{sinh}(d*x + c) + 3*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^2 + (a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*\text{sinh}(d*x + c)^3)*\log(-(a*\text{cosh}(d*x + c) + a*\text{sinh}(d*x + c) - (b*\text{cosh}(d*x + c) + b*\text{sinh}(d*x + c)))*\text{sqrt}((a^2 + b^2)/b^2) - b)/b) - 864*(a^3*f^2*\text{cosh}(d*x + c)^3 + 3*a^3*f^2*\text{cosh}(d*x + c)^2*\text{sinh}(d*x + c) + 3*a^3*f^2*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^2 + a^3*f^2*\text{sinh}(d*x + c)^3)*\text{polylog}(3, (a*\text{cosh}(d*x + c) + a*\text{sinh}(d*x + c) + (b*\text{cosh}(d*x + c) + b*\text{sinh}(d*x + c)))*\text{sqrt}((a^2 + b^2)/b^2))/b) - 864*(a^3*f^2*\text{cosh}(d*x + c)^3 + 3*a^3*f^2*\text{cosh}(d*x + c)^2*\text{sinh}(d*x + c) + 3*a^3*f^2*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^2 + a^3*f^2*\text{sinh}(d*x + c)^3)*\text{polylog}(3, (a*\text{cosh}(d*x + c) + a*\text{sinh}(d*x + c) - (b*\text{cosh}(d*x + c) + b*\text{sinh}(d*x + c)))*\text{sqrt}((a^2 + b^2)/b^2))/b) + 3*(18*a*b^2*d^2*f^2*x^2 + 18*a*b^2*d^2*e^2 + 18*a*b^2*d*e*f - 4*(9*b^3*d^2*
\end{aligned}$$

$$f^2x^2 + 9b^3d^2e^2 - 6b^3d^2ef + 2b^3f^2 + 6(3b^3d^2ef - b^3d^2f^2)x) \cosh(dx + c)^5 + 9ab^2f^2 + 45(2ab^2d^2f^2x^2 + 2ab^2d^2e^2 - 2ab^2d^2ef + ab^2f^2 + 2(2ab^2d^2ef - ab^2d^2f^2)x) \cosh(dx + c)^4 - 72((4a^2b - b^3)d^2f^2x^2 + (4a^2b - b^3)d^2e^2 - 2(4a^2b - b^3)d^2ef + 2(4a^2b - b^3)f^2 + 2((4a^2b - b^3)d^2ef - (4a^2b - b^3)d^2f^2)x) \cosh(dx + c)^3 - 144(a^3d^3f^2x^3 + 3a^3d^3efx^2 + 3a^3d^3e^2x + 6a^3cd^2e^2 - 6a^3c^2d^2ef + 2a^3c^3f^2) \cosh(dx + c)^2 + 18(2ab^2d^2ef + ab^2d^2f^2)x + 36((4a^2b - b^3)d^2f^2x^2 + (4a^2b - b^3)d^2e^2 + 2(4a^2b - b^3)d^2ef + 2(4a^2b - b^3)f^2 + 2((4a^2b - b^3)d^2ef + (4a^2b - b^3)d^2f^2)x) \cosh(dx + c) \sinh(dx + c) / (b^4d^3 \cosh(dx + c)^3 + 3b^4d^3 \cosh(dx + c)^2 \sinh(dx + c) + 3b^4d^3 \cosh(dx + c) \sinh(dx + c)^2 + b^4d^3 \sinh(dx + c)^3)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cosh(d\*x+c)\*sinh(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c) \sinh(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*cosh(d\*x + c)\*sinh(d\*x + c)^3/(b\*sinh(d\*x + c) + a), x)

$$3.393 \quad \int \frac{(e+fx) \cosh(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=348

$$\frac{a^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4 d^2} - \frac{a^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4 d^2} - \frac{a^2 f \cosh(c+dx)}{b^3 d^2} - \frac{a^3(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^4 d} - \frac{a^3(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^4 d}$$

[Out]  $-(a*f*x)/(4*b^2*d) + (a^3*(e+f*x)^2)/(2*b^4*f) - (a^2*f*\operatorname{Cosh}[c+d*x])/(b^3*d^2) + (f*\operatorname{Cosh}[c+d*x])/(3*b*d^2) - (f*\operatorname{Cosh}[c+d*x]^3)/(9*b*d^2) - (a^3*(e+f*x)*\operatorname{Log}[1+(b*E^{c+d*x})/(a-\operatorname{Sqrt}[a^2+b^2])])/(b^4*d) - (a^3*(e+f*x)*\operatorname{Log}[1+(b*E^{c+d*x})/(a+\operatorname{Sqrt}[a^2+b^2])])/(b^4*d) - (a^3*f*\operatorname{PolyLog}[2, -((b*E^{c+d*x})/(a-\operatorname{Sqrt}[a^2+b^2]))])/(b^4*d^2) - (a^3*f*\operatorname{PolyLog}[2, -((b*E^{c+d*x})/(a+\operatorname{Sqrt}[a^2+b^2]))])/(b^4*d^2) + (a^2*(e+f*x)*\operatorname{Sinh}[c+d*x])/(b^3*d) + (a*f*\operatorname{Cosh}[c+d*x]*\operatorname{Sinh}[c+d*x])/(4*b^2*d^2) - (a*(e+f*x)*\operatorname{Sinh}[c+d*x]^2)/(2*b^2*d) + ((e+f*x)*\operatorname{Sinh}[c+d*x]^3)/(3*b*d)$

**Rubi [A]** time = 0.52561, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$ , Rules used = {5579, 5446, 2633, 2635, 8, 3296, 2638, 5561, 2190, 2279, 2391}

$$\frac{a^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^4 d^2} - \frac{a^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^4 d^2} - \frac{a^2 f \cosh(c+dx)}{b^3 d^2} - \frac{a^3(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)}{b^4 d} - \frac{a^3(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right)}{b^4 d}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e+f*x)*\operatorname{Cosh}[c+d*x]*\operatorname{Sinh}[c+d*x]^3/(a+b*\operatorname{Sinh}[c+d*x]),x]$

[Out]  $-(a*f*x)/(4*b^2*d) + (a^3*(e+f*x)^2)/(2*b^4*f) - (a^2*f*\operatorname{Cosh}[c+d*x])/(b^3*d^2) + (f*\operatorname{Cosh}[c+d*x])/(3*b*d^2) - (f*\operatorname{Cosh}[c+d*x]^3)/(9*b*d^2) - (a^3*(e+f*x)*\operatorname{Log}[1+(b*E^{c+d*x})/(a-\operatorname{Sqrt}[a^2+b^2])])/(b^4*d) - (a^3*(e+f*x)*\operatorname{Log}[1+(b*E^{c+d*x})/(a+\operatorname{Sqrt}[a^2+b^2])])/(b^4*d) - (a^3*f*\operatorname{PolyLog}[2, -((b*E^{c+d*x})/(a-\operatorname{Sqrt}[a^2+b^2]))])/(b^4*d^2) - (a^3*f*\operatorname{PolyLog}[2, -((b*E^{c+d*x})/(a+\operatorname{Sqrt}[a^2+b^2]))])/(b^4*d^2) + (a^2*(e+f*x)*\operatorname{Sinh}[c+d*x])/(b^3*d) + (a*f*\operatorname{Cosh}[c+d*x]*\operatorname{Sinh}[c+d*x])/(4*b^2*d^2) - (a*(e+f*x)*\operatorname{Sinh}[c+d*x]^2)/(2*b^2*d) + ((e+f*x)*\operatorname{Sinh}[c+d*x]^3)/(3*b*d)$

Rule 5579



```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
(d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
0] && IGtQ[p, 0]
```

#### Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

#### Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

#### Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x
]*(b*Sinh[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sinh[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

#### Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

#### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

#### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

#### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx) \cosh(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx) \cosh(c+dx) \sinh^2(c+dx) dx}{b} - \frac{a \int \frac{(e+fx) \cosh(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= \frac{(e+fx) \sinh^3(c+dx)}{3bd} - \frac{a \int (e+fx) \cosh(c+dx) \sinh(c+dx) dx}{b^2} + \frac{a^2 \int (e+fx) \cosh(c+dx) dx}{b^3} \\
&= -\frac{a(e+fx) \sinh^2(c+dx)}{2b^2d} + \frac{(e+fx) \sinh^3(c+dx)}{3bd} + \frac{a^2 \int (e+fx) \cosh(c+dx) dx}{b^3} \\
&= \frac{a^3(e+fx)^2}{2b^4f} + \frac{f \cosh(c+dx)}{3bd^2} - \frac{f \cosh^3(c+dx)}{9bd^2} + \frac{a^2(e+fx) \sinh(c+dx)}{b^3d} \\
&= -\frac{afx}{4b^2d} + \frac{a^3(e+fx)^2}{2b^4f} - \frac{a^2f \cosh(c+dx)}{b^3d^2} + \frac{f \cosh(c+dx)}{3bd^2} - \frac{f \cosh^3(c+dx)}{9bd^2} \\
&= -\frac{afx}{4b^2d} + \frac{a^3(e+fx)^2}{2b^4f} - \frac{a^2f \cosh(c+dx)}{b^3d^2} + \frac{f \cosh(c+dx)}{3bd^2} - \frac{f \cosh^3(c+dx)}{9bd^2} \\
&= -\frac{afx}{4b^2d} + \frac{a^3(e+fx)^2}{2b^4f} - \frac{a^2f \cosh(c+dx)}{b^3d^2} + \frac{f \cosh(c+dx)}{3bd^2} - \frac{f \cosh^3(c+dx)}{9bd^2}
\end{aligned}$$

**Mathematica [A]** time = 1.72985, size = 447, normalized size = 1.28

$$72a^3f \text{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) + 72a^3f \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) + 72a^3cf \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right) + 72a^3dfx \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(-36*a^3*c^2*f - 72*a^3*c*d*f*x - 36*a^3*d^2*f*x^2 + 72*a^2*b*f*Cosh[c + d*x] - 18*b^3*f*Cosh[c + d*x] + 18*a*b^2*d*f*x*Cosh[2*(c + d*x)] + 2*b^3*f*Cosh[3*(c + d*x)] + 72*a^3*c*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 72*a^3*d*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 72*a^3*c*f*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 72*a^3*d*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 72*a^3*d*e*Log[a + b*Sinh[c + d*x]] - 72*a^3*c*f*Log[a + b*Sinh[c + d*x]] + 72*a^3*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 72*a^3*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 72*a^2*b*d*e*Sinh[c + d*x] - 72*a^2*b*d*f*x*Sinh[c + d*x] +
```

$$\frac{18b^3dfx \operatorname{Sinh}[c + dx] + 36ab^2de \operatorname{Sinh}[c + dx]^2 - 24b^3de \operatorname{Sinh}[c + dx]^3 - 9ab^2f \operatorname{Sinh}[2(c + dx)] - 6b^3dfx \operatorname{Sinh}[3(c + dx)]}{(72b^4d^2)}$$

**Maple [B]** time = 0.1, size = 671, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)`

[Out]  $\frac{1}{2}a^3fx^2/b^4 - a^3ex/b^4 + \frac{1}{72}(3dfx + 3de - f)/d^2/b \exp(3dx + 3c) - \frac{1}{16}a(2dfx + 2de - f)/b^2/d^2 \exp(2dx + 2c) + \frac{1}{8}(4a^2dfx - b^2dfx + 4a^2de - b^2de - 4a^2f + b^2f)/b^3/d^2 \exp(dx + c) - \frac{1}{8}(4a^2 - b^2)(dfx + de + f)/b^3/d^2 \exp(-dx - c) - \frac{1}{16}a(2dfx + 2de + f)/b^2/d^2 \exp(-2dx - 2c) - \frac{1}{72}(3dfx + 3de + f)/d^2/b \exp(-3dx - 3c) + a^3/b^4/d^2 f c \ln(b \exp(2dx + 2c) + 2a \exp(dx + c) - b) - 2a^3/b^4/d^2 f c \ln(\exp(dx + c)) - a^3/b^4/d^2 e \ln(b \exp(2dx + 2c) + 2a \exp(dx + c) - b) + 2a^3/b^4/d^2 e \ln(\exp(dx + c)) - a^3/b^4/d^2 f \ln((-b \exp(dx + c) + (a^2 + b^2)^{1/2} - a)/(-a + (a^2 + b^2)^{1/2})) * x - a^3/b^4/d^2 f \ln((-b \exp(dx + c) + (a^2 + b^2)^{1/2} - a)/(-a + (a^2 + b^2)^{1/2})) * c - a^3/b^4/d^2 f \ln((b \exp(dx + c) + (a^2 + b^2)^{1/2} + a)/(a + (a^2 + b^2)^{1/2})) * x - a^3/b^4/d^2 f \ln((b \exp(dx + c) + (a^2 + b^2)^{1/2} + a)/(a + (a^2 + b^2)^{1/2})) * c - a^3/b^4/d^2 f \operatorname{dilog}((b \exp(dx + c) + (a^2 + b^2)^{1/2} + a)/(a + (a^2 + b^2)^{1/2})) - a^3/b^4/d^2 f \operatorname{dilog}((-b \exp(dx + c) + (a^2 + b^2)^{1/2} - a)/(-a + (a^2 + b^2)^{1/2})) + 2a^3/b^4/d^2 f c x + a^3/b^4/d^2 f c^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{24}e \left( \frac{24(dx+c)a^3}{b^4d} + \frac{24a^3 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{b^4d} + \frac{(3abe^{(-dx-c)} - b^2 - 3(4a^2 - b^2)e^{(-2dx-2c)})e^{(3dx+3c)}}{b^3d} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out]  $-1/24e(24(dx+c)a^3/(b^4d) + 24a^3 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)/(b^4d) + (3a*be^{(-dx-c)} - b^2 - 3*(4a^2 - b^2)*e^{(-2dx-2c)})e^{(3dx+3c)}/b^3d + \dots)$

```
*d*x - 2*c))*e^(3*d*x + 3*c)/(b^3*d) + (3*a*b*e^(-2*d*x - 2*c) + b^2*e^(-3*
d*x - 3*c) + 3*(4*a^2 - b^2)*e^(-d*x - c))/(b^3*d)) - 1/144*f*((72*a^3*d^2*
x^2*e^(3*c) - 2*(3*b^3*d*x*e^(6*c) - b^3*e^(6*c))*e^(3*d*x) + 9*(2*a*b^2*d*
x*e^(5*c) - a*b^2*e^(5*c))*e^(2*d*x) + 18*(4*a^2*b*e^(4*c) - b^3*e^(4*c) -
(4*a^2*b*d*e^(4*c) - b^3*d*e^(4*c))*x)*e^(d*x) + 18*(4*a^2*b*e^(2*c) - b^3*
e^(2*c) + (4*a^2*b*d*e^(2*c) - b^3*d*e^(2*c))*x)*e^(-d*x) + 9*(2*a*b^2*d*x*
e^c + a*b^2*e^c)*e^(-2*d*x) + 2*(3*b^3*d*x + b^3)*e^(-3*d*x))*e^(-3*c)/(b^4
*d^2) - 9*integrate(32*(a^4*x*e^(d*x + c) - a^3*b*x)/(b^5*e^(2*d*x + 2*c) +
2*a*b^4*e^(d*x + c) - b^5), x))
```

---

**Fricas [B]** time = 2.53215, size = 5007, normalized size = 14.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/144*(2*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^6 + 2*(3*b^3*d*f*x
+ 3*b^3*d*e - b^3*f)*sinh(d*x + c)^6 - 6*b^3*d*f*x - 9*(2*a*b^2*d*f*x + 2*
a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^5 - 3*(6*a*b^2*d*f*x + 6*a*b^2*d*e - 3*a
*b^2*f - 4*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c))*sinh(d*x + c)^5
- 6*b^3*d*e + 18*((4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3)*d*e - (4*a^2*b -
b^3)*f)*cosh(d*x + c)^4 + 3*(6*(4*a^2*b - b^3)*d*f*x + 6*(4*a^2*b - b^3)*d
*e + 10*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^2 - 6*(4*a^2*b - b^
3)*f - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c))*sinh(d*x +
c)^4 - 2*b^3*f + 72*(a^3*d^2*f*x^2 + 2*a^3*d^2*e*x + 4*a^3*c*d*e - 2*a^3*c
^2*f)*cosh(d*x + c)^3 + 2*(36*a^3*d^2*f*x^2 + 72*a^3*d^2*e*x + 144*a^3*c*d*
e - 72*a^3*c^2*f + 20*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^3 - 4
5*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^2 + 36*((4*a^2*b -
b^3)*d*f*x + (4*a^2*b - b^3)*d*e - (4*a^2*b - b^3)*f)*cosh(d*x + c))*sinh(d
*x + c)^3 - 18*((4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3)*d*e + (4*a^2*b - b^
3)*f)*cosh(d*x + c)^2 + 6*(5*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c
)^4 - 3*(4*a^2*b - b^3)*d*f*x - 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*
cosh(d*x + c)^3 - 3*(4*a^2*b - b^3)*d*e + 18*((4*a^2*b - b^3)*d*f*x + (4*a^
2*b - b^3)*d*e - (4*a^2*b - b^3)*f)*cosh(d*x + c)^2 - 3*(4*a^2*b - b^3)*f +
36*(a^3*d^2*f*x^2 + 2*a^3*d^2*e*x + 4*a^3*c*d*e - 2*a^3*c^2*f)*cosh(d*x +
c))*sinh(d*x + c)^2 - 9*(2*a*b^2*d*f*x + 2*a*b^2*d*e + a*b^2*f)*cosh(d*x +
c) - 144*(a^3*f*cosh(d*x + c)^3 + 3*a^3*f*cosh(d*x + c)^2*sinh(d*x + c) + 3
*a^3*f*cosh(d*x + c)*sinh(d*x + c)^2 + a^3*f*sinh(d*x + c)^3)*dilog((a*cosh
(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
```

$$\begin{aligned}
& + b^2)/b^2) - b)/b + 1) - 144*(a^3*f*cosh(d*x + c)^3 + 3*a^3*f*cosh(d*x + \\
& c)^2*sinh(d*x + c) + 3*a^3*f*cosh(d*x + c)*sinh(d*x + c)^2 + a^3*f*sinh(d*x \\
& + c)^3)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*si \\
& nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 144*((a^3*d*e - a^3*c*f)*c \\
& osh(d*x + c)^3 + 3*(a^3*d*e - a^3*c*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a \\
& ^3*d*e - a^3*c*f)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*d*e - a^3*c*f)*sinh( \\
& d*x + c)^3)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2 \\
& )/b^2) + 2*a) - 144*((a^3*d*e - a^3*c*f)*cosh(d*x + c)^3 + 3*(a^3*d*e - a^3 \\
& *c*f)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^3*d*e - a^3*c*f)*cosh(d*x + c)*s \\
& inh(d*x + c)^2 + (a^3*d*e - a^3*c*f)*sinh(d*x + c)^3)*log(2*b*cosh(d*x + c) \\
& + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 144*((a^3*d*f*x + \\
& a^3*c*f)*cosh(d*x + c)^3 + 3*(a^3*d*f*x + a^3*c*f)*cosh(d*x + c)^2*sinh(d* \\
& x + c) + 3*(a^3*d*f*x + a^3*c*f)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3*d*f*x \\
& + a^3*c*f)*sinh(d*x + c)^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*c \\
& osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 144*((a^3*d \\
& *f*x + a^3*c*f)*cosh(d*x + c)^3 + 3*(a^3*d*f*x + a^3*c*f)*cosh(d*x + c)^2*s \\
& inh(d*x + c) + 3*(a^3*d*f*x + a^3*c*f)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^3 \\
& *d*f*x + a^3*c*f)*sinh(d*x + c)^3)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) \\
& - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 3*(6* \\
& a*b^2*d*f*x - 4*(3*b^3*d*f*x + 3*b^3*d*e - b^3*f)*cosh(d*x + c)^5 + 6*a*b^2 \\
& *d*e + 15*(2*a*b^2*d*f*x + 2*a*b^2*d*e - a*b^2*f)*cosh(d*x + c)^4 + 3*a*b^2 \\
& *f - 24*((4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3)*d*e - (4*a^2*b - b^3)*f)*c \\
& osh(d*x + c)^3 - 72*(a^3*d^2*f*x^2 + 2*a^3*d^2*e*x + 4*a^3*c*d*e - 2*a^3*c^ \\
& 2*f)*cosh(d*x + c)^2 + 12*((4*a^2*b - b^3)*d*f*x + (4*a^2*b - b^3)*d*e + (4 \\
& *a^2*b - b^3)*f)*cosh(d*x + c))*sinh(d*x + c))/(b^4*d^2*cosh(d*x + c)^3 + 3 \\
& *b^4*d^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^4*d^2*cosh(d*x + c)*sinh(d*x + \\
& c)^2 + b^4*d^2*sinh(d*x + c)^3)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)\*sinh(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \cosh(dx + c) \sinh(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```

$$3.394 \quad \int \frac{\cosh(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=76

$$\frac{a^2 \sinh(c+dx)}{b^3 d} - \frac{a^3 \log(a+b \sinh(c+dx))}{b^4 d} - \frac{a \sinh^2(c+dx)}{2b^2 d} + \frac{\sinh^3(c+dx)}{3bd}$$

[Out]  $-\left(\frac{a^3 \text{Log}[a + b \text{Sinh}[c + d*x]]}{b^4*d}\right) + \frac{a^2 \text{Sinh}[c + d*x]}{b^3*d} - \left(\frac{a \text{Sinh}[c + d*x]^2}{2*b^2*d} + \frac{\text{Sinh}[c + d*x]^3}{3*b*d}\right)$

**Rubi [A]** time = 0.0966879, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2833, 12, 43}

$$\frac{a^2 \sinh(c+dx)}{b^3 d} - \frac{a^3 \log(a+b \sinh(c+dx))}{b^4 d} - \frac{a \sinh^2(c+dx)}{2b^2 d} + \frac{\sinh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

[In] Int[(Cosh[c + d\*x]\*Sinh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out]  $-\left(\frac{a^3 \text{Log}[a + b \text{Sinh}[c + d*x]]}{b^4*d}\right) + \frac{a^2 \text{Sinh}[c + d*x]}{b^3*d} - \left(\frac{a \text{Sinh}[c + d*x]^2}{2*b^2*d} + \frac{\text{Sinh}[c + d*x]^3}{3*b*d}\right)$

### Rule 2833

Int[cos[(e\_.) + (f\_.)\*(x\_.)]\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Dist[1/(b\*f), Subst[Int[(a + x)^m\*(c + (d\*x)/b)^n, x], x, b\*Sin[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le



Q[7\*m + 4\*n + 4, 0] || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{\cosh(c + dx) \sinh^3(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\text{Subst}\left(\int \frac{x^3}{b^3(a+x)} dx, x, b \sinh(c + dx)\right)}{bd} \\
 &= \frac{\text{Subst}\left(\int \frac{x^3}{a+x} dx, x, b \sinh(c + dx)\right)}{b^4 d} \\
 &= \frac{\text{Subst}\left(\int \left(a^2 - ax + x^2 - \frac{a^3}{a+x}\right) dx, x, b \sinh(c + dx)\right)}{b^4 d} \\
 &= -\frac{a^3 \log(a + b \sinh(c + dx))}{b^4 d} + \frac{a^2 \sinh(c + dx)}{b^3 d} - \frac{a \sinh^2(c + dx)}{2b^2 d} + \frac{\sinh^3(c + dx)}{3bd}
 \end{aligned}$$

**Mathematica [A]** time = 0.15399, size = 66, normalized size = 0.87

$$\frac{6a^2 b \sinh(c + dx) - 6a^3 \log(a + b \sinh(c + dx)) - 3ab^2 \sinh^2(c + dx) + 2b^3 \sinh^3(c + dx)}{6b^4 d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cosh[c + d\*x]\*Sinh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] (-6\*a^3\*Log[a + b\*Sinh[c + d\*x]] + 6\*a^2\*b\*Sinh[c + d\*x] - 3\*a\*b^2\*Sinh[c + d\*x]^2 + 2\*b^3\*Sinh[c + d\*x]^3)/(6\*b^4\*d)

**Maple [A]** time = 0.016, size = 73, normalized size = 1.

$$-\frac{a^3 \ln(a + b \sinh(dx + c))}{b^4 d} + \frac{a^2 \sinh(dx + c)}{b^3 d} - \frac{(\sinh(dx + c))^2 a}{2b^2 d} + \frac{(\sinh(dx + c))^3}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

[Out] -a^3\*ln(a+b\*sinh(d\*x+c))/b^4/d+a^2\*sinh(d\*x+c)/b^3/d-1/2\*a\*sinh(d\*x+c)^2/b^2/d+1/3\*sinh(d\*x+c)^3/b/d

---

**Maxima [B]** time = 1.12151, size = 231, normalized size = 3.04

$$\frac{(dx+c)a^3}{b^4d} - \frac{a^3 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{b^4d} - \frac{(3abe^{(-dx-c)} - b^2 - 3(4a^2 - b^2)e^{(-2dx-2c)})e^{(3dx+3c)}}{24b^3d} - \frac{3abe^{(-2dx-2c)}}{b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-(d*x + c)*a^3/(b^4*d) - a^3*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/(b^4*d) - 1/24*(3*a*b*e^{(-d*x - c)} - b^2 - 3*(4*a^2 - b^2)*e^{(-2*d*x - 2*c)})*e^{(3*d*x + 3*c)}/(b^3*d) - 1/24*(3*a*b*e^{(-2*d*x - 2*c)} + b^2*e^{(-3*d*x - 3*c)} + 3*(4*a^2 - b^2)*e^{(-d*x - c)})/(b^3*d)$

---

**Fricas [B]** time = 2.49711, size = 1484, normalized size = 19.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $1/24*(b^3*\cosh(d*x + c)^6 + b^3*\sinh(d*x + c)^6 + 24*a^3*d*x*\cosh(d*x + c)^3 - 3*a*b^2*\cosh(d*x + c)^5 + 3*(2*b^3*\cosh(d*x + c) - a*b^2)*\sinh(d*x + c)^5 + 3*(4*a^2*b - b^3)*\cosh(d*x + c)^4 + 3*(5*b^3*\cosh(d*x + c)^2 - 5*a*b^2*\cosh(d*x + c) + 4*a^2*b - b^3)*\sinh(d*x + c)^4 - 3*a*b^2*\cosh(d*x + c) + 2*(10*b^3*\cosh(d*x + c)^3 + 12*a^3*d*x - 15*a*b^2*\cosh(d*x + c)^2 + 6*(4*a^2*b - b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 - b^3 - 3*(4*a^2*b - b^3)*\cosh(d*x + c)^2 + 3*(5*b^3*\cosh(d*x + c)^4 + 24*a^3*d*x*\cosh(d*x + c) - 10*a*b^2*\cosh(d*x + c)^3 - 4*a^2*b + b^3 + 6*(4*a^2*b - b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - 24*(a^3*\cosh(d*x + c)^3 + 3*a^3*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*a^3*\cosh(d*x + c)*\sinh(d*x + c)^2 + a^3*\sinh(d*x + c)^3)*\log(2*(b*\sinh(d*x + c) + a)/(\cosh(d*x + c) - \sinh(d*x + c))) + 3*(2*b^3*\cosh(d*x + c)^5 + 24*a^3*d*x*\cosh(d*x + c)^2 - 5*a*b^2*\cosh(d*x + c)^4 + 4*(4*a^2*b - b^3)*\cosh(d*x + c)^3 - a*b^2 - 2*(4*a^2*b - b^3)*\cosh(d*x + c))*\sinh(d*x + c))/(b^4*d*\cosh(d*x + c)^3 + 3*b^4*d*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*b^4*d*\cosh(d*x + c)*\sinh(d*x + c)^2 + b^4*d*\sinh(d*x + c)^3)$

---

**Sympy [A]** time = 2.83729, size = 105, normalized size = 1.38

$$\begin{cases} \frac{x \sinh^3(c) \cosh(c)}{\sinh^4(c+dx)} & \text{for } b = 0 \wedge d = 0 \\ \frac{4ad}{\sinh^4(c+dx)} & \text{for } b = 0 \\ \frac{x \sinh^3(c) \cosh(c)}{a+b \sinh(c)} & \text{for } d = 0 \\ -\frac{a^3 \log\left(\frac{a}{b} + \sinh(c+dx)\right)}{b^4 d} + \frac{a^2 \sinh(c+dx)}{b^3 d} - \frac{a \sinh^2(c+dx)}{2b^2 d} + \frac{\sinh^3(c+dx)}{3bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*sinh(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Piecewise((x\*sinh(c)\*\*3\*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (sinh(c + d\*x)\*\*4/(4\*a\*d), Eq(b, 0)), (x\*sinh(c)\*\*3\*cosh(c)/(a + b\*sinh(c)), Eq(d, 0)), (-a\*\*3\*log(a/b + sinh(c + d\*x))/(b\*\*4\*d) + a\*\*2\*sinh(c + d\*x)/(b\*\*3\*d) - a\*sinh(c + d\*x)\*\*2/(2\*b\*\*2\*d) + sinh(c + d\*x)\*\*3/(3\*b\*d), True))

---

**Giac [A]** time = 1.25732, size = 171, normalized size = 2.25

$$-\frac{a^3 \log\left(\left|b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 2a\right|\right)}{b^4 d} + \frac{b^2 d^2 \left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 - 3abd^2 \left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 12a^2 d^2 \left(e^{(dx+c)} - e^{(-dx-c)}\right)}{24b^3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] -a^3\*log(abs(b\*(e^(d\*x + c) - e^(-d\*x - c)) + 2\*a))/(b^4\*d) + 1/24\*(b^2\*d^2\*(e^(d\*x + c) - e^(-d\*x - c))^3 - 3\*a\*b\*d^2\*(e^(d\*x + c) - e^(-d\*x - c))^2 + 12\*a^2\*d^2\*(e^(d\*x + c) - e^(-d\*x - c)))/(b^3\*d^3)

$$3.395 \quad \int \frac{\cosh(c+dx) \sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable} \left( \frac{\sinh^3(c+dx) \cosh(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[(Cosh[c + d\*x]\*Sinh[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x]))], x]

**Rubi [A]** time = 0.0829812, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cosh(c+dx) \sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Cosh[c + d\*x]\*Sinh[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] Defer[Int] [(Cosh[c + d\*x]\*Sinh[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x]))], x]

Rubi steps

$$\int \frac{\cosh(c+dx) \sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\cosh(c+dx) \sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.003, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Cosh[c + d\*x]\*Sinh[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] \$Aborted

**Maple [A]** time = 0.162, size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c) (\sinh(dx+c))^3}{(fx+e)(a+b \sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)\*sinh(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(cosh(d\*x+c)\*sinh(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{e^{\left(-3c+\frac{3de}{f}\right)} E_1\left(\frac{3(fx+e)d}{f}\right)}{8bf} - \frac{ae^{\left(-2c+\frac{2de}{f}\right)} E_1\left(\frac{2(fx+e)d}{f}\right)}{4b^2f} + \frac{ae^{\left(2c-\frac{2de}{f}\right)} E_1\left(-\frac{2(fx+e)d}{f}\right)}{4b^2f} - \frac{e^{\left(3c-\frac{3de}{f}\right)} E_1\left(-\frac{3(fx+e)d}{f}\right)}{8bf} - \frac{(4a^2 - b^2)}{4b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*sinh(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-1/8*e^{(-3*c + 3*d*e/f)}*\exp\_integral\_e(1, 3*(f*x + e)*d/f)/(b*f) - 1/4*a*e^{(-2*c + 2*d*e/f)}*\exp\_integral\_e(1, 2*(f*x + e)*d/f)/(b^2*f) + 1/4*a*e^{(2*c - 2*d*e/f)}*\exp\_integral\_e(1, -2*(f*x + e)*d/f)/(b^2*f) - 1/8*e^{(3*c - 3*d*e/f)}*\exp\_integral\_e(1, -3*(f*x + e)*d/f)/(b*f) - 1/8*(4*a^2 - b^2)*e^{(-c + d*e/f)}*\exp\_integral\_e(1, (f*x + e)*d/f)/(b^3*f) - 1/8*(4*a^2*e^c - b^2*e^c)*e^{(-d*e/f)}*\exp\_integral\_e(1, -(f*x + e)*d/f)/(b^3*f) - a^3*\log(f*x + e)/(b^4*f) + 1/16*integrate(-32*(a^4*e^{(d*x + c)} - a^3*b)/(b^5*f*x + b^5*e - (b^5*f*x*e^{(2*c)} + b^5*e*e^{(2*c)})*e^{(2*d*x)} - 2*(a*b^4*f*x*e^c + a*b^4*e*e^c)*e^{(d*x)}), x)$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx+c) \sinh(dx+c)^3}{afx+ae+(bfx+be) \sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)*sinh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c) \sinh(dx + c)^3}{(fx + e)(b \sinh(dx + c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)*sinh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```

$$3.396 \quad \int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=1038

$$-\frac{(e+fx)^4}{32bf} + \frac{a^2(e+fx)^4}{8b^3f} + \frac{a^4(e+fx)^4}{4b^5f} - \frac{a \cosh^3(c+dx)(e+fx)^3}{3b^2d} - \frac{a^3 \cosh(c+dx)(e+fx)^3}{b^4d} - \frac{a^3 \sqrt{a^2+b^2} \log\left(\frac{e^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d}$$

```
[Out] (3*a^2*e*f^2*x)/(4*b^3*d^2) + (3*a^2*f^3*x^2)/(8*b^3*d^2) + (a^4*(e + f*x)^4)/(4*b^5*f) + (a^2*(e + f*x)^4)/(8*b^3*f) - (e + f*x)^4/(32*b*f) - (6*a^3*f^2*(e + f*x)*Cosh[c + d*x])/(b^4*d^3) - (4*a*f^2*(e + f*x)*Cosh[c + d*x])/(3*b^2*d^3) - (a^3*(e + f*x)^3*Cosh[c + d*x])/(b^4*d) - (3*a^2*f^3*Cosh[c + d*x]^2)/(8*b^3*d^4) - (3*a^2*f*(e + f*x)^2*Cosh[c + d*x]^2)/(4*b^3*d^2) - (2*a*f^2*(e + f*x)*Cosh[c + d*x]^3)/(9*b^2*d^3) - (a*(e + f*x)^3*Cosh[c + d*x]^3)/(3*b^2*d) - (3*f^3*Cosh[4*c + 4*d*x])/(1024*b*d^4) - (3*f*(e + f*x)^2*Cosh[4*c + 4*d*x])/(128*b*d^2) - (a^3*Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]))/(b^5*d) + (a^3*Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]))/(b^5*d) - (3*a^3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^2) + (3*a^3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^2) + (6*a^3*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^3) - (6*a^3*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^3) - (6*a^3*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^4) + (6*a^3*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^4) + (6*a^3*f^3*Sinh[c + d*x])/(b^4*d^4) + (14*a*f^3*Sinh[c + d*x])/(9*b^2*d^4) + (3*a^3*f*(e + f*x)^2*Sinh[c + d*x])/(b^4*d^2) + (2*a*f*(e + f*x)^2*Sinh[c + d*x])/(3*b^2*d^2) + (3*a^2*f^2*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^3*d^3) + (a^2*(e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^3*d) + (a*f*(e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b^2*d^2) + (2*a*f^3*Sinh[c + d*x]^3)/(27*b^2*d^4) + (3*f^2*(e + f*x)*Sinh[4*c + 4*d*x])/(256*b*d^3) + ((e + f*x)^3*Sinh[4*c + 4*d*x])/(32*b*d)
```

**Rubi [A]** time = 1.82668, antiderivative size = 1038, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 18, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5579, 5448, 3296, 2638, 5447, 3311, 2637, 2633, 32, 3310, 5565, 3322, 2264, 2190, 2531, 6609, 2282, 6589}

$$-\frac{(e+fx)^4}{32bf} + \frac{a^2(e+fx)^4}{8b^3f} + \frac{a^4(e+fx)^4}{4b^5f} - \frac{a \cosh^3(c+dx)(e+fx)^3}{3b^2d} - \frac{a^3 \cosh(c+dx)(e+fx)^3}{b^4d} - \frac{a^3 \sqrt{a^2+b^2} \log\left(\frac{e^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (3*a^2*e*f^2*x)/(4*b^3*d^2) + (3*a^2*f^3*x^2)/(8*b^3*d^2) + (a^4*(e + f*x)^4)/(4*b^5*f) + (a^2*(e + f*x)^4)/(8*b^3*f) - (e + f*x)^4/(32*b*f) - (6*a^3*f^2*(e + f*x)*Cosh[c + d*x])/(b^4*d^3) - (4*a*f^2*(e + f*x)*Cosh[c + d*x])/(3*b^2*d^3) - (a^3*(e + f*x)^3*Cosh[c + d*x])/(b^4*d) - (3*a^2*f^3*Cosh[c + d*x]^2)/(8*b^3*d^4) - (3*a^2*f*(e + f*x)^2*Cosh[c + d*x]^2)/(4*b^3*d^2) - (2*a*f^2*(e + f*x)*Cosh[c + d*x]^3)/(9*b^2*d^3) - (a*(e + f*x)^3*Cosh[c + d*x]^3)/(3*b^2*d) - (3*f^3*Cosh[4*c + 4*d*x])/(1024*b*d^4) - (3*f*(e + f*x)^2*Cosh[4*c + 4*d*x])/(128*b*d^2) - (a^3*Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(b^5*d) + (a^3*Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(b^5*d) - (3*a^3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^2) + (3*a^3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^2) + (6*a^3*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^3) - (6*a^3*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^3) - (6*a^3*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^4) + (6*a^3*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^4) + (6*a^3*f^3*Sinh[c + d*x])/(b^4*d^4) + (14*a*f^3*Sinh[c + d*x])/(9*b^2*d^4) + (3*a^3*f*(e + f*x)^2*Sinh[c + d*x])/(b^4*d^2) + (2*a*f*(e + f*x)^2*Sinh[c + d*x])/(3*b^2*d^2) + (3*a^2*f^2*(e + f*x)*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^3*d^3) + (a^2*(e + f*x)^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^3*d) + (a*f*(e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b^2*d^2) + (2*a*f^3*Sinh[c + d*x]^3)/(27*b^2*d^4) + (3*f^2*(e + f*x)*Sinh[4*c + 4*d*x])/(256*b*d^3) + ((e + f*x)^3*Sinh[4*c + 4*d*x])/(32*b*d)
```

### Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^(n)*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```



Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[  
 ((c + d\*x)^m\*cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[  
 e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ  
 [{c, d}, x]

Rule 5447

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sinh[(a\_.) +  
 (b\_.)\*(x\_)], x\_Symbol] := Simp[((c + d\*x)^m\*Cosh[a + b\*x]^(n + 1))/(b\*(n +  
 1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*Cosh[a + b\*x]^(n +  
 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3311

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] := Simp[(d\*m\*(c + d\*x)^(m - 1)\*(b\*Sine[e + f\*x])^n)/(f^2\*n^2), x] + (Dist  
 [(b^2\*(n - 1))/n, Int[(c + d\*x)^m\*(b\*Sine[e + f\*x])^(n - 2), x], x] - Dist[(  
 d^2\*m\*(m - 1))/(f^2\*n^2), Int[(c + d\*x)^(m - 2)\*(b\*Sine[e + f\*x])^n, x], x]  
 - Simp[(b\*(c + d\*x)^m\*cos[e + f\*x]\*(b\*Sine[e + f\*x])^(n - 1))/(f\*n), x]) /;  
 FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /;  
 FreeQ[{c, d}, x]

Rule 2633

Int[sin[(c\_.) + (d\_.)\*(x\_)]^(n\_.), x\_Symbol] := -Dist[d^(-1), Subst[Int[Expa  
 nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d\*x]], x] /; FreeQ[{c, d}, x]  
 && IGtQ[(n - 1)/2, 0]

Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m +  
 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
  Simp[(d*(b*SIN[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*SIN[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*SIN[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_))*((e_.) + (f_.)*(x_))^(m_.)/((a_.) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_))*((f_.) + (g_.)*(x_))^(m_.)/((a_.) + (b_.)*(F_)^(u_) + (c_.
)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.))*((c_.) + (d_.)*(x_))^(m_.)/
((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*x_))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \cosh^2(c+dx) \sinh^2(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= -\frac{a \int (e+fx)^3 \cosh^2(c+dx) \sinh(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)^3 \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
&= -\frac{(e+fx)^4}{32bf} - \frac{a(e+fx)^3 \cosh^3(c+dx)}{3b^2d} + \frac{a^2 \int (e+fx)^3 \cosh^2(c+dx) dx}{b^3} \\
&= -\frac{(e+fx)^4}{32bf} - \frac{3a^2 f(e+fx)^2 \cosh^2(c+dx)}{4b^3d^2} - \frac{2af^2(e+fx) \cosh^3(c+dx)}{9b^2d^3} \\
&= \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+fx)^4}{32bf} - \frac{a^3(e+fx)^3 \cosh(c+dx)}{b^4d} - \frac{3a^2f}{b^4d} \\
&= \frac{3a^2ef^2x}{4b^3d^2} + \frac{3a^2f^3x^2}{8b^3d^2} + \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+fx)^4}{32bf} - \frac{4af^2(e+fx)}{b^4d} \\
&= \frac{3a^2ef^2x}{4b^3d^2} + \frac{3a^2f^3x^2}{8b^3d^2} + \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+fx)^4}{32bf} - \frac{6a^3f^2(e+fx)}{b^4d} \\
&= \frac{3a^2ef^2x}{4b^3d^2} + \frac{3a^2f^3x^2}{8b^3d^2} + \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+fx)^4}{32bf} - \frac{6a^3f^2(e+fx)}{b^4d} \\
&= \frac{3a^2ef^2x}{4b^3d^2} + \frac{3a^2f^3x^2}{8b^3d^2} + \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+fx)^4}{32bf} - \frac{6a^3f^2(e+fx)}{b^4d} \\
&= \frac{3a^2ef^2x}{4b^3d^2} + \frac{3a^2f^3x^2}{8b^3d^2} + \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+fx)^4}{32bf} - \frac{6a^3f^2(e+fx)}{b^4d} \\
&= \frac{3a^2ef^2x}{4b^3d^2} + \frac{3a^2f^3x^2}{8b^3d^2} + \frac{a^4(e+fx)^4}{4b^5f} + \frac{a^2(e+fx)^4}{8b^3f} - \frac{(e+fx)^4}{32bf} - \frac{6a^3f^2(e+fx)}{b^4d}
\end{aligned}$$

**Mathematica [C]** time = 29.1008, size = 7520, normalized size = 7.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

[Out] Result too large to show

**Maple [F]** time = 0.238, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 (\cosh(dx + c))^2 (\sinh(dx + c))^3}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)`

[Out] `int((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [C]** time = 4.20574, size = 22876, normalized size = 22.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] 
$$-1/55296*(864*b^4*d^3*f^3*x^3 + 864*b^4*d^3*e^3 + 648*b^4*d^2*e^2*f - 27*(32*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f^3)*x)*\cosh(d*x + c)^8 - 27*(32*b^4*d^3*f^3*x^3 + 32$$

$$\begin{aligned}
& *b^4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f^3)*x)*\sinh(d*x + c)^8 + 324*b^4*d*e*f^2 + 256*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^3*d^2*e^2*f + 6*a*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + 3*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)*\cosh(d*x + c)^7 + 8*(288*a*b^3*d^3*f^3*x^3 + 288*a*b^3*d^3*e^3 - 288*a*b^3*d^2*e^2*f + 192*a*b^3*d*e*f^2 - 64*a*b^3*f^3 + 288*(3*a*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + 96*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x - 27*(32*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f^3)*x)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 81*b^4*f^3 - 1728*(4*a^2*b^2*d^3*f^3*x^3 + 4*a^2*b^2*d^3*e^3 - 6*a^2*b^2*d^2*e^2*f + 6*a^2*b^2*d*e*f^2 - 3*a^2*b^2*f^3 + 6*(2*a^2*b^2*d^3*e*f^2 - a^2*b^2*d^2*f^3)*x^2 + 6*(2*a^2*b^2*d^3*e^2*f - 2*a^2*b^2*d^2*e*f^2 + a^2*b^2*d*f^3)*x)*\cosh(d*x + c)^6 - 4*(1728*a^2*b^2*d^3*f^3*x^3 + 1728*a^2*b^2*d^3*e^3 - 2592*a^2*b^2*d^2*e^2*f + 2592*a^2*b^2*d*e*f^2 - 1296*a^2*b^2*f^3 + 2592*(2*a^2*b^2*d^3*e*f^2 - a^2*b^2*d^2*f^3)*x^2 + 189*(32*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f^3)*x)*\cosh(d*x + c)^2 + 2592*(2*a^2*b^2*d^3*e^2*f - 2*a^2*b^2*d^2*e*f^2 + a^2*b^2*d*f^3)*x - 448*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^3*d^2*e^2*f + 6*a*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + 3*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)*\cosh(d*x + c))*\sinh(d*x + c)^6 + 6912*((4*a^3*b + a*b^3)*d^3*f^3*x^3 + (4*a^3*b + a*b^3)*d^3*e^3 - 3*(4*a^3*b + a*b^3)*d^2*e^2*f + 6*(4*a^3*b + a*b^3)*d*e*f^2 - 6*(4*a^3*b + a*b^3)*f^3 + 3*((4*a^3*b + a*b^3)*d^3*e*f^2 - (4*a^3*b + a*b^3)*d^2*f^3)*x^2 + 3*((4*a^3*b + a*b^3)*d^3*e^2*f - 2*(4*a^3*b + a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + a*b^3)*d*f^3)*x)*\cosh(d*x + c)^5 + 24*(288*(4*a^3*b + a*b^3)*d^3*f^3*x^3 + 288*(4*a^3*b + a*b^3)*d^3*e^3 - 864*(4*a^3*b + a*b^3)*d^2*e^2*f + 1728*(4*a^3*b + a*b^3)*d*e*f^2 - 1728*(4*a^3*b + a*b^3)*f^3 - 63*(32*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f^3)*x)*\cosh(d*x + c))^3 + 864*((4*a^3*b + a*b^3)*d^3*e*f^2 - (4*a^3*b + a*b^3)*d^2*f^3)*x^2 + 224*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^3*d^2*e^2*f + 6*a*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + 3*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)*\cosh(d*x + c)^2 + 864*((4*a^3*b + a*b^3)*d^3*e^2*f - 2*(4*a^3*b + a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + a*b^3)*d*f^3)*x - 432*(4*a^2*b^2*d^3*f^3*x^3 + 4*a^2*b^2*d^3*e^3 - 6*a^2*b^2*d^2*e^2*f + 6*a^2*b^2*d*e*f^2 - 3*a^2*b^2*f^3 + 6*(2*a^2*b^2*d^3*e*f^2 - a^2*b^2*d^2*f^3)*x^2 + 6*(2*a^2*b^2*d^3*e^2*f - 2*a^2*b^2*d^2*e*f^2 + a^2*b^2*d*f^3)*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 1728*((8*a^4 + 4*a^2*b^2 - b^4)*d^4*f^3*x^4 + 4*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*e*f^2*x^3 + 6*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*e^2*f*x^2 + 4*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*e^3*x)*\cosh(d*x + c)^4 - 2*(864*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*f^3*x^4 + 3456*(8*a^4 + 4*a^
\end{aligned}$$

$$\begin{aligned}
& 2*b^2 - b^4)*d^4*e*f^2*x^3 + 5184*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*e^2*f*x^2 + \\
& 3456*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*e^3*x + 945*(32*b^4*d^3*f^3*x^3 + 32*b^4 \\
& 4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3*e \\
& *f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f^3 \\
& )*x)*\cosh(d*x + c)^4 - 4480*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^ \\
& 3*d^2*e^2*f + 6*a*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3* \\
& d^2*f^3)*x^2 + 3*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x) \\
& *\cosh(d*x + c)^3 + 12960*(4*a^2*b^2*d^3*f^3*x^3 + 4*a^2*b^2*d^3*e^3 - 6*a^2 \\
& *b^2*d^2*e^2*f + 6*a^2*b^2*d*e*f^2 - 3*a^2*b^2*f^3 + 6*(2*a^2*b^2*d^3*e*f^2 \\
& - a^2*b^2*d^2*f^3)*x^2 + 6*(2*a^2*b^2*d^3*e^2*f - 2*a^2*b^2*d^2*e*f^2 + a^ \\
& 2*b^2*d*f^3)*x)*\cosh(d*x + c)^2 - 17280*((4*a^3*b + a*b^3)*d^3*f^3*x^3 + (4 \\
& *a^3*b + a*b^3)*d^3*e^3 - 3*(4*a^3*b + a*b^3)*d^2*e^2*f + 6*(4*a^3*b + a*b^ \\
& 3)*d*e*f^2 - 6*(4*a^3*b + a*b^3)*f^3 + 3*((4*a^3*b + a*b^3)*d^3*e*f^2 - (4* \\
& a^3*b + a*b^3)*d^2*f^3)*x^2 + 3*((4*a^3*b + a*b^3)*d^3*e^2*f - 2*(4*a^3*b + \\
& a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + a*b^3)*d*f^3)*x)*\cosh(d*x + c)*\sinh(d*x + \\
& c)^4 + 6912*((4*a^3*b + a*b^3)*d^3*f^3*x^3 + (4*a^3*b + a*b^3)*d^3*e^3 + 3 \\
& *(4*a^3*b + a*b^3)*d^2*e^2*f + 6*(4*a^3*b + a*b^3)*d*e*f^2 + 6*(4*a^3*b + a \\
& *b^3)*f^3 + 3*((4*a^3*b + a*b^3)*d^3*e*f^2 + (4*a^3*b + a*b^3)*d^2*f^3)*x^2 \\
& + 3*((4*a^3*b + a*b^3)*d^3*e^2*f + 2*(4*a^3*b + a*b^3)*d^2*e*f^2 + 2*(4*a^ \\
& 3*b + a*b^3)*d*f^3)*x)*\cosh(d*x + c)^3 + 8*(864*(4*a^3*b + a*b^3)*d^3*f^3*x \\
& ^3 + 864*(4*a^3*b + a*b^3)*d^3*e^3 + 2592*(4*a^3*b + a*b^3)*d^2*e^2*f - 189 \\
& *(32*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*d^2*e^2*f + 12*b^4*d*e*f^2 - \\
& 3*b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)*x^2 + 12*(8*b^4*d^3*e^2*f - \\
& 4*b^4*d^2*e*f^2 + b^4*d*f^3)*x)*\cosh(d*x + c)^5 + 5184*(4*a^3*b + a*b^3)*d \\
& *e*f^2 + 1120*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^3*d^2*e^2*f + \\
& 6*a*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)*x^2 + \\
& 3*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)*\cosh(d*x + c) \\
& ^4 + 5184*(4*a^3*b + a*b^3)*f^3 - 4320*(4*a^2*b^2*d^3*f^3*x^3 + 4*a^2*b^2*d \\
& ^3*e^3 - 6*a^2*b^2*d^2*e^2*f + 6*a^2*b^2*d*e*f^2 - 3*a^2*b^2*f^3 + 6*(2*a^2 \\
& *b^2*d^3*e*f^2 - a^2*b^2*d^2*f^3)*x^2 + 6*(2*a^2*b^2*d^3*e^2*f - 2*a^2*b^2* \\
& d^2*e*f^2 + a^2*b^2*d*f^3)*x)*\cosh(d*x + c)^3 + 2592*((4*a^3*b + a*b^3)*d^3 \\
& *e*f^2 + (4*a^3*b + a*b^3)*d^2*f^3)*x^2 + 8640*((4*a^3*b + a*b^3)*d^3*f^3*x \\
& ^3 + (4*a^3*b + a*b^3)*d^3*e^3 - 3*(4*a^3*b + a*b^3)*d^2*e^2*f + 6*(4*a^3*b \\
& + a*b^3)*d*e*f^2 - 6*(4*a^3*b + a*b^3)*f^3 + 3*((4*a^3*b + a*b^3)*d^3*e*f^ \\
& 2 - (4*a^3*b + a*b^3)*d^2*f^3)*x^2 + 3*((4*a^3*b + a*b^3)*d^3*e^2*f - 2*(4* \\
& a^3*b + a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + a*b^3)*d*f^3)*x)*\cosh(d*x + c)^2 + \\
& 2592*((4*a^3*b + a*b^3)*d^3*e^2*f + 2*(4*a^3*b + a*b^3)*d^2*e*f^2 + 2*(4*a^ \\
& 3*b + a*b^3)*d*f^3)*x - 864*((8*a^4 + 4*a^2*b^2 - b^4)*d^4*f^3*x^4 + 4*(8*a \\
& ^4 + 4*a^2*b^2 - b^4)*d^4*e*f^2*x^3 + 6*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*e^2*f \\
& *x^2 + 4*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*e^3*x)*\cosh(d*x + c)*\sinh(d*x + c)^ \\
& 3 + 648*(4*b^4*d^3*e*f^2 + b^4*d^2*f^3)*x^2 + 1728*(4*a^2*b^2*d^3*f^3*x^3 + \\
& 4*a^2*b^2*d^3*e^3 + 6*a^2*b^2*d^2*e^2*f + 6*a^2*b^2*d*e*f^2 + 3*a^2*b^2*f^ \\
& 3 + 6*(2*a^2*b^2*d^3*e*f^2 + a^2*b^2*d^2*f^3)*x^2 + 6*(2*a^2*b^2*d^3*e^2*f \\
& + 2*a^2*b^2*d^2*e*f^2 + a^2*b^2*d*f^3)*x)*\cosh(d*x + c)^2 + 12*(576*a^2*b^2 \\
& *d^3*f^3*x^3 + 576*a^2*b^2*d^3*e^3 + 864*a^2*b^2*d^2*e^2*f + 864*a^2*b^2*d*
\end{aligned}$$

$$\begin{aligned}
& e^f^2 + 432a^2b^2f^3 - 63(32b^4d^3f^3x^3 + 32b^4d^3e^3 - 24b^4d^2e^2f + 12b^4d^2e^2f^2 - 3b^4d^2f^3 + 24(4b^4d^3e^2f^2 - b^4d^2f^3) \\
& *x^2 + 12(8b^4d^3e^2f - 4b^4d^2e^2f^2 + b^4d^2f^3)*x) * \cosh(dx + c)^6 + 448(9a^3b^3d^3f^3x^3 + 9a^3b^3d^3e^3 - 9a^3b^3d^2e^2f + 6a^3b^3d^2e^2f^2 - 2a^3b^3f^3 + 9(3a^3b^3d^3e^2f^2 - a^3b^3d^2f^3) *x^2 + 3(9a^3b^3d^3e^2f - 6a^3b^3d^2e^2f^2 + 2a^3b^3d^2f^3) *x) * \cosh(dx + c)^5 - 2 \\
& 160(4a^2b^2d^3f^3x^3 + 4a^2b^2d^3e^3 - 6a^2b^2d^2e^2f + 6a^2b^2d^2e^2f^2 - 3a^2b^2f^3 + 6(2a^2b^2d^3e^2f^2 - a^2b^2d^2f^3) *x^2 + 6(2a^2b^2d^3e^2f - 2a^2b^2d^2e^2f^2 + a^2b^2d^2f^3) *x) * \cosh(dx + c)^4 + 5760((4a^3b + a^3b^3) *d^3f^3x^3 + (4a^3b + a^3b^3) *d^3e^3 - 3(4a^3b + a^3b^3) *d^2e^2f + 6(4a^3b + a^3b^3) *d^2e^2f^2 - 6(4a^3b + a^3b^3) *d^2f^3 + 3((4a^3b + a^3b^3) *d^3e^2f^2 - (4a^3b + a^3b^3) *d^2f^3) *x^2 + 3((4a^3b + a^3b^3) *d^3e^2f - 2(4a^3b + a^3b^3) *d^2e^2f^2 + 2(4a^3b + a^3b^3) *d^2f^3) *x) * \cosh(dx + c)^3 + 864(2a^2b^2d^3e^2f^2 + a^2b^2d^2f^3) *x^2 - 864((8a^4 + 4a^2b^2 - b^4) *d^4f^3x^4 + 4(8a^4 + 4a^2b^2 - b^4) *d^4e^2f^2 *x^3 + 6(8a^4 + 4a^2b^2 - b^4) *d^4e^2f *x^2 + 4(8a^4 + 4a^2b^2 - b^4) *d^4e^3 *x) * \cosh(dx + c)^2 + 864(2a^2b^2d^3e^2f + 2a^2b^2d^2e^2f^2 + a^2b^2d^2f^3) *x + 1728((4a^3b + a^3b^3) *d^3f^3x^3 + (4a^3b + a^3b^3) *d^3e^3 + 3(4a^3b + a^3b^3) *d^2e^2f + 6(4a^3b + a^3b^3) *d^2e^2f^2 + 6(4a^3b + a^3b^3) *f^3 + 3((4a^3b + a^3b^3) *d^3e^2f^2 + (4a^3b + a^3b^3) *d^2f^3) *x^2 + 3((4a^3b + a^3b^3) *d^3e^2f + 2(4a^3b + a^3b^3) *d^2e^2f^2 + 2(4a^3b + a^3b^3) *d^2f^3) *x) * \cosh(dx + c) * \sinh(dx + c)^2 + 165888((a^3b *d^2f^3 *x^2 + 2a^3b *d^2e^2f^2 *x + a^3b *d^2e^2f) * \cosh(dx + c)^4 + 4(a^3b *d^2f^3 *x^2 + 2a^3b *d^2e^2f^2 *x + a^3b *d^2e^2f) * \cosh(dx + c)^3 * \sinh(dx + c) + 6(a^3b *d^2f^3 *x^2 + 2a^3b *d^2e^2f^2 *x + a^3b *d^2e^2f) * \cosh(dx + c)^2 * \sinh(dx + c)^2 + 4(a^3b *d^2f^3 *x^2 + 2a^3b *d^2e^2f^2 *x + a^3b *d^2e^2f) * \cosh(dx + c) * \sinh(dx + c)^3 + (a^3b *d^2f^3 *x^2 + 2a^3b *d^2e^2f^2 *x + a^3b *d^2e^2f) * \sinh(dx + c)^4) * \sqrt{(a^2 + b^2)/b^2} * \operatorname{dilog}((a * \cosh(dx + c) + a * \sinh(dx + c) + (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 165888((a^3b *d^2f^3 *x^2 + 2a^3b *d^2e^2f^2 *x + a^3b *d^2e^2f) * \cosh(dx + c)^4 + 4(a^3b *d^2f^3 *x^2 + 2a^3b *d^2e^2f^2 *x + a^3b *d^2e^2f) * \cosh(dx + c)^3 * \sinh(dx + c) + 6(a^3b *d^2f^3 *x^2 + 2a^3b *d^2e^2f^2 *x + a^3b *d^2e^2f) * \cosh(dx + c)^2 * \sinh(dx + c)^2 + 4(a^3b *d^2f^3 *x^2 + 2a^3b *d^2e^2f^2 *x + a^3b *d^2e^2f) * \cosh(dx + c) * \sinh(dx + c)^3 + (a^3b *d^2f^3 *x^2 + 2a^3b *d^2e^2f^2 *x + a^3b *d^2e^2f) * \sinh(dx + c)^4) * \sqrt{(a^2 + b^2)/b^2} * \operatorname{dilog}((a * \cosh(dx + c) + a * \sinh(dx + c) - (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 55296 * ((a^3b *d^3e^3 - 3a^3b *c *d^2e^2f + 3a^3b *c^2 *d^2e^2f^2 - a^3b *c^3 *f^3) * \cosh(dx + c)^4 + 4(a^3b *d^3e^3 - 3a^3b *c *d^2e^2f + 3a^3b *c^2 *d^2e^2f^2 - a^3b *c^3 *f^3) * \cosh(dx + c)^3 * \sinh(dx + c) + 6(a^3b *d^3e^3 - 3a^3b *c *d^2e^2f + 3a^3b *c^2 *d^2e^2f^2 - a^3b *c^3 *f^3) * \cosh(dx + c)^2 * \sinh(dx + c)^2 + 4(a^3b *d^3e^3 - 3a^3b *c *d^2e^2f + 3a^3b *c^2 *d^2e^2f^2 - a^3b *c^3 *f^3) * \cosh(dx + c) * \sinh(dx + c)^3 + (a^3b *d^3e^3 - 3a^3b *c *d^2e^2f + 3a^3b *c^2 *d^2e^2f^2 - a^3b *c^3 *f^3) * \sinh(dx + c)^4) * \sqrt{(a^2 + b^2)/b^2}
\end{aligned}$$



$$\begin{aligned}
& ((a^2 + b^2)/b^2) \log(2b \cosh(dx + c) + 2b \sinh(dx + c) + 2b \sqrt{(a^2 + b^2)/b^2} + 2a) + 55296 * ((a^3 b d^3 e^3 - 3a^3 b c d^2 e^2 f + 3a^3 b c^2 d e f^2 - a^3 b c^3 f^3) \cosh(dx + c)^4 + 4(a^3 b d^3 e^3 - 3a^3 b c d^2 e^2 f + 3a^3 b c^2 d e f^2 - a^3 b c^3 f^3) \cosh(dx + c)^3 \sinh(dx + c) + 6(a^3 b d^3 e^3 - 3a^3 b c d^2 e^2 f + 3a^3 b c^2 d e f^2 - a^3 b c^3 f^3) \cosh(dx + c)^2 \sinh(dx + c)^2 + 4(a^3 b d^3 e^3 - 3a^3 b c d^2 e^2 f + 3a^3 b c^2 d e f^2 - a^3 b c^3 f^3) \cosh(dx + c) \sinh(dx + c)^3 + (a^3 b d^3 e^3 - 3a^3 b c d^2 e^2 f + 3a^3 b c^2 d e f^2 - a^3 b c^3 f^3) \sinh(dx + c)^4) \sqrt{(a^2 + b^2)/b^2} \log(2b \cosh(dx + c) + 2b \sinh(dx + c) - 2b \sqrt{(a^2 + b^2)/b^2} + 2a) + 55296 * ((a^3 b d^3 f^3 x^3 + 3a^3 b d^3 e f^2 x^2 + 3a^3 b d^3 e^2 f x + 3a^3 b c d^2 e^2 f - 3a^3 b c^2 d e f^2 + a^3 b c^3 f^3) \cosh(dx + c)^4 + 4(a^3 b d^3 f^3 x^3 + 3a^3 b d^3 e f^2 x^2 + 3a^3 b d^3 e^2 f x + 3a^3 b c d^2 e^2 f - 3a^3 b c^2 d e f^2 + a^3 b c^3 f^3) \cosh(dx + c)^3 \sinh(dx + c) + 6(a^3 b d^3 f^3 x^3 + 3a^3 b d^3 e f^2 x^2 + 3a^3 b d^3 e^2 f x + 3a^3 b c d^2 e^2 f - 3a^3 b c^2 d e f^2 + a^3 b c^3 f^3) \cosh(dx + c)^2 \sinh(dx + c)^2 + 4(a^3 b d^3 f^3 x^3 + 3a^3 b d^3 e f^2 x^2 + 3a^3 b d^3 e^2 f x + 3a^3 b c d^2 e^2 f - 3a^3 b c^2 d e f^2 + a^3 b c^3 f^3) \cosh(dx + c) \sinh(dx + c)^3 + (a^3 b d^3 f^3 x^3 + 3a^3 b d^3 e f^2 x^2 + 3a^3 b d^3 e^2 f x + 3a^3 b c d^2 e^2 f - 3a^3 b c^2 d e f^2 + a^3 b c^3 f^3) \sinh(dx + c)^4) \sqrt{(a^2 + b^2)/b^2} \log(-a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b) - 55296 * ((a^3 b d^3 f^3 x^3 + 3a^3 b d^3 e f^2 x^2 + 3a^3 b d^3 e^2 f x + 3a^3 b c d^2 e^2 f - 3a^3 b c^2 d e f^2 + a^3 b c^3 f^3) \cosh(dx + c)^4 + 4(a^3 b d^3 f^3 x^3 + 3a^3 b d^3 e f^2 x^2 + 3a^3 b d^3 e^2 f x + 3a^3 b c d^2 e^2 f - 3a^3 b c^2 d e f^2 + a^3 b c^3 f^3) \cosh(dx + c)^3 \sinh(dx + c) + 6(a^3 b d^3 f^3 x^3 + 3a^3 b d^3 e f^2 x^2 + 3a^3 b d^3 e^2 f x + 3a^3 b c d^2 e^2 f - 3a^3 b c^2 d e f^2 + a^3 b c^3 f^3) \cosh(dx + c)^2 \sinh(dx + c)^2 + 4(a^3 b d^3 f^3 x^3 + 3a^3 b d^3 e f^2 x^2 + 3a^3 b d^3 e^2 f x + 3a^3 b c d^2 e^2 f - 3a^3 b c^2 d e f^2 + a^3 b c^3 f^3) \cosh(dx + c) \sinh(dx + c)^3 + (a^3 b d^3 f^3 x^3 + 3a^3 b d^3 e f^2 x^2 + 3a^3 b d^3 e^2 f x + 3a^3 b c d^2 e^2 f - 3a^3 b c^2 d e f^2 + a^3 b c^3 f^3) \sinh(dx + c)^4) \sqrt{(a^2 + b^2)/b^2} \log(-a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b) + 331776 * (a^3 b f^3 \cosh(dx + c)^4 + 4a^3 b f^3 \cosh(dx + c)^3 \sinh(dx + c) + 6a^3 b f^3 \cosh(dx + c)^2 \sinh(dx + c)^2 + 4a^3 b f^3 \cosh(dx + c) \sinh(dx + c)^3 + a^3 b f^3 \sinh(dx + c)^4) \sqrt{(a^2 + b^2)/b^2} \operatorname{polylog}(4, (a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2}))/b) - 331776 * (a^3 b f^3 \cosh(dx + c)^4 + 4a^3 b f^3 \cosh(dx + c)^3 \sinh(dx + c) + 6a^3 b f^3 \cosh(dx + c)^2 \sinh(dx + c)^2 + 4a^3 b f^3 \cosh(dx + c) \sinh(dx + c)^3 + a^3 b f^3 \sinh(dx + c)^4) \sqrt{(a^2 + b^2)/b^2} \operatorname{polylog}(4, (a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2}))/b) - 331776 * ((a^3 b d f^3 x + a^3 b d e f^2) \cosh(dx + c)^4 + 4(a^3 b d f^3 x + a^3 b d e f^2) \cosh(dx + c)^3 \sinh(dx + c) + 6(a^3 b d f^3 x + a^3 b d e f^2) \cosh(dx + c)^2
\end{aligned}$$

$$\begin{aligned}
& * \sinh(d*x + c)^2 + 4*(a^3*b*d*f^3*x + a^3*b*d*e*f^2)*\cosh(d*x + c)*\sinh(d*x \\
& + c)^3 + (a^3*b*d*f^3*x + a^3*b*d*e*f^2)*\sinh(d*x + c)^4*\sqrt{(a^2 + b^2)} \\
& /b^2)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b* \\
& \sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2))/b) + 331776*((a^3*b*d*f^3*x + a^3*b*d \\
& *e*f^2)*\cosh(d*x + c)^4 + 4*(a^3*b*d*f^3*x + a^3*b*d*e*f^2)*\cosh(d*x + c)^3 \\
& *\sinh(d*x + c) + 6*(a^3*b*d*f^3*x + a^3*b*d*e*f^2)*\cosh(d*x + c)^2*\sinh(d*x \\
& + c)^2 + 4*(a^3*b*d*f^3*x + a^3*b*d*e*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + \\
& (a^3*b*d*f^3*x + a^3*b*d*e*f^2)*\sinh(d*x + c)^4)*\sqrt{(a^2 + b^2)/b^2)*\text{pol} \\
& \text{ylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x \\
& + c))*\sqrt{(a^2 + b^2)/b^2))/b) + 324*(8*b^4*d^3*e^2*f + 4*b^4*d^2*e*f^2 + \\
& b^4*d*f^3)*x + 256*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 + 9*a*b^3*d^2*e^2 \\
& *f + 6*a*b^3*d*e*f^2 + 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 + a*b^3*d^2*f^3)* \\
& x^2 + 3*(9*a*b^3*d^3*e^2*f + 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)*\cosh(d*x \\
& + c) + 8*(288*a*b^3*d^3*f^3*x^3 + 288*a*b^3*d^3*e^3 + 288*a*b^3*d^2*e^2*f \\
& + 192*a*b^3*d*e*f^2 - 27*(32*b^4*d^3*f^3*x^3 + 32*b^4*d^3*e^3 - 24*b^4*d^2* \\
& e^2*f + 12*b^4*d*e*f^2 - 3*b^4*f^3 + 24*(4*b^4*d^3*e*f^2 - b^4*d^2*f^3)*x^2 \\
& + 12*(8*b^4*d^3*e^2*f - 4*b^4*d^2*e*f^2 + b^4*d*f^3)*x)*\cosh(d*x + c)^7 + \\
& 64*a*b^3*f^3 + 224*(9*a*b^3*d^3*f^3*x^3 + 9*a*b^3*d^3*e^3 - 9*a*b^3*d^2*e^2 \\
& *f + 6*a*b^3*d*e*f^2 - 2*a*b^3*f^3 + 9*(3*a*b^3*d^3*e*f^2 - a*b^3*d^2*f^3)* \\
& x^2 + 3*(9*a*b^3*d^3*e^2*f - 6*a*b^3*d^2*e*f^2 + 2*a*b^3*d*f^3)*x)*\cosh(d*x \\
& + c)^6 - 1296*(4*a^2*b^2*d^3*f^3*x^3 + 4*a^2*b^2*d^3*e^3 - 6*a^2*b^2*d^2*e \\
& ^2*f + 6*a^2*b^2*d*e*f^2 - 3*a^2*b^2*f^3 + 6*(2*a^2*b^2*d^3*e*f^2 - a^2*b^2 \\
& *d^2*f^3)*x^2 + 6*(2*a^2*b^2*d^3*e^2*f - 2*a^2*b^2*d^2*e*f^2 + a^2*b^2*d*f^ \\
& 3)*x)*\cosh(d*x + c)^5 + 4320*((4*a^3*b + a*b^3)*d^3*f^3*x^3 + (4*a^3*b + a \\
& b^3)*d^3*e^3 - 3*(4*a^3*b + a*b^3)*d^2*e^2*f + 6*(4*a^3*b + a*b^3)*d*e*f^2 \\
& - 6*(4*a^3*b + a*b^3)*f^3 + 3*((4*a^3*b + a*b^3)*d^3*e*f^2 - (4*a^3*b + a*b \\
& ^3)*d^2*f^3)*x^2 + 3*((4*a^3*b + a*b^3)*d^3*e^2*f - 2*(4*a^3*b + a*b^3)*d^2 \\
& *e*f^2 + 2*(4*a^3*b + a*b^3)*d*f^3)*x)*\cosh(d*x + c)^4 - 864*((8*a^4 + 4*a^ \\
& 2*b^2 - b^4)*d^4*f^3*x^4 + 4*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*e*f^2*x^3 + 6*(8 \\
& *a^4 + 4*a^2*b^2 - b^4)*d^4*e^2*f*x^2 + 4*(8*a^4 + 4*a^2*b^2 - b^4)*d^4*e^3 \\
& *x)*\cosh(d*x + c)^3 + 288*(3*a*b^3*d^3*e*f^2 + a*b^3*d^2*f^3)*x^2 + 2592*(( \\
& 4*a^3*b + a*b^3)*d^3*f^3*x^3 + (4*a^3*b + a*b^3)*d^3*e^3 + 3*(4*a^3*b + a*b \\
& ^3)*d^2*e^2*f + 6*(4*a^3*b + a*b^3)*d*e*f^2 + 6*(4*a^3*b + a*b^3)*f^3 + 3*( \\
& (4*a^3*b + a*b^3)*d^3*e*f^2 + (4*a^3*b + a*b^3)*d^2*f^3)*x^2 + 3*((4*a^3*b \\
& + a*b^3)*d^3*e^2*f + 2*(4*a^3*b + a*b^3)*d^2*e*f^2 + 2*(4*a^3*b + a*b^3)*d \\
& f^3)*x)*\cosh(d*x + c)^2 + 96*(9*a*b^3*d^3*e^2*f + 6*a*b^3*d^2*e*f^2 + 2*a*b \\
& ^3*d*f^3)*x + 432*(4*a^2*b^2*d^3*f^3*x^3 + 4*a^2*b^2*d^3*e^3 + 6*a^2*b^2*d^ \\
& 2*e^2*f + 6*a^2*b^2*d*e*f^2 + 3*a^2*b^2*f^3 + 6*(2*a^2*b^2*d^3*e*f^2 + a^2*b \\
& ^2*d^2*f^3)*x^2 + 6*(2*a^2*b^2*d^3*e^2*f + 2*a^2*b^2*d^2*e*f^2 + a^2*b^2*d \\
& *f^3)*x)*\cosh(d*x + c))*\sinh(d*x + c)/(b^5*d^4*\cosh(d*x + c)^4 + 4*b^5*d^4 \\
& *\cosh(d*x + c)^3*\sinh(d*x + c) + 6*b^5*d^4*\cosh(d*x + c)^2*\sinh(d*x + c)^2 \\
& + 4*b^5*d^4*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^5*d^4*\sinh(d*x + c)^4)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cosh(d\*x+c)\*\*2\*sinh(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c)^2 \sinh(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)^2\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*cosh(d\*x + c)^2\*sinh(d\*x + c)^3/(b\*sinh(d\*x + c) + a), x)

$$3.397 \quad \int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=755

$$-\frac{2a^3 f \sqrt{a^2 + b^2} (e + fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^5 d^2} + \frac{2a^3 f \sqrt{a^2 + b^2} (e + fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right)}{b^5 d^2} + \frac{2a^3 f^2 \sqrt{a^2 + b^2} \text{PolyLog}\left(3, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^5 d^2}$$

```
[Out] (a^2*f^2*x)/(4*b^3*d^2) + (a^4*(e + f*x)^3)/(3*b^5*f) + (a^2*(e + f*x)^3)/(6*b^3*f) - (e + f*x)^3/(24*b*f) - (2*a^3*f^2*Cosh[c + d*x])/(b^4*d^3) - (4*a*f^2*Cosh[c + d*x])/(9*b^2*d^3) - (a^3*(e + f*x)^2*Cosh[c + d*x])/(b^4*d) - (a^2*f*(e + f*x)*Cosh[c + d*x]^2)/(2*b^3*d^2) - (2*a*f^2*Cosh[c + d*x]^3)/(27*b^2*d^3) - (a*(e + f*x)^2*Cosh[c + d*x]^3)/(3*b^2*d) - (f*(e + f*x)*Cosh[4*c + 4*d*x])/(64*b*d^2) - (a^3*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]))/(b^5*d) + (a^3*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]))/(b^5*d) - (2*a^3*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^2) + (2*a^3*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^2) + (2*a^3*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^5*d^3) - (2*a^3*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^5*d^3) + (2*a^3*f*(e + f*x)*Sinh[c + d*x])/(b^4*d^2) + (4*a*f*(e + f*x)*Sinh[c + d*x])/(9*b^2*d^2) + (a^2*f^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^3*d^3) + (a^2*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*b^3*d) + (2*a*f*(e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(9*b^2*d^2) + (f^2*Sinh[4*c + 4*d*x])/(256*b*d^3) + ((e + f*x)^2*Sinh[4*c + 4*d*x])/(32*b*d)
```

**Rubi [A]** time = 1.51398, antiderivative size = 755, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 18, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5579, 5448, 3296, 2637, 5447, 3310, 2638, 3311, 32, 2635, 8, 5565, 3322, 2264, 2190, 2531, 2282, 6589}

$$-\frac{2a^3 f \sqrt{a^2 + b^2} (e + fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^5 d^2} + \frac{2a^3 f \sqrt{a^2 + b^2} (e + fx) \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right)}{b^5 d^2} + \frac{2a^3 f^2 \sqrt{a^2 + b^2} \text{PolyLog}\left(3, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^5 d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*f^2*x)/(4*b^3*d^2) + (a^4*(e + f*x)^3)/(3*b^5*f) + (a^2*(e + f*x)^3)/(6*b^3*f) - (e + f*x)^3/(24*b*f) - (2*a^3*f^2*Cosh[c + d*x])/(b^4*d^3) - (4*
```

$$\begin{aligned}
& a*f^2*\text{Cosh}[c + d*x]/(9*b^2*d^3) - (a^3*(e + f*x)^2*\text{Cosh}[c + d*x])/(b^4*d) \\
& - (a^2*f*(e + f*x)*\text{Cosh}[c + d*x]^2)/(2*b^3*d^2) - (2*a*f^2*\text{Cosh}[c + d*x]^3) \\
& /((27*b^2*d^3) - (a*(e + f*x)^2*\text{Cosh}[c + d*x]^3)/(3*b^2*d) - (f*(e + f*x)*\text{Co} \\
& \text{sh}[4*c + 4*d*x])/(64*b*d^2) - (a^3*\text{Sqrt}[a^2 + b^2]*(e + f*x)^2*\text{Log}[1 + (b*E \\
& ^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])])/(b^5*d) + (a^3*\text{Sqrt}[a^2 + b^2]*(e + f*x) \\
& )^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(b^5*d) - (2*a^3*\text{Sqrt}[a \\
& ^2 + b^2]*f*(e + f*x)*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))] \\
& )/(b^5*d^2) + (2*a^3*\text{Sqrt}[a^2 + b^2]*f*(e + f*x)*\text{PolyLog}[2, -((b*E^{(c + d*x)}) \\
& )/(a + \text{Sqrt}[a^2 + b^2])])/(b^5*d^2) + (2*a^3*\text{Sqrt}[a^2 + b^2]*f^2*\text{PolyLog}[3 \\
& , -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))] )/(b^5*d^3) - (2*a^3*\text{Sqrt}[a^2 + \\
& b^2]*f^2*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))] )/(b^5*d^3) + \\
& (2*a^3*f*(e + f*x)*\text{Sinh}[c + d*x])/(b^4*d^2) + (4*a*f*(e + f*x)*\text{Sinh}[c + d*x] \\
& )/(9*b^2*d^2) + (a^2*f^2*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(4*b^3*d^3) + (a^2*( \\
& e + f*x)^2*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(2*b^3*d) + (2*a*f*(e + f*x)*\text{Cosh}[c \\
& + d*x]^2*\text{Sinh}[c + d*x])/(9*b^2*d^2) + (f^2*\text{Sinh}[4*c + 4*d*x])/(256*b*d^3) \\
& + ((e + f*x)^2*\text{Sinh}[4*c + 4*d*x])/(32*b*d)
\end{aligned}$$

### Rule 5579

$$\begin{aligned}
& \text{Int}[(\text{Cosh}[(c_{.}) + (d_{.})*(x_{.})]^{\text{p}_{.}}*((e_{.}) + (f_{.})*(x_{.}))^{\text{m}_{.}}*\text{Sinh}[(c_{.}) + \\
& (d_{.})*(x_{.})]^{\text{n}_{.}})/(a_{.} + (b_{.})*\text{Sinh}[(c_{.}) + (d_{.})*(x_{.})]), x_{\text{Symbol}}] \text{:>} \text{D} \\
& \text{ist}[1/b, \text{Int}[(e + f*x)^m*\text{Cosh}[c + d*x]^p*\text{Sinh}[c + d*x]^{(n - 1)}, x], x] - \text{D} \\
& \text{ist}[a/b, \text{Int}[(e + f*x)^m*\text{Cosh}[c + d*x]^p*\text{Sinh}[c + d*x]^{(n - 1)}]/(a + b*\text{Sinh} \\
& [c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \text{IGtQ}\{m, 0\} \ \&\& \text{IGtQ}\{n, \\
& 0\} \ \&\& \text{IGtQ}\{p, 0\}
\end{aligned}$$

### Rule 5448

$$\begin{aligned}
& \text{Int}[\text{Cosh}[(a_{.}) + (b_{.})*(x_{.})]^{\text{p}_{.}}*((c_{.}) + (d_{.})*(x_{.}))^{\text{m}_{.}}*\text{Sinh}[(a_{.}) + \\
& (b_{.})*(x_{.})]^{\text{n}_{.}}], x_{\text{Symbol}}] \text{:>} \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sinh}[a + \\
& b*x]^n*\text{Cosh}[a + b*x]^p, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x \ \&\& \text{IGtQ}\{n, 0\} \ \& \\
& \ \& \text{IGtQ}\{p, 0\}
\end{aligned}$$

### Rule 3296

$$\begin{aligned}
& \text{Int}[(c_{.}) + (d_{.})*(x_{.})]^{\text{m}_{.}}*\text{sin}[(e_{.}) + (f_{.})*(x_{.})], x_{\text{Symbol}}] \text{:>} -\text{Simp}[ \\
& ((c + d*x)^m*\text{Cos}[e + f*x])/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m - 1)}*\text{Cos}[ \\
& e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x \ \&\& \text{GtQ}\{m, 0\}
\end{aligned}$$

### Rule 2637

$$\begin{aligned}
& \text{Int}[\text{sin}[\text{Pi}/2 + (c_{.}) + (d_{.})*(x_{.})], x_{\text{Symbol}}] \text{:>} \text{Simp}[\text{Sin}[c + d*x]/d, x] /; \\
& \text{FreeQ}\{c, d\}, x]
\end{aligned}$$

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] := Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=>
Simp[(d*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sine[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sine[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sine[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sine[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sine[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x
]*(b*Sine[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sine[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_.)]^(n_.)*((e_.) + (f_.)*(x_.))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := -Dist[a/b^2, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_.))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_])* (f_.)*(x_.)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(- (I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_.)*((f_.) + (g_.)*(x_.))^(m_.))/((a_.) + (b_.)*(F_)^(u_.) + (c_.)*(F_)^(v_.)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_.)))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_.)))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_.)))^(n_.)]*((f_.) + (g_.)*(x_.))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_.)*((a_.)*(v_)^(n_.))^(m_.) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \cosh^2(c+dx) \sinh^2(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= -\frac{a \int (e+fx)^2 \cosh^2(c+dx) \sinh(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)^2 \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
&= -\frac{(e+fx)^3}{24bf} - \frac{a(e+fx)^2 \cosh^3(c+dx)}{3b^2d} + \frac{a^2 \int (e+fx)^2 \cosh^2(c+dx) dx}{b^3} \\
&= -\frac{(e+fx)^3}{24bf} - \frac{a^2 f(e+fx) \cosh^2(c+dx)}{2b^3d^2} - \frac{2af^2 \cosh^3(c+dx)}{27b^2d^3} - \frac{a(e+fx)}{b^3} \\
&= \frac{a^4(e+fx)^3}{3b^5f} + \frac{a^2(e+fx)^3}{6b^3f} - \frac{(e+fx)^3}{24bf} - \frac{a^3(e+fx)^2 \cosh(c+dx)}{b^4d} - \frac{a^2 f(e+fx)}{b^3} \\
&= \frac{a^2 f^2 x}{4b^3 d^2} + \frac{a^4(e+fx)^3}{3b^5 f} + \frac{a^2(e+fx)^3}{6b^3 f} - \frac{(e+fx)^3}{24bf} - \frac{4af^2 \cosh(c+dx)}{9b^2 d^3} - \frac{a^3}{b^3} \\
&= \frac{a^2 f^2 x}{4b^3 d^2} + \frac{a^4(e+fx)^3}{3b^5 f} + \frac{a^2(e+fx)^3}{6b^3 f} - \frac{(e+fx)^3}{24bf} - \frac{2a^3 f^2 \cosh(c+dx)}{b^4 d^3} - \frac{4af^2}{b^3} \\
&= \frac{a^2 f^2 x}{4b^3 d^2} + \frac{a^4(e+fx)^3}{3b^5 f} + \frac{a^2(e+fx)^3}{6b^3 f} - \frac{(e+fx)^3}{24bf} - \frac{2a^3 f^2 \cosh(c+dx)}{b^4 d^3} - \frac{4af^2}{b^3} \\
&= \frac{a^2 f^2 x}{4b^3 d^2} + \frac{a^4(e+fx)^3}{3b^5 f} + \frac{a^2(e+fx)^3}{6b^3 f} - \frac{(e+fx)^3}{24bf} - \frac{2a^3 f^2 \cosh(c+dx)}{b^4 d^3} - \frac{4af^2}{b^3} \\
&= \frac{a^2 f^2 x}{4b^3 d^2} + \frac{a^4(e+fx)^3}{3b^5 f} + \frac{a^2(e+fx)^3}{6b^3 f} - \frac{(e+fx)^3}{24bf} - \frac{2a^3 f^2 \cosh(c+dx)}{b^4 d^3} - \frac{4af^2}{b^3}
\end{aligned}$$



**Mathematica [C]** time = 19.5215, size = 5115, normalized size = 6.77

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Cosh[c + d\*x]^2\*Sinh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] Result too large to show

**Maple [F]** time = 0.198, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\cosh(dx + c))^2 (\sinh(dx + c))^3}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*cosh(d\*x+c)^2\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*cosh(d\*x+c)^2\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)^2\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 3.29762, size = 14272, normalized size = 18.9

result too large to display



$$\begin{aligned}
& *b + a*b^3)*d^2*e^2 + 2*(4*a^3*b + a*b^3)*d*e*f + 2*(4*a^3*b + a*b^3)*f^2 + \\
& 2*((4*a^3*b + a*b^3)*d^2*e*f + (4*a^3*b + a*b^3)*d*f^2)*x)*\cosh(d*x + c)^3 \\
& + 8*(216*(4*a^3*b + a*b^3)*d^2*f^2*x^2 - 189*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^ \\
& 2*e^2 - 4*b^4*d*e*f + b^4*f^2 + 4*(4*b^4*d^2*e*f - b^4*d*f^2)*x)*\cosh(d*x + \\
& c)^5 + 216*(4*a^3*b + a*b^3)*d^2*e^2 + 280*(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3* \\
& d^2*e^2 - 6*a*b^3*d*e*f + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*e*f - a*b^3*d*f^2)*x \\
& )*\cosh(d*x + c)^4 + 432*(4*a^3*b + a*b^3)*d*e*f - 2160*(2*a^2*b^2*d^2*f^2*x \\
& ^2 + 2*a^2*b^2*d^2*e^2 - 2*a^2*b^2*d*e*f + a^2*b^2*f^2 + 2*(2*a^2*b^2*d^2*e \\
& *f - a^2*b^2*d*f^2)*x)*\cosh(d*x + c)^3 + 432*(4*a^3*b + a*b^3)*f^2 + 2160*( \\
& (4*a^3*b + a*b^3)*d^2*f^2*x^2 + (4*a^3*b + a*b^3)*d^2*e^2 - 2*(4*a^3*b + a* \\
& b^3)*d*e*f + 2*(4*a^3*b + a*b^3)*f^2 + 2*((4*a^3*b + a*b^3)*d^2*e*f - (4*a^ \\
& 3*b + a*b^3)*d*f^2)*x)*\cosh(d*x + c)^2 + 432*((4*a^3*b + a*b^3)*d^2*e*f + ( \\
& 4*a^3*b + a*b^3)*d*f^2)*x - 288*((8*a^4 + 4*a^2*b^2 - b^4)*d^3*f^2*x^3 + 3* \\
& (8*a^4 + 4*a^2*b^2 - b^4)*d^3*e*f*x^2 + 3*(8*a^4 + 4*a^2*b^2 - b^4)*d^3*e^2 \\
& *x)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 864*(2*a^2*b^2*d^2*f^2*x^2 + 2*a^2*b^2 \\
& *d^2*e^2 + 2*a^2*b^2*d*e*f + a^2*b^2*f^2 + 2*(2*a^2*b^2*d^2*e*f + a^2*b^2*d \\
& *f^2)*x)*\cosh(d*x + c)^2 + 12*(144*a^2*b^2*d^2*f^2*x^2 + 144*a^2*b^2*d^2*e^ \\
& 2 + 144*a^2*b^2*d*e*f - 63*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*f \\
& + b^4*f^2 + 4*(4*b^4*d^2*e*f - b^4*d*f^2)*x)*\cosh(d*x + c)^6 + 72*a^2*b^2* \\
& f^2 + 112*(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*d^2*e^2 - 6*a*b^3*d*e*f + 2*a*b^3* \\
& f^2 + 6*(3*a*b^3*d^2*e*f - a*b^3*d*f^2)*x)*\cosh(d*x + c)^5 - 1080*(2*a^2*b^ \\
& 2*d^2*f^2*x^2 + 2*a^2*b^2*d^2*e^2 - 2*a^2*b^2*d*e*f + a^2*b^2*f^2 + 2*(2*a^ \\
& 2*b^2*d^2*e*f - a^2*b^2*d*f^2)*x)*\cosh(d*x + c)^4 + 1440*((4*a^3*b + a*b^3) \\
& *d^2*f^2*x^2 + (4*a^3*b + a*b^3)*d^2*e^2 - 2*(4*a^3*b + a*b^3)*d*e*f + 2*(4 \\
& *a^3*b + a*b^3)*f^2 + 2*((4*a^3*b + a*b^3)*d^2*e*f - (4*a^3*b + a*b^3)*d*f^ \\
& 2)*x)*\cosh(d*x + c)^3 - 288*((8*a^4 + 4*a^2*b^2 - b^4)*d^3*f^2*x^3 + 3*(8*a \\
& ^4 + 4*a^2*b^2 - b^4)*d^3*e*f*x^2 + 3*(8*a^4 + 4*a^2*b^2 - b^4)*d^3*e^2*x)* \\
& \cosh(d*x + c)^2 + 144*(2*a^2*b^2*d^2*e*f + a^2*b^2*d*f^2)*x + 432*((4*a^3*b \\
& + a*b^3)*d^2*f^2*x^2 + (4*a^3*b + a*b^3)*d^2*e^2 + 2*(4*a^3*b + a*b^3)*d*e \\
& *f + 2*(4*a^3*b + a*b^3)*f^2 + 2*((4*a^3*b + a*b^3)*d^2*e*f + (4*a^3*b + a* \\
& b^3)*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 27648*((a^3*b*d*f^2*x + a^3 \\
& *b*d*e*f)*\cosh(d*x + c)^4 + 4*(a^3*b*d*f^2*x + a^3*b*d*e*f)*\cosh(d*x + c)^3 \\
& *sinh(d*x + c) + 6*(a^3*b*d*f^2*x + a^3*b*d*e*f)*\cosh(d*x + c)^2*sinh(d*x + \\
& c)^2 + 4*(a^3*b*d*f^2*x + a^3*b*d*e*f)*\cosh(d*x + c)*sinh(d*x + c)^3 + (a^ \\
& 3*b*d*f^2*x + a^3*b*d*e*f)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*dilog((a* \\
& \cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt( \\
& (a^2 + b^2)/b^2) - b)/b + 1) - 27648*((a^3*b*d*f^2*x + a^3*b*d*e*f)*\cosh(d* \\
& x + c)^4 + 4*(a^3*b*d*f^2*x + a^3*b*d*e*f)*\cosh(d*x + c)^3*sinh(d*x + c) + \\
& 6*(a^3*b*d*f^2*x + a^3*b*d*e*f)*\cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3*b* \\
& d*f^2*x + a^3*b*d*e*f)*\cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b*d*f^2*x + a^3 \\
& *b*d*e*f)*sinh(d*x + c)^4)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a \\
& *sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) \\
& - b)/b + 1) - 13824*((a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*cosh \\
& (d*x + c)^4 + 4*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*cosh(d*x \\
& + c)^3*sinh(d*x + c) + 6*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*
\end{aligned}$$

$$\begin{aligned}
& \cosh(dx + c)^2 \sinh(dx + c)^2 + 4*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*\cosh(dx + c)*\sinh(dx + c)^3 + (a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*\sinh(dx + c)^4*\sqrt{((a^2 + b^2)/b^2)}*\log(2*b*\cosh(dx + c) + 2*b*\sinh(dx + c) + 2*b*\sqrt{((a^2 + b^2)/b^2)} + 2*a) + 13824*((a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*\cosh(dx + c)^4 + 4*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*\cosh(dx + c)^3*\sinh(dx + c) + 6*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*(a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*\cosh(dx + c)*\sinh(dx + c)^3 + (a^3*b*d^2*e^2 - 2*a^3*b*c*d*e*f + a^3*b*c^2*f^2)*\sinh(dx + c)^4)*\sqrt{((a^2 + b^2)/b^2)}*\log(2*b*\cosh(dx + c) + 2*b*\sinh(dx + c) - 2*b*\sqrt{((a^2 + b^2)/b^2)} + 2*a) + 13824*((a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*\cosh(dx + c)^4 + 4*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*\cosh(dx + c)^3*\sinh(dx + c) + 6*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*\cosh(dx + c)*\sinh(dx + c)^3 + (a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*\sinh(dx + c)^4)*\sqrt{((a^2 + b^2)/b^2)}*\log(-(a*\cosh(dx + c) + a*\sinh(dx + c) + (b*\cosh(dx + c) + b*\sinh(dx + c))*\sqrt{((a^2 + b^2)/b^2)} - b)/b) - 13824*((a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*\cosh(dx + c)^4 + 4*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*\cosh(dx + c)^3*\sinh(dx + c) + 6*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*(a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*\cosh(dx + c)*\sinh(dx + c)^3 + (a^3*b*d^2*f^2*x^2 + 2*a^3*b*d^2*e*f*x + 2*a^3*b*c*d*e*f - a^3*b*c^2*f^2)*\sinh(dx + c)^4)*\sqrt{((a^2 + b^2)/b^2)}*\log(-(a*\cosh(dx + c) + a*\sinh(dx + c) - (b*\cosh(dx + c) + b*\sinh(dx + c))*\sqrt{((a^2 + b^2)/b^2)} - b)/b) - 27648*(a^3*b*f^2*\cosh(dx + c)^4 + 4*a^3*b*f^2*\cosh(dx + c)^3*\sinh(dx + c) + 6*a^3*b*f^2*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*a^3*b*f^2*\cosh(dx + c)*\sinh(dx + c)^3 + a^3*b*f^2*\sinh(dx + c)^4)*\sqrt{((a^2 + b^2)/b^2)}*\text{polylog}(3, (a*\cosh(dx + c) + a*\sinh(dx + c) + (b*\cosh(dx + c) + b*\sinh(dx + c))*\sqrt{((a^2 + b^2)/b^2)})))/b) + 27648*(a^3*b*f^2*\cosh(dx + c)^4 + 4*a^3*b*f^2*\cosh(dx + c)^3*\sinh(dx + c) + 6*a^3*b*f^2*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*a^3*b*f^2*\cosh(dx + c)*\sinh(dx + c)^3 + a^3*b*f^2*\sinh(dx + c)^4)*\sqrt{((a^2 + b^2)/b^2)}*\text{polylog}(3, (a*\cosh(dx + c) + a*\sinh(dx + c) - (b*\cosh(dx + c) + b*\sinh(dx + c))*\sqrt{((a^2 + b^2)/b^2)})))/b) + 108*(4*b^4*d^2*e*f + b^4*d*f^2)*x + 64*(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*d^2*e^2 + 6*a*b^3*d*e*f + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*e*f + a*b^3*d*f^2)*x)*\cosh(dx + c) + 8*(72*a*b^3*d^2*f^2*x^2 + 72*a*b^3*d^2*e^2 - 27*(8*b^4*d^2*f^2*x^2 + 8*b^4*d^2*e^2 - 4*b^4*d*e*f + b^4*f^2 + 4*(4*b^4*d^2*e*f - b^4*d*f^2)*x))*\cosh(dx + c)^7 + 48*a*b^3*d*e*f + 56*(9*a*b^3*d^2*f^2*x^2 + 9*a*b^3*d^2*e^2 - 6*a*b^3*d*e*f + 2*a*b^3*f^2 + 6*(3*a*b^3*d^2*e*f - a*b^3*d*f^2)*x)*\cosh(dx + c)^6 + 16*a*b^3*f^2 - 64*8*(2*a^2*b^2*d^2*f^2*x^2 + 2*a^2*b^2*d^2*e^2 - 2*a^2*b^2*d*e*f + a^2*b^2*f^2 + 2*(2*a^2*b^2*d^2*e*f - a^2*b^2*d*f^2)*x))*\cosh(dx + c)^5 + 1080*((4*a^3
\end{aligned}$$

```
*b + a*b^3)*d^2*f^2*x^2 + (4*a^3*b + a*b^3)*d^2*e^2 - 2*(4*a^3*b + a*b^3)*d
*e*f + 2*(4*a^3*b + a*b^3)*f^2 + 2*((4*a^3*b + a*b^3)*d^2*e*f - (4*a^3*b +
a*b^3)*d*f^2)*x)*cosh(d*x + c)^4 - 288*((8*a^4 + 4*a^2*b^2 - b^4)*d^3*f^2*x
^3 + 3*(8*a^4 + 4*a^2*b^2 - b^4)*d^3*e*f*x^2 + 3*(8*a^4 + 4*a^2*b^2 - b^4)*
d^3*e^2*x)*cosh(d*x + c)^3 + 648*((4*a^3*b + a*b^3)*d^2*f^2*x^2 + (4*a^3*b
+ a*b^3)*d^2*e^2 + 2*(4*a^3*b + a*b^3)*d*e*f + 2*(4*a^3*b + a*b^3)*f^2 + 2*
((4*a^3*b + a*b^3)*d^2*e*f + (4*a^3*b + a*b^3)*d*f^2)*x)*cosh(d*x + c)^2 +
48*(3*a*b^3*d^2*e*f + a*b^3*d*f^2)*x + 216*(2*a^2*b^2*d^2*f^2*x^2 + 2*a^2*b
^2*d^2*e^2 + 2*a^2*b^2*d*e*f + a^2*b^2*f^2 + 2*(2*a^2*b^2*d^2*e*f + a^2*b^2
*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))/(b^5*d^3*cosh(d*x + c)^4 + 4*b^5*d
^3*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^5*d^3*cosh(d*x + c)^2*sinh(d*x + c)^
2 + 4*b^5*d^3*cosh(d*x + c)*sinh(d*x + c)^3 + b^5*d^3*sinh(d*x + c)^4)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**2*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c)^2 \sinh(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algori
thm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)^2*sinh(d*x + c)^3/(b*sinh(d*x + c) + a)
, x)
```

$$3.398 \quad \int \frac{(e+fx) \cosh^2(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=474

$$-\frac{a^3 f \sqrt{a^2 + b^2} \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^5 d^2} + \frac{a^3 f \sqrt{a^2 + b^2} \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right)}{b^5 d^2} + \frac{a^3 f \sinh(c+dx)}{b^4 d^2} - \frac{a^2 f \cosh^2(c+dx)}{4b^3 d^2}$$

[Out] (a^4\*e\*x)/b^5 + (a^2\*e\*x)/(2\*b^3) + (a^4\*f\*x^2)/(2\*b^5) + (a^2\*f\*x^2)/(4\*b^3) - (e + f\*x)^2/(16\*b\*f) - (a^3\*(e + f\*x)\*Cosh[c + d\*x])/(b^4\*d) - (a^2\*f\*Cosh[c + d\*x]^2)/(4\*b^3\*d^2) - (a\*(e + f\*x)\*Cosh[c + d\*x]^3)/(3\*b^2\*d) - (f\*Cosh[4\*c + 4\*d\*x])/(128\*b\*d^2) - (a^3\*Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]))/(b^5\*d) + (a^3\*Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]))/(b^5\*d) - (a^3\*Sqrt[a^2 + b^2]\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^5\*d^2) + (a^3\*Sqrt[a^2 + b^2]\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^5\*d^2) + (a^3\*f\*Sinh[c + d\*x])/(b^4\*d^2) + (a\*f\*Sinh[c + d\*x])/(3\*b^2\*d^2) + (a^2\*(e + f\*x)\*Cosh[c + d\*x]\*Sinh[c + d\*x])/(2\*b^3\*d) + (a\*f\*Sinh[c + d\*x]^3)/(9\*b^2\*d^2) + ((e + f\*x)\*Sinh[4\*c + 4\*d\*x])/(32\*b\*d)

**Rubi [A]** time = 0.86452, antiderivative size = 474, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 14, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {5579, 5448, 3296, 2638, 5447, 2633, 3310, 5565, 2637, 3322, 2264, 2190, 2279, 2391}

$$-\frac{a^3 f \sqrt{a^2 + b^2} \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{b^5 d^2} + \frac{a^3 f \sqrt{a^2 + b^2} \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right)}{b^5 d^2} + \frac{a^3 f \sinh(c+dx)}{b^4 d^2} - \frac{a^2 f \cosh^2(c+dx)}{4b^3 d^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Cosh[c + d\*x]^2\*Sinh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] (a^4\*e\*x)/b^5 + (a^2\*e\*x)/(2\*b^3) + (a^4\*f\*x^2)/(2\*b^5) + (a^2\*f\*x^2)/(4\*b^3) - (e + f\*x)^2/(16\*b\*f) - (a^3\*(e + f\*x)\*Cosh[c + d\*x])/(b^4\*d) - (a^2\*f\*Cosh[c + d\*x]^2)/(4\*b^3\*d^2) - (a\*(e + f\*x)\*Cosh[c + d\*x]^3)/(3\*b^2\*d) - (f\*Cosh[4\*c + 4\*d\*x])/(128\*b\*d^2) - (a^3\*Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]))/(b^5\*d) + (a^3\*Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]))/(b^5\*d) - (a^3\*Sqrt[a^2 + b^2]\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^5\*d^2) + (a^3\*Sqrt[a^2 + b^2]\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^5\*d^2) + (a^3\*f\*Sinh[c + d\*x])/(b^4\*d^2) + (a\*f\*Sinh[c + d\*x])/(3\*b^2\*d^2)

$*d^2) + (a^2*(e + f*x)*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(2*b^3*d) + (a*f*\text{Sinh}[c + d*x]^3)/(9*b^2*d^2) + ((e + f*x)*\text{Sinh}[4*c + 4*d*x])/(32*b*d)$

### Rule 5579

$\text{Int}[(\text{Cosh}[c_.] + (d_.)*(x_.)]^{(p_.)*((e_.) + (f_.)*(x_.))^{(m_.)*\text{Sinh}[(c_.) + (d_.)*(x_.)]^{(n_.)}}/(a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_.)]), x\_Symbol] \rightarrow \text{Dist}[1/b, \text{Int}[(e + f*x)^m*\text{Cosh}[c + d*x]^p*\text{Sinh}[c + d*x]^{(n-1)}, x], x] - \text{Dist}[a/b, \text{Int}[(e + f*x)^m*\text{Cosh}[c + d*x]^p*\text{Sinh}[c + d*x]^{(n-1)})/(a + b*\text{Sinh}[c + d*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$

### Rule 5448

$\text{Int}[\text{Cosh}[(a_.) + (b_.)*(x_.)]^{(p_.)*((c_.) + (d_.)*(x_.))^{(m_.)*\text{Sinh}[(a_.) + (b_.)*(x_.)]^{(n_.)}}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[(c + d*x)^m, \text{Sinh}[a + b*x]^{n*\text{Cosh}[a + b*x]^p}, x], x] /; \text{FreeQ}[\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$

### Rule 3296

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)*\sin[(e_.) + (f_.)*(x_.)]}, x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$

### Rule 2638

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)]], x\_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

### Rule 5447

$\text{Int}[\text{Cosh}[(a_.) + (b_.)*(x_.)]^{(n_.)*((c_.) + (d_.)*(x_.))^{(m_.)*\text{Sinh}[(a_.) + (b_.)*(x_.)]}, x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Cosh}[a + b*x]^{(n+1)}/(b*(n+1)), x] - \text{Dist}[(d*m)/(b*(n+1)), \text{Int}[(c + d*x)^{(m-1)}*\text{Cosh}[a + b*x]^{(n+1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[n, -1]$

### Rule 2633

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)]^{(n_.)}], x\_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{(n-1)/2}], x], x], x, \text{Cos}[c + d*x]], x] /; \text{FreeQ}[\{c, d\}, x] \&\& \text{IGtQ}[(n-1)/2, 0]$

### Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
  Simp[(d*(b*SIN[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*SIN[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*SIN[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

### Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_))*((e_.) + (f_.)*(x_))^(m_.)/((a_.) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[SIN[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

### Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_]
*(f_.)*(x_))]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_))*((f_.) + (g_.)*(x_))^(m_.)/((a_.) + (b_.)*(F_)^(u_) + (c_.
)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2190

```
Int((((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))*((c_.) + (d_.)*(x_))^(m_.)/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279



```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx) \cosh^2(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx) \cosh^2(c+dx) \sinh^2(c+dx) dx}{b} - \frac{a \int \frac{(e+fx) \cosh^2(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= -\frac{a \int (e+fx) \cosh^2(c+dx) \sinh(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx) \cosh^2(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
&= -\frac{(e+fx)^2}{16bf} - \frac{a(e+fx) \cosh^3(c+dx)}{3b^2d} + \frac{a^2 \int (e+fx) \cosh^2(c+dx) dx}{b^3} - \dots \\
&= -\frac{(e+fx)^2}{16bf} - \frac{a^2 f \cosh^2(c+dx)}{4b^3d^2} - \frac{a(e+fx) \cosh^3(c+dx)}{3b^2d} + \frac{a^2(e+fx) \cosh(c+dx)}{b^4d} - \dots \\
&= \frac{a^4ex}{b^5} + \frac{a^2ex}{2b^3} + \frac{a^4fx^2}{2b^5} + \frac{a^2fx^2}{4b^3} - \frac{(e+fx)^2}{16bf} - \frac{a^3(e+fx) \cosh(c+dx)}{b^4d} - \dots \\
&= \frac{a^4ex}{b^5} + \frac{a^2ex}{2b^3} + \frac{a^4fx^2}{2b^5} + \frac{a^2fx^2}{4b^3} - \frac{(e+fx)^2}{16bf} - \frac{a^3(e+fx) \cosh(c+dx)}{b^4d} - \dots \\
&= \frac{a^4ex}{b^5} + \frac{a^2ex}{2b^3} + \frac{a^4fx^2}{2b^5} + \frac{a^2fx^2}{4b^3} - \frac{(e+fx)^2}{16bf} - \frac{a^3(e+fx) \cosh(c+dx)}{b^4d} - \dots \\
&= \frac{a^4ex}{b^5} + \frac{a^2ex}{2b^3} + \frac{a^4fx^2}{2b^5} + \frac{a^2fx^2}{4b^3} - \frac{(e+fx)^2}{16bf} - \frac{a^3(e+fx) \cosh(c+dx)}{b^4d} - \dots \\
&= \frac{a^4ex}{b^5} + \frac{a^2ex}{2b^3} + \frac{a^4fx^2}{2b^5} + \frac{a^2fx^2}{4b^3} - \frac{(e+fx)^2}{16bf} - \frac{a^3(e+fx) \cosh(c+dx)}{b^4d} - \dots
\end{aligned}$$

**Mathematica [C]** time = 11.8889, size = 2915, normalized size = 6.15

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)\*Cosh[c + d\*x]^2\*Sinh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] 
$$\begin{aligned} & -(e*(c/d + x - (2*a*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2])/((8*b) - (f*(x^2 + (2*a*((I*Pi*ArcTanh[(-b + a*Tanh[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (2*((-I)*c + ArcCos[(-I)*a/b])*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] + ((-2*I)*c + Pi - (2*I)*d*x)*ArcTanh[((a - I*b)*Tan[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] - (ArcCos[(-I)*a/b] + (2*I)*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]))*Log[((I*a + b)*(a + I*(b + Sqrt[-a^2 - b^2]))*(-I + Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a + b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])) - (ArcCos[(-I)*a/b] - (2*I)*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]))*Log[((I*a + b)*(I*a - b + Sqrt[-a^2 - b^2])*(I + Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(a - I*b + Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])) + (ArcCos[(-I)*a/b] - (2*I)*ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] - (2*I)*ArcTanh[((a - I*b)*Tan[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]))*Log[-(((-1)^(3/4)*Sqrt[-a^2 - b^2])*E^(-c/2 - (d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*Sqrt[a + b*Sinh[c + d*x]])] + (ArcCos[(-I)*a/b] + (2*I)*(ArcTanh[((a + I*b)*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]] + ArcTanh[((a - I*b)*Tan[((2*I)*c + Pi + (2*I)*d*x)/4])/Sqrt[-a^2 - b^2]))*Log[(-1)^(1/4)*Sqrt[-a^2 - b^2]*E^((c + d*x)/2))/(Sqrt[2]*Sqrt[(-I)*b]*Sqrt[a + b*Sinh[c + d*x]])] + I*(PolyLog[2, ((I*a + Sqrt[-a^2 - b^2])*(I*a + b - I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a + b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])) - PolyLog[2, ((a + I*Sqrt[-a^2 - b^2])*(-a + I*b + Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4]))/(b*(I*a + b + I*Sqrt[-a^2 - b^2]*Cot[((2*I)*c + Pi + (2*I)*d*x)/4])))]/Sqrt[-a^2 - b^2])/d^2)/(16*b) - (e*((4*a^2 + b^2)*(c + d*x) - (2*a*(4*a^2 + 3*b^2)*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 4*a*b*Cosh[c + d*x] + b^2*Sinh[2*(c + d*x)]))/(16*b^3*d) - (f*((4*a^2 + b^2)*(-c + d*x)*(c + d*x) - 8*a*b*d*x*Cosh[c + d*x] - b^2*Cosh[2*(c + d*x)] - (2*a*(4*a^2 + 3*b^2)*(2*c*ArcTanh[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + (c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])]) - (c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])]) + PolyLog[2, (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(-a + Sqrt[a^2 + b^2])]) - PolyLog[2, -(b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])])))/Sqrt[a^2 + b^2] + 8*a*b*Sinh[c + d*x] + 2*b^2*d*x*Sinh[2*(c + d*x)]))/(32*b^3*d^2) + (e*(6*(16*a^4 + 12*a^2*b^2 + b^4)*(c + d*x) - (12*a*(16*a^4 + 20*a^2*b^2 + 5*b^4)*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2] - 48*a*b*(2*a^2 + b^2)*Cosh[c + d*x] - 8*a*b^3*Cosh[3*(c + d*x)] + 6*b^2*(4*a^2 + b^2)*Sinh[2*(c + d*x)] + 3*b^4*Sinh[4*(c + d*x)]))/(96*b^5*d$$

$$\begin{aligned}
& ) + (f*(-576*a^4*\text{Sqrt}[a^2 + b^2]*c^2 - 432*a^2*b^2*\text{Sqrt}[a^2 + b^2]*c^2 - 36 \\
& *b^4*\text{Sqrt}[a^2 + b^2]*c^2 + 576*a^4*\text{Sqrt}[a^2 + b^2]*d^2*x^2 + 432*a^2*b^2*\text{Sqrt}[a^2 + b^2] \\
& *d^2*x^2 + 36*b^4*\text{Sqrt}[a^2 + b^2]*d^2*x^2 - 2304*a^5*c*\text{ArcTanh} \\
& [(a + b*\text{Cosh}[c + d*x] + b*\text{Sinh}[c + d*x])/ \text{Sqrt}[a^2 + b^2]] - 2880*a^3*b^2*c* \\
& \text{ArcTanh}[(a + b*\text{Cosh}[c + d*x] + b*\text{Sinh}[c + d*x])/ \text{Sqrt}[a^2 + b^2]] - 720*a*b^4*c* \\
& \text{ArcTanh}[(a + b*\text{Cosh}[c + d*x] + b*\text{Sinh}[c + d*x])/ \text{Sqrt}[a^2 + b^2]] - 1152 \\
& *a^3*b*\text{Sqrt}[a^2 + b^2]*d*x*\text{Cosh}[c + d*x] - 576*a*b^3*\text{Sqrt}[a^2 + b^2]*d*x*\text{Co} \\
& \text{sh}[c + d*x] - 144*a^2*b^2*\text{Sqrt}[a^2 + b^2]*\text{Cosh}[2*(c + d*x)] - 36*b^4*\text{Sqrt}[a \\
& ^2 + b^2]*\text{Cosh}[2*(c + d*x)] - 96*a*b^3*\text{Sqrt}[a^2 + b^2]*d*x*\text{Cosh}[3*(c + d*x) \\
& ] - 9*b^4*\text{Sqrt}[a^2 + b^2]*\text{Cosh}[4*(c + d*x)] - 1152*a^5*c*\text{Log}[1 + (b*(\text{Cosh}[c \\
& + d*x] + \text{Sinh}[c + d*x]))/(a - \text{Sqrt}[a^2 + b^2])] - 1440*a^3*b^2*c*\text{Log}[1 + ( \\
& b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a - \text{Sqrt}[a^2 + b^2])] - 360*a*b^4*c*\text{Log} \\
& [1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a - \text{Sqrt}[a^2 + b^2])] - 1152*a^5* \\
& d*x*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a - \text{Sqrt}[a^2 + b^2])] - 14 \\
& 40*a^3*b^2*d*x*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a - \text{Sqrt}[a^2 + \\
& b^2])] - 360*a*b^4*d*x*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a - \text{Sqr} \\
& \text{t}[a^2 + b^2])] + 1152*a^5*c*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a \\
& + \text{Sqrt}[a^2 + b^2])] + 1440*a^3*b^2*c*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d \\
& *x]))/(a + \text{Sqrt}[a^2 + b^2])] + 360*a*b^4*c*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh} \\
& [c + d*x]))/(a + \text{Sqrt}[a^2 + b^2])] + 1152*a^5*d*x*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] \\
& + \text{Sinh}[c + d*x]))/(a + \text{Sqrt}[a^2 + b^2])] + 1440*a^3*b^2*d*x*\text{Log}[1 + (b*(\text{Co} \\
& \text{sh}[c + d*x] + \text{Sinh}[c + d*x]))/(a + \text{Sqrt}[a^2 + b^2])] + 360*a*b^4*d*x*\text{Log}[1 \\
& + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a + \text{Sqrt}[a^2 + b^2])] - 72*a*(16*a^4 \\
& + 20*a^2*b^2 + 5*b^4)*\text{PolyLog}[2, (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(-a + \\
& \text{Sqrt}[a^2 + b^2])] + 72*a*(16*a^4 + 20*a^2*b^2 + 5*b^4)*\text{PolyLog}[2, -(b*(\text{Co} \\
& \text{sh}[c + d*x] + \text{Sinh}[c + d*x]))/(a + \text{Sqrt}[a^2 + b^2])] + 1152*a^3*b*\text{Sqrt}[a^2 \\
& + b^2]*\text{Sinh}[c + d*x] + 576*a*b^3*\text{Sqrt}[a^2 + b^2]*\text{Sinh}[c + d*x] + 288*a^2*b \\
& ^2*\text{Sqrt}[a^2 + b^2]*d*x*\text{Sinh}[2*(c + d*x)] + 72*b^4*\text{Sqrt}[a^2 + b^2]*d*x*\text{Sinh}[ \\
& 2*(c + d*x)] + 32*a*b^3*\text{Sqrt}[a^2 + b^2]*\text{Sinh}[3*(c + d*x)] + 36*b^4*\text{Sqrt}[a^2 \\
& + b^2]*d*x*\text{Sinh}[4*(c + d*x)]/(1152*b^5*\text{Sqrt}[a^2 + b^2]*d^2)
\end{aligned}$$


---

**Maple [B]** time = 0.106, size = 1213, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)*\cosh(d*x+c)^2*\sinh(d*x+c)^3/(a+b*\sinh(d*x+c)),x)$

[Out]  $-1/16*f*x^2/b-a^5/b^5/d^2*f/(a^2+b^2)^{(1/2)}*\text{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))+a^5/b^5/d^2*f/(a^2+b^2)^{(1/2)}*\text{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))+2*a^5/b^5/d*e/(a^2+b^2)^{(1/2)}$

$$\begin{aligned} & * \operatorname{arctanh}\left(\frac{1}{2} \cdot (2 \cdot b \cdot \exp(d \cdot x + c) + 2 \cdot a) / (a^2 + b^2)^{1/2}\right) + \frac{1}{2} \cdot a^4 \cdot f \cdot x^2 / b^5 + \frac{1}{4} \cdot a^2 \cdot f \cdot x^2 / b^3 - \frac{1}{8} \cdot a \cdot (4 \cdot a^2 + b^2) \cdot (d \cdot f \cdot x + d \cdot e + f) / b^4 / d^2 \cdot \exp(-d \cdot x - c) + a^4 \cdot e \cdot x / b^5 \\ & + \frac{1}{2} \cdot a^2 \cdot e \cdot x / b^3 - \frac{1}{8} \cdot e \cdot x / b + 2 \cdot a^3 / b^3 / d \cdot e / (a^2 + b^2)^{1/2} \cdot \operatorname{arctanh}\left(\frac{1}{2} \cdot (2 \cdot b \cdot \exp(d \cdot x + c) + 2 \cdot a) / (a^2 + b^2)^{1/2}\right) - a^3 / b^3 / d^2 \cdot f / (a^2 + b^2)^{1/2} \cdot \operatorname{dilog}\left(\frac{-b \cdot \exp(d \cdot x + c) + (a^2 + b^2)^{1/2} - a}{-a + (a^2 + b^2)^{1/2}}\right) + a^3 / b^3 / d^2 \cdot f / (a^2 + b^2)^{1/2} \cdot \operatorname{dilog}\left(\frac{b \cdot \exp(d \cdot x + c) + (a^2 + b^2)^{1/2} + a}{a + (a^2 + b^2)^{1/2}}\right) - 2 \cdot a^5 / b^5 / d^2 \cdot f \cdot c / (a^2 + b^2)^{1/2} \cdot \operatorname{arctanh}\left(\frac{1}{2} \cdot (2 \cdot b \cdot \exp(d \cdot x + c) + 2 \cdot a) / (a^2 + b^2)^{1/2}\right) - a^5 / b^5 / d \cdot f / (a^2 + b^2)^{1/2} \cdot \ln\left(\frac{-b \cdot \exp(d \cdot x + c) + (a^2 + b^2)^{1/2} - a}{-a + (a^2 + b^2)^{1/2}}\right) \cdot x - a^5 / b^5 / d^2 \cdot f / (a^2 + b^2)^{1/2} \cdot \ln\left(\frac{-b \cdot \exp(d \cdot x + c) + (a^2 + b^2)^{1/2} - a}{-a + (a^2 + b^2)^{1/2}}\right) \cdot c + a^5 / b^5 / d \cdot f / (a^2 + b^2)^{1/2} \cdot \ln\left(\frac{b \cdot \exp(d \cdot x + c) + (a^2 + b^2)^{1/2} + a}{a + (a^2 + b^2)^{1/2}}\right) \cdot x + a^5 / b^5 / d^2 \cdot f / (a^2 + b^2)^{1/2} \cdot \ln\left(\frac{b \cdot \exp(d \cdot x + c) + (a^2 + b^2)^{1/2} + a}{a + (a^2 + b^2)^{1/2}}\right) \cdot c - a^3 / b^3 / d \cdot f / (a^2 + b^2)^{1/2} \cdot \ln\left(\frac{-b \cdot \exp(d \cdot x + c) + (a^2 + b^2)^{1/2} - a}{-a + (a^2 + b^2)^{1/2}}\right) \cdot x - a^3 / b^3 / d^2 \cdot f / (a^2 + b^2)^{1/2} \cdot \ln\left(\frac{-b \cdot \exp(d \cdot x + c) + (a^2 + b^2)^{1/2} - a}{-a + (a^2 + b^2)^{1/2}}\right) \cdot c + a^3 / b^3 / d \cdot f / (a^2 + b^2)^{1/2} \cdot \ln\left(\frac{b \cdot \exp(d \cdot x + c) + (a^2 + b^2)^{1/2} + a}{a + (a^2 + b^2)^{1/2}}\right) \cdot x + a^3 / b^3 / d^2 \cdot f / (a^2 + b^2)^{1/2} \cdot \ln\left(\frac{b \cdot \exp(d \cdot x + c) + (a^2 + b^2)^{1/2} + a}{a + (a^2 + b^2)^{1/2}}\right) \cdot c - 2 \cdot a^3 / b^3 / d^2 \cdot f \cdot c / (a^2 + b^2)^{1/2} \cdot \operatorname{arctan} \\ & h\left(\frac{1}{2} \cdot (2 \cdot b \cdot \exp(d \cdot x + c) + 2 \cdot a) / (a^2 + b^2)^{1/2}\right) + \frac{1}{256} \cdot (4 \cdot d \cdot f \cdot x + 4 \cdot d \cdot e - f) / d^2 / b \cdot \exp(4 \cdot d \cdot x + 4 \cdot c) - \frac{1}{72} \cdot a \cdot (3 \cdot d \cdot f \cdot x + 3 \cdot d \cdot e - f) / b^2 / d^2 \cdot \exp(3 \cdot d \cdot x + 3 \cdot c) - \frac{1}{8} \cdot a \cdot (4 \cdot a^2 \cdot d \cdot f \cdot x + b^2 \cdot d \cdot f \cdot x + 4 \cdot a^2 \cdot d \cdot e + b^2 \cdot d \cdot e - 4 \cdot a^2 \cdot f - b^2 \cdot f) / b^4 / d^2 \cdot \exp(d \cdot x + c) - \frac{1}{256} \cdot (4 \cdot d \cdot f \cdot x + 4 \cdot d \cdot e + f) / d^2 / b \cdot \exp(-4 \cdot d \cdot x - 4 \cdot c) - \frac{1}{72} \cdot a \cdot (3 \cdot d \cdot f \cdot x + 3 \cdot d \cdot e + f) / b^2 / d^2 \cdot \exp(-3 \cdot d \cdot x - 3 \cdot c) + \frac{1}{16} \cdot a^2 \cdot (2 \cdot d \cdot f \cdot x + 2 \cdot d \cdot e - f) / b^3 / d^2 \cdot \exp(2 \cdot d \cdot x + 2 \cdot c) - \frac{1}{16} \cdot a^2 \cdot (2 \cdot d \cdot f \cdot x + 2 \cdot d \cdot e + f) / b^3 / d^2 \cdot \exp(-2 \cdot d \cdot x - 2 \cdot c) \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^2\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.79816, size = 7611, normalized size = 16.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^2\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm  
m="fricas")

[Out] 
$$\frac{1}{2304} \cdot (9 \cdot (4b^4 d f x + 4b^4 d e - b^4 f) \cosh(dx + c)^8 + 9 \cdot (4b^4 d f x + 4b^4 d e - b^4 f) \sinh(dx + c)^8 - 32 \cdot (3a^3 b^3 d f x + 3a^3 b^3 d e - a^3 b^3 f) \cosh(dx + c)^7 - 8 \cdot (12a^3 b^3 d f x + 12a^3 b^3 d e - 4a^3 b^3 f - 9 \cdot (4b^4 d f x + 4b^4 d e - b^4 f) \cosh(dx + c)) \sinh(dx + c)^7 - 36b^4 d f x + 144 \cdot (2a^2 b^2 d f x + 2a^2 b^2 d e - a^2 b^2 f) \cosh(dx + c)^6 + 4 \cdot (72a^2 b^2 d f x + 72a^2 b^2 d e - 36a^2 b^2 f + 63 \cdot (4b^4 d f x + 4b^4 d e - b^4 f) \cosh(dx + c)^2 - 56 \cdot (3a^3 b^3 d f x + 3a^3 b^3 d e - a^3 b^3 f) \cosh(dx + c)) \sinh(dx + c)^6 - 36b^4 d e - 288 \cdot ((4a^3 b + a^3 b^3) d f x + (4a^3 b + a^3 b^3) d e - (4a^3 b + a^3 b^3) f) \cosh(dx + c)^5 - 24 \cdot (12 \cdot (4a^3 b + a^3 b^3) d f x - 21 \cdot (4b^4 d f x + 4b^4 d e - b^4 f) \cosh(dx + c))^3 + 12 \cdot (4a^3 b + a^3 b^3) d e + 28 \cdot (3a^3 b^3 d f x + 3a^3 b^3 d e - a^3 b^3 f) \cosh(dx + c)^2 - 12 \cdot (4a^3 b + a^3 b^3) f - 36 \cdot (2a^2 b^2 d f x + 2a^2 b^2 d e - a^2 b^2 f) \cosh(dx + c)) \sinh(dx + c)^5 - 9b^4 f + 144 \cdot ((8a^4 + 4a^2 b^2 - b^4) d^2 f x^2 + 2 \cdot (8a^4 + 4a^2 b^2 - b^4) d^2 e x) \cosh(dx + c)^4 + 2 \cdot (72 \cdot (8a^4 + 4a^2 b^2 - b^4) d^2 f x^2 + 144 \cdot (8a^4 + 4a^2 b^2 - b^4) d^2 e x + 315 \cdot (4b^4 d f x + 4b^4 d e - b^4 f) \cosh(dx + c)^4 - 560 \cdot (3a^3 b^3 d f x + 3a^3 b^3 d e - a^3 b^3 f) \cosh(dx + c)^3 + 1080 \cdot (2a^2 b^2 d f x + 2a^2 b^2 d e - a^2 b^2 f) \cosh(dx + c)^2 - 720 \cdot ((4a^3 b + a^3 b^3) d f x + (4a^3 b + a^3 b^3) d e - (4a^3 b + a^3 b^3) f) \cosh(dx + c)) \sinh(dx + c)^4 - 288 \cdot ((4a^3 b + a^3 b^3) d f x + (4a^3 b + a^3 b^3) d e + (4a^3 b + a^3 b^3) f) \cosh(dx + c)^3 + 8 \cdot (63 \cdot (4b^4 d f x + 4b^4 d e - b^4 f) \cosh(dx + c)^5 - 140 \cdot (3a^3 b^3 d f x + 3a^3 b^3 d e - a^3 b^3 f) \cosh(dx + c)^4 - 36 \cdot (4a^3 b + a^3 b^3) d f x + 360 \cdot (2a^2 b^2 d f x + 2a^2 b^2 d e - a^2 b^2 f) \cosh(dx + c)^3 - 36 \cdot (4a^3 b + a^3 b^3) d e - 360 \cdot ((4a^3 b + a^3 b^3) d f x + (4a^3 b + a^3 b^3) d e - (4a^3 b + a^3 b^3) f) \cosh(dx + c)^2 - 36 \cdot (4a^3 b + a^3 b^3) f + 72 \cdot ((8a^4 + 4a^2 b^2 - b^4) d^2 f x^2 + 2 \cdot (8a^4 + 4a^2 b^2 - b^4) d^2 e x) \cosh(dx + c)) \sinh(dx + c)^3 - 144 \cdot (2a^2 b^2 d f x + 2a^2 b^2 d e + a^2 b^2 f) \cosh(dx + c)^2 - 12 \cdot (24a^2 b^2 d f x - 21 \cdot (4b^4 d f x + 4b^4 d e - b^4 f) \cosh(dx + c)^6 + 24a^2 b^2 d e + 56 \cdot (3a^3 b^3 d f x + 3a^3 b^3 d e - a^3 b^3 f) \cosh(dx + c)^5 + 12a^2 b^2 f - 180 \cdot (2a^2 b^2 d f x + 2a^2 b^2 d e - a^2 b^2 f) \cosh(dx + c)^4 + 240 \cdot ((4a^3 b + a^3 b^3) d f x + (4a^3 b + a^3 b^3) d e - (4a^3 b + a^3 b^3) f) \cosh(dx + c)^3 - 72 \cdot ((8a^4 + 4a^2 b^2 - b^4) d^2 f x^2 + 2 \cdot (8a^4 + 4a^2 b^2 - b^4) d^2 e x) \cosh(dx + c)^2 + 72 \cdot ((4a^3 b + a^3 b^3) d f x + (4a^3 b + a^3 b^3) d e + (4a^3 b + a^3 b^3) f) \cosh(dx + c)) \sinh(dx + c)^2 - 2304 \cdot (a^3 b^3 f \cosh(dx + c)^4 + 4a^3 b^3 f \cosh(dx + c)^3 \sinh(dx + c) + 6a^3 b^3 f \cosh(dx + c)^2 \sinh(dx + c)^2 + 4a^3 b^3 f \cosh(dx + c) \sinh(dx + c)^3 + a^3 b^3 f \sinh(dx + c)^4) \sqrt{(a^2 + b^2)/b^2} \operatorname{dilog}((a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c))) \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 2304 \cdot (a^3 b^3 f \cosh(dx + c)^4 + 4a^3 b^3 f \cosh(dx + c)^3 \sinh(dx + c) + 6a^3 b^3 f \cosh(dx + c)^2 \sinh(dx + c)^2 + 4a^3 b^3 f \cosh(dx + c) \sinh(dx + c)^3 + a^3 b^3 f \sinh(dx + c)^4)$$

$$\begin{aligned}
& c) \sinh(dx + c)^3 + a^3 b^* f \sinh(dx + c)^4 \sqrt{(a^2 + b^2)/b^2} \operatorname{dilog}((a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c))) \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 2304((a^3 b^* d^* e - a^3 b^* c^* f) \cosh(dx + c)^4 + 4(a^3 b^* d^* e - a^3 b^* c^* f) \cosh(dx + c)^3 \sinh(dx + c) + 6(a^3 b^* d^* e - a^3 b^* c^* f) \cosh(dx + c)^2 \sinh(dx + c)^2 + 4(a^3 b^* d^* e - a^3 b^* c^* f) \cosh(dx + c) \sinh(dx + c)^3 + (a^3 b^* d^* e - a^3 b^* c^* f) \sinh(dx + c)^4) \sqrt{(a^2 + b^2)/b^2} \log(2b \cosh(dx + c) + 2b \sinh(dx + c) + 2b \sqrt{(a^2 + b^2)/b^2} + 2a) - 2304((a^3 b^* d^* e - a^3 b^* c^* f) \cosh(dx + c)^4 + 4(a^3 b^* d^* e - a^3 b^* c^* f) \cosh(dx + c)^3 \sinh(dx + c) + 6(a^3 b^* d^* e - a^3 b^* c^* f) \cosh(dx + c)^2 \sinh(dx + c)^2 + 4(a^3 b^* d^* e - a^3 b^* c^* f) \cosh(dx + c) \sinh(dx + c)^3 + (a^3 b^* d^* e - a^3 b^* c^* f) \sinh(dx + c)^4) \sqrt{(a^2 + b^2)/b^2} \log(2b \cosh(dx + c) + 2b \sinh(dx + c) - 2b \sqrt{(a^2 + b^2)/b^2} + 2a) - 2304((a^3 b^* d^* f^* x + a^3 b^* c^* f) \cosh(dx + c)^4 + 4(a^3 b^* d^* f^* x + a^3 b^* c^* f) \cosh(dx + c)^3 \sinh(dx + c) + 6(a^3 b^* d^* f^* x + a^3 b^* c^* f) \cosh(dx + c)^2 \sinh(dx + c)^2 + 4(a^3 b^* d^* f^* x + a^3 b^* c^* f) \cosh(dx + c) \sinh(dx + c)^3 + (a^3 b^* d^* f^* x + a^3 b^* c^* f) \sinh(dx + c)^4) \sqrt{(a^2 + b^2)/b^2} \log(-(a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c))) \sqrt{(a^2 + b^2)/b^2} - b)/b) + 2304((a^3 b^* d^* f^* x + a^3 b^* c^* f) \cosh(dx + c)^4 + 4(a^3 b^* d^* f^* x + a^3 b^* c^* f) \cosh(dx + c)^3 \sinh(dx + c) + 6(a^3 b^* d^* f^* x + a^3 b^* c^* f) \cosh(dx + c)^2 \sinh(dx + c)^2 + 4(a^3 b^* d^* f^* x + a^3 b^* c^* f) \cosh(dx + c) \sinh(dx + c)^3 + (a^3 b^* d^* f^* x + a^3 b^* c^* f) \sinh(dx + c)^4) \sqrt{(a^2 + b^2)/b^2} \log(-(a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c))) \sqrt{(a^2 + b^2)/b^2} - b)/b) - 32(3a^3 b^3 d^* f^* x + 3a^3 b^3 d^* e + a^3 b^3 f) \cosh(dx + c) + 8(9(4b^4 d^* f^* x + 4b^4 d^* e - b^4 f) \cosh(dx + c)^7 - 12a^3 b^3 d^* f^* x - 28(3a^3 b^3 d^* f^* x + 3a^3 b^3 d^* e - a^3 b^3 f) \cosh(dx + c)^6 - 12a^3 b^3 d^* e + 108(2a^2 b^2 d^* f^* x + 2a^2 b^2 d^* e - a^2 b^2 f) \cosh(dx + c)^5 - 4a^3 b^3 f - 180((4a^3 b + a^3 b^3) d^* f^* x + (4a^3 b + a^3 b^3) d^* e - (4a^3 b + a^3 b^3) f) \cosh(dx + c)^4 + 72((8a^4 + 4a^2 b^2 - b^4) d^2 f^* x^2 + 2(8a^4 + 4a^2 b^2 - b^4) d^2 e^* x) \cosh(dx + c)^3 - 108((4a^3 b + a^3 b^3) d^* f^* x + (4a^3 b + a^3 b^3) d^* e + (4a^3 b + a^3 b^3) f) \cosh(dx + c)^2 - 36(2a^2 b^2 d^* f^* x + 2a^2 b^2 d^* e + a^2 b^2 f) \cosh(dx + c) \sinh(dx + c)) / (b^5 d^2 \cosh(dx + c)^4 + 4b^5 d^2 \cosh(dx + c)^3 \sinh(dx + c) + 6b^5 d^2 \cosh(dx + c)^2 \sinh(dx + c)^2 + 4b^5 d^2 \cosh(dx + c) \sinh(dx + c)^3 + b^5 d^2 \sinh(dx + c)^4)
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(dx+c)\*\*2\*sinh(dx+c)\*\*3/(a+b\*sinh(dx+c)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \cosh(dx + c)^2 \sinh(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^2\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)\*cosh(d\*x + c)^2\*sinh(d\*x + c)^3/(b\*sinh(d\*x + c) + a), x)

$$3.399 \quad \int \frac{\cosh^2(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=184

$$-\frac{a(3a^2 + b^2) \cosh(c + dx)}{3b^4d} + \frac{2a^3\sqrt{a^2 + b^2} \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^5d} + \frac{(4a^2 + b^2) \sinh(c + dx) \cosh(c + dx)}{8b^3d} + \frac{x(4a^2b^2)}{8b^4d}$$

[Out]  $((8*a^4 + 4*a^2*b^2 - b^4)*x)/(8*b^5) + (2*a^3*sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[(c + d*x)/2])/sqrt[a^2 + b^2]])/(b^5*d) - (a*(3*a^2 + b^2)*Cosh[c + d*x])/(3*b^4*d) + ((4*a^2 + b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(8*b^3*d) - (a*Cosh[c + d*x]*Sinh[c + d*x]^2)/(3*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*b*d)$

**Rubi [A]** time = 0.79298, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$ , Rules used = {2889, 3050, 3049, 3023, 2735, 2660, 618, 204}

$$-\frac{a(3a^2 + b^2) \cosh(c + dx)}{3b^4d} + \frac{2a^3\sqrt{a^2 + b^2} \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^5d} + \frac{(4a^2 + b^2) \sinh(c + dx) \cosh(c + dx)}{8b^3d} + \frac{x(4a^2b^2)}{8b^4d}$$

Antiderivative was successfully verified.

[In] Int[(Cosh[c + d\*x]^2\*Sinh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out]  $((8*a^4 + 4*a^2*b^2 - b^4)*x)/(8*b^5) + (2*a^3*sqrt[a^2 + b^2]*ArcTanh[(b - a*Tanh[(c + d*x)/2])/sqrt[a^2 + b^2]])/(b^5*d) - (a*(3*a^2 + b^2)*Cosh[c + d*x])/(3*b^4*d) + ((4*a^2 + b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(8*b^3*d) - (a*Cosh[c + d*x]*Sinh[c + d*x]^2)/(3*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*b*d)$

#### Rule 2889

Int[cos[(e\_.) + (f\_.)\*(x\_)]^2\*((d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(m\_), x\_Symbol] := Int[(d\*Sin[e + f\*x])^n\*(a + b\*Sin[e + f\*x])^m\*(1 - Sin[e + f\*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2\*m, 2\*n])

#### Rule 3050



```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)^2]), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0]
)))

```

### Rule 3049

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)^2]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] && !(IGtQ[n, 0] && (!IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))

```

### Rule 3023

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)^2]), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]

```

### Rule 2735

```

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

```

### Rule 2660

```

Int[((a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[

```

$a^2 - b^2, 0]$

### Rule 618

$\text{Int}[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2]^{-1}, x\_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c\}, x \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 204

$\text{Int}[(a_.) + (b_.)*(x_.)^2]^{-1}, x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

### Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^2(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx &= \int \frac{\sinh^3(c+dx) (1 + \sinh^2(c+dx))}{a+b \sinh(c+dx)} dx \\
 &= \frac{\cosh(c+dx) \sinh^3(c+dx)}{4bd} + \frac{\int \frac{\sinh^2(c+dx)(-3a+b \sinh(c+dx)-4a \sinh^2(c+dx))}{a+b \sinh(c+dx)} dx}{4b} \\
 &= -\frac{a \cosh(c+dx) \sinh^2(c+dx)}{3b^2d} + \frac{\cosh(c+dx) \sinh^3(c+dx)}{4bd} + \frac{\int \frac{\sinh(c+dx)(8a^2-ab \sinh(c+dx))}{a+b \sinh(c+dx)} dx}{4b} \\
 &= \frac{(4a^2 + b^2) \cosh(c+dx) \sinh(c+dx)}{8b^3d} - \frac{a \cosh(c+dx) \sinh^2(c+dx)}{3b^2d} + \frac{\cosh(c+dx) \sinh^3(c+dx)}{4bd} \\
 &= -\frac{a(3a^2 + b^2) \cosh(c+dx)}{3b^4d} + \frac{(4a^2 + b^2) \cosh(c+dx) \sinh(c+dx)}{8b^3d} - \frac{a \cosh(c+dx) \sinh^2(c+dx)}{3b^2d} \\
 &= \frac{(8a^4 + 4a^2b^2 - b^4)x}{8b^5} - \frac{a(3a^2 + b^2) \cosh(c+dx)}{3b^4d} + \frac{(4a^2 + b^2) \cosh(c+dx) \sinh(c+dx)}{8b^3d} \\
 &= \frac{(8a^4 + 4a^2b^2 - b^4)x}{8b^5} - \frac{a(3a^2 + b^2) \cosh(c+dx)}{3b^4d} + \frac{(4a^2 + b^2) \cosh(c+dx) \sinh(c+dx)}{8b^3d} \\
 &= \frac{(8a^4 + 4a^2b^2 - b^4)x}{8b^5} - \frac{a(3a^2 + b^2) \cosh(c+dx)}{3b^4d} + \frac{(4a^2 + b^2) \cosh(c+dx) \sinh(c+dx)}{8b^3d} \\
 &= \frac{(8a^4 + 4a^2b^2 - b^4)x}{8b^5} + \frac{2a^3 \sqrt{a^2 + b^2} \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b^5d} - \frac{a(3a^2 + b^2) \cosh(c+dx)}{3b^4d}
 \end{aligned}$$

**Mathematica [A]** time = 1.98925, size = 153, normalized size = 0.83

$$\frac{-24ab(4a^2 + b^2) \cosh(c + dx) + 3 \left( 4(4a^2b^2 + 8a^4 - b^4)(c + dx) + 8a^2b^2 \sinh(2(c + dx)) + 64a^3 \sqrt{-a^2 - b^2} \tan^{-1} \left( \frac{b - a \tanh\left(\frac{c + dx}{2}\right)}{\sqrt{-a^2 - b^2}} \right) \right)}{96b^5d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cosh[c + d\*x]^2\*Sinh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] (-24\*a\*b\*(4\*a^2 + b^2)\*Cosh[c + d\*x] - 8\*a\*b^3\*Cosh[3\*(c + d\*x)] + 3\*(4\*(8\*a^4 + 4\*a^2\*b^2 - b^4)\*(c + d\*x) + 64\*a^3\*sqrt[-a^2 - b^2]\*ArcTan[(b - a\*Tanh[(c + d\*x)/2])/sqrt[-a^2 - b^2]] + 8\*a^2\*b^2\*Sinh[2\*(c + d\*x)] + b^4\*Sinh[4\*(c + d\*x)])/(96\*b^5\*d)

**Maple [B]** time = 0.049, size = 624, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)^2\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

[Out] -1/8/d/b\*ln(tanh(1/2\*d\*x+1/2\*c)+1)+1/8/d/b\*ln(tanh(1/2\*d\*x+1/2\*c)-1)-3/8/d/b/(tanh(1/2\*d\*x+1/2\*c)+1)^2+3/8/d/b/(tanh(1/2\*d\*x+1/2\*c)-1)^2-1/d\*a^4/b^5\*ln(tanh(1/2\*d\*x+1/2\*c)-1)-1/2/d/b^3/(tanh(1/2\*d\*x+1/2\*c)+1)^2\*a^2-1/d/b^4/(tanh(1/2\*d\*x+1/2\*c)+1)\*a^3+1/d\*a^4/b^5\*ln(tanh(1/2\*d\*x+1/2\*c)+1)-1/3/d/b^2/(tanh(1/2\*d\*x+1/2\*c)+1)^3\*a+1/3/d/b^2/(tanh(1/2\*d\*x+1/2\*c)-1)^3\*a+1/2/d/b^3/(tanh(1/2\*d\*x+1/2\*c)-1)^2\*a^2+1/d/b^4/(tanh(1/2\*d\*x+1/2\*c)-1)\*a^3+1/2/d/b^2/(tanh(1/2\*d\*x+1/2\*c)+1)^2\*a+1/2/d/b^3/(tanh(1/2\*d\*x+1/2\*c)+1)\*a^2+1/2/d/b^2/(tanh(1/2\*d\*x+1/2\*c)-1)^2\*a+1/2/d/b^3/(tanh(1/2\*d\*x+1/2\*c)-1)\*a^2-1/2/d/b^2/(tanh(1/2\*d\*x+1/2\*c)+1)\*a+1/2/d/b^3\*ln(tanh(1/2\*d\*x+1/2\*c)+1)\*a^2+1/2/d/b^2/(tanh(1/2\*d\*x+1/2\*c)-1)\*a-1/2/d/b^3\*ln(tanh(1/2\*d\*x+1/2\*c)-1)\*a^2-2/d\*a^3\*(a^2+b^2)^(1/2)/b^5\*arctanh(1/2\*(2\*a\*tanh(1/2\*d\*x+1/2\*c)-2\*b)/(a^2+b^2)^(1/2))+1/4/d/b/(tanh(1/2\*d\*x+1/2\*c)-1)^4-1/4/d/b/(tanh(1/2\*d\*x+1/2\*c)+1)^4+1/2/d/b/(tanh(1/2\*d\*x+1/2\*c)+1)^3+1/2/d/b/(tanh(1/2\*d\*x+1/2\*c)-1)^3+1/8/d/b/(tanh(1/2\*d\*x+1/2\*c)+1)+1/8/d/b/(tanh(1/2\*d\*x+1/2\*c)-1)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [B]** time = 2.29543, size = 2803, normalized size = 15.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/192*(3*b^4*cosh(d*x + c)^8 + 3*b^4*sinh(d*x + c)^8 - 8*a*b^3*cosh(d*x + c)^7 + 24*a^2*b^2*cosh(d*x + c)^6 + 8*(3*b^4*cosh(d*x + c) - a*b^3)*sinh(d*x + c)^7 + 24*(8*a^4 + 4*a^2*b^2 - b^4)*d*x*cosh(d*x + c)^4 + 4*(21*b^4*cosh(d*x + c)^2 - 14*a*b^3*cosh(d*x + c) + 6*a^2*b^2)*sinh(d*x + c)^6 - 24*a^2*b^2*cosh(d*x + c)^2 - 24*(4*a^3*b + a*b^3)*cosh(d*x + c)^5 + 24*(7*b^4*cosh(d*x + c)^3 - 7*a*b^3*cosh(d*x + c)^2 + 6*a^2*b^2*cosh(d*x + c) - 4*a^3*b - a*b^3)*sinh(d*x + c)^5 - 8*a*b^3*cosh(d*x + c) + 2*(105*b^4*cosh(d*x + c)^4 - 140*a*b^3*cosh(d*x + c)^3 + 180*a^2*b^2*cosh(d*x + c)^2 + 12*(8*a^4 + 4*a^2*b^2 - b^4)*d*x - 60*(4*a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 3*b^4 - 24*(4*a^3*b + a*b^3)*cosh(d*x + c)^3 + 8*(21*b^4*cosh(d*x + c)^5 - 35*a*b^3*cosh(d*x + c)^4 + 60*a^2*b^2*cosh(d*x + c)^3 - 12*a^3*b - 3*a*b^3 + 12*(8*a^4 + 4*a^2*b^2 - b^4)*d*x*cosh(d*x + c) - 30*(4*a^3*b + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 12*(7*b^4*cosh(d*x + c)^6 - 14*a*b^3*cosh(d*x + c)^5 + 30*a^2*b^2*cosh(d*x + c)^4 + 12*(8*a^4 + 4*a^2*b^2 - b^4)*d*x*cosh(d*x + c)^2 - 2*a^2*b^2 - 20*(4*a^3*b + a*b^3)*cosh(d*x + c)^3 - 6*(4*a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 192*(a^3*cosh(d*x + c)^4 + 4*a^3*cosh(d*x + c)^3*sinh(d*x + c) + 6*a^3*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^3*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*sinh(d*x + c)^4)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 8*(3*b^4*cosh(d*x + c)^7 - 7*a*b^3*cosh(d*x + c)^6 + 18*a^2*b^2*cosh(d*x + c)^5 + 12*(8*a^4 + 4*a^2*b^2 - b^4)*d*x*cosh(d*x + c)^3 - 6*a^2*b^2*cosh(d*x + c) - 15*(4*a^3*b + a*b^3)*cosh(d*x + c)^4 - a*b^3 - 9*(4*a^3*b + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(b^5*d*cosh(d*x + c)^4 + 4*b^5*d*cosh(d*x + c
```

)<sup>3</sup>\*sinh(d\*x + c) + 6\*b<sup>5</sup>\*d\*cosh(d\*x + c)<sup>2</sup>\*sinh(d\*x + c)<sup>2</sup> + 4\*b<sup>5</sup>\*d\*cosh(d\*x + c)\*sinh(d\*x + c)<sup>3</sup> + b<sup>5</sup>\*d\*sinh(d\*x + c)<sup>4</sup>

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*\*2\*sinh(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)), x)

[Out] Timed out

**Giac [A]** time = 1.20274, size = 379, normalized size = 2.06

$$\frac{(8a^4 + 4a^2b^2 - b^4)(dx + c)}{8b^5d} - \frac{(24a^2b^2e^{2dx+2c} + 8ab^3e^{dx+c} + 3b^4 + 24(4a^3b + ab^3)e^{3dx+3c})e^{-4dx-4c}}{192b^5d} - \frac{(a^5 + a^3b^2)}{192b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)<sup>2</sup>\*sinh(d\*x+c)<sup>3</sup>/(a+b\*sinh(d\*x+c)), x, algorithm="giac")

[Out] 1/8\*(8\*a<sup>4</sup> + 4\*a<sup>2</sup>\*b<sup>2</sup> - b<sup>4</sup>)\*(d\*x + c)/(b<sup>5</sup>\*d) - 1/192\*(24\*a<sup>2</sup>\*b<sup>2</sup>\*e<sup>(2\*d\*x + 2\*c)</sup> + 8\*a\*b<sup>3</sup>\*e<sup>(d\*x + c)</sup> + 3\*b<sup>4</sup> + 24\*(4\*a<sup>3</sup>\*b + a\*b<sup>3</sup>)\*e<sup>(3\*d\*x + 3\*c)</sup>)\*e<sup>(-4\*d\*x - 4\*c)</sup>/(b<sup>5</sup>\*d) - (a<sup>5</sup> + a<sup>3</sup>\*b<sup>2</sup>)\*log(abs(2\*b\*e<sup>(d\*x + c)</sup> + 2\*a - 2\*sqrt(a<sup>2</sup> + b<sup>2</sup>))/abs(2\*b\*e<sup>(d\*x + c)</sup> + 2\*a + 2\*sqrt(a<sup>2</sup> + b<sup>2</sup>)))/(sqrt(a<sup>2</sup> + b<sup>2</sup>)\*b<sup>5</sup>\*d) + 1/192\*(3\*b<sup>3</sup>\*d<sup>3</sup>\*e<sup>(4\*d\*x + 4\*c)</sup> - 8\*a\*b<sup>2</sup>\*d<sup>3</sup>\*e<sup>(3\*d\*x + 3\*c)</sup> + 24\*a<sup>2</sup>\*b\*d<sup>3</sup>\*e<sup>(2\*d\*x + 2\*c)</sup> - 96\*a<sup>3</sup>\*d<sup>3</sup>\*e<sup>(d\*x + c)</sup> - 24\*a\*b<sup>2</sup>\*d<sup>3</sup>\*e<sup>(d\*x + c)</sup>)/(b<sup>4</sup>\*d<sup>4</sup>)

$$3.400 \quad \int \frac{\cosh^2(c+dx) \sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=38

$$\text{Unintegrable} \left( \frac{\sinh^3(c+dx) \cosh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[(Cosh[c + d\*x]^2\*Sinh[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.127051, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cosh^2(c+dx) \sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Cosh[c + d\*x]^2\*Sinh[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] Defer[Int] [(Cosh[c + d\*x]^2\*Sinh[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\cosh^2(c+dx) \sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\cosh^2(c+dx) \sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.001, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Cosh[c + d\*x]^2\*Sinh[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] \$Aborted

**Maple [A]** time = 0.123, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx + c))^2 (\sinh(dx + c))^3}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)^2\*sinh(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(cosh(d\*x+c)^2\*sinh(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-2(a^5 e^c + a^3 b^2 e^c) \int -\frac{e^{(dx)}}{b^6 f x + b^6 e - (b^6 f x e^{(2c)} + b^6 e e^{(2c)}) e^{(2dx)} - 2(ab^5 f x e^c + ab^5 e e^c) e^{(dx)}} dx - \frac{e^{(-4c + \frac{4de}{f})} E_1\left(\frac{4(fx+e)d}{f}\right)}{16bf}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2\*sinh(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm m="maxima")

[Out]  $-2*(a^5*e^c + a^3*b^2*e^c)*integrate(-e^{(d*x)}/(b^6*f*x + b^6*e - (b^6*f*x*e^{(2*c)} + b^6*e*e^{(2*c)})e^{(2*d*x)} - 2*(a*b^5*f*x*e^c + a*b^5*e*e^c)e^{(d*x)}), x) - 1/16*e^{(-4*c + 4*d*e/f)*exp\_integral\_e(1, 4*(f*x + e)*d/f)/(b*f) - 1/8*a*e^{(-3*c + 3*d*e/f)*exp\_integral\_e(1, 3*(f*x + e)*d/f)/(b^2*f) - 1/4*a^2*e^{(-2*c + 2*d*e/f)*exp\_integral\_e(1, 2*(f*x + e)*d/f)/(b^3*f) - 1/4*a^2*e^{(2*c - 2*d*e/f)*exp\_integral\_e(1, -2*(f*x + e)*d/f)/(b^3*f) + 1/8*a*e^{(3*c - 3*d*e/f)*exp\_integral\_e(1, -3*(f*x + e)*d/f)/(b^2*f) - 1/16*e^{(4*c - 4*d*e/f)*exp\_integral\_e(1, -4*(f*x + e)*d/f)/(b*f) - 1/8*(4*a^3 + a*b^2)*e^{(-c + d*e/f)*exp\_integral\_e(1, (f*x + e)*d/f)/(b^4*f) + 1/8*(4*a^3*e^c + a*b^2*e^c)*e^{(-d*e/f)*exp\_integral\_e(1, -(f*x + e)*d/f)/(b^4*f) + 1/8*(8*a^4 + 4*a^2*b^2 - b^4)*log(f*x + e)/(b^5*f)}$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\cosh(dx+c)^2 \sinh(dx+c)^3}{afx+ae+(bfx+be)\sinh(dx+c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^2*sinh(d*x + c)^3/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*sinh(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^2 \sinh(dx+c)^3}{(fx+e)(b \sinh(dx+c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*sinh(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^2*sinh(d*x + c)^3/((f*x + e)*(b*sinh(d*x + c) + a)), x)
```



$$3.401 \quad \int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=1443

result too large to display

```
[Out] (-3*a^3*f^3*x)/(8*b^4*d^3) + (45*a*f^3*x)/(256*b^2*d^3) - (a^3*(e + f*x)^3)/
/(4*b^4*d) + (3*a*(e + f*x)^3)/(32*b^2*d) + (a^3*(a^2 + b^2)*(e + f*x)^4)/(
4*b^6*f) - (6*a^4*f^3*Cosh[c + d*x])/(b^5*d^4) - (40*a^2*f^3*Cosh[c + d*x])
/(9*b^3*d^4) + (3*f^3*Cosh[c + d*x])/(4*b*d^4) - (3*a^4*f*(e + f*x)^2*Cosh[
c + d*x])/(b^5*d^2) - (2*a^2*f*(e + f*x)^2*Cosh[c + d*x])/(b^3*d^2) + (3*f*
(e + f*x)^2*Cosh[c + d*x])/(8*b*d^2) - (9*a*f^2*(e + f*x)*Cosh[c + d*x]^2)/
(32*b^2*d^3) - (2*a^2*f^3*Cosh[c + d*x]^3)/(27*b^3*d^4) - (a^2*f*(e + f*x)^
2*Cosh[c + d*x]^3)/(3*b^3*d^2) - (3*a*f^2*(e + f*x)*Cosh[c + d*x]^4)/(32*b^
2*d^3) - (a*(e + f*x)^3*Cosh[c + d*x]^4)/(4*b^2*d) - (f^3*Cosh[3*c + 3*d*x]
)/(216*b*d^4) - (f*(e + f*x)^2*Cosh[3*c + 3*d*x])/(48*b*d^2) - (3*f^3*Cosh[
5*c + 5*d*x])/(5000*b*d^4) - (3*f*(e + f*x)^2*Cosh[5*c + 5*d*x])/(400*b*d^2
) - (a^3*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2]])]/(b^6*d) - (a^3*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]])]/(b^6*d) - (3*a^3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(b^6*d^2) - (3*a^3*(a^2 + b^2)*f*(e
+ f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(b^6*d^2) +
(6*a^3*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))]/(b^6*d^3) + (6*a^3*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(b^6*d^3) - (6*a^3*(a^2 + b^2)*f^3*Pol
yLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(b^6*d^4) - (6*a^3*(a^2
+ b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(b^6*d^4)
+ (6*a^4*f^2*(e + f*x)*Sinh[c + d*x])/(b^5*d^3) + (40*a^2*f^2*(e + f*x)*Sin
h[c + d*x])/(9*b^3*d^3) - (3*f^2*(e + f*x)*Sinh[c + d*x])/(4*b*d^3) + (a^4*
(e + f*x)^3*Sinh[c + d*x])/(b^5*d) + (2*a^2*(e + f*x)^3*Sinh[c + d*x])/(3*b
^3*d) - ((e + f*x)^3*Sinh[c + d*x])/(8*b*d) + (3*a^3*f^3*Cosh[c + d*x]*Sinh
[c + d*x])/(8*b^4*d^4) + (45*a*f^3*Cosh[c + d*x]*Sinh[c + d*x])/(256*b^2*d^
4) + (3*a^3*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(4*b^4*d^2) + (9*a*f
*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x])/(32*b^2*d^2) + (2*a^2*f^2*(e + f*
x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(9*b^3*d^3) + (a^2*(e + f*x)^3*Cosh[c + d
*x]^2*Sinh[c + d*x])/(3*b^3*d) + (3*a*f^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(1
28*b^2*d^4) + (3*a*f*(e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(16*b^2*d^2
) - (3*a^3*f^2*(e + f*x)*Sinh[c + d*x]^2)/(4*b^4*d^3) - (a^3*(e + f*x)^3*Si
nh[c + d*x]^2)/(2*b^4*d) + (f^2*(e + f*x)*Sinh[3*c + 3*d*x])/(72*b*d^3) + (
(e + f*x)^3*Sinh[3*c + 3*d*x])/(48*b*d) + (3*f^2*(e + f*x)*Sinh[5*c + 5*d*x
])/(1000*b*d^3) + ((e + f*x)^3*Sinh[5*c + 5*d*x])/(80*b*d)
```

**Rubi [A]** time = 2.18695, antiderivative size = 1443, normalized size of antiderivative = 1., number of steps used = 55, number of rules used = 18, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5579, 5448, 3296, 2638, 5447, 3311, 32, 2635, 8, 3310, 5565, 5446, 5561, 2190, 2531, 6609, 2282, 6589}

$$\frac{6f^3 \cosh(c + dx)a^4}{b^5d^4} - \frac{3f(e + fx)^2 \cosh(c + dx)a^4}{b^5d^2} + \frac{(e + fx)^3 \sinh(c + dx)a^4}{b^5d} + \frac{6f^2(e + fx) \sinh(c + dx)a^4}{b^5d^3} + \frac{(a^2 + b^2)}{b^5d^3}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Cosh[c + d\*x]^3\*Sinh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] 
$$\begin{aligned} & (-3*a^3*f^3*x)/(8*b^4*d^3) + (45*a*f^3*x)/(256*b^2*d^3) - (a^3*(e + f*x)^3) / (4*b^4*d) + (3*a*(e + f*x)^3)/(32*b^2*d) + (a^3*(a^2 + b^2)*(e + f*x)^4) / (4*b^6*f) - (6*a^4*f^3*Cosh[c + d*x]) / (b^5*d^4) - (40*a^2*f^3*Cosh[c + d*x]) / (9*b^3*d^4) + (3*f^3*Cosh[c + d*x]) / (4*b*d^4) - (3*a^4*f*(e + f*x)^2*Cosh[c + d*x]) / (b^5*d^2) - (2*a^2*f*(e + f*x)^2*Cosh[c + d*x]) / (b^3*d^2) + (3*f*(e + f*x)^2*Cosh[c + d*x]) / (8*b*d^2) - (9*a*f^2*(e + f*x)*Cosh[c + d*x]^2) / (32*b^2*d^3) - (2*a^2*f^3*Cosh[c + d*x]^3) / (27*b^3*d^4) - (a^2*f*(e + f*x)^2*Cosh[c + d*x]^3) / (3*b^3*d^2) - (3*a*f^2*(e + f*x)*Cosh[c + d*x]^4) / (32*b^2*d^3) - (a*(e + f*x)^3*Cosh[c + d*x]^4) / (4*b^2*d) - (f^3*Cosh[3*c + 3*d*x]) / (216*b*d^4) - (f*(e + f*x)^2*Cosh[3*c + 3*d*x]) / (48*b*d^2) - (3*f^3*Cosh[5*c + 5*d*x]) / (5000*b*d^4) - (3*f*(e + f*x)^2*Cosh[5*c + 5*d*x]) / (400*b*d^2) - (a^3*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]) / (b^6*d) - (a^3*(a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]) / (b^6*d) - (3*a^3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]) / (b^6*d^2) - (3*a^3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) / (b^6*d^2) + (6*a^3*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]) / (b^6*d^3) + (6*a^3*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) / (b^6*d^3) - (6*a^3*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]) / (b^6*d^4) - (6*a^3*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]) / (b^6*d^4) + (6*a^4*f^2*(e + f*x)*Sinh[c + d*x]) / (b^5*d^3) + (40*a^2*f^2*(e + f*x)*Sinh[c + d*x]) / (9*b^3*d^3) - (3*f^2*(e + f*x)*Sinh[c + d*x]) / (4*b*d^3) + (a^4*(e + f*x)^3*Sinh[c + d*x]) / (b^5*d) + (2*a^2*(e + f*x)^3*Sinh[c + d*x]) / (3*b^3*d) - ((e + f*x)^3*Sinh[c + d*x]) / (8*b*d) + (3*a^3*f^3*Cosh[c + d*x]*Sinh[c + d*x]) / (8*b^4*d^4) + (45*a*f^3*Cosh[c + d*x]*Sinh[c + d*x]) / (256*b^2*d^4) + (3*a^3*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]) / (4*b^4*d^2) + (9*a*f*(e + f*x)^2*Cosh[c + d*x]*Sinh[c + d*x]) / (32*b^2*d^2) + (2*a^2*f^2*(e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x]) / (9*b^3*d^3) + (a^2*(e + f*x)^3*Cosh[c + d*x]^2*Sinh[c + d*x]) / (3*b^3*d) + (3*a*f^3*Cosh[c + d*x]^3*Sinh[c + d*x]) / (128*b^2*d^4) + (3*a*f*(e + f*x)^2*Cosh[c + d*x]^3*Sinh[c + d*x]) / (16*b^2*d^2) - (3*a^3*f^2*(e + f*x)*Sinh[c + d*x]^2) / (4*b^4*d^3) - (a^3*(e + f*x)^3*Sinh[c + d*x]) / (8*b^4*d^3) \end{aligned}$$

$$\frac{nh[c + d*x]^2}{(2*b^4*d)} + \frac{(f^2*(e + f*x)*\text{Sinh}[3*c + 3*d*x])}{(72*b*d^3)} + \frac{(e + f*x)^3*\text{Sinh}[3*c + 3*d*x]}{(48*b*d)} + \frac{(3*f^2*(e + f*x)*\text{Sinh}[5*c + 5*d*x])}{(1000*b*d^3)} + \frac{(e + f*x)^3*\text{Sinh}[5*c + 5*d*x]}{(80*b*d)}$$
Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
(d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
0] && IGtQ[p, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] := Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbo
l] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Sinh[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sinh[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sinh[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sinh[e + f*x])^(n - 1))/(f*n), x]) /;
```

FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x])\*(b\*SIN[c + d\*x])^(n - 1)/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*SIN[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rule 3310

Int[((c\_.) + (d\_.)\*(x\_))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(d\*(b\*SIN[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)\*(b\*SIN[e + f\*x])^(n - 2), x], x] - Simp[(b\*(c + d\*x)\*Cos[e + f\*x]\*(b\*SIN[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

### Rule 5565

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)])^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Dist[a/b^2, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

### Rule 5446

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sinh[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Simp[((c + d\*x)^m\*Sinh[a + b\*x]^(n + 1))/(b\*(n + 1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*Sinh[a + b\*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))^(m_.))/((a_.) + (b_.)*Sin
h[(c_.) + (d_.)*(x_.)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_.))))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_.))))^(n_.), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_.))))^(n_.)]*((f_.) + (g_.)
*(x_.))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_.))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \cosh^3(c+dx) \sinh^2(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= -\frac{a \int (e+fx)^3 \cosh^3(c+dx) \sinh(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)^3 \cosh^3(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
&= -\frac{a(e+fx)^3 \cosh^4(c+dx)}{4b^2d} + \frac{a^2 \int (e+fx)^3 \cosh^3(c+dx) dx}{b^3} - \frac{a^3 \int \frac{(e+fx)^3}{a+b \sinh(c+dx)} dx}{b^3} \\
&= -\frac{a^2 f (e+fx)^2 \cosh^3(c+dx)}{3b^3d^2} - \frac{3af^2(e+fx) \cosh^4(c+dx)}{32b^2d^3} - \frac{a(e+fx)^3 \cosh^2(c+dx)}{4b^3d} \\
&= \frac{a^3(a^2+b^2)(e+fx)^4}{4b^6f} + \frac{3f(e+fx)^2 \cosh(c+dx)}{8bd^2} - \frac{9af^2(e+fx) \cosh^2(c+dx)}{32b^2d^3} \\
&= \frac{3a(e+fx)^3}{32b^2d} + \frac{a^3(a^2+b^2)(e+fx)^4}{4b^6f} - \frac{3a^4f(e+fx)^2 \cosh(c+dx)}{b^5d^2} - \frac{2a^2f \cosh^2(c+dx)}{b^3d} \\
&= \frac{45af^3x}{256b^2d^3} - \frac{a^3(e+fx)^3}{4b^4d} + \frac{3a(e+fx)^3}{32b^2d} + \frac{a^3(a^2+b^2)(e+fx)^4}{4b^6f} - \frac{4a^2f^3 \cosh^2(c+dx)}{9b^3d} \\
&= -\frac{3a^3f^3x}{8b^4d^3} + \frac{45af^3x}{256b^2d^3} - \frac{a^3(e+fx)^3}{4b^4d} + \frac{3a(e+fx)^3}{32b^2d} + \frac{a^3(a^2+b^2)(e+fx)^4}{4b^6f} \\
&= -\frac{3a^3f^3x}{8b^4d^3} + \frac{45af^3x}{256b^2d^3} - \frac{a^3(e+fx)^3}{4b^4d} + \frac{3a(e+fx)^3}{32b^2d} + \frac{a^3(a^2+b^2)(e+fx)^4}{4b^6f} \\
&= -\frac{3a^3f^3x}{8b^4d^3} + \frac{45af^3x}{256b^2d^3} - \frac{a^3(e+fx)^3}{4b^4d} + \frac{3a(e+fx)^3}{32b^2d} + \frac{a^3(a^2+b^2)(e+fx)^4}{4b^6f}
\end{aligned}$$

**Mathematica [B]** time = 21.8732, size = 5157, normalized size = 3.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]), x]
```

[Out] Result too large to show

**Maple [F]** time = 0.23, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 (\cosh(dx + c))^3 (\sinh(dx + c))^3}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)`

[Out] `int((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `-1/960*e^3*((15*a*b^3*e^(-d*x - c) - 6*b^4 - 10*(4*a^2*b^2 + b^4)*e^(-2*d*x - 2*c) + 60*(2*a^3*b + a*b^3)*e^(-3*d*x - 3*c) - 60*(8*a^4 + 6*a^2*b^2 - b^4)*e^(-4*d*x - 4*c))*e^(5*d*x + 5*c)/(b^5*d) + 960*(a^5 + a^3*b^2)*(d*x + c)/(b^6*d) + (15*a*b^3*e^(-4*d*x - 4*c) + 6*b^4*e^(-5*d*x - 5*c) + 60*(8*a^4 + 6*a^2*b^2 - b^4)*e^(-d*x - c) + 60*(2*a^3*b + a*b^3)*e^(-2*d*x - 2*c) + 10*(4*a^2*b^2 + b^4)*e^(-3*d*x - 3*c))/(b^5*d) + 960*(a^5 + a^3*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^6*d)) - 1/34560000*(8640000*(a^5*d^4*f^3*e^(5*c) + a^3*b^2*d^4*f^3*e^(5*c))*x^4 + 34560000*(a^5*d^4*e*f^2*e^(5*c) + a^3*b^2*d^4*e*f^2*e^(5*c))*x^3 + 51840000*(a^5*d^4*e^2*f*e^(5*c) + a^3*b^2*d^4*e^2*f*e^(5*c))*x^2 - 1728*(125*b^5*d^3*f^3*x^3*e^(10*c) + 75*(5*d^3*e*f^2 - d^2*f^3)*b^5*x^2*e^(10*c) + 15*(25*d^3*e^2*f - 10*d^2*e*f^2 + 2*d*f^3)*b^5*x*e^(10*c) - 3*(25*d^2*e^2*f - 10*d*e*f^2 + 2*f^3)*b^5*e^(10*c))*e^(5*d*x) + 16875*(32*a*b^4*d^3*f^3*x^3*e^(9*c) + 24*(4*d^3*e*f^2 - d^2*f^3)*a*b^4*x^2*e^(9*c) + 12*(8*d^3*e^2*f - 4*d^2*e*f^2 + d*f^3)*a*b^4*x*e^(9*c) - 3*(8*d^2*e^2*f - 4*d*e*f^2 + f^3)*a*b^4*e^(9*c))*e^(4*d*x) + 40000*(4*(9*d^2*e^2*f - 6*d*e*f^2 + 2*f^3)*a^2*b^3*e^(8*c) + (9*d^2*e^2*f - 6*d*e*f^2 + 2*f^3)*b^5*e^(8*c) - 9*(4*a^2*b^3*d^3*f^3*e^(8*c) + b^5*d^3*f^3*e`

$$\begin{aligned}
& ^{(8c)} * x^3 - 9(4(3d^3 * e * f^2 - d^2 * f^3) * a^2 * b^3 * e^{(8c)} + (3d^3 * e * f^2 - \\
& d^2 * f^3) * b^5 * e^{(8c)}) * x^2 - 3(4(9d^3 * e^2 * f - 6d^2 * e * f^2 + 2d * f^3) * a^2 \\
& * b^3 * e^{(8c)} + (9d^3 * e^2 * f - 6d^2 * e * f^2 + 2d * f^3) * b^5 * e^{(8c)}) * x * e^{(3d \\
& * x)} - 540000(6(2d^2 * e^2 * f - 2d * e * f^2 + f^3) * a^3 * b^2 * e^{(7c)} + 3(2d^2 * \\
& e^2 * f - 2d * e * f^2 + f^3) * a * b^4 * e^{(7c)} - 4(2a^3 * b^2 * d^3 * f^3 * e^{(7c)} + a * b \\
& ^4 * d^3 * f^3 * e^{(7c)}) * x^3 - 6(2(2d^3 * e * f^2 - d^2 * f^3) * a^3 * b^2 * e^{(7c)} + (2 \\
& * d^3 * e * f^2 - d^2 * f^3) * a * b^4 * e^{(7c)}) * x^2 - 6(2(2d^3 * e^2 * f - 2d^2 * e * f^2 \\
& + d * f^3) * a^3 * b^2 * e^{(7c)} + (2d^3 * e^2 * f - 2d^2 * e * f^2 + d * f^3) * a * b^4 * e^{(7c)} \\
& )) * x * e^{(2d * x)} + 2160000(24(d^2 * e^2 * f - 2d * e * f^2 + 2 * f^3) * a^4 * b * e^{(6c)} \\
& + 18(d^2 * e^2 * f - 2d * e * f^2 + 2 * f^3) * a^2 * b^3 * e^{(6c)} - 3(d^2 * e^2 * f - 2d * \\
& e * f^2 + 2 * f^3) * b^5 * e^{(6c)} - (8a^4 * b * d^3 * f^3 * e^{(6c)} + 6a^2 * b^3 * d^3 * f^3 * e \\
& ^{(6c)} - b^5 * d^3 * f^3 * e^{(6c)}) * x^3 - 3(8(d^3 * e * f^2 - d^2 * f^3) * a^4 * b * e^{(6c)} \\
& ) + 6(d^3 * e * f^2 - d^2 * f^3) * a^2 * b^3 * e^{(6c)} - (d^3 * e * f^2 - d^2 * f^3) * b^5 * e^{( \\
& 6c)}) * x^2 - 3(8(d^3 * e^2 * f - 2d^2 * e * f^2 + 2d * f^3) * a^4 * b * e^{(6c)} + 6(d^3 \\
& * e^2 * f - 2d^2 * e * f^2 + 2d * f^3) * a^2 * b^3 * e^{(6c)} - (d^3 * e^2 * f - 2d^2 * e * f^2 \\
& + 2d * f^3) * b^5 * e^{(6c)}) * x * e^{(d * x)} + 2160000(24(d^2 * e^2 * f + 2d * e * f^2 + 2 \\
& * f^3) * a^4 * b * e^{(4c)} + 18(d^2 * e^2 * f + 2d * e * f^2 + 2 * f^3) * a^2 * b^3 * e^{(4c)} - \\
& 3(d^2 * e^2 * f + 2d * e * f^2 + 2 * f^3) * b^5 * e^{(4c)} + (8a^4 * b * d^3 * f^3 * e^{(4c)} + \\
& 6a^2 * b^3 * d^3 * f^3 * e^{(4c)} - b^5 * d^3 * f^3 * e^{(4c)}) * x^3 + 3(8(d^3 * e * f^2 + d^ \\
& 2 * f^3) * a^4 * b * e^{(4c)} + 6(d^3 * e * f^2 + d^2 * f^3) * a^2 * b^3 * e^{(4c)} - (d^3 * e * f^2 \\
& + d^2 * f^3) * b^5 * e^{(4c)}) * x^2 + 3(8(d^3 * e^2 * f + 2d^2 * e * f^2 + 2d * f^3) * a^4 \\
& * b * e^{(4c)} + 6(d^3 * e^2 * f + 2d^2 * e * f^2 + 2d * f^3) * a^2 * b^3 * e^{(4c)} - (d^3 * e \\
& ^2 * f + 2d^2 * e * f^2 + 2d * f^3) * b^5 * e^{(4c)}) * x * e^{(-d * x)} + 540000(6(2d^2 * e \\
& ^2 * f + 2d * e * f^2 + f^3) * a^3 * b^2 * e^{(3c)} + 3(2d^2 * e^2 * f + 2d * e * f^2 + f^3) \\
& * a * b^4 * e^{(3c)} + 4(2a^3 * b^2 * d^3 * f^3 * e^{(3c)} + a * b^4 * d^3 * f^3 * e^{(3c)}) * x^3 \\
& + 6(2(2d^3 * e * f^2 + d^2 * f^3) * a^3 * b^2 * e^{(3c)} + (2d^3 * e * f^2 + d^2 * f^3) * a * \\
& b^4 * e^{(3c)}) * x^2 + 6(2(2d^3 * e^2 * f + 2d^2 * e * f^2 + d * f^3) * a^3 * b^2 * e^{(3c)} \\
& + (2d^3 * e^2 * f + 2d^2 * e * f^2 + d * f^3) * a * b^4 * e^{(3c)}) * x * e^{(-2d * x)} + 40000 \\
& *(4(9d^2 * e^2 * f + 6d * e * f^2 + 2 * f^3) * a^2 * b^3 * e^{(2c)} + (9d^2 * e^2 * f + 6d * \\
& e * f^2 + 2 * f^3) * b^5 * e^{(2c)} + 9(4a^2 * b^3 * d^3 * f^3 * e^{(2c)} + b^5 * d^3 * f^3 * e^{( \\
& 2c)}) * x^3 + 9(4(3d^3 * e * f^2 + d^2 * f^3) * a^2 * b^3 * e^{(2c)} + (3d^3 * e * f^2 + d \\
& ^2 * f^3) * b^5 * e^{(2c)}) * x^2 + 3(4(9d^3 * e^2 * f + 6d^2 * e * f^2 + 2d * f^3) * a^2 * b \\
& ^3 * e^{(2c)} + (9d^3 * e^2 * f + 6d^2 * e * f^2 + 2d * f^3) * b^5 * e^{(2c)}) * x * e^{(-3d * \\
& x)} + 16875(32a * b^4 * d^3 * f^3 * x^3 * e^c + 24(4d^3 * e * f^2 + d^2 * f^3) * a * b^4 * x^2 \\
& * e^c + 12(8d^3 * e^2 * f + 4d^2 * e * f^2 + d * f^3) * a * b^4 * x * e^c + 3(8d^2 * e^2 * f \\
& + 4d * e * f^2 + f^3) * a * b^4 * e^c) * e^{(-4d * x)} + 1728(125b^5 * d^3 * f^3 * x^3 + 75( \\
& 5d^3 * e * f^2 + d^2 * f^3) * b^5 * x^2 + 15(25d^3 * e^2 * f + 10d^2 * e * f^2 + 2d * f^3) \\
& * b^5 * x + 3(25d^2 * e^2 * f + 10d * e * f^2 + 2 * f^3) * b^5) * e^{(-5d * x)} * e^{(-5c)} / (b \\
& ^6 * d^4) + \text{integrate}(-2((a^5 * b * f^3 + a^3 * b^3 * f^3) * x^3 + 3(a^5 * b * e * f^2 + a^ \\
& 3 * b^3 * e * f^2) * x^2 + 3(a^5 * b * e^2 * f + a^3 * b^3 * e^2 * f) * x - ((a^6 * f^3 * e^c + a^4 * \\
& b^2 * f^3 * e^c) * x^3 + 3(a^6 * e * f^2 * e^c + a^4 * b^2 * e * f^2 * e^c) * x^2 + 3(a^6 * e^2 * f \\
& * e^c + a^4 * b^2 * e^2 * f * e^c) * x) * e^{(d * x)}) / (b^7 * e^{(2d * x + 2c)} + 2a * b^6 * e^{(d * x \\
& + c)} - b^7), x)
\end{aligned}$$



---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)^3\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cosh(d\*x+c)\*\*3\*sinh(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c)^3 \sinh(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)^3\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*cosh(d\*x + c)^3\*sinh(d\*x + c)^3/(b\*sinh(d\*x + c) + a), x)

$$3.402 \quad \int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=1049

result too large to display

```
[Out] -(a^3*e*f*x)/(2*b^4*d) + (3*a*e*f*x)/(16*b^2*d) - (a^3*f^2*x^2)/(4*b^4*d) +
(3*a*f^2*x^2)/(32*b^2*d) + (a^3*(a^2 + b^2)*(e + f*x)^3)/(3*b^6*f) - (2*a^
4*f*(e + f*x)*Cosh[c + d*x])/(b^5*d^2) - (4*a^2*f*(e + f*x)*Cosh[c + d*x])/
(3*b^3*d^2) + (f*(e + f*x)*Cosh[c + d*x])/(4*b*d^2) - (3*a*f^2*Cosh[c + d*x
]^2)/(32*b^2*d^3) - (2*a^2*f*(e + f*x)*Cosh[c + d*x]^3)/(9*b^3*d^2) - (a*f^
2*Cosh[c + d*x]^4)/(32*b^2*d^3) - (a*(e + f*x)^2*Cosh[c + d*x]^4)/(4*b^2*d)
- (f*(e + f*x)*Cosh[3*c + 3*d*x])/(72*b*d^2) - (f*(e + f*x)*Cosh[5*c + 5*d
*x])/(200*b*d^2) - (a^3*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]])/(b^6*d) - (a^3*(a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]])/(b^6*d) - (2*a^3*(a^2 + b^2)*f*(e + f*x)*P
olyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(b^6*d^2) - (2*a^3*(a^
2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/
(b^6*d^2) + (2*a^3*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))]/(b^6*d^3) + (2*a^3*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x
)))/(a + Sqrt[a^2 + b^2]))]/(b^6*d^3) + (2*a^4*f^2*Sinh[c + d*x])/(b^5*d^3)
+ (14*a^2*f^2*Sinh[c + d*x])/(9*b^3*d^3) - (f^2*Sinh[c + d*x])/(4*b*d^3) +
(a^4*(e + f*x)^2*Sinh[c + d*x])/(b^5*d) + (2*a^2*(e + f*x)^2*Sinh[c + d*x]
)/(3*b^3*d) - ((e + f*x)^2*Sinh[c + d*x])/(8*b*d) + (a^3*f*(e + f*x)*Cosh[c
+ d*x]*Sinh[c + d*x])/(2*b^4*d^2) + (3*a*f*(e + f*x)*Cosh[c + d*x]*Sinh[c
+ d*x])/(16*b^2*d^2) + (a^2*(e + f*x)^2*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b
^3*d) + (a*f*(e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x])/(8*b^2*d^2) - (a^3*f^
2*Sinh[c + d*x]^2)/(4*b^4*d^3) - (a^3*(e + f*x)^2*Sinh[c + d*x]^2)/(2*b^4*d
) + (2*a^2*f^2*Sinh[c + d*x]^3)/(27*b^3*d^3) + (f^2*Sinh[3*c + 3*d*x])/(216
*b*d^3) + ((e + f*x)^2*Sinh[3*c + 3*d*x])/(48*b*d) + (f^2*Sinh[5*c + 5*d*x]
)/(1000*b*d^3) + ((e + f*x)^2*Sinh[5*c + 5*d*x])/(80*b*d)
```

**Rubi [A]** time = 1.62347, antiderivative size = 1049, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 15, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {5579, 5448, 3296, 2637, 5447, 3310, 3311, 2633, 5565, 5446, 5561, 2190, 2531, 2282, 6589}

$$-\frac{2f(e+fx) \cosh(c+dx)a^4}{b^5d^2} + \frac{2f^2 \sinh(c+dx)a^4}{b^5d^3} + \frac{(e+fx)^2 \sinh(c+dx)a^4}{b^5d} + \frac{(a^2+b^2)(e+fx)^3a^3}{3b^6f} - \frac{f^2x^2a^3}{4b^4d} - \frac{f^2 \sinh(c+dx)}{4b^4d}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Cosh[c + d\*x]^3\*Sinh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] 
$$-(a^3 e f x)/(2 b^4 d) + (3 a^2 e f x)/(16 b^2 d) - (a^3 f^2 x^2)/(4 b^4 d) + (3 a^2 f^2 x^2)/(32 b^2 d) + (a^3 (a^2 + b^2) (e + f x)^3)/(3 b^6 f) - (2 a^4 f (e + f x) \operatorname{Cosh}[c + d x])/(b^5 d^2) - (4 a^2 f (e + f x) \operatorname{Cosh}[c + d x])/(3 b^3 d^2) + (f (e + f x) \operatorname{Cosh}[c + d x])/(4 b d^2) - (3 a^2 f^2 \operatorname{Cosh}[c + d x]^2)/(32 b^2 d^3) - (2 a^2 f (e + f x) \operatorname{Cosh}[c + d x]^3)/(9 b^3 d^2) - (a^2 f^2 \operatorname{Cosh}[c + d x]^4)/(32 b^2 d^3) - (a (e + f x)^2 \operatorname{Cosh}[c + d x]^4)/(4 b^2 d) - (f (e + f x) \operatorname{Cosh}[3 c + 3 d x])/(72 b d^2) - (f (e + f x) \operatorname{Cosh}[5 c + 5 d x])/(200 b d^2) - (a^3 (a^2 + b^2) (e + f x)^2 \operatorname{Log}[1 + (b E^{(c + d x)})/(a - \sqrt{a^2 + b^2})])/(b^6 d) - (a^3 (a^2 + b^2) (e + f x)^2 \operatorname{Log}[1 + (b E^{(c + d x)})/(a + \sqrt{a^2 + b^2})])/(b^6 d) - (2 a^3 (a^2 + b^2) f (e + f x) \operatorname{PolyLog}[2, -(b E^{(c + d x)})/(a - \sqrt{a^2 + b^2})])/(b^6 d^2) - (2 a^3 (a^2 + b^2) f (e + f x) \operatorname{PolyLog}[2, -(b E^{(c + d x)})/(a + \sqrt{a^2 + b^2})])/(b^6 d^2) + (2 a^3 (a^2 + b^2) f^2 \operatorname{PolyLog}[3, -(b E^{(c + d x)})/(a - \sqrt{a^2 + b^2})])/(b^6 d^3) + (2 a^3 (a^2 + b^2) f^2 \operatorname{PolyLog}[3, -(b E^{(c + d x)})/(a + \sqrt{a^2 + b^2})])/(b^6 d^3) + (2 a^4 f^2 \operatorname{Sinh}[c + d x])/(b^5 d^3) + (14 a^2 f^2 \operatorname{Sinh}[c + d x])/(9 b^3 d^3) - (f^2 \operatorname{Sinh}[c + d x])/(4 b d^3) + (a^4 (e + f x)^2 \operatorname{Sinh}[c + d x])/(b^5 d) + (2 a^2 (e + f x)^2 \operatorname{Sinh}[c + d x])/(3 b^3 d) - ((e + f x)^2 \operatorname{Sinh}[c + d x])/(8 b d) + (a^3 f (e + f x) \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x])/(2 b^4 d^2) + (3 a^2 f (e + f x) \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x])/(16 b^2 d^2) + (a^2 (e + f x)^2 \operatorname{Cosh}[c + d x]^2 \operatorname{Sinh}[c + d x])/(3 b^3 d) + (a f (e + f x) \operatorname{Cosh}[c + d x]^3 \operatorname{Sinh}[c + d x])/(8 b^2 d^2) - (a^3 f^2 \operatorname{Sinh}[c + d x]^2)/(4 b^4 d^3) - (a^3 (e + f x)^2 \operatorname{Sinh}[c + d x]^2)/(2 b^4 d) + (2 a^2 f^2 \operatorname{Sinh}[c + d x]^3)/(27 b^3 d^3) + (f^2 \operatorname{Sinh}[3 c + 3 d x])/(216 b d^3) + ((e + f x)^2 \operatorname{Sinh}[3 c + 3 d x])/(48 b d) + (f^2 \operatorname{Sinh}[5 c + 5 d x])/(1000 b d^3) + ((e + f x)^2 \operatorname{Sinh}[5 c + 5 d x])/(80 b d)$$

### Rule 5579

Int[(Cosh[(c\_) + (d\_)\*(x\_)]^(p\_)\*((e\_) + (f\_)\*(x\_))^(m\_)\*Sinh[(c\_) + (d\_)\*(x\_)]^(n\_)]/((a\_) + (b\_)\*Sinh[(c\_) + (d\_)\*(x\_)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^p\*Sinh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Cosh[c + d\*x]^p\*Sinh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5448

Int[Cosh[(a\_) + (b\_)\*(x\_)]^(p\_)\*((c\_) + (d\_)\*(x\_))^(m\_)\*Sinh[(a\_) + (b\_)\*(x\_)]^(n\_), x\_Symbol] := Int[ExpandTrigReduce[(c + d\*x)^m, Sinh[a + b\*x]^n\*Cosh[a + b\*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] := Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=
Simp[(d*(b*sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*cos[e + f*x]*(b
*sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] := Simp[(d*m*(c + d*x)^(m - 1)*(b*sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*cos[e + f*x]*(b*sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Dist[a/b^2, Int[(e + f*x)^m*Cosh
```

```
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

#### Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

#### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

#### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

#### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)^v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]
```

**Rule 6589**

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

**Rubi steps**

$$\begin{aligned}
\int \frac{(e + fx)^2 \cosh^3(c + dx) \sinh^3(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\int (e + fx)^2 \cosh^3(c + dx) \sinh^2(c + dx) dx}{b} - \frac{a \int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh^2(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= -\frac{a \int (e + fx)^2 \cosh^3(c + dx) \sinh(c + dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)^2 \cosh^3(c+dx) \sinh(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} \\
&= -\frac{a(e + fx)^2 \cosh^4(c + dx)}{4b^2d} + \frac{a^2 \int (e + fx)^2 \cosh^3(c + dx) dx}{b^3} - \frac{a^3 \int \frac{(e+fx)^2}{a+b \sinh(c+dx)} dx}{b^3} \\
&= -\frac{2a^2 f(e + fx) \cosh^3(c + dx)}{9b^3d^2} - \frac{af^2 \cosh^4(c + dx)}{32b^2d^3} - \frac{a(e + fx)^2 \cosh^4(c + dx)}{4b^2d} \\
&= \frac{a^3(a^2 + b^2)(e + fx)^3}{3b^6f} + \frac{f(e + fx) \cosh(c + dx)}{4bd^2} - \frac{3af^2 \cosh^2(c + dx)}{32b^2d^3} - \frac{2a^4f(e + fx)}{b^5d^2} \\
&= \frac{3aefx}{16b^2d} + \frac{3af^2x^2}{32b^2d} + \frac{a^3(a^2 + b^2)(e + fx)^3}{3b^6f} - \frac{2a^4f(e + fx) \cosh(c + dx)}{b^5d^2} \\
&= -\frac{a^3efx}{2b^4d} + \frac{3aefx}{16b^2d} - \frac{a^3f^2x^2}{4b^4d} + \frac{3af^2x^2}{32b^2d} + \frac{a^3(a^2 + b^2)(e + fx)^3}{3b^6f} - \frac{2a^4f(e + fx) \cosh(c + dx)}{b^5d^2} \\
&= -\frac{a^3efx}{2b^4d} + \frac{3aefx}{16b^2d} - \frac{a^3f^2x^2}{4b^4d} + \frac{3af^2x^2}{32b^2d} + \frac{a^3(a^2 + b^2)(e + fx)^3}{3b^6f} - \frac{2a^4f(e + fx) \cosh(c + dx)}{b^5d^2}
\end{aligned}$$

**Mathematica [B]** time = 14.9595, size = 3179, normalized size = 3.03

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Cosh[c + d\*x]^3\*Sinh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] ((8\*a^3\*(a^2 + b^2)\*(6\*e^2\*E^(2\*c)\*x + 6\*e\*E^(2\*c)\*f\*x^2 + 2\*E^(2\*c)\*f^2\*x^3 + (6\*a\*Sqrt[a^2 + b^2]\*e^2\*ArcTan[(a + b\*E^(c + d\*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]\*d) + (6\*a\*Sqrt[-(a^2 + b^2)^2]\*e^2\*E^(2\*c)\*ArcTan[(a + b\*E^(c + d\*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)\*d) - (6\*a\*Sqrt[-(a^2 + b^2)^2]\*e^2\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)\*d) + (6\*a\*Sqrt[-(a^2 + b^2)^2]\*e^2\*E^(2\*c)\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)\*d) + (3\*e^2\*Log[2\*a\*E^(c + d\*x) + b\*(-1 + E^(2\*(c + d\*x)))]/d - (3\*e^2\*E^(2\*c)\*Log[2\*a\*E^(c + d\*x) + b\*(-1 + E^(2\*(c + d\*x)))]/d + (6\*e\*f\*x\*Log[1 + (b\*E^(2\*c + d\*x))]/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)]))/d - (6\*e\*E^(2\*c)\*f\*x\*Log[1 + (b\*E^(2\*c + d\*x))]/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)]))/d + (3\*f^2\*x^2\*Log[1 + (b\*E^(2\*c + d\*x))]/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)]))/d - (3\*E^(2\*c)\*f^2\*x^2\*Log[1 + (b\*E^(2\*c + d\*x))]/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)]))/d + (6\*e\*f\*x\*Log[1 + (b\*E^(2\*c + d\*x))]/(a\*E^c + Sqrt[(a^2 + b^2)\*E^(2\*c)]))/d - (6\*e\*E^(2\*c)\*f\*x\*Log[1 + (b\*E^(2\*c + d\*x))]/(a\*E^c + Sqrt[(a^2 + b^2)\*E^(2\*c)]))/d + (3\*f^2\*x^2\*Log[1 + (b\*E^(2\*c + d\*x))]/(a\*E^c + Sqrt[(a^2 + b^2)\*E^(2\*c)]))/d - (3\*E^(2\*c)\*f^2\*x^2\*Log[1 + (b\*E^(2\*c + d\*x))]/(a\*E^c + Sqrt[(a^2 + b^2)\*E^(2\*c)]))/d - (6\*(-1 + E^(2\*c))\*f\*(e + f\*x)\*PolyLog[2, -(b\*E^(2\*c + d\*x))/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)])])/d^2 - (6\*(-1 + E^(2\*c))\*f\*(e + f\*x)\*PolyLog[2, -(b\*E^(2\*c + d\*x))/(a\*E^c + Sqrt[(a^2 + b^2)\*E^(2\*c)])])/d^2 - (6\*f^2\*PolyLog[3, -(b\*E^(2\*c + d\*x))/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)])])/d^3 + (6\*E^(2\*c)\*f^2\*PolyLog[3, -(b\*E^(2\*c + d\*x))/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)])])/d^3 - (6\*f^2\*PolyLog[3, -(b\*E^(2\*c + d\*x))/(a\*E^c + Sqrt[(a^2 + b^2)\*E^(2\*c)])])/d^3 + (6\*E^(2\*c)\*f^2\*PolyLog[3, -(b\*E^(2\*c + d\*x))/(a\*E^c + Sqrt[(a^2 + b^2)\*E^(2\*c)])])/d^3)/(3\*b^6\*(-1 + E^(2\*c))) - (8\*a^3\*(a^2 + b^2)\*e^2\*x\*(1 + Cosh[2\*c] + Sinh[2\*c]))/(b^6\*(-1 + Cosh[2\*c] + Sinh[2\*c])) - (8\*a^3\*(a^2 + b^2)\*e\*f\*x^2\*(1 + Cosh[2\*c] + Sinh[2\*c]))/(b^6\*(-1 + Cosh[2\*c] + Sinh[2\*c])) - (8\*a^3\*(a^2 + b^2)\*f^2\*x^3\*(1 + Cosh[2\*c] + Sinh[2\*c]))/(3\*b^6\*(-1 + Cosh[2\*c] + Sinh[2\*c])) + ((-8\*a^4 - 6\*a^2\*b^2 + b^4)\*(d^2\*e^2 + 2\*d\*e\*f + 2\*f^2)\*(Cosh[c]/(2\*b^5\*d^3) - Sinh[c]/(2\*b^5\*d^3)) + (8\*a^4\*d\*e\*f + 6\*a^2\*b^2\*d\*e\*f - b^4\*d\*e\*f + 8\*a^4\*f^2 + 6\*a^2\*b^2\*f^2 - b^4\*f^2)\*(-(x\*Cosh[c])/(b^5\*d^2)) + (x\*Sinh[c])/(b^5\*d^2)) + (-8\*a^4 - 6\*a^2\*b^2 + b^4)\*((f^2\*x^2\*Cosh[c])/(2\*b^5\*d) - (f^2\*x^2\*Sinh[c])/(2\*b^5\*d))\*(Cosh[d\*x] - Sinh[d\*x]) + ((-8\*a^4 - 6\*a^2\*b^2 + b^4)\*(d^2\*e^2 - 2\*d\*e\*f + 2\*f^2)\*(-Cosh[c]/(2\*b^5\*d^3) - Sinh[c]/(2\*b^5\*d^3)) + (x\*(8\*a^4\*d\*e\*f\*Cosh[c] + 6\*a^2\*b^2\*d\*e\*f\*Cosh[c] - b^4\*d\*e\*f\*Cosh[c] - 8\*a^4\*f^2\*Cosh[c] - 6\*a^2\*b^2\*f^2\*Cosh[c] + b^4\*f^2\*Cosh[c] + 8\*a^4\*d\*e\*f\*Sinh[c] + 6\*a^2\*b^2\*d\*e\*f\*Sinh[c] - b^4\*d\*e\*f\*Sinh[c] - 8\*a^4\*f^2\*Sinh[c] - 6\*a^2\*b^2\*f^2\*Sinh[c] + b^4\*f^2\*Sinh[c]))/(b^5\*d^2) + (-8\*a^4 - 6\*a^2\*b^2 + b^4)\*(-(f^2\*x^2\*Cosh[c])/(2\*b^5\*d) - (f^2\*x^2\*Sinh[c])/(2\*b^5\*d))\*(Cosh[d\*x] + Sinh[d\*x]) + ((2\*a^2 + b^2)\*(2\*d^2\*e^2 + 2\*d\*e\*f + f^2)\*(-a\*Cosh[2\*c])/(4\*b^4\*d^3) + (a\*Sinh[2\*c])/

$$\begin{aligned}
& (4*b^4*d^3)) + (4*a^3*d*e*f + 2*a*b^2*d*e*f + 2*a^3*f^2 + a*b^2*f^2)*(- (x*\text{Cosh}[2*c]) / (2*b^4*d^2) + (x*\text{Sinh}[2*c]) / (2*b^4*d^2)) + (2*a^2 + b^2)*(- (a*f^2*x^2*\text{Cosh}[2*c]) / (2*b^4*d) + (a*f^2*x^2*\text{Sinh}[2*c]) / (2*b^4*d)) * (\text{Cosh}[2*d*x] - \text{Sinh}[2*d*x]) + ((2*a^2 + b^2)*(2*d^2*e^2 - 2*d*e*f + f^2)*(- (a*\text{Cosh}[2*c]) / (4*b^4*d^3) - (a*\text{Sinh}[2*c]) / (4*b^4*d^3)) + (x*(-4*a^3*d*e*f*\text{Cosh}[2*c] - 2*a*b^2*d*e*f*\text{Cosh}[2*c] + 2*a^3*f^2*\text{Cosh}[2*c] + a*b^2*f^2*\text{Cosh}[2*c] - 4*a^3*d*e*f*\text{Sinh}[2*c] - 2*a*b^2*d*e*f*\text{Sinh}[2*c] + 2*a^3*f^2*\text{Sinh}[2*c] + a*b^2*f^2*\text{Sinh}[2*c])) / (2*b^4*d^2) + (2*a^2 + b^2)*(- (a*f^2*x^2*\text{Cosh}[2*c]) / (2*b^4*d) - (a*f^2*x^2*\text{Sinh}[2*c]) / (2*b^4*d)) * (\text{Cosh}[2*d*x] + \text{Sinh}[2*d*x]) + ((4*a^2 + b^2)*(9*d^2*e^2 + 6*d*e*f + 2*f^2)*(- \text{Cosh}[3*c] / (108*b^3*d^3) + \text{Sinh}[3*c] / (108*b^3*d^3)) + (12*a^2*d*e*f + 3*b^2*d*e*f + 4*a^2*f^2 + b^2*f^2)*(- (x*\text{Cosh}[3*c]) / (18*b^3*d^2) + (x*\text{Sinh}[3*c]) / (18*b^3*d^2)) + (4*a^2 + b^2)*(- (f^2*x^2*\text{Cosh}[3*c]) / (12*b^3*d) + (f^2*x^2*\text{Sinh}[3*c]) / (12*b^3*d)) * (\text{Cosh}[3*d*x] - \text{Sinh}[3*d*x]) + ((4*a^2 + b^2)*(9*d^2*e^2 - 6*d*e*f + 2*f^2)*(\text{Cosh}[3*c] / (108*b^3*d^3) + \text{Sinh}[3*c] / (108*b^3*d^3)) + (x*(12*a^2*d*e*f*\text{Cosh}[3*c] + 3*b^2*d*e*f*\text{Cosh}[3*c] - 4*a^2*f^2*\text{Cosh}[3*c] - b^2*f^2*\text{Cosh}[3*c] + 12*a^2*d*e*f*\text{Sinh}[3*c] + 3*b^2*d*e*f*\text{Sinh}[3*c] - 4*a^2*f^2*\text{Sinh}[3*c] - b^2*f^2*\text{Sinh}[3*c])) / (18*b^3*d^2) + (4*a^2 + b^2)*((f^2*x^2*\text{Cosh}[3*c]) / (12*b^3*d) + (f^2*x^2*\text{Sinh}[3*c]) / (12*b^3*d)) * (\text{Cosh}[3*d*x] + \text{Sinh}[3*d*x]) + (- (a*f^2*x^2*\text{Cosh}[4*c]) / (8*b^2*d) + (a*f^2*x^2*\text{Sinh}[4*c]) / (8*b^2*d) + (8*d^2*e^2 + 4*d*e*f + f^2)*(- (a*\text{Cosh}[4*c]) / (64*b^2*d^3) + (a*\text{Sinh}[4*c]) / (64*b^2*d^3)) + (4*a*d*e*f + a*f^2)*(- (x*\text{Cosh}[4*c]) / (16*b^2*d^2) + (x*\text{Sinh}[4*c]) / (16*b^2*d^2))) * (\text{Cosh}[4*d*x] - \text{Sinh}[4*d*x]) + (- (a*f^2*x^2*\text{Cosh}[4*c]) / (8*b^2*d) - (a*f^2*x^2*\text{Sinh}[4*c]) / (8*b^2*d) + (8*d^2*e^2 - 4*d*e*f + f^2)*(- (a*\text{Cosh}[4*c]) / (64*b^2*d^3) - (a*\text{Sinh}[4*c]) / (64*b^2*d^3)) + (x*(-4*a*d*e*f*\text{Cosh}[4*c] + a*f^2*\text{Cosh}[4*c] - 4*a*d*e*f*\text{Sinh}[4*c] + a*f^2*\text{Sinh}[4*c])) / (16*b^2*d^2)) * (\text{Cosh}[4*d*x] + \text{Sinh}[4*d*x]) + (- (f^2*x^2*\text{Cosh}[5*c]) / (20*b*d) + (f^2*x^2*\text{Sinh}[5*c]) / (20*b*d) + (25*d^2*e^2 + 10*d*e*f + 2*f^2)*(- \text{Cosh}[5*c] / (500*b*d^3) + \text{Sinh}[5*c] / (500*b*d^3)) + (5*d*e*f + f^2)*(- (x*\text{Cosh}[5*c]) / (50*b*d^2) + (x*\text{Sinh}[5*c]) / (50*b*d^2))) * (\text{Cosh}[5*d*x] - \text{Sinh}[5*d*x]) + ((f^2*x^2*\text{Cosh}[5*c]) / (20*b*d) + (f^2*x^2*\text{Sinh}[5*c]) / (20*b*d) + (25*d^2*e^2 - 10*d*e*f + 2*f^2)*(\text{Cosh}[5*c] / (500*b*d^3) + \text{Sinh}[5*c] / (500*b*d^3)) + (x*(5*d*e*f*\text{Cosh}[5*c] - f^2*\text{Cosh}[5*c] + 5*d*e*f*\text{Sinh}[5*c] - f^2*\text{Sinh}[5*c])) / (50*b*d^2)) * (\text{Cosh}[5*d*x] + \text{Sinh}[5*d*x])) / 8
\end{aligned}$$

**Maple [F]** time = 0.211, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\cosh(dx + c))^3 (\sinh(dx + c))^3}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*cosh(d\*x+c)^3\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)



```
[Out] int((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/960*e^2*((15*a*b^3*e^(-d*x - c) - 6*b^4 - 10*(4*a^2*b^2 + b^4)*e^(-2*d*x - 2*c) + 60*(2*a^3*b + a*b^3)*e^(-3*d*x - 3*c) - 60*(8*a^4 + 6*a^2*b^2 - b^4)*e^(-4*d*x - 4*c))*e^(5*d*x + 5*c)/(b^5*d) + 960*(a^5 + a^3*b^2)*(d*x + c)/(b^6*d) + (15*a*b^3*e^(-4*d*x - 4*c) + 6*b^4*e^(-5*d*x - 5*c) + 60*(8*a^4 + 6*a^2*b^2 - b^4)*e^(-d*x - c) + 60*(2*a^3*b + a*b^3)*e^(-2*d*x - 2*c) + 10*(4*a^2*b^2 + b^4)*e^(-3*d*x - 3*c))/(b^5*d) + 960*(a^5 + a^3*b^2)*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(b^6*d)) - 1/1728000*(576000*(a^5*d^3*f^2*e^(5*c) + a^3*b^2*d^3*f^2*e^(5*c))*x^3 + 1728000*(a^5*d^3*e*f*e^(5*c) + a^3*b^2*d^3*e*f*e^(5*c))*x^2 - 432*(25*b^5*d^2*f^2*x^2*e^(10*c) + 10*(5*d^2*e*f - d*f^2)*b^5*x*e^(10*c) - 2*(5*d*e*f - f^2)*b^5*e^(10*c))*e^(5*d*x) + 3375*(8*a*b^4*d^2*f^2*x^2*e^(9*c) + 4*(4*d^2*e*f - d*f^2)*a*b^4*x*e^(9*c) - (4*d*e*f - f^2)*a*b^4*e^(9*c))*e^(4*d*x) + 2000*(8*(3*d*e*f - f^2)*a^2*b^3*e^(8*c) + 2*(3*d*e*f - f^2)*b^5*e^(8*c) - 9*(4*a^2*b^3*d^2*f^2*e^(8*c) + b^5*d^2*f^2*e^(8*c))*x^2 - 6*(4*(3*d^2*e*f - d*f^2)*a^2*b^3*e^(8*c) + (3*d^2*e*f - d*f^2)*b^5*e^(8*c))*x)*e^(3*d*x) - 54000*(2*(2*d*e*f - f^2)*a^3*b^2*e^(7*c) + (2*d*e*f - f^2)*a*b^4*e^(7*c) - 2*(2*a^3*b^2*d^2*f^2*e^(7*c) + a*b^4*d^2*f^2*e^(7*c))*x^2 - 2*(2*(2*d^2*e*f - d*f^2)*a^3*b^2*e^(7*c) + (2*d^2*e*f - d*f^2)*a*b^4*e^(7*c))*x)*e^(2*d*x) + 108000*(16*(d*e*f - f^2)*a^4*b*e^(6*c) + 12*(d*e*f - f^2)*a^2*b^3*e^(6*c) - 2*(d*e*f - f^2)*b^5*e^(6*c) - (8*a^4*b*d^2*f^2*e^(6*c) + 6*a^2*b^3*d^2*f^2*e^(6*c) - b^5*d^2*f^2*e^(6*c))*x^2 - 2*(8*(d^2*e*f - d*f^2)*a^4*b*e^(6*c) + 6*(d^2*e*f - d*f^2)*a^2*b^3*e^(6*c) - (d^2*e*f - d*f^2)*b^5*e^(6*c))*x)*e^(d*x) + 108000*(16*(d*e*f + f^2)*a^4*b*e^(4*c) + 12*(d*e*f + f^2)*a^2*b^3*e^(4*c) - 2*(d*e*f + f^2)*b^5*e^(4*c) + (8*a^4*b*d^2*f^2*e^(4*c) + 6*a^2*b^3*d^2*f^2*e^(4*c) - b^5*d^2*f^2*e^(4*c))*x^2 + 2*(8*(d^2*e*f + d*f^2)*a^4*b*e^(4*c) + 6*(d^2*e*f + d*f^2)*a^2*b^3*e^(4*c) - (d^2*e*f + d*f^2)*b^5*e^(4*c))*x)*e^(-d*x) + 54000*(2*(2*d*e*f + f^2)*a^3*b^2*e^(3*c) + (2*d*e*f + f^2)*a*b^4*e^(3*c) + 2*(2*a^3*b^2*d^2*f^2*e^(3*c) + a*b^4*d^2*f^2*e^(3*c))*x^2 + 2*(2*(2*d^2*e*f + d*f^2)*a^3*b^2*e^(3*c) + (2*d^2*e*f + d*f^2)*a*b^4*e^(3*c))*x)*e^(-2*d*x) + 2000*(8*(3*d*e*f + f^2)*a^2*b^3*e^(2*c) + 2*(3*d*e*f + f^2)*b^5*e^(2*c) + 9*(4*a^2*b^3*d^2*f^2*e^(2*c) + b^5*d^2*f^2*e^(2*c))*x^2 + 6*(4*(3*d^2*e*f +
```

$$d*f^2)*a^2*b^3*e^{(2*c)} + (3*d^2*e*f + d*f^2)*b^5*e^{(2*c)})*x)*e^{(-3*d*x)} + 3375*(8*a*b^4*d^2*f^2*x^2*e^c + 4*(4*d^2*e*f + d*f^2)*a*b^4*x*e^c + (4*d*e*f + f^2)*a*b^4*e^c)*e^{(-4*d*x)} + 432*(25*b^5*d^2*f^2*x^2 + 10*(5*d^2*e*f + d*f^2)*b^5*x + 2*(5*d*e*f + f^2)*b^5)*e^{(-5*d*x)})*e^{(-5*c)}/(b^6*d^3) + \text{integrate}(-2*((a^5*b*f^2 + a^3*b^3*f^2)*x^2 + 2*(a^5*b*e*f + a^3*b^3*e*f)*x - ((a^6*f^2*e^c + a^4*b^2*f^2*e^c)*x^2 + 2*(a^6*e*f*e^c + a^4*b^2*e*f*e^c)*x)*e^{(d*x)})/(b^7*e^{(2*d*x + 2*c)} + 2*a*b^6*e^{(d*x + c)} - b^7), x)$$

**Fricas [C]** time = 4.60862, size = 24563, normalized size = 23.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)^3\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $-1/1728000*(10800*b^5*d^2*f^2*x^2 - 432*(25*b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2*b^5*f^2 + 10*(5*b^5*d^2*e*f - b^5*d*f^2)*x)*\cosh(d*x + c)^{10} - 432*(25*b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2*b^5*f^2 + 10*(5*b^5*d^2*e*f - b^5*d*f^2)*x)*\sinh(d*x + c)^{10} + 3375*(8*a*b^4*d^2*f^2*x^2 + 8*a*b^4*d^2*e^2 - 4*a*b^4*d*e*f + a*b^4*f^2 + 4*(4*a*b^4*d^2*e*f - a*b^4*d*f^2)*x)*\cosh(d*x + c)^9 + 135*(200*a*b^4*d^2*f^2*x^2 + 200*a*b^4*d^2*e^2 - 100*a*b^4*d*e*f + 25*a*b^4*f^2 + 100*(4*a*b^4*d^2*e*f - a*b^4*d*f^2)*x - 32*(25*b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2*b^5*f^2 + 10*(5*b^5*d^2*e*f - b^5*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^9 + 10800*b^5*d^2*e^2 - 2000*(9*(4*a^2*b^3 + b^5)*d^2*f^2*x^2 + 9*(4*a^2*b^3 + b^5)*d^2*e^2 - 6*(4*a^2*b^3 + b^5)*d*e*f + 2*(4*a^2*b^3 + b^5)*f^2 + 6*(3*(4*a^2*b^3 + b^5)*d^2*e*f - (4*a^2*b^3 + b^5)*d*f^2)*x)*\cosh(d*x + c)^8 - 5*(3600*(4*a^2*b^3 + b^5)*d^2*f^2*x^2 + 3600*(4*a^2*b^3 + b^5)*d^2*e^2 - 2400*(4*a^2*b^3 + b^5)*d*e*f + 800*(4*a^2*b^3 + b^5)*f^2 + 3888*(25*b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2*b^5*f^2 + 10*(5*b^5*d^2*e*f - b^5*d*f^2)*x)*\cosh(d*x + c)^2 + 2400*(3*(4*a^2*b^3 + b^5)*d^2*e*f - (4*a^2*b^3 + b^5)*d*f^2)*x - 6075*(8*a*b^4*d^2*f^2*x^2 + 8*a*b^4*d^2*e^2 - 4*a*b^4*d*e*f + a*b^4*f^2 + 4*(4*a*b^4*d^2*e*f - a*b^4*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^8 + 4320*b^5*d*e*f + 54000*(2*(2*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^3*b^2 + a*b^4)*d^2*e^2 - 2*(2*a^3*b^2 + a*b^4)*d*e*f + (2*a^3*b^2 + a*b^4)*f^2 + 2*(2*(2*a^3*b^2 + a*b^4)*d^2*e*f - (2*a^3*b^2 + a*b^4)*d*f^2)*x)*\cosh(d*x + c)^7 + 20*(5400*(2*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 5400*(2*a^3*b^2 + a*b^4)*d^2*e^2 - 5400*(2*a^3*b^2 + a*b^4)*d*e*f - 2592*(25*b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2*b^5*f^2 + 10*(5*b^5*d^2*e*f - b^5*d*f^2)*x)*\cosh(d*x + c)^3 + 2700*(2*a^3*b^2 + a*b^4)*f^2 + 6075*(8*a*b^4*d^2*f^2*x$

$$\begin{aligned}
&^2 + 8*a*b^4*d^2*e^2 - 4*a*b^4*d*e*f + a*b^4*f^2 + 4*(4*a*b^4*d^2*e*f - a*b^4*d*f^2)*x)*\cosh(d*x + c)^2 + 5400*(2*(2*a^3*b^2 + a*b^4)*d^2*e*f - (2*a^3*b^2 + a*b^4)*d*f^2)*x - 800*(9*(4*a^2*b^3 + b^5)*d^2*f^2*x^2 + 9*(4*a^2*b^3 + b^5)*d^2*e^2 - 6*(4*a^2*b^3 + b^5)*d*e*f + 2*(4*a^2*b^3 + b^5)*f^2 + 6*(3*(4*a^2*b^3 + b^5)*d^2*e*f - (4*a^2*b^3 + b^5)*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 864*b^5*f^2 - 108000*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*f^2*x^2 + (8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2 - 2*(8*a^4*b + 6*a^2*b^3 - b^5)*d*e*f + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 + 2*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e*f - (8*a^4*b + 6*a^2*b^3 - b^5)*d*f^2)*x)*\cosh(d*x + c)^6 - 20*(5400*(8*a^4*b + 6*a^2*b^3 - b^5)*d^2*f^2*x^2 + 5400*(8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2 + 4536*(25*b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2*b^5*f^2 + 10*(5*b^5*d^2*e*f - b^5*d*f^2)*x)*\cosh(d*x + c)^4 - 10800*(8*a^4*b + 6*a^2*b^3 - b^5)*d*e*f - 14175*(8*a*b^4*d^2*f^2*x^2 + 8*a*b^4*d^2*e^2 - 4*a*b^4*d*e*f + a*b^4*f^2 + 4*(4*a*b^4*d^2*e*f - a*b^4*d*f^2)*x)*\cosh(d*x + c)^3 + 10800*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 + 2800*(9*(4*a^2*b^3 + b^5)*d^2*f^2*x^2 + 9*(4*a^2*b^3 + b^5)*d^2*e^2 - 6*(4*a^2*b^3 + b^5)*d*e*f + 2*(4*a^2*b^3 + b^5)*f^2 + 6*(3*(4*a^2*b^3 + b^5)*d^2*e*f - (4*a^2*b^3 + b^5)*d*f^2)*x)*\cosh(d*x + c)^2 + 10800*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e*f - (8*a^4*b + 6*a^2*b^3 - b^5)*d*f^2)*x - 18900*(2*(2*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^3*b^2 + a*b^4)*d^2*e^2 - 2*(2*a^3*b^2 + a*b^4)*d*e*f + (2*a^3*b^2 + a*b^4)*f^2 + 2*(2*(2*a^3*b^2 + a*b^4)*d^2*e*f - (2*a^3*b^2 + a*b^4)*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^6 - 576000*((a^5 + a^3*b^2)*d^3*f^2*x^3 + 3*(a^5 + a^3*b^2)*d^3*e*f*x^2 + 3*(a^5 + a^3*b^2)*d^3*e^2*x + 6*(a^5 + a^3*b^2)*c*d^2*e^2 - 6*(a^5 + a^3*b^2)*c^2*d*e*f + 2*(a^5 + a^3*b^2)*c^3*f^2)*\cosh(d*x + c)^5 - 2*(288000*(a^5 + a^3*b^2)*d^3*f^2*x^3 + 864000*(a^5 + a^3*b^2)*d^3*e*f*x^2 + 864000*(a^5 + a^3*b^2)*d^3*e^2*x + 1728000*(a^5 + a^3*b^2)*c*d^2*e^2 - 1728000*(a^5 + a^3*b^2)*c^2*d*e*f + 576000*(a^5 + a^3*b^2)*c^3*f^2 + 54432*(25*b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2*b^5*f^2 + 10*(5*b^5*d^2*e*f - b^5*d*f^2)*x)*\cosh(d*x + c)^5 - 212625*(8*a*b^4*d^2*f^2*x^2 + 8*a*b^4*d^2*e^2 - 4*a*b^4*d*e*f + a*b^4*f^2 + 4*(4*a*b^4*d^2*e*f - a*b^4*d*f^2)*x)*\cosh(d*x + c)^4 + 56000*(9*(4*a^2*b^3 + b^5)*d^2*f^2*x^2 + 9*(4*a^2*b^3 + b^5)*d^2*e^2 - 6*(4*a^2*b^3 + b^5)*d*e*f + 2*(4*a^2*b^3 + b^5)*f^2 + 6*(3*(4*a^2*b^3 + b^5)*d^2*e*f - (4*a^2*b^3 + b^5)*d*f^2)*x)*\cosh(d*x + c)^3 - 567000*(2*(2*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^3*b^2 + a*b^4)*d^2*e^2 - 2*(2*a^3*b^2 + a*b^4)*d*e*f + (2*a^3*b^2 + a*b^4)*f^2 + 2*(2*(2*a^3*b^2 + a*b^4)*d^2*e*f - (2*a^3*b^2 + a*b^4)*d*f^2)*x)*\cosh(d*x + c)^2 + 324000*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*f^2*x^2 + (8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2 - 2*(8*a^4*b + 6*a^2*b^3 - b^5)*d*e*f + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 + 2*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e*f - (8*a^4*b + 6*a^2*b^3 - b^5)*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 108000*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*f^2*x^2 + (8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2 + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*d*e*f + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 + 2*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e*f + (8*a^4*b + 6*a^2*b^3 - b^5)*d*f^2)*x)*\cosh(d*x + c)^4 + 10*(10800*(8*a^4*b + 6*a^2*b^3 - b^5)*d^2*f^2*x^2 - 9072*(25*b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2*b^5*f^2 + 10*(5*b^5*d^2
\end{aligned}$$

$$\begin{aligned}
& *e*f - b^5*d*f^2)*x)*\cosh(d*x + c)^6 + 42525*(8*a*b^4*d^2*f^2*x^2 + 8*a*b^4 \\
& *d^2*e^2 - 4*a*b^4*d*e*f + a*b^4*f^2 + 4*(4*a*b^4*d^2*e*f - a*b^4*d*f^2)*x) \\
& *\cosh(d*x + c)^5 + 10800*(8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2 - 14000*(9*(4*a \\
& a^2*b^3 + b^5)*d^2*f^2*x^2 + 9*(4*a^2*b^3 + b^5)*d^2*e^2 - 6*(4*a^2*b^3 + b \\
& ^5)*d*e*f + 2*(4*a^2*b^3 + b^5)*f^2 + 6*(3*(4*a^2*b^3 + b^5)*d^2*e*f - (4*a \\
& ^2*b^3 + b^5)*d*f^2)*x)*\cosh(d*x + c)^4 + 21600*(8*a^4*b + 6*a^2*b^3 - b^5) \\
& *d*e*f + 189000*(2*(2*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^3*b^2 + a*b^4)* \\
& d^2*e^2 - 2*(2*a^3*b^2 + a*b^4)*d*e*f + (2*a^3*b^2 + a*b^4)*f^2 + 2*(2*(2*a \\
& ^3*b^2 + a*b^4)*d^2*e*f - (2*a^3*b^2 + a*b^4)*d*f^2)*x)*\cosh(d*x + c)^3 + 2 \\
& 1600*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 - 162000*((8*a^4*b + 6*a^2*b^3 - b^5)* \\
& d^2*f^2*x^2 + (8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2 - 2*(8*a^4*b + 6*a^2*b^3 \\
& - b^5)*d*e*f + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 + 2*((8*a^4*b + 6*a^2*b^3 \\
& - b^5)*d^2*e*f - (8*a^4*b + 6*a^2*b^3 - b^5)*d*f^2)*x)*\cosh(d*x + c)^2 + 21 \\
& 600*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e*f + (8*a^4*b + 6*a^2*b^3 - b^5)*d*f^2) \\
& *x - 288000*((a^5 + a^3*b^2)*d^3*f^2*x^3 + 3*(a^5 + a^3*b^2)*d^3*e*f*x^2 \\
& + 3*(a^5 + a^3*b^2)*d^3*e^2*x + 6*(a^5 + a^3*b^2)*c*d^2*e^2 - 6*(a^5 + a^3*b \\
& ^2)*c^2*d*e*f + 2*(a^5 + a^3*b^2)*c^3*f^2)*\cosh(d*x + c))*\sinh(d*x + c)^4 \\
& + 54000*(2*(2*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^3*b^2 + a*b^4)*d^2*e^2 \\
& + 2*(2*a^3*b^2 + a*b^4)*d*e*f + (2*a^3*b^2 + a*b^4)*f^2 + 2*(2*(2*a^3*b^2 + \\
& a*b^4)*d^2*e*f + (2*a^3*b^2 + a*b^4)*d*f^2)*x)*\cosh(d*x + c)^3 - 20*(2592* \\
& (25*b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2*b^5*f^2 + 10*(5*b^5 \\
& *d^2*e*f - b^5*d*f^2)*x)*\cosh(d*x + c)^7 - 5400*(2*a^3*b^2 + a*b^4)*d^2*f^2 \\
& *x^2 - 14175*(8*a*b^4*d^2*f^2*x^2 + 8*a*b^4*d^2*e^2 - 4*a*b^4*d*e*f + a*b^4 \\
& *f^2 + 4*(4*a*b^4*d^2*e*f - a*b^4*d*f^2)*x)*\cosh(d*x + c)^6 + 5600*(9*(4*a^ \\
& 2*b^3 + b^5)*d^2*f^2*x^2 + 9*(4*a^2*b^3 + b^5)*d^2*e^2 - 6*(4*a^2*b^3 + b^5) \\
& )*d*e*f + 2*(4*a^2*b^3 + b^5)*f^2 + 6*(3*(4*a^2*b^3 + b^5)*d^2*e*f - (4*a^2 \\
& *b^3 + b^5)*d*f^2)*x)*\cosh(d*x + c)^5 - 5400*(2*a^3*b^2 + a*b^4)*d^2*e^2 - \\
& 94500*(2*(2*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^3*b^2 + a*b^4)*d^2*e^2 - \\
& 2*(2*a^3*b^2 + a*b^4)*d*e*f + (2*a^3*b^2 + a*b^4)*f^2 + 2*(2*(2*a^3*b^2 + a \\
& *b^4)*d^2*e*f - (2*a^3*b^2 + a*b^4)*d*f^2)*x)*\cosh(d*x + c)^4 - 5400*(2*a^3 \\
& *b^2 + a*b^4)*d*e*f + 108000*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*f^2*x^2 + (8* \\
& a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2 - 2*(8*a^4*b + 6*a^2*b^3 - b^5)*d*e*f + 2* \\
& (8*a^4*b + 6*a^2*b^3 - b^5)*f^2 + 2*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e*f - \\
& (8*a^4*b + 6*a^2*b^3 - b^5)*d*f^2)*x)*\cosh(d*x + c)^3 - 2700*(2*a^3*b^2 + a \\
& *b^4)*f^2 + 288000*((a^5 + a^3*b^2)*d^3*f^2*x^3 + 3*(a^5 + a^3*b^2)*d^3*e*f \\
& *x^2 + 3*(a^5 + a^3*b^2)*d^3*e^2*x + 6*(a^5 + a^3*b^2)*c*d^2*e^2 - 6*(a^5 + \\
& a^3*b^2)*c^2*d*e*f + 2*(a^5 + a^3*b^2)*c^3*f^2)*\cosh(d*x + c)^2 - 5400*(2* \\
& (2*a^3*b^2 + a*b^4)*d^2*e*f + (2*a^3*b^2 + a*b^4)*d*f^2)*x - 21600*((8*a^4*b \\
& + 6*a^2*b^3 - b^5)*d^2*f^2*x^2 + (8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2 + 2* \\
& (8*a^4*b + 6*a^2*b^3 - b^5)*d*e*f + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 + 2*( \\
& (8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e*f + (8*a^4*b + 6*a^2*b^3 - b^5)*d*f^2)*x) \\
& *\cosh(d*x + c))*\sinh(d*x + c)^3 + 2000*(9*(4*a^2*b^3 + b^5)*d^2*f^2*x^2 + 9 \\
& *(4*a^2*b^3 + b^5)*d^2*e^2 + 6*(4*a^2*b^3 + b^5)*d*e*f + 2*(4*a^2*b^3 + b^5) \\
& )*f^2 + 6*(3*(4*a^2*b^3 + b^5)*d^2*e*f + (4*a^2*b^3 + b^5)*d*f^2)*x)*\cosh(d \\
& *x + c)^2 - 20*(972*(25*b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2
\end{aligned}$$

$$\begin{aligned}
& *b^5*f^2 + 10*(5*b^5*d^2*e*f - b^5*d*f^2)*x)*\cosh(d*x + c)^8 - 6075*(8*a*b^4*d^2*f^2*x^2 + 8*a*b^4*d^2*e^2 - 4*a*b^4*d*e*f + a*b^4*f^2 + 4*(4*a*b^4*d^2*e*f - a*b^4*d*f^2)*x)*\cosh(d*x + c)^7 - 900*(4*a^2*b^3 + b^5)*d^2*f^2*x^2 \\
& + 2800*(9*(4*a^2*b^3 + b^5)*d^2*f^2*x^2 + 9*(4*a^2*b^3 + b^5)*d^2*e^2 - 6*(4*a^2*b^3 + b^5)*d*e*f + 2*(4*a^2*b^3 + b^5)*f^2 + 6*(3*(4*a^2*b^3 + b^5)*d^2*e*f - (4*a^2*b^3 + b^5)*d*f^2)*x)*\cosh(d*x + c)^6 - 56700*(2*(2*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^3*b^2 + a*b^4)*d^2*e^2 - 2*(2*a^3*b^2 + a*b^4)*d*e*f + (2*a^3*b^2 + a*b^4)*f^2 + 2*(2*(2*a^3*b^2 + a*b^4)*d^2*e*f - (2*a^3*b^2 + a*b^4)*d*f^2)*x)*\cosh(d*x + c)^5 - 900*(4*a^2*b^3 + b^5)*d^2*e^2 \\
& + 81000*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*f^2*x^2 + (8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2 - 2*(8*a^4*b + 6*a^2*b^3 - b^5)*d*e*f + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 + 2*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e*f - (8*a^4*b + 6*a^2*b^3 - b^5)*d*f^2)*x)*\cosh(d*x + c)^4 - 600*(4*a^2*b^3 + b^5)*d*e*f + 288000*((a^5 + a^3*b^2)*d^3*f^2*x^3 + 3*(a^5 + a^3*b^2)*d^3*e*f*x^2 + 3*(a^5 + a^3*b^2)*d^3*e^2*x + 6*(a^5 + a^3*b^2)*c*d^2*e^2 - 6*(a^5 + a^3*b^2)*c^2*d*e*f + 2*(a^5 + a^3*b^2)*c^3*f^2)*\cosh(d*x + c)^3 - 200*(4*a^2*b^3 + b^5)*f^2 - 32400*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*f^2*x^2 + (8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2 + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*d*e*f + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 + 2*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e*f + (8*a^4*b + 6*a^2*b^3 - b^5)*d*f^2)*x)*\cosh(d*x + c)^2 - 600*(3*(4*a^2*b^3 + b^5)*d^2*e*f + (4*a^2*b^3 + b^5)*d*f^2)*x - 8100*(2*(2*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^3*b^2 + a*b^4)*d^2*e^2 + 2*(2*a^3*b^2 + a*b^4)*d*e*f + (2*a^3*b^2 + a*b^4)*f^2 + 2*(2*(2*a^3*b^2 + a*b^4)*d^2*e*f + (2*a^3*b^2 + a*b^4)*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 4320*(5*b^5*d^2*e*f + b^5*d*f^2)*x + 3375*(8*a*b^4*d^2*f^2*x^2 + 8*a*b^4*d^2*e^2 + 4*a*b^4*d*e*f + a*b^4*f^2 + 4*(4*a*b^4*d^2*e*f + a*b^4*d*f^2)*x)*\cosh(d*x + c) + 3456000*((a^5 + a^3*b^2)*d*f^2*x + (a^5 + a^3*b^2)*d*e*f)*\cosh(d*x + c)^5 + 5*((a^5 + a^3*b^2)*d*f^2*x + (a^5 + a^3*b^2)*d*e*f)*\cosh(d*x + c)^4*\sinh(d*x + c) + 10*((a^5 + a^3*b^2)*d*f^2*x + (a^5 + a^3*b^2)*d*e*f)*\cosh(d*x + c)^3*\sinh(d*x + c)^2 + 10*((a^5 + a^3*b^2)*d*f^2*x + (a^5 + a^3*b^2)*d*e*f)*\cosh(d*x + c)^2*\sinh(d*x + c)^3 + 5*((a^5 + a^3*b^2)*d*f^2*x + (a^5 + a^3*b^2)*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c)^4 + ((a^5 + a^3*b^2)*d*f^2*x + (a^5 + a^3*b^2)*d*e*f)*\sinh(d*x + c)^5)*d \log((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 3456000*((a^5 + a^3*b^2)*d*f^2*x + (a^5 + a^3*b^2)*d*e*f)*\cosh(d*x + c)^5 + 5*((a^5 + a^3*b^2)*d*f^2*x + (a^5 + a^3*b^2)*d*e*f)*\cosh(d*x + c)^4*\sinh(d*x + c) + 10*((a^5 + a^3*b^2)*d*f^2*x + (a^5 + a^3*b^2)*d*e*f)*\cosh(d*x + c)^3*\sinh(d*x + c)^2 + 10*((a^5 + a^3*b^2)*d*f^2*x + (a^5 + a^3*b^2)*d*e*f)*\cosh(d*x + c)^2*\sinh(d*x + c)^3 + 5*((a^5 + a^3*b^2)*d*f^2*x + (a^5 + a^3*b^2)*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c)^4 + ((a^5 + a^3*b^2)*d*f^2*x + (a^5 + a^3*b^2)*d*e*f)*\sinh(d*x + c)^5)*d \operatorname{ilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 1728000*((a^5 + a^3*b^2)*d^2*e^2 - 2*(a^5 + a^3*b^2)*c*d*e*f + (a^5 + a^3*b^2)*c^2*f^2)*\cosh(d*x + c)^5 + 5*((a^5 + a^3*b^2)*d^2*e^2 - 2*(a^5 + a^3*b^2)*c*d*e*f + (a^5 + a^3*b^2)*c^2*f^2)*\cosh(d*x + c)^4*\sinh(d*x + c) + 10*((a^5 + a^3*b^2)*d^2*e^2 - 2*(a^5 + a
\end{aligned}$$

$$\begin{aligned}
& ^3*b^2)*c*d*e*f + (a^5 + a^3*b^2)*c^2*f^2)*\cosh(d*x + c)^3*\sinh(d*x + c)^2 \\
& + 10*((a^5 + a^3*b^2)*d^2*e^2 - 2*(a^5 + a^3*b^2)*c*d*e*f + (a^5 + a^3*b^2) \\
& *c^2*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^3 + 5*((a^5 + a^3*b^2)*d^2*e^2 - 2* \\
& (a^5 + a^3*b^2)*c*d*e*f + (a^5 + a^3*b^2)*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + \\
& c)^4 + ((a^5 + a^3*b^2)*d^2*e^2 - 2*(a^5 + a^3*b^2)*c*d*e*f + (a^5 + a^3*b \\
& ^2)*c^2*f^2)*\sinh(d*x + c)^5)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2 \\
& *b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 1728000*((a^5 + a^3*b^2)*d^2*e^2 - 2*(a^ \\
& 5 + a^3*b^2)*c*d*e*f + (a^5 + a^3*b^2)*c^2*f^2)*\cosh(d*x + c)^5 + 5*((a^5 + \\
& a^3*b^2)*d^2*e^2 - 2*(a^5 + a^3*b^2)*c*d*e*f + (a^5 + a^3*b^2)*c^2*f^2)*\co \\
& sh(d*x + c)^4*\sinh(d*x + c) + 10*((a^5 + a^3*b^2)*d^2*e^2 - 2*(a^5 + a^3*b^ \\
& 2)*c*d*e*f + (a^5 + a^3*b^2)*c^2*f^2)*\cosh(d*x + c)^3*\sinh(d*x + c)^2 + 10* \\
& ((a^5 + a^3*b^2)*d^2*e^2 - 2*(a^5 + a^3*b^2)*c*d*e*f + (a^5 + a^3*b^2)*c^2* \\
& f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^3 + 5*((a^5 + a^3*b^2)*d^2*e^2 - 2*(a^5 \\
& + a^3*b^2)*c*d*e*f + (a^5 + a^3*b^2)*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^4 \\
& + ((a^5 + a^3*b^2)*d^2*e^2 - 2*(a^5 + a^3*b^2)*c*d*e*f + (a^5 + a^3*b^2)*c \\
& ^2*f^2)*\sinh(d*x + c)^5)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sq \\
& rt((a^2 + b^2)/b^2) + 2*a) + 1728000*((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 \\
& + a^3*b^2)*d^2*e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2 \\
& )*\cosh(d*x + c)^5 + 5*((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2)*d^2* \\
& e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*\cosh(d*x + c)^ \\
& 4*\sinh(d*x + c) + 10*((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2)*d^2*e \\
& *f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*\cosh(d*x + c)^3 \\
& *\sinh(d*x + c)^2 + 10*((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2)*d^2* \\
& e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*\cosh(d*x + c)^ \\
& 2*\sinh(d*x + c)^3 + 5*((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2)*d^2* \\
& e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*\cosh(d*x + c)* \\
& \sinh(d*x + c)^4 + ((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2)*d^2*e*f* \\
& x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*\sinh(d*x + c)^5)*\l \\
& og(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c) \\
& )*\sqrt{(a^2 + b^2)/b^2} - b)/b) + 1728000*((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2 \\
& *(a^5 + a^3*b^2)*d^2*e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^ \\
& 2*f^2)*\cosh(d*x + c)^5 + 5*((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2) \\
& *d^2*e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*\cosh(d*x \\
& + c)^4*\sinh(d*x + c) + 10*((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2)* \\
& d^2*e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*\cosh(d*x + \\
& c)^3*\sinh(d*x + c)^2 + 10*((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2) \\
& *d^2*e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*\cosh(d*x \\
& + c)^2*\sinh(d*x + c)^3 + 5*((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2) \\
& *d^2*e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*\cosh(d*x \\
& + c)*\sinh(d*x + c)^4 + ((a^5 + a^3*b^2)*d^2*f^2*x^2 + 2*(a^5 + a^3*b^2)*d^2 \\
& *e*f*x + 2*(a^5 + a^3*b^2)*c*d*e*f - (a^5 + a^3*b^2)*c^2*f^2)*\sinh(d*x + c) \\
& ^5)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x \\
& + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 3456000*((a^5 + a^3*b^2)*f^2*\cosh(d*x \\
& + c)^5 + 5*(a^5 + a^3*b^2)*f^2*\cosh(d*x + c)^4*\sinh(d*x + c) + 10*(a^5 + \\
& a^3*b^2)*f^2*\cosh(d*x + c)^3*\sinh(d*x + c)^2 + 10*(a^5 + a^3*b^2)*f^2*\cosh(
\end{aligned}$$

$$\begin{aligned}
& d*x + c)^2*\sinh(d*x + c)^3 + 5*(a^5 + a^3*b^2)*f^2*\cosh(d*x + c)*\sinh(d*x + \\
& c)^4 + (a^5 + a^3*b^2)*f^2*\sinh(d*x + c)^5)*\text{polylog}(3, (a*\cosh(d*x + c) + \\
& a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\text{sqrt}((a^2 + b^2)/b^2) \\
& )/b) - 3456000*((a^5 + a^3*b^2)*f^2*\cosh(d*x + c)^5 + 5*(a^5 + a^3*b^2)*f^2 \\
& *\cosh(d*x + c)^4*\sinh(d*x + c) + 10*(a^5 + a^3*b^2)*f^2*\cosh(d*x + c)^3*\sin \\
& h(d*x + c)^2 + 10*(a^5 + a^3*b^2)*f^2*\cosh(d*x + c)^2*\sinh(d*x + c)^3 + 5*( \\
& a^5 + a^3*b^2)*f^2*\cosh(d*x + c)*\sinh(d*x + c)^4 + (a^5 + a^3*b^2)*f^2*\sinh \\
& (d*x + c)^5)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + \\
& c) + b*\sinh(d*x + c))*\text{sqrt}((a^2 + b^2)/b^2))/b) + 5*(5400*a*b^4*d^2*f^2*x^2 \\
& - 864*(25*b^5*d^2*f^2*x^2 + 25*b^5*d^2*e^2 - 10*b^5*d*e*f + 2*b^5*f^2 + 10 \\
& *(5*b^5*d^2*e*f - b^5*d*f^2)*x)*\cosh(d*x + c)^9 + 5400*a*b^4*d^2*e^2 + 6075 \\
& *(8*a*b^4*d^2*f^2*x^2 + 8*a*b^4*d^2*e^2 - 4*a*b^4*d*e*f + a*b^4*f^2 + 4*(4* \\
& a*b^4*d^2*e*f - a*b^4*d*f^2)*x)*\cosh(d*x + c)^8 + 2700*a*b^4*d*e*f - 3200*( \\
& 9*(4*a^2*b^3 + b^5)*d^2*f^2*x^2 + 9*(4*a^2*b^3 + b^5)*d^2*e^2 - 6*(4*a^2*b^ \\
& 3 + b^5)*d*e*f + 2*(4*a^2*b^3 + b^5)*f^2 + 6*(3*(4*a^2*b^3 + b^5)*d^2*e*f - \\
& (4*a^2*b^3 + b^5)*d*f^2)*x)*\cosh(d*x + c)^7 + 675*a*b^4*f^2 + 75600*(2*(2* \\
& a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^3*b^2 + a*b^4)*d^2*e^2 - 2*(2*a^3*b^2 \\
& + a*b^4)*d*e*f + (2*a^3*b^2 + a*b^4)*f^2 + 2*(2*(2*a^3*b^2 + a*b^4)*d^2*e* \\
& f - (2*a^3*b^2 + a*b^4)*d*f^2)*x)*\cosh(d*x + c)^6 - 129600*((8*a^4*b + 6*a^ \\
& 2*b^3 - b^5)*d^2*f^2*x^2 + (8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2 - 2*(8*a^4*b \\
& + 6*a^2*b^3 - b^5)*d*e*f + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 + 2*((8*a^4*b \\
& + 6*a^2*b^3 - b^5)*d^2*e*f - (8*a^4*b + 6*a^2*b^3 - b^5)*d*f^2)*x)*\cosh(d* \\
& x + c)^5 - 576000*((a^5 + a^3*b^2)*d^3*f^2*x^3 + 3*(a^5 + a^3*b^2)*d^3*e*f*x \\
& x^2 + 3*(a^5 + a^3*b^2)*d^3*e^2*x + 6*(a^5 + a^3*b^2)*c*d^2*e^2 - 6*(a^5 + \\
& a^3*b^2)*c^2*d*e*f + 2*(a^5 + a^3*b^2)*c^3*f^2)*\cosh(d*x + c)^4 + 86400*((8 \\
& *a^4*b + 6*a^2*b^3 - b^5)*d^2*f^2*x^2 + (8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e^2 \\
& + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*d*e*f + 2*(8*a^4*b + 6*a^2*b^3 - b^5)*f^2 \\
& + 2*((8*a^4*b + 6*a^2*b^3 - b^5)*d^2*e*f + (8*a^4*b + 6*a^2*b^3 - b^5)*d*f^ \\
& 2)*x)*\cosh(d*x + c)^3 + 32400*(2*(2*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^3 \\
& *b^2 + a*b^4)*d^2*e^2 + 2*(2*a^3*b^2 + a*b^4)*d*e*f + (2*a^3*b^2 + a*b^4)*f \\
& ^2 + 2*(2*(2*a^3*b^2 + a*b^4)*d^2*e*f + (2*a^3*b^2 + a*b^4)*d*f^2)*x)*\cosh( \\
& d*x + c)^2 + 2700*(4*a*b^4*d^2*e*f + a*b^4*d*f^2)*x + 800*(9*(4*a^2*b^3 + b \\
& ^5)*d^2*f^2*x^2 + 9*(4*a^2*b^3 + b^5)*d^2*e^2 + 6*(4*a^2*b^3 + b^5)*d*e*f + \\
& 2*(4*a^2*b^3 + b^5)*f^2 + 6*(3*(4*a^2*b^3 + b^5)*d^2*e*f + (4*a^2*b^3 + b^ \\
& 5)*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c))/(b^6*d^3*\cosh(d*x + c)^5 + 5*b^6 \\
& *d^3*\cosh(d*x + c)^4*\sinh(d*x + c) + 10*b^6*d^3*\cosh(d*x + c)^3*\sinh(d*x + \\
& c)^2 + 10*b^6*d^3*\cosh(d*x + c)^2*\sinh(d*x + c)^3 + 5*b^6*d^3*\cosh(d*x + c) \\
& *\sinh(d*x + c)^4 + b^6*d^3*\sinh(d*x + c)^5)
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*cosh(d*x+c)**3*sinh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c)^3 \sinh(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*cosh(d*x + c)^3*sinh(d*x + c)^3/(b*sinh(d*x + c) + a), x)
```



$$3.403 \quad \int \frac{(e+fx) \cosh^3(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=641

$$\frac{a^3 f (a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^6 d^2} - \frac{a^3 f (a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^6 d^2} - \frac{a^2 f \cosh^3(c+dx)}{9b^3 d^2} - \frac{a^4 f \cosh(c+dx)}{b^5 d^2}$$

```
[Out] -(a^3*f*x)/(4*b^4*d) + (3*a*f*x)/(32*b^2*d) + (a^3*(a^2 + b^2)*(e + f*x)^2)
/(2*b^6*f) - (a^4*f*Cosh[c + d*x])/(b^5*d^2) - (2*a^2*f*Cosh[c + d*x])/(3*b
^3*d^2) + (f*Cosh[c + d*x])/(8*b*d^2) - (a^2*f*Cosh[c + d*x]^3)/(9*b^3*d^2)
- (a*(e + f*x)*Cosh[c + d*x]^4)/(4*b^2*d) - (f*Cosh[3*c + 3*d*x])/(144*b*d
^2) - (f*Cosh[5*c + 5*d*x])/(400*b*d^2) - (a^3*(a^2 + b^2)*(e + f*x)*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(b^6*d) - (a^3*(a^2 + b^2)*(e + f
*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(b^6*d) - (a^3*(a^2 + b
^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^6*d^2) - (a^
3*(a^2 + b^2)*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^6*
d^2) + (a^4*(e + f*x)*Sinh[c + d*x])/(b^5*d) + (2*a^2*(e + f*x)*Sinh[c + d*
x])/(3*b^3*d) - ((e + f*x)*Sinh[c + d*x])/(8*b*d) + (a^3*f*Cosh[c + d*x]*Si
nh[c + d*x])/(4*b^4*d^2) + (3*a*f*Cosh[c + d*x]*Sinh[c + d*x])/(32*b^2*d^2)
+ (a^2*(e + f*x)*Cosh[c + d*x]^2*Sinh[c + d*x])/(3*b^3*d) + (a*f*Cosh[c +
d*x]^3*Sinh[c + d*x])/(16*b^2*d^2) - (a^3*(e + f*x)*Sinh[c + d*x]^2)/(2*b^4
*d) + ((e + f*x)*Sinh[3*c + 3*d*x])/(48*b*d) + ((e + f*x)*Sinh[5*c + 5*d*x]
)/(80*b*d)
```

**Rubi [A]** time = 0.944258, antiderivative size = 641, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 14, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {5579, 5448, 3296, 2638, 5447, 2635, 8, 3310, 5565, 5446, 5561, 2190, 2279, 2391}

$$\frac{a^3 f (a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^6 d^2} - \frac{a^3 f (a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^6 d^2} - \frac{a^2 f \cosh^3(c+dx)}{9b^3 d^2} - \frac{a^4 f \cosh(c+dx)}{b^5 d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(a^3*f*x)/(4*b^4*d) + (3*a*f*x)/(32*b^2*d) + (a^3*(a^2 + b^2)*(e + f*x)^2)
/(2*b^6*f) - (a^4*f*Cosh[c + d*x])/(b^5*d^2) - (2*a^2*f*Cosh[c + d*x])/(3*b
^3*d^2) + (f*Cosh[c + d*x])/(8*b*d^2) - (a^2*f*Cosh[c + d*x]^3)/(9*b^3*d^2)
- (a*(e + f*x)*Cosh[c + d*x]^4)/(4*b^2*d) - (f*Cosh[3*c + 3*d*x])/(144*b*d
```

$$\begin{aligned} &^2) - (f \cdot \text{Cosh}[5c + 5d \cdot x]) / (400 \cdot b \cdot d^2) - (a^3 \cdot (a^2 + b^2) \cdot (e + f \cdot x) \cdot \text{Log}[1 \\ &+ (b \cdot E^{(c + d \cdot x)}) / (a - \text{Sqrt}[a^2 + b^2])] / (b^6 \cdot d) - (a^3 \cdot (a^2 + b^2) \cdot (e + f \\ &\cdot x) \cdot \text{Log}[1 + (b \cdot E^{(c + d \cdot x)}) / (a + \text{Sqrt}[a^2 + b^2])] / (b^6 \cdot d) - (a^3 \cdot (a^2 + b \\ &^2) \cdot f \cdot \text{PolyLog}[2, -((b \cdot E^{(c + d \cdot x)}) / (a - \text{Sqrt}[a^2 + b^2]))] / (b^6 \cdot d^2) - (a^ \\ &3 \cdot (a^2 + b^2) \cdot f \cdot \text{PolyLog}[2, -((b \cdot E^{(c + d \cdot x)}) / (a + \text{Sqrt}[a^2 + b^2]))] / (b^6 \cdot \\ &d^2) + (a^4 \cdot (e + f \cdot x) \cdot \text{Sinh}[c + d \cdot x]) / (b^5 \cdot d) + (2 \cdot a^2 \cdot (e + f \cdot x) \cdot \text{Sinh}[c + d \cdot \\ &x]) / (3 \cdot b^3 \cdot d) - ((e + f \cdot x) \cdot \text{Sinh}[c + d \cdot x]) / (8 \cdot b \cdot d) + (a^3 \cdot f \cdot \text{Cosh}[c + d \cdot x] \cdot \text{Si} \\ &\text{nh}[c + d \cdot x]) / (4 \cdot b^4 \cdot d^2) + (3 \cdot a \cdot f \cdot \text{Cosh}[c + d \cdot x] \cdot \text{Sinh}[c + d \cdot x]) / (32 \cdot b^2 \cdot d^2) \\ &+ (a^2 \cdot (e + f \cdot x) \cdot \text{Cosh}[c + d \cdot x]^2 \cdot \text{Sinh}[c + d \cdot x]) / (3 \cdot b^3 \cdot d) + (a \cdot f \cdot \text{Cosh}[c + \\ &d \cdot x]^3 \cdot \text{Sinh}[c + d \cdot x]) / (16 \cdot b^2 \cdot d^2) - (a^3 \cdot (e + f \cdot x) \cdot \text{Sinh}[c + d \cdot x]^2) / (2 \cdot b^4 \\ &\cdot d) + ((e + f \cdot x) \cdot \text{Sinh}[3c + 3d \cdot x]) / (48 \cdot b \cdot d) + ((e + f \cdot x) \cdot \text{Sinh}[5c + 5d \cdot x]) \\ &/ (80 \cdot b \cdot d) \end{aligned}$$
Rule 5579

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) +
(d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1), x], x] - Di
st[a/b, Int[((e + f*x)^m*Cosh[c + d*x]^p*Sinh[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
0] && IGtQ[p, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5447

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n +
```

1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

### Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x] \* (b\*SIN[c + d\*x])^(n - 1))/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*SIN[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rule 3310

Int[((c\_.) + (d\_.)\*(x\_))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(d\*(b\*SIN[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)\*(b\*SIN[e + f\*x])^(n - 2), x], x] - Simp[(b\*(c + d\*x)\*Cos[e + f\*x]\*(b\*SIN[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

### Rule 5565

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)])^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Dist[a/b^2, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

### Rule 5446

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sinh[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Simp[((c + d\*x)^m\*Sinh[a + b\*x]^(n + 1))/(b\*(n + 1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*Sinh[a + b\*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

### Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps



$$\begin{aligned} & \text{Cosh}[4*(c + d*x)] + 900*a*b^4*d*f*x*\text{Cosh}[4*(c + d*x)] + 72*b^5*f*\text{Cosh}[5*(c \\ & + d*x)] + 28800*a^5*c*f*\text{Log}[1 + (b*E^(c + d*x))/(a - \text{Sqrt}[a^2 + b^2])] + 28 \\ & 800*a^3*b^2*c*f*\text{Log}[1 + (b*E^(c + d*x))/(a - \text{Sqrt}[a^2 + b^2])] + 28800*a^5* \\ & d*f*x*\text{Log}[1 + (b*E^(c + d*x))/(a - \text{Sqrt}[a^2 + b^2])] + 28800*a^3*b^2*d*f*x* \\ & \text{Log}[1 + (b*E^(c + d*x))/(a - \text{Sqrt}[a^2 + b^2])] + 28800*a^5*c*f*\text{Log}[1 + (b*E \\ & ^{(c + d*x))/(a + \text{Sqrt}[a^2 + b^2])] + 28800*a^3*b^2*c*f*\text{Log}[1 + (b*E^(c + d* \\ & x))/(a + \text{Sqrt}[a^2 + b^2])] + 28800*a^5*d*f*x*\text{Log}[1 + (b*E^(c + d*x))/(a + \text{S} \\ & \text{qrt}[a^2 + b^2])] + 28800*a^3*b^2*d*f*x*\text{Log}[1 + (b*E^(c + d*x))/(a + \text{Sqrt}[a^ \\ & 2 + b^2])] + 28800*a^5*d*e*\text{Log}[a + b*\text{Sinh}[c + d*x]] + 28800*a^3*b^2*d*e*\text{Log} \\ & [a + b*\text{Sinh}[c + d*x]] - 28800*a^5*c*f*\text{Log}[a + b*\text{Sinh}[c + d*x]] - 28800*a^3* \\ & b^2*c*f*\text{Log}[a + b*\text{Sinh}[c + d*x]] + 28800*a^3*(a^2 + b^2)*f*\text{PolyLog}[2, (b*E^ \\ & (c + d*x))/(-a + \text{Sqrt}[a^2 + b^2])] + 28800*a^3*(a^2 + b^2)*f*\text{PolyLog}[2, -(( \\ & b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b^2]))] - 28800*a^4*b*d*e*\text{Sinh}[c + d*x] - 21 \\ & 600*a^2*b^3*d*e*\text{Sinh}[c + d*x] + 3600*b^5*d*e*\text{Sinh}[c + d*x] - 28800*a^4*b*d* \\ & f*x*\text{Sinh}[c + d*x] - 21600*a^2*b^3*d*f*x*\text{Sinh}[c + d*x] + 3600*b^5*d*f*x*\text{Sinh} \\ & [c + d*x] - 3600*a^3*b^2*f*\text{Sinh}[2*(c + d*x)] - 1800*a*b^4*f*\text{Sinh}[2*(c + d*x \\ & )] - 2400*a^2*b^3*d*e*\text{Sinh}[3*(c + d*x)] - 600*b^5*d*e*\text{Sinh}[3*(c + d*x)] - 2 \\ & 400*a^2*b^3*d*f*x*\text{Sinh}[3*(c + d*x)] - 600*b^5*d*f*x*\text{Sinh}[3*(c + d*x)] - 225 \\ & *a*b^4*f*\text{Sinh}[4*(c + d*x)] - 360*b^5*d*e*\text{Sinh}[5*(c + d*x)] - 360*b^5*d*f*x* \\ & \text{Sinh}[5*(c + d*x)]/(28800*b^6*d^2) \end{aligned}$$

**Maple [B]** time = 0.115, size = 1363, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)*\cosh(d*x+c)^3*\sinh(d*x+c)^3/(a+b*\sinh(d*x+c)),x)$

[Out]  $a^5/b^6/d^2*f*c*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)-2*a^5/b^6/d^2*f*c*\ln(\exp(d*x+c))-a^5/b^6/d^2*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))$   
 $*x-a^5/b^6/d^2*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))$   
 $*c-a^5/b^6/d^2*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))$   
 $*x-a^5/b^6/d^2*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))$   
 $*c+2*a^5/b^6/d^2*f*c*x+1/2*a^3*f*x^2/b^4-1/32*a*(2*a^2+b^2)*(2*d*f*x+2*d*e+f)$   
 $/b^4/d^2*\exp(-2*d*x-2*c)-a^3/b^4/d^2*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/$   
 $(-a+(a^2+b^2)^{(1/2)}))$   
 $*x-a^3/b^4/d^2*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/$   
 $(-a+(a^2+b^2)^{(1/2)}))$   
 $*c-a^3/b^4/d^2*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))$   
 $*x-a^3/b^4/d^2*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))$   
 $*c+a^3/b^4/d^2*f*c*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)-2*a^3/b^4/d^2*f*c*\ln(\exp(d*x+c))+2*a^3/b^4/d^2*f*c*x-a^3*e*x/b^4+1/800*(5*d*f*x+5*d*e-f)/d^2/b*\exp(5*d*x+5*c)+1/288*(12*a^2*d*f*x+3*b^2*d*f*x+12*a^2*d*e+3*$

$$\begin{aligned}
& b^2 d e - 4 a^2 f - b^2 f) / b^3 / d^2 \exp(3 d x + 3 c) + a^5 / b^6 / d^2 f c^2 + 1 / 16 * (8 a^4 \\
& * d f x + 6 a^2 b^2 d f x - b^4 d f x + 8 a^4 d e + 6 a^2 b^2 d e - b^4 d e - 8 a^4 f - 6 \\
& a^2 b^2 f + b^4 f) / b^5 / d^2 \exp(d x + c) - 1 / 800 * (5 d f x + 5 d e + f) / d^2 / b \exp(-5 d x \\
& - 5 c) - 1 / 16 * (8 a^4 + 6 a^2 b^2 - b^4) * (d f x + d e + f) / b^5 / d^2 \exp(-d x - c) - 1 / 288 * ( \\
& 4 a^2 + b^2) * (3 d f x + 3 d e + f) / b^3 / d^2 \exp(-3 d x - 3 c) - 1 / 256 * a * (4 d f x + 4 d e \\
& + f) / b^2 / d^2 \exp(-4 d x - 4 c) - 1 / 256 * a * (4 d f x + 4 d e - f) / b^2 / d^2 \exp(4 d x + 4 c \\
& ) - 1 / 32 * a * (4 a^2 d f x + 2 b^2 d f x + 4 a^2 d e + 2 b^2 d e - 2 a^2 f - b^2 f) / b^4 / d^2 \\
& \exp(2 d x + 2 c) + a^3 / b^4 / d^2 f c^2 - a^3 / b^4 / d^2 f \operatorname{dilog}((b \exp(d x + c) + (a^2 + b \\
& ^2)^{(1/2)} + a) / (a + (a^2 + b^2)^{(1/2)})) - a^3 / b^4 / d^2 f \operatorname{dilog}((-b \exp(d x + c) + (a^2 + b \\
& ^2)^{(1/2)} - a) / (-a + (a^2 + b^2)^{(1/2)})) - a^3 / b^4 / d^2 e \ln(b \exp(2 d x + 2 c) + 2 a \exp( \\
& d x + c) - b) + 2 a^3 / b^4 / d^2 e \ln(\exp(d x + c)) + 1 / 2 a^5 / b^6 f x^2 - a^5 / b^6 e x - a^5 / b^6 \\
& / d^2 e \ln(b \exp(2 d x + 2 c) + 2 a \exp(d x + c) - b) + 2 a^5 / b^6 / d^2 e \ln(\exp(d x + c)) - a^5 \\
& / b^6 / d^2 f \operatorname{dilog}((b \exp(d x + c) + (a^2 + b^2)^{(1/2)} + a) / (a + (a^2 + b^2)^{(1/2)})) - a^5 \\
& / b^6 / d^2 f \operatorname{dilog}((-b \exp(d x + c) + (a^2 + b^2)^{(1/2)} - a) / (-a + (a^2 + b^2)^{(1/2)}))
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^3\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned}
& -1/960 * e * ((15 * a * b^3 * e^{(-d * x - c)} - 6 * b^4 - 10 * (4 * a^2 * b^2 + b^4) * e^{(-2 * d * x - 2 * c)} + 60 * (2 * a^3 * b + a * b^3) * e^{(-3 * d * x - 3 * c)} - 60 * (8 * a^4 + 6 * a^2 * b^2 - b^4) \\
& ) * e^{(-4 * d * x - 4 * c)}) * e^{(5 * d * x + 5 * c)} / (b^5 * d) + 960 * (a^5 + a^3 * b^2) * (d * x + c) \\
& / (b^6 * d) + (15 * a * b^3 * e^{(-4 * d * x - 4 * c)} + 6 * b^4 * e^{(-5 * d * x - 5 * c)} + 60 * (8 * a^4 \\
& + 6 * a^2 * b^2 - b^4) * e^{(-d * x - c)} + 60 * (2 * a^3 * b + a * b^3) * e^{(-2 * d * x - 2 * c)} + 1 \\
& 0 * (4 * a^2 * b^2 + b^4) * e^{(-3 * d * x - 3 * c)}) / (b^5 * d) + 960 * (a^5 + a^3 * b^2) * \log(-2 * \\
& a * e^{(-d * x - c)} + b * e^{(-2 * d * x - 2 * c)} - b) / (b^6 * d) - 1 / 57600 * f * ((28800 * (a^5 * \\
& d^2 * e^{(5 * c)} + a^3 * b^2 * d^2 * e^{(5 * c)}) * x^2 - 72 * (5 * b^5 * d * x * e^{(10 * c)} - b^5 * e^{(10 \\
& * c)}) * e^{(5 * d * x)} + 225 * (4 * a * b^4 * d * x * e^{(9 * c)} - a * b^4 * e^{(9 * c)}) * e^{(4 * d * x)} + 200 * \\
& (4 * a^2 * b^3 * e^{(8 * c)} + b^5 * e^{(8 * c)} - 3 * (4 * a^2 * b^3 * d * e^{(8 * c)} + b^5 * d * e^{(8 * c)}) * \\
& x) * e^{(3 * d * x)} - 1800 * (2 * a^3 * b^2 * e^{(7 * c)} + a * b^4 * e^{(7 * c)} - 2 * (2 * a^3 * b^2 * d * e^{(7 * c)} \\
& + a * b^4 * d * e^{(7 * c)}) * x) * e^{(2 * d * x)} + 3600 * (8 * a^4 * b * e^{(6 * c)} + 6 * a^2 * b^3 * e^{(6 * c)} \\
& - b^5 * e^{(6 * c)} - (8 * a^4 * b * d * e^{(6 * c)} + 6 * a^2 * b^3 * d * e^{(6 * c)} - b^5 * d * e^{(6 \\
& * c)}) * x) * e^{(d * x)} + 3600 * (8 * a^4 * b * e^{(4 * c)} + 6 * a^2 * b^3 * e^{(4 * c)} - b^5 * e^{(4 * c)} + \\
& (8 * a^4 * b * d * e^{(4 * c)} + 6 * a^2 * b^3 * d * e^{(4 * c)} - b^5 * d * e^{(4 * c)}) * x) * e^{(-d * x)} + 18 \\
& 00 * (2 * a^3 * b^2 * e^{(3 * c)} + a * b^4 * e^{(3 * c)} + 2 * (2 * a^3 * b^2 * d * e^{(3 * c)} + a * b^4 * d * e^{(3 * c)}) * x) * e^{(-2 * d * x)} + 200 * (4 * a^2 * b^3 * e^{(2 * c)} + b^5 * e^{(2 * c)} + 3 * (4 * a^2 * b^3 * \\
& d * e^{(2 * c)} + b^5 * d * e^{(2 * c)}) * x) * e^{(-3 * d * x)} + 225 * (4 * a * b^4 * d * x * e^c + a * b^4 * e^c
\end{aligned}$$

```
)e^(-4*d*x) + 72*(5*b^5*d*x + b^5)*e^(-5*d*x))*e^(-5*c)/(b^6*d^2) - 900*in
tegrate(128*((a^6*e^c + a^4*b^2*e^c)*x*e^(d*x) - (a^5*b + a^3*b^3)*x)/(b^7*
e^(2*d*x + 2*c) + 2*a*b^6*e^(d*x + c) - b^7), x))
```

**Fricas [B]** time = 4.19557, size = 12709, normalized size = 19.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorith
m="fricas")
```

```
[Out] 1/57600*(72*(5*b^5*d*f*x + 5*b^5*d*e - b^5*f)*cosh(d*x + c)^10 + 72*(5*b^5*
d*f*x + 5*b^5*d*e - b^5*f)*sinh(d*x + c)^10 - 225*(4*a*b^4*d*f*x + 4*a*b^4*
d*e - a*b^4*f)*cosh(d*x + c)^9 - 45*(20*a*b^4*d*f*x + 20*a*b^4*d*e - 5*a*b^
4*f - 16*(5*b^5*d*f*x + 5*b^5*d*e - b^5*f)*cosh(d*x + c))*sinh(d*x + c)^9 +
200*(3*(4*a^2*b^3 + b^5)*d*f*x + 3*(4*a^2*b^3 + b^5)*d*e - (4*a^2*b^3 + b^
5)*f)*cosh(d*x + c)^8 + 5*(120*(4*a^2*b^3 + b^5)*d*f*x + 120*(4*a^2*b^3 + b
^5)*d*e + 648*(5*b^5*d*f*x + 5*b^5*d*e - b^5*f)*cosh(d*x + c)^2 - 40*(4*a^2
*b^3 + b^5)*f - 405*(4*a*b^4*d*f*x + 4*a*b^4*d*e - a*b^4*f)*cosh(d*x + c))*
sinh(d*x + c)^8 - 360*b^5*d*f*x - 1800*(2*(2*a^3*b^2 + a*b^4)*d*f*x + 2*(2*
a^3*b^2 + a*b^4)*d*e - (2*a^3*b^2 + a*b^4)*f)*cosh(d*x + c)^7 - 20*(180*(2*
a^3*b^2 + a*b^4)*d*f*x - 432*(5*b^5*d*f*x + 5*b^5*d*e - b^5*f)*cosh(d*x + c
)^3 + 180*(2*a^3*b^2 + a*b^4)*d*e + 405*(4*a*b^4*d*f*x + 4*a*b^4*d*e - a*b^
4*f)*cosh(d*x + c)^2 - 90*(2*a^3*b^2 + a*b^4)*f - 80*(3*(4*a^2*b^3 + b^5)*d
*f*x + 3*(4*a^2*b^3 + b^5)*d*e - (4*a^2*b^3 + b^5)*f)*cosh(d*x + c))*sinh(d
*x + c)^7 - 360*b^5*d*e + 3600*((8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x + (8*a^4*
b + 6*a^2*b^3 - b^5)*d*e - (8*a^4*b + 6*a^2*b^3 - b^5)*f)*cosh(d*x + c)^6 +
20*(756*(5*b^5*d*f*x + 5*b^5*d*e - b^5*f)*cosh(d*x + c)^4 + 180*(8*a^4*b +
6*a^2*b^3 - b^5)*d*f*x - 945*(4*a*b^4*d*f*x + 4*a*b^4*d*e - a*b^4*f)*cosh(
d*x + c)^3 + 180*(8*a^4*b + 6*a^2*b^3 - b^5)*d*e + 280*(3*(4*a^2*b^3 + b^5)
*d*f*x + 3*(4*a^2*b^3 + b^5)*d*e - (4*a^2*b^3 + b^5)*f)*cosh(d*x + c)^2 - 1
80*(8*a^4*b + 6*a^2*b^3 - b^5)*f - 630*(2*(2*a^3*b^2 + a*b^4)*d*f*x + 2*(2*
a^3*b^2 + a*b^4)*d*e - (2*a^3*b^2 + a*b^4)*f)*cosh(d*x + c))*sinh(d*x + c)^
6 - 72*b^5*f + 28800*((a^5 + a^3*b^2)*d^2*f*x^2 + 2*(a^5 + a^3*b^2)*d^2*e*x
+ 4*(a^5 + a^3*b^2)*c*d*e - 2*(a^5 + a^3*b^2)*c^2*f)*cosh(d*x + c)^5 + 2*(
14400*(a^5 + a^3*b^2)*d^2*f*x^2 + 9072*(5*b^5*d*f*x + 5*b^5*d*e - b^5*f)*co
sh(d*x + c)^5 + 28800*(a^5 + a^3*b^2)*d^2*e*x - 14175*(4*a*b^4*d*f*x + 4*a*
b^4*d*e - a*b^4*f)*cosh(d*x + c)^4 + 57600*(a^5 + a^3*b^2)*c*d*e - 28800*(a
^5 + a^3*b^2)*c^2*f + 5600*(3*(4*a^2*b^3 + b^5)*d*f*x + 3*(4*a^2*b^3 + b^5)
*d*e - (4*a^2*b^3 + b^5)*f)*cosh(d*x + c)^3 - 18900*(2*(2*a^3*b^2 + a*b^4)*
```



$$\begin{aligned}
& d*f*x + 2*(2*a^3*b^2 + a*b^4)*d*e - (2*a^3*b^2 + a*b^4)*f)*\cosh(d*x + c)^2 \\
& + 10800*((8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x + (8*a^4*b + 6*a^2*b^3 - b^5)*d* \\
& e - (8*a^4*b + 6*a^2*b^3 - b^5)*f)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 3600*(( \\
& 8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x + (8*a^4*b + 6*a^2*b^3 - b^5)*d*e + (8*a^4 \\
& *b + 6*a^2*b^3 - b^5)*f)*\cosh(d*x + c)^4 + 10*(1512*(5*b^5*d*f*x + 5*b^5*d* \\
& e - b^5*f)*\cosh(d*x + c)^6 - 2835*(4*a*b^4*d*f*x + 4*a*b^4*d*e - a*b^4*f)*c \\
& osh(d*x + c)^5 + 1400*(3*(4*a^2*b^3 + b^5)*d*f*x + 3*(4*a^2*b^3 + b^5)*d*e \\
& - (4*a^2*b^3 + b^5)*f)*\cosh(d*x + c)^4 - 360*(8*a^4*b + 6*a^2*b^3 - b^5)*d* \\
& f*x - 6300*(2*(2*a^3*b^2 + a*b^4)*d*f*x + 2*(2*a^3*b^2 + a*b^4)*d*e - (2*a^ \\
& 3*b^2 + a*b^4)*f)*\cosh(d*x + c)^3 - 360*(8*a^4*b + 6*a^2*b^3 - b^5)*d*e + 5 \\
& 400*((8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x + (8*a^4*b + 6*a^2*b^3 - b^5)*d*e - \\
& (8*a^4*b + 6*a^2*b^3 - b^5)*f)*\cosh(d*x + c)^2 - 360*(8*a^4*b + 6*a^2*b^3 - \\
& b^5)*f + 14400*((a^5 + a^3*b^2)*d^2*f*x^2 + 2*(a^5 + a^3*b^2)*d^2*e*x + 4* \\
& (a^5 + a^3*b^2)*c*d*e - 2*(a^5 + a^3*b^2)*c^2*f)*\cosh(d*x + c))*\sinh(d*x + \\
& c)^4 - 1800*(2*(2*a^3*b^2 + a*b^4)*d*f*x + 2*(2*a^3*b^2 + a*b^4)*d*e + (2*a \\
& ^3*b^2 + a*b^4)*f)*\cosh(d*x + c)^3 + 20*(432*(5*b^5*d*f*x + 5*b^5*d*e - b^5 \\
& *f)*\cosh(d*x + c)^7 - 945*(4*a*b^4*d*f*x + 4*a*b^4*d*e - a*b^4*f)*\cosh(d*x \\
& + c)^6 + 560*(3*(4*a^2*b^3 + b^5)*d*f*x + 3*(4*a^2*b^3 + b^5)*d*e - (4*a^2* \\
& b^3 + b^5)*f)*\cosh(d*x + c)^5 - 3150*(2*(2*a^3*b^2 + a*b^4)*d*f*x + 2*(2*a^ \\
& 3*b^2 + a*b^4)*d*e - (2*a^3*b^2 + a*b^4)*f)*\cosh(d*x + c)^4 - 180*(2*a^3*b^ \\
& 2 + a*b^4)*d*f*x + 3600*((8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x + (8*a^4*b + 6*a \\
& ^2*b^3 - b^5)*d*e - (8*a^4*b + 6*a^2*b^3 - b^5)*f)*\cosh(d*x + c)^3 - 180*(2 \\
& *a^3*b^2 + a*b^4)*d*e + 14400*((a^5 + a^3*b^2)*d^2*f*x^2 + 2*(a^5 + a^3*b^2 \\
& )*d^2*e*x + 4*(a^5 + a^3*b^2)*c*d*e - 2*(a^5 + a^3*b^2)*c^2*f)*\cosh(d*x + c \\
& )^2 - 90*(2*a^3*b^2 + a*b^4)*f - 720*((8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x + ( \\
& 8*a^4*b + 6*a^2*b^3 - b^5)*d*e + (8*a^4*b + 6*a^2*b^3 - b^5)*f)*\cosh(d*x + \\
& c))*\sinh(d*x + c)^3 - 200*(3*(4*a^2*b^3 + b^5)*d*f*x + 3*(4*a^2*b^3 + b^5)* \\
& d*e + (4*a^2*b^3 + b^5)*f)*\cosh(d*x + c)^2 + 20*(162*(5*b^5*d*f*x + 5*b^5*d \\
& *e - b^5*f)*\cosh(d*x + c)^8 - 405*(4*a*b^4*d*f*x + 4*a*b^4*d*e - a*b^4*f)*c \\
& osh(d*x + c)^7 + 280*(3*(4*a^2*b^3 + b^5)*d*f*x + 3*(4*a^2*b^3 + b^5)*d*e - \\
& (4*a^2*b^3 + b^5)*f)*\cosh(d*x + c)^6 - 1890*(2*(2*a^3*b^2 + a*b^4)*d*f*x + \\
& 2*(2*a^3*b^2 + a*b^4)*d*e - (2*a^3*b^2 + a*b^4)*f)*\cosh(d*x + c)^5 + 2700* \\
& ((8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x + (8*a^4*b + 6*a^2*b^3 - b^5)*d*e - (8*a \\
& ^4*b + 6*a^2*b^3 - b^5)*f)*\cosh(d*x + c)^4 - 30*(4*a^2*b^3 + b^5)*d*f*x + 1 \\
& 4400*((a^5 + a^3*b^2)*d^2*f*x^2 + 2*(a^5 + a^3*b^2)*d^2*e*x + 4*(a^5 + a^3* \\
& b^2)*c*d*e - 2*(a^5 + a^3*b^2)*c^2*f)*\cosh(d*x + c)^3 - 30*(4*a^2*b^3 + b^5 \\
& )*d*e - 1080*((8*a^4*b + 6*a^2*b^3 - b^5)*d*f*x + (8*a^4*b + 6*a^2*b^3 - b^ \\
& 5)*d*e + (8*a^4*b + 6*a^2*b^3 - b^5)*f)*\cosh(d*x + c)^2 - 10*(4*a^2*b^3 + b \\
& ^5)*f - 270*(2*(2*a^3*b^2 + a*b^4)*d*f*x + 2*(2*a^3*b^2 + a*b^4)*d*e + (2*a \\
& ^3*b^2 + a*b^4)*f)*\cosh(d*x + c))*\sinh(d*x + c)^2 - 225*(4*a*b^4*d*f*x + 4* \\
& a*b^4*d*e + a*b^4*f)*\cosh(d*x + c) - 57600*((a^5 + a^3*b^2)*f*\cosh(d*x + c) \\
& ^5 + 5*(a^5 + a^3*b^2)*f*\cosh(d*x + c)^4*\sinh(d*x + c) + 10*(a^5 + a^3*b^2) \\
& *f*\cosh(d*x + c)^3*\sinh(d*x + c)^2 + 10*(a^5 + a^3*b^2)*f*\cosh(d*x + c)^2*s \\
& inh(d*x + c)^3 + 5*(a^5 + a^3*b^2)*f*\cosh(d*x + c)*\sinh(d*x + c)^4 + (a^5 + \\
& a^3*b^2)*f*\sinh(d*x + c)^5)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*
\end{aligned}$$

$$\begin{aligned}
& \cosh(dx + c) + b \sinh(dx + c) \sqrt{(a^2 + b^2)/b^2} - b/b + 1) - 57600 * \\
& ((a^5 + a^3 b^2) * f * \cosh(dx + c)^5 + 5 * (a^5 + a^3 b^2) * f * \cosh(dx + c)^4 * \sinh(dx + c) \\
& + 10 * (a^5 + a^3 b^2) * f * \cosh(dx + c)^3 * \sinh(dx + c)^2 + 10 * (a^5 + a^3 b^2) * f * \cosh(dx + c)^2 * \sinh(dx + c)^3 \\
& + 5 * (a^5 + a^3 b^2) * f * \cosh(dx + c) * \sinh(dx + c)^4 + (a^5 + a^3 b^2) * f * \sinh(dx + c)^5) * \operatorname{dilog}((a * \cosh(dx + c) \\
& + a * \sinh(dx + c) - (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2} - b/b + 1) - 57600 * ((a^5 + a^3 b^2) * d * e - (a^5 + a^3 b^2) * c * f) \\
& * \cosh(dx + c)^5 + 5 * ((a^5 + a^3 b^2) * d * e - (a^5 + a^3 b^2) * c * f) * \cosh(dx + c)^4 * \sinh(dx + c) + 10 * ((a^5 + a^3 b^2) * d * e - (a^5 + a^3 b^2) * c * f) * \cosh(dx + c)^3 * \sinh(dx + c)^2 \\
& + 10 * ((a^5 + a^3 b^2) * d * e - (a^5 + a^3 b^2) * c * f) * \cosh(dx + c)^2 * \sinh(dx + c)^3 + 5 * ((a^5 + a^3 b^2) * d * e - (a^5 + a^3 b^2) * c * f) * \cosh(dx + c) * \sinh(dx + c)^4 \\
& + ((a^5 + a^3 b^2) * d * e - (a^5 + a^3 b^2) * c * f) * \sinh(dx + c)^5) * \log(2 * b * \cosh(dx + c) + 2 * b * \sinh(dx + c) + 2 * b * \sqrt{(a^2 + b^2)/b^2} + 2 * a) - 57600 * ((a^5 + a^3 b^2) * d * e - (a^5 + a^3 b^2) * c * f) * \cosh(dx + c)^5 \\
& + 5 * ((a^5 + a^3 b^2) * d * e - (a^5 + a^3 b^2) * c * f) * \cosh(dx + c)^4 * \sinh(dx + c) + 10 * ((a^5 + a^3 b^2) * d * e - (a^5 + a^3 b^2) * c * f) * \cosh(dx + c)^3 * \sinh(dx + c)^2 \\
& + 10 * ((a^5 + a^3 b^2) * d * e - (a^5 + a^3 b^2) * c * f) * \cosh(dx + c)^2 * \sinh(dx + c)^3 + 5 * ((a^5 + a^3 b^2) * d * e - (a^5 + a^3 b^2) * c * f) * \cosh(dx + c) * \sinh(dx + c)^4 \\
& + ((a^5 + a^3 b^2) * d * e - (a^5 + a^3 b^2) * c * f) * \sinh(dx + c)^5) * \log(2 * b * \cosh(dx + c) + 2 * b * \sinh(dx + c) - 2 * b * \sqrt{(a^2 + b^2)/b^2} + 2 * a) - 57600 * ((a^5 + a^3 b^2) * d * f * x + (a^5 + a^3 b^2) * c * f) * \cosh(dx + c)^5 \\
& + 5 * ((a^5 + a^3 b^2) * d * f * x + (a^5 + a^3 b^2) * c * f) * \cosh(dx + c)^4 * \sinh(dx + c) + 10 * ((a^5 + a^3 b^2) * d * f * x + (a^5 + a^3 b^2) * c * f) * \cosh(dx + c)^3 * \sinh(dx + c)^2 \\
& + 10 * ((a^5 + a^3 b^2) * d * f * x + (a^5 + a^3 b^2) * c * f) * \cosh(dx + c)^2 * \sinh(dx + c)^3 + 5 * ((a^5 + a^3 b^2) * d * f * x + (a^5 + a^3 b^2) * c * f) * \cosh(dx + c) * \sinh(dx + c)^4 \\
& + ((a^5 + a^3 b^2) * d * f * x + (a^5 + a^3 b^2) * c * f) * \sinh(dx + c)^5) * \log(-(a * \cosh(dx + c) + a * \sinh(dx + c) + (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2} - b/b) - 57600 * ((a^5 + a^3 b^2) * d * f * x + (a^5 + a^3 b^2) * c * f) * \cosh(dx + c)^5 + 5 * ((a^5 + a^3 b^2) * d * f * x + (a^5 + a^3 b^2) * c * f) * \cosh(dx + c)^4 * \sinh(dx + c) + 10 * ((a^5 + a^3 b^2) * d * f * x + (a^5 + a^3 b^2) * c * f) * \cosh(dx + c)^3 * \sinh(dx + c)^2 + 10 * ((a^5 + a^3 b^2) * d * f * x + (a^5 + a^3 b^2) * c * f) * \cosh(dx + c)^2 * \sinh(dx + c)^3 + 5 * ((a^5 + a^3 b^2) * d * f * x + (a^5 + a^3 b^2) * c * f) * \cosh(dx + c) * \sinh(dx + c)^4 + ((a^5 + a^3 b^2) * d * f * x + (a^5 + a^3 b^2) * c * f) * \sinh(dx + c)^5) * \log(-(a * \cosh(dx + c) + a * \sinh(dx + c) - (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2} - b/b) + 5 * (144 * (5 * b^5 * d * f * x + 5 * b^5 * d * e - b^5 * f) * \cosh(dx + c)^9 - 405 * (4 * a * b^4 * d * f * x + 4 * a * b^4 * d * e - a * b^4 * f) * \cosh(dx + c)^8 - 180 * a * b^4 * d * f * x + 320 * (3 * (4 * a^2 * b^3 + b^5) * d * f * x + 3 * (4 * a^2 * b^3 + b^5) * d * e - (4 * a^2 * b^3 + b^5) * f) * \cosh(dx + c)^7 - 180 * a * b^4 * d * e - 2520 * (2 * (2 * a^3 * b^2 + a * b^4) * d * f * x + 2 * (2 * a^3 * b^2 + a * b^4) * d * e - (2 * a^3 * b^2 + a * b^4) * f) * \cosh(dx + c)^6 - 45 * a * b^4 * f + 4320 * ((8 * a^4 * b + 6 * a^2 * b^3 - b^5) * d * f * x + (8 * a^4 * b + 6 * a^2 * b^3 - b^5) * d * e - (8 * a^4 * b + 6 * a^2 * b^3 - b^5) * f) * \cosh(dx + c)^5 + 28800 * ((a^5 + a^3 b^2) * d^2 * f * x^2 + 2 * (a^5 + a^3 b^2) * d^2 * e * x + 4 * (a^5 + a^3 b^2) * c * d * e - 2 * (a^5 + a^3 b^2) * c^2 * f) * \cosh(dx + c)^4 - 2880 * ((8 * a^4 * b + 6 * a^2 * b^3 - b^5) * d * f * x + (8 * a^4 * b + 6 * a^2 * b^3 - b^5) * d * e + (
\end{aligned}$$

$$8*a^4*b + 6*a^2*b^3 - b^5)*f)*\cosh(d*x + c)^3 - 1080*(2*(2*a^3*b^2 + a*b^4)*d*f*x + 2*(2*a^3*b^2 + a*b^4)*d*e + (2*a^3*b^2 + a*b^4)*f)*\cosh(d*x + c)^2 - 80*(3*(4*a^2*b^3 + b^5)*d*f*x + 3*(4*a^2*b^3 + b^5)*d*e + (4*a^2*b^3 + b^5)*f)*\cosh(d*x + c))*\sinh(d*x + c))/(b^6*d^2*\cosh(d*x + c)^5 + 5*b^6*d^2*\cosh(d*x + c)^4*\sinh(d*x + c) + 10*b^6*d^2*\cosh(d*x + c)^3*\sinh(d*x + c)^2 + 10*b^6*d^2*\cosh(d*x + c)^2*\sinh(d*x + c)^3 + 5*b^6*d^2*\cosh(d*x + c)*\sinh(d*x + c)^4 + b^6*d^2*\sinh(d*x + c)^5)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)\*\*3\*sinh(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \cosh(dx + c)^3 \sinh(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^3\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)\*cosh(d\*x + c)^3\*sinh(d\*x + c)^3/(b\*sinh(d\*x + c) + a), x)

$$3.404 \quad \int \frac{\cosh^3(c+dx) \sinh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=141

$$\frac{(a^2 + b^2) \sinh^3(c + dx)}{3b^3d} - \frac{a(a^2 + b^2) \sinh^2(c + dx)}{2b^4d} + \frac{a^2(a^2 + b^2) \sinh(c + dx)}{b^5d} - \frac{a^3(a^2 + b^2) \log(a + b \sinh(c + dx))}{b^6d}$$

[Out]  $-(a^3(a^2 + b^2) \text{Log}[a + b \text{Sinh}[c + d*x]])/(b^6*d) + (a^2(a^2 + b^2) \text{Sinh}[c + d*x])/(b^5*d) - (a(a^2 + b^2) \text{Sinh}[c + d*x]^2)/(2*b^4*d) + ((a^2 + b^2) \text{Sinh}[c + d*x]^3)/(3*b^3*d) - (a \text{Sinh}[c + d*x]^4)/(4*b^2*d) + \text{Sinh}[c + d*x]^5/(5*b*d)$

**Rubi [A]** time = 0.223435, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {2837, 12, 894}

$$\frac{(a^2 + b^2) \sinh^3(c + dx)}{3b^3d} - \frac{a(a^2 + b^2) \sinh^2(c + dx)}{2b^4d} + \frac{a^2(a^2 + b^2) \sinh(c + dx)}{b^5d} - \frac{a^3(a^2 + b^2) \log(a + b \sinh(c + dx))}{b^6d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Cosh}[c + d*x]^3 \text{Sinh}[c + d*x]^3)/(a + b \text{Sinh}[c + d*x]), x]$

[Out]  $-(a^3(a^2 + b^2) \text{Log}[a + b \text{Sinh}[c + d*x]])/(b^6*d) + (a^2(a^2 + b^2) \text{Sinh}[c + d*x])/(b^5*d) - (a(a^2 + b^2) \text{Sinh}[c + d*x]^2)/(2*b^4*d) + ((a^2 + b^2) \text{Sinh}[c + d*x]^3)/(3*b^3*d) - (a \text{Sinh}[c + d*x]^4)/(4*b^2*d) + \text{Sinh}[c + d*x]^5/(5*b*d)$

### Rule 2837

$\text{Int}[\cos[(e_.) + (f_.)*(x_.)]^{(p_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(m_.)})*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[1/(b^p*f), \text{Subst}[\text{Int}[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^{((p - 1)/2)}, x], x, b*\text{Sin}[e + f*x]], x] /;$  FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 894

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))^(n\_)\*((a\_.) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*(f + g\*x)^n\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rubi steps

$$\int \frac{\cosh^3(c + dx) \sinh^3(c + dx)}{a + b \sinh(c + dx)} dx = -\frac{\text{Subst}\left(\int \frac{x^3(-b^2-x^2)}{b^3(a+x)} dx, x, b \sinh(c + dx)\right)}{b^3 d}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^3(-b^2-x^2)}{a+x} dx, x, b \sinh(c + dx)\right)}{b^6 d}$$

$$= -\frac{\text{Subst}\left(\int \left(-a^2(a^2 + b^2) + a(a^2 + b^2)x - (a^2 + b^2)x^2 + ax^3 - x^4 + \frac{a^3(a^2 + b^2)}{a+x}\right) dx, x, b \sinh(c + dx)\right)}{b^6 d}$$

$$= -\frac{a^3(a^2 + b^2) \log(a + b \sinh(c + dx))}{b^6 d} + \frac{a^2(a^2 + b^2) \sinh(c + dx)}{b^5 d} - \frac{a(a^2 + b^2) \sinh^2(c + dx)}{2b^4 d}$$

**Mathematica [A]** time = 0.387743, size = 123, normalized size = 0.87

$$\frac{-\frac{20(a^2+b^2)\sinh^3(c+dx)}{b^3} + \frac{30a(a^2+b^2)\sinh^2(c+dx)}{b^4} - \frac{60a^2(a^2+b^2)\sinh(c+dx)}{b^5} + \frac{60a^3(a^2+b^2)\log(a+b\sinh(c+dx))}{b^6} + \frac{15a\sinh^4(c+dx)}{b^2} - \frac{12\sinh^5(c+dx)}{b}}{60d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cosh[c + d\*x]^3\*Sinh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] -((60\*a^3\*(a^2 + b^2)\*Log[a + b\*Sinh[c + d\*x]])/b^6 - (60\*a^2\*(a^2 + b^2)\*Sinh[c + d\*x])/b^5 + (30\*a\*(a^2 + b^2)\*Sinh[c + d\*x]^2)/b^4 - (20\*(a^2 + b^2)\*Sinh[c + d\*x]^3)/b^3 + (15\*a\*Sinh[c + d\*x]^4)/b^2 - (12\*Sinh[c + d\*x]^5)/b)/(60\*d)

**Maple [B]** time = 0.052, size = 804, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x)`

[Out]  $\frac{3}{8} \frac{d}{b} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1)^2} - \frac{3}{8} \frac{d}{b} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1)^2} - \frac{1}{d} \frac{a^5}{b^6} \ln(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)^2 a - 2 \tanh(\frac{1}{2}d*x+\frac{1}{2}c) * b - a) + \frac{1}{d} \frac{a^5}{b^6} \ln(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1) - \frac{1}{4} \frac{d}{b^2} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1)^4} a - \frac{1}{3} \frac{d}{b^3} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1)^3} a^2 - \frac{1}{2} \frac{d}{b^4} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1)^2} a^3 - \frac{1}{4} \frac{d}{b^2} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1)^4} a - \frac{1}{3} \frac{d}{b^3} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1)^3} a^2 - \frac{1}{2} \frac{d}{b^4} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1)^2} a^3 - \frac{1}{d} \frac{a^4}{b^5} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1)} - \frac{1}{d} \frac{a^4}{b^5} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1)} + \frac{1}{d} \frac{a^5}{b^6} \ln(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1) + \frac{1}{2} \frac{d}{b^3} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1)^2} a^2 + \frac{1}{2} \frac{d}{b^4} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1)} a^3 + \frac{1}{2} \frac{d}{b^2} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1)^3} a - \frac{1}{2} \frac{d}{b^2} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1)^3} a - \frac{1}{2} \frac{d}{b^3} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1)^2} a^2 - \frac{1}{2} \frac{d}{b^4} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1)} a^3 - \frac{1}{5} \frac{d}{b} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1)^5} - \frac{1}{5} \frac{d}{b} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1)^5} - \frac{1}{d} \frac{a^3}{b^4} \ln(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)^2 a - 2 \tanh(\frac{1}{2}d*x+\frac{1}{2}c) * b - a) - \frac{5}{8} \frac{d}{b^2} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1)^2} a - \frac{1}{d} \frac{a^2}{b^3} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1)} a^2 + \frac{1}{d} \frac{a^3}{b^4} \ln(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1) - \frac{5}{8} \frac{d}{b^2} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1)^2} a - \frac{1}{d} \frac{a^2}{b^3} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1)} a^2 + \frac{1}{d} \frac{a^3}{b^4} \ln(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1) + \frac{3}{8} \frac{d}{b^2} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1)} a - \frac{3}{8} \frac{d}{b^2} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1)} a - \frac{1}{2} \frac{d}{b} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1)^4} + \frac{1}{2} \frac{d}{b} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1)^4} - \frac{7}{12} \frac{d}{b} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1)^3} - \frac{7}{12} \frac{d}{b} \frac{1}{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1)^3}$

**Maxima [B]** time = 1.31775, size = 405, normalized size = 2.87

$$\frac{(15ab^3e^{(-dx-c)} - 6b^4 - 10(4a^2b^2 + b^4)e^{(-2dx-2c)} + 60(2a^3b + ab^3)e^{(-3dx-3c)} - 60(8a^4 + 6a^2b^2 - b^4)e^{(-4dx-4c)})e^{(5dx+5c)}}{960b^5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^3*sinh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out]  $-\frac{1}{960} * (15 * a * b^3 * e^{(-d*x - c)} - 6 * b^4 - 10 * (4 * a^2 * b^2 + b^4) * e^{(-2*d*x - 2*c)} + 60 * (2 * a^3 * b + a * b^3) * e^{(-3*d*x - 3*c)} - 60 * (8 * a^4 + 6 * a^2 * b^2 - b^4) * e^{(-4*d*x - 4*c)}) * e^{(5*d*x + 5*c)} / (b^5 * d) - (a^5 + a^3 * b^2) * (d*x + c) / (b^6 * d) - \frac{1}{960} * (15 * a * b^3 * e^{(-4*d*x - 4*c)} + 6 * b^4 * e^{(-5*d*x - 5*c)} + 60 * (8 * a^4 + 6 * a^2 * b^2 - b^4) * e^{(-d*x - c)} + 60 * (2 * a^3 * b + a * b^3) * e^{(-2*d*x - 2*c)} + 10 * (4 * a^2 * b^2 + b^4) * e^{(-3*d*x - 3*c)}) / (b^5 * d) - (a^5 + a^3 * b^2) * \log(-2 * a * e^{(-d*x - c)} + b * e^{(-2*d*x - 2*c)} - b) / (b^6 * d)$

**Fricas [B]** time = 2.58708, size = 4018, normalized size = 28.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^3\*sinh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 1/960*(6*b^5*\cosh(d*x + c)^{10} + 6*b^5*\sinh(d*x + c)^{10} - 15*a*b^4*\cosh(d*x + c)^9 + 15*(4*b^5*\cosh(d*x + c) - a*b^4)*\sinh(d*x + c)^9 + 10*(4*a^2*b^3 + b^5)*\cosh(d*x + c)^8 + 5*(54*b^5*\cosh(d*x + c)^2 - 27*a*b^4*\cosh(d*x + c) + 8*a^2*b^3 + 2*b^5)*\sinh(d*x + c)^8 + 960*(a^5 + a^3*b^2)*d*x*\cosh(d*x + c)^5 - 60*(2*a^3*b^2 + a*b^4)*\cosh(d*x + c)^7 + 20*(36*b^5*\cosh(d*x + c)^3 - 27*a*b^4*\cosh(d*x + c)^2 - 6*a^3*b^2 - 3*a*b^4 + 4*(4*a^2*b^3 + b^5)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 60*(8*a^4*b + 6*a^2*b^3 - b^5)*\cosh(d*x + c)^6 + 20*(63*b^5*\cosh(d*x + c)^4 - 63*a*b^4*\cosh(d*x + c)^3 + 24*a^4*b + 18*a^2*b^3 - 3*b^5 + 14*(4*a^2*b^3 + b^5)*\cosh(d*x + c)^2 - 21*(2*a^3*b^2 + a*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^6 - 15*a*b^4*\cosh(d*x + c) + 2*(756*b^5*\cosh(d*x + c)^5 - 945*a*b^4*\cosh(d*x + c)^4 + 280*(4*a^2*b^3 + b^5)*\cosh(d*x + c)^3 + 480*(a^5 + a^3*b^2)*d*x - 630*(2*a^3*b^2 + a*b^4)*\cosh(d*x + c)^2 + 180*(8*a^4*b + 6*a^2*b^3 - b^5)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 6*b^5 - 60*(8*a^4*b + 6*a^2*b^3 - b^5)*\cosh(d*x + c)^4 + 10*(126*b^5*\cosh(d*x + c)^6 - 189*a*b^4*\cosh(d*x + c)^5 - 48*a^4*b - 36*a^2*b^3 + 6*b^5 + 70*(4*a^2*b^3 + b^5)*\cosh(d*x + c)^4 + 480*(a^5 + a^3*b^2)*d*x*\cosh(d*x + c) - 210*(2*a^3*b^2 + a*b^4)*\cosh(d*x + c)^3 + 90*(8*a^4*b + 6*a^2*b^3 - b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 - 60*(2*a^3*b^2 + a*b^4)*\cosh(d*x + c)^3 + 20*(36*b^5*\cosh(d*x + c)^7 - 63*a*b^4*\cosh(d*x + c)^6 + 28*(4*a^2*b^3 + b^5)*\cosh(d*x + c)^5 - 6*a^3*b^2 - 3*a*b^4 + 480*(a^5 + a^3*b^2)*d*x*\cosh(d*x + c)^2 - 105*(2*a^3*b^2 + a*b^4)*\cosh(d*x + c)^4 + 60*(8*a^4*b + 6*a^2*b^3 - b^5)*\cosh(d*x + c)^3 - 12*(8*a^4*b + 6*a^2*b^3 - b^5)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 10*(4*a^2*b^3 + b^5)*\cosh(d*x + c)^2 + 10*(27*b^5*\cosh(d*x + c)^8 - 54*a*b^4*\cosh(d*x + c)^7 + 28*(4*a^2*b^3 + b^5)*\cosh(d*x + c)^6 + 960*(a^5 + a^3*b^2)*d*x*\cosh(d*x + c)^3 - 126*(2*a^3*b^2 + a*b^4)*\cosh(d*x + c)^5 - 4*a^2*b^3 - b^5 + 90*(8*a^4*b + 6*a^2*b^3 - b^5)*\cosh(d*x + c)^4 - 36*(8*a^4*b + 6*a^2*b^3 - b^5)*\cosh(d*x + c)^2 - 18*(2*a^3*b^2 + a*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^2 - 960*((a^5 + a^3*b^2)*\cosh(d*x + c)^5 + 5*(a^5 + a^3*b^2)*\cosh(d*x + c)^4*\sinh(d*x + c) + 10*(a^5 + a^3*b^2)*\cosh(d*x + c)^3*\sinh(d*x + c)^2 + 10*(a^5 + a^3*b^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^3 + 5*(a^5 + a^3*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^4 + (a^5 + a^3*b^2)*\sinh(d*x + c)^5)*\log(2*(b*\sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 5*(12*b^5*\cosh(d*x + c)^9 - 27*a*b^4*\cosh(d*x + c)^8 + 16*(4*a^2*b^3 + b^5)*\cosh(d*x + c)^7 + 960*(a^5 + a^3*b^2)*d*x*\cosh(d*x + c)^4 - 84*(2*a^3*b^2 + a*b^4)*\cosh(d*x + c)^6 + 72*(8*a^4*b + 6*a^2*b^3 - b^5)*\cosh(d*x + c)^5 - 3*a*b^4 - 48*($$

$$8a^4b + 6a^2b^3 - b^5) \cosh(dx + c)^3 - 36(2a^3b^2 + ab^4) \cosh(dx + c)^2 - 4(4a^2b^3 + b^5) \cosh(dx + c) \sinh(dx + c) / (b^6 d \cosh(dx + c)^5 + 5b^6 d \cosh(dx + c)^4 \sinh(dx + c) + 10b^6 d \cosh(dx + c)^3 \sinh(dx + c)^2 + 10b^6 d \cosh(dx + c)^2 \sinh(dx + c)^3 + 5b^6 d \cosh(dx + c) \sinh(dx + c)^4 + b^6 d \sinh(dx + c)^5)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)\*\*3\*sinh(dx+c)\*\*3/(a+b\*sinh(dx+c)),x)

[Out] Timed out

**Giac [B]** time = 1.19815, size = 383, normalized size = 2.72

$$-\frac{(a^5 + a^3b^2) \log\left(\left|b(e^{dx+c}) - e^{-dx-c}\right| + 2a\right)}{b^6d} + \frac{6b^4d^4(e^{dx+c})^5 - 15ab^3d^4(e^{dx+c})^4 + 40a^2b^2d^4(e^{dx+c})^3 - 15a^2b^3d^4(e^{-dx-c})^5 + 15ab^3d^4(e^{-dx-c})^4 - 40a^2b^2d^4(e^{-dx-c})^3 + 120a^3b^2d^4(e^{-dx-c})^2 - 120a^2b^3d^4(e^{-dx-c})^2 + 480a^4d^4(e^{dx+c}) - 480a^2b^2d^4(e^{-dx-c}))}{b^5d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)^3\*sinh(dx+c)^3/(a+b\*sinh(dx+c)),x, algorithm="giac")

[Out]  $-(a^5 + a^3b^2) \log(\text{abs}(b(e^{dx+c}) - e^{-dx-c}) + 2a)) / (b^6d) + 1/960(6b^4d^4(e^{dx+c}) - e^{-dx-c})^5 - 15a^2b^3d^4(e^{dx+c}) - e^{-dx-c})^4 + 40a^2b^2d^4(e^{dx+c}) - e^{-dx-c})^3 + 40b^4d^4(e^{dx+c}) - e^{-dx-c})^3 - 120a^3b^2d^4(e^{dx+c}) - e^{-dx-c})^2 - 120a^2b^3d^4(e^{dx+c}) - e^{-dx-c})^2 + 480a^4d^4(e^{dx+c}) - e^{-dx-c}) + 480a^2b^2d^4(e^{dx+c}) - e^{-dx-c})) / (b^5d^5)$



$$3.405 \quad \int \frac{\cosh^3(c+dx) \sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=38

$$\text{Unintegrable} \left( \frac{\sinh^3(c+dx) \cosh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[(Cosh[c + d\*x]^3\*Sinh[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.126685, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cosh^3(c+dx) \sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Cosh[c + d\*x]^3\*Sinh[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] Defer[Int] [(Cosh[c + d\*x]^3\*Sinh[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\cosh^3(c+dx) \sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\cosh^3(c+dx) \sinh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.002, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Cosh[c + d\*x]^3\*Sinh[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] \$Aborted

**Maple [A]** time = 0.223, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx+c))^3 (\sinh(dx+c))^3}{(fx+e)(a+b \sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)^3\*sinh(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(cosh(d\*x+c)^3\*sinh(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^3\*sinh(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-1/32e^{(-5*c + 5*d*e/f)} \exp\_integral\_e(1, 5*(f*x + e)*d/f)/(b*f) - 1/16*a*e^{(-4*c + 4*d*e/f)} \exp\_integral\_e(1, 4*(f*x + e)*d/f)/(b^2*f) + 1/16*a*e^{(4*c - 4*d*e/f)} \exp\_integral\_e(1, -4*(f*x + e)*d/f)/(b^2*f) - 1/32e^{(5*c - 5*d*e/f)} \exp\_integral\_e(1, -5*(f*x + e)*d/f)/(b*f) - 1/32*(4*a^2 + b^2)*e^{(-3*c + 3*d*e/f)} \exp\_integral\_e(1, 3*(f*x + e)*d/f)/(b^3*f) - 1/32*(4*a^2*e^{(3*c)} + b^2*e^{(3*c)})*e^{(-3*d*e/f)} \exp\_integral\_e(1, -3*(f*x + e)*d/f)/(b^3*f) - 1/8*(2*a^3 + a*b^2)*e^{(-2*c + 2*d*e/f)} \exp\_integral\_e(1, 2*(f*x + e)*d/f)/(b^4*f) + 1/8*(2*a^3*e^{(2*c)} + a*b^2*e^{(2*c)})*e^{(-2*d*e/f)} \exp\_integral\_e(1, -2*(f*x + e)*d/f)/(b^4*f) - 1/16*(8*a^4 + 6*a^2*b^2 - b^4)*e^{(-c + d*e/f)} \exp\_integral\_e(1, (f*x + e)*d/f)/(b^5*f) - 1/16*(8*a^4*e^c + 6*a^2*b^2*e^c - b^4*e^c)*e^{(-d*e/f)} \exp\_integral\_e(1, -(f*x + e)*d/f)/(b^5*f) - (a^5 + a^3*b^2)*log(f*x + e)/(b^6*f) + 1/64*integrate(128*(a^5*b + a^3*b^3 - (a^6*e^c + a^4*b^2*e^c)*e^{(d*x)})/(b^7*f*x + b^7*e - (b^7*f*x*e^{(2*c)} + b^7*e*e^{(2*c)}))*e^{(2*d*x)} - 2*(a*b^6*f*x*e^c + a*b^6*e*e^c)*e^{(d*x)}, x)$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx+c)^3 \sinh(dx+c)^3}{afx+ae+(bfx+be)\sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^3\*sinh(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] integral(cosh(d\*x + c)^3\*sinh(d\*x + c)^3/(a\*f\*x + a\*e + (b\*f\*x + b\*e)\*sinh(d\*x + c)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*\*3\*sinh(d\*x+c)\*\*3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^3 \sinh(dx+c)^3}{(fx+e)(b\sinh(dx+c)+a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^3\*sinh(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate(cosh(d\*x + c)^3\*sinh(d\*x + c)^3/((f\*x + e)\*(b\*sinh(d\*x + c) + a)), x)

$$3.406 \quad \int \frac{(e+fx)^3 \sinh^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=1519

result too large to display

```
[Out] (a*(e + f*x)^4)/(4*b^2*f) + (2*a^2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b^3*d)
- (2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b*d) - (2*a^4*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b^3*(a^2 + b^2)*d) - (6*f^3*Cosh[c + d*x])/(b*d^4) - (3*f*(e + f*x)^2*Cosh[c + d*x])/(b*d^2) - (a^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) - (a^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) - (a*(e + f*x)^3*Log[1 + E^(2*(c + d*x))])/(b^2*d) + (a^3*(e + f*x)^3*Log[1 + E^(2*(c + d*x))])/(b^2*(a^2 + b^2)*d) - ((3*I)*a^2*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*d^2) + ((3*I)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + ((3*I)*a^4*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) + ((3*I)*a^2*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b^3*d^2) - ((3*I)*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - ((3*I)*a^4*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) - (3*a^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^2) - (3*a^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^2) - (3*a*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*b^2*d^2) + (3*a^3*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*b^2*(a^2 + b^2)*d^2) + ((6*I)*a^2*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b^3*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((6*I)*a^4*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) - ((6*I)*a^2*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(b^3*d^3) + ((6*I)*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(b*d^3) + ((6*I)*a^4*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) + (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^3) + (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^3) + (3*a*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*d^3) - (3*a^3*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*(a^2 + b^2)*d^3) - ((6*I)*a^2*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(b^3*d^4) + ((6*I)*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(b*d^4) + ((6*I)*a^4*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^4) + ((6*I)*a^2*f^3*PolyLog[4, I*E^(c + d*x)])/(b^3*d^4) - ((6*I)*f^3*PolyLog[4, I*E^(c + d*x)])/(b*d^4) - ((6*I)*a^4*f^3*PolyLog[4, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^4) - (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^4) - (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^4) - (3*a*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*b^2*d^4) + (3*a^3*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*b^2*(a^2 + b^2)*d^4) + (6*f^2*(e + f*x)*Sinh[c + d*x])/(b*d^3) + ((e + f*x)^3*Sinh[c + d*x])/(b*d)
```

**Rubi [A]** time = 2.15352, antiderivative size = 1519, normalized size of antiderivative = 1., number of steps used = 61, number of rules used = 15, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.441$ , Rules used = {5581, 5449, 3296, 2638, 4180, 2531, 6609, 2282, 6589, 3718, 2190, 5567, 5573, 5561, 6742}

result too large to display

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Sinh[c + d\*x]^2\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out]  $(a*(e + f*x)^4)/(4*b^2*f) + (2*a^2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b^3*d) - (2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b*d) - (2*a^4*(e + f*x)^3*ArcTan[E^(c + d*x)])/(b^3*(a^2 + b^2)*d) - (6*f^3*Cosh[c + d*x])/(b*d^4) - (3*f*(e + f*x)^2*Cosh[c + d*x])/(b*d^2) - (a^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) - (a^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) - (a*(e + f*x)^3*Log[1 + E^(2*(c + d*x))])/(b^2*d) + (a^3*(e + f*x)^3*Log[1 + E^(2*(c + d*x))])/(b^2*(a^2 + b^2)*d) - ((3*I)*a^2*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*d^2) + ((3*I)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + ((3*I)*a^4*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) + ((3*I)*a^2*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b^3*d^2) - ((3*I)*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - ((3*I)*a^4*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) - (3*a^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^2) - (3*a^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^2) - (3*a*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*b^2*d^2) + (3*a^3*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*b^2*(a^2 + b^2)*d^2) + ((6*I)*a^2*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b^3*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((6*I)*a^4*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) - ((6*I)*a^2*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(b^3*d^3) + ((6*I)*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(b*d^3) + ((6*I)*a^4*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) + (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^3) + (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b^2*(a^2 + b^2)*d^3) + (3*a*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*d^3) - (3*a^3*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*(a^2 + b^2)*d^3) - ((6*I)*a^2*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(b^3*d^4) + ((6*I)*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(b*d^4) + ((6*I)*a^4*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^4) + ((6*I)*a^2*f^3*PolyLog[4, I*E^(c + d*x)])/(b^3*d^4) - ((6*I)*f^3*PolyLog[4, I*E^(c + d*x)])/(b*d^4) - ((6*I)*a^4*f^3*PolyLog[4, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^4) - (6*a^3*f^3*PolyLog[4,$

$$-\left(\frac{b e^{c+dx}}{a - \sqrt{a^2 + b^2}}\right) / (b^2(a^2 + b^2)d^4) - (6a^3 f^3 \text{PolyLog}[4, -\left(\frac{b e^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)]) / (b^2(a^2 + b^2)d^4) - (3a^3 f^3 \text{PolyLog}[4, -E^{2(c+dx)}]) / (4b^2 d^4) + (3a^3 f^3 \text{PolyLog}[4, -E^{2(c+dx)}]) / (4b^2(a^2 + b^2)d^4) + (6f^2(e + fx) \text{Sinh}[c + dx]) / (b d^3) + ((e + fx)^3 \text{Sinh}[c + dx]) / (b d)$$
Rule 5581

```
Int[(((e_.) + (f_.)*(x_.))^(m_.)*Sinh[(c_.) + (d_.)*(x_.)]^(p_.)*Tanh[(c_.) + (d_.)*(x_.)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Dist[a/b, Int[(((e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n)/(a + b*Sinh[c + d*x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5449

```
Int[(((c_.) + (d_.)*(x_.))^(m_.)*Sinh[(a_.) + (b_.)*(x_.)]^(n_.)*Tanh[(a_.) + (b_.)*(x_.)]^(p_.), x_Symbol] := Int[(c + d*x)^m*Sinh[a + b*x]^n*Tanh[a + b*x]^(p - 2), x] - Int[(c + d*x)^m*Sinh[a + b*x]^(n - 2)*Tanh[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[(((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := -Simp[((c + d*x)^m * Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1) * Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_.)]*(((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m * ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1) * Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1) * Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_)^(m_.))*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 3718

```
Int[((c_.) + (d_.)*(x_)^(m_.))*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_)^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 5567

```
Int[(((e_.) + (f_.)*(x_.))^(m_.)*Tanh[(c_.) + (d_.)*(x_.)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

### Rule 5573

```
Int[(((e_.) + (f_.)*(x_.))^(m_.)*Sech[(c_.) + (d_.)*(x_.)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(e+fx)^3 \sinh^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \sinh(c+dx) \tanh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^3 \sinh(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= -\frac{a \int (e+fx)^3 \tanh(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)^3 \tanh(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} + \frac{\int (e+fx)^3 \sinh^2(c+dx) dx}{b} \\
&= \frac{a(e+fx)^4}{4b^2 f} - \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} + \frac{(e+fx)^3 \sinh(c+dx)}{bd} + \frac{a^2 \int (e+fx)^3 \tanh(c+dx) dx}{b^2} \\
&= \frac{a(e+fx)^4}{4b^2 f} + \frac{2a^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^3 d} - \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{3a^2 \int (e+fx)^3 \tanh(c+dx) dx}{b^2} \\
&= \frac{a(e+fx)^4}{4b^2 f} + \frac{a^3(e+fx)^4}{4b^2(a^2+b^2)f} + \frac{2a^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^3 d} - \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{3a^2 \int (e+fx)^3 \tanh(c+dx) dx}{b^2} \\
&= \frac{a(e+fx)^4}{4b^2 f} + \frac{a^3(e+fx)^4}{4b^2(a^2+b^2)f} + \frac{2a^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^3 d} - \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{3a^2 \int (e+fx)^3 \tanh(c+dx) dx}{b^2} \\
&= \frac{a(e+fx)^4}{4b^2 f} + \frac{2a^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^3 d} - \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2 \int (e+fx)^3 \tanh(c+dx) dx}{b^2} \\
&= \frac{a(e+fx)^4}{4b^2 f} + \frac{2a^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^3 d} - \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2 \int (e+fx)^3 \tanh(c+dx) dx}{b^2} \\
&= \frac{a(e+fx)^4}{4b^2 f} + \frac{2a^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^3 d} - \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2 \int (e+fx)^3 \tanh(c+dx) dx}{b^2} \\
&= \frac{a(e+fx)^4}{4b^2 f} + \frac{2a^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^3 d} - \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2 \int (e+fx)^3 \tanh(c+dx) dx}{b^2} \\
&= \frac{a(e+fx)^4}{4b^2 f} + \frac{2a^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^3 d} - \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2 \int (e+fx)^3 \tanh(c+dx) dx}{b^2} \\
&= \frac{a(e+fx)^4}{4b^2 f} + \frac{2a^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{b^3 d} - \frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^2 \int (e+fx)^3 \tanh(c+dx) dx}{b^2}
\end{aligned}$$

**Mathematica [A]** time = 26.2135, size = 2861, normalized size = 1.88

Result too large to show



$$\begin{aligned}
& bE^{(2c+dx)} / (aE^c + \sqrt{(a^2+b^2)E^{(2c)}}) / d - (2E^{(2c)}f^3x^3 \text{Log}[1 + (bE^{(2c+dx)}) / (aE^c + \sqrt{(a^2+b^2)E^{(2c)}})] / d - (6(-1 + E^{(2c)})f(e+fx)^2 \text{PolyLog}[2, -((bE^{(2c+dx)}) / (aE^c - \sqrt{(a^2+b^2)E^{(2c)}}))] / d^2 - (6(-1 + E^{(2c)})f(e+fx)^2 \text{PolyLog}[2, -((bE^{(2c+dx)}) / (aE^c + \sqrt{(a^2+b^2)E^{(2c)}}))] / d^2 - (12ef^2 \text{PolyLog}[3, -((bE^{(2c+dx)}) / (aE^c - \sqrt{(a^2+b^2)E^{(2c)}}))] / d^3 + (12eE^{(2c)}f^2 \text{PolyLog}[3, -((bE^{(2c+dx)}) / (aE^c - \sqrt{(a^2+b^2)E^{(2c)}}))] / d^3 - (12f^3x \text{PolyLog}[3, -((bE^{(2c+dx)}) / (aE^c - \sqrt{(a^2+b^2)E^{(2c)}}))] / d^3 + (12E^{(2c)}f^3x \text{PolyLog}[3, -((bE^{(2c+dx)}) / (aE^c - \sqrt{(a^2+b^2)E^{(2c)}}))] / d^3 - (12ef^2 \text{PolyLog}[3, -((bE^{(2c+dx)}) / (aE^c + \sqrt{(a^2+b^2)E^{(2c)}}))] / d^3 + (12eE^{(2c)}f^2 \text{PolyLog}[3, -((bE^{(2c+dx)}) / (aE^c + \sqrt{(a^2+b^2)E^{(2c)}}))] / d^3 - (12f^3x \text{PolyLog}[3, -((bE^{(2c+dx)}) / (aE^c + \sqrt{(a^2+b^2)E^{(2c)}}))] / d^3 + (12E^{(2c)}f^3x \text{PolyLog}[3, -((bE^{(2c+dx)}) / (aE^c + \sqrt{(a^2+b^2)E^{(2c)}}))] / d^3 + (12f^3 \text{PolyLog}[4, -((bE^{(2c+dx)}) / (aE^c - \sqrt{(a^2+b^2)E^{(2c)}}))] / d^4 - (12E^{(2c)}f^3 \text{PolyLog}[4, -((bE^{(2c+dx)}) / (aE^c - \sqrt{(a^2+b^2)E^{(2c)}}))] / d^4 + (12f^3 \text{PolyLog}[4, -((bE^{(2c+dx)}) / (aE^c + \sqrt{(a^2+b^2)E^{(2c)}}))] / d^4 - (12E^{(2c)}f^3 \text{PolyLog}[4, -((bE^{(2c+dx)}) / (aE^c + \sqrt{(a^2+b^2)E^{(2c)}}))] / d^4) / (2b^2(a^2+b^2)(-1+E^{(2c)})) - (ae^3x(a^2-b^2+(a^2+b^2)\text{Cosh}[2c])\text{Csch}[c]\text{Sech}[c]) / (2b^2(a^2+b^2)) - (3ae^2fx^2(a^2-b^2+(a^2+b^2)\text{Cosh}[2c])\text{Csch}[c]\text{Sech}[c]) / (4b^2(a^2+b^2)) - (aef^2x^3(a^2-b^2+(a^2+b^2)\text{Cosh}[2c])\text{Csch}[c]\text{Sech}[c]) / (2b^2(a^2+b^2)) - (af^3x^4(a^2-b^2+(a^2+b^2)\text{Cosh}[2c])\text{Csch}[c]\text{Sech}[c]) / (8b^2(a^2+b^2)) + ((6f^3+6df^2(e+fx)+3d^2f(e+fx)^2+d^3(e+fx)^3)(-\text{Cosh}[c+dx]+\text{Sinh}[c+dx])) / (2bd^4) + ((-6f^3+6df^2(e+fx)-3d^2f(e+fx)^2+d^3(e+fx)^3)(\text{Cosh}[c+dx]+\text{Sinh}[c+dx])) / (2bd^4)
\end{aligned}$$

**Maple [F]** time = 1.115, size = 0, normalized size = 0.

$$\int \frac{(fx+e)^3 (\sinh(dx+c))^2 \tanh(dx+c)}{a+b \sinh(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*sinh(d\*x+c)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*sinh(d\*x+c)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b^2 + b^4)*d) - 4*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + 2*a*log(e^(-2*d*x - 2*c) + 1)/((a^2 + b^2)*d) + 2*(d*x + c)*a/(b^2*d) - e^(d*x + c)/(b*d) + e^(-d*x - c)/(b*d))*e^3 - 1/4*(a*d^4*f^3*x^4*e^c + 4*a*d^4*e*f^2*x^3*e^c + 6*a*d^4*e^2*f*x^2*e^c - 2*(b*d^3*f^3*x^3*e^(2*c) + 3*(d^3*e*f^2 - d^2*f^3)*b*x^2*e^(2*c) + 3*(d^3*e^2*f - 2*d^2*e*f^2 + 2*d*f^3)*b*x*e^(2*c) - 3*(d^2*e^2*f - 2*d*e*f^2 + 2*f^3)*b*e^(2*c))*e^(d*x) + 2*(b*d^3*f^3*x^3 + 3*(d^3*e*f^2 + d^2*f^3)*b*x^2 + 3*(d^3*e^2*f + 2*d^2*e*f^2 + 2*d*f^3)*b*x + 3*(d^2*e^2*f + 2*d*e*f^2 + 2*f^3)*b)*e^(-d*x))*e^(-c)/(b^2*d^4) + integrate(2*(a^3*b*f^3*x^3 + 3*a^3*b*e*f^2*x^2 + 3*a^3*b*e^2*f*x - (a^4*f^3*x^3*e^c + 3*a^4*e*f^2*x^2*e^c + 3*a^4*e^2*f*x*e^c)*e^(d*x))/(a^2*b^3 + b^5 - (a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^3*b^2*e^c + a*b^4*e^c)*e^(d*x)), x) - integrate((-2*(a*f^3*x^3 + 3*a*e*f^2*x^2 + 3*a*e^2*f*x - (b*f^3*x^3*e^c + 3*b*e*f^2*x^2*e^c + 3*b*e^2*f*x*e^c)*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

**Fricas [C]** time = 4.30697, size = 10368, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(a^2*b + b^3)*d^3*f^3*x^3 + 2*(a^2*b + b^3)*d^3*e^3 + 6*(a^2*b + b^3)*d^2*e^2*f + 12*(a^2*b + b^3)*d*e*f^2 + 12*(a^2*b + b^3)*f^3 + 6*((a^2*b + b^3)*d^3*e*f^2 + (a^2*b + b^3)*d^2*f^3)*x^2 - 2*((a^2*b + b^3)*d^3*f^3*x^3 + (a^2*b + b^3)*d^3*e^3 - 3*(a^2*b + b^3)*d^2*e^2*f + 6*(a^2*b + b^3)*d*e*f^2 - 6*(a^2*b + b^3)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^2*b + b^3)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^2*b + b^3)*d^2*e*f^2 + 2*(a^2*b + b^3)*d*f^3)*x)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^3*f^3*x^3 + (a^2*b + b^3)*d^3*e^3 - 3*(a^2*b + b^3)*d^2*e^2*f + 6*(a^2*b + b^3)*d*e*f^2 - 6*(
```

$$\begin{aligned}
& a^2b + b^3)f^3 + 3*((a^2b + b^3)*d^3e^2f^2 - (a^2b + b^3)*d^2f^3)*x^2 \\
& + 3*((a^2b + b^3)*d^3e^2f - 2*(a^2b + b^3)*d^2e^2f^2 + 2*(a^2b + b^3)* \\
& d^2f^3)*x)*\sinh(dx + c)^2 + 6*((a^2b + b^3)*d^3e^2f + 2*(a^2b + b^3)*d^ \\
& 2e^2f^2 + 2*(a^2b + b^3)*d^2f^3)*x - ((a^3 + a*b^2)*d^4f^3*x^4 + 4*(a^3 + \\
& a*b^2)*d^4e^2f^2*x^3 + 6*(a^3 + a*b^2)*d^4e^2f*x^2 + 4*(a^3 + a*b^2)*d^4e \\
& ^3*x + 8*(a^3 + a*b^2)*c*d^3e^3 - 12*(a^3 + a*b^2)*c^2*d^2e^2f + 8*(a^3 \\
& + a*b^2)*c^3*d^2e^2f - 2*(a^3 + a*b^2)*c^4f^3)*\cosh(dx + c) + 12*((a^3*d \\
& ^2f^3*x^2 + 2*a^3*d^2e^2f^2*x + a^3*d^2e^2f)*\cosh(dx + c) + (a^3*d^2f^ \\
& 3*x^2 + 2*a^3*d^2e^2f^2*x + a^3*d^2e^2f)*\sinh(dx + c))*\operatorname{dilog}((a*\cosh(dx \\
& + c) + a*\sinh(dx + c) + (b*\cosh(dx + c) + b*\sinh(dx + c))*\sqrt{(a^2 + b \\
& ^2)/b^2} - b)/b + 1) + 12*((a^3*d^2f^3*x^2 + 2*a^3*d^2e^2f^2*x + a^3*d^2e \\
& ^2f)*\cosh(dx + c) + (a^3*d^2f^3*x^2 + 2*a^3*d^2e^2f^2*x + a^3*d^2e^2f) \\
& *\sinh(dx + c))*\operatorname{dilog}((a*\cosh(dx + c) + a*\sinh(dx + c) - (b*\cosh(dx + c) \\
& + b*\sinh(dx + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + ((12*a*b^2*d^2f^3* \\
& x^2 + 12*I*b^3*d^2f^3*x^2 + 24*a*b^2*d^2e^2f^2*x + 24*I*b^3*d^2e^2f^2*x + \\
& 12*a*b^2*d^2e^2f + 12*I*b^3*d^2e^2f)*\cosh(dx + c) + (12*a*b^2*d^2f^3* \\
& x^2 + 12*I*b^3*d^2f^3*x^2 + 24*a*b^2*d^2e^2f^2*x + 24*I*b^3*d^2e^2f^2*x + \\
& 12*a*b^2*d^2e^2f + 12*I*b^3*d^2e^2f)*\sinh(dx + c))*\operatorname{dilog}(I*\cosh(dx + \\
& c) + I*\sinh(dx + c)) + ((12*a*b^2*d^2f^3*x^2 - 12*I*b^3*d^2f^3*x^2 + 24* \\
& a*b^2*d^2e^2f^2*x - 24*I*b^3*d^2e^2f^2*x + 12*a*b^2*d^2e^2f - 12*I*b^3*d^ \\
& 2e^2f)*\cosh(dx + c) + (12*a*b^2*d^2f^3*x^2 - 12*I*b^3*d^2f^3*x^2 + 24* \\
& a*b^2*d^2e^2f^2*x - 24*I*b^3*d^2e^2f^2*x + 12*a*b^2*d^2e^2f - 12*I*b^3*d^ \\
& 2e^2f)*\sinh(dx + c))*\operatorname{dilog}(-I*\cosh(dx + c) - I*\sinh(dx + c)) + 4*((a^3 \\
& *d^3e^3 - 3*a^3*c*d^2e^2f + 3*a^3*c^2*d^2e^2f - a^3*c^3*f^3)*\cosh(dx + \\
& c) + (a^3*d^3e^3 - 3*a^3*c*d^2e^2f + 3*a^3*c^2*d^2e^2f - a^3*c^3*f^3)*\si \\
& nh(dx + c))*\log(2*b*\cosh(dx + c) + 2*b*\sinh(dx + c) + 2*b*\sqrt{(a^2 + b^ \\
& 2)/b^2} + 2*a) + 4*((a^3*d^3e^3 - 3*a^3*c*d^2e^2f + 3*a^3*c^2*d^2e^2f - \\
& a^3*c^3*f^3)*\cosh(dx + c) + (a^3*d^3e^3 - 3*a^3*c*d^2e^2f + 3*a^3*c^2*d \\
& *e^2f - a^3*c^3*f^3)*\sinh(dx + c))*\log(2*b*\cosh(dx + c) + 2*b*\sinh(dx + \\
& c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 4*((a^3*d^3f^3*x^3 + 3*a^3*d^3e^2 \\
& f^2*x^2 + 3*a^3*d^3e^2f*x + 3*a^3*c*d^2e^2f - 3*a^3*c^2*d^2e^2f + a^3*c^ \\
& ^3*f^3)*\cosh(dx + c) + (a^3*d^3f^3*x^3 + 3*a^3*d^3e^2f^2*x^2 + 3*a^3*d^3e^ \\
& 2f*x + 3*a^3*c*d^2e^2f - 3*a^3*c^2*d^2e^2f + a^3*c^3*f^3)*\sinh(dx + c \\
& ))*\log(-(a*\cosh(dx + c) + a*\sinh(dx + c) + (b*\cosh(dx + c) + b*\sinh(dx \\
& + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + 4*((a^3*d^3f^3*x^3 + 3*a^3*d^3e^2f^2 \\
& *x^2 + 3*a^3*d^3e^2f*x + 3*a^3*c*d^2e^2f - 3*a^3*c^2*d^2e^2f + a^3*c^3* \\
& f^3)*\cosh(dx + c) + (a^3*d^3f^3*x^3 + 3*a^3*d^3e^2f^2*x^2 + 3*a^3*d^3e^2 \\
& *f*x + 3*a^3*c*d^2e^2f - 3*a^3*c^2*d^2e^2f + a^3*c^3*f^3)*\sinh(dx + c))* \\
& \log(-(a*\cosh(dx + c) + a*\sinh(dx + c) - (b*\cosh(dx + c) + b*\sinh(dx + c \\
& ))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + ((4*a*b^2*d^3e^3 + 4*I*b^3*d^3e^3 - 12 \\
& *a*b^2*c*d^2e^2f - 12*I*b^3*c*d^2e^2f + 12*a*b^2*c^2*d^2e^2f + 12*I*b^3 \\
& *c^2*d^2e^2f - 4*a*b^2*c^3*f^3 - 4*I*b^3*c^3*f^3)*\cosh(dx + c) + (4*a*b^2* \\
& d^3e^3 + 4*I*b^3*d^3e^3 - 12*a*b^2*c*d^2e^2f - 12*I*b^3*c*d^2e^2f + 1 \\
& 2*a*b^2*c^2*d^2e^2f + 12*I*b^3*c^2*d^2e^2f - 4*a*b^2*c^3*f^3 - 4*I*b^3*c^3* \\
& f^3)*\sinh(dx + c))*\log(\cosh(dx + c) + \sinh(dx + c) + I) + ((4*a*b^2*d^3*
\end{aligned}$$

$$\begin{aligned}
& e^3 - 4*I*b^3*d^3*e^3 - 12*a*b^2*c*d^2*e^2*f + 12*I*b^3*c*d^2*e^2*f + 12*a* \\
& b^2*c^2*d*e*f^2 - 12*I*b^3*c^2*d*e*f^2 - 4*a*b^2*c^3*f^3 + 4*I*b^3*c^3*f^3) \\
& *cosh(d*x + c) + (4*a*b^2*d^3*e^3 - 4*I*b^3*d^3*e^3 - 12*a*b^2*c*d^2*e^2*f \\
& + 12*I*b^3*c*d^2*e^2*f + 12*a*b^2*c^2*d*e*f^2 - 12*I*b^3*c^2*d*e*f^2 - 4*a* \\
& b^2*c^3*f^3 + 4*I*b^3*c^3*f^3)*sinh(d*x + c)) *log(cosh(d*x + c) + sinh(d*x \\
& + c) - I) + ((4*a*b^2*d^3*f^3*x^3 - 4*I*b^3*d^3*f^3*x^3 + 12*a*b^2*d^3*e*f^ \\
& 2*x^2 - 12*I*b^3*d^3*e*f^2*x^2 + 12*a*b^2*d^3*e^2*f*x - 12*I*b^3*d^3*e^2*f* \\
& x + 12*a*b^2*c*d^2*e^2*f - 12*I*b^3*c*d^2*e^2*f - 12*a*b^2*c^2*d*e*f^2 + 12 \\
& *I*b^3*c^2*d*e*f^2 + 4*a*b^2*c^3*f^3 - 4*I*b^3*c^3*f^3)*cosh(d*x + c) + (4* \\
& a*b^2*d^3*f^3*x^3 - 4*I*b^3*d^3*f^3*x^3 + 12*a*b^2*d^3*e*f^2*x^2 - 12*I*b^3 \\
& *d^3*e*f^2*x^2 + 12*a*b^2*d^3*e^2*f*x - 12*I*b^3*d^3*e^2*f*x + 12*a*b^2*c*d \\
& ^2*e^2*f - 12*I*b^3*c*d^2*e^2*f - 12*a*b^2*c^2*d*e*f^2 + 12*I*b^3*c^2*d*e*f \\
& ^2 + 4*a*b^2*c^3*f^3 - 4*I*b^3*c^3*f^3)*sinh(d*x + c)) *log(I*cosh(d*x + c) \\
& + I*sinh(d*x + c) + 1) + ((4*a*b^2*d^3*f^3*x^3 + 4*I*b^3*d^3*f^3*x^3 + 12*a \\
& *b^2*d^3*e*f^2*x^2 + 12*I*b^3*d^3*e*f^2*x^2 + 12*a*b^2*d^3*e^2*f*x + 12*I*b \\
& ^3*d^3*e^2*f*x + 12*a*b^2*c*d^2*e^2*f + 12*I*b^3*c*d^2*e^2*f - 12*a*b^2*c^2 \\
& *d*e*f^2 - 12*I*b^3*c^2*d*e*f^2 + 4*a*b^2*c^3*f^3 + 4*I*b^3*c^3*f^3)*cosh(d \\
& *x + c) + (4*a*b^2*d^3*f^3*x^3 + 4*I*b^3*d^3*f^3*x^3 + 12*a*b^2*d^3*e*f^2*x \\
& ^2 + 12*I*b^3*d^3*e*f^2*x^2 + 12*a*b^2*d^3*e^2*f*x + 12*I*b^3*d^3*e^2*f*x + \\
& 12*a*b^2*c*d^2*e^2*f + 12*I*b^3*c*d^2*e^2*f - 12*a*b^2*c^2*d*e*f^2 - 12*I* \\
& b^3*c^2*d*e*f^2 + 4*a*b^2*c^3*f^3 + 4*I*b^3*c^3*f^3)*sinh(d*x + c)) *log(-I* \\
& cosh(d*x + c) - I*sinh(d*x + c) + 1) + 24*(a^3*f^3*cosh(d*x + c) + a^3*f^3* \\
& sinh(d*x + c)) *polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x \\
& + c) + b*sinh(d*x + c)) *sqrt((a^2 + b^2)/b^2))/b) + 24*(a^3*f^3*cosh(d*x + \\
& c) + a^3*f^3*sinh(d*x + c)) *polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - \\
& (b*cosh(d*x + c) + b*sinh(d*x + c)) *sqrt((a^2 + b^2)/b^2))/b) + ((24*a*b^2 \\
& *f^3 + 24*I*b^3*f^3)*cosh(d*x + c) + (24*a*b^2*f^3 + 24*I*b^3*f^3)*sinh(d*x \\
& + c)) *polylog(4, I*cosh(d*x + c) + I*sinh(d*x + c)) + ((24*a*b^2*f^3 - 24* \\
& I*b^3*f^3)*cosh(d*x + c) + (24*a*b^2*f^3 - 24*I*b^3*f^3)*sinh(d*x + c)) *pol \\
& ylog(4, -I*cosh(d*x + c) - I*sinh(d*x + c)) - 24*((a^3*d*f^3*x + a^3*d*e*f^ \\
& 2)*cosh(d*x + c) + (a^3*d*f^3*x + a^3*d*e*f^2)*sinh(d*x + c)) *polylog(3, (a \\
& *cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c)) *sqrt \\
& ((a^2 + b^2)/b^2))/b) - 24*((a^3*d*f^3*x + a^3*d*e*f^2)*cosh(d*x + c) + (a^ \\
& 3*d*f^3*x + a^3*d*e*f^2)*sinh(d*x + c)) *polylog(3, (a*cosh(d*x + c) + a*sin \\
& h(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c)) *sqrt((a^2 + b^2)/b^2))/b) \\
& - ((24*a*b^2*d*f^3*x + 24*I*b^3*d*f^3*x + 24*a*b^2*d*e*f^2 + 24*I*b^3*d*e*f \\
& ^2)*cosh(d*x + c) + (24*a*b^2*d*f^3*x + 24*I*b^3*d*f^3*x + 24*a*b^2*d*e*f^2 \\
& + 24*I*b^3*d*e*f^2)*sinh(d*x + c)) *polylog(3, I*cosh(d*x + c) + I*sinh(d*x \\
& + c)) - ((24*a*b^2*d*f^3*x - 24*I*b^3*d*f^3*x + 24*a*b^2*d*e*f^2 - 24*I*b^ \\
& 3*d*e*f^2)*cosh(d*x + c) + (24*a*b^2*d*f^3*x - 24*I*b^3*d*f^3*x + 24*a*b^2* \\
& d*e*f^2 - 24*I*b^3*d*e*f^2)*sinh(d*x + c)) *polylog(3, -I*cosh(d*x + c) - I* \\
& sinh(d*x + c)) - ((a^3 + a*b^2)*d^4*f^3*x^4 + 4*(a^3 + a*b^2)*d^4*e*f^2*x^3 \\
& + 6*(a^3 + a*b^2)*d^4*e^2*f*x^2 + 4*(a^3 + a*b^2)*d^4*e^3*x + 8*(a^3 + a*b \\
& ^2)*c*d^3*e^3 - 12*(a^3 + a*b^2)*c^2*d^2*e^2*f + 8*(a^3 + a*b^2)*c^3*d*e*f^ \\
& 2 - 2*(a^3 + a*b^2)*c^4*f^3 + 4*((a^2*b + b^3)*d^3*f^3*x^3 + (a^2*b + b^3)*
\end{aligned}$$

$$d^3e^3 - 3*(a^2*b + b^3)*d^2*e^2*f + 6*(a^2*b + b^3)*d*e*f^2 - 6*(a^2*b + b^3)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^2*b + b^3)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^2*b + b^3)*d^2*e*f^2 + 2*(a^2*b + b^3)*d*f^3)*x * \cosh(d*x + c) * \sinh(d*x + c) / ((a^2*b^2 + b^4)*d^4 * \cosh(d*x + c) + (a^2*b^2 + b^4)*d^4 * \sinh(d*x + c))$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^3 \sinh^2(c + dx) \tanh(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*sinh(d\*x+c)\*\*2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*\*3\*sinh(c + d\*x)\*\*2\*tanh(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*sinh(d\*x+c)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.407 \quad \int \frac{(e+fx)^2 \sinh^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=1067

result too large to display

```
[Out] (a*(e + f*x)^3)/(3*b^2*f) + (2*a^2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^3*d)
- (2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b*d) - (2*a^4*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^3*(a^2 + b^2)*d) - (2*f*(e + f*x)*Cosh[c + d*x])/(b*d^2) - (a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) - (a^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) - (a*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b^2*d) + (a^3*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b^2*(a^2 + b^2)*d) - ((2*I)*a^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*d^2) + ((2*I)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + ((2*I)*a^4*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) + ((2*I)*a^2*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b^3*d^2) - ((2*I)*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - ((2*I)*a^4*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) - (2*a^3*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d^2) - (2*a^3*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d^2) - (a*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(b^2*d^2) + (a^3*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(b^2*(a^2 + b^2)*d^2) + ((2*I)*a^2*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b^3*d^3) - ((2*I)*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((2*I)*a^4*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) - ((2*I)*a^2*f^2*PolyLog[3, I*E^(c + d*x)])/(b^3*d^3) + ((2*I)*f^2*PolyLog[3, I*E^(c + d*x)])/(b*d^3) + ((2*I)*a^4*f^2*PolyLog[3, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) + (2*a^3*f^2*PolyLog[3, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d^3) + (2*a^3*f^2*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d^3) + (a*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*d^3) - (a^3*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*b^2*(a^2 + b^2)*d^3) + (2*f^2*Sinh[c + d*x])/(b*d^3) + ((e + f*x)^2*Sinh[c + d*x])/(b*d)
```

**Rubi [A]** time = 1.69094, antiderivative size = 1067, normalized size of antiderivative = 1., number of steps used = 50, number of rules used = 14, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {5581, 5449, 3296, 2637, 4180, 2531, 2282, 6589, 3718, 2190, 5567, 5573, 5561, 6742}

$$\frac{2(e+fx)^2 \tan^{-1}(e^{c+dx}) a^4}{b^3(a^2+b^2)d} + \frac{2if(e+fx)\text{PolyLog}(2, -ie^{c+dx}) a^4}{b^3(a^2+b^2)d^2} - \frac{2if(e+fx)\text{PolyLog}(2, ie^{c+dx}) a^4}{b^3(a^2+b^2)d^2} - \frac{2if^2\text{PolyLog}(3, -E^{2(c+dx)})}{b^3(a^2+b^2)d^3} + \frac{2if^2\text{PolyLog}(3, E^{2(c+dx)})}{b^3(a^2+b^2)d^3} + \frac{2f^2\text{Sinh}(c+dx)}{b^3(a^2+b^2)d^3} + \frac{(e+fx)^2\text{Sinh}(c+dx)}{b^3(a^2+b^2)d^3}$$



Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Sinh[c + d\*x]^2\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (a\*(e + f\*x)^3)/(3\*b^2\*f) + (2\*a^2\*(e + f\*x)^2\*ArcTan[E^(c + d\*x)])/(b^3\*d) - (2\*(e + f\*x)^2\*ArcTan[E^(c + d\*x)])/(b\*d) - (2\*a^4\*(e + f\*x)^2\*ArcTan[E^(c + d\*x)])/(b^3\*(a^2 + b^2)\*d) - (2\*f\*(e + f\*x)\*Cosh[c + d\*x])/(b\*d^2) - (a^3\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b^2\*(a^2 + b^2)\*d) - (a^3\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b^2\*(a^2 + b^2)\*d) - (a\*(e + f\*x)^2\*Log[1 + E^(2\*(c + d\*x))])/(b^2\*d) + (a^3\*(e + f\*x)^2\*Log[1 + E^(2\*(c + d\*x))])/(b^2\*(a^2 + b^2)\*d) - ((2\*I)\*a^2\*f\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)])/(b^3\*d^2) + ((2\*I)\*f\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)])/(b\*d^2) + ((2\*I)\*a^4\*f\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)])/(b^3\*(a^2 + b^2)\*d^2) + ((2\*I)\*a^2\*f\*(e + f\*x)\*PolyLog[2, I\*E^(c + d\*x)])/(b^3\*d^2) - ((2\*I)\*f\*(e + f\*x)\*PolyLog[2, I\*E^(c + d\*x)])/(b\*d^2) - ((2\*I)\*a^4\*f\*(e + f\*x)\*PolyLog[2, I\*E^(c + d\*x)])/(b^3\*(a^2 + b^2)\*d^2) - (2\*a^3\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^2\*(a^2 + b^2)\*d^2) - (2\*a^3\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^2\*(a^2 + b^2)\*d^2) - (a\*f\*(e + f\*x)\*PolyLog[2, -E^(2\*(c + d\*x))])/(b^2\*d^2) + (a^3\*f\*(e + f\*x)\*PolyLog[2, -E^(2\*(c + d\*x))])/(b^2\*(a^2 + b^2)\*d^2) + ((2\*I)\*a^2\*f^2\*PolyLog[3, (-I)\*E^(c + d\*x)])/(b^3\*d^3) - ((2\*I)\*f^2\*PolyLog[3, (-I)\*E^(c + d\*x)])/(b\*d^3) - ((2\*I)\*a^4\*f^2\*PolyLog[3, (-I)\*E^(c + d\*x)])/(b^3\*(a^2 + b^2)\*d^3) - ((2\*I)\*a^2\*f^2\*PolyLog[3, I\*E^(c + d\*x)])/(b^3\*d^3) + ((2\*I)\*f^2\*PolyLog[3, I\*E^(c + d\*x)])/(b\*d^3) + ((2\*I)\*a^4\*f^2\*PolyLog[3, I\*E^(c + d\*x)])/(b^3\*(a^2 + b^2)\*d^3) + (2\*a^3\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b^2\*(a^2 + b^2)\*d^3) + (2\*a^3\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b^2\*(a^2 + b^2)\*d^3) + (a\*f^2\*PolyLog[3, -E^(2\*(c + d\*x))])/(2\*b^2\*d^3) - (a^3\*f^2\*PolyLog[3, -E^(2\*(c + d\*x))])/(2\*b^2\*(a^2 + b^2)\*d^3) + (2\*f^2\*Sinh[c + d\*x])/(b\*d^3) + ((e + f\*x)^2\*Sinh[c + d\*x])/(b\*d)

### Rule 5581

Int[(((e\_) + (f\_)\*(x\_))^(m\_)\*Sinh[(c\_) + (d\_)\*(x\_)]^(p\_)\*Tanh[(c\_) + (d\_)\*(x\_)]^(n\_))/((a\_) + (b\_)\*Sinh[(c\_) + (d\_)\*(x\_)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Sinh[c + d\*x]^(p - 1)\*Tanh[c + d\*x]^n, x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sinh[c + d\*x]^(p - 1)\*Tanh[c + d\*x]^n)/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5449

Int[((c\_) + (d\_)\*(x\_))^(m\_)\*Sinh[(a\_) + (b\_)\*(x\_)]^(n\_)\*Tanh[(a\_) + (b\_)\*(x\_)]^(p\_), x\_Symbol] := Int[(c + d\*x)^m\*Sinh[a + b\*x]^n\*Tanh[a + b\*x]^(p - 2), x] - Int[(c + d\*x)^m\*Sinh[a + b\*x]^(n - 2)\*Tanh[a + b\*x]^p, x]

/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[  
((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[  
e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /;  
FreeQ[{c, d}, x]

### Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))  
^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)/E^(  
I\*k\*Pi)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1  
- E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c +  
d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c,  
d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)  
\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x  
)))^n)]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m -  
1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f  
, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]  
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi  
onOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[  
{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*  
(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_S  
ymbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d  
, e, n, p}, x] && EqQ[b\*d, a\*e]

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x_
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-I*e) + f*fz*x)))/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_
_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sech[c
+ d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c +
d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_
_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \sinh^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \sinh(c+dx) \tanh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^2 \sinh(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= -\frac{a \int (e+fx)^2 \tanh(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)^2 \tanh(c+dx)}{a+b \sinh(c+dx)} dx}{b^2} + \frac{\int (e+fx)^2 \cosh(c+dx) dx}{b} \\
&= \frac{a(e+fx)^3}{3b^2 f} - \frac{2(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} + \frac{(e+fx)^2 \sinh(c+dx)}{bd} + \frac{a^2 \int (e+fx)^2 \tanh(c+dx) dx}{b^2} \\
&= \frac{a(e+fx)^3}{3b^2 f} + \frac{2a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^3 d} - \frac{2(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{2f(e+fx)^2 \sinh(c+dx)}{b^2} \\
&= \frac{a(e+fx)^3}{3b^2 f} + \frac{a^3(e+fx)^3}{3b^2(a^2+b^2)f} + \frac{2a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^3 d} - \frac{2(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} \\
&= \frac{a(e+fx)^3}{3b^2 f} + \frac{a^3(e+fx)^3}{3b^2(a^2+b^2)f} + \frac{2a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^3 d} - \frac{2(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} \\
&= \frac{a(e+fx)^3}{3b^2 f} + \frac{2a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^3 d} - \frac{2(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^4(e+fx)^2 \sinh(c+dx)}{b^3 d} \\
&= \frac{a(e+fx)^3}{3b^2 f} + \frac{2a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^3 d} - \frac{2(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^4(e+fx)^2 \sinh(c+dx)}{b^3 d} \\
&= \frac{a(e+fx)^3}{3b^2 f} + \frac{2a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^3 d} - \frac{2(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^4(e+fx)^2 \sinh(c+dx)}{b^3 d} \\
&= \frac{a(e+fx)^3}{3b^2 f} + \frac{2a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^3 d} - \frac{2(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^4(e+fx)^2 \sinh(c+dx)}{b^3 d} \\
&= \frac{a(e+fx)^3}{3b^2 f} + \frac{2a^2(e+fx)^2 \tan^{-1}(e^{c+dx})}{b^3 d} - \frac{2(e+fx)^2 \tan^{-1}(e^{c+dx})}{bd} - \frac{2a^4(e+fx)^2 \sinh(c+dx)}{b^3 d}
\end{aligned}$$

**Mathematica [B]** time = 19.6768, size = 3418, normalized size = 3.2

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Sinh[c + d\*x]^2\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] 
$$\begin{aligned} & -(-12*a*d^3*e^2*E^{(2*c)}*x + 12*a*d^3*e^2*(1 + E^{(2*c)})*x + 12*a*d^3*e*f*x^2 \\ & + 4*a*d^3*f^2*x^3 + 12*b*d^2*e^2*(1 + E^{(2*c)})*ArcTan[E^{(c + d*x)}] - 6*a*d \\ & ^2*e^2*(1 + E^{(2*c)})*(2*d*x - Log[1 + E^{(2*(c + d*x))}]) + (12*I)*b*d*e*(1 + \\ & E^{(2*c)})*f*(d*x*(Log[1 - I*E^{(c + d*x)}] - Log[1 + I*E^{(c + d*x)}]) - PolyLo \\ & g[2, (-I)*E^{(c + d*x)}] + PolyLog[2, I*E^{(c + d*x)}]) - 6*a*d*e*(1 + E^{(2*c)}) \\ & *f*(2*d*x*(d*x - Log[1 + E^{(2*(c + d*x))}]) - PolyLog[2, -E^{(2*(c + d*x))}]) \\ & + (6*I)*b*(1 + E^{(2*c)})*f^2*(d^2*x^2*Log[1 - I*E^{(c + d*x)}] - d^2*x^2*Log[1 \\ & + I*E^{(c + d*x)}] - 2*d*x*PolyLog[2, (-I)*E^{(c + d*x)}] + 2*d*x*PolyLog[2, I \\ & *E^{(c + d*x)}] + 2*PolyLog[3, (-I)*E^{(c + d*x)}] - 2*PolyLog[3, I*E^{(c + d*x)} \\ & ]) - a*(1 + E^{(2*c)})*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E^{(2*(c + d*x))}]) - \\ & 6*d*x*PolyLog[2, -E^{(2*(c + d*x))}] + 3*PolyLog[3, -E^{(2*(c + d*x))}]))/(6*(a \\ & ^2 + b^2)*d^3*(1 + E^{(2*c)})) + (a^3*(6*e^2*E^{(2*c)}*x + 6*e*E^{(2*c)}*f*x^2 + \\ & 2*E^{(2*c)}*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^{(c + d*x)})/Sqr \\ & t[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^{ \\ & (2*c)*ArcTan[(a + b*E^{(c + d*x)})/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - \\ & (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^{(c + d*x)})/Sqrt[a^2 + b^2]]) \\ & )/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^{(2*c)*ArcTanh[(a \\ & + b*E^{(c + d*x)})/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a \\ & *E^{(c + d*x)} + b*(-1 + E^{(2*(c + d*x))})])/d - (3*e^2*E^{(2*c)}*Log[2*a*E^{(c + \\ & d*x)} + b*(-1 + E^{(2*(c + d*x))})])/d + (6*e*f*x*Log[1 + (b*E^{(2*c + d*x)})/( \\ & a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d - (6*e*E^{(2*c)}*f*x*Log[1 + (b*E^{(2*c \\ & + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d + (3*f^2*x^2*Log[1 + (b*E^{ \\ & (2*c + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d - (3*E^{(2*c)}*f^2*x^2*L \\ & og[1 + (b*E^{(2*c + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d + (6*e*f*x \\ & *Log[1 + (b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d - (6*e*E \\ & ^{(2*c)}*f*x*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/ \\ & d + (3*f^2*x^2*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}] \\ & )])/d - (3*E^{(2*c)}*f^2*x^2*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b \\ & ^2)*E^{(2*c)}])])/d - (6*(-1 + E^{(2*c)})*f*(e + f*x)*PolyLog[2, -((b*E^{(2*c + \\ & d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^2 - (6*(-1 + E^{(2*c)})*f*(e + \\ & f*x)*PolyLog[2, -((b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/ \\ & /d^2 - (6*f^2*PolyLog[3, -((b*E^{(2*c + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2 \\ & *c)}])])/d^3 + (6*E^{(2*c)}*f^2*PolyLog[3, -((b*E^{(2*c + d*x)})/(a*E^c - Sqrt[ \\ & (a^2 + b^2)*E^{(2*c)}])])/d^3 - (6*f^2*PolyLog[3, -((b*E^{(2*c + d*x)})/(a*E^c \\ & + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^3 + (6*E^{(2*c)}*f^2*PolyLog[3, -((b*E^{(2* \\ & c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^3)/(3*b^2*(a^2 + b^2)*( \\ & -1 + E^{(2*c)})) + (Csch[c]*Sech[c]*(Cosh[c + d*x]/(24*b^2*d^3) - Sinh[c + d* \\ & x]/(24*b^2*d^3))*(-3*a^2*b*d^2*e^2*Cosh[2*d*x] - 3*b^3*d^2*e^2*Cosh[2*d*x] \\ & + 6*a^2*b*d*e*f*Cosh[2*d*x] + 6*b^3*d*e*f*Cosh[2*d*x] - 6*a^2*b*f^2*Cosh[2* \\ & d*x] - 6*b^3*f^2*Cosh[2*d*x] - 6*a^2*b*d^2*e*f*x*Cosh[2*d*x] - 6*b^3*d^2*e* \\ & f*x*Cosh[2*d*x] + 6*a^2*b*d*f^2*x*Cosh[2*d*x] + 6*b^3*d*f^2*x*Cosh[2*d*x] - \end{aligned}$$

$$\begin{aligned}
& 3a^2bd^2f^2x^2\text{Cosh}[2dx] - 3b^3d^2f^2x^2\text{Cosh}[2dx] - 6a^3d^3e^2xf^2\text{Cosh}[c-dx] - 6ab^2d^3e^2xf^2\text{Cosh}[c-dx] - 6a^3d^3efx^2\text{Cosh}[c-dx] - 6ab^2d^3efx^2\text{Cosh}[c-dx] - 2a^3d^3f^2x^3\text{Cosh}[c-dx] - 2ab^2d^3f^2x^3\text{Cosh}[c-dx] - 12a^3d^3e^2xf^2\text{Cosh}[c+dx] + 12ab^2d^3e^2xf^2\text{Cosh}[c+dx] - 12a^3d^3efx^2\text{Cosh}[c+dx] + 12ab^2d^3efx^2\text{Cosh}[c+dx] - 4a^3d^3f^2x^3\text{Cosh}[c+dx] + 4ab^2d^3f^2x^3\text{Cosh}[c+dx] - 6a^3d^3e^2xf^2\text{Cosh}[3c+dx] - 6ab^2d^3e^2xf^2\text{Cosh}[3c+dx] - 6a^3d^3efx^2\text{Cosh}[3c+dx] - 6ab^2d^3efx^2\text{Cosh}[3c+dx] - 2a^3d^3f^2x^3\text{Cosh}[3c+dx] - 2ab^2d^3f^2x^3\text{Cosh}[3c+dx] + 3a^2bd^2e^2\text{Cosh}[4c+2dx] + 3b^3d^2e^2\text{Cosh}[4c+2dx] - 6a^2bd^2ef\text{Cosh}[4c+2dx] - 6b^3d^2ef\text{Cosh}[4c+2dx] + 2d^2x] + 6a^2b^2f^2\text{Cosh}[4c+2dx] + 6b^3f^2\text{Cosh}[4c+2dx] + 6a^2bd^2efx\text{Cosh}[4c+2dx] + 6b^3d^2efx\text{Cosh}[4c+2dx] - 6a^2bd^2f^2x\text{Cosh}[4c+2dx] - 6b^3d^2f^2x\text{Cosh}[4c+2dx] + 3a^2bd^2f^2x^2\text{Cosh}[4c+2dx] + 3b^3d^2f^2x^2\text{Cosh}[4c+2dx] - 6a^2bd^2e^2\text{Sinh}[2c] - 6b^3d^2e^2\text{Sinh}[2c] - 12a^2bd^2ef\text{Sinh}[2c] - 12b^3d^2ef\text{Sinh}[2c] - 12a^2b^2f^2\text{Sinh}[2c] - 12b^3d^2e^2efx\text{Sinh}[2c] - 12b^3d^2efx\text{Sinh}[2c] - 12a^2bd^2f^2x\text{Sinh}[2c] - 12b^3d^2f^2x\text{Sinh}[2c] - 3a^2bd^2e^2\text{Sinh}[2dx] - 3b^3d^2e^2\text{Sinh}[2dx] + 6a^2bd^2ef\text{Sinh}[2dx] + 6b^3d^2ef\text{Sinh}[2dx] - 6a^2b^2f^2\text{Sinh}[2dx] - 6b^3f^2\text{Sinh}[2dx] - 6a^2bd^2efx\text{Sinh}[2dx] + 6a^2bd^2f^2x\text{Sinh}[2dx] + 6b^3d^2f^2x\text{Sinh}[2dx] - 3a^2bd^2f^2x^2\text{Sinh}[2dx] - 3b^3d^2f^2x^2\text{Sinh}[2dx] + 6a^3d^3e^2xf^2\text{Sinh}[c-dx] + 6ab^2d^3e^2xf^2\text{Sinh}[c-dx] + 6a^3d^3efx^2\text{Sinh}[c-dx] + 6ab^2d^3efx^2\text{Sinh}[c-dx] + 2a^3d^3f^2x^3\text{Sinh}[c-dx] + 2ab^2d^3f^2x^3\text{Sinh}[c-dx] - 12a^3d^3e^2xf^2\text{Sinh}[c+dx] + 12ab^2d^3e^2xf^2\text{Sinh}[c+dx] - 12a^3d^3efx^2\text{Sinh}[c+dx] + 12ab^2d^3efx^2\text{Sinh}[c+dx] - 4a^3d^3f^2x^3\text{Sinh}[c+dx] + 4ab^2d^3f^2x^3\text{Sinh}[c+dx] - 6a^3d^3e^2xf^2\text{Sinh}[3c+dx] - 6ab^2d^3e^2xf^2\text{Sinh}[3c+dx] - 6a^3d^3efx^2\text{Sinh}[3c+dx] - 6ab^2d^3efx^2\text{Sinh}[3c+dx] - 2a^3d^3f^2x^3\text{Sinh}[3c+dx] - 2ab^2d^3f^2x^3\text{Sinh}[3c+dx] + 3a^2bd^2e^2\text{Sinh}[4c+2dx] + 3b^3d^2e^2\text{Sinh}[4c+2dx] - 6a^2bd^2ef\text{Sinh}[4c+2dx] - 6b^3d^2ef\text{Sinh}[4c+2dx] + 6a^2b^2f^2\text{Sinh}[4c+2dx] + 6b^3f^2\text{Sinh}[4c+2dx] + 6a^2bd^2efx\text{Sinh}[4c+2dx] + 6b^3d^2efx\text{Sinh}[4c+2dx] - 6a^2bd^2f^2x\text{Sinh}[4c+2dx] - 6b^3d^2f^2x\text{Sinh}[4c+2dx] + 3a^2bd^2f^2x^2\text{Sinh}[4c+2dx] + 3b^3d^2f^2x^2\text{Sinh}[4c+2dx]
\end{aligned}$$

$$\text{)}}/(a^2 + b^2)$$

**Maple [F]** time = 0.853, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\sinh(dx + c))^2 \tanh(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*sinh(d\*x+c)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*sinh(d\*x+c)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2} \left( \frac{2a^3 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{(a^2b^2 + b^4)d} - \frac{4b \arctan(e^{(-dx-c)})}{(a^2 + b^2)d} + \frac{2a \log(e^{(-2dx-2c)} + 1)}{(a^2 + b^2)d} + \frac{2(dx+c)a}{b^2d} - \frac{e^{(dx+c)}}{bd} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sinh(d\*x+c)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$-1/2*(2*a^3*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/((a^2*b^2 + b^4)*d) - 4*b*\arctan(e^{(-d*x - c)})/((a^2 + b^2)*d) + 2*a*\log(e^{(-2*d*x - 2*c)} + 1)/((a^2 + b^2)*d) + 2*(d*x + c)*a/(b^2*d) - e^{(d*x + c)}/(b*d) + e^{(-d*x - c)}/(b*d)*e^2 - 1/6*(2*a*d^3*f^2*x^3*e^c + 6*a*d^3*e*f*x^2*e^c - 3*(b*d^2*f^2*x^2*e^{(2*c)} + 2*(d^2*e*f - d*f^2)*b*x*e^{(2*c)} - 2*(d*e*f - f^2)*b*e^{(2*c)})*e^{(d*x)} + 3*(b*d^2*f^2*x^2 + 2*(d^2*e*f + d*f^2)*b*x + 2*(d*e*f + f^2)*b)*e^{(-d*x)}*e^{(-c)}/(b^2*d^3) + integrate(2*(a^3*b*f^2*x^2 + 2*a^3*b*e*f*x - (a^4*f^2*x^2*e^c + 2*a^4*e*f*x*e^c)*e^{(d*x)})/(a^2*b^3 + b^5 - (a^2*b^3*e^{(2*c)} + b^5*e^{(2*c)})*e^{(2*d*x)} - 2*(a^3*b^2*e^c + a*b^4*e^c)*e^{(d*x)}), x) - integrate(-2*(a*f^2*x^2 + 2*a*e*f*x - (b*f^2*x^2*e^c + 2*b*e*f*x*e^c)*e^{(d*x)})/(a^2 + b^2 + (a^2*e^{(2*c)} + b^2*e^{(2*c)})*e^{(2*d*x)}), x)$$

**Fricas [C]** time = 3.65942, size = 6611, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sinh(d\*x+c)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-1/6*(3*(a^2*b + b^3)*d^2*f^2*x^2 + 3*(a^2*b + b^3)*d^2*e^2 + 6*(a^2*b + b^3)*d*e*f + 6*(a^2*b + b^3)*f^2 - 3*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f + 2*(a^2*b + b^3)*f^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*\cosh(d*x + c)^2 - 3*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f + 2*(a^2*b + b^3)*f^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*\sinh(d*x + c)^2 + 6*((a^2*b + b^3)*d^2*e*f + (a^2*b + b^3)*d*f^2)*x - 2*((a^3 + a*b^2)*d^3*f^2*x^3 + 3*(a^3 + a*b^2)*d^3*e*f*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*x + 6*(a^3 + a*b^2)*c*d^2*e^2 - 6*(a^3 + a*b^2)*c^2*d*e*f + 2*(a^3 + a*b^2)*c^3*f^2)*\cosh(d*x + c) + 12*((a^3*d*f^2*x + a^3*d*e*f)*\cosh(d*x + c) + (a^3*d*f^2*x + a^3*d*e*f)*\sinh(d*x + c))*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 12*((a^3*d*f^2*x + a^3*d*e*f)*\cosh(d*x + c) + (a^3*d*f^2*x + a^3*d*e*f)*\sinh(d*x + c))*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + ((12*a*b^2*d*f^2*x + 12*I*b^3*d*f^2*x + 12*a*b^2*d*e*f + 12*I*b^3*d*e*f)*\cosh(d*x + c) + (12*a*b^2*d*f^2*x + 12*I*b^3*d*f^2*x + 12*a*b^2*d*e*f + 12*I*b^3*d*e*f)*\sinh(d*x + c))*\operatorname{dilog}(I*\cosh(d*x + c) + I*\sinh(d*x + c)) + ((12*a*b^2*d*f^2*x - 12*I*b^3*d*f^2*x + 12*a*b^2*d*e*f - 12*I*b^3*d*e*f)*\cosh(d*x + c) + (12*a*b^2*d*f^2*x - 12*I*b^3*d*f^2*x + 12*a*b^2*d*e*f - 12*I*b^3*d*e*f)*\sinh(d*x + c))*\operatorname{dilog}(-I*\cosh(d*x + c) - I*\sinh(d*x + c)) + 6*((a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*\cosh(d*x + c) + (a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*\sinh(d*x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 6*((a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*\cosh(d*x + c) + (a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2)*\sinh(d*x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 6*((a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*\cosh(d*x + c) + (a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*\sinh(d*x + c))*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + 6*((a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*\cosh(d*x + c) + (a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2)*\sinh(d*x + c))*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + ((6*a*b^2*d^2*e^2 + 6*I*b^3*d^2*e^2 - 12*a*b^2*c*d*e*f - 12*I*b^3*c*d*e*f + 6*a*b^2*c^2*f^2 + 6*I*b^3*c^2*f^2)*\cosh(d*x + c) + (6*a*b^2*d^2*e^2 + 6*I*b^3*d^2*e^2 - 12*a*b^2*c*d*e*f - 12*I*b^3*c*d*e*f + 6*a*b^2*c^2*f^2 + 6*I*b^3*c^2*f^2)*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + I) + ((6*a*b^2*d^2*e^2 - 6*I*b^3*d^2*e^2 - 12*a*b^2*c*d*e*f + 12*I*b^3*c*d*e*f + 6*a*b^2*c^2*f^2 - 6*I*b^3*c^2*f^2)*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - I) + ((6*a*$$



$$\begin{aligned}
& b^2 d^2 f^2 x^2 - 6 I b^3 d^2 f^2 x^2 + 12 a b^2 d^2 e f x - 12 I b^3 d^2 e f x \\
& + 12 a b^2 c d e f - 12 I b^3 c d e f - 6 a b^2 c^2 f^2 + 6 I b^3 c^2 f^2 \\
& + (6 a b^2 d^2 f^2 x^2 - 6 I b^3 d^2 f^2 x^2 + 12 a b^2 d^2 e f x - 12 I b^3 d^2 e f x \\
& + 12 a b^2 c d e f - 12 I b^3 c d e f - 6 a b^2 c^2 f^2 + 6 I b^3 c^2 f^2) \sinh(d x + c) \\
& * \log(I \cosh(d x + c) + I \sinh(d x + c) + 1) + ((6 a b^2 d^2 f^2 x^2 + 6 I b^3 d^2 f^2 x^2 \\
& + 12 a b^2 d^2 e f x + 12 I b^3 d^2 e f x + 12 a b^2 c d e f + 12 I b^3 c d e f - 6 a b^2 c^2 f^2 \\
& - 6 I b^3 c^2 f^2) \cosh(d x + c) + (6 a b^2 d^2 f^2 x^2 + 6 I b^3 d^2 f^2 x^2 \\
& + 12 a b^2 d^2 e f x + 12 I b^3 d^2 e f x + 12 a b^2 c d e f + 12 I b^3 c d e f - 6 a b^2 c^2 f^2 \\
& - 6 I b^3 c^2 f^2) \sinh(d x + c)) * \log(-I \cosh(d x + c) - I \sinh(d x + c) + 1) \\
& - 12 (a^3 f^2 \cosh(d x + c) + a^3 f^2 \sinh(d x + c)) * \text{polylog}(3, (a \cosh(d x + c) + a \sinh(d x + c) \\
& + (b \cosh(d x + c) + b \sinh(d x + c)) * \sqrt{(a^2 + b^2)/b^2})/b) - 12 (a^3 f^2 \cosh(d x + c) \\
& + a^3 f^2 \sinh(d x + c)) * \text{polylog}(3, (a \cosh(d x + c) + a \sinh(d x + c) - (b \cosh(d x + c) \\
& + b \sinh(d x + c)) * \sqrt{(a^2 + b^2)/b^2})/b) - ((12 a b^2 f^2 + 12 I b^3 f^2) \cosh(d x + c) \\
& + (12 a b^2 f^2 + 12 I b^3 f^2) \sinh(d x + c)) * \text{polylog}(3, I \cosh(d x + c) + I \sinh(d x + c)) \\
& - ((12 a b^2 f^2 - 12 I b^3 f^2) \cosh(d x + c) + (12 a b^2 f^2 - 12 I b^3 f^2) \sinh(d x + c)) * \text{polylog}(3, \\
& -I \cosh(d x + c) - I \sinh(d x + c)) - 2 ((a^3 + a b^2) d^3 f^2 x^3 + 3 (a^3 + a b^2) d^3 e f x^2 \\
& + 3 (a^3 + a b^2) d^3 e^2 x + 6 (a^3 + a b^2) c d^2 e^2 - 6 (a^3 + a b^2) c^2 d e f + 2 (a^3 + a b^2) c^3 f^2 \\
& + 3 ((a^2 b + b^3) d^2 f^2 x^2 + (a^2 b + b^3) d^2 e^2 - 2 (a^2 b + b^3) d e f + 2 (a^2 b + b^3) f^2 \\
& + 2 ((a^2 b + b^3) d^2 e f - (a^2 b + b^3) d f^2) x) \cosh(d x + c) \sinh(d x + c) / ((a^2 b^2 + b^4) d^3 \cosh(d x + c) \\
& + (a^2 b^2 + b^4) d^3 \sinh(d x + c))
\end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + f x)^2 \sinh^2(c + d x) \tanh(c + d x)}{a + b \sinh(c + d x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*sinh(d\*x+c)\*\*2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*\*2\*sinh(c + d\*x)\*\*2\*tanh(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.408 \quad \int \frac{(e+fx) \sinh^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=631

$$\frac{ia^4 f \text{PolyLog}\left(2, -ie^{c+dx}\right)}{b^3 d^2 (a^2 + b^2)} - \frac{ia^4 f \text{PolyLog}\left(2, ie^{c+dx}\right)}{b^3 d^2 (a^2 + b^2)} - \frac{a^3 f \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d^2 (a^2 + b^2)} - \frac{a^3 f \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2 d^2 (a^2 + b^2)} + \dots$$

```
[Out] (a*(e + f*x)^2)/(2*b^2*f) + (2*a^2*(e + f*x)*ArcTan[E^(c + d*x)])/(b^3*d) -
(2*(e + f*x)*ArcTan[E^(c + d*x)])/(b*d) - (2*a^4*(e + f*x)*ArcTan[E^(c + d
*x)])/(b^3*(a^2 + b^2)*d) - (f*Cosh[c + d*x])/(b*d^2) - (a^3*(e + f*x)*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) - (a^3*(e +
f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) -
(a*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(b^2*d) + (a^3*(e + f*x)*Log[1 + E^
(2*(c + d*x))])/(b^2*(a^2 + b^2)*d) - (I*a^2*f*PolyLog[2, (-I)*E^(c + d*x)]
)/(b^3*d^2) + (I*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + (I*a^4*f*PolyLog
[2, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) + (I*a^2*f*PolyLog[2, I*E^(c +
d*x)])/(b^3*d^2) - (I*f*PolyLog[2, I*E^(c + d*x)])/(b*d^2) - (I*a^4*f*Poly
Log[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) - (a^3*f*PolyLog[2, -(b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d^2) - (a^3*f*PolyLog[2,
-(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d^2) - (a*f*Pol
yLog[2, -E^(2*(c + d*x))])/(2*b^2*d^2) + (a^3*f*PolyLog[2, -E^(2*(c + d*x))
])/(2*b^2*(a^2 + b^2)*d^2) + ((e + f*x)*Sinh[c + d*x])/(b*d)
```

**Rubi [A]** time = 0.957045, antiderivative size = 631, normalized size of antiderivative = 1., number of steps used = 39, number of rules used = 13, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$ , Rules used = {5581, 5449, 3296, 2638, 4180, 2279, 2391, 3718, 2190, 5567, 5573, 5561, 6742}

$$\frac{ia^4 f \text{PolyLog}\left(2, -ie^{c+dx}\right)}{b^3 d^2 (a^2 + b^2)} - \frac{ia^4 f \text{PolyLog}\left(2, ie^{c+dx}\right)}{b^3 d^2 (a^2 + b^2)} - \frac{a^3 f \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b^2 d^2 (a^2 + b^2)} - \frac{a^3 f \text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{b^2 d^2 (a^2 + b^2)} + \dots$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Sinh[c + d*x]^2*Tanh[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a*(e + f*x)^2)/(2*b^2*f) + (2*a^2*(e + f*x)*ArcTan[E^(c + d*x)])/(b^3*d) -
(2*(e + f*x)*ArcTan[E^(c + d*x)])/(b*d) - (2*a^4*(e + f*x)*ArcTan[E^(c + d
*x)])/(b^3*(a^2 + b^2)*d) - (f*Cosh[c + d*x])/(b*d^2) - (a^3*(e + f*x)*Log[
1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b^2*(a^2 + b^2)*d) - (a^3*(e +
```

$$\begin{aligned}
& f*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])]/(b^2*(a^2 + b^2)*d) - \\
& (a*(e + f*x)*\text{Log}[1 + E^{(2*(c + d*x))}]/(b^2*d) + (a^3*(e + f*x)*\text{Log}[1 + E^{(2*(c + d*x))}]/(b^2*(a^2 + b^2)*d) - (I*a^2*f*\text{PolyLog}[2, (-I)*E^{(c + d*x)}]/(b^3*d^2) + (I*f*\text{PolyLog}[2, (-I)*E^{(c + d*x)}]/(b*d^2) + (I*a^4*f*\text{PolyLog}[2, (-I)*E^{(c + d*x)}]/(b^3*(a^2 + b^2)*d^2) + (I*a^2*f*\text{PolyLog}[2, I*E^{(c + d*x)}]/(b^3*d^2) - (I*f*\text{PolyLog}[2, I*E^{(c + d*x)}]/(b*d^2) - (I*a^4*f*\text{PolyLog}[2, I*E^{(c + d*x)}]/(b^3*(a^2 + b^2)*d^2) - (a^3*f*\text{PolyLog}[2, -(b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])]/(b^2*(a^2 + b^2)*d^2) - (a^3*f*\text{PolyLog}[2, -(b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])]/(b^2*(a^2 + b^2)*d^2) - (a*f*\text{PolyLog}[2, -E^{(2*(c + d*x))}]/(2*b^2*d^2) + (a^3*f*\text{PolyLog}[2, -E^{(2*(c + d*x))}]/(2*b^2*(a^2 + b^2)*d^2) + ((e + f*x)*\text{Sinh}[c + d*x])/(b*d)
\end{aligned}$$
Rule 5581

$$\begin{aligned}
& \text{Int}[(((e_.) + (f_.)*(x_.))^{(m_.)*\text{Sinh}[(c_.) + (d_.)*(x_.)]^{(p_.)*\text{Tanh}[(c_.) + (d_.)*(x_.)]^{(n_.)}})/(a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_.)]), x\_Symbol] :> \text{Dist}[1/b, \text{Int}[(e + f*x)^m*\text{Sinh}[c + d*x]^{(p - 1)*\text{Tanh}[c + d*x]^n, x], x] - \text{Dist}[a/b, \text{Int}[(e + f*x)^m*\text{Sinh}[c + d*x]^{(p - 1)*\text{Tanh}[c + d*x]^n/(a + b*\text{Sinh}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]
\end{aligned}$$
Rule 5449

$$\begin{aligned}
& \text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)*\text{Sinh}[(a_.) + (b_.)*(x_.)]^{(n_.)*\text{Tanh}[(a_.) + (b_.)*(x_.)]^{(p_.)}], x\_Symbol] :> \text{Int}[(c + d*x)^m*\text{Sinh}[a + b*x]^n*\text{Tanh}[a + b*x]^{(p - 2)}, x] - \text{Int}[(c + d*x)^m*\text{Sinh}[a + b*x]^{(n - 2)*\text{Tanh}[a + b*x]^p, x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]
\end{aligned}$$
Rule 3296

$$\begin{aligned}
& \text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)*\sin[(e_.) + (f_.)*(x_.)]}, x\_Symbol] :> -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x]/f, x] + \text{Dist}[(d*m)/f, \text{Int}[(c + d*x)^{(m - 1)*\text{Cos}[e + f*x]}, x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]
\end{aligned}$$
Rule 2638

$$\begin{aligned}
& \text{Int}[\sin[(c_.) + (d_.)*(x_.)], x\_Symbol] :> -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}\{c, d\}, x]
\end{aligned}$$
Rule 4180

$$\begin{aligned}
& \text{Int}[\text{csc}[(e_.) + \text{Pi}*(k_.) + (\text{Complex}[0, fz\_])*(f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}], x\_Symbol] :> \text{Simp}[(-2*(c + d*x)^m*\text{ArcTanh}[E^{-(I*e) + f*fz*x}]/E^{(I*k*Pi)})/(f*fz*I), x] + (-\text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{(m - 1)*\text{Log}[1 - E^{-(I*e) + f*fz*x}]/E^{(I*k*Pi)}], x], x] + \text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c +
\end{aligned}$$

$d*x)^{(m-1)}*\text{Log}[1 + E^{-(I*e)} + f*fz*x)/E^{(I*k*Pi)}], x], x] /; \text{FreeQ}\{c, d, e, f, fz\}, x] \&\& \text{IntegerQ}[2*k] \&\& \text{IGtQ}[m, 0]$

### Rule 2279

$\text{Int}[\text{Log}[(a_) + (b_)*((F_)^{((e_)*((c_) + (d_)*(x_)))})^{(n_)}], x\_Symbol] \rightarrow \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x)})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \&\& \text{GtQ}[a, 0]$

### Rule 2391

$\text{Int}[\text{Log}[(c_)*((d_) + (e_)*(x_)^{(n_)})]/(x_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$

### Rule 3718

$\text{Int}[(c_ + (d_)*(x_))^{(m_)}*\text{tan}[(e_ + (\text{Complex}[0, fz_])*(f_)*(x_))], x\_Symbol] \rightarrow -\text{Simp}[(I*(c + d*x)^{(m+1)})/(d*(m+1)), x] + \text{Dist}[2*I, \text{Int}[(c + d*x)^m * E^{(2*(-I*e) + f*fz*x))}/(1 + E^{(2*(-I*e) + f*fz*x)}), x], x] /; \text{FreeQ}\{c, d, e, f, fz\}, x] \&\& \text{IGtQ}[m, 0]$

### Rule 2190

$\text{Int}[(F_)^{((g_)*((e_ + (f_)*(x_)))})^{(n_)}*((c_ + (d_)*(x_))^{(m_)})}/((a_ + (b_)*((F_)^{((g_)*((e_ + (f_)*(x_)))})^{(n_)}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m * \text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

### Rule 5567

$\text{Int}[(e_ + (f_)*(x_))^{(m_)}*\text{Tanh}[(c_ + (d_)*(x_))]^{(n_)}]/((a_ + (b_)*\text{Sinh}[(c_ + (d_)*(x_))]), x\_Symbol] \rightarrow \text{Dist}[1/b, \text{Int}[(e + f*x)^m * \text{Sech}[c + d*x]*\text{Tanh}[c + d*x]^{(n-1)}, x], x] - \text{Dist}[a/b, \text{Int}[(e + f*x)^m * \text{Sech}[c + d*x]*\text{Tanh}[c + d*x]^{(n-1)})/(a + b*\text{Sinh}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[n, 0]$

### Rule 5573

$\text{Int}[(e_ + (f_)*(x_))^{(m_)}*\text{Sech}[(c_ + (d_)*(x_))]^{(n_)}]/((a_ + (b_)*\text{Sinh}[(c_ + (d_)*(x_))]), x\_Symbol] \rightarrow \text{Dist}[b^2/(a^2 + b^2), \text{Int}[(e + f*x)^m * \text{Sech}[c + d*x]^{(n-2)}]/(a + b*\text{Sinh}[c + d*x]), x], x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(e + f*x)^m * \text{Sech}[c + d*x]^{(n-1)}*(a - b*\text{Sinh}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{IGtQ}[n, 0]$

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\sinh^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\sinh(c+dx)\tanh(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)\sinh(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)} dx}{b} \\
&= -\frac{a \int (e+fx)\tanh(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)} dx}{b^2} + \frac{\int (e+fx)\cosh(c+dx) dx}{b} \\
&= \frac{a(e+fx)^2}{2b^2f} - \frac{2(e+fx)\tan^{-1}(e^{c+dx})}{bd} + \frac{(e+fx)\sinh(c+dx)}{bd} + \frac{a^2 \int (e+fx)\tanh(c+dx) dx}{b^2} \\
&= \frac{a(e+fx)^2}{2b^2f} + \frac{2a^2(e+fx)\tan^{-1}(e^{c+dx})}{b^3d} - \frac{2(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{f\cosh(c+dx)}{b} \\
&= \frac{a(e+fx)^2}{2b^2f} + \frac{a^3(e+fx)^2}{2b^2(a^2+b^2)f} + \frac{2a^2(e+fx)\tan^{-1}(e^{c+dx})}{b^3d} - \frac{2(e+fx)\tan^{-1}(e^{c+dx})}{bd} \\
&= \frac{a(e+fx)^2}{2b^2f} + \frac{a^3(e+fx)^2}{2b^2(a^2+b^2)f} + \frac{2a^2(e+fx)\tan^{-1}(e^{c+dx})}{b^3d} - \frac{2(e+fx)\tan^{-1}(e^{c+dx})}{bd} \\
&= \frac{a(e+fx)^2}{2b^2f} + \frac{2a^2(e+fx)\tan^{-1}(e^{c+dx})}{b^3d} - \frac{2(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{2a^4(e+fx)}{b^3d} \\
&= \frac{a(e+fx)^2}{2b^2f} + \frac{2a^2(e+fx)\tan^{-1}(e^{c+dx})}{b^3d} - \frac{2(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{2a^4(e+fx)}{b^3d} \\
&= \frac{a(e+fx)^2}{2b^2f} + \frac{2a^2(e+fx)\tan^{-1}(e^{c+dx})}{b^3d} - \frac{2(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{2a^4(e+fx)}{b^3d} \\
&= \frac{a(e+fx)^2}{2b^2f} + \frac{2a^2(e+fx)\tan^{-1}(e^{c+dx})}{b^3d} - \frac{2(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{2a^4(e+fx)}{b^3d}
\end{aligned}$$

**Mathematica [A]** time = 4.78708, size = 481, normalized size = 0.76

$$\frac{a^3 \left( f \operatorname{PolyLog} \left( 2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a} \right) + f \operatorname{PolyLog} \left( 2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} \right) + f(c+dx) \log \left( \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1 \right) + f(c+dx) \log \left( \frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} + 1 \right) + de \log(a+b\sinh(c+dx)) - cf \log(a+b\sinh(c+dx)) \right)}{b^2(a^2+b^2)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)\*Sinh[c + d\*x]^2\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x

]

```
[Out] -(((f*Cosh[c + d*x])/b + (a^3*(-f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]) + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]]) + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))/(b^2*(a^2 + b^2)) + (-(a*d*e*(c + d*x) + a*c*f*(c + d*x) + (a*f*(c + d*x)^2)/2 + 2*b*d*e*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] - 2*b*c*f*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] + 2*b*f*(c + d*x)*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] + a*f*(c + d*x)*Log[2*Cosh[c + d*x]*(Cosh[c + d*x] - Sinh[c + d*x])] + a*d*e*Log[1 + Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] - a*c*f*Log[1 + Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] - I*b*f*PolyLog[2, (-I)*(Cosh[c + d*x] + Sinh[c + d*x])] + I*b*f*PolyLog[2, I*(Cosh[c + d*x] + Sinh[c + d*x])] - (a*f*PolyLog[2, -Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]])/2)/(a^2 + b^2) - (d*(e + f*x)*Sinh[c + d*x])/b/d^2)
```

---

**Maple [B]** time = 0.309, size = 4066, normalized size = 6.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/b^2/d^2*a^4*f/(a^2+b^2)^(3/2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/b^2/d^2*a^4*f/(a^2+b^2)^(3/2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/b^2/d^2*a^3*f/(a^2+b^2)*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))-1/b^2/d^2*a^3*f/(a^2+b^2)*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-2/d^2*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*c+1/d*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*a*x+1/d^2*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a*c+1/d*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a*x+1/d^2*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*a*c-2/d*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*a*x+4/d^2*f*c/(2*a^2+2*b^2)*b*arctan(exp(d*x+c))-1/d^2*f*c/(2*a^2+2*b^2)*a*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d^2*f*c/(2*a^2+2*b^2)*(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+2/d^2*f*c/(2*a^2+2*b^2)*a*ln(1+exp(2*d*x+2*c))-2/d^2*f/(2*a^2+2*b^2)*ln(1+I*exp(d*x+c))*a*c-2/d*f/(2*a^2+2*b^2)*ln(1-I*exp(d*x+c))*a*x+1/2*(d*f*x+d*e-f)/d^2/b*exp(d*x+c)-1/2*(d*f*x+d*e+f)/d^2/b*exp(-d*x-c)+1/2*a*f*x^2/b^2+2/d/(a^2+b^2)^(1/2)*b^2*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(
```



$$\begin{aligned}
& a^2+b^2)^{(1/2)}-a*e*x/b^2-2*a/b^2/d^2*f*c*\ln(\exp(d*x+c))+2*a/b^2/d*f*c*x+2/ \\
& d*a^2*e/(a^2+b^2)^{(3/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-1 \\
& /2/d^2*a*f/(a^2+b^2)*\operatorname{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) \\
& +1/d^2*a^2*f/(a^2+b^2)^{(3/2)}*\operatorname{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/( \\
& a+(a^2+b^2)^{(1/2)}))-1/d^2*a^2*f/(a^2+b^2)^{(3/2)}*\operatorname{dilog}((-b*\exp(d*x+c)+(a^2+ \\
& b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))-1/2/d^2*a*f/(a^2+b^2)*\operatorname{dilog}(b*\exp(d*x+ \\
& c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))-1/2/d*a*e/(a^2+b^2)*\ln(b*\exp(2*d \\
& *x+2*c))+2*a*\exp(d*x+c)-b)-2/d^2/(a^2+b^2)^{(1/2)}*b^2*f*c/(2*a^2+2*b^2)*\operatorname{arcta} \\
& \operatorname{nh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-2/d^2*f/(2*a^2+2*b^2)*\operatorname{dilog}(1- \\
& I*\exp(d*x+c))*a+1/d^2*f/(2*a^2+2*b^2)*\operatorname{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a) \\
& )/(a+(a^2+b^2)^{(1/2)}))*a+1/d^2*f/(2*a^2+2*b^2)*\operatorname{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)} \\
& )-a)/(-a+(a^2+b^2)^{(1/2)}))*a+a/b^2/d^2*f*c^2+2*a/b^2/d*e*\ln(\exp(d*x+ \\
& c))-2/b^2/d*a^4*f/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)} \\
& )+a)/(a+(a^2+b^2)^{(1/2)}))*x-2/b^2/d^2*a^4*f/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)} \\
& *\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*c+2/b^2/d^2*a^4 \\
& *f*c/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) \\
& +2/b^2/d^2*a^2*f*c/(2*a^2+2*b^2)*(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/2*(2*b* \\
& \exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+2/b^2/d*a^4*f/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)} \\
& )*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*x+2/b^2/d^2*a^4 \\
& *f/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+ \\
& (a^2+b^2)^{(1/2)}))*c-2/b^2/d^2*a^4*f/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)}*\operatorname{dilog}((b* \\
& \exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))+2/b^2/d^2*a^4*f/(2*a^2+2 \\
& *b^2)/(a^2+b^2)^{(1/2)}*\operatorname{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) \\
& -2/b^2/d^2*a^4*f*c/(a^2+b^2)^{(3/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a) \\
& )/(a^2+b^2)^{(1/2)})+1/b^2/d^2*a^4*f/(a^2+b^2)^{(3/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)} \\
& )+a)/(a+(a^2+b^2)^{(1/2)}))*c-1/b^2/d*a^3*f/(a^2+b^2)*\ln((b*\exp(d*x+c)+ \\
& (a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*x-1/b^2/d^2*a^3*f/(a^2+b^2)*\ln((b*e \\
& xp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*c-1/b^2/d*a^4*f/(a^2+b^2)^{(3/2)} \\
& *\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*x-1/b^2/d \\
& ^2*a^4*f/(a^2+b^2)^{(3/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) \\
& )*c+2/d*a^2*f/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)} \\
& )-a)/(-a+(a^2+b^2)^{(1/2)}))*x+2/d^2*a^2*f/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)} \\
& *\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*c-2/d*a^2*f/ \\
& (2*a^2+2*b^2)/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) \\
& )*x-2/d^2*a^2*f/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a) \\
& )/(a+(a^2+b^2)^{(1/2)}))*c-1/b^2/d*a^3*f/(a^2+b^2)*\ln((-b*\exp( \\
& d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*x-1/b^2/d^2*a^3*f/(a^2+b^2) \\
& *\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*c+1/b^2/d*a^4*f \\
& / (a^2+b^2)^{(3/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*x \\
& -2*I*b/d*f/(2*a^2+2*b^2)*\ln(1-I*\exp(d*x+c))*x-2*I*b/d^2*f/(2*a^2+2*b^2)*\ln( \\
& 1-I*\exp(d*x+c))*c+2*I*b/d*f/(2*a^2+2*b^2)*\ln(1+I*\exp(d*x+c))*x+2*I*b/d^2*f/ \\
& (2*a^2+2*b^2)*\ln(1+I*\exp(d*x+c))*c-2/d*e/(2*a^2+2*b^2)*a*\ln(1+\exp(2*d*x+2*c \\
& ))-4/d*e/(2*a^2+2*b^2)*b*\operatorname{arctan}(\exp(d*x+c))+1/d*e/(2*a^2+2*b^2)*a*\ln(b*\exp( \\
& 2*d*x+2*c))+2*a*\exp(d*x+c)-b)-2/d*e/(2*a^2+2*b^2)*(a^2+b^2)^{(1/2)}*\operatorname{arctanh}(1/ \\
& 2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-2/d^2*f/(2*a^2+2*b^2)*\operatorname{dilog}(1+I*\exp
\end{aligned}$$

$$\begin{aligned}
& (d*x+c)) * a - 2/b^2/d*a^2*e/(2*a^2+2*b^2)*(a^2+b^2)^{(1/2)} * \operatorname{arctanh}(1/2*(2*b*\exp \\
& (d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) - 2/b^2/d*a^4*e/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)} * a \\
& \operatorname{rctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) + 1/b^2/d^2*a^3*f*c/(a^2+b^2 \\
& ) * \ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b) + 2/b^2/d*a^4*e/(a^2+b^2)^{(3/2)} * \operatorname{arcta} \\
& \operatorname{nh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) - 1/b^2/d*a^3*e/(a^2+b^2) * \ln(b*e \\
& \exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b) - 2/d^2*a^2*f/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)} * d \\
& \operatorname{ilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) + 2/d^2*a^2*f/(2*a \\
& ^2+2*b^2)/(a^2+b^2)^{(1/2)} * \operatorname{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+ \\
& b^2)^{(1/2)})) + 1/2/d^2*a*f*c/(a^2+b^2) * \ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b) - \\
& 2/d^2*a^2*f*c/(a^2+b^2)^{(3/2)} * \operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1 \\
& /2)}) - 1/2/d*a*f/(a^2+b^2) * \ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2) \\
& ^{(1/2)})) * x - 1/2/d^2*a*f/(a^2+b^2) * \ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+( \\
& a^2+b^2)^{(1/2)})) * c - 1/2/d*a*f/(a^2+b^2) * \ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/ \\
& (a+(a^2+b^2)^{(1/2)})) * x - 1/2/d^2*a*f/(a^2+b^2) * \ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/ \\
& 2)}+a)/(a+(a^2+b^2)^{(1/2)})) * c + 1/d*a^2*f/(a^2+b^2)^{(3/2)} * \ln((b*\exp(d*x+c)+(a^ \\
& 2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) * x + 1/d^2*a^2*f/(a^2+b^2)^{(3/2)} * \ln((b*\exp \\
& (d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) * c - 1/d*a^2*f/(a^2+b^2)^{(3/2)} \\
& ) * \ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) * x - 1/d^2*a^2*f/ \\
& (a^2+b^2)^{(3/2)} * \ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) * \\
& c - 2*I*b/d^2*f/(2*a^2+2*b^2) * \operatorname{dilog}(1-I*\exp(d*x+c)) + 2*I*b/d^2*f/(2*a^2+2*b^2) \\
& * \operatorname{dilog}(1+I*\exp(d*x+c))
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2} \left( \frac{2a^3 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{(a^2b^2 + b^4)d} - \frac{4b \arctan(e^{(-dx-c)})}{(a^2 + b^2)d} + \frac{2a \log(e^{(-2dx-2c)} + 1)}{(a^2 + b^2)d} + \frac{2(dx+c)a}{b^2d} - \frac{e^{(dx+c)}}{bd} + \frac{e^{(-dx-c)}}{bd} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned}
& -1/2*(2*a^3*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/((a^2*b^2 + b^4 \\
& )*d) - 4*b*\arctan(e^{(-d*x - c)})/((a^2 + b^2)*d) + 2*a*\log(e^{(-2*d*x - 2*c)} \\
& + 1)/((a^2 + b^2)*d) + 2*(d*x + c)*a/(b^2*d) - e^{(d*x + c)}/(b*d) + e^{(-d*x \\
& - c)}/(b*d)) * e - 1/4*f*(2*(a*d^2*x^2*e^c - (b*d*x*e^{(2*c)} - b*e^{(2*c)})) * e^{(d* \\
& x) + (b*d*x + b)*e^{(-d*x)}} * e^{(-c)}/(b^2*d^2) - \operatorname{integrate}(-8*(a^4*x*e^{(d*x + \\
& c)} - a^3*b*x)/(a^2*b^3 + b^5 - (a^2*b^3*e^{(2*c)} + b^5*e^{(2*c)})) * e^{(2*d*x)} - \\
& 2*(a^3*b^2*e^c + a*b^4*e^c) * e^{(d*x)}), x) + \operatorname{integrate}(8*(b*x*e^{(d*x + c)} - a \\
& *x)/(a^2 + b^2 + (a^2*e^{(2*c)} + b^2*e^{(2*c)}) * e^{(2*d*x)}), x)
\end{aligned}$$

**Fricas [B]** time = 3.11505, size = 3537, normalized size = 5.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e - ((a^2*b + b^3)*d*f*x + (a^2 \\ & *b + b^3)*d*e - (a^2*b + b^3)*f)*\cosh(d*x + c)^2 - ((a^2*b + b^3)*d*f*x + ( \\ & a^2*b + b^3)*d*e - (a^2*b + b^3)*f)*\sinh(d*x + c)^2 + (a^2*b + b^3)*f - ((a \\ & ^3 + a*b^2)*d^2*f*x^2 + 2*(a^3 + a*b^2)*d^2*e*x + 4*(a^3 + a*b^2)*c*d*e - 2 \\ & *(a^3 + a*b^2)*c^2*f)*\cosh(d*x + c) + 2*(a^3*f*\cosh(d*x + c) + a^3*f*\sinh(d \\ & *x + c))*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\si \\ & nh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 2*(a^3*f*\cosh(d*x + c) + a \\ & ^3*f*\sinh(d*x + c))*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x \\ & + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + ((2*a*b^2*f + 2 \\ & *I*b^3*f)*\cosh(d*x + c) + (2*a*b^2*f + 2*I*b^3*f)*\sinh(d*x + c))*\operatorname{dilog}(I*\co \\ & sh(d*x + c) + I*\sinh(d*x + c)) + ((2*a*b^2*f - 2*I*b^3*f)*\cosh(d*x + c) + ( \\ & 2*a*b^2*f - 2*I*b^3*f)*\sinh(d*x + c))*\operatorname{dilog}(-I*\cosh(d*x + c) - I*\sinh(d*x + \\ & c)) + 2*((a^3*d*e - a^3*c*f)*\cosh(d*x + c) + (a^3*d*e - a^3*c*f)*\sinh(d*x \\ & + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} \\ & + 2*a) + 2*((a^3*d*e - a^3*c*f)*\cosh(d*x + c) + (a^3*d*e - a^3*c*f)*\sinh(d \\ & *x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b \\ & ^2} + 2*a) + 2*((a^3*d*f*x + a^3*c*f)*\cosh(d*x + c) + (a^3*d*f*x + a^3*c*f) \\ & *\sinh(d*x + c))*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) \\ & + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + 2*((a^3*d*f*x + a^3*c*f) \\ & *\cosh(d*x + c) + (a^3*d*f*x + a^3*c*f)*\sinh(d*x + c))*\log(-(a*\cosh(d*x + c) \\ & + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b \\ & ^2} - b)/b) + ((2*a*b^2*d*e + 2*I*b^3*d*e - 2*a*b^2*c*f - 2*I*b^3*c*f)*\cosh \\ & (d*x + c) + (2*a*b^2*d*e + 2*I*b^3*d*e - 2*a*b^2*c*f - 2*I*b^3*c*f)*\sinh(d*x \\ & + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + I) + ((2*a*b^2*d*e - 2*I*b^3*d* \\ & e - 2*a*b^2*c*f + 2*I*b^3*c*f)*\cosh(d*x + c) + (2*a*b^2*d*e - 2*I*b^3*d*e - \\ & 2*a*b^2*c*f + 2*I*b^3*c*f)*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c \\ & ) - I) + ((2*a*b^2*d*f*x - 2*I*b^3*d*f*x + 2*a*b^2*c*f - 2*I*b^3*c*f)*\cosh( \\ & d*x + c) + (2*a*b^2*d*f*x - 2*I*b^3*d*f*x + 2*a*b^2*c*f - 2*I*b^3*c*f)*\sinh \\ & (d*x + c))*\log(I*\cosh(d*x + c) + I*\sinh(d*x + c) + 1) + ((2*a*b^2*d*f*x + 2 \\ & *I*b^3*d*f*x + 2*a*b^2*c*f + 2*I*b^3*c*f)*\cosh(d*x + c) + (2*a*b^2*d*f*x + \\ & 2*I*b^3*d*f*x + 2*a*b^2*c*f + 2*I*b^3*c*f)*\sinh(d*x + c))*\log(-I*\cosh(d*x + \\ & c) - I*\sinh(d*x + c) + 1) - ((a^3 + a*b^2)*d^2*f*x^2 + 2*(a^3 + a*b^2)*d^2 \\ & *e*x + 4*(a^3 + a*b^2)*c*d*e - 2*(a^3 + a*b^2)*c^2*f + 2*((a^2*b + b^3)*d*f \\ & *x + (a^2*b + b^3)*d*e - (a^2*b + b^3)*f)*\cosh(d*x + c))*\sinh(d*x + c))/((a \\ & ^2*b^2 + b^4)*d^2*\cosh(d*x + c) + (a^2*b^2 + b^4)*d^2*\sinh(d*x + c)) \end{aligned}$$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \sinh^2(c + dx) \tanh(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*sinh(c + d*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.409 \quad \int \frac{\sinh^2(c+dx) \tanh(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=89

$$-\frac{a^3 \log(a + b \sinh(c + dx))}{b^2 d (a^2 + b^2)} - \frac{b \tan^{-1}(\sinh(c + dx))}{d (a^2 + b^2)} - \frac{a \log(\cosh(c + dx))}{d (a^2 + b^2)} + \frac{\sinh(c + dx)}{bd}$$

[Out] -((b\*ArcTan[Sinh[c + d\*x]])/((a^2 + b^2)\*d)) - (a\*Log[Cosh[c + d\*x]])/((a^2 + b^2)\*d) - (a^3\*Log[a + b\*Sinh[c + d\*x]])/(b^2\*(a^2 + b^2)\*d) + Sinh[c + d\*x]/(b\*d)

**Rubi [A]** time = 0.195858, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2837, 12, 1629, 635, 203, 260}

$$-\frac{a^3 \log(a + b \sinh(c + dx))}{b^2 d (a^2 + b^2)} - \frac{b \tan^{-1}(\sinh(c + dx))}{d (a^2 + b^2)} - \frac{a \log(\cosh(c + dx))}{d (a^2 + b^2)} + \frac{\sinh(c + dx)}{bd}$$

Antiderivative was successfully verified.

[In] Int[(Sinh[c + d\*x]^2\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] -((b\*ArcTan[Sinh[c + d\*x]])/((a^2 + b^2)\*d)) - (a\*Log[Cosh[c + d\*x]])/((a^2 + b^2)\*d) - (a^3\*Log[a + b\*Sinh[c + d\*x]])/(b^2\*(a^2 + b^2)\*d) + Sinh[c + d\*x]/(b\*d)

### Rule 2837

Int[cos[(e\_.) + (f\_.)\*(x\_.)]^(p\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Dist[1/(b^p\*f), Subst[Int[(a + x)^m\*(c + (d\*x)/b)^n\*(b^2 - x^2)^((p - 1)/2), x], x, b\*S in[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match Q[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 1629

```
Int[(Pq_)*((d_) + (e_)*(x_)^(m_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

### Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

### Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(c + dx) \tanh(c + dx)}{a + b \sinh(c + dx)} dx &= -\frac{b \operatorname{Subst}\left(\int \frac{x^3}{b^3(a+x)(-b^2-x^2)} dx, x, b \sinh(c + dx)\right)}{d} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{x^3}{(a+x)(-b^2-x^2)} dx, x, b \sinh(c + dx)\right)}{b^2 d} \\
&= -\frac{\operatorname{Subst}\left(\int \left(-1 + \frac{a^3}{(a^2+b^2)(a+x)} + \frac{b^4+ab^2x}{(a^2+b^2)(b^2+x^2)}\right) dx, x, b \sinh(c + dx)\right)}{b^2 d} \\
&= -\frac{a^3 \log(a + b \sinh(c + dx))}{b^2 (a^2 + b^2) d} + \frac{\sinh(c + dx)}{bd} - \frac{\operatorname{Subst}\left(\int \frac{b^4+ab^2x}{b^2+x^2} dx, x, b \sinh(c + dx)\right)}{b^2 (a^2 + b^2) d} \\
&= -\frac{a^3 \log(a + b \sinh(c + dx))}{b^2 (a^2 + b^2) d} + \frac{\sinh(c + dx)}{bd} - \frac{a \operatorname{Subst}\left(\int \frac{x}{b^2+x^2} dx, x, b \sinh(c + dx)\right)}{(a^2 + b^2) d} \\
&= -\frac{b \tan^{-1}(\sinh(c + dx))}{(a^2 + b^2) d} - \frac{a \log(\cosh(c + dx))}{(a^2 + b^2) d} - \frac{a^3 \log(a + b \sinh(c + dx))}{b^2 (a^2 + b^2) d} + \frac{\sinh(c + dx)}{bd}
\end{aligned}$$

**Mathematica [C]** time = 0.175769, size = 91, normalized size = 1.02

$$\frac{\frac{2a^3 \log(a+b \sinh(c+dx))}{b^2(a^2+b^2)} + \frac{\log(-\sinh(c+dx)+i)}{a+ib} + \frac{\log(\sinh(c+dx)+i)}{a-ib} - \frac{2 \sinh(c+dx)}{b}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Sinh[c + d\*x]^2\*Tanh[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] -(Log[I - Sinh[c + d\*x]]/(a + I\*b) + Log[I + Sinh[c + d\*x]]/(a - I\*b) + (2\*a^3\*Log[a + b\*Sinh[c + d\*x]])/(b^2\*(a^2 + b^2)) - (2\*Sinh[c + d\*x])/b)/(2\*d)

**Maple [B]** time = 0.06, size = 196, normalized size = 2.2

$$-\frac{1}{bd} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} + \frac{a}{b^2d} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) - \frac{1}{bd} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^{-1} + \frac{a}{b^2d} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) - 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d\*x+c)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] -1/d/b/(tanh(1/2\*d\*x+1/2\*c)+1)+1/d\*a/b^2\*ln(tanh(1/2\*d\*x+1/2\*c)+1)-1/d/b/(tanh(1/2\*d\*x+1/2\*c)-1)+1/d\*a/b^2\*ln(tanh(1/2\*d\*x+1/2\*c)-1)-1/d\*a^3/b^2/(a^2+b^2)\*ln(tanh(1/2\*d\*x+1/2\*c)^2\*a-2\*tanh(1/2\*d\*x+1/2\*c)\*b-a)-8/d/(8\*a^2+8\*b^2)\*a\*ln(tanh(1/2\*d\*x+1/2\*c)^2+1)-16/d/(8\*a^2+8\*b^2)\*b\*arctan(tanh(1/2\*d\*x+1/2\*c))

**Maxima [A]** time = 1.56604, size = 198, normalized size = 2.22

$$\frac{a^3 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{(a^2b^2 + b^4)d} + \frac{2b \arctan(e^{(-dx-c)})}{(a^2 + b^2)d} - \frac{a \log(e^{(-2dx-2c)} + 1)}{(a^2 + b^2)d} - \frac{(dx+c)a}{b^2d} + \frac{e^{(dx+c)}}{2bd} - \frac{e^{(-dx-c)}}{2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

```
[Out] -a^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^2*b^2 + b^4)*d) +
2*b*arctan(e^(-d*x - c))/((a^2 + b^2)*d) - a*log(e^(-2*d*x - 2*c) + 1)/((a^
2 + b^2)*d) - (d*x + c)*a/(b^2*d) + 1/2*e^(d*x + c)/(b*d) - 1/2*e^(-d*x - c
)/(b*d)
```

**Fricas [B]** time = 2.72294, size = 733, normalized size = 8.24

$$2(a^3 + ab^2)dx \cosh(dx + c) - a^2b - b^3 + (a^2b + b^3) \cosh(dx + c)^2 + (a^2b + b^3) \sinh(dx + c)^2 - 4(b^3 \cosh(dx + c) + b^3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] 1/2*(2*(a^3 + a*b^2)*d*x*cosh(d*x + c) - a^2*b - b^3 + (a^2*b + b^3)*cosh(d
*x + c)^2 + (a^2*b + b^3)*sinh(d*x + c)^2 - 4*(b^3*cosh(d*x + c) + b^3*sinh
(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(a^3*cosh(d*x + c) + a
^3*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c
))) - 2*(a*b^2*cosh(d*x + c) + a*b^2*sinh(d*x + c))*log(2*cosh(d*x + c)/(co
sh(d*x + c) - sinh(d*x + c))) + 2*((a^3 + a*b^2)*d*x + (a^2*b + b^3)*cosh(d
*x + c))*sinh(d*x + c)/((a^2*b^2 + b^4)*d*cosh(d*x + c) + (a^2*b^2 + b^4)*
d*sinh(d*x + c))
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(c + dx) \tanh(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*tanh(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)**2*tanh(c + d*x)/(a + b*sinh(c + d*x)), x)
```



**Giac [A]** time = 1.41449, size = 169, normalized size = 1.9

$$\frac{\frac{2a^3 \log(|be^{2dx+2c}+2ae^{dx+c}-b|)}{a^2b^2+b^4} - \frac{2adx}{b^2} + \frac{4b \arctan(e^{dx+c})}{a^2+b^2} + \frac{2a \log(e^{2dx+2c}+1)}{a^2+b^2} - \frac{e^{dx+c}}{b} + \frac{e^{-dx-c}}{b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)^2\*tanh(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] 
$$-1/2*(2*a^3*\log(\text{abs}(b*e^{(2*d*x + 2*c)} + 2*a*e^{(d*x + c)} - b)))/(a^2*b^2 + b^4) - 2*a*d*x/b^2 + 4*b*\arctan(e^{(d*x + c)})/(a^2 + b^2) + 2*a*\log(e^{(2*d*x + 2*c)} + 1)/(a^2 + b^2) - e^{(d*x + c)}/b + e^{(-d*x - c)}/b)/d$$

$$3.410 \quad \int \frac{\sinh^2(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable} \left( \frac{\sinh^2(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[(Sinh[c + d\*x]^2\*Tanh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0891297, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sinh^2(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Sinh[c + d\*x]^2\*Tanh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] Defer[Int] [(Sinh[c + d\*x]^2\*Tanh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\sinh^2(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\sinh^2(c+dx) \tanh(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.002, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Sinh[c + d\*x]^2\*Tanh[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] \$Aborted

**Maple [A]** time = 0.934, size = 0, normalized size = 0.

$$\int \frac{(\sinh(dx+c))^2 \tanh(dx+c)}{(fx+e)(a+b\sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d\*x+c)^2\*tanh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x)

[Out] int(sinh(d\*x+c)^2\*tanh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{e^{\left(-c+\frac{de}{f}\right)} E_1\left(\frac{(fx+e)d}{f}\right)}{2bf} - \frac{e^{\left(c-\frac{de}{f}\right)} E_1\left(-\frac{(fx+e)d}{f}\right)}{2bf} - \frac{a \log(fx+e)}{b^2f} + \frac{1}{4} \int \frac{dx}{a^2b^3e + b^5e + (a^2b^3f + b^5f)x - (a^2b^3ee^{(2c)} + b^5e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)^2\*tanh(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x, algorithm="maxima")

[Out] -1/2\*e^(-c + d\*e/f)\*exp\_integral\_e(1, (f\*x + e)\*d/f)/(b\*f) - 1/2\*e^(c - d\*e/f)\*exp\_integral\_e(1, -(f\*x + e)\*d/f)/(b\*f) - a\*log(f\*x + e)/(b^2\*f) + 1/4\*integrate(-8\*(a^4\*e^(d\*x + c) - a^3\*b)/(a^2\*b^3\*e + b^5\*e + (a^2\*b^3\*f + b^5\*f)\*x - (a^2\*b^3\*e\*e^(2\*c) + b^5\*e\*e^(2\*c) + (a^2\*b^3\*f\*e^(2\*c) + b^5\*f\*e^(2\*c))\*x)\*e^(2\*d\*x) - 2\*(a^3\*b^2\*e\*e^c + a\*b^4\*e\*e^c + (a^3\*b^2\*f\*e^c + a\*b^4\*f\*e^c)\*x)\*e^(d\*x)), x) - 1/4\*integrate(8\*(b\*e^(d\*x + c) - a)/(a^2\*e + b^2\*e + (a^2\*f + b^2\*f)\*x + (a^2\*e\*e^(2\*c) + b^2\*e\*e^(2\*c) + (a^2\*f\*e^(2\*c) + b^2\*f\*e^(2\*c))\*x)\*e^(2\*d\*x)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh(dx+c)^2 \tanh(dx+c)}{afx+ae+(bfx+be)\sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sinh(d*x + c)^2*tanh(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(c + dx) \tanh(c + dx)}{(a + b \sinh(c + dx))(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)**2*tanh(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*tanh(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.411 \quad \int \frac{(e+fx)^3 \sinh(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

Optimal. Leaf size=1294

result too large to display

```
[Out] (a^2*(e + f*x)^3)/(b^3*d) - (e + f*x)^3/(b*d) - (a^4*(e + f*x)^3)/(b^3*(a^2
+ b^2)*d) + (e + f*x)^4/(4*b*f) - (6*a*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/
(b^2*d^2) + (6*a^3*f*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^2*(a^2 + b^2)*d^2)
- (a^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(b*(a^2
+ b^2)^(3/2)*d) + (a^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2])])/(b*(a^2 + b^2)^(3/2)*d) - (3*a^2*f*(e + f*x)^2*Log[1 + E^(2*(c + d
*x))])/(b^3*d^2) + (3*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b*d^2) + (3*
a^4*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(b^3*(a^2 + b^2)*d^2) + ((6*I)*
a*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*d^3) - ((6*I)*a^3*f^2*(e
+ f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) - ((6*I)*a*f^2*
(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b^2*d^3) + ((6*I)*a^3*f^2*(e + f*x)*P
olyLog[2, I*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^3) - (3*a^3*f*(e + f*x)^2*Poly
Log[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^2)
+ (3*a^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
])/(b*(a^2 + b^2)^(3/2)*d^2) - (3*a^2*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d
*x))])/(b^3*d^3) + (3*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(b*d^3) +
(3*a^4*f^2*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(b^3*(a^2 + b^2)*d^3) -
((6*I)*a*f^3*PolyLog[3, (-I)*E^(c + d*x)])/(b^2*d^4) + ((6*I)*a^3*f^3*Poly
Log[3, (-I)*E^(c + d*x)])/(b^2*(a^2 + b^2)*d^4) + ((6*I)*a*f^3*PolyLog[3, I
*E^(c + d*x)])/(b^2*d^4) - ((6*I)*a^3*f^3*PolyLog[3, I*E^(c + d*x)])/(b^2*(
a^2 + b^2)*d^4) + (6*a^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) - (6*a^3*f^2*(e + f*x)*PolyLog[
3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^3) + (
3*a^2*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b^3*d^4) - (3*f^3*PolyLog[3, -E^
(2*(c + d*x))])/(2*b*d^4) - (3*a^4*f^3*PolyLog[3, -E^(2*(c + d*x))])/(2*b^3
*(a^2 + b^2)*d^4) - (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) + (6*a^3*f^3*PolyLog[4, -((b*E^(c + d*x
)))/(a + Sqrt[a^2 + b^2]))])/(b*(a^2 + b^2)^(3/2)*d^4) + (a*(e + f*x)^3*Sech
[c + d*x])/(b^2*d) - (a^3*(e + f*x)^3*Sech[c + d*x])/(b^2*(a^2 + b^2)*d) +
(a^2*(e + f*x)^3*Tanh[c + d*x])/(b^3*d) - ((e + f*x)^3*Tanh[c + d*x])/(b*d)
- (a^4*(e + f*x)^3*Tanh[c + d*x])/(b^3*(a^2 + b^2)*d)
```

---

**Rubi [A]** time = 2.45464, antiderivative size = 1294, normalized size of antiderivative = 1., number of steps used = 53, number of rules used = 18, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$ , Rules used = {5581, 3720, 3718, 2190, 2531, 2282, 6589, 32, 5567, 5451, 4180, 5583,

4184, 5573, 3322, 2264, 6609, 6742}

$$\frac{(e+fx)^3 a^4}{b^3(a^2+b^2)d} + \frac{3f(e+fx)^2 \log(1+e^{2(c+dx)}) a^4}{b^3(a^2+b^2)d^2} + \frac{3f^2(e+fx) \text{PolyLog}(2, -e^{2(c+dx)}) a^4}{b^3(a^2+b^2)d^3} - \frac{3f^3 \text{PolyLog}(3, -e^{2(c+dx)}) a^4}{2b^3(a^2+b^2)d^4}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Sinh[c + d\*x]\*Tanh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (a^2\*(e + f\*x)^3)/(b^3\*d) - (e + f\*x)^3/(b\*d) - (a^4\*(e + f\*x)^3)/(b^3\*(a^2 + b^2)\*d) + (e + f\*x)^4/(4\*b\*f) - (6\*a\*f\*(e + f\*x)^2\*ArcTan[E^(c + d\*x)])/(b^2\*d^2) + (6\*a^3\*f\*(e + f\*x)^2\*ArcTan[E^(c + d\*x)])/(b^2\*(a^2 + b^2)\*d^2) - (a^3\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b\*(a^2 + b^2)^(3/2)\*d) + (a^3\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b\*(a^2 + b^2)^(3/2)\*d) - (3\*a^2\*f\*(e + f\*x)^2\*Log[1 + E^(2\*(c + d\*x))])/(b^3\*d^2) + (3\*f\*(e + f\*x)^2\*Log[1 + E^(2\*(c + d\*x))])/(b\*d^2) + (3\*a^4\*f\*(e + f\*x)^2\*Log[1 + E^(2\*(c + d\*x))])/(b^3\*(a^2 + b^2)\*d^2) + ((6\*I)\*a\*f^2\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)])/(b^2\*d^3) - ((6\*I)\*a^3\*f^2\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)])/(b^2\*(a^2 + b^2)\*d^3) - ((6\*I)\*a\*f^2\*(e + f\*x)\*PolyLog[2, I\*E^(c + d\*x)])/(b^2\*d^3) + ((6\*I)\*a^3\*f^2\*(e + f\*x)\*PolyLog[2, I\*E^(c + d\*x)])/(b^2\*(a^2 + b^2)\*d^3) - (3\*a^3\*f\*(e + f\*x)^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*(a^2 + b^2)^(3/2)\*d^2) + (3\*a^3\*f\*(e + f\*x)^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*(a^2 + b^2)^(3/2)\*d^2) - (3\*a^2\*f^2\*(e + f\*x)\*PolyLog[2, -E^(2\*(c + d\*x))])/(b^3\*d^3) + (3\*f^2\*(e + f\*x)\*PolyLog[2, -E^(2\*(c + d\*x))])/(b\*d^3) + (3\*a^4\*f^2\*(e + f\*x)\*PolyLog[2, -E^(2\*(c + d\*x))])/(b^3\*(a^2 + b^2)\*d^3) - ((6\*I)\*a\*f^3\*PolyLog[3, (-I)\*E^(c + d\*x)])/(b^2\*d^4) + ((6\*I)\*a^3\*f^3\*PolyLog[3, (-I)\*E^(c + d\*x)])/(b^2\*(a^2 + b^2)\*d^4) + ((6\*I)\*a\*f^3\*PolyLog[3, I\*E^(c + d\*x)])/(b^2\*d^4) - ((6\*I)\*a^3\*f^3\*PolyLog[3, I\*E^(c + d\*x)])/(b^2\*(a^2 + b^2)\*d^4) + (6\*a^3\*f^2\*(e + f\*x)\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*(a^2 + b^2)^(3/2)\*d^3) - (6\*a^3\*f^2\*(e + f\*x)\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*(a^2 + b^2)^(3/2)\*d^3) + (3\*a^2\*f^3\*PolyLog[3, -E^(2\*(c + d\*x))])/(2\*b^3\*d^4) - (3\*f^3\*PolyLog[3, -E^(2\*(c + d\*x))])/(2\*b^3\*(a^2 + b^2)\*d^4) - (6\*a^3\*f^3\*PolyLog[4, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*(a^2 + b^2)^(3/2)\*d^4) + (6\*a^3\*f^3\*PolyLog[4, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*(a^2 + b^2)^(3/2)\*d^4) + (a\*(e + f\*x)^3\*Sech[c + d\*x])/(b^2\*d) - (a^3\*(e + f\*x)^3\*Sech[c + d\*x])/(b^2\*(a^2 + b^2)\*d) + (a^2\*(e + f\*x)^3\*Tanh[c + d\*x])/(b^3\*d) - ((e + f\*x)^3\*Tanh[c + d\*x])/(b\*d) - (a^4\*(e + f\*x)^3\*Tanh[c + d\*x])/(b^3\*(a^2 + b^2)\*d)

### Rule 5581

Int[(((e\_.) + (f\_.)\*(x\_.))^(m\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]^(p\_.)\*Tanh[(c\_.) +

```
(d_.)*(x_)^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n)/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

### Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))]/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x]
/; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol]
:> Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(((e + f*x)^m*Sech[c + d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n), x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol]
:> Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4184



```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

### Rule 3322

```
Int[(((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_]
*(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.
)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 6609

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^(c_.)*((a_.) + (b_.
)*(x_)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rubi steps



**Mathematica [A]** time = 13.0382, size = 1111, normalized size = 0.86

$$\left(2e^3 \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)d^3 - f^3x^3 \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}} + 1\right)d^3 - 3ef^2x^2 \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}} + 1\right)d^3 - 3e^2fx \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}} + 1\right)d^3 + f^3\right)$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^3\*Sinh[c + d\*x]\*Tanh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]), x]

[Out] (x\*(4\*e^3 + 6\*e^2\*f\*x + 4\*e\*f^2\*x^2 + f^3\*x^3))/(4\*b) - (f\*(12\*b\*d^3\*e^2\*E^(2\*c)\*x - 12\*b\*d^3\*e^2\*(1 + E^(2\*c))\*x - 12\*b\*d^3\*e\*f\*x^2 - 4\*b\*d^3\*f^2\*x^3 + 12\*a\*d^2\*e^2\*(1 + E^(2\*c))\*ArcTan[E^(c + d\*x)] + 6\*b\*d^2\*e^2\*(1 + E^(2\*c))\*(2\*d\*x - Log[1 + E^(2\*(c + d\*x))]) + (12\*I)\*a\*d\*e\*(1 + E^(2\*c))\*f\*(d\*x\*(Log[1 - I\*E^(c + d\*x)] - Log[1 + I\*E^(c + d\*x)]) - PolyLog[2, (-I)\*E^(c + d\*x)] + PolyLog[2, I\*E^(c + d\*x)]) + 6\*b\*d\*e\*(1 + E^(2\*c))\*f\*(2\*d\*x\*(d\*x - Log[1 + E^(2\*(c + d\*x))]) - PolyLog[2, -E^(2\*(c + d\*x))]) + (6\*I)\*a\*(1 + E^(2\*c))\*f^2\*(d^2\*x^2\*Log[1 - I\*E^(c + d\*x)] - d^2\*x^2\*Log[1 + I\*E^(c + d\*x)] - 2\*d\*x\*PolyLog[2, (-I)\*E^(c + d\*x)] + 2\*d\*x\*PolyLog[2, I\*E^(c + d\*x)] + 2\*PolyLog[3, (-I)\*E^(c + d\*x)] - 2\*PolyLog[3, I\*E^(c + d\*x)]) + b\*(1 + E^(2\*c))\*f^2\*(2\*d^2\*x^2\*(2\*d\*x - 3\*Log[1 + E^(2\*(c + d\*x))]) - 6\*d\*x\*PolyLog[2, -E^(2\*(c + d\*x))] + 3\*PolyLog[3, -E^(2\*(c + d\*x))]))/(2\*(a^2 + b^2)\*d^4\*(1 + E^(2\*c))) + (a^3\*(2\*d^3\*e^3\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]] - 3\*d^3\*e^2\*f\*x\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])] - 3\*d^3\*e\*f^2\*x^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])] - d^3\*f^3\*x^3\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])] + 3\*d^3\*e^2\*f\*x\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] + 3\*d^3\*e\*f^2\*x^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] + d^3\*f^3\*x^3\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] - 3\*d^2\*f\*(e + f\*x)^2\*PolyLog[2, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] + 3\*d^2\*f\*(e + f\*x)^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))] + 6\*d\*e\*f^2\*PolyLog[3, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] + 6\*d\*f^3\*x\*PolyLog[3, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] - 6\*d\*e\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))] - 6\*d\*f^3\*x\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))] - 6\*f^3\*PolyLog[4, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] + 6\*f^3\*PolyLog[4, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*(a^2 + b^2)^(3/2)\*d^4) + ((e + f\*x)^3\*Sech[c + d\*x]\*(a - b\*Sech[c]\*Sinh[d\*x]))/((a^2 + b^2)\*d)

**Maple [F]** time = 0.785, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \sinh(dx + c) (\tanh(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*sinh(d\*x+c)\*tanh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*sinh(d\*x+c)\*tanh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*sinh(d\*x+c)\*tanh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 5.39078, size = 16386, normalized size = 12.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*sinh(d\*x+c)\*tanh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 1/4\*((a^4 + 2\*a^2\*b^2 + b^4)\*d^4\*f^3\*x^4 + 4\*(a^4 + 2\*a^2\*b^2 + b^4)\*d^4\*e\*f^2\*x^3 + 6\*(a^4 + 2\*a^2\*b^2 + b^4)\*d^4\*e^2\*f\*x^2 + 4\*(a^4 + 2\*a^2\*b^2 + b^4)\*d^4\*e^3\*x + 8\*(a^2\*b^2 + b^4)\*d^3\*e^3 - 24\*(a^2\*b^2 + b^4)\*c\*d^2\*e^2\*f + 24\*(a^2\*b^2 + b^4)\*c^2\*d\*e\*f^2 - 8\*(a^2\*b^2 + b^4)\*c^3\*f^3 + ((a^4 + 2\*a^2\*b^2 + b^4)\*d^4\*f^3\*x^4 - 24\*(a^2\*b^2 + b^4)\*c\*d^2\*e^2\*f + 24\*(a^2\*b^2 + b^4)\*c^2\*d\*e\*f^2 - 8\*(a^2\*b^2 + b^4)\*c^3\*f^3 + 4\*((a^4 + 2\*a^2\*b^2 + b^4)\*d^4\*e\*f^2 - 2\*(a^2\*b^2 + b^4)\*d^3\*f^3)\*x^3 + 6\*((a^4 + 2\*a^2\*b^2 + b^4)\*d^4\*e^2\*f - 4\*(a^2\*b^2 + b^4)\*d^3\*e\*f^2)\*x^2 + 4\*((a^4 + 2\*a^2\*b^2 + b^4)\*d^4\*e^3

$$\begin{aligned}
& - 6*(a^2*b^2 + b^4)*d^3*e^2*f)*x)*\cosh(d*x + c)^2 + ((a^4 + 2*a^2*b^2 + b^4)*d^4*f^3*x^4 - 24*(a^2*b^2 + b^4)*c*d^2*e^2*f + 24*(a^2*b^2 + b^4)*c^2*d*e*f^2 - 8*(a^2*b^2 + b^4)*c^3*f^3 + 4*((a^4 + 2*a^2*b^2 + b^4)*d^4*e*f^2 - 2*(a^2*b^2 + b^4)*d^3*f^3)*x^3 + 6*((a^4 + 2*a^2*b^2 + b^4)*d^4*e^2*f - 4*(a^2*b^2 + b^4)*d^3*e*f^2)*x^2 + 4*((a^4 + 2*a^2*b^2 + b^4)*d^4*e^3 - 6*(a^2*b^2 + b^4)*d^3*e^2*f)*x)*\sinh(d*x + c)^2 - 12*(a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f + (a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f)*\cosh(d*x + c)^2 + 2*(a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f)*\cosh(d*x + c)*\sinh(d*x + c) + (a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f)*\sinh(d*x + c)^2)*\sqrt((a^2 + b^2)/b^2)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c)))*\sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*(a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f + (a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f)*\cosh(d*x + c)^2 + 2*(a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f)*\cosh(d*x + c)*\sinh(d*x + c) + (a^3*b*d^2*f^3*x^2 + 2*a^3*b*d^2*e*f^2*x + a^3*b*d^2*e^2*f)*\sinh(d*x + c)^2)*\sqrt((a^2 + b^2)/b^2)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c)))*\sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 4*(a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3 + (a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*\cosh(d*x + c)^2 + 2*(a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c) + (a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*\sinh(d*x + c)^2)*\sqrt((a^2 + b^2)/b^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3 + (a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*\cosh(d*x + c)^2 + 2*(a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c) + (a^3*b*d^3*e^3 - 3*a^3*b*c*d^2*e^2*f + 3*a^3*b*c^2*d*e*f^2 - a^3*b*c^3*f^3)*\sinh(d*x + c)^2)*\sqrt((a^2 + b^2)/b^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3 + (a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*\cosh(d*x + c)^2 + 2*(a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*\sinh(d*x + c)^2)*\sqrt((a^2 + b^2)/b^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c)))*\sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3 + (a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*\cosh(d*x + c)^2 + 2*(a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*\cosh(d*x + c)^2 + 2*(a^3*b*d^3*f^3*x^3 + 3*a^3*b*d^3*e*f^2*x^2 + 3*a^3*b*d^3*e^2*f*x + 3*a^3*b*c*d^2*e^2*f - 3*a^3*b*c^2*d*e*f^2 + a^3*b*c^3*f^3)*\sinh(d*x + c)^2)*\cos
\end{aligned}$$

$$\begin{aligned}
& h(dx + c) \sinh(dx + c) + (a^3 b d^3 f^3 x^3 + 3 a^3 b d^3 e f^2 x^2 + 3 a^3 b d^3 e^2 f x + 3 a^3 b c d^2 e^2 f - 3 a^3 b c^2 d e f^2 + a^3 b c^3 f^3) \sinh(dx + c)^2 \sqrt{(a^2 + b^2)/b^2} \log(-a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b \\
& - 24 (a^3 b f^3 \cosh(dx + c)^2 + 2 a^3 b f^3 \cosh(dx + c) \sinh(dx + c) + a^3 b f^3 \sinh(dx + c)^2 + a^3 b f^3) \sqrt{(a^2 + b^2)/b^2} \operatorname{polylog}(4, (a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2}))/b + 24 (a^3 b f^3 \cosh(dx + c)^2 + 2 a^3 b f^3 \cosh(dx + c) \sinh(dx + c) + a^3 b f^3 \sinh(dx + c)^2 + a^3 b f^3) \sqrt{(a^2 + b^2)/b^2} \operatorname{polylog}(4, (a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2}))/b + 24 (a^3 b d f^3 x + a^3 b d e f^2 + (a^3 b d f^3 x + a^3 b d e f^2) \cosh(dx + c)^2 + 2 (a^3 b d f^3 x + a^3 b d e f^2) \cosh(dx + c) \sinh(dx + c) + (a^3 b d f^3 x + a^3 b d e f^2) \sinh(dx + c)^2) \sqrt{(a^2 + b^2)/b^2} \operatorname{polylog}(3, (a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2}))/b) - 24 (a^3 b d f^3 x + a^3 b d e f^2 + (a^3 b d f^3 x + a^3 b d e f^2) \cosh(dx + c)^2 + 2 (a^3 b d f^3 x + a^3 b d e f^2) \cosh(dx + c) \sinh(dx + c) + (a^3 b d f^3 x + a^3 b d e f^2) \sinh(dx + c)^2) \sqrt{(a^2 + b^2)/b^2} \operatorname{polylog}(3, (a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2}))/b + 8 ((a^3 b + a b^3) d^3 f^3 x^3 + 3 (a^3 b + a b^3) d^3 e f^2 x^2 + 3 (a^3 b + a b^3) d^3 e^2 f x + (a^3 b + a b^3) d^3 e^3) \cosh(dx + c) + (-24 I (a^3 b + a b^3) d f^3 x + 24 (a^2 b^2 + b^4) d f^3 x - 24 I (a^3 b + a b^3) d e f^2 + 24 (a^2 b^2 + b^4) d e f^2 + (-24 I (a^3 b + a b^3) d f^3 x + 24 (a^2 b^2 + b^4) d f^3 x - 24 I (a^3 b + a b^3) d e f^2 + 24 (a^2 b^2 + b^4) d e f^2) \cosh(dx + c)^2 + (-48 I (a^3 b + a b^3) d f^3 x + 48 (a^2 b^2 + b^4) d f^3 x - 48 I (a^3 b + a b^3) d e f^2 + 48 (a^2 b^2 + b^4) d e f^2) \cosh(dx + c) \sinh(dx + c) + (-24 I (a^3 b + a b^3) d f^3 x + 24 (a^2 b^2 + b^4) d f^3 x - 24 I (a^3 b + a b^3) d e f^2 + 24 (a^2 b^2 + b^4) d e f^2) \sinh(dx + c)^2 \operatorname{dilog}(I \cosh(dx + c) + I \sinh(dx + c)) + (24 I (a^3 b + a b^3) d f^3 x + 24 (a^2 b^2 + b^4) d f^3 x + 24 I (a^3 b + a b^3) d e f^2 + 24 (a^2 b^2 + b^4) d e f^2 + (24 I (a^3 b + a b^3) d f^3 x + 24 (a^2 b^2 + b^4) d f^3 x + 24 I (a^3 b + a b^3) d e f^2 + 24 (a^2 b^2 + b^4) d e f^2) \cosh(dx + c)^2 + (48 I (a^3 b + a b^3) d f^3 x + 48 (a^2 b^2 + b^4) d f^3 x + 48 I (a^3 b + a b^3) d e f^2 + 48 (a^2 b^2 + b^4) d e f^2) \cosh(dx + c) \sinh(dx + c) + (24 I (a^3 b + a b^3) d f^3 x + 24 (a^2 b^2 + b^4) d f^3 x + 24 I (a^3 b + a b^3) d e f^2 + 24 (a^2 b^2 + b^4) d e f^2) \sinh(dx + c)^2 \operatorname{dilog}(-I \cosh(dx + c) - I \sinh(dx + c)) + (-12 I (a^3 b + a b^3) d^2 e^2 f + 12 (a^2 b^2 + b^4) d^2 e^2 f + 24 I (a^3 b + a b^3) c d e f^2 - 24 (a^2 b^2 + b^4) c d e f^2 - 12 I (a^3 b + a b^3) c^2 f^3 + 12 (a^2 b^2 + b^4) c^2 f^3 + (-12 I (a^3 b + a b^3) d^2 e^2 f + 12 (a^2 b^2 + b^4) d^2 e^2 f + 24 I (a^3 b + a b^3) c d e f^2 - 24 (a^2 b^2 + b^4) c d e f^2 - 12 I (a^3 b + a b^3) c^2 f^3 + 12 (a^2 b^2 + b^4) c^2 f^3) \cosh(dx + c)^2 + (-24 I (a^3 b + a b^3) d^2 e^2 f + 24 (a^2 b^2 + b^4) d^2 e^2 f + 48 I (a^3 b + a b^3) c d e f^2 - 48 (a^2 b^2 + b^4) c d e f^2 - 24 I (a^3 b + a b^3) c^2 f^3 + 24 (a^2 b^2 + b^4) c^2 f^3) \cosh
\end{aligned}$$

$$\begin{aligned}
& (d*x + c)*\sinh(d*x + c) + (-12*I*(a^3*b + a*b^3)*d^2*e^2*f + 12*(a^2*b^2 + b^4)*d^2*e^2*f + 24*I*(a^3*b + a*b^3)*c*d*e*f^2 - 24*(a^2*b^2 + b^4)*c*d*e*f^2 - 12*I*(a^3*b + a*b^3)*c^2*f^3 + 12*(a^2*b^2 + b^4)*c^2*f^3)*\sinh(d*x + c)^2 * \log(\cosh(d*x + c) + \sinh(d*x + c) + I) + (12*I*(a^3*b + a*b^3)*d^2*e^2*f + 12*(a^2*b^2 + b^4)*d^2*e^2*f - 24*I*(a^3*b + a*b^3)*c*d*e*f^2 - 24*(a^2*b^2 + b^4)*c*d*e*f^2 + 12*I*(a^3*b + a*b^3)*c^2*f^3 + 12*(a^2*b^2 + b^4)*c^2*f^3) * \cosh(d*x + c)^2 + (24*I*(a^3*b + a*b^3)*d^2*e^2*f + 24*(a^2*b^2 + b^4)*d^2*e^2*f - 48*I*(a^3*b + a*b^3)*c*d*e*f^2 - 48*(a^2*b^2 + b^4)*c*d*e*f^2 + 24*I*(a^3*b + a*b^3)*c^2*f^3 + 24*(a^2*b^2 + b^4)*c^2*f^3)*\cosh(d*x + c)*\sinh(d*x + c) + (12*I*(a^3*b + a*b^3)*d^2*e^2*f + 12*(a^2*b^2 + b^4)*d^2*e^2*f - 24*I*(a^3*b + a*b^3)*c*d*e*f^2 - 24*(a^2*b^2 + b^4)*c*d*e*f^2 + 12*I*(a^3*b + a*b^3)*c^2*f^3 + 12*(a^2*b^2 + b^4)*c^2*f^3)*\sinh(d*x + c)^2 * \log(\cosh(d*x + c) + \sinh(d*x + c) - I) + (12*I*(a^3*b + a*b^3)*d^2*f^3*x^2 + 12*(a^2*b^2 + b^4)*d^2*f^3*x^2 + 24*I*(a^3*b + a*b^3)*d^2*e*f^2*x + 24*(a^2*b^2 + b^4)*d^2*e*f^2*x + 24*I*(a^3*b + a*b^3)*c*d*e*f^2 + 24*(a^2*b^2 + b^4)*c*d*e*f^2 - 12*I*(a^3*b + a*b^3)*c^2*f^3 - 12*(a^2*b^2 + b^4)*c^2*f^3 + (12*I*(a^3*b + a*b^3)*d^2*f^3*x^2 + 12*(a^2*b^2 + b^4)*d^2*f^3*x^2 + 24*I*(a^3*b + a*b^3)*d^2*e*f^2*x + 24*(a^2*b^2 + b^4)*d^2*e*f^2*x + 24*I*(a^3*b + a*b^3)*c*d*e*f^2 + 24*(a^2*b^2 + b^4)*c*d*e*f^2 - 12*I*(a^3*b + a*b^3)*c^2*f^3 - 12*(a^2*b^2 + b^4)*c^2*f^3)*\cosh(d*x + c)^2 + (24*I*(a^3*b + a*b^3)*d^2*f^3*x^2 + 24*(a^2*b^2 + b^4)*d^2*f^3*x^2 + 48*I*(a^3*b + a*b^3)*d^2*e*f^2*x + 48*(a^2*b^2 + b^4)*d^2*e*f^2*x + 48*I*(a^3*b + a*b^3)*c*d*e*f^2 + 48*(a^2*b^2 + b^4)*c*d*e*f^2 - 24*I*(a^3*b + a*b^3)*c^2*f^3 - 24*(a^2*b^2 + b^4)*c^2*f^3)*\cosh(d*x + c)*\sinh(d*x + c) + (12*I*(a^3*b + a*b^3)*d^2*f^3*x^2 + 12*(a^2*b^2 + b^4)*d^2*f^3*x^2 + 24*I*(a^3*b + a*b^3)*d^2*e*f^2*x + 24*(a^2*b^2 + b^4)*d^2*e*f^2*x + 24*I*(a^3*b + a*b^3)*c*d*e*f^2 + 24*(a^2*b^2 + b^4)*c*d*e*f^2 - 12*I*(a^3*b + a*b^3)*c^2*f^3 - 12*(a^2*b^2 + b^4)*c^2*f^3)*\sinh(d*x + c)^2 * \log(I*\cosh(d*x + c) + I*\sinh(d*x + c) + 1) + (-12*I*(a^3*b + a*b^3)*d^2*f^3*x^2 + 12*(a^2*b^2 + b^4)*d^2*f^3*x^2 - 24*I*(a^3*b + a*b^3)*d^2*e*f^2*x + 24*(a^2*b^2 + b^4)*d^2*e*f^2*x - 24*I*(a^3*b + a*b^3)*c*d*e*f^2 + 24*(a^2*b^2 + b^4)*c*d*e*f^2 + 12*I*(a^3*b + a*b^3)*c^2*f^3 - 12*(a^2*b^2 + b^4)*c^2*f^3 + (-12*I*(a^3*b + a*b^3)*d^2*f^3*x^2 + 12*(a^2*b^2 + b^4)*d^2*f^3*x^2 - 24*I*(a^3*b + a*b^3)*d^2*e*f^2*x + 24*(a^2*b^2 + b^4)*d^2*e*f^2*x - 24*I*(a^3*b + a*b^3)*c*d*e*f^2 + 24*(a^2*b^2 + b^4)*c*d*e*f^2 + 12*I*(a^3*b + a*b^3)*c^2*f^3 - 12*(a^2*b^2 + b^4)*c^2*f^3)*\cosh(d*x + c)*\sinh(d*x + c) + (-12*I*(a^3*b + a*b^3)*d^2*f^3*x^2 + 12*(a^2*b^2 + b^4)*d^2*f^3*x^2 - 24*I*(a^3*b + a*b^3)*d^2*e*f^2*x + 24*(a^2*b^2 + b^4)*d^2*e*f^2*x - 24*I*(a^3*b + a*b^3)*c*d*e*f^2 + 24*(a^2*b^2 + b^4)*c*d*e*f^2 + 12*I*(a^3*b + a*b^3)*c^2*f^3 - 12*(a^2*b^2 + b^4)*c^2*f^3)
\end{aligned}$$

```

*sinh(d*x + c)^2)*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) + (24*I*(a^3*
b + a*b^3)*f^3 - 24*(a^2*b^2 + b^4)*f^3 + (24*I*(a^3*b + a*b^3)*f^3 - 24*(a
^2*b^2 + b^4)*f^3)*cosh(d*x + c)^2 + (48*I*(a^3*b + a*b^3)*f^3 - 48*(a^2*b^
2 + b^4)*f^3)*cosh(d*x + c)*sinh(d*x + c) + (24*I*(a^3*b + a*b^3)*f^3 - 24*
(a^2*b^2 + b^4)*f^3)*sinh(d*x + c)^2)*polylog(3, I*cosh(d*x + c) + I*sinh(d
*x + c)) + (-24*I*(a^3*b + a*b^3)*f^3 - 24*(a^2*b^2 + b^4)*f^3 + (-24*I*(a^
3*b + a*b^3)*f^3 - 24*(a^2*b^2 + b^4)*f^3)*cosh(d*x + c)^2 + (-48*I*(a^3*b
+ a*b^3)*f^3 - 48*(a^2*b^2 + b^4)*f^3)*cosh(d*x + c)*sinh(d*x + c) + (-24*I
*(a^3*b + a*b^3)*f^3 - 24*(a^2*b^2 + b^4)*f^3)*sinh(d*x + c)^2)*polylog(3,
-I*cosh(d*x + c) - I*sinh(d*x + c)) + 2*(4*(a^3*b + a*b^3)*d^3*f^3*x^3 + 12
*(a^3*b + a*b^3)*d^3*e*f^2*x^2 + 12*(a^3*b + a*b^3)*d^3*e^2*f*x + 4*(a^3*b
+ a*b^3)*d^3*e^3 + ((a^4 + 2*a^2*b^2 + b^4)*d^4*f^3*x^4 - 24*(a^2*b^2 + b^4
)*c*d^2*e^2*f + 24*(a^2*b^2 + b^4)*c^2*d*e*f^2 - 8*(a^2*b^2 + b^4)*c^3*f^3
+ 4*((a^4 + 2*a^2*b^2 + b^4)*d^4*e*f^2 - 2*(a^2*b^2 + b^4)*d^3*f^3)*x^3 + 6
*((a^4 + 2*a^2*b^2 + b^4)*d^4*e^2*f - 4*(a^2*b^2 + b^4)*d^3*e*f^2)*x^2 + 4*
((a^4 + 2*a^2*b^2 + b^4)*d^4*e^3 - 6*(a^2*b^2 + b^4)*d^3*e^2*f)*x)*cosh(d*x
+ c))*sinh(d*x + c))/((a^4*b + 2*a^2*b^3 + b^5)*d^4*cosh(d*x + c)^2 + 2*(a
^4*b + 2*a^2*b^3 + b^5)*d^4*cosh(d*x + c)*sinh(d*x + c) + (a^4*b + 2*a^2*b^
3 + b^5)*d^4*sinh(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*d^4)

```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^3 \sinh(c + dx) \tanh^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*sinh(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**3*sinh(c + d*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x)),
x)
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorith
m="giac")
```



[Out] Timed out

$$3.412 \quad \int \frac{(e+fx)^2 \sinh(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=904

$$-\frac{(e+fx)^2 a^4}{b^3(a^2+b^2)d} + \frac{2f(e+fx) \log(1+e^{2(c+dx)}) a^4}{b^3(a^2+b^2)d^2} + \frac{f^2 \text{PolyLog}(2, -e^{2(c+dx)}) a^4}{b^3(a^2+b^2)d^3} - \frac{(e+fx)^2 \tanh(c+dx) a^4}{b^3(a^2+b^2)d} + \frac{4f(e+fx)}{b^2}$$

[Out] (a^2\*(e + f\*x)^2)/(b^3\*d) - (e + f\*x)^2/(b\*d) - (a^4\*(e + f\*x)^2)/(b^3\*(a^2 + b^2)\*d) + (e + f\*x)^3/(3\*b\*f) - (4\*a\*f\*(e + f\*x)\*ArcTan[E^(c + d\*x)])/(b^2\*d^2) + (4\*a^3\*f\*(e + f\*x)\*ArcTan[E^(c + d\*x)])/(b^2\*(a^2 + b^2)\*d^2) - (a^3\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b\*(a^2 + b^2)^(3/2)\*d) + (a^3\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b\*(a^2 + b^2)^(3/2)\*d) - (2\*a^2\*f\*(e + f\*x)\*Log[1 + E^(2\*(c + d\*x))])/(b^3\*d^2) + (2\*f\*(e + f\*x)\*Log[1 + E^(2\*(c + d\*x))])/(b\*d^2) + (2\*a^4\*f\*(e + f\*x)\*Log[1 + E^(2\*(c + d\*x))])/(b^3\*(a^2 + b^2)\*d^2) + ((2\*I)\*a\*f^2\*PolyLog[2, (-I)\*E^(c + d\*x)])/(b^2\*d^3) - ((2\*I)\*a^3\*f^2\*PolyLog[2, (-I)\*E^(c + d\*x)])/(b^2\*(a^2 + b^2)\*d^3) - ((2\*I)\*a\*f^2\*PolyLog[2, I\*E^(c + d\*x)])/(b^2\*d^3) + ((2\*I)\*a^3\*f^2\*PolyLog[2, I\*E^(c + d\*x)])/(b^2\*(a^2 + b^2)\*d^3) - (2\*a^3\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*(a^2 + b^2)^(3/2)\*d^2) + (2\*a^3\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*(a^2 + b^2)^(3/2)\*d^2) - (a^2\*f^2\*PolyLog[2, -E^(2\*(c + d\*x))])/(b^3\*d^3) + (f^2\*PolyLog[2, -E^(2\*(c + d\*x))])/(b\*d^3) + (a^4\*f^2\*PolyLog[2, -E^(2\*(c + d\*x))])/(b^3\*(a^2 + b^2)\*d^3) + (2\*a^3\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*(a^2 + b^2)^(3/2)\*d^3) - (2\*a^3\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*(a^2 + b^2)^(3/2)\*d^3) + (a\*(e + f\*x)^2\*Sech[c + d\*x])/(b^2\*d) - (a^3\*(e + f\*x)^2\*Sech[c + d\*x])/(b^2\*(a^2 + b^2)\*d) + (a^2\*(e + f\*x)^2\*Tanh[c + d\*x])/(b^3\*d) - ((e + f\*x)^2\*Tanh[c + d\*x])/(b\*d) - (a^4\*(e + f\*x)^2\*Tanh[c + d\*x])/(b^3\*(a^2 + b^2)\*d)

**Rubi [A]** time = 1.82137, antiderivative size = 904, normalized size of antiderivative = 1., number of steps used = 44, number of rules used = 19, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.559$ , Rules used = {5581, 3720, 3718, 2190, 2279, 2391, 32, 5567, 5451, 4180, 5583, 4184, 5573, 3322, 2264, 2531, 2282, 6589, 6742}

$$-\frac{(e+fx)^2 a^4}{b^3(a^2+b^2)d} + \frac{2f(e+fx) \log(1+e^{2(c+dx)}) a^4}{b^3(a^2+b^2)d^2} + \frac{f^2 \text{PolyLog}(2, -e^{2(c+dx)}) a^4}{b^3(a^2+b^2)d^3} - \frac{(e+fx)^2 \tanh(c+dx) a^4}{b^3(a^2+b^2)d} + \frac{4f(e+fx)}{b^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Sinh[c + d\*x]\*Tanh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out]  $(a^2(e + fx)^2)/(b^3d) - (e + fx)^2/(bd) - (a^4(e + fx)^2)/(b^3(a^2 + b^2)d) + (e + fx)^3/(3bf) - (4a^3f(e + fx)*\text{ArcTan}[E^{(c + dx)}])/(b^2d^2) + (4a^3f(e + fx)*\text{ArcTan}[E^{(c + dx)}])/(b^2(a^2 + b^2)d^2) - (a^3(e + fx)^2*\text{Log}[1 + (bE^{(c + dx)})/(a - \text{Sqrt}[a^2 + b^2])])/(b(a^2 + b^2)^{3/2}d) + (a^3(e + fx)^2*\text{Log}[1 + (bE^{(c + dx)})/(a + \text{Sqrt}[a^2 + b^2])])/(b(a^2 + b^2)^{3/2}d) - (2a^2f(e + fx)*\text{Log}[1 + E^{(2(c + dx))}])/(b^3d^2) + (2f(e + fx)*\text{Log}[1 + E^{(2(c + dx))}])/(bd^2) + (2a^4f(e + fx)*\text{Log}[1 + E^{(2(c + dx))}])/(b^3(a^2 + b^2)d^2) + ((2I)a^4f^2*\text{PolyLog}[2, (-I)E^{(c + dx)}])/(b^2d^3) - ((2I)a^3f^2*\text{PolyLog}[2, (-I)E^{(c + dx)}])/(b^2(a^2 + b^2)d^3) - ((2I)a^3f^2*\text{PolyLog}[2, IE^{(c + dx)}])/(b^2d^3) + ((2I)a^3f^2*\text{PolyLog}[2, IE^{(c + dx)}])/(b^2(a^2 + b^2)d^3) - (2a^3f(e + fx)*\text{PolyLog}[2, -(bE^{(c + dx)})/(a - \text{Sqrt}[a^2 + b^2])])/(b(a^2 + b^2)^{3/2}d^2) + (2a^3f(e + fx)*\text{PolyLog}[2, -(bE^{(c + dx)})/(a + \text{Sqrt}[a^2 + b^2])])/(b(a^2 + b^2)^{3/2}d^2) - (a^2f^2*\text{PolyLog}[2, -E^{(2(c + dx))}])/(b^3d^3) + (f^2*\text{PolyLog}[2, -E^{(2(c + dx))}])/(bd^3) + (a^4f^2*\text{PolyLog}[2, -E^{(2(c + dx))}])/(b^3(a^2 + b^2)d^3) + (2a^3f^2*\text{PolyLog}[3, -(bE^{(c + dx)})/(a - \text{Sqrt}[a^2 + b^2])])/(b(a^2 + b^2)^{3/2}d^3) - (2a^3f^2*\text{PolyLog}[3, -(bE^{(c + dx)})/(a + \text{Sqrt}[a^2 + b^2])])/(b(a^2 + b^2)^{3/2}d^3) + (a(e + fx)^2*\text{Sech}[c + d*x])/(b^2d) - (a^3(e + fx)^2*\text{Sech}[c + d*x])/(b^2(a^2 + b^2)d) + (a^2(e + fx)^2*\text{Tanh}[c + d*x])/(b^3d) - ((e + fx)^2*\text{Tanh}[c + d*x])/(bd) - (a^4(e + fx)^2*\text{Tanh}[c + d*x])/(b^3(a^2 + b^2)d)$

### Rule 5581

Int[(((e\_) + (f\_)\*(x\_))^(m\_)\*Sinh[(c\_) + (d\_)\*(x\_)]^(p\_)\*Tanh[(c\_) + (d\_)\*(x\_)]^(n\_))/((a\_) + (b\_)\*Sinh[(c\_) + (d\_)\*(x\_)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Sinh[c + d\*x]^(p - 1)\*Tanh[c + d\*x]^n, x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sinh[c + d\*x]^(p - 1)\*Tanh[c + d\*x]^n)/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 3720

Int[((c\_) + (d\_)\*(x\_))^(m\_)\*((b\_)\*tan[(e\_) + (f\_)\*(x\_)]^(n\_)), x\_Symbol] := Simp[(b\*(c + d\*x)^m\*(b\*Tan[e + f\*x])^(n - 1))/(f\*(n - 1)), x] + (-Dist[(b\*d\*m)/(f\*(n - 1)), Int[(c + d\*x)^(m - 1)\*(b\*Tan[e + f\*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d\*x)^m\*(b\*Tan[e + f\*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

### Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol]
:> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))]/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol]
:> Simp[(((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n])/a)]/(b*f*g*n*Log[F]), x] - Dist[
(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n])/a], x], x] /;
FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /;
FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol]
:> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol]
:> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rule 5567

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[
(c_.) + (d_.)*(x_)]), x_Symbol]
:> Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[
((e + f*x)^m*Sech[c + d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

### Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol]
:> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n), x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[(((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```

2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*(a\_.) + (b\_.)\*(x\_)))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_) /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n]] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \sinh(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \tanh^2(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)^2 \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx}{b} \\
&= -\frac{(e+fx)^2 \tanh(c+dx)}{bd} - \frac{a \int (e+fx)^2 \operatorname{sech}(c+dx) \tanh(c+dx) dx}{b^2} + \frac{a^2 \int (e+fx)^2 \operatorname{sech}^2(c+dx) dx}{b^2} \\
&= -\frac{(e+fx)^2}{bd} + \frac{(e+fx)^3}{3bf} + \frac{a(e+fx)^2 \operatorname{sech}(c+dx)}{b^2 d} - \frac{(e+fx)^2 \tanh(c+dx)}{bd} + \frac{a^2 (e+fx)^2 \operatorname{sech}^2(c+dx)}{b^2} \\
&= -\frac{(e+fx)^2}{bd} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx) \tan^{-1}(e^{c+dx})}{b^2 d^2} + \frac{2f(e+fx) \log(1+e^{c+dx})}{bd^2} \\
&= \frac{a^2(e+fx)^2}{b^3 d} - \frac{(e+fx)^2}{bd} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx) \tan^{-1}(e^{c+dx})}{b^2 d^2} + \frac{2f(e+fx) \log(1+e^{c+dx})}{bd^2} \\
&= \frac{a^2(e+fx)^2}{b^3 d} - \frac{(e+fx)^2}{bd} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx) \tan^{-1}(e^{c+dx})}{b^2 d^2} - \frac{2a^2 f(e+fx) \log(1+e^{c+dx})}{b^2 d^2} \\
&= \frac{a^2(e+fx)^2}{b^3 d} - \frac{(e+fx)^2}{bd} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx) \tan^{-1}(e^{c+dx})}{b^2 d^2} - \frac{a^3(e+fx) \log(1+e^{c+dx})}{b^2 d^2} \\
&= \frac{a^2(e+fx)^2}{b^3 d} - \frac{(e+fx)^2}{bd} - \frac{a^4(e+fx)^2}{b^3(a^2+b^2)d} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx) \tan^{-1}(e^{c+dx})}{b^2 d^2} \\
&= \frac{a^2(e+fx)^2}{b^3 d} - \frac{(e+fx)^2}{bd} - \frac{a^4(e+fx)^2}{b^3(a^2+b^2)d} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx) \tan^{-1}(e^{c+dx})}{b^2 d^2} \\
&= \frac{a^2(e+fx)^2}{b^3 d} - \frac{(e+fx)^2}{bd} - \frac{a^4(e+fx)^2}{b^3(a^2+b^2)d} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx) \tan^{-1}(e^{c+dx})}{b^2 d^2} \\
&= \frac{a^2(e+fx)^2}{b^3 d} - \frac{(e+fx)^2}{bd} - \frac{a^4(e+fx)^2}{b^3(a^2+b^2)d} + \frac{(e+fx)^3}{3bf} - \frac{4af(e+fx) \tan^{-1}(e^{c+dx})}{b^2 d^2}
\end{aligned}$$

**Mathematica [A]** time = 8.54056, size = 937, normalized size = 1.04

$$\left(2e^2 \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right)d^2 - f^2x^2 \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}} + 1\right)d^2 - 2efx \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}} + 1\right)d^2 + f^2x^2 \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}} + 1\right)d^2 + 2efx \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}} + 1\right)d^2\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]), x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2))/(3*b) + (a^3*(2*d^2*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]) + 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*f^2*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(b*(a^2 + b^2)^(3/2)*d^3) + (2*b*e*f*Sech[c]*(Cosh[c]*Log[Cosh[c]*Cosh[d*x] + Sinh[c]*Sinh[d*x]] - d*x*Sinh[c]))/(a^2 + b^2)*d^2*(Cosh[c]^2 - Sinh[c]^2) - (4*a*e*f*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]])/((a^2 + b^2)*d^2*Sqrt[Cosh[c]^2 - Sinh[c]^2]) - (b*f^2*Csch[c]*(-(d^2*x^2)/E^ArcTanh[Coth[c]]) + (I*Coth[c]*(-d*x*(-Pi + (2*I)*ArcTanh[Coth[c]])) - Pi*Log[1 + E^(2*d*x)] - 2*(I*d*x + I*ArcTanh[Coth[c]])*Log[1 - E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]])]) + Pi*Log[Cosh[d*x]] + (2*I)*ArcTanh[Coth[c]]*Log[I*Sinh[d*x] + ArcTanh[Coth[c]]]) + I*PolyLog[2, E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]])])])/Sqrt[1 - Coth[c]^2])*Sech[c])/((a^2 + b^2)*d^3*Sqrt[Csch[c]^2*(-Cosh[c]^2 + Sinh[c]^2)]) - (2*a*f^2*(((I)*Csch[c]*(I*(d*x + ArcTanh[Coth[c]])*(Log[1 - E^(-(d*x) - ArcTanh[Coth[c]])] - Log[1 + E^(-(d*x) - ArcTanh[Coth[c]])]) + I*(PolyLog[2, -E^(-(d*x) - ArcTanh[Coth[c]])] - PolyLog[2, E^(-(d*x) - ArcTanh[Coth[c]])])])/Sqrt[1 - Coth[c]^2] - (2*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]]*ArcTanh[Coth[c]])/Sqrt[Cosh[c]^2 - Sinh[c]^2])/((a^2 + b^2)*d^3) + (Sech[c]*Sech[c + d*x]*(a*e^2*Cosh[c] + 2*a*e*f*x*Cosh[c] + a*f^2*x^2*Cosh[c] - b*e^2*Sinh[d*x] - 2*b*e*f*x*Sinh[d*x] - b*f^2*x^2*Sinh[d*x]))/((a^2 + b^2)*d
```



**Maple [F]** time = 0.651, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \sinh(dx + c) (\tanh(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*sinh(d\*x+c)\*tanh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*sinh(d\*x+c)\*tanh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sinh(d\*x+c)\*tanh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 4.13807, size = 9671, normalized size = 10.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*sinh(d\*x+c)\*tanh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 1/6\*(2\*(a^4 + 2\*a^2\*b^2 + b^4)\*d^3\*f^2\*x^3 + 6\*(a^4 + 2\*a^2\*b^2 + b^4)\*d^3\*e\*f\*x^2 + 6\*(a^4 + 2\*a^2\*b^2 + b^4)\*d^3\*e^2\*x + 12\*(a^2\*b^2 + b^4)\*d^2\*e^2 - 24\*(a^2\*b^2 + b^4)\*c\*d\*e\*f + 12\*(a^2\*b^2 + b^4)\*c^2\*f^2 + 2\*((a^4 + 2\*a^2\*b^2 + b^4)\*d^3\*f^2\*x^3 - 12\*(a^2\*b^2 + b^4)\*c\*d\*e\*f + 6\*(a^2\*b^2 + b^4)\*c^2\*f^2 + 3\*((a^4 + 2\*a^2\*b^2 + b^4)\*d^3\*e\*f - 2\*(a^2\*b^2 + b^4)\*d^2\*f^2)\*x^2 + 3\*((a^4 + 2\*a^2\*b^2 + b^4)\*d^3\*e^2 - 4\*(a^2\*b^2 + b^4)\*d^2\*e\*f)\*x)\*cosh(d\*x + c)^2 + 2\*((a^4 + 2\*a^2\*b^2 + b^4)\*d^3\*f^2\*x^3 - 12\*(a^2\*b^2 + b^4)\*c\*d\*e\*f + 6\*(a^2\*b^2 + b^4)\*c^2\*f^2 + 3\*((a^4 + 2\*a^2\*b^2 + b^4)\*d^3\*e\*f - 2\*

$$\begin{aligned}
& (a^2b^2 + b^4)d^2f^2)x^2 + 3*((a^4 + 2a^2b^2 + b^4)d^3e^2 - 4*(a^2b^2 + b^4)d^2ef)x) * \sinh(dx + c)^2 - 12*(a^3bdf^2x + a^3bdeff + \\
& (a^3bdf^2x + a^3bdeff) * \cosh(dx + c)^2 + 2*(a^3bdf^2x + a^3bdeff) * \cosh(dx + c) * \sinh(dx + c) + (a^3bdf^2x + a^3bdeff) * \sinh(dx + \\
& c)^2) * \sqrt{(a^2 + b^2)/b^2} * \operatorname{dilog}((a * \cosh(dx + c) + a * \sinh(dx + c) + (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 12*(a^3bdf^2x + a^3bdeff + (a^3bdf^2x + a^3bdeff) * \cosh(dx + c)^2 + \\
& 2*(a^3bdf^2x + a^3bdeff) * \cosh(dx + c) * \sinh(dx + c) + (a^3bdf^2x + a^3bdeff) * \sinh(dx + c)^2) * \sqrt{(a^2 + b^2)/b^2} * \operatorname{dilog}((a * \cosh(dx + \\
& c) + a * \sinh(dx + c) - (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 6*(a^3bd^2e^2 - 2a^3b * c * deff + a^3b * c^2f^2 + \\
& (a^3bd^2e^2 - 2a^3b * c * deff + a^3b * c^2f^2) * \cosh(dx + c)^2 + 2*(a^3bd^2e^2 - 2a^3b * c * deff + a^3b * c^2f^2) * \cosh(dx + c) * \sinh(dx + c) + \\
& (a^3bd^2e^2 - 2a^3b * c * deff + a^3b * c^2f^2) * \sinh(dx + c)^2) * \sqrt{(a^2 + b^2)/b^2} * \log(2 * b * \cosh(dx + c) + 2 * b * \sinh(dx + c) + 2 * b * \sqrt{(a^2 + b^2)/b^2} + 2 * a) - 6*(a^3bd^2e^2 - 2a^3b * c * deff + a^3b * c^2f^2 + (a^3bd^2e^2 - 2a^3b * c * deff + a^3b * c^2f^2) * \cosh(dx + c)^2 + 2*(a^3bd^2e^2 - 2a^3b * c * deff + a^3b * c^2f^2) * \cosh(dx + c) * \sinh(dx + c) + (a^3bd^2e^2 - 2a^3b * c * deff + a^3b * c^2f^2) * \sinh(dx + c)^2) * \sqrt{(a^2 + b^2)/b^2} * \log(2 * b * \cosh(dx + c) + 2 * b * \sinh(dx + c) - 2 * b * \sqrt{(a^2 + b^2)/b^2} + 2 * a) - 6*(a^3bd^2f^2x^2 + 2a^3bd^2 * e * f * x + 2a^3b * c * deff - a^3b * c^2f^2 + (a^3bd^2f^2x^2 + 2a^3bd^2 * e * f * x + 2a^3b * c * deff - a^3b * c^2f^2) * \cosh(dx + c)^2 + 2*(a^3bd^2f^2x^2 + 2a^3bd^2 * e * f * x + 2a^3b * c * deff - a^3b * c^2f^2) * \cosh(dx + c) * \sinh(dx + c) + (a^3bd^2f^2x^2 + 2a^3bd^2 * e * f * x + 2a^3b * c * deff - a^3b * c^2f^2) * \sinh(dx + c)^2) * \sqrt{(a^2 + b^2)/b^2} * \log(-(a * \cosh(dx + c) + a * \sinh(dx + c) + (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2} - b)/b) + 6*(a^3bd^2f^2x^2 + 2a^3bd^2 * e * f * x + 2a^3b * c * deff - a^3b * c^2f^2 + (a^3bd^2f^2x^2 + 2a^3bd^2 * e * f * x + 2a^3b * c * deff - a^3b * c^2f^2) * \cosh(dx + c)^2 + 2*(a^3bd^2f^2x^2 + 2a^3bd^2 * e * f * x + 2a^3b * c * deff - a^3b * c^2f^2) * \cosh(dx + c) * \sinh(dx + c) + (a^3bd^2f^2x^2 + 2a^3bd^2 * e * f * x + 2a^3b * c * deff - a^3b * c^2f^2) * \sinh(dx + c)^2) * \sqrt{(a^2 + b^2)/b^2} * \log(-(a * \cosh(dx + c) + a * \sinh(dx + c) - (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2} - b)/b) + 12*(a^3bf^2 * \cosh(dx + c)^2 + 2a^3bf^2 * \cosh(dx + c) * \sinh(dx + c) + a^3bf^2 * \sinh(dx + c)^2 + a^3bf^2) * \sqrt{(a^2 + b^2)/b^2} * \operatorname{polylog}(3, (a * \cosh(dx + c) + a * \sinh(dx + c) + (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2}))/b) - 12*(a^3bf^2 * \cosh(dx + c)^2 + 2a^3bf^2 * \cosh(dx + c) * \sinh(dx + c) + a^3bf^2 * \sinh(dx + c)^2 + a^3bf^2) * \sqrt{(a^2 + b^2)/b^2} * \operatorname{polylog}(3, (a * \cosh(dx + c) + a * \sinh(dx + c) - (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2}))/b) + 12*((a^3b + a * b^3) * d^2f^2x^2 + 2*(a^3b + a * b^3) * d^2 * e * f * x + (a^3b + a * b^3) * d^2 * e^2) * \cosh(dx + c) + (-12 * I * (a^3b + a * b^3) * f^2 + 12 * (a^2 * b^2 + b^4) * f^2 + (-12 * I * (a^3b + a * b^3) * f^2 + 12 * (a^2 * b^2 + b^4) * f^2) * \cosh(dx + c)^2 + (-24 * I * (a^3b + a * b^3) * f^2 + 24 * (a^2 * b^2 + b^4) * f^2) * \cosh(dx + c) * \sinh(dx + c) + (-12 * I * (a^3b + a * b^3) * f^2 + 12 * (a^2 * b^2 + b^4) * f^2) * \sinh(dx + c)
\end{aligned}$$

$$\begin{aligned}
& d*x + c)^2 * \operatorname{dilog}(I * \cosh(d*x + c) + I * \sinh(d*x + c)) + (12 * I * (a^3 * b + a * b^3) \\
& ) * f^2 + 12 * (a^2 * b^2 + b^4) * f^2 + (12 * I * (a^3 * b + a * b^3) * f^2 + 12 * (a^2 * b^2 + \\
& b^4) * f^2) * \cosh(d*x + c)^2 + (24 * I * (a^3 * b + a * b^3) * f^2 + 24 * (a^2 * b^2 + b^4) * \\
& f^2) * \cosh(d*x + c) * \sinh(d*x + c) + (12 * I * (a^3 * b + a * b^3) * f^2 + 12 * (a^2 * b^2 + \\
& b^4) * f^2) * \sinh(d*x + c)^2 * \operatorname{dilog}(-I * \cosh(d*x + c) - I * \sinh(d*x + c)) + (- \\
& 12 * I * (a^3 * b + a * b^3) * d * e * f + 12 * (a^2 * b^2 + b^4) * d * e * f + 12 * I * (a^3 * b + a * b^3) \\
& ) * c * f^2 - 12 * (a^2 * b^2 + b^4) * c * f^2 + (-12 * I * (a^3 * b + a * b^3) * d * e * f + 12 * (a^2 \\
& * b^2 + b^4) * d * e * f + 12 * I * (a^3 * b + a * b^3) * c * f^2 - 12 * (a^2 * b^2 + b^4) * c * f^2) * \\
& \cosh(d*x + c)^2 + (-24 * I * (a^3 * b + a * b^3) * d * e * f + 24 * (a^2 * b^2 + b^4) * d * e * f + \\
& 24 * I * (a^3 * b + a * b^3) * c * f^2 - 24 * (a^2 * b^2 + b^4) * c * f^2) * \cosh(d*x + c) * \sinh( \\
& d*x + c) + (-12 * I * (a^3 * b + a * b^3) * d * e * f + 12 * (a^2 * b^2 + b^4) * d * e * f + 12 * I * ( \\
& a^3 * b + a * b^3) * c * f^2 - 12 * (a^2 * b^2 + b^4) * c * f^2) * \sinh(d*x + c)^2 * \log(\cosh( \\
& d*x + c) + \sinh(d*x + c) + I) + (12 * I * (a^3 * b + a * b^3) * d * e * f + 12 * (a^2 * b^2 + \\
& b^4) * d * e * f - 12 * I * (a^3 * b + a * b^3) * c * f^2 - 12 * (a^2 * b^2 + b^4) * c * f^2 + (12 * I \\
& * (a^3 * b + a * b^3) * d * e * f + 12 * (a^2 * b^2 + b^4) * d * e * f - 12 * I * (a^3 * b + a * b^3) * c * \\
& f^2 - 12 * (a^2 * b^2 + b^4) * c * f^2) * \cosh(d*x + c)^2 + (24 * I * (a^3 * b + a * b^3) * d * e \\
& * f + 24 * (a^2 * b^2 + b^4) * d * e * f - 24 * I * (a^3 * b + a * b^3) * c * f^2 - 24 * (a^2 * b^2 + \\
& b^4) * c * f^2) * \cosh(d*x + c) * \sinh(d*x + c) + (12 * I * (a^3 * b + a * b^3) * d * e * f + 12 * \\
& (a^2 * b^2 + b^4) * d * e * f - 12 * I * (a^3 * b + a * b^3) * c * f^2 - 12 * (a^2 * b^2 + b^4) * c * f \\
& ^2) * \sinh(d*x + c)^2 * \log(\cosh(d*x + c) + \sinh(d*x + c) - I) + (12 * I * (a^3 * b \\
& + a * b^3) * d * f^2 * x + 12 * (a^2 * b^2 + b^4) * d * f^2 * x + 12 * I * (a^3 * b + a * b^3) * c * f^2 \\
& + 12 * (a^2 * b^2 + b^4) * c * f^2 + (12 * I * (a^3 * b + a * b^3) * d * f^2 * x + 12 * (a^2 * b^2 + \\
& b^4) * d * f^2 * x + 12 * I * (a^3 * b + a * b^3) * c * f^2 + 12 * (a^2 * b^2 + b^4) * c * f^2) * \cosh( \\
& d*x + c)^2 + (24 * I * (a^3 * b + a * b^3) * d * f^2 * x + 24 * (a^2 * b^2 + b^4) * d * f^2 * x + 2 \\
& 4 * I * (a^3 * b + a * b^3) * c * f^2 + 24 * (a^2 * b^2 + b^4) * c * f^2) * \cosh(d*x + c) * \sinh(d* \\
& x + c) + (12 * I * (a^3 * b + a * b^3) * d * f^2 * x + 12 * (a^2 * b^2 + b^4) * d * f^2 * x + 12 * I * \\
& (a^3 * b + a * b^3) * c * f^2 + 12 * (a^2 * b^2 + b^4) * c * f^2) * \sinh(d*x + c)^2 * \log(I * \co \\
& sh(d*x + c) + I * \sinh(d*x + c) + 1) + (-12 * I * (a^3 * b + a * b^3) * d * f^2 * x + 12 * (a \\
& ^2 * b^2 + b^4) * d * f^2 * x - 12 * I * (a^3 * b + a * b^3) * c * f^2 + 12 * (a^2 * b^2 + b^4) * c * f \\
& ^2 + (-12 * I * (a^3 * b + a * b^3) * d * f^2 * x + 12 * (a^2 * b^2 + b^4) * d * f^2 * x - 12 * I * (a^ \\
& 3 * b + a * b^3) * c * f^2 + 12 * (a^2 * b^2 + b^4) * c * f^2) * \cosh(d*x + c)^2 + (-24 * I * (a^ \\
& 3 * b + a * b^3) * d * f^2 * x + 24 * (a^2 * b^2 + b^4) * d * f^2 * x - 24 * I * (a^3 * b + a * b^3) * c * \\
& f^2 + 24 * (a^2 * b^2 + b^4) * c * f^2) * \cosh(d*x + c) * \sinh(d*x + c) + (-12 * I * (a^3 * b \\
& + a * b^3) * d * f^2 * x + 12 * (a^2 * b^2 + b^4) * d * f^2 * x - 12 * I * (a^3 * b + a * b^3) * c * f^2 \\
& + 12 * (a^2 * b^2 + b^4) * c * f^2) * \sinh(d*x + c)^2 * \log(-I * \cosh(d*x + c) - I * \sinh \\
& (d*x + c) + 1) + 4 * (3 * (a^3 * b + a * b^3) * d^2 * f^2 * x^2 + 6 * (a^3 * b + a * b^3) * d^2 * e \\
& * f * x + 3 * (a^3 * b + a * b^3) * d^2 * e^2 + ((a^4 + 2 * a^2 * b^2 + b^4) * d^3 * f^2 * x^3 - 1 \\
& 2 * (a^2 * b^2 + b^4) * c * d * e * f + 6 * (a^2 * b^2 + b^4) * c^2 * f^2 + 3 * ((a^4 + 2 * a^2 * b^2 \\
& + b^4) * d^3 * e * f - 2 * (a^2 * b^2 + b^4) * d^2 * f^2) * x^2 + 3 * ((a^4 + 2 * a^2 * b^2 + b^ \\
& 4) * d^3 * e^2 - 4 * (a^2 * b^2 + b^4) * d^2 * e * f) * x) * \cosh(d*x + c) * \sinh(d*x + c) / (( \\
& a^4 * b + 2 * a^2 * b^3 + b^5) * d^3 * \cosh(d*x + c)^2 + 2 * (a^4 * b + 2 * a^2 * b^3 + b^5) * \\
& d^3 * \cosh(d*x + c) * \sinh(d*x + c) + (a^4 * b + 2 * a^2 * b^3 + b^5) * d^3 * \sinh(d*x + \\
& c)^2 + (a^4 * b + 2 * a^2 * b^3 + b^5) * d^3)
\end{aligned}$$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^2 \sinh(c + dx) \tanh^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*sinh(d*x+c)*tanh(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*sinh(c + d*x)*tanh(c + d*x)**2/(a + b*sinh(c + d*x)),  
x)
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.413 \quad \int \frac{(e+fx) \sinh(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=454

$$-\frac{a^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2 (a^2+b^2)^{3/2}} + \frac{a^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2 (a^2+b^2)^{3/2}} + \frac{a^3 f \tan^{-1}(\sinh(c+dx))}{b^2 d^2 (a^2+b^2)} + \frac{a^4 f \log(\cosh(c+dx))}{b^3 d^2 (a^2+b^2)} - \frac{a^2 f}{b^2 d^2 (a^2+b^2)}$$

[Out] (e\*x)/b + (f\*x^2)/(2\*b) - (a\*f\*ArcTan[Sinh[c + d\*x]])/(b^2\*d^2) + (a^3\*f\*ArcTan[Sinh[c + d\*x]])/(b^2\*(a^2 + b^2)\*d^2) - (a^3\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b\*(a^2 + b^2)^(3/2)\*d) + (a^3\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b\*(a^2 + b^2)^(3/2)\*d) - (a^2\*f\*Log[Cosh[c + d\*x]])/(b^3\*d^2) + (f\*Log[Cosh[c + d\*x]])/(b\*d^2) + (a^4\*f\*Log[Cosh[c + d\*x]])/(b^3\*(a^2 + b^2)\*d^2) - (a^3\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*(a^2 + b^2)^(3/2)\*d^2) + (a^3\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*(a^2 + b^2)^(3/2)\*d^2) + (a\*(e + f\*x)\*Sech[c + d\*x])/(b^2\*d) - (a^3\*(e + f\*x)\*Sech[c + d\*x])/(b^2\*(a^2 + b^2)\*d) + (a^2\*(e + f\*x)\*Tanh[c + d\*x])/(b^3\*d) - ((e + f\*x)\*Tanh[c + d\*x])/(b\*d) - (a^4\*(e + f\*x)\*Tanh[c + d\*x])/(b^3\*(a^2 + b^2)\*d)

**Rubi [A]** time = 0.891763, antiderivative size = 454, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 15, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$ , Rules used = {5581, 3720, 3475, 5567, 5451, 3770, 5583, 4184, 5573, 3322, 2264, 2190, 2279, 2391, 6742}

$$-\frac{a^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{bd^2 (a^2+b^2)^{3/2}} + \frac{a^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{bd^2 (a^2+b^2)^{3/2}} + \frac{a^3 f \tan^{-1}(\sinh(c+dx))}{b^2 d^2 (a^2+b^2)} + \frac{a^4 f \log(\cosh(c+dx))}{b^3 d^2 (a^2+b^2)} - \frac{a^2 f}{b^2 d^2 (a^2+b^2)}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Sinh[c + d\*x]\*Tanh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (e\*x)/b + (f\*x^2)/(2\*b) - (a\*f\*ArcTan[Sinh[c + d\*x]])/(b^2\*d^2) + (a^3\*f\*ArcTan[Sinh[c + d\*x]])/(b^2\*(a^2 + b^2)\*d^2) - (a^3\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(b\*(a^2 + b^2)^(3/2)\*d) + (a^3\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(b\*(a^2 + b^2)^(3/2)\*d) - (a^2\*f\*Log[Cosh[c + d\*x]])/(b^3\*d^2) + (f\*Log[Cosh[c + d\*x]])/(b\*d^2) + (a^4\*f\*Log[Cosh[c + d\*x]])/(b^3\*(a^2 + b^2)\*d^2) - (a^3\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(b\*(a^2 + b^2)^(3/2)\*d^2) + (a^3\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(b\*(a^2 + b^2)^(3/2)\*d^2) + (

$$a*(e + f*x)*\text{Sech}[c + d*x]/(b^2*d) - (a^3*(e + f*x)*\text{Sech}[c + d*x])/(b^2*(a^2 + b^2)*d) + (a^2*(e + f*x)*\text{Tanh}[c + d*x])/(b^3*d) - ((e + f*x)*\text{Tanh}[c + d*x])/(b*d) - (a^4*(e + f*x)*\text{Tanh}[c + d*x])/(b^3*(a^2 + b^2)*d)$$
Rule 5581

```
Int[(((e_.) + (f_.)*(x_.))^(m_.)*Sinh[(c_.) + (d_.)*(x_.)]^(p_.)*Tanh[(c_.) + (d_.)*(x_.)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n, x], x] - Dist[a/b, Int[((e + f*x)^m*Sinh[c + d*x]^(p - 1)*Tanh[c + d*x]^n)/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.)), x_Symbol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5567

```
Int[(((e_.) + (f_.)*(x_.))^(m_.)*Tanh[(c_.) + (d_.)*(x_.)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sech[c + d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f*x)^m*Sech[c + d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 5451

```
Int[(((c_.) + (d_.)*(x_.))^(m_.)*Sech[(a_.) + (b_.)*(x_.)]^(n_.)*Tanh[(a_.) + (b_.)*(x_.)]^(p_.)), x_Symbol] := -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n), x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5583

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(p\_.)\*Tanh[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/b, Int[(e + f\*x)^m\*Sech[c + d\*x]^(p + 1)\*Tanh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[((e + f\*x)^m\*Sech[c + d\*x]^(p + 1)\*Tanh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(c + d\*x)^m\*Cot[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 5573

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f\*x)^m\*Sech[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f\*x)^m\*Sech[c + d\*x]^n\*(a - b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]

Rule 3322

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(-(I\*b) + 2\*a\*E^(-(I\*e) + f\*fz\*x) + I\*b\*E^(2\*(-(I\*e) + f\*fz\*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.)\*(F\_)^(v\_)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[((f + g\*x)^m\*F^u)/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[((f + g\*x)^m\*F^u)/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a)/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n], x], x]

```
)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(e+fx)\sinh(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\tanh^2(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)\tanh^2(c+dx)}{a+b\sinh(c+dx)} dx}{b} \\
&= -\frac{(e+fx)\tanh(c+dx)}{bd} - \frac{a \int (e+fx)\operatorname{sech}(c+dx)\tanh(c+dx) dx}{b^2} + \frac{a^2}{b^2} \int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)} dx \\
&= \frac{ex}{b} + \frac{fx^2}{2b} + \frac{f \log(\cosh(c+dx))}{bd^2} + \frac{a(e+fx)\operatorname{sech}(c+dx)}{b^2d} - \frac{(e+fx)\tanh(c+dx)}{bd} \\
&= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{af \tan^{-1}(\sinh(c+dx))}{b^2d^2} + \frac{f \log(\cosh(c+dx))}{bd^2} + \frac{a(e+fx)\operatorname{sech}(c+dx)}{b^2d} \\
&= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{af \tan^{-1}(\sinh(c+dx))}{b^2d^2} - \frac{a^2f \log(\cosh(c+dx))}{b^3d^2} + \frac{f \log(\cosh(c+dx))}{bd^2} \\
&= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{af \tan^{-1}(\sinh(c+dx))}{b^2d^2} - \frac{a^2f \log(\cosh(c+dx))}{b^3d^2} + \frac{f \log(\cosh(c+dx))}{bd^2} \\
&= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{af \tan^{-1}(\sinh(c+dx))}{b^2d^2} - \frac{a^3(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d} + \frac{a^3}{b^2} \int \frac{(e+fx)\tanh(c+dx)}{a+b\sinh(c+dx)} dx \\
&= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{af \tan^{-1}(\sinh(c+dx))}{b^2d^2} + \frac{a^3f \tan^{-1}(\sinh(c+dx))}{b^2(a^2+b^2)d^2} - \frac{a^3(e+fx)\tanh(c+dx)}{b^2(a^2+b^2)d} \\
&= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{af \tan^{-1}(\sinh(c+dx))}{b^2d^2} + \frac{a^3f \tan^{-1}(\sinh(c+dx))}{b^2(a^2+b^2)d^2} - \frac{a^3(e+fx)\tanh(c+dx)}{b^2(a^2+b^2)d}
\end{aligned}$$

**Mathematica [A]** time = 4.54347, size = 317, normalized size = 0.7

$$\frac{2a^3 \left( -f \operatorname{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) + f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) + 2de \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) - f(c+dx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}+1\right) + f(c+dx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}+1\right) - 2cf \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) \right)}{b(a^2+b^2)^{3/2}}$$


---


$$2d^2$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Sinh[c + d*x]*Tanh[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
]
```

```
[Out] (-(((c + d*x)*(c*f - d*(2*e + f*x)))/b) - (4*a*f*ArcTan[Tanh[(c + d*x)/2]])
/(a^2 + b^2) + (2*b*f*Log[Cosh[c + d*x]])/(a^2 + b^2) + (2*a^3*(2*d*e*ArcTan
h[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*c*f*ArcTanh[(a + b*E^(c + d*x))
/Sqrt[a^2 + b^2]] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
]]) + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) - f*PolyLo
g[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -(b*E^(c + d*x
))/(a + Sqrt[a^2 + b^2])]))/(b*(a^2 + b^2)^(3/2)) + (2*d*(e + f*x)*Sech[c
+ d*x]*(a - b*Sinh[c + d*x]))/(a^2 + b^2)/(2*d^2)
```

**Maple [B]** time = 0.214, size = 1897, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)), x)
```

```
[Out] 1/2*f*x^2/b+1/(a^2+b^2)^2/d^2*f*b*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)*a^2
+2/(a^2+b^2)/d^2*f*b^3/(2*a^2+2*b^2)*ln(1+exp(2*d*x+2*c))-1/(a^2+b^2)/d^2*f
*b^3/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-4/(a^2+b^2)/d^2*a^
3*f/(2*a^2+2*b^2)*arctan(exp(d*x+c))-2/(a^2+b^2)^(5/2)/d^2*f*b*arctanh(1/2*
(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^3-2/(a^2+b^2)^(5/2)/d^2*f*b^3*arcta
nh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^4/(a^2+b^2)/d^2*f*b^2/(2*a^2
+2*b^2)*a*arctan(exp(d*x+c))+2/(a^2+b^2)/d^2*a^2*b*f/(2*a^2+2*b^2)*ln(1+exp
(2*d*x+2*c))-2/(a^2+b^2)/d^2*a^2*b*f/(2*a^2+2*b^2)*ln(b*exp(2*d*x+2*c)+2*a*
exp(d*x+c)-b)+e*x/b+2/(a^2+b^2)^(3/2)/b/d^2*a^5*f/(2*a^2+2*b^2)*ln((b*exp(d
*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-2/(a^2+b^2)^(3/2)/b/d^2*a^5
*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-2/(a^2
+b^2)^(3/2)/d^2*a^3*b*f*c/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a
^2+b^2)^(1/2))-2/(a^2+b^2)^(3/2)/b/d*a^5*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+
(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+2/(a^2+b^2)^(3/2)/b/d*a^5*f/(2*a
^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x-2/(a^2
+b^2)^(3/2)/d^2*a^3*b*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/
(-a+(a^2+b^2)^(1/2)))*c+2/(a^2+b^2)^(3/2)/d^2*a^3*b*f/(2*a^2+2*b^2)*ln((b*
exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+2/(a^2+b^2)^(3/2)/d*a^3
*b*f/(2*a^2+2*b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))
*x-2/(a^2+b^2)^(3/2)/d*a^3*b*f/(2*a^2+2*b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1
/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-2/(a^2+b^2)^(3/2)/b/d^2*a^5*f/(2*a^2+2*b^2)*
ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c-2/b/d^2*a^3*f*
c/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(
1/2))+2/(a^2+b^2)^(3/2)/d*a^3*b*e/(2*a^2+2*b^2)*arctanh(1/2*(2*b*exp(d*x+c
)+2*a)/(a^2+b^2)^(1/2))+2/(a^2+b^2)^(3/2)/d^2*a^3*b*f/(2*a^2+2*b^2)*dilog((
```

$$\begin{aligned}
& b \exp(dx+c) + (a^2+b^2)^{1/2} + a / (a + (a^2+b^2)^{1/2}) - 2 / (a^2+b^2)^{3/2} / d^2 * \\
& a^3 * b * f / (2 * a^2 + 2 * b^2) * \operatorname{dilog}((-b \exp(dx+c) + (a^2+b^2)^{1/2} - a) / (-a + (a^2+b^2)^{1/2})) \\
& + 2 / (a^2+b^2)^{1/2} / d^2 * a * b * f / (2 * a^2 + 2 * b^2) * \operatorname{arctanh}(1/2 * (2 * b * \exp(dx+c) + 2 * a) / (a^2+b^2)^{1/2}) \\
& + 2 / (a^2+b^2)^{3/2} / d^2 * a * b^3 * f / (2 * a^2 + 2 * b^2) * \operatorname{arctanh}(1/2 * (2 * b * \exp(dx+c) + 2 * a) / (a^2+b^2)^{1/2}) \\
& + 2 / (a^2+b^2)^{3/2} / d^2 * a^3 * b * f / (2 * a^2 + 2 * b^2) * \operatorname{arctanh}(1/2 * (2 * b * \exp(dx+c) + 2 * a) / (a^2+b^2)^{1/2}) \\
& + 2 / (a^2+b^2)^{3/2} / b / d * a^5 * e / (2 * a^2 + 2 * b^2) * \operatorname{arctanh}(1/2 * (2 * b * \exp(dx+c) + 2 * a) / (a^2+b^2)^{1/2}) \\
& + 2 / (a^2+b^2)^{3/2} / b / d^2 * a^5 * f / (2 * a^2 + 2 * b^2) * \operatorname{dilog}((b \exp(dx+c) + (a^2+b^2)^{1/2} + a) / (a + (a^2+b^2)^{1/2})) \\
& - 2 / (a^2+b^2)^{3/2} / b / d^2 * a^5 * f / (2 * a^2 + 2 * b^2) * \operatorname{dilog}((-b \exp(dx+c) + (a^2+b^2)^{1/2} - a) / (-a + (a^2+b^2)^{1/2})) \\
& - 2 / (a^2+b^2)^{3/2} / d^2 * b * f * \ln(\exp(dx+c)) + 1/2 / (a^2+b^2)^2 / d^2 * f * b^3 * \ln(b \exp(2 * dx + 2 * c) + 2 * a * \exp(dx+c) - b) \\
& + 2 * (f * x + e) * (a * \exp(dx+c) + b) / d / (a^2+b^2) / (1 + \exp(2 * dx + 2 * c)) + 2 / b / d * a^3 * e / (2 * a^2 + 2 * b^2) / (a^2+b^2)^{1/2} * \operatorname{arctanh}(1/2 * (2 * b * \exp(dx+c) + 2 * a) / (a^2+b^2)^{1/2})
\end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(dx+c)\*tanh(dx+c)^2/(a+b\*sinh(dx+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 3.01406, size = 3753, normalized size = 8.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(dx+c)\*tanh(dx+c)^2/(a+b\*sinh(dx+c)),x, algorithm="fricas")

[Out]  $1/2 * ((a^4 + 2 * a^2 * b^2 + b^4) * d^2 * f * x^2 + 2 * (a^4 + 2 * a^2 * b^2 + b^4) * d^2 * e * x + 4 * (a^2 * b^2 + b^4) * d * e + ((a^4 + 2 * a^2 * b^2 + b^4) * d^2 * f * x^2 + 2 * ((a^4 + 2 * a^2 * b^2 + b^4) * d^2 * e - 2 * (a^2 * b^2 + b^4) * d * f) * x) * \cosh(dx + c)^2 + ((a^4 + 2 * a^2 * b^2 + b^4) * d^2 * f * x^2 + 2 * ((a^4 + 2 * a^2 * b^2 + b^4) * d^2 * e - 2 * (a^2 * b^2 + b^4) * d * f) * x) * \sinh(dx + c)^2 - 2 * (a^3 * b * f * \cosh(dx + c)^2 + 2 * a^3 * b * f * \cos$

```

h(d*x + c)*sinh(d*x + c) + a^3*b*f*sinh(d*x + c)^2 + a^3*b*f)*sqrt((a^2 + b
^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*si
nh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a^3*b*f*cosh(d*x + c)^2
+ 2*a^3*b*f*cosh(d*x + c)*sinh(d*x + c) + a^3*b*f*sinh(d*x + c)^2 + a^3*b*
f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a^3*b*d
*e - a^3*b*c*f + (a^3*b*d*e - a^3*b*c*f)*cosh(d*x + c)^2 + 2*(a^3*b*d*e - a
^3*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d*e - a^3*b*c*f)*sinh(d*x +
c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b
*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*(a^3*b*d*e - a^3*b*c*f + (a^3*b*d*e - a^3
*b*c*f)*cosh(d*x + c)^2 + 2*(a^3*b*d*e - a^3*b*c*f)*cosh(d*x + c)*sinh(d*x
+ c) + (a^3*b*d*e - a^3*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2
*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2
*(a^3*b*d*f*x + a^3*b*c*f + (a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)^2 + 2*(
a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d*f*x + a^3*b
*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh
(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/
b) + 2*(a^3*b*d*f*x + a^3*b*c*f + (a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)^2
+ 2*(a^3*b*d*f*x + a^3*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (a^3*b*d*f*x +
a^3*b*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) +
a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b) - 4*((a^3*b + a*b^3)*f*cosh(d*x + c)^2 + 2*(a^3*b + a*b^3)*f*cosh(
d*x + c)*sinh(d*x + c) + (a^3*b + a*b^3)*f*sinh(d*x + c)^2 + (a^3*b + a*b^3
)*f)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 4*((a^3*b + a*b^3)*d*f*x + (a^
3*b + a*b^3)*d*e)*cosh(d*x + c) + 2*((a^2*b^2 + b^4)*f*cosh(d*x + c)^2 + 2*
(a^2*b^2 + b^4)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2*b^2 + b^4)*f*sinh(d*x
+ c)^2 + (a^2*b^2 + b^4)*f)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x +
c))) + 2*(2*(a^3*b + a*b^3)*d*f*x + 2*(a^3*b + a*b^3)*d*e + ((a^4 + 2*a^2*
b^2 + b^4)*d^2*f*x^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*e - 2*(a^2*b^2 + b^4)
*d*f)*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b + 2*a^2*b^3 + b^5)*d^2*cosh(
d*x + c)^2 + 2*(a^4*b + 2*a^2*b^3 + b^5)*d^2*cosh(d*x + c)*sinh(d*x + c) +
(a^4*b + 2*a^2*b^3 + b^5)*d^2*sinh(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*d
^2)

```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \sinh(c + dx) \tanh^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*sinh(d\*x+c)\*tanh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out]  $\text{Integral}((e + f*x)*\sinh(c + d*x)*\tanh(c + d*x)**2/(a + b*\sinh(c + d*x)), x)$

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")`

[Out] Timed out

$$3.414 \quad \int \frac{\sinh(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=121

$$\frac{2a^3 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd(a^2+b^2)^{3/2}} - \frac{b \tanh(c+dx)}{d(a^2+b^2)} + \frac{a \operatorname{sech}(c+dx)}{d(a^2+b^2)} + \frac{a^2 x}{b(a^2+b^2)} + \frac{bx}{a^2+b^2}$$

[Out] (a^2\*x)/(b\*(a^2 + b^2)) + (b\*x)/(a^2 + b^2) + (2\*a^3\*ArcTanh[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[a^2 + b^2]])/(b\*(a^2 + b^2)^(3/2)\*d) + (a\*Sech[c + d\*x])/(a^2 + b^2)\*d - (b\*Tanh[c + d\*x])/((a^2 + b^2)\*d)

**Rubi [A]** time = 0.20893, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$ , Rules used = {2902, 2606, 8, 3473, 2735, 2660, 618, 204}

$$\frac{2a^3 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{bd(a^2+b^2)^{3/2}} - \frac{b \tanh(c+dx)}{d(a^2+b^2)} + \frac{a \operatorname{sech}(c+dx)}{d(a^2+b^2)} + \frac{a^2 x}{b(a^2+b^2)} + \frac{bx}{a^2+b^2}$$

Antiderivative was successfully verified.

[In] Int[(Sinh[c + d\*x]\*Tanh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (a^2\*x)/(b\*(a^2 + b^2)) + (b\*x)/(a^2 + b^2) + (2\*a^3\*ArcTanh[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[a^2 + b^2]])/(b\*(a^2 + b^2)^(3/2)\*d) + (a\*Sech[c + d\*x])/(a^2 + b^2)\*d - (b\*Tanh[c + d\*x])/((a^2 + b^2)\*d)

### Rule 2902

Int[((cos[(e\_.) + (f\_.)\*(x\_.)]\*(g\_.))^p)\*((d\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^n)/((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Dist[(a\*d^2)/(a^2 - b^2), Int[(g\*Cos[e + f\*x])^p\*(d\*Sin[e + f\*x])^(n - 2), x], x] + (-Dist[(b\*d)/(a^2 - b^2), Int[(g\*Cos[e + f\*x])^p\*(d\*Sin[e + f\*x])^(n - 1), x], x] - Dist[(a^2\*d^2)/(g^2\*(a^2 - b^2)), Int[((g\*Cos[e + f\*x])^(p + 2)\*(d\*Sin[e + f\*x])^(n - 2))/(a + b\*Sin[e + f\*x]), x], x]) /; FreeQ[{a, b, d, e, f, g}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2\*n, 2\*p] && LtQ[p, -1] && GtQ[n, 1]

### Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

### Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

### Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

### Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

### Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sinh(c+dx) \tanh^2(c+dx)}{a+b \sinh(c+dx)} dx &= -\frac{a \int \operatorname{sech}(c+dx) \tanh(c+dx) dx}{a^2+b^2} + \frac{a^2 \int \frac{\sinh(c+dx)}{a+b \sinh(c+dx)} dx}{a^2+b^2} + \frac{b \int \tanh^2(c+dx) dx}{a^2+b^2} \\
&= \frac{a^2 x}{b(a^2+b^2)} - \frac{b \tanh(c+dx)}{(a^2+b^2)d} - \frac{a^3 \int \frac{1}{a+b \sinh(c+dx)} dx}{b(a^2+b^2)} + \frac{b \int 1 dx}{a^2+b^2} + \frac{a \operatorname{Subst}\left(\int 1 dx, x, \frac{a+b \sinh(c+dx)}{a}\right)}{(a^2+b^2)d} \\
&= \frac{a^2 x}{b(a^2+b^2)} + \frac{bx}{a^2+b^2} + \frac{a \operatorname{sech}(c+dx)}{(a^2+b^2)d} - \frac{b \tanh(c+dx)}{(a^2+b^2)d} + \frac{(2ia^3) \operatorname{Subst}\left(\int \frac{1}{a-2ibx+a} dx, x, \frac{a+b \sinh(c+dx)}{a}\right)}{b(a^2+b^2)} \\
&= \frac{a^2 x}{b(a^2+b^2)} + \frac{bx}{a^2+b^2} + \frac{a \operatorname{sech}(c+dx)}{(a^2+b^2)d} - \frac{b \tanh(c+dx)}{(a^2+b^2)d} - \frac{(4ia^3) \operatorname{Subst}\left(\int \frac{1}{-4(a^2+b^2)} dx, x, \frac{a+b \sinh(c+dx)}{a}\right)}{b(a^2+b^2)} \\
&= \frac{a^2 x}{b(a^2+b^2)} + \frac{bx}{a^2+b^2} + \frac{2a^3 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}d} + \frac{a \operatorname{sech}(c+dx)}{(a^2+b^2)d} - \frac{b \tanh(c+dx)}{(a^2+b^2)d}
\end{aligned}$$

**Mathematica [A]** time = 0.502191, size = 96, normalized size = 0.79

$$\frac{2a^3 \tan^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{b(-a^2-b^2)^{3/2}} + \frac{\operatorname{sech}(c+dx)(a-b \sinh(c+dx))}{a^2+b^2} + \frac{c+dx}{b}$$

d

Antiderivative was successfully verified.

[In] Integrate[(Sinh[c + d\*x]\*Tanh[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] ((c + d\*x)/b + (2\*a^3\*ArcTan[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[-a^2 - b^2]])/(b\*(-a^2 - b^2)^(3/2)) + (Sech[c + d\*x]\*(a - b\*Sinh[c + d\*x]))/(a^2 + b^2))/d

**Maple [A]** time = 0.003, size = 158, normalized size = 1.3

$$\frac{1}{bd} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) - \frac{1}{bd} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) - 2 \frac{a^3}{bd(a^2+b^2)^{3/2}} \operatorname{Artanh}\left(\frac{1}{2} \frac{2a \tanh(1/2 dx + c/2) - 2b}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] int(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d/b*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/b*ln(tanh(1/2*d*x+1/2*c)-1)-2/d/b*a^3/(
a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-2
/d/(a^2+b^2)/(tanh(1/2*d*x+1/2*c)^2+1)*tanh(1/2*d*x+1/2*c)*b+2/d/(a^2+b^2)/
(tanh(1/2*d*x+1/2*c)^2+1)*a
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

**Fricas [B]** time = 2.53876, size = 1126, normalized size = 9.31

$$(a^4 + 2a^2b^2 + b^4)dx \cosh(dx + c)^2 + (a^4 + 2a^2b^2 + b^4)dx \sinh(dx + c)^2 + 2a^2b^2 + 2b^4 + (a^4 + 2a^2b^2 + b^4)dx + (a^3 c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas"
)
```

```
[Out] ((a^4 + 2*a^2*b^2 + b^4)*d*x*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d*x*
sinh(d*x + c)^2 + 2*a^2*b^2 + 2*b^4 + (a^4 + 2*a^2*b^2 + b^4)*d*x + (a^3*co
sh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*x + c)^2 + a
^3)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*
cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2
*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^
2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*
x + c) - b)) + 2*(a^3*b + a*b^3)*cosh(d*x + c) + 2*(a^3*b + a*b^3 + (a^4 +
2*a^2*b^2 + b^4)*d*x*cosh(d*x + c))*sinh(d*x + c))/((a^4*b + 2*a^2*b^3 + b
^5)*d*cosh(d*x + c)^2 + 2*(a^4*b + 2*a^2*b^3 + b^5)*d*cosh(d*x + c)*sinh(d*x
```

+ c) + (a<sup>4</sup>\*b + 2\*a<sup>2</sup>\*b<sup>3</sup> + b<sup>5</sup>)\*d\*sinh(d\*x + c)^2 + (a<sup>4</sup>\*b + 2\*a<sup>2</sup>\*b<sup>3</sup> + b<sup>5</sup>)\*d)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(c + dx) \tanh^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)\*tanh(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(sinh(c + d\*x)\*tanh(c + d\*x)\*\*2/(a + b\*sinh(c + d\*x)), x)

**Giac [A]** time = 1.43235, size = 182, normalized size = 1.5

$$\frac{a^3 \log\left(\frac{\left| \begin{array}{l} -2be^{(dx+2c)} - 2ae^c - 2\sqrt{a^2+b^2}e^c \\ -2be^{(dx+2c)} - 2ae^c + 2\sqrt{a^2+b^2}e^c \end{array} \right|}{(a^2b+b^3)\sqrt{a^2+b^2}}\right)}{d} + \frac{dx}{b} + \frac{2(ae^{(dx+c)}+b)}{(a^2+b^2)(e^{(2dx+2c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)\*tanh(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] (a<sup>3</sup>\*log(abs(-2\*b\*e<sup>(d\*x + 2\*c)</sup> - 2\*a\*e<sup>c</sup> - 2\*sqrt(a<sup>2</sup> + b<sup>2</sup>)\*e<sup>c</sup>)/abs(-2\*b\*e<sup>(d\*x + 2\*c)</sup> - 2\*a\*e<sup>c</sup> + 2\*sqrt(a<sup>2</sup> + b<sup>2</sup>)\*e<sup>c</sup>))/((a<sup>2</sup>\*b + b<sup>3</sup>)\*sqrt(a<sup>2</sup> + b<sup>2</sup>)) + d\*x/b + 2\*(a\*e<sup>(d\*x + c)</sup> + b)/((a<sup>2</sup> + b<sup>2</sup>)\*(e<sup>(2\*d\*x + 2\*c)</sup> + 1))/d

$$3.415 \quad \int \frac{\sinh(c+dx) \tanh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable} \left( \frac{\sinh(c+dx) \tanh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[(Sinh[c + d\*x]\*Tanh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0811154, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\sinh(c+dx) \tanh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Sinh[c + d\*x]\*Tanh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] Defer[Int] [(Sinh[c + d\*x]\*Tanh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\sinh(c+dx) \tanh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\sinh(c+dx) \tanh^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.003, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Sinh[c + d\*x]\*Tanh[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] \$Aborted

**Maple [A]** time = 0.753, size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c) \tanh(dx+c)^2}{(fx+e)(a+b\sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d\*x+c)\*tanh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(sinh(d\*x+c)\*tanh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-2a^3 \int \frac{e^{(dx+c)}}{a^2b^2e + b^4e + (a^2b^2f + b^4f)x - (a^2b^2ee^{(2c)} + b^4ee^{(2c)} + (a^2b^2fe^{(2c)} + b^4fe^{(2c)})x} e^{(2dx)} - 2(a^3bee^c + ab^3ee^c + (a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d\*x+c)\*tanh(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -2\*a^3\*integrate(-e^(d\*x + c)/(a^2\*b^2\*e + b^4\*e + (a^2\*b^2\*f + b^4\*f)\*x - (a^2\*b^2\*e\*e^(2\*c) + b^4\*e\*e^(2\*c) + (a^2\*b^2\*f\*e^(2\*c) + b^4\*f\*e^(2\*c))\*x)\*e^(2\*d\*x) - 2\*(a^3\*b\*e\*e^c + a\*b^3\*e\*e^c + (a^3\*b\*f\*e^c + a\*b^3\*f\*e^c)\*x)\*e^(d\*x)), x) + 2\*(a\*e^(d\*x + c) + b)/(a^2\*d\*e + b^2\*d\*e + (a^2\*d\*f + b^2\*d\*f)\*x + (a^2\*d\*e\*e^(2\*c) + b^2\*d\*e\*e^(2\*c) + (a^2\*d\*f\*e^(2\*c) + b^2\*d\*f\*e^(2\*c))\*x)\*e^(2\*d\*x)) + log(f\*x + e)/(b\*f) + 1/2\*integrate(4\*(a\*f\*e^(d\*x + c) + b\*f)/(a^2\*d\*e^2 + b^2\*d\*e^2 + (a^2\*d\*f^2 + b^2\*d\*f^2)\*x^2 + 2\*(a^2\*d\*e\*f + b^2\*d\*e\*f)\*x + (a^2\*d\*e^2\*e^(2\*c) + b^2\*d\*e^2\*e^(2\*c) + (a^2\*d\*f^2\*e^(2\*c) + b^2\*d\*f^2\*e^(2\*c))\*x^2 + 2\*(a^2\*d\*e\*f\*e^(2\*c) + b^2\*d\*e\*f\*e^(2\*c))\*x)\*e^(2\*d\*x)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh(dx+c) \tanh(dx+c)^2}{afx+ae+(bfx+be)\sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sinh(d*x + c)*tanh(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(c + dx) \tanh^2(c + dx)}{(a + b \sinh(c + dx))(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(sinh(c + d*x)*tanh(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*tanh(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.416 \quad \int \frac{(e+fx)^2 \tanh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=1479

result too large to display

```
[Out] (a^2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^3*d) + ((e + f*x)^2*ArcTan[E^(c +
d*x)])/(b*d) - (2*a^4*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b*(a^2 + b^2)^2*d)
- (a^4*(e + f*x)^2*ArcTan[E^(c + d*x)])/(b^3*(a^2 + b^2)*d) - (a^2*f^2*ArcT
an[Sinh[c + d*x]])/(b^3*d^3) + (f^2*ArcTan[Sinh[c + d*x]])/(b*d^3) + (a^4*f
^2*ArcTan[Sinh[c + d*x]])/(b^3*(a^2 + b^2)*d^3) - (a^3*(e + f*x)^2*Log[1 +
(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2 + b^2)^2*d - (a^3*(e + f*x)^
2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2 + b^2)^2*d + (a^3*
(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(a^2 + b^2)^2*d + (a*f^2*Log[Cosh[c
+ d*x]])/(b^2*d^3) - (a^3*f^2*Log[Cosh[c + d*x]])/(b^2*(a^2 + b^2)*d^3) -
(I*a^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*d^2) - (I*f*(e + f*x)
*PolyLog[2, (-I)*E^(c + d*x)])/(b*d^2) + ((2*I)*a^4*f*(e + f*x)*PolyLog[2,
(-I)*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^2) + (I*a^4*f*(e + f*x)*PolyLog[2, (-
I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) + (I*a^2*f*(e + f*x)*PolyLog[2, I*E^
(c + d*x)])/(b^3*d^2) + (I*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*d^2) -
((2*I)*a^4*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^2) -
(I*a^4*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) - (2*a^
3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2 +
b^2)^2*d^2) - (2*a^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^
2 + b^2]))])/(a^2 + b^2)^2*d^2) + (a^3*f*(e + f*x)*PolyLog[2, -E^(2*(c + d
*x))])/(a^2 + b^2)^2*d^2) + (I*a^2*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b^3*
d^3) + (I*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(b*d^3) - ((2*I)*a^4*f^2*PolyLo
g[3, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^3) - (I*a^4*f^2*PolyLog[3, (-I)*
E^(c + d*x)])/(b^3*(a^2 + b^2)*d^3) - (I*a^2*f^2*PolyLog[3, I*E^(c + d*x)]
)/(b^3*d^3) - (I*f^2*PolyLog[3, I*E^(c + d*x)])/(b*d^3) + ((2*I)*a^4*f^2*Pol
yLog[3, I*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^3) + (I*a^4*f^2*PolyLog[3, I*E^
(c + d*x)])/(b^3*(a^2 + b^2)*d^3) + (2*a^3*f^2*PolyLog[3, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/(a^2 + b^2)^2*d^3) + (2*a^3*f^2*PolyLog[3, -((b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2 + b^2)^2*d^3) - (a^3*f^2*PolyLog
[3, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^3) + (a^2*f*(e + f*x)*Sech[c + d*
x])/(b^3*d^2) - (f*(e + f*x)*Sech[c + d*x])/(b*d^2) - (a^4*f*(e + f*x)*Sech
[c + d*x])/(b^3*(a^2 + b^2)*d^2) + (a*(e + f*x)^2*Sech[c + d*x]^2)/(2*b^2*d
) - (a^3*(e + f*x)^2*Sech[c + d*x]^2)/(2*b^2*(a^2 + b^2)*d) - (a*f*(e + f*x)
)*Tanh[c + d*x])/(b^2*d^2) + (a^3*f*(e + f*x)*Tanh[c + d*x])/(b^2*(a^2 + b^
2)*d^2) + (a^2*(e + f*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*b^3*d) - ((e + f
*x)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*b*d) - (a^4*(e + f*x)^2*Sech[c + d*x]
*Tanh[c + d*x])/(2*b^3*(a^2 + b^2)*d)
```

**Rubi [A]** time = 2.45324, antiderivative size = 1479, normalized size of antiderivative = 1., number of steps used = 71, number of rules used = 17, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$ , Rules used = {5567, 5455, 4180, 2531, 2282, 6589, 4186, 3770, 5583, 5451, 4184, 3475, 5573, 5561, 2190, 6742, 3718}

result too large to display

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Tanh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out]  $(a^2(e + f*x)^2 \operatorname{ArcTan}[E^{(c + d*x)}]) / (b^3*d) + ((e + f*x)^2 \operatorname{ArcTan}[E^{(c + d*x)}]) / (b*d) - (2*a^4*(e + f*x)^2 \operatorname{ArcTan}[E^{(c + d*x)}]) / (b*(a^2 + b^2)^2*d) - (a^4*(e + f*x)^2 \operatorname{ArcTan}[E^{(c + d*x)}]) / (b^3*(a^2 + b^2)*d) - (a^2*f^2 \operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]]) / (b^3*d^3) + (f^2 \operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]]) / (b*d^3) + (a^4*f^2 \operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]]) / (b^3*(a^2 + b^2)*d^3) - (a^3*(e + f*x)^2 \operatorname{Log}[1 + (b*E^{(c + d*x)}) / (a - \operatorname{Sqrt}[a^2 + b^2])]) / ((a^2 + b^2)^2*d) - (a^3*(e + f*x)^2 \operatorname{Log}[1 + (b*E^{(c + d*x)}) / (a + \operatorname{Sqrt}[a^2 + b^2])]) / ((a^2 + b^2)^2*d) + (a^3*(e + f*x)^2 \operatorname{Log}[1 + E^{(2*(c + d*x))}]) / ((a^2 + b^2)^2*d) + (a*f^2 \operatorname{Log}[\operatorname{Cosh}[c + d*x]]) / (b^2*d^3) - (a^3*f^2 \operatorname{Log}[\operatorname{Cosh}[c + d*x]]) / (b^2*(a^2 + b^2)*d^3) - (I*a^2*f*(e + f*x)*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x)}]) / (b^3*d^2) - (I*f*(e + f*x)*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x)}]) / (b*d^2) + ((2*I)*a^4*f*(e + f*x)*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x)}]) / (b*(a^2 + b^2)^2*d^2) + (I*a^4*f*(e + f*x)*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x)}]) / (b^3*(a^2 + b^2)*d^2) + (I*a^2*f*(e + f*x)*\operatorname{PolyLog}[2, I*E^{(c + d*x)}]) / (b^3*d^2) + (I*f*(e + f*x)*\operatorname{PolyLog}[2, I*E^{(c + d*x)}]) / (b*d^2) - ((2*I)*a^4*f*(e + f*x)*\operatorname{PolyLog}[2, I*E^{(c + d*x)}]) / (b*(a^2 + b^2)^2*d^2) - (I*a^4*f*(e + f*x)*\operatorname{PolyLog}[2, I*E^{(c + d*x)}]) / (b^3*(a^2 + b^2)*d^2) - (2*a^3*f*(e + f*x)*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)}) / (a - \operatorname{Sqrt}[a^2 + b^2])]) / ((a^2 + b^2)^2*d^2) - (2*a^3*f*(e + f*x)*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)}) / (a + \operatorname{Sqrt}[a^2 + b^2])]) / ((a^2 + b^2)^2*d^2) + (a^3*f*(e + f*x)*\operatorname{PolyLog}[2, -E^{(2*(c + d*x))}]) / ((a^2 + b^2)^2*d^2) + (I*a^2*f^2*\operatorname{PolyLog}[3, (-I)*E^{(c + d*x)}]) / (b^3*d^3) + (I*f^2*\operatorname{PolyLog}[3, (-I)*E^{(c + d*x)}]) / (b*d^3) - ((2*I)*a^4*f^2*\operatorname{PolyLog}[3, (-I)*E^{(c + d*x)}]) / (b*(a^2 + b^2)^2*d^3) - (I*a^4*f^2*\operatorname{PolyLog}[3, (-I)*E^{(c + d*x)}]) / (b^3*(a^2 + b^2)*d^3) - (I*a^2*f^2*\operatorname{PolyLog}[3, I*E^{(c + d*x)}]) / (b^3*d^3) - (I*f^2*\operatorname{PolyLog}[3, I*E^{(c + d*x)}]) / (b*d^3) + ((2*I)*a^4*f^2*\operatorname{PolyLog}[3, I*E^{(c + d*x)}]) / (b*(a^2 + b^2)^2*d^3) + (I*a^4*f^2*\operatorname{PolyLog}[3, I*E^{(c + d*x)}]) / (b^3*(a^2 + b^2)*d^3) + (2*a^3*f^2*\operatorname{PolyLog}[3, -((b*E^{(c + d*x)}) / (a - \operatorname{Sqrt}[a^2 + b^2])]) / ((a^2 + b^2)^2*d^3) + (2*a^3*f^2*\operatorname{PolyLog}[3, -((b*E^{(c + d*x)}) / (a + \operatorname{Sqrt}[a^2 + b^2])]) / ((a^2 + b^2)^2*d^3) - (a^3*f^2*\operatorname{PolyLog}[3, -E^{(2*(c + d*x))}]) / (2*(a^2 + b^2)^2*d^3) + (a^2*f*(e + f*x)*\operatorname{Sech}[c + d*x]) / (b^3*d^2) - (f*(e + f*x)*\operatorname{Sech}[c + d*x]) / (b*d^2) - (a^4*f*(e + f*x)*\operatorname{Sech}[c + d*x]) / (b^3*(a^2 + b^2)*d^2) + (a*(e + f*x)^2*\operatorname{Sech}[c + d*x]^2) / (2*b^2*d) - (a^3*(e + f*x)^2*\operatorname{Sech}[c + d*x]^2) / (2*b^2*(a^2 + b^2)*d) - (a*f*(e + f*x)$

) $\tanh[c + d*x]$ )/( $b^2*d^2$ ) + ( $a^3*f*(e + f*x)*\tanh[c + d*x]$ )/( $b^2*(a^2 + b^2)*d^2$ ) + ( $a^2*(e + f*x)^2*\operatorname{sech}[c + d*x]*\tanh[c + d*x]$ )/( $2*b^3*d$ ) - (( $e + f*x$ ) $^2*\operatorname{sech}[c + d*x]*\tanh[c + d*x]$ )/( $2*b*d$ ) - ( $a^4*(e + f*x)^2*\operatorname{sech}[c + d*x]*\tanh[c + d*x]$ )/( $2*b^3*(a^2 + b^2)*d$ )

### Rule 5567

$\operatorname{Int}[\frac{((e_{.}) + (f_{.})*(x_{.}))^{(m_{.})}*\tanh[(c_{.}) + (d_{.})*(x_{.})]^{(n_{.})}}{(a_{.}) + (b_{.})*\sinh[(c_{.}) + (d_{.})*(x_{.})]}, x\_Symbol] \rightarrow \operatorname{Dist}[1/b, \operatorname{Int}[(e + f*x)^m*\operatorname{sech}[c + d*x]*\tanh[c + d*x]^{(n-1)}, x], x] - \operatorname{Dist}[a/b, \operatorname{Int}[\frac{(e + f*x)^m*\operatorname{sech}[c + d*x]*\tanh[c + d*x]^{(n-1)}}{(a + b*\sinh[c + d*x])}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \operatorname{IGtQ}[m, 0] \&\& \operatorname{IGtQ}[n, 0]$

### Rule 5455

$\operatorname{Int}[\frac{((c_{.}) + (d_{.})*(x_{.}))^{(m_{.})}*\operatorname{sech}[(a_{.}) + (b_{.})*(x_{.})]*\tanh[(a_{.}) + (b_{.})*(x_{.})]^{(p_{.})}}{x\_Symbol] \rightarrow \operatorname{Int}[(c + d*x)^m*\operatorname{sech}[a + b*x]*\tanh[a + b*x]^{(p-2)}, x] - \operatorname{Int}[(c + d*x)^m*\operatorname{sech}[a + b*x]^3*\tanh[a + b*x]^{(p-2)}, x] /; \operatorname{FreeQ}\{a, b, c, d, m\}, x] \&\& \operatorname{IGtQ}[p/2, 0]$

### Rule 4180

$\operatorname{Int}[\operatorname{csc}[(e_{.}) + \pi*(k_{.}) + (\operatorname{Complex}[0, fz_{.}])*(f_{.})*(x_{.})]*((c_{.}) + (d_{.})*(x_{.}))^{(m_{.})}, x\_Symbol] \rightarrow \operatorname{Simp}[(-2*(c + d*x)^m*\operatorname{ArcTanh}[E^{-(I*e) + f*fz*x}/E^{(I*k*\pi)}])/(f*fz*I), x] + (-\operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 - E^{-(I*e) + f*fz*x}/E^{(I*k*\pi)}], x], x] + \operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 + E^{-(I*e) + f*fz*x}/E^{(I*k*\pi)}], x], x]) /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x] \&\& \operatorname{IntegerQ}[2*k] \&\& \operatorname{IGtQ}[m, 0]$

### Rule 2531

$\operatorname{Int}[\operatorname{Log}[1 + (e_{.})*((F_{.})^{((c_{.})*((a_{.}) + (b_{.})*(x_{.})))})^{(n_{.})}]*((f_{.}) + (g_{.})*(x_{.}))^{(m_{.})}, x\_Symbol] \rightarrow -\operatorname{Simp}[\frac{(f + g*x)^m*\operatorname{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n]}{(b*c*n*\operatorname{Log}[F])}, x] + \operatorname{Dist}[(g*m)/(b*c*n*\operatorname{Log}[F]), \operatorname{Int}[(f + g*x)^{(m-1)}*\operatorname{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n}], x], x] /; \operatorname{FreeQ}\{F, a, b, c, e, f, g, n\}, x] \&\& \operatorname{GtQ}[m, 0]$

### Rule 2282

$\operatorname{Int}[u_{.}, x\_Symbol] \rightarrow \operatorname{With}\{v = \operatorname{FunctionOfExponential}[u, x]\}, \operatorname{Dist}[v/D[v, x], \operatorname{Subst}[\operatorname{Int}[\operatorname{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \operatorname{FunctionOfExponentialQ}[u, x] \&\& \operatorname{!MatchQ}[u, (w_{.})*((a_{.})*(v_{.})^{(n_{.})})^{(m_{.})} /; \operatorname{FreeQ}\{a, m, n\}, x] \&\& \operatorname{IntegerQ}[m*n] \&\& \operatorname{!MatchQ}[u, E^{((c_{.})*((a_{.}) + (b_{.})*x))* (F_{.})[v_{.}] /; \operatorname{FreeQ}\{a, b, c\}, x] \&\& \operatorname{InverseFunctionQ}[F[x]]]$



Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x]
+ (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*
(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^
m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*
(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x]
&& GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rule 5583

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)*Tanh[(c_.) +
(d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/b, Int[(e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1), x], x]
- Dist[a/b, Int[(((e + f*x)^m*Sech[c + d*x]^(p + 1)*Tanh[c + d*x]^(n - 1))
/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0]
&& IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5451

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[p, 1] && GtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Simp
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

### Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[(((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 3718

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[(((c
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rubi steps



**Mathematica [B]** time = 30.8825, size = 3368, normalized size = 2.28

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Tanh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] 
$$\begin{aligned} & (-12a^3d^3e^2E^{(2c)}x - 12a^3d^3eE^{(2c)}f^2x - 12ab^2d^3E^{(2c)}f^2x - 12a^3d^3eE^{(2c)}f^2x^2 - 4a^3d^3E^{(2c)}f^2x^3 + 18a^2b^2d^2 \\ & e^2\text{ArcTan}[E^{(c+d*x)}] + 6b^3d^2e^2\text{ArcTan}[E^{(c+d*x)}] + 18a^2b^2d^2 \\ & e^2E^{(2c)}\text{ArcTan}[E^{(c+d*x)}] + 6b^3d^2e^2E^{(2c)}\text{ArcTan}[E^{(c+d*x)}] \\ & + 12a^2b^2f^2\text{ArcTan}[E^{(c+d*x)}] + 12b^3f^2\text{ArcTan}[E^{(c+d*x)}] + 12a^2b^2E^{(2c)}f^2\text{ArcTan}[E^{(c+d*x)}] \\ & + 12b^3E^{(2c)}f^2\text{ArcTan}[E^{(c+d*x)}] + (18I)a^2b^2d^2efx\text{Log}[1 - IE^{(c+d*x)}] + (6I)b^3d^2efx\text{Log}[1 - IE^{(c+d*x)}] \\ & + (18I)a^2b^2d^2eE^{(2c)}fx\text{Log}[1 - IE^{(c+d*x)}] + (6I)b^3d^2eE^{(2c)}fx\text{Log}[1 - IE^{(c+d*x)}] \\ & + (9I)a^2b^2d^2f^2x^2\text{Log}[1 - IE^{(c+d*x)}] + (3I)b^3d^2f^2x^2\text{Log}[1 - IE^{(c+d*x)}] \\ & + (9I)a^2b^2d^2E^{(2c)}f^2x^2\text{Log}[1 - IE^{(c+d*x)}] + (3I)b^3d^2E^{(2c)}f^2x^2\text{Log}[1 - IE^{(c+d*x)}] \\ & - (18I)a^2b^2d^2efx\text{Log}[1 + IE^{(c+d*x)}] - (6I)b^3d^2efx\text{Log}[1 + IE^{(c+d*x)}] - (18I)a^2b^2d^2 \\ & eE^{(2c)}fx\text{Log}[1 + IE^{(c+d*x)}] - (6I)b^3d^2eE^{(2c)}fx\text{Log}[1 + IE^{(c+d*x)}] - (9I)a^2b^2d^2f^2x^2\text{Log}[1 + IE^{(c+d*x)}] \\ & - (3I)b^3d^2f^2x^2\text{Log}[1 + IE^{(c+d*x)}] - (9I)a^2b^2d^2E^{(2c)}f^2x^2\text{Log}[1 + IE^{(c+d*x)}] - (3I)b^3d^2 \\ & E^{(2c)}f^2x^2\text{Log}[1 + IE^{(c+d*x)}] + 6a^3d^2e^2\text{Log}[1 + E^{(2(c+d*x))}] + 6a^3d^2e^2E^{(2c)}\text{Log}[1 + E^{(2(c+d*x))}] \\ & + 6ab^2f^2\text{Log}[1 + E^{(2(c+d*x))}] + 6a^3E^{(2c)}f^2\text{Log}[1 + E^{(2(c+d*x))}] + 6ab^2E^{(2c)}f^2\text{Log}[1 + E^{(2(c+d*x))}] \\ & + 12a^3d^2efx\text{Log}[1 + E^{(2(c+d*x))}] + 12a^3d^2E^{(2c)}fx\text{Log}[1 + E^{(2(c+d*x))}] + 6a^3d^2f^2x^2\text{Log}[1 + E^{(2(c+d*x))}] \\ & + 6a^3d^2E^{(2c)}f^2x^2\text{Log}[1 + E^{(2(c+d*x))}] - (6I)b(3a^2 + b^2)d(1 + E^{(2c)})f(e + f*x)\text{PolyLog}[2, (-I)E^{(c+d*x)}] \\ & + (6I)b(3a^2 + b^2)d(1 + E^{(2c)})f(e + f*x)\text{PolyLog}[2, IE^{(c+d*x)}] + 6a^3d^2ef\text{PolyLog}[2, -E^{(2(c+d*x))}] \\ & + 6a^3d^2eE^{(2c)}f\text{PolyLog}[2, -E^{(2(c+d*x))}] + 6a^3d^2f^2x\text{PolyLog}[2, -E^{(2(c+d*x))}] + 6a^3d^2E^{(2c)}f^2x\text{PolyLog}[2, -E^{(2(c+d*x))}] \\ & + (18I)a^2b^2f^2\text{PolyLog}[3, (-I)E^{(c+d*x)}] + (6I)b^3f^2\text{PolyLog}[3, (-I)E^{(c+d*x)}] + (18I)a^2b^2E^{(2c)}f^2\text{PolyLog}[3, (-I)E^{(c+d*x)}] \\ & + (6I)b^3E^{(2c)}f^2\text{PolyLog}[3, (-I)E^{(c+d*x)}] - (18I)a^2b^2f^2\text{PolyLog}[3, IE^{(c+d*x)}] - (6I)b^3f^2\text{PolyLog}[3, IE^{(c+d*x)}] \\ & - (18I)a^2b^2E^{(2c)}f^2\text{PolyLog}[3, IE^{(c+d*x)}] - (6I)b^3E^{(2c)}f^2\text{PolyLog}[3, IE^{(c+d*x)}] - 3a^3f^2\text{PolyLog}[3, -E^{(2(c+d*x))}] \\ & - 3a^3E^{(2c)}f^2\text{PolyLog}[3, -E^{(2(c+d*x))}])]/(6(a^2 + b^2)^2d^3(1 + E^{(2c)})) + (a^3(6e^2E^{(2c)}x + 6eE^{(2c)}f^2x^2 + 2E^{(2c)}f^2x^3 + (6a\text{Sqrt}[a^2 + b^2])e^2\text{ArcTan}[(a + b \end{aligned}$$

$$\begin{aligned}
& *E^{(c + d*x)} / \text{Sqrt}[-a^2 - b^2]] / (\text{Sqrt}[-(a^2 + b^2)^2] * d) + (6*a*\text{Sqrt}[-(a^2 + b^2)^2] * e^{2*c} * \text{ArcTan}[(a + b * E^{(c + d*x)}) / \text{Sqrt}[-a^2 - b^2]]) / ((a^2 + b^2)^{(3/2)} * d) - (6*a*\text{Sqrt}[-(a^2 + b^2)^2] * e^{2*c} * \text{ArcTanh}[(a + b * E^{(c + d*x)}) / \text{Sqrt}[a^2 + b^2]]) / ((-a^2 - b^2)^{(3/2)} * d) + (6*a*\text{Sqrt}[-(a^2 + b^2)^2] * e^{2*c} * \text{ArcTanh}[(a + b * E^{(c + d*x)}) / \text{Sqrt}[a^2 + b^2]]) / ((-a^2 - b^2)^{(3/2)} * d) + (3*e^{2*c} * \text{Log}[2*a * E^{(c + d*x)} + b * (-1 + E^{(2*(c + d*x))})]) / d - (3*e^{2*c} * E^{(2*c)} * \text{Log}[2*a * E^{(c + d*x)} + b * (-1 + E^{(2*(c + d*x))})]) / d + (6*e*f*x * \text{Log}[1 + (b * E^{(2*c + d*x)}) / (a * E^c - \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}])]) / d - (6*e * E^{(2*c)} * f * x * \text{Log}[1 + (b * E^{(2*c + d*x)}) / (a * E^c - \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}])]) / d + (3*f^2 * x^2 * \text{Log}[1 + (b * E^{(2*c + d*x)}) / (a * E^c - \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}])]) / d - (3 * E^{(2*c)} * f^2 * x^2 * \text{Log}[1 + (b * E^{(2*c + d*x)}) / (a * E^c - \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}])]) / d + (6 * e * f * x * \text{Log}[1 + (b * E^{(2*c + d*x)}) / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}])]) / d - (6 * e * E^{(2*c)} * f * x * \text{Log}[1 + (b * E^{(2*c + d*x)}) / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}])]) / d + (3 * f^2 * x^2 * \text{Log}[1 + (b * E^{(2*c + d*x)}) / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}])]) / d - (3 * E^{(2*c)} * f^2 * x^2 * \text{Log}[1 + (b * E^{(2*c + d*x)}) / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}])]) / d - (6 * (-1 + E^{(2*c)}) * f * (e + f * x) * \text{PolyLog}[2, -((b * E^{(2*c + d*x)}) / (a * E^c - \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}])])]) / d^2 - (6 * (-1 + E^{(2*c)}) * f * (e + f * x) * \text{PolyLog}[2, -((b * E^{(2*c + d*x)}) / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}])])]) / d^2 - (6 * f^2 * \text{PolyLog}[3, -((b * E^{(2*c + d*x)}) / (a * E^c - \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}])])]) / d^3 + (6 * E^{(2*c)} * f^2 * \text{PolyLog}[3, -((b * E^{(2*c + d*x)}) / (a * E^c - \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}])])]) / d^3 - (6 * f^2 * \text{PolyLog}[3, -((b * E^{(2*c + d*x)}) / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}])])]) / d^3 + (6 * E^{(2*c)} * f^2 * \text{PolyLog}[3, -((b * E^{(2*c + d*x)}) / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{(2*c)}])])]) / d^3) / (3 * (a^2 + b^2)^2 * (-1 + E^{(2*c)})) + (\text{Csch}[c] * \text{Sech}[c] * \text{Sech}[c + d*x]^2 * (-6 * a^3 * e * f - 6 * a * b^2 * e * f - 12 * a^3 * d^2 * e^2 * x - 6 * a^3 * f^2 * x - 6 * a * b^2 * f^2 * x - 12 * a^3 * d^2 * e * f * x^2 - 4 * a^3 * d^2 * f^2 * x^3 + 6 * a^3 * e * f * \text{Cosh}[2*c] + 6 * a * b^2 * e * f * \text{Cosh}[2*c] + 6 * a^3 * f^2 * x * \text{Cosh}[2*c] + 6 * a * b^2 * f^2 * x * \text{Cosh}[2*c] + 6 * a^3 * e * f * \text{Cosh}[2*d*x] + 6 * a * b^2 * e * f * \text{Cosh}[2*d*x] + 6 * a^3 * f^2 * x * \text{Cosh}[2*d*x] + 6 * a * b^2 * f^2 * x * \text{Cosh}[2*d*x] + 3 * a^2 * b * d * e^2 * \text{Cosh}[c - d*x] + 3 * b^3 * d * e^2 * \text{Cosh}[c - d*x] + 6 * a^2 * b * d * e * f * x * \text{Cosh}[c - d*x] + 6 * b^3 * d * e * f * x * \text{Cosh}[c - d*x] + 3 * a^2 * b * d * f^2 * x^2 * \text{Cosh}[c - d*x] + 3 * b^3 * d * f^2 * x^2 * \text{Cosh}[c - d*x] - 3 * a^2 * b * d * e^2 * \text{Cosh}[3*c + d*x] - 3 * b^3 * d * e^2 * \text{Cosh}[3*c + d*x] - 6 * a^2 * b * d * e * f * x * \text{Cosh}[3*c + d*x] - 6 * b^3 * d * e * f * x * \text{Cosh}[3*c + d*x] - 3 * a^2 * b * d * f^2 * x^2 * \text{Cosh}[3*c + d*x] - 3 * b^3 * d * f^2 * x^2 * \text{Cosh}[3*c + d*x] - 6 * a^3 * e * f * \text{Cosh}[2*c + 2*d*x] - 6 * a * b^2 * e * f * \text{Cosh}[2*c + 2*d*x] - 12 * a^3 * d^2 * e^2 * x * \text{Cosh}[2*c + 2*d*x] - 6 * a^3 * f^2 * x * \text{Cosh}[2*c + 2*d*x] - 6 * a * b^2 * f^2 * x * \text{Cosh}[2*c + 2*d*x] - 12 * a^3 * d^2 * e * f * x^2 * \text{Cosh}[2*c + 2*d*x] - 4 * a^3 * d^2 * f^2 * x^3 * \text{Cosh}[2*c + 2*d*x] + 6 * a^3 * d * e^2 * \text{Sinh}[2*c] + 6 * a * b^2 * d * e^2 * \text{Sinh}[2*c] + 12 * a^3 * d * e * f * x * \text{Sinh}[2*c] + 12 * a * b^2 * d * e * f * x * \text{Sinh}[2*c] + 6 * a^3 * d * f^2 * x^2 * \text{Sinh}[2*c] + 6 * a * b^2 * d * f^2 * x^2 * \text{Sinh}[2*c] - 6 * a^2 * b * e * f * \text{Sinh}[c - d*x] - 6 * b^3 * e * f * \text{Sinh}[c - d*x] - 6 * a^2 * b * f^2 * x * \text{Sinh}[c - d*x] - 6 * b^3 * f^2 * x * \text{Sinh}[c - d*x] - 6 * a^2 * b * e * f * \text{Sinh}[3*c + d*x] - 6 * b^3 * e * f * \text{Sinh}[3*c + d*x] - 6 * a^2 * b * f^2 * x * \text{Sinh}[3*c + d*x] - 6 * b^3 * f^2 * x * \text{Sinh}[3*c + d*x])) / (24 * (a^2 + b^2)^2 * d^2)
\end{aligned}$$



$$+ c))/((a^4 + 2*a^2*b^2 + b^4)*d^3) + 2*b^3*f^2*\arctan(e^{(d*x + c)})/((a^4 + 2*a^2*b^2 + b^4)*d^3) + (2*a*f^2*x + 2*a*e*f - (b*d*f^2*x^2*e^{(3*c)} + 2*b*e*f*e^{(3*c)} + 2*(d*e*f + f^2)*b*x*e^{(3*c)}))*e^{(3*d*x)} + 2*(a*d*f^2*x^2*e^{(2*c)} + a*e*f*e^{(2*c)} + (2*d*e*f + f^2)*a*x*e^{(2*c)}))*e^{(2*d*x)} + (b*d*f^2*x^2*e^c - 2*b*e*f*e^c + 2*(d*e*f - f^2)*b*x*e^c)*e^{(d*x)}/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^{(4*c)} + b^2*d^2*e^{(4*c)}))*e^{(4*d*x)} + 2*(a^2*d^2*e^{(2*c)} + b^2*d^2*e^{(2*c)}))*e^{(2*d*x)} + \int (2*(a^3*b*f^2*x^2 + 2*a^3*b*e*f*x - (a^4*f^2*x^2*e^c + 2*a^4*e*f*x*e^c))*e^{(d*x)})/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b*e^{(2*c)} + 2*a^2*b^3*e^{(2*c)} + b^5*e^{(2*c)}))*e^{(2*d*x)} - 2*(a^5*e^c + 2*a^3*b^2*e^c + a*b^4*e^c)*e^{(d*x)}, x$$

**Fricas [C]** time = 5.78727, size = 24035, normalized size = 16.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*tanh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-1/2*(4*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*\cosh(d*x + c)^4 + 4*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*\sinh(d*x + c)^4 - 4*(a^3 + a*b^2)*d*e*f + 4*(a^3 + a*b^2)*c*f^2 + 2*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 + 2*(a^2*b + b^3)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (a^2*b + b^3)*d*f^2)*x)*\cosh(d*x + c)^3 + 2*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 + 2*(a^2*b + b^3)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (a^2*b + b^3)*d*f^2)*x + 8*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 4*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a*b^2)*d^2*e^2 + (a^3 + a*b^2)*d*e*f - 2*(a^3 + a*b^2)*c*f^2 + (2*(a^3 + a*b^2)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x)*\cosh(d*x + c)^2 - 2*(2*(a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f - 4*(a^3 + a*b^2)*c*f^2 - 12*((a^3 + a*b^2)*d*f^2*x + (a^3 + a*b^2)*c*f^2)*\cosh(d*x + c)^2 + 2*(2*(a^3 + a*b^2)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x - 3*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 + 2*(a^2*b + b^3)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (a^2*b + b^3)*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c)^2 - 2*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f - (a^2*b + b^3)*d*f^2)*x)*\cosh(d*x + c) + 4*(a^3*d*f^2*x + a^3*d*e*f + (a^3*d*f^2*x + a^3*d*e*f)*\cosh(d*x + c))^4 + 4*(a^3*d*f^2*x + a^3*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^3*d*f^2*x + a^3*d*e*f)*\sinh(d*x + c)^4 + 2*(a^3*d*f^2*x + a^3*d*e*f)*\cosh(d*x + c)^2 + 2*(a^3*d*f^2*x + a^3*d*e*f + 3*(a^3*d*f^2*x + a^3*d*e*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((a^3*d*f^2*x + a^3*d*e*f)*\cosh(d*x + c)^3 + (a^3*d*f^2*x + a^3*d*e*f)*\cosh(d*x + c))*\sinh(d*x + c))*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c)))$$





$$\begin{aligned}
& ^2) \cosh(dx + c)^4 + 4*(a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2) \cosh(dx \\
& x + c) \sinh(dx + c)^3 + (a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2) \sinh(dx \\
& *x + c)^4 + 2*(a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2) \cosh(dx + c)^2 + \\
& 2*(a^3*d^2*e^2 - 2*a^3*c*d*e*f + a^3*c^2*f^2 + 3*(a^3*d^2*e^2 - 2*a^3*c*d* \\
& e*f + a^3*c^2*f^2) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4*((a^3*d^2*e^2 - 2*a \\
& ^3*c*d*e*f + a^3*c^2*f^2) \cosh(dx + c)^3 + (a^3*d^2*e^2 - 2*a^3*c*d*e*f + \\
& a^3*c^2*f^2) \cosh(dx + c)) \sinh(dx + c)) \log(2*b \cosh(dx + c) + 2*b \sinh \\
& (dx + c) - 2*b \sqrt{(a^2 + b^2)/b^2} + 2*a) + 2*(a^3*d^2*f^2*x^2 + 2*a^3*d \\
& ^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2 + (a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x \\
& + 2*a^3*c*d*e*f - a^3*c^2*f^2) \cosh(dx + c)^4 + 4*(a^3*d^2*f^2*x^2 + 2*a^ \\
& 3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2) \cosh(dx + c) \sinh(dx + c)^3 + \\
& (a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2) \sinh(dx \\
& + c)^4 + 2*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2 \\
& ) \cosh(dx + c)^2 + 2*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - \\
& a^3*c^2*f^2 + 3*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^ \\
& 2*f^2) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4*((a^3*d^2*f^2*x^2 + 2*a^3*d^2*e \\
& *f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2) \cosh(dx + c)^3 + (a^3*d^2*f^2*x^2 + 2* \\
& a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2) \cosh(dx + c)) \sinh(dx + c)) \\
& \log(-(a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c \\
& )) \sqrt{(a^2 + b^2)/b^2} - b)/b) + 2*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2 \\
& *a^3*c*d*e*f - a^3*c^2*f^2 + (a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d \\
& *e*f - a^3*c^2*f^2) \cosh(dx + c)^4 + 4*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x \\
& + 2*a^3*c*d*e*f - a^3*c^2*f^2) \cosh(dx + c) \sinh(dx + c)^3 + (a^3*d^2*f^2 \\
& *x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2) \sinh(dx + c)^4 + 2*( \\
& a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2) \cosh(dx + \\
& c)^2 + 2*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2 \\
& + 3*(a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3*c*d*e*f - a^3*c^2*f^2) \cosh( \\
& dx + c)^2) \sinh(dx + c)^2 + 4*((a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f*x + 2*a^3 \\
& *c*d*e*f - a^3*c^2*f^2) \cosh(dx + c)^3 + (a^3*d^2*f^2*x^2 + 2*a^3*d^2*e*f* \\
& x + 2*a^3*c*d*e*f - a^3*c^2*f^2) \cosh(dx + c)) \sinh(dx + c)) \log(-(a \cosh \\
& (dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 \\
& + b^2)/b^2} - b)/b) - (2*a^3*d^2*e^2 - 4*a^3*c*d*e*f + I*(3*a^2*b + b^3)*d \\
& ^2*e^2 - 2*I*(3*a^2*b + b^3)*c*d*e*f + (2*a^3*d^2*e^2 - 4*a^3*c*d*e*f + I*( \\
& 3*a^2*b + b^3)*d^2*e^2 - 2*I*(3*a^2*b + b^3)*c*d*e*f + 2*(a^3*c^2 + a^3 + a \\
& *b^2)*f^2 + I*(2*a^2*b + 2*b^3 + (3*a^2*b + b^3)*c^2)*f^2) \cosh(dx + c)^4 \\
& + (8*a^3*d^2*e^2 - 16*a^3*c*d*e*f + 4*I*(3*a^2*b + b^3)*d^2*e^2 - 8*I*(3*a^ \\
& 2*b + b^3)*c*d*e*f + 8*(a^3*c^2 + a^3 + a*b^2)*f^2 + 4*I*(2*a^2*b + 2*b^3 + \\
& (3*a^2*b + b^3)*c^2)*f^2) \cosh(dx + c) \sinh(dx + c)^3 + (2*a^3*d^2*e^2 - \\
& 4*a^3*c*d*e*f + I*(3*a^2*b + b^3)*d^2*e^2 - 2*I*(3*a^2*b + b^3)*c*d*e*f + \\
& 2*(a^3*c^2 + a^3 + a*b^2)*f^2 + I*(2*a^2*b + 2*b^3 + (3*a^2*b + b^3)*c^2)*f \\
& ^2) \sinh(dx + c)^4 + 2*(a^3*c^2 + a^3 + a*b^2)*f^2 + I*(2*a^2*b + 2*b^3 + \\
& (3*a^2*b + b^3)*c^2)*f^2 + (4*a^3*d^2*e^2 - 8*a^3*c*d*e*f + 2*I*(3*a^2*b + \\
& b^3)*d^2*e^2 - 4*I*(3*a^2*b + b^3)*c*d*e*f + 4*(a^3*c^2 + a^3 + a*b^2)*f^2 \\
& + 2*I*(2*a^2*b + 2*b^3 + (3*a^2*b + b^3)*c^2)*f^2) \cosh(dx + c)^2 + (4*a^3 \\
& *d^2*e^2 - 8*a^3*c*d*e*f + 2*I*(3*a^2*b + b^3)*d^2*e^2 - 4*I*(3*a^2*b + b^3)
\end{aligned}$$

$$\begin{aligned}
& ) * c * d * e * f + 4 * (a^3 * c^2 + a^3 + a * b^2) * f^2 + 2 * I * (2 * a^2 * b + 2 * b^3 + (3 * a^2 * b \\
& + b^3) * c^2) * f^2 + (12 * a^3 * d^2 * e^2 - 24 * a^3 * c * d * e * f + 6 * I * (3 * a^2 * b + b^3) * d \\
& ^2 * e^2 - 12 * I * (3 * a^2 * b + b^3) * c * d * e * f + 12 * (a^3 * c^2 + a^3 + a * b^2) * f^2 + 6 * \\
& I * (2 * a^2 * b + 2 * b^3 + (3 * a^2 * b + b^3) * c^2) * f^2) * \cosh(d * x + c)^2 * \sinh(d * x + \\
& c)^2 + ((8 * a^3 * d^2 * e^2 - 16 * a^3 * c * d * e * f + 4 * I * (3 * a^2 * b + b^3) * d^2 * e^2 - 8 * I \\
& * (3 * a^2 * b + b^3) * c * d * e * f + 8 * (a^3 * c^2 + a^3 + a * b^2) * f^2 + 4 * I * (2 * a^2 * b + 2 \\
& * b^3 + (3 * a^2 * b + b^3) * c^2) * f^2) * \cosh(d * x + c)^3 + (8 * a^3 * d^2 * e^2 - 16 * a^3 * \\
& c * d * e * f + 4 * I * (3 * a^2 * b + b^3) * d^2 * e^2 - 8 * I * (3 * a^2 * b + b^3) * c * d * e * f + 8 * (a^ \\
& 3 * c^2 + a^3 + a * b^2) * f^2 + 4 * I * (2 * a^2 * b + 2 * b^3 + (3 * a^2 * b + b^3) * c^2) * f^2) \\
& * \cosh(d * x + c) * \sinh(d * x + c) * \log(\cosh(d * x + c) + \sinh(d * x + c) + I) - (2 * \\
& a^3 * d^2 * e^2 - 4 * a^3 * c * d * e * f - I * (3 * a^2 * b + b^3) * d^2 * e^2 + 2 * I * (3 * a^2 * b + b^ \\
& 3) * c * d * e * f + (2 * a^3 * d^2 * e^2 - 4 * a^3 * c * d * e * f - I * (3 * a^2 * b + b^3) * d^2 * e^2 + 2 \\
& * I * (3 * a^2 * b + b^3) * c * d * e * f + 2 * (a^3 * c^2 + a^3 + a * b^2) * f^2 - I * (2 * a^2 * b + 2 \\
& * b^3 + (3 * a^2 * b + b^3) * c^2) * f^2) * \cosh(d * x + c)^4 + (8 * a^3 * d^2 * e^2 - 16 * a^3 * \\
& c * d * e * f - 4 * I * (3 * a^2 * b + b^3) * d^2 * e^2 + 8 * I * (3 * a^2 * b + b^3) * c * d * e * f + 8 * (a^ \\
& 3 * c^2 + a^3 + a * b^2) * f^2 - 4 * I * (2 * a^2 * b + 2 * b^3 + (3 * a^2 * b + b^3) * c^2) * f^2) \\
& * \cosh(d * x + c) * \sinh(d * x + c)^3 + (2 * a^3 * d^2 * e^2 - 4 * a^3 * c * d * e * f - I * (3 * a^2 * \\
& b + b^3) * d^2 * e^2 + 2 * I * (3 * a^2 * b + b^3) * c * d * e * f + 2 * (a^3 * c^2 + a^3 + a * b^2) * \\
& f^2 - I * (2 * a^2 * b + 2 * b^3 + (3 * a^2 * b + b^3) * c^2) * f^2) * \sinh(d * x + c)^4 + 2 * (a \\
& ^3 * c^2 + a^3 + a * b^2) * f^2 - I * (2 * a^2 * b + 2 * b^3 + (3 * a^2 * b + b^3) * c^2) * f^2 + \\
& (4 * a^3 * d^2 * e^2 - 8 * a^3 * c * d * e * f - 2 * I * (3 * a^2 * b + b^3) * d^2 * e^2 + 4 * I * (3 * a^2 * \\
& b + b^3) * c * d * e * f + 4 * (a^3 * c^2 + a^3 + a * b^2) * f^2 - 2 * I * (2 * a^2 * b + 2 * b^3 + ( \\
& 3 * a^2 * b + b^3) * c^2) * f^2) * \cosh(d * x + c)^2 + (4 * a^3 * d^2 * e^2 - 8 * a^3 * c * d * e * f - \\
& 2 * I * (3 * a^2 * b + b^3) * d^2 * e^2 + 4 * I * (3 * a^2 * b + b^3) * c * d * e * f + 4 * (a^3 * c^2 + a \\
& ^3 + a * b^2) * f^2 - 2 * I * (2 * a^2 * b + 2 * b^3 + (3 * a^2 * b + b^3) * c^2) * f^2 + (12 * a^3 \\
& * d^2 * e^2 - 24 * a^3 * c * d * e * f - 6 * I * (3 * a^2 * b + b^3) * d^2 * e^2 + 12 * I * (3 * a^2 * b + b \\
& ^3) * c * d * e * f + 12 * (a^3 * c^2 + a^3 + a * b^2) * f^2 - 6 * I * (2 * a^2 * b + 2 * b^3 + (3 * a^ \\
& 2 * b + b^3) * c^2) * f^2) * \cosh(d * x + c)^2 * \sinh(d * x + c)^2 + ((8 * a^3 * d^2 * e^2 - 1 \\
& 6 * a^3 * c * d * e * f - 4 * I * (3 * a^2 * b + b^3) * d^2 * e^2 + 8 * I * (3 * a^2 * b + b^3) * c * d * e * f + \\
& 8 * (a^3 * c^2 + a^3 + a * b^2) * f^2 - 4 * I * (2 * a^2 * b + 2 * b^3 + (3 * a^2 * b + b^3) * c^2 \\
& ) * f^2) * \cosh(d * x + c)^3 + (8 * a^3 * d^2 * e^2 - 16 * a^3 * c * d * e * f - 4 * I * (3 * a^2 * b + b \\
& ^3) * d^2 * e^2 + 8 * I * (3 * a^2 * b + b^3) * c * d * e * f + 8 * (a^3 * c^2 + a^3 + a * b^2) * f^2 - \\
& 4 * I * (2 * a^2 * b + 2 * b^3 + (3 * a^2 * b + b^3) * c^2) * f^2) * \cosh(d * x + c) * \sinh(d * x + \\
& c) * \log(\cosh(d * x + c) + \sinh(d * x + c) - I) - (2 * a^3 * d^2 * f^2 * x^2 + 4 * a^3 * d^ \\
& 2 * e * f * x + 4 * a^3 * c * d * e * f - 2 * a^3 * c^2 * f^2 - I * (3 * a^2 * b + b^3) * d^2 * f^2 * x^2 - 2 \\
& * I * (3 * a^2 * b + b^3) * d^2 * e * f * x - 2 * I * (3 * a^2 * b + b^3) * c * d * e * f + I * (3 * a^2 * b + b \\
& ^3) * c^2 * f^2 + (2 * a^3 * d^2 * f^2 * x^2 + 4 * a^3 * d^2 * e * f * x + 4 * a^3 * c * d * e * f - 2 * a^3 * \\
& c^2 * f^2 - I * (3 * a^2 * b + b^3) * d^2 * f^2 * x^2 - 2 * I * (3 * a^2 * b + b^3) * d^2 * e * f * x - 2 \\
& * I * (3 * a^2 * b + b^3) * c * d * e * f + I * (3 * a^2 * b + b^3) * c^2 * f^2) * \cosh(d * x + c)^4 + ( \\
& 8 * a^3 * d^2 * f^2 * x^2 + 16 * a^3 * d^2 * e * f * x + 16 * a^3 * c * d * e * f - 8 * a^3 * c^2 * f^2 - 4 * I \\
& * (3 * a^2 * b + b^3) * d^2 * f^2 * x^2 - 8 * I * (3 * a^2 * b + b^3) * d^2 * e * f * x - 8 * I * (3 * a^2 * b \\
& + b^3) * c * d * e * f + 4 * I * (3 * a^2 * b + b^3) * c^2 * f^2) * \cosh(d * x + c) * \sinh(d * x + c)^ \\
& 3 + (2 * a^3 * d^2 * f^2 * x^2 + 4 * a^3 * d^2 * e * f * x + 4 * a^3 * c * d * e * f - 2 * a^3 * c^2 * f^2 - \\
& I * (3 * a^2 * b + b^3) * d^2 * f^2 * x^2 - 2 * I * (3 * a^2 * b + b^3) * d^2 * e * f * x - 2 * I * (3 * a^2 * \\
& b + b^3) * c * d * e * f + I * (3 * a^2 * b + b^3) * c^2 * f^2) * \sinh(d * x + c)^4 + (4 * a^3 * d^2 *
\end{aligned}$$

$$\begin{aligned}
& f^2x^2 + 8a^3d^2efx + 8a^3cd^2ef - 4a^3c^2f^2 - 2I(3a^2b + b^3)d^2f^2x^2 - 4I(3a^2b + b^3)d^2efx - 4I(3a^2b + b^3)cd^2ef + 2I(3a^2b + b^3)c^2f^2) \cosh(dx + c)^2 + (4a^3d^2f^2x^2 + 8a^3d^2efx + 8a^3cd^2ef - 4a^3c^2f^2 - 2I(3a^2b + b^3)d^2f^2x^2 - 4I(3a^2b + b^3)d^2efx - 4I(3a^2b + b^3)cd^2ef + 2I(3a^2b + b^3)c^2f^2 + (12a^3d^2f^2x^2 + 24a^3d^2efx + 24a^3cd^2ef - 12a^3c^2f^2 - 6I(3a^2b + b^3)d^2f^2x^2 - 12I(3a^2b + b^3)d^2efx - 12I(3a^2b + b^3)cd^2ef + 6I(3a^2b + b^3)c^2f^2) \cosh(dx + c)^2) \sinh(dx + c)^2 + ((8a^3d^2f^2x^2 + 16a^3d^2efx + 16a^3cd^2ef - 8a^3c^2f^2 - 4I(3a^2b + b^3)d^2f^2x^2 - 8I(3a^2b + b^3)d^2efx - 8I(3a^2b + b^3)cd^2ef + 4I(3a^2b + b^3)c^2f^2) \cosh(dx + c)^3 + (8a^3d^2f^2x^2 + 16a^3d^2efx + 16a^3cd^2ef - 8a^3c^2f^2 - 4I(3a^2b + b^3)d^2f^2x^2 - 8I(3a^2b + b^3)d^2efx - 8I(3a^2b + b^3)cd^2ef + 4I(3a^2b + b^3)c^2f^2) \cosh(dx + c)) \sinh(dx + c) \log(I \cosh(dx + c) + I \sinh(dx + c) + 1) - (2a^3d^2f^2x^2 + 4a^3d^2efx + 4a^3cd^2ef - 2a^3c^2f^2 + I(3a^2b + b^3)d^2f^2x^2 + 2I(3a^2b + b^3)d^2efx + 2I(3a^2b + b^3)cd^2ef - I(3a^2b + b^3)c^2f^2 + (2a^3d^2f^2x^2 + 4a^3d^2efx + 4a^3cd^2ef - 2a^3c^2f^2 + I(3a^2b + b^3)d^2f^2x^2 + 2I(3a^2b + b^3)d^2efx + 2I(3a^2b + b^3)cd^2ef - I(3a^2b + b^3)c^2f^2) \cosh(dx + c)^4 + (8a^3d^2f^2x^2 + 16a^3d^2efx + 16a^3cd^2ef - 8a^3c^2f^2 + 4I(3a^2b + b^3)d^2f^2x^2 + 8I(3a^2b + b^3)d^2efx + 8I(3a^2b + b^3)cd^2ef - 4I(3a^2b + b^3)c^2f^2) \cosh(dx + c) \sinh(dx + c)^3 + (2a^3d^2f^2x^2 + 4a^3d^2efx + 4a^3cd^2ef - 2a^3c^2f^2 + I(3a^2b + b^3)d^2f^2x^2 + 2I(3a^2b + b^3)d^2efx + 2I(3a^2b + b^3)cd^2ef - I(3a^2b + b^3)c^2f^2) \sinh(dx + c)^4 + (4a^3d^2f^2x^2 + 8a^3d^2efx + 8a^3cd^2ef - 4a^3c^2f^2 + 2I(3a^2b + b^3)d^2f^2x^2 + 4I(3a^2b + b^3)d^2efx + 4I(3a^2b + b^3)cd^2ef - 2I(3a^2b + b^3)c^2f^2) \cosh(dx + c)^2 + (4a^3d^2f^2x^2 + 8a^3d^2efx + 8a^3cd^2ef - 4a^3c^2f^2 + 2I(3a^2b + b^3)d^2f^2x^2 + 4I(3a^2b + b^3)d^2efx + 4I(3a^2b + b^3)cd^2ef - 2I(3a^2b + b^3)c^2f^2 + (12a^3d^2f^2x^2 + 24a^3d^2efx + 24a^3cd^2ef - 12a^3c^2f^2 + 6I(3a^2b + b^3)d^2f^2x^2 + 12I(3a^2b + b^3)d^2efx + 12I(3a^2b + b^3)cd^2ef - 6I(3a^2b + b^3)c^2f^2) \cosh(dx + c)^2) \sinh(dx + c)^2 + ((8a^3d^2f^2x^2 + 16a^3d^2efx + 16a^3cd^2ef - 8a^3c^2f^2 + 4I(3a^2b + b^3)d^2f^2x^2 + 8I(3a^2b + b^3)d^2efx + 8I(3a^2b + b^3)cd^2ef - 4I(3a^2b + b^3)c^2f^2) \cosh(dx + c)^3 + (8a^3d^2f^2x^2 + 16a^3d^2efx + 16a^3cd^2ef - 8a^3c^2f^2 + 4I(3a^2b + b^3)d^2f^2x^2 + 8I(3a^2b + b^3)d^2efx + 8I(3a^2b + b^3)cd^2ef - 4I(3a^2b + b^3)c^2f^2) \cosh(dx + c)) \sinh(dx + c) \log(-I \cosh(dx + c) - I \sinh(dx + c) + 1) - 4(a^3f^2 \cosh(dx + c))^4 + 4a^3f^2 \cosh(dx + c) \sinh(dx + c)^3 + a^3f^2 \sinh(dx + c)^4 + 2a^3f^2 \cosh(dx + c)^2 + a^3f^2 + 2(3a^3f^2 \cosh(dx + c))^2 + a^3f^2) \sinh(dx + c)^2 + 4(a^3f^2 \cosh(dx + c))^3 + a^3f^2 \cosh(dx + c) \sinh(dx + c) \text{polylo}
\end{aligned}$$

```

g(3, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c
))*sqrt((a^2 + b^2)/b^2))/b) - 4*(a^3*f^2*cosh(d*x + c)^4 + 4*a^3*f^2*cosh(
d*x + c)*sinh(d*x + c)^3 + a^3*f^2*sinh(d*x + c)^4 + 2*a^3*f^2*cosh(d*x + c
)^2 + a^3*f^2 + 2*(3*a^3*f^2*cosh(d*x + c)^2 + a^3*f^2)*sinh(d*x + c)^2 + 4
*(a^3*f^2*cosh(d*x + c)^3 + a^3*f^2*cosh(d*x + c))*sinh(d*x + c))*polylog(3
, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*
sqrt((a^2 + b^2)/b^2))/b) + (4*a^3*f^2 + (4*a^3*f^2 + 2*I*(3*a^2*b + b^3)*f
^2)*cosh(d*x + c)^4 + (16*a^3*f^2 + 8*I*(3*a^2*b + b^3)*f^2)*cosh(d*x + c)*
sinh(d*x + c)^3 + (4*a^3*f^2 + 2*I*(3*a^2*b + b^3)*f^2)*sinh(d*x + c)^4 + 2
*I*(3*a^2*b + b^3)*f^2 + (8*a^3*f^2 + 4*I*(3*a^2*b + b^3)*f^2)*cosh(d*x + c
)^2 + (8*a^3*f^2 + 4*I*(3*a^2*b + b^3)*f^2 + (24*a^3*f^2 + 12*I*(3*a^2*b +
b^3)*f^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((16*a^3*f^2 + 8*I*(3*a^2*b +
b^3)*f^2)*cosh(d*x + c)^3 + (16*a^3*f^2 + 8*I*(3*a^2*b + b^3)*f^2)*cosh(d*x
+ c))*sinh(d*x + c))*polylog(3, I*cosh(d*x + c) + I*sinh(d*x + c)) + (4*a^
3*f^2 + (4*a^3*f^2 - 2*I*(3*a^2*b + b^3)*f^2)*cosh(d*x + c)^4 + (16*a^3*f^2
- 8*I*(3*a^2*b + b^3)*f^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a^3*f^2 - 2*
I*(3*a^2*b + b^3)*f^2)*sinh(d*x + c)^4 - 2*I*(3*a^2*b + b^3)*f^2 + (8*a^3*f
^2 - 4*I*(3*a^2*b + b^3)*f^2)*cosh(d*x + c)^2 + (8*a^3*f^2 - 4*I*(3*a^2*b +
b^3)*f^2 + (24*a^3*f^2 - 12*I*(3*a^2*b + b^3)*f^2)*cosh(d*x + c)^2)*sinh(d
*x + c)^2 + ((16*a^3*f^2 - 8*I*(3*a^2*b + b^3)*f^2)*cosh(d*x + c)^3 + (16*a
^3*f^2 - 8*I*(3*a^2*b + b^3)*f^2)*cosh(d*x + c))*sinh(d*x + c))*polylog(3,
-I*cosh(d*x + c) - I*sinh(d*x + c)) - 2*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b
+ b^3)*d^2*e^2 - 2*(a^2*b + b^3)*d*e*f - 8*((a^3 + a*b^2)*d*f^2*x + (a^3 +
a*b^2)*c*f^2)*cosh(d*x + c)^3 - 3*((a^2*b + b^3)*d^2*f^2*x^2 + (a^2*b + b^
3)*d^2*e^2 + 2*(a^2*b + b^3)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + (a^2*b + b^
3)*d*f^2)*x)*cosh(d*x + c)^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^2*b + b^3)*d*f
^2)*x + 4*((a^3 + a*b^2)*d^2*f^2*x^2 + (a^3 + a*b^2)*d^2*e^2 + (a^3 + a*b^2
)*d*e*f - 2*(a^3 + a*b^2)*c*f^2 + (2*(a^3 + a*b^2)*d^2*e*f - (a^3 + a*b^2)*
d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d
*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)*sinh(d*x + c)^3 + (
a^4 + 2*a^2*b^2 + b^4)*d^3*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^3*
cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^3 + 2*(3*(a^4 + 2*a^2*b^2 + b^4
)*d^3*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*d^3)*sinh(d*x + c)^2 + 4*((
a^4 + 2*a^2*b^2 + b^4)*d^3*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*d^3*co
sh(d*x + c))*sinh(d*x + c))

```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^2 \tanh^3(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*tanh(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*tanh(c + d*x)**3/(a + b*sinh(c + d*x)), x)
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*tanh(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.417 \quad \int \frac{(e+fx) \tanh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=894

result too large to display

```
[Out] (a^2*(e + f*x)*ArcTan[E^(c + d*x)])/(b^3*d) + ((e + f*x)*ArcTan[E^(c + d*x)
])/ (b*d) - (2*a^4*(e + f*x)*ArcTan[E^(c + d*x)])/(b*(a^2 + b^2)^2*d) - (a^4
*(e + f*x)*ArcTan[E^(c + d*x)])/(b^3*(a^2 + b^2)*d) - (a^3*(e + f*x)*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/((a^2 + b^2)^2*d) - (a^3*(e + f*x
)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/((a^2 + b^2)^2*d) + (a^3*
(e + f*x)*Log[1 + E^(2*(c + d*x))])/((a^2 + b^2)^2*d) - ((I/2)*a^2*f*PolyLo
g[2, (-I)*E^(c + d*x)])/(b^3*d^2) - ((I/2)*f*PolyLog[2, (-I)*E^(c + d*x)])/
(b*d^2) + (I*a^4*f*PolyLog[2, (-I)*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^2) + ((
I/2)*a^4*f*PolyLog[2, (-I)*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) + ((I/2)*a^2
*f*PolyLog[2, I*E^(c + d*x)])/(b^3*d^2) + ((I/2)*f*PolyLog[2, I*E^(c + d*x)
])/ (b*d^2) - (I*a^4*f*PolyLog[2, I*E^(c + d*x)])/(b*(a^2 + b^2)^2*d^2) - ((
I/2)*a^4*f*PolyLog[2, I*E^(c + d*x)])/(b^3*(a^2 + b^2)*d^2) - (a^3*f*PolyLo
g[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2) - (a^3*
f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/((a^2 + b^2)^2*d^2)
+ (a^3*f*PolyLog[2, -E^(2*(c + d*x))])/((2*(a^2 + b^2)^2*d^2) + (a^2*f*Sech
[c + d*x])/(2*b^3*d^2) - (f*Sech[c + d*x])/(2*b*d^2) - (a^4*f*Sech[c + d*x]
)/(2*b^3*(a^2 + b^2)*d^2) + (a*(e + f*x)*Sech[c + d*x]^2)/(2*b^2*d) - (a^3*
(e + f*x)*Sech[c + d*x]^2)/(2*b^2*(a^2 + b^2)*d) - (a*f*Tanh[c + d*x])/(2*b
^2*d^2) + (a^3*f*Tanh[c + d*x])/(2*b^2*(a^2 + b^2)*d^2) + (a^2*(e + f*x)*Se
ch[c + d*x]*Tanh[c + d*x])/(2*b^3*d) - ((e + f*x)*Sech[c + d*x]*Tanh[c + d*
x])/(2*b*d) - (a^4*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b^3*(a^2 + b^2
)*d)
```

**Rubi [A]** time = 1.41524, antiderivative size = 894, normalized size of antiderivative = 1., number of steps used = 55, number of rules used = 15, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$ , Rules used = {5567, 5455, 4180, 2279, 2391, 4185, 5583, 5451, 3767, 8, 5573, 5561, 2190, 6742, 3718}

$$-\frac{(e+fx) \tan^{-1}(e^{c+dx}) a^4}{b^3(a^2+b^2)d} - \frac{2(e+fx) \tan^{-1}(e^{c+dx}) a^4}{b(a^2+b^2)^2 d} + \frac{if \text{PolyLog}(2, -ie^{c+dx}) a^4}{2b^3(a^2+b^2)d^2} + \frac{if \text{PolyLog}(2, -ie^{c+dx}) a^4}{b(a^2+b^2)^2 d^2} - \frac{if \text{Po}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Tanh[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (a^2*(e + f*x)*ArcTan[E^(c + d*x)]/(b^3*d) + ((e + f*x)*ArcTan[E^(c + d*x)
])/ (b*d) - (2*a^4*(e + f*x)*ArcTan[E^(c + d*x)]/(b*(a^2 + b^2)^2*d) - (a^4
*(e + f*x)*ArcTan[E^(c + d*x)]/(b^3*(a^2 + b^2)*d) - (a^3*(e + f*x)*Log[1
+ (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2 + b^2)^2*d) - (a^3*(e + f*x
)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2 + b^2)^2*d) + (a^3*
(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a^2 + b^2)^2*d) - ((I/2)*a^2*f*PolyLo
g[2, (-I)*E^(c + d*x)]/(b^3*d^2) - ((I/2)*f*PolyLog[2, (-I)*E^(c + d*x)]/
(b*d^2) + (I*a^4*f*PolyLog[2, (-I)*E^(c + d*x)]/(b*(a^2 + b^2)^2*d^2) + ((
I/2)*a^4*f*PolyLog[2, (-I)*E^(c + d*x)]/(b^3*(a^2 + b^2)*d^2) + ((I/2)*a^2
*f*PolyLog[2, I*E^(c + d*x)]/(b^3*d^2) + ((I/2)*f*PolyLog[2, I*E^(c + d*x)
])/ (b*d^2) - (I*a^4*f*PolyLog[2, I*E^(c + d*x)]/(b*(a^2 + b^2)^2*d^2) - ((
I/2)*a^4*f*PolyLog[2, I*E^(c + d*x)]/(b^3*(a^2 + b^2)*d^2) - (a^3*f*PolyLo
g[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2 + b^2)^2*d^2) - (a^3*
f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2 + b^2)^2*d^2
+ (a^3*f*PolyLog[2, -E^(2*(c + d*x))])/(2*(a^2 + b^2)^2*d^2) + (a^2*f*Sech
[c + d*x])/(2*b^3*d^2) - (f*Sech[c + d*x])/(2*b*d^2) - (a^4*f*Sech[c + d*x]
)/(2*b^3*(a^2 + b^2)*d^2) + (a*(e + f*x)*Sech[c + d*x]^2)/(2*b^2*d) - (a^3*
(e + f*x)*Sech[c + d*x]^2)/(2*b^2*(a^2 + b^2)*d) - (a*f*Tanh[c + d*x])/(2*b
^2*d^2) + (a^3*f*Tanh[c + d*x])/(2*b^2*(a^2 + b^2)*d^2) + (a^2*(e + f*x)*Se
ch[c + d*x]*Tanh[c + d*x])/(2*b^3*d) - ((e + f*x)*Sech[c + d*x]*Tanh[c + d*
x])/(2*b*d) - (a^4*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*b^3*(a^2 + b^2
)*d)
```

### Rule 5567

```
Int[(((e_.) + (f_.)*(x_.))^m_.)*Tanh[(c_.) + (d_.)*(x_.)]^(n_.))/((a_.) + (b_
_.)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := Dist[1/b, Int[(e + f*x)^m*Sech[c
+ d*x]*Tanh[c + d*x]^(n - 1), x], x] - Dist[a/b, Int[(((e + f*x)^m*Sech[c +
d*x]*Tanh[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b,
c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

### Rule 5455

```
Int[(((c_.) + (d_.)*(x_.))^m_.)*Sech[(a_.) + (b_.)*(x_.)]*Tanh[(a_.) + (b_.)*
(x_.)]^(p_.), x_Symbol] := Int[(c + d*x)^m*Sech[a + b*x]*Tanh[a + b*x]^(p - 2
), x] - Int[(c + d*x)^m*Sech[a + b*x]^3*Tanh[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]
```

### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_.)]*((c_.) + (d_.)*(x_
))^m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 4185

Int[(csc[(e\_) + (f\_)\*(x\_)]\*(b\_))^(n\_)\*((c\_) + (d\_)\*(x\_)), x\_Symbol] :> -Simp[(b^2\*(c + d\*x)\*Cot[e + f\*x]\*(b\*Csc[e + f\*x])^(n - 2))/(f\*(n - 1)), x] + (Dist[(b^2\*(n - 2))/(n - 1), Int[(c + d\*x)\*(b\*Csc[e + f\*x])^(n - 2), x], x] - Simp[(b^2\*d\*(b\*Csc[e + f\*x])^(n - 2))/(f^2\*(n - 1)\*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]

### Rule 5583

Int[(((e\_) + (f\_)\*(x\_))^(m\_)\*Sech[(c\_) + (d\_)\*(x\_)]^(p\_)\*Tanh[(c\_) + (d\_)\*(x\_)]^(n\_))/((a\_) + (b\_)\*Sinh[(c\_) + (d\_)\*(x\_)]), x\_Symbol] :> Dist[1/b, Int[(e + f\*x)^m\*Sech[c + d\*x]^(p + 1)\*Tanh[c + d\*x]^(n - 1), x], x] - Dist[a/b, Int[(((e + f\*x)^m\*Sech[c + d\*x]^(p + 1)\*Tanh[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5451

Int[(((c\_) + (d\_)\*(x\_))^(m\_)\*Sech[(a\_) + (b\_)\*(x\_)]^(n\_)\*Tanh[(a\_) + (b\_)\*(x\_)]^(p\_)), x\_Symbol] :> -Simp[((c + d\*x)^m\*Sech[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Sech[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

### Rule 3767

Int[csc[(c\_) + (d\_)\*(x\_)]^(n\_), x\_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

### Rule 8



Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rule 5573

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[b^2/(a^2 + b^2), Int[(e + f\*x)^m\*Sech[c + d\*x]^(n - 2)/(a + b\*Sinh[c + d\*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f\*x)^m\*Sech[c + d\*x]^n\*(a - b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]

### Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[(e + f\*x)^m\*E^(c + d\*x)/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[(e + f\*x)^m\*E^(c + d\*x)/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

### Rule 2190

Int[(((F)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_) + (b\_.)\*((F)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 3718

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]), x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x)))/(1 + E^(2\*(-I\*e) + f\*fz\*x)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\tanh^3(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\operatorname{sech}(c+dx)\tanh^2(c+dx) dx}{b} - \frac{a \int \frac{(e+fx)\operatorname{sech}(c+dx)\tanh^2(c+dx)}{a+b\sinh(c+dx)} dx}{b} \\
&= -\frac{a \int (e+fx)\operatorname{sech}^2(c+dx)\tanh(c+dx) dx}{b^2} + \frac{a^2 \int \frac{(e+fx)\operatorname{sech}^2(c+dx)\tanh(c+dx)}{a+b\sinh(c+dx)} dx}{b^2} + \frac{\int (e+fx) dx}{b} \\
&= \frac{2(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{f\operatorname{sech}(c+dx)}{2bd^2} + \frac{a(e+fx)\operatorname{sech}^2(c+dx)}{2b^2d} - \frac{(e+fx)\operatorname{sech}(c+dx)}{2bd} \\
&= \frac{(e+fx)\tan^{-1}(e^{c+dx})}{bd} + \frac{a^2 f\operatorname{sech}(c+dx)}{2b^3d^2} - \frac{f\operatorname{sech}(c+dx)}{2bd^2} + \frac{a(e+fx)\operatorname{sech}^2(c+dx)}{2b^2d} + \frac{fx}{b} \\
&= \frac{a^2(e+fx)\tan^{-1}(e^{c+dx})}{b^3d} + \frac{(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{if\operatorname{Li}_2(-ie^{c+dx})}{bd^2} + \frac{if\operatorname{Li}_2(ie^{c+dx})}{bd^2} + \frac{a^2 f}{b^3} \\
&= \frac{a^3(e+fx)^2}{2(a^2+b^2)^2 f} + \frac{a^2(e+fx)\tan^{-1}(e^{c+dx})}{b^3d} + \frac{(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{if\operatorname{Li}_2(-ie^{c+dx})}{2bd^2} + \frac{if\operatorname{Li}_2(ie^{c+dx})}{2bd^2} \\
&= \frac{a^3(e+fx)^2}{2(a^2+b^2)^2 f} + \frac{a^2(e+fx)\tan^{-1}(e^{c+dx})}{b^3d} + \frac{(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{a^3(e+fx)\log(1+ie^{c+dx})}{(a^2+b^2)^2 f} \\
&= \frac{a^2(e+fx)\tan^{-1}(e^{c+dx})}{b^3d} + \frac{(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{2a^4(e+fx)\tan^{-1}(e^{c+dx})}{b(a^2+b^2)^2 d} - \frac{a^4(e+fx)}{b^3} \\
&= \frac{a^2(e+fx)\tan^{-1}(e^{c+dx})}{b^3d} + \frac{(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{2a^4(e+fx)\tan^{-1}(e^{c+dx})}{b(a^2+b^2)^2 d} - \frac{a^4(e+fx)}{b^3} \\
&= \frac{a^2(e+fx)\tan^{-1}(e^{c+dx})}{b^3d} + \frac{(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{2a^4(e+fx)\tan^{-1}(e^{c+dx})}{b(a^2+b^2)^2 d} - \frac{a^4(e+fx)}{b^3} \\
&= \frac{a^2(e+fx)\tan^{-1}(e^{c+dx})}{b^3d} + \frac{(e+fx)\tan^{-1}(e^{c+dx})}{bd} - \frac{2a^4(e+fx)\tan^{-1}(e^{c+dx})}{b(a^2+b^2)^2 d} - \frac{a^4(e+fx)}{b^3}
\end{aligned}$$

**Mathematica [A]** time = 7.86361, size = 588, normalized size = 0.66

$$-2a^3 f \operatorname{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2-a}}\right) - 2a^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2+a}}\right) - ibf(3a^2+b^2) \operatorname{PolyLog}(2, -ie^{c+dx}) + ibf(3a^2+b^2) \operatorname{PolyLog}(2, ie^{c+dx})$$

Antiderivative was successfully verified.

[In] Integrate(((e + f\*x)\*Tanh[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x)

[Out]  $(-2*a^3*d*e*(c + d*x) + 2*a^3*c*f*(c + d*x) + 6*a^2*b*d*e*\text{ArcTan}[E^{(c + d*x)}] + 2*b^3*d*e*\text{ArcTan}[E^{(c + d*x)}] - 6*a^2*b*c*f*\text{ArcTan}[E^{(c + d*x)}] - 2*b^3*c*f*\text{ArcTan}[E^{(c + d*x)}] + (3*I)*a^2*b*f*(c + d*x)*\text{Log}[1 - I*E^{(c + d*x)}] + I*b^3*f*(c + d*x)*\text{Log}[1 - I*E^{(c + d*x)}] - (3*I)*a^2*b*f*(c + d*x)*\text{Log}[1 + I*E^{(c + d*x)}] - I*b^3*f*(c + d*x)*\text{Log}[1 + I*E^{(c + d*x)}] - 2*a^3*f*(c + d*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])] - 2*a^3*f*(c + d*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])] + 2*a^3*d*e*\text{Log}[1 + E^{(2*(c + d*x))}] - 2*a^3*c*f*\text{Log}[1 + E^{(2*(c + d*x))}] + 2*a^3*f*(c + d*x)*\text{Log}[1 + E^{(2*(c + d*x))}] - 2*a^3*d*e*\text{Log}[a + b*\text{Sinh}[c + d*x]] + 2*a^3*c*f*\text{Log}[a + b*\text{Sinh}[c + d*x]] - I*b*(3*a^2 + b^2)*f*\text{PolyLog}[2, (-I)*E^{(c + d*x)}] + I*b*(3*a^2 + b^2)*f*\text{PolyLog}[2, I*E^{(c + d*x)}] - 2*a^3*f*\text{PolyLog}[2, (b*E^{(c + d*x)})/(-a + \text{Sqrt}[a^2 + b^2])] - 2*a^3*f*\text{PolyLog}[2, -(b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])] + a^3*f*\text{PolyLog}[2, -E^{(2*(c + d*x))}] - (a^2 + b^2)*f*\text{Sech}[c + d*x]*(b + a*\text{Sinh}[c + d*x]) + (a^2 + b^2)*d*(e + f*x)*\text{Sech}[c + d*x]^2*(a - b*\text{Sinh}[c + d*x]))/(2*(a^2 + b^2)^2*d^2)$

---

**Maple [B]** time = 0.202, size = 2284, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)\*tanh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

[Out]  $3/d^2/(a^2+b^2)^{(3/2)}*b^2*f*c/(2*a^2+2*b^2)*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})*a^2+2/d*e/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})*a^2-2/(a^2+b^2)^{(3/2)}/d*a^4*e/(2*a^2+2*b^2)*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-2/d^2*f*c/(2*a^2+2*b^2)/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})*a^2-1/d/(a^2+b^2)^{(3/2)}*b^4*e/(2*a^2+2*b^2)*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+1/d/(a^2+b^2)^{(1/2)}*b^2*e/(2*a^2+2*b^2)*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-2/(a^2+b^2)/d*a^3*f/(2*a^2+2*b^2)*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*x-2/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*c-2/(a^2+b^2)/d*a^3*f/(2*a^2+2*b^2)*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*x-2/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*c+2/(a^2+b^2)/d*a^3*f/(2*a^2+2*b^2)*\ln(1+I*\exp(d*x+c))*x+2/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*\ln(1+I*\exp(d*x+c))*c-2/(a^2$

$$\begin{aligned}
& +b^2)/d^2*a^3*f*c/(2*a^2+2*b^2)*\ln(1+\exp(2*d*x+2*c))+2/(a^2+b^2)/d^2*a^3*f*c/ \\
& (2*a^2+2*b^2)*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)+2/(a^2+b^2)/d^2*a^3*f/c/ \\
& (2*a^2+2*b^2)*\ln(1-I*\exp(d*x+c))*x+2/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*\ln(1- \\
& I*\exp(d*x+c))*c-3*I*b/(a^2+b^2)/d^2*a^2*f/(2*a^2+2*b^2)*\ln(1+I*\exp(d*x+c))*x- \\
& 3*I*b/(a^2+b^2)/d^2*a^2*f/(2*a^2+2*b^2)*\ln(1+I*\exp(d*x+c))*c+3*I/(a^2+b^2)/ \\
& d^2*a^2*f/(2*a^2+2*b^2)*\ln(1-I*\exp(d*x+c))*b*x+3*I/(a^2+b^2)/d^2*a^2*f/(2*a^2 \\
& +2*b^2)*\ln(1-I*\exp(d*x+c))*b*c-I*b^3/(a^2+b^2)/d^2*f/(2*a^2+2*b^2)*\ln(1+I*\exp \\
& (d*x+c))*x-I*b^3/(a^2+b^2)/d^2*f/(2*a^2+2*b^2)*\ln(1+I*\exp(d*x+c))*c+I*b^3/( \\
& a^2+b^2)/d^2*f/(2*a^2+2*b^2)*\ln(1-I*\exp(d*x+c))*x+I*b^3/(a^2+b^2)/d^2*f/(2*a^ \\
& 2+2*b^2)*\ln(1-I*\exp(d*x+c))*c-6*b/(a^2+b^2)/d^2*a^2*f*c/(2*a^2+2*b^2)*\arctan \\
& (\exp(d*x+c))-2*b^3/(a^2+b^2)/d^2*f*c/(2*a^2+2*b^2)*\arctan(\exp(d*x+c))+I*b^ \\
& 3/(a^2+b^2)/d^2*f/(2*a^2+2*b^2)*\operatorname{dilog}(1-I*\exp(d*x+c))+6*b/(a^2+b^2)/d^2*a^2*e \\
& / (2*a^2+2*b^2)*\arctan(\exp(d*x+c))-I*b^3/(a^2+b^2)/d^2*f/(2*a^2+2*b^2)*\operatorname{dilog} \\
& (1+I*\exp(d*x+c))+3*I/(a^2+b^2)/d^2*a^2*f/(2*a^2+2*b^2)*\operatorname{dilog}(1-I*\exp(d*x+c) \\
& )*b-3*I*b/(a^2+b^2)/d^2*a^2*f/(2*a^2+2*b^2)*\operatorname{dilog}(1+I*\exp(d*x+c))+1/d^2/(a^ \\
& 2+b^2)^{(3/2)}*b^4*f*c/(2*a^2+2*b^2)*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^ \\
& 2)^{(1/2)})-3/d/(a^2+b^2)^{(3/2)}*b^2*e/(2*a^2+2*b^2)*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+ \\
& c)+2*a)/(a^2+b^2)^{(1/2)})*a^2-1/d^2/(a^2+b^2)^{(1/2)}*b^2*f*c/(2*a^2+2*b^2)*\ar \\
& ctanh(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+2/(a^2+b^2)^{(3/2)}/d^2*a^4*f \\
& *c/(2*a^2+2*b^2)*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+(-b*d*f* \\
& x*\exp(3*d*x+3*c)+2*a*d*f*x*\exp(2*d*x+2*c)-b*d*e*\exp(3*d*x+3*c)+2*a*d*e*\exp( \\
& 2*d*x+2*c)+b*d*f*x*\exp(d*x+c)-b*f*\exp(3*d*x+3*c)+a*f*\exp(2*d*x+2*c)+b*d*e*e \\
& xp(d*x+c)-f*b*\exp(d*x+c)+a*f)/d^2/(a^2+b^2)/(1+\exp(2*d*x+2*c))^2+2/(a^2+b^2 \\
& )/d^2*a^3*f/(2*a^2+2*b^2)*\operatorname{dilog}(1+I*\exp(d*x+c))+2*b^3/(a^2+b^2)/d^2*e/(2*a^2+ \\
& 2*b^2)*\arctan(\exp(d*x+c))+2/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*\operatorname{dilog}(1-I*\exp \\
& (d*x+c))-2/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*\operatorname{dilog}((b*\exp(d*x+c)+(a^2+b^2)^ \\
& (1/2)+a)/(a+(a^2+b^2)^{(1/2)}))-2/(a^2+b^2)/d^2*a^3*f/(2*a^2+2*b^2)*\operatorname{dilog}((-b \\
& *\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))+2/(a^2+b^2)/d^2*a^3*e/(2 \\
& *a^2+2*b^2)*\ln(1+\exp(2*d*x+2*c))-2/(a^2+b^2)/d^2*a^3*e/(2*a^2+2*b^2)*\ln(b*\exp \\
& (2*d*x+2*c)+2*a*\exp(d*x+c)-b)
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\left( \frac{a^3 \log(-2ae^{-dx-c}) + be^{(-2dx-2c)} - b}{(a^4 + 2a^2b^2 + b^4)d} - \frac{a^3 \log(e^{(-2dx-2c)} + 1)}{(a^4 + 2a^2b^2 + b^4)d} + \frac{(3a^2b + b^3) \arctan(e^{-dx-c})}{(a^4 + 2a^2b^2 + b^4)d} + \frac{be^{(-dx-c)} - 2}{(a^2 + b^2 + 2(a^2 + b^2))} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*tanh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-(a^3 \log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b))/((a^4 + 2*a^2*b^2 + b^4)*d) - a^3 \log(e^{(-2*d*x - 2*c)} + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) + (3*a^2$

```

*b + b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (b*e^(-d*x - c)
) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*
e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d))*e - f*((b*d*x*e^(3*c)
+ b*e^(3*c))*e^(3*d*x) - (2*a*d*x*e^(2*c) + a*e^(2*c))*e^(2*d*x) - (b*d*x*
e^c - b*e^c)*e^(d*x) - a)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) + b^2*d^2*e
^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x)) - inte
grate(-2*(a^4*x*e^(d*x + c) - a^3*b*x)/(a^4*b + 2*a^2*b^3 + b^5 - (a^4*b*e^
(2*c) + 2*a^2*b^3*e^(2*c) + b^5*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c + 2*a^3*b^2
*e^c + a*b^4*e^c)*e^(d*x)), x) - integrate(-(2*a^3*x - (3*a^2*b*e^c + b^3*e
^c)*x*e^(d*x))/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c) + 2*a^2*b^2*e^(2*c) +
b^4*e^(2*c))*e^(2*d*x)), x)

```

---

**Fricas [B]** time = 4.00183, size = 11386, normalized size = 12.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*tanh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

```

[Out] -1/2*(2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^2*b + b^3)*f)*cosh(d*
x + c)^3 + 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^2*b + b^3)*f)*si
nh(d*x + c)^3 - 2*(2*(a^3 + a*b^2)*d*f*x + 2*(a^3 + a*b^2)*d*e + (a^3 + a*b
^2)*f)*cosh(d*x + c)^2 - 2*(2*(a^3 + a*b^2)*d*f*x + 2*(a^3 + a*b^2)*d*e + (
a^3 + a*b^2)*f - 3*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^2*b + b^3)
*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(a^3 + a*b^2)*f - 2*((a^2*b + b^3)*d
*f*x + (a^2*b + b^3)*d*e - (a^2*b + b^3)*f)*cosh(d*x + c) + 2*(a^3*f*cosh(d
*x + c)^4 + 4*a^3*f*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*f*sinh(d*x + c)^4 +
2*a^3*f*cosh(d*x + c)^2 + a^3*f + 2*(3*a^3*f*cosh(d*x + c)^2 + a^3*f)*sinh
(d*x + c)^2 + 4*(a^3*f*cosh(d*x + c)^3 + a^3*f*cosh(d*x + c))*sinh(d*x + c)
)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 2*(a^3*f*cosh(d*x + c)^4 + 4*a^3*
f*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*f*sinh(d*x + c)^4 + 2*a^3*f*cosh(d*x
+ c)^2 + a^3*f + 2*(3*a^3*f*cosh(d*x + c)^2 + a^3*f)*sinh(d*x + c)^2 + 4*(a
^3*f*cosh(d*x + c)^3 + a^3*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2) - b)/b + 1) - ((2*a^3*f + I*(3*a^2*b + b^3)*f)*cosh(d*x + c)^4 +
(8*a^3*f + 4*I*(3*a^2*b + b^3)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^3*f
+ I*(3*a^2*b + b^3)*f)*sinh(d*x + c)^4 + 2*a^3*f + (4*a^3*f + 2*I*(3*a^2*b
+ b^3)*f)*cosh(d*x + c)^2 + (4*a^3*f + (12*a^3*f + 6*I*(3*a^2*b + b^3)*f)*c
osh(d*x + c)^2 + 2*I*(3*a^2*b + b^3)*f)*sinh(d*x + c)^2 + I*(3*a^2*b + b^3)
*f + ((8*a^3*f + 4*I*(3*a^2*b + b^3)*f)*cosh(d*x + c)^3 + (8*a^3*f + 4*I*(3

```

$$\begin{aligned}
& *a^2*b + b^3)*f)*\cosh(d*x + c))*\sinh(d*x + c))*\operatorname{dilog}(I*\cosh(d*x + c) + I*\sinh(d*x + c)) - ((2*a^3*f - I*(3*a^2*b + b^3)*f)*\cosh(d*x + c)^4 + (8*a^3*f - 4*I*(3*a^2*b + b^3)*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (2*a^3*f - I*(3*a^2*b + b^3)*f)*\sinh(d*x + c)^4 + 2*a^3*f + (4*a^3*f - 2*I*(3*a^2*b + b^3)*f)*\cosh(d*x + c)^2 + (4*a^3*f + (12*a^3*f - 6*I*(3*a^2*b + b^3)*f)*\cosh(d*x + c)^2 - 2*I*(3*a^2*b + b^3)*f)*\sinh(d*x + c)^2 - I*(3*a^2*b + b^3)*f + ((8*a^3*f - 4*I*(3*a^2*b + b^3)*f)*\cosh(d*x + c)^3 + (8*a^3*f - 4*I*(3*a^2*b + b^3)*f)*\cosh(d*x + c))*\sinh(d*x + c))*\operatorname{dilog}(-I*\cosh(d*x + c) - I*\sinh(d*x + c)) + 2*(a^3*d*e - a^3*c*f + (a^3*d*e - a^3*c*f)*\cosh(d*x + c)^4 + 4*(a^3*d*e - a^3*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^3*d*e - a^3*c*f)*\sinh(d*x + c)^4 + 2*(a^3*d*e - a^3*c*f)*\cosh(d*x + c)^2 + 2*(a^3*d*e - a^3*c*f + 3*(a^3*d*e - a^3*c*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((a^3*d*e - a^3*c*f)*\cosh(d*x + c)^3 + (a^3*d*e - a^3*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 2*(a^3*d*e - a^3*c*f + (a^3*d*e - a^3*c*f)*\cosh(d*x + c)^4 + 4*(a^3*d*e - a^3*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^3*d*e - a^3*c*f)*\sinh(d*x + c)^4 + 2*(a^3*d*e - a^3*c*f)*\cosh(d*x + c)^2 + 2*(a^3*d*e - a^3*c*f + 3*(a^3*d*e - a^3*c*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((a^3*d*e - a^3*c*f)*\cosh(d*x + c)^3 + (a^3*d*e - a^3*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 2*(a^3*d*f*x + a^3*c*f + (a^3*d*f*x + a^3*c*f)*\cosh(d*x + c)^4 + 4*(a^3*d*f*x + a^3*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^3*d*f*x + a^3*c*f)*\sinh(d*x + c)^4 + 2*(a^3*d*f*x + a^3*c*f)*\cosh(d*x + c)^2 + 2*(a^3*d*f*x + a^3*c*f + 3*(a^3*d*f*x + a^3*c*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((a^3*d*f*x + a^3*c*f)*\cosh(d*x + c)^3 + (a^3*d*f*x + a^3*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + 2*(a^3*d*f*x + a^3*c*f + (a^3*d*f*x + a^3*c*f)*\cosh(d*x + c)^4 + 4*(a^3*d*f*x + a^3*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^3*d*f*x + a^3*c*f)*\sinh(d*x + c)^4 + 2*(a^3*d*f*x + a^3*c*f)*\cosh(d*x + c)^2 + 2*(a^3*d*f*x + a^3*c*f + 3*(a^3*d*f*x + a^3*c*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((a^3*d*f*x + a^3*c*f)*\cosh(d*x + c)^3 + (a^3*d*f*x + a^3*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - (2*a^3*d*e - 2*a^3*c*f + (2*a^3*d*e - 2*a^3*c*f + I*(3*a^2*b + b^3)*d*e - I*(3*a^2*b + b^3)*c*f)*\cosh(d*x + c)^4 + (8*a^3*d*e - 8*a^3*c*f + 4*I*(3*a^2*b + b^3)*d*e - 4*I*(3*a^2*b + b^3)*c*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (2*a^3*d*e - 2*a^3*c*f + I*(3*a^2*b + b^3)*d*e - I*(3*a^2*b + b^3)*c*f)*\sinh(d*x + c)^4 + I*(3*a^2*b + b^3)*d*e - I*(3*a^2*b + b^3)*c*f + (4*a^3*d*e - 4*a^3*c*f + 2*I*(3*a^2*b + b^3)*d*e - 2*I*(3*a^2*b + b^3)*c*f)*\cosh(d*x + c)^2 + (4*a^3*d*e - 4*a^3*c*f + 2*I*(3*a^2*b + b^3)*d*e - 2*I*(3*a^2*b + b^3)*c*f + (12*a^3*d*e - 12*a^3*c*f + 6*I*(3*a^2*b + b^3)*d*e - 6*I*(3*a^2*b + b^3)*c*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + ((8*a^3*d*e - 8*a^3*c*f + 4*I*(3*a^2*b + b^3)*d*e - 4*I*(3*a^2*b + b^3)*c*f)*\cosh(d*x + c)^3 + (8*a^3*d*e - 8*a^3*c*f + 4*I*(3*a^2*b + b^3)*d*e - 4*I*(3*a^2*b + b^3)*c*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + I) - (2*a^3*d*e
\end{aligned}$$

$$\begin{aligned}
& - 2a^3c^2f + (2a^3d^2e - 2a^3c^2f - I(3a^2b + b^3)d^2e + I(3a^2b + b^3)c^2f) \cosh(dx + c)^4 + (8a^3d^2e - 8a^3c^2f - 4I(3a^2b + b^3)d^2e + 4I(3a^2b + b^3)c^2f) \cosh(dx + c) \sinh(dx + c)^3 + (2a^3d^2e - 2a^3c^2f - I(3a^2b + b^3)d^2e + I(3a^2b + b^3)c^2f) \sinh(dx + c)^4 - I(3a^2b + b^3)d^2e + I(3a^2b + b^3)c^2f + (4a^3d^2e - 4a^3c^2f - 2I(3a^2b + b^3)d^2e + 2I(3a^2b + b^3)c^2f) \cosh(dx + c)^2 + (4a^3d^2e - 4a^3c^2f - 2I(3a^2b + b^3)d^2e + 2I(3a^2b + b^3)c^2f + (12a^3d^2e - 12a^3c^2f - 6I(3a^2b + b^3)d^2e + 6I(3a^2b + b^3)c^2f) \cosh(dx + c)^2) \sinh(dx + c)^2 + ((8a^3d^2e - 8a^3c^2f - 4I(3a^2b + b^3)d^2e + 4I(3a^2b + b^3)c^2f) \cosh(dx + c)^3 + (8a^3d^2e - 8a^3c^2f - 4I(3a^2b + b^3)d^2e + 4I(3a^2b + b^3)c^2f) \cosh(dx + c)) \sinh(dx + c) \log(\cosh(dx + c) + \sinh(dx + c) - I) - (2a^3d^2f^2x + 2a^3c^2f^2x + (2a^3d^2f^2x + 2a^3c^2f^2x - I(3a^2b + b^3)d^2f^2x - I(3a^2b + b^3)c^2f^2x) \cosh(dx + c)^4 + (8a^3d^2f^2x + 8a^3c^2f^2x - 4I(3a^2b + b^3)d^2f^2x - 4I(3a^2b + b^3)c^2f^2x) \cosh(dx + c) \sinh(dx + c)^3 + (2a^3d^2f^2x + 2a^3c^2f^2x - I(3a^2b + b^3)d^2f^2x - I(3a^2b + b^3)c^2f^2x) \sinh(dx + c)^4 - I(3a^2b + b^3)d^2f^2x - I(3a^2b + b^3)c^2f^2x + (4a^3d^2f^2x + 4a^3c^2f^2x - 2I(3a^2b + b^3)d^2f^2x - 2I(3a^2b + b^3)c^2f^2x) \cosh(dx + c)^2 + (4a^3d^2f^2x + 4a^3c^2f^2x - 2I(3a^2b + b^3)d^2f^2x - 2I(3a^2b + b^3)c^2f^2x) \cosh(dx + c)^2) \sinh(dx + c)^2 + ((8a^3d^2f^2x + 8a^3c^2f^2x - 4I(3a^2b + b^3)d^2f^2x - 4I(3a^2b + b^3)c^2f^2x) \cosh(dx + c)^3 + (8a^3d^2f^2x + 8a^3c^2f^2x - 4I(3a^2b + b^3)d^2f^2x - 4I(3a^2b + b^3)c^2f^2x) \cosh(dx + c)) \sinh(dx + c) \log(I \cosh(dx + c) + I \sinh(dx + c) + 1) - (2a^3d^2f^2x + 2a^3c^2f^2x + (2a^3d^2f^2x + 2a^3c^2f^2x + I(3a^2b + b^3)d^2f^2x + I(3a^2b + b^3)c^2f^2x) \cosh(dx + c)^4 + (8a^3d^2f^2x + 8a^3c^2f^2x + 4I(3a^2b + b^3)d^2f^2x + 4I(3a^2b + b^3)c^2f^2x) \cosh(dx + c) \sinh(dx + c)^3 + (2a^3d^2f^2x + 2a^3c^2f^2x + I(3a^2b + b^3)d^2f^2x + I(3a^2b + b^3)c^2f^2x) \sinh(dx + c)^4 + I(3a^2b + b^3)d^2f^2x + I(3a^2b + b^3)c^2f^2x + (4a^3d^2f^2x + 4a^3c^2f^2x + 2I(3a^2b + b^3)d^2f^2x + 2I(3a^2b + b^3)c^2f^2x) \cosh(dx + c)^2 + (4a^3d^2f^2x + 4a^3c^2f^2x + 2I(3a^2b + b^3)d^2f^2x + 2I(3a^2b + b^3)c^2f^2x + (12a^3d^2f^2x + 12a^3c^2f^2x + 6I(3a^2b + b^3)d^2f^2x + 6I(3a^2b + b^3)c^2f^2x) \cosh(dx + c)^2) \sinh(dx + c)^2 + ((8a^3d^2f^2x + 8a^3c^2f^2x + 4I(3a^2b + b^3)d^2f^2x + 4I(3a^2b + b^3)c^2f^2x) \cosh(dx + c)^3 + (8a^3d^2f^2x + 8a^3c^2f^2x + 4I(3a^2b + b^3)d^2f^2x + 4I(3a^2b + b^3)c^2f^2x) \cosh(dx + c)) \sinh(dx + c) \log(-I \cosh(dx + c) - I \sinh(dx + c) + 1) - 2((a^2b + b^3)d^2f^2x + (a^2b + b^3)d^2e - 3((a^2b + b^3)d^2f^2x + (a^2b + b^3)d^2e + (a^2b + b^3)f) \cosh(dx + c))^2 - (a^2b + b^3)f + 2(2(a^3 + ab^2)d^2f^2x + 2(a^3 + ab^2)d^2e + (a^3 + ab^2)f) \cosh(dx + c) \sinh(dx + c)) / ((a^4 + 2a^2b^2 + b^4)d^2 \cosh(dx + c)^4 + 4(a^4 + 2a^2b^2 + b^4)d^2 \cosh(dx + c) \sinh(dx + c)^3 + (a^4 + 2a^2b^2 + b^4)d^2 \sinh(dx + c)^4 + 2(a^4 + 2a^2b^2 + b^4)d^2 \cosh(dx + c)^2 + (a^4 + 2a^2b^2 + b^4)d^2 + 2(3(a^4 + 2a^2b^2 + b^4)d^2 \cosh(dx + c)^2 + (a^4 + 2a^2b^2 + b^4)d^2) \sinh(dx + c)^2 + 4((a^4 + 2a^2b^2 + b^4)d^2 \cosh(dx + c)^3 + (a^4 + 2a^2b^2 + b^4)d^2 \sinh(dx + c)^3)
\end{aligned}$$

$$^2 * \cosh(dx + c) * \sinh(dx + c)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \tanh^3(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*tanh(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*tanh(c + d\*x)\*\*3/(a + b\*sinh(c + d\*x)), x)

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*tanh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out



$$3.418 \quad \int \frac{\tanh^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=120

$$-\frac{a^3 \log(a + b \sinh(c + dx))}{d(a^2 + b^2)^2} + \frac{b(3a^2 + b^2) \tan^{-1}(\sinh(c + dx))}{2d(a^2 + b^2)^2} + \frac{a^3 \log(\cosh(c + dx))}{d(a^2 + b^2)^2} + \frac{\operatorname{sech}^2(c + dx)(a - b \sinh(c + dx))}{2d(a^2 + b^2)}$$

[Out] (b\*(3\*a^2 + b^2)\*ArcTan[Sinh[c + d\*x]])/(2\*(a^2 + b^2)^2\*d) + (a^3\*Log[Cosh[c + d\*x]])/((a^2 + b^2)^2\*d) - (a^3\*Log[a + b\*Sinh[c + d\*x]])/((a^2 + b^2)^2\*d) + (Sech[c + d\*x]^2\*(a - b\*Sinh[c + d\*x]))/(2\*(a^2 + b^2)\*d)

**Rubi [A]** time = 0.201183, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2721, 1647, 801, 635, 203, 260}

$$-\frac{a^3 \log(a + b \sinh(c + dx))}{d(a^2 + b^2)^2} + \frac{b(3a^2 + b^2) \tan^{-1}(\sinh(c + dx))}{2d(a^2 + b^2)^2} + \frac{a^3 \log(\cosh(c + dx))}{d(a^2 + b^2)^2} + \frac{\operatorname{sech}^2(c + dx)(a - b \sinh(c + dx))}{2d(a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d\*x]^3/(a + b\*Sinh[c + d\*x]),x]

[Out] (b\*(3\*a^2 + b^2)\*ArcTan[Sinh[c + d\*x]])/(2\*(a^2 + b^2)^2\*d) + (a^3\*Log[Cosh[c + d\*x]])/((a^2 + b^2)^2\*d) - (a^3\*Log[a + b\*Sinh[c + d\*x]])/((a^2 + b^2)^2\*d) + (Sech[c + d\*x]^2\*(a - b\*Sinh[c + d\*x]))/(2\*(a^2 + b^2)\*d)

### Rule 2721

Int[((a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)])^(m\_)\*tan[(e\_) + (f\_)\*(x\_)]^(p\_), x\_Symbol] :> Dist[1/f, Subst[Int[(x^p\*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b\*Sin[e + f\*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

### Rule 1647

Int[(Pq\_)\*((d\_) + (e\_)\*(x\_))^(m\_)\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> With[{Q = PolynomialQuotient[(d + e\*x)^m\*Pq, a + c\*x^2, x], f = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + c\*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(d + e\*x)^m\*Pq, a + c\*x^2, x], x, 1]}, Simp[((a\*g - c\*f\*x)\*(a + c

```
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
+ 3))/(d + e*x)^m, x], x] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

### Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
 x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

### Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

### Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 260

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(a+x)(-b^2-x^2)^2} dx, x, b\sinh(c+dx)\right)}{d} \\
&= \frac{\text{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)d} - \frac{\text{Subst}\left(\int \frac{\frac{ab^4}{a^2+b^2} + \frac{b^2(2a^2+b^2)x}{a^2+b^2}}{(a+x)(-b^2-x^2)} dx, x, b\sinh(c+dx)\right)}{2b^2d} \\
&= \frac{\text{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)d} - \frac{\text{Subst}\left(\int \left(\frac{2a^3b^2}{(a^2+b^2)^2(a+x)} - \frac{b^2(3a^2b^2+b^4+2a^3x)}{(a^2+b^2)^2(b^2+x^2)}\right) dx, x, b\sinh(c+dx)\right)}{2b^2d} \\
&= -\frac{a^3 \log(a+b\sinh(c+dx))}{(a^2+b^2)^2 d} + \frac{\text{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)d} + \frac{\text{Subst}\left(\int \frac{3a^2b^2+b^4+2a^3x}{b^2+x^2} dx, x, b\sinh(c+dx)\right)}{2(a^2+b^2)d} \\
&= -\frac{a^3 \log(a+b\sinh(c+dx))}{(a^2+b^2)^2 d} + \frac{\text{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)d} + \frac{a^3 \text{Subst}\left(\int \frac{x}{b^2+x^2} dx, x, b\sinh(c+dx)\right)}{(a^2+b^2)^2 d} \\
&= \frac{b(3a^2+b^2)\tan^{-1}(\sinh(c+dx))}{2(a^2+b^2)^2 d} + \frac{a^3 \log(\cosh(c+dx))}{(a^2+b^2)^2 d} - \frac{a^3 \log(a+b\sinh(c+dx))}{(a^2+b^2)^2 d} + \frac{\text{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)d}
\end{aligned}$$

**Mathematica [C]** time = 0.402208, size = 152, normalized size = 1.27

$$\frac{-a(a^2+b^2)\text{sech}^2(c+dx) - (a^3 - i(2a^2b+b^3))\log(-\sinh(c+dx)+i) - (a^3 + i(2a^2b+b^3))\log(\sinh(c+dx)+i) + \frac{b(3a^2+b^2)\tan^{-1}(\sinh(c+dx))}{2(a^2+b^2)^2 d} + \frac{a^3 \log(\cosh(c+dx))}{(a^2+b^2)^2 d} - \frac{a^3 \log(a+b\sinh(c+dx))}{(a^2+b^2)^2 d} + \frac{\text{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)d}}{2d(a^2+b^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d\*x]^3/(a + b\*Sinh[c + d\*x]),x]

[Out] -(b\*(a^2 + b^2)\*ArcTan[Sinh[c + d\*x]] - (a^3 - I\*(2\*a^2\*b + b^3))\*Log[I - Sinh[c + d\*x]] - (a^3 + I\*(2\*a^2\*b + b^3))\*Log[I + Sinh[c + d\*x]] + 2\*a^3\*Log[a + b\*Sinh[c + d\*x]] - a\*(a^2 + b^2)\*Sech[c + d\*x]^2 + b\*(a^2 + b^2)\*Sech[c + d\*x]\*Tanh[c + d\*x])/(2\*(a^2 + b^2)^2\*d)

**Maple [B]** time = 0.002, size = 472, normalized size = 3.9

$$-8 \frac{a^3 \ln\left(\left(\tanh\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2 a - 2 \tanh\left(\frac{1}{2}dx + \frac{c}{2}\right) b - a\right)}{d(8a^4 + 16a^2b^2 + 8b^4)} + \frac{a^2b}{d(a^4 + 2a^2b^2 + b^4)} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

[Out] 
$$-8/d*a^3/(8*a^4+16*a^2*b^2+8*b^4)*\ln(\tanh(1/2*d*x+1/2*c)^2*a-2*\tanh(1/2*d*x+1/2*c)*b-a)+1/d/(a^4+2*a^2*b^2+b^4)/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)^3*a^2*b+1/d/(a^4+2*a^2*b^2+b^4)/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)^3*b^3-2/d/(a^4+2*a^2*b^2+b^4)/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)^2*a^3-2/d/(a^4+2*a^2*b^2+b^4)/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)^2*a*b^2-1/d/(a^4+2*a^2*b^2+b^4)/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)*a^2*b-1/d/(a^4+2*a^2*b^2+b^4)/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)*b^3+1/d/(a^4+2*a^2*b^2+b^4)*a^3*\ln(\tanh(1/2*d*x+1/2*c)^2+1)+3/d/(a^4+2*a^2*b^2+b^4)*\arctan(\tanh(1/2*d*x+1/2*c))*a^2*b+1/d/(a^4+2*a^2*b^2+b^4)*\arctan(\tanh(1/2*d*x+1/2*c))*b^3$$

**Maxima [A]** time = 1.67531, size = 293, normalized size = 2.44

$$\frac{a^3 \log\left(-2ae^{-dx-c} + be^{-2dx-2c} - b\right)}{(a^4 + 2a^2b^2 + b^4)d} + \frac{a^3 \log\left(e^{-2dx-2c} + 1\right)}{(a^4 + 2a^2b^2 + b^4)d} - \frac{(3a^2b + b^3) \arctan\left(e^{-dx-c}\right)}{(a^4 + 2a^2b^2 + b^4)d} - \frac{be^{-dx-c} - 2}{(a^2 + b^2 + 2(a^2 + b^2))d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$-a^3*\log(-2*a*e^{-d*x-c} + b*e^{-2*d*x-2*c} - b)/((a^4 + 2*a^2*b^2 + b^4)*d) + a^3*\log(e^{-2*d*x-2*c} + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (3*a^2*b + b^3)*\arctan(e^{-d*x-c})/((a^4 + 2*a^2*b^2 + b^4)*d) - (b*e^{-d*x-c} - 2*a*e^{-2*d*x-2*c} - b*e^{-3*d*x-3*c})/((a^2 + b^2 + 2*(a^2 + b^2))*e^{-2*d*x-2*c} + (a^2 + b^2)*e^{-4*d*x-4*c})*d$$

**Fricas [B]** time = 2.90572, size = 2217, normalized size = 18.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-\left((a^2b + b^3)\cosh(dx + c)^3 + (a^2b + b^3)\sinh(dx + c)^3 - 2(a^3 + a^2b)\cosh(dx + c)^2 - (2a^3 + 2a^2b - 3(a^2b + b^3)\cosh(dx + c))\sinh(dx + c)^2 - ((3a^2b + b^3)\cosh(dx + c)^4 + 4(3a^2b + b^3)\cosh(dx + c)\sinh(dx + c)^3 + (3a^2b + b^3)\sinh(dx + c)^4 + 3a^2b + b^3 + 2(3a^2b + b^3)\cosh(dx + c)^2 + 2(3a^2b + b^3 + 3(3a^2b + b^3)\cosh(dx + c)^2)\sinh(dx + c)^2 + 4((3a^2b + b^3)\cosh(dx + c)^3 + (3a^2b + b^3)\cosh(dx + c))\sinh(dx + c))\arctan(\cosh(dx + c) + \sinh(dx + c)) - (a^2b + b^3)\cosh(dx + c) + (a^3\cosh(dx + c)^4 + 4a^3\cosh(dx + c)\sinh(dx + c)^3 + a^3\sinh(dx + c)^4 + 2a^3\cosh(dx + c)^2 + a^3 + 2(3a^3\cosh(dx + c)^2 + a^3)\sinh(dx + c)^2 + 4(a^3\cosh(dx + c)^3 + a^3\cosh(dx + c))\sinh(dx + c))\log(2(b\sinh(dx + c) + a)/(\cosh(dx + c) - \sinh(dx + c))) - (a^3\cosh(dx + c)^4 + 4a^3\cosh(dx + c)\sinh(dx + c)^3 + a^3\sinh(dx + c)^4 + 2a^3\cosh(dx + c)^2 + a^3 + 2(3a^3\cosh(dx + c)^2 + a^3)\sinh(dx + c)^2 + 4(a^3\cosh(dx + c)^3 + a^3\cosh(dx + c))\sinh(dx + c))\log(2\cosh(dx + c)/(\cosh(dx + c) - \sinh(dx + c))) - (a^2b + b^3 - 3(a^2b + b^3)\cosh(dx + c)^2 + 4(a^3 + a^2b)\cosh(dx + c))\sinh(dx + c)\right)/((a^4 + 2a^2b^2 + b^4)d\cosh(dx + c)^4 + 4(a^4 + 2a^2b^2 + b^4)d\cosh(dx + c)\sinh(dx + c)^3 + (a^4 + 2a^2b^2 + b^4)d\sinh(dx + c)^4 + 2(a^4 + 2a^2b^2 + b^4)d\cosh(dx + c)^2 + 2(3(a^4 + 2a^2b^2 + b^4)d\cosh(dx + c)^2 + (a^4 + 2a^2b^2 + b^4)d)\sinh(dx + c)^2 + (a^4 + 2a^2b^2 + b^4)d + 4((a^4 + 2a^2b^2 + b^4)d\cosh(dx + c)^3 + (a^4 + 2a^2b^2 + b^4)d\cosh(dx + c))\sinh(dx + c))$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(tanh(c + d\*x)\*\*3/(a + b\*sinh(c + d\*x)), x)

**Giac [A]** time = 1.4486, size = 301, normalized size = 2.51

$$\frac{\frac{a^3 \log(e^{(2dx+2c)+1})}{a^4+2a^2b^2+b^4} - \frac{a^3 \log(|-be^{(2dx+2c)}-2ae^{(dx+c)}+b|)}{a^4+2a^2b^2+b^4} + \frac{(3a^2be^c+b^3e^c) \arctan(e^{(dx+c)})e^{-c}}{a^4+2a^2b^2+b^4} - \frac{(a^2be^{(3c)}+b^3e^{(3c)})e^{(3dx)}-2(a^3e^{(2c)}+ab^2e^{(2c)})e^{(2dx)}-(a^2be^{(2c)}+b^3e^{(2c)})e^{(dx)}}{(a^2+b^2)^2(e^{(2dx+2c)+1})^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] (a^3\*log(e^(2\*d\*x + 2\*c) + 1)/(a^4 + 2\*a^2\*b^2 + b^4) - a^3\*log(abs(-b\*e^(2\*d\*x + 2\*c) - 2\*a\*e^(d\*x + c) + b))/(a^4 + 2\*a^2\*b^2 + b^4) + (3\*a^2\*b\*e^c + b^3\*e^c)\*arctan(e^(d\*x + c))\*e^(-c)/(a^4 + 2\*a^2\*b^2 + b^4) - ((a^2\*b\*e^(3\*c) + b^3\*e^(3\*c))\*e^(3\*d\*x) - 2\*(a^3\*e^(2\*c) + a\*b^2\*e^(2\*c))\*e^(2\*d\*x) - (a^2\*b\*e^c + b^3\*e^c)\*e^(d\*x))/((a^2 + b^2)^2\*(e^(2\*d\*x + 2\*c) + 1)^2))/d

$$3.419 \quad \int \frac{\tanh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Optimal. Leaf size=30

$$\text{Unintegrable} \left( \frac{\tanh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[Tanh[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0757226, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Tanh[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] Defer[Int][Tanh[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\tanh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\tanh^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.001, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[Tanh[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] \$Aborted

---

**Maple [A]** time = 0.905, size = 0, normalized size = 0.

$$\int \frac{(\tanh(dx + c))^3}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(tanh(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -(a*f + (b*d*f*x*e^{(3*c)} + (d*e - f)*b*e^{(3*c)})*e^{(3*d*x)} - (2*a*d*f*x*e^{(2*c)} + (2*d*e - f)*a*e^{(2*c)})*e^{(2*d*x)} - (b*d*f*x*e^c + (d*e + f)*b*e^c)*e^{(d*x)}) / (a^2*d^2*e^2 + b^2*d^2*e^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*(a^2*d^2*e*f + b^2*d^2*e*f)*x + (a^2*d^2*e^2*e^{(4*c)} + b^2*d^2*e^2*e^{(4*c)} + (a^2*d^2*f^2*e^{(4*c)} + b^2*d^2*f^2*e^{(4*c)})*x^2 + 2*(a^2*d^2*e*f*e^{(4*c)} + b^2*d^2*e*f*e^{(4*c)})*x)*e^{(4*d*x)} + 2*(a^2*d^2*e^2*e^{(2*c)} + b^2*d^2*e^2*e^{(2*c)} + (a^2*d^2*f^2*e^{(2*c)} + b^2*d^2*f^2*e^{(2*c)})*x^2 + 2*(a^2*d^2*e*f*e^{(2*c)} + b^2*d^2*e*f*e^{(2*c)})*x)*e^{(2*d*x)}) + \text{integrate}(- (2*a^3*d^2*f^2*x^2 + 4*a^3*d^2*e*f*x + 2*a*b^2*f^2 + 2*(d^2*e^2 + f^2)*a^3 - ((3*d^2*e^2 + 2*f^2)*a^2*b*e^c + (d^2*e^2 + 2*f^2)*b^3*e^c + (3*a^2*b*d^2*f^2*e^c + b^3*d^2*f^2*e^c)*x^2 + 2*(3*a^2*b*d^2*e*f*e^c + b^3*d^2*e*f*e^c)*x)*e^{(d*x)}) / (a^4*d^2*e^3 + 2*a^2*b^2*d^2*e^3 + b^4*d^2*e^3 + (a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*e*f^2 + 2*a^2*b^2*d^2*e*f^2 + b^4*d^2*e*f^2)*x^2 + 3*(a^4*d^2*e^2*f + 2*a^2*b^2*d^2*e^2*f + b^4*d^2*e^2*f)*x + (a^4*d^2*e^3*e^{(2*c)} + 2*a^2*b^2*d^2*e^3*e^{(2*c)} + b^4*d^2*e^3*e^{(2*c)} + (a^4*d^2*f^3*e^{(2*c)} + 2*a^2*b^2*d^2*f^3*e^{(2*c)} + b^4*d^2*f^3*e^{(2*c)})*x^3 + 3*(a^4*d^2*e*f^2*e^{(2*c)} + 2*a^2*b^2*d^2*e*f^2*e^{(2*c)} + b^4*d^2*e*f^2*e^{(2*c)})*x^2 + 3*(a^4*d^2*e^2*f*e^{(2*c)} + 2*a^2*b^2*d^2*e^2*f*e^{(2*c)} + b^4*d^2*e^2*f*e^{(2*c)})*x)*e^{(2*d*x)}), x) + \text{integrate}(-2*(a^4*e^{(d*x + c)} - a^3*b) / (a^4*b*e + 2*a^2*b^3*e + b^5*e + (a^4*b*f + 2*a^2*b^3*f + b^5*f)*x - (a^4*b*e*e^{(2*c)} + 2*a^2*b^3*e*e^{(2*c)} + b^5*e*e^{(2*c)} + (a^4*b*f*e^{(2*c)} + 2*a^2*b^3*f \end{aligned}$$



$*e^{(2*c)} + b^5*f*e^{(2*c)})*x)*e^{(2*d*x)} - 2*(a^5*e*e^c + 2*a^3*b^2*e*e^c + a*b^4*e*e^c + (a^5*f*e^c + 2*a^3*b^2*f*e^c + a*b^4*f*e^c)*x)*e^{(d*x)}, x)$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tanh(dx+c)^3}{afx+ae+(bfx+be)\sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] integral(tanh(d\*x + c)^3/(a\*f\*x + a\*e + (b\*f\*x + b\*e)\*sinh(d\*x + c)), x)

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(c+dx)}{(a+b\sinh(c+dx))(e+fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d\*x+c)\*\*3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(tanh(c + d\*x)\*\*3/((a + b\*sinh(c + d\*x))\*(e + f\*x)), x)

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.420 \quad \int \frac{(e+fx)^3 \coth(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=451

$$\frac{6f^2(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3} + \frac{6f^2(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^3} - \frac{3f(e+fx)^2\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2} - \frac{3f(e+fx)^2\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2} - \frac{3f(e+fx)^2\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2} - \frac{3f(e+fx)^2\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2}$$

[Out] -(((e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/(a\*d)) - ((e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/(a\*d) + ((e + f\*x)^3\*Log[1 - E^(2\*(c + d\*x))])/(a\*d) - (3\*f\*(e + f\*x)^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a\*d^2) - (3\*f\*(e + f\*x)^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a\*d^2) + (3\*f\*(e + f\*x)^2\*PolyLog[2, E^(2\*(c + d\*x))])/(2\*a\*d^2) + (6\*f^2\*(e + f\*x)\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a\*d^3) + (6\*f^2\*(e + f\*x)\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a\*d^3) - (3\*f^2\*(e + f\*x)\*PolyLog[3, E^(2\*(c + d\*x))])/(2\*a\*d^3) - (6\*f^3\*PolyLog[4, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a\*d^4) - (6\*f^3\*PolyLog[4, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a\*d^4) + (3\*f^3\*PolyLog[4, E^(2\*(c + d\*x))])/(4\*a\*d^4)

**Rubi [A]** time = 0.769999, antiderivative size = 451, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$ , Rules used = {5569, 3716, 2190, 2531, 6609, 2282, 6589, 5561}

$$\frac{6f^2(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3} + \frac{6f^2(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^3} - \frac{3f(e+fx)^2\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2} - \frac{3f(e+fx)^2\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2} - \frac{3f(e+fx)^2\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2} - \frac{3f(e+fx)^2\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Coth[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] -(((e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/(a\*d)) - ((e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/(a\*d) + ((e + f\*x)^3\*Log[1 - E^(2\*(c + d\*x))])/(a\*d) - (3\*f\*(e + f\*x)^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a\*d^2) - (3\*f\*(e + f\*x)^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a\*d^2) + (3\*f\*(e + f\*x)^2\*PolyLog[2, E^(2\*(c + d\*x))])/(2\*a\*d^2) + (6\*f^2\*(e + f\*x)\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a\*d^3) + (6\*f^2\*(e + f\*x)\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a\*d^3) - (3\*f^2\*(e + f\*x)\*PolyLog[3, E^(2\*(c + d\*x))])/(2\*a\*d^3) - (6\*f^3\*PolyLog[4, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a\*d^4) - (6\*f^3\*PolyLog[4, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a\*d^4) + (3\*f^3\*PolyLog[4, E^(2\*(c + d\*x))])/(4\*a\*d^4)

$b^2]))]/(a*d^4) + (3*f^3*PolyLog[4, E^(2*(c + d*x))]/(4*a*d^4)$

### Rule 5569

$\text{Int}[(\text{Coth}[c_.] + (d_.)*(x_.)]^{(n_.)}*((e_.) + (f_.)*(x_.))^{(m_.)}]/((a_.) + (b_.)*\text{Sinh}[c_.] + (d_.)*(x_.)], x\_Symbol] \rightarrow \text{Dist}[1/a, \text{Int}[(e + f*x)^m*\text{Coth}[c + d*x]^n, x], x] - \text{Dist}[b/a, \text{Int}[(e + f*x)^m*\text{Cosh}[c + d*x]*\text{Coth}[c + d*x]^{(n - 1)}/(a + b*\text{Sinh}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[n, 0]$

### Rule 3716

$\text{Int}[(c_. + (d_.)*(x_.))^{(m_.)}*\tan[(e_.) + \text{Pi}*(k_.) + (\text{Complex}[0, fz\_])*(f_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[(I*(c + d*x)^{(m + 1)})/(d*(m + 1)), x] + \text{Dist}[2*I, \text{Int}[(c + d*x)^m*E^{(2*(-(I*e) + f*fz*x))}]/(E^{(2*I*k*Pi)}*(1 + E^{(2*(-(I*e) + f*fz*x))/E^{(2*I*k*Pi))}), x], x] /; \text{FreeQ}\{c, d, e, f, fz\}, x] \&\& \text{IntegerQ}[4*k] \&\& \text{IGtQ}[m, 0]$

### Rule 2190

$\text{Int}[(F_)^{((g_.)*((e_.) + (f_.)*(x_.)))^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}}]/((a_.) + (b_.)*((F_)^{((g_.)*((e_.) + (f_.)*(x_.)))^{(n_.)}}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x)))^n})/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

### Rule 2531

$\text{Int}[\text{Log}[1 + (e_.)*((F_)^{((c_.)*((a_.) + (b_.)*(x_.)))^{(n_.)}}]*(f_.) + (g_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow -\text{Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n})]/(b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{(m - 1)}*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))^n})], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0]$

### Rule 6609

$\text{Int}[(e_. + (f_.)*(x_.))^{(m_.)}*PolyLog[n_, (d_.)*((F_)^{((c_.)*((a_.) + (b_.)*(x_.)))^{(p_.)}}], x\_Symbol] \rightarrow \text{Simp}[(e + f*x)^m*PolyLog[n + 1, d*(F^{(c*(a + b*x)))^p}]/(b*c*p*\text{Log}[F]), x] - \text{Dist}[(f*m)/(b*c*p*\text{Log}[F]), \text{Int}[(e + f*x)^{(m - 1)}*PolyLog[n + 1, d*(F^{(c*(a + b*x)))^p}], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, n, p\}, x] \&\& \text{GtQ}[m, 0]$

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 5561

```
Int[(Cosh[(c_) + (d_)*(x_)]*((e_) + (f_)*(x_))^(m_))/((a_) + (b_)*Sin
h[(c_) + (d_)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e + fx)^3 \coth(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\int (e + fx)^3 \coth(c + dx) dx}{a} - \frac{b \int \frac{(e + fx)^3 \cosh(c + dx)}{a + b \sinh(c + dx)} dx}{a} \\
&= -\frac{2 \int \frac{e^{2(c + dx)}(e + fx)^3}{1 - e^{2(c + dx)}} dx}{a} - \frac{b \int \frac{e^{c + dx}(e + fx)^3}{a - \sqrt{a^2 + b^2} + be^{c + dx}} dx}{a} - \frac{b \int \frac{e^{c + dx}(e + fx)^3}{a + \sqrt{a^2 + b^2} + be^{c + dx}} dx}{a} \\
&= -\frac{(e + fx)^3 \log\left(1 + \frac{be^{c + dx}}{a - \sqrt{a^2 + b^2}}\right)}{ad} - \frac{(e + fx)^3 \log\left(1 + \frac{be^{c + dx}}{a + \sqrt{a^2 + b^2}}\right)}{ad} + \frac{(e + fx)^3 \log(1 - e^{2(c + dx)})}{ad} \\
&= -\frac{(e + fx)^3 \log\left(1 + \frac{be^{c + dx}}{a - \sqrt{a^2 + b^2}}\right)}{ad} - \frac{(e + fx)^3 \log\left(1 + \frac{be^{c + dx}}{a + \sqrt{a^2 + b^2}}\right)}{ad} + \frac{(e + fx)^3 \log(1 - e^{2(c + dx)})}{ad} \\
&= -\frac{(e + fx)^3 \log\left(1 + \frac{be^{c + dx}}{a - \sqrt{a^2 + b^2}}\right)}{ad} - \frac{(e + fx)^3 \log\left(1 + \frac{be^{c + dx}}{a + \sqrt{a^2 + b^2}}\right)}{ad} + \frac{(e + fx)^3 \log(1 - e^{2(c + dx)})}{ad} \\
&= -\frac{(e + fx)^3 \log\left(1 + \frac{be^{c + dx}}{a - \sqrt{a^2 + b^2}}\right)}{ad} - \frac{(e + fx)^3 \log\left(1 + \frac{be^{c + dx}}{a + \sqrt{a^2 + b^2}}\right)}{ad} + \frac{(e + fx)^3 \log(1 - e^{2(c + dx)})}{ad} \\
&= -\frac{(e + fx)^3 \log\left(1 + \frac{be^{c + dx}}{a - \sqrt{a^2 + b^2}}\right)}{ad} - \frac{(e + fx)^3 \log\left(1 + \frac{be^{c + dx}}{a + \sqrt{a^2 + b^2}}\right)}{ad} + \frac{(e + fx)^3 \log(1 - e^{2(c + dx)})}{ad}
\end{aligned}$$

**Mathematica [B]** time = 19.0461, size = 1924, normalized size = 4.27

result too large to display

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^3\*Coth[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] 
$$-(E^{(2*c)}*((e + f*x)^4/(E^{(2*c)}*f) - (2*(1 - E^{(-2*c)}))*(e + f*x)^3*\text{Log}[1 - E^{(-c - d*x)}])/d - (2*(1 - E^{(-2*c)})*(e + f*x)^3*\text{Log}[1 + E^{(-c - d*x)}])/d + (6*(-1 + E^{(2*c)})*f*(d^2*(e + f*x)^2*\text{PolyLog}[2, -E^{(-c - d*x)}] + 2*f*(d*(e + f*x)*\text{PolyLog}[3, -E^{(-c - d*x)}] + f*\text{PolyLog}[4, -E^{(-c - d*x)}]))/(d^4*E^{(2*c)}) + (6*(-1 + E^{(2*c)})*f*(d^2*(e + f*x)^2*\text{PolyLog}[2, E^{(-c - d*x)}] + 2*f*(d*(e + f*x)*\text{PolyLog}[3, E^{(-c - d*x)}] + f*\text{PolyLog}[4, E^{(-c - d*x)}]))/(d^4*E^{(2*c)})))/(2*a*(-1 + E^{(2*c)})) + (4*e^3*E^{(2*c)}*x + 6*e^2*E^{(2*c)}*f*x^2 + 4*e*E^{(2*c)}*f^2*x^3 + E^{(2*c)}*f^3*x^4 + (4*a*\text{Sqrt}[-(a^2 + b^2)^2]*e^3*E^{(2*c)}*\text{ArcTan}[(a + b*E^{(c + d*x)})/\text{Sqrt}[-a^2 - b^2]])/((a^2 + b^2)^{(3/2)}*d) + (4*a*\text{Sqrt}[-(a^2 + b^2)^2]*e^3*E^{(2*c)}*\text{ArcTanh}[(a + b*E^{(c + d*x)})/\text{Sqrt}[a^2 + b^2]])/((-a^2 - b^2)^{(3/2)}*d) + (2*e^3*\text{Log}[b - 2*a*E^{(c + d*x)} - b*E^{(2*(c + d*x))}])/d - (2*e^3*E^{(2*c)}*\text{Log}[2*a*E^{(c + d*x)} + b*(-1 + E^{(2*(c + d*x))})])/d + (6*e^2*f*x*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d - (6*e^2*E^{(2*c)}*f*x*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d + (6*e*f^2*x^2*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d - (6*e*E^{(2*c)}*f^2*x^2*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d + (2*f^3*x^3*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d - (2*E^{(2*c)}*f^3*x^3*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d + (6*e^2*f*x*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c + \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d - (6*e^2*E^{(2*c)}*f*x*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c + \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d + (6*e*f^2*x^2*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c + \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d - (6*e*E^{(2*c)}*f^2*x^2*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c + \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d + (2*f^3*x^3*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c + \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d - (2*E^{(2*c)}*f^3*x^3*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c + \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d - (6*(-1 + E^{(2*c)})*f*(e + f*x)^2*\text{PolyLog}[2, -((b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])])/d^2 - (6*(-1 + E^{(2*c)})*f*(e + f*x)^2*\text{PolyLog}[2, -((b*E^{(2*c + d*x)})/(a*E^c + \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])])/d^2 - (12*e*f^2*\text{PolyLog}[3, -((b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])])/d^3 + (12*e*E^{(2*c)}*f^2*\text{PolyLog}[3, -((b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])])/d^3 - (12*f^3*x*\text{PolyLog}[3, -((b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])])/d^3 + (12*E^{(2*c)}*f^3*x*\text{PolyLog}[3, -((b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])])/d^3 - (12*e*f^2*\text{PolyLog}[3, -((b*E^{(2*c + d*x)})/(a*E^c + \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])])/d^3 + (12*e*E^{(2*c)}*f^2*\text{PolyLog}[3, -((b*E^{(2*c + d*x)})/(a*E^c + \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])])/d^3 - (12*f^3*x*\text{PolyLog}[3, -((b*E^{(2*c + d*x)})/(a*E^c + \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])])/d^3$$

+ d\*x))/(a\*E^c + Sqrt[(a^2 + b^2)\*E^(2\*c)])))]/d^3 + (12\*E^(2\*c)\*f^3\*x\*PolyLog[3, -((b\*E^(2\*c + d\*x))/(a\*E^c + Sqrt[(a^2 + b^2)\*E^(2\*c)])))]/d^3 + (12\*f^3\*PolyLog[4, -((b\*E^(2\*c + d\*x))/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)])))]/d^4 - (12\*E^(2\*c)\*f^3\*PolyLog[4, -((b\*E^(2\*c + d\*x))/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)])))]/d^4 + (12\*f^3\*PolyLog[4, -((b\*E^(2\*c + d\*x))/(a\*E^c + Sqrt[(a^2 + b^2)\*E^(2\*c)])))]/d^4 - (12\*E^(2\*c)\*f^3\*PolyLog[4, -((b\*E^(2\*c + d\*x))/(a\*E^c + Sqrt[(a^2 + b^2)\*E^(2\*c)])))]/d^4)/(2\*a\*(-1 + E^(2\*c)))

**Maple [F]** time = 0.495, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \coth(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-e^3 \left( \frac{\log(-2ae^{-dx-c}) + be^{-2dx-2c} - b}{ad} - \frac{\log(e^{-dx-c} + 1)}{ad} - \frac{\log(e^{-dx-c} - 1)}{ad} \right) + \frac{3(dx \log(e^{dx+c} + 1) + \text{Li}_2(-e^{dx+c}))}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -e^3\*(log(-2\*a\*e^(-d\*x - c) + b\*e^(-2\*d\*x - 2\*c) - b)/(a\*d) - log(e^(-d\*x - c) + 1)/(a\*d) - log(e^(-d\*x - c) - 1)/(a\*d)) + 3\*(d\*x\*log(e^(d\*x + c) + 1) + dilog(-e^(d\*x + c)))\*e^2\*f/(a\*d^2) + 3\*(d\*x\*log(-e^(d\*x + c) + 1) + dilog(e^(d\*x + c)))\*e^2\*f/(a\*d^2) + 3\*(d^2\*x^2\*log(e^(d\*x + c) + 1) + 2\*d\*x\*dilog(-e^(d\*x + c)) - 2\*polylog(3, -e^(d\*x + c)))\*e\*f^2/(a\*d^3) + 3\*(d^2\*x^2\*log(-e^(d\*x + c) + 1) + 2\*d\*x\*dilog(e^(d\*x + c)) - 2\*polylog(3, e^(d\*x + c)))\*e\*f^2/(a\*d^3) + (d^3\*x^3\*log(e^(d\*x + c) + 1) + 3\*d^2\*x^2\*dilog(-e^(d\*x + c)) - 6\*d\*x\*polylog(3, -e^(d\*x + c)) + 6\*polylog(4, -e^(d\*x + c)))\*f^3/(a\*d^4) + (d^3\*x^3\*log(-e^(d\*x + c) + 1) + 3\*d^2\*x^2\*dilog(e^(d\*x + c)) - 6\*d\*x\*polylog(3, e^(d\*x + c)) + 6\*polylog(4, e^(d\*x + c)))\*f^3/(a\*d^4) - 1/2\*(d^4\*f^3\*x^4 + 4\*d^4\*e\*f^2\*x^3 + 6\*d^4\*e^2\*f\*x^2)/(a\*d^4) + integrate(-2\*(b\*f

$$\frac{f^3 x^3 + 3 b e f^2 x^2 + 3 b e^2 f x - (a f^3 x^3 e^c + 3 a e f^2 x^2 e^c + 3 a e^2 f x e^c) e^{d x}}{(a b e^{2 d x + 2 c} + 2 a^2 e^{d x + c} - a b), x}$$

**Fricas [C]** time = 2.85818, size = 2996, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -(6 f^3 \operatorname{polylog}(4, (a \cosh(d x + c) + a \sinh(d x + c) + (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2}))/b + 6 f^3 \operatorname{polylog}(4, (a \cosh(d x + c) + a \sinh(d x + c) - (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2}))/b - 6 f^3 \operatorname{polylog}(4, \cosh(d x + c) + \sinh(d x + c)) - 6 f^3 \operatorname{polylog}(4, -\cosh(d x + c) - \sinh(d x + c)) + 3(d^2 f^3 x^2 + 2 d^2 e f^2 x + d^2 e^2 f) \operatorname{dilog}((a \cosh(d x + c) + a \sinh(d x + c) + (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 3(d^2 f^3 x^2 + 2 d^2 e f^2 x + d^2 e^2 f) \operatorname{dilog}((a \cosh(d x + c) + a \sinh(d x + c) - (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 3(d^2 f^3 x^2 + 2 d^2 e f^2 x + d^2 e^2 f) \operatorname{dilog}(\cosh(d x + c) + \sinh(d x + c)) - 3(d^2 f^3 x^2 + 2 d^2 e f^2 x + d^2 e^2 f) \operatorname{dilog}(-\cosh(d x + c) - \sinh(d x + c)) + (d^3 e^3 - 3 c d^2 e^2 f + 3 c^2 d e f^2 - c^3 f^3) \log(2 b \cosh(d x + c) + 2 b \sinh(d x + c) + 2 b \sqrt{(a^2 + b^2)/b^2} + 2 a) + (d^3 e^3 - 3 c d^2 e^2 f + 3 c^2 d e f^2 - c^3 f^3) \log(2 b \cosh(d x + c) + 2 b \sinh(d x + c) - 2 b \sqrt{(a^2 + b^2)/b^2} + 2 a) + (d^3 f^3 x^3 + 3 d^3 e f^2 x^2 + 3 d^3 e^2 f x + 3 c d^2 e^2 f - 3 c^2 d e f^2 + c^3 f^3) \log(-a \cosh(d x + c) + a \sinh(d x + c) + (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b + (d^3 f^3 x^3 + 3 d^3 e f^2 x^2 + 3 d^3 e^2 f x + 3 c d^2 e^2 f - 3 c^2 d e f^2 + c^3 f^3) \log(-a \cosh(d x + c) + a \sinh(d x + c) - (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b - (d^3 f^3 x^3 + 3 d^3 e f^2 x^2 + 3 d^3 e^2 f x + d^3 e^3) \log(\cosh(d x + c) + \sinh(d x + c) + 1) - (d^3 e^3 - 3 c d^2 e^2 f + 3 c^2 d e f^2 - c^3 f^3) \log(\cosh(d x + c) + \sinh(d x + c) - 1) - (d^3 f^3 x^3 + 3 d^3 e f^2 x^2 + 3 d^3 e^2 f x + 3 c d^2 e^2 f - 3 c^2 d e f^2 + c^3 f^3) \log(-\cosh(d x + c) - \sinh(d x + c) + 1) - 6(d f^3 x + d e f^2) \operatorname{polylog}(3, (a \cosh(d x + c) + a \sinh(d x + c) + (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2}))/b - 6(d f^3 x + d e f^2) \operatorname{polylog}(3, (a \cosh(d x + c) + a \sinh(d x + c) - (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2}))/b + 6(d f^3 x + d e f^2) \operatorname{polylog}(3, \cosh(d x + c) + \sinh(d x + c)) + 6(d f^3 x + d e f^2) \operatorname{polylog}(3, -\cosh(d x + c) - \sinh(d x + c)))/(a d^4 \end{aligned}$$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^3 \coth(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*\*3\*coth(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \coth(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*coth(d\*x + c)/(b\*sinh(d\*x + c) + a), x)



$$3.421 \quad \int \frac{(e+fx)^2 \coth(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=325

$$\frac{2f(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2} - \frac{2f(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2} + \frac{2f^2\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3} + \frac{2f^2\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^3}$$

```
[Out] -(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]))/(a*d)) - ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]))/(a*d) + ((e + f*x)^2*Log[1 - E^(2*(c + d*x))]/(a*d) - (2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a*d^2) - (2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a*d^2) + (f*(e + f*x)*PolyLog[2, E^(2*(c + d*x))]/(a*d^2) + (2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a*d^3) + (2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a*d^3) - (f^2*PolyLog[3, E^(2*(c + d*x))]/(2*a*d^3))
```

**Rubi [A]** time = 0.653868, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$ , Rules used = {5569, 3716, 2190, 2531, 2282, 6589, 5561}

$$\frac{2f(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2} - \frac{2f(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2} + \frac{2f^2\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^3} + \frac{2f^2\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]))/(a*d)) - ((e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]))/(a*d) + ((e + f*x)^2*Log[1 - E^(2*(c + d*x))]/(a*d) - (2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a*d^2) - (2*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a*d^2) + (f*(e + f*x)*PolyLog[2, E^(2*(c + d*x))]/(a*d^2) + (2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a*d^3) + (2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a*d^3) - (f^2*PolyLog[3, E^(2*(c + d*x))]/(2*a*d^3))
```

**Rule 5569**

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Coth[c
```

+ d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Cosh[c + d\*x]\*Coth[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 3716

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*tan[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_.)], x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x)))/(E^(2\*I\*k\*Pi)\*(1 + E^(2\*(-I\*e) + f\*fz\*x)))/E^(2\*I\*k\*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_.))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_.))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_.))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_.))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_.)\*((a\_.)\*(v\_)^(n\_.))^(m\_.) /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\* (F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.)]/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))^(m_.))/((a_.) + (b_.)*Sin
h[(c_.) + (d_.)*(x_.)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{(e + fx)^2 \coth(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\int (e + fx)^2 \coth(c + dx) dx}{a} - \frac{b \int \frac{(e + fx)^2 \cosh(c + dx)}{a + b \sinh(c + dx)} dx}{a} \\ &= -\frac{2 \int \frac{e^{2(c+dx)}(e+fx)^2}{1-e^{2(c+dx)}} dx}{a} - \frac{b \int \frac{e^{c+dx}(e+fx)^2}{a-\sqrt{a^2+b^2}+be^{c+dx}} dx}{a} - \frac{b \int \frac{e^{c+dx}(e+fx)^2}{a+\sqrt{a^2+b^2}+be^{c+dx}} dx}{a} \\ &= -\frac{(e + fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e + fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e + fx)^2 \log(1 - e^{2(c+dx)})}{ad} \\ &= -\frac{(e + fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e + fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e + fx)^2 \log(1 - e^{2(c+dx)})}{ad} \\ &= -\frac{(e + fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e + fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e + fx)^2 \log(1 - e^{2(c+dx)})}{ad} \\ &= -\frac{(e + fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e + fx)^2 \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e + fx)^2 \log(1 - e^{2(c+dx)})}{ad} \end{aligned}$$

**Mathematica [B]** time = 14.1565, size = 1296, normalized size = 3.99

$$2e^{2c} f^2 x^3 + 6e^{2c} f x^2 - \frac{3e^{2c} f^2 \log\left(\frac{e^{2c+dx} b}{a e^c - \sqrt{(a^2+b^2)} e^{2c}} + 1\right) x^2}{d} + \frac{3f^2 \log\left(\frac{e^{2c+dx} b}{a e^c - \sqrt{(a^2+b^2)} e^{2c}} + 1\right) x^2}{d} - \frac{3e^{2c} f^2 \log\left(\frac{e^{2c+dx} b}{e^c a + \sqrt{(a^2+b^2)} e^{2c}} + 1\right) x^2}{d} + \frac{3f^2 \log\left(\frac{e^{2c+dx} b}{e^c a + \sqrt{(a^2+b^2)} e^{2c}} + 1\right) x^2}{d}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Coth[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (6\*e^2\*E^(2\*c)\*x + 6\*e\*E^(2\*c)\*f\*x^2 + 2\*E^(2\*c)\*f^2\*x^3 - (2\*(e + f\*x)^3)/f + (6\*a\*Sqrt[a^2 + b^2]\*e^2\*ArcTan[(a + b\*E^(c + d\*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]\*d) + (6\*a\*Sqrt[-(a^2 + b^2)^2]\*e^2\*E^(2\*c)\*ArcTan[(a

$$\begin{aligned}
& + b \cdot E^{(c + d \cdot x)} / \sqrt{-a^2 - b^2} / ((a^2 + b^2)^{(3/2)} \cdot d) - (6 \cdot a \cdot \sqrt{-(a^2 + b^2)^2} \cdot e^{2 \cdot c} \cdot \operatorname{ArcTanh}[(a + b \cdot E^{(c + d \cdot x)}) / \sqrt{a^2 + b^2}] / ((-a^2 - b^2)^{(3/2)} \cdot d) \\
& + (6 \cdot a \cdot \sqrt{-(a^2 + b^2)^2} \cdot e^{2 \cdot c} \cdot E^{(2 \cdot c)} \cdot \operatorname{ArcTanh}[(a + b \cdot E^{(c + d \cdot x)}) / \sqrt{a^2 + b^2}] / \sqrt{a^2 + b^2}) / ((-a^2 - b^2)^{(3/2)} \cdot d) \\
& + (3 \cdot (-1 + E^{(2 \cdot c)}) \cdot (e + f \cdot x)^2 \cdot \operatorname{Log}[1 - E^{(-c - d \cdot x)}]) / d + (3 \cdot (-1 + E^{(2 \cdot c)}) \cdot (e + f \cdot x)^2 \cdot \operatorname{Log}[1 + E^{(-c - d \cdot x)}]) / d \\
& + (3 \cdot e^{2 \cdot c} \cdot \operatorname{Log}[2 \cdot a \cdot E^{(c + d \cdot x)} + b \cdot (-1 + E^{(2 \cdot (c + d \cdot x))})]) / d - (3 \cdot e^{2 \cdot c} \cdot E^{(2 \cdot c)} \cdot \operatorname{Log}[2 \cdot a \cdot E^{(c + d \cdot x)} + b \cdot (-1 + E^{(2 \cdot (c + d \cdot x))})]) / d \\
& + (6 \cdot e \cdot f \cdot x \cdot \operatorname{Log}[1 + (b \cdot E^{(2 \cdot c + d \cdot x)}) / (a \cdot E^c - \sqrt{(a^2 + b^2) \cdot E^{(2 \cdot c)}})]) / d - (6 \cdot e \cdot E^{(2 \cdot c)} \cdot f \cdot x \cdot \operatorname{Log}[1 + (b \cdot E^{(2 \cdot c + d \cdot x)}) / (a \cdot E^c - \sqrt{(a^2 + b^2) \cdot E^{(2 \cdot c)}})]) / d \\
& + (3 \cdot f^2 \cdot x^2 \cdot \operatorname{Log}[1 + (b \cdot E^{(2 \cdot c + d \cdot x)}) / (a \cdot E^c - \sqrt{(a^2 + b^2) \cdot E^{(2 \cdot c)}})]) / d - (3 \cdot E^{(2 \cdot c)} \cdot f^2 \cdot x^2 \cdot \operatorname{Log}[1 + (b \cdot E^{(2 \cdot c + d \cdot x)}) / (a \cdot E^c - \sqrt{(a^2 + b^2) \cdot E^{(2 \cdot c)}})]) / d \\
& + (6 \cdot e \cdot f \cdot x \cdot \operatorname{Log}[1 + (b \cdot E^{(2 \cdot c + d \cdot x)}) / (a \cdot E^c + \sqrt{(a^2 + b^2) \cdot E^{(2 \cdot c)}})]) / d - (6 \cdot e \cdot E^{(2 \cdot c)} \cdot f \cdot x \cdot \operatorname{Log}[1 + (b \cdot E^{(2 \cdot c + d \cdot x)}) / (a \cdot E^c + \sqrt{(a^2 + b^2) \cdot E^{(2 \cdot c)}})]) / d \\
& + (3 \cdot f^2 \cdot x^2 \cdot \operatorname{Log}[1 + (b \cdot E^{(2 \cdot c + d \cdot x)}) / (a \cdot E^c + \sqrt{(a^2 + b^2) \cdot E^{(2 \cdot c)}})]) / d - (3 \cdot E^{(2 \cdot c)} \cdot f^2 \cdot x^2 \cdot \operatorname{Log}[1 + (b \cdot E^{(2 \cdot c + d \cdot x)}) / (a \cdot E^c + \sqrt{(a^2 + b^2) \cdot E^{(2 \cdot c)}})]) / d \\
& - (6 \cdot (-1 + E^{(2 \cdot c)}) \cdot f \cdot (e + f \cdot x) \cdot \operatorname{PolyLog}[2, -((b \cdot E^{(2 \cdot c + d \cdot x)}) / (a \cdot E^c - \sqrt{(a^2 + b^2) \cdot E^{(2 \cdot c)}})])]) / d^2 - (6 \cdot (-1 + E^{(2 \cdot c)}) \cdot f \cdot (e + f \cdot x) \cdot \operatorname{PolyLog}[2, -((b \cdot E^{(2 \cdot c + d \cdot x)}) / (a \cdot E^c + \sqrt{(a^2 + b^2) \cdot E^{(2 \cdot c)}})])]) / d^2 \\
& - (6 \cdot (-1 + E^{(2 \cdot c)}) \cdot f \cdot (d \cdot (e + f \cdot x) \cdot \operatorname{PolyLog}[2, -E^{(-c - d \cdot x)}] + f \cdot \operatorname{PolyLog}[3, -E^{(-c - d \cdot x)}])) / d^3 - (6 \cdot (-1 + E^{(2 \cdot c)}) \cdot f \cdot (d \cdot (e + f \cdot x) \cdot \operatorname{PolyLog}[2, E^{(-c - d \cdot x)}] + f \cdot \operatorname{PolyLog}[3, E^{(-c - d \cdot x)}])) / d^3 \\
& - (6 \cdot f^2 \cdot \operatorname{PolyLog}[3, -((b \cdot E^{(2 \cdot c + d \cdot x)}) / (a \cdot E^c - \sqrt{(a^2 + b^2) \cdot E^{(2 \cdot c)}})])]) / d^3 + (6 \cdot E^{(2 \cdot c)} \cdot f^2 \cdot \operatorname{PolyLog}[3, -((b \cdot E^{(2 \cdot c + d \cdot x)}) / (a \cdot E^c - \sqrt{(a^2 + b^2) \cdot E^{(2 \cdot c)}})])]) / d^3 \\
& - (6 \cdot f^2 \cdot \operatorname{PolyLog}[3, -((b \cdot E^{(2 \cdot c + d \cdot x)}) / (a \cdot E^c + \sqrt{(a^2 + b^2) \cdot E^{(2 \cdot c)}})])]) / d^3 + (6 \cdot E^{(2 \cdot c)} \cdot f^2 \cdot \operatorname{PolyLog}[3, -((b \cdot E^{(2 \cdot c + d \cdot x)}) / (a \cdot E^c + \sqrt{(a^2 + b^2) \cdot E^{(2 \cdot c)}})])]) / d^3 / (3 \cdot a \cdot (-1 + E^{(2 \cdot c)}))
\end{aligned}$$

**Maple [F]** time = 0.39, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \coth(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-e^2 \left( \frac{\log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{ad} - \frac{\log(e^{(-dx-c)} + 1)}{ad} - \frac{\log(e^{(-dx-c)} - 1)}{ad} \right) + \frac{2(dx \log(e^{(dx+c)} + 1) + \text{Li}_2(-e^{(dx+c)}))}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-e^2 * (\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/(a*d) - \log(e^{(-d*x - c)} + 1)/(a*d) - \log(e^{(-d*x - c)} - 1)/(a*d)) + 2*(d*x*\log(e^{(d*x + c)} + 1) + \text{dilog}(-e^{(d*x + c)}))*e*f/(a*d^2) + 2*(d*x*\log(-e^{(d*x + c)} + 1) + \text{dilog}(e^{(d*x + c)}))*e*f/(a*d^2) + (d^2*x^2*\log(e^{(d*x + c)} + 1) + 2*d*x*\text{dilog}(-e^{(d*x + c)}) - 2*\text{polylog}(3, -e^{(d*x + c)}))*f^2/(a*d^3) + (d^2*x^2*\log(-e^{(d*x + c)} + 1) + 2*d*x*\text{dilog}(e^{(d*x + c)}) - 2*\text{polylog}(3, e^{(d*x + c)}))*f^2/(a*d^3) - 2/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^2)/(a*d^3) + \text{integrate}(-2*(b*f^2*x^2 + 2*b*e*f*x - (a*f^2*x^2*e^c + 2*a*e*f*x*e^c))*e^{(d*x)})/(a*b*e^{(2*d*x + 2*c)} + 2*a^2*e^{(d*x + c)} - a*b), x)$

**Fricas [C]** time = 2.61739, size = 2053, normalized size = 6.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $(2*f^2*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) + 2*f^2*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b - 2*f^2*\text{polylog}(3, \cosh(d*x + c) + \sinh(d*x + c)) - 2*f^2*\text{polylog}(3, -\cosh(d*x + c) - \sinh(d*x + c)) - 2*(d*f^2*x + d*e*f)*\text{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 2*(d*f^2*x + d*e*f)*\text{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 2*(d*f^2*x + d*e*f)*\text{dilog}(\cosh(d*x + c) + \sinh(d*x + c)) + 2*(d*f^2*x + d*e*f)*\text{dilog}(-\cosh(d*x + c) - \sinh(d*x + c)) - (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - (d^2*e^2 - 2*c*d*e*f + c^2*f^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - (d^2*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d$

```
*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (d^2*f^2*x^2 + 2
*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) -
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (d^2*f
^2*x^2 + 2*d^2*e*f*x + d^2*e^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (d
^2*e^2 - 2*c*d*e*f + c^2*f^2)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + (d^2
*f^2*x^2 + 2*d^2*e*f*x + 2*c*d*e*f - c^2*f^2)*log(-cosh(d*x + c) - sinh(d*x
+ c) + 1))/(a*d^3)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^2 \coth(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \coth(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)^2*coth(d*x + c)/(b*sinh(d*x + c) + a), x)
```

$$3.422 \quad \int \frac{(e+fx) \coth(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=205

$$\frac{f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2} - \frac{f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2} + \frac{f \operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{2ad^2} - \frac{(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{ad} - \frac{(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} + 1\right)}{ad}$$

```
[Out] -(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]))/(a*d)) - ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]))/(a*d) + ((e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d) - (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*d^2) - (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*d^2) + (f*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^2)
```

**Rubi [A]** time = 0.380062, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {5569, 3716, 2190, 2279, 2391, 5561}

$$\frac{f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2} - \frac{f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2} + \frac{f \operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{2ad^2} - \frac{(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{ad} - \frac{(e+fx) \log\left(\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} + 1\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Coth[c + d*x])/(a + b*Sinh[c + d*x]), x]
```

```
[Out] -(((e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]))/(a*d)) - ((e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]))/(a*d) + ((e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d) - (f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*d^2) - (f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*d^2) + (f*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^2)
```

### Rule 5569

```
Int[(Coth[(c_) + (d_)*(x_)]^(n_)*((e_) + (f_)*(x_))^(m_)]/((a_) + (b_) * Sinh[(c_) + (d_)*(x_)], x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Coth[c + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

### Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(e+fx) \coth(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx) \coth(c+dx) dx}{a} - \frac{b \int \frac{(e+fx) \cosh(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= -\frac{2 \int \frac{e^{2(c+dx)}(e+fx)}{1-e^{2(c+dx)}} dx}{a} - \frac{b \int \frac{e^{c+dx}(e+fx)}{a-\sqrt{a^2+b^2+be^{c+dx}}} dx}{a} - \frac{b \int \frac{e^{c+dx}(e+fx)}{a+\sqrt{a^2+b^2+be^{c+dx}}} dx}{a} \\
&= -\frac{(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e+fx) \log(1 - e^{2(c+dx)})}{ad} \\
&= -\frac{(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e+fx) \log(1 - e^{2(c+dx)})}{ad} \\
&= -\frac{(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad} - \frac{(e+fx) \log\left(1 + \frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{ad} + \frac{(e+fx) \log(1 - e^{2(c+dx)})}{ad}
\end{aligned}$$

**Mathematica [A]** time = 0.945634, size = 236, normalized size = 1.15

$$f \operatorname{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) + f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) + \frac{1}{2} f \operatorname{PolyLog}\left(2, e^{-2(c+dx)}\right) + f(c+dx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right) + f(c$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Coth[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] -(((f\*(c + d\*x)^2) - f\*(c + d\*x)\*Log[1 - E^(-2\*(c + d\*x))]) + f\*(c + d\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]) + f\*(c + d\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]) - d\*e\*Log[Sinh[c + d\*x]] + c\*f\*Log[Sinh[c + d\*x]] + d\*e\*Log[a + b\*Sinh[c + d\*x]] - c\*f\*Log[a + b\*Sinh[c + d\*x]] + (f\*PolyLog[2, E^(-2\*(c + d\*x))])/2 + f\*PolyLog[2, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2]]) + f\*PolyLog[2, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a\*d^2)

**Maple [B]** time = 0.134, size = 451, normalized size = 2.2

$$\frac{e \ln(e^{dx+c} - 1)}{da} - \frac{e \ln(be^{2dx+2c} + 2ae^{dx+c} - b)}{da} + \frac{e \ln(e^{dx+c} + 1)}{da} - \frac{f \operatorname{dilog}(e^{dx+c})}{ad^2} - \frac{f}{ad^2} \operatorname{dilog}\left(\left(-be^{dx+c} + \sqrt{a^2 + b^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)`

[Out]  $\frac{1}{d} \frac{1}{a} e \ln(\exp(d*x+c)-1) - \frac{1}{d} \frac{1}{e} \frac{1}{a} \ln(b \exp(2*d*x+2*c) + 2*a \exp(d*x+c) - b) + \frac{1}{d} \frac{1}{a} e \ln(\exp(d*x+c)+1) - \frac{1}{d^2} \frac{f}{a} \operatorname{dilog}(\exp(d*x+c)) - \frac{1}{d^2} \frac{f}{a} \operatorname{dilog}\left(\frac{-b \exp(d*x+c) + (a^2+b^2)^{1/2} - a}{-a + (a^2+b^2)^{1/2}}\right) - \frac{1}{d^2} \frac{f}{a} \operatorname{dilog}\left(\frac{b \exp(d*x+c) + (a^2+b^2)^{1/2} + a}{a + (a^2+b^2)^{1/2}}\right) + \frac{1}{d^2} \frac{f}{a} \operatorname{dilog}(\exp(d*x+c)+1) - \frac{1}{d} \frac{f}{a} \ln\left(\frac{-b \exp(d*x+c) + (a^2+b^2)^{1/2} - a}{-a + (a^2+b^2)^{1/2}}\right) * x - \frac{1}{d^2} \frac{f}{a} \ln\left(\frac{-b \exp(d*x+c) + (a^2+b^2)^{1/2} - a}{-a + (a^2+b^2)^{1/2}}\right) * c - \frac{1}{d} \frac{f}{a} \ln\left(\frac{b \exp(d*x+c) + (a^2+b^2)^{1/2} + a}{a + (a^2+b^2)^{1/2}}\right) * x - \frac{1}{d^2} \frac{f}{a} \ln\left(\frac{b \exp(d*x+c) + (a^2+b^2)^{1/2} + a}{a + (a^2+b^2)^{1/2}}\right) * c + \frac{1}{d} \frac{1}{a} \ln(\exp(d*x+c)+1) * f * x - \frac{1}{d^2} \frac{1}{a} * f * c * \ln(\exp(d*x+c)-1) + \frac{1}{d^2} \frac{1}{a} * f * c * \ln(b \exp(2*d*x+2*c) + 2*a \exp(d*x+c) - b)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-e \left( \frac{\log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{ad} - \frac{\log(e^{(-dx-c)} + 1)}{ad} - \frac{\log(e^{(-dx-c)} - 1)}{ad} \right) + f \int \frac{2x(e^{(dx+c)} + e^{(-dx-c)})}{(b(e^{(dx+c)} - e^{(-dx-c)}) + 2a)(e^{(dx+c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out]  $-e * (\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/(a*d) - \log(e^{(-d*x - c)} + 1)/(a*d) - \log(e^{(-d*x - c)} - 1)/(a*d)) + f * \operatorname{integrate}(2*x*(e^{(d*x + c)} + e^{(-d*x - c)})/((b*(e^{(d*x + c)} - e^{(-d*x - c)}) + 2*a)*(e^{(d*x + c)} - e^{(-d*x - c)})), x)$

**Fricas [B]** time = 2.48589, size = 1243, normalized size = 6.06

$$f \operatorname{Li}_2 \left( \frac{a \cosh(dx+c) + a \sinh(dx+c) + (b \cosh(dx+c) + b \sinh(dx+c)) \sqrt{\frac{a^2+b^2}{b^2}} - b}{b} + 1 \right) + f \operatorname{Li}_2 \left( \frac{a \cosh(dx+c) + a \sinh(dx+c) - (b \cosh(dx+c) + b \sinh(dx+c)) \sqrt{\frac{a^2+b^2}{b^2}}}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

```
[Out] -(f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + f*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - f*dilog(cosh(d*x + c) + sinh(d*x + c)) - f*dilog(-cosh(d*x + c) - sinh(d*x + c)) + (d*e - c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (d*e - c*f)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (d*f*x + c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (d*f*x + c*f)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (d*f*x + d*e)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (d*e - c*f)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - (d*f*x + c*f)*log(-cosh(d*x + c) - sinh(d*x + c) + 1))/(a*d^2)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \coth(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \coth(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*coth(d*x + c)/(b*sinh(d*x + c) + a), x)
```

$$3.423 \quad \int \frac{\coth(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=34

$$\frac{\log(\sinh(c+dx))}{ad} - \frac{\log(a+b \sinh(c+dx))}{ad}$$

[Out] Log[Sinh[c + d\*x]]/(a\*d) - Log[a + b\*Sinh[c + d\*x]]/(a\*d)

**Rubi [A]** time = 0.0473075, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$ , Rules used = {2721, 36, 29, 31}

$$\frac{\log(\sinh(c+dx))}{ad} - \frac{\log(a+b \sinh(c+dx))}{ad}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d\*x]/(a + b\*Sinh[c + d\*x]),x]

[Out] Log[Sinh[c + d\*x]]/(a\*d) - Log[a + b\*Sinh[c + d\*x]]/(a\*d)

#### Rule 2721

Int[((a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)])^(m\_)\*tan[(e\_) + (f\_)\*(x\_)]^(p\_), x\_Symbol] :> Dist[1/f, Subst[Int[(x^p\*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b\*Sin[e + f\*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

#### Rule 36

Int[1/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))), x\_Symbol] :> Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 29

Int[(x\_)^(-1), x\_Symbol] :> Simp[Log[x], x]

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rubi steps

$$\begin{aligned} \int \frac{\coth(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+x)} dx, x, b \sinh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{1}{x} dx, x, b \sinh(c + dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{1}{a+x} dx, x, b \sinh(c + dx)\right)}{ad} \\ &= \frac{\log(\sinh(c + dx))}{ad} - \frac{\log(a + b \sinh(c + dx))}{ad} \end{aligned}$$

**Mathematica [A]** time = 0.0213791, size = 28, normalized size = 0.82

$$\frac{\log(\sinh(c + dx)) - \log(a + b \sinh(c + dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (Log[Sinh[c + d*x]] - Log[a + b*Sinh[c + d*x]])/(a*d)
```

**Maple [A]** time = 0.001, size = 35, normalized size = 1.

$$\frac{\ln(\sinh(dx + c))}{da} - \frac{\ln(a + b \sinh(dx + c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] ln(sinh(d*x+c))/a/d-ln(a+b*sinh(d*x+c))/a/d
```

**Maxima [B]** time = 1.08305, size = 101, normalized size = 2.97

$$-\frac{\log(-2ae^{-dx-c} + be^{-2dx-2c} - b)}{ad} + \frac{\log(e^{-dx-c} + 1)}{ad} + \frac{\log(e^{-dx-c} - 1)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-\log(-2*a*e^{-(d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/(a*d) + \log(e^{-(d*x - c)} + 1)/(a*d) + \log(e^{-(d*x - c)} - 1)/(a*d)$

**Fricas [A]** time = 2.39192, size = 170, normalized size = 5.

$$\frac{\log\left(\frac{2(b\sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right) - \log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $-(\log(2*(b*\sinh(d*x + c) + a)/(\cosh(d*x + c) - \sinh(d*x + c)))) - \log(2*\sinh(d*x + c)/(\cosh(d*x + c) - \sinh(d*x + c)))/(a*d)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(coth(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

**Giac [A]** time = 1.32442, size = 85, normalized size = 2.5

$$\frac{\log(e^{(dx+c)+1})}{a} - \frac{\log(|be^{(2dx+2c)+2ae^{(dx+c)}-b|)}}{a} + \frac{\log(|e^{(dx+c)}-1|)}{a}$$

$$d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(e^(d*x + c) + 1)/a - log(abs(b*e^(2*d*x + 2*c) + 2*a*e^(d*x + c) - b))  
/a + log(abs(e^(d*x + c) - 1))/a)/d
```

$$3.424 \quad \int \frac{\coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=28

$$\text{Unintegrable} \left( \frac{\coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[Coth[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.049565, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Coth[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Coth[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [A]** time = 29.0795, size = 0, normalized size = 0.

$$\int \frac{\coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[Coth[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Integrate[Coth[c + d\*x]/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]



---

**Maple [A]** time = 0.379, size = 0, normalized size = 0.

$$\int \frac{\coth(dx + c)}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x)

[Out] int(coth(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(dx + c)}{(fx + e)(b \sinh(dx + c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x, algorithm="maxima")

[Out] integrate(coth(d\*x + c)/((f\*x + e)\*(b\*sinh(d\*x + c) + a)), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth(dx + c)}{afx + ae + (bfx + be) \sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x, algorithm="fricas")

[Out] integral(coth(d\*x + c)/(a\*f\*x + a\*e + (b\*f\*x + b\*e)\*sinh(d\*x + c)), x)

---

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(c + dx)}{(a + b \sinh(c + dx))(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(coth(c + d\*x)/((a + b\*sinh(c + d\*x))\*(e + f\*x)), x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(dx + c)}{(fx + e)(b \sinh(dx + c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate(coth(d\*x + c)/((f\*x + e)\*(b\*sinh(d\*x + c) + a)), x)

$$3.425 \quad \int \frac{(e+fx)^3 \cosh(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=638

$$\frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^3} - \frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{abd^3} - \frac{3f\sqrt{a^2+b^2}(e+fx)^2\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^3} + \frac{3f\sqrt{a^2+b^2}(e+fx)^2\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{abd^3}$$

```
[Out] (e + f*x)^4/(4*b*f) - (2*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(a*b*d) + (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(a*b*d) - (3*f*(e + f*x)^2*PolyLog[2, -E^(c + d*x)]/(a*d^2) + (3*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)]/(a*d^2) - (3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a*b*d^2) + (3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a*b*d^2) + (6*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)]/(a*d^3) - (6*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)]/(a*d^3) + (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a*b*d^3) - (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a*b*d^3) - (6*f^3*PolyLog[4, -E^(c + d*x)]/(a*d^4) + (6*f^3*PolyLog[4, E^(c + d*x)]/(a*d^4) - (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a*b*d^4) + (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a*b*d^4)
```

**Rubi [A]** time = 1.27521, antiderivative size = 638, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 14, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$ , Rules used = {5585, 5450, 3296, 2637, 4182, 2531, 6609, 2282, 6589, 5565, 32, 3322, 2264, 2190}

$$\frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^3} - \frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{abd^3} - \frac{3f\sqrt{a^2+b^2}(e+fx)^2\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^3} + \frac{3f\sqrt{a^2+b^2}(e+fx)^2\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{abd^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e + f*x)^4/(4*b*f) - (2*(e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(a*b*d) + (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(a*b*d) - (3*f*(e + f*x)^2*PolyLog[2, -E^(c + d*x)]/(a*d^2) + (3*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)]/(a*d^2) - (3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a*b*d^2) + (3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a*b*d^2) + (6*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)]/(a*d^3) - (6*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)]/(a*d^3) + (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a*b*d^3) - (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a*b*d^3) - (6*f^3*PolyLog[4, -E^(c + d*x)]/(a*d^4) + (6*f^3*PolyLog[4, E^(c + d*x)]/(a*d^4) - (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a*b*d^4) + (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a*b*d^4)
```

```
)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a*b*d^2) + (3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a*b*d^2) + (6*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)]/(a*d^3) - (6*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)]/(a*d^3) + (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a*b*d^3) - (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a*b*d^3) - (6*f^3*PolyLog[4, -E^(c + d*x)]/(a*d^4) + (6*f^3*PolyLog[4, E^(c + d*x)]/(a*d^4) - (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a*b*d^4) + (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a*b*d^4)
```

### Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/(a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

$f*Fz*x]], x], x]) /; \text{FreeQ}\{c, d, e, f, Fz\}, x\} \&\& \text{IGtQ}[m, 0]$

### Rule 2531

$\text{Int}[\text{Log}[1 + (e\_)*(F\_)^{(c\_)*((a\_)+(b\_)*(x\_))})^{(n\_)}]*((f\_)+(g\_)*(x\_))^{(m\_)}, x\_Symbol] := -\text{Simp}[(f + g*x)^m * \text{PolyLog}[2, -(e*(F^{c*(a + b*x)}))^{(n)}]] / (b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m) / (b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{(m-1)} * \text{PolyLog}[2, -(e*(F^{c*(a + b*x)}))^{(n)}]], x], x] /; \text{FreeQ}\{F, a, b, c, e, f, g, n\}, x\} \&\& \text{GtQ}[m, 0]$

### Rule 6609

$\text{Int}[(e\_)+(f\_)*(x\_))^{(m\_)} * \text{PolyLog}[n, (d\_)*(F\_)^{(c\_)*((a\_)+(b\_)*(x\_))})^{(p\_)}], x\_Symbol] := \text{Simp}[(e + f*x)^m * \text{PolyLog}[n + 1, d*(F^{c*(a + b*x)})^{(p)}]] / (b*c*p*\text{Log}[F]), x] - \text{Dist}[(f*m) / (b*c*p*\text{Log}[F]), \text{Int}[(e + f*x)^{(m-1)} * \text{PolyLog}[n + 1, d*(F^{c*(a + b*x)})^{(p)}]], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, n, p\}, x\} \&\& \text{GtQ}[m, 0]$

### Rule 2282

$\text{Int}[u, x\_Symbol] := \text{With}\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \&\& !\text{MatchQ}[u, (w\_)*((a\_)*(v\_))^{(n\_)}]^{(m\_)} /; \text{FreeQ}\{a, m, n\}, x\} \&\& \text{IntegerQ}[m*n] \&\& !\text{MatchQ}[u, E^{(c\_)*((a\_)+(b\_)*x)}*(F\_)[v_] /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{InverseFunctionQ}[F[x]]]$

### Rule 6589

$\text{Int}[\text{PolyLog}[n, (c\_)*((a\_)+(b\_)*(x\_))^{(p\_)}]] / ((d\_)+(e\_)*(x\_)), x\_Symbol] := \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p] / (e*p), x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x\} \&\& \text{EqQ}[b*d, a*e]$

### Rule 5565

$\text{Int}[(\text{Cosh}[c\_)+(d\_)*(x\_)]^{(n\_)}*((e\_)+(f\_)*(x\_))^{(m\_)}] / ((a\_)+(b\_)*\text{Sinh}[c\_)+(d\_)*(x\_)]), x\_Symbol] := -\text{Dist}[a/b^2, \text{Int}[(e + f*x)^m * \text{Cosh}[c + d*x]^{(n-2)}, x], x] + (\text{Dist}[1/b, \text{Int}[(e + f*x)^m * \text{Cosh}[c + d*x]^{(n-2)} * \text{Sinh}[c + d*x], x], x] + \text{Dist}[(a^2 + b^2)/b^2, \text{Int}[(e + f*x)^m * \text{Cosh}[c + d*x]^{(n-2)}] / (a + b*\text{Sinh}[c + d*x]), x], x)) /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{IGtQ}[n, 1] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{IGtQ}[m, 0]$

### Rule 32

$\text{Int}[(a\_)+(b\_)*(x\_))^{(m\_)}, x\_Symbol] := \text{Simp}[(a + b*x)^{(m+1)} / (b*(m+1)), x] /; \text{FreeQ}\{a, b, m\}, x\} \&\& \text{NeQ}[m, -1]$

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_] *
(f_.)*(x_))], x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps



$$\begin{aligned}
& + d*x))/(a - \text{Sqrt}[a^2 + b^2]) - d^3*f^3*x^3*\text{Log}[1 + (b*E^(c + d*x))/(a - \\
& \text{Sqrt}[a^2 + b^2])] + 3*d^3*e^2*f*x*\text{Log}[1 + (b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b \\
& ^2])] + 3*d^3*e*f^2*x^2*\text{Log}[1 + (b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b^2])] + d^ \\
& 3*f^3*x^3*\text{Log}[1 + (b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b^2])] - 3*d^2*f*(e + f*x) \\
& )^2*\text{PolyLog}[2, (b*E^(c + d*x))/(-a + \text{Sqrt}[a^2 + b^2])] + 3*d^2*f*(e + f*x) \\
& ^2*\text{PolyLog}[2, -((b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b^2]))] + 6*d*e*f^2*\text{PolyLog}[ \\
& 3, (b*E^(c + d*x))/(-a + \text{Sqrt}[a^2 + b^2])] + 6*d*f^3*x*\text{PolyLog}[3, (b*E^(c + \\
& d*x))/(-a + \text{Sqrt}[a^2 + b^2])] - 6*d*e*f^2*\text{PolyLog}[3, -((b*E^(c + d*x))/(a \\
& + \text{Sqrt}[a^2 + b^2]))] - 6*d*f^3*x*\text{PolyLog}[3, -((b*E^(c + d*x))/(a + \text{Sqrt}[a^2 \\
& + b^2]))] - 6*f^3*\text{PolyLog}[4, (b*E^(c + d*x))/(-a + \text{Sqrt}[a^2 + b^2])] + 6*f \\
& ^3*\text{PolyLog}[4, -((b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b^2]))] - 4*b*(2*d^3*(e + \\
& f*x)^3*\text{ArcTanh}[\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]] + 3*f*(d^2*(e + f*x)^2*\text{PolyLo \\
& g}[2, -\text{Cosh}[c + d*x] - \text{Sinh}[c + d*x]] - 2*d*f*(e + f*x)*\text{PolyLog}[3, -\text{Cosh}[c + \\
& d*x] - \text{Sinh}[c + d*x]] + 2*f^2*\text{PolyLog}[4, -\text{Cosh}[c + d*x] - \text{Sinh}[c + d*x]]) \\
& - 3*f*(d^2*(e + f*x)^2*\text{PolyLog}[2, \text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]] - 2*d*f*(e \\
& + f*x)*\text{PolyLog}[3, \text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]] + 2*f^2*\text{PolyLog}[4, \text{Cosh}[c \\
& + d*x] + \text{Sinh}[c + d*x]])))/(4*a*b*d^4)
\end{aligned}$$

**Maple [F]** time = 0.734, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c) \coth(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*cosh(d\*x+c)\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*cosh(d\*x+c)\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError



**Fricas [C]** time = 3.41837, size = 3588, normalized size = 5.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^4*f^3*x^4 + 4*a*d^4*e*f^2*x^3 + 6*a*d^4*e^2*f*x^2 + 4*a*d^4*e^3*x
- 24*b*f^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x +
c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*b*
f^3*sqrt((a^2 + b^2)/b^2)*polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 24*b*f^3*pol
ylog(4, cosh(d*x + c) + sinh(d*x + c)) - 24*b*f^3*polylog(4, -cosh(d*x + c)
- sinh(d*x + c)) - 12*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*sqrt
((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 12*(b*d^2*f^3*x^2
+ 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x +
c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2
)/b^2) - b)/b + 1) + 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c
^3*f^3)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2
*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*
d*e*f^2 - b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh
(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 4*(b*d^3*f^3*x^3 + 3*b*d^3*e
*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)
*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*
x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 4*(b*d^3*f^3*x^3
+ 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 +
b*c^3*f^3)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) -
(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 24*(b*
d*f^3*x + b*d*e*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*
sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/
b) - 24*(b*d*f^3*x + b*d*e*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*
x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 +
b^2)/b^2))/b) + 12*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*dilog(co
sh(d*x + c) + sinh(d*x + c)) - 12*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*
e^2*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - 4*(b*d^3*f^3*x^3 + 3*b*d^3*e
*f^2*x^2 + 3*b*d^3*e^2*f*x + b*d^3*e^3)*log(cosh(d*x + c) + sinh(d*x + c) +
1) + 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(cos
h(d*x + c) + sinh(d*x + c) - 1) + 4*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*
b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3)*log(-cosh(d*
x + c) - sinh(d*x + c) + 1) - 24*(b*d*f^3*x + b*d*e*f^2)*polylog(3, cosh(d*
```

$$x + c) + \sinh(d*x + c)) + 24*(b*d*f^3*x + b*d*e*f^2)*\text{polylog}(3, -\cosh(d*x + c) - \sinh(d*x + c))/(a*b*d^4)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cosh(d\*x+c)\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.426 \quad \int \frac{(e+fx)^2 \cosh(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=462

$$\frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^2} + \frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{abd^2} + \frac{2f^2\sqrt{a^2+b^2}\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^3} + \frac{2f^2\sqrt{a^2+b^2}\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{abd^3}$$

```
[Out] (e + f*x)^3/(3*b*f) - (2*(e + f*x)^2*ArcTanh[E^(c + d*x)]/(a*d) - (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(a*b*d) + (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(a*b*d) - (2*f*(e + f*x)*PolyLog[2, -E^(c + d*x)]/(a*d^2) + (2*f*(e + f*x)*PolyLog[2, E^(c + d*x)]/(a*d^2) - (2*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a*b*d^2) + (2*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a*b*d^2) + (2*f^2*PolyLog[3, -E^(c + d*x)]/(a*d^3) - (2*f^2*PolyLog[3, E^(c + d*x)]/(a*d^3) + (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a*b*d^3) - (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a*b*d^3))
```

**Rubi [A]** time = 1.04022, antiderivative size = 462, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 13, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$ , Rules used = {5585, 5450, 3296, 2638, 4182, 2531, 2282, 6589, 5565, 32, 3322, 2264, 2190}

$$\frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^2} + \frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{abd^2} + \frac{2f^2\sqrt{a^2+b^2}\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^3} + \frac{2f^2\sqrt{a^2+b^2}\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{abd^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e + f*x)^3/(3*b*f) - (2*(e + f*x)^2*ArcTanh[E^(c + d*x)]/(a*d) - (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(a*b*d) + (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(a*b*d) - (2*f*(e + f*x)*PolyLog[2, -E^(c + d*x)]/(a*d^2) + (2*f*(e + f*x)*PolyLog[2, E^(c + d*x)]/(a*d^2) - (2*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a*b*d^2) + (2*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a*b*d^2) + (2*f^2*PolyLog[3, -E^(c + d*x)]/(a*d^3) - (2*f^2*PolyLog[3, E^(c + d*x)]/(a*d^3) + (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a*b*d^3) - (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a*b*d^3))
```

$$- \text{Sqrt}[a^2 + b^2])]) / (a*b*d^3) - (2*\text{Sqrt}[a^2 + b^2]*f^2*\text{PolyLog}[3, -(b*E^{(c + d*x)}) / (a + \text{Sqrt}[a^2 + b^2])]) / (a*b*d^3)$$

### Rule 5585

$$\text{Int}[(\text{Cosh}[(c_.) + (d_.)*(x_)]^{(p_.)}*\text{Coth}[(c_.) + (d_.)*(x_)]^{(n_.)}*((e_.) + (f_.)*(x_))^{(m_.)}) / ((a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_)]), x\_Symbol] \rightarrow \text{Dist}[1/a, \text{Int}[(e + f*x)^m*\text{Cosh}[c + d*x]^p*\text{Coth}[c + d*x]^n, x] - \text{Dist}[b/a, \text{Int}[(e + f*x)^m*\text{Cosh}[c + d*x]^{(p + 1)}*\text{Coth}[c + d*x]^{(n - 1)} / (a + b*\text{Sinh}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$$

### Rule 5450

$$\text{Int}[\text{Cosh}[(a_.) + (b_.)*(x_)]^{(n_.)}*\text{Coth}[(a_.) + (b_.)*(x_)]^{(p_.)}*((c_.) + (d_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \text{Int}[(c + d*x)^m*\text{Cosh}[a + b*x]^n*\text{Coth}[a + b*x]^{(p - 2)}, x] + \text{Int}[(c + d*x)^m*\text{Cosh}[a + b*x]^{(n - 2)}*\text{Coth}[a + b*x]^p, x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$$

### Rule 3296

$$\text{Int}[(c_.) + (d_.)*(x_)]^{(m_.)}*\sin[(e_.) + (f_.)*(x_)], x\_Symbol] \rightarrow -\text{Simp}[(c + d*x)^m*\text{Cos}[e + f*x] / f, x] + \text{Dist}[(d*m) / f, \text{Int}[(c + d*x)^{(m - 1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$$

### Rule 2638

$$\text{Int}[\sin[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x] / d, x] /; \text{FreeQ}\{c, d\}, x]$$

### Rule 4182

$$\text{Int}[\text{csc}[(e_.) + (\text{Complex}[0, fz\_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(-2*(c + d*x)^m*\text{ArcTanh}[E^{-(I*e)} + f*fz*x]) / (f*fz*I), x] + (-\text{Dist}[(d*m) / (f*fz*I), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 - E^{-(I*e)} + f*fz*x], x], x] + \text{Dist}[(d*m) / (f*fz*I), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + E^{-(I*e)} + f*fz*x], x], x]) /; \text{FreeQ}\{c, d, e, f, fz\}, x] \&\& \text{IGtQ}[m, 0]$$

### Rule 2531

$$\text{Int}[\text{Log}[1 + (e_.)*((F_)^{((c_.)*((a_.) + (b_.)*(x_)))})^{(n_.)}]*((f_.) + (g_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow -\text{Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))})^n]) / (b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m) / (b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{(m - 1)}*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x)))})^n], x], x] /; \text{FreeQ}\{F, a, b, c, e, f$$

, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*(a\_.) + (b\_.)\*x))\*(F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 5565

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Dist[a/b^2, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

### Rule 3322

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*sin[(e\_.) + (Complex[0, fz\_])\* (f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(-(I\*b) + 2\*a\*E^(-(I\*e) + f\*fz\*x) + I\*b\*E^(2\*(-(I\*e) + f\*fz\*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

### Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.)\*(F\_)^(v\_)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[((f + g\*x)^m\*F^u)/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[((f + g\*x)^m\*F^u)/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e + fx)^2 \cosh(c + dx) \coth(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\int (e + fx)^2 \cosh(c + dx) \coth(c + dx) dx}{a} - \frac{b \int \frac{(e + fx)^2 \cosh^2(c + dx)}{a + b \sinh(c + dx)} dx}{a} \\
&= \frac{\int (e + fx)^2 \operatorname{csch}(c + dx) dx}{a} + \frac{\int (e + fx)^2 dx}{b} - \frac{(a^2 + b^2) \int \frac{(e + fx)^2}{a + b \sinh(c + dx)} dx}{ab} \\
&= \frac{(e + fx)^3}{3bf} - \frac{2(e + fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{(2(a^2 + b^2)) \int \frac{e^{c+dx}(e + fx)^2}{-b + 2ae^{c+dx} + be^{2(c+dx)}} dx}{ab} \\
&= \frac{(e + fx)^3}{3bf} - \frac{2(e + fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2f(e + fx) \operatorname{Li}_2(-e^{c+dx})}{ad^2} + \frac{2f(e + fx)}{a} \\
&= \frac{(e + fx)^3}{3bf} - \frac{2(e + fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{\sqrt{a^2 + b^2}(e + fx)^2 \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{abd} \\
&= \frac{(e + fx)^3}{3bf} - \frac{2(e + fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{\sqrt{a^2 + b^2}(e + fx)^2 \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{abd} \\
&= \frac{(e + fx)^3}{3bf} - \frac{2(e + fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{\sqrt{a^2 + b^2}(e + fx)^2 \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{abd} \\
&= \frac{(e + fx)^3}{3bf} - \frac{2(e + fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{\sqrt{a^2 + b^2}(e + fx)^2 \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{abd} \\
&= \frac{(e + fx)^3}{3bf} - \frac{2(e + fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{\sqrt{a^2 + b^2}(e + fx)^2 \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{abd}
\end{aligned}$$

**Mathematica [A]** time = 1.87117, size = 489, normalized size = 1.06

$$\frac{\sqrt{a^2 + b^2} \left( -2df(e + fx) \operatorname{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2 + b^2} - a}\right) + 2df(e + fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right) + 2f^2 \operatorname{PolyLog}\left(3, \frac{be^{c+dx}}{\sqrt{a^2 + b^2} - a}\right) - 2f^2 \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right) \right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (x*(3*e^2 + 3*e*f*x + f^2*x^2))/(3*b) + ((e + f*x)^2*Log[1 - E^(c + d*x)] -
(e + f*x)^2*Log[1 + E^(c + d*x)] - (2*f*(d*(e + f*x)*PolyLog[2, -E^(c + d*
x)] - f*PolyLog[3, -E^(c + d*x)]))/d^2 + (2*f*(d*(e + f*x)*PolyLog[2, E^(c
+ d*x)] - f*PolyLog[3, E^(c + d*x)]))/d^2)/(a*d) + (Sqrt[a^2 + b^2]*(2*d^2*
e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*Log[1 + (b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]]) - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a
- Sqrt[a^2 + b^2]]) + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]]) + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) - 2*d*f
*(e + f*x)*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]]) + 2*d*f*(e +
f*x)*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) + 2*f^2*PolyLog[3
, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]]) - 2*f^2*PolyLog[3, -(b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2])]))/(a*b*d^3)
```

**Maple [F]** time = 0.594, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c) \coth(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [C]** time = 2.73447, size = 2483, normalized size = 5.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\frac{1}{3} \left( a^3 d^3 f^2 x^3 + 3 a^2 d^3 e f x^2 + 3 a d^3 e^2 x + 6 b f^2 \sqrt{\frac{a^2 + b^2}{b^2}} \operatorname{polylog}\left(3, \frac{a \cosh(dx+c) + a \sinh(dx+c) + (b \cosh(dx+c) + b \sinh(dx+c)) \sqrt{\frac{a^2 + b^2}{b^2}}}{b}\right) - 6 b f^2 \sqrt{\frac{a^2 + b^2}{b^2}} \operatorname{polylog}\left(3, \frac{a \cosh(dx+c) + a \sinh(dx+c) - (b \cosh(dx+c) + b \sinh(dx+c)) \sqrt{\frac{a^2 + b^2}{b^2}}}{b}\right) - 6 b f^2 \operatorname{polylog}\left(3, \cosh(dx+c) + \sinh(dx+c)\right) + 6 b f^2 \operatorname{polylog}\left(3, -\cosh(dx+c) - \sinh(dx+c)\right) - 6 (b d f^2 x + b d e f) \sqrt{\frac{a^2 + b^2}{b^2}} \operatorname{dilog}\left(\frac{a \cosh(dx+c) + a \sinh(dx+c) + (b \cosh(dx+c) + b \sinh(dx+c)) \sqrt{\frac{a^2 + b^2}{b^2}}}{b} + 1\right) + 6 (b d f^2 x + b d e f) \sqrt{\frac{a^2 + b^2}{b^2}} \operatorname{dilog}\left(\frac{a \cosh(dx+c) + a \sinh(dx+c) - (b \cosh(dx+c) + b \sinh(dx+c)) \sqrt{\frac{a^2 + b^2}{b^2}}}{b} + 1\right) + 3 (b d^2 e^2 - 2 b c d e f + b c^2 f^2) \sqrt{\frac{a^2 + b^2}{b^2}} \log\left(2 b \cosh(dx+c) + 2 b \sinh(dx+c) + 2 b \sqrt{\frac{a^2 + b^2}{b^2}} + 2 a\right) - 3 (b d^2 e^2 - 2 b c d e f + b c^2 f^2) \sqrt{\frac{a^2 + b^2}{b^2}} \log\left(2 b \cosh(dx+c) + 2 b \sinh(dx+c) - 2 b \sqrt{\frac{a^2 + b^2}{b^2}} + 2 a\right) - 3 (b d^2 f^2 x^2 + 2 b d^2 e f x + 2 b c d e f - b c^2 f^2) \sqrt{\frac{a^2 + b^2}{b^2}} \log\left(-\frac{a \cosh(dx+c) + a \sinh(dx+c) + (b \cosh(dx+c) + b \sinh(dx+c)) \sqrt{\frac{a^2 + b^2}{b^2}}}{b} + 1\right) + 3 (b d^2 f^2 x^2 + 2 b d^2 e f x + 2 b c d e f - b c^2 f^2) \sqrt{\frac{a^2 + b^2}{b^2}} \log\left(-\frac{a \cosh(dx+c) + a \sinh(dx+c) - (b \cosh(dx+c) + b \sinh(dx+c)) \sqrt{\frac{a^2 + b^2}{b^2}}}{b} + 1\right) + 6 (b d f^2 x + b d e f) \operatorname{dilog}(\cosh(dx+c) + \sinh(dx+c)) - 6 (b d f^2 x + b d e f) \operatorname{dilog}(-\cosh(dx+c) - \sinh(dx+c)) - 3 (b d^2 f^2 x^2 + 2 b d^2 e f x + b d^2 e^2) \log(\cosh(dx+c) + \sinh(dx+c) + 1) + 3 (b d^2 e^2 - 2 b c d e f + b c^2 f^2) \log(\cosh(dx+c) + \sinh(dx+c) - 1) + 3 (b d^2 f^2 x^2 + 2 b d^2 e f x + 2 b c d e f - b c^2 f^2) \log(-\cosh(dx+c) - \sinh(dx+c) + 1) \right) / (a b d^3)$$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^2 \cosh(c + dx) \coth(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((f*x+e)**2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral((e + f*x)**2*cosh(c + d*x)*coth(c + d*x)/(a + b*sinh(c + d*x)), x)
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.427 \quad \int \frac{(e+fx) \cosh(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=286

$$-\frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^2} + \frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{abd^2} - \frac{f\text{PolyLog}\left(2, -e^{c+dx}\right)}{ad^2} + \frac{f\text{PolyLog}\left(2, e^{c+dx}\right)}{ad^2}$$

[Out] (e\*x)/b + (f\*x^2)/(2\*b) - (2\*(e + f\*x)\*ArcTanh[E^(c + d\*x)]/(a\*d) - (Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/(a\*b\*d) + (Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]))/(a\*b\*d) - (f\*PolyLog[2, -E^(c + d\*x)]/(a\*d^2) + (f\*PolyLog[2, E^(c + d\*x)]/(a\*d^2) - (Sqrt[a^2 + b^2]\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))]/(a\*b\*d^2) + (Sqrt[a^2 + b^2]\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))]/(a\*b\*d^2)))/(a\*b\*d^2)

**Rubi [A]** time = 0.579288, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 11, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$ , Rules used = {5585, 5450, 3296, 2637, 4182, 2279, 2391, 5565, 3322, 2264, 2190}

$$-\frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd^2} + \frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{abd^2} - \frac{f\text{PolyLog}\left(2, -e^{c+dx}\right)}{ad^2} + \frac{f\text{PolyLog}\left(2, e^{c+dx}\right)}{ad^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Cosh[c + d\*x]\*Coth[c + d\*x])/(a + b\*Sinh[c + d\*x]), x]

[Out] (e\*x)/b + (f\*x^2)/(2\*b) - (2\*(e + f\*x)\*ArcTanh[E^(c + d\*x)]/(a\*d) - (Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/(a\*b\*d) + (Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]))/(a\*b\*d) - (f\*PolyLog[2, -E^(c + d\*x)]/(a\*d^2) + (f\*PolyLog[2, E^(c + d\*x)]/(a\*d^2) - (Sqrt[a^2 + b^2]\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))]/(a\*b\*d^2) + (Sqrt[a^2 + b^2]\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))]/(a\*b\*d^2)))/(a\*b\*d^2)

**Rule 5585**

Int[(Cosh[(c\_.) + (d\_.)\*(x\_.)]^(p\_.)\*Coth[(c\_.) + (d\_.)\*(x\_.)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.)]/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] :> Dist[1/a, Int[(e + f\*x)^m\*Cosh[c + d\*x]^p\*Coth[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(p + 1)\*Coth[c + d\*x]^(n - 1))/(a + b\*Sinh

$[c + d*x]), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5450

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*Coth[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[(c + d\*x)^m\*Cosh[a + b\*x]^n\*Coth[a + b\*x]^(p - 2), x] + Int[(c + d\*x)^m\*Cosh[a + b\*x]^(n - 2)\*Coth[a + b\*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[(c + d\*x)^m\*cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rule 4182

Int[csc[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x))]/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 5565

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Dist[a/b^2, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)

```
) * Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m * Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

### Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m * E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a * E^(-(I*e) + f*fz*x) + I*b * E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m * F^u)/(b - q + 2*c * F^u), x], x] - Dist[(2*c)/q, Int[
((f + g*x)^m * F^u)/(b + q + 2*c * F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m * Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n * Log[F]), x] - Di
st[(d*m)/(b*f*g*n * Log[F]), Int[(c + d*x)^(m - 1) * Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx) \cosh(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx) \cosh(c+dx) \coth(c+dx) dx}{a} - \frac{b \int \frac{(e+fx) \cosh^2(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= \frac{\int (e+fx) \operatorname{csch}(c+dx) dx}{a} + \frac{\int (e+fx) dx}{b} - \frac{(a^2+b^2) \int \frac{e+fx}{a+b \sinh(c+dx)} dx}{ab} \\
&= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{2(e+fx) \tanh^{-1}(e^{c+dx})}{ad} - \frac{(2(a^2+b^2)) \int \frac{e^{c+dx}(e+fx)}{-b+2ae^{c+dx}+be^{2(c+dx)}} dx}{ab} \\
&= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{2(e+fx) \tanh^{-1}(e^{c+dx})}{ad} - \frac{(2\sqrt{a^2+b^2}) \int \frac{e^{c+dx}(e+fx)}{2a-2\sqrt{a^2+b^2}+2be^{c+dx}} dx}{a} \\
&= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{2(e+fx) \tanh^{-1}(e^{c+dx})}{ad} - \frac{\sqrt{a^2+b^2}(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd} \\
&= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{2(e+fx) \tanh^{-1}(e^{c+dx})}{ad} - \frac{\sqrt{a^2+b^2}(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd} \\
&= \frac{ex}{b} + \frac{fx^2}{2b} - \frac{2(e+fx) \tanh^{-1}(e^{c+dx})}{ad} - \frac{\sqrt{a^2+b^2}(e+fx) \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{abd}
\end{aligned}$$

**Mathematica [A]** time = 1.83532, size = 339, normalized size = 1.19

$$2\sqrt{a^2+b^2} \left( -f \operatorname{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) + f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) + 2de \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) - f(c+dx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((e+f\*x)\*Cosh[c+d\*x]\*Coth[c+d\*x])/(a+b\*Sinh[c+d\*x]),x]

[Out]  $(-(a*(c+d*x)*(c*f-d*(2*e+f*x))) + 2*b*d*e*\operatorname{Log}[\operatorname{Tanh}[(c+d*x)/2]] - 2*b*c*f*\operatorname{Log}[\operatorname{Tanh}[(c+d*x)/2]] + 2*b*f*((c+d*x)*(\operatorname{Log}[1-E^{-(c+d*x)}] - \operatorname{Log}[1+E^{-(c+d*x)}]) + \operatorname{PolyLog}[2, -E^{-(c+d*x)}] - \operatorname{PolyLog}[2, E^{-(c+d*x)}]) + 2*\operatorname{Sqrt}[a^2+b^2]*(2*d*e*\operatorname{ArcTanh}[(a+b*E^{(c+d*x)})/\operatorname{Sqrt}[a^2+b^2]] - 2*c*f*\operatorname{ArcTanh}[(a+b*E^{(c+d*x)})/\operatorname{Sqrt}[a^2+b^2]] - f*(c+d*x)*\operatorname{Log}[1+(b*E^{(c+d*x)})/(a-\operatorname{Sqrt}[a^2+b^2])] + f*(c+d*x)*\operatorname{Log}[1+(b*E^{(c+d*x)})/(a+\operatorname{Sqrt}[a^2+b^2])] - f*\operatorname{PolyLog}[2, (b*E^{(c+d*x)})/(-a+\operatorname{Sqrt}[a^2+b^2])] + f*\operatorname{PolyLog}[2, -(b*E^{(c+d*x)})/(a+\operatorname{Sqrt}[a^2+b^2])]))/(2*a*b*d^2)$

**Maple [B]** time = 0.187, size = 970, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)*\cosh(d*x+c)*\coth(d*x+c)/(a+b*\sinh(d*x+c)),x)$

[Out]  $\frac{1}{2}f*x^2/b+e*x/b-1/d*f*b/a/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*x-1/d^2*f*b/a/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*c+1/d*f*b/a/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*x+1/d^2*f*b/a/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*c+1/d^2*f*b/a/(a^2+b^2)^{(1/2)}*\text{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))-1/d/a*\ln(\exp(d*x+c)+1)*f*x-1/d^2*f*b/a/(a^2+b^2)^{(1/2)}*\text{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))-1/d^2/a*f*c*\ln(\exp(d*x+c)-1)-2*a/b/d^2*f*c/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))-2/d^2*f*c*b/a/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))-a/b/d*f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*x-a/b/d^2*f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*c+a/b/d*f/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*x+a/b/d^2*f/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*c+1/d/a*e*\ln(\exp(d*x+c)-1)-1/d/a*e*\ln(\exp(d*x+c)+1)-a/b/d^2*f/(a^2+b^2)^{(1/2)}*\text{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))+a/b/d^2*f/(a^2+b^2)^{(1/2)}*\text{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))+2*a/b/d*e/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))+2/d*e*b/a/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))-1/d^2*f/a*\text{dilog}(\exp(d*x+c))-1/d^2*f/a*\text{dilog}(\exp(d*x+c)+1)$

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x+e)*\cosh(d*x+c)*\coth(d*x+c)/(a+b*\sinh(d*x+c)),x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

---

**Fricas [B]** time = 2.72461, size = 1544, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\frac{1}{2} * (a * d^2 * f * x^2 + 2 * a * d^2 * e * x - 2 * b * f * \sqrt{(a^2 + b^2) / b^2} * \operatorname{dilog}((a * \cosh(d * x + c) + a * \sinh(d * x + c) + (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b + 1) + 2 * b * f * \sqrt{(a^2 + b^2) / b^2} * \operatorname{dilog}((a * \cosh(d * x + c) + a * \sinh(d * x + c) - (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b + 1) + 2 * b * f * \operatorname{dilog}(\cosh(d * x + c) + \sinh(d * x + c)) - 2 * b * f * \operatorname{dilog}(-\cosh(d * x + c) - \sinh(d * x + c)) + 2 * (b * d * e - b * c * f) * \sqrt{(a^2 + b^2) / b^2} * \log(2 * b * \cosh(d * x + c) + 2 * b * \sinh(d * x + c) + 2 * b * \sqrt{(a^2 + b^2) / b^2} + 2 * a) - 2 * (b * d * e - b * c * f) * \sqrt{(a^2 + b^2) / b^2} * \log(2 * b * \cosh(d * x + c) + 2 * b * \sinh(d * x + c) - 2 * b * \sqrt{(a^2 + b^2) / b^2} + 2 * a) - 2 * (b * d * f * x + b * c * f) * \sqrt{(a^2 + b^2) / b^2} * \log(-(a * \cosh(d * x + c) + a * \sinh(d * x + c) + (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b) + 2 * (b * d * f * x + b * c * f) * \sqrt{(a^2 + b^2) / b^2} * \log(-(a * \cosh(d * x + c) + a * \sinh(d * x + c) - (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b) - 2 * (b * d * f * x + b * d * e) * \log(\cosh(d * x + c) + \sinh(d * x + c) + 1) + 2 * (b * d * e - b * c * f) * \log(\cosh(d * x + c) + \sinh(d * x + c) - 1) + 2 * (b * d * f * x + b * c * f) * \log(-\cosh(d * x + c) - \sinh(d * x + c) + 1)) / (a * b * d^2)$$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \cosh(c + dx) \coth(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*cosh(c + d\*x)\*coth(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="g  
iac")
```

```
[Out] Timed out
```



$$3.428 \quad \int \frac{\cosh(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=71

$$\frac{2\sqrt{a^2+b^2} \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{abd} - \frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{x}{b}$$

[Out] x/b - ArcTanh[Cosh[c + d\*x]]/(a\*d) + (2\*Sqrt[a^2 + b^2]\*ArcTanh[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[a^2 + b^2]])/(a\*b\*d)

**Rubi [A]** time = 0.212942, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$ , Rules used = {2889, 3058, 2660, 618, 204, 3770}

$$\frac{2\sqrt{a^2+b^2} \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{abd} - \frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{x}{b}$$

Antiderivative was successfully verified.

[In] Int[(Cosh[c + d\*x]\*Coth[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] x/b - ArcTanh[Cosh[c + d\*x]]/(a\*d) + (2\*Sqrt[a^2 + b^2]\*ArcTanh[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[a^2 + b^2]])/(a\*b\*d)

### Rule 2889

Int[cos[(e\_.) + (f\_.)\*(x\_.)]^2\*((d\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]^(n\_))\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]^(m\_)), x\_Symbol] :> Int[(d\*Sin[e + f\*x])^n\*(a + b\*Sin[e + f\*x])^m\*(1 - Sin[e + f\*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2\*m, 2\*n])

### Rule 3058

Int[((A\_.) + (C\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]^2)/(((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]\*(c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Simp[(C\*x)/(b\*d), x] + (Dist[(A\*b^2 + a^2\*C)/(b\*(b\*c - a\*d)), Int[1/(a + b\*Sin[e + f\*x]), x], x] - Dist[(c^2\*C + A\*d^2)/(d\*(b\*c - a\*d)), Int[1/(c + d\*Sin[e + f\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b\*c - a\*d, 0] &&

NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]

### Rule 2660

Int[((a\_) + (b\_)\*sin[(c\_) + (d\_)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + 2\*b\*e\*x + a\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

### Rule 618

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 3770

Int[csc[(c\_) + (d\_)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{\cosh(c + dx) \coth(c + dx)}{a + b \sinh(c + dx)} dx &= \int \frac{\operatorname{csch}(c + dx) (1 + \sinh^2(c + dx))}{a + b \sinh(c + dx)} dx \\
 &= \frac{x}{b} + \frac{\int \operatorname{csch}(c + dx) dx}{a} - \frac{(a^2 + b^2) \int \frac{1}{a + b \sinh(c + dx)} dx}{ab} \\
 &= \frac{x}{b} - \frac{\tanh^{-1}(\cosh(c + dx))}{ad} + \frac{(2i(a^2 + b^2)) \operatorname{Subst}\left(\int \frac{1}{a - 2ibx + ax^2} dx, x, \tan\left(\frac{1}{2}(ic + idx)\right)\right)}{abd} \\
 &= \frac{x}{b} - \frac{\tanh^{-1}(\cosh(c + dx))}{ad} - \frac{(4i(a^2 + b^2)) \operatorname{Subst}\left(\int \frac{1}{-4(a^2 + b^2) - x^2} dx, x, -2ib + 2a \tan\left(\frac{1}{2}(ic + idx)\right)\right)}{abd} \\
 &= \frac{x}{b} - \frac{\tanh^{-1}(\cosh(c + dx))}{ad} + \frac{2\sqrt{a^2 + b^2} \tanh^{-1}\left(\frac{b - a \tanh\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a^2 + b^2}}\right)}{abd}
 \end{aligned}$$

**Mathematica [A]** time = 0.146125, size = 80, normalized size = 1.13

$$\frac{2\sqrt{-a^2 - b^2} \tan^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right) + a(c+dx) + b \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{abd}$$

Antiderivative was successfully verified.

[In] Integrate[(Cosh[c + d\*x]\*Coth[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (a\*(c + d\*x) + 2\*Sqrt[-a^2 - b^2]\*ArcTan[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[-a^2 - b^2]]) + b\*Log[Tanh[(c + d\*x)/2]]/(a\*b\*d)

**Maple [B]** time = 0.002, size = 150, normalized size = 2.1

$$\frac{1}{bd} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) - \frac{1}{bd} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) + \frac{1}{da} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 2 \frac{a}{bd\sqrt{a^2 + b^2}} \operatorname{Arctanh}\left(\frac{1}{2} \frac{2a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + b}{a + b \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] 1/d/b\*ln(tanh(1/2\*d\*x+1/2\*c)+1)-1/d/b\*ln(tanh(1/2\*d\*x+1/2\*c)-1)+1/d/a\*ln(tanh(1/2\*d\*x+1/2\*c))-2/d\*a/b/(a^2+b^2)^(1/2)\*arctanh(1/2\*(2\*a\*tanh(1/2\*d\*x+1/2\*c)-2\*b)/(a^2+b^2)^(1/2))-2/d\*b/a/(a^2+b^2)^(1/2)\*arctanh(1/2\*(2\*a\*tanh(1/2\*d\*x+1/2\*c)-2\*b)/(a^2+b^2)^(1/2))

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.58144, size = 559, normalized size = 7.87

$$\frac{adx - b \log(\cosh(dx + c) + \sinh(dx + c) + 1) + b \log(\cosh(dx + c) + \sinh(dx + c) - 1) + \sqrt{a^2 + b^2} \log\left(\frac{b^2 \cosh(dx+c)^2 + b^2}{abd}\right)}{abd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] (a\*d\*x - b\*log(cosh(d\*x + c) + sinh(d\*x + c) + 1) + b\*log(cosh(d\*x + c) + sinh(d\*x + c) - 1) + sqrt(a^2 + b^2)\*log((b^2\*cosh(d\*x + c)^2 + b^2\*sinh(d\*x + c)^2 + 2\*a\*b\*cosh(d\*x + c) + 2\*a^2 + b^2 + 2\*(b^2\*cosh(d\*x + c) + a\*b)\*sinh(d\*x + c) + 2\*sqrt(a^2 + b^2)\*(b\*cosh(d\*x + c) + b\*sinh(d\*x + c) + a))/(b\*cosh(d\*x + c)^2 + b\*sinh(d\*x + c)^2 + 2\*a\*cosh(d\*x + c) + 2\*(b\*cosh(d\*x + c) + a)\*sinh(d\*x + c) - b)))/(a\*b\*d)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c + dx) \coth(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(cosh(c + d\*x)\*coth(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

**Giac [B]** time = 2.39724, size = 188, normalized size = 2.65

$$\frac{\frac{dx}{b} - \frac{(a^2 e^c + b^2 e^c) e^{(-c)} \log\left(\frac{2 b e^{(dx+2c)} + 2 a e^c - 2 \sqrt{a^2 + b^2} e^c}{2 b e^{(dx+2c)} + 2 a e^c + 2 \sqrt{a^2 + b^2} e^c}\right)}{\sqrt{a^2 + b^2} a b} - \frac{\log(e^{(dx+c)} + 1)}{a} + \frac{\log(|e^{(dx+c)} - 1|)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

```
[Out] (d*x/b - (a^2*e^c + b^2*e^c)*e^(-c)*log(abs(2*b*e^(d*x + 2*c) + 2*a*e^c - 2
*sqrt(a^2 + b^2)*e^c)/abs(2*b*e^(d*x + 2*c) + 2*a*e^c + 2*sqrt(a^2 + b^2)*e
^c))/(sqrt(a^2 + b^2)*a*b) - log(e^(d*x + c) + 1)/a + log(abs(e^(d*x + c) -
1))/a)/d
```

$$3.429 \quad \int \frac{\cosh(c+dx) \coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable} \left( \frac{\cosh(c+dx) \coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[(Cosh[c + d\*x]\*Coth[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.059309, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cosh(c+dx) \coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Cosh[c + d\*x]\*Coth[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int] [(Cosh[c + d\*x]\*Coth[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\cosh(c+dx) \coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\cosh(c+dx) \coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [A]** time = 57.9944, size = 0, normalized size = 0.

$$\int \frac{\cosh(c+dx) \coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Cosh[c + d\*x]\*Coth[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Integrate[(Cosh[c + d\*x]\*Coth[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Maple [A]** time = 0.826, size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c) \coth(dx + c)}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)\*coth(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x)

[Out] int(cosh(d\*x+c)\*coth(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-2(a^2e^c + b^2e^c) \int -\frac{e^{(dx)}}{ab^2fx + ab^2e - (ab^2fxe^{(2c)} + ab^2ee^{(2c)})e^{(2dx)} - 2(a^2bfxe^c + a^2bee^c)e^{(dx)}} dx + \frac{\log(fx + e)}{bf} + \int \frac{1}{afx + ae + (bfx + be) \sinh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*coth(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x, algorithm="maxima")

[Out] -2\*(a^2\*e^c + b^2\*e^c)\*integrate(-e^(d\*x)/(a\*b^2\*f\*x + a\*b^2\*e - (a\*b^2\*f\*x\*e^(2\*c) + a\*b^2\*e\*e^(2\*c))\*e^(2\*d\*x) - 2\*(a^2\*b\*f\*x\*e^c + a^2\*b\*e\*e^c)\*e^(d\*x)), x) + log(f\*x + e)/(b\*f) + integrate(1/(a\*f\*x + a\*e + (a\*f\*x\*e^c + a\*e\*e^c)\*e^(d\*x)), x) + integrate(-1/(a\*f\*x + a\*e - (a\*f\*x\*e^c + a\*e\*e^c)\*e^(d\*x)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx + c) \coth(dx + c)}{afx + ae + (bfx + be) \sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] integral(cosh(d*x + c)*coth(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
+ c)), x)
```

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c + dx) \coth(c + dx)}{(a + b \sinh(c + dx))(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(cosh(c + d*x)*coth(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] Timed out
```



$$3.430 \quad \int \frac{(e+fx)^3 \cosh^2(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=656

$$\frac{6f^2(a^2+b^2)(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^3} + \frac{6f^2(a^2+b^2)(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ab^2d^3} - \frac{3f(a^2+b^2)(e+fx)}{a}$$

```
[Out] -(e + f*x)^4/(4*a*f) + ((a^2 + b^2)*(e + f*x)^4)/(4*a*b^2*f) - (6*f^3*Cosh[
c + d*x])/(b*d^4) - (3*f*(e + f*x)^2*Cosh[c + d*x])/(b*d^2) - ((a^2 + b^2)*
(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*b^2*d) - ((a
^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*b
^2*d) + ((e + f*x)^3*Log[1 - E^(2*(c + d*x))])/(a*d) - (3*(a^2 + b^2)*f*(e +
f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b^2*d^2) -
(3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))])/(a*b^2*d^2) + (3*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*x))])/(2*a*d
^2) + (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(a*b^2*d^3) + (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E
^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b^2*d^3) - (3*f^2*(e + f*x)*PolyLog[
3, E^(2*(c + d*x))])/(2*a*d^3) - (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]))])/(a*b^2*d^4) - (6*(a^2 + b^2)*f^3*PolyLog[4,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*b^2*d^4) + (3*f^3*PolyLog[4,
E^(2*(c + d*x))])/(4*a*d^4) + (6*f^2*(e + f*x)*Sinh[c + d*x])/(b*d^3) + ((e
+ f*x)^3*Sinh[c + d*x])/(b*d)
```

**Rubi [A]** time = 1.22967, antiderivative size = 656, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 17, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5585, 5450, 5446, 3311, 32, 2635, 8, 3716, 2190, 2531, 6609, 2282, 6589, 5565, 3296, 2638, 5561}

$$\frac{6f^2(a^2+b^2)(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^3} + \frac{6f^2(a^2+b^2)(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ab^2d^3} - \frac{3f(a^2+b^2)(e+fx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(e + f*x)^4/(4*a*f) + ((a^2 + b^2)*(e + f*x)^4)/(4*a*b^2*f) - (6*f^3*Cosh[
c + d*x])/(b*d^4) - (3*f*(e + f*x)^2*Cosh[c + d*x])/(b*d^2) - ((a^2 + b^2)*
(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*b^2*d) - ((a
^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*b^
```

$$\begin{aligned}
& 2*d) + ((e + f*x)^3*\text{Log}[1 - E^{(2*(c + d*x))}]/(a*d) - (3*(a^2 + b^2)*f*(e + \\
& f*x)^2*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))]/(a*b^2*d^2) - \\
& (3*(a^2 + b^2)*f*(e + f*x)^2*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + \\
& b^2]))]/(a*b^2*d^2) + (3*f*(e + f*x)^2*\text{PolyLog}[2, E^{(2*(c + d*x))}]/(2*a*d \\
& ^2) + (6*(a^2 + b^2)*f^2*(e + f*x)*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a \\
& ^2 + b^2]))]/(a*b^2*d^3) + (6*(a^2 + b^2)*f^2*(e + f*x)*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))]/(a*b^2*d^3) - (3*f^2*(e + f*x)*\text{PolyLog}[3, E^{(2*(c + d*x))}]/(2*a*d^3) - (6*(a^2 + b^2)*f^3*\text{PolyLog}[4, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))]/(a*b^2*d^4) - (6*(a^2 + b^2)*f^3*\text{PolyLog}[4, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))]/(a*b^2*d^4) + (3*f^3*\text{PolyLog}[4, E^{(2*(c + d*x))}]/(4*a*d^4) + (6*f^2*(e + f*x)*\text{Sinh}[c + d*x])/(b*d^3) + ((e + f*x)^3*\text{Sinh}[c + d*x])/(b*d)
\end{aligned}$$

### Rule 5585

$$\begin{aligned}
& \text{Int}[(\text{Cosh}[(c_.) + (d_.)*(x_.)]^{(p_.)}*\text{Coth}[(c_.) + (d_.)*(x_.)]^{(n_.)}*((e_.) + \\
& (f_.)*(x_.))^{(m_.)})/((a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_.)]), x\_Symbol] \text{:>} \text{Dist}[1/a, \text{Int}[(e + f*x)^m*\text{Cosh}[c + d*x]^p*\text{Coth}[c + d*x]^n, x], x] - \text{Dist}[b/a, \text{Int}[(e + f*x)^m*\text{Cosh}[c + d*x]^{(p + 1)}*\text{Coth}[c + d*x]^{(n - 1)}/(a + b*\text{Sinh}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]
\end{aligned}$$

### Rule 5450

$$\begin{aligned}
& \text{Int}[\text{Cosh}[(a_.) + (b_.)*(x_.)]^{(n_.)}*\text{Coth}[(a_.) + (b_.)*(x_.)]^{(p_.)}*((c_.) + \\
& (d_.)*(x_.))^{(m_.)}, x\_Symbol] \text{:>} \text{Int}[(c + d*x)^m*\text{Cosh}[a + b*x]^n*\text{Coth}[a + b*x]^{(p - 2)}, x] + \text{Int}[(c + d*x)^m*\text{Cosh}[a + b*x]^{(n - 2)}*\text{Coth}[a + b*x]^p, x] /; \text{FreeQ}\{a, b, c, d, m\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]
\end{aligned}$$

### Rule 5446

$$\begin{aligned}
& \text{Int}[\text{Cosh}[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}*\text{Sinh}[(a_.) + (b_.)* \\
& (x_.)]^{(n_.)}, x\_Symbol] \text{:>} \text{Simp}[(c + d*x)^m*\text{Sinh}[a + b*x]^{(n + 1)}/(b*(n + 1)), x] - \text{Dist}[(d*m)/(b*(n + 1)), \text{Int}[(c + d*x)^{(m - 1)}*\text{Sinh}[a + b*x]^{(n + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x\} \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[n, -1]
\end{aligned}$$

### Rule 3311

$$\begin{aligned}
& \text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*((b_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^{(n_.)}, x\_Symbol] \text{:>} \text{Simp}[(d*m*(c + d*x)^{(m - 1)}*(b*\text{Sin}[e + f*x])^n)/(f^2*n^2), x] + (\text{Dist}[(b^2*(n - 1))/n, \text{Int}[(c + d*x)^m*(b*\text{Sin}[e + f*x])^{(n - 2)}, x], x] - \text{Dist}[(d^2*m*(m - 1))/(f^2*n^2), \text{Int}[(c + d*x)^{(m - 2)}*(b*\text{Sin}[e + f*x])^n, x], x] - \text{Simp}[(b*(c + d*x)^m*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(n - 1)})/(f*n), x]) /; \text{FreeQ}\{b, c, d, e, f\}, x\} \&\& \text{GtQ}[n, 1] \&\& \text{GtQ}[m, 1]
\end{aligned}$$

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*e) + f*fz*x))/E^(2*I*k*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && GtQ[m, 0]
```

$(m - 1) \cdot \text{PolyLog}[n + 1, d \cdot (F^{c \cdot (a + b \cdot x)})^p], x], x] /;$  FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 5565

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Dist[a/b^2, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(e + fx)^3 \cosh^2(c + dx) \coth(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\int (e + fx)^3 \cosh^2(c + dx) \coth(c + dx) dx}{a} - \frac{b \int \frac{(e + fx)^3 \cosh^3(c + dx)}{a + b \sinh(c + dx)} dx}{a} \\
&= \frac{\int (e + fx)^3 \coth(c + dx) dx}{a} + \frac{\int (e + fx)^3 \cosh(c + dx) dx}{b} - \frac{(a^2 + b^2) \int \frac{e^{2(c + dx)}(e + fx)^3}{1 - e^{2(c + dx)}} dx}{a} \\
&= -\frac{(e + fx)^4}{4af} + \frac{(a^2 + b^2)(e + fx)^4}{4ab^2f} + \frac{(e + fx)^3 \sinh(c + dx)}{bd} - \frac{2 \int \frac{e^{2(c + dx)}(e + fx)^3}{1 - e^{2(c + dx)}} dx}{a} \\
&= -\frac{(e + fx)^4}{4af} + \frac{(a^2 + b^2)(e + fx)^4}{4ab^2f} - \frac{3f(e + fx)^2 \cosh(c + dx)}{bd^2} - \frac{(a^2 + b^2) \int \frac{e^{2(c + dx)}(e + fx)^3}{1 - e^{2(c + dx)}} dx}{a} \\
&= -\frac{(e + fx)^4}{4af} + \frac{(a^2 + b^2)(e + fx)^4}{4ab^2f} - \frac{3f(e + fx)^2 \cosh(c + dx)}{bd^2} - \frac{(a^2 + b^2) \int \frac{e^{2(c + dx)}(e + fx)^3}{1 - e^{2(c + dx)}} dx}{a} \\
&= -\frac{(e + fx)^4}{4af} + \frac{(a^2 + b^2)(e + fx)^4}{4ab^2f} - \frac{6f^3 \cosh(c + dx)}{bd^4} - \frac{3f(e + fx)^2 \cosh(c + dx)}{bd^2} \\
&= -\frac{(e + fx)^4}{4af} + \frac{(a^2 + b^2)(e + fx)^4}{4ab^2f} - \frac{6f^3 \cosh(c + dx)}{bd^4} - \frac{3f(e + fx)^2 \cosh(c + dx)}{bd^2} \\
&= -\frac{(e + fx)^4}{4af} + \frac{(a^2 + b^2)(e + fx)^4}{4ab^2f} - \frac{6f^3 \cosh(c + dx)}{bd^4} - \frac{3f(e + fx)^2 \cosh(c + dx)}{bd^2}
\end{aligned}$$

**Mathematica [B]** time = 18.4519, size = 2574, normalized size = 3.92

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^3\*Cosh[c + d\*x]^2\*Coth[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] ((a^2 + b^2)\*E^(2\*c)\*x\*(4\*e^3 + 6\*e^2\*f\*x + 4\*e\*f^2\*x^2 + f^3\*x^3))/(2\*a\*b^2\*(-1 + E^(2\*c))) - (E^(2\*c))\*((e + f\*x)^4/(E^(2\*c)\*f) - (2\*(1 - E^(-2\*c)))\*(e + f\*x)^3\*Log[1 - E^(-c - d\*x)])/d - (2\*(1 - E^(-2\*c)))\*(e + f\*x)^3\*Log[1 + E^(-c - d\*x)])/d + (6\*(-1 + E^(2\*c))\*f\*(d^2\*(e + f\*x)^2\*PolyLog[2, -E^(-c - d\*x)] + 2\*f\*(d\*(e + f\*x)\*PolyLog[3, -E^(-c - d\*x)] + f\*PolyLog[4, -E^(-c - d\*x)])))/(d^4\*E^(2\*c)) + (6\*(-1 + E^(2\*c))\*f\*(d^2\*(e + f\*x)^2\*PolyLog[2,

$$\begin{aligned}
& E^{-c-dx}] + 2f*(d*(e+fx)*\text{PolyLog}[3, E^{-c-dx}] + f*\text{PolyLog}[4, E^{-c-dx}]))/(d^4E^{(2c)})))/(2a*(-1+E^{(2c)})) - ((a^2+b^2)*(2a*\text{Sqrt}[a^2+b^2]*d^3e^3*\text{ArcTan}[(a+bE^{(c+dx)})/\text{Sqrt}[-a^2-b^2]] + 2a*\text{Sqrt}[-a^2-b^2]*d^3e^3*\text{ArcTanh}[(a+bE^{(c+dx)})/\text{Sqrt}[a^2+b^2]] + \text{Sqrt}[-(a^2+b^2)^2]*d^3e^3*\text{Log}[2aE^{(c+dx)}+b*(-1+E^{(2(c+dx))})]) + 3*\text{Sqrt}[-(a^2+b^2)^2]*d^3e^2*f*x*\text{Log}[1+(bE^{(2c+dx)})/(aE^c-\text{Sqrt}[(a^2+b^2)*E^{(2c)}])]) + 3*\text{Sqrt}[-(a^2+b^2)^2]*d^3e*f^2*x^2*\text{Log}[1+(bE^{(2c+dx)})/(aE^c-\text{Sqrt}[(a^2+b^2)*E^{(2c)}])]) + \text{Sqrt}[-(a^2+b^2)^2]*d^3*f^3*x^3*\text{Log}[1+(bE^{(2c+dx)})/(aE^c-\text{Sqrt}[(a^2+b^2)*E^{(2c)}])]) + 3*\text{Sqrt}[-(a^2+b^2)^2]*d^3e^2*f*x*\text{Log}[1+(bE^{(2c+dx)})/(aE^c+\text{Sqrt}[(a^2+b^2)*E^{(2c)}])]) + 3*\text{Sqrt}[-(a^2+b^2)^2]*d^3e*f^2*x^2*\text{Log}[1+(bE^{(2c+dx)})/(aE^c+\text{Sqrt}[(a^2+b^2)*E^{(2c)}])]) + \text{Sqrt}[-(a^2+b^2)^2]*d^3*f^3*x^3*\text{Log}[1+(bE^{(2c+dx)})/(aE^c+\text{Sqrt}[(a^2+b^2)*E^{(2c)}])]) + 3*\text{Sqrt}[-(a^2+b^2)^2]*d^2*f*(e+fx)^2*\text{PolyLog}[2, -((bE^{(2c+dx)})/(aE^c-\text{Sqrt}[(a^2+b^2)*E^{(2c)}])])]) + 3*\text{Sqrt}[-(a^2+b^2)^2]*d^2*f*(e+fx)^2*\text{PolyLog}[2, -((bE^{(2c+dx)})/(aE^c+\text{Sqrt}[(a^2+b^2)*E^{(2c)}])])]) - 6*\text{Sqrt}[-(a^2+b^2)^2]*d*e*f^2*\text{PolyLog}[3, -((bE^{(2c+dx)})/(aE^c-\text{Sqrt}[(a^2+b^2)*E^{(2c)}])])]) - 6*\text{Sqrt}[-(a^2+b^2)^2]*d*f^3*x*\text{PolyLog}[3, -((bE^{(2c+dx)})/(aE^c-\text{Sqrt}[(a^2+b^2)*E^{(2c)}])])]) - 6*\text{Sqrt}[-(a^2+b^2)^2]*d*e*f^2*\text{PolyLog}[3, -((bE^{(2c+dx)})/(aE^c+\text{Sqrt}[(a^2+b^2)*E^{(2c)}])])]) - 6*\text{Sqrt}[-(a^2+b^2)^2]*d*f^3*x*\text{PolyLog}[3, -((bE^{(2c+dx)})/(aE^c+\text{Sqrt}[(a^2+b^2)*E^{(2c)}])])]) + 6*\text{Sqrt}[-(a^2+b^2)^2]*f^3*\text{PolyLog}[4, -((bE^{(2c+dx)})/(aE^c-\text{Sqrt}[(a^2+b^2)*E^{(2c)}])])]) + 6*\text{Sqrt}[-(a^2+b^2)^2]*f^3*\text{PolyLog}[4, -((bE^{(2c+dx)})/(aE^c+\text{Sqrt}[(a^2+b^2)*E^{(2c)}])])])]) / (a*b^2*\text{Sqrt}[-(a^2+b^2)^2]*d^4) + \text{Csch}[c]*(\text{Cosh}[c+dx]/(8*b^2*d^4) - \text{Sinh}[c+dx]/(8*b^2*d^4))*(-4*a*d^4*e^3*x*\text{Cosh}[dx] - 6*a*d^4*e^2*f*x^2*\text{Cosh}[dx] - 4*a*d^4*e*f^2*x^3*\text{Cosh}[dx] - a*d^4*f^3*x^4*\text{Cosh}[dx] - 4*a*d^4*e^3*x*\text{Cosh}[2c+dx] - 6*a*d^4*e^2*f*x^2*\text{Cosh}[2c+dx] - 4*a*d^4*e*f^2*x^3*\text{Cosh}[2c+dx] - a*d^4*f^3*x^4*\text{Cosh}[2c+dx] - 2*b*d^3*e^3*\text{Cosh}[c+2dx] + 6*b*d^2*e^2*f*\text{Cosh}[c+2dx] - 12*b*d*e*f^2*\text{Cosh}[c+2dx] + 12*b*f^3*\text{Cosh}[c+2dx] - 6*b*d^3*e^2*f*x*\text{Cosh}[c+2dx] + 12*b*d^2*e*f^2*x*\text{Cosh}[c+2dx] - 12*b*d*f^3*x*\text{Cosh}[c+2dx] - 6*b*d^3*e*f^2*x^2*\text{Cosh}[c+2dx] + 6*b*d^2*f^3*x^2*\text{Cosh}[c+2dx] - 2*b*d^3*f^3*x^3*\text{Cosh}[c+2dx] + 2*b*d^3*e^3*\text{Cosh}[3c+2dx] - 6*b*d^2*e^2*f*\text{Cosh}[3c+2dx] + 12*b*d*e*f^2*\text{Cosh}[3c+2dx] - 12*b*f^3*\text{Cosh}[3c+2dx] + 6*b*d^3*e^2*f*x*\text{Cosh}[3c+2dx] - 12*b*d^2*e*f^2*x*\text{Cosh}[3c+2dx] + 12*b*d*f^3*x*\text{Cosh}[3c+2dx] + 6*b*d^3*e*f^2*x^2*\text{Cosh}[3c+2dx] - 6*b*d^2*f^3*x^2*\text{Cosh}[3c+2dx] + 2*b*d^3*f^3*x^3*\text{Cosh}[3c+2dx] - 4*b*d^3*e^3*\text{Sinh}[c] - 12*b*d^2*e^2*f*\text{Sinh}[c] - 24*b*d*e*f^2*\text{Sinh}[c] - 24*b*f^3*\text{Sinh}[c] - 12*b*d^3*e^2*f*x*\text{Sinh}[c] - 24*b*d^2*e*f^2*x*\text{Sinh}[c] - 24*b*d*f^3*x*\text{Sinh}[c] - 12*b*d^3*e*f^2*x^2*\text{Sinh}[c] - 12*b*d^2*f^3*x^2*\text{Sinh}[c] - 4*b*d^3*f^3*x^3*\text{Sinh}[c] - 4*a*d^4*e^3*x*\text{Sinh}[dx] - 6*a*d^4*e^2*f*x^2*\text{Sinh}[dx] - 4*a*d^4*e*f^2*x^3*\text{Sinh}[dx] - a*d^4*f^3*x^4*\text{Sinh}[dx] - 4*a*d^4*e^3*x*\text{Sinh}[2c+dx] - 6*a*d^4*e^2*f*x^2*\text{Sinh}[2c+dx] - 4*a*d^4*e*f^2*x^3*\text{Sinh}[2c+dx] - a*d^4*f^3*x^4*\text{Sinh}[2c+dx] - 2*b*d^3*e^3*\text{Sinh}[c+2dx] + 6*b*d^2*e^2*f*\text{Sinh}[c+2dx] -
\end{aligned}$$

$12*b*d*e*f^2*\text{Sinh}[c + 2*d*x] + 12*b*f^3*\text{Sinh}[c + 2*d*x] - 6*b*d^3*e^2*f*x*\text{Sinh}[c + 2*d*x] + 12*b*d^2*e*f^2*x*\text{Sinh}[c + 2*d*x] - 12*b*d*f^3*x*\text{Sinh}[c + 2*d*x] - 6*b*d^3*e*f^2*x^2*\text{Sinh}[c + 2*d*x] + 6*b*d^2*f^3*x^2*\text{Sinh}[c + 2*d*x] - 2*b*d^3*f^3*x^3*\text{Sinh}[c + 2*d*x] + 2*b*d^3*e^3*\text{Sinh}[3*c + 2*d*x] - 6*b*d^2*e^2*f*\text{Sinh}[3*c + 2*d*x] + 12*b*d*e*f^2*\text{Sinh}[3*c + 2*d*x] - 12*b*f^3*\text{Sinh}[3*c + 2*d*x] + 6*b*d^3*e^2*f*x*\text{Sinh}[3*c + 2*d*x] - 12*b*d^2*e*f^2*x*\text{Sinh}[3*c + 2*d*x] + 12*b*d*f^3*x*\text{Sinh}[3*c + 2*d*x] + 6*b*d^3*e*f^2*x^2*\text{Sinh}[3*c + 2*d*x] - 6*b*d^2*f^3*x^2*\text{Sinh}[3*c + 2*d*x] + 2*b*d^3*f^3*x^3*\text{Sinh}[3*c + 2*d*x])$

**Maple [F]** time = 1.22, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 (\cosh(dx + c))^2 \coth(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*cosh(d\*x+c)^2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*cosh(d\*x+c)^2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)^2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-1/2*e^3*(2*(d*x + c)*a/(b^2*d) - e^{(d*x + c)}/(b*d) + e^{(-d*x - c)}/(b*d) - 2*\log(e^{(-d*x - c)} + 1)/(a*d) - 2*\log(e^{(-d*x - c)} - 1)/(a*d) + 2*(a^2 + b^2)*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/(a*b^2*d)) + 3*(d*x*\log(e^{(d*x + c)} + 1) + \text{dilog}(-e^{(d*x + c)}))*e^{2*f}/(a*d^2) + 3*(d*x*\log(-e^{(d*x + c)} + 1) + \text{dilog}(e^{(d*x + c)}))*e^{2*f}/(a*d^2) + 3*(d^2*x^2*\log(e^{(d*x + c)} + 1) + 2*d*x*\text{dilog}(-e^{(d*x + c)}) - 2*\text{polylog}(3, -e^{(d*x + c)}))*e^{f^2}/(a*d^3) + 3*(d^2*x^2*\log(-e^{(d*x + c)} + 1) + 2*d*x*\text{dilog}(e^{(d*x + c)}) - 2*\text{polylog}(3, e^{(d*x + c)}))*e^{f^2}/(a*d^3) + (d^3*x^3*\log(e^{(d*x + c)} + 1) + 3*d^2*x^2*\text{dilog}(-e^{(d*x + c)}) - 6*d*x*\text{polylog}(3, -e^{(d*x + c)}) + 6*\text{polylog}(4, -e^{(d*x + c)}))$

$$\begin{aligned}
& x + c))) * f^3 / (a * d^4) + (d^3 * x^3 * \log(-e^{(d * x + c)} + 1) + 3 * d^2 * x^2 * \operatorname{dilog}(e^{(d * x + c)}) - 6 * d * x * \operatorname{polylog}(3, e^{(d * x + c)}) + 6 * \operatorname{polylog}(4, e^{(d * x + c)})) * f^3 / \\
& (a * d^4) - 1 / 2 * (d^4 * f^3 * x^4 + 4 * d^4 * e * f^2 * x^3 + 6 * d^4 * e^2 * f * x^2) / (a * d^4) - 1 \\
& / 4 * (a * d^4 * f^3 * x^4 * e^c + 4 * a * d^4 * e * f^2 * x^3 * e^c + 6 * a * d^4 * e^2 * f * x^2 * e^c - 2 * ( \\
& b * d^3 * f^3 * x^3 * e^{(2 * c)} + 3 * (d^3 * e * f^2 - d^2 * f^3) * b * x^2 * e^{(2 * c)} + 3 * (d^3 * e^2 * \\
& f - 2 * d^2 * e * f^2 + 2 * d * f^3) * b * x * e^{(2 * c)} - 3 * (d^2 * e^2 * f - 2 * d * e * f^2 + 2 * f^3) * \\
& b * e^{(2 * c)}) * e^{(d * x)} + 2 * (b * d^3 * f^3 * x^3 + 3 * (d^3 * e * f^2 + d^2 * f^3) * b * x^2 + 3 * ( \\
& d^3 * e^2 * f + 2 * d^2 * e * f^2 + 2 * d * f^3) * b * x + 3 * (d^2 * e^2 * f + 2 * d * e * f^2 + 2 * f^3) * \\
& b) * e^{(-d * x)}) * e^{(-c)} / (b^2 * d^4) + \operatorname{integrate}(-2 * ((a^2 * b * f^3 + b^3 * f^3) * x^3 + 3 \\
& * (a^2 * b * e * f^2 + b^3 * e * f^2) * x^2 + 3 * (a^2 * b * e^2 * f + b^3 * e^2 * f) * x - ((a^3 * f^3 * \\
& e^c + a * b^2 * f^3 * e^c) * x^3 + 3 * (a^3 * e * f^2 * e^c + a * b^2 * e * f^2 * e^c) * x^2 + 3 * (a^3 \\
& * e^2 * f * e^c + a * b^2 * e^2 * f * e^c) * x) * e^{(d * x)}) / (a * b^3 * e^{(2 * d * x + 2 * c)} + 2 * a^2 * b^2 \\
& * e^{(d * x + c)} - a * b^3), x)
\end{aligned}$$

**Fricas [C]** time = 3.65833, size = 7715, normalized size = 11.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] 
$$\begin{aligned}
& -1/4 * (2 * a * b * d^3 * f^3 * x^3 + 2 * a * b * d^3 * e^3 + 6 * a * b * d^2 * e^2 * f + 12 * a * b * d * e * f^2 \\
& + 12 * a * b * f^3 + 6 * (a * b * d^3 * e * f^2 + a * b * d^2 * f^3) * x^2 - 2 * (a * b * d^3 * f^3 * x^3 + a \\
& * b * d^3 * e^3 - 3 * a * b * d^2 * e^2 * f + 6 * a * b * d * e * f^2 - 6 * a * b * f^3 + 3 * (a * b * d^3 * e * f^2 \\
& - a * b * d^2 * f^3) * x^2 + 3 * (a * b * d^3 * e^2 * f - 2 * a * b * d^2 * e * f^2 + 2 * a * b * d * f^3) * x) * \\
& \cosh(d * x + c)^2 - 2 * (a * b * d^3 * f^3 * x^3 + a * b * d^3 * e^3 - 3 * a * b * d^2 * e^2 * f + 6 * a * \\
& b * d * e * f^2 - 6 * a * b * f^3 + 3 * (a * b * d^3 * e * f^2 - a * b * d^2 * f^3) * x^2 + 3 * (a * b * d^3 * e^2 * \\
& 2 * f - 2 * a * b * d^2 * e * f^2 + 2 * a * b * d * f^3) * x) * \sinh(d * x + c)^2 + 6 * (a * b * d^3 * e^2 * f \\
& + 2 * a * b * d^2 * e * f^2 + 2 * a * b * d * f^3) * x - (a^2 * d^4 * f^3 * x^4 + 4 * a^2 * d^4 * e * f^2 * x^3 \\
& + 6 * a^2 * d^4 * e^2 * f * x^2 + 4 * a^2 * d^4 * e^3 * x + 8 * a^2 * c * d^3 * e^3 - 12 * a^2 * c^2 * d^2 \\
& * e^2 * f + 8 * a^2 * c^3 * d * e * f^2 - 2 * a^2 * c^4 * f^3) * \cosh(d * x + c) + 12 * (((a^2 + b^2) \\
& ) * d^2 * f^3 * x^2 + 2 * (a^2 + b^2) * d^2 * e * f^2 * x + (a^2 + b^2) * d^2 * e^2 * f) * \cosh(d * x \\
& + c) + ((a^2 + b^2) * d^2 * f^3 * x^2 + 2 * (a^2 + b^2) * d^2 * e * f^2 * x + (a^2 + b^2) * \\
& d^2 * e^2 * f) * \sinh(d * x + c)) * \operatorname{dilog}((a * \cosh(d * x + c) + a * \sinh(d * x + c) + (b * \cos \\
& h(d * x + c) + b * \sinh(d * x + c)) * \operatorname{sqrt}((a^2 + b^2) / b^2) - b) / b + 1) + 12 * (((a^2 \\
& + b^2) * d^2 * f^3 * x^2 + 2 * (a^2 + b^2) * d^2 * e * f^2 * x + (a^2 + b^2) * d^2 * e^2 * f) * \co \\
& sh(d * x + c) + ((a^2 + b^2) * d^2 * f^3 * x^2 + 2 * (a^2 + b^2) * d^2 * e * f^2 * x + (a^2 + \\
& b^2) * d^2 * e^2 * f) * \sinh(d * x + c)) * \operatorname{dilog}((a * \cosh(d * x + c) + a * \sinh(d * x + c) - \\
& (b * \cosh(d * x + c) + b * \sinh(d * x + c)) * \operatorname{sqrt}((a^2 + b^2) / b^2) - b) / b + 1) - 12 * \\
& ((b^2 * d^2 * f^3 * x^2 + 2 * b^2 * d^2 * e * f^2 * x + b^2 * d^2 * e^2 * f) * \cosh(d * x + c) + (b^2
\end{aligned}$$



$$\begin{aligned}
& *d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*\sinh(d*x + c))*\operatorname{dilog}(\cosh \\
& (d*x + c) + \sinh(d*x + c)) - 12*((b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2 \\
& *d^2*e^2*f)*\cosh(d*x + c) + (b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2* \\
& e^2*f)*\sinh(d*x + c))*\operatorname{dilog}(-\cosh(d*x + c) - \sinh(d*x + c)) + 4*((a^2 + b^ \\
& 2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + \\
& b^2)*c^3*f^3)*\cosh(d*x + c) + ((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e \\
& ^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*\sinh(d*x + c))*\log( \\
& 2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + \\
& 4*((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e \\
& *f^2 - (a^2 + b^2)*c^3*f^3)*\cosh(d*x + c) + ((a^2 + b^2)*d^3*e^3 - 3*(a^2 + \\
& b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*\sinh(d \\
& *x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b \\
& ^2} + 2*a) + 4*((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3* \\
& (a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e \\
& *f^2 + (a^2 + b^2)*c^3*f^3)*\cosh(d*x + c) + ((a^2 + b^2)*d^3*f^3*x^3 + 3*(a \\
& ^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e \\
& ^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*\sinh(d*x + c))*\log( \\
& -(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{ \\
& (a^2 + b^2)/b^2} - b)/b) + 4*((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)* \\
& d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*( \\
& a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*\cosh(d*x + c) + ((a^2 + b^2)* \\
& d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*( \\
& a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*\sinh(d*x \\
& + c))*\log(- (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + \\
& b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 4*((b^2*d^3*f^3*x^3 + 3*b^ \\
& 2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + b^2*d^3*e^3)*\cosh(d*x + c) + (b^2*d^3 \\
& *f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + b^2*d^3*e^3)*\sinh(d*x \\
& + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) - 4*((b^2*d^3*e^3 - 3*b^2*c*d^ \\
& 2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*\cosh(d*x + c) + (b^2*d^3*e^3 - 3 \\
& *b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*\sinh(d*x + c))*\log(\cosh \\
& (d*x + c) + \sinh(d*x + c) - 1) - 4*((b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 \\
& + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)* \\
& \cosh(d*x + c) + (b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x \\
& + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*\sinh(d*x + c))*\log(- \\
& \cosh(d*x + c) - \sinh(d*x + c) + 1) + 24*((a^2 + b^2)*f^3*\cosh(d*x + c) + (a \\
& ^2 + b^2)*f^3*\sinh(d*x + c))*\operatorname{polylog}(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) \\
& + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) + 24*((a^2 \\
& + b^2)*f^3*\cosh(d*x + c) + (a^2 + b^2)*f^3*\sinh(d*x + c))*\operatorname{polylog}(4, (a*\cos \\
& h(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^ \\
& 2 + b^2)/b^2}))/b) - 24*(b^2*f^3*\cosh(d*x + c) + b^2*f^3*\sinh(d*x + c))*\operatorname{poly} \\
& \log(4, \cosh(d*x + c) + \sinh(d*x + c)) - 24*(b^2*f^3*\cosh(d*x + c) + b^2*f^3 \\
& *\sinh(d*x + c))*\operatorname{polylog}(4, -\cosh(d*x + c) - \sinh(d*x + c)) - 24*((a^2 + b^ \\
& 2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*\cosh(d*x + c) + ((a^2 + b^2)*d*f^3*x + (a \\
& ^2 + b^2)*d*e*f^2)*\sinh(d*x + c))*\operatorname{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x \\
& + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) - 24*(
\end{aligned}$$

$$\begin{aligned} & ((a^2 + b^2)d^3f^3x + (a^2 + b^2)d^2ef^2)cosh(dx + c) + ((a^2 + b^2)d^3f^3x + (a^2 + b^2)d^2ef^2)sinh(dx + c)) * polylog(3, (a*cosh(dx + c) + a * sinh(dx + c) - (b*cosh(dx + c) + b*sinh(dx + c)) * sqrt((a^2 + b^2)/b^2)) / b) \\ & + 24*((b^2*d^3f^3x + b^2*d^2ef^2)*cosh(dx + c) + (b^2*d^3f^3x + b^2*d^2ef^2)*sinh(dx + c)) * polylog(3, cosh(dx + c) + sinh(dx + c)) \\ & + 24*((b^2*d^3f^3x + b^2*d^2ef^2)*cosh(dx + c) + (b^2*d^3f^3x + b^2*d^2ef^2)*sinh(dx + c)) * polylog(3, -cosh(dx + c) - sinh(dx + c)) \\ & - (a^2*d^4f^3x^4 + 4*a^2*d^4*ef^2*x^3 + 6*a^2*d^4*e^2*f*x^2 + 4*a^2*d^4*e^3*x + 8*a^2*c*d^3*e^3 - 12*a^2*c^2*d^2*e^2*f + 8*a^2*c^3*d*ef^2 - 2*a^2*c^4*f^3 + 4*(a*b*d^3*f^3*x^3 + a*b*d^3*e^3 - 3*a*b*d^2*e^2*f + 6*a*b*d*ef^2 - 6*a*b*f^3 + 3*(a*b*d^3*ef^2 - a*b*d^2*f^3)*x^2 + 3*(a*b*d^3*e^2*f - 2*a*b*d^2*ef^2 + 2*a*b*d*f^3)*x) * cosh(dx + c)) * sinh(dx + c)) / (a*b^2*d^4*cosh(dx + c) + a*b^2*d^4*sinh(dx + c)) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cosh(d\*x+c)\*\*2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)^2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.431 \quad \int \frac{(e+fx)^2 \cosh^2(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=486

$$\frac{2f(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^2} - \frac{2f(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ab^2d^2} + \frac{2f^2(a^2+b^2)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^3}$$

[Out]  $-(e+fx)^3/(3af) + ((a^2+b^2)(e+fx)^3)/(3ab^2f) - (2f(e+fx)\cosh[c+dx])/(bd^2) - ((a^2+b^2)(e+fx)^2\text{Log}[1+(bE^{c+dx})/(a-\sqrt{a^2+b^2})])/(ab^2d) - ((a^2+b^2)(e+fx)^2\text{Log}[1+(bE^{c+dx})/(a+\sqrt{a^2+b^2})])/(ab^2d) + ((e+fx)^2\text{Log}[1-E^{2(c+dx)}])/(ad) - (2(a^2+b^2)f(e+fx)\text{PolyLog}[2, -(bE^{c+dx})/(a-\sqrt{a^2+b^2})])/(ab^2d^2) - (2(a^2+b^2)f(e+fx)\text{PolyLog}[2, -(bE^{c+dx})/(a+\sqrt{a^2+b^2})])/(ab^2d^2) + (f(e+fx)\text{PolyLog}[2, E^{2(c+dx)}])/(ad^2) + (2(a^2+b^2)f^2\text{PolyLog}[3, -(bE^{c+dx})/(a-\sqrt{a^2+b^2})])/(ab^2d^3) + (2(a^2+b^2)f^2\text{PolyLog}[3, -(bE^{c+dx})/(a+\sqrt{a^2+b^2})])/(ab^2d^3) - (f^2\text{PolyLog}[3, E^{2(c+dx)}])/(2ad^3) + (2f^2\sinh[c+dx])/(bd^3) + ((e+fx)^2\sinh[c+dx])/(bd)$

**Rubi [A]** time = 1.02531, antiderivative size = 486, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 13, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$ , Rules used = {5585, 5450, 5446, 3310, 3716, 2190, 2531, 2282, 6589, 5565, 3296, 2637, 5561}

$$\frac{2f(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^2} - \frac{2f(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ab^2d^2} + \frac{2f^2(a^2+b^2)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ab^2d^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(e+fx)^2\cosh[c+dx]^2\coth[c+dx]/(a+b\sinh[c+dx]), x]$

[Out]  $-(e+fx)^3/(3af) + ((a^2+b^2)(e+fx)^3)/(3ab^2f) - (2f(e+fx)\cosh[c+dx])/(bd^2) - ((a^2+b^2)(e+fx)^2\text{Log}[1+(bE^{c+dx})/(a-\sqrt{a^2+b^2})])/(ab^2d) - ((a^2+b^2)(e+fx)^2\text{Log}[1+(bE^{c+dx})/(a+\sqrt{a^2+b^2})])/(ab^2d) + ((e+fx)^2\text{Log}[1-E^{2(c+dx)}])/(ad) - (2(a^2+b^2)f(e+fx)\text{PolyLog}[2, -(bE^{c+dx})/(a-\sqrt{a^2+b^2})])/(ab^2d^2) - (2(a^2+b^2)f(e+fx)\text{PolyLog}[2, -(bE^{c+dx})/(a+\sqrt{a^2+b^2})])/(ab^2d^2) + (f(e+fx)\text{PolyLog}[2, E^{2(c+dx)}])/(ad^2) + (2(a^2+b^2)f^2\text{PolyLog}[3, -(bE^{c+dx})/(a-\sqrt{a^2+b^2})])/(ab^2d^3) + (2(a^2+b^2)f^2\text{PolyLog}[3, -(bE^{c+dx})/(a+\sqrt{a^2+b^2})])/(ab^2d^3) - (f^2\text{PolyLog}[3, E^{2(c+dx)}])/(2ad^3) + (2f^2\sinh[c+dx])/(bd^3) + ((e+fx)^2\sinh[c+dx])/(bd)$

$$\frac{c + d*x)}{(a - \sqrt{a^2 + b^2})}]/(a*b^2*d^3) + (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^{(c + d*x)})/(a + \sqrt{a^2 + b^2})))]/(a*b^2*d^3) - (f^2*PolyLog[3, E^{(2*(c + d*x))}])/(2*a*d^3) + (2*f^2*Sinh[c + d*x])/(b*d^3) + ((e + f*x)^2*Sinh[c + d*x])/(b*d)$$

### Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := Simp[(d*(b*Sine + f*x)^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*Sine + f*x)^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*Sine + f*x)^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]
```

### Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*e) + f*fz*x)))/E^(2*I*k*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5565

```
Int[(Cosh[(c_) + (d_)*(x_)]^(n_))*((e_) + (f_)*(x_))^(m_)]/((a_) + (b_
)*Sinh[(c_) + (d_)*(x_)]), x_Symbol] := -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3296

```
Int[(((c_) + (d_)*(x_))^(m_))*sin[(e_) + (f_)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e + fx)^2 \cosh^2(c + dx) \coth(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\int (e + fx)^2 \cosh^2(c + dx) \coth(c + dx) dx}{a} - \frac{b \int \frac{(e + fx)^2 \cosh^3(c + dx)}{a + b \sinh(c + dx)} dx}{a} \\
&= \frac{\int (e + fx)^2 \coth(c + dx) dx}{a} + \frac{\int (e + fx)^2 \cosh(c + dx) dx}{b} - \frac{(a^2 + b^2) \int \frac{(e + fx)^2 \cosh^3(c + dx)}{a + b \sinh(c + dx)} dx}{a} \\
&= -\frac{(e + fx)^3}{3af} + \frac{(a^2 + b^2)(e + fx)^3}{3ab^2f} + \frac{(e + fx)^2 \sinh(c + dx)}{bd} - \frac{2 \int \frac{e^{2(c + dx)}(e + fx)^2}{1 - e^{2(c + dx)}} dx}{a} \\
&= -\frac{(e + fx)^3}{3af} + \frac{(a^2 + b^2)(e + fx)^3}{3ab^2f} - \frac{2f(e + fx) \cosh(c + dx)}{bd^2} - \frac{(a^2 + b^2)(e + fx)^2 \sinh(c + dx)}{a} \\
&= -\frac{(e + fx)^3}{3af} + \frac{(a^2 + b^2)(e + fx)^3}{3ab^2f} - \frac{2f(e + fx) \cosh(c + dx)}{bd^2} - \frac{(a^2 + b^2)(e + fx)^2 \sinh(c + dx)}{a} \\
&= -\frac{(e + fx)^3}{3af} + \frac{(a^2 + b^2)(e + fx)^3}{3ab^2f} - \frac{2f(e + fx) \cosh(c + dx)}{bd^2} - \frac{(a^2 + b^2)(e + fx)^2 \sinh(c + dx)}{a} \\
&= -\frac{(e + fx)^3}{3af} + \frac{(a^2 + b^2)(e + fx)^3}{3ab^2f} - \frac{2f(e + fx) \cosh(c + dx)}{bd^2} - \frac{(a^2 + b^2)(e + fx)^2 \sinh(c + dx)}{a}
\end{aligned}$$

**Mathematica [B]** time = 14.3014, size = 1521, normalized size = 3.13

result too large to display

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^2\*Cosh[c + d\*x]^2\*Coth[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] 
$$-(E^{(2*c)}*((2*(e + f*x)^3)/(E^{(2*c)}*f) - (3*(1 - E^{(-2*c)})*(e + f*x)^2*\text{Log}[1 - E^{(-c - d*x)}])/d - (3*(1 - E^{(-2*c)})*(e + f*x)^2*\text{Log}[1 + E^{(-c - d*x)}])/d + (6*(-1 + E^{(2*c)})*f*(d*(e + f*x)*\text{PolyLog}[2, -E^{(-c - d*x)}] + f*\text{PolyLog}[3, -E^{(-c - d*x)}]))/(d^3*E^{(2*c)}) + (6*(-1 + E^{(2*c)})*f*(d*(e + f*x)*\text{PolyLog}[2, E^{(-c - d*x)}] + f*\text{PolyLog}[3, E^{(-c - d*x)}]))/(d^3*E^{(2*c)})))/(3*a*(-1 + E^{(2*c)})) + ((a^2 + b^2)*(6*e^2*E^{(2*c)}*x + 6*e*E^{(2*c)}*f*x^2 + 2*E^{(2*c)}*f^2*x^3 + (6*a*\text{Sqrt}[a^2 + b^2]*e^2*\text{ArcTan}[(a + b*E^{(c + d*x)})/\text{Sqrt}[-a^2 - b^2]])/(\text{Sqrt}[-(a^2 + b^2)^2]*d) + (6*a*\text{Sqrt}[-(a^2 + b^2)^2]*e^2*E^{(2*c)}*\text{ArcTan}[(a + b*E^{(c + d*x)})/\text{Sqrt}[-a^2 - b^2]])/((a^2 + b^2)^{(3/2)}*d) - (6*a*\text{Sqrt}[-(a^2 + b^2)^2]*e^2*\text{ArcTanh}[(a + b*E^{(c + d*x)})/\text{Sqrt}[a^2 + b^2]])/((-a^2 - b^2)^{(3/2)}*d) + (6*a*\text{Sqrt}[-(a^2 + b^2)^2]*e^2*E^{(2*c)}*\text{ArcTanh}[(a + b*E^{(c + d*x)})/\text{Sqrt}[a^2 + b^2]])/((-a^2 - b^2)^{(3/2)}*d) + (3*e^2*\text{Log}[2*a*E^{(c + d*x)} + b*(-1 + E^{(2*(c + d*x)}))])/d - (3*e^2*E^{(2*c)}*\text{Log}[2*a*E^{(c + d*x)} + b*(-1 + E^{(2*(c + d*x)}))])/d + (6*e*f*x*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d - (6*e*E^{(2*c)}*f*x*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d + (3*f^2*x^2*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d - (3*E^{(2*c)}*f^2*x^2*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d + (6*e*f*x*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c + \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d - (6*e*E^{(2*c)}*f*x*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c + \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d + (3*f^2*x^2*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c + \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d - (3*E^{(2*c)}*f^2*x^2*\text{Log}[1 + (b*E^{(2*c + d*x)})/(a*E^c + \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d - (6*(-1 + E^{(2*c)})*f*(e + f*x)*\text{PolyLog}[2, -((b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d^2 - (6*(-1 + E^{(2*c)})*f*(e + f*x)*\text{PolyLog}[2, -((b*E^{(2*c + d*x)})/(a*E^c + \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d^2 - (6*f^2*\text{PolyLog}[3, -((b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d^3 + (6*E^{(2*c)}*f^2*\text{PolyLog}[3, -((b*E^{(2*c + d*x)})/(a*E^c - \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d^3 - (6*f^2*\text{PolyLog}[3, -((b*E^{(2*c + d*x)})/(a*E^c + \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d^3 + (6*E^{(2*c)}*f^2*\text{PolyLog}[3, -((b*E^{(2*c + d*x)})/(a*E^c + \text{Sqrt}[(a^2 + b^2)*E^{(2*c)}])])/d^3))/(3*a*b^2*(-1 + E^{(2*c)})) - (a*x*(3*e^2 + 3*e*f*x + f^2*x^2)*\text{Cosh}[c]*\text{Csch}[c/2]*\text{Sech}[c/2])/(6*b^2) + (\text{Cosh}[d*x]*(-2*d*e*f*\text{Cosh}[c] - 2*d*f^2*x*\text{Cosh}[c] + d^2*e^2*\text{Sinh}[c] + 2*f^2*\text{Sinh}[c] + 2*d^2*e*f*x*\text{Sinh}[c] + d^2*f^2*x^2*\text{Sinh}[c]))/(b*d^3) + ((d^2*e^2*\text{Cosh}[c] + 2*f^2*\text{Cosh}[c] + 2*d^2*e*f*x*\text{Cosh}[c] + d^2*f^2*x^2*\text{Cosh}[c] - 2*d*e*f*\text{Sinh}[c] - 2*d*f^2*x*\text{Sinh}[c])*Sinh[d*x])/(b*d^3)$$

**Maple [F]** time = 1.063, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\cosh(dx + c))^2 \coth(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*cosh(d\*x+c)^2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*cosh(d\*x+c)^2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)^2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/2*e^{2*(d*x + c)}*a/(b^2*d) - e^{(d*x + c)}/(b*d) + e^{-(d*x - c)}/(b*d) - \\ & 2*\log(e^{-(d*x - c)} + 1)/(a*d) - 2*\log(e^{-(d*x - c)} - 1)/(a*d) + 2*(a^2 + b^2)*\log(-2*a*e^{-(d*x - c)} + b*e^{-(2*d*x - 2*c)} - b)/(a*b^2*d) + 2*(d*x*\log(e^{(d*x + c)} + 1) + \operatorname{dilog}(-e^{(d*x + c)}))*e^f/(a*d^2) + 2*(d*x*\log(-e^{(d*x + c)} + 1) + \operatorname{dilog}(e^{(d*x + c)}))*e^f/(a*d^2) + (d^2*x^2*\log(e^{(d*x + c)} + 1) + 2*d*x*\operatorname{dilog}(-e^{(d*x + c)})) * f^2/(a*d^3) + (d^2*x^2*\log(-e^{(d*x + c)} + 1) + 2*d*x*\operatorname{dilog}(e^{(d*x + c)})) * f^2/(a*d^3) - 2/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^2)/(a*d^3) - 1/6*(2*a*d^3*f^2*x^3*e^c + 6*a*d^3*e*f*x^2*e^c - 3*(b*d^2*f^2*x^2*e^{(2*c)} + 2*(d^2*e*f - d*f^2)*b*x*e^{(2*c)} - 2*(d*e*f - f^2)*b*e^{(2*c)}) * e^{(d*x)} + 3*(b*d^2*f^2*x^2 + 2*(d^2*e*f + d*f^2)*b*x + 2*(d*e*f + f^2)*b) * e^{(-d*x)} * e^{(-c)}/(b^2*d^3) + \operatorname{integrate}(-2*((a^2*b*f^2 + b^3*f^2)*x^2 + 2*(a^2*b*e*f + b^3*e*f)*x - ((a^3*f^2*e^c + a*b^2*f^2*e^c)*x^2 + 2*(a^3*e*f*e^c + a*b^2*e*f*e^c)*x) * e^{(d*x)})/(a*b^3*e^{(2*d*x + 2*c)} + 2*a^2*b^2*e^{(d*x + c)} - a*b^3), x) \end{aligned}$$

**Fricas [C]** time = 2.86187, size = 5030, normalized size = 10.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((f\*x+e)^2\*cosh(d\*x+c)^2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm  
m="fricas")

[Out] 
$$-1/6*(3*a*b*d^2*f^2*x^2 + 3*a*b*d^2*e^2 + 6*a*b*d*e*f + 6*a*b*f^2 - 3*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*\cosh(d*x + c)^2 - 3*(a*b*d^2*f^2*x^2 + a*b*d^2*e^2 - 2*a*b*d*e*f + 2*a*b*f^2 + 2*(a*b*d^2*e*f - a*b*d*f^2)*x)*\sinh(d*x + c)^2 + 6*(a*b*d^2*e*f + a*b*d*f^2)*x - 2*(a^2*d^3*f^2*x^3 + 3*a^2*d^3*e*f*x^2 + 3*a^2*d^3*e^2*x + 6*a^2*c*d^2*e^2 - 6*a^2*c^2*d*e*f + 2*a^2*c^3*f^2)*\cosh(d*x + c) + 12*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*\cosh(d*x + c) + ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*\sinh(d*x + c))*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 12*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*\cosh(d*x + c) + ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*\sinh(d*x + c))*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 12*((b^2*d*f^2*x + b^2*d*e*f)*\cosh(d*x + c) + (b^2*d*f^2*x + b^2*d*e*f)*\sinh(d*x + c))*\operatorname{dilog}(\cosh(d*x + c) + \sinh(d*x + c)) - 12*((b^2*d*f^2*x + b^2*d*e*f)*\cosh(d*x + c) + (b^2*d*f^2*x + b^2*d*e*f)*\sinh(d*x + c))*\operatorname{dilog}(-\cosh(d*x + c) - \sinh(d*x + c)) + 6*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c) + ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\sinh(d*x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 6*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c) + ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\sinh(d*x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 6*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c) + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\sinh(d*x + c))*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + 6*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c) + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) - 6*((b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*\cosh(d*x + c) + (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) - 6*((b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*\cosh(d*x + c) + (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*\sinh(d*x + c))*\log(-\cosh(d*x + c) - \sinh(d*x + c) + 1) - 12*((a^2 + b^2)*f^2*\cosh(d*x + c) + (a^2 + b^2)*f^2*\sinh(d*x + c))*\operatorname{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) - 12*(($$

$$a^2 + b^2)f^2 \cosh(dx + c) + (a^2 + b^2)f^2 \sinh(dx + c)) \operatorname{polylog}(3, (a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c))) \sqrt{((a^2 + b^2)/b^2)})/b + 12(b^2 f^2 \cosh(dx + c) + b^2 f^2 \sinh(dx + c)) \operatorname{polylog}(3, \cosh(dx + c) + \sinh(dx + c)) + 12(b^2 f^2 \cosh(dx + c) + b^2 f^2 \sinh(dx + c)) \operatorname{polylog}(3, -\cosh(dx + c) - \sinh(dx + c)) - 2(a^2 d^3 f^2 x^3 + 3a^2 d^3 e f x^2 + 3a^2 d^3 e^2 x + 6a^2 c d^2 e^2 - 6a^2 c^2 d e f + 2a^2 c^3 f^2 + 3(a b d^2 f^2 x^2 + a b d^2 e^2 - 2a b d e f + 2a b f^2 + 2(a b d^2 e f - a b d f^2) x) \cosh(dx + c)) \sinh(dx + c)) / (a b^2 d^3 \cosh(dx + c) + a b^2 d^3 \sinh(dx + c))$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cosh(d\*x+c)\*\*2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)^2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.432 \quad \int \frac{(e+fx) \cosh^2(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=322

$$\frac{f(a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{ab^2 d^2} - \frac{f(a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right)}{ab^2 d^2} + \frac{f \operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{2ad^2} - \frac{(a^2 + b^2)(e + f)}{2ad^2}$$

[Out]  $-(e + f*x)^2/(2*a*f) + ((a^2 + b^2)*(e + f*x)^2)/(2*a*b^2*f) - (f*\operatorname{Cosh}[c + d*x])/(b*d^2) - ((a^2 + b^2)*(e + f*x)*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])])/(a*b^2*d) - ((a^2 + b^2)*(e + f*x)*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])/(a*b^2*d) + ((e + f*x)*\operatorname{Log}[1 - E^{(2*(c + d*x))}])/(a*d) - ((a^2 + b^2)*f*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])])/(a*b^2*d^2) - ((a^2 + b^2)*f*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])/(a*b^2*d^2) + (f*\operatorname{PolyLog}[2, E^{(2*(c + d*x))}])/(2*a*d^2) + ((e + f*x)*\operatorname{Sinh}[c + d*x])/(b*d)$

**Rubi [A]** time = 0.591985, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 13, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$ , Rules used = {5585, 5450, 5446, 2635, 8, 3716, 2190, 2279, 2391, 5565, 3296, 2638, 5561}

$$\frac{f(a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{ab^2 d^2} - \frac{f(a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right)}{ab^2 d^2} + \frac{f \operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{2ad^2} - \frac{(a^2 + b^2)(e + f)}{2ad^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\frac{(e + f*x)*\operatorname{Cosh}[c + d*x]^2*\operatorname{Coth}[c + d*x]}{(a + b*\operatorname{Sinh}[c + d*x])}, x]$

[Out]  $-(e + f*x)^2/(2*a*f) + ((a^2 + b^2)*(e + f*x)^2)/(2*a*b^2*f) - (f*\operatorname{Cosh}[c + d*x])/(b*d^2) - ((a^2 + b^2)*(e + f*x)*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])])/(a*b^2*d) - ((a^2 + b^2)*(e + f*x)*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])/(a*b^2*d) + ((e + f*x)*\operatorname{Log}[1 - E^{(2*(c + d*x))}])/(a*d) - ((a^2 + b^2)*f*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])])/(a*b^2*d^2) - ((a^2 + b^2)*f*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])/(a*b^2*d^2) + (f*\operatorname{PolyLog}[2, E^{(2*(c + d*x))}])/(2*a*d^2) + ((e + f*x)*\operatorname{Sinh}[c + d*x])/(b*d)$

Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
(f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
0] && IGtQ[p, 0]
```

### Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] := Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_.), x_Symbol] := -Simp[(b*cos[c + d*x]
*(b*sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

### Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

### Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
```

st[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_)\*((F\_)^((e\_)\*((c\_) + (d\_)\*(x\_))))^(n\_)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 5565

Int[(Cosh[(c\_) + (d\_)\*(x\_)]^(n\_)\*((e\_) + (f\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*Sinh[(c\_) + (d\_)\*(x\_)]), x\_Symbol] :> -Dist[a/b^2, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x)) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

### Rule 3296

Int[((c\_) + (d\_)\*(x\_))^(m\_)\*sin[(e\_) + (f\_)\*(x\_)], x\_Symbol] :> -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2638

Int[sin[(c\_) + (d\_)\*(x\_)], x\_Symbol] :> -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rule 5561

Int[(Cosh[(c\_) + (d\_)\*(x\_)]\*((e\_) + (f\_)\*(x\_))^(m\_))/((a\_) + (b\_)\*Sinh[(c\_) + (d\_)\*(x\_)]), x\_Symbol] :> -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx) \cosh^2(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx) \cosh^2(c+dx) \coth(c+dx) dx}{a} - \frac{b \int \frac{(e+fx) \cosh^3(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= \frac{\int (e+fx) \coth(c+dx) dx}{a} + \frac{\int (e+fx) \cosh(c+dx) dx}{b} - \frac{(a^2+b^2) \int \frac{(e+fx)}{a+b \sinh(c+dx)} dx}{ab} \\
&= -\frac{(e+fx)^2}{2af} + \frac{(a^2+b^2)(e+fx)^2}{2ab^2f} + \frac{(e+fx) \sinh(c+dx)}{bd} - \frac{2 \int \frac{e^{2(c+dx)}(e+fx)}{1-e^{2(c+dx)}} dx}{a} \\
&= -\frac{(e+fx)^2}{2af} + \frac{(a^2+b^2)(e+fx)^2}{2ab^2f} - \frac{f \cosh(c+dx)}{bd^2} - \frac{(a^2+b^2)(e+fx) \log}{ab^2d} \\
&= -\frac{(e+fx)^2}{2af} + \frac{(a^2+b^2)(e+fx)^2}{2ab^2f} - \frac{f \cosh(c+dx)}{bd^2} - \frac{(a^2+b^2)(e+fx) \log}{ab^2d} \\
&= -\frac{(e+fx)^2}{2af} + \frac{(a^2+b^2)(e+fx)^2}{2ab^2f} - \frac{f \cosh(c+dx)}{bd^2} - \frac{(a^2+b^2)(e+fx) \log}{ab^2d}
\end{aligned}$$

**Mathematica [A]** time = 1.92519, size = 296, normalized size = 0.92

$$(a^2+b^2) \left( -f \operatorname{PolyLog} \left( 2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a} \right) - f \operatorname{PolyLog} \left( 2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} \right) - f(c+dx) \log \left( \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1 \right) - f(c+dx) \log \left( \frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]^2*Coth[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(a*b*f*Cosh[c + d*x]) + b^2*d*e*Log[Sinh[c + d*x]] - b^2*c*f*Log[Sinh[c + d*x]] + (b^2*f*((c + d*x)*(c + d*x + 2*Log[1 - E^(-2*(c + d*x))]) - PolyLog[2, E^(-2*(c + d*x))]))/2 + (a^2 + b^2)*((f*(c + d*x)^2)/2 - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])]) - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - d*e*Log[a + b*Sinh[c + d*x]] + c*f*Log[a + b*Sinh[c + d*x]] - f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - f*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + a*b*d*(e + f*x)*Sinh[c + d*x]/(a*b^2*d^2)
```

**Maple [B]** time = 0.255, size = 932, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x)`

[Out] 
$$\begin{aligned} & 1/2*(d*f*x+d*e-f)/d^2/b*\exp(d*x+c)-1/2*(d*f*x+d*e+f)/d^2/b*\exp(-d*x-c)+1/2* \\ & a*f*x^2/b^2-1/d*f/a*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) \\ & ))*x-1/d^2*f/a*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))* \\ & c-1/d*f/a*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*x-1/d^2* \\ & f/a*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*c+1/d^2*f*c/a* \\ & \ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)-2*a/b^2/d^2*f*c*\ln(b*\exp(2*d* \\ & x+2*c)+2*a*\exp(d*x+c)-b)-2*a/b^2/d^2*f*c*\ln(\exp(d*x+c))-a/b^2/d^2*f*\ln((-b*\exp \\ & (d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*x-a/b^2/d^2*f*\ln((-b*\exp \\ & (d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*c-a/b^2/d^2*f*\ln((b*\exp(d*x+c) \\ & )+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*x-a/b^2/d^2*f*\ln((b*\exp(d*x+c)+(a \\ & ^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*c+2*a/b^2/d^2*f*c*x-1/d^2*f/a*dilog((-b \\ & *\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))-1/d^2*f/a*dilog((b*\exp \\ & (d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))-1/d^2*f/a*dilog(\exp(d*x+c)) \\ & +1/d^2*f/a*dilog(\exp(d*x+c)+1)+1/d/a*e*\ln(\exp(d*x+c)-1)+1/d/a*e*\ln(\exp(d*x+ \\ & c)+1)+a/b^2/d^2*f*c^2-a/b^2/d*e*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)+2*a/b \\ & ^2/d*e*\ln(\exp(d*x+c))-a/b^2/d^2*f*dilog((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a \\ & +(a^2+b^2)^{(1/2)}))-a/b^2/d^2*f*dilog((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+ \\ & (a^2+b^2)^{(1/2)}))+1/d/a*\ln(\exp(d*x+c)+1)*f*x-1/d^2/a*f*c*\ln(\exp(d*x+c)-1)-1 \\ & /d*e/a*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}e \left( \frac{2(dx+c)a}{b^2d} - \frac{e^{(dx+c)}}{bd} + \frac{e^{(-dx-c)}}{bd} - \frac{2 \log(e^{(-dx-c)}+1)}{ad} - \frac{2 \log(e^{(-dx-c)}-1)}{ad} + \frac{2(a^2+b^2) \log(-2ae^{(-dx-c)}+be^{(-2dxc)})}{ab^2d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*cosh(d*x+c)^2*coth(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & -1/2*e*(2*(d*x+c)*a/(b^2*d) - e^{(d*x+c)}/(b*d) + e^{(-d*x-c)}/(b*d) - 2* \\ & \log(e^{(-d*x-c)}+1)/(a*d) - 2*\log(e^{(-d*x-c)}-1)/(a*d) + 2*(a^2+b^2) \\ & * \log(-2*a*e^{(-d*x-c)}+b*e^{(-2*d*x-2*c)}-b)/(a*b^2*d) - 1/4*f*(2*(a*d \end{aligned}$$

$$\begin{aligned} & ^2*x^2*e^c - (b*d*x*e^{(2*c)} - b*e^{(2*c)})*e^{(d*x)} + (b*d*x + b)*e^{(-d*x)}*e^{(-c)}/(b^2*d^2) - \text{integrate}(8*((a^3*e^c + a*b^2*e^c)*x*e^{(d*x)} - (a^2*b + b^3)*x)/(a*b^3*e^{(2*d*x + 2*c)} + 2*a^2*b^2*e^{(d*x + c)} - a*b^3), x) + 4*\text{integrate}(x/(a*e^{(d*x + c)} + a), x) - 4*\text{integrate}(x/(a*e^{(d*x + c)} - a), x) \end{aligned}$$

**Fricas [B]** time = 2.99753, size = 2799, normalized size = 8.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/2*(a*b*d*f*x + a*b*d*e + a*b*f - (a*b*d*f*x + a*b*d*e - a*b*f)*\cosh(d*x + c)^2 - (a*b*d*f*x + a*b*d*e - a*b*f)*\sinh(d*x + c)^2 - (a^2*d^2*f*x^2 + 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f)*\cosh(d*x + c) + 2*((a^2 + b^2)*f*\cosh(d*x + c) + (a^2 + b^2)*f*\sinh(d*x + c))*\text{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 2*((a^2 + b^2)*f*\cosh(d*x + c) + (a^2 + b^2)*f*\sinh(d*x + c))*\text{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 2*(b^2*f*\cosh(d*x + c) + b^2*f*\sinh(d*x + c))*\text{dilog}(\cosh(d*x + c) + \sinh(d*x + c)) - 2*(b^2*f*\cosh(d*x + c) + b^2*f*\sinh(d*x + c))*\text{dilog}(-\cosh(d*x + c) - \sinh(d*x + c)) + 2*(((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*\cosh(d*x + c) + ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*\sinh(d*x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 2*(((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*\cosh(d*x + c) + ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*\sinh(d*x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 2*(((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*\cosh(d*x + c) + ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*\sinh(d*x + c))*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + 2*(((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*\cosh(d*x + c) + ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*\sinh(d*x + c))*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 2*((b^2*d*f*x + b^2*d*e)*\cosh(d*x + c) + (b^2*d*f*x + b^2*d*e)*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) - 2*((b^2*d*f*x + b^2*c*f)*\cosh(d*x + c) + (b^2*d*f*x + b^2*c*f)*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) - 2*((b^2*d*f*x + b^2*c*f)*\cosh(d*x + c) + (b^2*d*f*x + b^2*c*f)*\sinh(d*x + c))*\log(-\cosh(d*x + c) - \sinh(d*x + c) + 1) - (a^2*d^2*f*x^2 + 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*a^2*c^2*f + 2*(a*b*d*f*x + a*b*d*e - a*b*f)*\cosh(d*x + c))*\sinh(d*x + c))/(a*b^2*d^2*\cosh(d*x + c) + a*b^2*d^2*\sinh(d*x + c)) \end{aligned}$$



---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \cosh^2(c + dx) \coth(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)\*\*2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*cosh(c + d\*x)\*\*2\*coth(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)^2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.433 \quad \int \frac{\cosh^2(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=57

$$-\frac{(a^2 + b^2) \log(a + b \sinh(c + dx))}{ab^2d} + \frac{\log(\sinh(c + dx))}{ad} + \frac{\sinh(c + dx)}{bd}$$

[Out] Log[Sinh[c + d\*x]]/(a\*d) - ((a^2 + b^2)\*Log[a + b\*Sinh[c + d\*x]])/(a\*b^2\*d) + Sinh[c + d\*x]/(b\*d)

**Rubi [A]** time = 0.128075, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2837, 12, 894}

$$-\frac{(a^2 + b^2) \log(a + b \sinh(c + dx))}{ab^2d} + \frac{\log(\sinh(c + dx))}{ad} + \frac{\sinh(c + dx)}{bd}$$

Antiderivative was successfully verified.

[In] Int[(Cosh[c + d\*x]^2\*Coth[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] Log[Sinh[c + d\*x]]/(a\*d) - ((a^2 + b^2)\*Log[a + b\*Sinh[c + d\*x]])/(a\*b^2\*d) + Sinh[c + d\*x]/(b\*d)

### Rule 2837

Int[cos[(e\_.) + (f\_.)\*(x\_)]^(p\_)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(m\_.))\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_.), x\_Symbol] := Dist[1/(b^p\*f), Subst[Int[(a + x)^m\*(c + (d\*x)/b)^n\*(b^2 - x^2)^((p - 1)/2), x], x, b\*S in[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 894

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))^(n\_)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] := Int[ExpandIntegrand[(d + e\*x)^m\*(f + g\*x)^n\*(a + c\*x

$\wedge 2) \wedge p, x], x] /; \text{FreeQ}[\{a, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[e*f - d*g, 0] \ \&\& \ \text{NeQ}[c * d^2 + a*e^2, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ ((\text{EqQ}[p, 1] \ \&\& \ \text{IntegersQ}[m, n]) \ || \ (\text{ILtQ}[m, 0] \ \&\& \ \text{ILtQ}[n, 0]))$

### Rubi steps

$$\begin{aligned} \int \frac{\cosh^2(c + dx) \coth(c + dx)}{a + b \sinh(c + dx)} dx &= -\frac{\text{Subst}\left(\int \frac{b(-b^2-x^2)}{x(a+x)} dx, x, b \sinh(c + dx)\right)}{b^3 d} \\ &= -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{x(a+x)} dx, x, b \sinh(c + dx)\right)}{b^2 d} \\ &= -\frac{\text{Subst}\left(\int \left(-1 - \frac{b^2}{ax} + \frac{a^2+b^2}{a(a+x)}\right) dx, x, b \sinh(c + dx)\right)}{b^2 d} \\ &= \frac{\log(\sinh(c + dx))}{ad} - \frac{(a^2 + b^2) \log(a + b \sinh(c + dx))}{ab^2 d} + \frac{\sinh(c + dx)}{bd} \end{aligned}$$

**Mathematica [A]** time = 0.0817595, size = 48, normalized size = 0.84

$$\frac{-\left(\frac{a}{b^2} + \frac{1}{a}\right) \log(a + b \sinh(c + dx)) + \frac{\log(\sinh(c+dx))}{a} + \frac{\sinh(c+dx)}{b}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(Cosh[c + d\*x]^2\*Coth[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (Log[Sinh[c + d\*x]]/a - (a^(-1) + a/b^2)\*Log[a + b\*Sinh[c + d\*x]] + Sinh[c + d\*x]/b)/d

**Maple [B]** time = 0.078, size = 178, normalized size = 3.1

$$-\frac{1}{bd} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} + \frac{a}{b^2 d} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) - \frac{1}{bd} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^{-1} + \frac{a}{b^2 d} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) - 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)^2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out]  $-1/d/b/(\tanh(1/2*d*x+1/2*c)+1)+1/d*a/b^2*\ln(\tanh(1/2*d*x+1/2*c)+1)-1/d/b/(\tanh(1/2*d*x+1/2*c)-1)+1/d*a/b^2*\ln(\tanh(1/2*d*x+1/2*c)-1)+1/d*a*\ln(\tanh(1/2*d*x+1/2*c))-1/d*a/b^2*\ln(\tanh(1/2*d*x+1/2*c)^{2*a-2}*\tanh(1/2*d*x+1/2*c)*b-a)-1/d/a*\ln(\tanh(1/2*d*x+1/2*c)^{2*a-2}*\tanh(1/2*d*x+1/2*c)*b-a)$

**Maxima [B]** time = 1.08799, size = 176, normalized size = 3.09

$$-\frac{(dx+c)a}{b^2d} + \frac{e^{(dx+c)}}{2bd} - \frac{e^{(-dx-c)}}{2bd} + \frac{\log(e^{(-dx-c)}+1)}{ad} + \frac{\log(e^{(-dx-c)}-1)}{ad} - \frac{(a^2+b^2)\log(-2ae^{(-dx-c)}+be^{(-2dx-2c)}-b)}{ab^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-(d*x+c)*a/(b^2*d) + 1/2*e^{(d*x+c)}/(b*d) - 1/2*e^{(-d*x-c)}/(b*d) + \log(e^{(-d*x-c)}+1)/(a*d) + \log(e^{(-d*x-c)}-1)/(a*d) - (a^2+b^2)*\log(-a*e^{(-d*x-c)}+b*e^{(-2*d*x-2*c)}-b)/(a*b^2*d)$

**Fricas [B]** time = 3.20018, size = 533, normalized size = 9.35

$$\frac{2a^2dx \cosh(dx+c) + ab \cosh(dx+c)^2 + ab \sinh(dx+c)^2 - ab - 2((a^2+b^2) \cosh(dx+c) + (a^2+b^2) \sinh(dx+c)) \log(2(b \sinh(dx+c) + a)/(\cosh(dx+c) - \sinh(dx+c))) + 2(b^2 \cosh(dx+c) + b^2 \sinh(dx+c)) \log(2 \sinh(dx+c)/(\cosh(dx+c) - \sinh(dx+c))) + 2(a^2 d x + a b \cosh(dx+c)) \sinh(dx+c)}{2(ab^2d \cosh(dx+c) + a^2 d x + a b \cosh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $1/2*(2*a^2*d*x*\cosh(d*x+c) + a*b*\cosh(d*x+c)^2 + a*b*\sinh(d*x+c)^2 - a*b - 2*((a^2+b^2)*\cosh(d*x+c) + (a^2+b^2)*\sinh(d*x+c))*\log(2*(b*\sinh(d*x+c) + a)/(\cosh(d*x+c) - \sinh(d*x+c))) + 2*(b^2*\cosh(d*x+c) + b^2*\sinh(d*x+c))*\log(2*\sinh(d*x+c)/(\cosh(d*x+c) - \sinh(d*x+c))) + 2*(a^2*d*x + a*b*\cosh(d*x+c))*\sinh(d*x+c)/(a*b^2*d*\cosh(d*x+c) + a*b^2*d*\sinh(d*x+c))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(c + dx) \coth(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*\*2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(cosh(c + d\*x)\*\*2\*coth(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

**Giac [A]** time = 1.5483, size = 146, normalized size = 2.56

$$\frac{\frac{2adx}{b^2} + \frac{e^{(dx+c)}}{b} - \frac{e^{(-dx-c)}}{b} + \frac{2 \log(e^{(dx+c)}+1)}{a} + \frac{2 \log(|e^{(dx+c)}-1|)}{a} - \frac{2(a^2+b^2) \log(|be^{(2dx+2c)}+2ae^{(dx+c)}-b|)}{ab^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2\*coth(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] 1/2\*(2\*a\*d\*x/b^2 + e^(d\*x + c)/b - e^(-d\*x - c)/b + 2\*log(e^(d\*x + c) + 1)/a + 2\*log(abs(e^(d\*x + c) - 1))/a - 2\*(a^2 + b^2)\*log(abs(b\*e^(2\*d\*x + 2\*c) + 2\*a\*e^(d\*x + c) - b))/(a\*b^2))/d

$$3.434 \quad \int \frac{\cosh^2(c+dx) \coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable} \left( \frac{\cosh^2(c+dx) \coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[(Cosh[c + d\*x]^2\*Coth[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x]))], x]

**Rubi [A]** time = 0.0834457, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cosh^2(c+dx) \coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Cosh[c + d\*x]^2\*Coth[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] Defer[Int] [(Cosh[c + d\*x]^2\*Coth[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x]))], x]

Rubi steps

$$\int \frac{\cosh^2(c+dx) \coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\cosh^2(c+dx) \coth(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.002, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Cosh[c + d\*x]^2\*Coth[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] \$Aborted

**Maple [A]** time = 1.079, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx + c))^2 \coth(dx + c)}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)^2\*coth(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(cosh(d\*x+c)^2\*coth(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{e^{\left(-c + \frac{de}{f}\right)} E_1\left(\frac{(fx+e)d}{f}\right)}{2bf} - \frac{e^{\left(c - \frac{de}{f}\right)} E_1\left(-\frac{(fx+e)d}{f}\right)}{2bf} - \frac{a \log(fx + e)}{b^2 f} + \frac{1}{4} \int \frac{8(a^2 b + b^3 - (a^3 e^c + ab^2 e^e))e^{(d*x)}}{ab^3 fx + ab^3 e - (ab^3 fxe^{(2c)} + ab^3 ee^{(2c)})e^{(2dx)} - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)^2\*coth(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -1/2\*e^(-c + d\*e/f)\*exp\_integral\_e(1, (f\*x + e)\*d/f)/(b\*f) - 1/2\*e^(c - d\*e/f)\*exp\_integral\_e(1, -(f\*x + e)\*d/f)/(b\*f) - a\*log(f\*x + e)/(b^2\*f) + 1/4\*integrate(8\*(a^2\*b + b^3 - (a^3\*e^c + a\*b^2\*e^e)\*e^(d\*x))/(a\*b^3\*f\*x + a\*b^3\*e - (a\*b^3\*f\*x\*e^(2\*c) + a\*b^3\*e\*e^(2\*c))\*e^(2\*d\*x) - 2\*(a^2\*b^2\*f\*x\*e^c + a^2\*b^2\*e\*e^c)\*e^(d\*x)), x) - integrate(1/(a\*f\*x + a\*e + (a\*f\*x\*e^c + a\*e\*e^c)\*e^(d\*x)), x) + integrate(-1/(a\*f\*x + a\*e - (a\*f\*x\*e^c + a\*e\*e^c)\*e^(d\*x)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx + c)^2 \coth(dx + c)}{afx + ae + (bfx + be) \sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^2*coth(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(c + dx) \coth(c + dx)}{(a + b \sinh(c + dx))(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(cosh(c + d*x)**2*coth(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

sage<sub>0</sub>x

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*coth(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] sage0*x
```



$$3.435 \quad \int \frac{(e+fx)^3 \operatorname{csch}(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx$$

Optimal. Leaf size=1049

result too large to display

```
[Out] (-2*b*(e + f*x)^3*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (2*(e + f*x)^3*ArcTanh[E^(2*c + 2*d*x)])/(a*d) - (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) - (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) + (b^2*(e + f*x)^3*Log[1 + E^(2*(c + d*x))])/(a*(a^2 + b^2)*d) + ((3*I)*b*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) - ((3*I)*b*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d^2) + (3*b^2*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*a*(a^2 + b^2)*d^2) - (3*f*(e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a*d^2) + (3*f*(e + f*x)^2*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2) - ((6*I)*b*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) + ((6*I)*b*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/((a^2 + b^2)*d^3) + (6*b^2*f^2*(e + f*x)*PolyLog[3, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d^3) + (6*b^2*f^2*(e + f*x)*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d^3) - (3*b^2*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*a*(a^2 + b^2)*d^3) + (3*f^2*(e + f*x)*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a*d^3) - (3*f^2*(e + f*x)*PolyLog[3, E^(2*c + 2*d*x)])/(2*a*d^3) + ((6*I)*b*f^3*PolyLog[4, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^4) - ((6*I)*b*f^3*PolyLog[4, I*E^(c + d*x)])/((a^2 + b^2)*d^4) - (6*b^2*f^3*PolyLog[4, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d^4) - (6*b^2*f^3*PolyLog[4, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d^4) + (3*b^2*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*a*(a^2 + b^2)*d^4) - (3*f^3*PolyLog[4, -E^(2*c + 2*d*x)])/(4*a*d^4) + (3*f^3*PolyLog[4, E^(2*c + 2*d*x)])/(4*a*d^4)
```

**Rubi [A]** time = 1.34083, antiderivative size = 1049, normalized size of antiderivative = 1., number of steps used = 40, number of rules used = 13, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$ , Rules used = {5589, 5461, 4182, 2531, 6609, 2282, 6589, 5573, 5561, 2190, 6742, 4180, 3718}

$$\frac{6ib \operatorname{PolyLog}\left(4, -ie^{c+dx}\right) f^3}{(a^2 + b^2) d^4} - \frac{6ib \operatorname{PolyLog}\left(4, ie^{c+dx}\right) f^3}{(a^2 + b^2) d^4} - \frac{6b^2 \operatorname{PolyLog}\left(4, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right) f^3}{a (a^2 + b^2) d^4} - \frac{6b^2 \operatorname{PolyLog}\left(4, -\frac{be^{c+dx}}{a + \sqrt{a^2 + b^2}}\right) f^3}{a (a^2 + b^2) d^4}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Csch[c + d\*x]\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] 
$$\begin{aligned} & (-2*b*(e + f*x)^3*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (2*(e + f*x)^3*Arc \\ & Tanh[E^(2*c + 2*d*x)])/(a*d) - (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a \\ & - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) - (b^2*(e + f*x)^3*Log[1 + (b*E^(c + \\ & d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) + (b^2*(e + f*x)^3*Log[1 + \\ & E^(2*(c + d*x))]/(a*(a^2 + b^2)*d) + ((3*I)*b*f*(e + f*x)^2*PolyLog[2, (- \\ & I)*E^(c + d*x)])/((a^2 + b^2)*d^2) - ((3*I)*b*f*(e + f*x)^2*PolyLog[2, I*E^( \\ & (c + d*x)])/((a^2 + b^2)*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + \\ & d*x))/(a - Sqrt[a^2 + b^2]))]/(a*(a^2 + b^2)*d^2) - (3*b^2*f*(e + f*x)^2* \\ & PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a*(a^2 + b^2)*d^2) + \\ & (3*b^2*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))]/(2*a*(a^2 + b^2)*d^2) - \\ & (3*f*(e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a*d^2) + (3*f*(e + f*x)^ \\ & 2*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2) - ((6*I)*b*f^2*(e + f*x)*PolyLog[3 \\ & , (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) + ((6*I)*b*f^2*(e + f*x)*PolyLog[3, \\ & I*E^(c + d*x)])/((a^2 + b^2)*d^3) + (6*b^2*f^2*(e + f*x)*PolyLog[3, -((b*E^( \\ & (c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a*(a^2 + b^2)*d^3) + (6*b^2*f^2*(e + f \\ & *x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a*(a^2 + b^2)*d^ \\ & 3) - (3*b^2*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))]/(2*a*(a^2 + b^2)*d^ \\ & 3) + (3*f^2*(e + f*x)*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a*d^3) - (3*f^2*(e + \\ & f*x)*PolyLog[3, E^(2*c + 2*d*x)])/(2*a*d^3) + ((6*I)*b*f^3*PolyLog[4, (-I) \\ & *E^(c + d*x)])/((a^2 + b^2)*d^4) - ((6*I)*b*f^3*PolyLog[4, I*E^(c + d*x)])/ \\ & ((a^2 + b^2)*d^4) - (6*b^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + \\ & b^2]))]/(a*(a^2 + b^2)*d^4) - (6*b^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a \\ & + Sqrt[a^2 + b^2]))]/(a*(a^2 + b^2)*d^4) + (3*b^2*f^3*PolyLog[4, -E^(2*(c \\ & + d*x))]/(4*a*(a^2 + b^2)*d^4) - (3*f^3*PolyLog[4, -E^(2*c + 2*d*x)])/(4*a \\ & *d^4) + (3*f^3*PolyLog[4, E^(2*c + 2*d*x)])/(4*a*d^4) \end{aligned}$$

### Rule 5589

Int[(Csch[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(p\_.)]/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Sech[c + d\*x]^p\*Csch[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Sech[c + d\*x]^p\*Csch[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5461

Int[Csch[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sech[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Dist[2^n, Int[(c + d\*x)^m\*Csch[2\*a + 2\*b\*x]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^m, x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3718

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[(((c
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \operatorname{csch}(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \operatorname{csch}(c+dx) \operatorname{sech}(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^3 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= \frac{2 \int (e+fx)^3 \operatorname{csch}(2c+2dx) dx}{a} - \frac{b \int (e+fx)^3 \operatorname{sech}(c+dx) (a-b \sinh(c+dx)) dx}{a(a^2+b^2)} \\
&= \frac{b^2(e+fx)^4}{4a(a^2+b^2)f} - \frac{2(e+fx)^3 \tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b \int (a(e+fx)^3 \operatorname{sech}(c+dx)) dx}{a(a^2+b^2)} \\
&= \frac{b^2(e+fx)^4}{4a(a^2+b^2)f} - \frac{2(e+fx)^3 \tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)d} \\
&= -\frac{2b(e+fx)^3 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^3 \tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)d} \\
&= -\frac{2b(e+fx)^3 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^3 \tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)d} \\
&= -\frac{2b(e+fx)^3 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^3 \tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)d} \\
&= -\frac{2b(e+fx)^3 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^3 \tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)d} \\
&= -\frac{2b(e+fx)^3 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^3 \tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)d} \\
&= -\frac{2b(e+fx)^3 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^3 \tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^3 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)d}
\end{aligned}$$

**Mathematica [B]** time = 32.8475, size = 3000, normalized size = 2.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
]
```

```

[Out] 2*((a*E^c*((e + f*x)^4/(4*E^c*f) + ((1 + E^(-c))*(e + f*x)^3*Log[1 + E^(-c - d*x)]))/d - (3*(1 + E^c)*f*(d^2*(e + f*x)^2*PolyLog[2, -E^(-c - d*x)] + 2*f*(d*(e + f*x)*PolyLog[3, -E^(-c - d*x)] + f*PolyLog[4, -E^(-c - d*x)])))/(d^4*E^c)))/(2*(a^2 + b^2)*(1 + E^c)) + ((I/2)*a*E^c*((e + f*x)^4/(4*E^c*f) + ((I + E^(-c))*(e + f*x)^3*Log[1 - I*E^(-c - d*x)]))/d - (3*(1 + I*E^c)*f*(d^2*(e + f*x)^2*PolyLog[2, I*E^(-c - d*x)] + 2*f*(d*(e + f*x)*PolyLog[3, I*E^(-c - d*x)] + f*PolyLog[4, I*E^(-c - d*x)])))/(d^4*E^c)))/((a^2 + b^2)*(-I + E^c)) - (b^2*E^(2*c))*((e + f*x)^4/(E^(2*c)*f) - (2*(1 - E^(-2*c)))*(e + f*x)^3*Log[1 - E^(-c - d*x)]))/d - (2*(1 - E^(-2*c)))*(e + f*x)^3*Log[1 + E^(-c - d*x)]))/d + (6*(-1 + E^(2*c))*f*(d^2*(e + f*x)^2*PolyLog[2, -E^(-c - d*x)] + 2*f*(d*(e + f*x)*PolyLog[3, -E^(-c - d*x)] + f*PolyLog[4, -E^(-c - d*x)])))/(d^4*E^(2*c)) + (6*(-1 + E^(2*c))*f*(d^2*(e + f*x)^2*PolyLog[2, E^(-c - d*x)] + 2*f*(d*(e + f*x)*PolyLog[3, E^(-c - d*x)] + f*PolyLog[4, E^(-c - d*x)])))/(d^4*E^(2*c)))/(4*a*(a^2 + b^2)*(-1 + E^(2*c))) - ((I/2)*b*((-2*I)*d^3*e^3*ArcTan[E^(c + d*x)] + 3*d^3*e^2*f*x*Log[1 - I*E^(c + d*x)] + 3*d^3*e*f^2*x^2*Log[1 - I*E^(c + d*x)] - 3*d^3*e^2*f*x*Log[1 + I*E^(c + d*x)] - 3*d^3*f^3*x^3*Log[1 - I*E^(c + d*x)] - 3*d^3*f^2*x^2*Log[1 + I*E^(c + d*x)] - 3*d^3*f^3*x^3*Log[1 + I*E^(c + d*x)] - 3*d^2*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)] + 3*d^2*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)] + 6*d*e*f^2*PolyLog[3, (-I)*E^(c + d*x)] + 6*d*f^3*x*PolyLog[3, (-I)*E^(c + d*x)] - 6*d*e*f^2*PolyLog[3, I*E^(c + d*x)] - 6*d*f^3*x*PolyLog[3, I*E^(c + d*x)] - 6*f^3*PolyLog[4, (-I)*E^(c + d*x)] + 6*f^3*PolyLog[4, I*E^(c + d*x)])))/(a^2 + b^2)*d^4) - (a*(-((e + f*x)^3*Log[1 - E^(c + d*x)] + (e + f*x)^3*Log[1 - I*E^(c + d*x)] + (3*f*(d^2*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)] - 2*d*f*(e + f*x)*PolyLog[3, I*E^(c + d*x)] + 2*f^2*PolyLog[4, I*E^(c + d*x)])))/d^3 - (3*f*(d^2*(e + f*x)^2*PolyLog[2, E^(c + d*x)] - 2*d*f*(e + f*x)*PolyLog[3, E^(c + d*x)] + 2*f^2*PolyLog[4, E^(c + d*x)]))/d^3))/(2*(a^2 + b^2)*d + (b^2*((E^(2*c))*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3))/(-1 + E^(2*c)) - (2*(d^3*e^3*Log[b - 2*a*E^(c + d*x)] - b*E^(2*(c + d*x))] + 3*d^3*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]]] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]]] + d^3*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]]] + 3*d^3*e^2*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]]] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]]] + d^3*f^3*x^3*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]]] + 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]]]) + 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]]]) - 6*d*e*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]]]) - 6*d*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]]]) - 6*d*e*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]]]) - 6*d*f^3*x*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]]]) + 6*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]]]) + 6*f^3*PolyLog[4, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]]])])))/d^4)/(4*a*(a^2 + b^2)) - (b^2*x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*x^3)

```

```

*Csch[c/2]*Sech[c/2]*Sech[c])/(32*a*(a^2 + b^2)) + (3*a*e^2*f*x^2*Csch[c/2]
*Sech[c/2])/(16*(a^2 + b^2)*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c
/2])) + (3*b^2*e^2*f*x^2*Csch[c/2]*Sech[c/2])/(16*a*(a^2 + b^2)*(Cosh[c/2]
- I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c/2])) + (a*e*f^2*x^3*Csch[c/2]*Sech[c/2
])/(8*(a^2 + b^2)*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c/2])) + (b
^2*e*f^2*x^3*Csch[c/2]*Sech[c/2])/(8*a*(a^2 + b^2)*(Cosh[c/2] - I*Sinh[c/2]
)*(Cosh[c/2] + I*Sinh[c/2])) + (a*f^3*x^4*Csch[c/2]*Sech[c/2])/(32*(a^2 + b
^2)*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c/2])) + (b^2*f^3*x^4*Csco
h[c/2]*Sech[c/2])/(32*a*(a^2 + b^2)*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] +
I*Sinh[c/2])) - (3*a*e^2*f*x^2*Cosh[c]*Csch[c/2]*Sech[c/2])/(16*(a^2 + b^2)
*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c/2])) - (a*e*f^2*x^3*Cosh[c
]*Csch[c/2]*Sech[c/2])/(8*(a^2 + b^2)*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2]
+ I*Sinh[c/2])) - (a*f^3*x^4*Cosh[c]*Csch[c/2]*Sech[c/2])/(32*(a^2 + b^2)*(
Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c/2])) - (((3*I)/16)*a*e^2*f*x
^2*Csch[c/2]*Sech[c/2]*Sinh[c])/((a^2 + b^2)*(Cosh[c/2] - I*Sinh[c/2])*(Cos
h[c/2] + I*Sinh[c/2])) - ((I/8)*a*e*f^2*x^3*Csch[c/2]*Sech[c/2]*Sinh[c])/((
a^2 + b^2)*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c/2])) - ((I/32)*a
*f^3*x^4*Csch[c/2]*Sech[c/2]*Sinh[c])/((a^2 + b^2)*(Cosh[c/2] - I*Sinh[c/2]
)*(Cosh[c/2] + I*Sinh[c/2])) - (e^3*x*Csch[c/2]*Sech[c/2]*(-a^2 - b^2 + a^2
*Cosh[c] + I*a^2*Sinh[c]))/(8*a*(a^2 + b^2)*(Cosh[c/2] - I*Sinh[c/2])*(Cosh
[c/2] + I*Sinh[c/2]))

```

**Maple [F]** time = 0.756, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \operatorname{csch}(dx + c) \operatorname{sech}(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*csch(d\*x+c)\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$-e^3*(b^2*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/((a^3 + a*b^2)*d) - 2*b*\arctan(e^{(-d*x - c)})/((a^2 + b^2)*d) + a*\log(e^{(-2*d*x - 2*c)} + 1)/((a^2 + b^2)*d) - \log(e^{(-d*x - c)} + 1)/(a*d) - \log(e^{(-d*x - c)} - 1)/(a*d) + 3*(d*x*\log(e^{(d*x + c)} + 1) + \operatorname{dilog}(-e^{(d*x + c)})) * e^2*f/(a*d^2) + 3*(d*x*\log(-e^{(d*x + c)} + 1) + \operatorname{dilog}(e^{(d*x + c)})) * e^2*f/(a*d^2) + 3*(d^2*x^2*\log(e^{(d*x + c)} + 1) + 2*d*x*\operatorname{dilog}(-e^{(d*x + c)}) - 2*\operatorname{polylog}(3, -e^{(d*x + c)})) * e*f^2/(a*d^3) + 3*(d^2*x^2*\log(-e^{(d*x + c)} + 1) + 2*d*x*\operatorname{dilog}(e^{(d*x + c)})) - 2*\operatorname{polylog}(3, e^{(d*x + c)}) * e*f^2/(a*d^3) + (d^3*x^3*\log(e^{(d*x + c)} + 1) + 3*d^2*x^2*\operatorname{dilog}(-e^{(d*x + c)}) - 6*d*x*\operatorname{polylog}(3, -e^{(d*x + c)}) + 6*\operatorname{polylog}(4, -e^{(d*x + c)})) * f^3/(a*d^4) + (d^3*x^3*\log(-e^{(d*x + c)} + 1) + 3*d^2*x^2*\operatorname{dilog}(e^{(d*x + c)}) - 6*d*x*\operatorname{polylog}(3, e^{(d*x + c)}) + 6*\operatorname{polylog}(4, e^{(d*x + c)})) * f^3/(a*d^4) - 1/2*(d^4*f^3*x^4 + 4*d^4*e*f^2*x^3 + 6*d^4*e^2*f*x^2)/(a*d^4) + \operatorname{integrate}(2*(b^3*f^3*x^3 + 3*b^3*e*f^2*x^2 + 3*b^3*e^2*f*x - (a*b^2*f^3*x^3*e^c + 3*a*b^2*e*f^2*x^2*e^c + 3*a*b^2*e^2*f*x*e^c)) * e^{(d*x)})/(a^3*b + a*b^3 - (a^3*b*e^{(2*c)} + a*b^3*e^{(2*c)})) * e^{(2*d*x)} - 2*(a^4*e^c + a^2*b^2*e^c) * e^{(d*x)}, x) - \operatorname{integrate}(-2*(a*f^3*x^3 + 3*a*e*f^2*x^2 + 3*a*e^2*f*x - (b*f^3*x^3*e^c + 3*b*e*f^2*x^2*e^c + 3*b*e^2*f*x*e^c)) * e^{(d*x)})/(a^2 + b^2 + (a^2*e^{(2*c)} + b^2*e^{(2*c)})) * e^{(2*d*x)}, x)$$

---

**Fricas [C]** time = 3.69805, size = 5839, normalized size = 5.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*csch(d\*x+c)\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-(6*b^2*f^3*\operatorname{polylog}(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) + 6*b^2*f^3*\operatorname{polylog}(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) - 6*(a^2 + b^2)*f^3*\operatorname{polylog}(4, \cosh(d*x + c) + \sinh(d*x + c)) - 6*(a^2 + b^2)*f^3*\operatorname{polylog}(4, -\cosh(d*x + c) - \sinh(d*x + c)) + 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 3*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*\operatorname{dilog}(\cosh(d*x + c) + \sinh(d*x + c))$$



$$\begin{aligned}
& c)) + (3a^2d^2f^3x^2 + 3Iab^2d^2f^3x^2 + 6a^2d^2e^2f^2x + 6Iab^2d^2e^2f^2x + 3a^2d^2e^2f + 3Iab^2d^2e^2f) \operatorname{dilog}(I \cosh(dx + c) \\
& + I \sinh(dx + c)) + (3a^2d^2f^3x^2 - 3Iab^2d^2f^3x^2 + 6a^2d^2e^2f^2x - 6Iab^2d^2e^2f^2x + 3a^2d^2e^2f - 3Iab^2d^2e^2f) \operatorname{dilog}(-I \cosh(dx + c) - I \sinh(dx + c)) - 3((a^2 + b^2)d^2f^3x^2 + 2(a^2 + \\
& b^2)d^2e^2f^2x + (a^2 + b^2)d^2e^2f) \operatorname{dilog}(-\cosh(dx + c) - \sinh(dx + c)) + (b^2d^3e^3 - 3b^2cd^2e^2f + 3b^2c^2d^2e^2f - b^2c^3f^3) \\
& * \log(2b \cosh(dx + c) + 2b \sinh(dx + c) + 2b \sqrt{(a^2 + b^2)/b^2} + 2a) + (b^2d^3e^3 - 3b^2cd^2e^2f + 3b^2c^2d^2e^2f - b^2c^3f^3) * \log(2b \cosh(dx + c) + 2b \sinh(dx + c) - 2b \sqrt{(a^2 + b^2)/b^2} + 2a) \\
& + (b^2d^3f^3x^3 + 3b^2d^3e^2f^2x^2 + 3b^2d^3e^2f^2x + 3b^2cd^2e^2f - 3b^2c^2d^2e^2f + b^2c^3f^3) * \log(-(a \cosh(dx + c) + a \sinh(dx + c)) + (b \cosh(dx + c) + b \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2} - b)/b) + \\
& (b^2d^3f^3x^3 + 3b^2d^3e^2f^2x^2 + 3b^2d^3e^2f^2x + 3b^2cd^2e^2f - 3b^2c^2d^2e^2f + b^2c^3f^3) * \log(-(a \cosh(dx + c) + a \sinh(dx + c)) - (b \cosh(dx + c) + b \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2} - b)/b) - \\
& ((a^2 + b^2)d^3f^3x^3 + 3(a^2 + b^2)d^3e^2f^2x^2 + 3(a^2 + b^2)d^3e^2f^2x + (a^2 + b^2)d^3e^3) * \log(\cosh(dx + c) + \sinh(dx + c) + 1) + (a^2d^3e^3 + Iab^2d^3e^3 - 3a^2cd^2e^2f - 3Iab^2cd^2e^2f + 3a^2c^2d^2e^2f + 3Iab^2c^2d^2e^2f - a^2c^3f^3 - Iab^2c^3f^3) * \log(\cosh(dx + c) + \sinh(dx + c) + 1) + (a^2d^3e^3 - Iab^2d^3e^3 - 3a^2cd^2e^2f + 3Iab^2cd^2e^2f + 3a^2c^2d^2e^2f - 3Iab^2c^2d^2e^2f - a^2c^3f^3 + Iab^2c^3f^3) * \log(\cosh(dx + c) + \sinh(dx + c) - 1) - ((a^2 + b^2)d^3e^3 - 3(a^2 + b^2)cd^2e^2f + 3(a^2 + b^2)c^2d^2e^2f - (a^2 + b^2)c^3f^3) * \log(\cosh(dx + c) + \sinh(dx + c) - 1) + (a^2d^3f^3x^3 - Iab^2d^3f^3x^3 + 3a^2d^3e^2f^2x^2 - 3Iab^2d^3e^2f^2x^2 + 3a^2d^3e^2f^2x - 3Iab^2d^3e^2f^2x + 3a^2cd^2e^2f - 3Iab^2cd^2e^2f - 3a^2c^2d^2e^2f + 3Iab^2c^2d^2e^2f + a^2c^3f^3 - Iab^2c^3f^3) * \log(I \cosh(dx + c) + I \sinh(dx + c) + 1) + (a^2d^3f^3x^3 + Iab^2d^3f^3x^3 + 3a^2d^3e^2f^2x^2 + 3Iab^2d^3e^2f^2x^2 + 3a^2d^3e^2f^2x + 3Iab^2d^3e^2f^2x + 3a^2cd^2e^2f + 3Iab^2cd^2e^2f - 3a^2c^2d^2e^2f - 3Iab^2c^2d^2e^2f + a^2c^3f^3 + Iab^2c^3f^3) * \log(-I \cosh(dx + c) - I \sinh(dx + c) + 1) - ((a^2 + b^2)d^3f^3x^3 + 3(a^2 + b^2)d^3e^2f^2x^2 + 3(a^2 + b^2)d^3e^2f^2x + 3(a^2 + b^2)cd^2e^2f - 3(a^2 + b^2)c^2d^2e^2f + (a^2 + b^2)c^3f^3) * \log(-\cosh(dx + c) - \sinh(dx + c) + 1) + (6a^2f^3 + 6Iab^2f^3) * \operatorname{polylog}(4, I \cosh(dx + c) + I \sinh(dx + c)) + (6a^2f^3 - 6Iab^2f^3) * \operatorname{polylog}(4, -I \cosh(dx + c) - I \sinh(dx + c)) - 6(b^2d^2f^3x + b^2d^2e^2f^2) * \operatorname{polylog}(3, (a \cosh(dx + c) + a \sinh(dx + c)) + (b \cosh(dx + c) + b \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2})/b) - 6(b^2d^2f^3x + b^2d^2e^2f^2) * \operatorname{polylog}(3, (a \cosh(dx + c) + a \sinh(dx + c)) - (b \cosh(dx + c) + b \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2})/b) + 6((a^2 + b^2)d^2f^3x + (a^2 + b^2)d^2e^2f^2) * \operatorname{polylog}(3, \cosh(dx + c) + \sinh(dx + c)) - (6a^2d^2f^3x + 6Iab^2d^2f^3x + 6a^2d^2e^2f^2 + 6Iab^2d^2e^2f^2) * \operatorname{polylog}(3, I \cosh(dx + c) + I \sinh(dx + c)) - (6a^2d^2f^3x - 6Iab^2d^2f^3x + 6a^2d^2e^2f^2 - 6Iab^2d^2e^2f^2) * \operatorname{polylog}(3, -I \cosh(dx + c) - I \sinh(dx + c))
\end{aligned}$$

$h(d*x + c)) + 6*((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*\text{polylog}(3, -\cos$   
 $h(d*x + c) - \sinh(d*x + c)))/((a^3 + a*b^2)*d^4)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cscsch(d\*x+c)\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \operatorname{csch}(dx + c) \operatorname{sech}(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cscsch(d\*x+c)\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^3\*cscsch(d\*x + c)\*sech(d\*x + c)/(b\*sinh(d\*x + c) + a), x)

$$3.436 \quad \int \frac{(e+fx)^2 \operatorname{csch}(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=734

$$\frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2(a^2+b^2)} - \frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2(a^2+b^2)} + \frac{b^2 f(e+fx) \operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{ad^2(a^2+b^2)} +$$

[Out]  $(-2*b*(e + f*x)^2*\operatorname{ArcTan}[E^{(c + d*x)}])/((a^2 + b^2)*d) - (2*(e + f*x)^2*\operatorname{ArcTanh}[E^{(2*c + 2*d*x)}])/(a*d) - (b^2*(e + f*x)^2*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2])])/(a*(a^2 + b^2)*d) - (b^2*(e + f*x)^2*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2])])/(a*(a^2 + b^2)*d) + (b^2*(e + f*x)^2*\operatorname{Log}[1 + E^{(2*(c + d*x))}])/(a*(a^2 + b^2)*d) + ((2*I)*b*f*(e + f*x)*\operatorname{PolyLog}[2, (-I)*E^{(c + d*x)}])/((a^2 + b^2)*d^2) - ((2*I)*b*f*(e + f*x)*\operatorname{PolyLog}[2, I*E^{(c + d*x)}])/((a^2 + b^2)*d^2) - (2*b^2*f*(e + f*x)*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) - (2*b^2*f*(e + f*x)*\operatorname{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) + (b^2*f*(e + f*x)*\operatorname{PolyLog}[2, -E^{(2*(c + d*x))}])/(a*(a^2 + b^2)*d^2) - (f*(e + f*x)*\operatorname{PolyLog}[2, -E^{(2*c + 2*d*x)}])/(a*d^2) + (f*(e + f*x)*\operatorname{PolyLog}[2, E^{(2*c + 2*d*x)}])/(a*d^2) - ((2*I)*b*f^2*\operatorname{PolyLog}[3, (-I)*E^{(c + d*x)}])/((a^2 + b^2)*d^3) + ((2*I)*b*f^2*\operatorname{PolyLog}[3, I*E^{(c + d*x)}])/((a^2 + b^2)*d^3) + (2*b^2*f^2*\operatorname{PolyLog}[3, -((b*E^{(c + d*x)})/(a - \operatorname{Sqrt}[a^2 + b^2]))])/(a*(a^2 + b^2)*d^3) + (2*b^2*f^2*\operatorname{PolyLog}[3, -((b*E^{(c + d*x)})/(a + \operatorname{Sqrt}[a^2 + b^2]))])/(a*(a^2 + b^2)*d^3) - (b^2*f^2*\operatorname{PolyLog}[3, -E^{(2*(c + d*x))}])/(2*a*(a^2 + b^2)*d^3) + (f^2*\operatorname{PolyLog}[3, -E^{(2*c + 2*d*x)}])/(2*a*d^3) - (f^2*\operatorname{PolyLog}[3, E^{(2*c + 2*d*x)}])/(2*a*d^3)$

**Rubi [A]** time = 1.09728, antiderivative size = 734, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 12, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5589, 5461, 4182, 2531, 2282, 6589, 5573, 5561, 2190, 6742, 4180, 3718}

$$\frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2(a^2+b^2)} - \frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2(a^2+b^2)} + \frac{b^2 f(e+fx) \operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{ad^2(a^2+b^2)} +$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e + f*x)^2*\operatorname{CsCh}[c + d*x]*\operatorname{SeCh}[c + d*x])/(a + b*\operatorname{Sinh}[c + d*x]), x]$

[Out]  $(-2*b*(e + f*x)^2*\operatorname{ArcTan}[E^{(c + d*x)}])/((a^2 + b^2)*d) - (2*(e + f*x)^2*\operatorname{ArcTanh}[E^{(2*c + 2*d*x)}])/(a*d) - (b^2*(e + f*x)^2*\operatorname{Log}[1 + (b*E^{(c + d*x)})/(a$

$$\begin{aligned}
& - \text{Sqrt}[a^2 + b^2]]]/(a*(a^2 + b^2)*d) - (b^2*(e + f*x)^2*\text{Log}[1 + (b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b^2])]]/(a*(a^2 + b^2)*d) + (b^2*(e + f*x)^2*\text{Log}[1 + E^(2*(c + d*x))]]/(a*(a^2 + b^2)*d) + ((2*I)*b*f*(e + f*x)*\text{PolyLog}[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) - ((2*I)*b*f*(e + f*x)*\text{PolyLog}[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) - (2*b^2*f*(e + f*x)*\text{PolyLog}[2, -((b*E^(c + d*x))/(a - \text{Sqrt}[a^2 + b^2]))]]/(a*(a^2 + b^2)*d^2) - (2*b^2*f*(e + f*x)*\text{PolyLog}[2, -((b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b^2]))]]/(a*(a^2 + b^2)*d^2) + (b^2*f*(e + f*x)*\text{PolyLog}[2, -E^(2*(c + d*x))]]/(a*(a^2 + b^2)*d^2) - (f*(e + f*x)*\text{PolyLog}[2, -E^(2*c + 2*d*x)])/(a*d^2) + (f*(e + f*x)*\text{PolyLog}[2, E^(2*c + 2*d*x)])/(a*d^2) - ((2*I)*b*f^2*\text{PolyLog}[3, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^3) + ((2*I)*b*f^2*\text{PolyLog}[3, I*E^(c + d*x)])/((a^2 + b^2)*d^3) + (2*b^2*f^2*\text{PolyLog}[3, -((b*E^(c + d*x))/(a - \text{Sqrt}[a^2 + b^2]))]]/(a*(a^2 + b^2)*d^3) + (2*b^2*f^2*\text{PolyLog}[3, -((b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b^2]))]]/(a*(a^2 + b^2)*d^3) - (b^2*f^2*\text{PolyLog}[3, -E^(2*(c + d*x))]]/(2*a*(a^2 + b^2)*d^3) + (f^2*\text{PolyLog}[3, -E^(2*c + 2*d*x)])/(2*a*d^3) - (f^2*\text{PolyLog}[3, E^(2*c + 2*d*x)])/(2*a*d^3)
\end{aligned}$$

### Rule 5589

```

Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)]/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

```

### Rule 5461

```

Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]

```

### Rule 4182

```

Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

```

### Rule 2531

```

Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))]^(n_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*\text{PolyLog}[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*\text{Log}[F]), x] + Dist[(g*m)/(b*c*n*\text{Log}[F]), Int[(f + g*x)^(m -

```

1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 5573

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f\*x)^m\*Sech[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f\*x)^m\*Sech[c + d\*x]^n\*(a - b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]

### Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \operatorname{csch}(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \operatorname{csch}(c+dx) \operatorname{sech}(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= \frac{2 \int (e+fx)^2 \operatorname{csch}(2c+2dx) dx}{a} - \frac{b \int (e+fx)^2 \operatorname{sech}(c+dx) (a-b \sinh(c+dx)) dx}{a(a^2+b^2)} \\
&= \frac{b^2(e+fx)^3}{3a(a^2+b^2)f} - \frac{2(e+fx)^2 \tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b \int (a(e+fx)^2 \operatorname{sech}(c+dx)) dx}{a(a^2+b^2)} \\
&= \frac{b^2(e+fx)^3}{3a(a^2+b^2)f} - \frac{2(e+fx)^2 \tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)d} \\
&= -\frac{2b(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^2 \tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)d} \\
&= -\frac{2b(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^2 \tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)d} \\
&= -\frac{2b(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^2 \tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)d} \\
&= -\frac{2b(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^2 \tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)d} \\
&= -\frac{2b(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)^2 \tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)d}
\end{aligned}$$

**Mathematica [B]** time = 33.66, size = 3268, normalized size = 4.45

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Csch[c + d\*x]\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]),x  
]

[Out] 2\*(-(b^2\*x\*(3\*e^2 + 3\*e\*f\*x + f^2\*x^2)\*Csch[2\*c])/(6\*a\*(a^2 + b^2)) + (a\*E^c\*((e + f\*x)^3/(3\*E^c\*f) + ((1 + E^(-c))\*(e + f\*x)^2\*Log[1 + E^(-c - d\*x)]))

$$\begin{aligned}
& /d - (2*(1 + E^c)*f*(d*(e + f*x)*PolyLog[2, -E^(-c - d*x)] + f*PolyLog[3, - \\
& E^(-c - d*x)])))/(d^3*E^c))/(2*(a^2 + b^2)*(1 + E^c)) + (d^2*(d*x*((-3*I)*b \\
& *e*f*x + a*((-3*I)*e^2*E^c + 3*e*f*x + f^2*x^2)) + 3*(1 + I*E^c)*f*x*(2*a*e \\
& - (2*I)*b*e + a*f*x)*Log[1 - I*E^(-c - d*x)] + 3*a*e^2*(1 + I*E^c)*Log[I - \\
& E^c + d*x]) - (6*I)*d*(-I + E^c)*f*((-I)*b*e + a*(e + f*x))*PolyLog[2, I \\
& *E^(-c - d*x)] - (6*I)*a*(-I + E^c)*f^2*PolyLog[3, I*E^(-c - d*x)]/(6*(a - \\
& I*b)*((-I)*a + b)*d^3*(-I + E^c) - (b^2*E^(2*c))*((2*(e + f*x)^3)/(E^(2*c) \\
& *f) - (3*(1 - E^(-2*c))*(e + f*x)^2*Log[1 - E^(-c - d*x)]))/d - (3*(1 - E^(- \\
& 2*c))*e + f*x)^2*Log[1 + E^(-c - d*x)]/d + (6*(-1 + E^(2*c))*f*(d*(e + f* \\
& x)*PolyLog[2, -E^(-c - d*x)] + f*PolyLog[3, -E^(-c - d*x)]))/(d^3*E^(2*c)) \\
& + (6*(-1 + E^(2*c))*f*(d*(e + f*x)*PolyLog[2, E^(-c - d*x)] + f*PolyLog[3, \\
& E^(-c - d*x)]))/(d^3*E^(2*c)))/(6*a*(a^2 + b^2)*(-1 + E^(2*c))) - ((I/2)*b \\
& *((-2*I)*d^2*e^2*ArcTan[E^c + d*x] + d^2*f^2*x^2*Log[1 - I*E^c + d*x] - \\
& d^2*f^2*x^2*Log[1 + I*E^c + d*x] - 2*d*f^2*x*PolyLog[2, (-I)*E^c + d*x] \\
& ] + 2*d*f^2*x*PolyLog[2, I*E^c + d*x] + 2*f^2*PolyLog[3, (-I)*E^c + d*x] \\
& ] - 2*f^2*PolyLog[3, I*E^c + d*x]))/((a^2 + b^2)*d^3) - ((-I)*b*d^3*e*E^c \\
& (2*c)*f*x^2 + 2*a*d^2*e^2*ArcTan[1 - (1 + I)*E^c + d*x] + (2*I)*a*d^2*e^2* \\
& E^(2*c)*ArcTan[1 - (1 + I)*E^c + d*x] + (2*I)*a*d^2*e*f*x*Log[1 - E^c + \\
& d*x] - 2*a*d^2*e*E^(2*c)*f*x*Log[1 - E^c + d*x] + I*a*d^2*f^2*x^2*Log[1 \\
& - E^c + d*x] - a*d^2*E^(2*c)*f^2*x^2*Log[1 - E^c + d*x] - (2*I)*a*d^2*e \\
& *f*x*Log[1 - I*E^c + d*x] + 2*b*d^2*e*f*x*Log[1 - I*E^c + d*x] + 2*a*d^ \\
& 2*e*E^(2*c)*f*x*Log[1 - I*E^c + d*x] + (2*I)*b*d^2*e*E^(2*c)*f*x*Log[1 - \\
& I*E^c + d*x] - I*a*d^2*f^2*x^2*Log[1 - I*E^c + d*x] + a*d^2*E^(2*c)*f^2 \\
& *x^2*Log[1 - I*E^c + d*x] + 2*d*(-I + E^(2*c))*f*(I*b*e + a*(e + f*x))*Po \\
& lyLog[2, I*E^c + d*x] - 2*a*d*(-I + E^(2*c))*f*(e + f*x)*PolyLog[2, E^c + \\
& d*x] + (2*I)*a*f^2*PolyLog[3, I*E^c + d*x] - 2*a*E^(2*c)*f^2*PolyLog[3 \\
& , I*E^c + d*x] - (2*I)*a*f^2*PolyLog[3, E^c + d*x] + 2*a*E^(2*c)*f^2*Po \\
& lyLog[3, E^c + d*x]]/(2*(a^2 + b^2)*d^3*(-I + E^(2*c))) + (b^2*(6*e^2*E^c \\
& (2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*sqrt[a^2 + b^2]*e^2*A \\
& rcTan[(a + b*E^c + d*x))/sqrt[-a^2 - b^2]]/sqrt[-(a^2 + b^2)^2]*d) + (6* \\
& a*sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^c + d*x))/sqrt[-a^2 - b \\
& ^2]]/((a^2 + b^2)^(3/2)*d) - (6*a*sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b* \\
& E^c + d*x))/sqrt[a^2 + b^2]]/((-a^2 - b^2)^(3/2)*d) + (6*a*sqrt[-(a^2 + b \\
& ^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^c + d*x))/sqrt[a^2 + b^2]]/((-a^2 - b \\
& ^2)^(3/2)*d) + (3*e^2*Log[2*a*E^c + d*x] + b*(-1 + E^(2*(c + d*x))))/d - \\
& (3*e^2*E^(2*c)*Log[2*a*E^c + d*x] + b*(-1 + E^(2*(c + d*x))))/d + (6*e*f* \\
& x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - sqrt[(a^2 + b^2)*E^(2*c)]])/d - (6*e* \\
& E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - sqrt[(a^2 + b^2)*E^(2*c)]]) \\
& /d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - sqrt[(a^2 + b^2)*E^(2*c) \\
& ]])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - sqrt[(a^2 + \\
& b^2)*E^(2*c)]])/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + sqrt[(a^2 \\
& + b^2)*E^(2*c)]])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + \\
& sqrt[(a^2 + b^2)*E^(2*c)]])/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^ \\
& c + sqrt[(a^2 + b^2)*E^(2*c)]])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + \\
& d*x))/(a*E^c + sqrt[(a^2 + b^2)*E^(2*c)]])/d - (6*(-1 + E^(2*c))*f*(e + f
\end{aligned}$$



```

*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])))/d
^2 - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c +
Sqrt[(a^2 + b^2)*E^(2*c)])))]/d^2 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(
a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])))]/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*
E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])))]/d^3 - (6*f^2*PolyLog[
3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])))]/d^3 + (6*E^(2
*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))
])/d^3)/(6*a*(a^2 + b^2)*(-1 + E^(2*c))) + (b^2*e*f*x^2)/(2*a*(a^2 + b^2)*
(Cosh[c] + Sinh[c])^2) + (b^2*f^2*x^3)/(6*a*(a^2 + b^2)*(Cosh[c] + Sinh[c])
^2) + (b^2*f^2*x^3*Coth[2*c])/(6*a*(a^2 + b^2)*(Cosh[c] + Sinh[c])^2) + (b^
2*e*f*x^2*Cosh[2*c]*Csch[c]*Sech[c])/(4*a*(a^2 + b^2)*(Cosh[c] + Sinh[c])^2
) + (e^2*x*Csch[c]^2*(-a^2*Coth[c]) + Csch[c]*(a^2 + b^2 - I*a^2*Sinh[c]))
)/(a*(a^2 + b^2)*(Csch[c/2] - I*Sech[c/2])*(Csch[c/2] + I*Sech[c/2])) - ((1
/4 - I/4)*a*e*f*x^2*Cosh[c])/((a^2 + b^2)*(Cosh[c] + Sinh[c])^2*(-I - I*Cos
h[c] + Sinh[c] + Sinh[2*c])) + (b*e*f*x^2*Cosh[c])/(4*(a^2 + b^2)*(Cosh[c]
+ Sinh[c])^2*(-I - I*Cosh[c] + Sinh[c] + Sinh[2*c])) - ((1/12 - I/12)*a*f^2
*x^3*Cosh[c])/((a^2 + b^2)*(Cosh[c] + Sinh[c])^2*(-I - I*Cosh[c] + Sinh[c]
+ Sinh[2*c])) - (b*e*f*x^2*Cosh[3*c])/(4*(a^2 + b^2)*(Cosh[c] + Sinh[c])^2*
(-I - I*Cosh[c] + Sinh[c] + Sinh[2*c])) - ((1/4 - I/4)*a*e*f*x^2*Sinh[c])/
(a^2 + b^2)*(Cosh[c] + Sinh[c])^2*(-I - I*Cosh[c] + Sinh[c] + Sinh[2*c])) +
(b*e*f*x^2*Sinh[c])/(4*(a^2 + b^2)*(Cosh[c] + Sinh[c])^2*(-I - I*Cosh[c] +
Sinh[c] + Sinh[2*c])) - ((1/12 - I/12)*a*f^2*x^3*Sinh[c])/((a^2 + b^2)*(Co
sh[c] + Sinh[c])^2*(-I - I*Cosh[c] + Sinh[c] + Sinh[2*c])) + ((1/4 - I/4)*a
*e*f*x^2*Cosh[3*c])/((a^2 + b^2)*(Cosh[c] + Sinh[c])^2*(Cosh[c] + I*(-I + S
inh[c] + Sinh[2*c]))) + ((1/12 - I/12)*a*f^2*x^3*Cosh[3*c])/((a^2 + b^2)*(C
osh[c] + Sinh[c])^2*(Cosh[c] + I*(-I + Sinh[c] + Sinh[2*c]))) - (b*e*f*x^2*
Sinh[3*c])/(4*(a^2 + b^2)*(Cosh[c] + Sinh[c])^2*(-I - I*Cosh[c] + Sinh[c] +
Sinh[2*c])) + ((1/4 - I/4)*a*e*f*x^2*Sinh[3*c])/((a^2 + b^2)*(Cosh[c] + Si
nh[c])^2*(Cosh[c] + I*(-I + Sinh[c] + Sinh[2*c]))) + ((1/12 - I/12)*a*f^2*x
^3*Sinh[3*c])/((a^2 + b^2)*(Cosh[c] + Sinh[c])^2*(Cosh[c] + I*(-I + Sinh[c]
+ Sinh[2*c])))

```

---

**Maple [F]** time = 0.58, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \operatorname{csch}(dx + c) \operatorname{sech}(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*csch(d\*x+c)\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out]  $\text{int}((f*x+e)^2*\text{csch}(d*x+c)*\text{sech}(d*x+c)/(a+b*\sinh(d*x+c)),x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-e^2 \left( \frac{b^2 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{(a^3 + ab^2)d} - \frac{2b \arctan(e^{(-dx-c)})}{(a^2 + b^2)d} + \frac{a \log(e^{(-2dx-2c)} + 1)}{(a^2 + b^2)d} - \frac{\log(e^{(-dx-c)} + 1)}{ad} - \frac{\log(e^{(-dx-c)} - 1)}{ad} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x+e)^2*\text{csch}(d*x+c)*\text{sech}(d*x+c)/(a+b*\sinh(d*x+c)),x, \text{algorithm}="maxima")$

[Out]  $-e^2*(b^2*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/((a^3 + a*b^2)*d) - 2*b*\arctan(e^{(-d*x - c)})/((a^2 + b^2)*d) + a*\log(e^{(-2*d*x - 2*c)} + 1)/((a^2 + b^2)*d) - \log(e^{(-d*x - c)} + 1)/(a*d) - \log(e^{(-d*x - c)} - 1)/(a*d) + 2*(d*x*\log(e^{(d*x + c)} + 1) + \text{dilog}(-e^{(d*x + c)})))*e*f/(a*d^2) + 2*(d*x*\log(-e^{(d*x + c)} + 1) + \text{dilog}(e^{(d*x + c)})))*e*f/(a*d^2) + (d^2*x^2*\log(e^{(d*x + c)} + 1) + 2*d*x*\text{dilog}(-e^{(d*x + c)}) - 2*\text{polylog}(3, -e^{(d*x + c)}))*f^2/(a*d^3) + (d^2*x^2*\log(-e^{(d*x + c)} + 1) + 2*d*x*\text{dilog}(e^{(d*x + c)}) - 2*\text{polylog}(3, e^{(d*x + c)}))*f^2/(a*d^3) - 2/3*(d^3*f^2*x^3 + 3*d^3*e*f*x^2)/(a*d^3) + \text{integrate}(2*(b^3*f^2*x^2 + 2*b^3*e*f*x - (a*b^2*f^2*x^2*e^c + 2*a*b^2*e*f*x*e^c)*e^{(d*x)})/(a^3*b + a*b^3 - (a^3*b*e^{(2*c)} + a*b^3*e^{(2*c)})*e^{(2*d*x)} - 2*(a^4*e^c + a^2*b^2*e^c)*e^{(d*x)}), x) - \text{integrate}(-2*(a*f^2*x^2 + 2*a*e*f*x - (b*f^2*x^2*e^c + 2*b*e*f*x*e^c)*e^{(d*x)})/(a^2 + b^2 + (a^2*e^{(2*c)} + b^2*e^{(2*c)})*e^{(2*d*x)}), x)$

**Fricas [C]** time = 3.05557, size = 3794, normalized size = 5.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x+e)^2*\text{csch}(d*x+c)*\text{sech}(d*x+c)/(a+b*\sinh(d*x+c)),x, \text{algorithm}="fricas")$

[Out]  $(2*b^2*f^2*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b + 2*b^2*f^2*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b - 2*(a^2 + b^2)*f^2*\text{polylog}(3, \cosh(d*x + c) + \sinh(d*x + c))$

$$\begin{aligned}
& c)) - 2*(a^2 + b^2)*f^2*\text{polylog}(3, -\cosh(dx + c) - \sinh(dx + c)) - 2*(b^2*d*f^2*x + b^2*d*e*f)*\text{dilog}((a*\cosh(dx + c) + a*\sinh(dx + c) + (b*\cosh(dx + c) + b*\sinh(dx + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 2*(b^2*d*f^2*x + b^2*d*e*f)*\text{dilog}((a*\cosh(dx + c) + a*\sinh(dx + c) - (b*\cosh(dx + c) + b*\sinh(dx + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*\text{dilog}(\cosh(dx + c) + \sinh(dx + c)) - (2*a^2*d*f^2*x + 2*I*a*b*d*f^2*x + 2*a^2*d*e*f + 2*I*a*b*d*e*f)*\text{dilog}(I*\cosh(dx + c) + I*\sinh(dx + c)) - (2*a^2*d*f^2*x - 2*I*a*b*d*f^2*x + 2*a^2*d*e*f - 2*I*a*b*d*e*f)*\text{dilog}(-I*\cosh(dx + c) - I*\sinh(dx + c)) + 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*\text{dilog}(-\cosh(dx + c) - \sinh(dx + c)) - (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*\log(2*b*\cosh(dx + c) + 2*b*\sinh(dx + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*\log(2*b*\cosh(dx + c) + 2*b*\sinh(dx + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*\log(-(a*\cosh(dx + c) + a*\sinh(dx + c) + (b*\cosh(dx + c) + b*\sinh(dx + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*\log(-(a*\cosh(dx + c) + a*\sinh(dx + c) - (b*\cosh(dx + c) + b*\sinh(dx + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + (a^2 + b^2)*d^2*e^2)*\log(\cosh(dx + c) + \sinh(dx + c) + 1) - (a^2*d^2*e^2 + I*a*b*d^2*e^2 - 2*a^2*c*d*e*f - 2*I*a*b*c*d*e*f + a^2*c^2*f^2 + I*a*b*c^2*f^2)*\log(\cosh(dx + c) + \sinh(dx + c) + I) - (a^2*d^2*e^2 - I*a*b*d^2*e^2 - 2*a^2*c*d*e*f + 2*I*a*b*c*d*e*f + a^2*c^2*f^2 - I*a*b*c^2*f^2)*\log(\cosh(dx + c) + \sinh(dx + c) - I) + ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\log(\cosh(dx + c) + \sinh(dx + c) - 1) - (a^2*d^2*f^2*x^2 - I*a*b*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x - 2*I*a*b*d^2*e*f*x + 2*a^2*c*d*e*f - 2*I*a*b*c*d*e*f - a^2*c^2*f^2 + I*a*b*c^2*f^2)*\log(I*\cosh(dx + c) + I*\sinh(dx + c) + 1) - (a^2*d^2*f^2*x^2 + I*a*b*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + 2*I*a*b*d^2*e*f*x + 2*a^2*c*d*e*f + 2*I*a*b*c*d*e*f - a^2*c^2*f^2 - I*a*b*c^2*f^2)*\log(-I*\cosh(dx + c) - I*\sinh(dx + c) + 1) + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\log(-\cosh(dx + c) - \sinh(dx + c) + 1) + (2*a^2*f^2 + 2*I*a*b*f^2)*\text{polylog}(3, I*\cosh(dx + c) + I*\sinh(dx + c)) + (2*a^2*f^2 - 2*I*a*b*f^2)*\text{polylog}(3, -I*\cosh(dx + c) - I*\sinh(dx + c))/((a^3 + a*b^2)*d^3)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cscsch(dx+c)\*sech(dx+c)/(a+b\*sinh(dx+c)),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \operatorname{csch}(dx + c) \operatorname{sech}(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*csch(d\*x+c)\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)^2\*csch(d\*x + c)\*sech(d\*x + c)/(b\*sinh(d\*x + c) + a), x)

$$3.437 \quad \int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=439

$$\frac{b^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2(a^2+b^2)} - \frac{b^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2(a^2+b^2)} + \frac{b^2 f \operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{2ad^2(a^2+b^2)} + \frac{ibf \operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{d^2(a^2+b^2)}$$

```
[Out] (-2*b*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (2*(e + f*x)*ArcTanh[E^(2*c + 2*d*x)])/(a*d) - (b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) - (b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) + (b^2*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a*(a^2 + b^2)*d) + (I*b*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) - (I*b*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) - (b^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) - (b^2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) + (b^2*f*PolyLog[2, -E^(2*(c + d*x))])/(2*a*(a^2 + b^2)*d^2) - (f*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a*d^2) + (f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2)
```

**Rubi [A]** time = 0.636683, antiderivative size = 439, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 11, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$ , Rules used = {5589, 5461, 4182, 2279, 2391, 5573, 5561, 2190, 6742, 4180, 3718}

$$\frac{b^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2(a^2+b^2)} - \frac{b^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2(a^2+b^2)} + \frac{b^2 f \operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{2ad^2(a^2+b^2)} + \frac{ibf \operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{d^2(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*b*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (2*(e + f*x)*ArcTanh[E^(2*c + 2*d*x)])/(a*d) - (b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) - (b^2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)*d) + (b^2*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a*(a^2 + b^2)*d) + (I*b*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) - (I*b*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)*d^2) - (b^2*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) - (b^2*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)*d^2) + (b^2*f*PolyLog[2, -E^(2*(c + d*x))])/(2*a*(a^2 + b^2)*d^2) - (f*PolyL
```

$\text{og}[2, -E^{(2*c + 2*d*x)}]/(2*a*d^2) + (f*\text{PolyLog}[2, E^{(2*c + 2*d*x)}]/(2*a*d^2))$

### Rule 5589

$\text{Int}[(\text{Csch}[(c_.) + (d_.)*(x_)]^{(n_.)}*((e_.) + (f_.)*(x_))^{(m_.)}*\text{Sech}[(c_.) + (d_.)*(x_)]^{(p_.)})/((a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_)]), x\_Symbol] \rightarrow \text{Dist}[1/a, \text{Int}[(e + f*x)^m*\text{Sech}[c + d*x]^p*\text{Csch}[c + d*x]^n, x], x] - \text{Dist}[b/a, \text{Int}[(e + f*x)^m*\text{Sech}[c + d*x]^p*\text{Csch}[c + d*x]^{(n-1)}]/(a + b*\text{Sinh}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$

### Rule 5461

$\text{Int}[\text{Csch}[(a_.) + (b_.)*(x_)]^{(n_.)}*((c_.) + (d_.)*(x_))^{(m_.)}*\text{Sech}[(a_.) + (b_.)*(x_)]^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[2^n, \text{Int}[(c + d*x)^m*\text{Csch}[2*a + 2*b*x]^n, x], x] /; \text{FreeQ}\{a, b, c, d\}, x \&\& \text{RationalQ}[m] \&\& \text{IntegerQ}[n]$

### Rule 4182

$\text{Int}[\text{csc}[(e_.) + (\text{Complex}[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(-2*(c + d*x)^m*\text{ArcTanh}[E^{-(I*e)} + f*fz*x)]/(f*fz*I), x] + (-\text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 - E^{-(I*e)} + f*fz*x]], x], x] + \text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + E^{-(I*e)} + f*fz*x]], x], x) /; \text{FreeQ}\{c, d, e, f, fz\}, x \&\& \text{IGtQ}[m, 0]$

### Rule 2279

$\text{Int}[\text{Log}[(a_.) + (b_.)*((F_.)^{((e_.)*((c_.) + (d_.)*(x_)))})^{(n_.)}], x\_Symbol] \rightarrow \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x \&\& \text{GtQ}[a, 0]$

### Rule 2391

$\text{Int}[\text{Log}[(c_.)*((d_.) + (e_.)*(x_))^{(n_.)}]/(x_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x \&\& \text{EqQ}[c*d, 1]$

### Rule 5573

$\text{Int}[(e_.) + (f_.)*(x_)]^{(m_.)}*\text{Sech}[(c_.) + (d_.)*(x_)]^{(n_.)}/((a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_)]), x\_Symbol] \rightarrow \text{Dist}[b^2/(a^2 + b^2), \text{Int}[(e + f*x)^m*\text{Sech}[c + d*x]^{(n-2)}]/(a + b*\text{Sinh}[c + d*x]), x], x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(e + f*x)^m*\text{Sech}[c + d*x]^n*(a - b*\text{Sinh}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{IGtQ}[n, 0]$

Rule 5561

```
Int[(Cosh[c_.] + (d_.)*(x_.))*((e_.) + (f_.)*(x_.))^(m_.)]/((a_.) + (b_.)*Sin
h[(c_.) + (d_.)*(x_.)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_.))))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_.))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_.)]*((c_.) + (d_.)*(x_
))^m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)]]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3718

```
Int[(((c_.) + (d_.)*(x_.))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)]), x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx}{a} \\
&= \frac{2 \int (e+fx)\operatorname{csch}(2c+2dx) dx}{a} - \frac{b \int (e+fx)\operatorname{sech}(c+dx)(a-b\sinh(c+dx)) dx}{a(a^2+b^2)} \\
&= \frac{b^2(e+fx)^2}{2a(a^2+b^2)f} - \frac{2(e+fx)\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b \int (a(e+fx)\operatorname{sech}(c+dx) - b(e+fx)\sinh(c+dx)) dx}{a(a^2+b^2)} \\
&= \frac{b^2(e+fx)^2}{2a(a^2+b^2)f} - \frac{2(e+fx)\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)\log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)d} \\
&= -\frac{2b(e+fx)\tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)\log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)} \\
&= -\frac{2b(e+fx)\tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)\log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)} \\
&= -\frac{2b(e+fx)\tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)\log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)} \\
&= -\frac{2b(e+fx)\tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2(e+fx)\tanh^{-1}(e^{2c+2dx})}{ad} - \frac{b^2(e+fx)\log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)}
\end{aligned}$$

**Mathematica [B]** time = 2.68984, size = 1540, normalized size = 3.51

result too large to display

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Csch[c + d\*x]\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] 2\*(((I/4)\*(a^2 - b^2)\*(d\*e - c\*f)\*(c + d\*x))/(a\*(a^2 + b^2)\*d^2) - ((I/8)\*(a^2 - b^2)\*f\*(c + d\*x)^2)/(a\*(a^2 + b^2)\*d^2) - (e\*ArcTanh[1 - (2\*I)\*Tanh[(c + d\*x)/2]])/((a - I\*b)\*d) + (I\*b\*e\*ArcTanh[1 - (2\*I)\*Tanh[(c + d\*x)/2]])/(a\*(a - I\*b)\*d) + (c\*f\*ArcTanh[1 - (2\*I)\*Tanh[(c + d\*x)/2]])/((a - I\*b)\*d^2) - (I\*b\*c\*f\*ArcTanh[1 - (2\*I)\*Tanh[(c + d\*x)/2]])/(a\*(a - I\*b)\*d^2) + (e\*Log[Cosh[(c + d\*x)/2]])/(2\*a\*d) - (c\*f\*Log[Cosh[(c + d\*x)/2]])/(2\*a\*d^2) -



$$\begin{aligned}
& (e^{(-I/2)(c+dx)} + \text{Log}[\text{Cosh}[(c+dx)/2] + I\text{Sinh}[(c+dx)/2]]) / (2(a + I*b)*d) + (c*f^{(-I/2)(c+dx)} + \text{Log}[\text{Cosh}[(c+dx)/2] + I\text{Sinh}[(c+dx)/2]]) / (2(a + I*b)*d^2) - ((I/4)*b*e^{(-I)(c+dx)} + 2*\text{ArcTanh}[1 - (2*I)*\text{Tanh}[(c+dx)/2]] + \text{Log}[-1 + \text{Cosh}[c+dx] + I\text{Sinh}[c+dx]]) / (a*(a - I*b)*d) + ((I/4)*b*c*f^{(-I)(c+dx)} + 2*\text{ArcTanh}[1 - (2*I)*\text{Tanh}[(c+dx)/2]] + \text{Log}[-1 + \text{Cosh}[c+dx] + I\text{Sinh}[c+dx]]) / (a*(a - I*b)*d^2) + (I*f^{(-I/8)(c+dx)^2} - (I/2)*(c+dx)*\text{Log}[1 + E^{-(c+dx)}] + (I/2)*\text{PolyLog}[2, -E^{-(c+dx)}]) / (a*d^2) - ((I/2)*b*f^{(-I/2)(c+dx)^2} + (I/4)*(3*\text{Pi}*(c+dx) + (1 - I)*(c+dx)^2 + 2*(\text{Pi} - (2*I)*(c+dx))*\text{Log}[1 + I*E^{-(c+dx)}] - 4*\text{Pi}*\text{Log}[1 + E^{(c+dx)}] - 2*\text{Pi}*\text{Log}[-\text{Cos}[(\text{Pi} + (2*I)*(c+dx))/4]] + 4*\text{Pi}*\text{Log}[\text{Cosh}[(c+dx)/2]] + (4*I)*\text{PolyLog}[2, (-I)*E^{-(c+dx)}]) / (a*(a - I*b)*d^2) + ((I/2)*f^{((c+dx)^2/4} + (-3*\text{Pi}*(c+dx) - (1 - I)*(c+dx)^2 - 2*(\text{Pi} - (2*I)*(c+dx))*\text{Log}[1 + I*E^{-(c+dx)}] + 4*\text{Pi}*\text{Log}[1 + E^{(c+dx)}] + 2*\text{Pi}*\text{Log}[-\text{Cos}[(\text{Pi} + (2*I)*(c+dx))/4]] - 4*\text{Pi}*\text{Log}[\text{Cosh}[(c+dx)/2]] - (4*I)*\text{PolyLog}[2, (-I)*E^{-(c+dx)}]) / 4 - (I/2)*(-(c+dx)^2/2 + 2*(c+dx)*\text{Log}[1 - E^{(c+dx)}] + 2*\text{PolyLog}[2, E^{(c+dx)}])) / ((a - I*b)*d^2) + (b*f^{((c+dx)^2/4} + (-3*\text{Pi}*(c+dx) - (1 - I)*(c+dx)^2 - 2*(\text{Pi} - (2*I)*(c+dx))*\text{Log}[1 + I*E^{-(c+dx)}] + 4*\text{Pi}*\text{Log}[1 + E^{(c+dx)}] + 2*\text{Pi}*\text{Log}[-\text{Cos}[(\text{Pi} + (2*I)*(c+dx))/4]] - 4*\text{Pi}*\text{Log}[\text{Cosh}[(c+dx)/2]] - (4*I)*\text{PolyLog}[2, (-I)*E^{-(c+dx)}]) / 4 - (I/2)*(-(c+dx)^2/2 + 2*(c+dx)*\text{Log}[1 - E^{(c+dx)}] + 2*\text{PolyLog}[2, E^{(c+dx)}])) / (2*a*(a - I*b)*d^2) - (b^2*(-f*(c+dx)^2)/2 + f*(c+dx)*\text{Log}[1 + (b*E^{(c+dx)})/(a + \text{Sqrt}[a^2 + b^2])] + f*(c+dx)*\text{Log}[a + b*\text{Sinh}[c+dx]] - c*f*\text{Log}[a + b*\text{Sinh}[c+dx]] + f*\text{PolyLog}[2, (b*E^{(c+dx)})/(-a + \text{Sqrt}[a^2 + b^2])] + f*\text{PolyLog}[2, -(b*E^{(c+dx)})/(a + \text{Sqrt}[a^2 + b^2])]) / (2*a*(a^2 + b^2)*d^2) - (I*f*(-E^{((I/4)*\text{Pi})*(c+dx)^2}/4 + ((\text{Pi}*(c+dx))/4 - \text{Pi}*\text{Log}[1 + E^{(c+dx)}] - 2*(\text{Pi}/4 + (I/2)*(c+dx))*\text{Log}[1 - E^{((2*I)*(\text{Pi}/4 + (I/2)*(c+dx))}] + \text{Pi}*\text{Log}[\text{Cosh}[(c+dx)/2]] + (\text{Pi}*\text{Log}[\text{Sin}[\text{Pi}/4 + (I/2)*(c+dx)]])/2 + I*\text{PolyLog}[2, E^{((2*I)*(\text{Pi}/4 + (I/2)*(c+dx))}]) / \text{Sqrt}[2])]) / (\text{Sqrt}[2]*(a + I*b)*d^2)
\end{aligned}$$

**Maple [B]** time = 0.171, size = 1065, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)*\text{csch}(d*x+c)*\text{sech}(d*x+c)/(a+b*\text{sinh}(d*x+c)),x)$

[Out]  $1/d^2*f*c*b^2/(a^2+b^2)/a*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)-1/d^2*f*b^2/(a^2+b^2)/a*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*c-1/d*f*b^2/(a^2+b^2)/a*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})$

2)))\*x-1/d\*f\*b^2/(a^2+b^2)/a\*ln((b\*exp(d\*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))\*x-1/d^2\*f\*b^2/(a^2+b^2)/a\*ln((b\*exp(d\*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))\*c+4\*I/d^2\*f/(4\*a^2+4\*b^2)\*ln(1+I\*exp(d\*x+c))\*b\*c-4\*I/d\*f/(4\*a^2+4\*b^2)\*ln(1-I\*exp(d\*x+c))\*b\*x-4\*I/d^2\*f/(4\*a^2+4\*b^2)\*ln(1-I\*exp(d\*x+c))\*b\*c+4\*I/d\*f/(4\*a^2+4\*b^2)\*ln(1+I\*exp(d\*x+c))\*b\*x-4/d^2\*f/(4\*a^2+4\*b^2)\*dilog(1+I\*exp(d\*x+c))\*a-4/d^2\*f/(4\*a^2+4\*b^2)\*dilog(1-I\*exp(d\*x+c))\*a-8/d\*e/(4\*a^2+4\*b^2)\*b\*arctan(exp(d\*x+c))-4/d\*e/(4\*a^2+4\*b^2)\*a\*ln(1+exp(2\*d\*x+2\*c))-1/d^2\*f/a\*dilog(exp(d\*x+c))+1/d^2\*f/a\*dilog(exp(d\*x+c)+1)-1/d^2\*f\*b^2/(a^2+b^2)/a\*dilog((-b\*exp(d\*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))-1/d^2\*f\*b^2/(a^2+b^2)/a\*dilog((b\*exp(d\*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+8/d^2\*f\*c/(4\*a^2+4\*b^2)\*b\*arctan(exp(d\*x+c))-4/d\*f/(4\*a^2+4\*b^2)\*ln(1+I\*exp(d\*x+c))\*a\*x-4/d^2\*f/(4\*a^2+4\*b^2)\*ln(1+I\*exp(d\*x+c))\*a\*c-4/d\*f/(4\*a^2+4\*b^2)\*ln(1-I\*exp(d\*x+c))\*a\*x-4/d^2\*f/(4\*a^2+4\*b^2)\*ln(1-I\*exp(d\*x+c))\*a\*c-1/d\*e\*b^2/(a^2+b^2)/a\*ln(b\*exp(2\*d\*x+2\*c)+2\*a\*exp(d\*x+c)-b)+4/d^2\*f\*c/(4\*a^2+4\*b^2)\*a\*ln(1+exp(2\*d\*x+2\*c))+4\*I/d^2\*f/(4\*a^2+4\*b^2)\*dilog(1+I\*exp(d\*x+c))\*b-4\*I/d^2\*f/(4\*a^2+4\*b^2)\*dilog(1-I\*exp(d\*x+c))\*b+1/d/a\*e\*ln(exp(d\*x+c)-1)+1/d/a\*e\*ln(exp(d\*x+c)+1)+1/d/a\*ln(exp(d\*x+c)+1)\*f\*x-1/d^2/a\*f\*c\*ln(exp(d\*x+c)-1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-e \left( \frac{b^2 \log(-2ae^{-dx-c}) + be^{(-2dx-2c)} - b}{(a^3 + ab^2)d} - \frac{2b \arctan(e^{-dx-c})}{(a^2 + b^2)d} + \frac{a \log(e^{(-2dx-2c)} + 1)}{(a^2 + b^2)d} - \frac{\log(e^{-dx-c} + 1)}{ad} - \frac{\log(e^{-dx-c} - 1)}{ad} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -e\*(b^2\*log(-2\*a\*e^(-d\*x - c) + b\*e^(-2\*d\*x - 2\*c) - b)/((a^3 + a\*b^2)\*d) - 2\*b\*arctan(e^(-d\*x - c))/((a^2 + b^2)\*d) + a\*log(e^(-2\*d\*x - 2\*c) + 1)/((a^2 + b^2)\*d) - log(e^(-d\*x - c) + 1)/(a\*d) - log(e^(-d\*x - c) - 1)/(a\*d) + 4\*f\*integrate(2\*x/((b\*(e^(d\*x + c) - e^(-d\*x - c)) + 2\*a)\*(e^(d\*x + c) + e^(-d\*x - c))\*(e^(d\*x + c) - e^(-d\*x - c))), x)

**Fricas [B]** time = 2.52182, size = 2091, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-(b^2*f*dilog((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + b^2*f*dilog((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - (a^2 + b^2)*f*dilog(\cosh(d*x + c) + \sinh(d*x + c)) - (a^2 + b^2)*f*dilog(-\cosh(d*x + c) - \sinh(d*x + c)) + (a^2*f + I*a*b*f)*dilog(I*\cosh(d*x + c) + I*\sinh(d*x + c)) + (a^2*f - I*a*b*f)*dilog(-I*\cosh(d*x + c) - I*\sinh(d*x + c)) + (b^2*d*e - b^2*c*f)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + (b^2*d*e - b^2*c*f)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + (b^2*d*f*x + b^2*c*f)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + (b^2*d*f*x + b^2*c*f)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - ((a^2 + b^2)*d*f*x + (a^2 + b^2)*d*e)*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) + (a^2*d*e + I*a*b*d*e - a^2*c*f - I*a*b*c*f)*\log(\cosh(d*x + c) + \sinh(d*x + c) + I) + (a^2*d*e - I*a*b*d*e - a^2*c*f + I*a*b*c*f)*\log(\cosh(d*x + c) + \sinh(d*x + c) - I) - ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + (a^2*d*f*x - I*a*b*d*f*x + a^2*c*f - I*a*b*c*f)*\log(I*\cosh(d*x + c) + I*\sinh(d*x + c) + 1) + (a^2*d*f*x + I*a*b*d*f*x + a^2*c*f + I*a*b*c*f)*\log(-I*\cosh(d*x + c) - I*\sinh(d*x + c) + 1) - ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*\log(-\cosh(d*x + c) - \sinh(d*x + c) + 1))/((a^3 + a*b^2)*d^2)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \operatorname{csch}(dx + c) \operatorname{sech}(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*csch(d*x + c)*sech(d*x + c)/(b*sinh(d*x + c) + a), x)
```

$$3.438 \quad \int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=90

$$-\frac{b^2 \log(a+b\sinh(c+dx))}{ad(a^2+b^2)} - \frac{b \tan^{-1}(\sinh(c+dx))}{d(a^2+b^2)} - \frac{a \log(\cosh(c+dx))}{d(a^2+b^2)} + \frac{\log(\sinh(c+dx))}{ad}$$

[Out] -((b\*ArcTan[Sinh[c + d\*x]])/((a^2 + b^2)\*d)) - (a\*Log[Cosh[c + d\*x]])/((a^2 + b^2)\*d) + Log[Sinh[c + d\*x]]/(a\*d) - (b^2\*Log[a + b\*Sinh[c + d\*x]])/(a\*(a^2 + b^2)\*d)

**Rubi [A]** time = 0.169638, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$ , Rules used = {2837, 12, 894, 635, 203, 260}

$$-\frac{b^2 \log(a+b\sinh(c+dx))}{ad(a^2+b^2)} - \frac{b \tan^{-1}(\sinh(c+dx))}{d(a^2+b^2)} - \frac{a \log(\cosh(c+dx))}{d(a^2+b^2)} + \frac{\log(\sinh(c+dx))}{ad}$$

Antiderivative was successfully verified.

[In] Int[(Csch[c + d\*x]\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] -((b\*ArcTan[Sinh[c + d\*x]])/((a^2 + b^2)\*d)) - (a\*Log[Cosh[c + d\*x]])/((a^2 + b^2)\*d) + Log[Sinh[c + d\*x]]/(a\*d) - (b^2\*Log[a + b\*Sinh[c + d\*x]])/(a\*(a^2 + b^2)\*d)

### Rule 2837

Int[cos[(e\_.) + (f\_.)\*(x\_.)]^(p\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Dist[1/(b^p\*f), Subst[Int[(a + x)^m\*(c + (d\*x)/b)^n\*(b^2 - x^2)^((p - 1)/2), x], x, b\*S in[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match Q[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

### Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

### Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

### Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx &= -\frac{b \operatorname{Subst}\left(\int \frac{b}{x(a+x)(-b^2-x^2)} dx, x, b\sinh(c+dx)\right)}{d} \\
&= -\frac{b^2 \operatorname{Subst}\left(\int \frac{1}{x(a+x)(-b^2-x^2)} dx, x, b\sinh(c+dx)\right)}{d} \\
&= -\frac{b^2 \operatorname{Subst}\left(\int \left(-\frac{1}{ab^2x} + \frac{1}{a(a^2+b^2)(a+x)} + \frac{b^2+ax}{b^2(a^2+b^2)(b^2+x^2)}\right) dx, x, b\sinh(c+dx)\right)}{d} \\
&= \frac{\log(\sinh(c+dx))}{ad} - \frac{b^2 \log(a+b\sinh(c+dx))}{a(a^2+b^2)d} - \frac{\operatorname{Subst}\left(\int \frac{b^2+ax}{b^2+x^2} dx, x, b\sinh(c+dx)\right)}{(a^2+b^2)d} \\
&= \frac{\log(\sinh(c+dx))}{ad} - \frac{b^2 \log(a+b\sinh(c+dx))}{a(a^2+b^2)d} - \frac{a \operatorname{Subst}\left(\int \frac{x}{b^2+x^2} dx, x, b\sinh(c+dx)\right)}{(a^2+b^2)d} \\
&= -\frac{b \tan^{-1}(\sinh(c+dx))}{(a^2+b^2)d} - \frac{a \log(\cosh(c+dx))}{(a^2+b^2)d} + \frac{\log(\sinh(c+dx))}{ad} - \frac{b^2 \log(a+b\sinh(c+dx))}{a(a^2+b^2)d}
\end{aligned}$$

**Mathematica [C]** time = 0.133677, size = 92, normalized size = 1.02

$$\frac{\frac{2b^2 \log(a+b\sinh(c+dx))}{a(a^2+b^2)} + \frac{\log(-\sinh(c+dx)+i)}{a+ib} + \frac{\log(\sinh(c+dx)+i)}{a-ib} - \frac{2 \log(\sinh(c+dx))}{a}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Csch[c + d\*x]\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] -(Log[I - Sinh[c + d\*x]]/(a + I\*b) - (2\*Log[Sinh[c + d\*x]])/a + Log[I + Sinh[c + d\*x]]/(a - I\*b) + (2\*b^2\*Log[a + b\*Sinh[c + d\*x]])/(a\*(a^2 + b^2)))/(2\*d)

**Maple [A]** time = 0.003, size = 123, normalized size = 1.4

$$\frac{1}{da} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \frac{b^2}{d(a^2+b^2)a} \ln\left(\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2 a - 2 \tanh(1/2 dx + c/2) b - a\right) - \frac{a}{d(a^2+b^2)} \ln\left(\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2 a + 2 \tanh(1/2 dx + c/2) b - a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x)`

[Out]  $\frac{1}{d} \frac{1}{a} \ln(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)) - \frac{1}{d} \frac{b^2}{(a^2 + b^2)} \frac{1}{a} \ln(\tanh(\frac{1}{2}d*x + \frac{1}{2}c))^2 - \frac{a - 2 \tanh(\frac{1}{2}d*x + \frac{1}{2}c) * b - a}{(a^2 + b^2)} - \frac{1}{d} \frac{1}{(a^2 + b^2)} * a \ln(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 1) - \frac{2}{d} \frac{1}{(a^2 + b^2)} * b * \arctan(\tanh(\frac{1}{2}d*x + \frac{1}{2}c))$

**Maxima [A]** time = 1.72629, size = 186, normalized size = 2.07

$$\frac{b^2 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{(a^3 + ab^2)d} + \frac{2b \arctan(e^{(-dx-c)})}{(a^2 + b^2)d} - \frac{a \log(e^{(-2dx-2c)} + 1)}{(a^2 + b^2)d} + \frac{\log(e^{(-dx-c)} + 1)}{ad} + \frac{\log(e^{(-dx-c)})}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out]  $-b^2 \log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b) / ((a^3 + a*b^2)*d) + 2*b*\arctan(e^{(-d*x - c)}) / ((a^2 + b^2)*d) - a*\log(e^{(-2*d*x - 2*c)} + 1) / ((a^2 + b^2)*d) + \log(e^{(-d*x - c)} + 1) / (a*d) + \log(e^{(-d*x - c)} - 1) / (a*d)$

**Fricas [A]** time = 2.6949, size = 350, normalized size = 3.89

$$\frac{2ab \arctan(\cosh(dx+c) + \sinh(dx+c)) + b^2 \log\left(\frac{2(b \sinh(dx+c) + a)}{\cosh(dx+c) - \sinh(dx+c)}\right) + a^2 \log\left(\frac{2 \cosh(dx+c)}{\cosh(dx+c) - \sinh(dx+c)}\right) - (a^2 + b^2) \log\left(\frac{2 \cosh(dx+c)}{\cosh(dx+c) - \sinh(dx+c)}\right)}{(a^3 + ab^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out]  $-(2*a*b*\arctan(\cosh(d*x + c) + \sinh(d*x + c)) + b^2*\log(2*(b*\sinh(d*x + c) + a)/(\cosh(d*x + c) - \sinh(d*x + c)))) + a^2*\log(2*\cosh(d*x + c)/(\cosh(d*x + c) - \sinh(d*x + c))) - (a^2 + b^2)*\log(2*\sinh(d*x + c)/(\cosh(d*x + c) - \sinh(d*x + c)))/((a^3 + a*b^2)*d)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [A]** time = 1.27236, size = 211, normalized size = 2.34

$$\frac{b^3 \log\left(\left|b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 2a\right|\right)}{a^3bd + ab^3d} - \frac{\left(\pi + 2 \arctan\left(\frac{1}{2}\left(e^{2dx+2c} - 1\right)e^{(-dx-c)}\right)\right)b}{2\left(a^2d + b^2d\right)} - \frac{a \log\left(\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4\right)}{2\left(a^2d + b^2d\right)} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out]  $-b^3 \log(\text{abs}(b*(e^{(d*x + c)} - e^{(-d*x - c)}) + 2*a)) / (a^3*b*d + a*b^3*d) - 1/2*(\pi + 2*\arctan(1/2*(e^{(2*d*x + 2*c)} - 1)*e^{(-d*x - c)})) * b / (a^2*d + b^2*d) - 1/2*a*\log((e^{(d*x + c)} - e^{(-d*x - c)})^2 + 4) / (a^2*d + b^2*d) + \log(\text{abs}(e^{(d*x + c)} - e^{(-d*x - c)})) / (a*d)$

$$3.439 \quad \int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable}\left(\frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}, x\right)$$

[Out] Unintegrable[(Csch[c + d\*x]\*Sech[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0632793, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Csch[c + d\*x]\*Sech[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int] [(Csch[c + d\*x]\*Sech[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Mathematica [A]** time = 27.5799, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csch[c + d\*x]\*Sech[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Integrate[(Csch[c + d\*x]\*Sech[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Maple [A]** time = 0.098, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx + c) \operatorname{sech}(dx + c)}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d\*x+c)\*sech(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x)

[Out] int(csch(d\*x+c)\*sech(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx + c) \operatorname{sech}(dx + c)}{(fx + e)(b \sinh(dx + c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)\*sech(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x, algorithm="maxima")

[Out] integrate(csch(d\*x + c)\*sech(d\*x + c)/((f\*x + e)\*(b\*sinh(d\*x + c) + a)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(dx + c) \operatorname{sech}(dx + c)}{afx + ae + (bfx + be) \sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)\*sech(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)), x, algorithm="fricas")

[Out] `integral(csch(d*x + c)*sech(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)), x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx + c) \operatorname{sech}(dx + c)}{(fx + e)(b \sinh(dx + c) + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)), x, algorithm="giac")`

[Out] `integrate(csch(d*x + c)*sech(d*x + c)/((f*x + e)*(b*sinh(d*x + c) + a)), x)`

$$3.440 \quad \int \frac{(e+fx)^3 \operatorname{csch}(c+dx) \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=1164

result too large to display

```
[Out] -((b*(e + f*x)^3)/((a^2 + b^2)*d)) - (6*f*(e + f*x)^2*ArcTan[E^(c + d*x)]/
(a*d^2) + (6*b^2*f*(e + f*x)^2*ArcTan[E^(c + d*x)]/(a*(a^2 + b^2)*d^2) - (
2*(e + f*x)^3*ArcTanh[E^(c + d*x)]/(a*d) - (b^3*(e + f*x)^3*Log[1 + (b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^(3/2)*d) + (b^3*(e + f*x)^
3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^(3/2)*d) +
(3*b*f*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(a*(a^2 + b^2)*d^2) - (3*f*(e +
f*x)^2*PolyLog[2, -E^(c + d*x)]/(a*d^2) + ((6*I)*f^2*(e + f*x)*PolyLog[2,
(-I)*E^(c + d*x)]/(a*d^3) - ((6*I)*b^2*f^2*(e + f*x)*PolyLog[2, (-I)*E^(c
+ d*x)]/(a*(a^2 + b^2)*d^3) - ((6*I)*f^2*(e + f*x)*PolyLog[2, I*E^(c + d*
x)]/(a*d^3) + ((6*I)*b^2*f^2*(e + f*x)*PolyLog[2, I*E^(c + d*x)]/(a*(a^2
+ b^2)*d^3) + (3*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)]/(a*d^2) - (3*b^3*f*
(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 +
b^2)^(3/2)*d^2) + (3*b^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + S
qrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^2) + (3*b*f^2*(e + f*x)*PolyLog[2
, -E^(2*(c + d*x))])/(a*(a^2 + b^2)*d^3) + (6*f^2*(e + f*x)*PolyLog[3, -E^(c
+ d*x)]/(a*d^3) - ((6*I)*f^3*PolyLog[3, (-I)*E^(c + d*x)]/(a*d^4) + ((6*I
)*b^2*f^3*PolyLog[3, (-I)*E^(c + d*x)]/(a*(a^2 + b^2)*d^4) + ((6*I)*f^3*Po
lyLog[3, I*E^(c + d*x)]/(a*d^4) - ((6*I)*b^2*f^3*PolyLog[3, I*E^(c + d*x)]
)/(a*(a^2 + b^2)*d^4) - (6*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)]/(a*d^3) +
(6*b^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])
/(a*(a^2 + b^2)^(3/2)*d^3) - (6*b^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*
x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^3) - (3*b*f^3*PolyLog[3
, -E^(2*(c + d*x))])/(2*(a^2 + b^2)*d^4) - (6*f^3*PolyLog[4, -E^(c + d*x)]
)/(a*d^4) + (6*f^3*PolyLog[4, E^(c + d*x)]/(a*d^4) - (6*b^3*f^3*PolyLog[4,
-((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^4) + (6*b
^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)
^(3/2)*d^4) + ((e + f*x)^3*Sech[c + d*x])/(a*d) - (b^2*(e + f*x)^3*Sech[c +
d*x])/(a*(a^2 + b^2)*d) - (b*(e + f*x)^3*Tanh[c + d*x])/(a*(a^2 + b^2)*d)
```

---

**Rubi [A]** time = 2.19887, antiderivative size = 1164, normalized size of antiderivative = 1., number of steps used = 53, number of rules used = 22, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$ , Rules used = {5589, 2622, 321, 207, 5462, 6741, 12, 6742, 6273, 4182, 2531, 6609,

2282, 6589, 4180, 5573, 3322, 2264, 2190, 4184, 3718, 5451}

$$\frac{(e+fx)^3 \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^3}{a(a^2+b^2)^{3/2}d} + \frac{(e+fx)^3 \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^3}{a(a^2+b^2)^{3/2}d} - \frac{3f(e+fx)^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)b^3}{a(a^2+b^2)^{3/2}d^2} + \frac{3f(e+fx)^2 \text{PolyLog}\left(2, -\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)b^3}{a(a^2+b^2)^{3/2}d^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Csch[c + d\*x]\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] -((b\*(e + f\*x)^3)/((a^2 + b^2)\*d)) - (6\*f\*(e + f\*x)^2\*ArcTan[E^(c + d\*x)])/(a\*d^2) + (6\*b^2\*f\*(e + f\*x)^2\*ArcTan[E^(c + d\*x)])/(a\*(a^2 + b^2)\*d^2) - (2\*(e + f\*x)^3\*ArcTanh[E^(c + d\*x)])/(a\*d) - (b^3\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a\*(a^2 + b^2)^(3/2)\*d) + (b^3\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a\*(a^2 + b^2)^(3/2)\*d) + (3\*b\*f\*(e + f\*x)^2\*Log[1 + E^(2\*(c + d\*x))])/(a^2 + b^2)\*d^2) - (3\*f\*(e + f\*x)^2\*PolyLog[2, -E^(c + d\*x)])/(a\*d^2) + ((6\*I)\*f^2\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*d^3) - ((6\*I)\*b^2\*f^2\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*(a^2 + b^2)\*d^3) - ((6\*I)\*f^2\*(e + f\*x)\*PolyLog[2, I\*E^(c + d\*x)])/(a\*d^3) + ((6\*I)\*b^2\*f^2\*(e + f\*x)\*PolyLog[2, I\*E^(c + d\*x)])/(a\*(a^2 + b^2)\*d^3) + (3\*f\*(e + f\*x)^2\*PolyLog[2, E^(c + d\*x)])/(a\*d^2) - (3\*b^3\*f\*(e + f\*x)^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a\*(a^2 + b^2)^(3/2)\*d^2) + (3\*b^3\*f\*(e + f\*x)^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a\*(a^2 + b^2)^(3/2)\*d^2) + (3\*b\*f^2\*(e + f\*x)\*PolyLog[2, -E^(2\*(c + d\*x))])/(a^2 + b^2)\*d^3) + (6\*f^2\*(e + f\*x)\*PolyLog[3, -E^(c + d\*x)])/(a\*d^3) - ((6\*I)\*f^3\*PolyLog[3, (-I)\*E^(c + d\*x)])/(a\*d^4) + ((6\*I)\*b^2\*f^3\*PolyLog[3, (-I)\*E^(c + d\*x)])/(a\*(a^2 + b^2)\*d^4) + ((6\*I)\*f^3\*PolyLog[3, I\*E^(c + d\*x)])/(a\*d^4) - ((6\*I)\*b^2\*f^3\*PolyLog[3, I\*E^(c + d\*x)])/(a\*(a^2 + b^2)\*d^4) - (6\*f^2\*(e + f\*x)\*PolyLog[3, E^(c + d\*x)])/(a\*d^3) + (6\*b^3\*f^2\*(e + f\*x)\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a\*(a^2 + b^2)^(3/2)\*d^3) - (6\*b^3\*f^2\*(e + f\*x)\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a\*(a^2 + b^2)^(3/2)\*d^3) - (3\*b\*f^3\*PolyLog[3, -E^(2\*(c + d\*x))])/(2\*(a^2 + b^2)\*d^4) - (6\*f^3\*PolyLog[4, -E^(c + d\*x)])/(a\*d^4) + (6\*f^3\*PolyLog[4, E^(c + d\*x)])/(a\*d^4) - (6\*b^3\*f^3\*PolyLog[4, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a\*(a^2 + b^2)^(3/2)\*d^4) + (6\*b^3\*f^3\*PolyLog[4, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a\*(a^2 + b^2)^(3/2)\*d^4) + ((e + f\*x)^3\*Sech[c + d\*x])/(a\*d) - (b^2\*(e + f\*x)^3\*Sech[c + d\*x])/(a\*(a^2 + b^2)\*d) - (b\*(e + f\*x)^3\*Tanh[c + d\*x])/(a^2 + b^2)\*d)

### Rule 5589

Int[(Csch[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(p\_.)]/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Sech[c + d\*x]^p\*Csch[c + d\*x]^n, x] - Dist[b/a, Int[((e + f\*x)^m\*Sech[c + d\*x]^p\*Csch[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d

\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 2622

Int[csc[(e\_.) + (f\_.)\*(x\_)]^(n\_.)\*((a\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(m\_), x\_Symbol] := Dist[1/(f\*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a\*Sec[e + f\*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

### Rule 5462

Int[Csch[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sech[(a\_.) + (b\_.)\*(x\_)]^(p\_.), x\_Symbol] := With[{u = IntHide[Csch[a + b\*x]^n\*Sech[a + b\*x]^p, x]}, Dist[(c + d\*x)^m, u, x] - Dist[d\*m, Int[(c + d\*x)^(m - 1)\*u, x], x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]

### Rule 6741

Int[u\_, x\_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_)^(m_.), x_Symbol] := Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x]
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&& !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_)^(m_.), x
_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_)^(m_.))*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```



Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)])/((f\*fz\*I)), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

Rule 5573

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[b^2/(a^2 + b^2), Int[(e + f\*x)^m\*Sech[c + d\*x]^(n - 2)/(a + b\*Sinh[c + d\*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f\*x)^m\*Sech[c + d\*x]^n\*(a - b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]

Rule 3322

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(-(I\*b) + 2\*a\*E^(-(I\*e) + f\*fz\*x) + I\*b\*E^(2\*(-(I\*e) + f\*fz\*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.)\*(F\_)^(v\_)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a)/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n], x]]

)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[(c + d\*x)^m\*Cot[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3718

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)], x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x))]/(1 + E^(2\*(-I\*e) + f\*fz\*x)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

#### Rule 5451

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sech[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*Tanh[(a\_.) + (b\_.)\*(x\_)]^(p\_.), x\_Symbol] := -Simp[(c + d\*x)^m\*Sech[a + b\*x]^n/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Sech[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

#### Rubi steps



**Mathematica [A]** time = 16.9409, size = 1467, normalized size = 1.26

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x]*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x])
,x]
```

```
[Out] 4*(-(f*Csch[c + d*x]*(12*b*d^3*e^2*E^(2*c)*x - 12*b*d^3*e^2*(1 + E^(2*c))*x
- 12*b*d^3*e*f*x^2 - 4*b*d^3*f^2*x^3 + 12*a*d^2*e^2*(1 + E^(2*c))*ArcTan[E
^(c + d*x)] + 6*b*d^2*e^2*(1 + E^(2*c))*(2*d*x - Log[1 + E^(2*(c + d*x))])
+ (12*I)*a*d*e*(1 + E^(2*c))*f*(d*x*(Log[1 - I*E^(c + d*x)] - Log[1 + I*E^(
c + d*x)]) - PolyLog[2, (-I)*E^(c + d*x)] + PolyLog[2, I*E^(c + d*x)]) + 6*
b*d*e*(1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 + E^(2*(c + d*x))]) - PolyLog[2,
-E^(2*(c + d*x))]) + (6*I)*a*(1 + E^(2*c))*f^2*(d^2*x^2*Log[1 - I*E^(c + d*
x)] - d^2*x^2*Log[1 + I*E^(c + d*x)] - 2*d*x*PolyLog[2, (-I)*E^(c + d*x)] +
2*d*x*PolyLog[2, I*E^(c + d*x)] + 2*PolyLog[3, (-I)*E^(c + d*x)] - 2*PolyL
og[3, I*E^(c + d*x)]) + b*(1 + E^(2*c))*f^2*(2*d^2*x^2*(2*d*x - 3*Log[1 + E
^(2*(c + d*x))]) - 6*d*x*PolyLog[2, -E^(2*(c + d*x))]) + 3*PolyLog[3, -E^(2*
(c + d*x))])*(a + b*Sinh[c + d*x]))/(8*(a^2 + b^2)*d^4*(1 + E^(2*c))*(b +
a*Csch[c + d*x])) + (b^3*Csch[c + d*x]*(2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x
))/Sqrt[a^2 + b^2]] - 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2]]) - 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]) -
d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]) + 3*d^3*e^2*f*x*
Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) + 3*d^3*e*f^2*x^2*Log[1 + (b
*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]]) - 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a
+ Sqrt[a^2 + b^2]]) + 3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 + b^2]))] + 6*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2
+ b^2]]) + 6*d*f^3*x*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]]) - 6
*d*e*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*f^3*x*P
olyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*f^3*PolyLog[4, (b*E
^(c + d*x))/(-a + Sqrt[a^2 + b^2]]) + 6*f^3*PolyLog[4, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])*(a + b*Sinh[c + d*x]))/(4*a*(a^2 + b^2)^(3/2)*d^4*(b
+ a*Csch[c + d*x])) + (Csch[c + d*x]*(-2*(e + f*x)^3*ArcTanh[Cosh[c + d*x]
+ Sinh[c + d*x]] - (3*f*(d^2*(e + f*x)^2*PolyLog[2, -Cosh[c + d*x] - Sinh[
c + d*x]] - 2*d*f*(e + f*x)*PolyLog[3, -Cosh[c + d*x] - Sinh[c + d*x]] + 2*
f^2*PolyLog[4, -Cosh[c + d*x] - Sinh[c + d*x]]))/d^3 + (3*f*(d^2*(e + f*x)^
2*PolyLog[2, Cosh[c + d*x] + Sinh[c + d*x]] - 2*d*f*(e + f*x)*PolyLog[3, Co
sh[c + d*x] + Sinh[c + d*x]] + 2*f^2*PolyLog[4, Cosh[c + d*x] + Sinh[c + d*
x]]))/d^3)*(a + b*Sinh[c + d*x]))/(4*a*d*(b + a*Csch[c + d*x])) + (Csch[c +
d*x]*Sech[c]*Sech[c + d*x]*(a*e^3*Cosh[c] + 3*a*e^2*f*x*Cosh[c] + 3*a*e*f^
2*x^2*Cosh[c] + a*f^3*x^3*Cosh[c] - b*e^3*Sinh[d*x] - 3*b*e^2*f*x*Sinh[d*x]
```

$$- 3*b*e*f^2*x^2*\text{Sinh}[d*x] - b*f^3*x^3*\text{Sinh}[d*x])*(a + b*\text{Sinh}[c + d*x])/(4*(a^2 + b^2)*d*(b + a*\text{Csch}[c + d*x]))$$

**Maple [F]** time = 1.674, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \operatorname{csch}(dx + c) (\operatorname{sech}(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)`

[Out] `int((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [C]** time = 5.446, size = 21639, normalized size = 18.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] 
$$\frac{1}{2}*(4*(a^3*b + a*b^3)*d^3*e^3 - 12*(a^3*b + a*b^3)*c*d^2*e^2*f + 12*(a^3*b + a*b^3)*c^2*d*e*f^2 - 4*(a^3*b + a*b^3)*c^3*f^3 - 4*((a^3*b + a*b^3)*d^3*f^3*x^3 + 3*(a^3*b + a*b^3)*d^3*e*f^2*x^2 + 3*(a^3*b + a*b^3)*d^3*e^2*f*x +$$

$$\begin{aligned}
& 3*(a^3*b + a*b^3)*c*d^2*e^2*f - 3*(a^3*b + a*b^3)*c^2*d*e*f^2 + (a^3*b + a \\
& *b^3)*c^3*f^3)*\cosh(d*x + c)^2 - 4*((a^3*b + a*b^3)*d^3*f^3*x^3 + 3*(a^3*b \\
& + a*b^3)*d^3*e*f^2*x^2 + 3*(a^3*b + a*b^3)*d^3*e^2*f*x + 3*(a^3*b + a*b^3)* \\
& c*d^2*e^2*f - 3*(a^3*b + a*b^3)*c^2*d*e*f^2 + (a^3*b + a*b^3)*c^3*f^3)*\sinh \\
& (d*x + c)^2 - 6*(b^4*d^2*f^3*x^2 + 2*b^4*d^2*e*f^2*x + b^4*d^2*e^2*f + (b^4 \\
& *d^2*f^3*x^2 + 2*b^4*d^2*e*f^2*x + b^4*d^2*e^2*f)*\cosh(d*x + c)^2 + 2*(b^4* \\
& d^2*f^3*x^2 + 2*b^4*d^2*e*f^2*x + b^4*d^2*e^2*f)*\cosh(d*x + c)*\sinh(d*x + c \\
& ) + (b^4*d^2*f^3*x^2 + 2*b^4*d^2*e*f^2*x + b^4*d^2*e^2*f)*\sinh(d*x + c)^2)* \\
& \sqrt{((a^2 + b^2)/b^2)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d* \\
& x + c) + b*\sinh(d*x + c)))*\sqrt{((a^2 + b^2)/b^2) - b}/b + 1) + 6*(b^4*d^2*f^ \\
& 3*x^2 + 2*b^4*d^2*e*f^2*x + b^4*d^2*e^2*f + (b^4*d^2*f^3*x^2 + 2*b^4*d^2*e* \\
& f^2*x + b^4*d^2*e^2*f)*\cosh(d*x + c)^2 + 2*(b^4*d^2*f^3*x^2 + 2*b^4*d^2*e*f \\
& ^2*x + b^4*d^2*e^2*f)*\cosh(d*x + c)*\sinh(d*x + c) + (b^4*d^2*f^3*x^2 + 2*b^ \\
& 4*d^2*e*f^2*x + b^4*d^2*e^2*f)*\sinh(d*x + c)^2)*\sqrt{((a^2 + b^2)/b^2)*\operatorname{dilog} \\
& ((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))* \\
& \sqrt{((a^2 + b^2)/b^2) - b}/b + 1) + 2*(b^4*d^3*e^3 - 3*b^4*c*d^2*e^2*f + 3*b \\
& ^4*c^2*d*e*f^2 - b^4*c^3*f^3 + (b^4*d^3*e^3 - 3*b^4*c*d^2*e^2*f + 3*b^4*c^2 \\
& *d*e*f^2 - b^4*c^3*f^3)*\cosh(d*x + c)^2 + 2*(b^4*d^3*e^3 - 3*b^4*c*d^2*e^2* \\
& f + 3*b^4*c^2*d*e*f^2 - b^4*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c) + (b^4*d^3 \\
& *e^3 - 3*b^4*c*d^2*e^2*f + 3*b^4*c^2*d*e*f^2 - b^4*c^3*f^3)*\sinh(d*x + c)^2 \\
& )*\sqrt{((a^2 + b^2)/b^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{ \\
& ((a^2 + b^2)/b^2) + 2*a) - 2*(b^4*d^3*e^3 - 3*b^4*c*d^2*e^2*f + 3*b^4*c^2* \\
& d*e*f^2 - b^4*c^3*f^3 + (b^4*d^3*e^3 - 3*b^4*c*d^2*e^2*f + 3*b^4*c^2*d*e*f^2 \\
& - b^4*c^3*f^3)*\cosh(d*x + c)^2 + 2*(b^4*d^3*e^3 - 3*b^4*c*d^2*e^2*f + 3*b \\
& ^4*c^2*d*e*f^2 - b^4*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c) + (b^4*d^3*e^3 - \\
& 3*b^4*c*d^2*e^2*f + 3*b^4*c^2*d*e*f^2 - b^4*c^3*f^3)*\sinh(d*x + c)^2)*\sqrt{ \\
& (a^2 + b^2)/b^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{((a^2 \\
& + b^2)/b^2) + 2*a) - 2*(b^4*d^3*f^3*x^3 + 3*b^4*d^3*e*f^2*x^2 + 3*b^4*d^3*e \\
& ^2*f*x + 3*b^4*c*d^2*e^2*f - 3*b^4*c^2*d*e*f^2 + b^4*c^3*f^3 + (b^4*d^3*f^3 \\
& *x^3 + 3*b^4*d^3*e*f^2*x^2 + 3*b^4*d^3*e^2*f*x + 3*b^4*c*d^2*e^2*f - 3*b^4*c \\
& ^2*d*e*f^2 + b^4*c^3*f^3)*\cosh(d*x + c)^2 + 2*(b^4*d^3*f^3*x^3 + 3*b^4*d^3 \\
& *e*f^2*x^2 + 3*b^4*d^3*e^2*f*x + 3*b^4*c*d^2*e^2*f - 3*b^4*c^2*d*e*f^2 + b^ \\
& 4*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c) + (b^4*d^3*f^3*x^3 + 3*b^4*d^3*e*f^2 \\
& *x^2 + 3*b^4*d^3*e^2*f*x + 3*b^4*c*d^2*e^2*f - 3*b^4*c^2*d*e*f^2 + b^4*c^3* \\
& f^3)*\sinh(d*x + c)^2)*\sqrt{((a^2 + b^2)/b^2)*\log(-(a*\cosh(d*x + c) + a*\sinh( \\
& d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c)))*\sqrt{((a^2 + b^2)/b^2) - b}/b \\
& ) + 2*(b^4*d^3*f^3*x^3 + 3*b^4*d^3*e*f^2*x^2 + 3*b^4*d^3*e^2*f*x + 3*b^4*c* \\
& d^2*e^2*f - 3*b^4*c^2*d*e*f^2 + b^4*c^3*f^3 + (b^4*d^3*f^3*x^3 + 3*b^4*d^3* \\
& e*f^2*x^2 + 3*b^4*d^3*e^2*f*x + 3*b^4*c*d^2*e^2*f - 3*b^4*c^2*d*e*f^2 + b^4 \\
& *c^3*f^3)*\cosh(d*x + c)^2 + 2*(b^4*d^3*f^3*x^3 + 3*b^4*d^3*e*f^2*x^2 + 3*b^ \\
& 4*d^3*e^2*f*x + 3*b^4*c*d^2*e^2*f - 3*b^4*c^2*d*e*f^2 + b^4*c^3*f^3)*\cosh(d \\
& *x + c)*\sinh(d*x + c) + (b^4*d^3*f^3*x^3 + 3*b^4*d^3*e*f^2*x^2 + 3*b^4*d^3* \\
& e^2*f*x + 3*b^4*c*d^2*e^2*f - 3*b^4*c^2*d*e*f^2 + b^4*c^3*f^3)*\sinh(d*x + c \\
& )^2)*\sqrt{((a^2 + b^2)/b^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cos \\
& h(d*x + c) + b*\sinh(d*x + c)))*\sqrt{((a^2 + b^2)/b^2) - b}/b) - 12*(b^4*f^3*c
\end{aligned}$$

$$\begin{aligned}
& \text{osh}(d*x + c)^2 + 2*b^4*f^3*\cosh(d*x + c)*\sinh(d*x + c) + b^4*f^3*\sinh(d*x + \\
& c)^2 + b^4*f^3)*\sqrt{(a^2 + b^2)/b^2)*\text{polylog}(4, (a*\cosh(d*x + c) + a*\sinh \\
& (d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2))/b) + \\
& 12*(b^4*f^3*\cosh(d*x + c)^2 + 2*b^4*f^3*\cosh(d*x + c)*\sinh(d*x + c) + b^4* \\
& f^3*\sinh(d*x + c)^2 + b^4*f^3)*\sqrt{(a^2 + b^2)/b^2)*\text{polylog}(4, (a*\cosh(d*x \\
& + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b \\
& ^2)/b^2))/b) + 12*(b^4*d*f^3*x + b^4*d*e*f^2 + (b^4*d*f^3*x + b^4*d*e*f^2)* \\
& \cosh(d*x + c)^2 + 2*(b^4*d*f^3*x + b^4*d*e*f^2)*\cosh(d*x + c)*\sinh(d*x + c) \\
& + (b^4*d*f^3*x + b^4*d*e*f^2)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2)*\text{polyl} \\
& \text{og}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + \\
& c))*\sqrt{(a^2 + b^2)/b^2))/b) - 12*(b^4*d*f^3*x + b^4*d*e*f^2 + (b^4*d*f^3*x \\
& + b^4*d*e*f^2)*\cosh(d*x + c)^2 + 2*(b^4*d*f^3*x + b^4*d*e*f^2)*\cosh(d*x + \\
& c)*\sinh(d*x + c) + (b^4*d*f^3*x + b^4*d*e*f^2)*\sinh(d*x + c)^2)*\sqrt{(a^2 \\
& + b^2)/b^2)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c \\
& ) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2))/b) + 4*((a^4 + a^2*b^2)*d^3*f^3 \\
& *x^3 + 3*(a^4 + a^2*b^2)*d^3*e*f^2*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*f*x + (a \\
& ^4 + a^2*b^2)*d^3*e^3)*\cosh(d*x + c) + 6*((a^4 + 2*a^2*b^2 + b^4)*d^2*f^3*x \\
& ^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d^2*e^ \\
& 2*f + ((a^4 + 2*a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2* \\
& e*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d^2*e^2*f)*\cosh(d*x + c)^2 + 2*((a^4 + 2* \\
& a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f^2*x + (a^4 + \\
& 2*a^2*b^2 + b^4)*d^2*e^2*f)*\cosh(d*x + c)*\sinh(d*x + c) + ((a^4 + 2*a^2*b^ \\
& 2 + b^4)*d^2*f^3*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f^2*x + (a^4 + 2*a^2 \\
& *b^2 + b^4)*d^2*e^2*f)*\sinh(d*x + c)^2)*\text{dilog}(\cosh(d*x + c) + \sinh(d*x + c \\
& )) + (-12*I*(a^4 + a^2*b^2)*d*f^3*x + 12*(a^3*b + a*b^3)*d*f^3*x - 12*I*(a^4 \\
& + a^2*b^2)*d*e*f^2 + 12*(a^3*b + a*b^3)*d*e*f^2 + (-12*I*(a^4 + a^2*b^2)*d \\
& *f^3*x + 12*(a^3*b + a*b^3)*d*f^3*x - 12*I*(a^4 + a^2*b^2)*d*e*f^2 + 12*(a^ \\
& 3*b + a*b^3)*d*e*f^2)*\cosh(d*x + c)^2 + (-24*I*(a^4 + a^2*b^2)*d*f^3*x + 24 \\
& *(a^3*b + a*b^3)*d*f^3*x - 24*I*(a^4 + a^2*b^2)*d*e*f^2 + 24*(a^3*b + a*b^3 \\
& )*d*e*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (-12*I*(a^4 + a^2*b^2)*d*f^3*x + 1 \\
& 2*(a^3*b + a*b^3)*d*f^3*x - 12*I*(a^4 + a^2*b^2)*d*e*f^2 + 12*(a^3*b + a*b^ \\
& 3)*d*e*f^2)*\sinh(d*x + c)^2)*\text{dilog}(I*\cosh(d*x + c) + I*\sinh(d*x + c)) + (12 \\
& *I*(a^4 + a^2*b^2)*d*f^3*x + 12*(a^3*b + a*b^3)*d*f^3*x + 12*I*(a^4 + a^2*b \\
& ^2)*d*e*f^2 + 12*(a^3*b + a*b^3)*d*e*f^2 + (12*I*(a^4 + a^2*b^2)*d*f^3*x + \\
& 12*(a^3*b + a*b^3)*d*f^3*x + 12*I*(a^4 + a^2*b^2)*d*e*f^2 + 12*(a^3*b + a*b \\
& ^3)*d*e*f^2)*\cosh(d*x + c)^2 + (24*I*(a^4 + a^2*b^2)*d*f^3*x + 24*(a^3*b + \\
& a*b^3)*d*f^3*x + 24*I*(a^4 + a^2*b^2)*d*e*f^2 + 24*(a^3*b + a*b^3)*d*e*f^2) \\
& *\cosh(d*x + c)*\sinh(d*x + c) + (12*I*(a^4 + a^2*b^2)*d*f^3*x + 12*(a^3*b + \\
& a*b^3)*d*f^3*x + 12*I*(a^4 + a^2*b^2)*d*e*f^2 + 12*(a^3*b + a*b^3)*d*e*f^2) \\
& *\sinh(d*x + c)^2)*\text{dilog}(-I*\cosh(d*x + c) - I*\sinh(d*x + c)) - 6*((a^4 + 2*a \\
& ^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*d^2*e*f^2*x + (a^4 + \\
& 2*a^2*b^2 + b^4)*d^2*e^2*f + ((a^4 + 2*a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^4 \\
& + 2*a^2*b^2 + b^4)*d^2*e*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d^2*e^2*f)*\cosh(d* \\
& x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d^2*f^3*x^2 + 2*(a^4 + 2*a^2*b^2 + b^ \\
& 4)*d^2*e*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d^2*e^2*f)*\cosh(d*x + c)*\sinh(d*x
\end{aligned}$$

$$\begin{aligned}
& + c) + ((a^4 + 2a^2b^2 + b^4)d^2f^3x^2 + 2(a^4 + 2a^2b^2 + b^4)d^2 \\
& *ef^2x + (a^4 + 2a^2b^2 + b^4)d^2e^2f) * \sinh(dx + c)^2 * \operatorname{dilog}(-\cosh(dx + c) - \sinh(dx + c)) - 2((a^4 + 2a^2b^2 + b^4)d^3f^3x^3 + 3(a^4 \\
& + 2a^2b^2 + b^4)d^3ef^2x^2 + 3(a^4 + 2a^2b^2 + b^4)d^3e^2fx + (a^4 + 2a^2b^2 + b^4)d^3e^3 + ((a^4 + 2a^2b^2 + b^4)d^3f^3x^3 + 3 \\
& (a^4 + 2a^2b^2 + b^4)d^3ef^2x^2 + 3(a^4 + 2a^2b^2 + b^4)d^3e^2fx + (a^4 + 2a^2b^2 + b^4)d^3e^3) * \cosh(dx + c)^2 + 2((a^4 + 2a^2b^2 + b^4)d^3f^3x^3 + 3(a^4 + 2 \\
& a^2b^2 + b^4)d^3e^2fx + (a^4 + 2a^2b^2 + b^4)d^3e^3) * \cosh(dx + c) * \sinh(dx + c) + ((a^4 + 2a^2b^2 + b^4)d^3f^3x^3 + 3(a^4 + 2a^2b^2 \\
& + b^4)d^3ef^2x^2 + 3(a^4 + 2a^2b^2 + b^4)d^3e^2fx + (a^4 + 2a^2b^2 + b^4)d^3e^3) * \sinh(dx + c)^2 * \log(\cosh(dx + c) + \sinh(dx + c) + \\
& 1) + (-6I(a^4 + a^2b^2)d^2e^2f + 6(a^3b + ab^3)d^2e^2f + 12I(a^4 + a^2b^2) * cd * ef^2 - 12(a^3b + ab^3) * cd * ef^2 - 6I(a^4 + a^2b^2) * c^2f^3 + 6(a^3b + ab^3) * c^2f^3 + (-6I(a^4 + a^2b^2)d^2e^2f + \\
& 6(a^3b + ab^3)d^2e^2f + 12I(a^4 + a^2b^2) * cd * ef^2 - 12(a^3b + ab^3) * cd * ef^2 - 6I(a^4 + a^2b^2) * c^2f^3 + 6(a^3b + ab^3) * c^2f^3) * \cosh(dx + c)^2 + (-12I(a^4 + a^2b^2)d^2e^2f + 12(a^3b + ab^3)d^2 \\
& e^2f + 24I(a^4 + a^2b^2) * cd * ef^2 - 24(a^3b + ab^3) * cd * ef^2 - 12I(a^4 + a^2b^2) * c^2f^3 + 12(a^3b + ab^3) * c^2f^3) * \cosh(dx + c) * \sinh(dx + c) + (-6I(a^4 + a^2b^2)d^2e^2f + 6(a^3b + ab^3)d^2e^2f \\
& + 12I(a^4 + a^2b^2) * cd * ef^2 - 12(a^3b + ab^3) * cd * ef^2 - 6I(a^4 + a^2b^2) * c^2f^3 + 6(a^3b + ab^3) * c^2f^3) * \sinh(dx + c)^2 * \log(\cosh(dx + c) + \sinh(dx + c) + I) + (6I(a^4 + a^2b^2)d^2e^2f + 6(a^3b + \\
& ab^3)d^2e^2f - 12I(a^4 + a^2b^2) * cd * ef^2 - 12(a^3b + ab^3) * cd * ef^2 + 6I(a^4 + a^2b^2) * c^2f^3 + 6(a^3b + ab^3) * c^2f^3 + (6I(a^4 + a^2b^2)d^2e^2f + 6(a^3b + ab^3)d^2e^2f - 12I(a^4 + a^2b^2) * \\
& cd * ef^2 - 12(a^3b + ab^3) * cd * ef^2 + 6I(a^4 + a^2b^2) * c^2f^3 + 6(a^3b + ab^3) * c^2f^3) * \cosh(dx + c)^2 + (12I(a^4 + a^2b^2)d^2e^2f + 12(a^3b + ab^3)d^2e^2f - 24I(a^4 + a^2b^2) * cd * ef^2 - 24(a^3b \\
& + ab^3) * cd * ef^2 + 12I(a^4 + a^2b^2) * c^2f^3 + 12(a^3b + ab^3) * c^2f^3) * \cosh(dx + c) * \sinh(dx + c) + (6I(a^4 + a^2b^2)d^2e^2f + 6(a^3b + ab^3)d^2e^2f - 12I(a^4 + a^2b^2) * cd * ef^2 - 12(a^3b + ab^3) \\
& * cd * ef^2 + 6I(a^4 + a^2b^2) * c^2f^3 + 6(a^3b + ab^3) * c^2f^3) * \sinh(dx + c)^2 * \log(\cosh(dx + c) + \sinh(dx + c) - I) + 2((a^4 + 2a^2b^2 + b^4)d^3e^3 - 3(a^4 + 2a^2b^2 + b^4) * cd^2e^2f + 3(a^4 + 2a^2b^2 + \\
& b^4) * c^2d * ef^2 - (a^4 + 2a^2b^2 + b^4) * c^3f^3 + ((a^4 + 2a^2b^2 + b^4)d^3e^3 - 3(a^4 + 2a^2b^2 + b^4) * cd^2e^2f + 3(a^4 + 2a^2b^2 + b^4) * c^2d * ef^2 - (a^4 + 2a^2b^2 + b^4) * c^3f^3) * \cosh(dx + c)^2 + 2((a^4 + 2a^2b^2 + b^4)d^3e^3 - 3(a^4 + 2a^2b^2 + b^4) * cd^2e^2f + 3(a^4 + 2a^2b^2 + b^4) * c^2d * ef^2 - (a^4 + 2a^2b^2 + b^4) * c^3f^3) * \cosh(dx + c) * \sinh(dx + c) + ((a^4 + 2a^2b^2 + b^4)d^3e^3 - 3(a^4 + 2a^2b^2 + b^4) * cd^2e^2f + 3(a^4 + 2a^2b^2 + b^4) * c^2d * ef^2 - (a^4 + 2a^2b^2 + b^4) * c^3f^3) * \sinh(dx + c)^2 * \log(\cosh(dx + c) + \sinh(dx + c) - 1) + (6I(a^4 + a^2b^2)d^2f^3x^2 + 6(a^3b + ab^3)d^2f^3x^2 + 12
\end{aligned}$$



$$\begin{aligned}
& *I*(a^4 + a^2*b^2)*d^2*e*f^2*x + 12*(a^3*b + a*b^3)*d^2*e*f^2*x + 12*I*(a^4 \\
& + a^2*b^2)*c*d*e*f^2 + 12*(a^3*b + a*b^3)*c*d*e*f^2 - 6*I*(a^4 + a^2*b^2)* \\
& c^2*f^3 - 6*(a^3*b + a*b^3)*c^2*f^3 + (6*I*(a^4 + a^2*b^2)*d^2*f^3*x^2 + 6* \\
& (a^3*b + a*b^3)*d^2*f^3*x^2 + 12*I*(a^4 + a^2*b^2)*d^2*e*f^2*x + 12*(a^3*b \\
& + a*b^3)*d^2*e*f^2*x + 12*I*(a^4 + a^2*b^2)*c*d*e*f^2 + 12*(a^3*b + a*b^3)* \\
& c*d*e*f^2 - 6*I*(a^4 + a^2*b^2)*c^2*f^3 - 6*(a^3*b + a*b^3)*c^2*f^3)*\cosh(d \\
& *x + c)^2 + (12*I*(a^4 + a^2*b^2)*d^2*f^3*x^2 + 12*(a^3*b + a*b^3)*d^2*f^3* \\
& x^2 + 24*I*(a^4 + a^2*b^2)*d^2*e*f^2*x + 24*(a^3*b + a*b^3)*d^2*e*f^2*x + 2 \\
& 4*I*(a^4 + a^2*b^2)*c*d*e*f^2 + 24*(a^3*b + a*b^3)*c*d*e*f^2 - 12*I*(a^4 + \\
& a^2*b^2)*c^2*f^3 - 12*(a^3*b + a*b^3)*c^2*f^3)*\cosh(d*x + c)*\sinh(d*x + c) \\
& + (6*I*(a^4 + a^2*b^2)*d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d^2*f^3*x^2 + 12*I*( \\
& a^4 + a^2*b^2)*d^2*e*f^2*x + 12*(a^3*b + a*b^3)*d^2*e*f^2*x + 12*I*(a^4 + a \\
& ^2*b^2)*c*d*e*f^2 + 12*(a^3*b + a*b^3)*c*d*e*f^2 - 6*I*(a^4 + a^2*b^2)*c^2* \\
& f^3 - 6*(a^3*b + a*b^3)*c^2*f^3)*\sinh(d*x + c)^2*\log(I*\cosh(d*x + c) + I*s \\
& \sinh(d*x + c) + 1) + (-6*I*(a^4 + a^2*b^2)*d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d \\
& ^2*f^3*x^2 - 12*I*(a^4 + a^2*b^2)*d^2*e*f^2*x + 12*(a^3*b + a*b^3)*d^2*e*f^ \\
& 2*x - 12*I*(a^4 + a^2*b^2)*c*d*e*f^2 + 12*(a^3*b + a*b^3)*c*d*e*f^2 + 6*I*( \\
& a^4 + a^2*b^2)*c^2*f^3 - 6*(a^3*b + a*b^3)*c^2*f^3 + (-6*I*(a^4 + a^2*b^2)* \\
& d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d^2*f^3*x^2 - 12*I*(a^4 + a^2*b^2)*d^2*e*f^ \\
& 2*x + 12*(a^3*b + a*b^3)*d^2*e*f^2*x - 12*I*(a^4 + a^2*b^2)*c*d*e*f^2 + 12* \\
& (a^3*b + a*b^3)*c*d*e*f^2 + 6*I*(a^4 + a^2*b^2)*c^2*f^3 - 6*(a^3*b + a*b^3) \\
& *c^2*f^3)*\cosh(d*x + c)^2 + (-12*I*(a^4 + a^2*b^2)*d^2*f^3*x^2 + 12*(a^3*b \\
& + a*b^3)*d^2*f^3*x^2 - 24*I*(a^4 + a^2*b^2)*d^2*e*f^2*x + 24*(a^3*b + a*b^3 \\
& )*d^2*e*f^2*x - 24*I*(a^4 + a^2*b^2)*c*d*e*f^2 + 24*(a^3*b + a*b^3)*c*d*e*f \\
& ^2 + 12*I*(a^4 + a^2*b^2)*c^2*f^3 - 12*(a^3*b + a*b^3)*c^2*f^3)*\cosh(d*x + \\
& c)*\sinh(d*x + c) + (-6*I*(a^4 + a^2*b^2)*d^2*f^3*x^2 + 6*(a^3*b + a*b^3)*d \\
& ^2*f^3*x^2 - 12*I*(a^4 + a^2*b^2)*d^2*e*f^2*x + 12*(a^3*b + a*b^3)*d^2*e*f^ \\
& 2*x - 12*I*(a^4 + a^2*b^2)*c*d*e*f^2 + 12*(a^3*b + a*b^3)*c*d*e*f^2 + 6*I*(a \\
& ^4 + a^2*b^2)*c^2*f^3 - 6*(a^3*b + a*b^3)*c^2*f^3)*\sinh(d*x + c)^2*\log(-I* \\
& \cosh(d*x + c) - I*\sinh(d*x + c) + 1) + 2*((a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x \\
& ^3 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^3*e*f^2*x^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^ \\
& 3*e^2*f*x + 3*(a^4 + 2*a^2*b^2 + b^4)*c*d^2*e^2*f - 3*(a^4 + 2*a^2*b^2 + b^ \\
& 4)*c^2*d*e*f^2 + (a^4 + 2*a^2*b^2 + b^4)*c^3*f^3 + ((a^4 + 2*a^2*b^2 + b^4) \\
& *d^3*f^3*x^3 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^3*e*f^2*x^2 + 3*(a^4 + 2*a^2*b^2 \\
& + b^4)*d^3*e^2*f*x + 3*(a^4 + 2*a^2*b^2 + b^4)*c*d^2*e^2*f - 3*(a^4 + 2*a^ \\
& 2*b^2 + b^4)*c^2*d*e*f^2 + (a^4 + 2*a^2*b^2 + b^4)*c^3*f^3)*\cosh(d*x + c)^2 \\
& + 2*((a^4 + 2*a^2*b^2 + b^4)*d^3*f^3*x^3 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^3*e \\
& *f^2*x^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^3*e^2*f*x + 3*(a^4 + 2*a^2*b^2 + b^4 \\
& )*c*d^2*e^2*f - 3*(a^4 + 2*a^2*b^2 + b^4)*c^2*d*e*f^2 + (a^4 + 2*a^2*b^2 + \\
& b^4)*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c) + ((a^4 + 2*a^2*b^2 + b^4)*d^3*f^ \\
& 3*x^3 + 3*(a^4 + 2*a^2*b^2 + b^4)*d^3*e*f^2*x^2 + 3*(a^4 + 2*a^2*b^2 + b^4) \\
& *d^3*e^2*f*x + 3*(a^4 + 2*a^2*b^2 + b^4)*c*d^2*e^2*f - 3*(a^4 + 2*a^2*b^2 + \\
& b^4)*c^2*d*e*f^2 + (a^4 + 2*a^2*b^2 + b^4)*c^3*f^3)*\sinh(d*x + c)^2*\log(- \\
& \cosh(d*x + c) - \sinh(d*x + c) + 1) + 12*((a^4 + 2*a^2*b^2 + b^4)*f^3*\cosh(d \\
& *x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*f^3*\cosh(d*x + c)*\sinh(d*x + c) + (a^
\end{aligned}$$

$$\begin{aligned}
& 4 + 2a^2b^2 + b^4)f^3\sinh(dx + c)^2 + (a^4 + 2a^2b^2 + b^4)f^3)\text{polylog}(4, \cosh(dx + c) + \sinh(dx + c)) - 12*((a^4 + 2a^2b^2 + b^4)f^3\cosh(dx + c)^2 + 2*(a^4 + 2a^2b^2 + b^4)f^3\cosh(dx + c)*\sinh(dx + c) + (a^4 + 2a^2b^2 + b^4)f^3\sinh(dx + c)^2 + (a^4 + 2a^2b^2 + b^4)f^3) \\
& * \text{polylog}(4, -\cosh(dx + c) - \sinh(dx + c)) - 12*((a^4 + 2a^2b^2 + b^4)*d*f^3*x + (a^4 + 2a^2b^2 + b^4)*d*e*f^2 + ((a^4 + 2a^2b^2 + b^4)*d*f^3*x + (a^4 + 2a^2b^2 + b^4)*d*e*f^2)*\cosh(dx + c)^2 + 2*((a^4 + 2a^2b^2 + b^4)*d*f^3*x + (a^4 + 2a^2b^2 + b^4)*d*e*f^2)*\cosh(dx + c)*\sinh(dx + c) \\
& ) + ((a^4 + 2a^2b^2 + b^4)*d*f^3*x + (a^4 + 2a^2b^2 + b^4)*d*e*f^2)*\sinh(dx + c)^2)*\text{polylog}(3, \cosh(dx + c) + \sinh(dx + c)) + (12*I*(a^4 + a^2*b^2)*f^3 - 12*(a^3*b + a*b^3)*f^3) * \cosh(dx + c)^2 + (24*I*(a^4 + a^2*b^2)*f^3 - 24*(a^3*b + a*b^3)*f^3) * \cosh(dx + c)*\sinh(dx + c) + (12*I*(a^4 + a^2*b^2)*f^3 - 12*(a^3*b + a*b^3)*f^3) * \sinh(dx + c)^2) * \text{polylog}(3, I*\cosh(dx + c) + I*\sinh(dx + c)) + (-12*I*(a^4 + a^2*b^2)*f^3 - 12*(a^3*b + a*b^3)*f^3 + (-12*I*(a^4 + a^2*b^2)*f^3 - 12*(a^3*b + a*b^3)*f^3) * \cosh(dx + c)^2 + (-24*I*(a^4 + a^2*b^2)*f^3 - 24*(a^3*b + a*b^3)*f^3) * \cosh(dx + c)*\sinh(dx + c) + (-12*I*(a^4 + a^2*b^2)*f^3 - 12*(a^3*b + a*b^3)*f^3) * \sinh(dx + c)^2) * \text{polylog}(3, -I*\cosh(dx + c) - I*\sinh(dx + c)) + 12*((a^4 + 2a^2b^2 + b^4)*d*f^3*x + (a^4 + 2a^2b^2 + b^4)*d*e*f^2 + ((a^4 + 2a^2b^2 + b^4)*d*f^3*x + (a^4 + 2a^2b^2 + b^4)*d*e*f^2)*\cosh(dx + c)^2 + 2*((a^4 + 2a^2b^2 + b^4)*d*f^3*x + (a^4 + 2a^2b^2 + b^4)*d*e*f^2)*\cosh(dx + c)*\sinh(dx + c) + ((a^4 + 2a^2b^2 + b^4)*d*f^3*x + (a^4 + 2a^2b^2 + b^4)*d*e*f^2)*\sinh(dx + c)^2) * \text{polylog}(3, -\cosh(dx + c) - \sinh(dx + c)) + 4*((a^4 + a^2*b^2)*d^3*f^3*x^3 + 3*(a^4 + a^2*b^2)*d^3*e*f^2*x^2 + 3*(a^4 + a^2*b^2)*d^3*e^2*f*x + (a^4 + a^2*b^2)*d^3*e^3 - 2*((a^3*b + a*b^3)*d^3*f^3*x^3 + 3*(a^3*b + a*b^3)*d^3*e*f^2*x^2 + 3*(a^3*b + a*b^3)*d^3*e^2*f*x + 3*(a^3*b + a*b^3)*c*d^2*e^2*f - 3*(a^3*b + a*b^3)*c^2*d*e*f^2 + (a^3*b + a*b^3)*c^3*f^3) * \cosh(dx + c)) * \sinh(dx + c)) / ((a^5 + 2a^3*b^2 + a*b^4)*d^4*\cosh(dx + c)^2 + 2*(a^5 + 2a^3*b^2 + a*b^4)*d^4*\cosh(dx + c)*\sinh(dx + c) + (a^5 + 2a^3*b^2 + a*b^4)*d^4*\sinh(dx + c)^2 + (a^5 + 2a^3*b^2 + a*b^4)*d^4)
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cscsh(dx+c)\*sech(dx+c)\*\*2/(a+b\*sinh(dx+c)),x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm  
m="giac")
```

```
[Out] Timed out
```

$$3.441 \quad \int \frac{(e+fx)^2 \operatorname{csch}(c+dx) \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=795

$$-\frac{(e+fx)^2 \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^3}{a(a^2+b^2)^{3/2}d} + \frac{(e+fx)^2 \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^3}{a(a^2+b^2)^{3/2}d} - \frac{2f(e+fx)\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)b^3}{a(a^2+b^2)^{3/2}d^2} + \frac{2f(e+fx)}{d^2}$$

[Out]  $-\left(\frac{(b(e+fx)^2)}{(a^2+b^2)d} - (4f(e+fx)\operatorname{ArcTan}[E^{(c+dx)}])\right)/(a^2d^2) + (4b^2f(e+fx)\operatorname{ArcTan}[E^{(c+dx)}])/(a(a^2+b^2)d^2) - (2(e+fx)^2\operatorname{ArcTanh}[E^{(c+dx)}])/(ad) - (b^3(e+fx)^2\log[1+(bE^{(c+dx)})/(a-\sqrt{a^2+b^2})])/(a(a^2+b^2)^{3/2}d) + (b^3(e+fx)^2\log[1+(bE^{(c+dx)})/(a+\sqrt{a^2+b^2})])/(a(a^2+b^2)^{3/2}d) + (2b^2f(e+fx)\log[1+E^{2(c+dx)}])/(a(a^2+b^2)d^2) - (2f(e+fx)\operatorname{PolyLog}[2, -E^{(c+dx)}])/(a^2d^2) + ((2I)f^2\operatorname{PolyLog}[2, (-I)E^{(c+dx)}])/(a^2d^3) - ((2I)b^2f^2\operatorname{PolyLog}[2, (-I)E^{(c+dx)}])/(a(a^2+b^2)d^3) - ((2I)f^2\operatorname{PolyLog}[2, IE^{(c+dx)}])/(a^2d^3) + ((2I)b^2f^2\operatorname{PolyLog}[2, IE^{(c+dx)}])/(a(a^2+b^2)d^3) + (2f(e+fx)\operatorname{PolyLog}[2, E^{(c+dx)}])/(a^2d^2) - (2b^3f(e+fx)\operatorname{PolyLog}[2, -(bE^{(c+dx)})/(a-\sqrt{a^2+b^2})])/(a(a^2+b^2)^{3/2}d^2) + (2b^3f(e+fx)\operatorname{PolyLog}[2, -(bE^{(c+dx)})/(a+\sqrt{a^2+b^2})])/(a(a^2+b^2)^{3/2}d^2) + (b^2f^2\operatorname{PolyLog}[2, -E^{2(c+dx)}])/(a(a^2+b^2)d^3) + (2f^2\operatorname{PolyLog}[3, -E^{(c+dx)}])/(a^2d^3) - (2f^2\operatorname{PolyLog}[3, E^{(c+dx)}])/(a^2d^3) + (2b^3f^2\operatorname{PolyLog}[3, -(bE^{(c+dx)})/(a-\sqrt{a^2+b^2})])/(a(a^2+b^2)^{3/2}d^3) - (2b^3f^2\operatorname{PolyLog}[3, -(bE^{(c+dx)})/(a+\sqrt{a^2+b^2})])/(a(a^2+b^2)^{3/2}d^3) + ((e+fx)^2\operatorname{Sech}[c+dx])/(ad) - (b^2(e+fx)^2\operatorname{Sech}[c+dx])/(a(a^2+b^2)d) - (b(e+fx)^2\operatorname{Tanh}[c+dx])/(a(a^2+b^2)d)$

**Rubi [A]** time = 1.61409, antiderivative size = 795, normalized size of antiderivative = 1., number of steps used = 44, number of rules used = 23, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.676$ , Rules used = {5589, 2622, 321, 207, 5462, 6741, 12, 6742, 6273, 4182, 2531, 2282, 6589, 4180, 2279, 2391, 5573, 3322, 2264, 2190, 4184, 3718, 5451}

$$-\frac{(e+fx)^2 \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^3}{a(a^2+b^2)^{3/2}d} + \frac{(e+fx)^2 \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^3}{a(a^2+b^2)^{3/2}d} - \frac{2f(e+fx)\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)b^3}{a(a^2+b^2)^{3/2}d^2} + \frac{2f(e+fx)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[((e+f\*x)^2\*Csch[c+d\*x]\*Sech[c+d\*x]^2)/(a+b\*Sinh[c+d\*x]),x]

```
[Out] -((b*(e + f*x)^2)/((a^2 + b^2)*d)) - (4*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a
*d^2) + (4*b^2*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a*(a^2 + b^2)*d^2) - (2*(e
+ f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (b^3*(e + f*x)^2*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^(3/2)*d) + (b^3*(e + f*x)^2*Lo
g[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^(3/2)*d) + (2*
b*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a*(a^2 + b^2)*d^2) - (2*f*(e + f*x)*
PolyLog[2, -E^(c + d*x)])/(a*d^2) + ((2*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)]
)/(a*d^3) - ((2*I)*b^2*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^3
) - ((2*I)*f^2*PolyLog[2, I*E^(c + d*x)])/(a*d^3) + ((2*I)*b^2*f^2*PolyLog[
2, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^3) + (2*f*(e + f*x)*PolyLog[2, E^(c + d
*x)])/(a*d^2) - (2*b^3*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a
^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^2) + (2*b^3*f*(e + f*x)*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^2) + (b*f^2*
PolyLog[2, -E^(2*(c + d*x))])/(a*(a^2 + b^2)*d^3) + (2*f^2*PolyLog[3, -E^(c +
d*x)])/(a*d^3) - (2*f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) + (2*b^3*f^2*Poly
Log[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^(3/2)*d^3)
- (2*b^3*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2
+ b^2)^(3/2)*d^3) + ((e + f*x)^2*Sech[c + d*x])/(a*d) - (b^2*(e + f*x)^2*S
ech[c + d*x])/(a*(a^2 + b^2)*d) - (b*(e + f*x)^2*Tanh[c + d*x])/(a*(a^2 + b^2
)*d)
```

### Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
(d_.)*(x_)]^(p_.)]/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
IGtQ[p, 0]
```

### Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_.), x_S
ymbol] := Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2
], x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

### Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_))^(n_.)]^(p_.), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)^(n_.)*((c_.) + (d_.)*(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(p_.)], x_Symbol] := With[{u = IntHide[Csch[a + b*x]^n*Sech[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6741

```
Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_)^(m_.), x_Symbol] := Simp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m + 1, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])*((c_.) + (d_.)*(x_)^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

$f*Fz*x]$ ,  $x]$ ,  $x]$ ) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\* (F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)]]/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[(((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-I*e) + f*fz*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))]/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5451



```

Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \operatorname{csch}(c+dx) \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \operatorname{csch}(c+dx) \operatorname{sech}^2(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^2 \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= -\frac{(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{ad} - \frac{b \int (e+fx)^2 \operatorname{sech}^2(c+dx) dx}{a} \\
&= -\frac{(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{ad} - \frac{b \int (a(e+fx)^2 \operatorname{sech}^2(c+dx) dx)}{a} \\
&= -\frac{(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)^2 \operatorname{sech}(c+dx)}{ad} - \frac{(2b^4) \int \frac{e^{c+dx}}{2a-2\sqrt{a^2+b^2}} dx}{a(a^2+b^2)} \\
&= -\frac{(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{ad} - \frac{b^3(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{3/2}d} + \frac{b^3(e+fx)^2}{a(a^2+b^2)} \\
&= -\frac{b(e+fx)^2}{(a^2+b^2)d} + \frac{4b^2 f(e+fx) \tan^{-1}(e^{c+dx})}{a(a^2+b^2)d^2} - \frac{(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{ad} \\
&= -\frac{b(e+fx)^2}{(a^2+b^2)d} - \frac{4f(e+fx) \tan^{-1}(e^{c+dx})}{ad^2} + \frac{4b^2 f(e+fx) \tan^{-1}(e^{c+dx})}{a(a^2+b^2)d^2} - \frac{b^3(e+fx)^2}{a(a^2+b^2)} \\
&= -\frac{b(e+fx)^2}{(a^2+b^2)d} - \frac{4f(e+fx) \tan^{-1}(e^{c+dx})}{ad^2} + \frac{4b^2 f(e+fx) \tan^{-1}(e^{c+dx})}{a(a^2+b^2)d^2} - \frac{b^3(e+fx)^2}{a(a^2+b^2)} \\
&= -\frac{b(e+fx)^2}{(a^2+b^2)d} - \frac{4f(e+fx) \tan^{-1}(e^{c+dx})}{ad^2} + \frac{4b^2 f(e+fx) \tan^{-1}(e^{c+dx})}{a(a^2+b^2)d^2} - \frac{2(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{ad} \\
&= -\frac{b(e+fx)^2}{(a^2+b^2)d} - \frac{4f(e+fx) \tan^{-1}(e^{c+dx})}{ad^2} + \frac{4b^2 f(e+fx) \tan^{-1}(e^{c+dx})}{a(a^2+b^2)d^2} - \frac{2(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{ad} \\
&= -\frac{b(e+fx)^2}{(a^2+b^2)d} - \frac{4f(e+fx) \tan^{-1}(e^{c+dx})}{ad^2} + \frac{4b^2 f(e+fx) \tan^{-1}(e^{c+dx})}{a(a^2+b^2)d^2} - \frac{2(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{ad}
\end{aligned}$$

**Mathematica [A]** time = 11.5366, size = 1245, normalized size = 1.57

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Csch[c + d\*x]\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out]  $4*((\text{Csch}[c + d*x]*((e + f*x)^2*\text{Log}[1 - E^{(c + d*x)}] - (e + f*x)^2*\text{Log}[1 + E^{(c + d*x)}] - (2*f*(d*(e + f*x)*\text{PolyLog}[2, -E^{(c + d*x)}] - f*\text{PolyLog}[3, -E^{(c + d*x)}])))/d^2 + (2*f*(d*(e + f*x)*\text{PolyLog}[2, E^{(c + d*x)}] - f*\text{PolyLog}[3, E^{(c + d*x)}]))/d^2)*(a + b*\text{Sinh}[c + d*x]))/(4*a*d*(b + a*\text{Csch}[c + d*x])) + (b^3*\text{Csch}[c + d*x]*(2*d^2*e^2*\text{ArcTanh}[(a + b*E^{(c + d*x)})/\text{Sqrt}[a^2 + b^2]] - 2*d^2*e*f*x*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])] - d^2*f^2*x^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])] + 2*d^2*e*f*x*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])] + d^2*f^2*x^2*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])] - 2*d*f*(e + f*x)*\text{PolyLog}[2, (b*E^{(c + d*x)})/(-a + \text{Sqrt}[a^2 + b^2])] + 2*d*f*(e + f*x)*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))] + 2*f^2*\text{PolyLog}[3, (b*E^{(c + d*x)})/(-a + \text{Sqrt}[a^2 + b^2])] - 2*f^2*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])*(a + b*\text{Sinh}[c + d*x]))/(4*a*(a^2 + b^2)^(3/2)*d^3*(b + a*\text{Csch}[c + d*x])) + (b*e*f*\text{Csch}[c + d*x]*\text{Sech}[c]*(\text{Cosh}[c]*\text{Log}[\text{Cosh}[c]*\text{Cosh}[d*x] + \text{Sinh}[c]*\text{Sinh}[d*x]] - d*x*\text{Sinh}[c])*(a + b*\text{Sinh}[c + d*x]))/(2*(a^2 + b^2)*d^2*(b + a*\text{Csch}[c + d*x])*(\text{Cosh}[c]^2 - \text{Sinh}[c]^2)) - (a*e*f*\text{ArcTan}[(\text{Sinh}[c] + \text{Cosh}[c]*\text{Tanh}[(d*x)/2])/\text{Sqrt}[\text{Cosh}[c]^2 - \text{Sinh}[c]^2]]*\text{Csch}[c + d*x]*(a + b*\text{Sinh}[c + d*x]))/((a^2 + b^2)*d^2*(b + a*\text{Csch}[c + d*x])* \text{Sqrt}[\text{Cosh}[c]^2 - \text{Sinh}[c]^2]) - (b*f^2*\text{Csch}[c]*\text{Csch}[c + d*x]*(-((d^2*x^2)/E^{\text{ArcTanh}[\text{Coth}[c]])} + (I*\text{Coth}[c]*(-d*x*(-\text{Pi} + (2*I)*\text{ArcTanh}[\text{Coth}[c]))] - \text{Pi}*\text{Log}[1 + E^{(2*d*x)}] - 2*(I*d*x + I*\text{ArcTanh}[\text{Coth}[c]])*\text{Log}[1 - E^{((2*I)*(I*d*x + I*\text{ArcTanh}[\text{Coth}[c]))}]) + \text{Pi}*\text{Log}[\text{Cosh}[d*x]] + (2*I)*\text{ArcTanh}[\text{Coth}[c]]*\text{Log}[I*\text{Sinh}[d*x] + \text{ArcTanh}[\text{Coth}[c]]]) + I*\text{PolyLog}[2, E^{((2*I)*(I*d*x + I*\text{ArcTanh}[\text{Coth}[c]))}])))/\text{Sqrt}[1 - \text{Coth}[c]^2])* \text{Sech}[c]*(a + b*\text{Sinh}[c + d*x]))/(4*(a^2 + b^2)*d^3*(b + a*\text{Csch}[c + d*x])* \text{Sqrt}[\text{Csch}[c]^2*(-\text{Cosh}[c]^2 + \text{Sinh}[c]^2)] - (a*f^2*\text{Csch}[c + d*x]*((-I)*\text{Csch}[c]*(I*(d*x + \text{ArcTanh}[\text{Coth}[c]))*(\text{Log}[1 - E^{-(d*x)} - \text{ArcTanh}[\text{Coth}[c]])] - \text{Log}[1 + E^{-(d*x)} - \text{ArcTanh}[\text{Coth}[c]])] + I*(\text{PolyLog}[2, -E^{-(d*x)} - \text{ArcTanh}[\text{Coth}[c]])] - \text{PolyLog}[2, E^{-(d*x)} - \text{ArcTanh}[\text{Coth}[c]])])))/\text{Sqrt}[1 - \text{Coth}[c]^2] - (2*\text{ArcTan}[(\text{Sinh}[c] + \text{Cosh}[c]*\text{Tanh}[(d*x)/2])/\text{Sqrt}[\text{Cosh}[c]^2 - \text{Sinh}[c]^2]]*\text{ArcTanh}[\text{Coth}[c]])/\text{Sqrt}[\text{Cosh}[c]^2 - \text{Sinh}[c]^2]*(a + b*\text{Sinh}[c + d*x]))/(2*(a^2 + b^2)*d^3*(b + a*\text{Csch}[c + d*x])) + (\text{Csch}[c + d*x]*\text{Sech}[c]*\text{Sech}[c + d*x]*(a*e^2*\text{Cosh}[c] + 2*a*e*f*x*\text{Cosh}[c] + a*f^2*x^2*\text{Cosh}[c] - b*e^2*\text{Sinh}[d*x] - 2*b*e*f*x*\text{Sinh}[d*x] - b*f^2*x^2*\text{Sinh}[d*x])*(a + b*\text{Sinh}[c + d*x]))/(4*(a^2 + b^2)*d*(b + a*\text{Csch}[c + d*x]))$

---

**Maple [F]** time = 2.283, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \operatorname{csch}(dx + c) (\operatorname{sech}(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*csch(d\*x+c)\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*csch(d\*x+c)\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*csch(d\*x+c)\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [C]** time = 3.99185, size = 12713, normalized size = 15.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*csch(d\*x+c)\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{2} * (4 * (a^3 * b + a * b^3) * d^2 * e^2 - 8 * (a^3 * b + a * b^3) * c * d * e * f + 4 * (a^3 * b + a * b^3) * c^2 * f^2 - 4 * ((a^3 * b + a * b^3) * d^2 * f^2 * x^2 + 2 * (a^3 * b + a * b^3) * d^2 * e * f * x + 2 * (a^3 * b + a * b^3) * c * d * e * f - (a^3 * b + a * b^3) * c^2 * f^2) * \cosh(d * x + c)^2 - 4 * ((a^3 * b + a * b^3) * d^2 * f^2 * x^2 + 2 * (a^3 * b + a * b^3) * d^2 * e * f * x + 2 * (a^3 * b + a * b^3) * c * d * e * f - (a^3 * b + a * b^3) * c^2 * f^2) * \sinh(d * x + c)^2 - 4 * (b^4 * d * f^2 * x + b^4 * d * e * f + (b^4 * d * f^2 * x + b^4 * d * e * f) * \cosh(d * x + c)^2 + 2 * (b^4 * d * f^2 * x + b^4$

$$\begin{aligned}
& *d*e*f)*\cosh(d*x + c)*\sinh(d*x + c) + (b^4*d*f^2*x + b^4*d*e*f)*\sinh(d*x + \\
& c)^2)*\sqrt{(a^2 + b^2)/b^2)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*c \\
& \cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2) - b)/b + 1) + 4*(b^4*d \\
& *f^2*x + b^4*d*e*f + (b^4*d*f^2*x + b^4*d*e*f)*\cosh(d*x + c)^2 + 2*(b^4*d \\
& f^2*x + b^4*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c) + (b^4*d*f^2*x + b^4*d*e*f)* \\
& \sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x \\
& + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2) - b)/b + 1 \\
& ) + 2*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2 + (b^4*d^2*e^2 - 2*b^4*c*d \\
& *e*f + b^4*c^2*f^2)*\cosh(d*x + c)^2 + 2*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4* \\
& c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c \\
& ^2*f^2)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2)*\log(2*b*\cosh(d*x + c) + 2*b* \\
& \sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2) + 2*a) - 2*(b^4*d^2*e^2 - 2*b^4*c \\
& *d*e*f + b^4*c^2*f^2 + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*\cosh(d*x \\
& + c)^2 + 2*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*\cosh(d*x + c)*\sinh( \\
& d*x + c) + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*\sinh(d*x + c)^2)*\sqrt{ \\
& (a^2 + b^2)/b^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^ \\
& 2 + b^2)/b^2) + 2*a) - 2*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f \\
& - b^4*c^2*f^2 + (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c \\
& ^2*f^2)*\cosh(d*x + c)^2 + 2*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d* \\
& e*f - b^4*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (b^4*d^2*f^2*x^2 + 2*b^4*d \\
& ^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b \\
& ^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x \\
& + c))*\sqrt{(a^2 + b^2)/b^2) - b)/b) + 2*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x \\
& + 2*b^4*c*d*e*f - b^4*c^2*f^2 + (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4 \\
& *c*d*e*f - b^4*c^2*f^2)*\cosh(d*x + c)^2 + 2*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e* \\
& f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (b^4*d^2*f \\
& ^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*\sinh(d*x + c)^2)*\sqrt{ \\
& (a^2 + b^2)/b^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + \\
& c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2) - b)/b) + 4*(b^4*f^2*\cosh(d*x \\
& + c)^2 + 2*b^4*f^2*\cosh(d*x + c)*\sinh(d*x + c) + b^4*f^2*\sinh(d*x + c)^2 + \\
& b^4*f^2)*\sqrt{(a^2 + b^2)/b^2)*\operatorname{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c \\
& ) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2))/b) - 4*(b^4* \\
& f^2*\cosh(d*x + c)^2 + 2*b^4*f^2*\cosh(d*x + c)*\sinh(d*x + c) + b^4*f^2*\sinh( \\
& d*x + c)^2 + b^4*f^2)*\sqrt{(a^2 + b^2)/b^2)*\operatorname{polylog}(3, (a*\cosh(d*x + c) + a \\
& *\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2)) \\
& /b) + 4*((a^4 + a^2*b^2)*d^2*f^2*x^2 + 2*(a^4 + a^2*b^2)*d^2*e*f*x + (a^4 + \\
& a^2*b^2)*d^2*e^2)*\cosh(d*x + c) + 4*((a^4 + 2*a^2*b^2 + b^4)*d*f^2*x + (a^ \\
& 4 + 2*a^2*b^2 + b^4)*d*e*f + ((a^4 + 2*a^2*b^2 + b^4)*d*f^2*x + (a^4 + 2*a^ \\
& 2*b^2 + b^4)*d*e*f)*\cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d*f^2*x + \\
& (a^4 + 2*a^2*b^2 + b^4)*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c) + ((a^4 + 2*a^2* \\
& b^2 + b^4)*d*f^2*x + (a^4 + 2*a^2*b^2 + b^4)*d*e*f)*\sinh(d*x + c)^2)*\operatorname{dilog}( \\
& \cosh(d*x + c) + \sinh(d*x + c)) + (-4*I*(a^4 + a^2*b^2)*f^2 + 4*(a^3*b + a*b \\
& ^3)*f^2 + (-4*I*(a^4 + a^2*b^2)*f^2 + 4*(a^3*b + a*b^3)*f^2)*\cosh(d*x + c)^ \\
& 2 + (-8*I*(a^4 + a^2*b^2)*f^2 + 8*(a^3*b + a*b^3)*f^2)*\cosh(d*x + c)*\sinh(d \\
& *x + c) + (-4*I*(a^4 + a^2*b^2)*f^2 + 4*(a^3*b + a*b^3)*f^2)*\sinh(d*x + c)^
\end{aligned}$$

$$\begin{aligned}
& 2) * \operatorname{dilog}(I * \cosh(dx + c) + I * \sinh(dx + c)) + (4 * I * (a^4 + a^2 * b^2) * f^2 + 4 * \\
& (a^3 * b + a * b^3) * f^2 + (4 * I * (a^4 + a^2 * b^2) * f^2 + 4 * (a^3 * b + a * b^3) * f^2) * \cos \\
& h(dx + c)^2 + (8 * I * (a^4 + a^2 * b^2) * f^2 + 8 * (a^3 * b + a * b^3) * f^2) * \cosh(dx + \\
& c) * \sinh(dx + c) + (4 * I * (a^4 + a^2 * b^2) * f^2 + 4 * (a^3 * b + a * b^3) * f^2) * \sinh( \\
& dx + c)^2) * \operatorname{dilog}(-I * \cosh(dx + c) - I * \sinh(dx + c)) - 4 * ((a^4 + 2 * a^2 * b^2 \\
& + b^4) * d * f^2 * x + (a^4 + 2 * a^2 * b^2 + b^4) * d * e * f + ((a^4 + 2 * a^2 * b^2 + b^4) * \\
& d * f^2 * x + (a^4 + 2 * a^2 * b^2 + b^4) * d * e * f) * \cosh(dx + c)^2 + 2 * ((a^4 + 2 * a^2 * \\
& b^2 + b^4) * d * f^2 * x + (a^4 + 2 * a^2 * b^2 + b^4) * d * e * f) * \cosh(dx + c) * \sinh(dx \\
& + c) + ((a^4 + 2 * a^2 * b^2 + b^4) * d * f^2 * x + (a^4 + 2 * a^2 * b^2 + b^4) * d * e * f) * \sinh \\
& (dx + c)^2) * \operatorname{dilog}(-\cosh(dx + c) - \sinh(dx + c)) - 2 * ((a^4 + 2 * a^2 * b^2 \\
& + b^4) * d^2 * f^2 * x^2 + 2 * (a^4 + 2 * a^2 * b^2 + b^4) * d^2 * e * f * x + (a^4 + 2 * a^2 * b^2 \\
& + b^4) * d^2 * e^2 + ((a^4 + 2 * a^2 * b^2 + b^4) * d^2 * f^2 * x^2 + 2 * (a^4 + 2 * a^2 * b^2 \\
& + b^4) * d^2 * e * f * x + (a^4 + 2 * a^2 * b^2 + b^4) * d^2 * e^2) * \cosh(dx + c)^2 + 2 * (( \\
& a^4 + 2 * a^2 * b^2 + b^4) * d^2 * f^2 * x^2 + 2 * (a^4 + 2 * a^2 * b^2 + b^4) * d^2 * e * f * x + \\
& (a^4 + 2 * a^2 * b^2 + b^4) * d^2 * e^2) * \cosh(dx + c) * \sinh(dx + c) + ((a^4 + 2 * a^2 * \\
& b^2 + b^4) * d^2 * f^2 * x^2 + 2 * (a^4 + 2 * a^2 * b^2 + b^4) * d^2 * e * f * x + (a^4 + 2 * a^ \\
& ^2 * b^2 + b^4) * d^2 * e^2) * \sinh(dx + c)^2) * \log(\cosh(dx + c) + \sinh(dx + c) + \\
& 1) + (-4 * I * (a^4 + a^2 * b^2) * d * e * f + 4 * (a^3 * b + a * b^3) * d * e * f + 4 * I * (a^4 + a^ \\
& 2 * b^2) * c * f^2 - 4 * (a^3 * b + a * b^3) * c * f^2 + (-4 * I * (a^4 + a^2 * b^2) * d * e * f + 4 * (a \\
& ^3 * b + a * b^3) * d * e * f + 4 * I * (a^4 + a^2 * b^2) * c * f^2 - 4 * (a^3 * b + a * b^3) * c * f^2) * \\
& \cosh(dx + c)^2 + (-8 * I * (a^4 + a^2 * b^2) * d * e * f + 8 * (a^3 * b + a * b^3) * d * e * f + 8 \\
& * I * (a^4 + a^2 * b^2) * c * f^2 - 8 * (a^3 * b + a * b^3) * c * f^2) * \cosh(dx + c) * \sinh(dx \\
& + c) + (-4 * I * (a^4 + a^2 * b^2) * d * e * f + 4 * (a^3 * b + a * b^3) * d * e * f + 4 * I * (a^4 + a \\
& ^2 * b^2) * c * f^2 - 4 * (a^3 * b + a * b^3) * c * f^2) * \sinh(dx + c)^2) * \log(\cosh(dx + c) \\
& + \sinh(dx + c) + I) + (4 * I * (a^4 + a^2 * b^2) * d * e * f + 4 * (a^3 * b + a * b^3) * d * e * \\
& f - 4 * I * (a^4 + a^2 * b^2) * c * f^2 - 4 * (a^3 * b + a * b^3) * c * f^2 + (4 * I * (a^4 + a^2 * b \\
& ^2) * d * e * f + 4 * (a^3 * b + a * b^3) * d * e * f - 4 * I * (a^4 + a^2 * b^2) * c * f^2 - 4 * (a^3 * b \\
& + a * b^3) * c * f^2) * \cosh(dx + c)^2 + (8 * I * (a^4 + a^2 * b^2) * d * e * f + 8 * (a^3 * b + a \\
& * b^3) * d * e * f - 8 * I * (a^4 + a^2 * b^2) * c * f^2 - 8 * (a^3 * b + a * b^3) * c * f^2) * \cosh(dx \\
& + c) * \sinh(dx + c) + (4 * I * (a^4 + a^2 * b^2) * d * e * f + 4 * (a^3 * b + a * b^3) * d * e * f \\
& - 4 * I * (a^4 + a^2 * b^2) * c * f^2 - 4 * (a^3 * b + a * b^3) * c * f^2) * \sinh(dx + c)^2) * \log \\
& (\cosh(dx + c) + \sinh(dx + c) - I) + 2 * ((a^4 + 2 * a^2 * b^2 + b^4) * d^2 * e^2 - \\
& 2 * (a^4 + 2 * a^2 * b^2 + b^4) * c * d * e * f + (a^4 + 2 * a^2 * b^2 + b^4) * c^2 * f^2 + ((a^4 \\
& + 2 * a^2 * b^2 + b^4) * d^2 * e^2 - 2 * (a^4 + 2 * a^2 * b^2 + b^4) * c * d * e * f + (a^4 + 2 * \\
& a^2 * b^2 + b^4) * c^2 * f^2) * \cosh(dx + c)^2 + 2 * ((a^4 + 2 * a^2 * b^2 + b^4) * d^2 * e^ \\
& 2 - 2 * (a^4 + 2 * a^2 * b^2 + b^4) * c * d * e * f + (a^4 + 2 * a^2 * b^2 + b^4) * c^2 * f^2) * \co \\
& sh(dx + c) * \sinh(dx + c) + ((a^4 + 2 * a^2 * b^2 + b^4) * d^2 * e^2 - 2 * (a^4 + 2 * a \\
& ^2 * b^2 + b^4) * c * d * e * f + (a^4 + 2 * a^2 * b^2 + b^4) * c^2 * f^2) * \sinh(dx + c)^2) * \log \\
& (\cosh(dx + c) + \sinh(dx + c) - 1) + (4 * I * (a^4 + a^2 * b^2) * d * f^2 * x + 4 * (a \\
& ^3 * b + a * b^3) * d * f^2 * x + 4 * I * (a^4 + a^2 * b^2) * c * f^2 + 4 * (a^3 * b + a * b^3) * c * f^2 \\
& + (4 * I * (a^4 + a^2 * b^2) * d * f^2 * x + 4 * (a^3 * b + a * b^3) * d * f^2 * x + 4 * I * (a^4 + a^ \\
& 2 * b^2) * c * f^2 + 4 * (a^3 * b + a * b^3) * c * f^2) * \cosh(dx + c)^2 + (8 * I * (a^4 + a^2 * b \\
& ^2) * d * f^2 * x + 8 * (a^3 * b + a * b^3) * d * f^2 * x + 8 * I * (a^4 + a^2 * b^2) * c * f^2 + 8 * (a^ \\
& 3 * b + a * b^3) * c * f^2) * \cosh(dx + c) * \sinh(dx + c) + (4 * I * (a^4 + a^2 * b^2) * d * f^ \\
& 2 * x + 4 * (a^3 * b + a * b^3) * d * f^2 * x + 4 * I * (a^4 + a^2 * b^2) * c * f^2 + 4 * (a^3 * b + a *
\end{aligned}$$

$$\begin{aligned}
& b^3 * c * f^2 * \sinh(d * x + c)^2 * \log(I * \cosh(d * x + c) + I * \sinh(d * x + c) + 1) + (-4 * I * (a^4 + a^2 * b^2) * d * f^2 * x + 4 * (a^3 * b + a * b^3) * d * f^2 * x - 4 * I * (a^4 + a^2 * b^2) * c * f^2 + 4 * (a^3 * b + a * b^3) * c * f^2 + (-4 * I * (a^4 + a^2 * b^2) * d * f^2 * x + 4 * (a^3 * b + a * b^3) * d * f^2 * x - 4 * I * (a^4 + a^2 * b^2) * c * f^2 + 4 * (a^3 * b + a * b^3) * c * f^2) * \cosh(d * x + c)^2 + (-8 * I * (a^4 + a^2 * b^2) * d * f^2 * x + 8 * (a^3 * b + a * b^3) * d * f^2 * x - 8 * I * (a^4 + a^2 * b^2) * c * f^2 + 8 * (a^3 * b + a * b^3) * c * f^2) * \cosh(d * x + c) * \sinh(d * x + c) + (-4 * I * (a^4 + a^2 * b^2) * d * f^2 * x + 4 * (a^3 * b + a * b^3) * d * f^2 * x - 4 * I * (a^4 + a^2 * b^2) * c * f^2 + 4 * (a^3 * b + a * b^3) * c * f^2) * \sinh(d * x + c)^2 * \log(-I * \cosh(d * x + c) - I * \sinh(d * x + c) + 1) + 2 * ((a^4 + 2 * a^2 * b^2 + b^4) * d^2 * f^2 * x^2 + 2 * (a^4 + 2 * a^2 * b^2 + b^4) * d^2 * e * f * x + 2 * (a^4 + 2 * a^2 * b^2 + b^4) * c * d * e * f - (a^4 + 2 * a^2 * b^2 + b^4) * c^2 * f^2 + ((a^4 + 2 * a^2 * b^2 + b^4) * d^2 * f^2 * x^2 + 2 * (a^4 + 2 * a^2 * b^2 + b^4) * d^2 * e * f * x + 2 * (a^4 + 2 * a^2 * b^2 + b^4) * c * d * e * f - (a^4 + 2 * a^2 * b^2 + b^4) * c^2 * f^2) * \cosh(d * x + c)^2 + 2 * ((a^4 + 2 * a^2 * b^2 + b^4) * d^2 * f^2 * x^2 + 2 * (a^4 + 2 * a^2 * b^2 + b^4) * d^2 * e * f * x + 2 * (a^4 + 2 * a^2 * b^2 + b^4) * c * d * e * f - (a^4 + 2 * a^2 * b^2 + b^4) * c^2 * f^2) * \cosh(d * x + c) * \sinh(d * x + c) + ((a^4 + 2 * a^2 * b^2 + b^4) * d^2 * f^2 * x^2 + 2 * (a^4 + 2 * a^2 * b^2 + b^4) * d^2 * e * f * x + 2 * (a^4 + 2 * a^2 * b^2 + b^4) * c * d * e * f - (a^4 + 2 * a^2 * b^2 + b^4) * c^2 * f^2) * \sinh(d * x + c)^2 * \log(-\cosh(d * x + c) - \sinh(d * x + c) + 1) - 4 * ((a^4 + 2 * a^2 * b^2 + b^4) * f^2 * \cosh(d * x + c)^2 + 2 * (a^4 + 2 * a^2 * b^2 + b^4) * f^2 * \cosh(d * x + c) * \sinh(d * x + c) + (a^4 + 2 * a^2 * b^2 + b^4) * f^2 * \sinh(d * x + c)^2 + (a^4 + 2 * a^2 * b^2 + b^4) * f^2) * \text{polylog}(3, \cosh(d * x + c) + \sinh(d * x + c)) + 4 * ((a^4 + 2 * a^2 * b^2 + b^4) * f^2 * \cosh(d * x + c)^2 + 2 * (a^4 + 2 * a^2 * b^2 + b^4) * f^2 * \cosh(d * x + c) * \sinh(d * x + c) + (a^4 + 2 * a^2 * b^2 + b^4) * f^2 * \sinh(d * x + c)^2 + (a^4 + 2 * a^2 * b^2 + b^4) * f^2) * \text{polylog}(3, -\cosh(d * x + c) - \sinh(d * x + c)) + 4 * ((a^4 + a^2 * b^2) * d^2 * f^2 * x^2 + 2 * (a^4 + a^2 * b^2) * d^2 * e * f * x + (a^4 + a^2 * b^2) * d^2 * e^2 - 2 * ((a^3 * b + a * b^3) * d^2 * f^2 * x^2 + 2 * (a^3 * b + a * b^3) * d^2 * e * f * x + 2 * (a^3 * b + a * b^3) * c * d * e * f - (a^3 * b + a * b^3) * c^2 * f^2) * \cosh(d * x + c)) * \sinh(d * x + c) / ((a^5 + 2 * a^3 * b^2 + a * b^4) * d^3 * \cosh(d * x + c)^2 + 2 * (a^5 + 2 * a^3 * b^2 + a * b^4) * d^3 * \cosh(d * x + c) * \sinh(d * x + c) + (a^5 + 2 * a^3 * b^2 + a * b^4) * d^3 * \sinh(d * x + c)^2 + (a^5 + 2 * a^3 * b^2 + a * b^4) * d^3)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*csc(d\*x+c)\*sech(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```



$$3.442 \quad \int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=442

$$-\frac{b^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2 (a^2+b^2)^{3/2}} + \frac{b^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2 (a^2+b^2)^{3/2}} - \frac{f \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{ad^2} + \frac{f \operatorname{PolyLog}\left(2, e^{c+dx}\right)}{ad^2} + \frac{b^2 f \operatorname{tan}}{a}$$

[Out]  $-\left(\frac{f \operatorname{ArcTan}[\operatorname{Sinh}[c+d*x]]}{a*d^2}\right) + \left(\frac{b^2*f \operatorname{ArcTan}[\operatorname{Sinh}[c+d*x]]}{a*(a^2+b^2)*d^2}\right) - \left(\frac{2*f*x \operatorname{ArcTanh}[E^{(c+d*x)}]}{a*d}\right) + \left(\frac{f*x \operatorname{ArcTanh}[\operatorname{Cosh}[c+d*x]]}{a*d}\right) - \left(\frac{(e+f*x) \operatorname{ArcTanh}[\operatorname{Cosh}[c+d*x]]}{a*d}\right) - \left(\frac{b^3*(e+f*x)*\operatorname{Log}[1+(b*E^{(c+d*x)})/(a-\operatorname{Sqrt}[a^2+b^2])]}{a*(a^2+b^2)^{(3/2)*d}}\right) + \left(\frac{b^3*(e+f*x)*\operatorname{Log}[1+(b*E^{(c+d*x)})/(a+\operatorname{Sqrt}[a^2+b^2])]}{a*(a^2+b^2)^{(3/2)*d}}\right) + \left(\frac{b*f*\operatorname{Log}[\operatorname{Cosh}[c+d*x]]}{(a^2+b^2)*d^2}\right) - \left(\frac{f*\operatorname{PolyLog}[2, -E^{(c+d*x)}]}{a*d^2}\right) + \left(\frac{f*\operatorname{PolyLog}[2, E^{(c+d*x)}]}{a*d^2}\right) - \left(\frac{b^3*f*\operatorname{PolyLog}[2, -(b*E^{(c+d*x)})/(a-\operatorname{Sqrt}[a^2+b^2])]}{a*(a^2+b^2)^{(3/2)*d^2}}\right) + \left(\frac{b^3*f*\operatorname{PolyLog}[2, -(b*E^{(c+d*x)})/(a+\operatorname{Sqrt}[a^2+b^2])]}{a*(a^2+b^2)^{(3/2)*d^2}}\right) + \left(\frac{(e+f*x)*\operatorname{Sech}[c+d*x]}{a*d}\right) - \left(\frac{b^2*(e+f*x)*\operatorname{Sech}[c+d*x]}{a*(a^2+b^2)*d}\right) - \left(\frac{b*(e+f*x)*\operatorname{Tanh}[c+d*x]}{(a^2+b^2)*d}\right)$

**Rubi [A]** time = 0.805706, antiderivative size = 442, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 19, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.594$ , Rules used = {5589, 2622, 321, 207, 5462, 6271, 12, 4182, 2279, 2391, 3770, 5573, 3322, 2264, 2190, 6742, 4184, 3475, 5451}

$$-\frac{b^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2 (a^2+b^2)^{3/2}} + \frac{b^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2 (a^2+b^2)^{3/2}} - \frac{f \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{ad^2} + \frac{f \operatorname{PolyLog}\left(2, e^{c+dx}\right)}{ad^2} + \frac{b^2 f \operatorname{tan}}{a}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[\left(\frac{(e+f*x)*\operatorname{Csch}[c+d*x]*\operatorname{Sech}[c+d*x]^2}{a+b*\operatorname{Sinh}[c+d*x]}\right), x]$

[Out]  $-\left(\frac{f \operatorname{ArcTan}[\operatorname{Sinh}[c+d*x]]}{a*d^2}\right) + \left(\frac{b^2*f \operatorname{ArcTan}[\operatorname{Sinh}[c+d*x]]}{a*(a^2+b^2)*d^2}\right) - \left(\frac{2*f*x \operatorname{ArcTanh}[E^{(c+d*x)}]}{a*d}\right) + \left(\frac{f*x \operatorname{ArcTanh}[\operatorname{Cosh}[c+d*x]]}{a*d}\right) - \left(\frac{(e+f*x) \operatorname{ArcTanh}[\operatorname{Cosh}[c+d*x]]}{a*d}\right) - \left(\frac{b^3*(e+f*x)*\operatorname{Log}[1+(b*E^{(c+d*x)})/(a-\operatorname{Sqrt}[a^2+b^2])]}{a*(a^2+b^2)^{(3/2)*d}}\right) + \left(\frac{b^3*(e+f*x)*\operatorname{Log}[1+(b*E^{(c+d*x)})/(a+\operatorname{Sqrt}[a^2+b^2])]}{a*(a^2+b^2)^{(3/2)*d}}\right) + \left(\frac{b*f*\operatorname{Log}[\operatorname{Cosh}[c+d*x]]}{(a^2+b^2)*d^2}\right) - \left(\frac{f*\operatorname{PolyLog}[2, -E^{(c+d*x)}]}{a*d^2}\right) + \left(\frac{f*\operatorname{PolyLog}[2, E^{(c+d*x)}]}{a*d^2}\right) - \left(\frac{b^3*f*\operatorname{PolyLog}[2, -(b*E^{(c+d*x)})/(a-\operatorname{Sqrt}[a^2+b^2])]}{a*(a^2+b^2)^{(3/2)*d^2}}\right) + \left(\frac{b^3*f*\operatorname{PolyLog}[2, -(b*E^{(c+d*x)})/(a+\operatorname{Sqrt}[a^2+b^2])]}{a*(a^2+b^2)^{(3/2)*d^2}}\right) + \left(\frac{(e+f*x)*\operatorname{Sech}[c+d*x]}{a*d}\right) - \left(\frac{b^2*(e+f*x)*\operatorname{Sech}[c+d*x]}{a*(a^2+b^2)*d}\right) - \left(\frac{b*(e+f*x)*\operatorname{Tanh}[c+d*x]}{(a^2+b^2)*d}\right)$

$$3f \cdot \text{PolyLog}[2, -((bE^{(c+dx)})/(a + \sqrt{a^2 + b^2}))]/(a(a^2 + b^2)^{(3/2)d^2) + ((e + fx) \cdot \text{Sech}[c + dx])/(ad) - (b^2(e + fx) \cdot \text{Sech}[c + dx])/(a(a^2 + b^2)d) - (b(e + fx) \cdot \text{Tanh}[c + dx])/((a^2 + b^2)d)$$
Rule 5589

```
Int[(Csch[(c_) + (d_)*(x_)]^(n_)*((e_) + (f_)*(x_))^(m_)*Sech[(c_) +
(d_)*(x_)]^(p_)]/((a_) + (b_)*Sinh[(c_) + (d_)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2622

```
Int[csc[(e_) + (f_)*(x_)]^(n_)*((a_)*sec[(e_) + (f_)*(x_)]^(m_)), x_Symbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2)], x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_))^(n_)]^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_) + (b_)*(x_)]^(n_)*((c_) + (d_)*(x_))^(m_)*Sech[(a_) + (b_)*(x_)]^(p_), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

Int[ArcTanh[u\_], x\_Symbol] := Simp[x\*ArcTanh[u], x] - Int[SimplifyIntegrand[(x\*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 4182

Int[csc[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

### Rule 5573

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f\*x)^m\*Sech[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f\*x)^m\*Sech[c + d\*x]^n\*(a - b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]

### Rule 3322

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_) + (b\_.)\*sin[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(-(I\*b) + 2\*a\*E^(-(I\*e) + f\*fz\*x) + I\*b\*E^(2\*(-(I\*e) + f\*fz\*x))), x], x] /; F

reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

#### Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.)\*(F\_)^(v\_)), x\_Symbol] :=> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

#### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.), x\_Symbol] :=> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

#### Rule 6742

Int[u\_, x\_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :=> -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

#### Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :=> -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

#### Rule 5451

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sech[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*Tanh[(a\_.) + (b\_.)\*(x\_)]^(p\_.), x\_Symbol] :=> -Simp[((c + d\*x)^m\*Sech[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Sech[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx}{a} \\
&= -\frac{(e+fx)\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)\operatorname{sech}(c+dx)}{ad} - \frac{b \int (e+fx)\operatorname{sech}^2(c+dx) dx}{a} \\
&= -\frac{(e+fx)\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(e+fx)\operatorname{sech}(c+dx)}{ad} - \frac{b \int (a+fx)\operatorname{sech}^2(c+dx) dx}{a} \\
&= -\frac{f \tan^{-1}(\sinh(c+dx))}{ad^2} + \frac{fx \tanh^{-1}(\cosh(c+dx))}{ad} - \frac{(e+fx)\tanh^{-1}(\cosh(c+dx))}{ad} \\
&= -\frac{f \tan^{-1}(\sinh(c+dx))}{ad^2} + \frac{fx \tanh^{-1}(\cosh(c+dx))}{ad} - \frac{(e+fx)\tanh^{-1}(\cosh(c+dx))}{ad} \\
&= -\frac{f \tan^{-1}(\sinh(c+dx))}{ad^2} + \frac{b^2 f \tan^{-1}(\sinh(c+dx))}{a(a^2+b^2)d^2} - \frac{2fx \tanh^{-1}(e^{c+dx})}{ad} \\
&= -\frac{f \tan^{-1}(\sinh(c+dx))}{ad^2} + \frac{b^2 f \tan^{-1}(\sinh(c+dx))}{a(a^2+b^2)d^2} - \frac{2fx \tanh^{-1}(e^{c+dx})}{ad} \\
&= -\frac{f \tan^{-1}(\sinh(c+dx))}{ad^2} + \frac{b^2 f \tan^{-1}(\sinh(c+dx))}{a(a^2+b^2)d^2} - \frac{2fx \tanh^{-1}(e^{c+dx})}{ad}
\end{aligned}$$

**Mathematica [A]** time = 7.2462, size = 459, normalized size = 1.04

$$\operatorname{csch}(c+dx)(a+b\sinh(c+dx)) \left( \frac{b^3(-f\operatorname{PolyLog}\left(2, \frac{b(\sinh(c+dx)+\cosh(c+dx))}{\sqrt{a^2+b^2}-a}\right) + f\operatorname{PolyLog}\left(2, -\frac{b(\sinh(c+dx)+\cosh(c+dx))}{\sqrt{a^2+b^2}+a}\right) + 2de \tanh^{-1}\left(\frac{a+b\sinh(c+dx)}{\sqrt{a^2+b^2}}\right)}{\dots} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Csch[c + d\*x]\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]), x]

[Out] (Csch[c + d\*x]\*(a + b\*Sinh[c + d\*x])\*((-2\*a\*f\*ArcTan[Tanh[(c + d\*x)/2]]))/(a^2 + b^2) + (b\*f\*Log[Cosh[c + d\*x]])/(a^2 + b^2) + (d\*e\*Log[Tanh[(c + d\*x)/2]])/(a^2 + b^2)

$$\begin{aligned} & 2]]/a - (c*f*\text{Log}[\text{Tanh}[(c + d*x)/2]])/a + (f*((c + d*x)*(\text{Log}[1 - E^{(-c - d*x)}] - \text{Log}[1 + E^{(-c - d*x)}]) + \text{PolyLog}[2, -E^{(-c - d*x)}] - \text{PolyLog}[2, E^{(-c - d*x)}]))/a + (b^3*(2*d*e*\text{ArcTanh}[(a + b*\text{Cosh}[c + d*x] + b*\text{Sinh}[c + d*x])/ \text{Sqrt}[a^2 + b^2]] - 2*c*f*\text{ArcTanh}[(a + b*\text{Cosh}[c + d*x] + b*\text{Sinh}[c + d*x])/ \text{Sqrt}[a^2 + b^2]] - f*(c + d*x)*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a - \text{Sqrt}[a^2 + b^2])] + f*(c + d*x)*\text{Log}[1 + (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a + \text{Sqrt}[a^2 + b^2])] - f*\text{PolyLog}[2, (b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(-a + \text{Sqrt}[a^2 + b^2])] + f*\text{PolyLog}[2, -(b*(\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x]))/(a + \text{Sqrt}[a^2 + b^2])]))/(a*(a^2 + b^2)^(3/2)) + (d*(e + f*x)*\text{Sech}[c + d*x]*(a - b*\text{Sinh}[c + d*x]))/(a^2 + b^2))/(d^2*(b + a*\text{Csch}[c + d*x])) \end{aligned}$$

**Maple [B]** time = 0.279, size = 1815, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)`

[Out] 
$$\begin{aligned} & -1/(a^2+b^2)^(5/2)/d^2*f*b*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^(1/2)) * a^3-2/(a^2+b^2)^(5/2)/d^2*f*b^3*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^(1/2)) * a+1/(a^2+b^2)/d*a*e*\ln(\exp(d*x+c)-1)-1/(a^2+b^2)/d*a*e*\ln(\exp(d*x+c)+1)-1/(a^2+b^2)/d^2*a*f*\text{dilog}(\exp(d*x+c))-1/(a^2+b^2)/d^2*a*f*\text{dilog}(\exp(d*x+c)+1)+1/(a^2+b^2)^(5/2)/d^2*a*b^3*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))) * c-1/(a^2+b^2)^(5/2)/d^2*a*b^3*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))) * c+1/(a^2+b^2)^(5/2)/d*f*b^5/a*\ln((b*\exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))) * x-1/(a^2+b^2)^(3/2)/d^2*b^3*f*c/a*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2)^(5/2)/d^2*f*a^3*b*c*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^(5/2)/d^2*b^5*f*c/a*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^(5/2)/d*a*b^3*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))) * x-1/(a^2+b^2)^(5/2)/d*f*b^5/a*\ln((-b*\exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))) * x+1/(a^2+b^2)^(5/2)/d*a*b^3*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))) * x+1/(a^2+b^2)^(5/2)/d^2*f*b^5/a*\ln((b*\exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))) * c-1/(a^2+b^2)^(5/2)/d^2*f*b^5/a*\ln((-b*\exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))) * c-2/(a^2+b^2)/d^2*b*f*\ln(\exp(d*x+c))-1/(a^2+b^2)/d*\ln(\exp(d*x+c)+1)*a*f*x-8/(a^2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*\text{arctan}(\exp(d*x+c))+4/(a^2+b^2)/d^2*f*b^3/(4*a^2+4*b^2)*\ln(1+\exp(2*d*x+2*c))-1/(a^2+b^2)/d^2*a*f*c*\ln(\exp(d*x+c)-1)-1/(a^2+b^2)/d^2*b^2*f/a*\text{dilog}(\exp(d*x+c))-1/(a^2+b^2)/d^2*b^2*f/a*\text{dilog}(\exp(d*x+c)+1)+1/(a^2+b^2)/d*b^2*e/a*\ln(\exp(d*x+c)-1)-1/(a^2+b^2)/d*b^2*e/a*\ln(\exp(d*x+c)+1)+b/d*e/(a^2+b^2)^(3/2)*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^(1/2)) \end{aligned}$$

$$\begin{aligned}
& *a+2*(f*x+e)*(a*\exp(d*x+c)+b)/d/(a^2+b^2)/(1+\exp(2*d*x+2*c))-1/(a^2+b^2)/d^2 \\
& *b^2*f*c/a*\ln(\exp(d*x+c)-1)+4/(a^2+b^2)/d^2*a^2*b*f/(4*a^2+4*b^2)*\ln(1+\exp \\
& (2*d*x+2*c))-1/(a^2+b^2)^{(5/2)}/d^2*a*b^3*f*dilog((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)} \\
& -a)/(-a+(a^2+b^2)^{(1/2)}))+1/(a^2+b^2)^{(5/2)}/d^2*a*b^3*f*dilog((b*\exp(d*x+c) \\
& +(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))-8/(a^2+b^2)/d^2*a*b^2*f/(4*a^2 \\
& +4*b^2)*\arctan(\exp(d*x+c))+1/(a^2+b^2)^{(3/2)}/d^2*f*b^3/a*\operatorname{arctanh}(1/2*(2*b* \\
& \exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))-1/(a^2+b^2)^{(5/2)}/d^2*f*b^5/a*dilog((-b*\exp \\
& (d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))+1/(a^2+b^2)^{(5/2)}/d^2*f*b^5/a \\
& *dilog((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))-1/(a^2+b^2)^{(5/2)}/d^2*f*b^5/a \\
& *\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))+1/(a^2+b^2)^{(3/2)}/d^2*a*f*b \\
& *\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))+1/(a^2+b^2)^{(3/2)}/d*b^3*e/a \\
& *\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))-1/(a^2+b^2)^{(5/2)}/d*a^3*b*e \\
& *\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))+1/(a^2+b^2)^{(5/2)}/d*b^5*e/a \\
& *\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))-1/(a^2+b^2)/d*b^2*f/a \\
& *\ln(\exp(d*x+c)+1)*x-b/d^2*f*c/(a^2+b^2)^{(3/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))*a
\end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.63917, size = 5233, normalized size = 11.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $-(2*(a^3*b + a*b^3)*d*f*x*\cosh(d*x + c)^2 + 2*(a^3*b + a*b^3)*d*f*x*\sinh(d*x + c)^2 - 2*(a^3*b + a*b^3)*d*e + (b^4*f*\cosh(d*x + c)^2 + 2*b^4*f*\cosh(d*x + c)*\sinh(d*x + c) + b^4*f*\sinh(d*x + c)^2 + b^4*f)*\sqrt{(a^2 + b^2)/b^2}$

```

*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x +
c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^4*f*cosh(d*x + c)^2 + 2*b^4*f*c
osh(d*x + c)*sinh(d*x + c) + b^4*f*sinh(d*x + c)^2 + b^4*f)*sqrt((a^2 + b^2
)/b^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^4*d*e - b^4*c*f + (b^4*d*
e - b^4*c*f)*cosh(d*x + c)^2 + 2*(b^4*d*e - b^4*c*f)*cosh(d*x + c)*sinh(d*x
+ c) + (b^4*d*e - b^4*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(2*b*
cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^4
*d*e - b^4*c*f + (b^4*d*e - b^4*c*f)*cosh(d*x + c)^2 + 2*(b^4*d*e - b^4*c*f
)*cosh(d*x + c)*sinh(d*x + c) + (b^4*d*e - b^4*c*f)*sinh(d*x + c)^2)*sqrt((
a^2 + b^2)/b^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 +
b^2)/b^2) + 2*a) + (b^4*d*f*x + b^4*c*f + (b^4*d*f*x + b^4*c*f)*cosh(d*x +
c)^2 + 2*(b^4*d*f*x + b^4*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b^4*d*f*x +
b^4*c*f)*sinh(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*s
inh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
b)/b) - (b^4*d*f*x + b^4*c*f + (b^4*d*f*x + b^4*c*f)*cosh(d*x + c)^2 + 2*(b
^4*d*f*x + b^4*c*f)*cosh(d*x + c)*sinh(d*x + c) + (b^4*d*f*x + b^4*c*f)*sin
h(d*x + c)^2)*sqrt((a^2 + b^2)/b^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c)
- (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + 2*((
a^4 + a^2*b^2)*f*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*f*cosh(d*x + c)*sinh(d
*x + c) + (a^4 + a^2*b^2)*f*sinh(d*x + c)^2 + (a^4 + a^2*b^2)*f)*arctan(cos
h(d*x + c) + sinh(d*x + c)) - 2*((a^4 + a^2*b^2)*d*f*x + (a^4 + a^2*b^2)*d*
e)*cosh(d*x + c) - ((a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)^2 + 2*(a^4 + 2*
a^2*b^2 + b^4)*f*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*f*si
nh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*f)*dilog(cosh(d*x + c) + sinh(d*x +
c)) + ((a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^
4)*f*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*f*sinh(d*x + c)^
2 + (a^4 + 2*a^2*b^2 + b^4)*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - ((a^
3*b + a*b^3)*f*cosh(d*x + c)^2 + 2*(a^3*b + a*b^3)*f*cosh(d*x + c)*sinh(d*x
+ c) + (a^3*b + a*b^3)*f*sinh(d*x + c)^2 + (a^3*b + a*b^3)*f)*log(2*cosh(d
*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + ((a^4 + 2*a^2*b^2 + b^4)*d*f*x +
(a^4 + 2*a^2*b^2 + b^4)*d*e + ((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^
2*b^2 + b^4)*d*e)*cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4
+ 2*a^2*b^2 + b^4)*d*e)*cosh(d*x + c)*sinh(d*x + c) + ((a^4 + 2*a^2*b^2 +
b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*e)*sinh(d*x + c)^2)*log(cosh(d*x + c
) + sinh(d*x + c) + 1) - ((a^4 + 2*a^2*b^2 + b^4)*d*e - (a^4 + 2*a^2*b^2 +
b^4)*c*f + ((a^4 + 2*a^2*b^2 + b^4)*d*e - (a^4 + 2*a^2*b^2 + b^4)*c*f)*cosh
(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d*e - (a^4 + 2*a^2*b^2 + b^4)*c*f)
*cosh(d*x + c)*sinh(d*x + c) + ((a^4 + 2*a^2*b^2 + b^4)*d*e - (a^4 + 2*a^2*
b^2 + b^4)*c*f)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - (
a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*c*f + ((a^4 + 2*a^2
*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*c*f)*cosh(d*x + c)^2 + 2*((a^4
+ 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*c*f)*cosh(d*x + c)*sinh(
d*x + c) + ((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*c*f)*si
nh(d*x + c)^2)*log(-cosh(d*x + c) - sinh(d*x + c) + 1) + 2*(2*(a^3*b + a*b^

```



$$3)*d*f*x*cosh(d*x + c) - (a^4 + a^2*b^2)*d*f*x - (a^4 + a^2*b^2)*d*e)*sinh(d*x + c))/((a^5 + 2*a^3*b^2 + a*b^4)*d^2*cosh(d*x + c)^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d^2*cosh(d*x + c)*sinh(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4)*d^2*sinh(d*x + c)^2 + (a^5 + 2*a^3*b^2 + a*b^4)*d^2)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)\*sech(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.443 \quad \int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=113

$$\frac{2b^3 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad(a^2+b^2)^{3/2}} - \frac{b \operatorname{sech}(c+dx)(a \sinh(c+dx)+b)}{ad(a^2+b^2)} + \frac{\operatorname{sech}(c+dx)}{ad} - \frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

[Out]  $-(\operatorname{ArcTanh}[\operatorname{Cosh}[c+d*x]]/(a*d)) + (2*b^3*\operatorname{ArcTanh}[(b-a*\operatorname{Tanh}[(c+d*x)/2])/ \operatorname{Sqrt}[a^2+b^2]])/(a*(a^2+b^2)^{(3/2)*d}) + \operatorname{Sech}[c+d*x]/(a*d) - (b*\operatorname{Sech}[c+d*x]*(b+a*\operatorname{Sinh}[c+d*x]))/(a*(a^2+b^2)*d)$

**Rubi [A]** time = 0.259943, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2898, 2622, 321, 207, 2696, 12, 2660, 618, 204}

$$\frac{2b^3 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad(a^2+b^2)^{3/2}} - \frac{b \operatorname{sech}(c+dx)(a \sinh(c+dx)+b)}{ad(a^2+b^2)} + \frac{\operatorname{sech}(c+dx)}{ad} - \frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Csch}[c+d*x]*\operatorname{Sech}[c+d*x]^2)/(a+b*\operatorname{Sinh}[c+d*x]),x]$

[Out]  $-(\operatorname{ArcTanh}[\operatorname{Cosh}[c+d*x]]/(a*d)) + (2*b^3*\operatorname{ArcTanh}[(b-a*\operatorname{Tanh}[(c+d*x)/2])/ \operatorname{Sqrt}[a^2+b^2]])/(a*(a^2+b^2)^{(3/2)*d}) + \operatorname{Sech}[c+d*x]/(a*d) - (b*\operatorname{Sech}[c+d*x]*(b+a*\operatorname{Sinh}[c+d*x]))/(a*(a^2+b^2)*d)$

### Rule 2898

$\operatorname{Int}[(\cos[(e_.) + (f_.)*(x_)]*(g_.)^{(p_)}*\sin[(e_.) + (f_.)*(x_)]^{(n_)}]/((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]), x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrig}[(g*\cos[e + f*x])^p, \sin[e + f*x]^n/(a + b*\sin[e + f*x]), x], x] /; \operatorname{FreeQ}\{a, b, e, f, g, p\}, x] \&\& \operatorname{NeQ}[a^2 - b^2, 0] \&\& \operatorname{IntegerQ}[n] \&\& (\operatorname{LtQ}[n, 0] \|\ \operatorname{IGtQ}[p + 1/2, 0])$

### Rule 2622

$\operatorname{Int}[\operatorname{csc}[(e_.) + (f_.)*(x_)]^{(n_)}*((a_.)*\operatorname{sec}[(e_.) + (f_.)*(x_)]^{(m_)}), x\_Symbol] \rightarrow \operatorname{Dist}[1/(f*a^n), \operatorname{Subst}[\operatorname{Int}[x^{(m+n-1)}/(-1+x^2/a^2)^{(n+1)/2}$

), x], x, a\*Sec[e + f\*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

### Rule 2696

Int[(cos[(e\_.) + (f\_.)\*(x\_)]\*(g\_.))^(p\_)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(m\_), x\_Symbol] := Simp[(g\*cos[e + f\*x])^(p + 1)\*(a + b\*sin[e + f\*x])^(m + 1)\*(b - a\*sin[e + f\*x])/(f\*g\*(a^2 - b^2)\*(p + 1)), x] + Dist[1/(g^2\*(a^2 - b^2)\*(p + 1)), Int[(g\*cos[e + f\*x])^(p + 2)\*(a + b\*sin[e + f\*x])^m\*(a^2\*(p + 2) - b^2\*(m + p + 2) + a\*b\*(m + p + 3)\*sin[e + f\*x]), x], x] /; FreeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[2\*m, 2\*p]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 2660

Int[((a\_) + (b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(-1), x\_Symbol] := With[{e = FreeFactors[Tan[(c + d\*x)/2], x]}, Dist[(2\*e)/d, Subst[Int[1/(a + 2\*b\*e\*x + a\*e^2\*x^2), x], x, Tan[(c + d\*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

### Rule 618

Int[((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4\*a\*c - x^2, x], x], x, b + 2\*c\*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx &= i \int \left( -\frac{i\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a} + \frac{ib\operatorname{sech}^2(c+dx)}{a(a+b\sinh(c+dx))} \right) dx \\
&= \frac{\int \operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx) dx}{a} - \frac{b \int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx}{a} \\
&= -\frac{b\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a(a^2+b^2)d} - \frac{b \int \frac{b^2}{a+b\sinh(c+dx)} dx}{a(a^2+b^2)} + \frac{\operatorname{Subst}\left(\int \frac{x^2}{-1+x^2} dx, x, \operatorname{sech}(c+dx)\right)}{ad} \\
&= \frac{\operatorname{sech}(c+dx)}{ad} - \frac{b\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a(a^2+b^2)d} - \frac{b^3 \int \frac{1}{a+b\sinh(c+dx)} dx}{a(a^2+b^2)} + \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \operatorname{sech}(c+dx)\right)}{ad} \\
&= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{\operatorname{sech}(c+dx)}{ad} - \frac{b\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a(a^2+b^2)d} + \frac{(2ib^3 \operatorname{sech}(c+dx))}{ad} \\
&= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{\operatorname{sech}(c+dx)}{ad} - \frac{b\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a(a^2+b^2)d} - \frac{(4ib^3 \operatorname{sech}(c+dx))}{ad} \\
&= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{2b^3 \tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{3/2}d} + \frac{\operatorname{sech}(c+dx)}{ad} - \frac{b\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a(a^2+b^2)d}
\end{aligned}$$

**Mathematica [A]** time = 0.264734, size = 171, normalized size = 1.51

$$\frac{-ab\sqrt{-a^2-b^2}\tanh(c+dx) + a^2\sqrt{-a^2-b^2}\operatorname{sech}(c+dx) + b^2\sqrt{-a^2-b^2}\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + a^2\sqrt{-a^2-b^2}\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad(-a^2-b^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Csch[c + d\*x]\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]), x]

```
[Out] -((-2*b^3*ArcTan[(b - a*Tanh[(c + d*x)/2])/Sqrt[-a^2 - b^2]] + a^2*Sqrt[-a^2 - b^2]*Log[Tanh[(c + d*x)/2]] + b^2*Sqrt[-a^2 - b^2]*Log[Tanh[(c + d*x)/2]]) + a^2*Sqrt[-a^2 - b^2]*Sech[c + d*x] - a*b*Sqrt[-a^2 - b^2]*Tanh[c + d*x])/((a*(-a^2 - b^2)^(3/2)*d))
```

**Maple [A]** time = 0.001, size = 136, normalized size = 1.2

$$\frac{1}{da} \ln \left( \tanh \left( \frac{dx}{2} + \frac{c}{2} \right) \right) - 2 \frac{b^3}{da (a^2 + b^2)^{3/2}} \operatorname{Artanh} \left( \frac{1}{2} \frac{2a \tanh(1/2 dx + c/2) - 2b}{\sqrt{a^2 + b^2}} \right) - 2 \frac{\tanh(1/2 dx + c/2)}{d (a^2 + b^2) \left( \tanh(1/2 dx + c/2) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/d/a*ln(tanh(1/2*d*x+1/2*c))-2/d/a*b^3/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-2/d/(a^2+b^2)/(tanh(1/2*d*x+1/2*c)^(2+1))*tanh(1/2*d*x+1/2*c)*b+2/d/(a^2+b^2)/(tanh(1/2*d*x+1/2*c)^(2+1))*a
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [B]** time = 3.11662, size = 1455, normalized size = 12.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*a^3*b + 2*a*b^3 + (b^3*cosh(d*x + c)^2 + 2*b^3*cosh(d*x + c)*sinh(d*x + c) + b^3*sinh(d*x + c)^2 + b^3)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 2*(a^4 + a^2*b^2)*cosh(d*x + c) - (a^4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (a^4 + 2*a^2*b^2 + b^4 + (a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)*sinh(d*x + c) + (a^4 + 2*a^2*b^2 + b^4)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(a^4 + a^2*b^2)*sinh(d*x + c))/((a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c)^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^5 + 2*a^3*b^2 + a*b^4)*d*sinh(d*x + c)^2 + (a^5 + 2*a^3*b^2 + a*b^4)*d)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 1.21899, size = 209, normalized size = 1.85

$$-\frac{b^3 \log\left(\frac{|2be^{(dx+c)}+2a-2\sqrt{a^2+b^2}|}{|2be^{(dx+c)}+2a+2\sqrt{a^2+b^2}|}\right)}{(a^3d+ab^2d)\sqrt{a^2+b^2}} + \frac{2(ae^{(dx+c)}+b)}{(a^2d+b^2d)(e^{2dx+2c}+1)} - \frac{\log(e^{(dx+c)}+1)}{ad} + \frac{\log(|e^{(dx+c)}-1|)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -b^3*log(abs(2*b*e^(d*x + c) + 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*e^(d*x + c) + 2*a + 2*sqrt(a^2 + b^2)))/((a^3*d + a*b^2*d)*sqrt(a^2 + b^2)) + 2*(a*e^(d*x + c) + b)/((a^2*d + b^2*d)*(e^(2*d*x + 2*c) + 1)) - log(e^(d*x + c) + 1
```

$$)/(a*d) + \log(\text{abs}(e^{(d*x + c)} - 1))/(a*d)$$

$$3.444 \quad \int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable}\left(\frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}, x\right)$$

[Out] Unintegrable[(Csch[c + d\*x]\*Sech[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0893257, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Csch[c + d\*x]\*Sech[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int] [(Csch[c + d\*x]\*Sech[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.001, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Csch[c + d\*x]\*Sech[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]



[Out] \$Aborted

**Maple [A]** time = 1.864, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c) (\operatorname{sech}(dx+c))^2}{(fx+e)(a+b\sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)`

[Out] `int(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$-8b^3 \int \frac{e^{(dx+c)}}{4(a^3be + ab^3e + (a^3bf + ab^3f)x - (a^3bee^{2c} + ab^3ee^{2c} + (a^3bfe^{2c} + ab^3fe^{2c})x)e^{2dx} - 2(a^4ee^c + a^2b^2ee^c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `-8*b^3*integrate(-1/4*e^(d*x + c)/(a^3*b*e + a*b^3*e + (a^3*b*f + a*b^3*f)*x - (a^3*b*e*e^(2*c) + a*b^3*e*e^(2*c) + (a^3*b*f*e^(2*c) + a*b^3*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^4*e*e^c + a^2*b^2*e*e^c + (a^4*f*e^c + a^2*b^2*f*e^c)*x)*e^(d*x)), x) + 2*(a*e^(d*x + c) + b)/(a^2*d*e + b^2*d*e + (a^2*d*f + b^2*d*f)*x + (a^2*d*e*e^(2*c) + b^2*d*e*e^(2*c) + (a^2*d*f*e^(2*c) + b^2*d*f*e^(2*c))*x)*e^(2*d*x)) + 8*integrate(1/4*(a*f*e^(d*x + c) + b*f)/(a^2*d*e^2 + b^2*d*e^2 + (a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(a^2*d*e*f + b^2*d*e*f)*x + (a^2*d*e^2*e^(2*c) + b^2*d*e^2*e^(2*c) + (a^2*d*f^2*e^(2*c) + b^2*d*f^2*e^(2*c))*x^2 + 2*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e^(2*c))*x)*e^(2*d*x)), x) + 8*integrate(1/8/(a*f*x + a*e + (a*f*x*e^c + a*e*e^c)*e^(d*x)), x) + 8*integrate(-1/8/(a*f*x + a*e - (a*f*x*e^c + a*e*e^c)*e^(d*x)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{csch}(dx+c)\text{sech}(dx+c)^2}{(afx+ae+(bfx+be)\sinh(dx+c))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csch(d*x + c)*sech(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.445 \quad \int \frac{(e+fx)^2 \operatorname{csch}(c+dx) \operatorname{sech}^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=1185

result too large to display

```
[Out] (e*f*x)/(a*d) + (f^2*x^2)/(2*a*d) - (2*b^3*(e + f*x)^2*ArcTan[E^(c + d*x)])
/((a^2 + b^2)^2*d) - (b*(e + f*x)^2*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) +
(b*f^2*ArcTan[Sinh[c + d*x]])/((a^2 + b^2)*d^3) - (2*(e + f*x)^2*ArcTanh[E^
(2*c + 2*d*x)]/(a*d) - (b^4*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2]])/(a*(a^2 + b^2)^2*d) - (b^4*(e + f*x)^2*Log[1 + (b*E^(c + d*x)
)/(a + Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)^2*d) + (b^4*(e + f*x)^2*Log[1 + E^
(2*(c + d*x))]/(a*(a^2 + b^2)^2*d) + (f^2*Log[Cosh[c + d*x]])/(a*d^3) - (b
^2*f^2*Log[Cosh[c + d*x]])/(a*(a^2 + b^2)*d^3) + ((2*I)*b^3*f*(e + f*x)*Pol
yLog[2, (-I)*E^(c + d*x)]/((a^2 + b^2)^2*d^2) + (I*b*f*(e + f*x)*PolyLog[2
, (-I)*E^(c + d*x)]/((a^2 + b^2)*d^2) - ((2*I)*b^3*f*(e + f*x)*PolyLog[2,
I*E^(c + d*x)]/((a^2 + b^2)^2*d^2) - (I*b*f*(e + f*x)*PolyLog[2, I*E^(c +
d*x)]/((a^2 + b^2)*d^2) - (2*b^4*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^2*d^2) - (2*b^4*f*(e + f*x)*PolyLog
[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^2*d^2) + (b^4
*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))]/(a*(a^2 + b^2)^2*d^2) - (f*(e +
f*x)*PolyLog[2, -E^(2*c + 2*d*x)]/(a*d^2) + (f*(e + f*x)*PolyLog[2, E^(2*c
+ 2*d*x)]/(a*d^2) - ((2*I)*b^3*f^2*PolyLog[3, (-I)*E^(c + d*x)]/((a^2 +
b^2)^2*d^3) - (I*b*f^2*PolyLog[3, (-I)*E^(c + d*x)]/((a^2 + b^2)*d^3) + ((
2*I)*b^3*f^2*PolyLog[3, I*E^(c + d*x)]/((a^2 + b^2)^2*d^3) + (I*b*f^2*Poly
Log[3, I*E^(c + d*x)]/((a^2 + b^2)*d^3) + (2*b^4*f^2*PolyLog[3, -(b*E^(c
+ d*x))/(a - Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^2*d^3) + (2*b^4*f^2*PolyLog
[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^2*d^3) - (b^4
*f^2*PolyLog[3, -E^(2*(c + d*x))]/(2*a*(a^2 + b^2)^2*d^3) + (f^2*PolyLog[3
, -E^(2*c + 2*d*x)]/(2*a*d^3) - (f^2*PolyLog[3, E^(2*c + 2*d*x)]/(2*a*d^3
) - (b*f*(e + f*x)*Sech[c + d*x])/((a^2 + b^2)*d^2) - (b^2*(e + f*x)^2*Sech
[c + d*x]^2)/(2*a*(a^2 + b^2)*d) - (f*(e + f*x)*Tanh[c + d*x])/(a*d^2) + (b
^2*f*(e + f*x)*Tanh[c + d*x])/(a*(a^2 + b^2)*d^2) - (b*(e + f*x)^2*Sech[c +
d*x]*Tanh[c + d*x])/(2*(a^2 + b^2)*d) - ((e + f*x)^2*Tanh[c + d*x]^2)/(2*a
*d)
```

---

**Rubi [A]** time = 2.2249, antiderivative size = 1185, normalized size of antiderivative = 1., number of steps used = 57, number of rules used = 23, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.676$ , Rules used = {5589, 2620, 14, 5462, 6741, 12, 6742, 2551, 4182, 2531, 2282, 6589,

3720, 3475, 5573, 5561, 2190, 4180, 3718, 4186, 3770, 5451, 4184}

$$\frac{(e+fx)^2 \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^4}{a(a^2+b^2)^2 d} - \frac{(e+fx)^2 \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^4}{a(a^2+b^2)^2 d} + \frac{(e+fx)^2 \log(1+e^{2(c+dx)})b^4}{a(a^2+b^2)^2 d} - \frac{2f(e+fx)\text{PolyLog}}{a(a^2+b^2)}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Csch[c + d\*x]\*Sech[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] (e\*f\*x)/(a\*d) + (f^2\*x^2)/(2\*a\*d) - (2\*b^3\*(e + f\*x)^2\*ArcTan[E^(c + d\*x)]) / ((a^2 + b^2)^2\*d) - (b\*(e + f\*x)^2\*ArcTan[E^(c + d\*x)]) / ((a^2 + b^2)\*d) + (b\*f^2\*ArcTan[Sinh[c + d\*x]]) / ((a^2 + b^2)\*d^3) - (2\*(e + f\*x)^2\*ArcTanh[E^(2\*c + 2\*d\*x)]) / (a\*d) - (b^4\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])]) / (a\*(a^2 + b^2)^2\*d) - (b^4\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])]) / (a\*(a^2 + b^2)^2\*d) + (b^4\*(e + f\*x)^2\*Log[1 + E^(2\*(c + d\*x))]) / (a\*(a^2 + b^2)^2\*d) + (f^2\*Log[Cosh[c + d\*x]]) / (a\*d^3) - (b^2\*f^2\*Log[Cosh[c + d\*x]]) / (a\*(a^2 + b^2)\*d^3) + ((2\*I)\*b^3\*f\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)]) / ((a^2 + b^2)^2\*d^2) + (I\*b\*f\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)]) / ((a^2 + b^2)\*d^2) - ((2\*I)\*b^3\*f\*(e + f\*x)\*PolyLog[2, I\*E^(c + d\*x)]) / ((a^2 + b^2)^2\*d^2) - (I\*b\*f\*(e + f\*x)\*PolyLog[2, I\*E^(c + d\*x)]) / ((a^2 + b^2)\*d^2) - (2\*b^4\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))]) / (a\*(a^2 + b^2)^2\*d^2) - (2\*b^4\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))]) / (a\*(a^2 + b^2)^2\*d^2) + (b^4\*f\*(e + f\*x)\*PolyLog[2, -E^(2\*(c + d\*x))]) / (a\*(a^2 + b^2)^2\*d^2) - (f\*(e + f\*x)\*PolyLog[2, -E^(2\*c + 2\*d\*x)]) / (a\*d^2) + (f\*(e + f\*x)\*PolyLog[2, E^(2\*c + 2\*d\*x)]) / (a\*d^2) - ((2\*I)\*b^3\*f^2\*PolyLog[3, (-I)\*E^(c + d\*x)]) / ((a^2 + b^2)^2\*d^3) - (I\*b\*f^2\*PolyLog[3, (-I)\*E^(c + d\*x)]) / ((a^2 + b^2)\*d^3) + ((2\*I)\*b^3\*f^2\*PolyLog[3, I\*E^(c + d\*x)]) / ((a^2 + b^2)^2\*d^3) + (I\*b\*f^2\*PolyLog[3, I\*E^(c + d\*x)]) / ((a^2 + b^2)\*d^3) + (2\*b^4\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))]) / (a\*(a^2 + b^2)^2\*d^3) + (2\*b^4\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))]) / (a\*(a^2 + b^2)^2\*d^3) - (b^4\*f^2\*PolyLog[3, -E^(2\*(c + d\*x))]) / (2\*a\*(a^2 + b^2)^2\*d^3) + (f^2\*PolyLog[3, -E^(2\*c + 2\*d\*x)]) / (2\*a\*d^3) - (f^2\*PolyLog[3, E^(2\*c + 2\*d\*x)]) / (2\*a\*d^3) - (b\*f\*(e + f\*x)\*Sech[c + d\*x]) / ((a^2 + b^2)\*d^2) - (b^2\*(e + f\*x)^2\*Sech[c + d\*x]^2) / (2\*a\*(a^2 + b^2)\*d) - (f\*(e + f\*x)\*Tanh[c + d\*x]) / (a\*d^2) + (b^2\*f\*(e + f\*x)\*Tanh[c + d\*x]) / (a\*(a^2 + b^2)\*d^2) - (b\*(e + f\*x)^2\*Sech[c + d\*x]\*Tanh[c + d\*x]) / (2\*(a^2 + b^2)\*d) - ((e + f\*x)^2\*Tanh[c + d\*x]^2) / (2\*a\*d)

### Rule 5589

Int[(Csch[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(p\_.)]/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Sech[c + d\*x]^p\*Csch[c + d\*x]^n, x], x] - Dist[b/a

```
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
IGtQ[p, 0]
```

### Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

### Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_
+ (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

### Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

### Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v !=
= u]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_.)*(v_)] /; FreeQ[b, x]
```

### Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
```

ionFreeQ[u, x] && NeQ[m, -1]

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)]], x], x]
+ Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)]], x], x)
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol]
:> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*Log[F]), x]
+ Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x]
/; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_)), x_Symbol]
:> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

### Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 3718

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
```

$(m - 2) \cdot (b \cdot \text{Csc}[e + f \cdot x])^{(n - 2)}, x, x] + \text{Dist}[(b^2 \cdot (n - 2)) / (n - 1), \text{Int}[(c + d \cdot x)^m \cdot (b \cdot \text{Csc}[e + f \cdot x])^{(n - 2)}, x, x] - \text{Simp}[(b^2 \cdot d \cdot m \cdot (c + d \cdot x)^{(m - 1)} \cdot (b \cdot \text{Csc}[e + f \cdot x])^{(n - 2)}) / (f^2 \cdot (n - 1) \cdot (n - 2)), x]] /;$  FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

### Rule 3770

$\text{Int}[\text{csc}[(c \cdot) + (d \cdot) \cdot (x \cdot)], x\_Symbol] \text{ :> } -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d \cdot x]] / d, x] /;$  FreeQ[{c, d}, x]

### Rule 5451

$\text{Int}[(c \cdot) + (d \cdot) \cdot (x \cdot)]^{(m \cdot)} \cdot \text{Sech}[(a \cdot) + (b \cdot) \cdot (x \cdot)]^{(n \cdot)} \cdot \text{Tanh}[(a \cdot) + (b \cdot) \cdot (x \cdot)]^{(p \cdot)}, x\_Symbol] \text{ :> } -\text{Simp}[(c + d \cdot x)^m \cdot \text{Sech}[a + b \cdot x]^n / (b \cdot n), x] + \text{Dist}[(d \cdot m) / (b \cdot n), \text{Int}[(c + d \cdot x)^{(m - 1)} \cdot \text{Sech}[a + b \cdot x]^n, x]] /;$  FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

### Rule 4184

$\text{Int}[\text{csc}[(e \cdot) + (f \cdot) \cdot (x \cdot)]^2 \cdot ((c \cdot) + (d \cdot) \cdot (x \cdot))^{(m \cdot)}, x\_Symbol] \text{ :> } -\text{Simp}[(c + d \cdot x)^m \cdot \text{Cot}[e + f \cdot x] / f, x] + \text{Dist}[(d \cdot m) / f, \text{Int}[(c + d \cdot x)^{(m - 1)} \cdot \text{Cot}[e + f \cdot x], x], x] /;$  FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rubi steps



$$\begin{aligned}
\int \frac{(e+fx)^2 \operatorname{csch}(c+dx) \operatorname{sech}^3(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \operatorname{csch}(c+dx) \operatorname{sech}^3(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^2 \operatorname{sech}^3(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= \frac{(e+fx)^2 \log(\tanh(c+dx))}{ad} - \frac{(e+fx)^2 \tanh^2(c+dx)}{2ad} - \frac{b \int (e+fx)^2 \operatorname{sech}^3(c+dx) dx}{a} \\
&= \frac{(e+fx)^2 \log(\tanh(c+dx))}{ad} - \frac{(e+fx)^2 \tanh^2(c+dx)}{2ad} - \frac{b^3 \int (e+fx)^2 \operatorname{sech}^3(c+dx) dx}{a} \\
&= \frac{b^4(e+fx)^3}{3a(a^2+b^2)^2 f} + \frac{(e+fx)^2 \log(\tanh(c+dx))}{ad} - \frac{(e+fx)^2 \tanh^2(c+dx)}{2ad} \\
&= \frac{b^4(e+fx)^3}{3a(a^2+b^2)^2 f} - \frac{b^4(e+fx)^2 \log\left(1 + \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^2 d} - \frac{b^4(e+fx)^2 \log\left(1 + \frac{b}{a+\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^2 d} \\
&= -\frac{2b^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} - \frac{b(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2) d} + \frac{bf^2 \tan^{-1}(\sinh(c+dx))}{(a^2+b^2)} \\
&= -\frac{2b^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} - \frac{b(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2) d} + \frac{bf^2 \tan^{-1}(\sinh(c+dx))}{(a^2+b^2)} \\
&= \frac{efx}{ad} + \frac{f^2 x^2}{2ad} - \frac{2b^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} - \frac{b(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2) d} + \frac{bf^2 \tan^{-1}(\sinh(c+dx))}{(a^2+b^2)} \\
&= \frac{efx}{ad} + \frac{f^2 x^2}{2ad} - \frac{2b^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} - \frac{b(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2) d} + \frac{bf^2 \tan^{-1}(\sinh(c+dx))}{(a^2+b^2)} \\
&= \frac{efx}{ad} + \frac{f^2 x^2}{2ad} - \frac{2b^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} - \frac{b(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2) d} + \frac{bf^2 \tan^{-1}(\sinh(c+dx))}{(a^2+b^2)} \\
&= \frac{efx}{ad} + \frac{f^2 x^2}{2ad} - \frac{2b^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} - \frac{b(e+fx)^2 \tan^{-1}(e^{c+dx})}{(a^2+b^2) d} + \frac{bf^2 \tan^{-1}(\sinh(c+dx))}{(a^2+b^2)}
\end{aligned}$$

**Mathematica [B]** time = 36.4513, size = 4072, normalized size = 3.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(E^(2*c)*((2*(e + f*x)^3)/(E^(2*c)*f) - (3*(1 - E^(-2*c))*(e + f*x)^2*Log[1 - E^(-c - d*x)]))/d - (3*(1 - E^(-2*c))*(e + f*x)^2*Log[1 - E^(-c - d*x)])/d + (6*(-1 + E^(2*c))*f*(d*(e + f*x)*PolyLog[2, -E^(-c - d*x)] + f*PolyLog[3, -E^(-c - d*x)]))/(d^3*E^(2*c)) + (6*(-1 + E^(2*c))*f*(d*(e + f*x)*PolyLog[2, E^(-c - d*x)] + f*PolyLog[3, E^(-c - d*x)]))/(d^3*E^(2*c)))/(3*a*(-1 + E^(2*c))) - (-12*a^3*d^3*e^2*E^(2*c)*x - 24*a*b^2*d^3*e^2*E^(2*c)*x + 12*a^3*d*E^(2*c)*f^2*x + 12*a*b^2*d*E^(2*c)*f^2*x - 12*a^3*d^3*e*E^(2*c)*f*x^2 - 24*a*b^2*d^3*e*E^(2*c)*f*x^2 - 4*a^3*d^3*E^(2*c)*f^2*x^3 - 8*a*b^2*d^3*E^(2*c)*f^2*x^3 + 6*a^2*b*d^2*e^2*ArcTan[E^(c + d*x)] + 18*b^3*d^2*e^2*ArcTan[E^(c + d*x)] + 6*a^2*b*d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] + 18*b^3*d^2*e^2*E^(2*c)*ArcTan[E^(c + d*x)] - 12*a^2*b*f^2*ArcTan[E^(c + d*x)] - 12*b^3*f^2*ArcTan[E^(c + d*x)] - 12*a^2*b*E^(2*c)*f^2*ArcTan[E^(c + d*x)] - 12*b^3*E^(2*c)*f^2*ArcTan[E^(c + d*x)] + (6*I)*a^2*b*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (18*I)*b^3*d^2*e*f*x*Log[1 - I*E^(c + d*x)] + (6*I)*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (18*I)*b^3*d^2*e*E^(2*c)*f*x*Log[1 - I*E^(c + d*x)] + (3*I)*a^2*b*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] + (9*I)*b^3*d^2*f^2*x^2*Log[1 - I*E^(c + d*x)] + (3*I)*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] + (9*I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 - I*E^(c + d*x)] - (6*I)*a^2*b*d^2*e*f*x*Log[1 + I*E^(c + d*x)] - (18*I)*b^3*d^2*e*f*x*Log[1 + I*E^(c + d*x)] - (6*I)*a^2*b*d^2*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (18*I)*b^3*d^2*e*E^(2*c)*f*x*Log[1 + I*E^(c + d*x)] - (3*I)*a^2*b*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (9*I)*b^3*d^2*f^2*x^2*Log[1 + I*E^(c + d*x)] - (3*I)*a^2*b*d^2*E^(2*c)*f^2*x^2*Log[1 + I*E^(c + d*x)] - (9*I)*b^3*d^2*E^(2*c)*f^2*x^2*Log[1 + I*E^(c + d*x)] + 6*a^3*d^2*e^2*Log[1 + E^(2*(c + d*x))] + 12*a*b^2*d^2*e^2*Log[1 + E^(2*(c + d*x))] + 6*a^3*d^2*e^2*E^(2*c)*Log[1 + E^(2*(c + d*x))] + 12*a*b^2*d^2*e^2*E^(2*c)*Log[1 + E^(2*(c + d*x))] - 6*a^3*f^2*Log[1 + E^(2*(c + d*x))] - 6*a*b^2*f^2*Log[1 + E^(2*(c + d*x))] - 6*a^3*E^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] - 6*a*b^2*E^(2*c)*f^2*Log[1 + E^(2*(c + d*x))] + 12*a^3*d^2*e*f*x*Log[1 + E^(2*(c + d*x))] + 24*a*b^2*d^2*e*f*x*Log[1 + E^(2*(c + d*x))] + 12*a^3*d^2*e*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))] + 24*a*b^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(2*(c + d*x))] + 6*a^3*d^2*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 12*a*b^2*d^2*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 6*a^3*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(2*(c + d*x))] + 12*a*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E^(2*(c + d*x))] - (6*I)*b*(a^2 + 3*b^2)*d*(1 + E^(2*c))*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)] + (6*I)*b*(a^2 + 3*b^2)*d*(1 + E^(2*c))
```

$$\begin{aligned}
& ) * f * (e + f * x) * \text{PolyLog}[2, I * E^{(c + d * x)}] + 6 * a^3 * d * e * f * \text{PolyLog}[2, -E^{(2 * (c + d * x))}] + 12 * a * b^2 * d * e * f * \text{PolyLog}[2, -E^{(2 * (c + d * x))}] + 6 * a^3 * d * e * E^{(2 * c)} * f \\
& * \text{PolyLog}[2, -E^{(2 * (c + d * x))}] + 12 * a * b^2 * d * e * E^{(2 * c)} * f * \text{PolyLog}[2, -E^{(2 * (c + d * x))}] + 6 * a^3 * d * f^2 * x * \text{PolyLog}[2, -E^{(2 * (c + d * x))}] + 12 * a * b^2 * d * f^2 * x * \text{PolyLog}[2, -E^{(2 * (c + d * x))}] + 6 * a^3 * d * E^{(2 * c)} * f^2 * x * \text{PolyLog}[2, -E^{(2 * (c + d * x))}] + 12 * a * b^2 * d * E^{(2 * c)} * f^2 * x * \text{PolyLog}[2, -E^{(2 * (c + d * x))}] + (6 * I) * a^2 * b * f^2 * \text{PolyLog}[3, (-I) * E^{(c + d * x)}] + (18 * I) * b^3 * f^2 * \text{PolyLog}[3, (-I) * E^{(c + d * x)}] + (6 * I) * a^2 * b * E^{(2 * c)} * f^2 * \text{PolyLog}[3, (-I) * E^{(c + d * x)}] + (18 * I) * b^3 * E^{(2 * c)} * f^2 * \text{PolyLog}[3, (-I) * E^{(c + d * x)}] - (6 * I) * a^2 * b * f^2 * \text{PolyLog}[3, I * E^{(c + d * x)}] - (18 * I) * b^3 * f^2 * \text{PolyLog}[3, I * E^{(c + d * x)}] - (6 * I) * a^2 * b * E^{(2 * c)} * f^2 * \text{PolyLog}[3, I * E^{(c + d * x)}] - (18 * I) * b^3 * E^{(2 * c)} * f^2 * \text{PolyLog}[3, I * E^{(c + d * x)}] - 3 * a^3 * f^2 * \text{PolyLog}[3, -E^{(2 * (c + d * x))}] - 6 * a * b^2 * f^2 * \text{PolyLog}[3, -E^{(2 * (c + d * x))}] - 3 * a^3 * E^{(2 * c)} * f^2 * \text{PolyLog}[3, -E^{(2 * (c + d * x))}] - 6 * a * b^2 * E^{(2 * c)} * f^2 * \text{PolyLog}[3, -E^{(2 * (c + d * x))}]) / (6 * (a^2 + b^2)^2 * d^3 * (1 + E^{(2 * c)})) + (b^4 * (6 * e^2 * E^{(2 * c)} * x + 6 * e * E^{(2 * c)} * f * x^2 + 2 * E^{(2 * c)} * f^2 * x^3 + (6 * a * \text{Sqrt}[a^2 + b^2] * e^2 * \text{ArcTan}[(a + b * E^{(c + d * x)}) / \text{Sqrt}[-a^2 - b^2]]) / (\text{Sqrt}[-(a^2 + b^2)^2] * d) + (6 * a * \text{Sqrt}[-(a^2 + b^2)^2] * e^2 * E^{(2 * c)} * \text{ArcTan}[(a + b * E^{(c + d * x)}) / \text{Sqrt}[-a^2 - b^2]]) / ((a^2 + b^2)^(3/2) * d) - (6 * a * \text{Sqrt}[-(a^2 + b^2)^2] * e^2 * \text{ArcTanh}[(a + b * E^{(c + d * x)}) / \text{Sqrt}[a^2 + b^2]]) / ((-a^2 - b^2)^(3/2) * d) + (6 * a * \text{Sqrt}[-(a^2 + b^2)^2] * e^2 * E^{(2 * c)} * \text{ArcTanh}[(a + b * E^{(c + d * x)}) / \text{Sqrt}[a^2 + b^2]]) / ((-a^2 - b^2)^(3/2) * d) + (3 * e^2 * \text{Log}[2 * a * E^{(c + d * x)} + b * (-1 + E^{(2 * (c + d * x))})]) / d - (3 * e^2 * E^{(2 * c)} * \text{Log}[2 * a * E^{(c + d * x)} + b * (-1 + E^{(2 * (c + d * x))})]) / d + (6 * e * f * x * \text{Log}[1 + (b * E^{(2 * c + d * x)}) / (a * E^c - \text{Sqrt}[(a^2 + b^2) * E^{(2 * c)}])]) / d - (6 * e * E^{(2 * c)} * f * x * \text{Log}[1 + (b * E^{(2 * c + d * x)}) / (a * E^c - \text{Sqrt}[(a^2 + b^2) * E^{(2 * c)}])]) / d + (3 * f^2 * x^2 * \text{Log}[1 + (b * E^{(2 * c + d * x)}) / (a * E^c - \text{Sqrt}[(a^2 + b^2) * E^{(2 * c)}])]) / d - (3 * E^{(2 * c)} * f^2 * x^2 * \text{Log}[1 + (b * E^{(2 * c + d * x)}) / (a * E^c - \text{Sqrt}[(a^2 + b^2) * E^{(2 * c)}])]) / d + (6 * e * f * x * \text{Log}[1 + (b * E^{(2 * c + d * x)}) / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{(2 * c)}])]) / d - (6 * e * E^{(2 * c)} * f * x * \text{Log}[1 + (b * E^{(2 * c + d * x)}) / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{(2 * c)}])]) / d + (3 * f^2 * x^2 * \text{Log}[1 + (b * E^{(2 * c + d * x)}) / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{(2 * c)}])]) / d - (3 * E^{(2 * c)} * f^2 * x^2 * \text{Log}[1 + (b * E^{(2 * c + d * x)}) / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{(2 * c)}])]) / d - (6 * (-1 + E^{(2 * c)}) * f * (e + f * x) * \text{PolyLog}[2, -((b * E^{(2 * c + d * x)}) / (a * E^c - \text{Sqrt}[(a^2 + b^2) * E^{(2 * c)}])])]) / d^2 - (6 * (-1 + E^{(2 * c)}) * f * (e + f * x) * \text{PolyLog}[2, -((b * E^{(2 * c + d * x)}) / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{(2 * c)}])])]) / d^2 - (6 * f^2 * \text{PolyLog}[3, -((b * E^{(2 * c + d * x)}) / (a * E^c - \text{Sqrt}[(a^2 + b^2) * E^{(2 * c)}])])]) / d^3 + (6 * E^{(2 * c)} * f^2 * \text{PolyLog}[3, -((b * E^{(2 * c + d * x)}) / (a * E^c - \text{Sqrt}[(a^2 + b^2) * E^{(2 * c)}])])]) / d^3 - (6 * f^2 * \text{PolyLog}[3, -((b * E^{(2 * c + d * x)}) / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{(2 * c)}])])]) / d^3 + (6 * E^{(2 * c)} * f^2 * \text{PolyLog}[3, -((b * E^{(2 * c + d * x)}) / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{(2 * c)}])])]) / d^3 - (6 * f^2 * \text{PolyLog}[3, -((b * E^{(2 * c + d * x)}) / (a * E^c + \text{Sqrt}[(a^2 + b^2) * E^{(2 * c)}])])]) / d^3)) / (3 * a * (a^2 + b^2)^2 * (-1 + E^{(2 * c)})) + (\text{Csch}[c] * \text{Sech}[c] * \text{Sech}[c + d * x]^2 * (-6 * a^3 * e * f - 6 * a * b^2 * e * f + 12 * a^3 * d^2 * e^2 * x + 24 * a * b^2 * d^2 * e^2 * x - 6 * a^3 * f^2 * x - 6 * a * b^2 * f^2 * x + 12 * a^3 * d^2 * e * f * x^2 + 24 * a * b^2 * d^2 * e * f * x^2 + 4 * a^3 * d^2 * f^2 * x^3 + 8 * a * b^2 * d^2 * f^2 * x^3 + 6 * a^3 * e * f * \text{Cosh}[2 * c] + 6 * a * b^2 * e * f * \text{Cosh}[2 * c] + 6 * a^3 * f^2 * x * \text{Cosh}[2 * c] + 6 * a * b^2 * f^2 * x * \text{Cosh}[2 * c] + 6 * a^3 * e * f * \text{Cosh}[2 * d * x] + 6 * a * b^2 * e * f * \text{Cosh}[2 * d * x] + 6 * a^3 * f^2 * x * \text{Cosh}[2 * d * x] + 6 * a * b^2 * f^2 * x * \text{Cosh}[2 * d * x] + 3 * a^2 * b * d * e^2 * \text{Cosh}[c - d * x] + 3 * b^3 * d * e^2 * \text{Cosh}
\end{aligned}$$

$$\begin{aligned} & [c - dx] + 6a^2bde^fx \cosh[c - dx] + 6b^3d^2e^fx \cosh[c - dx] + 3 \\ & a^2b^2d^2f^2x^2 \cosh[c - dx] + 3b^3d^2f^2x^2 \cosh[c - dx] - 3a^2b^2d^2 \\ & e^2 \cosh[3c + dx] - 3b^3d^2e^2 \cosh[3c + dx] - 6a^2b^2d^2e^fx \cosh[3c \\ & + dx] - 6b^3d^2e^fx \cosh[3c + dx] - 3a^2b^2d^2f^2x^2 \cosh[3c + dx] \\ & - 3b^3d^2f^2x^2 \cosh[3c + dx] - 6a^3e^fx \cosh[2c + 2dx] - 6a^2b^2 \\ & e^fx \cosh[2c + 2dx] + 12a^3d^2e^2x \cosh[2c + 2dx] + 24a^2b^2d^2 \\ & e^2x \cosh[2c + 2dx] - 6a^3f^2x \cosh[2c + 2dx] - 6a^2b^2f^2x \cosh \\ & [2c + 2dx] + 12a^3d^2e^fx^2 \cosh[2c + 2dx] + 24a^2b^2d^2e^fx^2 \\ & \cosh[2c + 2dx] + 4a^3d^2f^2x^3 \cosh[2c + 2dx] + 8a^2b^2d^2f^2x^3 \\ & \cosh[2c + 2dx] + 6a^3d^2e^2 \sinh[2c] + 6a^2b^2d^2e^2 \sinh[2c] + \\ & 12a^3d^2e^fx \sinh[2c] + 12a^2b^2d^2e^fx \sinh[2c] + 6a^3d^2f^2x^2 \sinh \\ & [2c] + 6a^2b^2d^2f^2x^2 \sinh[2c] - 6a^2b^2e^fx \sinh[c - dx] - 6b^3e^fx \\ & \sinh[c - dx] - 6a^2b^2f^2x \sinh[c - dx] - 6b^3f^2x \sinh[c - dx] - \\ & 6a^2b^2e^fx \sinh[3c + dx] - 6b^3e^fx \sinh[3c + dx] - 6a^2b^2f^2x \sinh \\ & [3c + dx] - 6b^3f^2x \sinh[3c + dx]) / (24(a^2 + b^2)^2d^2) \end{aligned}$$

**Maple [F]** time = 1.372, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \operatorname{csch}(dx + c) (\operatorname{sech}(dx + c))^3}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*csch(d\*x+c)\*sech(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*csch(d\*x+c)\*sech(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*csch(d\*x+c)\*sech(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -a^2\*b\*d^2\*f^2\*integrate(x^2\*e^(d\*x + c)/(a^4\*d^2\*e^(2\*d\*x + 2\*c) + 2\*a^2\*b^2\*d^2\*e^(2\*d\*x + 2\*c) + b^4\*d^2\*e^(2\*d\*x + 2\*c) + a^4\*d^2 + 2\*a^2\*b^2\*d^2 + b^4\*d^2), x) - 3\*b^3\*d^2\*f^2\*integrate(x^2\*e^(d\*x + c)/(a^4\*d^2\*e^(2\*d\*x

$$\begin{aligned}
& + 2*c) + 2*a^2*b^2*d^2*e^{(2*d*x + 2*c)} + b^4*d^2*e^{(2*d*x + 2*c)} + a^4*d^2 \\
& + 2*a^2*b^2*d^2 + b^4*d^2), x) + 2*a^3*d^2*f^2*\integrate(x^2/(a^4*d^2*e^{(2* \\
& d*x + 2*c)} + 2*a^2*b^2*d^2*e^{(2*d*x + 2*c)} + b^4*d^2*e^{(2*d*x + 2*c)} + a^4* \\
& d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 4*a*b^2*d^2*f^2*\integrate(x^2/(a^4*d^2 \\
& *e^{(2*d*x + 2*c)} + 2*a^2*b^2*d^2*e^{(2*d*x + 2*c)} + b^4*d^2*e^{(2*d*x + 2*c)} \\
& + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 2*a^2*b*d^2*e*f*\integrate(x*e^{(d \\
& *x + c)/(a^4*d^2*e^{(2*d*x + 2*c)} + 2*a^2*b^2*d^2*e^{(2*d*x + 2*c)} + b^4*d^2* \\
& e^{(2*d*x + 2*c)} + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) - 6*b^3*d^2*e*f*\int \\
& tegrate(x*e^{(d*x + c)/(a^4*d^2*e^{(2*d*x + 2*c)} + 2*a^2*b^2*d^2*e^{(2*d*x + 2 \\
& *c)} + b^4*d^2*e^{(2*d*x + 2*c)} + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + 4* \\
& a^3*d^2*e*f*\integrate(x/(a^4*d^2*e^{(2*d*x + 2*c)} + 2*a^2*b^2*d^2*e^{(2*d*x + \\
& 2*c)} + b^4*d^2*e^{(2*d*x + 2*c)} + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x) + \\
& 8*a*b^2*d^2*e*f*\integrate(x/(a^4*d^2*e^{(2*d*x + 2*c)} + 2*a^2*b^2*d^2*e^{(2*d \\
& *x + 2*c)} + b^4*d^2*e^{(2*d*x + 2*c)} + a^4*d^2 + 2*a^2*b^2*d^2 + b^4*d^2), x \\
& ) - a^3*f^2*(2*(d*x + c)/((a^4 + 2*a^2*b^2 + b^4)*d^3) - \log(e^{(2*d*x + 2*c)} \\
& + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) - a*b^2*f^2*(2*(d*x + c)/((a^4 + 2*a^ \\
& 2*b^2 + b^4)*d^3) - \log(e^{(2*d*x + 2*c)} + 1)/((a^4 + 2*a^2*b^2 + b^4)*d^3)) \\
& - (b^4*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/((a^5 + 2*a^3*b^2 + \\
& a*b^4)*d) - (a^2*b + 3*b^3)*\arctan(e^{(-d*x - c)})/((a^4 + 2*a^2*b^2 + b^4)* \\
& d) + (a^3 + 2*a*b^2)*\log(e^{(-2*d*x - 2*c)} + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) \\
& + (b*e^{(-d*x - c)} - 2*a*e^{(-2*d*x - 2*c)} - b*e^{(-3*d*x - 3*c)})/((a^2 + b^2 \\
& + 2*(a^2 + b^2)*e^{(-2*d*x - 2*c)} + (a^2 + b^2)*e^{(-4*d*x - 4*c)})*d) - \log(e \\
& ^{(-d*x - c)} + 1)/(a*d) - \log(e^{(-d*x - c)} - 1)/(a*d))*e^2 + 2*a^2*b*f^2*\arc \\
& tan(e^{(d*x + c)})/((a^4 + 2*a^2*b^2 + b^4)*d^3) + 2*b^3*f^2*\arctan(e^{(d*x + \\
& c)})/((a^4 + 2*a^2*b^2 + b^4)*d^3) + (2*a*f^2*x + 2*a*e*f - (b*d*f^2*x^2*e^{( \\
& 3*c)} + 2*b*e*f*e^{(3*c)} + 2*(d*e*f + f^2)*b*x*e^{(3*c)})*e^{(3*d*x)} + 2*(a*d*f^ \\
& 2*x^2*e^{(2*c)} + a*e*f*e^{(2*c)} + (2*d*e*f + f^2)*a*x*e^{(2*c)})*e^{(2*d*x)} + (b \\
& *d*f^2*x^2*e^c - 2*b*e*f*e^c + 2*(d*e*f - f^2)*b*x*e^c)*e^{(d*x)}/(a^2*d^2 + \\
& b^2*d^2 + (a^2*d^2*e^{(4*c)} + b^2*d^2*e^{(4*c)})*e^{(4*d*x)} + 2*(a^2*d^2*e^{(2* \\
& c)} + b^2*d^2*e^{(2*c)})*e^{(2*d*x)}) + 2*(d*x*\log(e^{(d*x + c)} + 1) + \operatorname{dilog}(-e^{( \\
& d*x + c)})))*e*f/(a*d^2) + 2*(d*x*\log(-e^{(d*x + c)} + 1) + \operatorname{dilog}(e^{(d*x + c)})) \\
& *e*f/(a*d^2) + (d^2*x^2*\log(e^{(d*x + c)} + 1) + 2*d*x*\operatorname{dilog}(-e^{(d*x + c)}) - \\
& 2*\operatorname{polylog}(3, -e^{(d*x + c)}))*f^2/(a*d^3) + (d^2*x^2*\log(-e^{(d*x + c)} + 1) + \\
& 2*d*x*\operatorname{dilog}(e^{(d*x + c)}) - 2*\operatorname{polylog}(3, e^{(d*x + c)}))*f^2/(a*d^3) - 2/3*(d^ \\
& 3*f^2*x^3 + 3*d^3*e*f*x^2)/(a*d^3) + \integrate(2*(b^5*f^2*x^2 + 2*b^5*e*f*x \\
& - (a*b^4*f^2*x^2*e^c + 2*a*b^4*e*f*x*e^c)*e^{(d*x)})/(a^5*b + 2*a^3*b^3 + a* \\
& b^5 - (a^5*b*e^{(2*c)} + 2*a^3*b^3*e^{(2*c)} + a*b^5*e^{(2*c)})*e^{(2*d*x)} - 2*(a^ \\
& 6*e^c + 2*a^4*b^2*e^c + a^2*b^4*e^c)*e^{(d*x)}), x)
\end{aligned}$$


---

**Fricas [F-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.446 \quad \int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=746

$$\frac{b^4 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2(a^2+b^2)^2} - \frac{b^4 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2(a^2+b^2)^2} + \frac{b^4 f \operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{2ad^2(a^2+b^2)^2} + \frac{ib^3 f \operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{d^2(a^2+b^2)^2}$$

```
[Out] (f*x)/(2*a*d) - (2*b^3*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)^2*d) - (
b*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (2*f*x*ArcTanh[E^(2*c +
2*d*x)])/(a*d) - (b^4*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])])/(a*(a^2 + b^2)^2*d) - (b^4*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])])/(a*(a^2 + b^2)^2*d) + (b^4*(e + f*x)*Log[1 + E^(2*(c + d*x))
])/(a*(a^2 + b^2)^2*d) - (f*x*Log[Tanh[c + d*x]])/(a*d) + ((e + f*x)*Log[Ta
nh[c + d*x]])/(a*d) + (I*b^3*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2
*d^2) + ((I/2)*b*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) - (I*b^3
*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - ((I/2)*b*f*PolyLog[2, I
*E^(c + d*x)])/((a^2 + b^2)*d^2) - (b^4*f*PolyLog[2, -(b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^2*d^2) - (b^4*f*PolyLog[2, -(b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^2*d^2) + (b^4*f*PolyLog[2, -E
^(2*(c + d*x))])/(2*a*(a^2 + b^2)^2*d^2) - (f*PolyLog[2, -E^(2*c + 2*d*x)])
/(2*a*d^2) + (f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2) - (b*f*Sech[c + d*x]
)/(2*(a^2 + b^2)*d^2) - (b^2*(e + f*x)*Sech[c + d*x]^2)/(2*a*(a^2 + b^2)*d
- (f*Tanh[c + d*x])/(2*a*d^2) + (b^2*f*Tanh[c + d*x])/(2*a*(a^2 + b^2)*d^2
) - (b*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*(a^2 + b^2)*d) - ((e + f*x
)*Tanh[c + d*x]^2)/(2*a*d)
```

**Rubi [A]** time = 1.05909, antiderivative size = 746, normalized size of antiderivative = 1., number of steps used = 43, number of rules used = 20, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$ , Rules used = {5589, 2620, 14, 5462, 2548, 12, 4182, 2279, 2391, 3473, 8, 5573, 5561, 2190, 6742, 4180, 3718, 4185, 5451, 3767}

$$\frac{b^4 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{ad^2(a^2+b^2)^2} - \frac{b^4 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{ad^2(a^2+b^2)^2} + \frac{b^4 f \operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{2ad^2(a^2+b^2)^2} + \frac{ib^3 f \operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{d^2(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (f*x)/(2*a*d) - (2*b^3*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)^2*d) - (
b*(e + f*x)*ArcTan[E^(c + d*x)])/((a^2 + b^2)*d) - (2*f*x*ArcTanh[E^(2*c +
2*d*x)])/(a*d) - (b^4*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2
])])/(a*(a^2 + b^2)^2*d) - (b^4*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])])/(a*(a^2 + b^2)^2*d) + (b^4*(e + f*x)*Log[1 + E^(2*(c + d*x))
])/(a*(a^2 + b^2)^2*d) - (f*x*Log[Tanh[c + d*x]])/(a*d) + ((e + f*x)*Log[Ta
nh[c + d*x]])/(a*d) + (I*b^3*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)^2
*d^2) + ((I/2)*b*f*PolyLog[2, (-I)*E^(c + d*x)])/((a^2 + b^2)*d^2) - (I*b^3
*f*PolyLog[2, I*E^(c + d*x)])/((a^2 + b^2)^2*d^2) - ((I/2)*b*f*PolyLog[2, I
*E^(c + d*x)])/((a^2 + b^2)*d^2) - (b^4*f*PolyLog[2, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^2) - (b^4*f*PolyLog[2, -((b*E^(c +
d*x))/(a + Sqrt[a^2 + b^2]))])/(a*(a^2 + b^2)^2*d^2) + (b^4*f*PolyLog[2, -E
^(2*(c + d*x))])/(2*a*(a^2 + b^2)^2*d^2) - (f*PolyLog[2, -E^(2*c + 2*d*x)])
/(2*a*d^2) + (f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2) - (b*f*Sech[c + d*x]
)/(2*(a^2 + b^2)*d^2) - (b^2*(e + f*x)*Sech[c + d*x]^2)/(2*a*(a^2 + b^2)*d
) - (f*Tanh[c + d*x])/(2*a*d^2) + (b^2*f*Tanh[c + d*x])/(2*a*(a^2 + b^2)*d^2
) - (b*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*(a^2 + b^2)*d) - ((e + f*x
)*Tanh[c + d*x]^2)/(2*a*d)
```

### Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
(d_.)*(x_)]^(p_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
IGtQ[p, 0]
```

### Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:= Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

### Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

### Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] := With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
```



], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]

### Rule 2548

Int[Log[u\_], x\_Symbol] := Simp[x\*Log[u], x] - Int[SimplifyIntegrand[(x\*D[u, x])/u, x], x] /; InverseFunctionFreeQ[u, x]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 4182

Int[csc[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 3473

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*(b\*Tan[c + d\*x])^(n - 1))/(d\*(n - 1)), x] - Dist[b^2, Int[(b\*Tan[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 3718

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 4185

```

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :=
-Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]

```

### Rule 5451

```

Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] := -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

```

### Rule 3767

```

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx}{a} \\
&= \frac{(e+fx)\log(\tanh(c+dx))}{ad} - \frac{(e+fx)\tanh^2(c+dx)}{2ad} - \frac{b \int (e+fx)\operatorname{sech}^3(c+dx) dx}{a} \\
&= \frac{(e+fx)\log(\tanh(c+dx))}{ad} - \frac{(e+fx)\tanh^2(c+dx)}{2ad} - \frac{b^3 \int (e+fx)\operatorname{sech}(c+dx) dx}{a} \\
&= \frac{b^4(e+fx)^2}{2a(a^2+b^2)^2 f} - \frac{fx \log(\tanh(c+dx))}{ad} + \frac{(e+fx)\log(\tanh(c+dx))}{ad} - \frac{f \tanh^2(c+dx)}{2a} \\
&= \frac{fx}{2ad} + \frac{b^4(e+fx)^2}{2a(a^2+b^2)^2 f} - \frac{b^4(e+fx)\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^2 d} - \frac{b^4(e+fx)\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^2 d} \\
&= \frac{fx}{2ad} - \frac{2b^3(e+fx)\tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} - \frac{b(e+fx)\tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2fx \tanh^{-1}(e^{2c+2dx})}{ad} \\
&= \frac{fx}{2ad} - \frac{2b^3(e+fx)\tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} - \frac{b(e+fx)\tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2fx \tanh^{-1}(e^{2c+2dx})}{ad} \\
&= \frac{fx}{2ad} - \frac{2b^3(e+fx)\tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} - \frac{b(e+fx)\tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2fx \tanh^{-1}(e^{2c+2dx})}{ad} \\
&= \frac{fx}{2ad} - \frac{2b^3(e+fx)\tan^{-1}(e^{c+dx})}{(a^2+b^2)^2 d} - \frac{b(e+fx)\tan^{-1}(e^{c+dx})}{(a^2+b^2)d} - \frac{2fx \tanh^{-1}(e^{2c+2dx})}{ad}
\end{aligned}$$

**Mathematica [A]** time = 10.8415, size = 886, normalized size = 1.19

$$\frac{\left(-\frac{1}{2}f(c+dx)^2 + f \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}} + 1\right)(c+dx) + f \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}} + 1\right)(c+dx) + de \log(a+b\sinh(c+dx)) - cf \log(a+b\sinh(c+dx))\right)}{a(a^2+b^2)^2 d^2}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Csch[c + d\*x]\*Sech[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]), x]

```
[Out] (e*Log[Sinh[c + d*x]])/(a*d) - (c*f*Log[Sinh[c + d*x]])/(a*d^2) - (I*f*(I*(c + d*x)*Log[1 - E^(-2*(c + d*x))] - (I/2)*(-(c + d*x)^2 + PolyLog[2, E^(-2*(c + d*x))])))/(a*d^2) - (b^4*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]) + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]]) + f*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a*(a^2 + b^2)^2*d^2) - (-2*a^3*d*e*(c + d*x) - 4*a*b^2*d*e*(c + d*x) + 2*a^3*c*f*(c + d*x) + 4*a*b^2*c*f*(c + d*x) - a^3*f*(c + d*x)^2 - 2*a*b^2*f*(c + d*x)^2 + 2*a^2*b*d*e*ArcTan[E^(c + d*x)] + 6*b^3*d*e*ArcTan[E^(c + d*x)] - 2*a^2*b*c*f*ArcTan[E^(c + d*x)] - 6*b^3*c*f*ArcTan[E^(c + d*x)] + I*a^2*b*f*(c + d*x)*Log[1 - I*E^(c + d*x)] + (3*I)*b^3*f*(c + d*x)*Log[1 - I*E^(c + d*x)] - I*a^2*b*f*(c + d*x)*Log[1 + I*E^(c + d*x)] - (3*I)*b^3*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + 2*a^3*d*e*Log[1 + E^(2*(c + d*x))] + 4*a*b^2*d*e*Log[1 + E^(2*(c + d*x))] - 2*a^3*c*f*Log[1 + E^(2*(c + d*x))] - 4*a*b^2*c*f*Log[1 + E^(2*(c + d*x))] + 2*a^3*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] + 4*a*b^2*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] - I*b*(a^2 + 3*b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] + I*b*(a^2 + 3*b^2)*f*PolyLog[2, I*E^(c + d*x)] + a^3*f*PolyLog[2, -E^(2*(c + d*x))] + 2*a*b^2*f*PolyLog[2, -E^(2*(c + d*x))]/(2*(a^2 + b^2)^2*d^2) + (Sech[c + d*x]*(-(b*f) - a*f*Sinh[c + d*x]))/(2*(a^2 + b^2)*d^2) + (Sech[c + d*x]^2*(a*d*e - a*c*f + a*f*(c + d*x) - b*d*e*Sinh[c + d*x] + b*c*f*Sinh[c + d*x] - b*f*(c + d*x)*Sinh[c + d*x]))/(2*(a^2 + b^2)*d^2)
```

**Maple [B]** time = 0.244, size = 2580, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)), x)
```

```
[Out] 1/(a^2+b^2)/d*a*e*ln(exp(d*x+c)-1)+1/(a^2+b^2)/d*a*e*ln(exp(d*x+c)+1)-1/(a^2+b^2)/d^2*a*f*dilog(exp(d*x+c))+1/(a^2+b^2)/d^2*a*f*dilog(exp(d*x+c)+1)-8/(a^2+b^2)/d*b^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*a*x-8/(a^2+b^2)/d^2*b^2*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*a*c+8/(a^2+b^2)/d^2*b^2*f*c/(4*a^2+4*b^2)*a*ln(1+exp(2*d*x+2*c))-6*I/(a^2+b^2)/d*b^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x-6*I/(a^2+b^2)/d^2*b^3*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*c+6*I/(a^2+b^2)/d*b^3*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x+6*I/(a^2+b^2)/d^2*b^3*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*c+2*I/(a^2+b^2)/d^2*a^2*f/(4*a^2+4*b^2)*dilog(1+I*exp(d*x+c))*b-2*I/(a^2+b^2)/d^2*a^2*f/(4*a^2+4*b^2)*dilog(1-I*exp(d*x+c))*b-1/2/(a^2+b^2)^(5/2)/d^2*a^2*b^2*f*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+4/(a^2+b^2)/d^2*a^2*f*c/(4*a^2+4*b^2)*b*arctan(exp(d*x+c))
```

$$\begin{aligned}
& -8/(a^2+b^2)/d*b^2*f/(4*a^2+4*b^2)*\ln(1+I*\exp(d*x+c))*a*x-8/(a^2+b^2)/d^2*b \\
& ^2*f/(4*a^2+4*b^2)*\ln(1+I*\exp(d*x+c))*a*c+(-b*d*f*x*\exp(3*d*x+3*c)+2*a*d*f* \\
& x*\exp(2*d*x+2*c)-b*d*e*\exp(3*d*x+3*c)+2*a*d*e*\exp(2*d*x+2*c)+b*d*f*x*\exp(d* \\
& x+c)-b*f*\exp(3*d*x+3*c)+a*f*\exp(2*d*x+2*c)+b*d*e*\exp(d*x+c)-f*b*\exp(d*x+c)+ \\
& a*f)/d^2/(a^2+b^2)/(1+\exp(2*d*x+2*c))^2+1/(a^2+b^2)/d*\ln(\exp(d*x+c)+1)*a*f* \\
& x-1/(a^2+b^2)/d^2*a*f*c*\ln(\exp(d*x+c)-1)-1/(a^2+b^2)/d^2*b^2*f/a*dilog(\exp( \\
& d*x+c))+1/(a^2+b^2)/d^2*b^2*f/a*dilog(\exp(d*x+c)+1)+1/(a^2+b^2)/d*b^2*e/a*1 \\
& n(\exp(d*x+c)-1)+1/(a^2+b^2)/d*b^2*e/a*\ln(\exp(d*x+c)+1)-1/(a^2+b^2)/d^2*b^2* \\
& f*c/a*\ln(\exp(d*x+c)-1)+1/(a^2+b^2)/d*b^2*f/a*\ln(\exp(d*x+c)+1)*x+2*I/(a^2+b^ \\
& 2)/d*a^2*f/(4*a^2+4*b^2)*\ln(1+I*\exp(d*x+c))*b*x+2*I/(a^2+b^2)/d^2*a^2*f/(4* \\
& a^2+4*b^2)*\ln(1+I*\exp(d*x+c))*b*c-2*I/(a^2+b^2)/d*a^2*f/(4*a^2+4*b^2)*\ln(1- \\
& I*\exp(d*x+c))*b*x-2*I/(a^2+b^2)/d^2*a^2*f/(4*a^2+4*b^2)*\ln(1-I*\exp(d*x+c))* \\
& b*c-1/(a^2+b^2)^2/d^2*b^4*f/a*dilog((-b*\exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+( \\
& a^2+b^2)^(1/2)))-1/(a^2+b^2)^2/d^2*b^4*f/a*dilog((b*\exp(d*x+c)+(a^2+b^2)^(1 \\
& /2)+a)/(a+(a^2+b^2)^(1/2)))-1/2/(a^2+b^2)^(3/2)/d*b^2*e*arctanh(1/2*(2*b*ex \\
& p(d*x+c)+2*a)/(a^2+b^2)^(1/2))-12/(a^2+b^2)/d*b^3*e/(4*a^2+4*b^2)*arctan(ex \\
& p(d*x+c))+1/2/(a^2+b^2)^(5/2)/d*b^4*e*arctanh(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2 \\
& +b^2)^(1/2))-4/(a^2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*dilog(1+I*\exp(d*x+c))-4/(a \\
& ^2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*dilog(1-I*\exp(d*x+c))-1/(a^2+b^2)^2/d*b^4*e \\
& /a*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)-4/(a^2+b^2)/d*a^3*e/(4*a^2+4*b^2)* \\
& \ln(1+\exp(2*d*x+2*c))-1/(a^2+b^2)^2/d^2*b^4*f/a*\ln((b*\exp(d*x+c)+(a^2+b^2)^( \\
& 1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+6*I/(a^2+b^2)/d^2*b^3*f/(4*a^2+4*b^2)*dilog( \\
& 1+I*\exp(d*x+c))-6*I/(a^2+b^2)/d^2*b^3*f/(4*a^2+4*b^2)*dilog(1-I*\exp(d*x+c)) \\
& +1/2/(a^2+b^2)^(3/2)/d^2*b^2*f*c*arctanh(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2) \\
& ^2)+12/(a^2+b^2)/d^2*b^3*f*c/(4*a^2+4*b^2)*arctan(\exp(d*x+c))-1/2/(a^2+ \\
& b^2)^(5/2)/d^2*b^4*f*c*arctanh(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-8/ \\
& (a^2+b^2)/d^2*b^2*f/(4*a^2+4*b^2)*dilog(1+I*\exp(d*x+c))*a-8/(a^2+b^2)/d^2*b \\
& ^2*f/(4*a^2+4*b^2)*dilog(1-I*\exp(d*x+c))*a+4/(a^2+b^2)/d^2*a^3*f*c/(4*a^2+4 \\
& *b^2)*\ln(1+\exp(2*d*x+2*c))+1/(a^2+b^2)^2/d^2*b^4*f*c/a*\ln(b*\exp(2*d*x+2*c)+ \\
& 2*a*\exp(d*x+c)-b)-4/(a^2+b^2)/d*a^2*e/(4*a^2+4*b^2)*b*arctan(\exp(d*x+c))-8/ \\
& (a^2+b^2)/d*b^2*e/(4*a^2+4*b^2)*a*\ln(1+\exp(2*d*x+2*c))+1/2/(a^2+b^2)^(5/2)/ \\
& d*a^2*b^2*e*arctanh(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-4/(a^2+b^2)/d \\
& *a^3*f/(4*a^2+4*b^2)*\ln(1+I*\exp(d*x+c))*x-4/(a^2+b^2)/d^2*a^3*f/(4*a^2+4*b^ \\
& 2)*\ln(1+I*\exp(d*x+c))*c-4/(a^2+b^2)/d*a^3*f/(4*a^2+4*b^2)*\ln(1-I*\exp(d*x+c) \\
& )*x-4/(a^2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*\ln(1-I*\exp(d*x+c))*c-1/(a^2+b^2)^2/ \\
& d*b^4*f/a*\ln((-b*\exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x-1/(a \\
& ^2+b^2)^2/d^2*b^4*f/a*\ln((-b*\exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1 \\
& /2)))*c-1/(a^2+b^2)^2/d*b^4*f/a*\ln((b*\exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2 \\
& +b^2)^(1/2)))*x
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b^4*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^5 + 2*a^3*b^2 + a*b^4)*d) - (a^2*b + 3*b^3)*arctan(e^(-d*x - c))/((a^4 + 2*a^2*b^2 + b^4)*d) + (a^3 + 2*a*b^2)*log(e^(-2*d*x - 2*c) + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) + (b*e^(-d*x - c) - 2*a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c))/((a^2 + b^2 + 2*(a^2 + b^2)*e^(-2*d*x - 2*c) + (a^2 + b^2)*e^(-4*d*x - 4*c))*d) - log(e^(-d*x - c) + 1)/(a*d) - log(e^(-d*x - c) - 1)/(a*d))*e - f*((b*d*x*e^(3*c) + b*e^(3*c))*e^(3*d*x) - (2*a*d*x*e^(2*c) + a*e^(2*c))*e^(2*d*x) - (b*d*x*e^c - b*e^c)*e^(d*x) - a)/(a^2*d^2 + b^2*d^2 + (a^2*d^2*e^(4*c) + b^2*d^2*e^(4*c))*e^(4*d*x) + 2*(a^2*d^2*e^(2*c) + b^2*d^2*e^(2*c))*e^(2*d*x)) - 16*integrate(-1/8*(a*b^4*x*e^(d*x + c) - b^5*x)/(a^5*b + 2*a^3*b^3 + a*b^5 - (a^5*b*e^(2*c) + 2*a^3*b^3*e^(2*c) + a*b^5*e^(2*c))*e^(2*d*x) - 2*(a^6*e^c + 2*a^4*b^2*e^c + a^2*b^4*e^c)*e^(d*x)), x) + 16*integrate(1/16*((a^2*b*e^c + 3*b^3*e^c)*x*e^(d*x) - 2*(a^3 + 2*a*b^2)*x)/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c) + 2*a^2*b^2*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x) + 16*integrate(1/16*x/(a*e^(d*x + c) + a), x) - 16*integrate(1/16*x/(a*e^(d*x + c) - a), x))
```

---

**Fricas [B]** time = 4.2291, size = 17816, normalized size = 23.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e + (a^3*b + a*b^3)*f)*cosh(d*x + c)^3 + 2*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e + (a^3*b + a*b^3)*f)*sinh(d*x + c)^3 - 2*(2*(a^4 + a^2*b^2)*d*f*x + 2*(a^4 + a^2*b^2)*d*e + (a^4 + a^2*b^2)*f)*cosh(d*x + c)^2 - 2*(2*(a^4 + a^2*b^2)*d*f*x + 2*(a^4 + a^2*b^2)*d*e + (a^4 + a^2*b^2)*f - 3*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e + (a^3*b + a*b^3)*f)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(a^4 + a^2*b^2)*f - 2*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e - (a^3*b + a*b^3)*f)*cosh(d*x + c) + 2*(b^4*f*cosh(d*x + c)^4 + 4*b^4*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*f*sinh(d*x + c)^4 + 2*b^4*f*cosh(d*x + c)^2 + b^4*f + 2*(3*b^4*f*cosh(d*x + c)^2 + b^4*f)*sinh(d*x + c)^2 + 4*(b^4*f*cosh(d*x + c)^3 + b^4*f*cosh(d*x + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b +
```

$$\begin{aligned}
& 1) + 2*(b^4*f*cosh(d*x + c)^4 + 4*b^4*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^4 \\
& *f*sinh(d*x + c)^4 + 2*b^4*f*cosh(d*x + c)^2 + b^4*f + 2*(3*b^4*f*cosh(d*x \\
& + c)^2 + b^4*f)*sinh(d*x + c)^2 + 4*(b^4*f*cosh(d*x + c)^3 + b^4*f*cosh(d*x \\
& + c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d* \\
& x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 2*((a^4 + 2*a \\
& ^2*b^2 + b^4)*f*cosh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c) \\
& *sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*f*sinh(d*x + c)^4 + 2*(a^4 + 2*a \\
& ^2*b^2 + b^4)*f*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + \\
& c)^2 + (a^4 + 2*a^2*b^2 + b^4)*f)*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4 \\
& )*f + 4*((a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4 \\
& )*f*cosh(d*x + c))*sinh(d*x + c))*dilog(cosh(d*x + c) + sinh(d*x + c)) + (( \\
& 2*(a^4 + 2*a^2*b^2)*f + I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^4 + 4*(2*(a^4 \\
& + 2*a^2*b^2)*f + I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2* \\
& (a^4 + 2*a^2*b^2)*f + I*(a^3*b + 3*a*b^3)*f)*sinh(d*x + c)^4 + 2*(2*(a^4 + \\
& 2*a^2*b^2)*f + I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^2 + 2*(3*(2*(a^4 + 2*a^ \\
& 2*b^2)*f + I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2)*f + \\
& I*(a^3*b + 3*a*b^3)*f)*sinh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2)*f + I*(a^3*b \\
& + 3*a*b^3)*f + 4*((2*(a^4 + 2*a^2*b^2)*f + I*(a^3*b + 3*a*b^3)*f)*cosh(d*x \\
& + c)^3 + (2*(a^4 + 2*a^2*b^2)*f + I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c))*sin \\
& h(d*x + c))*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) + ((2*(a^4 + 2*a^2*b^2 \\
& )*f - I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^4 + 4*(2*(a^4 + 2*a^2*b^2)*f - I \\
& *(a^3*b + 3*a*b^3)*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*(a^4 + 2*a^2*b^2)* \\
& f - I*(a^3*b + 3*a*b^3)*f)*sinh(d*x + c)^4 + 2*(2*(a^4 + 2*a^2*b^2)*f - I*( \\
& a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^2 + (6*(2*(a^4 + 2*a^2*b^2)*f - I*(a^3*b \\
& + 3*a*b^3)*f)*cosh(d*x + c)^2 + 4*(a^4 + 2*a^2*b^2)*f - 2*I*(a^3*b + 3*a*b^ \\
& 3)*f)*sinh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2)*f - I*(a^3*b + 3*a*b^3)*f + 4*( \\
& (2*(a^4 + 2*a^2*b^2)*f - I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c)^3 + (2*(a^4 + \\
& 2*a^2*b^2)*f - I*(a^3*b + 3*a*b^3)*f)*cosh(d*x + c))*sinh(d*x + c))*dilog( \\
& -I*cosh(d*x + c) - I*sinh(d*x + c)) - 2*((a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x \\
& + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 \\
& + 2*a^2*b^2 + b^4)*f*sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x \\
& + c)^2 + 2*(3*(a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c)^2 + (a^4 + 2*a^2*b^2 \\
& + b^4)*f)*sinh(d*x + c)^2 + (a^4 + 2*a^2*b^2 + b^4)*f + 4*((a^4 + 2*a^2*b^ \\
& 2 + b^4)*f*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*f*cosh(d*x + c))*sinh( \\
& d*x + c))*dilog(-cosh(d*x + c) - sinh(d*x + c)) + 2*(b^4*d*e - b^4*c*f + (b \\
& ^4*d*e - b^4*c*f)*cosh(d*x + c)^4 + 4*(b^4*d*e - b^4*c*f)*cosh(d*x + c)*sin \\
& h(d*x + c)^3 + (b^4*d*e - b^4*c*f)*sinh(d*x + c)^4 + 2*(b^4*d*e - b^4*c*f)* \\
& cosh(d*x + c)^2 + 2*(b^4*d*e - b^4*c*f + 3*(b^4*d*e - b^4*c*f)*cosh(d*x + c \\
& )^2)*sinh(d*x + c)^2 + 4*((b^4*d*e - b^4*c*f)*cosh(d*x + c)^3 + (b^4*d*e - \\
& b^4*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x \\
& + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + 2*(b^4*d*e - b^4*c*f + (b^4*d*e \\
& - b^4*c*f)*cosh(d*x + c)^4 + 4*(b^4*d*e - b^4*c*f)*cosh(d*x + c)*sinh(d*x + \\
& c)^3 + (b^4*d*e - b^4*c*f)*sinh(d*x + c)^4 + 2*(b^4*d*e - b^4*c*f)*cosh(d* \\
& x + c)^2 + 2*(b^4*d*e - b^4*c*f + 3*(b^4*d*e - b^4*c*f)*cosh(d*x + c)^2)*si \\
& nh(d*x + c)^2 + 4*((b^4*d*e - b^4*c*f)*cosh(d*x + c)^3 + (b^4*d*e - b^4*c*f
\end{aligned}$$



$$\begin{aligned}
& ) * \cosh(dx + c) * \sinh(dx + c) * \log(2b * \cosh(dx + c) + 2b * \sinh(dx + c) - \\
& 2b * \sqrt{(a^2 + b^2)/b^2} + 2a) + 2*(b^4 * d * f * x + b^4 * c * f + (b^4 * d * f * x + b \\
& ^4 * c * f) * \cosh(dx + c)^4 + 4*(b^4 * d * f * x + b^4 * c * f) * \cosh(dx + c) * \sinh(dx + \\
& c)^3 + (b^4 * d * f * x + b^4 * c * f) * \sinh(dx + c)^4 + 2*(b^4 * d * f * x + b^4 * c * f) * \cosh \\
& (dx + c)^2 + 2*(b^4 * d * f * x + b^4 * c * f + 3*(b^4 * d * f * x + b^4 * c * f) * \cosh(dx + c \\
& )^2) * \sinh(dx + c)^2 + 4*((b^4 * d * f * x + b^4 * c * f) * \cosh(dx + c)^3 + (b^4 * d * f * \\
& x + b^4 * c * f) * \cosh(dx + c) * \sinh(dx + c)) * \log(-(a * \cosh(dx + c) + a * \sinh(d \\
& * x + c) + (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2} - b)/b) \\
& + 2*(b^4 * d * f * x + b^4 * c * f + (b^4 * d * f * x + b^4 * c * f) * \cosh(dx + c)^4 + 4*(b^4 * \\
& d * f * x + b^4 * c * f) * \cosh(dx + c) * \sinh(dx + c)^3 + (b^4 * d * f * x + b^4 * c * f) * \sinh \\
& (dx + c)^4 + 2*(b^4 * d * f * x + b^4 * c * f) * \cosh(dx + c)^2 + 2*(b^4 * d * f * x + b^4 * \\
& c * f + 3*(b^4 * d * f * x + b^4 * c * f) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 4*((b^4 * d * \\
& f * x + b^4 * c * f) * \cosh(dx + c)^3 + (b^4 * d * f * x + b^4 * c * f) * \cosh(dx + c) * \sinh( \\
& dx + c)) * \log(-(a * \cosh(dx + c) + a * \sinh(dx + c) - (b * \cosh(dx + c) + b * \si \\
& nh(dx + c)) * \sqrt{(a^2 + b^2)/b^2} - b)/b) - 2*(((a^4 + 2*a^2*b^2 + b^4)*d* \\
& f*x + (a^4 + 2*a^2*b^2 + b^4)*d*e)*\cosh(dx + c)^4 + 4*((a^4 + 2*a^2*b^2 + \\
& b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*e)*\cosh(dx + c)*\sinh(dx + c)^3 + ( \\
& (a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*e)*\sinh(dx + c)^ \\
& 4 + (a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*e + 2*((a^4 + \\
& 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*e)*\cosh(dx + c)^2 + 2* \\
& ((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*e + 3*((a^4 + 2* \\
& a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*e)*\cosh(dx + c)^2)*\sinh(d \\
& * x + c)^2 + 4*(((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*d*e \\
& ) * \cosh(dx + c)^3 + ((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4) \\
& ) * d * e) * \cosh(dx + c) * \sinh(dx + c)) * \log(\cosh(dx + c) + \sinh(dx + c) + 1) \\
& + ((2*(a^4 + 2*a^2*b^2)*d*e + I*(a^3*b + 3*a*b^3)*d*e - 2*(a^4 + 2*a^2*b^2 \\
& ) * c * f - I*(a^3*b + 3*a*b^3) * c * f) * \cosh(dx + c)^4 + (8*(a^4 + 2*a^2*b^2)*d*e \\
& + 4*I*(a^3*b + 3*a*b^3)*d*e - 8*(a^4 + 2*a^2*b^2)*c*f - 4*I*(a^3*b + 3*a*b \\
& ^3) * c * f) * \cosh(dx + c) * \sinh(dx + c)^3 + (2*(a^4 + 2*a^2*b^2)*d*e + I*(a^3*b \\
& + 3*a*b^3)*d*e - 2*(a^4 + 2*a^2*b^2)*c*f - I*(a^3*b + 3*a*b^3)*c*f) * \sinh( \\
& dx + c)^4 + 2*(a^4 + 2*a^2*b^2)*d*e + I*(a^3*b + 3*a*b^3)*d*e - 2*(a^4 + 2 \\
& * a^2 * b^2) * c * f - I * (a^3 * b + 3 * a * b^3) * c * f + (4 * (a^4 + 2 * a^2 * b^2) * d * e + 2 * I * (a \\
& ^3 * b + 3 * a * b^3) * d * e - 4 * (a^4 + 2 * a^2 * b^2) * c * f - 2 * I * (a^3 * b + 3 * a * b^3) * c * f) * \\
& \cosh(dx + c)^2 + (4 * (a^4 + 2 * a^2 * b^2) * d * e + 2 * I * (a^3 * b + 3 * a * b^3) * d * e - 4 * \\
& (a^4 + 2 * a^2 * b^2) * c * f - 2 * I * (a^3 * b + 3 * a * b^3) * c * f + (12 * (a^4 + 2 * a^2 * b^2) * d \\
& * e + 6 * I * (a^3 * b + 3 * a * b^3) * d * e - 12 * (a^4 + 2 * a^2 * b^2) * c * f - 6 * I * (a^3 * b + 3 * \\
& a * b^3) * c * f) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + ((8 * (a^4 + 2 * a^2 * b^2) * d * e + \\
& 4 * I * (a^3 * b + 3 * a * b^3) * d * e - 8 * (a^4 + 2 * a^2 * b^2) * c * f - 4 * I * (a^3 * b + 3 * a * b^3) \\
& * c * f) * \cosh(dx + c)^3 + (8 * (a^4 + 2 * a^2 * b^2) * d * e + 4 * I * (a^3 * b + 3 * a * b^3) * d * \\
& e - 8 * (a^4 + 2 * a^2 * b^2) * c * f - 4 * I * (a^3 * b + 3 * a * b^3) * c * f) * \cosh(dx + c) * \sin \\
& h(dx + c)) * \log(\cosh(dx + c) + \sinh(dx + c) + I) + ((2 * (a^4 + 2 * a^2 * b^2) * \\
& d * e - I * (a^3 * b + 3 * a * b^3) * d * e - 2 * (a^4 + 2 * a^2 * b^2) * c * f + I * (a^3 * b + 3 * a * b^ \\
& 3) * c * f) * \cosh(dx + c)^4 + (8 * (a^4 + 2 * a^2 * b^2) * d * e - 4 * I * (a^3 * b + 3 * a * b^3) * \\
& d * e - 8 * (a^4 + 2 * a^2 * b^2) * c * f + 4 * I * (a^3 * b + 3 * a * b^3) * c * f) * \cosh(dx + c) * \si \\
& nh(dx + c)^3 + (2 * (a^4 + 2 * a^2 * b^2) * d * e - I * (a^3 * b + 3 * a * b^3) * d * e - 2 * (a^4
\end{aligned}$$

$$\begin{aligned}
& + 2*a^2*b^2)*c*f + I*(a^3*b + 3*a*b^3)*c*f)*\sinh(d*x + c)^4 + 2*(a^4 + 2*a^2*b^2)*d*e - I*(a^3*b + 3*a*b^3)*d*e - 2*(a^4 + 2*a^2*b^2)*c*f + I*(a^3*b \\
& + 3*a*b^3)*c*f + (4*(a^4 + 2*a^2*b^2)*d*e - 2*I*(a^3*b + 3*a*b^3)*d*e - 4*(a^4 + 2*a^2*b^2)*c*f + 2*I*(a^3*b + 3*a*b^3)*c*f) * \cosh(d*x + c)^2 + (4*(a^4 \\
& + 2*a^2*b^2)*d*e - 2*I*(a^3*b + 3*a*b^3)*d*e - 4*(a^4 + 2*a^2*b^2)*c*f + 2 \\
& *I*(a^3*b + 3*a*b^3)*c*f + (12*(a^4 + 2*a^2*b^2)*d*e - 6*I*(a^3*b + 3*a*b^3) \\
& ) * d*e - 12*(a^4 + 2*a^2*b^2)*c*f + 6*I*(a^3*b + 3*a*b^3)*c*f) * \cosh(d*x + c) \\
& ^2) * \sinh(d*x + c)^2 + ((8*(a^4 + 2*a^2*b^2)*d*e - 4*I*(a^3*b + 3*a*b^3)*d*e \\
& - 8*(a^4 + 2*a^2*b^2)*c*f + 4*I*(a^3*b + 3*a*b^3)*c*f) * \cosh(d*x + c)^3 + ( \\
& 8*(a^4 + 2*a^2*b^2)*d*e - 4*I*(a^3*b + 3*a*b^3)*d*e - 8*(a^4 + 2*a^2*b^2)*c \\
& *f + 4*I*(a^3*b + 3*a*b^3)*c*f) * \cosh(d*x + c)) * \sinh(d*x + c)) * \log(\cosh(d*x \\
& + c) + \sinh(d*x + c) - I) - 2*((a^4 + 2*a^2*b^2 + b^4)*d*e - (a^4 + 2*a^2*b^2 \\
& + b^4)*c*f) * \cosh(d*x + c)^4 + 4*((a^4 + 2*a^2*b^2 + b^4)*d*e - (a^4 + 2 \\
& *a^2*b^2 + b^4)*c*f) * \cosh(d*x + c) * \sinh(d*x + c)^3 + ((a^4 + 2*a^2*b^2 + b^4) \\
& *d*e - (a^4 + 2*a^2*b^2 + b^4)*c*f) * \sinh(d*x + c)^4 + (a^4 + 2*a^2*b^2 + \\
& b^4)*d*e - (a^4 + 2*a^2*b^2 + b^4)*c*f + 2*((a^4 + 2*a^2*b^2 + b^4)*d*e - ( \\
& a^4 + 2*a^2*b^2 + b^4)*c*f) * \cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d \\
& e - (a^4 + 2*a^2*b^2 + b^4)*c*f + 3*((a^4 + 2*a^2*b^2 + b^4)*d*e - (a^4 + 2 \\
& *a^2*b^2 + b^4)*c*f) * \cosh(d*x + c)^2) * \sinh(d*x + c)^2 + 4*((a^4 + 2*a^2*b^2 \\
& + b^4)*d*e - (a^4 + 2*a^2*b^2 + b^4)*c*f) * \cosh(d*x + c)^3 + ((a^4 + 2*a^2 \\
& *b^2 + b^4)*d*e - (a^4 + 2*a^2*b^2 + b^4)*c*f) * \cosh(d*x + c)) * \sinh(d*x + c) \\
& ) * \log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + ((2*(a^4 + 2*a^2*b^2)*d*f*x - I* \\
& (a^3*b + 3*a*b^3)*d*f*x + 2*(a^4 + 2*a^2*b^2)*c*f - I*(a^3*b + 3*a*b^3)*c*f \\
& ) * \cosh(d*x + c)^4 + (8*(a^4 + 2*a^2*b^2)*d*f*x - 4*I*(a^3*b + 3*a*b^3)*d*f* \\
& x + 8*(a^4 + 2*a^2*b^2)*c*f - 4*I*(a^3*b + 3*a*b^3)*c*f) * \cosh(d*x + c) * \sinh \\
& (d*x + c)^3 + (2*(a^4 + 2*a^2*b^2)*d*f*x - I*(a^3*b + 3*a*b^3)*d*f*x + 2*(a \\
& ^4 + 2*a^2*b^2)*c*f - I*(a^3*b + 3*a*b^3)*c*f) * \sinh(d*x + c)^4 + 2*(a^4 + 2 \\
& *a^2*b^2)*d*f*x - I*(a^3*b + 3*a*b^3)*d*f*x + 2*(a^4 + 2*a^2*b^2)*c*f - I*( \\
& a^3*b + 3*a*b^3)*c*f + (4*(a^4 + 2*a^2*b^2)*d*f*x - 2*I*(a^3*b + 3*a*b^3)*d \\
& *f*x + 4*(a^4 + 2*a^2*b^2)*c*f - 2*I*(a^3*b + 3*a*b^3)*c*f) * \cosh(d*x + c)^2 \\
& + (4*(a^4 + 2*a^2*b^2)*d*f*x - 2*I*(a^3*b + 3*a*b^3)*d*f*x + 4*(a^4 + 2*a^2 \\
& *b^2)*c*f - 2*I*(a^3*b + 3*a*b^3)*c*f + (12*(a^4 + 2*a^2*b^2)*d*f*x - 6*I* \\
& (a^3*b + 3*a*b^3)*d*f*x + 12*(a^4 + 2*a^2*b^2)*c*f - 6*I*(a^3*b + 3*a*b^3)* \\
& c*f) * \cosh(d*x + c)^2) * \sinh(d*x + c)^2 + ((8*(a^4 + 2*a^2*b^2)*d*f*x - 4*I*( \\
& a^3*b + 3*a*b^3)*d*f*x + 8*(a^4 + 2*a^2*b^2)*c*f - 4*I*(a^3*b + 3*a*b^3)*c \\
& f) * \cosh(d*x + c)^3 + (8*(a^4 + 2*a^2*b^2)*d*f*x - 4*I*(a^3*b + 3*a*b^3)*d*f \\
& *x + 8*(a^4 + 2*a^2*b^2)*c*f - 4*I*(a^3*b + 3*a*b^3)*c*f) * \cosh(d*x + c)) * \si \\
& nh(d*x + c)) * \log(I * \cosh(d*x + c) + I * \sinh(d*x + c) + 1) + ((2*(a^4 + 2*a^2* \\
& b^2)*d*f*x + I*(a^3*b + 3*a*b^3)*d*f*x + 2*(a^4 + 2*a^2*b^2)*c*f + I*(a^3*b \\
& + 3*a*b^3)*c*f) * \cosh(d*x + c)^4 + (8*(a^4 + 2*a^2*b^2)*d*f*x + 4*I*(a^3*b \\
& + 3*a*b^3)*d*f*x + 8*(a^4 + 2*a^2*b^2)*c*f + 4*I*(a^3*b + 3*a*b^3)*c*f) * \cos \\
& h(d*x + c) * \sinh(d*x + c)^3 + (2*(a^4 + 2*a^2*b^2)*d*f*x + I*(a^3*b + 3*a*b^ \\
& 3)*d*f*x + 2*(a^4 + 2*a^2*b^2)*c*f + I*(a^3*b + 3*a*b^3)*c*f) * \sinh(d*x + c) \\
& ^4 + 2*(a^4 + 2*a^2*b^2)*d*f*x + I*(a^3*b + 3*a*b^3)*d*f*x + 2*(a^4 + 2*a^2 \\
& *b^2)*c*f + I*(a^3*b + 3*a*b^3)*c*f + (4*(a^4 + 2*a^2*b^2)*d*f*x + 2*I*(a^3
\end{aligned}$$

```

*b + 3*a*b^3)*d*f*x + 4*(a^4 + 2*a^2*b^2)*c*f + 2*I*(a^3*b + 3*a*b^3)*c*f)*
cosh(d*x + c)^2 + (4*(a^4 + 2*a^2*b^2)*d*f*x + 2*I*(a^3*b + 3*a*b^3)*d*f*x
+ 4*(a^4 + 2*a^2*b^2)*c*f + 2*I*(a^3*b + 3*a*b^3)*c*f + (12*(a^4 + 2*a^2*b^
2)*d*f*x + 6*I*(a^3*b + 3*a*b^3)*d*f*x + 12*(a^4 + 2*a^2*b^2)*c*f + 6*I*(a^
3*b + 3*a*b^3)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*(a^4 + 2*a^2*b^2
)*d*f*x + 4*I*(a^3*b + 3*a*b^3)*d*f*x + 8*(a^4 + 2*a^2*b^2)*c*f + 4*I*(a^3*
b + 3*a*b^3)*c*f)*cosh(d*x + c)^3 + (8*(a^4 + 2*a^2*b^2)*d*f*x + 4*I*(a^3*b
+ 3*a*b^3)*d*f*x + 8*(a^4 + 2*a^2*b^2)*c*f + 4*I*(a^3*b + 3*a*b^3)*c*f)*co
sh(d*x + c))*sinh(d*x + c))*log(-I*cosh(d*x + c) - I*sinh(d*x + c) + 1) - 2
*(((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*c*f)*cosh(d*x +
c)^4 + 4*((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)*c*f)*cosh
(d*x + c))*sinh(d*x + c)^3 + ((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b
^2 + b^4)*c*f)*sinh(d*x + c)^4 + (a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a
^2*b^2 + b^4)*c*f + 2*((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b
^4)*c*f)*cosh(d*x + c)^2 + 2*((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*
b^2 + b^4)*c*f + 3*((a^4 + 2*a^2*b^2 + b^4)*d*f*x + (a^4 + 2*a^2*b^2 + b^4)
*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(((a^4 + 2*a^2*b^2 + b^4)*d*f*x
+ (a^4 + 2*a^2*b^2 + b^4)*c*f)*cosh(d*x + c)^3 + ((a^4 + 2*a^2*b^2 + b^4)*d
*f*x + (a^4 + 2*a^2*b^2 + b^4)*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-cosh
(d*x + c) - sinh(d*x + c) + 1) - 2*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)
*d*e - 3*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e + (a^3*b + a*b^3)*f)*
cosh(d*x + c)^2 - (a^3*b + a*b^3)*f + 2*(2*(a^4 + a^2*b^2)*d*f*x + 2*(a^4 +
a^2*b^2)*d*e + (a^4 + a^2*b^2)*f)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 2*
a^3*b^2 + a*b^4)*d^2*cosh(d*x + c)^4 + 4*(a^5 + 2*a^3*b^2 + a*b^4)*d^2*cosh
(d*x + c))*sinh(d*x + c)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*d^2*sinh(d*x + c)^4 +
2*(a^5 + 2*a^3*b^2 + a*b^4)*d^2*cosh(d*x + c)^2 + (a^5 + 2*a^3*b^2 + a*b^4
)*d^2 + 2*(3*(a^5 + 2*a^3*b^2 + a*b^4)*d^2*cosh(d*x + c)^2 + (a^5 + 2*a^3*b
^2 + a*b^4)*d^2)*sinh(d*x + c)^2 + 4*((a^5 + 2*a^3*b^2 + a*b^4)*d^2*cosh(d*
x + c)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*d^2*cosh(d*x + c))*sinh(d*x + c))

```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)\*sech(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.447 \quad \int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=160

$$\frac{b^4 \log(a + b \sinh(c + dx))}{ad(a^2 + b^2)^2} - \frac{b^3 \tan^{-1}(\sinh(c + dx))}{d(a^2 + b^2)^2} - \frac{b \tan^{-1}(\sinh(c + dx))}{2d(a^2 + b^2)} - \frac{a(a^2 + 2b^2) \log(\cosh(c + dx))}{d(a^2 + b^2)^2} + \frac{\operatorname{sech}^3(c + dx)}{a + b \sinh(c + dx)}$$

[Out] -((b^3\*ArcTan[Sinh[c + d\*x]])/((a^2 + b^2)^2\*d)) - (b\*ArcTan[Sinh[c + d\*x]])/(2\*(a^2 + b^2)\*d) - (a\*(a^2 + 2\*b^2)\*Log[Cosh[c + d\*x]])/((a^2 + b^2)^2\*d) + Log[Sinh[c + d\*x]]/(a\*d) - (b^4\*Log[a + b\*Sinh[c + d\*x]])/(a\*(a^2 + b^2)^2\*d) + (Sech[c + d\*x]^2\*(a - b\*Sinh[c + d\*x]))/(2\*(a^2 + b^2)\*d)

**Rubi [A]** time = 0.270547, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$ , Rules used = {2837, 12, 894, 639, 203, 635, 260}

$$\frac{b^4 \log(a + b \sinh(c + dx))}{ad(a^2 + b^2)^2} - \frac{b^3 \tan^{-1}(\sinh(c + dx))}{d(a^2 + b^2)^2} - \frac{b \tan^{-1}(\sinh(c + dx))}{2d(a^2 + b^2)} - \frac{a(a^2 + 2b^2) \log(\cosh(c + dx))}{d(a^2 + b^2)^2} + \frac{\operatorname{sech}^3(c + dx)}{a + b \sinh(c + dx)}$$

Antiderivative was successfully verified.

[In] Int[(Csch[c + d\*x]\*Sech[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] -((b^3\*ArcTan[Sinh[c + d\*x]])/((a^2 + b^2)^2\*d)) - (b\*ArcTan[Sinh[c + d\*x]])/(2\*(a^2 + b^2)\*d) - (a\*(a^2 + 2\*b^2)\*Log[Cosh[c + d\*x]])/((a^2 + b^2)^2\*d) + Log[Sinh[c + d\*x]]/(a\*d) - (b^4\*Log[a + b\*Sinh[c + d\*x]])/(a\*(a^2 + b^2)^2\*d) + (Sech[c + d\*x]^2\*(a - b\*Sinh[c + d\*x]))/(2\*(a^2 + b^2)\*d)

### Rule 2837

Int[cos[(e\_.) + (f\_.)\*(x\_)]^(p\_)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(m\_.)\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[1/(b^p\*f), Subst[Int[(a + x)^m\*(c + (d\*x)/b)^n\*(b^2 - x^2)^((p - 1)/2), x], x, b\*Sinh[e + f\*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{b^3 \operatorname{Subst}\left(\int \frac{b}{x(a+x)(-b^2-x^2)^2} dx, x, b\sinh(c+dx)\right)}{d} \\
&= \frac{b^4 \operatorname{Subst}\left(\int \frac{1}{x(a+x)(-b^2-x^2)^2} dx, x, b\sinh(c+dx)\right)}{d} \\
&= \frac{b^4 \operatorname{Subst}\left(\int \left(\frac{1}{ab^4x} - \frac{1}{a(a^2+b^2)^2(a+x)} + \frac{-b^2-ax}{b^2(a^2+b^2)(b^2+x^2)^2} + \frac{-b^4-a(a^2+2b^2)x}{b^4(a^2+b^2)^2(b^2+x^2)}\right) dx, x, b\sinh(c+dx)\right)}{d} \\
&= \frac{\log(\sinh(c+dx))}{ad} - \frac{b^4 \log(a+b\sinh(c+dx))}{a(a^2+b^2)^2 d} + \frac{\operatorname{Subst}\left(\int \frac{-b^4-a(a^2+2b^2)x}{b^2+x^2} dx, x, b\sinh(c+dx)\right)}{(a^2+b^2)^2 d} \\
&= \frac{\log(\sinh(c+dx))}{ad} - \frac{b^4 \log(a+b\sinh(c+dx))}{a(a^2+b^2)^2 d} + \frac{\operatorname{sech}^2(c+dx)(a-b\sinh(c+dx))}{2(a^2+b^2)d} \\
&= -\frac{b^3 \tan^{-1}(\sinh(c+dx))}{(a^2+b^2)^2 d} - \frac{b \tan^{-1}(\sinh(c+dx))}{2(a^2+b^2)d} - \frac{a(a^2+2b^2) \log(\cosh(c+dx))}{(a^2+b^2)^2 d}
\end{aligned}$$

**Mathematica [A]** time = 0.793089, size = 196, normalized size = 1.22

$$\frac{-a^2(a^2+b^2)\operatorname{sech}^2(c+dx) - 2(a^2+b^2)^2 \log(\sinh(c+dx)) + a(a^3+2ab^2+(-b^2)^{3/2}) \log(\sqrt{-b^2}-b\sinh(c+dx))}{(a^2+b^2)^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[(Csch[c + d\*x]\*Sech[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] -(a\*b\*(a^2 + b^2)\*ArcTan[Sinh[c + d\*x]] - 2\*(a^2 + b^2)^2\*Log[Sinh[c + d\*x]] + a\*(a^3 + 2\*a\*b^2 + (-b^2)^(3/2))\*Log[Sqrt[-b^2] - b\*Sinh[c + d\*x]] + 2\*b^4\*Log[a + b\*Sinh[c + d\*x]] + a\*(a^3 + 2\*a\*b^2 - (-b^2)^(3/2))\*Log[Sqrt[-b^2] + b\*Sinh[c + d\*x]] - a^2\*(a^2 + b^2)\*Sech[c + d\*x]^2 + a\*b\*(a^2 + b^2)\*Sech[c + d\*x]\*Tanh[c + d\*x])/(2\*a\*(a^2 + b^2)^2\*d)

**Maple [B]** time = 0.003, size = 530, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)`

[Out]  $\frac{1}{d} \frac{1}{a} \ln(\tanh(\frac{1}{2}dx + \frac{1}{2}c)) - \frac{1}{d} \frac{b^4}{a} \frac{1}{(a^4 + 2a^2b^2 + b^4)} \ln(\tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 a - 2 \tanh(\frac{1}{2}dx + \frac{1}{2}c) b - a) + \frac{1}{d} \frac{1}{(a^4 + 2a^2b^2 + b^4)} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1)^2 \tanh(\frac{1}{2}dx + \frac{1}{2}c)^3 a^2 b + 1} \frac{1}{(a^4 + 2a^2b^2 + b^4)} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1)^2 \tanh(\frac{1}{2}dx + \frac{1}{2}c)^3 b^3} - \frac{2}{d} \frac{1}{(a^4 + 2a^2b^2 + b^4)} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1)^2 \tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 a^3} - \frac{2}{d} \frac{1}{(a^4 + 2a^2b^2 + b^4)} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1)^2 \tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 a^2 b^2} - \frac{1}{d} \frac{1}{(a^4 + 2a^2b^2 + b^4)} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1)^2 \tanh(\frac{1}{2}dx + \frac{1}{2}c) a^2 b} - \frac{1}{d} \frac{1}{(a^4 + 2a^2b^2 + b^4)} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1)^2 \tanh(\frac{1}{2}dx + \frac{1}{2}c) b^3} - \frac{1}{d} \frac{1}{(a^4 + 2a^2b^2 + b^4)} \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1)^2 \tanh(\frac{1}{2}dx + \frac{1}{2}c) a^3} \ln(\tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1) - \frac{2}{d} \frac{1}{(a^4 + 2a^2b^2 + b^4)} \ln(\tanh(\frac{1}{2}dx + \frac{1}{2}c)^2 + 1) a^2 b^2 - \frac{1}{d} \frac{1}{(a^4 + 2a^2b^2 + b^4)} \arctan(\tanh(\frac{1}{2}dx + \frac{1}{2}c)) a^2 b^3 - \frac{3}{d} \frac{1}{(a^4 + 2a^2b^2 + b^4)} \arctan(\tanh(\frac{1}{2}dx + \frac{1}{2}c)) b^3$

**Maxima [A]** time = 1.81423, size = 358, normalized size = 2.24

$$-\frac{b^4 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{(a^5 + 2a^3b^2 + ab^4)d} + \frac{(a^2b + 3b^3) \arctan(e^{(-dx-c)})}{(a^4 + 2a^2b^2 + b^4)d} - \frac{(a^3 + 2ab^2) \log(e^{(-2dx-2c)} + 1)}{(a^4 + 2a^2b^2 + b^4)d} - \frac{be^{(-2dx-2c)}}{(a^2 + b^2 + 2ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out]  $-b^4 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b) / ((a^5 + 2a^3b^2 + ab^4)d) + (a^2b + 3b^3) \arctan(e^{(-dx-c)}) / ((a^4 + 2a^2b^2 + b^4)d) - (a^3 + 2ab^2) \log(e^{(-2dx-2c)} + 1) / ((a^4 + 2a^2b^2 + b^4)d) - (be^{(-dx-c)} - 2ae^{(-2dx-2c)} - be^{(-3dx-3c)}) / ((a^2 + b^2 + 2ab) * (a^2 + b^2) * e^{(-2dx-2c)} + (a^2 + b^2) * e^{(-4dx-4c)}) * d + \log(e^{(-dx-c)} + 1) / (a*d) + \log(e^{(-dx-c)} - 1) / (a*d)$

**Fricas [B]** time = 4.2508, size = 3066, normalized size = 19.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -((a^3*b + a*b^3)*cosh(d*x + c)^3 + (a^3*b + a*b^3)*sinh(d*x + c)^3 - 2*(a^4 + a^2*b^2)*cosh(d*x + c)^2 - (2*a^4 + 2*a^2*b^2 - 3*(a^3*b + a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + ((a^3*b + 3*a*b^3)*cosh(d*x + c)^4 + 4*(a^3*b + 3*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b + 3*a*b^3)*sinh(d*x + c)^4 + a^3*b + 3*a*b^3 + 2*(a^3*b + 3*a*b^3)*cosh(d*x + c)^2 + 2*(a^3*b + 3*a*b^3)^3 + 3*(a^3*b + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3*b + 3*a*b^3)*cosh(d*x + c)^3 + (a^3*b + 3*a*b^3)*cosh(d*x + c))*sinh(d*x + c))*arc tan(cosh(d*x + c) + sinh(d*x + c)) - (a^3*b + a*b^3)*cosh(d*x + c) + (b^4*cosh(d*x + c)^4 + 4*b^4*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*sinh(d*x + c)^4 + 2*b^4*cosh(d*x + c)^2 + b^4 + 2*(3*b^4*cosh(d*x + c)^2 + b^4)*sinh(d*x + c)^2 + 4*(b^4*cosh(d*x + c)^3 + b^4*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + ((a^4 + 2*a^2*b^2)*cosh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2)*sinh(d*x + c)^4 + a^4 + 2*a^2*b^2 + 2*(a^4 + 2*a^2*b^2)*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + 3*(a^4 + 2*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^4 + 2*a^2*b^2)*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*sinh(d*x + c)^4 + a^4 + 2*a^2*b^2 + b^4 + 2*(a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c)^3 + (a^4 + 2*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))) - (a^3*b + a*b^3 - 3*(a^3*b + a*b^3)*cosh(d*x + c)^2 + 4*(a^4 + a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c)^4 + 4*(a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*d*sinh(d*x + c)^4 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 + 2*a^3*b^2 + a*b^4)*d)*sinh(d*x + c)^2 + (a^5 + 2*a^3*b^2 + a*b^4)*d + 4*((a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c))
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

[Out] Timed out

**Giac [B]** time = 1.76585, size = 485, normalized size = 3.03

$$\frac{b^5 \log\left(\left|b(e^{(dx+c)} - e^{(-dx-c)}) + 2a\right|\right)}{a^5bd + 2a^3b^3d + ab^5d} - \frac{\left(\pi + 2 \arctan\left(\frac{1}{2}(e^{(2dx+2c)} - 1)e^{(-dx-c)}\right)\right)(a^2b + 3b^3)}{4(a^4d + 2a^2b^2d + b^4d)} - \frac{(a^3 + 2ab^2) \log\left(\left|e^{(dx+c)}\right|\right)}{2(a^4d + 2a^2b^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")`

[Out] `-b^5*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^5*b*d + 2*a^3*b^3*d + a*b^5*d) - 1/4*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a^2*b + 3*b^3)/(a^4*d + 2*a^2*b^2*d + b^4*d) - 1/2*(a^3 + 2*a*b^2)*log((e^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^4*d + 2*a^2*b^2*d + b^4*d) + 1/2*(a^3*(e^(d*x + c) - e^(-d*x - c))^2 + 2*a*b^2*(e^(d*x + c) - e^(-d*x - c))^2 - 2*a^2*b*(e^(d*x + c) - e^(-d*x - c)) - 2*b^3*(e^(d*x + c) - e^(-d*x - c)) + 8*a^3 + 12*a*b^2)/((a^4*d + 2*a^2*b^2*d + b^4*d)*((e^(d*x + c) - e^(-d*x - c))^2 + 4)) + log(abs(e^(d*x + c) - e^(-d*x - c)))/(a*d)`

$$3.448 \quad \int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable}\left(\frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}, x\right)$$

[Out] Unintegrable[(Csch[c + d\*x]\*Sech[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0894066, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Csch[c + d\*x]\*Sech[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int] [(Csch[c + d\*x]\*Sech[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Mathematica [A]** time = 174.592, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csch[c + d\*x]\*Sech[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

```
[Out] Integrate[(Csch[c + d*x]*Sech[c + d*x]^3)/((e + f*x)*(a + b*Sinh[c + d*x]))
, x]
```

**Maple [A]** time = 3.421, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx + c) (\operatorname{sech}(dx + c))^3}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -(a*f + (b*d*f*x*e^(3*c) + (d*e - f)*b*e^(3*c))*e^(3*d*x) - (2*a*d*f*x*e^(2
*c) + (2*d*e - f)*a*e^(2*c))*e^(2*d*x) - (b*d*f*x*e^c + (d*e + f)*b*e^c)*e^
(d*x))/(a^2*d^2*e^2 + b^2*d^2*e^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + 2*(a^
2*d^2*e*f + b^2*d^2*e*f)*x + (a^2*d^2*e^2*e^(4*c) + b^2*d^2*e^2*e^(4*c) + (
a^2*d^2*f^2*e^(4*c) + b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^2*d^2*e*f*e^(4*c) + b
^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x) + 2*(a^2*d^2*e^2*e^(2*c) + b^2*d^2*e^2*e^(
2*c) + (a^2*d^2*f^2*e^(2*c) + b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^2*d^2*e*f*e^(
2*c) + b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) + 16*integrate(-1/8*(a*b^4*e^(d*x
+ c) - b^5)/(a^5*b*e + 2*a^3*b^3*e + a*b^5*e + (a^5*b*f + 2*a^3*b^3*f + a
b^5*f)*x - (a^5*b*e*e^(2*c) + 2*a^3*b^3*e*e^(2*c) + a*b^5*e*e^(2*c) + (a^5*
b*f*e^(2*c) + 2*a^3*b^3*f*e^(2*c) + a*b^5*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^6*
e*e^c + 2*a^4*b^2*e*e^c + a^2*b^4*e*e^c + (a^6*f*e^c + 2*a^4*b^2*f*e^c + a^
2*b^4*f*e^c)*x)*e^(d*x)), x) - 16*integrate(-1/16*(2*(d^2*e^2 - f^2)*a^3 +
2*(2*d^2*e^2 - f^2)*a*b^2 + 2*(a^3*d^2*f^2 + 2*a*b^2*d^2*f^2)*x^2 + 4*(a^3*
d^2*e*f + 2*a*b^2*d^2*e*f)*x - ((d^2*e^2 - 2*f^2)*a^2*b*e^c + (3*d^2*e^2 -
2*f^2)*b^3*e^c + (a^2*b*d^2*f^2*e^c + 3*b^3*d^2*f^2*e^c)*x^2 + 2*(a^2*b*d^2
```

```
*e*f*e^c + 3*b^3*d^2*e*f*e^c)*x)*e^(d*x))/(a^4*d^2*e^3 + 2*a^2*b^2*d^2*e^3
+ b^4*d^2*e^3 + (a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^
4*d^2*e*f^2 + 2*a^2*b^2*d^2*e*f^2 + b^4*d^2*e*f^2)*x^2 + 3*(a^4*d^2*e^2*f +
2*a^2*b^2*d^2*e^2*f + b^4*d^2*e^2*f)*x + (a^4*d^2*e^3*e^(2*c) + 2*a^2*b^2*
d^2*e^3*e^(2*c) + b^4*d^2*e^3*e^(2*c) + (a^4*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^
2*f^3*e^(2*c) + b^4*d^2*f^3*e^(2*c))*x^3 + 3*(a^4*d^2*e*f^2*e^(2*c) + 2*a^2
*b^2*d^2*e*f^2*e^(2*c) + b^4*d^2*e*f^2*e^(2*c))*x^2 + 3*(a^4*d^2*e^2*f*e^(2
*c) + 2*a^2*b^2*d^2*e^2*f*e^(2*c) + b^4*d^2*e^2*f*e^(2*c))*x)*e^(2*d*x)), x
) - 16*integrate(1/16/(a*f*x + a*e + (a*f*x*e^c + a*e*e^c)*e^(d*x)), x) + 1
6*integrate(-1/16/(a*f*x + a*e - (a*f*x*e^c + a*e*e^c)*e^(d*x)), x)
```

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.449 \quad \int \frac{(e+fx)^3 \coth(c+dx) \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=601

$$\frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3} - \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^3} + \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2}$$

```
[Out] (-6*f*(e + f*x)^2*ArcTanh[E^(c + d*x)]/(a*d^2) - ((e + f*x)^3*Csch[c + d*x
])/ (a*d) + (b*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
a^2*d) + (b*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^
2*d) - (b*(e + f*x)^3*Log[1 - E^(2*(c + d*x))])/(a^2*d) - (6*f^2*(e + f*x)*
PolyLog[2, -E^(c + d*x)]/(a*d^3) + (6*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)
])/ (a*d^3) + (3*b*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2]))])/(a^2*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]))])/(a^2*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*
x))])/(2*a^2*d^2) + (6*f^3*PolyLog[3, -E^(c + d*x)]/(a*d^4) - (6*f^3*PolyL
og[3, E^(c + d*x)]/(a*d^4) - (6*b*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x
)))/(a - Sqrt[a^2 + b^2])])/(a^2*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*d^3) + (3*b*f^2*(e + f*x)*PolyLo
g[3, E^(2*(c + d*x))])/(2*a^2*d^3) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/(a^2*d^4) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/(a^2*d^4) - (3*b*f^3*PolyLog[4, E^(2*(c + d*x))])/(
4*a^2*d^4)
```

**Rubi [A]** time = 1.00297, antiderivative size = 601, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 11, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$ , Rules used = {5587, 5452, 4182, 2531, 2282, 6589, 5569, 3716, 2190, 6609, 5561}

$$\frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3} - \frac{6bf^2(e+fx)\operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^3} + \frac{3bf(e+fx)^2\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-6*f*(e + f*x)^2*ArcTanh[E^(c + d*x)]/(a*d^2) - ((e + f*x)^3*Csch[c + d*x
])/ (a*d) + (b*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
a^2*d) + (b*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^
2*d) - (b*(e + f*x)^3*Log[1 - E^(2*(c + d*x))])/(a^2*d) - (6*f^2*(e + f*x)*
PolyLog[2, -E^(c + d*x)]/(a*d^3) + (6*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)
])/ (a*d^3)
```

$$\begin{aligned} & ]/(a*d^3) + (3*b*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a^2*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a^2*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*x))]/(2*a^2*d^2) + (6*f^3*PolyLog[3, -E^(c + d*x)]/(a*d^4) - (6*f^3*PolyLog[3, E^(c + d*x)]/(a*d^4) - (6*b*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a^2*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a^2*d^3) + (3*b*f^2*(e + f*x)*PolyLog[3, E^(2*(c + d*x))]/(2*a^2*d^3) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a^2*d^4) + (6*b*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a^2*d^4) - (3*b*f^3*PolyLog[4, E^(2*(c + d*x))]/(4*a^2*d^4) \end{aligned}$$

### Rule 5587

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Csch[c + d*x]^(p - 1)*Coth[c + d*x]^n)/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n), x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))]^(n_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```



Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Coth[c + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*e) + f*fz*x))/E^(2*I*k*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_.)]\*((e\_.) + (f\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*Sin h[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] :> -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(e + fx)^3 \coth(c + dx) \operatorname{csch}(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\int (e + fx)^3 \coth(c + dx) \operatorname{csch}(c + dx) dx}{a} - \frac{b \int \frac{(e + fx)^3 \coth(c + dx)}{a + b \sinh(c + dx)} dx}{a} \\
 &= -\frac{(e + fx)^3 \operatorname{csch}(c + dx)}{ad} - \frac{b \int (e + fx)^3 \coth(c + dx) dx}{a^2} + \frac{b^2 \int \frac{(e + fx)^3 \cosh(c + dx)}{a + b \sinh(c + dx)} dx}{a^2} \\
 &= -\frac{6f(e + fx)^2 \tanh^{-1}(e^{c + dx})}{ad^2} - \frac{(e + fx)^3 \operatorname{csch}(c + dx)}{ad} + \frac{(2b) \int \frac{e^{2(c + dx)}(e + fx)^3}{1 - e^{2(c + dx)}} dx}{a^2} \\
 &= -\frac{6f(e + fx)^2 \tanh^{-1}(e^{c + dx})}{ad^2} - \frac{(e + fx)^3 \operatorname{csch}(c + dx)}{ad} + \frac{b(e + fx)^3 \log(1 + e^{2(c + dx)})}{a^2 d} \\
 &= -\frac{6f(e + fx)^2 \tanh^{-1}(e^{c + dx})}{ad^2} - \frac{(e + fx)^3 \operatorname{csch}(c + dx)}{ad} + \frac{b(e + fx)^3 \log(1 + e^{2(c + dx)})}{a^2 d} \\
 &= -\frac{6f(e + fx)^2 \tanh^{-1}(e^{c + dx})}{ad^2} - \frac{(e + fx)^3 \operatorname{csch}(c + dx)}{ad} + \frac{b(e + fx)^3 \log(1 + e^{2(c + dx)})}{a^2 d} \\
 &= -\frac{6f(e + fx)^2 \tanh^{-1}(e^{c + dx})}{ad^2} - \frac{(e + fx)^3 \operatorname{csch}(c + dx)}{ad} + \frac{b(e + fx)^3 \log(1 + e^{2(c + dx)})}{a^2 d} \\
 &= -\frac{6f(e + fx)^2 \tanh^{-1}(e^{c + dx})}{ad^2} - \frac{(e + fx)^3 \operatorname{csch}(c + dx)}{ad} + \frac{b(e + fx)^3 \log(1 + e^{2(c + dx)})}{a^2 d}
 \end{aligned}$$

**Mathematica [C]** time = 48.7021, size = 5638, normalized size = 9.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Result too large to show
```

**Maple [F]** time = 0.909, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \coth(dx + c) \operatorname{csch}(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] e^3*(2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) + b*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*d) - b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d) - 2*(f^3*x^3*e^c + 3*e*f^2*x^2*e^c + 3*e^2*f*x*e^c)*e^(d*x)/(a*d*e^(2*d*x + 2*c) - a*d) - 3*e^2*f*log(e^(d*x + c) + 1)/(a*d^2) + 3*e^2*f*log(e^(d*x + c) - 1)/(a*d^2) - (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*b*f^3/(a^2*d^4) - (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*b*f^3/(a^2*d^4) - 3*(b*d*e^2*f + 2*a*e*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 3*(b*d*e^2*f - 2*a*e*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3) - 3*(b*d*e*f^2 + a*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^2*d^4) - 3*(b*d*e*f^2 - a*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))/(a^2*d^4) + 1/4
```

```

*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 + a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f + 2*a*d*e*
f^2)*d^2*x^2)/(a^2*d^4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 - a*f^3)*d^3*x^
3 + 6*(b*d^2*e^2*f - 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^4) - integrate(-2*(b^2*f^
3*x^3 + 3*b^2*e*f^2*x^2 + 3*b^2*e^2*f*x - (a*b*f^3*x^3*e^c + 3*a*b*e*f^2*x^
2*e^c + 3*a*b*e^2*f*x*e^c)*e^(d*x))/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x +
c) - a^2*b), x)

```

**Fricas [C]** time = 3.62349, size = 9829, normalized size = 16.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")

```

```

[Out] -(2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*cosh(
d*x + c) + 3*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f - (b*d^2*f^3*x^
2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c)^2 - 2*(b*d^2*f^3*x^2 + 2*b
*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*f^3*x^2 +
2*b*d^2*e*f^2*x + b*d^2*e^2*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*
sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) -
b)/b + 1) + 3*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f - (b*d^2*f^3*x^
x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c)^2 - 2*(b*d^2*f^3*x^2 + 2
*b*d^2*e*f^2*x + b*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*f^3*x^2
+ 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) +
a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)
- b)/b + 1) - 3*(b*d^2*f^3*x^2 + b*d^2*e^2*f - 2*a*d*e*f^2 - (b*d^2*f^3*x^
2 + b*d^2*e^2*f - 2*a*d*e*f^2 + 2*(b*d^2*e*f^2 - a*d*f^3)*x)*cosh(d*x + c)^
2 - 2*(b*d^2*f^3*x^2 + b*d^2*e^2*f - 2*a*d*e*f^2 + 2*(b*d^2*e*f^2 - a*d*f^3
)*x)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*f^3*x^2 + b*d^2*e^2*f - 2*a*d*e*f
^2 + 2*(b*d^2*e*f^2 - a*d*f^3)*x)*sinh(d*x + c)^2 + 2*(b*d^2*e*f^2 - a*d*f^
3)*x)*dilog(cosh(d*x + c) + sinh(d*x + c)) - 3*(b*d^2*f^3*x^2 + b*d^2*e^2*f
+ 2*a*d*e*f^2 - (b*d^2*f^3*x^2 + b*d^2*e^2*f + 2*a*d*e*f^2 + 2*(b*d^2*e*f^
2 + a*d*f^3)*x)*cosh(d*x + c)^2 - 2*(b*d^2*f^3*x^2 + b*d^2*e^2*f + 2*a*d*e*
f^2 + 2*(b*d^2*e*f^2 + a*d*f^3)*x)*cosh(d*x + c)*sinh(d*x + c) - (b*d^2*f^3
*x^2 + b*d^2*e^2*f + 2*a*d*e*f^2 + 2*(b*d^2*e*f^2 + a*d*f^3)*x)*sinh(d*x +
c)^2 + 2*(b*d^2*e*f^2 + a*d*f^3)*x)*dilog(-cosh(d*x + c) - sinh(d*x + c)) +
(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3 - (b*d^3*e^3 -
3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*cosh(d*x + c)^2 - 2*(b*d^3*e
^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*cosh(d*x + c)*sinh(d*x
+ c) - (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sinh(d*x

```

$$\begin{aligned}
& + c)^2) * \log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3 - ( \\
& b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3) * \cosh(d*x + c)^2 \\
& - 2*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3) * \cosh(d*x + \\
& c) * \sinh(d*x + c) - (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f \\
& ^3) * \sinh(d*x + c)^2) * \log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{( \\
& a^2 + b^2)/b^2} + 2*a) + (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f \\
& *x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3 - (b*d^3*f^3*x^3 + 3*b*d \\
& ^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f \\
& ^3) * \cosh(d*x + c)^2 - 2*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f \\
& *x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3) * \cosh(d*x + c) * \sinh(d*x \\
& + c) - (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2 \\
& *f - 3*b*c^2*d*e*f^2 + b*c^3*f^3) * \sinh(d*x + c)^2) * \log(-(a*\cosh(d*x + c) + \\
& a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c)) * \sqrt{(a^2 + b^2)/b^2} \\
& - b)/b) + (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2 \\
& *e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3 - (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + \\
& 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3) * \cosh(d*x \\
& + c)^2 - 2*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2 \\
& *e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3) * \cosh(d*x + c) * \sinh(d*x + c) - (b*d^3*f \\
& ^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d \\
& *e*f^2 + b*c^3*f^3) * \sinh(d*x + c)^2) * \log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c \\
& )) - (b*\cosh(d*x + c) + b*\sinh(d*x + c)) * \sqrt{(a^2 + b^2)/b^2} - b)/b) - (b* \\
& d^3*f^3*x^3 + b*d^3*e^3 + 3*a*d^2*e^2*f + 3*(b*d^3*e*f^2 + a*d^2*f^3)*x^2 - \\
& (b*d^3*f^3*x^3 + b*d^3*e^3 + 3*a*d^2*e^2*f + 3*(b*d^3*e*f^2 + a*d^2*f^3)*x \\
& ^2 + 3*(b*d^3*e^2*f + 2*a*d^2*e*f^2)*x) * \cosh(d*x + c)^2 - 2*(b*d^3*f^3*x^3 \\
& + b*d^3*e^3 + 3*a*d^2*e^2*f + 3*(b*d^3*e*f^2 + a*d^2*f^3)*x^2 + 3*(b*d^3*e^ \\
& 2*f + 2*a*d^2*e*f^2)*x) * \cosh(d*x + c) * \sinh(d*x + c) - (b*d^3*f^3*x^3 + b*d \\
& ^3*e^3 + 3*a*d^2*e^2*f + 3*(b*d^3*e*f^2 + a*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f + \\
& 2*a*d^2*e*f^2)*x) * \sinh(d*x + c)^2 + 3*(b*d^3*e^2*f + 2*a*d^2*e*f^2)*x) * \log( \\
& \cosh(d*x + c) + \sinh(d*x + c) + 1) - (b*d^3*e^3 - 3*(b*c + a)*d^2*e^2*f + 3 \\
& *(b*c^2 + 2*a*c)*d*e*f^2 - (b*c^3 + 3*a*c^2)*f^3 - (b*d^3*e^3 - 3*(b*c + a) \\
& *d^2*e^2*f + 3*(b*c^2 + 2*a*c)*d*e*f^2 - (b*c^3 + 3*a*c^2)*f^3) * \cosh(d*x + \\
& c)^2 - 2*(b*d^3*e^3 - 3*(b*c + a)*d^2*e^2*f + 3*(b*c^2 + 2*a*c)*d*e*f^2 - ( \\
& b*c^3 + 3*a*c^2)*f^3) * \cosh(d*x + c) * \sinh(d*x + c) - (b*d^3*e^3 - 3*(b*c + a) \\
& ) * d^2 * e^2 * f + 3 * (b * c^2 + 2 * a * c) * d * e * f^2 - (b * c^3 + 3 * a * c^2) * f^3) * \sinh(d * x + \\
& c)^2) * \log(\cosh(d*x + c) + \sinh(d*x + c) - 1) - (b*d^3*f^3*x^3 + 3*b*c*d^2* \\
& e^2*f - 3*(b*c^2 + 2*a*c)*d*e*f^2 + (b*c^3 + 3*a*c^2)*f^3 + 3*(b*d^3*e*f^2 \\
& - a*d^2*f^3)*x^2 - (b*d^3*f^3*x^3 + 3*b*c*d^2*e^2*f - 3*(b*c^2 + 2*a*c)*d*e \\
& *f^2 + (b*c^3 + 3*a*c^2)*f^3 + 3*(b*d^3*e*f^2 - a*d^2*f^3)*x^2 + 3*(b*d^3*e \\
& ^2*f - 2*a*d^2*e*f^2)*x) * \cosh(d*x + c)^2 - 2*(b*d^3*f^3*x^3 + 3*b*c*d^2*e^2 \\
& *f - 3*(b*c^2 + 2*a*c)*d*e*f^2 + (b*c^3 + 3*a*c^2)*f^3 + 3*(b*d^3*e*f^2 - a \\
& *d^2*f^3)*x^2 + 3*(b*d^3*e^2*f - 2*a*d^2*e*f^2)*x) * \cosh(d*x + c) * \sinh(d*x + \\
& c) - (b*d^3*f^3*x^3 + 3*b*c*d^2*e^2*f - 3*(b*c^2 + 2*a*c)*d*e*f^2 + (b*c^3 \\
& + 3*a*c^2)*f^3 + 3*(b*d^3*e*f^2 - a*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f - 2*a*d \\
& ^2*e*f^2)*x) * \sinh(d*x + c)^2 + 3*(b*d^3*e^2*f - 2*a*d^2*e*f^2)*x) * \log(-\cosh(
\end{aligned}$$

$$\begin{aligned}
& d*x + c) - \sinh(d*x + c) + 1) - 6*(b*f^3*\cosh(d*x + c)^2 + 2*b*f^3*\cosh(d*x \\
& + c)*\sinh(d*x + c) + b*f^3*\sinh(d*x + c)^2 - b*f^3)*\text{polylog}(4, (a*\cosh(d*x \\
& + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\text{sqrt}((a^2 + b \\
& ^2)/b^2))/b) - 6*(b*f^3*\cosh(d*x + c)^2 + 2*b*f^3*\cosh(d*x + c)*\sinh(d*x + \\
& c) + b*f^3*\sinh(d*x + c)^2 - b*f^3)*\text{polylog}(4, (a*\cosh(d*x + c) + a*\sinh(d*x \\
& + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\text{sqrt}((a^2 + b^2)/b^2))/b) + 6* \\
& (b*f^3*\cosh(d*x + c)^2 + 2*b*f^3*\cosh(d*x + c)*\sinh(d*x + c) + b*f^3*\sinh(d \\
& *x + c)^2 - b*f^3)*\text{polylog}(4, \cosh(d*x + c) + \sinh(d*x + c)) + 6*(b*f^3*\cos \\
& h(d*x + c)^2 + 2*b*f^3*\cosh(d*x + c)*\sinh(d*x + c) + b*f^3*\sinh(d*x + c)^2 \\
& - b*f^3)*\text{polylog}(4, -\cosh(d*x + c) - \sinh(d*x + c)) - 6*(b*d*f^3*x + b*d*e*f \\
& ^2 - (b*d*f^3*x + b*d*e*f^2)*\cosh(d*x + c)^2 - 2*(b*d*f^3*x + b*d*e*f^2)*c \\
& osh(d*x + c)*\sinh(d*x + c) - (b*d*f^3*x + b*d*e*f^2)*\sinh(d*x + c)^2)*\text{polyl} \\
& og(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + \\
& c))*\text{sqrt}((a^2 + b^2)/b^2))/b) - 6*(b*d*f^3*x + b*d*e*f^2 - (b*d*f^3*x + b*d \\
& *e*f^2)*\cosh(d*x + c)^2 - 2*(b*d*f^3*x + b*d*e*f^2)*\cosh(d*x + c)*\sinh(d*x \\
& + c) - (b*d*f^3*x + b*d*e*f^2)*\sinh(d*x + c)^2)*\text{polylog}(3, (a*\cosh(d*x + c) \\
& + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\text{sqrt}((a^2 + b^2)/b \\
& ^2))/b) + 6*(b*d*f^3*x + b*d*e*f^2 - a*f^3 - (b*d*f^3*x + b*d*e*f^2 - a*f^3) \\
& )*\cosh(d*x + c)^2 - 2*(b*d*f^3*x + b*d*e*f^2 - a*f^3)*\cosh(d*x + c)*\sinh(d*x \\
& + c) - (b*d*f^3*x + b*d*e*f^2 - a*f^3)*\sinh(d*x + c)^2)*\text{polylog}(3, \cosh(d \\
& *x + c) + \sinh(d*x + c)) + 6*(b*d*f^3*x + b*d*e*f^2 + a*f^3 - (b*d*f^3*x + \\
& b*d*e*f^2 + a*f^3)*\cosh(d*x + c)^2 - 2*(b*d*f^3*x + b*d*e*f^2 + a*f^3)*\cosh \\
& (d*x + c)*\sinh(d*x + c) - (b*d*f^3*x + b*d*e*f^2 + a*f^3)*\sinh(d*x + c)^2)* \\
& \text{polylog}(3, -\cosh(d*x + c) - \sinh(d*x + c)) + 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f \\
& ^2*x^2 + 3*a*d^3*e^2*f*x + a*d^3*e^3)*\sinh(d*x + c))/(a^2*d^4*\cosh(d*x + c) \\
& ^2 + 2*a^2*d^4*\cosh(d*x + c)*\sinh(d*x + c) + a^2*d^4*\sinh(d*x + c)^2 - a^2* \\
& d^4)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*coth(d\*x+c)\*csch(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.450 \quad \int \frac{(e+fx)^2 \coth(c+dx) \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=419

$$\frac{2bf(e+fx)\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2} + \frac{2bf(e+fx)\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2} - \frac{2bf^2\operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3} - \frac{2bf^2\operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^3}$$

[Out]  $(-4*f*(e+f*x)*\operatorname{ArcTanh}[E^{(c+d*x)}])/(a*d^2) - ((e+f*x)^2*\operatorname{Csch}[c+d*x])/(a*d) + (b*(e+f*x)^2*\operatorname{Log}[1+(b*E^{(c+d*x)})/(a-\operatorname{Sqrt}[a^2+b^2])])/(a^2*d) + (b*(e+f*x)^2*\operatorname{Log}[1+(b*E^{(c+d*x)})/(a+\operatorname{Sqrt}[a^2+b^2])])/(a^2*d) - (b*(e+f*x)^2*\operatorname{Log}[1-E^{(2*(c+d*x))}])/(a^2*d) - (2*f^2*\operatorname{PolyLog}[2, -E^{(c+d*x)}])/(a*d^3) + (2*f^2*\operatorname{PolyLog}[2, E^{(c+d*x)}])/(a*d^3) + (2*b*f*(e+f*x)*\operatorname{PolyLog}[2, -((b*E^{(c+d*x)})/(a-\operatorname{Sqrt}[a^2+b^2]))])/(a^2*d^2) + (2*b*f*(e+f*x)*\operatorname{PolyLog}[2, -((b*E^{(c+d*x)})/(a+\operatorname{Sqrt}[a^2+b^2]))])/(a^2*d^2) - (b*f*(e+f*x)*\operatorname{PolyLog}[2, E^{(2*(c+d*x))}])/(a^2*d^2) - (2*b*f^2*\operatorname{PolyLog}[3, -((b*E^{(c+d*x)})/(a-\operatorname{Sqrt}[a^2+b^2]))])/(a^2*d^3) - (2*b*f^2*\operatorname{PolyLog}[3, -((b*E^{(c+d*x)})/(a+\operatorname{Sqrt}[a^2+b^2]))])/(a^2*d^3) + (b*f^2*\operatorname{PolyLog}[3, E^{(2*(c+d*x))}])/(2*a^2*d^3)$

**Rubi [A]** time = 0.817859, antiderivative size = 419, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 12, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {5587, 5452, 4182, 2279, 2391, 5569, 3716, 2190, 2531, 2282, 6589, 5561}

$$\frac{2bf(e+fx)\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2} + \frac{2bf(e+fx)\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2} - \frac{2bf^2\operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3} - \frac{2bf^2\operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(e+f*x)^2*\operatorname{Coth}[c+d*x]*\operatorname{Csch}[c+d*x])/(a+b*\operatorname{Sinh}[c+d*x]),x]$

[Out]  $(-4*f*(e+f*x)*\operatorname{ArcTanh}[E^{(c+d*x)}])/(a*d^2) - ((e+f*x)^2*\operatorname{Csch}[c+d*x])/(a*d) + (b*(e+f*x)^2*\operatorname{Log}[1+(b*E^{(c+d*x)})/(a-\operatorname{Sqrt}[a^2+b^2])])/(a^2*d) + (b*(e+f*x)^2*\operatorname{Log}[1+(b*E^{(c+d*x)})/(a+\operatorname{Sqrt}[a^2+b^2])])/(a^2*d) - (b*(e+f*x)^2*\operatorname{Log}[1-E^{(2*(c+d*x))}])/(a^2*d) - (2*f^2*\operatorname{PolyLog}[2, -E^{(c+d*x)}])/(a*d^3) + (2*f^2*\operatorname{PolyLog}[2, E^{(c+d*x)}])/(a*d^3) + (2*b*f*(e+f*x)*\operatorname{PolyLog}[2, -((b*E^{(c+d*x)})/(a-\operatorname{Sqrt}[a^2+b^2]))])/(a^2*d^2) + (2*b*f*(e+f*x)*\operatorname{PolyLog}[2, -((b*E^{(c+d*x)})/(a+\operatorname{Sqrt}[a^2+b^2]))])/(a^2*d^2) - (b*f*(e+f*x)*\operatorname{PolyLog}[2, E^{(2*(c+d*x))}])/(a^2*d^2) - (2*b*f^2*\operatorname{PolyLog}[3, -((b*E^{(c+d*x)})/(a-\operatorname{Sqrt}[a^2+b^2]))])/(a^2*d^3) - (2*b*f^2*\operatorname{PolyLog}[3, -((b*E^{(c+d*x)})/(a+\operatorname{Sqrt}[a^2+b^2]))])/(a^2*d^3) + (b*f^2*\operatorname{PolyLog}[3, E^{(2*(c+d*x))}])/(2*a^2*d^3)$



$\log[3, E^{(2*(c + d*x))}]/(2*a^2*d^3)$

### Rule 5587

Int[(Coth[(c\_.) + (d\_.)\*(x\_.)]^(n\_.)\*Csch[(c\_.) + (d\_.)\*(x\_.)]^(p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] :> Dist[1/a, Int[(e + f\*x)^m\*Csch[c + d\*x]^p\*Coth[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Csch[c + d\*x]^(p - 1)\*Coth[c + d\*x]^n)/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5452

Int[Coth[(a\_.) + (b\_.)\*(x\_.)]^(p\_.)\*Csch[(a\_.) + (b\_.)\*(x\_.)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] :> -Simp[((c + d\*x)^m\*Csch[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Csch[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

### Rule 4182

Int[csc[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_.)]\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)])/((f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_.) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_.))))^(n\_.)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.)^(n\_.))]/(x\_.), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 5569

Int[(Coth[(c\_.) + (d\_.)\*(x\_.)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] :> Dist[1/a, Int[(e + f\*x)^m\*Coth[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Cosh[c + d\*x]\*Coth[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_.)*((a_.)*(v_)^(n_.))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
```

, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(e+fx)^2 \coth(c+dx) \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \coth(c+dx) \operatorname{csch}(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^2 \coth(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
 &= -\frac{(e+fx)^2 \operatorname{csch}(c+dx)}{ad} - \frac{b \int (e+fx)^2 \coth(c+dx) dx}{a^2} + \frac{b^2 \int \frac{(e+fx)^2 \cosh(c+dx)}{a+b \sinh(c+dx)} dx}{a^2} \\
 &= -\frac{4f(e+fx) \tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+fx)^2 \operatorname{csch}(c+dx)}{ad} + \frac{(2b) \int \frac{e^{2(c+dx)}(e+fx)^2}{1-e^{2(c+dx)}} dx}{a^2} \\
 &= -\frac{4f(e+fx) \tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+fx)^2 \operatorname{csch}(c+dx)}{ad} + \frac{b(e+fx)^2 \log\left(1 + \frac{e^{2(c+dx)}}{1-e^{2(c+dx)}}\right)}{a^2 d} \\
 &= -\frac{4f(e+fx) \tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+fx)^2 \operatorname{csch}(c+dx)}{ad} + \frac{b(e+fx)^2 \log\left(1 + \frac{e^{2(c+dx)}}{1-e^{2(c+dx)}}\right)}{a^2 d} \\
 &= -\frac{4f(e+fx) \tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+fx)^2 \operatorname{csch}(c+dx)}{ad} + \frac{b(e+fx)^2 \log\left(1 + \frac{e^{2(c+dx)}}{1-e^{2(c+dx)}}\right)}{a^2 d} \\
 &= -\frac{4f(e+fx) \tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+fx)^2 \operatorname{csch}(c+dx)}{ad} + \frac{b(e+fx)^2 \log\left(1 + \frac{e^{2(c+dx)}}{1-e^{2(c+dx)}}\right)}{a^2 d}
 \end{aligned}$$

**Mathematica [B]** time = 20.2357, size = 1595, normalized size = 3.81

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -(((e + f*x)^2*Csch[c])/(a*d)) - (b*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a
```

```

*e^(c + d*x) + b*(-1 + E^(2*(c + d*x))))/d - (3*e^2*E^(2*c)*Log[2*a*E^(c +
d*x) + b*(-1 + E^(2*(c + d*x)))]/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/
a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]])/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c
+ d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]])/d + (3*f^2*x^2*Log[1 + (b*E^
(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]])/d - (3*E^(2*c)*f^2*x^2*L
og[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]])/d + (6*e*f*x
*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]])/d - (6*e*E
^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]])/
d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)
]])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b
^2)*E^(2*c)]])/d - (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c +
d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]])]/d^2 - (6*(-1 + E^(2*c))*f*(e +
f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]])]
)/d^2 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2
*c)]])]/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[
(a^2 + b^2)*E^(2*c)]])]/d^3 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c
+ Sqrt[(a^2 + b^2)*E^(2*c)]])]/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^(2*
c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]])]/d^3)/(3*a^2*(-1 + E^(2*c)
)) + (b*d^3*(e + f*x)^3*(-1 + Coth[c]) + 3*d*e*f*(b*d*e - 2*a*f)*(d*x - Log
[1 - Cosh[c + d*x] - Sinh[c + d*x]]) - 6*d*f^2*(b*d*e + a*f)*x*Log[1 + Cosh
[c + d*x] - Sinh[c + d*x]] - 3*b*d^2*f^3*x^2*Log[1 + Cosh[c + d*x] - Sinh[
c + d*x]] - 6*d*f^2*(b*d*e - a*f)*x*Log[1 - Cosh[c + d*x] + Sinh[c + d*x]] -
3*b*d^2*f^3*x^2*Log[1 - Cosh[c + d*x] + Sinh[c + d*x]] + 3*d*e*f*(b*d*e +
2*a*f)*(d*x - Log[1 + Cosh[c + d*x] + Sinh[c + d*x]]) + 6*f^2*(b*d*e - a*f)
*PolyLog[2, Cosh[c + d*x] - Sinh[c + d*x]] + 6*f^2*(b*d*e + a*f)*PolyLog[2,
-Cosh[c + d*x] + Sinh[c + d*x]] + 6*b*f^3*(d*x*PolyLog[2, Cosh[c + d*x] -
Sinh[c + d*x]] + PolyLog[3, Cosh[c + d*x] - Sinh[c + d*x]]) + 6*b*f^3*(d*x*
PolyLog[2, -Cosh[c + d*x] + Sinh[c + d*x]] + PolyLog[3, -Cosh[c + d*x] + Si
nh[c + d*x]]))/(3*a^2*d^3*f) + ((e + f*x)^2*Csch[c/2]*Csch[(c + d*x)/2]*Sin
h[(d*x)/2])/(2*a*d) + ((e + f*x)^2*Sech[c/2]*Sech[(c + d*x)/2]*Sinh[(d*x)/2
])/ (2*a*d)

```

---

**Maple [F]** time = 0.668, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \coth(dx + c) \operatorname{csch}(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*coth(d\*x+c)\*csch(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out]  $\int ((f*x+e)^2*\coth(d*x+c)*\csch(d*x+c)/(a+b*\sinh(d*x+c)), x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$e^2 \left( \frac{2e^{(-dx-c)}}{(ae^{(-2dx-2c)} - a)d} + \frac{b \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{a^2d} - \frac{b \log(e^{(-dx-c)} + 1)}{a^2d} - \frac{b \log(e^{(-dx-c)} - 1)}{a^2d} \right) - \frac{2(f^2x^2e^c + \dots)}{ade^{(2dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x+e)^2*\coth(d*x+c)*\csch(d*x+c)/(a+b*\sinh(d*x+c)), x, \text{algorithm}="maxima")$

[Out]  $e^2*(2*e^{(-d*x - c)} / ((a*e^{(-2*d*x - 2*c)} - a)*d) + b*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b) / (a^2*d) - b*\log(e^{(-d*x - c)} + 1) / (a^2*d) - b*\log(e^{(-d*x - c)} - 1) / (a^2*d) - 2*(f^2*x^2*e^c + 2*e*f*x*e^c)*e^{(d*x)} / (a*d*e^{(2*d*x + 2*c)} - a*d) - 2*e*f*\log(e^{(d*x + c)} + 1) / (a*d^2) + 2*e*f*\log(e^{(d*x + c)} - 1) / (a*d^2) - (d^2*x^2*\log(e^{(d*x + c)} + 1) + 2*d*x*\text{dilog}(-e^{(d*x + c)}) - 2*\text{polylog}(3, -e^{(d*x + c)})) * b*f^2 / (a^2*d^3) - (d^2*x^2*\log(-e^{(d*x + c)} + 1) + 2*d*x*\text{dilog}(e^{(d*x + c)}) - 2*\text{polylog}(3, e^{(d*x + c)})) * b*f^2 / (a^2*d^3) - 2*(b*d*e*f + a*f^2)*(d*x*\log(e^{(d*x + c)} + 1) + \text{dilog}(-e^{(d*x + c)})) / (a^2*d^3) - 2*(b*d*e*f - a*f^2)*(d*x*\log(-e^{(d*x + c)} + 1) + \text{dilog}(e^{(d*x + c)})) / (a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*e*f + a*f^2)*d^2*x^2) / (a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*e*f - a*f^2)*d^2*x^2) / (a^2*d^3) - \text{integrate}(-2*(b^2*f^2*x^2 + 2*b^2*e*f*x - (a*b*f^2*x^2*e^c + 2*a*b*e*f*x*e^c)*e^{(d*x)}) / (a^2*b*e^{(2*d*x + 2*c)} + 2*a^3*e^{(d*x + c)} - a^2*b), x)$

**Fricas [C]** time = 2.93885, size = 6053, normalized size = 14.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x+e)^2*\coth(d*x+c)*\csch(d*x+c)/(a+b*\sinh(d*x+c)), x, \text{algorithm}="fricas")$

[Out]  $-(2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + a*d^2*e^2)*\cosh(d*x + c) + 2*(b*d*f^2*x + b*d*e*f - (b*d*f^2*x + b*d*e*f)*\cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d*f^2*x + b*d*e*f)*\sinh(d*x + c)^2)*\text{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c)))$

$$\begin{aligned}
& \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 2*(b*d*f^2*x + b*d*e*f - (b*d*f^2*x + \\
& b*d*e*f)*\cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*e*f)*\cosh(d*x + c)*\sinh(d*x + \\
& c) - (b*d*f^2*x + b*d*e*f)*\sinh(d*x + c)^2)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - \\
& (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b) \\
& /b + 1) - 2*(b*d*f^2*x + b*d*e*f - a*f^2 - (b*d*f^2*x + b*d*e*f - a*f^2)*\cosh(d*x + c)^2 - \\
& 2*(b*d*f^2*x + b*d*e*f - a*f^2)*\cosh(d*x + c)*\sinh(d*x + c) \\
& - (b*d*f^2*x + b*d*e*f - a*f^2)*\sinh(d*x + c)^2)*\operatorname{dilog}(\cosh(d*x + c) + \sinh(d*x + c)) - \\
& 2*(b*d*f^2*x + b*d*e*f + a*f^2 - (b*d*f^2*x + b*d*e*f + a*f^2)*\cosh(d*x + c)^2 - \\
& 2*(b*d*f^2*x + b*d*e*f + a*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d*f^2*x + b*d*e*f + \\
& a*f^2)*\sinh(d*x + c)^2)*\operatorname{dilog}(-\cosh(d*x + c) - \sinh(d*x + c)) + (b*d^2*e^2 - 2*b*c*d*e*f + \\
& b*c^2*f^2 - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*\cosh(d*x + c)^2 - 2*(b*d^2*e^2 - 2*b*c*d*e*f + \\
& b*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*\sinh(d*x + c)^2)* \\
& \log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + (b*d^2*e^2 - 2*b*c*d*e*f + \\
& b*c^2*f^2 - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*\cosh(d*x + c)^2 - 2*(b*d^2*e^2 - 2*b*c*d*e*f + \\
& b*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*\sinh(d*x + c)^2)* \\
& \log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + \\
& 2*b*c*d*e*f - b*c^2*f^2 - (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*\cosh(d*x + c)^2 - \\
& 2*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d^2*f^2*x^2 + \\
& 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*\sinh(d*x + c)^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + \\
& (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + \\
& 2*b*c*d*e*f - b*c^2*f^2 - (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*\cosh(d*x + c)^2 - \\
& 2*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d^2*f^2*x^2 + \\
& 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*\sinh(d*x + c)^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - \\
& (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - (b*d^2*f^2*x^2 + b*d^2*e^2 + 2*a*d*e*f - \\
& (b*d^2*f^2*x^2 + b*d^2*e^2 + 2*a*d*e*f + 2*(b*d^2*e*f + a*d*f^2)*x)*\cosh(d*x + c)^2 - 2*(b*d^2*f^2*x^2 + \\
& b*d^2*e^2 + 2*a*d*e*f + 2*(b*d^2*e*f + a*d*f^2)*x)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d^2*f^2*x^2 + \\
& b*d^2*e^2 + 2*a*d*e*f + 2*(b*d^2*e*f + a*d*f^2)*x)*\sinh(d*x + c)^2 + 2*(b*d^2*e*f + a*d*f^2)*x)*\log(\cosh(d*x + c) + \\
& \sinh(d*x + c) + 1) - (b*d^2*e^2 - 2*(b*c + a)*d*e*f + (b*c^2 + 2*a*c)*f^2 - (b*d^2*e^2 - 2*(b*c + a)*d*e*f + \\
& (b*c^2 + 2*a*c)*f^2)*\cosh(d*x + c)^2 - 2*(b*d^2*e^2 - 2*(b*c + a)*d*e*f + (b*c^2 + 2*a*c)*f^2)*\cosh(d*x + c)* \\
& \sinh(d*x + c) - (b*d^2*e^2 - 2*(b*c + a)*d*e*f + (b*c^2 + 2*a*c)*f^2)*\sinh(d*x + c)^2)*\log(\cosh(d*x + c) + \\
& \sinh(d*x + c) - 1) - (b*d^2*f^2*x^2 + 2*b*c*d*e*f - (b*c^2 + 2*a*c)*f^2 - (b*d^2*f^2*x^2 + 2*b*c*d*e*f - \\
& (b*c^2 + 2*a*c)*f^2 + 2*(b*d^2*e*f - a*d*f^2)*x)*\cosh(d*x + c)^2 - 2*(b*d^2*f^2*x^2 + 2*b*c*d*e*f - \\
& (b*c^2 + 2*a*c)*f^2 + 2*(b*d^2*e*f - a*d*f^2)*x)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d^2*f^2*x^2 + 2*b*c*d*e*f - \\
& (b*c^2 + 2*a*c)*f^2 + 2*(b*d^2*e*f - a*d*f^2)*x)*\sinh(d*x + c)^2 + 2*(b*d^2*e*f - a*d*f^2)*x)*\log(-\cosh(d*x + c) \\
& - \sinh(d*x + c) + 1) + 2*(b*f^2*\cosh(d*x + c)^2 + 2*b*f^2*\cosh(d*x + c)*\sin
\end{aligned}$$

$$\begin{aligned}
& h(dx + c) + b*f^2*\sinh(dx + c)^2 - b*f^2)*\text{polylog}(3, (a*\cosh(dx + c) + a \\
& * \sinh(dx + c) + (b*\cosh(dx + c) + b*\sinh(dx + c))*\sqrt{(a^2 + b^2)/b^2)) \\
& /b) + 2*(b*f^2*\cosh(dx + c)^2 + 2*b*f^2*\cosh(dx + c)*\sinh(dx + c) + b*f^2 \\
& * \sinh(dx + c)^2 - b*f^2)*\text{polylog}(3, (a*\cosh(dx + c) + a*\sinh(dx + c) - \\
& (b*\cosh(dx + c) + b*\sinh(dx + c))*\sqrt{(a^2 + b^2)/b^2))/b) - 2*(b*f^2*\co \\
& sh(dx + c)^2 + 2*b*f^2*\cosh(dx + c)*\sinh(dx + c) + b*f^2*\sinh(dx + c)^2 \\
& - b*f^2)*\text{polylog}(3, \cosh(dx + c) + \sinh(dx + c)) - 2*(b*f^2*\co \\
& sh(dx + c)^2 + 2*b*f^2*\cosh(dx + c)*\sinh(dx + c) + b*f^2*\sinh(dx + c)^2 - b*f^2)* \\
& \text{polylog}(3, -\cosh(dx + c) - \sinh(dx + c)) + 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f \\
& *x + a*d^2*e^2)*\sinh(dx + c))/(a^2*d^3*\cosh(dx + c)^2 + 2*a^2*d^3*\cosh(dx \\
& *x + c)*\sinh(dx + c) + a^2*d^3*\sinh(dx + c)^2 - a^2*d^3)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*coth(dx+c)\*csch(dx+c)/(a+b\*sinh(dx+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*coth(dx+c)\*csch(dx+c)/(a+b\*sinh(dx+c)),x, algorithm="giac")

[Out] Timed out

$$3.451 \quad \int \frac{(e+fx) \coth(c+dx) \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=243

$$\frac{bf \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2 d^2} + \frac{bf \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2 d^2} - \frac{bf \operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{2a^2 d^2} + \frac{b(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{a^2 d} + \dots$$

[Out]  $-\left(\frac{f \operatorname{ArcTanh}[\operatorname{Cosh}[c+dx]]}{a d^2}\right) - \left(\frac{(e+fx) \operatorname{Csch}[c+dx]}{a d}\right) + \left(\frac{b(e+fx) \operatorname{Log}\left[1 + \frac{b E^{c+dx}}{a - \sqrt{a^2+b^2}}\right]}{a^2 d}\right) + \left(\frac{b(e+fx) \operatorname{Log}\left[1 + \frac{b E^{c+dx}}{a + \sqrt{a^2+b^2}}\right]}{a^2 d}\right) - \left(\frac{b(e+fx) \operatorname{Log}\left[1 - E^{2(c+dx)}\right]}{a^2 d}\right) + \left(\frac{b f \operatorname{PolyLog}\left[2, -\frac{b E^{c+dx}}{a - \sqrt{a^2+b^2}}\right]}{a^2 d^2}\right) + \left(\frac{b f \operatorname{PolyLog}\left[2, -\frac{b E^{c+dx}}{a + \sqrt{a^2+b^2}}\right]}{a^2 d^2}\right) - \left(\frac{b f \operatorname{PolyLog}\left[2, E^{2(c+dx)}\right]}{2 a^2 d^2}\right)$

**Rubi [A]** time = 0.46413, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$ , Rules used = {5587, 5452, 3770, 5569, 3716, 2190, 2279, 2391, 5561}

$$\frac{bf \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2 d^2} + \frac{bf \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2 d^2} - \frac{bf \operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{2a^2 d^2} + \frac{b(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{a^2 d} + \dots$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{(e+fx) \operatorname{Coth}[c+dx] \operatorname{Csch}[c+dx]}{a+b \operatorname{Sinh}[c+dx]}, x\right]$

[Out]  $-\left(\frac{f \operatorname{ArcTanh}[\operatorname{Cosh}[c+dx]]}{a d^2}\right) - \left(\frac{(e+fx) \operatorname{Csch}[c+dx]}{a d}\right) + \left(\frac{b(e+fx) \operatorname{Log}\left[1 + \frac{b E^{c+dx}}{a - \sqrt{a^2+b^2}}\right]}{a^2 d}\right) + \left(\frac{b(e+fx) \operatorname{Log}\left[1 + \frac{b E^{c+dx}}{a + \sqrt{a^2+b^2}}\right]}{a^2 d}\right) - \left(\frac{b(e+fx) \operatorname{Log}\left[1 - E^{2(c+dx)}\right]}{a^2 d}\right) + \left(\frac{b f \operatorname{PolyLog}\left[2, -\frac{b E^{c+dx}}{a - \sqrt{a^2+b^2}}\right]}{a^2 d^2}\right) + \left(\frac{b f \operatorname{PolyLog}\left[2, -\frac{b E^{c+dx}}{a + \sqrt{a^2+b^2}}\right]}{a^2 d^2}\right) - \left(\frac{b f \operatorname{PolyLog}\left[2, E^{2(c+dx)}\right]}{2 a^2 d^2}\right)$

**Rule 5587**

$\operatorname{Int}\left[\frac{\operatorname{Coth}\left[\frac{c}{d} + (x)\right]^n \operatorname{Csch}\left[\frac{c}{d} + (x)\right]^p \left(\frac{e}{f} + \frac{f}{e}\right)^m}{(a + b \operatorname{Sinh}\left[\frac{c}{d} + (x)\right])}, x_{\text{Symbol}}\right] := \operatorname{Dist}\left[\frac{1}{a}, \operatorname{Int}\left[\frac{(e+fx)^m \operatorname{Csch}[c+dx]^p \operatorname{Coth}[c+dx]^n}{(a+b \operatorname{Sinh}[c+dx])}, x\right] - \operatorname{Dist}\left[\frac{b}{a}, \operatorname{Int}\left[\frac{(e+fx)^m \operatorname{Csch}[c+dx]^p \operatorname{Coth}[c+dx]^n}{(a+b \operatorname{Sinh}[c+dx])}, x\right]\right]$



\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5452

Int[Coth[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*Csch[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> -Simp[((c + d\*x)^m\*Csch[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Csch[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

### Rule 5569

Int[(Coth[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] :> Dist[1/a, Int[(e + f\*x)^m\*Coth[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Cosh[c + d\*x]\*Coth[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 3716

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)], x\_Symbol] :> -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x))/(E^(2\*I\*k\*Pi)\*(1 + E^(2\*(-I\*e) + f\*fz\*x)))/E^(2\*I\*k\*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_)))^(n\_.), x\_Symbol] :> Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_.) + (b\_.)\*((F\_)^(e\_.)\*((c\_.) + (d\_.)\*(x\_)))^(n\_.)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_)^(m_.)))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(e + fx) \coth(c + dx) \operatorname{csch}(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\int (e + fx) \coth(c + dx) \operatorname{csch}(c + dx) dx}{a} - \frac{b \int \frac{(e + fx) \coth(c + dx)}{a + b \sinh(c + dx)} dx}{a} \\ &= -\frac{(e + fx) \operatorname{csch}(c + dx)}{ad} - \frac{b \int (e + fx) \coth(c + dx) dx}{a^2} + \frac{b^2 \int \frac{(e + fx) \cosh(c + dx)}{a + b \sinh(c + dx)} dx}{a^2} \\ &= -\frac{f \tanh^{-1}(\cosh(c + dx))}{ad^2} - \frac{(e + fx) \operatorname{csch}(c + dx)}{ad} + \frac{(2b) \int \frac{e^{2(c + dx)}(e + fx)}{1 - e^{2(c + dx)}} dx}{a^2} + \dots \\ &= -\frac{f \tanh^{-1}(\cosh(c + dx))}{ad^2} - \frac{(e + fx) \operatorname{csch}(c + dx)}{ad} + \frac{b(e + fx) \log\left(1 + \frac{be^{c + dx}}{a - \sqrt{a^2 + b^2}}\right)}{a^2 d} \\ &= -\frac{f \tanh^{-1}(\cosh(c + dx))}{ad^2} - \frac{(e + fx) \operatorname{csch}(c + dx)}{ad} + \frac{b(e + fx) \log\left(1 + \frac{be^{c + dx}}{a - \sqrt{a^2 + b^2}}\right)}{a^2 d} \\ &= -\frac{f \tanh^{-1}(\cosh(c + dx))}{ad^2} - \frac{(e + fx) \operatorname{csch}(c + dx)}{ad} + \frac{b(e + fx) \log\left(1 + \frac{be^{c + dx}}{a - \sqrt{a^2 + b^2}}\right)}{a^2 d} \end{aligned}$$

**Mathematica [A]** time = 1.89411, size = 416, normalized size = 1.71

$$2bf \operatorname{PolyLog}\left(2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a}\right) + 2bf \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right) + bf \operatorname{PolyLog}\left(2, e^{-2(c+dx)}\right) + 2bcf \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right) + 2bcf \log\left(\frac{be^{c+dx}}{a+\sqrt{a^2+b^2}} + 1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Coth[c + d*x]*Csch[c + d*x])/(a + b*Sinh[c + d*x]), x]
```

```
[Out] (-2*b*c^2*f - 4*b*c*d*f*x - 2*b*d^2*f*x^2 - a*d*e*Coth[(c + d*x)/2] - a*d*f*x*Coth[(c + d*x)/2] - 2*b*c*f*Log[1 - E^(-2*(c + d*x))] - 2*b*d*f*x*Log[1 - E^(-2*(c + d*x))] + 2*b*c*f*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*b*d*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 2*b*c*f*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + 2*b*d*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - 2*b*d*e*Log[Sinh[c + d*x]] + 2*b*c*f*Log[Sinh[c + d*x]] + 2*b*d*e*Log[a + b*Sinh[c + d*x]] - 2*b*c*f*Log[a + b*Sinh[c + d*x]] + 2*a*f*Log[Tanh[(c + d*x)/2]] + b*f*PolyLog[2, E^(-2*(c + d*x))] + 2*b*f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 2*b*f*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] + a*d*e*Tanh[(c + d*x)/2] + a*d*f*x*Tanh[(c + d*x)/2]/(2*a^2*d^2)
```

**Maple [B]** time = 0.138, size = 528, normalized size = 2.2

$$-2 \frac{(fx + e)e^{dx+c}}{da(e^{2dx+2c} - 1)} + \frac{bfc \ln(e^{dx+c} - 1)}{a^2 d^2} - \frac{bfc \ln(be^{2dx+2c} + 2ae^{dx+c} - b)}{a^2 d^2} - \frac{f \ln(e^{dx+c} + 1)}{ad^2} + \frac{f \ln(e^{dx+c} - 1)}{ad^2} + \frac{bfd}{ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)), x)
```

```
[Out] -2/d*(f*x+e)/a*exp(d*x+c)/(exp(2*d*x+2*c)-1)+1/a^2/d^2*b*f*c*ln(exp(d*x+c)-1)-1/a^2/d^2*b*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d^2/a*f*ln(exp(d*x+c)+1)+1/d^2/a*f*ln(exp(d*x+c)-1)+1/a^2/d^2*b*f*dilog(exp(d*x+c))+1/a^2/d^2*b*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))))+1/a^2/d^2*b*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))))-1/a^2/d^2*b*f*dilog(exp(d*x+c)+1)-1/a^2/d*b*e*ln(exp(d*x+c)-1)+1/a^2/d*b*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/a^2/d*b*e*ln(exp(d*x+c)+1)+1/a^2/d*b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))))*x+1/a^2/d^2*b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))))*c+1/a^2/d*b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))))*x+1/a^2/d^2*b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))))*c-1/a^2/d*b*f*ln(exp(d*x+c)+1)*x
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left( 2bd \int \frac{x}{2(a^2 de^{(dx+c)} + a^2 d)} dx - 2bd \int \frac{x}{2(a^2 de^{(dx+c)} - a^2 d)} dx + a \left( \frac{dx+c}{a^2 d^2} - \frac{\log(e^{(dx+c)} + 1)}{a^2 d^2} \right) - a \left( \frac{dx+c}{a^2 d^2} - \frac{\log(e^{(dx+c)} - 1)}{a^2 d^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] (2*b*d*integrate(1/2*x/(a^2*d*e^(d*x + c) + a^2*d), x) - 2*b*d*integrate(1/2*x/(a^2*d*e^(d*x + c) - a^2*d), x) + a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) + 1)/(a^2*d^2)) - a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2*d^2)) - 2*x*e^(d*x + c)/(a*d*e^(2*d*x + 2*c) - a*d) - 2*integrate((a*b*x*e^(d*x + c) - b^2*x)/(a^2*b*e^(2*d*x + 2*c) + 2*a^3*e^(d*x + c) - a^2*b), x)*f + e*(2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) + b*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^2*d) - b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d))
```

**Fricas [B]** time = 2.40876, size = 3152, normalized size = 12.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*(a*d*f*x + a*d*e)*cosh(d*x + c) - (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) + b*f*sinh(d*x + c)^2 - b*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) + b*f*sinh(d*x + c)^2 - b*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) + b*f*sinh(d*x + c)^2 - b*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) + (b*f*cosh(d*x + c)^2 + 2*b*f*cosh(d*x + c)*sinh(d*x + c) + b*f*sinh(d*x + c)^2 - b*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) + (b*d*e - b*c*f - (b*d*e - b*c*f)*cosh(d*x + c)^2 - 2*(b*d*e - b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*e - b*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*d*e - b*c*f - (b*d*e - b*c*f)*cosh(d*x + c)^2 - 2*(b*d*e - b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*e - b*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*cosh(d*x + c)^2 - 2*(b*d*f*x + b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f*x + b*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*cosh(d*x + c)^2 - 2*(b*d*f*x + b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f*x + b*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*cosh(d*x + c)^2 - 2*(b*d*f*x + b*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f*x + b*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b)
```

```

+ c))*sqrt((a^2 + b^2)/b^2) - b)/b) - (b*d*f*x + b*d*e - (b*d*f*x + b*d*e +
a*f)*cosh(d*x + c)^2 - 2*(b*d*f*x + b*d*e + a*f)*cosh(d*x + c)*sinh(d*x +
c) - (b*d*f*x + b*d*e + a*f)*sinh(d*x + c)^2 + a*f)*log(cosh(d*x + c) + sin
h(d*x + c) + 1) - (b*d*e - (b*d*e - (b*c + a)*f)*cosh(d*x + c)^2 - 2*(b*d*e
- (b*c + a)*f)*cosh(d*x + c)*sinh(d*x + c) - (b*d*e - (b*c + a)*f)*sinh(d*
x + c)^2 - (b*c + a)*f)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - (b*d*f*x +
b*c*f - (b*d*f*x + b*c*f)*cosh(d*x + c)^2 - 2*(b*d*f*x + b*c*f)*cosh(d*x +
c)*sinh(d*x + c) - (b*d*f*x + b*c*f)*sinh(d*x + c)^2)*log(-cosh(d*x + c) -
sinh(d*x + c) + 1) + 2*(a*d*f*x + a*d*e)*sinh(d*x + c))/(a^2*d^2*cosh(d*x
+ c)^2 + 2*a^2*d^2*cosh(d*x + c)*sinh(d*x + c) + a^2*d^2*sinh(d*x + c)^2 -
a^2*d^2)

```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \coth(dx + c) \operatorname{csch}(dx + c)}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] integrate((f*x + e)*coth(d*x + c)*csch(d*x + c)/(b*sinh(d*x + c) + a), x)
```

$$3.452 \quad \int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=50

$$-\frac{b \log(\sinh(c+dx))}{a^2d} + \frac{b \log(a+b\sinh(c+dx))}{a^2d} - \frac{\operatorname{csch}(c+dx)}{ad}$$

[Out]  $-(\operatorname{Csch}[c+d*x]/(a*d)) - (b*\operatorname{Log}[\operatorname{Sinh}[c+d*x]])/(a^2*d) + (b*\operatorname{Log}[a+b*\operatorname{Sinh}[c+d*x]])/(a^2*d)$

**Rubi [A]** time = 0.0749684, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$ , Rules used = {2833, 12, 44}

$$-\frac{b \log(\sinh(c+dx))}{a^2d} + \frac{b \log(a+b\sinh(c+dx))}{a^2d} - \frac{\operatorname{csch}(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Coth}[c+d*x]*\operatorname{Csch}[c+d*x])/(a+b*\operatorname{Sinh}[c+d*x]),x]$

[Out]  $-(\operatorname{Csch}[c+d*x]/(a*d)) - (b*\operatorname{Log}[\operatorname{Sinh}[c+d*x]])/(a^2*d) + (b*\operatorname{Log}[a+b*\operatorname{Sinh}[c+d*x]])/(a^2*d)$

### Rule 2833

$\operatorname{Int}[\cos[(e_.) + (f_.)*(x_.)]*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[1/(b*f), \operatorname{Subst}[\operatorname{Int}[(a+x)^m*(c+(d*x)/b)^n, x], x, b*\operatorname{Sin}[e+f*x]], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, m, n\}, x]$

### Rule 12

$\operatorname{Int}[(a_)*(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{MatchQ}[u, (b_)*(v_)] /; \operatorname{FreeQ}[b, x]$

### Rule 44

$\operatorname{Int}[(a_)+(b_.)*(x_.))^{(m_.)}*((c_.)+(d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a+b*x)^m*(c+d*x)^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{ILtQ}[m, 0] \ \&\& \ \operatorname{IntegerQ}[n] \ \&\& \ !(\operatorname{IGtQ}[n, 0] \ \&\& \ \operatorname{LtQ}[m$

+ n + 2, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{b^2}{x^2(a+x)} dx, x, b\sinh(c+dx)\right)}{bd} \\
 &= \frac{b \operatorname{Subst}\left(\int \frac{1}{x^2(a+x)} dx, x, b\sinh(c+dx)\right)}{d} \\
 &= \frac{b \operatorname{Subst}\left(\int \left(\frac{1}{ax^2} - \frac{1}{a^2x} + \frac{1}{a^2(a+x)}\right) dx, x, b\sinh(c+dx)\right)}{d} \\
 &= -\frac{\operatorname{csch}(c+dx)}{ad} - \frac{b \log(\sinh(c+dx))}{a^2d} + \frac{b \log(a+b\sinh(c+dx))}{a^2d}
 \end{aligned}$$

**Mathematica [A]** time = 0.0414655, size = 50, normalized size = 1.

$$-\frac{b \log(\sinh(c+dx))}{a^2d} + \frac{b \log(a+b\sinh(c+dx))}{a^2d} - \frac{\operatorname{csch}(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Integrate[(Coth[c + d\*x]\*Csch[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] -(Csch[c + d\*x]/(a\*d)) - (b\*Log[Sinh[c + d\*x]]/(a^2\*d) + (b\*Log[a + b\*Sinh[c + d\*x]]/(a^2\*d))

**Maple [A]** time = 0.001, size = 35, normalized size = 0.7

$$-\frac{\operatorname{csch}(dx+c)}{da} + \frac{b \ln(\operatorname{acsch}(dx+c)+b)}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d\*x+c)\*csch(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] -csch(d\*x+c)/a/d+1/d\*b/a^2\*ln(a\*csch(d\*x+c)+b)

**Maxima [B]** time = 1.17884, size = 149, normalized size = 2.98

$$\frac{2e^{(-dx-c)}}{(ae^{(-2dx-2c)} - a)d} + \frac{b \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{a^2d} - \frac{b \log(e^{(-dx-c)} + 1)}{a^2d} - \frac{b \log(e^{(-dx-c)} - 1)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)\*csch(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $2e^{(-d*x - c)} / ((a*e^{(-2*d*x - 2*c)} - a)*d) + b*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b) / (a^2*d) - b*\log(e^{(-d*x - c)} + 1) / (a^2*d) - b*\log(e^{(-d*x - c)} - 1) / (a^2*d)$

**Fricas [B]** time = 2.1464, size = 554, normalized size = 11.08

$$\frac{2a \cosh(dx+c) - (b \cosh(dx+c)^2 + 2b \cosh(dx+c) \sinh(dx+c) + b \sinh(dx+c)^2 - b) \log\left(\frac{2(b \sinh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right) + a^2d \cosh(dx+c)^2 + 2a^2d \cosh(dx+c) \sinh(dx+c)}{a^2d \cosh(dx+c)^2 + 2a^2d \cosh(dx+c) \sinh(dx+c) + a^2d \sinh(dx+c)^2 - a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)\*csch(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $-(2*a*\cosh(d*x + c) - (b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 - b)*\log(2*(b*\sinh(d*x + c) + a)/(\cosh(d*x + c) - \sinh(d*x + c))) + (b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 - b)*\log(2*\sinh(d*x + c)/(\cosh(d*x + c) - \sinh(d*x + c))) + 2*a*\sinh(d*x + c)/(a^2*d*\cosh(d*x + c)^2 + 2*a^2*d*\cosh(d*x + c)*\sinh(d*x + c) + a^2*d*\sinh(d*x + c)^2 - a^2*d)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(c+dx) \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)\*csch(d\*x+c)/(a+b\*sinh(d\*x+c)),x)



[Out] Integral(coth(c + d\*x)\*csch(c + d\*x)/(a + b\*sinh(c + d\*x)), x)

**Giac [A]** time = 1.33678, size = 132, normalized size = 2.64

$$\frac{\frac{b \log(e^{(dx+c)+1})}{a^2} - \frac{b \log(|be^{(2dx+2c)+2ae^{(dx+c)}-b|)}{a^2} + \frac{b \log(|e^{(dx+c)}-1|)}{a^2} + \frac{2e^{(dx+c)}}{a(e^{(dx+c)+1})(e^{(dx+c)}-1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)\*csch(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out]  $-(b \cdot \log(e^{(d \cdot x + c)} + 1)/a^2 - b \cdot \log(\text{abs}(b \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + 2 \cdot a \cdot e^{(d \cdot x + c)} - b))/a^2 + b \cdot \log(\text{abs}(e^{(d \cdot x + c)} - 1))/a^2 + 2 \cdot e^{(d \cdot x + c)}/(a \cdot (e^{(d \cdot x + c)} + 1) \cdot (e^{(d \cdot x + c)} - 1)))/d$

$$3.453 \quad \int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable}\left(\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}, x\right)$$

[Out] Unintegrable[(Coth[c + d\*x]\*Csch[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0663352, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Coth[c + d\*x]\*Csch[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int] [(Coth[c + d\*x]\*Csch[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.001, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Coth[c + d\*x]\*Csch[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] \$Aborted

**Maple [A]** time = 0.916, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c) \operatorname{coth}(dx+c)}{(fx+e)(a+b \sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)`

[Out] `int(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{2e^{(dx+c)}}{adf x + ade - (adf x e^{(2c)} + ade e^{(2c)})e^{(2dx)}} - 2 \int -\frac{bdf x + bde + af}{2(a^2 d f^2 x^2 + 2 a^2 d e f x + a^2 d e^2 - (a^2 d f^2 x^2 e^c + 2 a^2 d e f x e^c + a^2 d e^2 e^c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `2*e^(d*x + c)/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e*e^(2*c))*e^(2*d*x)) - 2*integrate(-1/2*(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) + 2*integrate(1/2*(b*d*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) - 2*integrate(-(a*b*e^(d*x + c) - b^2)/(a^2*b*f*x + a^2*b*e - (a^2*b*f*x*e^(2*c) + a^2*b*e*e^(2*c))*e^(2*d*x) - 2*(a^3*f*x*e^c + a^3*e^c)*e^(d*x)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{coth}(dx+c) \operatorname{csch}(dx+c)}{afx+ae+(bfx+be) \sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="f
ricas")
```

```
[Out] integral(coth(d*x + c)*csch(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x
+ c)), x)
```

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(c + dx) \operatorname{csch}(c + dx)}{(a + b \sinh(c + dx))(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)*csch(c + d*x)/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*csch(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="g
iac")
```

```
[Out] Timed out
```

$$3.454 \quad \int \frac{(e+fx)^3 \coth^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=721

$$\frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3} + \frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^3} + \frac{3f\sqrt{a^2+b^2}(e+fx)^2}{a^2d^3}$$

```
[Out] -((e + f*x)^3/(a*d)) + (2*b*(e + f*x)^3*ArcTanh[E^(c + d*x)]/(a^2*d) - ((e + f*x)^3*Coth[c + d*x])/(a*d) + (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(a^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(a^2*d) + (3*f*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -E^(c + d*x)]/(a^2*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)]/(a^2*d^2) + (3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^2) - (3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^2) + (3*f^2*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)]/(a^2*d^3) + (6*b*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)]/(a^2*d^3) - (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^3) + (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^3) - (3*f^3*PolyLog[3, E^(2*(c + d*x))])/(2*a*d^4) + (6*b*f^3*PolyLog[4, -E^(c + d*x)]/(a^2*d^4) - (6*b*f^3*PolyLog[4, E^(c + d*x)]/(a^2*d^4) + (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^4) - (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^4)
```

**Rubi [A]** time = 1.63628, antiderivative size = 721, normalized size of antiderivative = 1., number of steps used = 41, number of rules used = 17, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.607$ , Rules used = {5569, 3720, 3716, 2190, 2531, 2282, 6589, 32, 5585, 5450, 3296, 2637, 4182, 6609, 5565, 3322, 2264}

$$\frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3} + \frac{6f^2\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^3} + \frac{3f\sqrt{a^2+b^2}(e+fx)^2}{a^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]), x]
```

```
[Out] -((e + f*x)^3/(a*d)) + (2*b*(e + f*x)^3*ArcTanh[E^(c + d*x)]/(a^2*d) - ((e + f*x)^3*Coth[c + d*x])/(a*d) + (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(a^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(a^2*d) + (3*f*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -E^(c + d*x)]/(a^2*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, E^(c + d*x)]/(a^2*d^2) + (3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^2) - (3*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^2) + (3*f^2*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a*d^3) - (6*b*f^2*(e + f*x)*PolyLog[3, -E^(c + d*x)]/(a^2*d^3) + (6*b*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)]/(a^2*d^3) - (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^3) + (6*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^3) - (3*f^3*PolyLog[3, E^(2*(c + d*x))])/(2*a*d^4) + (6*b*f^3*PolyLog[4, -E^(c + d*x)]/(a^2*d^4) - (6*b*f^3*PolyLog[4, E^(c + d*x)]/(a^2*d^4) + (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^4) - (6*Sqrt[a^2 + b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^4)
```

$$\begin{aligned} & (c + d*x)/(a - \text{Sqrt}[a^2 + b^2])]/(a^2*d) - (\text{Sqrt}[a^2 + b^2]*(e + f*x)^3 * \text{Log}[1 + (b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b^2])])/(a^2*d) + (3*f*(e + f*x)^2 * \text{Log}[1 - E^(2*(c + d*x))])/(a*d^2) + (3*b*f*(e + f*x)^2 * \text{PolyLog}[2, -E^(c + d*x)])/(a^2*d^2) - (3*b*f*(e + f*x)^2 * \text{PolyLog}[2, E^(c + d*x)])/(a^2*d^2) + (3 * \text{Sqrt}[a^2 + b^2]*f*(e + f*x)^2 * \text{PolyLog}[2, -((b*E^(c + d*x))/(a - \text{Sqrt}[a^2 + b^2]))])/(a^2*d^2) - (3*\text{Sqrt}[a^2 + b^2]*f*(e + f*x)^2 * \text{PolyLog}[2, -((b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b^2]))])/(a^2*d^2) + (3*f^2*(e + f*x)*\text{PolyLog}[2, E^(2*(c + d*x))])/(a*d^3) - (6*b*f^2*(e + f*x)*\text{PolyLog}[3, -E^(c + d*x)])/(a^2*d^3) + (6*b*f^2*(e + f*x)*\text{PolyLog}[3, E^(c + d*x)])/(a^2*d^3) - (6*\text{Sqrt}[a^2 + b^2]*f^2*(e + f*x)*\text{PolyLog}[3, -((b*E^(c + d*x))/(a - \text{Sqrt}[a^2 + b^2]))])/(a^2*d^3) + (6*\text{Sqrt}[a^2 + b^2]*f^2*(e + f*x)*\text{PolyLog}[3, -((b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b^2]))])/(a^2*d^3) - (3*f^3*\text{PolyLog}[3, E^(2*(c + d*x))])/(2*a*d^4) + (6*b*f^3*\text{PolyLog}[4, -E^(c + d*x)])/(a^2*d^4) - (6*b*f^3*\text{PolyLog}[4, E^(c + d*x)])/(a^2*d^4) + (6*\text{Sqrt}[a^2 + b^2]*f^3*\text{PolyLog}[4, -((b*E^(c + d*x))/(a - \text{Sqrt}[a^2 + b^2]))])/(a^2*d^4) - (6*\text{Sqrt}[a^2 + b^2]*f^3*\text{PolyLog}[4, -((b*E^(c + d*x))/(a + \text{Sqrt}[a^2 + b^2]))])/(a^2*d^4) \end{aligned}$$

### Rule 5569

```
Int[((Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Coth[c + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

### Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Dist[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

### Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))]/(E^(2*I*k*Pi)*(1 + E^(2*(-I*e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp
```

```

[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist
[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

### Rule 2531

```

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]

```

### Rule 2282

```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

### Rule 6589

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]

```

### Rule 32

```

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

```

### Rule 5585

```

Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
(f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
0] && IGtQ[p, 0]

```

### Rule 5450

```

Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] := Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]

```

/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[  
((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[  
e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /;  
FreeQ[{c, d}, x]

### Rule 4182

Int[csc[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x  
\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)])/(f\*fz\*I), x]  
+ (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)]  
, x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) +  
f\*fz\*x)], x], x)) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.  
)\*(x\_)))^(p\_.)], x\_Symbol] := Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a  
+ b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)  
(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c,  
d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 5565

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_) + (b\_.  
) \* Sinh[(c\_.) + (d\_.)\*(x\_)])], x\_Symbol] := -Dist[a/b^2, Int[(e + f\*x)^m\*Cosh  
[c + d\*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)  
)\*Sinh[c + d\*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f\*x)^m\*Cosh[c + d  
\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]  
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

### Rule 3322

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_) + (b\_.)\*sin[(e\_.) + (Complex[0, fz\_])\*(  
f\_.)\*(x\_)])], x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(-  
(I\*b) + 2\*a\*E^(-(I\*e) + f\*fz\*x) + I\*b\*E^(2\*(-(I\*e) + f\*fz\*x))), x], x] /; F  
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]



Rule 2264

Int[((F\_)^(u\_)\*((f\_.) + (g\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_) + (c\_.) \* (F\_)^(v\_)), x\_Symbol] :> With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u]/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[((f + g\*x)^m\*F^u)/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \coth^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \coth^2(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^3 \cosh(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= -\frac{(e+fx)^3 \coth(c+dx)}{ad} + \frac{\int (e+fx)^3 dx}{a} - \frac{b \int (e+fx)^3 \cosh(c+dx) \coth(c+dx) dx}{a^2} \\
&= -\frac{(e+fx)^3}{ad} + \frac{(e+fx)^4}{4af} - \frac{(e+fx)^3 \coth(c+dx)}{ad} - \frac{\int (e+fx)^3 dx}{a} - \frac{b \int (e+fx)^3 \operatorname{csch}(c+dx) dx}{a^2} \\
&= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^2 d} - \frac{(e+fx)^3 \coth(c+dx)}{ad} + \frac{3f(e+fx)^2 \log(e^{c+dx})}{ad^2} \\
&= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^2 d} - \frac{(e+fx)^3 \coth(c+dx)}{ad} + \frac{3f(e+fx)^2 \log(e^{c+dx})}{ad^2} \\
&= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^2 d} - \frac{(e+fx)^3 \coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)}{a} \\
&= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^2 d} - \frac{(e+fx)^3 \coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)}{a} \\
&= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^2 d} - \frac{(e+fx)^3 \coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)}{a} \\
&= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^2 d} - \frac{(e+fx)^3 \coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)}{a} \\
&= -\frac{(e+fx)^3}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^2 d} - \frac{(e+fx)^3 \coth(c+dx)}{ad} + \frac{\sqrt{a^2+b^2}(e+fx)}{a}
\end{aligned}$$

**Mathematica [A]** time = 8.68733, size = 1350, normalized size = 1.87

$$\sqrt{a^2+b^2} \left( -2e^3 \tanh^{-1} \left( \frac{a+be^{c+dx}}{\sqrt{a^2+b^2}} \right) d^3 + f^3 x^3 \log \left( \frac{e^{c+dx} b}{a-\sqrt{a^2+b^2}} + 1 \right) d^3 + 3ef^2 x^2 \log \left( \frac{e^{c+dx} b}{a-\sqrt{a^2+b^2}} + 1 \right) d^3 + 3e^2 f x \log \left( \frac{e^{c+dx} b}{a-\sqrt{a^2+b^2}} + 1 \right) d^3 \right)$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)^3\*Coth[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (Sqrt[a^2 + b^2]\*(-2\*d^3\*e^3\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]] + 3\*d^3\*e^2\*f\*x\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])] + 3\*d^3\*e\*f^2\*x^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])] + d^3\*f^3\*x^3\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]) - 3\*d^3\*e^2\*f\*x\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] - 3\*d^3\*e\*f^2\*x^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])] - d^3\*f^3\*x^3\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]) + 3\*d^2\*f\*(e + f\*x)^2\*PolyLog[2, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] - 3\*d^2\*f\*(e + f\*x)^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))] - 6\*d\*e\*f^2\*PolyLog[3, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] - 6\*d\*f^3\*x\*PolyLog[3, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] + 6\*d\*e\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))] + 6\*d\*f^3\*x\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))] + 6\*f^3\*PolyLog[4, (b\*E^(c + d\*x))/(-a + Sqrt[a^2 + b^2])] - 6\*f^3\*PolyLog[4, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^2\*d^4) - (a\*d^3\*(e + f\*x)^3\*(-1 + Coth[c]) - d^2\*e^2\*(b\*d\*e - 3\*a\*f)\*(d\*x - Log[1 - Cosh[c + d\*x] - Sinh[c + d\*x]]) - 3\*d^2\*e\*f\*(b\*d\*e + 2\*a\*f)\*x\*Log[1 + Cosh[c + d\*x] - Sinh[c + d\*x]] - 3\*d^2\*f^2\*(b\*d\*e + a\*f)\*x^2\*Log[1 + Cosh[c + d\*x] - Sinh[c + d\*x]] - b\*d^3\*f^3\*x^3\*Log[1 + Cosh[c + d\*x] - Sinh[c + d\*x]] + 3\*d^2\*e\*f\*(b\*d\*e - 2\*a\*f)\*x\*Log[1 - Cosh[c + d\*x] + Sinh[c + d\*x]] + 3\*d^2\*f^2\*(b\*d\*e - a\*f)\*x^2\*Log[1 - Cosh[c + d\*x] + Sinh[c + d\*x]] + b\*d^3\*f^3\*x^3\*Log[1 - Cosh[c + d\*x] + Sinh[c + d\*x]] + d^2\*e^2\*(b\*d\*e + 3\*a\*f)\*(d\*x - Log[1 + Cosh[c + d\*x] + Sinh[c + d\*x]]) - 3\*d\*e\*f\*(b\*d\*e - 2\*a\*f)\*PolyLog[2, Cosh[c + d\*x] - Sinh[c + d\*x]] + 3\*d\*e\*f\*(b\*d\*e + 2\*a\*f)\*PolyLog[2, -Cosh[c + d\*x] + Sinh[c + d\*x]] + 6\*f^2\*(-(b\*d\*e) + a\*f)\*(d\*x\*PolyLog[2, Cosh[c + d\*x] - Sinh[c + d\*x]] + PolyLog[3, Cosh[c + d\*x] - Sinh[c + d\*x]]) + 6\*f^2\*(b\*d\*e + a\*f)\*(d\*x\*PolyLog[2, -Cosh[c + d\*x] + Sinh[c + d\*x]] + PolyLog[3, -Cosh[c + d\*x] + Sinh[c + d\*x]]) + PolyLog[3, -Cosh[c + d\*x] + Sinh[c + d\*x]]) - 3\*b\*f^3\*(d^2\*x^2\*PolyLog[2, Cosh[c + d\*x] - Sinh[c + d\*x]] + 2\*(d\*x\*PolyLog[3, Cosh[c + d\*x] - Sinh[c + d\*x]] + PolyLog[4, Cosh[c + d\*x] - Sinh[c + d\*x]])) + 3\*b\*f^3\*(d^2\*x^2\*PolyLog[2, -Cosh[c + d\*x] + Sinh[c + d\*x]] + 2\*(d\*x\*PolyLog[3, -Cosh[c + d\*x] + Sinh[c + d\*x]] + PolyLog[4, -Cosh[c + d\*x] + Sinh[c + d\*x]])))/(a^2\*d^4) + (Sech[c/2]\*Sech[c/2 + (d\*x)/2]\*(-(e^3\*Sinh[(d\*x)/2]) - 3\*e^2\*f\*x\*Sinh[(d\*x)/2] - 3\*e\*f^2\*x^2\*Sinh[(d\*x)/2] - f^3\*x^3\*Sinh[(d\*x)/2]))/(2\*a\*d) + (Csch[c/2]\*Csch[c/2 + (d\*x)/2]\*(e^3\*Sinh[(d\*x)/2] + 3\*e^2\*f\*x\*Sinh[(d\*x)/2] + 3\*e\*f^2\*x^2\*Sinh[(d\*x)/2] + f^3\*x^3\*Sinh[(d\*x)/2]))/(2\*a\*d)

**Maple [F]** time = 0.827, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 (\coth(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*coth(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*coth(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*coth(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [C]** time = 3.23768, size = 10496, normalized size = 14.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*coth(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-(2*a*d^3*e^3 - 6*a*c*d^2*e^2*f + 6*a*c^2*d*e*f^2 - 2*a*c^3*f^3 + 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + 3*a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*\cosh(d*x + c)^2 + 4*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + 3*a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c) + 2*(a*d^3*f^3*x^3 + 3*a*d^3*e*f^2*x^2 + 3*a*d^3*e^2*f*x + 3*a*c*d^2*e^2*f - 3*a*c^2*d*e*f^2 + a*c^3*f^3)*\sinh(d*x + c)^2 + 3*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f - (b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*\cosh(d*x + c)^2 - 2*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2}*dilog((a*\cosh(d*x + c)$$

$$\begin{aligned}
& + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2 - b}/b + 1) - 3*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f - (b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f) * \cosh(dx + c)^2 - 2*(b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f) * \cosh(dx + c) * \sinh(dx + c) - (b*d^2*f^3*x^2 + 2*b*d^2*e*f^2*x + b*d^2*e^2*f) * \sinh(dx + c)^2) * \sqrt{(a^2 + b^2)/b^2}) * \operatorname{dilog}((a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c))) * \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3) * \cosh(dx + c)^2 - 2*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3) * \cosh(dx + c) * \sinh(dx + c) - (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3) * \sinh(dx + c)^2) * \sqrt{(a^2 + b^2)/b^2} * \log(2*b*\cosh(dx + c) + 2*b*\sinh(dx + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3 - (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3) * \cosh(dx + c)^2 - 2*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3) * \cosh(dx + c) * \sinh(dx + c) - (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3) * \sinh(dx + c)^2) * \sqrt{(a^2 + b^2)/b^2} * \log(2*b*\cosh(dx + c) + 2*b*\sinh(dx + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3 - (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3) * \cosh(dx + c)^2 - 2*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3) * \cosh(dx + c) * \sinh(dx + c) - (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3) * \sinh(dx + c)^2) * \sqrt{(a^2 + b^2)/b^2} * \log(-(a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c))) * \sqrt{(a^2 + b^2)/b^2} - b)/b) - (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3 - (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3) * \cosh(dx + c)^2 - 2*(b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3) * \cosh(dx + c) * \sinh(dx + c) - (b*d^3*f^3*x^3 + 3*b*d^3*e*f^2*x^2 + 3*b*d^3*e^2*f*x + 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + b*c^3*f^3) * \sinh(dx + c)^2) * \sqrt{(a^2 + b^2)/b^2} * \log(-(a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c))) * \sqrt{(a^2 + b^2)/b^2} - b)/b) - 6*(b*f^3*\cosh(dx + c)^2 + 2*b*f^3*\cosh(dx + c) * \sinh(dx + c) + b*f^3*\sinh(dx + c)^2 - b*f^3) * \sqrt{(a^2 + b^2)/b^2} * \operatorname{polylog}(4, (a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c))) * \sqrt{(a^2 + b^2)/b^2})/b) + 6*(b*f^3*\cosh(dx + c)^2 + 2*b*f^3*\cosh(dx + c) * \sinh(dx + c) + b*f^3*\sinh(dx + c)^2 - b*f^3) * \sqrt{(a^2 + b^2)/b^2} * \operatorname{polylog}(4, (a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c))) * \sqrt{(a^2 + b^2)/b^2})/b) - 6*(b*d*f^3*x + b*d*e*f^2 - (b*d*f^3*x + b*d*e*f^2) * \cosh(dx + c)^2 - 2*(b*d*f^3*x + b*d*e*f^2) * \cosh(dx + c) * \sinh(dx + c) - (b*d*f^3*x + b*d*e*f^2) * \sinh(dx + c)^2) * \sqrt{(a^2 + b^2)/b^2} * \operatorname{polylog}(3, (a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c))) * \sqrt{(a^2 + b^2)/b^2})/b) + 6*(b*d*f^3*x + b*d*e*f^2 - (b*d*f^3*x + b*d*e*f^2) * \cosh(dx + c)^2 - 2*(b*d*f^3*x + b*d*e
\end{aligned}$$

$$\begin{aligned}
& *f^2) * \cosh(dx + c) * \sinh(dx + c) - (b*d*f^3*x + b*d*e*f^2) * \sinh(dx + c)^2 \\
& ) * \sqrt{(a^2 + b^2)/b^2} * \text{polylog}(3, (a * \cosh(dx + c) + a * \sinh(dx + c) - (b * \\
& \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2)/b^2})/b) - 3 * (b*d^2*f^3*x \\
& ^2 + b*d^2*e^2*f - 2*a*d*e*f^2 - (b*d^2*f^3*x^2 + b*d^2*e^2*f - 2*a*d*e*f^2 \\
& + 2*(b*d^2*e*f^2 - a*d*f^3)*x) * \cosh(dx + c)^2 - 2*(b*d^2*f^3*x^2 + b*d^2* \\
& e^2*f - 2*a*d*e*f^2 + 2*(b*d^2*e*f^2 - a*d*f^3)*x) * \cosh(dx + c) * \sinh(dx + \\
& c) - (b*d^2*f^3*x^2 + b*d^2*e^2*f - 2*a*d*e*f^2 + 2*(b*d^2*e*f^2 - a*d*f^3 \\
& ) * x) * \sinh(dx + c)^2 + 2*(b*d^2*e*f^2 - a*d*f^3)*x * \text{dilog}(\cosh(dx + c) + \text{sinh}(dx + c)) \\
& ) + 3*(b*d^2*f^3*x^2 + b*d^2*e^2*f + 2*a*d*e*f^2 - (b*d^2*f^3*x^2 + b*d^2*e^2*f + 2*a*d*e*f^2 \\
& + 2*(b*d^2*e*f^2 + a*d*f^3)*x) * \cosh(dx + c)^2 - 2*(b*d^2*f^3*x^2 + b*d^2*e^2*f + 2*a*d*e*f^2 \\
& + 2*(b*d^2*e*f^2 + a*d*f^3)*x) * \cosh(dx + c) * \sinh(dx + c) - (b*d^2*f^3*x^2 + b*d^2*e^2*f + 2*a*d*e* \\
& f^2 + 2*(b*d^2*e*f^2 + a*d*f^3)*x) * \sinh(dx + c)^2 + 2*(b*d^2*e*f^2 + a*d*f^ \\
& ^3)*x * \text{dilog}(-\cosh(dx + c) - \sinh(dx + c)) + (b*d^3*f^3*x^3 + b*d^3*e^3 + \\
& 3*a*d^2*e^2*f + 3*(b*d^3*e*f^2 + a*d^2*f^3)*x^2 - (b*d^3*f^3*x^3 + b*d^3*e^ \\
& ^3 + 3*a*d^2*e^2*f + 3*(b*d^3*e*f^2 + a*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f + 2*a \\
& *d^2*e*f^2)*x) * \cosh(dx + c)^2 - 2*(b*d^3*f^3*x^3 + b*d^3*e^3 + 3*a*d^2*e^2 \\
& *f + 3*(b*d^3*e*f^2 + a*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f + 2*a*d^2*e*f^2)*x) * \c \\
& osh(dx + c) * \sinh(dx + c) - (b*d^3*f^3*x^3 + b*d^3*e^3 + 3*a*d^2*e^2*f + 3 \\
& *(b*d^3*e*f^2 + a*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f + 2*a*d^2*e*f^2)*x) * \sinh(dx \\
& + c)^2 + 3*(b*d^3*e^2*f + 2*a*d^2*e*f^2)*x * \log(\cosh(dx + c) + \sinh(dx \\
& + c) + 1) - (b*d^3*e^3 - 3*(b*c + a)*d^2*e^2*f + 3*(b*c^2 + 2*a*c)*d*e*f^2 \\
& - (b*c^3 + 3*a*c^2)*f^3 - (b*d^3*e^3 - 3*(b*c + a)*d^2*e^2*f + 3*(b*c^2 + 2 \\
& *a*c)*d*e*f^2 - (b*c^3 + 3*a*c^2)*f^3) * \cosh(dx + c)^2 - 2*(b*d^3*e^3 - 3*( \\
& b*c + a)*d^2*e^2*f + 3*(b*c^2 + 2*a*c)*d*e*f^2 - (b*c^3 + 3*a*c^2)*f^3) * \cos \\
& h(dx + c) * \sinh(dx + c) - (b*d^3*e^3 - 3*(b*c + a)*d^2*e^2*f + 3*(b*c^2 + \\
& 2*a*c)*d*e*f^2 - (b*c^3 + 3*a*c^2)*f^3) * \sinh(dx + c)^2 * \log(\cosh(dx + c) \\
& + \sinh(dx + c) - 1) - (b*d^3*f^3*x^3 + 3*b*c*d^2*e^2*f - 3*(b*c^2 + 2*a*c) \\
& *d*e*f^2 + (b*c^3 + 3*a*c^2)*f^3 + 3*(b*d^3*e*f^2 - a*d^2*f^3)*x^2 - (b*d^3 \\
& *f^3*x^3 + 3*b*c*d^2*e^2*f - 3*(b*c^2 + 2*a*c)*d*e*f^2 + (b*c^3 + 3*a*c^2)* \\
& f^3 + 3*(b*d^3*e*f^2 - a*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f - 2*a*d^2*e*f^2)*x) * \\
& \cosh(dx + c)^2 - 2*(b*d^3*f^3*x^3 + 3*b*c*d^2*e^2*f - 3*(b*c^2 + 2*a*c)*d* \\
& e*f^2 + (b*c^3 + 3*a*c^2)*f^3 + 3*(b*d^3*e*f^2 - a*d^2*f^3)*x^2 + 3*(b*d^3* \\
& e^2*f - 2*a*d^2*e*f^2)*x) * \cosh(dx + c) * \sinh(dx + c) - (b*d^3*f^3*x^3 + 3* \\
& b*c*d^2*e^2*f - 3*(b*c^2 + 2*a*c)*d*e*f^2 + (b*c^3 + 3*a*c^2)*f^3 + 3*(b*d^ \\
& 3*e*f^2 - a*d^2*f^3)*x^2 + 3*(b*d^3*e^2*f - 2*a*d^2*e*f^2)*x) * \sinh(dx + c) \\
& ^2 + 3*(b*d^3*e^2*f - 2*a*d^2*e*f^2)*x * \log(-\cosh(dx + c) - \sinh(dx + c) \\
& + 1) + 6*(b*f^3 * \cosh(dx + c)^2 + 2*b*f^3 * \cosh(dx + c) * \sinh(dx + c) + b*f \\
& ^3 * \sinh(dx + c)^2 - b*f^3) * \text{polylog}(4, \cosh(dx + c) + \sinh(dx + c)) - 6*( \\
& b*f^3 * \cosh(dx + c)^2 + 2*b*f^3 * \cosh(dx + c) * \sinh(dx + c) + b*f^3 * \sinh(dx \\
& + c)^2 - b*f^3) * \text{polylog}(4, -\cosh(dx + c) - \sinh(dx + c)) + 6*(b*d*f^3*x \\
& + b*d*e*f^2 - a*f^3 - (b*d*f^3*x + b*d*e*f^2 - a*f^3) * \cosh(dx + c)^2 - 2* \\
& (b*d*f^3*x + b*d*e*f^2 - a*f^3) * \cosh(dx + c) * \sinh(dx + c) - (b*d*f^3*x + \\
& b*d*e*f^2 - a*f^3) * \sinh(dx + c)^2) * \text{polylog}(3, \cosh(dx + c) + \sinh(dx + c \\
& )) - 6*(b*d*f^3*x + b*d*e*f^2 + a*f^3 - (b*d*f^3*x + b*d*e*f^2 + a*f^3) * \cos
\end{aligned}$$

$$\frac{h(dx + c)^2 - 2*(b*d*f^3*x + b*d*e*f^2 + a*f^3)*\cosh(dx + c)*\sinh(dx + c) - (b*d*f^3*x + b*d*e*f^2 + a*f^3)*\sinh(dx + c)^2*\text{polylog}(3, -\cosh(dx + c) - \sinh(dx + c))}{a^2*d^4*\cosh(dx + c)^2 + 2*a^2*d^4*\cosh(dx + c)*\sinh(dx + c) + a^2*d^4*\sinh(dx + c)^2 - a^2*d^4}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*coth(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*coth(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.455 \quad \int \frac{(e+fx)^2 \coth^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=517

$$\frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2} - \frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2} - \frac{2f^2\sqrt{a^2+b^2}\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3}$$

```
[Out] -((e + f*x)^2/(a*d)) + (2*b*(e + f*x)^2*ArcTanh[E^(c + d*x)]/(a^2*d) - ((e + f*x)^2*Coth[c + d*x])/(a*d) + (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(a^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(a^2*d) + (2*f*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d^2) + (2*b*f*(e + f*x)*PolyLog[2, -E^(c + d*x)]/(a^2*d^2) - (2*b*f*(e + f*x)*PolyLog[2, E^(c + d*x)]/(a^2*d^2) + (2*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^2) - (2*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^2) + (f^2*PolyLog[2, E^(2*(c + d*x))])/(a*d^3) - (2*b*f^2*PolyLog[3, -E^(c + d*x)]/(a^2*d^3) + (2*b*f^2*PolyLog[3, E^(c + d*x)]/(a^2*d^3) - (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^3) + (2*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^3)
```

**Rubi [A]** time = 1.29593, antiderivative size = 517, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 18, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$ , Rules used = {5569, 3720, 3716, 2190, 2279, 2391, 32, 5585, 5450, 3296, 2638, 4182, 2531, 2282, 6589, 5565, 3322, 2264}

$$\frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2} - \frac{2f\sqrt{a^2+b^2}(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2} - \frac{2f^2\sqrt{a^2+b^2}\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]), x]
```

```
[Out] -((e + f*x)^2/(a*d)) + (2*b*(e + f*x)^2*ArcTanh[E^(c + d*x)]/(a^2*d) - ((e + f*x)^2*Coth[c + d*x])/(a*d) + (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(a^2*d) - (Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(a^2*d) + (2*f*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d^2) + (2*b*f*(e + f*x)*PolyLog[2, -E^(c + d*x)]/(a^2*d^2) - (2*b*f*(e + f*x)*PolyLog[2, E^(c + d*x)]/(a^2*d^2) + (2*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*d^2) - (2*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*d^2)
```

$$\begin{aligned} &)/(a^2*d^2) - (2*\text{Sqrt}[a^2 + b^2]*f*(e + f*x)*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))]/(a^2*d^2) + (f^2*\text{PolyLog}[2, E^{(2*(c + d*x))}]/(a*d^3) - (2*b*f^2*\text{PolyLog}[3, -E^{(c + d*x)}]/(a^2*d^3) + (2*b*f^2*\text{PolyLog}[3, E^{(c + d*x)}]/(a^2*d^3) - (2*\text{Sqrt}[a^2 + b^2]*f^2*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))]/(a^2*d^3) + (2*\text{Sqrt}[a^2 + b^2]*f^2*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))]/(a^2*d^3) \end{aligned}$$
Rule 5569

$$\text{Int}[(\text{Coth}[(c_.) + (d_.)*(x_.)]^{(n_.)}*((e_.) + (f_.)*(x_.))^{(m_.)})/((a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_.)]), x\_Symbol] \rightarrow \text{Dist}[1/a, \text{Int}[(e + f*x)^m*\text{Coth}[c + d*x]^n, x], x] - \text{Dist}[b/a, \text{Int}[(e + f*x)^m*\text{Cosh}[c + d*x]*\text{Coth}[c + d*x]^{(n - 1)}]/(a + b*\text{Sinh}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[n, 0]$$
Rule 3720

$$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*((b_.)*\tan[(e_.) + (f_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[(b*(c + d*x)^m*(b*\tan[e + f*x])^{(n - 1)})/(f*(n - 1)), x] + (-\text{Dist}[(b*d*m)/(f*(n - 1)), \text{Int}[(c + d*x)^{(m - 1)}*(b*\tan[e + f*x])^{(n - 1)}, x], x] - \text{Dist}[b^2, \text{Int}[(c + d*x)^m*(b*\tan[e + f*x])^{(n - 2)}, x], x]) /; \text{FreeQ}\{b, c, d, e, f\}, x\} \&\& \text{GtQ}[n, 1] \&\& \text{GtQ}[m, 0]$$
Rule 3716

$$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\tan[(e_.) + \text{Pi}*(k_.) + (\text{Complex}[0, fz\_])*(f_.)*(x_.)], x\_Symbol] \rightarrow -\text{Simp}[(I*(c + d*x)^{(m + 1)})/(d*(m + 1)), x] + \text{Dist}[2*I, \text{Int}[(c + d*x)^m*E^{(2*(-I*e) + f*fz*x))}/(E^{(2*I*k*Pi)}*(1 + E^{(2*(-I*e) + f*fz*x))}/E^{(2*I*k*Pi)}), x], x] /; \text{FreeQ}\{c, d, e, f, fz\}, x\} \&\& \text{IntegerQ}[4*k] \&\& \text{IGtQ}[m, 0]$$
Rule 2190

$$\text{Int}[(F_.)^{((g_.)*((e_.) + (f_.)*(x_.)))^{(n_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}}/((a_.) + (b_.)*((F_.)^{((g_.)*((e_.) + (f_.)*(x_.)))^{(n_.)}}), x\_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x))})^n)/a]/(b*f*g^n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g^n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + (b*(F^{(g*(e + f*x))})^n)/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \&\& \text{IGtQ}[m, 0]$$
Rule 2279

$$\text{Int}[\text{Log}[(a_.) + (b_.)*((F_.)^{((e_.)*((c_.) + (d_.)*(x_.)))^{(n_.)}})], x\_Symbol] \rightarrow \text{Dist}[1/(d*e^n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \&\& \text{GtQ}[a, 0]$$



Rule 2391

Int[Log[(c\_.)\*(d\_) + (e\_.)\*(x\_)^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 5585

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]^(p\_.)\*Coth[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Cosh[c + d\*x]^p\*Coth[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(p + 1)\*Coth[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 5450

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*Coth[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[(c + d\*x)^m\*Cosh[a + b\*x]^n\*Coth[a + b\*x]^(p - 2), x] + Int[(c + d\*x)^m\*Cosh[a + b\*x]^(n - 2)\*Coth[a + b\*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[((c + d\*x)^m\*cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4182

Int[csc[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)])/ (f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5565

```
Int[((Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)
)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] := -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_]
*(f_.)*(x_))], x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
```



```

g[1 + E^(-c - d*x)] - b*d^2*f^2*x^2*Log[1 + E^(-c - d*x)] - d*e*(b*d*e - 2*
a*f)*(d*x - Log[1 - E^(c + d*x)]) + d*e*(b*d*e + 2*a*f)*(d*x - Log[1 + E^(c
+ d*x)]) + 2*f*(b*d*e + a*f)*PolyLog[2, -E^(-c - d*x)] + 2*f*(-(b*d*e) + a
*f)*PolyLog[2, E^(-c - d*x)] + 2*b*f^2*(d*x*PolyLog[2, -E^(-c - d*x)] + Pol
yLog[3, -E^(-c - d*x)]) - 2*b*f^2*(d*x*PolyLog[2, E^(-c - d*x)] + PolyLog[3
, E^(-c - d*x)])))/(a^2*d^3) + (Sqrt[a^2 + b^2]*(-2*d^2*e^2*ArcTanh[(a + b*
E^(c + d*x))/Sqrt[a^2 + b^2]] + 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 + b^2]]) + d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]
] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) - d^2*f^2*x^
2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) + 2*d*f*(e + f*x)*PolyLog[
2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-
a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2]))])))/(a^2*d^3) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(-(e^2*Sinh[(d*x)/2]
- 2*e*f*x*Sinh[(d*x)/2] - f^2*x^2*Sinh[(d*x)/2]))/(2*a*d) + (Csch[c/2]*Csc
h[c/2 + (d*x)/2]*(e^2*Sinh[(d*x)/2] + 2*e*f*x*Sinh[(d*x)/2] + f^2*x^2*Sinh[
(d*x)/2]))/(2*a*d)

```

**Maple [F]** time = 0.713, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\coth(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [C]** time = 2.81707, size = 6512, normalized size = 12.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*coth(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-(2*a*d^2*e^2 - 4*a*c*d*e*f + 2*a*c^2*f^2 + 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*\cosh(d*x + c)^2 + 4*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + 2*(a*d^2*f^2*x^2 + 2*a*d^2*e*f*x + 2*a*c*d*e*f - a*c^2*f^2)*\sinh(d*x + c)^2 + 2*(b*d*f^2*x + b*d*e*f - (b*d*f^2*x + b*d*e*f)*\cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d*f^2*x + b*d*e*f)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2}*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 2*(b*d*f^2*x + b*d*e*f - (b*d*f^2*x + b*d*e*f)*\cosh(d*x + c)^2 - 2*(b*d*f^2*x + b*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d*f^2*x + b*d*e*f)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2}*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2 - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*\cosh(d*x + c)^2 - 2*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2}*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2 - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*\cosh(d*x + c)^2 - 2*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2}*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*\cosh(d*x + c)^2 - 2*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2}*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2 - (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*\cosh(d*x + c)^2 - 2*(b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d^2*f^2*x^2 + 2*b*d^2*e*f*x + 2*b*c*d*e*f - b*c^2*f^2)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2}*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + 2*(b*f^2*\cosh(d*x + c)^2 + 2*b*f^2*\cosh(d*x + c)*\sinh(d*x + c) + b*f^2*\sinh(d*x + c)^2 - b*f^2)*\sqrt{(a^2 + b^2)/b^2}*\operatorname{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) - 2*(b*f^2*$$

```

cosh(d*x + c)^2 + 2*b*f^2*cosh(d*x + c)*sinh(d*x + c) + b*f^2*sinh(d*x + c)
^2 - b*f^2)*sqrt((a^2 + b^2)/b^2)*polylog(3, (a*cosh(d*x + c) + a*sinh(d*x
+ c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 2*(b
*d*f^2*x + b*d*e*f - a*f^2 - (b*d*f^2*x + b*d*e*f - a*f^2)*cosh(d*x + c)^2
- 2*(b*d*f^2*x + b*d*e*f - a*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f^2*x
+ b*d*e*f - a*f^2)*sinh(d*x + c)^2)*dilog(cosh(d*x + c) + sinh(d*x + c)) +
2*(b*d*f^2*x + b*d*e*f + a*f^2 - (b*d*f^2*x + b*d*e*f + a*f^2)*cosh(d*x + c)
)^2 - 2*(b*d*f^2*x + b*d*e*f + a*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b*d*f^
2*x + b*d*e*f + a*f^2)*sinh(d*x + c)^2)*dilog(-cosh(d*x + c) - sinh(d*x + c
)) + (b*d^2*f^2*x^2 + b*d^2*e^2 + 2*a*d*e*f - (b*d^2*f^2*x^2 + b*d^2*e^2 +
2*a*d*e*f + 2*(b*d^2*e*f + a*d*f^2)*x)*cosh(d*x + c)^2 - 2*(b*d^2*f^2*x^2 +
b*d^2*e^2 + 2*a*d*e*f + 2*(b*d^2*e*f + a*d*f^2)*x)*cosh(d*x + c)*sinh(d*x
+ c) - (b*d^2*f^2*x^2 + b*d^2*e^2 + 2*a*d*e*f + 2*(b*d^2*e*f + a*d*f^2)*x)*
sinh(d*x + c)^2 + 2*(b*d^2*e*f + a*d*f^2)*x)*log(cosh(d*x + c) + sinh(d*x +
c) + 1) - (b*d^2*e^2 - 2*(b*c + a)*d*e*f + (b*c^2 + 2*a*c)*f^2 - (b*d^2*e^
2 - 2*(b*c + a)*d*e*f + (b*c^2 + 2*a*c)*f^2)*cosh(d*x + c)^2 - 2*(b*d^2*e^2
- 2*(b*c + a)*d*e*f + (b*c^2 + 2*a*c)*f^2)*cosh(d*x + c)*sinh(d*x + c) - (
b*d^2*e^2 - 2*(b*c + a)*d*e*f + (b*c^2 + 2*a*c)*f^2)*sinh(d*x + c)^2)*log(c
osh(d*x + c) + sinh(d*x + c) - 1) - (b*d^2*f^2*x^2 + 2*b*c*d*e*f - (b*c^2 +
2*a*c)*f^2 - (b*d^2*f^2*x^2 + 2*b*c*d*e*f - (b*c^2 + 2*a*c)*f^2 + 2*(b*d^2
*e*f - a*d*f^2)*x)*cosh(d*x + c)^2 - 2*(b*d^2*f^2*x^2 + 2*b*c*d*e*f - (b*c^
2 + 2*a*c)*f^2 + 2*(b*d^2*e*f - a*d*f^2)*x)*cosh(d*x + c)*sinh(d*x + c) - (
b*d^2*f^2*x^2 + 2*b*c*d*e*f - (b*c^2 + 2*a*c)*f^2 + 2*(b*d^2*e*f - a*d*f^2)
*x)*sinh(d*x + c)^2 + 2*(b*d^2*e*f - a*d*f^2)*x)*log(-cosh(d*x + c) - sinh(
d*x + c) + 1) - 2*(b*f^2*cosh(d*x + c)^2 + 2*b*f^2*cosh(d*x + c)*sinh(d*x +
c) + b*f^2*sinh(d*x + c)^2 - b*f^2)*polylog(3, cosh(d*x + c) + sinh(d*x +
c)) + 2*(b*f^2*cosh(d*x + c)^2 + 2*b*f^2*cosh(d*x + c)*sinh(d*x + c) + b*f^
2*sinh(d*x + c)^2 - b*f^2)*polylog(3, -cosh(d*x + c) - sinh(d*x + c)))/(a^2
*d^3*cosh(d*x + c)^2 + 2*a^2*d^3*cosh(d*x + c)*sinh(d*x + c) + a^2*d^3*sinh
(d*x + c)^2 - a^2*d^3)

```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^2 \coth^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*coth(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*\*2\*coth(c + d\*x)\*\*2/(a + b\*sinh(c + d\*x)), x)

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.456 \quad \int \frac{(e+fx) \coth^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=294

$$\frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2} - \frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2} + \frac{bf\text{PolyLog}\left(2, -e^{c+dx}\right)}{a^2d^2} - \frac{bf\text{PolyLog}\left(2, e^{c+dx}\right)}{a^2d^2}$$

[Out] (2\*b\*(e + f\*x)\*ArcTanh[E^(c + d\*x)]/(a^2\*d) - ((e + f\*x)\*Coth[c + d\*x])/(a\*d) + (Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/(a^2\*d) - (Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/(a^2\*d) + (f\*Log[Sinh[c + d\*x]])/(a\*d^2) + (b\*f\*PolyLog[2, -E^(c + d\*x)]/(a^2\*d^2) - (b\*f\*PolyLog[2, E^(c + d\*x)]/(a^2\*d^2) + (Sqrt[a^2 + b^2]\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))]/(a^2\*d^2) - (Sqrt[a^2 + b^2]\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))]/(a^2\*d^2)))/(a^2\*d^2)

**Rubi [A]** time = 0.688467, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 14, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$ , Rules used = {5569, 3720, 3475, 5585, 5450, 3296, 2637, 4182, 2279, 2391, 5565, 3322, 2264, 2190}

$$\frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2d^2} - \frac{f\sqrt{a^2+b^2}\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2d^2} + \frac{bf\text{PolyLog}\left(2, -e^{c+dx}\right)}{a^2d^2} - \frac{bf\text{PolyLog}\left(2, e^{c+dx}\right)}{a^2d^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Coth[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]), x]

[Out] (2\*b\*(e + f\*x)\*ArcTanh[E^(c + d\*x)]/(a^2\*d) - ((e + f\*x)\*Coth[c + d\*x])/(a\*d) + (Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]])/(a^2\*d) - (Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]])/(a^2\*d) + (f\*Log[Sinh[c + d\*x]])/(a\*d^2) + (b\*f\*PolyLog[2, -E^(c + d\*x)]/(a^2\*d^2) - (b\*f\*PolyLog[2, E^(c + d\*x)]/(a^2\*d^2) + (Sqrt[a^2 + b^2]\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))]/(a^2\*d^2) - (Sqrt[a^2 + b^2]\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))]/(a^2\*d^2)))/(a^2\*d^2)

**Rule 5569**

Int[(Coth[(c\_.) + (d\_.)\*(x\_)]^(n\_.))\*((e\_.) + (f\_.)\*(x\_))^(m\_.)]/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Coth[c



+ d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Cosh[c + d\*x]\*Coth[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 3720

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] :> Simp[(b\*(c + d\*x)^m\*(b\*Tan[e + f\*x])^(n - 1))/(f\*(n - 1)), x] + (-Dist[(b\*d\*m)/(f\*(n - 1)), Int[(c + d\*x)^(m - 1)\*(b\*Tan[e + f\*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d\*x)^m\*(b\*Tan[e + f\*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

### Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

### Rule 5585

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)])^(p\_.)\*Coth[(c\_.) + (d\_.)\*(x\_)])^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] :> Dist[1/a, Int[(e + f\*x)^m\*Cosh[c + d\*x]^p\*Coth[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(p + 1)\*Coth[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5450

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*Coth[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :> Int[(c + d\*x)^m\*Cosh[a + b\*x]^n\*Coth[a + b\*x]^(p - 2), x] + Int[(c + d\*x)^m\*Cosh[a + b\*x]^(n - 2)\*Coth[a + b\*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] :> -Simp[((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x)] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2)
]*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x)] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*(
f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
)))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e + fx) \coth^2(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\int (e + fx) \coth^2(c + dx) dx}{a} - \frac{b \int \frac{(e + fx) \cosh(c + dx) \coth(c + dx)}{a + b \sinh(c + dx)} dx}{a} \\
&= -\frac{(e + fx) \coth(c + dx)}{ad} + \frac{\int (e + fx) dx}{a} - \frac{b \int (e + fx) \cosh(c + dx) \coth(c + dx) dx}{a^2} + \frac{b \int (e + fx) dx}{a} \\
&= \frac{ex}{a} + \frac{fx^2}{2a} - \frac{(e + fx) \coth(c + dx)}{ad} + \frac{f \log(\sinh(c + dx))}{ad^2} - \frac{\int (e + fx) dx}{a} - \frac{b \int (e + fx) dx}{a} \\
&= \frac{2b(e + fx) \tanh^{-1}(e^{c+dx})}{a^2d} - \frac{(e + fx) \coth(c + dx)}{ad} + \frac{f \log(\sinh(c + dx))}{ad^2} + \frac{(2(a^2 + b^2) \int (e + fx) dx)}{a^2d} \\
&= \frac{2b(e + fx) \tanh^{-1}(e^{c+dx})}{a^2d} - \frac{(e + fx) \coth(c + dx)}{ad} + \frac{f \log(\sinh(c + dx))}{ad^2} + \frac{(2b\sqrt{a^2 + b^2} \int (e + fx) dx)}{a^2d} \\
&= \frac{2b(e + fx) \tanh^{-1}(e^{c+dx})}{a^2d} - \frac{(e + fx) \coth(c + dx)}{ad} + \frac{\sqrt{a^2 + b^2}(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{a^2d} \\
&= \frac{2b(e + fx) \tanh^{-1}(e^{c+dx})}{a^2d} - \frac{(e + fx) \coth(c + dx)}{ad} + \frac{\sqrt{a^2 + b^2}(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{a^2d} \\
&= \frac{2b(e + fx) \tanh^{-1}(e^{c+dx})}{a^2d} - \frac{(e + fx) \coth(c + dx)}{ad} + \frac{\sqrt{a^2 + b^2}(e + fx) \log\left(1 + \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{a^2d}
\end{aligned}$$

**Mathematica [A]** time = 3.66865, size = 364, normalized size = 1.24

$$\frac{2\sqrt{a^2 + b^2} \left( f \text{PolyLog} \left( 2, \frac{be^{c+dx}}{\sqrt{a^2 + b^2} - a} \right) - f \text{PolyLog} \left( 2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a} \right) - 2de \tanh^{-1} \left( \frac{a + be^{c+dx}}{\sqrt{a^2 + b^2}} \right) + f(c + dx) \log \left( \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}} + 1 \right) \right)}{a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Coth[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

```
[Out] 
$$\begin{aligned} & -(a*d*(e + f*x)*\text{Coth}[(c + d*x)/2]) + 2*a*f*\text{Log}[\text{Sinh}[c + d*x]] - 2*b*d*e*\text{Log}[\text{Tanh}[(c + d*x)/2]] \\ & + 2*b*c*f*\text{Log}[\text{Tanh}[(c + d*x)/2]] + 2*b*f*(-((c + d*x)*\text{Log}[1 - E^{-(c + d*x)}] - \text{Log}[1 + E^{-(c + d*x)}])) - \text{PolyLog}[2, -E^{-(c + d*x)}] \\ & + \text{PolyLog}[2, E^{-(c + d*x)}]) + 2*\text{Sqrt}[a^2 + b^2]*(-2*d*e*\text{ArcTanh}[(a + b*E^{(c + d*x)})/\text{Sqrt}[a^2 + b^2]] \\ & + 2*c*f*\text{ArcTanh}[(a + b*E^{(c + d*x)})/\text{Sqrt}[a^2 + b^2]] + f*(c + d*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])] - f*(c + d*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])] + f*\text{PolyLog}[2, (b*E^{(c + d*x)})/(-a + \text{Sqrt}[a^2 + b^2])] - f*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))]) - a*d*(e + f*x)*\text{Tanh}[(c + d*x)/2])/(2*a^2*d^2) \end{aligned}$$

```

**Maple [B]** time = 0.213, size = 1017, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] 
$$\begin{aligned} & -2/d*e/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2}))+1/d^2*f/(a^2+b^2)^{(1/2)}*\text{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2}))) \\ & -1/d^2*f/(a^2+b^2)^{(1/2)}*\text{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2}))) \\ & -2/a^2/d*b^2*e/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2}))+1/a^2/d^2*b*f*c*\ln(\exp(d*x+c)-1)+1/a^2/d^2*b*f/(a^2+b^2)^{(1/2)}*\text{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2}))) \\ & -1/a^2/d^2*b^2*f/(a^2+b^2)^{(1/2)}*\text{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2}))) \\ & +1/a^2/d*b*f*\ln(\exp(d*x+c)+1)*x-1/a^2/d*b^2*f/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2}))) \\ & *c+2/a^2/d^2*b^2*f*c/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2}))+1/a^2/d*b^2*f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2}))) \\ & *x+1/a^2/d^2*b^2*f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2}))) \\ & *c-2/d*(f*x+e)/a/(\exp(2*d*x+2*c)-1)-1/a^2/d*b*e*\ln(\exp(d*x+c)-1)+1/a^2/d*b*e*\ln(\exp(d*x+c)+1)+1/a^2/d^2*b*f*\text{dilog}(\exp(d*x+c))+1/a^2/d^2*b*f*\text{dilog}(\exp(d*x+c)+1)-2/d^2/a*f*\ln(\exp(d*x+c))+1/d^2/a*f*\ln(\exp(d*x+c)-1)+1/d^2/a*f*\ln(\exp(d*x+c)+1)+1/d*f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2}))) \\ & *x+1/d^2*f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2}))) \\ & *c-1/d*f/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2}))) \\ & *x-1/d^2*f/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2}))) \\ & *c+2/d^2*f*c/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2})) \end{aligned}$$

```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*coth(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [B]** time = 2.35579, size = 3430, normalized size = 11.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*coth(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -(2*a*d*e - 2*a*c*f + 2*(a*d*f*x + a*c*f)*\cosh(d*x + c)^2 + 4*(a*d*f*x + a*c*f)*\cosh(d*x + c)*\sinh(d*x + c) + 2*(a*d*f*x + a*c*f)*\sinh(d*x + c)^2 - (b*f*\cosh(d*x + c)^2 + 2*b*f*\cosh(d*x + c)*\sinh(d*x + c) + b*f*\sinh(d*x + c)^2 - b*f)*\sqrt{(a^2 + b^2)/b^2}*dilog((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + (b*f*\cosh(d*x + c)^2 + 2*b*f*\cosh(d*x + c)*\sinh(d*x + c) + b*f*\sinh(d*x + c)^2 - b*f)*\sqrt{(a^2 + b^2)/b^2}*dilog((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - (b*d*e - b*c*f - (b*d*e - b*c*f)*\cosh(d*x + c)^2 - 2*(b*d*e - b*c*f)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d*e - b*c*f)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2}*log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + (b*d*e - b*c*f - (b*d*e - b*c*f)*\cosh(d*x + c)^2 - 2*(b*d*e - b*c*f)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d*e - b*c*f)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2}*log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + (b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*\cosh(d*x + c)^2 - 2*(b*d*f*x + b*c*f)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d*f*x + b*c*f)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2}*log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - (b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*\cosh(d*x + c)^2 - 2*(b*d*f*x + b*c*f)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d*f*x + b*c*f)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2}*log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + (b*f*\cosh(d*x + c)^2 + 2*b*f*\cosh(d*x + c)*\sinh(d*x + c) - (b*d*f*x + b*c*f)*\cosh(d*x + c)^2 - 2*(b*d*f*x + b*c*f)*\cosh(d*x + c)*\sinh(d*x + c) - (b*d*f*x + b*c*f)*\sinh(d*x + c)^2)*\sqrt{(a^2 + b^2)/b^2} \end{aligned}$$

$$\begin{aligned}
& h(dx + c) + b*f*\sinh(dx + c)^2 - b*f*\operatorname{dilog}(\cosh(dx + c) + \sinh(dx + c)) \\
& - (b*f*\cosh(dx + c)^2 + 2*b*f*\cosh(dx + c)*\sinh(dx + c) + b*f*\sinh(dx + c)^2 - b*f)*\operatorname{dilog}(-\cosh(dx + c) - \sinh(dx + c)) \\
& + (b*d*f*x + b*d*e - (b*d*f*x + b*d*e + a*f)*\cosh(dx + c)^2 - 2*(b*d*f*x + b*d*e + a*f)*\cosh(dx + c)*\sinh(dx + c) \\
& - (b*d*f*x + b*d*e + a*f)*\sinh(dx + c)^2 + a*f)*\log(\cosh(dx + c) + \sinh(dx + c) + 1) \\
& - (b*d*e - (b*d*e - (b*c + a)*f)*\cosh(dx + c)^2 - 2*(b*d*e - (b*c + a)*f)*\cosh(dx + c)*\sinh(dx + c) \\
& - (b*d*e - (b*c + a)*f)*\sinh(dx + c)^2 - (b*c + a)*f)*\log(\cosh(dx + c) + \sinh(dx + c) - 1) \\
& - (b*d*f*x + b*c*f - (b*d*f*x + b*c*f)*\cosh(dx + c)^2 - 2*(b*d*f*x + b*c*f)*\cosh(dx + c)*\sinh(dx + c) \\
& - (b*d*f*x + b*c*f)*\sinh(dx + c)^2)*\log(-\cosh(dx + c) - \sinh(dx + c) + 1) \\
& / (a^2*d^2*\cosh(dx + c)^2 + 2*a^2*d^2*\cosh(dx + c)*\sinh(dx + c) + a^2*d^2*\sinh(dx + c)^2 - a^2*d^2)
\end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \coth^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*coth(dx+c)\*\*2/(a+b\*sinh(dx+c)),x)

[Out] Integral((e + f\*x)\*coth(c + d\*x)\*\*2/(a + b\*sinh(c + d\*x)), x)

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*coth(dx+c)^2/(a+b\*sinh(dx+c)),x, algorithm="giac")

[Out] Timed out

$$3.457 \quad \int \frac{\coth^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=77

$$-\frac{2\sqrt{a^2+b^2} \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d} + \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{\coth(c+dx)}{ad}$$

[Out] (b\*ArcTanh[Cosh[c + d\*x]])/(a^2\*d) - (2\*Sqrt[a^2 + b^2]\*ArcTanh[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[a^2 + b^2]])/(a^2\*d) - Coth[c + d\*x]/(a\*d)

**Rubi [A]** time = 0.266282, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {2723, 3056, 3001, 3770, 2660, 618, 204}

$$-\frac{2\sqrt{a^2+b^2} \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d} + \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d\*x]^2/(a + b\*Sinh[c + d\*x]),x]

[Out] (b\*ArcTanh[Cosh[c + d\*x]])/(a^2\*d) - (2\*Sqrt[a^2 + b^2]\*ArcTanh[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[a^2 + b^2]])/(a^2\*d) - Coth[c + d\*x]/(a\*d)

### Rule 2723

Int[((a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)])^(m\_)/tan[(e\_) + (f\_)\*(x\_)]^2, x\_Symbol] :> Int[((a + b\*Sin[e + f\*x])^m\*(1 - Sin[e + f\*x]^2))/Sin[e + f\*x]^2, x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]

### Rule 3056

Int[((a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)])^(m\_)\*((c\_) + (d\_)\*sin[(e\_) + (f\_)\*(x\_)])^(n\_)\*((A\_) + (C\_)\*sin[(e\_) + (f\_)\*(x\_)]^2), x\_Symbol] :> -Simp[(A\*b^2 + a^2\*C)\*Cos[e + f\*x]\*(a + b\*Sin[e + f\*x])^(m + 1)\*(c + d\*Sin[e + f\*x])^(n + 1)/(f\*(m + 1)\*(b\*c - a\*d)\*(a^2 - b^2)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(a^2 - b^2)), Int[(a + b\*Sin[e + f\*x])^(m + 1)\*(c + d\*Sin[e + f\*x])^n\*Simp[a\*(m + 1)\*(b\*c - a\*d)\*(A + C) + d\*(A\*b^2 + a^2\*C)\*(m + n + 2

```
) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A
*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, c, d,
e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d
^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(
IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0]
)))
```

### Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :=> Dist[(A*b
- a*B)/(b*c - a*d), Int[1/(a + b*SIN[e + f*x]), x], x] + Dist[(B*c - A*d)/(
b*c - a*d), Int[1/(c + d*SIN[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f,
A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :=> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

### Rule 2660

```
Int[((a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :=> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

### Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :=> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :=> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

### Rubi steps



$$\begin{aligned}
\int \frac{\coth^2(c+dx)}{a+b\sinh(c+dx)} dx &= \int \frac{\operatorname{csch}^2(c+dx)(1+\sinh^2(c+dx))}{a+b\sinh(c+dx)} dx \\
&= -\frac{\coth(c+dx)}{ad} + \frac{i \int \frac{\operatorname{csch}(c+dx)(ib-ia\sinh(c+dx))}{a+b\sinh(c+dx)} dx}{a} \\
&= -\frac{\coth(c+dx)}{ad} - \frac{b \int \operatorname{csch}(c+dx) dx}{a^2} + \frac{(a^2+b^2) \int \frac{1}{a+b\sinh(c+dx)} dx}{a^2} \\
&= \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2 d} - \frac{\coth(c+dx)}{ad} - \frac{(2i(a^2+b^2)) \operatorname{Subst}\left(\int \frac{1}{a-2ibx+ax^2} dx, x, \tan\left(\frac{1}{2}(ic+dx)\right)\right)}{a^2 d} \\
&= \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2 d} - \frac{\coth(c+dx)}{ad} + \frac{(4i(a^2+b^2)) \operatorname{Subst}\left(\int \frac{1}{-4(a^2+b^2)-x^2} dx, x, -2ib + \frac{2(a^2+b^2)\tanh\left(\frac{1}{2}(ic+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2 d} \\
&= \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2 d} - \frac{2\sqrt{a^2+b^2} \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2 d} - \frac{\coth(c+dx)}{ad}
\end{aligned}$$

**Mathematica [A]** time = 0.441296, size = 98, normalized size = 1.27

$$\frac{4\sqrt{-a^2-b^2} \tan^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right) + a \tanh\left(\frac{1}{2}(c+dx)\right) + a \coth\left(\frac{1}{2}(c+dx)\right) + 2b \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2a^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d\*x]^2/(a + b\*Sinh[c + d\*x]),x]

[Out] -(4\*sqrt[-a^2 - b^2]\*ArcTan[(b - a\*Tanh[(c + d\*x)/2])/sqrt[-a^2 - b^2]] + a\*Coth[(c + d\*x)/2] + 2\*b\*Log[Tanh[(c + d\*x)/2]] + a\*Tanh[(c + d\*x)/2])/(2\*a^2\*d)

**Maple [B]** time = 0.003, size = 147, normalized size = 1.9

$$-\frac{1}{2da} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{2da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-1} - \frac{b}{da^2} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2 \frac{1}{d\sqrt{a^2+b^2}} \operatorname{Arctanh}\left(\frac{1}{2} \frac{2a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/2/d/a*tanh(1/2*d*x+1/2*c)-1/2/d/a/tanh(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tanh(
1/2*d*x+1/2*c))+2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*
b)/(a^2+b^2)^(1/2))+2/d/a^2*b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tanh(1/2*d
*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [B]** time = 2.25519, size = 975, normalized size = 12.66

$$\sqrt{a^2 + b^2} \left( \cosh(dx + c)^2 + 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c)^2 - 1 \right) \log \left( \frac{b^2 \cosh(dx+c)^2 + b^2 \sinh(dx+c)^2 + 2ab \cosh(dx+c)}{b \cosh(dx+c)^2 + b \sinh(dx+c)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2 + b^2)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*
x + c)^2 - 1)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d
*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(
a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*
sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c)
- b)) + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
+ c)^2 - b)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (b*cosh(d*x + c)^2 + 2
*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*log(cosh(d*x + c) +
sinh(d*x + c) - 1) - 2*a)/(a^2*d*cosh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c)*s
inh(d*x + c) + a^2*d*sinh(d*x + c)^2 - a^2*d)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)), x)

[Out] Integral(coth(c + d\*x)\*\*2/(a + b\*sinh(c + d\*x)), x)

**Giac [B]** time = 1.83974, size = 201, normalized size = 2.61

$$\frac{(a^2 e^c + b^2 e^c) e^{-c} \log\left(\frac{2 b e^{(dx+2c)} + 2 a e^c - 2 \sqrt{a^2 + b^2} e^c}{2 b e^{(dx+2c)} + 2 a e^c + 2 \sqrt{a^2 + b^2} e^c}\right)}{\sqrt{a^2 + b^2} a^2} + \frac{b \log(e^{(dx+c)} + 1)}{a^2} - \frac{b \log(|e^{(dx+c)} - 1|)}{a^2} - \frac{2}{a(e^{(2dx+2c)} - 1)}$$

$d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)^2/(a+b\*sinh(d\*x+c)), x, algorithm="giac")

[Out] ((a^2\*e^c + b^2\*e^c)\*e^(-c)\*log(abs(2\*b\*e^(d\*x + 2\*c) + 2\*a\*e^c - 2\*sqrt(a^2 + b^2)\*e^c)/abs(2\*b\*e^(d\*x + 2\*c) + 2\*a\*e^c + 2\*sqrt(a^2 + b^2)\*e^c))/(sqrt(a^2 + b^2)\*a^2) + b\*log(e^(d\*x + c) + 1)/a^2 - b\*log(abs(e^(d\*x + c) - 1))/a^2 - 2/(a\*(e^(2\*d\*x + 2\*c) - 1))/d

$$3.458 \quad \int \frac{\coth^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Optimal. Leaf size=30

$$\text{Unintegrable} \left( \frac{\coth^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[Coth[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0734448, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Coth[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Coth[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\coth^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\coth^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180., size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[Coth[c + d\*x]^2/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] \$Aborted

---

**Maple [A]** time = 0.999, size = 0, normalized size = 0.

$$\int \frac{(\coth(dx + c))^2}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)`

[Out] `int(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$2(a^2e^c + b^2e^c) \int -\frac{e^{(dx)}}{a^2bfx + a^2be - (a^2bfxe^{(2c)} + a^2bee^{(2c)})e^{(2dx)} - 2(a^3fxe^c + a^3ee^c)e^{(dx)}} dx + \frac{2}{adfx + ade - (adfxe^{(2c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `2*(a^2*e^c + b^2*e^c)*integrate(-e^(d*x)/(a^2*b*f*x + a^2*b*e - (a^2*b*f*x*e^(2*c) + a^2*b*e*e^(2*c))*e^(2*d*x) - 2*(a^3*f*x*e^c + a^3*e*e^c)*e^(d*x)), x) + 2/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e*e^(2*c))*e^(2*d*x)) - integrate(-(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) - integrate((b*d*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth(dx + c)^2}{afx + ae + (bfx + be) \sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] `integral(coth(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)`

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(c + dx)}{(a + b \sinh(c + dx))(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)`

[Out] `Integral(coth(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)`

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")`

[Out] Timed out

$$3.459 \quad \int \frac{(e+fx)^3 \cosh(c+dx) \coth^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=718

$$\frac{6f^2(a^2+b^2)(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^3} - \frac{6f^2(a^2+b^2)(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2bd^3} + \frac{3f(a^2+b^2)(e+fx)}{a^2bd^3}$$

```
[Out] (b*(e + f*x)^4)/(4*a^2*f) - ((a^2 + b^2)*(e + f*x)^4)/(4*a^2*b*f) - (6*f*(e
+ f*x)^2*ArcTanh[E^(c + d*x)])/(a*d^2) - ((e + f*x)^3*Csch[c + d*x])/(a*d)
+ ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])
/(a^2*b*d) + ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2
+ b^2])])/(a^2*b*d) - (b*(e + f*x)^3*Log[1 - E^(2*(c + d*x))])/(a^2*d) - (
6*f^2*(e + f*x)*PolyLog[2, -E^(c + d*x)]/(a*d^3) + (6*f^2*(e + f*x)*PolyLo
g[2, E^(c + d*x)]/(a*d^3) + (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E
^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*b*d^2) + (3*(a^2 + b^2)*f*(e + f*
x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^2) - (3
*b*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*x))])/(2*a^2*d^2) + (6*f^3*PolyLog[
3, -E^(c + d*x)]/(a*d^4) - (6*f^3*PolyLog[3, E^(c + d*x)]/(a*d^4) - (6*(a
^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))
])/(a^2*b*d^3) - (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^3) + (3*b*f^2*(e + f*x)*PolyLog[3, E^(2*(
c + d*x))])/(2*a^2*d^3) + (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/
(a - Sqrt[a^2 + b^2]))])/(a^2*b*d^4) + (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E
(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^4) - (3*b*f^3*PolyLog[4, E^(2*
(c + d*x))])/(4*a^2*d^4)
```

**Rubi [A]** time = 1.64475, antiderivative size = 718, normalized size of antiderivative = 1., number of steps used = 48, number of rules used = 19, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.559$ , Rules used = {5585, 5450, 3296, 2638, 5452, 4182, 2531, 2282, 6589, 5446, 3311, 32, 2635, 8, 3716, 2190, 6609, 5565, 5561}

$$\frac{6f^2(a^2+b^2)(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^3} - \frac{6f^2(a^2+b^2)(e+fx)\text{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2bd^3} + \frac{3f(a^2+b^2)(e+fx)}{a^2bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*(e + f*x)^4)/(4*a^2*f) - ((a^2 + b^2)*(e + f*x)^4)/(4*a^2*b*f) - (6*f*(e
+ f*x)^2*ArcTanh[E^(c + d*x)])/(a*d^2) - ((e + f*x)^3*Csch[c + d*x])/(a*d)
```

$$\begin{aligned}
& + ((a^2 + b^2)(e + fx)^3 \text{Log}[1 + (bE^{(c + dx)})/(a - \text{Sqrt}[a^2 + b^2])]) / (a^2 b d) \\
& + ((a^2 + b^2)(e + fx)^3 \text{Log}[1 + (bE^{(c + dx)})/(a + \text{Sqrt}[a^2 + b^2])]) / (a^2 b d) \\
& - (b(e + fx)^3 \text{Log}[1 - E^{(2(c + dx))}] / (a^2 d) - (6f^2(e + fx) \text{PolyLog}[2, -E^{(c + dx)}]) / (a d^3) + (6f^2(e + fx) \text{PolyLog}[2, E^{(c + dx)}]) / (a d^3) \\
& + (3(a^2 + b^2) f (e + fx)^2 \text{PolyLog}[2, -(bE^{(c + dx)})/(a - \text{Sqrt}[a^2 + b^2])]) / (a^2 b d^2) + (3(a^2 + b^2) f (e + fx)^2 \text{PolyLog}[2, -(bE^{(c + dx)})/(a + \text{Sqrt}[a^2 + b^2])]) / (a^2 b d^2) \\
& - (3 b f (e + fx)^2 \text{PolyLog}[2, E^{(2(c + dx))}] / (2 a^2 d^2) + (6f^3 \text{PolyLog}[3, -E^{(c + dx)}]) / (a d^4) - (6f^3 \text{PolyLog}[3, E^{(c + dx)}]) / (a d^4) \\
& - (6(a^2 + b^2) f^2 (e + fx) \text{PolyLog}[3, -(bE^{(c + dx)})/(a - \text{Sqrt}[a^2 + b^2])]) / (a^2 b d^3) - (6(a^2 + b^2) f^2 (e + fx) \text{PolyLog}[3, -(bE^{(c + dx)})/(a + \text{Sqrt}[a^2 + b^2])]) / (a^2 b d^3) \\
& + (3 b f^2 (e + fx) \text{PolyLog}[3, E^{(2(c + dx))}] / (2 a^2 d^3) + (6(a^2 + b^2) f^3 \text{PolyLog}[4, -(bE^{(c + dx)})/(a - \text{Sqrt}[a^2 + b^2])]) / (a^2 b d^4) \\
& + (6(a^2 + b^2) f^3 \text{PolyLog}[4, -(bE^{(c + dx)})/(a + \text{Sqrt}[a^2 + b^2])]) / (a^2 b d^4) - (3 b f^3 \text{PolyLog}[4, E^{(2(c + dx))}] / (4 a^2 d^4)
\end{aligned}$$

### Rule 5585

```

Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

```

### Rule 5450

```

Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

```

### Rule 3296

```

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

```

### Rule 2638

```

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

```



Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol]
:> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^(m - 1)*(b*Sine[e + f*x])^(n - 2), x], x] - Dist
[d^2*m*(m - 1)/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sine[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sine[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]
*(b*Sine[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sine[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

### Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

### Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_
.)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_.)]^(n_.)*((e_.) + (f_.)*(x_.))^(m_.))/((a_.) + (b_.
)*Sinh[(c_.) + (d_.)*(x_.)]), x_Symbol] := -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))^(m_.))/((a_.) + (b_.)*Sin
h[(c_.) + (d_.)*(x_.)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \cosh(c+dx) \coth^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \cosh(c+dx) \coth^2(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^3 \cosh^2(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= \frac{\int (e+fx)^3 \cosh(c+dx) dx}{a} + \frac{\int (e+fx)^3 \coth(c+dx) \operatorname{csch}(c+dx) dx}{a} - \frac{b}{a} \int \frac{(e+fx)^3 \cosh(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx \\
&= -\frac{(e+fx)^3 \operatorname{csch}(c+dx)}{ad} + \frac{(e+fx)^3 \sinh(c+dx)}{ad} - \frac{\int (e+fx)^3 \cosh(c+dx) \coth(c+dx) dx}{a} \\
&= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad^2} - \frac{3f(e+fx)^3}{ad} \\
&= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+fx)^3}{ad} \\
&= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad^2} - \frac{6f^3 \cosh(c+dx)}{ad} \\
&= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+fx)^3}{ad} \\
&= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+fx)^3}{ad} \\
&= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+fx)^3}{ad} \\
&= \frac{b(e+fx)^4}{4a^2f} - \frac{(a^2+b^2)(e+fx)^4}{4a^2bf} - \frac{6f(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+fx)^3}{ad}
\end{aligned}$$

**Mathematica [C]** time = 13.512, size = 10534, normalized size = 14.67

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Result too large to show
```

---

**Maple [F]** time = 0.986, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \cosh(dx + c) (\coth(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*cosh(d\*x+c)\*coth(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*cosh(d\*x+c)\*coth(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)\*coth(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$e^3 * ((d*x + c)/(b*d) + 2*e^{(-d*x - c)} / ((a*e^{(-2*d*x - 2*c)} - a)*d) - b*\log(e^{(-d*x - c)} + 1)/(a^2*d) - b*\log(e^{(-d*x - c)} - 1)/(a^2*d) + (a^2 + b^2)*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/(a^2*b*d)) - 3*e^2*f*\log(e^{(d*x + c)} + 1)/(a*d^2) + 3*e^2*f*\log(e^{(d*x + c)} - 1)/(a*d^2) - 1/4*(a*d*f^3*x^4 + 4*a*d*e*f^2*x^3 + 6*a*d*e^2*f*x^2 - (a*d*f^3*x^4*e^{(2*c)} + 4*a*d*e*f^2*x^3*e^{(2*c)} + 6*a*d*e^2*f*x^2*e^{(2*c)}))*e^{(2*d*x)} + 8*(b*f^3*x^3*e^c + 3*b*e*f^2*x^2*e^c + 3*b*e^2*f*x*e^c)*e^{(d*x)} / (a*b*d*e^{(2*d*x + 2*c)} - a*b*d) - (d^3*x^3*\log(e^{(d*x + c)} + 1) + 3*d^2*x^2*dilog(-e^{(d*x + c)}) - 6*d*x*polylog(3, -e^{(d*x + c)}) + 6*polylog(4, -e^{(d*x + c)}))*b*f^3/(a^2*d^4) - (d^3*x^3*\log(-e^{(d*x + c)} + 1) + 3*d^2*x^2*dilog(e^{(d*x + c)}) - 6*d*x*polylog(3, e^{(d*x + c)}) + 6*polylog(4, e^{(d*x + c)}))*b*f^3/(a^2*d^4) - 3*(b*d*e^2*f + 2*a*e*f^2)*(d*x*\log(e^{(d*x + c)} + 1) + dilog(-e^{(d*x + c)})) / (a^2*d^3) - 3*(b*d*e^2*f - 2*a*e*f^2)*(d*x*\log(-e^{(d*x + c)} + 1) + dilog(e^{(d*x + c)})) / (a^2*d^3) - 3*(b*d*e*f^2 + a*f^3)*(d^2*x^2*\log(e^{(d*x + c)} + 1) + 2*d*x*dilog(-e^{(d*x + c)}) - 2*polylog(3, -e^{(d*x + c)})) / (a^2*d^4) - 3*(b*d*e*f^2 - a*f^3)*(d^2*x^2*\log(-e^{(d*x + c)} + 1) + 2*d*x*dilog(e^{(d*x + c)}) - 2*polylog(3, e^{(d*x + c)})) / (a^2*d^4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 + a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f + 2*a*d*e*f^2)*d^2*x^2) / (a^2*d^4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 - a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f - 2*a*d*e*f^2)*d^2*x^2) / (a^2*d^4) - integrate(-2*((a^2*b*f^3 + b^3*f^3)*x^3 + 3*(a^2*b*e*f^2 + b^3*e*f^2)*x^2 + 3*(a^2*b*e^2*f + b^3*e^2*f)*x - ((a^3*f^3*e^c + a*b^2*f^3*e^c)$$

$$*x^3 + 3*(a^3*e*f^2*e^c + a*b^2*e*f^2*e^c)*x^2 + 3*(a^3*e^2*f*e^c + a*b^2*e^2*f*e^c)*x)*e^{(d*x)} / (a^2*b^2*e^{(2*d*x + 2*c)} + 2*a^3*b*e^{(d*x + c)} - a^2*b^2), x)$$

**Fricas [C]** time = 3.42364, size = 13009, normalized size = 18.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)\*coth(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\frac{1}{4}*(a^2*d^4*f^3*x^4 + 4*a^2*d^4*e*f^2*x^3 + 6*a^2*d^4*e^2*f*x^2 + 4*a^2*d^4*e^3*x + 8*a^2*c*d^3*e^3 - 12*a^2*c^2*d^2*e^2*f + 8*a^2*c^3*d*e*f^2 - 2*a^2*c^4*f^3 - (a^2*d^4*f^3*x^4 + 4*a^2*d^4*e*f^2*x^3 + 6*a^2*d^4*e^2*f*x^2 + 4*a^2*d^4*e^3*x + 8*a^2*c*d^3*e^3 - 12*a^2*c^2*d^2*e^2*f + 8*a^2*c^3*d*e*f^2 - 2*a^2*c^4*f^3)*\cosh(d*x + c)^2 - (a^2*d^4*f^3*x^4 + 4*a^2*d^4*e*f^2*x^3 + 6*a^2*d^4*e^2*f*x^2 + 4*a^2*d^4*e^3*x + 8*a^2*c*d^3*e^3 - 12*a^2*c^2*d^2*e^2*f + 8*a^2*c^3*d*e*f^2 - 2*a^2*c^4*f^3)*\sinh(d*x + c)^2 - 8*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*x + a*b*d^3*e^3)*\cosh(d*x + c) - 12*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f - ((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*\cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*\cosh(d*x + c)*\sinh(d*x + c) - ((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*\sinh(d*x + c)^2)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 12*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f - ((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*\cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*\cosh(d*x + c)*\sinh(d*x + c) - ((a^2 + b^2)*d^2*f^3*x^2 + 2*(a^2 + b^2)*d^2*e*f^2*x + (a^2 + b^2)*d^2*e^2*f)*\sinh(d*x + c)^2)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 12*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*f^2 - (b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*f^2 + 2*(b^2*d^2*e*f^2 - a*b*d*f^3)*x)*\cosh(d*x + c)^2 - 2*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*f^2 + 2*(b^2*d^2*e*f^2 - a*b*d*f^3)*x)*\sinh(d*x + c)^2 + 2*(b^2*d^2*e*f^2 - a*b*d*f^3)*x)*\operatorname{dilog}(\cosh(d*x + c) + \sinh(d*x + c)) + 12*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f + 2*a*b*d*e*f^2 - (b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f + 2*a*b*d*e*f^2 + 2*(b^2*d^2*e*f^2 + a*b*d$$

$$\begin{aligned}
& *f^3*x)*\cosh(d*x + c)^2 - 2*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f + 2*a*b*d*e*f \\
& ^2 + 2*(b^2*d^2*e*f^2 + a*b*d*f^3)*x)*\cosh(d*x + c)*\sinh(d*x + c) - (b^2*d^ \\
& 2*f^3*x^2 + b^2*d^2*e^2*f + 2*a*b*d*e*f^2 + 2*(b^2*d^2*e*f^2 + a*b*d*f^3)*x \\
& )*\sinh(d*x + c)^2 + 2*(b^2*d^2*e*f^2 + a*b*d*f^3)*x)*\operatorname{dilog}(-\cosh(d*x + c) - \\
& \sinh(d*x + c)) - 4*((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a \\
& ^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3 - ((a^2 + b^2)*d^3*e^3 - 3*(a^2 \\
& + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*\cosh \\
& (d*x + c)^2 - 2*((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + \\
& b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c) - ((a^ \\
& 2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - \\
& (a^2 + b^2)*c^3*f^3)*\sinh(d*x + c)^2*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x \\
& + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - 4*((a^2 + b^2)*d^3*e^3 - 3*(a^2 + \\
& b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3 - ((a^2 \\
& + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - ( \\
& a^2 + b^2)*c^3*f^3)*\cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2 \\
& )*c*d^2*e^2*f + 3*(a^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*\cosh(d*x + \\
& c)*\sinh(d*x + c) - ((a^2 + b^2)*d^3*e^3 - 3*(a^2 + b^2)*c*d^2*e^2*f + 3*(a \\
& ^2 + b^2)*c^2*d*e*f^2 - (a^2 + b^2)*c^3*f^3)*\sinh(d*x + c)^2*\log(2*b*\cosh( \\
& d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - 4*((a^2 + \\
& b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x \\
& + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3* \\
& f^3 - ((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2 \\
& )*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^ \\
& 2 + b^2)*c^3*f^3)*\cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b \\
& ^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - \\
& 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + \\
& c) - ((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2) \\
& *d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 \\
& + b^2)*c^3*f^3)*\sinh(d*x + c)^2*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + \\
& (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 4*((a^ \\
& 2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2* \\
& f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c \\
& ^3*f^3 - ((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + \\
& b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + \\
& (a^2 + b^2)*c^3*f^3)*\cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 \\
& + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2* \\
& f - 3*(a^2 + b^2)*c^2*d*e*f^2 + (a^2 + b^2)*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x \\
& + c) - ((a^2 + b^2)*d^3*f^3*x^3 + 3*(a^2 + b^2)*d^3*e*f^2*x^2 + 3*(a^2 + b \\
& ^2)*d^3*e^2*f*x + 3*(a^2 + b^2)*c*d^2*e^2*f - 3*(a^2 + b^2)*c^2*d*e*f^2 + ( \\
& a^2 + b^2)*c^3*f^3)*\sinh(d*x + c)^2*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c \\
& ) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + 4*( \\
& b^2*d^3*f^3*x^3 + b^2*d^3*e^3 + 3*a*b*d^2*e^2*f + 3*(b^2*d^3*e*f^2 + a*b*d^ \\
& 2*f^3)*x^2 - (b^2*d^3*f^3*x^3 + b^2*d^3*e^3 + 3*a*b*d^2*e^2*f + 3*(b^2*d^3* \\
& e*f^2 + a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f + 2*a*b*d^2*e*f^2)*x)*\cosh(d*x \\
& + c)^2 - 2*(b^2*d^3*f^3*x^3 + b^2*d^3*e^3 + 3*a*b*d^2*e^2*f + 3*(b^2*d^3*e*
\end{aligned}$$

$$\begin{aligned}
& f^2 + a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f + 2*a*b*d^2*e*f^2)*x)*\cosh(d*x + \\
& c)*\sinh(d*x + c) - (b^2*d^3*f^3*x^3 + b^2*d^3*e^3 + 3*a*b*d^2*e^2*f + 3*(b^ \\
& 2*d^3*e*f^2 + a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f + 2*a*b*d^2*e*f^2)*x)*\sin \\
& h(d*x + c)^2 + 3*(b^2*d^3*e^2*f + 2*a*b*d^2*e*f^2)*x)*\log(\cosh(d*x + c) + s \\
& inh(d*x + c) + 1) + 4*(b^2*d^3*e^3 - 3*(b^2*c + a*b)*d^2*e^2*f + 3*(b^2*c^2 \\
& + 2*a*b*c)*d*e*f^2 - (b^2*c^3 + 3*a*b*c^2)*f^3 - (b^2*d^3*e^3 - 3*(b^2*c + \\
& a*b)*d^2*e^2*f + 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 - (b^2*c^3 + 3*a*b*c^2)*f^3 \\
& )*\cosh(d*x + c)^2 - 2*(b^2*d^3*e^3 - 3*(b^2*c + a*b)*d^2*e^2*f + 3*(b^2*c^2 \\
& + 2*a*b*c)*d*e*f^2 - (b^2*c^3 + 3*a*b*c^2)*f^3)*\cosh(d*x + c)*\sinh(d*x + c \\
& ) - (b^2*d^3*e^3 - 3*(b^2*c + a*b)*d^2*e^2*f + 3*(b^2*c^2 + 2*a*b*c)*d*e*f^ \\
& 2 - (b^2*c^3 + 3*a*b*c^2)*f^3)*\sinh(d*x + c)^2)*\log(\cosh(d*x + c) + \sinh(d* \\
& x + c) - 1) + 4*(b^2*d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c \\
& )*d*e*f^2 + (b^2*c^3 + 3*a*b*c^2)*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 \\
& - (b^2*d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 + ( \\
& b^2*c^3 + 3*a*b*c^2)*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(b^2*d^3 \\
& *e^2*f - 2*a*b*d^2*e*f^2)*x)*\cosh(d*x + c)^2 - 2*(b^2*d^3*f^3*x^3 + 3*b^2*c \\
& *d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 + (b^2*c^3 + 3*a*b*c^2)*f^3 + 3* \\
& (b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f - 2*a*b*d^2*e*f^2)*x)* \\
& \cosh(d*x + c)*\sinh(d*x + c) - (b^2*d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2 \\
& *c^2 + 2*a*b*c)*d*e*f^2 + (b^2*c^3 + 3*a*b*c^2)*f^3 + 3*(b^2*d^3*e*f^2 - a* \\
& b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f - 2*a*b*d^2*e*f^2)*x)*\sinh(d*x + c)^2 + 3 \\
& *(b^2*d^3*e^2*f - 2*a*b*d^2*e*f^2)*x)*\log(-\cosh(d*x + c) - \sinh(d*x + c) + \\
& 1) + 24*((a^2 + b^2)*f^3*\cosh(d*x + c)^2 + 2*(a^2 + b^2)*f^3*\cosh(d*x + c)* \\
& \sinh(d*x + c) + (a^2 + b^2)*f^3*\sinh(d*x + c)^2 - (a^2 + b^2)*f^3)*\text{polylog}( \\
& 4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c)) \\
& )*\sqrt{(a^2 + b^2)/b^2})/b) + 24*((a^2 + b^2)*f^3*\cosh(d*x + c)^2 + 2*(a^2 + \\
& b^2)*f^3*\cosh(d*x + c)*\sinh(d*x + c) + (a^2 + b^2)*f^3*\sinh(d*x + c)^2 - ( \\
& a^2 + b^2)*f^3)*\text{polylog}(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x \\
& + c) + b*\sinh(d*x + c)))*\sqrt{(a^2 + b^2)/b^2})/b) - 24*(b^2*f^3*\cosh(d*x + \\
& c)^2 + 2*b^2*f^3*\cosh(d*x + c)*\sinh(d*x + c) + b^2*f^3*\sinh(d*x + c)^2 - b \\
& ^2*f^3)*\text{polylog}(4, \cosh(d*x + c) + \sinh(d*x + c)) - 24*(b^2*f^3*\cosh(d*x + \\
& c)^2 + 2*b^2*f^3*\cosh(d*x + c)*\sinh(d*x + c) + b^2*f^3*\sinh(d*x + c)^2 - b \\
& ^2*f^3)*\text{polylog}(4, -\cosh(d*x + c) - \sinh(d*x + c)) + 24*((a^2 + b^2)*d*f^3*x \\
& + (a^2 + b^2)*d*e*f^2 - ((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*\cosh(d \\
& *x + c)^2 - 2*((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*\cosh(d*x + c)*\sin \\
& h(d*x + c) - ((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*\sinh(d*x + c)^2)*\text{p} \\
& olylog(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d* \\
& x + c)))*\sqrt{(a^2 + b^2)/b^2})/b) + 24*((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d \\
& *e*f^2 - ((a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*\cosh(d*x + c)^2 - 2*(( \\
& a^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - ((a \\
& ^2 + b^2)*d*f^3*x + (a^2 + b^2)*d*e*f^2)*\sinh(d*x + c)^2)*\text{polylog}(3, (a*\cos \\
& h(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c)))*\sqrt{(a^ \\
& 2 + b^2)/b^2})/b) - 24*(b^2*d*f^3*x + b^2*d*e*f^2 - a*b*f^3 - (b^2*d*f^3*x \\
& + b^2*d*e*f^2 - a*b*f^3)*\cosh(d*x + c)^2 - 2*(b^2*d*f^3*x + b^2*d*e*f^2 - a \\
& *b*f^3)*\cosh(d*x + c)*\sinh(d*x + c) - (b^2*d*f^3*x + b^2*d*e*f^2 - a*b*f^3)
\end{aligned}$$



$$\begin{aligned} & * \sinh(dx + c)^2 * \text{polylog}(3, \cosh(dx + c) + \sinh(dx + c)) - 24 * (b^2 * d * f^3 * x + b^2 * d * e * f^2 + a * b * f^3 - (b^2 * d * f^3 * x + b^2 * d * e * f^2 + a * b * f^3) * \cosh(dx + c))^2 - 2 * (b^2 * d * f^3 * x + b^2 * d * e * f^2 + a * b * f^3) * \cosh(dx + c) * \sinh(dx + c) - (b^2 * d * f^3 * x + b^2 * d * e * f^2 + a * b * f^3) * \sinh(dx + c)^2 * \text{polylog}(3, -\cosh(dx + c) - \sinh(dx + c)) - 2 * (4 * a * b * d^3 * f^3 * x^3 + 12 * a * b * d^3 * e * f^2 * x^2 + 12 * a * b * d^3 * e^2 * f * x + 4 * a * b * d^3 * e^3 + (a^2 * d^4 * f^3 * x^4 + 4 * a^2 * d^4 * e * f^2 * x^3 + 6 * a^2 * d^4 * e^2 * f * x^2 + 4 * a^2 * d^4 * e^3 * x + 8 * a^2 * c * d^3 * e^3 - 12 * a^2 * c^2 * d^2 * e^2 * f + 8 * a^2 * c^3 * d * e * f^2 - 2 * a^2 * c^4 * f^3) * \cosh(dx + c)) * \sinh(dx + c)) / (a^2 * b * d^4 * \cosh(dx + c)^2 + 2 * a^2 * b * d^4 * \cosh(dx + c) * \sinh(dx + c) + a^2 * b * d^4 * \sinh(dx + c)^2 - a^2 * b * d^4) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cosh(d\*x+c)\*coth(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cosh(d\*x+c)\*coth(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.460 \quad \int \frac{(e+fx)^2 \cosh(c+dx) \coth^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=518

$$\frac{2f(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^2} + \frac{2f(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2bd^2} - \frac{2f^2(a^2+b^2)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2bd^3}$$

```
[Out] (b*(e + f*x)^3)/(3*a^2*f) - ((a^2 + b^2)*(e + f*x)^3)/(3*a^2*b*f) - (4*f*(e
+ f*x)*ArcTanh[E^(c + d*x)]/(a*d^2) - ((e + f*x)^2*Csch[c + d*x])/(a*d) +
((a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
a^2*b*d) + ((a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2])])/(a^2*b*d) - (b*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a^2*d) - (2*
f^2*PolyLog[2, -E^(c + d*x)]/(a*d^3) + (2*f^2*PolyLog[2, E^(c + d*x)]/(a*
d^3) + (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/(a^2*b*d^2) + (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^2) - (b*f*(e + f*x)*PolyLog[2, E^
(2*(c + d*x))])/(a^2*d^2) - (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/(a^2*b*d^3) - (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^3) + (b*f^2*PolyLog[3, E^(2*
(c + d*x))])/(2*a^2*d^3)
```

**Rubi [A]** time = 1.28673, antiderivative size = 518, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 17, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$ , Rules used = {5585, 5450, 3296, 2637, 5452, 4182, 2279, 2391, 5446, 3310, 3716, 2190, 2531, 2282, 6589, 5565, 5561}

$$\frac{2f(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2bd^2} + \frac{2f(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2bd^2} - \frac{2f^2(a^2+b^2)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*(e + f*x)^3)/(3*a^2*f) - ((a^2 + b^2)*(e + f*x)^3)/(3*a^2*b*f) - (4*f*(e
+ f*x)*ArcTanh[E^(c + d*x)]/(a*d^2) - ((e + f*x)^2*Csch[c + d*x])/(a*d) +
((a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
a^2*b*d) + ((a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 +
b^2])])/(a^2*b*d) - (b*(e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a^2*d) - (2*
f^2*PolyLog[2, -E^(c + d*x)]/(a*d^3) + (2*f^2*PolyLog[2, E^(c + d*x)]/(a*
d^3) + (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/(a^2*b*d^2) + (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^2) - (b*f*(e + f*x)*PolyLog[2, E^
(2*(c + d*x))])/(a^2*d^2) - (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))
/(a - Sqrt[a^2 + b^2]))])/(a^2*b*d^3) - (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*b*d^3) + (b*f^2*PolyLog[3, E^(2*
(c + d*x))])/(2*a^2*d^3)
```

$$\frac{2 + b^2)}}{(a^2 b d^2) + (2(a^2 + b^2) f (e + f x) \text{PolyLog}[2, -((b E^{(c + d x)}) / (a + \sqrt{a^2 + b^2}))]) / (a^2 b d^2) - (b f (e + f x) \text{PolyLog}[2, E^{(2(c + d x))}] / (a^2 d^2) - (2(a^2 + b^2) f^2 \text{PolyLog}[3, -((b E^{(c + d x)}) / (a - \sqrt{a^2 + b^2}))]) / (a^2 b d^3) - (2(a^2 + b^2) f^2 \text{PolyLog}[3, -((b E^{(c + d x)}) / (a + \sqrt{a^2 + b^2}))]) / (a^2 b d^3) + (b f^2 \text{PolyLog}[3, E^{(2(c + d x))}] / (2 a^2 d^3))$$

### Rule 5585

$$\text{Int}[(\text{Cosh}[(c_.) + (d_.)(x_.)]^{(p_.)} \text{Coth}[(c_.) + (d_.)(x_.)]^{(n_.)} ((e_.) + (f_.)(x_.))^{(m_.)}) / ((a_.) + (b_.) \text{Sinh}[(c_.) + (d_.)(x_.)]), x\_Symbol] \rightarrow \text{Dist}[1/a, \text{Int}[(e + f x)^m \text{Cosh}[c + d x]^p \text{Coth}[c + d x]^n, x], x] - \text{Dist}[b/a, \text{Int}[(e + f x)^m \text{Cosh}[c + d x]^{(p + 1)} \text{Coth}[c + d x]^{(n - 1)} / (a + b \text{Sinh}[c + d x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$$

### Rule 5450

$$\text{Int}[\text{Cosh}[(a_.) + (b_.)(x_.)]^{(n_.)} \text{Coth}[(a_.) + (b_.)(x_.)]^{(p_.)} ((c_.) + (d_.)(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Int}[(c + d x)^m \text{Cosh}[a + b x]^n \text{Coth}[a + b x]^{(p - 2)}, x] + \text{Int}[(c + d x)^m \text{Cosh}[a + b x]^{(n - 2)} \text{Coth}[a + b x]^p, x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$$

### Rule 3296

$$\text{Int}[(c_.) + (d_.)(x_.)]^{(m_.)} \sin[(e_.) + (f_.)(x_.)], x\_Symbol] \rightarrow -\text{Simp}[(c + d x)^m \text{Cos}[e + f x] / f, x] + \text{Dist}[(d m) / f, \text{Int}[(c + d x)^{(m - 1)} \text{Cos}[e + f x], x], x] /; \text{FreeQ}\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$$

### Rule 2637

$$\text{Int}[\sin[\text{Pi}/2 + (c_.) + (d_.)(x_.)], x\_Symbol] \rightarrow \text{Simp}[\sin[c + d x] / d, x] /; \text{FreeQ}\{c, d\}, x]$$

### Rule 5452

$$\text{Int}[\text{Coth}[(a_.) + (b_.)(x_.)]^{(p_.)} \text{Csch}[(a_.) + (b_.)(x_.)]^{(n_.)} ((c_.) + (d_.)(x_.))^{(m_.)}, x\_Symbol] \rightarrow -\text{Simp}[(c + d x)^m \text{Csch}[a + b x]^n / (b n), x] + \text{Dist}[(d m) / (b n), \text{Int}[(c + d x)^{(m - 1)} \text{Csch}[a + b x]^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[p, 1] \&\& \text{GtQ}[m, 0]$$

### Rule 4182

$$\text{Int}[\text{csc}[(e_.) + (\text{Complex}[0, fz\_]) (f_.)(x_.)] ((c_.) + (d_.)(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(-2(c + d x)^m \text{ArcTanh}[E^{-(I e) + f fz x}]) / (f fz I), x]$$

+ (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 5446

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sinh[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Simp[((c + d\*x)^m\*Sinh[a + b\*x]^(n + 1))/(b\*(n + 1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*Sinh[a + b\*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

### Rule 3310

Int[((c\_.) + (d\_.)\*(x\_))\*((b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(d\*(b\*Sine[e + f\*x])^n)/(f^2\*n^2), x] + (Dist[(b^2\*(n - 1))/n, Int[(c + d\*x)\*(b\*Sine[e + f\*x])^(n - 2), x], x] - Simp[(b\*(c + d\*x)\*Cos[e + f\*x]\*(b\*Sine[e + f\*x])^(n - 1))/(f\*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

### Rule 3716

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)], x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-(I\*e) + f\*fz\*x)))/(E^(2\*I\*k\*Pi)\*(1 + E^(2\*(-(I\*e) + f\*fz\*x)))/E^(2\*I\*k\*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]]/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a]], x]

))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_))))^(n\_.)]\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)][v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 5565

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Dist[a/b^2, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

### Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \cosh(c+dx) \coth^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \cosh(c+dx) \coth^2(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^2 \cosh^2(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= \frac{\int (e+fx)^2 \cosh(c+dx) dx}{a} + \frac{\int (e+fx)^2 \coth(c+dx) \operatorname{csch}(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^2 \cosh^2(c+dx) \coth(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= -\frac{(e+fx)^2 \operatorname{csch}(c+dx)}{ad} + \frac{(e+fx)^2 \sinh(c+dx)}{ad} - \frac{\int (e+fx)^2 \cosh(c+dx) dx}{a} \\
&= \frac{b(e+fx)^3}{3a^2f} - \frac{(a^2+b^2)(e+fx)^3}{3a^2bf} - \frac{4f(e+fx) \tanh^{-1}(e^{c+dx})}{ad^2} - \frac{2f(e+fx)^2 c}{a} \\
&= \frac{b(e+fx)^3}{3a^2f} - \frac{(a^2+b^2)(e+fx)^3}{3a^2bf} - \frac{4f(e+fx) \tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+fx)^2 c}{a} \\
&= \frac{b(e+fx)^3}{3a^2f} - \frac{(a^2+b^2)(e+fx)^3}{3a^2bf} - \frac{4f(e+fx) \tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+fx)^2 c}{a} \\
&= \frac{b(e+fx)^3}{3a^2f} - \frac{(a^2+b^2)(e+fx)^3}{3a^2bf} - \frac{4f(e+fx) \tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+fx)^2 c}{a} \\
&= \frac{b(e+fx)^3}{3a^2f} - \frac{(a^2+b^2)(e+fx)^3}{3a^2bf} - \frac{4f(e+fx) \tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+fx)^2 c}{a} \\
&= \frac{b(e+fx)^3}{3a^2f} - \frac{(a^2+b^2)(e+fx)^3}{3a^2bf} - \frac{4f(e+fx) \tanh^{-1}(e^{c+dx})}{ad^2} - \frac{(e+fx)^2 c}{a}
\end{aligned}$$

**Mathematica [B]** time = 14.8317, size = 1720, normalized size = 3.32

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] -((a^2 + b^2)*(6*e^2*E^(2*c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 +
```

$$\begin{aligned}
& E^{(2*(c + d*x))})/d - (3*e^{2*E^{(2*c)}}*Log[2*a*E^{(c + d*x)} + b*(-1 + E^{(2*(c + d*x))})])/d + (6*e*f*x*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d - (6*e*e^{(2*c)}*f*x*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d + (3*f^2*x^2*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d - (3*E^{(2*c)}*f^2*x^2*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d + (6*e*f*x*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d - (6*e*e^{(2*c)}*f*x*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d + (3*f^2*x^2*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d - (3*E^{(2*c)}*f^2*x^2*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d - (6*(-1 + E^{(2*c)})*f*(e + f*x)*PolyLog[2, -((b*E^{(2*c + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^2 - (6*(-1 + E^{(2*c)})*f*(e + f*x)*PolyLog[2, -((b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^2 - (6*f^2*PolyLog[3, -((b*E^{(2*c + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^3 + (6*E^{(2*c)}*f^2*PolyLog[3, -((b*E^{(2*c + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^3 - (6*f^2*PolyLog[3, -((b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^3 + (6*E^{(2*c)}*f^2*PolyLog[3, -((b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^3)))/(3*a^2*b*(-1 + E^{(2*c)})) + (b*d^3*(e + f*x)^3*(-1 + Coth[c]) + 3*d*e*f*(b*d*e - 2*a*f)*(d*x - Log[1 - Cosh[c + d*x] - Sinh[c + d*x]]) - 6*d*f^2*(b*d*e + a*f)*x*Log[1 + Cosh[c + d*x] - Sinh[c + d*x]] - 3*b*d^2*f^3*x^2*Log[1 + Cosh[c + d*x] - Sinh[c + d*x]] - 6*d*f^2*(b*d*e - a*f)*x*Log[1 - Cosh[c + d*x] + Sinh[c + d*x]] - 3*b*d^2*f^3*x^2*Log[1 - Cosh[c + d*x] + Sinh[c + d*x]] + 3*d*e*f*(b*d*e + 2*a*f)*(d*x - Log[1 + Cosh[c + d*x] + Sinh[c + d*x]]) + 6*f^2*(b*d*e - a*f)*PolyLog[2, Cosh[c + d*x] - Sinh[c + d*x]] + 6*f^2*(b*d*e + a*f)*PolyLog[2, -Cosh[c + d*x] + Sinh[c + d*x]] + 6*b*f^3*(d*x*PolyLog[2, Cosh[c + d*x] - Sinh[c + d*x]] + PolyLog[3, Cosh[c + d*x] - Sinh[c + d*x]]) + 6*b*f^3*(d*x*PolyLog[2, -Cosh[c + d*x] + Sinh[c + d*x]] + PolyLog[3, -Cosh[c + d*x] + Sinh[c + d*x]]))/(3*a^2*d^3*f) + ((-3*b*e^2 - 6*b*e*f*x - 3*b*f^2*x^2 + 3*a*d*e^2*x*Cosh[c] + 3*a*d*e*f*x^2*Cosh[c] + a*d*f^2*x^3*Cosh[c])*Csch[c/2]*Sech[c/2])/(6*a*b*d) + (Csch[c/2]*Csch[c/2 + (d*x)/2]*(e^2*Sinh[(d*x)/2] + 2*e*f*x*Sinh[(d*x)/2] + f^2*x^2*Sinh[(d*x)/2]))/(2*a*d) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(e^2*Sinh[(d*x)/2] + 2*e*f*x*Sinh[(d*x)/2] + f^2*x^2*Sinh[(d*x)/2]))/(2*a*d)
\end{aligned}$$

**Maple [F]** time = 0.81, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \cosh(dx + c) (\coth(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*cosh(d\*x+c)\*coth(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out]  $\int (f*x+e)^2 * \cosh(d*x+c) * \coth(d*x+c)^2 / (a+b*\sinh(d*x+c)), x$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] 
$$e^{2*((d*x + c)/(b*d) + 2*e^{-d*x - c}/((a*e^{-2*d*x - 2*c}) - a)*d) - b*\log(e^{-d*x - c} + 1)/(a^2*d) - b*\log(e^{-d*x - c} - 1)/(a^2*d) + (a^2 + b^2)*\log(-2*a*e^{-d*x - c} + b*e^{-2*d*x - 2*c} - b)/(a^2*b*d) - 1/3*(a*d*f^2*x^3 + 3*a*d*e*f*x^2 - (a*d*f^2*x^3*e^{2*c} + 3*a*d*e*f*x^2*e^{2*c}))*e^{2*d*x} + 6*(b*f^2*x^2*e^c + 2*b*e*f*x*e^c)*e^{d*x})/(a*b*d*e^{(2*d*x + 2*c)} - a*b*d) - 2*e*f*\log(e^{d*x + c} + 1)/(a*d^2) + 2*e*f*\log(e^{d*x + c} - 1)/(a*d^2) - (d^2*x^2*\log(e^{d*x + c} + 1) + 2*d*x*dilog(-e^{d*x + c})) - 2*polylog(3, -e^{d*x + c}))*b*f^2/(a^2*d^3) - (d^2*x^2*\log(-e^{d*x + c} + 1) + 2*d*x*dilog(e^{d*x + c})) - 2*polylog(3, e^{d*x + c}))*b*f^2/(a^2*d^3) - 2*(b*d*e*f + a*f^2)*(d*x*\log(e^{d*x + c} + 1) + dilog(-e^{d*x + c}))/a^2*d^3 - 2*(b*d*e*f - a*f^2)*(d*x*\log(-e^{d*x + c} + 1) + dilog(e^{d*x + c}))/a^2*d^3 + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*e*f + a*f^2)*d^2*x^2)/a^2*d^3 + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*e*f - a*f^2)*d^2*x^2)/a^2*d^3 - \int (-2*((a^2*b*f^2 + b^3*f^2)*x^2 + 2*(a^2*b*e*f + b^3*e*f)*x - ((a^3*f^2*e^c + a*b^2*f^2*e^c)*x^2 + 2*(a^3*e*f*e^c + a*b^2*e*f*e^c)*x)*e^{d*x})/(a^2*b^2*e^{(2*d*x + 2*c)} + 2*a^3*b*e^{d*x + c} - a^2*b^2), x$$

**Fricas [C]** time = 2.94505, size = 8101, normalized size = 15.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] 
$$1/3*(a^2*d^3*f^2*x^3 + 3*a^2*d^3*e*f*x^2 + 3*a^2*d^3*e^2*x + 6*a^2*c*d^2*e^2 - 6*a^2*c^2*d*e*f + 2*a^2*c^3*f^2 - (a^2*d^3*f^2*x^3 + 3*a^2*d^3*e*f*x^2 + 3*a^2*d^3*e^2*x + 6*a^2*c*d^2*e^2 - 6*a^2*c^2*d*e*f + 2*a^2*c^3*f^2)*\cosh$$



$$\begin{aligned}
& (d*x + c)^2 - (a^2*d^3*f^2*x^3 + 3*a^2*d^3*e*f*x^2 + 3*a^2*d^3*e^2*x + 6*a^2*c*d^2*e^2 - 6*a^2*c^2*d*e*f + 2*a^2*c^3*f^2)*\sinh(d*x + c)^2 - 6*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + a*b*d^2*e^2)*\cosh(d*x + c) - 6*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f - ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*\cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c) - ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*\sinh(d*x + c)^2)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 6*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f - ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*\cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c) - ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f)*\sinh(d*x + c)^2)*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 6*(b^2*d*f^2*x + b^2*d*e*f - a*b*f^2 - (b^2*d*f^2*x + b^2*d*e*f - a*b*f^2)*\cosh(d*x + c)^2 - 2*(b^2*d*f^2*x + b^2*d*e*f - a*b*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - (b^2*d*f^2*x + b^2*d*e*f - a*b*f^2)*\sinh(d*x + c)^2)*\operatorname{dilog}(\cosh(d*x + c) + \sinh(d*x + c)) + 6*(b^2*d*f^2*x + b^2*d*e*f + a*b*f^2 - (b^2*d*f^2*x + b^2*d*e*f + a*b*f^2)*\cosh(d*x + c)^2 - 2*(b^2*d*f^2*x + b^2*d*e*f + a*b*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - (b^2*d*f^2*x + b^2*d*e*f + a*b*f^2)*\sinh(d*x + c)^2)*\operatorname{dilog}(-\cosh(d*x + c) - \sinh(d*x + c)) - 3*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2 - ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\sinh(d*x + c)^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - 3*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2 - ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\sinh(d*x + c)^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) - 3*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2 - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\sinh(d*x + c)^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 3*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2 - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\sinh(d*x + c)^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 3*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2 - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\sinh(d*x + c)^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b)
\end{aligned}$$

$$\begin{aligned} & \sqrt{(a^2 + b^2)/b^2} - b)/b) + 3*(b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d* \\ & e*f - (b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d*e*f + 2*(b^2*d^2*e*f + a*b*d \\ & *f^2)*x)*\cosh(d*x + c)^2 - 2*(b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d*e*f + \\ & 2*(b^2*d^2*e*f + a*b*d*f^2)*x)*\cosh(d*x + c)*\sinh(d*x + c) - (b^2*d^2*f^2*x^2 \\ & + b^2*d^2*e^2 + 2*a*b*d*e*f + 2*(b^2*d^2*e*f + a*b*d*f^2)*x)*\sinh(d*x + \\ & c)^2 + 2*(b^2*d^2*e*f + a*b*d*f^2)*x)*\log(\cosh(d*x + c) + \sinh(d*x + c) + \\ & 1) + 3*(b^2*d^2*e^2 - 2*(b^2*c + a*b)*d*e*f + (b^2*c^2 + 2*a*b*c)*f^2 - (b^2 \\ & *d^2*e^2 - 2*(b^2*c + a*b)*d*e*f + (b^2*c^2 + 2*a*b*c)*f^2)*\cosh(d*x + c)^2 \\ & - 2*(b^2*d^2*e^2 - 2*(b^2*c + a*b)*d*e*f + (b^2*c^2 + 2*a*b*c)*f^2)*\cosh(d \\ & *x + c)*\sinh(d*x + c) - (b^2*d^2*e^2 - 2*(b^2*c + a*b)*d*e*f + (b^2*c^2 + \\ & 2*a*b*c)*f^2)*\sinh(d*x + c)^2*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 3*( \\ & b^2*d^2*f^2*x^2 + 2*b^2*c*d*e*f - (b^2*c^2 + 2*a*b*c)*f^2 - (b^2*d^2*f^2*x^2 \\ & + 2*b^2*c*d*e*f - (b^2*c^2 + 2*a*b*c)*f^2 + 2*(b^2*d^2*e*f - a*b*d*f^2)*x) \\ & )*\cosh(d*x + c)^2 - 2*(b^2*d^2*f^2*x^2 + 2*b^2*c*d*e*f - (b^2*c^2 + 2*a*b*c) \\ & )*f^2 + 2*(b^2*d^2*e*f - a*b*d*f^2)*x)*\cosh(d*x + c)*\sinh(d*x + c) - (b^2*d \\ & ^2*f^2*x^2 + 2*b^2*c*d*e*f - (b^2*c^2 + 2*a*b*c)*f^2 + 2*(b^2*d^2*e*f - a*b \\ & *d*f^2)*x)*\sinh(d*x + c)^2 + 2*(b^2*d^2*e*f - a*b*d*f^2)*x)*\log(-\cosh(d*x + \\ & c) - \sinh(d*x + c) + 1) - 6*((a^2 + b^2)*f^2*\cosh(d*x + c)^2 + 2*(a^2 + b^2) \\ & *f^2*\cosh(d*x + c)*\sinh(d*x + c) + (a^2 + b^2)*f^2*\sinh(d*x + c)^2 - (a^2 \\ & + b^2)*f^2)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + \\ & c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) - 6*((a^2 + b^2)*f^2*\cosh(d \\ & *x + c)^2 + 2*(a^2 + b^2)*f^2*\cosh(d*x + c)*\sinh(d*x + c) + (a^2 + b^2)*f^2 \\ & *\sinh(d*x + c)^2 - (a^2 + b^2)*f^2)*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d* \\ & x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) + 6* \\ & (b^2*f^2*\cosh(d*x + c)^2 + 2*b^2*f^2*\cosh(d*x + c)*\sinh(d*x + c) + b^2*f^2* \\ & \sinh(d*x + c)^2 - b^2*f^2)*\text{polylog}(3, \cosh(d*x + c) + \sinh(d*x + c)) + 6*(b \\ & ^2*f^2*\cosh(d*x + c)^2 + 2*b^2*f^2*\cosh(d*x + c)*\sinh(d*x + c) + b^2*f^2*si \\ & nh(d*x + c)^2 - b^2*f^2)*\text{polylog}(3, -\cosh(d*x + c) - \sinh(d*x + c)) - 2*(3* \\ & a*b*d^2*f^2*x^2 + 6*a*b*d^2*e*f*x + 3*a*b*d^2*e^2 + (a^2*d^3*f^2*x^3 + 3*a^2 \\ & *d^3*e*f*x^2 + 3*a^2*d^3*e^2*x + 6*a^2*c*d^2*e^2 - 6*a^2*c^2*d*e*f + 2*a^2 \\ & *c^3*f^2)*\cosh(d*x + c))*\sinh(d*x + c))/(a^2*b*d^3*\cosh(d*x + c)^2 + 2*a^2* \\ & b*d^3*\cosh(d*x + c)*\sinh(d*x + c) + a^2*b*d^3*\sinh(d*x + c)^2 - a^2*b*d^3) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*cosh(d\*x+c)\*coth(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm  
m="giac")
```

```
[Out] Timed out
```

$$3.461 \quad \int \frac{(e+fx) \cosh(c+dx) \coth^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=324

$$\frac{f(a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{a^2 b d^2} + \frac{f(a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right)}{a^2 b d^2} - \frac{b f \operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{2 a^2 d^2} + \frac{(a^2 + b^2)(e + f x)}{a^2 b d^2}$$

[Out] (b\*(e + f\*x)^2)/(2\*a^2\*f) - ((a^2 + b^2)\*(e + f\*x)^2)/(2\*a^2\*b\*f) - (f\*ArcTanh[Cosh[c + d\*x]])/(a\*d^2) - ((e + f\*x)\*Csch[c + d\*x])/(a\*d) + ((a^2 + b^2)\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^2\*b\*d) + ((a^2 + b^2)\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^2\*b\*d) - (b\*(e + f\*x)\*Log[1 - E^(2\*(c + d\*x))])/(a^2\*d) + ((a^2 + b^2)\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^2\*b\*d^2) + ((a^2 + b^2)\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^2\*b\*d^2) - (b\*f\*PolyLog[2, E^(2\*(c + d\*x))])/(2\*a^2\*d^2)

**Rubi [A]** time = 0.742632, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 15, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.469$ , Rules used = {5585, 5450, 3296, 2638, 5452, 3770, 5446, 2635, 8, 3716, 2190, 2279, 2391, 5565, 5561}

$$\frac{f(a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{a^2 b d^2} + \frac{f(a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right)}{a^2 b d^2} - \frac{b f \operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{2 a^2 d^2} + \frac{(a^2 + b^2)(e + f x)}{a^2 b d^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Cosh[c + d\*x]\*Coth[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (b\*(e + f\*x)^2)/(2\*a^2\*f) - ((a^2 + b^2)\*(e + f\*x)^2)/(2\*a^2\*b\*f) - (f\*ArcTanh[Cosh[c + d\*x]])/(a\*d^2) - ((e + f\*x)\*Csch[c + d\*x])/(a\*d) + ((a^2 + b^2)\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^2\*b\*d) + ((a^2 + b^2)\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^2\*b\*d) - (b\*(e + f\*x)\*Log[1 - E^(2\*(c + d\*x))])/(a^2\*d) + ((a^2 + b^2)\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^2\*b\*d^2) + ((a^2 + b^2)\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^2\*b\*d^2) - (b\*f\*PolyLog[2, E^(2\*(c + d\*x))])/(2\*a^2\*d^2)

Rule 5585

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(p_.)*Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) +
(f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
0] && IGtQ[p, 0]
```

### Rule 5450

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*Coth[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] := Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

### Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

### Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

### Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

### Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_.)]\*((e\_.) + (f\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*Sin h[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(e + fx) \cosh(c + dx) \coth^2(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\int (e + fx) \cosh(c + dx) \coth^2(c + dx) dx}{a} - \frac{b \int \frac{(e + fx) \cosh^2(c + dx) \coth(c + dx)}{a + b \sinh(c + dx)} dx}{a} \\
 &= \frac{\int (e + fx) \cosh(c + dx) dx}{a} + \frac{\int (e + fx) \coth(c + dx) \operatorname{csch}(c + dx) dx}{a} - \frac{b \int \frac{(e + fx) \cosh^2(c + dx) \coth(c + dx)}{a + b \sinh(c + dx)} dx}{a} \\
 &= -\frac{(e + fx) \operatorname{csch}(c + dx)}{ad} + \frac{(e + fx) \sinh(c + dx)}{ad} - \frac{\int (e + fx) \cosh(c + dx) dx}{a} \\
 &= \frac{b(e + fx)^2}{2a^2 f} - \frac{(a^2 + b^2)(e + fx)^2}{2a^2 bf} - \frac{f \tanh^{-1}(\cosh(c + dx))}{ad^2} - \frac{f \cosh(c + dx)}{ad^2} \\
 &= \frac{b(e + fx)^2}{2a^2 f} - \frac{(a^2 + b^2)(e + fx)^2}{2a^2 bf} - \frac{f \tanh^{-1}(\cosh(c + dx))}{ad^2} - \frac{(e + fx) \operatorname{csch}(c + dx)}{ad} \\
 &= \frac{b(e + fx)^2}{2a^2 f} - \frac{(a^2 + b^2)(e + fx)^2}{2a^2 bf} - \frac{f \tanh^{-1}(\cosh(c + dx))}{ad^2} - \frac{(e + fx) \operatorname{csch}(c + dx)}{ad} \\
 &= \frac{b(e + fx)^2}{2a^2 f} - \frac{(a^2 + b^2)(e + fx)^2}{2a^2 bf} - \frac{f \tanh^{-1}(\cosh(c + dx))}{ad^2} - \frac{(e + fx) \operatorname{csch}(c + dx)}{ad}
 \end{aligned}$$

**Mathematica [A]** time = 2.5848, size = 313, normalized size = 0.97

$$\frac{2(a^2 + b^2) \left( f \operatorname{PolyLog} \left( 2, \frac{be^{c+dx}}{\sqrt{a^2 + b^2 - a}} \right) + f \operatorname{PolyLog} \left( 2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2 + a}} \right) + f(c+dx) \log \left( \frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}} + 1 \right) + f(c+dx) \log \left( \frac{be^{c+dx}}{\sqrt{a^2 + b^2 + a}} + 1 \right) + de \log(a + b \sinh(c + dx)) - cf \log(a + b \sinh(c + dx)) \right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Cosh[c + d*x]*Coth[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-(a*d*(e + f*x)*Coth[(c + d*x)/2]) - 2*b*d*e*Log[Sinh[c + d*x]] + 2*b*c*f*
Log[Sinh[c + d*x]] + 2*a*f*Log[Tanh[(c + d*x)/2]] + b*f*(-((c + d*x)*(c + d
*x + 2*Log[1 - E^(-2*(c + d*x))])) + PolyLog[2, E^(-2*(c + d*x))]) + (2*(a^
2 + b^2)*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 + b^2]]) + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])
+ d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[
2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]]) + f*PolyLog[2, -((b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2]))))/b + a*d*(e + f*x)*Tanh[(c + d*x)/2]/(2*a^2*d^2)
```

**Maple [B]** time = 0.23, size = 938, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] -1/2*f*x^2/b+1/a^2/d^2*b*f*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2
+b^2)^(1/2)))+1/a^2/d^2*b*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+
b^2)^(1/2)))+1/a^2/d*b*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/d^2/b*f*c^
2+1/d/b*e*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-2/d/b*e*ln(exp(d*x+c))+1/d^
2/b*f*dilog((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))+1/d^2/b*f
*dilog((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))+1/a^2/d*b*f*
ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1/a^2/d^2*b*f*ln
((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/a^2/d^2*b*f*c*ln
(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+1/a^2/d*b*f*ln((-b*exp(d*x+c)+(a^2+b^2)
^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*x+1/a^2/d^2*b*f*ln((-b*exp(d*x+c)+(a^2+b^2)
^(1/2)-a)/(-a+(a^2+b^2)^(1/2)))*c+1/a^2/d^2*b*f*c*ln(exp(d*x+c)-1)-1/a^2/d*
b*f*ln(exp(d*x+c)+1)*x+e*x/b-1/a^2/d*b*e*ln(exp(d*x+c)-1)-1/a^2/d*b*e*ln(ex
p(d*x+c)+1)+1/a^2/d^2*b*f*dilog(exp(d*x+c))-1/a^2/d^2*b*f*dilog(exp(d*x+c)+
1)-2/d/b*f*c*x-1/d^2/b*f*c*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)+2/d^2/b*f*
c*ln(exp(d*x+c))+1/d/b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)
^(1/2)))*x+1/d^2/b*f*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/
2)))*c+1/d/b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*x+1
/d^2/b*f*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c+1/d^2/a
*f*ln(exp(d*x+c)-1)-1/d^2/a*f*ln(exp(d*x+c)+1)-2/d*(f*x+e)/a*exp(d*x+c)/(ex
p(2*d*x+2*c)-1)
```





```

x + c)^2 - b^2*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) - 2*(b^2*f*cosh(d*x
+ c)^2 + 2*b^2*f*cosh(d*x + c)*sinh(d*x + c) + b^2*f*sinh(d*x + c)^2 - b^2*
f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) - 2*((a^2 + b^2)*d*e - (a^2 + b^2)
*c*f - ((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)^2 - 2*((a^2 + b^2)
*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2 + b^2)*d*e - (a
^2 + b^2)*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) +
2*b*sqrt((a^2 + b^2)/b^2) + 2*a) - 2*((a^2 + b^2)*d*e - (a^2 + b^2)*c*f -
((a^2 + b^2)*d*e - (a^2 + b^2)*c*f)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*e -
(a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2 + b^2)*d*e - (a^2 + b^
2)*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sq
rt((a^2 + b^2)/b^2) + 2*a) - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f - ((a^2
+ b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f*x + (
a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2 + b^2)*d*f*x + (a^2 + b
^2)*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - 2*((a^2 + b^2)
*d*f*x + (a^2 + b^2)*c*f - ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x +
c)^2 - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)
- ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x
+ c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^
2)/b^2) - b)/b) + 2*(b^2*d*f*x + b^2*d*e + a*b*f - (b^2*d*f*x + b^2*d*e + a
*b*f)*cosh(d*x + c)^2 - 2*(b^2*d*f*x + b^2*d*e + a*b*f)*cosh(d*x + c)*sinh(
d*x + c) - (b^2*d*f*x + b^2*d*e + a*b*f)*sinh(d*x + c)^2)*log(cosh(d*x + c)
+ sinh(d*x + c) + 1) + 2*(b^2*d*e - (b^2*d*e - (b^2*c + a*b)*f)*cosh(d*x +
c)^2 - 2*(b^2*d*e - (b^2*c + a*b)*f)*cosh(d*x + c)*sinh(d*x + c) - (b^2*d*
e - (b^2*c + a*b)*f)*sinh(d*x + c)^2 - (b^2*c + a*b)*f)*log(cosh(d*x + c) +
sinh(d*x + c) - 1) + 2*(b^2*d*f*x + b^2*c*f - (b^2*d*f*x + b^2*c*f)*cosh(d
*x + c)^2 - 2*(b^2*d*f*x + b^2*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b^2*d*f*
x + b^2*c*f)*sinh(d*x + c)^2)*log(-cosh(d*x + c) - sinh(d*x + c) + 1) - 2*(
2*a*b*d*f*x + 2*a*b*d*e + (a^2*d^2*f*x^2 + 2*a^2*d^2*e*x + 4*a^2*c*d*e - 2*
a^2*c^2*f)*cosh(d*x + c))*sinh(d*x + c))/(a^2*b*d^2*cosh(d*x + c)^2 + 2*a^2
*b*d^2*cosh(d*x + c)*sinh(d*x + c) + a^2*b*d^2*sinh(d*x + c)^2 - a^2*b*d^2)

```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \cosh(c + dx) \coth^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*cosh(d\*x+c)\*coth(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*cosh(c + d\*x)\*coth(c + d\*x)\*\*2/(a + b\*sinh(c + d\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \cosh(dx + c) \coth(dx + c)^2}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*cosh(d*x+c)*coth(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*cosh(d*x + c)*coth(d*x + c)^2/(b*sinh(d*x + c) + a), x)
```

$$3.462 \quad \int \frac{\cosh(c+dx) \coth^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=59

$$\frac{(a^2 + b^2) \log(a + b \sinh(c + dx))}{a^2 b d} - \frac{b \log(\sinh(c + dx))}{a^2 d} - \frac{\operatorname{csch}(c + dx)}{a d}$$

[Out] -(Csch[c + d\*x]/(a\*d)) - (b\*Log[Sinh[c + d\*x]]/(a^2\*d) + ((a^2 + b^2)\*Log[a + b\*Sinh[c + d\*x]]/(a^2\*b\*d)

**Rubi [A]** time = 0.122573, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2837, 12, 894}

$$\frac{(a^2 + b^2) \log(a + b \sinh(c + dx))}{a^2 b d} - \frac{b \log(\sinh(c + dx))}{a^2 d} - \frac{\operatorname{csch}(c + dx)}{a d}$$

Antiderivative was successfully verified.

[In] Int[(Cosh[c + d\*x]\*Coth[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] -(Csch[c + d\*x]/(a\*d)) - (b\*Log[Sinh[c + d\*x]]/(a^2\*d) + ((a^2 + b^2)\*Log[a + b\*Sinh[c + d\*x]]/(a^2\*b\*d)

### Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_.)]^(p_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)
*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

### Rule 12

```
Int[(a_.)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_.)*(v_) /; FreeQ[b, x]]
```

### Rule 894

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))^(n_.)*((a_.) + (c_.)*(x_.)
^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

$\int \frac{\cosh^2(c+dx) \coth^2(c+dx)}{a+b \sinh(c+dx)} dx$  ; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

### Rubi steps

$$\begin{aligned} \int \frac{\cosh(c+dx) \coth^2(c+dx)}{a+b \sinh(c+dx)} dx &= -\frac{\text{Subst}\left(\int \frac{b^2(-b^2-x^2)}{x^2(a+x)} dx, x, b \sinh(c+dx)\right)}{b^3 d} \\ &= -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{x^2(a+x)} dx, x, b \sinh(c+dx)\right)}{bd} \\ &= -\frac{\text{Subst}\left(\int \left(-\frac{b^2}{ax^2} + \frac{b^2}{a^2x} + \frac{-a^2-b^2}{a^2(a+x)}\right) dx, x, b \sinh(c+dx)\right)}{bd} \\ &= -\frac{\text{csch}(c+dx)}{ad} - \frac{b \log(\sinh(c+dx))}{a^2 d} + \frac{(a^2+b^2) \log(a+b \sinh(c+dx))}{a^2 bd} \end{aligned}$$

**Mathematica [A]** time = 0.0859398, size = 52, normalized size = 0.88

$$\frac{(a^2 + b^2) \log(a + b \sinh(c + dx)) - ab \text{csch}(c + dx) + b^2(-\log(\sinh(c + dx)))}{a^2 bd}$$

Antiderivative was successfully verified.

[In] Integrate[(Cosh[c + d\*x]\*Coth[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (-(a\*b\*Csch[c + d\*x]) - b^2\*Log[Sinh[c + d\*x]] + (a^2 + b^2)\*Log[a + b\*Sinh[c + d\*x]])/(a^2\*b\*d)

**Maple [B]** time = 0.001, size = 172, normalized size = 2.9

$$\frac{1}{2da} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{bd} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) - \frac{1}{bd} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) - \frac{1}{2da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-1} - \frac{b}{da^2} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d\*x+c)\*coth(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out]  $\frac{1}{2} \frac{d}{a} \tanh\left(\frac{1}{2} dx + \frac{1}{2} c\right) - \frac{1}{d} \frac{b}{b} \ln\left(\tanh\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right) - \frac{1}{d} \frac{b}{b} \ln\left(\tanh\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right) - \frac{1}{2} \frac{d}{a} \frac{1}{\tanh\left(\frac{1}{2} dx + \frac{1}{2} c\right)} - \frac{1}{d} \frac{a^2 b}{a^2 b} \ln\left(\tanh\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right) + \frac{1}{d} \frac{b}{b} \ln\left(\tanh\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)^2 a - 2 \tanh\left(\frac{1}{2} dx + \frac{1}{2} c\right) * b - a + \frac{1}{d} \frac{a^2 b}{a^2 b} \ln\left(\tanh\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)^2 a - 2 \tanh\left(\frac{1}{2} dx + \frac{1}{2} c\right) * b - a$

**Maxima [B]** time = 1.22645, size = 177, normalized size = 3.

$$\frac{dx + c}{bd} + \frac{2e^{(-dx-c)}}{(ae^{(-2dx-2c)} - a)d} - \frac{b \log(e^{(-dx-c)} + 1)}{a^2 d} - \frac{b \log(e^{(-dx-c)} - 1)}{a^2 d} + \frac{(a^2 + b^2) \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{a^2 bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*coth(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $(d*x + c)/(b*d) + 2*e^{(-d*x - c)} / ((a*e^{(-2*d*x - 2*c)} - a)*d) - b*\log(e^{(-d*x - c)} + 1)/(a^2*d) - b*\log(e^{(-d*x - c)} - 1)/(a^2*d) + (a^2 + b^2)*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/(a^2*b*d)$

**Fricas [B]** time = 2.2924, size = 751, normalized size = 12.73

$$a^2 dx \cosh(dx + c)^2 + a^2 dx \sinh(dx + c)^2 - a^2 dx + 2ab \cosh(dx + c) - ((a^2 + b^2) \cosh(dx + c)^2 + 2(a^2 + b^2) \cosh(dx + c) \sinh(dx + c) + (a^2 + b^2) \sinh(dx + c)^2 - a^2 - b^2) \log(2(b \sinh(dx + c) + a) / (\cosh(dx + c) - \sinh(dx + c))) + (b^2 \cosh(dx + c)^2 + 2b^2 \cosh(dx + c) \sinh(dx + c) + b^2 \sinh(dx + c)^2 - b^2) \log(2 \sinh(dx + c) / (\cosh(dx + c) - \sinh(dx + c))) + 2(a^2 d x \cosh(dx + c) + a b) \sinh(dx + c) / (a^2 b d \cosh(dx + c)^2 + 2a^2 b d \cosh(dx + c) \sinh(dx + c) + a^2 b d \sinh(dx + c)^2 - a^2 b d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*coth(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $-(a^2*d*x*\cosh(d*x + c)^2 + a^2*d*x*\sinh(d*x + c)^2 - a^2*d*x + 2*a*b*\cosh(d*x + c) - ((a^2 + b^2)*\cosh(d*x + c)^2 + 2*(a^2 + b^2)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2 + b^2)*\sinh(d*x + c)^2 - a^2 - b^2)*\log(2*(b*\sinh(d*x + c) + a)/(\cosh(d*x + c) - \sinh(d*x + c))) + (b^2*\cosh(d*x + c)^2 + 2*b^2*\cosh(d*x + c)*\sinh(d*x + c) + b^2*\sinh(d*x + c)^2 - b^2)*\log(2*\sinh(d*x + c)/(\cosh(d*x + c) - \sinh(d*x + c))) + 2*(a^2*d*x*\cosh(d*x + c) + a*b)*\sinh(d*x + c)/(a^2*b*d*\cosh(d*x + c)^2 + 2*a^2*b*d*\cosh(d*x + c)*\sinh(d*x + c) + a^2*b*d*\sinh(d*x + c)^2 - a^2*b*d)$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c + dx) \coth^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*coth(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)), x)

[Out] Integral(cosh(c + d\*x)\*coth(c + d\*x)\*\*2/(a + b\*sinh(c + d\*x)), x)

---

**Giac [A]** time = 1.38761, size = 153, normalized size = 2.59

$$\frac{\frac{dx}{b} + \frac{b \log(e^{(dx+c)+1})}{a^2} + \frac{b \log(|e^{(dx+c)}-1|)}{a^2} - \frac{(a^2+b^2) \log(|be^{(2dx+2c)}+2ae^{(dx+c)}-b|)}{a^2b} + \frac{2e^{(dx+c)}}{a(e^{(dx+c)}+1)(e^{(dx+c)}-1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d\*x+c)\*coth(d\*x+c)^2/(a+b\*sinh(d\*x+c)), x, algorithm="giac")

[Out]  $-(d*x/b + b*\log(e^{(d*x + c) + 1})/a^2 + b*\log(\text{abs}(e^{(d*x + c)} - 1))/a^2 - (a^2 + b^2)*\log(\text{abs}(b*e^{(2*d*x + 2*c)} + 2*a*e^{(d*x + c)} - b))/(a^2*b) + 2*e^{(d*x + c)}/(a*(e^{(d*x + c)} + 1)*(e^{(d*x + c)} - 1)))/d$

$$3.463 \quad \int \frac{\cosh(c+dx) \coth^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable} \left( \frac{\cosh(c+dx) \coth^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[(Cosh[c + d\*x]\*Coth[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0854025, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\cosh(c+dx) \coth^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Cosh[c + d\*x]\*Coth[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int] [(Cosh[c + d\*x]\*Coth[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\cosh(c+dx) \coth^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\cosh(c+dx) \coth^2(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.002, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Cosh[c + d\*x]\*Coth[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]



[Out] \$Aborted

**Maple [A]** time = 0.944, size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c) (\coth(dx + c))^2}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)`

[Out] `int(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{2e^{(dx+c)}}{adf x + ade - (adf x e^{2c} + a d e e^{2c})e^{2dx}} + \frac{\log(fx + e)}{bf} - \frac{1}{2} \int -\frac{2(bdf x + bde + af)}{a^2 d f^2 x^2 + 2 a^2 d e f x + a^2 d e^2 - (a^2 d f^2 x^2 e^c + 2 a^2 d e f x e^c + a^2 d e^2 e^c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `2*e^(d*x + c)/(a*d*f*x + a*d*e - (a*d*f*x*e^(2*c) + a*d*e*e^(2*c)))*e^(2*d*x) + log(f*x + e)/(b*f) - 1/2*integrate(-2*(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) + 1/2*integrate(2*(b*d*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) - 1/2*integrate(4*(a^2*b + b^3 - (a^3*e^c + a*b^2*e^c)*e^(d*x))/(a^2*b^2*f*x + a^2*b^2*e - (a^2*b^2*f*x*e^(2*c) + a^2*b^2*e*e^(2*c))*e^(2*d*x) - 2*(a^3*b*f*x*e^c + a^3*b*e*e^c)*e^(d*x)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx + c) \coth(dx + c)^2}{afx + ae + (bf x + be) \sinh(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)*coth(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(c + dx) \coth^2(c + dx)}{(a + b \sinh(c + dx))(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(cosh(c + d*x)*coth(c + d*x)**2/((a + b*sinh(c + d*x))*(e + f*x)), x)
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*coth(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.464 \quad \int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=1428

result too large to display

```
[Out] (-2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(a*d) + (2*b^2*(e + f*x)^3*ArcTan[E^(c + d*x)])/(a*(a^2 + b^2)*d) - (6*f*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d^2) + (2*b*(e + f*x)^3*ArcTanh[E^(2*c + 2*d*x)])/(a^2*d) - ((e + f*x)^3*Csch[c + d*x])/(a*d) + (b^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d) + (b^3*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d) - (b^3*(e + f*x)^3*Log[1 + E^(2*(c + d*x))])/(a^2*(a^2 + b^2)*d) - (6*f^2*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^3) + ((3*I)*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) - ((3*I)*b^2*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) - ((3*I)*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + ((3*I)*b^2*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) + (6*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^3) + (3*b^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^2) + (3*b^3*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^2) - (3*b^3*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*a^2*(a^2 + b^2)*d^2) + (3*b*f*(e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a^2*d^2) - (3*b*f*(e + f*x)^2*PolyLog[2, E^(2*c + 2*d*x)])/(2*a^2*d^2) + (6*f^3*PolyLog[3, -E^(c + d*x)])/(a*d^4) - ((6*I)*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^3) + ((6*I)*b^2*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^3) + ((6*I)*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a*d^3) - ((6*I)*b^2*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^3) - (6*f^3*PolyLog[3, E^(c + d*x)])/(a*d^4) - (6*b^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^3) - (6*b^3*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^3) + (3*b^3*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*a^2*(a^2 + b^2)*d^3) - (3*b*f^2*(e + f*x)*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a^2*d^3) + (3*b*f^2*(e + f*x)*PolyLog[3, E^(2*c + 2*d*x)])/(2*a^2*d^3) + ((6*I)*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(a*d^4) - ((6*I)*b^2*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^4) - ((6*I)*f^3*PolyLog[4, I*E^(c + d*x)])/(a*d^4) + ((6*I)*b^2*f^3*PolyLog[4, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^4) + (6*b^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^4) + (6*b^3*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^4) - (3*b^3*f^3*PolyLog[4, -E^(2*(c + d*x))])/(4*a^2*(a^2 + b^2)*d^4) + (3*b*f^3*PolyLog[4, -E^(2*c + 2*d*x)])/(4*a^2*d^4) - (3*b*f^3*PolyLog[4, E^(2*c + 2*d*x)])/(4*a^2*d^4)
```

**Rubi [A]** time = 2.28375, antiderivative size = 1428, normalized size of antiderivative = 1., number of steps used = 64, number of rules used = 20, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$ , Rules used = {5589, 2621, 321, 207, 5462, 6741, 12, 6742, 5205, 4180, 2531, 6609, 2282, 6589, 4182, 5461, 5573, 5561, 2190, 3718}

result too large to display

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Csch[c + d\*x]^2\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] 
$$\begin{aligned} & (-2*(e + f*x)^3*\text{ArcTan}[E^{(c + d*x)}])/(a*d) + (2*b^2*(e + f*x)^3*\text{ArcTan}[E^{(c + d*x)}])/(a*(a^2 + b^2)*d) - (6*f*(e + f*x)^2*\text{ArcTanh}[E^{(c + d*x)}])/(a*d^2) \\ & + (2*b*(e + f*x)^3*\text{ArcTanh}[E^{(2*c + 2*d*x)}])/(a^2*d) - ((e + f*x)^3*\text{Csch}[c + d*x])/(a*d) + (b^3*(e + f*x)^3*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])])/(a^2*(a^2 + b^2)*d) \\ & + (b^3*(e + f*x)^3*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(a^2*(a^2 + b^2)*d) - (b^3*(e + f*x)^3*\text{Log}[1 + E^{(2*(c + d*x))}])/(a^2*(a^2 + b^2)*d) \\ & - (6*f^2*(e + f*x)*\text{PolyLog}[2, -E^{(c + d*x)}])/(a*d^3) + ((3*I)*f*(e + f*x)^2*\text{PolyLog}[2, (-I)*E^{(c + d*x)}])/(a*d^2) - ((3*I)*b^2*f*(e + f*x)^2*\text{PolyLog}[2, (-I)*E^{(c + d*x)}])/(a*(a^2 + b^2)*d^2) \\ & - ((3*I)*f*(e + f*x)^2*\text{PolyLog}[2, I*E^{(c + d*x)}])/(a*d^2) + ((3*I)*b^2*f*(e + f*x)^2*\text{PolyLog}[2, I*E^{(c + d*x)}])/(a*(a^2 + b^2)*d^2) + (6*f^2*(e + f*x)*\text{PolyLog}[2, E^{(c + d*x)}])/(a*d^3) \\ & + (3*b^3*f*(e + f*x)^2*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^2) + (3*b^3*f*(e + f*x)^2*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^2) \\ & - (3*b^3*f*(e + f*x)^2*\text{PolyLog}[2, -E^{(2*(c + d*x))}])/(2*a^2*(a^2 + b^2)*d^2) + (3*b*f*(e + f*x)^2*\text{PolyLog}[2, -E^{(2*c + 2*d*x)}])/(2*a^2*d^2) - (3*b*f*(e + f*x)^2*\text{PolyLog}[2, E^{(2*c + 2*d*x)}])/(2*a^2*d^2) \\ & + (6*f^3*\text{PolyLog}[3, -E^{(c + d*x)}])/(a*d^4) - ((6*I)*f^2*(e + f*x)*\text{PolyLog}[3, (-I)*E^{(c + d*x)}])/(a*d^3) + ((6*I)*b^2*f^2*(e + f*x)*\text{PolyLog}[3, (-I)*E^{(c + d*x)}])/(a*(a^2 + b^2)*d^3) \\ & + ((6*I)*f^2*(e + f*x)*\text{PolyLog}[3, I*E^{(c + d*x)}])/(a*d^3) - ((6*I)*b^2*f^2*(e + f*x)*\text{PolyLog}[3, I*E^{(c + d*x)}])/(a*(a^2 + b^2)*d^3) - (6*f^3*\text{PolyLog}[3, E^{(c + d*x)}])/(a*d^4) \\ & - (6*b^3*f^2*(e + f*x)*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^3) - (6*b^3*f^2*(e + f*x)*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^3) \\ & + (3*b^3*f^2*(e + f*x)*\text{PolyLog}[3, -E^{(2*(c + d*x))}])/(2*a^2*(a^2 + b^2)*d^3) - (3*b*f^2*(e + f*x)*\text{PolyLog}[3, -E^{(2*c + 2*d*x)}])/(2*a^2*d^3) + (3*b*f^2*(e + f*x)*\text{PolyLog}[3, E^{(2*c + 2*d*x)}])/(2*a^2*d^3) \\ & + ((6*I)*f^3*\text{PolyLog}[4, (-I)*E^{(c + d*x)}])/(a*d^4) - ((6*I)*b^2*f^3*\text{PolyLog}[4, (-I)*E^{(c + d*x)}])/(a*(a^2 + b^2)*d^4) - ((6*I)*f^3*\text{PolyLog}[4, I*E^{(c + d*x)}])/(a*d^4) \\ & + ((6*I)*b^2*f^3*\text{PolyLog}[4, I*E^{(c + d*x)}])/(a*(a^2 + b^2)*d^4) + (6*b^3*f^3*\text{PolyLog}[4, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^4) \\ & + (6*b^3*f^3*\text{PolyLog}[4, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))])/(a^2*(a^2 + b^2)*d^4) - (3*b^3*f^3*\text{PolyLog}[4, -E^{(2*(c + d*x))}])/(4*a^2*(a^2 + \end{aligned}$$

$b^2*d^4) + (3*b*f^3*PolyLog[4, -E^{(2*c + 2*d*x)}])/(4*a^2*d^4) - (3*b*f^3*PolyLog[4, E^{(2*c + 2*d*x)}])/(4*a^2*d^4)$

### Rule 5589

$\text{Int}[(\text{Csch}[c_.] + (d_.)(x_.)]^{(n_.)}*((e_.) + (f_.)(x_.))^{(m_.)}\text{Sech}[(c_.) + (d_.)(x_.)]^{(p_.)}/((a_.) + (b_.)\text{Sinh}[(c_.) + (d_.)(x_.)]), x\_Symbol] \text{ :> Dist}[1/a, \text{Int}[(e + f*x)^m*\text{Sech}[c + d*x]^p*\text{Csch}[c + d*x]^n, x], x] - \text{Dist}[b/a, \text{Int}[((e + f*x)^m*\text{Sech}[c + d*x]^p*\text{Csch}[c + d*x]^{(n - 1)})/(a + b*\text{Sinh}[c + d*x]), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$

### Rule 2621

$\text{Int}[(\text{csc}[e_.] + (f_.)(x_.)]*(a_.)]^{(m_.)}*\text{sec}[(e_.) + (f_.)(x_.)]^{(n_.)}, x\_Symbol] \text{ :> -Dist}[(f*a^n)^{-1}, \text{Subst}[\text{Int}[x^{(m + n - 1)}/(-1 + x^2/a^2)^{(n + 1)/2}], x], x, a*\text{Csc}[e + f*x]], x] /; \text{FreeQ}[\{a, e, f, m\}, x] \&\& \text{IntegerQ}[(n + 1)/2] \&\& \text{!(IntegerQ}[(m + 1)/2] \&\& \text{LtQ}[0, m, n])$

### Rule 321

$\text{Int}[(c_.)(x_.)]^{(m_.)}*((a_.) + (b_.)(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \text{ :> Simp}[(c^{(n - 1)}*(c*x)^{(m - n + 1)}*(a + b*x^n)^{(p + 1)})/(b*(m + n*p + 1)), x] - \text{Dist}[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), \text{Int}[(c*x)^{(m - n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[m, n - 1] \&\& \text{NeQ}[m + n*p + 1, 0] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 207

$\text{Int}[(a_.) + (b_.)(x_.)^2]^{-1}, x\_Symbol] \text{ :> -Simp}[\text{ArcTanh}[(\text{Rt}[b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{LtQ}[a, 0] \text{ || GtQ}[b, 0])$

### Rule 5462

$\text{Int}[\text{Csch}[a_.] + (b_.)(x_.)]^{(n_.)}*((c_.) + (d_.)(x_.))^{(m_.)}\text{Sech}[(a_.) + (b_.)(x_.)]^{(p_.)}, x\_Symbol] \text{ :> With}[\{u = \text{IntHide}[\text{Csch}[a + b*x]^n*\text{Sech}[a + b*x]^p, x]\}, \text{Dist}[(c + d*x)^m, u, x] - \text{Dist}[d*m, \text{Int}[(c + d*x)^{(m - 1)}*u, x], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{IntegersQ}[n, p] \&\& \text{GtQ}[m, 0] \&\& \text{NeQ}[n, p]$

### Rule 6741

$\text{Int}[u_, x\_Symbol] \text{ :> With}[\{v = \text{NormalizeIntegrand}[u, x]\}, \text{Int}[v, x] /; v \neq u]$

Rule 12

```
Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :=> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
!FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :=> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :=> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :=> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x))]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```

Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_)^(m_))/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

### Rule 3718

```

Int[((c_) + (d_)*(x_)^(m_))*tan[(e_) + (Complex[0, fz_])*(f_)*(x_)], x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-I*e) + f*fz*x))/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

```

### Rubi steps



$$\begin{aligned}
\int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \operatorname{csch}^2(c+dx) \operatorname{sech}(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^3 \operatorname{csch}(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= -\frac{(e+fx)^3 \tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)^3 \operatorname{csch}(c+dx)}{ad} - \frac{b \int (e+fx)^3 \operatorname{csch}(c+dx) \operatorname{sech}(c+dx) dx}{a} \\
&= -\frac{(e+fx)^3 \tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)^3 \operatorname{csch}(c+dx)}{ad} - \frac{(2b) \int (e+fx)^3 \operatorname{csch}(c+dx) \operatorname{sech}(c+dx) dx}{a} \\
&= -\frac{b^3(e+fx)^4}{4a^2(a^2+b^2)f} - \frac{(e+fx)^3 \tan^{-1}(\sinh(c+dx))}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^2d} \\
&= -\frac{b^3(e+fx)^4}{4a^2(a^2+b^2)f} - \frac{(e+fx)^3 \tan^{-1}(\sinh(c+dx))}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^2d} \\
&= \frac{2b^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{a(a^2+b^2)d} - \frac{(e+fx)^3 \tan^{-1}(\sinh(c+dx))}{ad} + \frac{2b(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^2d} \\
&= \frac{2b^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{a(a^2+b^2)d} - \frac{6f(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad^2} + \frac{2b(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^2d} \\
&= \frac{2b^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{a(a^2+b^2)d} - \frac{6f(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad^2} + \frac{2b(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^2d} \\
&= -\frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{ad} + \frac{2b^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{a(a^2+b^2)d} - \frac{6f(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad^2} \\
&= -\frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{ad} + \frac{2b^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{a(a^2+b^2)d} - \frac{6f(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad^2} \\
&= -\frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{ad} + \frac{2b^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{a(a^2+b^2)d} - \frac{6f(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad^2} \\
&= -\frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{ad} + \frac{2b^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{a(a^2+b^2)d} - \frac{6f(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad^2} \\
&= -\frac{2(e+fx)^3 \tan^{-1}(e^{c+dx})}{ad} + \frac{2b^2(e+fx)^3 \tan^{-1}(e^{c+dx})}{a(a^2+b^2)d} - \frac{6f(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad^2}
\end{aligned}$$

**Mathematica [B]** time = 14.0157, size = 7234, normalized size = 5.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Result too large to show
```

**Maple [F]** time = 1.48, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 (\operatorname{csch}(dx + c))^2 \operatorname{sech}(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + a^2*b^2)*d) + 2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^2 + b^2)*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-d*x - c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d))*e^3 - 2*(f^3*x^3*e^c + 3*e*f^2*x^2*e^c + 3*e^2*f*x*e^c)*e^(d*x)/(a*d*e^(2*d*x + 2*c) - a*d) - 3*e^2*f*log(e^(d*x + c) + 1)/(a*d^2) + 3*e^2*f*log(e^(d*x + c) - 1)/(a*d^2) - (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c))) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c))*b*f^3/(a^2*d^4) - (d^3*x^3
```

```

*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c))*b*f^3/(a^2*d^4) - 3*(b*d*e^2*f + 2*a*e*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^2*d^3) - 3*(b*d*e^2*f - 2*a*e*f^2)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)))/(a^2*d^3) - 3*(b*d*e*f^2 + a*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^2*d^4) - 3*(b*d*e*f^2 - a*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))/(a^2*d^4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 + a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f + 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^4) + 1/4*(b*d^4*f^3*x^4 + 4*(b*d*e*f^2 - a*f^3)*d^3*x^3 + 6*(b*d^2*e^2*f - 2*a*d*e*f^2)*d^2*x^2)/(a^2*d^4) - integrate(2*(b^4*f^3*x^3 + 3*b^4*e*f^2*x^2 + 3*b^4*e^2*f*x - (a*b^3*f^3*x^3*e^c + 3*a*b^3*e*f^2*x^2*e^c + 3*a*b^3*e^2*f*x*e^c))*e^(d*x))/(a^4*b + a^2*b^3 - (a^4*b*e^(2*c) + a^2*b^3*e^(2*c))*e^(2*d*x) - 2*(a^5*e^c + a^3*b^2*e^c)*e^(d*x)), x) - integrate(2*(b*f^3*x^3 + 3*b*e*f^2*x^2 + 3*b*e^2*f*x + (a*f^3*x^3*e^c + 3*a*e*f^2*x^2*e^c + 3*a*e^2*f*x*e^c))*e^(d*x))/(a^2 + b^2 + (a^2*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)

```

---

**Fricas [C]** time = 5.38672, size = 21464, normalized size = 15.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((f*x+e)^3*cosh(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")

```

```

[Out] -(2*((a^3 + a*b^2)*d^3*f^3*x^3 + 3*(a^3 + a*b^2)*d^3*e*f^2*x^2 + 3*(a^3 + a*b^2)*d^3*e^2*f*x + (a^3 + a*b^2)*d^3*e^3)*cosh(d*x + c) + 3*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f - (b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*cosh(d*x + c)^2 - 2*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + 3*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f - (b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*cosh(d*x + c)^2 - 2*(b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d^2*f^3*x^2 + 2*b^3*d^2*e*f^2*x + b^3*d^2*e^2*f)*sinh(d*x + c)^2)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - 3*((a^2*b + b^3)*d^2*f^3*x^2 + (a^2*b + b^3)*d^2*e^2*f - 2*(a^3 + a*b^2)*d*e*f^2 - ((a^2*b + b^3)*d^2*f^3*x^2 + (a^2*b + b^3)*d^2*e^2*f - 2*(a^3 + a*b^2)*d*e*f^2 + 2*((a^2*b + b^3)*d^2*e*f^2 - (a^3 + a*b^2)*d*f^3)*x)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^2*f^3*x^2

```

$$\begin{aligned}
& + (a^2b + b^3)d^2e^2f - 2(a^3 + ab^2)d^2e^2f + 2((a^2b + b^3)d^2e^2f \\
& e^2f - (a^3 + ab^2)d^2f^3)x) \cosh(dx + c) \sinh(dx + c) - ((a^2b + b^3) \\
& )d^2f^3x^2 + (a^2b + b^3)d^2e^2f - 2(a^3 + ab^2)d^2e^2f + 2((a^2 \\
& *b + b^3)d^2e^2f - (a^3 + ab^2)d^2f^3)x) \sinh(dx + c)^2 + 2((a^2b + \\
& b^3)d^2e^2f - (a^3 + ab^2)d^2f^3)x) \operatorname{dilog}(\cosh(dx + c) + \sinh(dx + \\
& c)) - (3Ia^3d^2f^3x^2 - 3a^2bd^2f^3x^2 + 6Ia^3d^2e^2fx - 6 \\
& a^2bd^2e^2fx + 3Ia^3d^2e^2f - 3a^2bd^2e^2f + (-3Ia^3d^2f^3 \\
& ^3x^2 + 3a^2bd^2f^3x^2 - 6Ia^3d^2e^2fx + 6a^2bd^2e^2fx - \\
& 3Ia^3d^2e^2f + 3a^2bd^2e^2f) \cosh(dx + c)^2 + (-6Ia^3d^2f^3x \\
& x^2 + 6a^2bd^2f^3x^2 - 12Ia^3d^2e^2fx + 12a^2bd^2e^2fx - 6 \\
& *Ia^3d^2e^2f + 6a^2bd^2e^2f) \cosh(dx + c) \sinh(dx + c) + (-3Ia \\
& ^3d^2f^3x^2 + 3a^2bd^2f^3x^2 - 6Ia^3d^2e^2fx + 6a^2bd^2e^2 \\
& f^2fx - 3Ia^3d^2e^2f + 3a^2bd^2e^2f) \sinh(dx + c)^2 \operatorname{dilog}(I \cos \\
& h(dx + c) + I \sinh(dx + c)) - (-3Ia^3d^2f^3x^2 - 3a^2bd^2f^3x^2 \\
& - 6Ia^3d^2e^2fx - 6a^2bd^2e^2fx - 3Ia^3d^2e^2f - 3a^2bd^2e^2 \\
& d^2e^2f + (3Ia^3d^2f^3x^2 + 3a^2bd^2f^3x^2 + 6Ia^3d^2e^2fx \\
& x + 6a^2bd^2e^2fx + 3Ia^3d^2e^2f + 3a^2bd^2e^2f) \cosh(dx + \\
& c)^2 + (6Ia^3d^2f^3x^2 + 6a^2bd^2f^3x^2 + 12Ia^3d^2e^2fx + \\
& 12a^2bd^2e^2fx + 6Ia^3d^2e^2f + 6a^2bd^2e^2f) \cosh(dx + c \\
& ) \sinh(dx + c) + (3Ia^3d^2f^3x^2 + 3a^2bd^2f^3x^2 + 6Ia^3d^2e^2 \\
& e^2fx + 6a^2bd^2e^2fx + 3Ia^3d^2e^2f + 3a^2bd^2e^2f) \sinh \\
& (dx + c)^2 \operatorname{dilog}(-I \cosh(dx + c) - I \sinh(dx + c)) - 3((a^2b + b^3)d \\
& ^2f^3x^2 + (a^2b + b^3)d^2e^2f + 2(a^3 + ab^2)d^2e^2f - ((a^2b + \\
& b^3)d^2f^3x^2 + (a^2b + b^3)d^2e^2f + 2(a^3 + ab^2)d^2e^2f + 2(( \\
& a^2b + b^3)d^2e^2f + (a^3 + ab^2)d^2f^3)x) \cosh(dx + c)^2 - 2((a^2b \\
& b + b^3)d^2f^3x^2 + (a^2b + b^3)d^2e^2f + 2(a^3 + ab^2)d^2e^2f + \\
& 2((a^2b + b^3)d^2e^2f + (a^3 + ab^2)d^2f^3)x) \cosh(dx + c) \sinh(dx \\
& + c) - ((a^2b + b^3)d^2f^3x^2 + (a^2b + b^3)d^2e^2f + 2(a^3 + ab \\
& ^2)d^2e^2f + 2((a^2b + b^3)d^2e^2f + (a^3 + ab^2)d^2f^3)x) \sinh(dx \\
& + c)^2 + 2((a^2b + b^3)d^2e^2f + (a^3 + ab^2)d^2f^3)x) \operatorname{dilog}(-\cosh( \\
& dx + c) - \sinh(dx + c)) + (b^3d^3e^3 - 3b^3cd^2e^2f + 3b^3c^2d^2 \\
& e^2f - b^3c^3f^3 - (b^3d^3e^3 - 3b^3cd^2e^2f + 3b^3c^2d^2e^2f \\
& - b^3c^3f^3) \cosh(dx + c)^2 - 2(b^3d^3e^3 - 3b^3cd^2e^2f + 3b^3 \\
& c^2d^2e^2f - b^3c^3f^3) \cosh(dx + c) \sinh(dx + c) - (b^3d^3e^3 - 3 \\
& b^3cd^2e^2f + 3b^3c^2d^2e^2f - b^3c^3f^3) \sinh(dx + c)^2) \log(2b \\
& * \cosh(dx + c) + 2b \sinh(dx + c) + 2b \sqrt{(a^2 + b^2)/b^2} + 2a) + (b^3 \\
& d^3e^3 - 3b^3cd^2e^2f + 3b^3c^2d^2e^2f - b^3c^3f^3 - (b^3d^3e^3 \\
& e^3 - 3b^3cd^2e^2f + 3b^3c^2d^2e^2f - b^3c^3f^3) \cosh(dx + c)^2 \\
& - 2(b^3d^3e^3 - 3b^3cd^2e^2f + 3b^3c^2d^2e^2f - b^3c^3f^3) \cos \\
& h(dx + c) \sinh(dx + c) - (b^3d^3e^3 - 3b^3cd^2e^2f + 3b^3c^2d^2e^2 \\
& f^2 - b^3c^3f^3) \sinh(dx + c)^2) \log(2b \cosh(dx + c) + 2b \sinh(dx + \\
& c) - 2b \sqrt{(a^2 + b^2)/b^2} + 2a) + (b^3d^3f^3x^3 + 3b^3d^3e^2f^2 \\
& *x^2 + 3b^3d^3e^2f*x + 3b^3cd^2e^2f - 3b^3c^2d^2e^2f + b^3c^3f^3 \\
& f^3 - (b^3d^3f^3x^3 + 3b^3d^3e^2f*x^2 + 3b^3d^3e^2f*x + 3b^3c^2 \\
& d^2e^2f - 3b^3c^2d^2e^2f + b^3c^3f^3) \cosh(dx + c)^2 - 2(b^3d^3f
\end{aligned}$$

$$\begin{aligned}
& 3^3x^3 + 3b^3d^3e^2f^2x^2 + 3b^3d^3e^2f^2x + 3b^3c^2d^2e^2f - 3b^3c^2d^2e^2f + b^3c^3f^3) \cosh(dx + c) \sinh(dx + c) - (b^3d^3f^3x^3 + 3b^3d^3e^2f^2x^2 + 3b^3d^3e^2f^2x + 3b^3c^2d^2e^2f - 3b^3c^2d^2e^2f + b^3c^3f^3) \sinh(dx + c)^2 \log(-(\cosh(dx + c) + \sinh(dx + c)) + (\cosh(dx + c) + \sinh(dx + c)) \sqrt{((a^2 + b^2)/b^2) - b}/b) + \\
& (b^3d^3f^3x^3 + 3b^3d^3e^2f^2x^2 + 3b^3d^3e^2f^2x + 3b^3c^2d^2e^2f - 3b^3c^2d^2e^2f + b^3c^3f^3 - (b^3d^3f^3x^3 + 3b^3d^3e^2f^2x^2 + 3b^3d^3e^2f^2x + 3b^3c^2d^2e^2f - 3b^3c^2d^2e^2f + b^3c^3f^3) \cosh(dx + c)^2 - 2(b^3d^3f^3x^3 + 3b^3d^3e^2f^2x^2 + 3b^3d^3e^2f^2x + 3b^3c^2d^2e^2f - 3b^3c^2d^2e^2f + b^3c^3f^3) \cosh(dx + c) \sinh(dx + c) - (b^3d^3f^3x^3 + 3b^3d^3e^2f^2x^2 + 3b^3d^3e^2f^2x + 3b^3c^2d^2e^2f - 3b^3c^2d^2e^2f + b^3c^3f^3) \sinh(dx + c)^2) \log(-(\cosh(dx + c) + \sinh(dx + c)) + (\cosh(dx + c) + \sinh(dx + c)) \sqrt{((a^2 + b^2)/b^2) - b}/b) - ((a^2b + b^3)d^3f^3x^3 + (a^2b + b^3)d^3e^3 + 3(a^3 + ab^2)d^2e^2f + 3((a^2b + b^3)d^3e^2f + (a^3 + ab^2)d^2f^3)x^2 - ((a^2b + b^3)d^3f^3x^3 + (a^2b + b^3)d^3e^3 + 3(a^3 + ab^2)d^2e^2f + 3((a^2b + b^3)d^3e^2f + (a^3 + ab^2)d^2f^3)x^2 + 3((a^2b + b^3)d^3e^2f + 2(a^3 + ab^2)d^2e^2f)x) \cosh(dx + c)^2 - 2((a^2b + b^3)d^3f^3x^3 + (a^2b + b^3)d^3e^3 + 3(a^3 + ab^2)d^2e^2f + 3((a^2b + b^3)d^3e^2f + (a^3 + ab^2)d^2f^3)x^2 + 3((a^2b + b^3)d^3e^2f + 2(a^3 + ab^2)d^2e^2f)x) \cosh(dx + c) \sinh(dx + c) - ((a^2b + b^3)d^3f^3x^3 + (a^2b + b^3)d^3e^3 + 3(a^3 + ab^2)d^2e^2f + 3((a^2b + b^3)d^3e^2f + (a^3 + ab^2)d^2f^3)x^2 + 3((a^2b + b^3)d^3e^2f + 2(a^3 + ab^2)d^2e^2f)x) \sinh(dx + c)^2 + 3((a^2b + b^3)d^3e^2f + 2(a^3 + ab^2)d^2e^2f)x) \log(\cosh(dx + c) + \sinh(dx + c) + 1) - (Ia^3d^3e^3 - a^2bd^3e^3 - 3Ia^3c^2d^2e^2f + 3a^2b^2cd^2e^2f + 3Ia^3c^2d^2e^2f - 3a^2b^2c^2d^2e^2f - Ia^3c^3f^3 + a^2b^2c^3f^3 + (-Ia^3d^3e^3 + a^2bd^3e^3 + 3Ia^3c^2d^2e^2f - 3a^2b^2cd^2e^2f - 3Ia^3c^2d^2e^2f + 3a^2b^2c^2d^2e^2f + Ia^3c^3f^3 - a^2b^2c^3f^3) \cosh(dx + c)^2 + (-2Ia^3d^3e^3 + 2a^2bd^3e^3 + 6Ia^3c^2d^2e^2f - 6a^2b^2cd^2e^2f - 6Ia^3c^2d^2e^2f + 6a^2b^2c^2d^2e^2f + 2Ia^3c^3f^3 - 2a^2b^2c^3f^3) \cosh(dx + c) \sinh(dx + c) + (-Ia^3d^3e^3 + a^2bd^3e^3 + 3Ia^3c^2d^2e^2f + 3a^2b^2cd^2e^2f - 3Ia^3c^2d^2e^2f - 3a^2b^2c^2d^2e^2f + Ia^3c^3f^3 + a^2b^2c^3f^3 + (Ia^3d^3e^3 + a^2bd^3e^3 - 3Ia^3c^2d^2e^2f - 3a^2b^2cd^2e^2f + 3Ia^3c^2d^2e^2f + 3a^2b^2c^2d^2e^2f - Ia^3c^3f^3 - a^2b^2c^3f^3) \cosh(dx + c)^2 + (2Ia^3d^3e^3 + 2a^2bd^3e^3 - 6Ia^3c^2d^2e^2f - 6a^2b^2cd^2e^2f + 6Ia^3c^2d^2e^2f + 6a^2b^2c^2d^2e^2f - 2Ia^3c^3f^3 - 2a^2b^2c^3f^3) \cosh(dx + c) \sinh(dx + c) + (Ia^3d^3e^3 + a^2bd^3e^3 - 3Ia^3c^2d^2e^2f - 3a^2b^2cd^2e^2f + 3Ia^3c^2d^2e^2f + 3a^2b^2c^2d^2e^2f - Ia^3c^3f^3 - a^2b^2c^3f^3) \sinh(dx + c)^2) \log(\cosh(dx + c) + \sinh(dx + c) - I) - ((a^2b + b^3) *
\end{aligned}$$

$$\begin{aligned}
& d^3e^3 - 3*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d^2e^2*f + 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 - ((a^2*b + b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3 - ((a^2*b + b^3)*d^3e^3 - 3*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d^2e^2*f + 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 - ((a^2*b + b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3)*\cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^3e^3 - 3*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d^2e^2*f + 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 - ((a^2*b + b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3)*\sinh(d*x + c) - ((a^2*b + b^3)*d^3e^3 - 3*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d^2e^2*f + 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 - ((a^2*b + b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3)*\sinh(d*x + c)^2*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) - (-I*a^3*d^3*f^3*x^3 - a^2*b*d^3*f^3*x^3 - 3*I*a^3*d^3*e*f^2*x^2 - 3*a^2*b*d^3*e*f^2*x^2 - 3*I*a^3*d^3*e^2*f*x - 3*a^2*b*d^3*e^2*f*x - 3*I*a^3*c*d^2*e^2*f - 3*a^2*b*c*d^2*e^2*f + 3*I*a^3*c^2*d*e*f^2 + 3*a^2*b*c^2*d*e*f^2 - I*a^3*c^3*f^3 - a^2*b*c^3*f^3 + (I*a^3*d^3*f^3*x^3 + a^2*b*d^3*f^3*x^3 + 3*I*a^3*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e*f^2*x^2 + 3*I*a^3*d^3*e^2*f*x + 3*a^2*b*d^3*e^2*f*x + 3*I*a^3*c*d^2*e^2*f + 3*a^2*b*c*d^2*e^2*f - 3*I*a^3*c^2*d*e*f^2 - 3*a^2*b*c^2*d*e*f^2 + I*a^3*c^3*f^3 + a^2*b*c^3*f^3)*\cosh(d*x + c)^2 + (2*I*a^3*d^3*f^3*x^3 + 2*a^2*b*d^3*f^3*x^3 + 6*I*a^3*d^3*e*f^2*x^2 + 6*a^2*b*d^3*e*f^2*x^2 + 6*I*a^3*d^3*e^2*f*x + 6*a^2*b*d^3*e^2*f*x + 6*I*a^3*c*d^2*e^2*f + 6*a^2*b*c*d^2*e^2*f - 6*I*a^3*c^2*d*e*f^2 - 6*a^2*b*c^2*d*e*f^2 + 2*I*a^3*c^3*f^3 + 2*a^2*b*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c) + (I*a^3*d^3*f^3*x^3 + a^2*b*d^3*f^3*x^3 + 3*I*a^3*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e*f^2*x^2 + 3*I*a^3*d^3*e^2*f*x + 3*a^2*b*d^3*e^2*f*x + 3*I*a^3*c*d^2*e^2*f + 3*a^2*b*c*d^2*e^2*f - 3*I*a^3*c^2*d*e*f^2 - 3*a^2*b*c^2*d*e*f^2 + I*a^3*c^3*f^3 + a^2*b*c^3*f^3)*\sinh(d*x + c)^2*\log(I*\cosh(d*x + c) + I*\sinh(d*x + c) + 1) - (I*a^3*d^3*f^3*x^3 - a^2*b*d^3*f^3*x^3 + 3*I*a^3*d^3*e*f^2*x^2 - 3*a^2*b*d^3*e*f^2*x^2 + 3*I*a^3*d^3*e^2*f*x - 3*a^2*b*d^3*e^2*f*x + 3*I*a^3*c*d^2*e^2*f - 3*a^2*b*c*d^2*e^2*f - 3*I*a^3*c^2*d*e*f^2 + 3*a^2*b*c^2*d*e*f^2 + I*a^3*c^3*f^3 - a^2*b*c^3*f^3 + (-I*a^3*d^3*f^3*x^3 + a^2*b*d^3*f^3*x^3 - 3*I*a^3*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e*f^2*x^2 - 3*I*a^3*d^3*e^2*f*x + 3*a^2*b*d^3*e^2*f*x - 3*I*a^3*c*d^2*e^2*f + 3*a^2*b*c*d^2*e^2*f + 3*I*a^3*c^2*d*e*f^2 - 3*a^2*b*c^2*d*e*f^2 - I*a^3*c^3*f^3 + a^2*b*c^3*f^3)*\cosh(d*x + c)^2 + (-2*I*a^3*d^3*f^3*x^3 + 2*a^2*b*d^3*f^3*x^3 - 6*I*a^3*d^3*e*f^2*x^2 + 6*a^2*b*d^3*e*f^2*x^2 - 6*I*a^3*d^3*e^2*f*x + 6*a^2*b*d^3*e^2*f*x - 6*I*a^3*c*d^2*e^2*f + 6*a^2*b*c*d^2*e^2*f + 6*I*a^3*c^2*d*e*f^2 - 6*a^2*b*c^2*d*e*f^2 - 2*I*a^3*c^3*f^3 + 2*a^2*b*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c) + (-I*a^3*d^3*f^3*x^3 + a^2*b*d^3*f^3*x^3 - 3*I*a^3*d^3*e*f^2*x^2 + 3*a^2*b*d^3*e*f^2*x^2 - 3*I*a^3*d^3*e^2*f*x + 3*a^2*b*d^3*e^2*f*x - 3*I*a^3*c*d^2*e^2*f + 3*a^2*b*c*d^2*e^2*f + 3*I*a^3*c^2*d*e*f^2 - 3*a^2*b*c^2*d*e*f^2 - I*a^3*c^3*f^3 + a^2*b*c^3*f^3)*\sinh(d*x + c)^2*\log(-I*\cosh(d*x + c) - I*\sinh(d*x + c) + 1) - ((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*c*d^2*e^2*f - 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 + ((a^2*b + b^3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^3 + a*b^2)*d^2*f^3)*x^2 - ((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*c*d^2*e^2*f - 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 + ((a^2*b + b^3)
\end{aligned}$$

$$\begin{aligned}
& *c^3 + 3*(a^3 + a*b^2)*c^2)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^3 + a*b^2) \\
& )*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^3 + a*b^2)*d^2*e*f^2)*x \\
& *cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*c*d^2*e^2 \\
& *f - 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 + ((a^2*b + b^3)*c^3 \\
& + 3*(a^3 + a*b^2)*c^2)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^3 + a*b^2)*d^ \\
& 2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^3 + a*b^2)*d^2*e*f^2)*x)*cos \\
& h(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d^3*f^3*x^3 + 3*(a^2*b + b^3)*c*d \\
& ^2*e^2*f - 3*((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*d*e*f^2 + ((a^2*b + b^ \\
& 3)*c^3 + 3*(a^3 + a*b^2)*c^2)*f^3 + 3*((a^2*b + b^3)*d^3*e*f^2 - (a^3 + a*b \\
& ^2)*d^2*f^3)*x^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^3 + a*b^2)*d^2*e*f^2)* \\
& x)*sinh(d*x + c)^2 + 3*((a^2*b + b^3)*d^3*e^2*f - 2*(a^3 + a*b^2)*d^2*e*f^2 \\
& )*x)*log(-cosh(d*x + c) - sinh(d*x + c) + 1) - 6*(b^3*f^3*cosh(d*x + c)^2 + \\
& 2*b^3*f^3*cosh(d*x + c)*sinh(d*x + c) + b^3*f^3*sinh(d*x + c)^2 - b^3*f^3) \\
& *polylog(4, (a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh( \\
& d*x + c))*sqrt((a^2 + b^2)/b^2))/b) - 6*(b^3*f^3*cosh(d*x + c)^2 + 2*b^3*f^ \\
& 3*cosh(d*x + c)*sinh(d*x + c) + b^3*f^3*sinh(d*x + c)^2 - b^3*f^3)*polylog( \\
& 4, (a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c)) \\
& *sqrt((a^2 + b^2)/b^2))/b) + 6*((a^2*b + b^3)*f^3*cosh(d*x + c)^2 + 2*(a^2* \\
& b + b^3)*f^3*cosh(d*x + c)*sinh(d*x + c) + (a^2*b + b^3)*f^3*sinh(d*x + c)^ \\
& 2 - (a^2*b + b^3)*f^3)*polylog(4, cosh(d*x + c) + sinh(d*x + c)) + 6*(-I*a^ \\
& 3*f^3 + a^2*b*f^3 + (I*a^3*f^3 - a^2*b*f^3)*cosh(d*x + c)^2 + 2*(I*a^3*f^3 \\
& - a^2*b*f^3)*cosh(d*x + c)*sinh(d*x + c) + (I*a^3*f^3 - a^2*b*f^3)*sinh(d*x \\
& + c)^2)*polylog(4, I*cosh(d*x + c) + I*sinh(d*x + c)) + 6*(I*a^3*f^3 + a^2 \\
& *b*f^3 + (-I*a^3*f^3 - a^2*b*f^3)*cosh(d*x + c)^2 + 2*(-I*a^3*f^3 - a^2*b*f \\
& ^3)*cosh(d*x + c)*sinh(d*x + c) + (-I*a^3*f^3 - a^2*b*f^3)*sinh(d*x + c)^2) \\
& *polylog(4, -I*cosh(d*x + c) - I*sinh(d*x + c)) + 6*((a^2*b + b^3)*f^3*cosh \\
& (d*x + c)^2 + 2*(a^2*b + b^3)*f^3*cosh(d*x + c)*sinh(d*x + c) + (a^2*b + b^ \\
& 3)*f^3*sinh(d*x + c)^2 - (a^2*b + b^3)*f^3)*polylog(4, -cosh(d*x + c) - sin \\
& h(d*x + c)) - 6*(b^3*d*f^3*x + b^3*d*e*f^2 - (b^3*d*f^3*x + b^3*d*e*f^2)*co \\
& sh(d*x + c)^2 - 2*(b^3*d*f^3*x + b^3*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) - \\
& (b^3*d*f^3*x + b^3*d*e*f^2)*sinh(d*x + c)^2)*polylog(3, (a*cosh(d*x + c) + \\
& a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2 \\
& ))/b) - 6*(b^3*d*f^3*x + b^3*d*e*f^2 - (b^3*d*f^3*x + b^3*d*e*f^2)*cosh(d*x \\
& + c)^2 - 2*(b^3*d*f^3*x + b^3*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) - (b^3* \\
& d*f^3*x + b^3*d*e*f^2)*sinh(d*x + c)^2)*polylog(3, (a*cosh(d*x + c) + a*sin \\
& h(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) \\
& + 6*((a^2*b + b^3)*d*f^3*x + (a^2*b + b^3)*d*e*f^2 - (a^3 + a*b^2)*f^3 - (( \\
& a^2*b + b^3)*d*f^3*x + (a^2*b + b^3)*d*e*f^2 - (a^3 + a*b^2)*f^3)*cosh(d*x \\
& + c)^2 - 2*((a^2*b + b^3)*d*f^3*x + (a^2*b + b^3)*d*e*f^2 - (a^3 + a*b^2)*f \\
& ^3)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d*f^3*x + (a^2*b + b^3)*d* \\
& e*f^2 - (a^3 + a*b^2)*f^3)*sinh(d*x + c)^2)*polylog(3, cosh(d*x + c) + sinh \\
& (d*x + c)) - (-6*I*a^3*d*f^3*x + 6*a^2*b*d*f^3*x - 6*I*a^3*d*e*f^2 + 6*a^2* \\
& b*d*e*f^2 + (6*I*a^3*d*f^3*x - 6*a^2*b*d*f^3*x + 6*I*a^3*d*e*f^2 - 6*a^2*b* \\
& d*e*f^2)*cosh(d*x + c)^2 + (12*I*a^3*d*f^3*x - 12*a^2*b*d*f^3*x + 12*I*a^3* \\
& d*e*f^2 - 12*a^2*b*d*e*f^2)*cosh(d*x + c)*sinh(d*x + c) + (6*I*a^3*d*f^3*x
\end{aligned}$$

$$\begin{aligned}
& - 6a^2bdf^3x + 6Ia^3d*ef^2 - 6a^2b*d*ef^2) \sinh(dx + c)^2 \operatorname{polylog}(3, I \cosh(dx + c) + I \sinh(dx + c)) - (6Ia^3d*f^3x + 6a^2b*d*f^3x + 6Ia^3d*ef^2 + 6a^2b*d*ef^2 + (-6Ia^3d*f^3x - 6a^2b*d*f^3x - 6Ia^3d*ef^2 - 6a^2b*d*ef^2) \cosh(dx + c)^2 + (-12Ia^3d*f^3x - 12a^2b*d*f^3x - 12Ia^3d*ef^2 - 12a^2b*d*ef^2) \cosh(dx + c) \sinh(dx + c) + (-6Ia^3d*f^3x - 6a^2b*d*f^3x - 6Ia^3d*ef^2 - 6a^2b*d*ef^2) \sinh(dx + c)^2) \operatorname{polylog}(3, -I \cosh(dx + c) - I \sinh(dx + c)) + 6((a^2b + b^3)d*f^3x + (a^2b + b^3)d*ef^2 + (a^3 + a*b^2)*f^3 - ((a^2b + b^3)d*f^3x + (a^2b + b^3)d*ef^2 + (a^3 + a*b^2)*f^3) \cosh(dx + c)^2 - 2((a^2b + b^3)d*f^3x + (a^2b + b^3)d*ef^2 + (a^3 + a*b^2)*f^3) \cosh(dx + c) \sinh(dx + c) - ((a^2b + b^3)d*f^3x + (a^2b + b^3)d*ef^2 + (a^3 + a*b^2)*f^3) \sinh(dx + c)^2) \operatorname{polylog}(3, -\cosh(dx + c) - \sinh(dx + c)) + 2((a^3 + a*b^2)d^3f^3x^3 + 3(a^3 + a*b^2)d^3*ef^2*x^2 + 3(a^3 + a*b^2)d^3*e^2*f*x + (a^3 + a*b^2)d^3*e^3) \sinh(dx + c)) / ((a^4 + a^2*b^2)d^4 \cosh(dx + c)^2 + 2(a^4 + a^2*b^2)d^4 \cosh(dx + c) \sinh(dx + c) + (a^4 + a^2*b^2)d^4 \sinh(dx + c)^2 - (a^4 + a^2*b^2)d^4)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*cscsh(d\*x+c)\*\*2\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*cscsh(d\*x+c)^2\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out



$$3.465 \quad \int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=982

result too large to display

```
[Out] (-2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(a*d) + (2*b^2*(e + f*x)^2*ArcTan[E^(c + d*x)])/(a*(a^2 + b^2)*d) - (4*f*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^2) + (2*b*(e + f*x)^2*ArcTanh[E^(2*c + 2*d*x)])/(a^2*d) - ((e + f*x)^2*Csch[c + d*x])/(a*d) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d) + (b^3*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d) - (b^3*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(a^2*(a^2 + b^2)*d) - (2*f^2*PolyLog[2, -E^(c + d*x)])/(a*d^3) + ((2*I)*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) - ((2*I)*b^2*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) - ((2*I)*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + ((2*I)*b^2*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) + (2*f^2*PolyLog[2, E^(c + d*x)])/(a*d^3) + (2*b^3*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d^2) + (2*b^3*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d^2) - (b^3*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(a^2*(a^2 + b^2)*d^2) + (b*f*(e + f*x)*PolyLog[2, -E^(2*c + 2*d*x)])/(a^2*d^2) - (b*f*(e + f*x)*PolyLog[2, E^(2*c + 2*d*x)])/(a^2*d^2) - ((2*I)*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(a*d^3) + ((2*I)*b^2*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^3) + ((2*I)*f^2*PolyLog[3, I*E^(c + d*x)])/(a*d^3) - ((2*I)*b^2*f^2*PolyLog[3, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^3) - (2*b^3*f^2*PolyLog[3, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d^3) - (2*b^3*f^2*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)*d^3) + (b^3*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*a^2*(a^2 + b^2)*d^3) - (b*f^2*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a^2*d^3) + (b*f^2*PolyLog[3, E^(2*c + 2*d*x)])/(2*a^2*d^3)
```

**Rubi [A]** time = 1.66756, antiderivative size = 982, normalized size of antiderivative = 1., number of steps used = 53, number of rules used = 21, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.618$ , Rules used = {5589, 2621, 321, 207, 5462, 6741, 12, 6742, 5205, 4180, 2531, 2282, 6589, 4182, 2279, 2391, 5461, 5573, 5561, 2190, 3718}

$$\frac{(e+fx)^2 \log\left(\frac{e^{c+dx} b}{a-\sqrt{a^2+b^2}} + 1\right) b^3}{a^2 (a^2 + b^2) d} + \frac{(e+fx)^2 \log\left(\frac{e^{c+dx} b}{a+\sqrt{a^2+b^2}} + 1\right) b^3}{a^2 (a^2 + b^2) d} - \frac{(e+fx)^2 \log(1 + e^{2(c+dx)}) b^3}{a^2 (a^2 + b^2) d} + \frac{2f(e+fx) \operatorname{PolyLog}}{a^2 (a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Csch[c + d\*x]^2\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (-2\*(e + f\*x)^2\*ArcTan[E^(c + d\*x)])/(a\*d) + (2\*b^2\*(e + f\*x)^2\*ArcTan[E^(c + d\*x)])/(a\*(a^2 + b^2)\*d) - (4\*f\*(e + f\*x)\*ArcTanh[E^(c + d\*x)])/(a\*d^2) + (2\*b\*(e + f\*x)^2\*ArcTanh[E^(2\*c + 2\*d\*x)])/(a^2\*d) - ((e + f\*x)^2\*Csch[c + d\*x])/(a\*d) + (b^3\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^2\*(a^2 + b^2)\*d) + (b^3\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^2\*(a^2 + b^2)\*d) - (b^3\*(e + f\*x)^2\*Log[1 + E^(2\*(c + d\*x))])/(a^2\*(a^2 + b^2)\*d) - (2\*f^2\*PolyLog[2, -E^(c + d\*x)])/(a\*d^3) + ((2\*I)\*f\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*d^2) - ((2\*I)\*b^2\*f\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*(a^2 + b^2)\*d^2) - ((2\*I)\*f\*(e + f\*x)\*PolyLog[2, I\*E^(c + d\*x)])/(a\*d^2) + ((2\*I)\*b^2\*f\*(e + f\*x)\*PolyLog[2, I\*E^(c + d\*x)])/(a\*(a^2 + b^2)\*d^2) + (2\*f^2\*PolyLog[2, E^(c + d\*x)])/(a\*d^3) + (2\*b^3\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^2\*(a^2 + b^2)\*d^2) + (2\*b^3\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^2\*(a^2 + b^2)\*d^2) - (b^3\*f\*(e + f\*x)\*PolyLog[2, -E^(2\*(c + d\*x))])/(a^2\*(a^2 + b^2)\*d^2) + (b\*f\*(e + f\*x)\*PolyLog[2, -E^(2\*c + 2\*d\*x)])/(a^2\*d^2) - (b\*f\*(e + f\*x)\*PolyLog[2, E^(2\*c + 2\*d\*x)])/(a^2\*d^2) - ((2\*I)\*f^2\*PolyLog[3, (-I)\*E^(c + d\*x)])/(a\*d^3) + ((2\*I)\*b^2\*f^2\*PolyLog[3, (-I)\*E^(c + d\*x)])/(a\*(a^2 + b^2)\*d^3) + ((2\*I)\*f^2\*PolyLog[3, I\*E^(c + d\*x)])/(a\*d^3) - ((2\*I)\*b^2\*f^2\*PolyLog[3, I\*E^(c + d\*x)])/(a\*(a^2 + b^2)\*d^3) - (2\*b^3\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^2\*(a^2 + b^2)\*d^3) - (2\*b^3\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^2\*(a^2 + b^2)\*d^3) + (b^3\*f^2\*PolyLog[3, -E^(2\*(c + d\*x))])/(2\*a^2\*(a^2 + b^2)\*d^3) - (b\*f^2\*PolyLog[3, -E^(2\*c + 2\*d\*x)])/(2\*a^2\*d^3) + (b\*f^2\*PolyLog[3, E^(2\*c + 2\*d\*x)])/(2\*a^2\*d^3)

### Rule 5589

Int[(Csch[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(p\_.)]/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] :> Dist[1/a, Int[(e + f\*x)^m\*Sech[c + d\*x]^p\*Csch[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Sech[c + d\*x]^p\*Csch[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 2621

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_.)\*sec[(e\_.) + (f\_.)\*(x\_)]^(n\_.), x\_Symbol] :> -Dist[(f\*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)]^(n + 1)/2, x], x, a\*Csc[e + f\*x], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

### Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

### Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] := With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

### Rule 6741

```
Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v !=
= u]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Sim
p[((c + d*x)^(m + 1)*(a + b*ArcTan[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 + u^2), x], x], x]
/; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] &&
!FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +
1, x]]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*(d_) + (e_.)*(x_)^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] := Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^(n - 2)/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[(e + f*x)^m*E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x] + Int[(e + f*x)^m*E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 3718

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[(c + d*x)^m*E^(2*(-(I*e) + f*fz*x))]/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \operatorname{csch}^2(c+dx) \operatorname{sech}(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^2 \operatorname{csch}(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= -\frac{(e+fx)^2 \tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)^2 \operatorname{csch}(c+dx)}{ad} - \frac{b \int (e+fx)^2 \operatorname{csch}^2(c+dx) \operatorname{sech}(c+dx) dx}{a} \\
&= -\frac{(e+fx)^2 \tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)^2 \operatorname{csch}(c+dx)}{ad} - \frac{(2b) \int (e+fx)^2 \operatorname{csch}^2(c+dx) \operatorname{sech}(c+dx) dx}{a} \\
&= -\frac{b^3(e+fx)^3}{3a^2(a^2+b^2)f} - \frac{(e+fx)^2 \tan^{-1}(\sinh(c+dx))}{ad} + \frac{2b(e+fx)^2 \tanh^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{a^2d} \\
&= -\frac{b^3(e+fx)^3}{3a^2(a^2+b^2)f} - \frac{(e+fx)^2 \tan^{-1}(\sinh(c+dx))}{ad} + \frac{2b(e+fx)^2 \tanh^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{a^2d} \\
&= \frac{2b^2(e+fx)^2 \tan^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{a(a^2+b^2)d} - \frac{(e+fx)^2 \tan^{-1}(\sinh(c+dx))}{ad} + \frac{2b(e+fx)^2 \tanh^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{a^2d} \\
&= \frac{2b^2(e+fx)^2 \tan^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{a(a^2+b^2)d} - \frac{4f(e+fx) \tanh^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{ad^2} + \frac{2b(e+fx)^2 \tanh^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{a^2d} \\
&= \frac{2b^2(e+fx)^2 \tan^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{a(a^2+b^2)d} - \frac{4f(e+fx) \tanh^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{ad^2} + \frac{2b(e+fx)^2 \tanh^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{a^2d} \\
&= -\frac{2(e+fx)^2 \tan^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{ad} + \frac{2b^2(e+fx)^2 \tan^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{a(a^2+b^2)d} - \frac{4f(e+fx) \tanh^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{ad^2} \\
&= -\frac{2(e+fx)^2 \tan^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{ad} + \frac{2b^2(e+fx)^2 \tan^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{a(a^2+b^2)d} - \frac{4f(e+fx) \tanh^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{ad^2} \\
&= -\frac{2(e+fx)^2 \tan^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{ad} + \frac{2b^2(e+fx)^2 \tan^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{a(a^2+b^2)d} - \frac{4f(e+fx) \tanh^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{ad^2} \\
&= -\frac{2(e+fx)^2 \tan^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{ad} + \frac{2b^2(e+fx)^2 \tan^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{a(a^2+b^2)d} - \frac{4f(e+fx) \tanh^{-1}\left(\frac{e^{c+dx}}{a+b \sinh(c+dx)}\right)}{ad^2}
\end{aligned}$$

**Mathematica [B]** time = 13.7961, size = 2107, normalized size = 2.15

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Csch[c + d\*x]^2\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] -(((e + f\*x)^2\*Csch[c])/(a\*d)) - (12\*b\*d^3\*e^2\*E^(2\*c)\*x - 12\*b\*d^3\*e^2\*(1 + E^(2\*c))\*x - 12\*b\*d^3\*e\*f\*x^2 - 4\*b\*d^3\*f^2\*x^3 + 12\*a\*d^2\*e^2\*(1 + E^(2\*c))\*ArcTan[E^(c + d\*x)] + 6\*b\*d^2\*e^2\*(1 + E^(2\*c))\*(2\*d\*x - Log[1 + E^(2\*(c + d\*x))]) + (12\*I)\*a\*d\*e\*(1 + E^(2\*c))\*f\*(d\*x\*(Log[1 - I\*E^(c + d\*x)] - Log[1 + I\*E^(c + d\*x)]) - PolyLog[2, (-I)\*E^(c + d\*x)] + PolyLog[2, I\*E^(c + d\*x)]) + 6\*b\*d\*e\*(1 + E^(2\*c))\*f\*(2\*d\*x\*(d\*x - Log[1 + E^(2\*(c + d\*x))]) - PolyLog[2, -E^(2\*(c + d\*x))]) + (6\*I)\*a\*(1 + E^(2\*c))\*f^2\*(d^2\*x^2\*Log[1 - I\*E^(c + d\*x)] - d^2\*x^2\*Log[1 + I\*E^(c + d\*x)] - 2\*d\*x\*PolyLog[2, (-I)\*E^(c + d\*x)] + 2\*d\*x\*PolyLog[2, I\*E^(c + d\*x)] + 2\*PolyLog[3, (-I)\*E^(c + d\*x)] - 2\*PolyLog[3, I\*E^(c + d\*x)]) + b\*(1 + E^(2\*c))\*f^2\*(2\*d^2\*x^2\*(2\*d\*x - 3\*Log[1 + E^(2\*(c + d\*x))]) - 6\*d\*x\*PolyLog[2, -E^(2\*(c + d\*x))]) + 3\*PolyLog[3, -E^(2\*(c + d\*x))])/(6\*(a^2 + b^2)\*d^3\*(1 + E^(2\*c))) - (b^3\*(6\*e^2\*E^(2\*c)\*x + 6\*e\*E^(2\*c)\*f\*x^2 + 2\*E^(2\*c)\*f^2\*x^3 + (6\*a\*Sqrt[a^2 + b^2]\*e^2\*ArcTan[(a + b\*E^(c + d\*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]\*d) + (6\*a\*Sqrt[-(a^2 + b^2)^2]\*e^2\*E^(2\*c)\*ArcTan[(a + b\*E^(c + d\*x))/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^(3/2)\*d) - (6\*a\*Sqrt[-(a^2 + b^2)^2]\*e^2\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)\*d) + (6\*a\*Sqrt[-(a^2 + b^2)^2]\*e^2\*E^(2\*c)\*ArcTanh[(a + b\*E^(c + d\*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)\*d) + (3\*e^2\*Log[2\*a\*E^(c + d\*x) + b\*(-1 + E^(2\*(c + d\*x)))]/d - (3\*e^2\*E^(2\*c)\*Log[2\*a\*E^(c + d\*x) + b\*(-1 + E^(2\*(c + d\*x)))]/d + (6\*e\*f\*x\*Log[1 + (b\*E^(2\*c + d\*x))/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)]]))/d - (6\*e\*E^(2\*c)\*f\*x\*Log[1 + (b\*E^(2\*c + d\*x))/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)]]))/d + (3\*f^2\*x^2\*Log[1 + (b\*E^(2\*c + d\*x))/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)]]))/d - (3\*E^(2\*c)\*f^2\*x^2\*Log[1 + (b\*E^(2\*c + d\*x))/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)]]))/d + (6\*e\*f\*x\*Log[1 + (b\*E^(2\*c + d\*x))/(a\*E^c + Sqrt[(a^2 + b^2)\*E^(2\*c)]]))/d - (6\*e\*E^(2\*c)\*f\*x\*Log[1 + (b\*E^(2\*c + d\*x))/(a\*E^c + Sqrt[(a^2 + b^2)\*E^(2\*c)]]))/d + (3\*f^2\*x^2\*Log[1 + (b\*E^(2\*c + d\*x))/(a\*E^c + Sqrt[(a^2 + b^2)\*E^(2\*c)]]))/d - (3\*E^(2\*c)\*f^2\*x^2\*Log[1 + (b\*E^(2\*c + d\*x))/(a\*E^c + Sqrt[(a^2 + b^2)\*E^(2\*c)]]))/d - (6\*(-1 + E^(2\*c))\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(2\*c + d\*x))/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)]])))/d^2 - (6\*(-1 + E^(2\*c))\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(2\*c + d\*x))/(a\*E^c + Sqrt[(a^2 + b^2)\*E^(2\*c)]])))/d^2 - (6\*f^2\*PolyLog[3, -((b\*E^(2\*c + d\*x))/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)]])))/d^3 + (6\*E^(2\*c)\*f^2\*PolyLog[3, -((b\*E^(2\*c + d\*x))/(a\*E^c - Sqrt[(a^2 + b^2)\*E^(2\*c)]])))/d^3 - (6\*f^2\*PolyLog[3, -((b\*E^(2\*c + d\*x))/(a\*E^c + Sqrt[(a^2 + b^2)\*E^(2\*c)]])))/d^3 + (6\*E^(2\*c)\*f^2\*PolyLog[3, -((b\*E^(2\*c + d\*x))/(a\*E^c + Sqrt[(a^2 + b^2)\*E^(2\*c)]])))/d^3)/(3\*a^2\*(a^2 + b^2)\*(-1 + E^(2\*c))) + (b\*d^3\*(e + f\*x)^3\*(-1 + Coth[c]) + 3\*d\*e\*f\*(b\*d\*e - 2\*a\*f)\*(d\*x - Log[1 - Cosh[c + d\*x] - Sinh[c + d\*x]]) - 6\*d\*f^2\*(b\*d\*e + a\*f)\*x\*Log[1 + Cosh[c + d\*x] - Sinh[c + d\*x]] - 3\*b

```

*d^2*f^3*x^2*Log[1 + Cosh[c + d*x] - Sinh[c + d*x]] - 6*d*f^2*(b*d*e - a*f)
*x*Log[1 - Cosh[c + d*x] + Sinh[c + d*x]] - 3*b*d^2*f^3*x^2*Log[1 - Cosh[c
+ d*x] + Sinh[c + d*x]] + 3*d*e*f*(b*d*e + 2*a*f)*(d*x - Log[1 + Cosh[c + d
*x] + Sinh[c + d*x]]) + 6*f^2*(b*d*e - a*f)*PolyLog[2, Cosh[c + d*x] - Sinh
[c + d*x]] + 6*f^2*(b*d*e + a*f)*PolyLog[2, -Cosh[c + d*x] + Sinh[c + d*x]]
+ 6*b*f^3*(d*x*PolyLog[2, Cosh[c + d*x] - Sinh[c + d*x]] + PolyLog[3, Cosh
[c + d*x] - Sinh[c + d*x]]) + 6*b*f^3*(d*x*PolyLog[2, -Cosh[c + d*x] + Sinh
[c + d*x]] + PolyLog[3, -Cosh[c + d*x] + Sinh[c + d*x]])/(3*a^2*d^3*f) - (
b*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Csch[c]*Sech[c])/(3*(a^2 + b^2)) + ((e + f*
x)^2*Csch[c/2]*Csch[(c + d*x)/2]*Sinh[(d*x)/2])/(2*a*d) + ((e + f*x)^2*Sech
[c/2]*Sech[(c + d*x)/2]*Sinh[(d*x)/2])/(2*a*d)

```

**Maple [F]** time = 1.095, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\operatorname{csch}(dx + c))^2 \operatorname{sech}(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b^3*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/((a^4 + a^2*b^2)*d) +
2*a*arctan(e^(-d*x - c))/((a^2 + b^2)*d) + b*log(e^(-2*d*x - 2*c) + 1)/((a^
2 + b^2)*d) + 2*e^(-d*x - c)/((a*e^(-2*d*x - 2*c) - a)*d) - b*log(e^(-d*x -
c) + 1)/(a^2*d) - b*log(e^(-d*x - c) - 1)/(a^2*d))*e^2 - 2*(f^2*x^2*e^c +
2*e*f*x*e^c)*e^(d*x)/(a*d*e^(2*d*x + 2*c) - a*d) - 2*e*f*log(e^(d*x + c) +
1)/(a*d^2) + 2*e*f*log(e^(d*x + c) - 1)/(a*d^2) - (d^2*x^2*log(e^(d*x + c)
+ 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*b*f^2/(a^2*d

```



$$\begin{aligned} &^3) - (d^2*x^2*\log(-e^{(d*x + c)} + 1) + 2*d*x*dilog(e^{(d*x + c)}) - 2*polylog \\ &(3, e^{(d*x + c)}))*b*f^2/(a^2*d^3) - 2*(b*d*e*f + a*f^2)*(d*x*\log(e^{(d*x + c)} \\ &+ 1) + dilog(-e^{(d*x + c)}))/(a^2*d^3) - 2*(b*d*e*f - a*f^2)*(d*x*\log(-e^{(d*x + c)} \\ &+ 1) + dilog(e^{(d*x + c)}))/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d \\ &*e*f + a*f^2)*d^2*x^2)/(a^2*d^3) + 1/3*(b*d^3*f^2*x^3 + 3*(b*d*e*f - a*f^2) \\ &*d^2*x^2)/(a^2*d^3) - integrate(2*(b^4*f^2*x^2 + 2*b^4*e*f*x - (a*b^3*f^2*x \\ &^2*e^c + 2*a*b^3*e*f*x*e^c)*e^{(d*x)})/(a^4*b + a^2*b^3 - (a^4*b*e^{(2*c)} + a^ \\ &2*b^3*e^{(2*c)})*e^{(2*d*x)} - 2*(a^5*e^c + a^3*b^2*e^c)*e^{(d*x)}), x) - integra \\ &te(2*(b*f^2*x^2 + 2*b*e*f*x + (a*f^2*x^2*e^c + 2*a*e*f*x*e^c)*e^{(d*x)})/(a^2 \\ &+ b^2 + (a^2*e^{(2*c)} + b^2*e^{(2*c)})*e^{(2*d*x)}), x) \end{aligned}$$


---

**Fricas [C]** time = 4.03983, size = 12925, normalized size = 13.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*cosh(d\*x+c)^2\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} &-(2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^2*e*f*x + (a^3 + a*b^2)* \\ &d^2*e^2)*\cosh(d*x + c) + 2*(b^3*d*f^2*x + b^3*d*e*f - (b^3*d*f^2*x + b^3*d* \\ &e*f)*\cosh(d*x + c)^2 - 2*(b^3*d*f^2*x + b^3*d*e*f)*\cosh(d*x + c)*\sinh(d*x + \\ &c) - (b^3*d*f^2*x + b^3*d*e*f)*\sinh(d*x + c)^2)*dilog((a*\cosh(d*x + c) + a \\ &*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} \\ &- b)/b + 1) + 2*(b^3*d*f^2*x + b^3*d*e*f - (b^3*d*f^2*x + b^3*d*e*f)*\cosh(d \\ &x + c)^2 - 2*(b^3*d*f^2*x + b^3*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c) - (b^3* \\ &d*f^2*x + b^3*d*e*f)*\sinh(d*x + c)^2)*dilog((a*\cosh(d*x + c) + a*\sinh(d*x + \\ &c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) \\ &- 2*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*e*f - (a^3 + a*b^2)*f^2 - ((a \\ &^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*e*f - (a^3 + a*b^2)*f^2)*\cosh(d*x + c \\ &)^2 - 2*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*e*f - (a^3 + a*b^2)*f^2)*\c \\ &osh(d*x + c)*\sinh(d*x + c) - ((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*e*f - \\ &(a^3 + a*b^2)*f^2)*\sinh(d*x + c)^2)*dilog(\cosh(d*x + c) + \sinh(d*x + c)) - \\ &(2*I*a^3*d*f^2*x - 2*a^2*b*d*f^2*x + 2*I*a^3*d*e*f - 2*a^2*b*d*e*f + (-2*I \\ &a^3*d*f^2*x + 2*a^2*b*d*f^2*x - 2*I*a^3*d*e*f + 2*a^2*b*d*e*f)*\cosh(d*x + \\ &c)^2 + (-4*I*a^3*d*f^2*x + 4*a^2*b*d*f^2*x - 4*I*a^3*d*e*f + 4*a^2*b*d*e*f) \\ &*\cosh(d*x + c)*\sinh(d*x + c) + (-2*I*a^3*d*f^2*x + 2*a^2*b*d*f^2*x - 2*I*a^ \\ &3*d*e*f + 2*a^2*b*d*e*f)*\sinh(d*x + c)^2)*dilog(I*\cosh(d*x + c) + I*\sinh(d \\ &x + c)) - (-2*I*a^3*d*f^2*x - 2*a^2*b*d*f^2*x - 2*I*a^3*d*e*f - 2*a^2*b*d*e \\ &*f + (2*I*a^3*d*f^2*x + 2*a^2*b*d*f^2*x + 2*I*a^3*d*e*f + 2*a^2*b*d*e*f)*\co \\ &sh(d*x + c)^2 + (4*I*a^3*d*f^2*x + 4*a^2*b*d*f^2*x + 4*I*a^3*d*e*f + 4*a^2* \end{aligned}$$

$$\begin{aligned}
& b*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c) + (2*I*a^3*d*f^2*x + 2*a^2*b*d*f^2*x + \\
& 2*I*a^3*d*e*f + 2*a^2*b*d*e*f)*\sinh(d*x + c)^2)*\operatorname{dilog}(-I*\cosh(d*x + c) - I \\
& *\sinh(d*x + c)) - 2*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*e*f + (a^3 + a \\
& *b^2)*f^2 - ((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*e*f + (a^3 + a*b^2)*f^2 \\
& ^2)*\cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d*f^2*x + (a^2*b + b^3)*d*e*f + (a^3 \\
& + a*b^2)*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - ((a^2*b + b^3)*d*f^2*x + (a^2*b \\
& + b^3)*d*e*f + (a^3 + a*b^2)*f^2)*\sinh(d*x + c)^2)*\operatorname{dilog}(-\cosh(d*x + c) - \\
& \sinh(d*x + c)) + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2 - (b^3*d^2*e^2 \\
& - 2*b^3*c*d*e*f + b^3*c^2*f^2)*\cosh(d*x + c)^2 - 2*(b^3*d^2*e^2 - 2*b^3*c*d \\
& *e*f + b^3*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - (b^3*d^2*e^2 - 2*b^3*c*d* \\
& e*f + b^3*c^2*f^2)*\sinh(d*x + c)^2)*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + \\
& c) + 2*b*\sqrt{((a^2 + b^2)/b^2) + 2*a}) + (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2 \\
& *f^2 - (b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*\cosh(d*x + c)^2 - 2*( \\
& b^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - (b \\
& ^3*d^2*e^2 - 2*b^3*c*d*e*f + b^3*c^2*f^2)*\sinh(d*x + c)^2)*\log(2*b*\cosh(d*x \\
& + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{((a^2 + b^2)/b^2) + 2*a}) + (b^3*d^2*f^2 \\
& *x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2 - (b^3*d^2*f^2*x^2 + 2 \\
& *b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*\cosh(d*x + c)^2 - 2*(b^3*d^2* \\
& f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*\cosh(d*x + c)*\sinh \\
& (d*x + c) - (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2 \\
& ^2)*\sinh(d*x + c)^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + \\
& c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2) - b}/b) + (b^3*d^2*f^2*x^2 + 2 \\
& *b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2 - (b^3*d^2*f^2*x^2 + 2*b^3*d^2 \\
& *e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*\cosh(d*x + c)^2 - 2*(b^3*d^2*f^2*x^2 \\
& + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c \\
& ) - (b^3*d^2*f^2*x^2 + 2*b^3*d^2*e*f*x + 2*b^3*c*d*e*f - b^3*c^2*f^2)*\sinh \\
& (d*x + c)^2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b* \\
& \sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2) - b}/b) - ((a^2*b + b^3)*d^2*f^2*x^2 + \\
& (a^2*b + b^3)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f - ((a^2*b + b^3)*d^2*f^2*x^2 \\
& + (a^2*b + b^3)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f \\
& + (a^3 + a*b^2)*d*f^2)*x)*\cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d^2*f^2*x^2 + \\
& (a^2*b + b^3)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^2*b + b^3)*d^2*e*f + \\
& (a^3 + a*b^2)*d*f^2)*x)*\cosh(d*x + c)*\sinh(d*x + c) - ((a^2*b + b^3)*d^2*f^2 \\
& ^2*x^2 + (a^2*b + b^3)*d^2*e^2 + 2*(a^3 + a*b^2)*d*e*f + 2*((a^2*b + b^3)*d^2 \\
& *e*f + (a^3 + a*b^2)*d*f^2)*x)*\sinh(d*x + c)^2 + 2*((a^2*b + b^3)*d^2*e*f \\
& + (a^3 + a*b^2)*d*f^2)*x)*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) - (I*a^3* \\
& d^2*e^2 - a^2*b*d^2*e^2 - 2*I*a^3*c*d*e*f + 2*a^2*b*c*d*e*f + I*a^3*c^2*f^2 \\
& - a^2*b*c^2*f^2 + (-I*a^3*d^2*e^2 + a^2*b*d^2*e^2 + 2*I*a^3*c*d*e*f - 2*a^2 \\
& *b*c*d*e*f - I*a^3*c^2*f^2 + a^2*b*c^2*f^2)*\cosh(d*x + c)^2 + (-2*I*a^3*d^2 \\
& *e^2 + 2*a^2*b*d^2*e^2 + 4*I*a^3*c*d*e*f - 4*a^2*b*c*d*e*f - 2*I*a^3*c^2*f^2 \\
& ^2 + 2*a^2*b*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (-I*a^3*d^2*e^2 + a^2*b \\
& *d^2*e^2 + 2*I*a^3*c*d*e*f - 2*a^2*b*c*d*e*f - I*a^3*c^2*f^2 + a^2*b*c^2*f^2 \\
& ^2)*\sinh(d*x + c)^2)*\log(\cosh(d*x + c) + \sinh(d*x + c) + I) - (-I*a^3*d^2*e^2 \\
& - a^2*b*d^2*e^2 + 2*I*a^3*c*d*e*f + 2*a^2*b*c*d*e*f - I*a^3*c^2*f^2 - a^2 \\
& *b*c^2*f^2 + (I*a^3*d^2*e^2 + a^2*b*d^2*e^2 - 2*I*a^3*c*d*e*f - 2*a^2*b*c*d
\end{aligned}$$

$$\begin{aligned}
& *e^f + I*a^3*c^2*f^2 + a^2*b*c^2*f^2)*\cosh(d*x + c)^2 + (2*I*a^3*d^2*e^2 + \\
& 2*a^2*b*d^2*e^2 - 4*I*a^3*c*d*e*f - 4*a^2*b*c*d*e*f + 2*I*a^3*c^2*f^2 + 2*a \\
& ^2*b*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (I*a^3*d^2*e^2 + a^2*b*d^2*e^2 \\
& - 2*I*a^3*c*d*e*f - 2*a^2*b*c*d*e*f + I*a^3*c^2*f^2 + a^2*b*c^2*f^2)*\sinh(d \\
& *x + c)^2*\log(\cosh(d*x + c) + \sinh(d*x + c) - I) - ((a^2*b + b^3)*d^2*e^2 \\
& - 2*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d*e*f + ((a^2*b + b^3)*c^2 + 2*(a^3 + a \\
& *b^2)*c)*f^2 - ((a^2*b + b^3)*d^2*e^2 - 2*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d \\
& *e*f + ((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2)*\cosh(d*x + c)^2 - 2*((a \\
& ^2*b + b^3)*d^2*e^2 - 2*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d*e*f + ((a^2*b + b \\
& ^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2)*\cosh(d*x + c)*\sinh(d*x + c) - ((a^2*b + b \\
& ^3)*d^2*e^2 - 2*(a^3 + a*b^2 + (a^2*b + b^3)*c)*d*e*f + ((a^2*b + b^3)*c^2 \\
& + 2*(a^3 + a*b^2)*c)*f^2)*\sinh(d*x + c)^2*\log(\cosh(d*x + c) + \sinh(d*x + c \\
& ) - 1) - (-I*a^3*d^2*f^2*x^2 - a^2*b*d^2*f^2*x^2 - 2*I*a^3*d^2*e*f*x - 2*a^ \\
& 2*b*d^2*e*f*x - 2*I*a^3*c*d*e*f - 2*a^2*b*c*d*e*f + I*a^3*c^2*f^2 + a^2*b*c \\
& ^2*f^2 + (I*a^3*d^2*f^2*x^2 + a^2*b*d^2*f^2*x^2 + 2*I*a^3*d^2*e*f*x + 2*a^2 \\
& *b*d^2*e*f*x + 2*I*a^3*c*d*e*f + 2*a^2*b*c*d*e*f - I*a^3*c^2*f^2 - a^2*b*c^ \\
& 2*f^2)*\cosh(d*x + c)^2 + (2*I*a^3*d^2*f^2*x^2 + 2*a^2*b*d^2*f^2*x^2 + 4*I*a \\
& ^3*d^2*e*f*x + 4*a^2*b*d^2*e*f*x + 4*I*a^3*c*d*e*f + 4*a^2*b*c*d*e*f - 2*I \\
& a^3*c^2*f^2 - 2*a^2*b*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c) + (I*a^3*d^2*f^2 \\
& *x^2 + a^2*b*d^2*f^2*x^2 + 2*I*a^3*d^2*e*f*x + 2*a^2*b*d^2*e*f*x + 2*I*a^3 \\
& c*d*e*f + 2*a^2*b*c*d*e*f - I*a^3*c^2*f^2 - a^2*b*c^2*f^2)*\sinh(d*x + c)^2) \\
& *log(I*\cosh(d*x + c) + I*\sinh(d*x + c) + 1) - (I*a^3*d^2*f^2*x^2 - a^2*b*d^ \\
& 2*f^2*x^2 + 2*I*a^3*d^2*e*f*x - 2*a^2*b*d^2*e*f*x + 2*I*a^3*c*d*e*f - 2*a^2 \\
& *b*c*d*e*f - I*a^3*c^2*f^2 + a^2*b*c^2*f^2 + (-I*a^3*d^2*f^2*x^2 + a^2*b*d^ \\
& 2*f^2*x^2 - 2*I*a^3*d^2*e*f*x + 2*a^2*b*d^2*e*f*x - 2*I*a^3*c*d*e*f + 2*a^2 \\
& *b*c*d*e*f + I*a^3*c^2*f^2 - a^2*b*c^2*f^2)*\cosh(d*x + c)^2 + (-2*I*a^3*d^2 \\
& *f^2*x^2 + 2*a^2*b*d^2*f^2*x^2 - 4*I*a^3*d^2*e*f*x + 4*a^2*b*d^2*e*f*x - 4 \\
& I*a^3*c*d*e*f + 4*a^2*b*c*d*e*f + 2*I*a^3*c^2*f^2 - 2*a^2*b*c^2*f^2)*\cosh(d \\
& *x + c)*\sinh(d*x + c) + (-I*a^3*d^2*f^2*x^2 + a^2*b*d^2*f^2*x^2 - 2*I*a^3*d \\
& ^2*e*f*x + 2*a^2*b*d^2*e*f*x - 2*I*a^3*c*d*e*f + 2*a^2*b*c*d*e*f + I*a^3*c^ \\
& 2*f^2 - a^2*b*c^2*f^2)*\sinh(d*x + c)^2)*log(-I*\cosh(d*x + c) - I*\sinh(d*x + \\
& c) + 1) - ((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*c*d*e*f - ((a^2*b + \\
& b^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2 - ((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b \\
& + b^3)*c*d*e*f - ((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2 + 2*((a^2*b + \\
& b^3)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x)*\cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d \\
& ^2*f^2*x^2 + 2*(a^2*b + b^3)*c*d*e*f - ((a^2*b + b^3)*c^2 + 2*(a^3 + a*b^2) \\
& *c)*f^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^3 + a*b^2)*d*f^2)*x)*\cosh(d*x + c) \\
& *\sinh(d*x + c) - ((a^2*b + b^3)*d^2*f^2*x^2 + 2*(a^2*b + b^3)*c*d*e*f - ((a^ \\
& 2*b + b^3)*c^2 + 2*(a^3 + a*b^2)*c)*f^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^3 + \\
& a*b^2)*d*f^2)*x)*\sinh(d*x + c)^2 + 2*((a^2*b + b^3)*d^2*e*f - (a^3 + a*b^2 \\
& )*d*f^2)*x)*log(-\cosh(d*x + c) - \sinh(d*x + c) + 1) + 2*(b^3*f^2*\cosh(d*x + \\
& c)^2 + 2*b^3*f^2*\cosh(d*x + c)*\sinh(d*x + c) + b^3*f^2*\sinh(d*x + c)^2 - b \\
& ^3*f^2)*polylog(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + \\
& b*\sinh(d*x + c))*sqrt((a^2 + b^2)/b^2))/b) + 2*(b^3*f^2*\cosh(d*x + c)^2 + 2 \\
& *b^3*f^2*\cosh(d*x + c)*\sinh(d*x + c) + b^3*f^2*\sinh(d*x + c)^2 - b^3*f^2)*p
\end{aligned}$$

$$\begin{aligned} & \text{olylog}(3, (a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx \\ & x + c)) \sqrt{(a^2 + b^2)/b^2})/b) - 2*((a^2*b + b^3)*f^2*\cosh(dx + c)^2 + \\ & 2*(a^2*b + b^3)*f^2*\cosh(dx + c)*\sinh(dx + c) + (a^2*b + b^3)*f^2*\sinh(dx \\ & x + c)^2 - (a^2*b + b^3)*f^2)*\text{polylog}(3, \cosh(dx + c) + \sinh(dx + c)) + 2 \\ & *(I*a^3*f^2 - a^2*b*f^2 + (-I*a^3*f^2 + a^2*b*f^2)*\cosh(dx + c)^2 + 2*(-I* \\ & a^3*f^2 + a^2*b*f^2)*\cosh(dx + c)*\sinh(dx + c) + (-I*a^3*f^2 + a^2*b*f^2) \\ & *\sinh(dx + c)^2)*\text{polylog}(3, I*\cosh(dx + c) + I*\sinh(dx + c)) - (2*I*a^3*f^ \\ & f^2 + 2*a^2*b*f^2 - 2*(I*a^3*f^2 + a^2*b*f^2)*\cosh(dx + c)^2 - 4*(I*a^3*f^ \\ & 2 + a^2*b*f^2)*\cosh(dx + c)*\sinh(dx + c) - 2*(I*a^3*f^2 + a^2*b*f^2)*\sinh \\ & (dx + c)^2)*\text{polylog}(3, -I*\cosh(dx + c) - I*\sinh(dx + c)) - 2*((a^2*b + b \\ & ^3)*f^2*\cosh(dx + c)^2 + 2*(a^2*b + b^3)*f^2*\cosh(dx + c)*\sinh(dx + c) + \\ & (a^2*b + b^3)*f^2*\sinh(dx + c)^2 - (a^2*b + b^3)*f^2)*\text{polylog}(3, -\cosh(dx \\ & x + c) - \sinh(dx + c)) + 2*((a^3 + a*b^2)*d^2*f^2*x^2 + 2*(a^3 + a*b^2)*d^ \\ & 2*e*f*x + (a^3 + a*b^2)*d^2*e^2)*\sinh(dx + c))/((a^4 + a^2*b^2)*d^3*\cosh(d \\ & *x + c)^2 + 2*(a^4 + a^2*b^2)*d^3*\cosh(dx + c)*\sinh(dx + c) + (a^4 + a^2* \\ & b^2)*d^3*\sinh(dx + c)^2 - (a^4 + a^2*b^2)*d^3) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*csch(d\*x+c)\*\*2\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

sage<sub>0</sub>x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*csch(d\*x+c)^2\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] sage<sub>0</sub>\*x

$$3.466 \quad \int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=591

$$\frac{b^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2 d^2 (a^2 + b^2)} + \frac{b^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2 d^2 (a^2 + b^2)} - \frac{b^3 f \operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{2a^2 d^2 (a^2 + b^2)} - \frac{ib^2 f \operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^2 (a^2 + b^2)} +$$

```
[Out] (-2*f*x*ArcTan[E^(c + d*x)])/(a*d) + (2*b^2*(e + f*x)*ArcTan[E^(c + d*x)])/(
(a*(a^2 + b^2)*d) + (f*x*ArcTan[Sinh[c + d*x]])/(a*d) - ((e + f*x)*ArcTan[S
inh[c + d*x]])/(a*d) + (2*b*(e + f*x)*ArcTanh[E^(2*c + 2*d*x)])/(a^2*d) - (
f*ArcTanh[Cosh[c + d*x]])/(a*d^2) - ((e + f*x)*Csch[c + d*x])/(a*d) + (b^3*
(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(a^2*(a^2 + b^2)*
d) + (b^3*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(a^2*(a
^2 + b^2)*d) - (b^3*(e + f*x)*Log[1 + E^(2*(c + d*x))]/(a^2*(a^2 + b^2)*d)
+ (I*f*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) - (I*b^2*f*PolyLog[2, (-I)*E^
(c + d*x)])/(a*(a^2 + b^2)*d^2) - (I*f*PolyLog[2, I*E^(c + d*x)])/(a*d^2) +
(I*b^2*f*PolyLog[2, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) + (b^3*f*PolyLog[2
, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a^2*(a^2 + b^2)*d^2) + (b^3*f
*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a^2*(a^2 + b^2)*d^2
) - (b^3*f*PolyLog[2, -E^(2*(c + d*x))]/(2*a^2*(a^2 + b^2)*d^2) + (b*f*Pol
yLog[2, -E^(2*c + 2*d*x)])/(2*a^2*d^2) - (b*f*PolyLog[2, E^(2*c + 2*d*x)]/
(2*a^2*d^2)
```

**Rubi [A]** time = 0.900204, antiderivative size = 591, normalized size of antiderivative = 1., number of steps used = 37, number of rules used = 18, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$ , Rules used = {5589, 2621, 321, 207, 5462, 5203, 12, 4180, 2279, 2391, 3770, 5461, 4182, 5573, 5561, 2190, 6742, 3718}

$$\frac{b^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2 d^2 (a^2 + b^2)} + \frac{b^3 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2 d^2 (a^2 + b^2)} - \frac{b^3 f \operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{2a^2 d^2 (a^2 + b^2)} - \frac{ib^2 f \operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{ad^2 (a^2 + b^2)} +$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-2*f*x*ArcTan[E^(c + d*x)])/(a*d) + (2*b^2*(e + f*x)*ArcTan[E^(c + d*x)])/(
(a*(a^2 + b^2)*d) + (f*x*ArcTan[Sinh[c + d*x]])/(a*d) - ((e + f*x)*ArcTan[S
inh[c + d*x]])/(a*d) + (2*b*(e + f*x)*ArcTanh[E^(2*c + 2*d*x)])/(a^2*d) - (
f*ArcTanh[Cosh[c + d*x]])/(a*d^2) - ((e + f*x)*Csch[c + d*x])/(a*d) + (b^3*
```

$$\begin{aligned} & (e + f*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])]/(a^2*(a^2 + b^2)*d) \\ & + (b^3*(e + f*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])]/(a^2*(a^2 + b^2)*d) \\ & - (b^3*(e + f*x)*\text{Log}[1 + E^{(2*(c + d*x))}]/(a^2*(a^2 + b^2)*d) \\ & + (I*f*\text{PolyLog}[2, (-I)*E^{(c + d*x)}]/(a*d^2) - (I*b^2*f*\text{PolyLog}[2, (-I)*E^{(c + d*x)}]/(a*(a^2 + b^2)*d^2) \\ & - (I*f*\text{PolyLog}[2, I*E^{(c + d*x)}]/(a*d^2) + (I*b^2*f*\text{PolyLog}[2, I*E^{(c + d*x)}]/(a*(a^2 + b^2)*d^2) \\ & + (b^3*f*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2]))]/(a^2*(a^2 + b^2)*d^2) \\ & + (b^3*f*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2]))]/(a^2*(a^2 + b^2)*d^2) \\ & - (b^3*f*\text{PolyLog}[2, -E^{(2*(c + d*x))}]/(2*a^2*(a^2 + b^2)*d^2) + (b*f*\text{PolyLog}[2, -E^{(2*c + 2*d*x)}]/(2*a^2*d^2) \\ & - (b*f*\text{PolyLog}[2, E^{(2*c + 2*d*x)}]/(2*a^2*d^2) \end{aligned}$$
Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)]/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
```

```
(b_.)*(x_)^(p_), x_Symbol] := With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

### Rule 5203

```
Int[ArcTan[u_], x_Symbol] := Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^m, x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

### Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^m*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] := Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
```

$^n, x], x] /;$  FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]

### Rule 4182

Int[csc[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 5573

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f\*x)^m\*Sech[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f\*x)^m\*Sech[c + d\*x]^n\*(a - b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]

### Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

### Rule 2190

Int[(((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^((g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a)/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 3718

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]), x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-(I\*e) + f\*fz\*x)))/(1 + E^(2\*(-(I\*e) + f\*fz\*x))), x], x] /;



FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx}{a} \\
 &= -\frac{(e+fx)\tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)\operatorname{csch}(c+dx)}{ad} - \frac{b \int (e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx) dx}{a^2} \\
 &= -\frac{(e+fx)\tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)\operatorname{csch}(c+dx)}{ad} - \frac{(2b) \int (e+fx)\operatorname{csch}(c+dx)\operatorname{sech}(c+dx) dx}{a^2} \\
 &= -\frac{b^3(e+fx)^2}{2a^2(a^2+b^2)f} + \frac{fx \tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)\tan^{-1}(\sinh(c+dx))}{ad} \\
 &= -\frac{b^3(e+fx)^2}{2a^2(a^2+b^2)f} + \frac{fx \tan^{-1}(\sinh(c+dx))}{ad} - \frac{(e+fx)\tan^{-1}(\sinh(c+dx))}{ad} \\
 &= -\frac{2fx \tan^{-1}(e^{c+dx})}{ad} + \frac{2b^2(e+fx)\tan^{-1}(e^{c+dx})}{a(a^2+b^2)d} + \frac{fx \tan^{-1}(\sinh(c+dx))}{ad} \\
 &= -\frac{2fx \tan^{-1}(e^{c+dx})}{ad} + \frac{2b^2(e+fx)\tan^{-1}(e^{c+dx})}{a(a^2+b^2)d} + \frac{fx \tan^{-1}(\sinh(c+dx))}{ad} \\
 &= -\frac{2fx \tan^{-1}(e^{c+dx})}{ad} + \frac{2b^2(e+fx)\tan^{-1}(e^{c+dx})}{a(a^2+b^2)d} + \frac{fx \tan^{-1}(\sinh(c+dx))}{ad} \\
 &= -\frac{2fx \tan^{-1}(e^{c+dx})}{ad} + \frac{2b^2(e+fx)\tan^{-1}(e^{c+dx})}{a(a^2+b^2)d} + \frac{fx \tan^{-1}(\sinh(c+dx))}{ad}
 \end{aligned}$$

**Mathematica [A]** time = 6.52814, size = 535, normalized size = 0.91

$$\frac{2iaf\operatorname{PolyLog}(2,-i(\sinh(c+dx)+\cosh(c+dx)))-2iaf\operatorname{PolyLog}(2,i(\sinh(c+dx)+\cosh(c+dx)))+bf\operatorname{PolyLog}(2,-\sinh(2(c+dx))-\cosh(2(c+dx)))-4ade \tan^{-1}(\sinh(c+dx))}{a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-((d*(e + f*x)*Coth[(c + d*x)/2])/a) - (2*b*d*e*Log[Sinh[c + d*x]])/a^2 +
(2*b*c*f*Log[Sinh[c + d*x]])/a^2 + (2*f*Log[Tanh[(c + d*x)/2]])/a + (b*f*(-
((c + d*x)*(c + d*x + 2*Log[1 - E^(-2*(c + d*x))])) + PolyLog[2, E^(-2*(c +
d*x))]))/a^2 + (2*b^3*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c +
d*x))/(a - Sqrt[a^2 + b^2]]) + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 + b^2]]) + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]
] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]]) + f*PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])))]/(a^2*(a^2 + b^2)) + (-2*b*c*d*e +
b*c^2*f - 2*b*d^2*e*x - b*d^2*f*x^2 - 4*a*d*e*ArcTan[Cosh[c + d*x] + Sinh[c
+ d*x]] - 4*a*d*f*x*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] + 2*b*d*e*Log[1
+ Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] + 2*b*d*f*x*Log[1 + Cosh[2*(c + d*
x)] + Sinh[2*(c + d*x)]] + (2*I)*a*f*PolyLog[2, (-I)*(Cosh[c + d*x] + Sinh[
c + d*x])] - (2*I)*a*f*PolyLog[2, I*(Cosh[c + d*x] + Sinh[c + d*x])] + b*f*
PolyLog[2, -Cosh[2*(c + d*x)] - Sinh[2*(c + d*x)]]/(a^2 + b^2) + (d*(e + f
*x)*Tanh[(c + d*x)/2])/a)/(2*d^2)
```

**Maple [B]** time = 0.315, size = 1529, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/a^2/d^2*b*f*c*ln(exp(d*x+c)-1)-1/a^2/d*b*f*ln(exp(d*x+c)+1)*x+1/d*e*b/a/(
a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))+1/(a^2+b^2
)^(3/2)/d^2*b^3*f*c/a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/d
^2*f*c*b/a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2)
)-1/a^2/d*b*e*ln(exp(d*x+c)-1)-1/a^2/d*b*e*ln(exp(d*x+c)+1)+1/a^2/d^2*b*f*d
ilog(exp(d*x+c))-1/a^2/d^2*b*f*dilog(exp(d*x+c)+1)+1/d^2/a*f*ln(exp(d*x+c)-
1)-1/d^2/a*f*ln(exp(d*x+c)+1)-2/d*(f*x+e)/a*exp(d*x+c)/(exp(2*d*x+2*c)-1)-b
/d*e/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a+1/
a^2/d*b^3*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1
/2)))*x+1/a^2/d*b^3*f/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2
+b^2)^(1/2)))*x+1/a^2/d^2*b^3*f/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)
-a)/(-a+(a^2+b^2)^(1/2)))*c+1/a^2/d^2*b^3*f/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2
+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2)))*c-1/a^2/d^2*b^3*f*c/(a^2+b^2)*ln(b*exp(
2*d*x+2*c)+2*a*exp(d*x+c)-b)+4*I*a/d*f/(4*a^2+4*b^2)*ln(1+I*exp(d*x+c))*x-4
*I*a/d*f/(4*a^2+4*b^2)*ln(1-I*exp(d*x+c))*x+4*I*a/d^2*f/(4*a^2+4*b^2)*ln(1+
```

$$\begin{aligned}
& I \exp(dx+c) * c - 4 * I * a / d^2 * f / (4 * a^2 + 4 * b^2) * \ln(1 - I \exp(dx+c)) * c + 4 / d * b * f / (4 * a^2 + 4 * b^2) * \ln(1 + I \exp(dx+c)) * x + 4 / d * b * f / (4 * a^2 + 4 * b^2) * \ln(1 - I \exp(dx+c)) * x + 4 / d^2 * b * f / (4 * a^2 + 4 * b^2) * \ln(1 + I \exp(dx+c)) * c + 4 / d^2 * b * f / (4 * a^2 + 4 * b^2) * \ln(1 - I \exp(dx+c)) * c - 4 / d^2 * b * f * c / (4 * a^2 + 4 * b^2) * \ln(1 + \exp(2 * dx + 2 * c)) + 1 / a / d^2 * b * f / (a^2 + b^2)^{(1/2)} * \operatorname{arctanh}(1/2 * (2 * b * \exp(dx+c) + 2 * a) / (a^2 + b^2)^{(1/2)}) + 1 / a^2 / d^2 * b^3 * f / (a^2 + b^2) * \operatorname{dilog}((-b * \exp(dx+c) + (a^2 + b^2)^{(1/2)} - a) / (-a + (a^2 + b^2)^{(1/2)})) + 1 / a^2 / d^2 * b^3 * f / (a^2 + b^2) * \operatorname{dilog}((b * \exp(dx+c) + (a^2 + b^2)^{(1/2)} + a) / (a + (a^2 + b^2)^{(1/2)})) + 8 * a / d^2 * f * c / (4 * a^2 + 4 * b^2) * \operatorname{arctan}(\exp(dx+c)) + 1 / a^2 / d * b^3 * e / (a^2 + b^2) * \ln(b * \exp(2 * dx + 2 * c) + 2 * a * \exp(dx+c) - b) - 4 * I * a / d^2 * f / (4 * a^2 + 4 * b^2) * \operatorname{dilog}(1 - I \exp(dx+c)) - 1 / (a^2 + b^2)^{(3/2)} / d^2 * f * b^3 / a * \operatorname{arctanh}(1/2 * (2 * b * \exp(dx+c) + 2 * a) / (a^2 + b^2)^{(1/2)}) - 1 / (a^2 + b^2)^{(3/2)} / d^2 * a * f * b * \operatorname{arctanh}(1/2 * (2 * b * \exp(dx+c) + 2 * a) / (a^2 + b^2)^{(1/2)}) - 1 / (a^2 + b^2)^{(3/2)} / d * b^3 * e / a * \operatorname{arctanh}(1/2 * (2 * b * \exp(dx+c) + 2 * a) / (a^2 + b^2)^{(1/2)}) + 4 * I * a / d^2 * f / (4 * a^2 + 4 * b^2) * \operatorname{dilog}(1 + I \exp(dx+c)) + b / d^2 * f * c / (a^2 + b^2)^{(3/2)} * \operatorname{arctanh}(1/2 * (2 * b * \exp(dx+c) + 2 * a) / (a^2 + b^2)^{(1/2)}) * a + 4 / d^2 * b * f / (4 * a^2 + 4 * b^2) * \operatorname{dilog}(1 + I \exp(dx+c)) + 4 / d^2 * b * f / (4 * a^2 + 4 * b^2) * \operatorname{dilog}(1 - I \exp(dx+c)) + 4 / d * b * e / (4 * a^2 + 4 * b^2) * \ln(1 + \exp(2 * dx + 2 * c)) - 8 * a / d * e / (4 * a^2 + 4 * b^2) * \operatorname{arctan}(\exp(dx+c))
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left( \frac{b^3 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{(a^4 + a^2b^2)d} + \frac{2a \arctan(e^{(-dx-c)})}{(a^2 + b^2)d} + \frac{b \log(e^{(-2dx-2c)} + 1)}{(a^2 + b^2)d} + \frac{2e^{(-dx-c)}}{(ae^{(-2dx-2c)} - a)d} - \frac{b \log(e^{(-dx-c)})}{a^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)^2\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] (b^3\*log(-2\*a\*e^(-d\*x - c) + b\*e^(-2\*d\*x - 2\*c) - b)/((a^4 + a^2\*b^2)\*d) + 2\*a\*arctan(e^(-d\*x - c))/((a^2 + b^2)\*d) + b\*log(e^(-2\*d\*x - 2\*c) + 1)/((a^2 + b^2)\*d) + 2\*e^(-d\*x - c)/((a\*e^(-2\*d\*x - 2\*c) - a)\*d) - b\*log(e^(-d\*x - c) + 1)/(a^2\*d) - b\*log(e^(-d\*x - c) - 1)/(a^2\*d))\*e + (8\*b\*d\*integrate(1/8\*x/(a^2\*d\*e^(d\*x + c) + a^2\*d), x) - 8\*b\*d\*integrate(1/8\*x/(a^2\*d\*e^(d\*x + c) - a^2\*d), x) + a\*((d\*x + c)/(a^2\*d^2) - log(e^(d\*x + c) + 1)/(a^2\*d^2)) - a\*((d\*x + c)/(a^2\*d^2) - log(e^(d\*x + c) - 1)/(a^2\*d^2)) - 2\*x\*e^(d\*x + c)/(a\*d\*e^(2\*d\*x + 2\*c) - a\*d) - 8\*integrate(-1/4\*(a\*b^3\*x\*e^(d\*x + c) - b^4\*x)/(a^4\*b + a^2\*b^3 - (a^4\*b\*e^(2\*c) + a^2\*b^3\*e^(2\*c))\*e^(2\*d\*x) - 2\*(a^5\*e^c + a^3\*b^2\*e^c)\*e^(d\*x)), x) - 8\*integrate(1/4\*(a\*x\*e^(d\*x + c) + b\*x)/(a^2 + b^2 + (a^2\*e^(2\*c) + b^2\*e^(2\*c))\*e^(2\*d\*x)), x))\*f

**Fricas [B]** time = 3.03856, size = 6273, normalized size = 10.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e)*cosh(d*x + c) - (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cosh(d*x + c)*sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 - b^3*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^3*f*cosh(d*x + c)^2 + 2*b^3*f*cosh(d*x + c)*sinh(d*x + c) + b^3*f*sinh(d*x + c)^2 - b^3*f)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + ((a^2*b + b^3)*f*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2*b + b^3)*f*sinh(d*x + c)^2 - (a^2*b + b^3)*f)*dilog(cosh(d*x + c) + sinh(d*x + c)) - (I*a^3*f - a^2*b*f + (-I*a^3*f + a^2*b*f)*cosh(d*x + c)^2 - 2*(I*a^3*f - a^2*b*f)*cosh(d*x + c)*sinh(d*x + c) + (-I*a^3*f + a^2*b*f)*sinh(d*x + c)^2)*dilog(I*cosh(d*x + c) + I*sinh(d*x + c)) - (-I*a^3*f - a^2*b*f + (I*a^3*f + a^2*b*f)*cosh(d*x + c)^2 - 2*(-I*a^3*f - a^2*b*f)*cosh(d*x + c)*sinh(d*x + c) + (I*a^3*f + a^2*b*f)*sinh(d*x + c)^2)*dilog(-I*cosh(d*x + c) - I*sinh(d*x + c)) + ((a^2*b + b^3)*f*cosh(d*x + c)^2 + 2*(a^2*b + b^3)*f*cosh(d*x + c)*sinh(d*x + c) + (a^2*b + b^3)*f*sinh(d*x + c)^2 - (a^2*b + b^3)*f)*dilog(-cosh(d*x + c) - sinh(d*x + c)) + (b^3*d*e - b^3*c*f - (b^3*d*e - b^3*c*f)*cosh(d*x + c)^2 - 2*(b^3*d*e - b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*e - b^3*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d*e - b^3*c*f - (b^3*d*e - b^3*c*f)*cosh(d*x + c)^2 - 2*(b^3*d*e - b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*e - b^3*c*f)*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 + b^2)/b^2) + 2*a) + (b^3*d*f*x + b^3*c*f - (b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^2 - 2*(b^3*d*f*x + b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*f*x + b^3*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (b^3*d*f*x + b^3*c*f - (b^3*d*f*x + b^3*c*f)*cosh(d*x + c)^2 - 2*(b^3*d*f*x + b^3*c*f)*cosh(d*x + c)*sinh(d*x + c) - (b^3*d*f*x + b^3*c*f)*sinh(d*x + c)^2)*log(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) - ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e - ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^3 + a*b^2)*f)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^3 + a*b^2)*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*d*e + (a^3 + a*b^2)*f)*sinh(d*x + c)^2 + (a^3 + a*b^2)*f)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (I*a^3*d*e - a^2*b*d*e - I*a^3*c*f + a^2*b*c*f + (-I*a^3*d*e + a^2*b*d*e + I
```

```

*a^3*c*f - a^2*b*c*f)*cosh(d*x + c)^2 + (-2*I*a^3*d*e + 2*a^2*b*d*e + 2*I*a
^3*c*f - 2*a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (-I*a^3*d*e + a^2*b*d*e
+ I*a^3*c*f - a^2*b*c*f)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c
) + I) - (-I*a^3*d*e - a^2*b*d*e + I*a^3*c*f + a^2*b*c*f + (I*a^3*d*e + a^2
*b*d*e - I*a^3*c*f - a^2*b*c*f)*cosh(d*x + c)^2 + (2*I*a^3*d*e + 2*a^2*b*d*
e - 2*I*a^3*c*f - 2*a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (I*a^3*d*e + a
^2*b*d*e - I*a^3*c*f - a^2*b*c*f)*sinh(d*x + c)^2)*log(cosh(d*x + c) + sinh
(d*x + c) - I) - ((a^2*b + b^3)*d*e - ((a^2*b + b^3)*d*e - (a^3 + a*b^2 + (
a^2*b + b^3)*c)*f)*cosh(d*x + c)^2 - 2*((a^2*b + b^3)*d*e - (a^3 + a*b^2 +
(a^2*b + b^3)*c)*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)*d*e - (a^3
+ a*b^2 + (a^2*b + b^3)*c)*f)*sinh(d*x + c)^2 - (a^3 + a*b^2 + (a^2*b + b^
3)*c)*f)*log(cosh(d*x + c) + sinh(d*x + c) - 1) - (-I*a^3*d*f*x - a^2*b*d*f
*x - I*a^3*c*f - a^2*b*c*f + (I*a^3*d*f*x + a^2*b*d*f*x + I*a^3*c*f + a^2*b
*c*f)*cosh(d*x + c)^2 + (2*I*a^3*d*f*x + 2*a^2*b*d*f*x + 2*I*a^3*c*f + 2*a^
2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (I*a^3*d*f*x + a^2*b*d*f*x + I*a^3*c
*f + a^2*b*c*f)*sinh(d*x + c)^2)*log(I*cosh(d*x + c) + I*sinh(d*x + c) + 1)
- (I*a^3*d*f*x - a^2*b*d*f*x + I*a^3*c*f - a^2*b*c*f + (-I*a^3*d*f*x + a^2
*b*d*f*x - I*a^3*c*f + a^2*b*c*f)*cosh(d*x + c)^2 + (-2*I*a^3*d*f*x + 2*a^2
*b*d*f*x - 2*I*a^3*c*f + 2*a^2*b*c*f)*cosh(d*x + c)*sinh(d*x + c) + (-I*a^3
*d*f*x + a^2*b*d*f*x - I*a^3*c*f + a^2*b*c*f)*sinh(d*x + c)^2)*log(-I*cosh(
d*x + c) - I*sinh(d*x + c) + 1) - ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*c*f
- ((a^2*b + b^3)*d*f*x + (a^2*b + b^3)*c*f)*cosh(d*x + c)^2 - 2*((a^2*b + b
^3)*d*f*x + (a^2*b + b^3)*c*f)*cosh(d*x + c)*sinh(d*x + c) - ((a^2*b + b^3)
*d*f*x + (a^2*b + b^3)*c*f)*sinh(d*x + c)^2)*log(-cosh(d*x + c) - sinh(d*x
+ c) + 1) + 2*((a^3 + a*b^2)*d*f*x + (a^3 + a*b^2)*d*e)*sinh(d*x + c))/((a^
4 + a^2*b^2)*d^2*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*d^2*cosh(d*x + c)*sinh
(d*x + c) + (a^4 + a^2*b^2)*d^2*sinh(d*x + c)^2 - (a^4 + a^2*b^2)*d^2)

```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)\*\*2\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.467 \quad \int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=104

$$\frac{b^3 \log(a + b \sinh(c + dx))}{a^2 d (a^2 + b^2)} - \frac{a \tan^{-1}(\sinh(c + dx))}{d (a^2 + b^2)} + \frac{b \log(\cosh(c + dx))}{d (a^2 + b^2)} - \frac{b \log(\sinh(c + dx))}{a^2 d} - \frac{\operatorname{csch}(c + dx)}{ad}$$

[Out] -((a\*ArcTan[Sinh[c + d\*x]])/((a^2 + b^2)\*d)) - Csch[c + d\*x]/(a\*d) + (b\*Log[Cosh[c + d\*x]])/((a^2 + b^2)\*d) - (b\*Log[Sinh[c + d\*x]])/(a^2\*d) + (b^3\*Log[a + b\*Sinh[c + d\*x]])/(a^2\*(a^2 + b^2)\*d)

**Rubi [A]** time = 0.169645, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2837, 12, 894, 635, 203, 260}

$$\frac{b^3 \log(a + b \sinh(c + dx))}{a^2 d (a^2 + b^2)} - \frac{a \tan^{-1}(\sinh(c + dx))}{d (a^2 + b^2)} + \frac{b \log(\cosh(c + dx))}{d (a^2 + b^2)} - \frac{b \log(\sinh(c + dx))}{a^2 d} - \frac{\operatorname{csch}(c + dx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[(Csch[c + d\*x]^2\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] -((a\*ArcTan[Sinh[c + d\*x]])/((a^2 + b^2)\*d)) - Csch[c + d\*x]/(a\*d) + (b\*Log[Cosh[c + d\*x]])/((a^2 + b^2)\*d) - (b\*Log[Sinh[c + d\*x]])/(a^2\*d) + (b^3\*Log[a + b\*Sinh[c + d\*x]])/(a^2\*(a^2 + b^2)\*d)

### Rule 2837

Int[cos[(e\_.) + (f\_.)\*(x\_.)]^(p\_.)\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Dist[1/(b^p\*f), Subst[Int[(a + x)^m\*(c + (d\*x)/b)^n\*(b^2 - x^2)^((p - 1)/2), x], x, b\*Sinh[e + f\*x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

### Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

### Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

### Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

### Rubi steps



$$\begin{aligned}
\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx &= -\frac{b \operatorname{Subst}\left(\int \frac{b^2}{x^2(a+x)(-b^2-x^2)} dx, x, b\sinh(c+dx)\right)}{d} \\
&= -\frac{b^3 \operatorname{Subst}\left(\int \frac{1}{x^2(a+x)(-b^2-x^2)} dx, x, b\sinh(c+dx)\right)}{d} \\
&= -\frac{b^3 \operatorname{Subst}\left(\int \left(-\frac{1}{ab^2x^2} + \frac{1}{a^2b^2x} - \frac{1}{a^2(a^2+b^2)(a+x)} + \frac{a-x}{b^2(a^2+b^2)(b^2+x^2)}\right) dx, x, b\sinh(c+dx)\right)}{d} \\
&= -\frac{\operatorname{csch}(c+dx)}{ad} - \frac{b \log(\sinh(c+dx))}{a^2d} + \frac{b^3 \log(a+b\sinh(c+dx))}{a^2(a^2+b^2)d} - \frac{b \operatorname{Subst}\left(\int \frac{a}{b^2}\right)}{d} \\
&= -\frac{\operatorname{csch}(c+dx)}{ad} - \frac{b \log(\sinh(c+dx))}{a^2d} + \frac{b^3 \log(a+b\sinh(c+dx))}{a^2(a^2+b^2)d} + \frac{b \operatorname{Subst}\left(\int \frac{a}{b^2}\right)}{d} \\
&= -\frac{a \tan^{-1}(\sinh(c+dx))}{(a^2+b^2)d} - \frac{\operatorname{csch}(c+dx)}{ad} + \frac{b \log(\cosh(c+dx))}{(a^2+b^2)d} - \frac{b \log(\sinh(c+dx))}{a^2d}
\end{aligned}$$

**Mathematica [A]** time = 0.593664, size = 160, normalized size = 1.54

$$\frac{b^3 \left( \frac{\log(\sinh(c+dx))}{a^2b^2} - \frac{(a\sqrt{-b^2+b^2}) \log(\sqrt{-b^2}-b\sinh(c+dx))}{2b^4(a^2+b^2)} - \frac{\log(a+b\sinh(c+dx))}{a^2(a^2+b^2)} - \frac{\left(\frac{a}{\sqrt{-b^2}}+1\right) \log(\sqrt{-b^2}+b\sinh(c+dx))}{2b^2(a^2+b^2)} + \frac{\operatorname{csch}(c+dx)}{ab^3} \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(Csch[c + d\*x]^2\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] -((b^3\*(Csch[c + d\*x]/(a\*b^3) + Log[Sinh[c + d\*x]]/(a^2\*b^2) - ((b^2 + a\*sqrt[-b^2])\*Log[Sqrt[-b^2] - b\*Sinh[c + d\*x]])/(2\*b^4\*(a^2 + b^2)) - Log[a + b\*Sinh[c + d\*x]]/(a^2\*(a^2 + b^2)) - ((1 + a/Sqrt[-b^2])\*Log[Sqrt[-b^2] + b\*Sinh[c + d\*x]])/(2\*b^2\*(a^2 + b^2))))/d

**Maple [A]** time = 0.003, size = 159, normalized size = 1.5

$$\frac{1}{2da} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{2da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-1} - \frac{b}{da^2} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \frac{b^3}{da^2(a^2+b^2)} \ln\left(\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2 a - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x)`

[Out]  $\frac{1}{2} \frac{d}{a} \tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right) - \frac{1}{2} \frac{d}{a} \frac{1}{\tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)} - \frac{1}{d} \frac{b \ln\left(\tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right) + \frac{1}{d} \frac{b^3}{a^2} \frac{1}{(a^2 + b^2)} \ln\left(\tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 a - 2 \tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right) b - a\right) + \frac{1}{d} \frac{1}{(a^2 + b^2)} b \ln\left(\tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 + 1\right) - \frac{2}{d} \frac{1}{(a^2 + b^2)} a \arctan\left(\tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right)}{a^4 + a^2 b^2} + \frac{2 a \arctan\left(e^{-d x - c}\right)}{(a^2 + b^2) d} + \frac{b \log\left(e^{-2 d x - 2 c} + 1\right)}{(a^2 + b^2) d} + \frac{2 e^{-d x - c}}{(a e^{-2 d x - 2 c} - a) d} - \frac{b \log\left(e^{-d x - c}\right)}{a^2 d}$

**Maxima [A]** time = 1.74286, size = 234, normalized size = 2.25

$$\frac{b^3 \log\left(-2 a e^{(-d x - c)} + b e^{(-2 d x - 2 c)} - b\right)}{(a^4 + a^2 b^2) d} + \frac{2 a \arctan\left(e^{(-d x - c)}\right)}{(a^2 + b^2) d} + \frac{b \log\left(e^{(-2 d x - 2 c)} + 1\right)}{(a^2 + b^2) d} + \frac{2 e^{(-d x - c)}}{(a e^{(-2 d x - 2 c)} - a) d} - \frac{b \log\left(e^{(-d x - c)}\right)}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out]  $b^3 \log\left(-2 a e^{(-d x - c)} + b e^{(-2 d x - 2 c)} - b\right) / ((a^4 + a^2 b^2) d) + 2 a \arctan\left(e^{(-d x - c)}\right) / ((a^2 + b^2) d) + b \log\left(e^{(-2 d x - 2 c)} + 1\right) / ((a^2 + b^2) d) + 2 e^{(-d x - c)} / ((a e^{(-2 d x - 2 c)} - a) d) - b \log\left(e^{(-d x - c)} + 1\right) / (a^2 d) - b \log\left(e^{(-d x - c)} - 1\right) / (a^2 d)$

**Fricas [B]** time = 2.8194, size = 1098, normalized size = 10.56

$$2 \left( a^3 \cosh(dx + c)^2 + 2 a^3 \cosh(dx + c) \sinh(dx + c) + a^3 \sinh(dx + c)^2 - a^3 \right) \arctan(\cosh(dx + c) + \sinh(dx + c)) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out]  $-(2(a^3 \cosh(dx + c)^2 + 2 a^3 \cosh(dx + c) \sinh(dx + c) + a^3 \sinh(dx + c)^2 - a^3) \arctan(\cosh(dx + c) + \sinh(dx + c)) + 2(a^3 + a b^2) \cosh(dx + c) - (b^3 \cosh(dx + c)^2 + 2 b^3 \cosh(dx + c) \sinh(dx + c) + b^3 \sinh(dx + c)^2 - b^3) \log(2(b \sinh(dx + c) + a) / (\cosh(dx + c) - \sinh(dx + c))))$

$x + c))) - (a^2 b \cosh(dx + c)^2 + 2 a^2 b \cosh(dx + c) \sinh(dx + c) + a^2 b \sinh(dx + c)^2 - a^2 b) \log(2 \cosh(dx + c) / (\cosh(dx + c) - \sinh(dx + c))) - (a^2 b + b^3 - (a^2 b + b^3) \cosh(dx + c)^2 - 2 (a^2 b + b^3) \cosh(dx + c) \sinh(dx + c) - (a^2 b + b^3) \sinh(dx + c)^2) \log(2 \sinh(dx + c) / (\cosh(dx + c) - \sinh(dx + c))) + 2 (a^3 + a b^2) \sinh(dx + c) / ((a^4 + a^2 b^2) d \cosh(dx + c)^2 + 2 (a^4 + a^2 b^2) d \cosh(dx + c) \sinh(dx + c) + (a^4 + a^2 b^2) d \sinh(dx + c)^2 - (a^4 + a^2 b^2) d)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)\*\*2\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [B]** time = 1.31908, size = 284, normalized size = 2.73

$$\frac{b^4 \log\left(\left|b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 2a\right|\right)}{a^4 b d + a^2 b^3 d} - \frac{\left(\pi + 2 \arctan\left(\frac{1}{2}\left(e^{2dx+2c} - 1\right)e^{(-dx-c)}\right)\right)a}{2\left(a^2 d + b^2 d\right)} + \frac{b \log\left(\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4\right)}{2\left(a^2 d + b^2 d\right)} - b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^2\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out]  $b^4 \log(\text{abs}(b(e^{(dx + c)} - e^{(-dx - c)}) + 2a)) / (a^4 b d + a^2 b^3 d) - 1/2(\pi + 2 \arctan(1/2(e^{(2dx + 2c)} - 1)e^{(-dx - c)})) * a / (a^2 d + b^2 d) + 1/2 b \log((e^{(dx + c)} - e^{(-dx - c)})^2 + 4) / (a^2 d + b^2 d) - b \log(\text{abs}(e^{(dx + c)} - e^{(-dx - c)})) / (a^2 d) + (b(e^{(dx + c)} - e^{(-dx - c)}) - 2a) / (a^2 d * (e^{(dx + c)} - e^{(-dx - c)}))$

$$3.468 \quad \int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}, x\right)$$

[Out] Unintegrable[(Csch[c + d\*x]^2\*Sech[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x]))], x]

**Rubi [A]** time = 0.0924191, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Csch[c + d\*x]^2\*Sech[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int] [(Csch[c + d\*x]^2\*Sech[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x]))], x]

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Mathematica [A]** time = 76.3949, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csch[c + d\*x]^2\*Sech[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Integrate[(Csch[c + d\*x]^2\*Sech[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Maple [A]** time = 1.513, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx + c))^2 \operatorname{sech}(dx + c)}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d\*x+c)^2\*sech(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(csch(d\*x+c)^2\*sech(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{2e^{(dx+c)}}{adf x + ade - (adf x e^{(2c)} + a d e e^{(2c)})e^{(2dx)}} - 8 \int -\frac{ab^3 e^{(2c)}}{4(a^4 b e + a^2 b^3 e + (a^4 b f + a^2 b^3 f)x - (a^4 b e e^{(2c)} + a^2 b^3 e e^{(2c)} + (a^4 b f e^{(2c)} + a^2 b^3 f e^{(2c)})x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^2\*sech(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $2e^{(dx + c)}/(a*d*f*x + a*d*e - (a*d*f*x*e^{(2*c)} + a*d*e*e^{(2*c)}))e^{(2*d*x)}$   
 $- 8*\integrate(-1/4*(a*b^3*e^{(d*x + c)} - b^4)/(a^4*b*e + a^2*b^3*e + (a^4*b*f + a^2*b^3*f)*x - (a^4*b*e*e^{(2*c)} + a^2*b^3*e*e^{(2*c)} + (a^4*b*f*e^{(2*c)} + a^2*b^3*f*e^{(2*c)})*x))*e^{(2*d*x)} - 2*(a^5*e*e^c + a^3*b^2*e*e^c + (a^5*f*e^c + a^3*b^2*f*e^c)*x)*e^{(d*x)}, x) - 8*\integrate(-1/8*(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 - (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c))*e^{(d*x)}, x) + 8*\integrate(1/8*(b*d*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c))*e^{(d*x)}, x) - 8*\integrate(1/4*(a*e^{(d*x + c)} + b)/(a^2*e + b^2*e + (a^2*f + b^2*f)*x + (a^2*e*e^{(2*c)} + b^2*e*e^{(2*c)} + (a^2*f*e^{(2*c)} + b^2*f*e^{(2*c)})*x))*e^{(2*d*x)}, x)$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{csch}(dx+c)^2 \text{sech}(dx+c)}{afx+ae+(bfx+be)\sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csch(d*x + c)^2*sech(d*x + c)/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.469 \quad \int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx) \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=914

result too large to display

```
[Out] (-2*(e + f*x)^2)/(a*d) + (b^2*(e + f*x)^2)/(a*(a^2 + b^2)*d) + (4*b*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a^2*d^2) - (4*b^3*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) + (2*b*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d) - (2*(e + f*x)^2*Coth[2*c + 2*d*x])/(a*d) + (b^4*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d) - (b^4*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d) - (2*b^2*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a*(a^2 + b^2)*d^2) + (2*f*(e + f*x)*Log[1 - E^(4*(c + d*x))])/(a*d^2) + (2*b*f*(e + f*x)*PolyLog[2, -E^(c + d*x)]/(a^2*d^2) - ((2*I)*b*f^2*PolyLog[2, (-I)*E^(c + d*x)]/(a^2*d^3) + ((2*I)*b^3*f^2*PolyLog[2, (-I)*E^(c + d*x)]/(a^2*(a^2 + b^2)*d^3) + ((2*I)*b*f^2*PolyLog[2, I*E^(c + d*x)]/(a^2*d^3) - ((2*I)*b^3*f^2*PolyLog[2, I*E^(c + d*x)]/(a^2*(a^2 + b^2)*d^3) - (2*b*f*(e + f*x)*PolyLog[2, E^(c + d*x)]/(a^2*d^2) + (2*b^4*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d^2) - (2*b^4*f*(e + f*x)*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d^2) - (b^2*f^2*PolyLog[2, -E^(2*(c + d*x))])/(a*(a^2 + b^2)*d^3) + (f^2*PolyLog[2, E^(4*(c + d*x))])/(2*a*d^3) - (2*b*f^2*PolyLog[3, -E^(c + d*x)]/(a^2*d^3) + (2*b*f^2*PolyLog[3, E^(c + d*x)]/(a^2*d^3) - (2*b^4*f^2*PolyLog[3, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d^3) + (2*b^4*f^2*PolyLog[3, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^(3/2)*d^3) - (b*(e + f*x)^2*Sech[c + d*x])/(a^2*d) + (b^3*(e + f*x)^2*Sech[c + d*x])/(a^2*(a^2 + b^2)*d) + (b^2*(e + f*x)^2*Tanh[c + d*x])/(a*(a^2 + b^2)*d)
```

**Rubi [A]** time = 2.03739, antiderivative size = 914, normalized size of antiderivative = 1., number of steps used = 51, number of rules used = 25, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.694$ , Rules used = {5589, 5461, 4184, 3716, 2190, 2279, 2391, 2622, 321, 207, 5462, 6741, 12, 6742, 6273, 4182, 2531, 2282, 6589, 4180, 5573, 3322, 2264, 3718, 5451}

$$\frac{(e+fx)^2 \log\left(\frac{e^{c+dx} b}{a-\sqrt{a^2+b^2}} + 1\right) b^4}{a^2 (a^2 + b^2)^{3/2} d} - \frac{(e+fx)^2 \log\left(\frac{e^{c+dx} b}{a+\sqrt{a^2+b^2}} + 1\right) b^4}{a^2 (a^2 + b^2)^{3/2} d} + \frac{2f(e+fx) \operatorname{PolyLog}\left(2, -\frac{b e^{c+dx}}{a-\sqrt{a^2+b^2}}\right) b^4}{a^2 (a^2 + b^2)^{3/2} d^2} - \frac{2f(e+fx) \operatorname{PolyLog}\left(2, -\frac{b e^{c+dx}}{a+\sqrt{a^2+b^2}}\right) b^4}{a^2 (a^2 + b^2)^{3/2} d^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Csch[c + d\*x]^2\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (-2\*(e + f\*x)^2)/(a\*d) + (b^2\*(e + f\*x)^2)/(a\*(a^2 + b^2)\*d) + (4\*b\*f\*(e + f\*x)\*ArcTan[E^(c + d\*x)]/(a^2\*d^2) - (4\*b^3\*f\*(e + f\*x)\*ArcTan[E^(c + d\*x)]/(a^2\*(a^2 + b^2)\*d^2) + (2\*b\*(e + f\*x)^2\*ArcTanh[E^(c + d\*x)]/(a^2\*d) - (2\*(e + f\*x)^2\*Coth[2\*c + 2\*d\*x])/(a\*d) + (b^4\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^2\*(a^2 + b^2)^(3/2)\*d) - (b^4\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^2\*(a^2 + b^2)^(3/2)\*d) - (2\*b^2\*f\*(e + f\*x)\*Log[1 + E^(2\*(c + d\*x))])/(a\*(a^2 + b^2)\*d^2) + (2\*f\*(e + f\*x)\*Log[1 - E^(4\*(c + d\*x))])/(a\*d^2) + (2\*b\*f\*(e + f\*x)\*PolyLog[2, -E^(c + d\*x)]/(a^2\*d^2) - ((2\*I)\*b\*f^2\*PolyLog[2, (-I)\*E^(c + d\*x)]/(a^2\*d^3) + ((2\*I)\*b^3\*f^2\*PolyLog[2, (-I)\*E^(c + d\*x)]/(a^2\*(a^2 + b^2)\*d^3) + ((2\*I)\*b\*f^2\*PolyLog[2, I\*E^(c + d\*x)]/(a^2\*d^3) - ((2\*I)\*b^3\*f^2\*PolyLog[2, I\*E^(c + d\*x)]/(a^2\*(a^2 + b^2)\*d^3) - (2\*b\*f\*(e + f\*x)\*PolyLog[2, E^(c + d\*x)]/(a^2\*d^2) + (2\*b^4\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^2\*(a^2 + b^2)^(3/2)\*d^2) - (2\*b^4\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^2\*(a^2 + b^2)^(3/2)\*d^2) - (b^2\*f^2\*PolyLog[2, -E^(2\*(c + d\*x))])/(a\*(a^2 + b^2)\*d^3) + (f^2\*PolyLog[2, E^(4\*(c + d\*x))])/(2\*a\*d^3) - (2\*b\*f^2\*PolyLog[3, -E^(c + d\*x)]/(a^2\*d^3) + (2\*b\*f^2\*PolyLog[3, E^(c + d\*x)]/(a^2\*d^3) - (2\*b^4\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^2\*(a^2 + b^2)^(3/2)\*d^3) + (2\*b^4\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^2\*(a^2 + b^2)^(3/2)\*d^3) - (b\*(e + f\*x)^2\*Sech[c + d\*x])/(a^2\*d) + (b^3\*(e + f\*x)^2\*Sech[c + d\*x])/(a^2\*(a^2 + b^2)\*d) + (b^2\*(e + f\*x)^2\*Tanh[c + d\*x])/(a\*(a^2 + b^2)\*d)

### Rule 5589

Int[(Csch[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(p\_.)]/((a\_) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Sech[c + d\*x]^p\*Csch[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Sech[c + d\*x]^p\*Csch[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5461

Int[Csch[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sech[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Dist[2^n, Int[(c + d\*x)^m\*Csch[2\*a + 2\*b\*x]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]

### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Sim



$p[(c + dx)^m \cot[e + fx]/f, x] + \text{Dist}[(d \cdot m)/f, \text{Int}[(c + dx)^{m-1} \cot[e + fx], x], x] /; \text{FreeQ}\{c, d, e, f\}, x\} \ \&\& \ \text{GtQ}[m, 0]$

### Rule 3716

$\text{Int}[(c + dx)^m \tan[e + \text{Pi} \cdot k + \text{Complex}[0, fz] \cdot (f + dx)], x\_Symbol] \rightarrow -\text{Simp}[(I \cdot (c + dx)^{m+1})/(d \cdot (m + 1)), x] + \text{Dist}[2 \cdot I, \text{Int}[(c + dx)^m \cdot E^{2 \cdot (-I \cdot e + f \cdot fz \cdot x)}]/(E^{2 \cdot I \cdot k \cdot \text{Pi}} \cdot (1 + E^{2 \cdot (-I \cdot e + f \cdot fz \cdot x)})/E^{2 \cdot I \cdot k \cdot \text{Pi}})], x], x] /; \text{FreeQ}\{c, d, e, f, fz\}, x\} \ \&\& \ \text{IntegerQ}[4 \cdot k] \ \&\& \ \text{IGtQ}[m, 0]$

### Rule 2190

$\text{Int}[(F + (g + (e + f \cdot x)))^n \cdot (c + dx)^m / ((a + b \cdot (F + (g + (e + f \cdot x))))^n)], x\_Symbol] \rightarrow \text{Simp}[(c + dx)^m \cdot \text{Log}[1 + (b \cdot (F + (g + (e + f \cdot x))))^n/a] / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), x] - \text{Dist}[(d \cdot m) / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), \text{Int}[(c + dx)^{m-1} \cdot \text{Log}[1 + (b \cdot (F + (g + (e + f \cdot x))))^n/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x\} \ \&\& \ \text{IGtQ}[m, 0]$

### Rule 2279

$\text{Int}[\text{Log}[a + b \cdot (F + (e + (c + dx)))^n], x\_Symbol] \rightarrow \text{Dist}[1/(d \cdot e \cdot n \cdot \text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b \cdot x]/x, x], x, (F + e \cdot (c + dx))^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x\} \ \&\& \ \text{GtQ}[a, 0]$

### Rule 2391

$\text{Int}[\text{Log}[c + (d + (e + x)^n)]/x, x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c \cdot e \cdot x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x\} \ \&\& \ \text{EqQ}[c \cdot d, 1]$

### Rule 2622

$\text{Int}[\text{csc}[e + (f + x)^n] \cdot (a + b \cdot (e + (f + x)^n))^m], x\_Symbol] \rightarrow \text{Dist}[1/(f \cdot a^n), \text{Subst}[\text{Int}[x^{m+n-1}/(-1 + x^2/a^2)^{(n+1)/2}], x], x, a \cdot \text{Sec}[e + f \cdot x], x] /; \text{FreeQ}\{a, e, f, m\}, x\} \ \&\& \ \text{IntegerQ}[(n+1)/2] \ \&\& \ \text{IntegerQ}[(m+1)/2] \ \&\& \ \text{LtQ}[0, m, n]$

### Rule 321

$\text{Int}[(c + dx)^m \cdot (a + b \cdot (c + dx)^n)^p], x\_Symbol] \rightarrow \text{Simp}[(c + dx)^{m-1} \cdot (c \cdot x)^{m-n+1} \cdot (a + b \cdot x^n)^{p+1} / (b \cdot (m + n \cdot p + 1)), x] - \text{Dist}[(a \cdot c^n \cdot (m - n + 1)) / (b \cdot (m + n \cdot p + 1)), \text{Int}[(c \cdot x)^{m-n} \cdot (a + b \cdot x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, p\}, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{GtQ}[m, n-1] \ \&\& \ \text{NeQ}[m + n \cdot p + 1, 0] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)^(n_.)*((c_.) + (d_.)*(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(p_.)], x_Symbol] := With[{u = IntHide[Csch[a + b*x]^n*Sech[a + b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x], x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6741

```
Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_)^(m_.), x_Symbol] := Simp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m + 1, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)])*((c_.) + (d_.)*(x_)^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
```

$f*Fz*x]$ ,  $x]$ ,  $x]$ ) /; FreeQ[{c, d, e, f, Fz}, x] && IGtQ[m, 0]

### Rule 2531

Int[Log[1 + (e\_.)\*((F\_)^((c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(n\_.))\*((f\_.) + (g\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((f + g\*x)^m\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n])]/(b\*c\*n\*Log[F]), x] + Dist[(g\*m)/(b\*c\*n\*Log[F]), Int[(f + g\*x)^(m - 1)\*PolyLog[2, -(e\*(F^(c\*(a + b\*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\* (F\_) [v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, Fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*Fz\*x)/E^(I\*k\*Pi)])]/(f\*Fz\*I), x] + (-Dist[(d\*m)/(f\*Fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*Fz\*x)/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*Fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*Fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c, d, e, f, Fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 5573

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f\*x)^m\*Sech[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f\*x)^m\*Sech[c + d\*x]^n\*(a - b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]

### Rule 3322

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*sin[(e\_.) + (Complex[0, Fz\_])\*(f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[(c + d\*x)^m\*E^(-(I\*e) + f\*Fz\*x)]/(-

```
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x)), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[(f + g*x)^m*F^u]/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[(f + g*x)^m*F^u]/(b + q + 2*c*F^u), x], x] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n), x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx) \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \operatorname{csch}^2(c+dx) \operatorname{sech}^2(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^2 \operatorname{csch}(c+dx) \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= \frac{4 \int (e+fx)^2 \operatorname{csch}^2(2c+2dx) dx}{a} - \frac{b \int (e+fx)^2 \operatorname{csch}(c+dx) \operatorname{sech}^2(c+dx) dx}{a^2} \\
&= \frac{b(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{a^2 d} - \frac{2(e+fx)^2 \coth(2c+2dx)}{ad} - \frac{b(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{a^2 d} \\
&= -\frac{2(e+fx)^2}{ad} + \frac{b(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{a^2 d} - \frac{2(e+fx)^2 \coth(2c+2dx)}{ad} \\
&= -\frac{2(e+fx)^2}{ad} + \frac{b(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{a^2 d} - \frac{2(e+fx)^2 \coth(2c+2dx)}{ad} \\
&= -\frac{2(e+fx)^2}{ad} + \frac{b(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{a^2 d} - \frac{2(e+fx)^2 \coth(2c+2dx)}{ad} \\
&= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a(a^2+b^2)d} - \frac{4b^3 f(e+fx) \tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)d^2} + \frac{b(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{a^2 d} \\
&= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a(a^2+b^2)d} + \frac{4bf(e+fx) \tan^{-1}(e^{c+dx})}{a^2 d^2} - \frac{4b^3 f(e+fx)}{a^2(a^2+b^2)d} \\
&= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a(a^2+b^2)d} + \frac{4bf(e+fx) \tan^{-1}(e^{c+dx})}{a^2 d^2} - \frac{4b^3 f(e+fx)}{a^2(a^2+b^2)d} \\
&= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a(a^2+b^2)d} + \frac{4bf(e+fx) \tan^{-1}(e^{c+dx})}{a^2 d^2} - \frac{4b^3 f(e+fx)}{a^2(a^2+b^2)d} \\
&= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a(a^2+b^2)d} + \frac{4bf(e+fx) \tan^{-1}(e^{c+dx})}{a^2 d^2} - \frac{4b^3 f(e+fx)}{a^2(a^2+b^2)d} \\
&= -\frac{2(e+fx)^2}{ad} + \frac{b^2(e+fx)^2}{a(a^2+b^2)d} + \frac{4bf(e+fx) \tan^{-1}(e^{c+dx})}{a^2 d^2} - \frac{4b^3 f(e+fx)}{a^2(a^2+b^2)d}
\end{aligned}$$

**Mathematica [B]** time = 28.6303, size = 2677, normalized size = 2.93

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Csch[c + d\*x]^2\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out]  $4 * (((I/2) * a * (-d * (e + f * x) * (d * (e + f * x) + (1 + I * E^{(2 * c)}) * f * \text{Log}[1 - E^{(-c - d * x)}]) + (1 + I * E^{(2 * c)}) * f * \text{Log}[1 + I * E^{(-c - d * x)}])) + (1 + I * E^{(2 * c)}) * f^2 * \text{PolyLog}[2, (-I) * E^{(-c - d * x)} + (1 + I * E^{(2 * c)}) * f^2 * \text{PolyLog}[2, E^{(-c - d * x)}])]) / ((a^2 + b^2) * d^3 * (-I + E^{(2 * c)})) - (b * (4 * a * b * d^2 * e * E^{(2 * c)} * f * x + 2 * a * b * d^2 * E^{(2 * c)} * f^2 * x^2 + 2 * a^2 * d^2 * e^2 * \text{ArcTanh}[E^{(c + d * x)}] + 2 * b^2 * d^2 * e^2 * \text{ArcTanh}[E^{(c + d * x)}] - 2 * a^2 * d^2 * e^2 * E^{(2 * c)} * \text{ArcTanh}[E^{(c + d * x)}] - 2 * b^2 * d^2 * e^2 * E^{(2 * c)} * \text{ArcTanh}[E^{(c + d * x)}] - 2 * a^2 * d^2 * e * f * x * \text{Log}[1 - E^{(c + d * x)}] - 2 * b^2 * d^2 * e * f * x * \text{Log}[1 - E^{(c + d * x)}] + 2 * a^2 * d^2 * e * E^{(2 * c)} * f * x * \text{Log}[1 - E^{(c + d * x)}] + 2 * b^2 * d^2 * e * E^{(2 * c)} * f * x * \text{Log}[1 - E^{(c + d * x)}] - a^2 * d^2 * f^2 * x^2 * \text{Log}[1 - E^{(c + d * x)}] - b^2 * d^2 * f^2 * x^2 * \text{Log}[1 - E^{(c + d * x)}] + a^2 * d^2 * E^{(2 * c)} * f^2 * x^2 * \text{Log}[1 - E^{(c + d * x)}] + b^2 * d^2 * E^{(2 * c)} * f^2 * x^2 * \text{Log}[1 - E^{(c + d * x)}]) + 2 * a^2 * d^2 * e * f * x * \text{Log}[1 + E^{(c + d * x)}] + 2 * b^2 * d^2 * e * f * x * \text{Log}[1 + E^{(c + d * x)}] - 2 * a^2 * d^2 * e * E^{(2 * c)} * f * x * \text{Log}[1 + E^{(c + d * x)}] - 2 * b^2 * d^2 * e * E^{(2 * c)} * f * x * \text{Log}[1 + E^{(c + d * x)}] + a^2 * d^2 * f^2 * x^2 * \text{Log}[1 + E^{(c + d * x)}] + b^2 * d^2 * f^2 * x^2 * \text{Log}[1 + E^{(c + d * x)}] - a^2 * d^2 * E^{(2 * c)} * f^2 * x^2 * \text{Log}[1 + E^{(c + d * x)}] - b^2 * d^2 * E^{(2 * c)} * f^2 * x^2 * \text{Log}[1 + E^{(c + d * x)}] + 2 * a * b * d * e * f * \text{Log}[1 - E^{(2 * (c + d * x))}] - 2 * a * b * d * e * E^{(2 * c)} * f * \text{Log}[1 - E^{(2 * (c + d * x))}] + 2 * a * b * d * f^2 * x * \text{Log}[1 - E^{(2 * (c + d * x))}] - 2 * a * b * d * E^{(2 * c)} * f^2 * x * \text{Log}[1 - E^{(2 * (c + d * x))}] - 2 * (a^2 + b^2) * d * (-1 + E^{(2 * c)}) * f * (e + f * x) * \text{PolyLog}[2, -E^{(c + d * x)}] + 2 * (a^2 + b^2) * d * (-1 + E^{(2 * c)}) * f * (e + f * x) * \text{PolyLog}[2, E^{(c + d * x)}] + a * b * f^2 * \text{PolyLog}[2, E^{(2 * (c + d * x))}] - a * b * E^{(2 * c)} * f^2 * \text{PolyLog}[2, E^{(2 * (c + d * x))}] - 2 * a^2 * f^2 * \text{PolyLog}[3, -E^{(c + d * x)}] - 2 * b^2 * f^2 * \text{PolyLog}[3, -E^{(c + d * x)}] + 2 * a^2 * E^{(2 * c)} * f^2 * \text{PolyLog}[3, -E^{(c + d * x)}] + 2 * b^2 * E^{(2 * c)} * f^2 * \text{PolyLog}[3, -E^{(c + d * x)}] + 2 * a^2 * f^2 * \text{PolyLog}[3, E^{(c + d * x)}] + 2 * b^2 * f^2 * \text{PolyLog}[3, E^{(c + d * x)}] - 2 * a^2 * E^{(2 * c)} * f^2 * \text{PolyLog}[3, E^{(c + d * x)}] - 2 * b^2 * E^{(2 * c)} * f^2 * \text{PolyLog}[3, E^{(c + d * x)}]) / (4 * a^2 * (a^2 + b^2) * d^3 * (-1 + E^{(2 * c)})) + (b^4 * (-2 * d^2 * e^2 * \text{ArcTanh}[(a + b * E^{(c + d * x)})] / \text{Sqrt}[a^2 + b^2]) + 2 * d^2 * e * f * x * \text{Log}[1 + (b * E^{(c + d * x)}) / (a - \text{Sqrt}[a^2 + b^2])] + d^2 * f^2 * x^2 * \text{Log}[1 + (b * E^{(c + d * x)}) / (a - \text{Sqrt}[a^2 + b^2])] - 2 * d^2 * e * f * x * \text{Log}[1 + (b * E^{(c + d * x)}) / (a + \text{Sqrt}[a^2 + b^2])] - d^2 * f^2 * x^2 * \text{Log}[1 + (b * E^{(c + d * x)}) / (a + \text{Sqrt}[a^2 + b^2])] + 2 * d * f * (e + f * x) * \text{PolyLog}[2, (b * E^{(c + d * x)}) / (-a + \text{Sqrt}[a^2 + b^2])] - 2 * d * f * (e + f * x) * \text{PolyLog}[2, -((b * E^{(c + d * x)}) / (a + \text{Sqrt}[a^2 + b^2]))] - 2 * f^2 * \text{PolyLog}[3, (b * E^{(c + d * x)}) / (-a + \text{Sqrt}[a^2 + b^2])] + 2 * f^2 * \text{PolyLog}[3, -((b * E^{(c + d * x)}) / (a + \text{Sqrt}[a^2 + b^2]))]) / (4 * a^2 * (a^2 + b^2)^(3/2) * d^3) + (a * e * f * \text{Sech}[c/2] * (\text{Cosh}[c/2] * \text{Log}[\text{Cosh}[c/2] * \text{Cosh}[(d * x)/2] + \text{Sinh}[c/2] * \text{Sinh}[(d * x)/2]] - (d$

```

*x*Sinh[c/2])/2))/(2*(a^2 + b^2)*d^2*(Cosh[c/2]^2 - Sinh[c/2]^2)) - (a*f^2*
Csch[c/2]*(-(d^2*x^2)/(4*E^ArcTanh[Coth[c/2]])) + (I*Coth[c/2]*(-(d*x*(-Pi +
(2*I)*ArcTanh[Coth[c/2]])))/2 - Pi*Log[1 + E^(d*x)] - 2*((I/2)*d*x + I*ArcT
anh[Coth[c/2]])*Log[1 - E^((2*I)*((I/2)*d*x + I*ArcTanh[Coth[c/2]])]) + Pi*
Log[Cosh[(d*x)/2]] + (2*I)*ArcTanh[Coth[c/2]]*Log[I*Sinh[(d*x)/2 + ArcTanh[
Coth[c/2]]]]) + I*PolyLog[2, E^((2*I)*((I/2)*d*x + I*ArcTanh[Coth[c/2]])])])
/Sqrt[1 - Coth[c/2]^2]*Sech[c/2])/(2*(a^2 + b^2)*d^3*Sqrt[Csch[c/2]^2*(-Co
sh[c/2]^2 + Sinh[c/2]^2)) - (e*f*x*Csch[c/2]*Sech[c/2]*(a^2*Cosh[c] - b^2*
Cosh[c] + a^2*Cosh[2*c] - I*a^2*Sinh[c] - I*b^2*Sinh[c]))/(8*a*(a^2 + b^2)*
d*(Cosh[c/2] - I*Sinh[c/2])*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c] + I*Sinh[c])
) - (f^2*x^2*Csch[c/2]*Sech[c/2]*(a^2*Cosh[c] - b^2*Cosh[c] + a^2*Cosh[2*c]
- I*a^2*Sinh[c] - I*b^2*Sinh[c]))/(16*a*(a^2 + b^2)*d*(Cosh[c/2] - I*Sinh[
c/2])*(Cosh[c/2] + I*Sinh[c/2])*(Cosh[c] + I*Sinh[c])) + (b*e*f*ArcTan[(Sin
h[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]])/((a^2 + b^2)*d^
2*Sqrt[Cosh[c]^2 - Sinh[c]^2]) - ((I/2)*a*f*(Cosh[c] + Sinh[c])*((e + f*x)
^2*(Cosh[c] - Sinh[c]))/(2*f) + ((e + f*x)*Log[1 - I*Cosh[c + d*x] + I*Sinh
[c + d*x]]*(I + Cosh[c] - Sinh[c]))/d - (I*f*PolyLog[2, I*(Cosh[c + d*x] -
Sinh[c + d*x]))*(Cosh[c] - Sinh[c])*(-I + Cosh[c] + Sinh[c]))/d^2)/((a^2 +
b^2)*d*(-I + Cosh[c] + Sinh[c])) + (b*f^2*(((I)*Csch[c]*(I*(d*x + ArcTanh
[Coth[c]]))*(Log[1 - E^(-(d*x) - ArcTanh[Coth[c]])]) - Log[1 + E^(-(d*x) - Ar
cTanh[Coth[c]])]) + I*(PolyLog[2, -E^(-(d*x) - ArcTanh[Coth[c]])]) - PolyLog
[2, E^(-(d*x) - ArcTanh[Coth[c]])])))/Sqrt[1 - Coth[c]^2] - (2*ArcTan[(Sinh
[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]]*ArcTanh[Coth[c]])
/Sqrt[Cosh[c]^2 - Sinh[c]^2]))/(2*(a^2 + b^2)*d^3) + (Csch[2*c]*Csch[2*c +
2*d*x]*(a*b*e^2*Cosh[c - d*x] + 2*a*b*e*f*x*Cosh[c - d*x] + a*b*f^2*x^2*Cos
h[c - d*x] - a*b*e^2*Cosh[3*c + d*x] - 2*a*b*e*f*x*Cosh[3*c + d*x] - a*b*f^
2*x^2*Cosh[3*c + d*x] - b^2*e^2*Sinh[2*c] - 2*b^2*e*f*x*Sinh[2*c] - b^2*f^2
*x^2*Sinh[2*c] + 2*a^2*e^2*Sinh[2*d*x] + b^2*e^2*Sinh[2*d*x] + 4*a^2*e*f*x*
Sinh[2*d*x] + 2*b^2*e*f*x*Sinh[2*d*x] + 2*a^2*f^2*x^2*Sinh[2*d*x] + b^2*f^2
*x^2*Sinh[2*d*x]))/(4*a*(a^2 + b^2)*d)

```

---

**Maple [F]** time = 1.658, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\operatorname{csch}(dx + c))^2 (\operatorname{sech}(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*csch(d\*x+c)^2\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)), x)

[Out] int((f\*x+e)^2\*csch(d\*x+c)^2\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)), x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*csch(d\*x+c)^2\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [C]** time = 5.02976, size = 23058, normalized size = 25.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*csch(d\*x+c)^2\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/2*(4*(2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*e^2 - 8*(2*a^5 + 3*a^3*b^2 + a*b^4) \\ & *c*d*e*f + 4*(2*a^5 + 3*a^3*b^2 + a*b^4)*c^2*f^2 + 4*((2*a^5 + 3*a^3*b^2 + \\ & a*b^4)*d^2*f^2*x^2 + 2*(2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*e*f*x + 2*(2*a^5 + 3 \\ & *a^3*b^2 + a*b^4)*c*d*e*f - (2*a^5 + 3*a^3*b^2 + a*b^4)*c^2*f^2)*\cosh(d*x + \\ & c)^4 + 4*((2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^5 + 3*a^3*b^2 + \\ & a*b^4)*d^2*e*f*x + 2*(2*a^5 + 3*a^3*b^2 + a*b^4)*c*d*e*f - (2*a^5 + 3*a^3*b \\ & ^2 + a*b^4)*c^2*f^2)*\sinh(d*x + c)^4 + 4*((a^4*b + a^2*b^3)*d^2*f^2*x^2 + \\ & 2*(a^4*b + a^2*b^3)*d^2*e*f*x + (a^4*b + a^2*b^3)*d^2*e^2)*\cosh(d*x + c)^3 \\ & + 4*((a^4*b + a^2*b^3)*d^2*f^2*x^2 + 2*(a^4*b + a^2*b^3)*d^2*e*f*x + (a^4*b \\ & + a^2*b^3)*d^2*e^2 + 4*((2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(2*a^5 \\ & + 3*a^3*b^2 + a*b^4)*d^2*e*f*x + 2*(2*a^5 + 3*a^3*b^2 + a*b^4)*c*d*e*f - ( \\ & 2*a^5 + 3*a^3*b^2 + a*b^4)*c^2*f^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*((a^ \\ & 3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(a^3*b^2 + a*b^4)*d^2*e*f*x + (a^3*b^2 + a*b \\ & ^4)*d^2*e^2)*\cosh(d*x + c)^2 + 4*((a^3*b^2 + a*b^4)*d^2*f^2*x^2 + 2*(a^3*b^ \\ & 2 + a*b^4)*d^2*e*f*x + (a^3*b^2 + a*b^4)*d^2*e^2 + 6*((2*a^5 + 3*a^3*b^2 + \\ & a*b^4)*d^2*f^2*x^2 + 2*(2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*e*f*x + 2*(2*a^5 + 3 \\ & *a^3*b^2 + a*b^4)*c*d*e*f - (2*a^5 + 3*a^3*b^2 + a*b^4)*c^2*f^2)*\cosh(d*x + \\ & c)^2 + 3*((a^4*b + a^2*b^3)*d^2*f^2*x^2 + 2*(a^4*b + a^2*b^3)*d^2*e*f*x + \\ & (a^4*b + a^2*b^3)*d^2*e^2)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 4*(b^5*d*f^2*x \\ & + b^5*d*e*f - (b^5*d*f^2*x + b^5*d*e*f)*\cosh(d*x + c)^4 - 4*(b^5*d*f^2*x + \end{aligned}$$



$$\begin{aligned}
& b^5 d e f) \cosh(d x + c)^3 \sinh(d x + c) - 6(b^5 d f^2 x + b^5 d e f) \cosh \\
& (d x + c)^2 \sinh(d x + c)^2 - 4(b^5 d f^2 x + b^5 d e f) \cosh(d x + c) \sin \\
& h(d x + c)^3 - (b^5 d f^2 x + b^5 d e f) \sinh(d x + c)^4 \sqrt{(a^2 + b^2) / \\
& b^2} \operatorname{dilog}((a \cosh(d x + c) + a \sinh(d x + c) + (b \cosh(d x + c) + b \sinh(d \\
& x + c)) \sqrt{(a^2 + b^2) / b^2} - b) / b + 1) - 4(b^5 d f^2 x + b^5 d e f - ( \\
& b^5 d f^2 x + b^5 d e f) \cosh(d x + c)^4 - 4(b^5 d f^2 x + b^5 d e f) \cosh \\
& (d x + c)^3 \sinh(d x + c) - 6(b^5 d f^2 x + b^5 d e f) \cosh(d x + c)^2 \sin \\
& h(d x + c)^2 - 4(b^5 d f^2 x + b^5 d e f) \cosh(d x + c) \sinh(d x + c)^3 - \\
& (b^5 d f^2 x + b^5 d e f) \sinh(d x + c)^4 \sqrt{(a^2 + b^2) / b^2} \operatorname{dilog}((a \cosh(d x + c) + a \sinh(d x + c) - (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2) / b^2} - b) / b + 1) - 2(b^5 d^2 e^2 - 2 b^5 c d e f + b^5 c^2 f^2 - (b^5 d^2 e^2 - 2 b^5 c d e f + b^5 c^2 f^2) \cosh(d x + c)^4 - 4(b^5 d^2 e^2 - 2 b^5 c d e f + b^5 c^2 f^2) \cosh(d x + c)^3 \sinh(d x + c) - 6(b^5 d^2 e^2 - 2 b^5 c d e f + b^5 c^2 f^2) \cosh(d x + c)^2 \sinh(d x + c)^2 - 4(b^5 d^2 e^2 - 2 b^5 c d e f + b^5 c^2 f^2) \cosh(d x + c) \sinh(d x + c)^3 - (b^5 d^2 e^2 - 2 b^5 c d e f + b^5 c^2 f^2) \sinh(d x + c)^4) \sqrt{(a^2 + b^2) / b^2} \log(2 b \cosh(d x + c) + 2 b \sinh(d x + c) + 2 b \sqrt{(a^2 + b^2) / b^2} + 2 a) + 2(b^5 d^2 e^2 - 2 b^5 c d e f + b^5 c^2 f^2 - (b^5 d^2 e^2 - 2 b^5 c d e f + b^5 c^2 f^2) \cosh(d x + c)^4 - 4(b^5 d^2 e^2 - 2 b^5 c d e f + b^5 c^2 f^2) \cosh(d x + c)^3 \sinh(d x + c) - 6(b^5 d^2 e^2 - 2 b^5 c d e f + b^5 c^2 f^2) \cosh(d x + c)^2 \sinh(d x + c)^2 - 4(b^5 d^2 e^2 - 2 b^5 c d e f + b^5 c^2 f^2) \cosh(d x + c) \sinh(d x + c)^3 - (b^5 d^2 e^2 - 2 b^5 c d e f + b^5 c^2 f^2) \sinh(d x + c)^4) \sqrt{(a^2 + b^2) / b^2} \log(2 b \cosh(d x + c) + 2 b \sinh(d x + c) - 2 b \sqrt{(a^2 + b^2) / b^2} + 2 a) + 2(b^5 d^2 f^2 x^2 + 2 b^5 d^2 e f x + 2 b^5 c d e f - b^5 c^2 f^2 - (b^5 d^2 f^2 x^2 + 2 b^5 d^2 e f x + 2 b^5 c d e f - b^5 c^2 f^2) \cosh(d x + c)^4 - 4(b^5 d^2 f^2 x^2 + 2 b^5 d^2 e f x + 2 b^5 c d e f - b^5 c^2 f^2) \cosh(d x + c)^3 \sinh(d x + c) - 6(b^5 d^2 f^2 x^2 + 2 b^5 d^2 e f x + 2 b^5 c d e f - b^5 c^2 f^2) \cosh(d x + c)^2 \sinh(d x + c)^2 - 4(b^5 d^2 f^2 x^2 + 2 b^5 d^2 e f x + 2 b^5 c d e f - b^5 c^2 f^2) \cosh(d x + c) \sinh(d x + c)^3 - (b^5 d^2 f^2 x^2 + 2 b^5 d^2 e f x + 2 b^5 c d e f - b^5 c^2 f^2) \sinh(d x + c)^4) \sqrt{(a^2 + b^2) / b^2} \log(-(a \cosh(d x + c) + a \sinh(d x + c) + (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2) / b^2} - b) / b) - 2(b^5 d^2 f^2 x^2 + 2 b^5 d^2 e f x + 2 b^5 c d e f - b^5 c^2 f^2 - (b^5 d^2 f^2 x^2 + 2 b^5 d^2 e f x + 2 b^5 c d e f - b^5 c^2 f^2) \cosh(d x + c)^4 - 4(b^5 d^2 f^2 x^2 + 2 b^5 d^2 e f x + 2 b^5 c d e f - b^5 c^2 f^2) \cosh(d x + c)^3 \sinh(d x + c) - 6(b^5 d^2 f^2 x^2 + 2 b^5 d^2 e f x + 2 b^5 c d e f - b^5 c^2 f^2) \cosh(d x + c)^2 \sinh(d x + c)^2 - 4(b^5 d^2 f^2 x^2 + 2 b^5 d^2 e f x + 2 b^5 c d e f - b^5 c^2 f^2) \cosh(d x + c) \sinh(d x + c)^3 - (b^5 d^2 f^2 x^2 + 2 b^5 d^2 e f x + 2 b^5 c d e f - b^5 c^2 f^2) \sinh(d x + c)^4) \sqrt{(a^2 + b^2) / b^2} \log(-(a \cosh(d x + c) + a \sinh(d x + c) - (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2) / b^2} - b) / b) + 4(b^5 f^2 \cosh(d x + c)^4 + 4 b^5 f^2 \cosh(d x + c)^3 \sinh(d x + c) + 6 b^5 f^2 \cosh(d x + c)^2 \sinh(d x + c)^2 + 4 b^5 f^2 \cosh(d x + c) \sinh(d x + c)^3 + b^5 f^2 \sinh(d x + c)^4 - b^5 f^2) \sqrt{(a^2 + b^2) / b^2} \operatorname{polylog}(3, (a \cosh(d x + c)
\end{aligned}$$

$$\begin{aligned}
& + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b} \\
& - 4*(b^5*f^2*\cosh(d*x + c)^4 + 4*b^5*f^2*\cosh(d*x + c)^3*\sinh(d*x + c) \\
& + 6*b^5*f^2*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*b^5*f^2*\cosh(d*x + c)*\sinh(d*x + c)^3 \\
& + b^5*f^2*\sinh(d*x + c)^4 - b^5*f^2)*\sqrt{(a^2 + b^2)/b^2}*polylog(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) - 4*((a^4*b + a^2*b^3)*d^2*f^2*x^2 + 2*(a^4*b + a^2*b^3)*d^2*e*f*x + (a^4*b + a^2*b^3)*d^2*e^2)*\cosh(d*x + c) - 4*((a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x - ((a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e*f - (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*\cosh(d*x + c)^4 - 4*((a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e*f - (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*\cosh(d*x + c)^3*\sinh(d*x + c) - 6*((a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e*f - (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 - 4*((a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e*f - (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 - ((a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e*f - (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*\sinh(d*x + c)^4 + (a^4*b + 2*a^2*b^3 + b^5)*d*e*f - (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*dilog(\cosh(d*x + c) + \sinh(d*x + c)) - (4*((a^5 + a^3*b^2)*f^2 + I*(a^4*b + a^2*b^3)*f^2)*\cosh(d*x + c)^4 + 16*((a^5 + a^3*b^2)*f^2 + I*(a^4*b + a^2*b^3)*f^2)*\cosh(d*x + c)^3*\sinh(d*x + c) + 24*((a^5 + a^3*b^2)*f^2 + I*(a^4*b + a^2*b^3)*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 16*((a^5 + a^3*b^2)*f^2 + I*(a^4*b + a^2*b^3)*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + 4*((a^5 + a^3*b^2)*f^2 + I*(a^4*b + a^2*b^3)*f^2)*\sinh(d*x + c)^4 - 4*(a^5 + a^3*b^2)*f^2 - 4*I*(a^4*b + a^2*b^3)*f^2)*dilog(I*\cosh(d*x + c) + I*\sinh(d*x + c)) - (4*((a^5 + a^3*b^2)*f^2 - I*(a^4*b + a^2*b^3)*f^2)*\cosh(d*x + c)^4 + 16*((a^5 + a^3*b^2)*f^2 - I*(a^4*b + a^2*b^3)*f^2)*\cosh(d*x + c)^3*\sinh(d*x + c) + 24*((a^5 + a^3*b^2)*f^2 - I*(a^4*b + a^2*b^3)*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 16*((a^5 + a^3*b^2)*f^2 - I*(a^4*b + a^2*b^3)*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + 4*((a^5 + a^3*b^2)*f^2 - I*(a^4*b + a^2*b^3)*f^2)*\sinh(d*x + c)^4 - 4*(a^5 + a^3*b^2)*f^2 + 4*I*(a^4*b + a^2*b^3)*f^2)*dilog(-I*\cosh(d*x + c) - I*\sinh(d*x + c)) + 4*((a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x - ((a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e*f + (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*\cosh(d*x + c)^4 - 4*((a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e*f + (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*\cosh(d*x + c)^3*\sinh(d*x + c) - 6*((a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e*f + (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 - 4*((a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e*f + (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 - ((a^4*b + 2*a^2*b^3 + b^5)*d*f^2*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e*f + (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*\sinh(d*x + c)^4 + (a^4*b + 2*a^2*b^3 + b^5)*d*e*f + (a^5 + 2*a^3*b^2 + a*b^4)*f^2)*dilog(-\cosh(d*x + c) - \sinh(d*x + c)) + 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*f^2*x^2 + (a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2 - ((a^4*b + 2*a^2*b^3 + b^5)*d^2*f^2*x^2 + (a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d*e*f + 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e*f + (a^5 + 2*a^3*b^2 + a*b^4)*d*f^2)*x)*\cosh(d*x + c)^4 - 4*((a^4*b + 2*a^2
\end{aligned}$$

$$\begin{aligned}
& *b^3 + b^5)*d^2*f^2*x^2 + (a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d*e*f + 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e*f + (a^5 + 2*a^3*b^2 + a*b^4)*d*f^2)*x)*\cosh(d*x + c)^3*\sinh(d*x + c) - 6*((a^4*b + 2*a^2*b^3 + b^5)*d^2*f^2*x^2 + (a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d*e*f + 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e*f + (a^5 + 2*a^3*b^2 + a*b^4)*d*f^2)*x)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 - 4*((a^4*b + 2*a^2*b^3 + b^5)*d^2*f^2*x^2 + (a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d*e*f + 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e*f + (a^5 + 2*a^3*b^2 + a*b^4)*d*f^2)*x)*\cosh(d*x + c)*\sinh(d*x + c)^3 - ((a^4*b + 2*a^2*b^3 + b^5)*d^2*f^2*x^2 + (a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d*e*f + 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e*f + (a^5 + 2*a^3*b^2 + a*b^4)*d*f^2)*x)*\sinh(d*x + c)^4 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*d*e*f + 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e*f + (a^5 + 2*a^3*b^2 + a*b^4)*d*f^2)*x)*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) - ((4*(a^5 + a^3*b^2)*d*e*f + 4*I*(a^4*b + a^2*b^3)*d*e*f - 4*(a^5 + a^3*b^2)*c*f^2 - 4*I*(a^4*b + a^2*b^3)*c*f^2)*\cosh(d*x + c)^4 + (16*(a^5 + a^3*b^2)*d*e*f + 16*I*(a^4*b + a^2*b^3)*d*e*f - 16*(a^5 + a^3*b^2)*c*f^2 - 16*I*(a^4*b + a^2*b^3)*c*f^2)*\cosh(d*x + c)^3*\sinh(d*x + c) + (24*(a^5 + a^3*b^2)*d*e*f + 24*I*(a^4*b + a^2*b^3)*d*e*f - 24*(a^5 + a^3*b^2)*c*f^2 - 24*I*(a^4*b + a^2*b^3)*c*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + (16*(a^5 + a^3*b^2)*d*e*f + 16*I*(a^4*b + a^2*b^3)*d*e*f - 16*(a^5 + a^3*b^2)*c*f^2 - 16*I*(a^4*b + a^2*b^3)*c*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (4*(a^5 + a^3*b^2)*d*e*f + 4*I*(a^4*b + a^2*b^3)*d*e*f - 4*(a^5 + a^3*b^2)*c*f^2 - 4*I*(a^4*b + a^2*b^3)*c*f^2)*\sinh(d*x + c)^4 - 4*(a^5 + a^3*b^2)*d*e*f - 4*I*(a^4*b + a^2*b^3)*d*e*f + 4*(a^5 + a^3*b^2)*c*f^2 + 4*I*(a^4*b + a^2*b^3)*c*f^2)*\log(\cosh(d*x + c) + \sinh(d*x + c) + I) - ((4*(a^5 + a^3*b^2)*d*e*f - 4*I*(a^4*b + a^2*b^3)*d*e*f - 4*(a^5 + a^3*b^2)*c*f^2 + 4*I*(a^4*b + a^2*b^3)*c*f^2)*\cosh(d*x + c)^4 + (16*(a^5 + a^3*b^2)*d*e*f - 16*I*(a^4*b + a^2*b^3)*d*e*f - 16*(a^5 + a^3*b^2)*c*f^2 + 16*I*(a^4*b + a^2*b^3)*c*f^2)*\cosh(d*x + c)^3*\sinh(d*x + c) + (24*(a^5 + a^3*b^2)*d*e*f - 24*I*(a^4*b + a^2*b^3)*d*e*f - 24*(a^5 + a^3*b^2)*c*f^2 + 24*I*(a^4*b + a^2*b^3)*c*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + (16*(a^5 + a^3*b^2)*d*e*f - 16*I*(a^4*b + a^2*b^3)*d*e*f - 16*(a^5 + a^3*b^2)*c*f^2 + 16*I*(a^4*b + a^2*b^3)*c*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (4*(a^5 + a^3*b^2)*d*e*f - 4*I*(a^4*b + a^2*b^3)*d*e*f - 4*(a^5 + a^3*b^2)*c*f^2 + 4*I*(a^4*b + a^2*b^3)*c*f^2)*\sinh(d*x + c)^4 - 4*(a^5 + a^3*b^2)*d*e*f + 4*I*(a^4*b + a^2*b^3)*d*e*f + 4*(a^5 + a^3*b^2)*c*f^2 - 4*I*(a^4*b + a^2*b^3)*c*f^2)*\log(\cosh(d*x + c) + \sinh(d*x + c) - I) - 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2 - ((a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2 - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*c)*d*e*f + ((a^4*b + 2*a^2*b^3 + b^5)*c^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*c)*f^2)*\cosh(d*x + c)^4 - 4*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e^2 - 2*(a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*c)*d*e*f + ((a^4*b + 2*a^2*b^3 + b^5)*c^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*c)*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 - 4*((a
\end{aligned}$$



$$\begin{aligned}
&^5 + 2*a^3*b^2 + a*b^4)*d*f^2)*x)*\cosh(d*x + c)*\sinh(d*x + c)^3 - ((a^4*b + \\
&2*a^2*b^3 + b^5)*d^2*f^2*x^2 + 2*(a^4*b + 2*a^2*b^3 + b^5)*c*d*e*f - ((a^4 \\
&*b + 2*a^2*b^3 + b^5)*c^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*c)*f^2 + 2*((a^4*b \\
&+ 2*a^2*b^3 + b^5)*d^2*e*f - (a^5 + 2*a^3*b^2 + a*b^4)*d*f^2)*x)*\sinh(d*x + \\
&c)^4 - ((a^4*b + 2*a^2*b^3 + b^5)*c^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*c)*f^2 \\
&+ 2*((a^4*b + 2*a^2*b^3 + b^5)*d^2*e*f - (a^5 + 2*a^3*b^2 + a*b^4)*d*f^2)* \\
&x)*\log(-\cosh(d*x + c) - \sinh(d*x + c) + 1) - 4*((a^4*b + 2*a^2*b^3 + b^5)*f \\
&^2*\cosh(d*x + c)^4 + 4*(a^4*b + 2*a^2*b^3 + b^5)*f^2*\cosh(d*x + c)^3*\sinh(d \\
&*x + c) + 6*(a^4*b + 2*a^2*b^3 + b^5)*f^2*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + \\
&4*(a^4*b + 2*a^2*b^3 + b^5)*f^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^4*b + 2 \\
&a^2*b^3 + b^5)*f^2*\sinh(d*x + c)^4 - (a^4*b + 2*a^2*b^3 + b^5)*f^2)*\text{polylo} \\
&g(3, \cosh(d*x + c) + \sinh(d*x + c)) + 4*((a^4*b + 2*a^2*b^3 + b^5)*f^2*\cosh \\
&(d*x + c)^4 + 4*(a^4*b + 2*a^2*b^3 + b^5)*f^2*\cosh(d*x + c)^3*\sinh(d*x + c) \\
&+ 6*(a^4*b + 2*a^2*b^3 + b^5)*f^2*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*(a^4 \\
&*b + 2*a^2*b^3 + b^5)*f^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^4*b + 2*a^2*b^ \\
&3 + b^5)*f^2*\sinh(d*x + c)^4 - (a^4*b + 2*a^2*b^3 + b^5)*f^2)*\text{polylog}(3, -c \\
&osh(d*x + c) - \sinh(d*x + c)) - 4*((a^4*b + a^2*b^3)*d^2*f^2*x^2 + 2*(a^4*b \\
&+ a^2*b^3)*d^2*e*f*x + (a^4*b + a^2*b^3)*d^2*e^2 - 4*((2*a^5 + 3*a^3*b^2 + \\
&a*b^4)*d^2*f^2*x^2 + 2*(2*a^5 + 3*a^3*b^2 + a*b^4)*d^2*e*f*x + 2*(2*a^5 + \\
&3*a^3*b^2 + a*b^4)*c*d*e*f - (2*a^5 + 3*a^3*b^2 + a*b^4)*c^2*f^2)*\cosh(d*x \\
&+ c)^3 - 3*((a^4*b + a^2*b^3)*d^2*f^2*x^2 + 2*(a^4*b + a^2*b^3)*d^2*e*f*x + \\
&(a^4*b + a^2*b^3)*d^2*e^2)*\cosh(d*x + c)^2 - 2*((a^3*b^2 + a*b^4)*d^2*f^2* \\
&x^2 + 2*(a^3*b^2 + a*b^4)*d^2*e*f*x + (a^3*b^2 + a*b^4)*d^2*e^2)*\cosh(d*x + \\
&c))*\sinh(d*x + c))/((a^6 + 2*a^4*b^2 + a^2*b^4)*d^3*\cosh(d*x + c)^4 + 4*(a \\
&^6 + 2*a^4*b^2 + a^2*b^4)*d^3*\cosh(d*x + c)^3*\sinh(d*x + c) + 6*(a^6 + 2*a^ \\
&4*b^2 + a^2*b^4)*d^3*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*(a^6 + 2*a^4*b^2 + \\
&a^2*b^4)*d^3*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d \\
&^3*\sinh(d*x + c)^4 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d^3)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*csc(d\*x+c)\*\*2\*sech(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.470 \quad \int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=499

$$\frac{b^4 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2 d^2 (a^2+b^2)^{3/2}} - \frac{b^4 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2 d^2 (a^2+b^2)^{3/2}} + \frac{bf \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{a^2 d^2} - \frac{bf \operatorname{PolyLog}\left(2, e^{c+dx}\right)}{a^2 d^2} - \frac{b^3 f \operatorname{tanh}\left(\frac{c+dx}{a}\right)}{a^2 d^2}$$

[Out] (b\*f\*ArcTan[Sinh[c + d\*x]])/(a^2\*d^2) - (b^3\*f\*ArcTan[Sinh[c + d\*x]])/(a^2\*(a^2 + b^2)\*d^2) + (2\*b\*f\*x\*ArcTanh[E^(c + d\*x)])/(a^2\*d) - (b\*f\*x\*ArcTanh[Cosh[c + d\*x]])/(a^2\*d) + (b\*(e + f\*x)\*ArcTanh[Cosh[c + d\*x]])/(a^2\*d) - (2\*(e + f\*x)\*Coth[2\*c + 2\*d\*x])/(a\*d) + (b^4\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^2\*(a^2 + b^2)^(3/2)\*d) - (b^4\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^2\*(a^2 + b^2)^(3/2)\*d) - (b^2\*f\*Log[Cosh[c + d\*x]])/(a\*(a^2 + b^2)\*d^2) + (f\*Log[Sinh[2\*c + 2\*d\*x]])/(a\*d^2) + (b\*f\*PolyLog[2, -E^(c + d\*x)])/(a^2\*d^2) - (b\*f\*PolyLog[2, E^(c + d\*x)])/(a^2\*d^2) + (b^4\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^2\*(a^2 + b^2)^(3/2)\*d^2) - (b^4\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^2\*(a^2 + b^2)^(3/2)\*d^2) - (b\*(e + f\*x)\*Sech[c + d\*x])/(a^2\*d) + (b^3\*(e + f\*x)\*Sech[c + d\*x])/(a^2\*(a^2 + b^2)\*d) + (b^2\*(e + f\*x)\*Tanh[c + d\*x])/(a\*(a^2 + b^2)\*d)

**Rubi [A]** time = 0.977582, antiderivative size = 499, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 20, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$ , Rules used = {5589, 5461, 4184, 3475, 2622, 321, 207, 5462, 6271, 12, 4182, 2279, 2391, 3770, 5573, 3322, 2264, 2190, 6742, 5451}

$$\frac{b^4 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2 d^2 (a^2+b^2)^{3/2}} - \frac{b^4 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^2 d^2 (a^2+b^2)^{3/2}} + \frac{bf \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{a^2 d^2} - \frac{bf \operatorname{PolyLog}\left(2, e^{c+dx}\right)}{a^2 d^2} - \frac{b^3 f \operatorname{tanh}\left(\frac{c+dx}{a}\right)}{a^2 d^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Csch[c + d\*x]^2\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (b\*f\*ArcTan[Sinh[c + d\*x]])/(a^2\*d^2) - (b^3\*f\*ArcTan[Sinh[c + d\*x]])/(a^2\*(a^2 + b^2)\*d^2) + (2\*b\*f\*x\*ArcTanh[E^(c + d\*x)])/(a^2\*d) - (b\*f\*x\*ArcTanh[Cosh[c + d\*x]])/(a^2\*d) + (b\*(e + f\*x)\*ArcTanh[Cosh[c + d\*x]])/(a^2\*d) - (2\*(e + f\*x)\*Coth[2\*c + 2\*d\*x])/(a\*d) + (b^4\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^2\*(a^2 + b^2)^(3/2)\*d) - (b^4\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^2\*(a^2 + b^2)^(3/2)\*d) - (b^2\*f\*Log[Cosh[c + d\*x]])/(a\*(a^2 + b^2)\*d^2) + (f\*Log[Sinh[2\*c + 2\*d\*x]])/(a\*d^2) + (b\*f\*PolyLog[2, -E^(c + d\*x)])/(a^2\*d^2) - (b\*f\*PolyLog[2, E^(c + d\*x)])/(a^2\*d^2) + (b^4\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^2\*(a^2 + b^2)^(3/2)\*d^2) - (b^4\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^2\*(a^2 + b^2)^(3/2)\*d^2) - (b\*(e + f\*x)\*Sech[c + d\*x])/(a^2\*d) + (b^3\*(e + f\*x)\*Sech[c + d\*x])/(a^2\*(a^2 + b^2)\*d) + (b^2\*(e + f\*x)\*Tanh[c + d\*x])/(a\*(a^2 + b^2)\*d)

$$\begin{aligned} & f \cdot \text{Log}[\text{Cosh}[c + d \cdot x]] / (a \cdot (a^2 + b^2) \cdot d^2) + (f \cdot \text{Log}[\text{Sinh}[2 \cdot c + 2 \cdot d \cdot x]]) / (a \cdot d^2) \\ & + (b \cdot f \cdot \text{PolyLog}[2, -E^{(c + d \cdot x)}]) / (a^2 \cdot d^2) - (b \cdot f \cdot \text{PolyLog}[2, E^{(c + d \cdot x)}]) / (a^2 \cdot d^2) \\ & + (b^4 \cdot f \cdot \text{PolyLog}[2, -((b \cdot E^{(c + d \cdot x)}) / (a - \text{Sqrt}[a^2 + b^2]))]) / (a^2 \cdot (a^2 + b^2)^{(3/2)} \cdot d^2) \\ & - (b^4 \cdot f \cdot \text{PolyLog}[2, -((b \cdot E^{(c + d \cdot x)}) / (a + \text{Sqrt}[a^2 + b^2]))]) / (a^2 \cdot (a^2 + b^2)^{(3/2)} \cdot d^2) \\ & - (b \cdot (e + f \cdot x) \cdot \text{Sech}[c + d \cdot x]) / (a^2 \cdot d) + (b^3 \cdot (e + f \cdot x) \cdot \text{Sech}[c + d \cdot x]) / (a^2 \cdot (a^2 + b^2) \cdot d) \\ & + (b^2 \cdot (e + f \cdot x) \cdot \text{Tanh}[c + d \cdot x]) / (a \cdot (a^2 + b^2) \cdot d) \end{aligned}$$
Rule 5589

$$\begin{aligned} & \text{Int}[(\text{Csch}[(c\_.) + (d\_.) \cdot (x\_)]^{(n\_.)} \cdot ((e\_.) + (f\_.) \cdot (x\_))^{(m\_.)} \cdot \text{Sech}[(c\_.) + (d\_.) \cdot (x\_)]^{(p\_.)}) / ((a\_.) + (b\_.) \cdot \text{Sinh}[(c\_.) + (d\_.) \cdot (x\_)]), x\_Symbol] \rightarrow \text{Dist}[1/a, \\ & \text{Int}[(e + f \cdot x)^m \cdot \text{Sech}[c + d \cdot x]^p \cdot \text{Csch}[c + d \cdot x]^n, x] - \text{Dist}[b/a, \text{Int}[(e + f \cdot x)^m \cdot \text{Sech}[c + d \cdot x]^p \cdot \text{Csch}[c + d \cdot x]^{(n - 1)} / (a + b \cdot \text{Sinh}[c + d \cdot x]), x], x] /; \\ & \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0] \end{aligned}$$
Rule 5461

$$\begin{aligned} & \text{Int}[\text{Csch}[(a\_.) + (b\_.) \cdot (x\_)]^{(n\_.)} \cdot ((c\_.) + (d\_.) \cdot (x\_))^{(m\_.)} \cdot \text{Sech}[(a\_.) + (b\_.) \cdot (x\_)]^{(n\_.)}, x\_Symbol] \rightarrow \text{Dist}[2^n, \text{Int}[(c + d \cdot x)^m \cdot \text{Csch}[2 \cdot a + 2 \cdot b \cdot x]^n, x], x] /; \\ & \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{RationalQ}[m] \&\& \text{IntegerQ}[n] \end{aligned}$$
Rule 4184

$$\begin{aligned} & \text{Int}[\text{csc}[(e\_.) + (f\_.) \cdot (x\_)]^2 \cdot ((c\_.) + (d\_.) \cdot (x\_))^{(m\_.)}, x\_Symbol] \rightarrow -\text{Simp}[(c + d \cdot x)^m \cdot \text{Cot}[e + f \cdot x] / f, x] + \text{Dist}[(d \cdot m) / f, \text{Int}[(c + d \cdot x)^{(m - 1)} \cdot \text{Cot}[e + f \cdot x], x], x] /; \\ & \text{FreeQ}\{c, d, e, f\}, x\} \&\& \text{GtQ}[m, 0] \end{aligned}$$
Rule 3475

$$\begin{aligned} & \text{Int}[\tan[(c\_.) + (d\_.) \cdot (x\_)], x\_Symbol] \rightarrow -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d \cdot x], x]] / d, x] /; \\ & \text{FreeQ}\{c, d\}, x\} \end{aligned}$$
Rule 2622

$$\begin{aligned} & \text{Int}[\text{csc}[(e\_.) + (f\_.) \cdot (x\_)]^{(n\_.)} \cdot ((a\_.) \cdot \text{sec}[(e\_.) + (f\_.) \cdot (x\_)]^{(m\_.)}), x\_Symbol] \rightarrow \text{Dist}[1 / (f \cdot a^n), \text{Subst}[\text{Int}[x^{(m + n - 1)} / (-1 + x^2 / a^2)^{(n + 1) / 2}], x], x, a \cdot \text{Sec}[e + f \cdot x], x] /; \\ & \text{FreeQ}\{a, e, f, m\}, x\} \&\& \text{IntegerQ}[(n + 1) / 2] \&\& \text{!(IntegerQ}[(m + 1) / 2] \&\& \text{LtQ}[0, m, n]) \end{aligned}$$
Rule 321

$$\begin{aligned} & \text{Int}[(c \cdot (c \cdot x)^{(m - n + 1)} \cdot (a + b \cdot x^n)^{(p + 1)}) / (b \cdot (m + n \cdot p + 1)), x] - \text{Dist}[ \end{aligned}$$



$(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), \text{Int}[(c*x)^{(m - n)}*(a + b*x^n)^p, x], x] /;$  FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 207

$\text{Int}[(a + b*x^2)^{-1}, x\_Symbol] := -\text{Simp}[\text{ArcTanh}[\text{Rt}[b, 2]*x]/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[b, 2]), x] /;$  FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

### Rule 5462

$\text{Int}[\text{Csch}[(a + b*x)^n]*((c + d*x)^m)*\text{Sech}[(a + b*x)^p], x\_Symbol] := \text{With}[\{u = \text{IntHide}[\text{Csch}[a + b*x]^n*\text{Sech}[a + b*x]^p, x]\}, \text{Dist}[(c + d*x)^m, u, x] - \text{Dist}[d*m, \text{Int}[(c + d*x)^{(m - 1)}*u, x], x] /;$  FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]

### Rule 6271

$\text{Int}[\text{ArcTanh}[u], x\_Symbol] := \text{Simp}[x*\text{ArcTanh}[u], x] - \text{Int}[\text{SimplifyIntegrand}[(x*D[u, x])/(1 - u^2), x], x] /;$  InverseFunctionFreeQ[u, x]

### Rule 12

$\text{Int}[(a)*(u), x\_Symbol] := \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b)\*(v) /; FreeQ[b, x]]

### Rule 4182

$\text{Int}[\text{csc}[(e + \text{Complex}[0, fz])*f*x]^m]*((c + d*x)^m), x\_Symbol] := \text{Simp}[(-2*(c + d*x)^m*\text{ArcTanh}[E^{-(I*e) + f*fz*x}]]/(f*fz*I), x] + (-\text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 - E^{-(I*e) + f*fz*x}], x], x] + \text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + E^{-(I*e) + f*fz*x}], x], x) /;$  FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 2279

$\text{Int}[\text{Log}[(a + b*x)*((F)^{(e*(c + d*x)})^n)], x\_Symbol] := \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{e*(c + d*x)})^n], x] /;$  FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

### Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

### Rule 3322

```
Int[(((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

### Rule 2264

```
Int[(((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rule 5451

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sech[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*Tanh[(a\_.) + (b\_.)\*(x\_)]^(p\_.), x\_Symbol] :> -Simp[((c + d\*x)^m\*Sech[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Sech[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(e + fx) \operatorname{csch}^2(c + dx) \operatorname{sech}^2(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\int (e + fx) \operatorname{csch}^2(c + dx) \operatorname{sech}^2(c + dx) dx}{a} - \frac{b \int \frac{(e + fx) \operatorname{csch}(c + dx) \operatorname{sech}^2(c + dx)}{a + b \sinh(c + dx)} dx}{a} \\
&= \frac{4 \int (e + fx) \operatorname{csch}^2(2c + 2dx) dx}{a} - \frac{b \int (e + fx) \operatorname{csch}(c + dx) \operatorname{sech}^2(c + dx) dx}{a^2} \\
&= \frac{b(e + fx) \tanh^{-1}(\cosh(c + dx))}{a^2 d} - \frac{2(e + fx) \operatorname{coth}(2c + 2dx)}{ad} - \frac{b(e + fx) \operatorname{sech}^2(c + dx)}{a^2} \\
&= \frac{b(e + fx) \tanh^{-1}(\cosh(c + dx))}{a^2 d} - \frac{2(e + fx) \operatorname{coth}(2c + 2dx)}{ad} + \frac{f \log(\sinh(c + dx))}{a^2 d} \\
&= \frac{bf \tan^{-1}(\sinh(c + dx))}{a^2 d^2} - \frac{bfx \tanh^{-1}(\cosh(c + dx))}{a^2 d} + \frac{b(e + fx) \tanh^{-1}(\cosh(c + dx))}{a^2 d} \\
&= \frac{bf \tan^{-1}(\sinh(c + dx))}{a^2 d^2} - \frac{bfx \tanh^{-1}(\cosh(c + dx))}{a^2 d} + \frac{b(e + fx) \tanh^{-1}(\cosh(c + dx))}{a^2 d} \\
&= \frac{bf \tan^{-1}(\sinh(c + dx))}{a^2 d^2} - \frac{b^3 f \tan^{-1}(\sinh(c + dx))}{a^2 (a^2 + b^2) d^2} + \frac{2bfx \tanh^{-1}(e^{c+dx})}{a^2 d} \\
&= \frac{bf \tan^{-1}(\sinh(c + dx))}{a^2 d^2} - \frac{b^3 f \tan^{-1}(\sinh(c + dx))}{a^2 (a^2 + b^2) d^2} + \frac{2bfx \tanh^{-1}(e^{c+dx})}{a^2 d} \\
&= \frac{bf \tan^{-1}(\sinh(c + dx))}{a^2 d^2} - \frac{b^3 f \tan^{-1}(\sinh(c + dx))}{a^2 (a^2 + b^2) d^2} + \frac{2bfx \tanh^{-1}(e^{c+dx})}{a^2 d}
\end{aligned}$$

**Mathematica [C]** time = 8.95006, size = 1862, normalized size = 3.73

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)\*Csch[c + d\*x]^2\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] 
$$4*(-(f*(c + d*x))/(8*(a + I*b)*d^2) + ((I/8)*((2 - I)*a^3*d*f + (3*I)*a^2*b*d*f - I*a*b^2*d*f + I*b^3*d*f + a^2*b*c*d*f + I*a*b^2*c*d*f)*(c + d*x))/(a*(a + I*b)*(a^2 + b^2)*d^3) - ((I/16)*b*f*(c + d*x)^2)/((a^2 + b^2)*d^2) + ((I/4)*f*ArcTan[(a*Cosh[(c + d*x)/2] - b*Cosh[(c + d*x)/2] + a*Sinh[(c + d*x)/2] + b*Sinh[(c + d*x)/2])/(a*Cosh[(c + d*x)/2] + b*Cosh[(c + d*x)/2] - a*Sinh[(c + d*x)/2] + b*Sinh[(c + d*x)/2])])/(a*(a + I*b)*d^2) - (a*f*ArcTan[1 - (2*I)*Tanh[(c + d*x)/2]])/(2*(a^2 + b^2)*d^2) - (b^2*f*ArcTan[1 - (2*I)*Tanh[(c + d*x)/2]])/(2*a*(a^2 + b^2)*d^2) - (b*c*f*ArcTan[1 - (2*I)*Tanh[(c + d*x)/2]])/(2*(a^2 + b^2)*d^2) + ((-d*e*Cosh[(c + d*x)/2]) + c*f*Cosh[(c + d*x)/2] - f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2]/(8*a*d^2) + (a*f*Log[Cosh[(c + d*x)/2]])/(4*(a^2 + b^2)*d^2) + (b^2*f*Log[Cosh[(c + d*x)/2]])/(4*a*(a^2 + b^2)*d^2) - (b*c*f*Log[Cosh[(c + d*x)/2]])/(4*(a^2 + b^2)*d^2) + (f*Log[Cosh[c + d*x]])/(8*(a + I*b)*d^2) + (a*f*((-I)*(c + d*x) + 2*ArcTan[1 - (2*I)*Tanh[(c + d*x)/2]] + Log[-1 + Cosh[c + d*x] + I*Sinh[c + d*x]]))/(4*(a^2 + b^2)*d^2) + ((I/8)*b*f*((-I)*(c + d*x) + 2*ArcTan[1 - (2*I)*Tanh[(c + d*x)/2]] + Log[-1 + Cosh[c + d*x] + I*Sinh[c + d*x]]))/(a^2 + b^2)*d^2 + (b^2*f*((-I)*(c + d*x) + 2*ArcTan[1 - (2*I)*Tanh[(c + d*x)/2]] + Log[-1 + Cosh[c + d*x] + I*Sinh[c + d*x]]))/(8*a*(a^2 + b^2)*d^2) + (b*c*f*((-I)*(c + d*x) + 2*ArcTan[1 - (2*I)*Tanh[(c + d*x)/2]] + Log[-1 + Cosh[c + d*x] + I*Sinh[c + d*x]]))/(8*(a^2 + b^2)*d^2) - (b*e*Log[Tanh[(c + d*x)/2]])/(4*(a^2 + b^2)*d) - (b^3*e*Log[Tanh[(c + d*x)/2]])/(4*a^2*(a^2 + b^2)*d) + (b^3*c*f*Log[Tanh[(c + d*x)/2]])/(4*a^2*(a^2 + b^2)*d^2) + ((I/2)*b*f*((-I/8)*(c + d*x)^2 - (I/2)*(c + d*x)*Log[1 + E^(-c - d*x)] + (I/2)*PolyLog[2, -E^(-c - d*x)]))/(a^2 + b^2)*d^2 - (b*f*((-I/2)*(c + d*x)^2 + (I/4)*(3*Pi*(c + d*x) + (1 - I)*(c + d*x)^2 + 2*(Pi - (2*I)*(c + d*x))*Log[1 + I*E^(-c - d*x)] - 4*Pi*Log[1 + E^(c + d*x)] - 2*Pi*Log[-Cos[(Pi + (2*I)*(c + d*x))/4]] + 4*Pi*Log[Cosh[(c + d*x)/2]] + (4*I)*PolyLog[2, (-I)*E^(-c - d*x)])))/(4*(a^2 + b^2)*d^2) + ((I/4)*b^3*f*(I*(c + d*x)*(Log[1 - E^(-c - d*x)] - Log[1 + E^(-c - d*x)]) + I*(PolyLog[2, -E^(-c - d*x)] - PolyLog[2, E^(-c - d*x)])))/(a^2*(a^2 + b^2)*d^2) - ((I/4)*b*f*((c + d*x)^2/4 + (-3*Pi*(c + d*x) - (1 - I)*(c + d*x)^2 - 2*(Pi - (2*I)*(c + d*x))*Log[1 + I*E^(-c - d*x)] + 4*Pi*Log[1 + E^(c + d*x)] + 2*Pi*Log[-Cos[(Pi + (2*I)*(c + d*x))/4]] - 4*Pi*Log[Cosh[(c + d*x)/2]] - (4*I)*PolyLog[2, (-I)*E^(-c - d*x)])/4 - (I/2)*(-(c + d*x)^2/2 + 2*(c + d*x)*Log[1 - E^(c + d*x)] + 2*PolyLog[2, E^(c + d*x)])))/(a^2 + b^2)*d^2) + (b^4*(-2*d*e*ArcTan[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + 2*c*f*ArcTan[(a + b*Cosh[c + d*x] + b*Sinh[c + d*x])/Sqrt[a^2 + b^2]] + f*(c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a - Sqrt[a^2 + b^2])] - f*(c + d*x)*Log[1 + (b*(Cosh[c + d*x] + Sinh[c + d*x]))/(a + Sqrt[a^2 + b^2])] + f*PolyLog[2, (b*(Cos$$

$$\frac{h[c + d*x] + \text{Sinh}[c + d*x]}{(-a + \text{Sqrt}[a^2 + b^2])} - f \cdot \text{PolyLog}[2, -((b \cdot (\text{Cosh}[c + d*x] + \text{Sinh}[c + d*x])) / (a + \text{Sqrt}[a^2 + b^2])))] / (4 \cdot a^2 \cdot (a^2 + b^2)^{3/2} \cdot d^2) + (\text{Sech}[(c + d*x)/2] \cdot (-(d \cdot e \cdot \text{Sinh}[(c + d*x)/2]) + c \cdot f \cdot \text{Sinh}[(c + d*x)/2] - f \cdot (c + d*x) \cdot \text{Sinh}[(c + d*x)/2])) / (8 \cdot a \cdot d^2) + (\text{Sech}[c + d*x] \cdot (-(b \cdot d \cdot e) + b \cdot c \cdot f - b \cdot f \cdot (c + d*x) - a \cdot d \cdot e \cdot \text{Sinh}[c + d*x] + a \cdot c \cdot f \cdot \text{Sinh}[c + d*x] - a \cdot f \cdot (c + d*x) \cdot \text{Sinh}[c + d*x])) / (4 \cdot (a^2 + b^2) \cdot d^2)$$

**Maple [B]** time = 0.268, size = 1771, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)*\text{csch}(d*x+c)^2*\text{sech}(d*x+c)^2/(a+b*\text{sinh}(d*x+c)),x)$

[Out] 
$$\begin{aligned} & -2*(f*x+e)*(a*b*\exp(3*d*x+3*c)+b^2*\exp(2*d*x+2*c)-a*b*\exp(d*x+c)+2*a^2+b^2) \\ & /d/(a^2+b^2)/(1+\exp(2*d*x+2*c))/a/(\exp(2*d*x+2*c)-1)-1/(a^2+b^2)^{(5/2)}/d^2* \\ & a^2*b^2*f*c*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))+1/a^2/(a^2+b^2) \\ & /d^2*b^3*f*c*\ln(\exp(d*x+c)-1)+1/a^2/(a^2+b^2)/d*b^3*f*\ln(\exp(d*x+c)+1)*x- \\ & 1/a^2/(a^2+b^2)^{(5/2)}/d*b^6*e*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) \\ & )-1/a^2/(a^2+b^2)^{(3/2)}/d*b^4*e*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) \\ & )+4*a/(a^2+b^2)/d^2*b^2*f/(4*a^2+4*b^2)*\ln(1+\exp(2*d*x+2*c))+8*a^2/ \\ & (a^2+b^2)/d^2*b*f/(4*a^2+4*b^2)*\text{arctan}(\exp(d*x+c))+1/a^2/(a^2+b^2)^{(5/2)}/d^2 \\ & *b^6*f*\text{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))-1/a^2 \\ & /((a^2+b^2)^{(5/2)}/d^2*b^6*f*\text{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))) \\ & )+1/(a^2+b^2)^{(5/2)}/d*b^4*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))) \\ & )*x-1/(a^2+b^2)^{(5/2)}/d*b^4*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))) \\ & )*x+1/(a^2+b^2)^{(5/2)}/d^2*b^4*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))) \\ & )*c-1/(a^2+b^2)^{(5/2)}/d^2*b^4*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))) \\ & )*c+1/a^2/(a^2+b^2)^{(3/2)}/d^2*b^4*f*c*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))+1/a^2/ \\ & (a^2+b^2)^{(5/2)}/d*b^6*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))) \\ & )*x-1/a^2/(a^2+b^2)^{(5/2)}/d*b^6*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))) \\ & )*x+1/a^2/(a^2+b^2)^{(5/2)}/d^2*b^6*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))) \\ & )*c-1/a^2/(a^2+b^2)^{(5/2)}/d^2*b^6*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))) \\ & )*c+1/a^2/(a^2+b^2)^{(5/2)}/d^2*b^6*f*c*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))-4/d^2/(a^2+b^2) \\ & *a*f*\ln(\exp(d*x+c))+a/(a^2+b^2)/d^2*f*\ln(\exp(d*x+c)-1)+a/(a^2+b^2)/d^2*f*\ln(\exp(d*x+c)+1) \\ & )-1/(a^2+b^2)/d*b*e*\ln(\exp(d*x+c)-1)+1/(a^2+b^2)/d*b*e*\ln(\exp(d*x+c)+1)+1/(a^2+b^2)/d^2*b*f*\text{dilog}(\exp(d*x+c)) \\ & )+1/(a^2+b^2)/d^2*b*f*\text{dilog}(\exp(d*x+c)+1)+1/(a^2+b^2)/d*\ln(\exp(d*x+c)+1)*b*f*x+1/(a^2+b^2)^{(5/2)}/d^2*b^4*f*\text{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))-1/( \end{aligned}$$

$$\begin{aligned}
& a^2+b^2)^{(5/2)}/d^2*b^4*f*dilog((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)+a)/(a+(a^2+b^2)^{(1/2)}))+1/(a^2+b^2)/d^2*b*f*c*\ln(\exp(d*x+c)-1)+8/(a^2+b^2)/d^2*b^3*f/(4*a^2+4*b^2)*\arctan(\exp(d*x+c))-1/a^2/(a^2+b^2)/d*b^3*e*\ln(\exp(d*x+c)-1)+1/a^2/(a^2+b^2)/d*b^3*e*\ln(\exp(d*x+c)+1)+1/a/(a^2+b^2)/d^2*b^2*f*\ln(\exp(d*x+c)-1)+1/a/(a^2+b^2)/d^2*b^2*f*\ln(\exp(d*x+c)+1)-2/a/(a^2+b^2)/d^2*b^2*f*\ln(\exp(d*x+c))+4*a^3/(a^2+b^2)/d^2*f/(4*a^2+4*b^2)*\ln(1+\exp(2*d*x+2*c))+1/a^2/(a^2+b^2)/d^2*b^3*f*dilog(\exp(d*x+c))+1/a^2/(a^2+b^2)/d^2*b^3*f*dilog(\exp(d*x+c)+1)-1/(a^2+b^2)^{(3/2)}/d*b^2*e*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))+1/(a^2+b^2)^{(3/2)}/d^2*b^2*f*c*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))+1/(a^2+b^2)^{(5/2)}/d*a^2*b^2*e*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})
\end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)^2\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 3.12856, size = 9453, normalized size = 18.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)^2\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned}
& -(2*((2*a^5 + 3*a^3*b^2 + a*b^4)*d*f*x + (a^5 + 2*a^3*b^2 + a*b^4)*c*f))*\cosh(d*x + c)^4 + 2*((2*a^5 + 3*a^3*b^2 + a*b^4)*d*f*x + (a^5 + 2*a^3*b^2 + a*b^4)*c*f)*\sinh(d*x + c)^4 + 2*((a^4*b + a^2*b^3)*d*f*x + (a^4*b + a^2*b^3)*d*e)*\cosh(d*x + c)^3 + 2*((a^4*b + a^2*b^3)*d*f*x + (a^4*b + a^2*b^3)*d*e + 4*((2*a^5 + 3*a^3*b^2 + a*b^4)*d*f*x + (a^5 + 2*a^3*b^2 + a*b^4)*c*f))*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*(2*a^5 + 3*a^3*b^2 + a*b^4)*d*e - 2*(a^5 + 2*a^3*b^2 + a*b^4)*c*f + 2*((a^3*b^2 + a*b^4)*d*f*x + (a^3*b^2 + a*b^4)*d*e)*\cosh(d*x + c)^2 + 2*((a^3*b^2 + a*b^4)*d*f*x + (a^3*b^2 + a*b^4)*d*e + 6*(
\end{aligned}$$

$$\begin{aligned}
& (2a^5 + 3a^3b^2 + ab^4)d^2f^2x + (a^5 + 2a^3b^2 + ab^4)cf \cosh(dx + c)^2 + 3((a^4b + a^2b^3)d^2f^2x + (a^4b + a^2b^3)de) \cosh(dx + c) \\
& ) \sinh(dx + c)^2 - (b^5f \cosh(dx + c)^4 + 4b^5f \cosh(dx + c)^3 \sinh(dx + c) + 6b^5f \cosh(dx + c)^2 \sinh(dx + c)^2 + 4b^5f \cosh(dx + c) \sinh(dx + c)^3 + b^5f \sinh(dx + c)^4 - b^5f) \sqrt{(a^2 + b^2)/b^2} \operatorname{dilog} \\
& ((a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c))) \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + (b^5f \cosh(dx + c)^4 + 4b^5f \cosh(dx + c)^3 \sinh(dx + c) + 6b^5f \cosh(dx + c)^2 \sinh(dx + c)^2 + 4b^5f \cosh(dx + c) \sinh(dx + c)^3 + b^5f \sinh(dx + c)^4 - b^5f) \sqrt{(a^2 + b^2)/b^2} \operatorname{dilog} \\
& ((a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c))) \sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - (b^5de - b^5cf - (b^5de - b^5cf) \cosh(dx + c)^4 - 4(b^5de - b^5cf) \cosh(dx + c)^3 \sinh(dx + c) - 6(b^5de - b^5cf) \cosh(dx + c)^2 \sinh(dx + c)^2 - 4(b^5de - b^5cf) \cosh(dx + c) \sinh(dx + c)^3 - (b^5de - b^5cf) \sinh(dx + c)^4) \sqrt{(a^2 + b^2)/b^2} \log(2b \cosh(dx + c) + 2b \sinh(dx + c) + 2b \sqrt{(a^2 + b^2)/b^2} + 2a) + (b^5de - b^5cf - (b^5de - b^5cf) \cosh(dx + c)^4 - 4(b^5de - b^5cf) \cosh(dx + c)^3 \sinh(dx + c) - 6(b^5de - b^5cf) \cosh(dx + c)^2 \sinh(dx + c)^2 - 4(b^5de - b^5cf) \cosh(dx + c) \sinh(dx + c)^3 - (b^5de - b^5cf) \sinh(dx + c)^4) \sqrt{(a^2 + b^2)/b^2} \log(2b \cosh(dx + c) + 2b \sinh(dx + c) - 2b \sqrt{(a^2 + b^2)/b^2} + 2a) + (b^5d^2f^2x + b^5cf - (b^5d^2f^2x + b^5cf) \cosh(dx + c)^4 - 4(b^5d^2f^2x + b^5cf) \cosh(dx + c)^3 \sinh(dx + c) - 6(b^5d^2f^2x + b^5cf) \cosh(dx + c)^2 \sinh(dx + c)^2 - 4(b^5d^2f^2x + b^5cf) \cosh(dx + c) \sinh(dx + c)^3 - (b^5d^2f^2x + b^5cf) \sinh(dx + c)^4) \sqrt{(a^2 + b^2)/b^2} \log(-(a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c))) \sqrt{(a^2 + b^2)/b^2} - b)/b - (b^5d^2f^2x + b^5cf - (b^5d^2f^2x + b^5cf) \cosh(dx + c)^4 - 4(b^5d^2f^2x + b^5cf) \cosh(dx + c)^3 \sinh(dx + c) - 6(b^5d^2f^2x + b^5cf) \cosh(dx + c)^2 \sinh(dx + c)^2 - 4(b^5d^2f^2x + b^5cf) \cosh(dx + c) \sinh(dx + c)^3 - (b^5d^2f^2x + b^5cf) \sinh(dx + c)^4) \sqrt{(a^2 + b^2)/b^2} \log(-(a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c))) \sqrt{(a^2 + b^2)/b^2} - b)/b) - 2((a^4b + a^2b^3) f \cosh(dx + c)^4 + 4(a^4b + a^2b^3) f \cosh(dx + c)^3 \sinh(dx + c) + 6(a^4b + a^2b^3) f \cosh(dx + c)^2 \sinh(dx + c)^2 + 4(a^4b + a^2b^3) f \cosh(dx + c) \sinh(dx + c)^3 + (a^4b + a^2b^3) f \sinh(dx + c)^4 - (a^4b + a^2b^3) f) \arctan(\cosh(dx + c) + \sinh(dx + c)) - 2((a^4b + a^2b^3) d^2f^2x + (a^4b + a^2b^3) de) \cosh(dx + c) + ((a^4b + 2a^2b^3 + b^5) f \cosh(dx + c)^4 + 4(a^4b + 2a^2b^3 + b^5) f \cosh(dx + c)^3 \sinh(dx + c) + 6(a^4b + 2a^2b^3 + b^5) f \cosh(dx + c)^2 \sinh(dx + c)^2 + 4(a^4b + 2a^2b^3 + b^5) f \cosh(dx + c) \sinh(dx + c)^3 + (a^4b + 2a^2b^3 + b^5) f \sinh(dx + c)^4 - (a^4b + 2a^2b^3 + b^5) f) \operatorname{dilog}(\cosh(dx + c) + \sinh(dx + c)) - ((a^4b + 2a^2b^3 + b^5) f \cosh(dx + c)^4 + 4(a^4b + 2a^2b^3 + b^5) f \cosh(dx + c)^3 \sinh(dx + c) + 6(a^4b + 2a^2b^3 + b^5) f \cosh(dx + c)^2 \sinh(dx + c)^2 + 4(a^4b + 2a^2b^3 + b^5) f \cosh(dx + c) \sinh(dx + c)^3 + (a^4b + 2a^2b^3 + b^5) f \sinh(dx + c)^4 - (a^4b + 2a^2b^3 + b^5) f) \operatorname{dilog}(-\cosh(dx + c) + \sinh(dx + c))
\end{aligned}$$

$$\begin{aligned}
& x + c) - \sinh(dx + c)) - ((a^5 + a^3b^2)*f*\cosh(dx + c)^4 + 4*(a^5 + a^3b^2)*b^2*f*\cosh(dx + c)^3*\sinh(dx + c) + 6*(a^5 + a^3b^2)*f*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*(a^5 + a^3b^2)*f*\cosh(dx + c)*\sinh(dx + c)^3 + (a^5 + a^3b^2)*f*\sinh(dx + c)^4 - (a^5 + a^3b^2)*f)*\log(2*\cosh(dx + c)/(\cosh(dx + c) - \sinh(dx + c))) - (((a^4*b + 2*a^2*b^3 + b^5)*d*f*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e + (a^5 + 2*a^3*b^2 + a*b^4)*f)*\cosh(dx + c)^4 + 4*((a^4*b + 2*a^2*b^3 + b^5)*d*f*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e + (a^5 + 2*a^3*b^2 + a*b^4)*f)*\cosh(dx + c)^3*\sinh(dx + c) + 6*((a^4*b + 2*a^2*b^3 + b^5)*d*f*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e + (a^5 + 2*a^3*b^2 + a*b^4)*f)*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*((a^4*b + 2*a^2*b^3 + b^5)*d*f*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e + (a^5 + 2*a^3*b^2 + a*b^4)*f)*\cosh(dx + c)*\sinh(dx + c)^3 + (((a^4*b + 2*a^2*b^3 + b^5)*d*f*x + (a^4*b + 2*a^2*b^3 + b^5)*d*e + (a^5 + 2*a^3*b^2 + a*b^4)*f)*\sinh(dx + c)^4 - (a^4*b + 2*a^2*b^3 + b^5)*d*f*x - (a^4*b + 2*a^2*b^3 + b^5)*d*e - (a^5 + 2*a^3*b^2 + a*b^4)*f)*\log(\cosh(dx + c) + \sinh(dx + c) + 1) + (((a^4*b + 2*a^2*b^3 + b^5)*d*e - (a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*c)*f)*\cosh(dx + c)^4 + 4*((a^4*b + 2*a^2*b^3 + b^5)*d*e - (a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*c)*f)*\cosh(dx + c)^3*\sinh(dx + c) + 6*((a^4*b + 2*a^2*b^3 + b^5)*d*e - (a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*c)*f)*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*((a^4*b + 2*a^2*b^3 + b^5)*d*e - (a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*c)*f)*\cosh(dx + c)*\sinh(dx + c)^3 + ((a^4*b + 2*a^2*b^3 + b^5)*d*e - (a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*c)*f)*\sinh(dx + c)^4 - (a^4*b + 2*a^2*b^3 + b^5)*d*e + (a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*c)*f)*\log(\cosh(dx + c) + \sinh(dx + c) - 1) + (((a^4*b + 2*a^2*b^3 + b^5)*d*f*x + (a^4*b + 2*a^2*b^3 + b^5)*c*f)*\cosh(dx + c)^4 + 4*((a^4*b + 2*a^2*b^3 + b^5)*d*f*x + (a^4*b + 2*a^2*b^3 + b^5)*c*f)*\cosh(dx + c)^3*\sinh(dx + c) + 6*((a^4*b + 2*a^2*b^3 + b^5)*d*f*x + (a^4*b + 2*a^2*b^3 + b^5)*c*f)*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*((a^4*b + 2*a^2*b^3 + b^5)*d*f*x + (a^4*b + 2*a^2*b^3 + b^5)*c*f)*\cosh(dx + c)*\sinh(dx + c)^3 + ((a^4*b + 2*a^2*b^3 + b^5)*d*f*x + (a^4*b + 2*a^2*b^3 + b^5)*c*f)*\sinh(dx + c)^4 - (a^4*b + 2*a^2*b^3 + b^5)*d*f*x - (a^4*b + 2*a^2*b^3 + b^5)*c*f)*\log(-\cosh(dx + c) - \sinh(dx + c) + 1) - 2*((a^4*b + a^2*b^3)*d*f*x - 4*((2*a^5 + 3*a^3*b^2 + a*b^4)*d*f*x + (a^5 + 2*a^3*b^2 + a*b^4)*c*f)*\cosh(dx + c)^3 + (a^4*b + a^2*b^3)*d*e - 3*((a^4*b + a^2*b^3)*d*f*x + (a^4*b + a^2*b^3)*d*e)*\cosh(dx + c)^2 - 2*((a^3*b^2 + a*b^4)*d*f*x + (a^3*b^2 + a*b^4)*d*e)*\cosh(dx + c))/((a^6 + 2*a^4*b^2 + a^2*b^4)*d^2*\cosh(dx + c)^4 + 4*(a^6 + 2*a^4*b^2 + a^2*b^4)*d^2*\cosh(dx + c)^3*\sinh(dx + c) + 6*(a^6 + 2*a^4*b^2 + a^2*b^4)*d^2*\cosh(dx + c)^2*\sinh(dx + c)^2 + 4*(a^6 + 2*a^4*b^2 + a^2*b^4)*d^2*\cosh(dx + c)*\sinh(dx + c)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d^2*\sinh(dx + c)^4 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d^2)
\end{aligned}$$



**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**2*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.471 \quad \int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=144

$$-\frac{2b^4 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2 d (a^2+b^2)^{3/2}} + \frac{b^2 \operatorname{sech}(c+dx)(a \sinh(c+dx)+b)}{a^2 d (a^2+b^2)} - \frac{b \operatorname{sech}(c+dx)}{a^2 d} + \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2 d} - \frac{\tanh(c+dx)}{a d}$$

[Out] (b\*ArcTanh[Cosh[c + d\*x]])/(a^2\*d) - (2\*b^4\*ArcTanh[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[a^2 + b^2]])/(a^2\*(a^2 + b^2)^(3/2)\*d) - Coth[c + d\*x]/(a\*d) - (b\*Sech[c + d\*x])/(a^2\*d) + (b^2\*Sech[c + d\*x]\*(b + a\*Sinh[c + d\*x]))/(a^2\*(a^2 + b^2)\*d) - Tanh[c + d\*x]/(a\*d)

**Rubi [A]** time = 0.310752, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$ , Rules used = {2898, 2622, 321, 207, 2620, 14, 2696, 12, 2660, 618, 204}

$$-\frac{2b^4 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2 d (a^2+b^2)^{3/2}} + \frac{b^2 \operatorname{sech}(c+dx)(a \sinh(c+dx)+b)}{a^2 d (a^2+b^2)} - \frac{b \operatorname{sech}(c+dx)}{a^2 d} + \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2 d} - \frac{\tanh(c+dx)}{a d}$$

Antiderivative was successfully verified.

[In] Int[(Csch[c + d\*x]^2\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (b\*ArcTanh[Cosh[c + d\*x]])/(a^2\*d) - (2\*b^4\*ArcTanh[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[a^2 + b^2]])/(a^2\*(a^2 + b^2)^(3/2)\*d) - Coth[c + d\*x]/(a\*d) - (b\*Sech[c + d\*x])/(a^2\*d) + (b^2\*Sech[c + d\*x]\*(b + a\*Sinh[c + d\*x]))/(a^2\*(a^2 + b^2)\*d) - Tanh[c + d\*x]/(a\*d)

### Rule 2898

Int[((cos[(e\_.) + (f\_.)\*(x\_.)]\*(g\_.))^p)\*sin[(e\_.) + (f\_.)\*(x\_.)]^n)/((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] := Int[ExpandTrig[(g\*cos[e + f\*x])^p, sin[e + f\*x]^n/(a + b\*sin[e + f\*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[n, 0] || IGtQ[p + 1/2, 0])

### Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_Symbol]
:> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

### Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol]
:> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

### Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

### Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol]
:> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

### Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol]
:> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[2*m, 2*p]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol]
:> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_.)*(v_)] /; FreeQ[b, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx &= -\int \left( \frac{b\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a^2} - \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a} - \frac{b^2\operatorname{sech}^2(c+dx)}{a^2(a+b\sinh(c+dx))} \right) dx \\
&= \frac{\int \operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx) dx}{a} - \frac{b \int \operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx) dx}{a^2} + \frac{b^2 \int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx}{a^2} \\
&= \frac{b^2\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a^2(a^2+b^2)d} + \frac{b^2 \int \frac{b^2}{a+b\sinh(c+dx)} dx}{a^2(a^2+b^2)} + \frac{i \operatorname{Subst}\left(\int \frac{1+x^2}{x^2} dx, x, \frac{b+a\sinh(c+dx)}{a}\right)}{ad} \\
&= -\frac{b\operatorname{sech}(c+dx)}{a^2d} + \frac{b^2\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a^2(a^2+b^2)d} + \frac{b^4 \int \frac{1}{a+b\sinh(c+dx)} dx}{a^2(a^2+b^2)} + \frac{i \operatorname{Subst}\left(\int \frac{1+x^2}{x^2} dx, x, \frac{b+a\sinh(c+dx)}{a}\right)}{ad} \\
&= \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{\operatorname{coth}(c+dx)}{ad} - \frac{b\operatorname{sech}(c+dx)}{a^2d} + \frac{b^2\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a^2(a^2+b^2)d} \\
&= \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{\operatorname{coth}(c+dx)}{ad} - \frac{b\operatorname{sech}(c+dx)}{a^2d} + \frac{b^2\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a^2(a^2+b^2)d} \\
&= \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{2b^4 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^{3/2}d} - \frac{\operatorname{coth}(c+dx)}{ad} - \frac{b\operatorname{sech}(c+dx)}{a^2d}
\end{aligned}$$

**Mathematica [A]** time = 2.64522, size = 135, normalized size = 0.94

$$\frac{4b^4 \tan^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{a^2(-a^2-b^2)^{3/2}} + \frac{2\operatorname{sech}(c+dx)(a\sinh(c+dx)+b)}{a^2+b^2} + \frac{2b \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a^2} + \frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{a} + \frac{\operatorname{coth}\left(\frac{1}{2}(c+dx)\right)}{a}$$


---


$$2d$$

Antiderivative was successfully verified.

[In] Integrate[(Csch[c + d\*x]^2\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] -((4\*b^4\*ArcTan[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[-a^2 - b^2]])/(a^2\*(-a^2 - b^2)^(3/2)) + Coth[(c + d\*x)/2]/a + (2\*b\*Log[Tanh[(c + d\*x)/2]])/a^2 + (2\*Sech[c + d\*x]\*(b + a\*Sinh[c + d\*x]))/(a^2 + b^2) + Tanh[(c + d\*x)/2]/a)/(2\*d)

**Maple [A]** time = 0.003, size = 174, normalized size = 1.2

$$-\frac{1}{2da} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{2da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-1} - \frac{b}{da^2} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + 2 \frac{b^4}{da^2 (a^2 + b^2)^{3/2}} \operatorname{Artanh}\left(\frac{1}{2} \frac{2a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{a^2 + b^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)`

[Out] `-1/2/d/a*tanh(1/2*d*x+1/2*c)-1/2/d/a/tanh(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tanh(1/2*d*x+1/2*c))+2/d/a^2*b^4/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-2/d/(a^2+b^2)/(tanh(1/2*d*x+1/2*c)^2+1)*a*tanh(1/2*d*x+1/2*c)-2/d/(a^2+b^2)/(tanh(1/2*d*x+1/2*c)^2+1)*b`

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [B]** time = 3.11544, size = 2483, normalized size = 17.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] `-(4*a^5 + 6*a^3*b^2 + 2*a*b^4 + 2*(a^4*b + a^2*b^3)*cosh(d*x + c)^3 + 2*(a^4*b + a^2*b^3)*sinh(d*x + c)^3 + 2*(a^3*b^2 + a*b^4)*cosh(d*x + c)^2 + 2*(a^3*b^2 + a*b^4 + 3*(a^4*b + a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - (b^4*cosh(d*x + c)^4 + 4*b^4*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^4*cosh(d*x + c)`

$$\begin{aligned} &^2 \sinh(dx + c)^2 + 4b^4 \cosh(dx + c) \sinh(dx + c)^3 + b^4 \sinh(dx + c) \\ &^4 - b^4 \sqrt{a^2 + b^2} \log((b^2 \cosh(dx + c)^2 + b^2 \sinh(dx + c)^2 + \\ & 2ab \cosh(dx + c) + 2a^2 + b^2 + 2(b^2 \cosh(dx + c) + ab) \sinh(dx + \\ & c) - 2\sqrt{a^2 + b^2}(b \cosh(dx + c) + b \sinh(dx + c) + a)) / (b \cosh(dx \\ & x + c)^2 + b \sinh(dx + c)^2 + 2a \cosh(dx + c) + 2(b \cosh(dx + c) + a) \sinh(dx + c) - b)) - 2(a^4 b + a^2 b^3) \cosh(dx + c) + (a^4 b + 2a^2 b^3 \\ & 3 + b^5 - (a^4 b + 2a^2 b^3 + b^5) \cosh(dx + c)^4 - 4(a^4 b + 2a^2 b^3 \\ & + b^5) \cosh(dx + c)^3 \sinh(dx + c) - 6(a^4 b + 2a^2 b^3 + b^5) \cosh(dx \\ & + c)^2 \sinh(dx + c)^2 - 4(a^4 b + 2a^2 b^3 + b^5) \cosh(dx + c) \sinh(dx \\ & x + c)^3 - (a^4 b + 2a^2 b^3 + b^5) \sinh(dx + c)^4) \log(\cosh(dx + c) + \sinh(dx + c) + 1) - (a^4 b + 2a^2 b^3 + b^5 - (a^4 b + 2a^2 b^3 + b^5) \cosh(dx + c)^4 - 4(a^4 b + 2a^2 b^3 + b^5) \cosh(dx + c)^3 \sinh(dx + c) - 6(a^4 b + 2a^2 b^3 + b^5) \cosh(dx + c)^2 \sinh(dx + c)^2 - 4(a^4 b + 2a^2 b^3 + b^5) \cosh(dx + c) \sinh(dx + c)^3 - (a^4 b + 2a^2 b^3 + b^5) \sinh(dx + c)^4) \log(\cosh(dx + c) + \sinh(dx + c) - 1) - 2(a^4 b + a^2 b^3 - 3(a^4 b + a^2 b^3) \cosh(dx + c)^2 - 2(a^3 b^2 + a b^4) \cosh(dx + c)) \sinh(dx + c)) / ((a^6 + 2a^4 b^2 + a^2 b^4) d \cosh(dx + c)^4 + 4(a^6 + 2a^4 b^2 + a^2 b^4) d \cosh(dx + c)^3 \sinh(dx + c) + 6(a^6 + 2a^4 b^2 + a^2 b^4) d \cosh(dx + c)^2 \sinh(dx + c)^2 + 4(a^6 + 2a^4 b^2 + a^2 b^4) d \cosh(dx + c) \sinh(dx + c)^3 + (a^6 + 2a^4 b^2 + a^2 b^4) d \sinh(dx + c)^4 - (a^6 + 2a^4 b^2 + a^2 b^4) d) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)\*\*2\*sech(dx+c)\*\*2/(a+b\*sinh(dx+c)), x)

[Out] Timed out

**Giac [A]** time = 1.72368, size = 262, normalized size = 1.82

$$\frac{b^4 \log\left(\frac{-2be^{(dx+c)} - 2a - 2\sqrt{a^2+b^2}}{-2be^{(dx+c)} - 2a + 2\sqrt{a^2+b^2}}\right)}{(a^4 d + a^2 b^2 d) \sqrt{a^2 + b^2}} - \frac{2(ab e^{(3dx+3c)} + b^2 e^{(2dx+2c)} - ab e^{(dx+c)} + 2a^2 + b^2)}{(a^3 d + ab^2 d)(e^{(4dx+4c)} - 1)} + \frac{b \log(e^{(dx+c)} + 1)}{a^2 d} - \frac{b \log(|e^{(dx+c)} - 1|)}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] -b^4*log(abs(-2*b*e^(d*x + c) - 2*a - 2*sqrt(a^2 + b^2))/abs(-2*b*e^(d*x + c) - 2*a + 2*sqrt(a^2 + b^2)))/((a^4*d + a^2*b^2*d)*sqrt(a^2 + b^2)) - 2*(a*b*e^(3*d*x + 3*c) + b^2*e^(2*d*x + 2*c) - a*b*e^(d*x + c) + 2*a^2 + b^2)/((a^3*d + a*b^2*d)*(e^(4*d*x + 4*c) - 1)) + b*log(e^(d*x + c) + 1)/(a^2*d) - b*log(abs(e^(d*x + c) - 1))/(a^2*d)
```



$$3.472 \quad \int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Optimal.** Leaf size=38

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}, x\right)$$

[Out] Unintegrable[(Csch[c + d\*x]^2\*Sech[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.129061, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Csch[c + d\*x]^2\*Sech[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int] [(Csch[c + d\*x]^2\*Sech[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Mathematica [A]** time = 154.862, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csch[c + d\*x]^2\*Sech[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Integrate[(Csch[c + d\*x]^2\*Sech[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Maple [A]** time = 1.777, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx + c))^2 (\operatorname{sech}(dx + c))^2}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d\*x+c)^2\*sech(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(csch(d\*x+c)^2\*sech(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$16b^4 \int -\frac{e^{(dx+c)}}{8(a^4be + a^2b^3e + (a^4bf + a^2b^3f)x - (a^4bee^{(2c)} + a^2b^3ee^{(2c)} + (a^4bfe^{(2c)} + a^2b^3fe^{(2c)})x)e^{(2dx)} - 2(a^5ee^c + a^3b^2e^c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^2\*sech(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 16\*b^4\*integrate(-1/8\*e^(d\*x + c)/(a^4\*b\*e + a^2\*b^3\*e + (a^4\*b\*f + a^2\*b^3\*f)\*x - (a^4\*b\*e\*e^(2\*c) + a^2\*b^3\*e\*e^(2\*c) + (a^4\*b\*f\*e^(2\*c) + a^2\*b^3\*f\*e^(2\*c))\*x)\*e^(2\*d\*x) - 2\*(a^5\*e\*e^c + a^3\*b^2\*e\*e^c + (a^5\*f\*e^c + a^3\*b^2\*f\*e^c)\*x)\*e^(d\*x)), x) + 2\*(a\*b\*e^(3\*d\*x + 3\*c) + b^2\*e^(2\*d\*x + 2\*c) - a\*b\*e^(d\*x + c) + 2\*a^2 + b^2)/(a^3\*d\*e + a\*b^2\*d\*e + (a^3\*d\*f + a\*b^2\*d\*f)\*x - (a^3\*d\*e\*e^(4\*c) + a\*b^2\*d\*e\*e^(4\*c) + (a^3\*d\*f\*e^(4\*c) + a\*b^2\*d\*f\*e^(4\*c))\*x)\*e^(4\*d\*x)) - 16\*integrate(-1/16\*(b\*d\*f\*x + b\*d\*e + a\*f)/(a^2\*d\*f^2\*x^2 + 2\*a^2\*d\*e\*f\*x + a^2\*d\*e^2 - (a^2\*d\*f^2\*x^2\*e^c + 2\*a^2\*d\*e\*f\*x\*e^c + a^2\*d\*e^2\*e^c)\*e^(d\*x)), x) - 16\*integrate(1/16\*(b\*d\*f\*x + b\*d\*e - a\*f)/(a^2\*d\*f^2\*x^2 + 2\*a^2\*d\*e\*f\*x + a^2\*d\*e^2 + (a^2\*d\*f^2\*x^2\*e^c + 2\*a^2\*d\*e\*f\*x\*e^c + a^2\*d\*e^2\*e^c)\*e^(d\*x)), x) - 16\*integrate(1/8\*(b\*f\*e^(d\*x + c) - a\*f)/(a^2\*d\*e^2 + b^2\*d\*e^2 + (a^2\*d\*f^2 + b^2\*d\*f^2)\*x^2 + 2\*(a^2\*d\*e\*f + b^2\*d\*e\*f)\*x + (a^2\*d\*e^2\*e^(2\*c) + b^2\*d\*e^2\*e^(2\*c) + (a^2\*d\*f^2\*e^(2\*c) + b^2\*d\*f^2\*e^(2\*c))\*x^2 + 2\*(a^2\*d\*e\*f\*e^(2\*c) + b^2\*d\*e\*f\*e^(2\*c))\*x)\*e^(

2\*d\*x)), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{csch}(dx+c)^2 \text{sech}(dx+c)^2}{afx+ae+(bfx+be)\sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^2\*sech(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] integral(csch(d\*x + c)^2\*sech(d\*x + c)^2/(a\*f\*x + a\*e + (b\*f\*x + b\*e)\*sinh(d\*x + c)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)\*\*2\*sech(d\*x+c)\*\*2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^2\*sech(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.473 \quad \int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=978

result too large to display

```
[Out] -(b*f*x)/(2*a^2*d) - (3*f*x*ArcTan[E^(c + d*x)])/(a*d) + (2*b^4*(e + f*x)*ArcTan[E^(c + d*x)])/(a*(a^2 + b^2)^2*d) + (b^2*(e + f*x)*ArcTan[E^(c + d*x)])/(a*(a^2 + b^2)*d) + (3*f*x*ArcTan[Sinh[c + d*x]])/(2*a*d) - (3*(e + f*x)*ArcTan[Sinh[c + d*x]])/(2*a*d) + (2*b*f*x*ArcTanh[E^(2*c + 2*d*x)])/(a^2*d) - (f*ArcTanh[Cosh[c + d*x]])/(a*d^2) - (3*(e + f*x)*Csch[c + d*x])/(2*a*d) + (b^5*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^2*d) + (b^5*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^2*d) - (b^5*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a^2*(a^2 + b^2)^2*d) + (b*f*x*Log[Tanh[c + d*x]])/(a^2*d) - (b*(e + f*x)*Log[Tanh[c + d*x]])/(a^2*d) + (((3*I)/2)*f*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^2) - (I*b^4*f*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)^2*d^2) - ((I/2)*b^2*f*PolyLog[2, (-I)*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) - (((3*I)/2)*f*PolyLog[2, I*E^(c + d*x)])/(a*d^2) + (I*b^4*f*PolyLog[2, I*E^(c + d*x)])/(a*(a^2 + b^2)^2*d^2) + ((I/2)*b^2*f*PolyLog[2, I*E^(c + d*x)])/(a*(a^2 + b^2)*d^2) + (b^5*f*PolyLog[2, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^2*d^2) + (b^5*f*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^2*d^2) - (b^5*f*PolyLog[2, -E^(2*(c + d*x))])/(2*a^2*(a^2 + b^2)^2*d^2) + (b*f*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a^2*d^2) - (b*f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a^2*d^2) - (f*Sech[c + d*x])/(2*a*d^2) + (b^2*f*Sech[c + d*x])/(2*a*(a^2 + b^2)*d^2) + (b^3*(e + f*x)*Sech[c + d*x]^2)/(2*a^2*(a^2 + b^2)*d) + ((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2)/(2*a*d) + (b*f*Tanh[c + d*x])/(2*a^2*d^2) - (b^3*f*Tanh[c + d*x])/(2*a^2*(a^2 + b^2)*d^2) + (b^2*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*a*(a^2 + b^2)*d) + (b*(e + f*x)*Tanh[c + d*x]^2)/(2*a^2*d)
```

---

**Rubi [A]** time = 1.411, antiderivative size = 978, normalized size of antiderivative = 1., number of steps used = 57, number of rules used = 27, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.794$ , Rules used = {5589, 2621, 288, 321, 207, 5462, 5203, 12, 4180, 2279, 2391, 3770, 2622, 2620, 14, 2548, 4182, 3473, 8, 5573, 5561, 2190, 6742, 3718, 4185, 5451, 3767}

$$\frac{(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^5}{a^2(a^2+b^2)^2d} + \frac{(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^5}{a^2(a^2+b^2)^2d} - \frac{(e+fx)\log(1+e^{2(c+dx)})b^5}{a^2(a^2+b^2)^2d} + \frac{f\operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^2(a^2+b^2)^2d^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Csch[c + d\*x]^2\*Sech[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out]  $-(b*f*x)/(2*a^2*d) - (3*f*x*ArcTan[E^{(c + d*x)}])/(a*d) + (2*b^4*(e + f*x)*ArcTan[E^{(c + d*x)}])/(a*(a^2 + b^2)^2*d) + (b^2*(e + f*x)*ArcTan[E^{(c + d*x)}])/(a*(a^2 + b^2)*d) + (3*f*x*ArcTan[Sinh[c + d*x]])/(2*a*d) - (3*(e + f*x)*ArcTan[Sinh[c + d*x]])/(2*a*d) + (2*b*f*x*ArcTanh[E^{(2*c + 2*d*x)}])/(a^2*d) - (f*ArcTanh[Cosh[c + d*x]])/(a*d^2) - (3*(e + f*x)*Csch[c + d*x])/(2*a*d) + (b^5*(e + f*x)*Log[1 + (b*E^{(c + d*x)})/(a - Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^2*d) + (b^5*(e + f*x)*Log[1 + (b*E^{(c + d*x)})/(a + Sqrt[a^2 + b^2])])/(a^2*(a^2 + b^2)^2*d) - (b^5*(e + f*x)*Log[1 + E^{(2*(c + d*x))}])/(a^2*(a^2 + b^2)^2*d) + (b*f*x*Log[Tanh[c + d*x]])/(a^2*d) - (b*(e + f*x)*Log[Tanh[c + d*x]])/(a^2*d) + (((3*I)/2)*f*PolyLog[2, (-I)*E^{(c + d*x)}])/(a*d^2) - (I*b^4*f*PolyLog[2, (-I)*E^{(c + d*x)}])/(a*(a^2 + b^2)^2*d^2) - ((I/2)*b^2*f*PolyLog[2, (-I)*E^{(c + d*x)}])/(a*(a^2 + b^2)*d^2) - (((3*I)/2)*f*PolyLog[2, I*E^{(c + d*x)}])/(a*d^2) + (I*b^4*f*PolyLog[2, I*E^{(c + d*x)}])/(a*(a^2 + b^2)^2*d^2) + ((I/2)*b^2*f*PolyLog[2, I*E^{(c + d*x)}])/(a*(a^2 + b^2)*d^2) + (b^5*f*PolyLog[2, -((b*E^{(c + d*x)})/(a - Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^2*d^2) + (b^5*f*PolyLog[2, -((b*E^{(c + d*x)})/(a + Sqrt[a^2 + b^2]))])/(a^2*(a^2 + b^2)^2*d^2) - (b^5*f*PolyLog[2, -E^{(2*(c + d*x))}])/(2*a^2*(a^2 + b^2)^2*d^2) + (b*f*PolyLog[2, -E^{(2*c + 2*d*x)}])/(2*a^2*d^2) - (b*f*PolyLog[2, E^{(2*c + 2*d*x)}])/(2*a^2*d^2) - (f*Sech[c + d*x])/(2*a*d^2) + (b^2*f*Sech[c + d*x])/(2*a*(a^2 + b^2)*d^2) + (b^3*(e + f*x)*Sech[c + d*x]^2)/(2*a^2*(a^2 + b^2)*d) + ((e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2)/(2*a*d) + (b*f*Tanh[c + d*x])/(2*a^2*d^2) - (b^3*f*Tanh[c + d*x])/(2*a^2*(a^2 + b^2)*d^2) + (b^2*(e + f*x)*Sech[c + d*x]*Tanh[c + d*x])/(2*a*(a^2 + b^2)*d) + (b*(e + f*x)*Tanh[c + d*x]^2)/(2*a^2*d)$

### Rule 5589

Int[(Csch[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(p\_.)]/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Sech[c + d\*x]^p\*Csch[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Sech[c + d\*x]^p\*Csch[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 2621

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_.)\*sec[(e\_.) + (f\_.)\*(x\_)]^(n\_.), x\_Symbol] := -Dist[(f\*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a\*Csc[e + f\*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

### Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

### Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] := With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

### Rule 5203

```
Int[ArcTan[u_], x_Symbol] := Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^m_., x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi))]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] :> Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] :> -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

### Rule 2622

Int[csc[(e\_.) + (f\_.)\*(x\_)^(n\_.)\*((a\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(m\_.), x\_Symbol] :> Dist[1/(f\*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a\*Sec[e + f\*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

### Rule 2620

Int[csc[(e\_.) + (f\_.)\*(x\_)^(m\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(n\_.), x\_Symbol] :> Dist[1/f, Subst[Int[(1 + x^2)^(m + n)/2 - 1)/x^m, x], x, Tan[e + f\*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]

### Rule 14

Int[(u\_)\*((c\_.)\*(x\_)^(m\_.), x\_Symbol] :> Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_) + (b\_.)\*(v\_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

### Rule 2548

Int[Log[u\_], x\_Symbol] :> Simp[x\*Log[u], x] - Int[SimplifyIntegrand[(x\*D[u, x])/u, x], x] /; InverseFunctionFreeQ[u, x]

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

### Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

### Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```



Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-I*e) + f*fz*x)))/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_)), x_Symbol] :=
-Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] := -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx}{a} \\
&= -\frac{3(e+fx)\tan^{-1}(\sinh(c+dx))}{2ad} - \frac{3(e+fx)\operatorname{csch}(c+dx)}{2ad} + \frac{(e+fx)\operatorname{csch}(c+dx)}{a} \\
&= -\frac{3(e+fx)\tan^{-1}(\sinh(c+dx))}{2ad} - \frac{3(e+fx)\operatorname{csch}(c+dx)}{2ad} - \frac{b(e+fx)\log(\tanh(\frac{c+dx}{2}))}{a^2d} \\
&= \frac{3fx\tan^{-1}(\sinh(c+dx))}{2ad} - \frac{3(e+fx)\tan^{-1}(\sinh(c+dx))}{2ad} - \frac{3f\tanh^{-1}(\cosh(c+dx))}{2ad^2} \\
&= -\frac{b^5(e+fx)^2}{2a^2(a^2+b^2)^2f} + \frac{3fx\tan^{-1}(\sinh(c+dx))}{2ad} - \frac{3(e+fx)\tan^{-1}(\sinh(c+dx))}{2ad} \\
&= -\frac{bfx}{2a^2d} - \frac{b^5(e+fx)^2}{2a^2(a^2+b^2)^2f} - \frac{3fx\tan^{-1}(e^{c+dx})}{ad} + \frac{3fx\tan^{-1}(\sinh(c+dx))}{2ad} \\
&= -\frac{bfx}{2a^2d} - \frac{3fx\tan^{-1}(e^{c+dx})}{ad} + \frac{2b^4(e+fx)\tan^{-1}(e^{c+dx})}{a(a^2+b^2)^2d} + \frac{b^2(e+fx)\tan^{-1}(e^{c+dx})}{a(a^2+b^2)} \\
&= -\frac{bfx}{2a^2d} - \frac{3fx\tan^{-1}(e^{c+dx})}{ad} + \frac{2b^4(e+fx)\tan^{-1}(e^{c+dx})}{a(a^2+b^2)^2d} + \frac{b^2(e+fx)\tan^{-1}(e^{c+dx})}{a(a^2+b^2)} \\
&= -\frac{bfx}{2a^2d} - \frac{3fx\tan^{-1}(e^{c+dx})}{ad} + \frac{2b^4(e+fx)\tan^{-1}(e^{c+dx})}{a(a^2+b^2)^2d} + \frac{b^2(e+fx)\tan^{-1}(e^{c+dx})}{a(a^2+b^2)} \\
&= -\frac{bfx}{2a^2d} - \frac{3fx\tan^{-1}(e^{c+dx})}{ad} + \frac{2b^4(e+fx)\tan^{-1}(e^{c+dx})}{a(a^2+b^2)^2d} + \frac{b^2(e+fx)\tan^{-1}(e^{c+dx})}{a(a^2+b^2)}
\end{aligned}$$

**Mathematica [A]** time = 10.9538, size = 1337, normalized size = 1.37

result too large to display

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Csch[c + d\*x]^2\*Sech[c + d\*x]^3)/(a + b\*Sinh[c + d\*x])

,x]

```
[Out] 8*(((-(d*e*Cosh[(c + d*x)/2]) + c*f*Cosh[(c + d*x)/2] - f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2]*Csch[c + d*x]*(a + b*Sinh[c + d*x]))/(16*a*d^2*(b + a*Csch[c + d*x])) - (b*e*Csch[c + d*x]*Log[Sinh[c + d*x]]*(a + b*Sinh[c + d*x]))/(8*a^2*d*(b + a*Csch[c + d*x])) + (b*c*f*Csch[c + d*x]*Log[Sinh[c + d*x]]*(a + b*Sinh[c + d*x]))/(8*a^2*d^2*(b + a*Csch[c + d*x])) + (f*Csch[c + d*x]*Log[Tanh[(c + d*x)/2]]*(a + b*Sinh[c + d*x]))/(8*a*d^2*(b + a*Csch[c + d*x])) + ((I/8)*b*f*Csch[c + d*x]*(I*(c + d*x)*Log[1 - E^(-2*(c + d*x))]) - (I/2)*(-(c + d*x)^2 + PolyLog[2, E^(-2*(c + d*x))]))*(a + b*Sinh[c + d*x]))/(a^2*d^2*(b + a*Csch[c + d*x])) + (b^5*Csch[c + d*x]*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]) + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]]) + f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]))*(a + b*Sinh[c + d*x]))/(8*a^2*(a^2 + b^2)^2*d^2*(b + a*Csch[c + d*x])) + (Csch[c + d*x]*(-2*a^2*b*d*e*(c + d*x) - 4*b^3*d*e*(c + d*x) + 2*a^2*b*c*f*(c + d*x) + 4*b^3*c*f*(c + d*x) - a^2*b*f*(c + d*x)^2 - 2*b^3*f*(c + d*x)^2 - 6*a^3*d*e*ArcTan[E^(c + d*x)] - 10*a*b^2*d*e*ArcTan[E^(c + d*x)] + 6*a^3*c*f*ArcTan[E^(c + d*x)] + 10*a*b^2*c*f*ArcTan[E^(c + d*x)] - (3*I)*a^3*f*(c + d*x)*Log[1 - I*E^(c + d*x)] - (5*I)*a*b^2*f*(c + d*x)*Log[1 - I*E^(c + d*x)] + (3*I)*a^3*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + (5*I)*a*b^2*f*(c + d*x)*Log[1 + I*E^(c + d*x)] + 2*a^2*b*d*e*Log[1 + E^(2*(c + d*x))] + 4*b^3*d*e*Log[1 + E^(2*(c + d*x))] - 2*a^2*b*c*f*Log[1 + E^(2*(c + d*x))] - 4*b^3*c*f*Log[1 + E^(2*(c + d*x))] + 2*a^2*b*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] + 4*b^3*f*(c + d*x)*Log[1 + E^(2*(c + d*x))] + I*a*(3*a^2 + 5*b^2)*f*PolyLog[2, (-I)*E^(c + d*x)] - I*a*(3*a^2 + 5*b^2)*f*PolyLog[2, I*E^(c + d*x)] + a^2*b*f*PolyLog[2, -E^(2*(c + d*x))] + 2*b^3*f*PolyLog[2, -E^(2*(c + d*x))]))*(a + b*Sinh[c + d*x]))/(16*(a^2 + b^2)^2*d^2*(b + a*Csch[c + d*x])) + (Csch[c + d*x]*Sech[(c + d*x)/2]*(d*e*Sinh[(c + d*x)/2] - c*f*Sinh[(c + d*x)/2] + f*(c + d*x)*Sinh[(c + d*x)/2]))*(a + b*Sinh[c + d*x]))/(16*a*d^2*(b + a*Csch[c + d*x])) + (Csch[c + d*x]*Sech[c + d*x]*(a + b*Sinh[c + d*x])*(-(a*f) + b*f*Sinh[c + d*x]))/(16*(a^2 + b^2)*d^2*(b + a*Csch[c + d*x])) + (Csch[c + d*x]*Sech[c + d*x]^2*(a + b*Sinh[c + d*x])*(-(b*d*e) + b*c*f - b*f*(c + d*x) - a*d*e*Sinh[c + d*x] + a*c*f*Sinh[c + d*x] - a*f*(c + d*x)*Sinh[c + d*x]))/(16*(a^2 + b^2)*d^2*(b + a*Csch[c + d*x]))
```

---

**Maple [B]** time = 0.273, size = 3280, normalized size = 3.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)*\text{csch}(d*x+c)^2*\text{sech}(d*x+c)^3/(a+b*\sinh(d*x+c)),x)$

[Out] 
$$\begin{aligned} & -1/(a^2+b^2)^{(5/2)}/d^2*f*b*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) \\ & *a^3-2/(a^2+b^2)^{(5/2)}/d^2*f*b^3*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) \\ & *a-2/(a^2+b^2)^{(3/2)}/d^2*b^3*f*c/a*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) \\ & +3/2/(a^2+b^2)^{(5/2)}/d^2*f*a^3*b*c*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) \\ & +2/(a^2+b^2)^{(5/2)}/d^2*b^5*f*c/a*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) \\ & +1/a^2/(a^2+b^2)/d^2*b^3*f*c*\ln(\exp(d*x+c)-1)-1/a^2/(a^2+b^2)/d*b^3*f*\ln(\exp(d*x+c)+1) \\ & *x+7/2*a/d^2/(a^2+b^2)^{(5/2)}*b^3*f*c*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) \\ & +4*a^2/d^2/(a^2+b^2)*f/(4*a^2+4*b^2)*\ln(1+I*\exp(d*x+c))*b*c+4*a^2/d^2/(a^2+b^2)*f/(4*a^2+4*b^2) \\ & *\ln(1-I*\exp(d*x+c))*b*c+4*a^2/d/(a^2+b^2)*f/(4*a^2+4*b^2)*\ln(1+I*\exp(d*x+c)) \\ & *b*x+4*a^2/d/(a^2+b^2)*f/(4*a^2+4*b^2)*\ln(1-I*\exp(d*x+c))*b*x-4*a^2/d^2/(a^2+b^2) \\ & *f*c/(4*a^2+4*b^2)*b*\ln(1+\exp(2*d*x+2*c))-8/d^2/(a^2+b^2)*b^3*f*c/(4*a^2+4*b^2) \\ & *\ln(1+\exp(2*d*x+2*c))+12*a^3/d^2/(a^2+b^2)*f*c/(4*a^2+4*b^2)*\text{arctan}(\exp(d*x+c)) \\ & -20*a/d/(a^2+b^2)*b^2*e/(4*a^2+4*b^2)*\text{arctan}(\exp(d*x+c))+20*a/d^2/(a^2+b^2)*b^2*f*c \\ & /((4*a^2+4*b^2)*\text{arctan}(\exp(d*x+c))+10*I*a/d^2/(a^2+b^2)*b^2*f/(4*a^2+4*b^2) \\ & *dilog(1+I*\exp(d*x+c))-10*I*a/d^2/(a^2+b^2)*b^2*f/(4*a^2+4*b^2) \\ & *dilog(1-I*\exp(d*x+c))+6*I*a^3/d^2/(a^2+b^2)*f/(4*a^2+4*b^2)*\ln(1+I*\exp(d*x+c)) \\ & *c-6*I*a^3/d^2/(a^2+b^2)*f/(4*a^2+4*b^2)*\ln(1-I*\exp(d*x+c))*c+6*I*a^3/d \\ & /((a^2+b^2)*f/(4*a^2+4*b^2)*\ln(1+I*\exp(d*x+c)))*x-6*I*a^3/d/(a^2+b^2) \\ & *f/(4*a^2+4*b^2)*\ln(1-I*\exp(d*x+c))*x+3/2*b/d*e/(a^2+b^2)^{(3/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a) \\ & /((a^2+b^2)^{(1/2)})*a+a/(a^2+b^2)/d^2*f*\ln(\exp(d*x+c)-1)-a/(a^2+b^2)/d^2*f*\ln(\exp(d*x+c)+1) \\ & -1/(a^2+b^2)/d*b*e*\ln(\exp(d*x+c)-1)-1/(a^2+b^2)/d*b*e*\ln(\exp(d*x+c)+1) \\ & +1/(a^2+b^2)/d^2*b*f*dilog(\exp(d*x+c))-1/(a^2+b^2)/d^2*b*f*dilog(\exp(d*x+c)+1) \\ & +1/(a^2+b^2)^{(3/2)}/d^2*f*b^3/a*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) \\ & -1/(a^2+b^2)^{(5/2)}/d^2*f*b^5/a*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) \\ & +1/(a^2+b^2)^{(3/2)}/d^2*a*f*b*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) \\ & +2/(a^2+b^2)^{(3/2)}/d*b^3*e/a*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) \\ & -3/2/(a^2+b^2)^{(5/2)}/d*a^3*b*e*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) \\ & -2/(a^2+b^2)^{(5/2)}/d*b^5*e/a*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) \\ & -1/(a^2+b^2)/d*\ln(\exp(d*x+c)+1)*b*f*x+1/(a^2+b^2)/d^2*b*f*c*\ln(\exp(d*x+c)-1) \\ & -1/a^2/(a^2+b^2)/d*b^3*e*\ln(\exp(d*x+c)-1)-1/a^2/(a^2+b^2)/d*b^3*e*\ln(\exp(d*x+c)+1) \\ & +1/a/(a^2+b^2)/d^2*b^2*f*\ln(\exp(d*x+c)-1)-1/a/(a^2+b^2)/d^2*b^2*f*\ln(\exp(d*x+c)+1) \\ & +1/a^2/(a^2+b^2)/d^2*b^3*f*dilog(\exp(d*x+c))-1/a^2/(a^2+b^2)/d^2*b^3*f*dilog(\exp(d*x+c)+1) \\ & +10*I*a/d/(a^2+b^2)*b^2*f/(4*a^2+4*b^2)*\ln(1+I*\exp(d*x+c))*x-10*I*a/d \\ & /((a^2+b^2)*b^2*f/(4*a^2+4*b^2)*\ln(1-I*\exp(d*x+c)))*x+10*I*a/d^2/(a^2+b^2) \\ & *b^2*f/(4*a^2+4*b^2)*\ln(1+I*\exp(d*x+c))*c-10*I*a/d^2/(a^2+b^2)*b^2*f/(4*a^2+4*b^2) \\ & *\ln(1-I*\exp(d*x+c))*c+1/a^2/d^2/(a^2+b^2)^2*b^5*f*dilog((-b*\exp(d*x+c) \\ & +(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))+1/a^2/d^2/(a^2+b^2)^2*b^5*f*dilog((b*\exp(d*x+c) \\ & +(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))+8/d/(a^2+b^2)*b^3*e/(4*a^2+4*b^2) \\ & *\ln(1+\exp(2*d*x+2*c))+8/d^2/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*dilog(1+I*\exp(d*x+c))+8/d^2/(a^2+b^2) \\ & *b^3*f/(4*a^2+4*b^2)*dilog(1-I*\exp(d*x+c))+8/d^2/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*dilog(1-I*\exp(d*x+c)) \end{aligned}$$

$$\begin{aligned}
& d*x+c)) - 12*a^3/d/(a^2+b^2)*e/(4*a^2+4*b^2)*\arctan(\exp(d*x+c))+1/a^2/d/(a^2+ \\
& b^2)^2*b^5*e*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)-3/2*b/d^2*f*c/(a^2+b^2)^{ \\
& (3/2)*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})*a-(3*a^2*d*f*x*\exp( \\
& 5*d*x+5*c)+2*b^2*d*f*x*\exp(5*d*x+5*c)+3*a^2*d*e*\exp(5*d*x+5*c)+2*a*b*d*f*x* \\
& \exp(4*d*x+4*c)+2*b^2*d*e*\exp(5*d*x+5*c)+2*a^2*d*f*x*\exp(3*d*x+3*c)+a^2*f*\exp \\
& (5*d*x+5*c)+2*a*b*d*e*\exp(4*d*x+4*c)+4*b^2*d*f*x*\exp(3*d*x+3*c)+2*a^2*d*e* \\
& \exp(3*d*x+3*c)-2*a*b*d*f*x*\exp(2*d*x+2*c)+a*b*f*\exp(4*d*x+4*c)+4*b^2*d*e*\exp \\
& (3*d*x+3*c)+3*a^2*d*f*x*\exp(d*x+c)-2*a*b*d*e*\exp(2*d*x+2*c)+2*b^2*d*f*x*\exp \\
& (d*x+c)+3*a^2*d*e*\exp(d*x+c)+2*b^2*d*e*\exp(d*x+c)-a^2*f*\exp(d*x+c)-a*b*f)/ \\
& d^2/(a^2+b^2)/(1+\exp(2*d*x+2*c))^2/a/(\exp(2*d*x+2*c)-1)-7/2*a/d/(a^2+b^2)^{( \\
& 5/2)*b^3*e*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-1/a^2/d^2/(a^2 \\
& +b^2)^2*b^5*f*c*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)+8/d^2/(a^2+b^2)*b^3*f \\
& /(4*a^2+4*b^2)*\ln(1-I*\exp(d*x+c))*c+8/d/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*\ln(1+ \\
& I*\exp(d*x+c))*x+8/d/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*\ln(1-I*\exp(d*x+c))*x+8/d^ \\
& 2/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*\ln(1+I*\exp(d*x+c))*c+4*a^2/d/(a^2+b^2)*e/(4 \\
& *a^2+4*b^2)*b*\ln(1+\exp(2*d*x+2*c))+1/a^2/d^2/(a^2+b^2)^2*b^5*f*\ln((b*\exp(d* \\
& x+c)+(a^2+b^2)^{(1/2)+a)/(a+(a^2+b^2)^{(1/2)}))*c+1/a^2/d^2/(a^2+b^2)^2*b^5*f* \\
& \ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)-a)/(-a+(a^2+b^2)^{(1/2)}))*c+1/a^2/d/(a^2+b \\
& ^2)^2*b^5*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)+a)/(a+(a^2+b^2)^{(1/2)}))*x+1/a^ \\
& 2/d/(a^2+b^2)^2*b^5*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)-a)/(-a+(a^2+b^2)^{(1 \\
& /2)}))*x+4*a^2/d^2/(a^2+b^2)*f/(4*a^2+4*b^2)*\operatorname{dilog}(1+I*\exp(d*x+c))*b+4*a^2/d \\
& ^2/(a^2+b^2)*f/(4*a^2+4*b^2)*\operatorname{dilog}(1-I*\exp(d*x+c))*b-6*I*a^3/d^2/(a^2+b^2)* \\
& f/(4*a^2+4*b^2)*\operatorname{dilog}(1-I*\exp(d*x+c))+6*I*a^3/d^2/(a^2+b^2)*f/(4*a^2+4*b^2) \\
& *\operatorname{dilog}(1+I*\exp(d*x+c))
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)^2\*sech(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $(b^5*\log(-2*a*e^{-d*x-c} + b*e^{-2*d*x-2*c} - b)/((a^6 + 2*a^4*b^2 + a^2*b^4)*d) + (3*a^3 + 5*a*b^2)*\arctan(e^{-d*x-c})/((a^4 + 2*a^2*b^2 + b^4)*d) + (a^2*b + 2*b^3)*\log(e^{-2*d*x-2*c} + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (2*a*b*e^{-2*d*x-2*c} - 2*a*b*e^{-4*d*x-4*c} + (3*a^2 + 2*b^2)*e^{-d*x-c} + 2*(a^2 + 2*b^2)*e^{-3*d*x-3*c} + (3*a^2 + 2*b^2)*e^{-5*d*x-5*c})/((a^3 + a*b^2 + (a^3 + a*b^2)*e^{-2*d*x-2*c} - (a^3 + a*b^2)*e^{-4*d*x-4*c} - (a^3 + a*b^2)*e^{-6*d*x-6*c}))*d - b*\log(e^{-d*x-c} + 1)/(a^2*d) - b*\log(e^{-d*x-c} - 1)/(a^2*d))*e + (32*b*d*\operatorname{integrate}(1/32*x/(a^2*d$

```
*e^(d*x + c) + a^2*d), x) - 32*b*d*integrate(1/32*x/(a^2*d*e^(d*x + c) - a^
2*d), x) + a*((d*x + c)/(a^2*d^2) - log(e^(d*x + c) + 1)/(a^2*d^2)) - a*((d
*x + c)/(a^2*d^2) - log(e^(d*x + c) - 1)/(a^2*d^2)) - (2*a*b*d*x*e^(2*d*x +
2*c) - 2*(a^2*d*e^(3*c) + 2*b^2*d*e^(3*c))*x*e^(3*d*x) + a*b - (a^2*e^(5*c
) + (3*a^2*d*e^(5*c) + 2*b^2*d*e^(5*c))*x)*e^(5*d*x) - (2*a*b*d*x*e^(4*c) +
a*b*e^(4*c))*e^(4*d*x) + (a^2*e^c - (3*a^2*d*e^c + 2*b^2*d*e^c)*x)*e^(d*x)
)/(a^3*d^2 + a*b^2*d^2 - (a^3*d^2*e^(6*c) + a*b^2*d^2*e^(6*c))*e^(6*d*x) -
(a^3*d^2*e^(4*c) + a*b^2*d^2*e^(4*c))*e^(4*d*x) + (a^3*d^2*e^(2*c) + a*b^2*
d^2*e^(2*c))*e^(2*d*x)) - 32*integrate(-1/16*(a*b^5*x*e^(d*x + c) - b^6*x)/
(a^6*b + 2*a^4*b^3 + a^2*b^5 - (a^6*b*e^(2*c) + 2*a^4*b^3*e^(2*c) + a^2*b^5
*e^(2*c))*e^(2*d*x) - 2*(a^7*e^c + 2*a^5*b^2*e^c + a^3*b^4*e^c)*e^(d*x)), x
) - 32*integrate(1/32*((3*a^3*e^c + 5*a*b^2*e^c)*x*e^(d*x) + 2*(a^2*b + 2*b
^3)*x)/(a^4 + 2*a^2*b^2 + b^4 + (a^4*e^(2*c) + 2*a^2*b^2*e^(2*c) + b^4*e^(2
*c))*e^(2*d*x)), x))*f
```

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**2*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.474 \quad \int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=180

$$\frac{b^5 \log(a + b \sinh(c + dx))}{a^2 d (a^2 + b^2)^2} - \frac{a (a^2 + 2b^2) \tan^{-1}(\sinh(c + dx))}{d (a^2 + b^2)^2} - \frac{a \tan^{-1}(\sinh(c + dx))}{2d (a^2 + b^2)} + \frac{b (a^2 + 2b^2) \log(\cosh(c + dx))}{d (a^2 + b^2)^2}$$

[Out]  $-(a*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(2*(a^2 + b^2)*d) - (a*(a^2 + 2*b^2)*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/((a^2 + b^2)^2*d) - \operatorname{Csch}[c + d*x]/(a*d) + (b*(a^2 + 2*b^2)*\operatorname{Log}[\operatorname{Cosh}[c + d*x]])/((a^2 + b^2)^2*d) - (b*\operatorname{Log}[\operatorname{Sinh}[c + d*x]])/(a^2*d) + (b^5*\operatorname{Log}[a + b*\operatorname{Sinh}[c + d*x]])/(a^2*(a^2 + b^2)^2*d) - (\operatorname{Sech}[c + d*x]^2*(b + a*\operatorname{Sinh}[c + d*x]))/(2*(a^2 + b^2)*d)$

**Rubi [A]** time = 0.259158, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$ , Rules used = {2837, 12, 894, 639, 203, 635, 260}

$$\frac{b^5 \log(a + b \sinh(c + dx))}{a^2 d (a^2 + b^2)^2} - \frac{a (a^2 + 2b^2) \tan^{-1}(\sinh(c + dx))}{d (a^2 + b^2)^2} - \frac{a \tan^{-1}(\sinh(c + dx))}{2d (a^2 + b^2)} + \frac{b (a^2 + 2b^2) \log(\cosh(c + dx))}{d (a^2 + b^2)^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Csch}[c + d*x]^2*\operatorname{Sech}[c + d*x]^3)/(a + b*\operatorname{Sinh}[c + d*x]), x]$

[Out]  $-(a*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(2*(a^2 + b^2)*d) - (a*(a^2 + 2*b^2)*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/((a^2 + b^2)^2*d) - \operatorname{Csch}[c + d*x]/(a*d) + (b*(a^2 + 2*b^2)*\operatorname{Log}[\operatorname{Cosh}[c + d*x]])/((a^2 + b^2)^2*d) - (b*\operatorname{Log}[\operatorname{Sinh}[c + d*x]])/(a^2*d) + (b^5*\operatorname{Log}[a + b*\operatorname{Sinh}[c + d*x]])/(a^2*(a^2 + b^2)^2*d) - (\operatorname{Sech}[c + d*x]^2*(b + a*\operatorname{Sinh}[c + d*x]))/(2*(a^2 + b^2)*d)$

### Rule 2837

$\operatorname{Int}[\cos[(e_.) + (f_.)*(x_.)]^{(p_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(m_.)})*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Dist}[1/(b^p*f), \operatorname{Subst}[\operatorname{Int}[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^{(p-1)/2}, x], x, b*\sin[e + f*x]], x] /;$   $\operatorname{FreeQ}\{a, b, c, d, e, f, m, n\}, x \&\& \operatorname{IntegerQ}[(p-1)/2] \&\& \operatorname{NeQ}[a^2 - b^2, 0]$

### Rule 12



Int[(a\_)\*(u\_), x\_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 894

Int[((d\_.) + (e\_.)\*(x\_))^(m\_)\*((f\_.) + (g\_.)\*(x\_))^(n\_)\*((a\_) + (c\_.)\*(x\_)^2)^(p\_.), x\_Symbol] :=> Int[ExpandIntegrand[(d + e\*x)^m\*(f + g\*x)^n\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

### Rule 639

Int[((d\_) + (e\_.)\*(x\_))\*((a\_) + (c\_.)\*(x\_)^2)^(p\_), x\_Symbol] :=> Simp[((a\*e - c\*d\*x)\*(a + c\*x^2)^(p + 1))/(2\*a\*c\*(p + 1)), x] + Dist[(d\*(2\*p + 3))/(2\*a\*(p + 1)), Int[(a + c\*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && LtQ[p, -1] && NeQ[p, -3/2]

### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :=> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

### Rule 635

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (c\_.)\*(x\_)^2), x\_Symbol] :=> Dist[d, Int[1/(a + c\*x^2), x], x] + Dist[e, Int[x/(a + c\*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a\*c)]

### Rule 260

Int[(x\_)^(m\_.)/((a\_) + (b\_.)\*(x\_)^(n\_)), x\_Symbol] :=> Simp[Log[RemoveContent[a + b\*x^n, x]]/(b\*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

### Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{b^3 \operatorname{Subst}\left(\int \frac{b^2}{x^2(a+x)(-b^2-x^2)^2} dx, x, b\sinh(c+dx)\right)}{d} \\
&= \frac{b^5 \operatorname{Subst}\left(\int \frac{1}{x^2(a+x)(-b^2-x^2)^2} dx, x, b\sinh(c+dx)\right)}{d} \\
&= \frac{b^5 \operatorname{Subst}\left(\int \left(\frac{1}{ab^4x^2} - \frac{1}{a^2b^4x} + \frac{1}{a^2(a^2+b^2)^2(a+x)} + \frac{-a+x}{b^2(a^2+b^2)(b^2+x^2)^2} - \frac{(a^2+2b^2)(a-x)}{b^4(a^2+b^2)^2(b^2+x^2)}\right) dx, x, b\sinh(c+dx)\right)}{d} \\
&= -\frac{\operatorname{csch}(c+dx)}{ad} - \frac{b \log(\sinh(c+dx))}{a^2d} + \frac{b^5 \log(a+b\sinh(c+dx))}{a^2(a^2+b^2)^2d} + \frac{b^3 \operatorname{Subst}\left(\int \frac{1}{b^2(a^2+b^2)(b^2+x^2)^2} dx, x, b\sinh(c+dx)\right)}{d} \\
&= -\frac{\operatorname{csch}(c+dx)}{ad} - \frac{b \log(\sinh(c+dx))}{a^2d} + \frac{b^5 \log(a+b\sinh(c+dx))}{a^2(a^2+b^2)^2d} - \frac{\operatorname{sech}^2(c+dx)}{2(a^2+b^2)d} \\
&= -\frac{a \tan^{-1}(\sinh(c+dx))}{2(a^2+b^2)d} - \frac{a(a^2+2b^2) \tan^{-1}(\sinh(c+dx))}{(a^2+b^2)^2d} - \frac{\operatorname{csch}(c+dx)}{ad} + \frac{b(a^2+b^2)}{2d(a^2+b^2)^2}
\end{aligned}$$

**Mathematica [C]** time = 0.93015, size = 227, normalized size = 1.26

$$\frac{\operatorname{csch}(c+dx)(a+b\sinh(c+dx))\left(\frac{b\operatorname{sech}^2(c+dx)}{a^2+b^2} - \frac{2b^5 \log(a+b\sinh(c+dx))}{a^2(a^2+b^2)^2} - \frac{(b+ia)(a^2+2b^2) \log(-\sinh(c+dx)+i)}{(a^2+b^2)^2} + \frac{(-b+ia)(a^2+2b^2) \log(\sinh(c+dx)+i)}{(a^2+b^2)^2}\right)}{2d(\operatorname{acsch}(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] Integrate[(Csch[c + d\*x]^2\*Sech[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] -(Csch[c + d\*x]\*(a + b\*Sinh[c + d\*x])\*((a\*ArcTan[Sinh[c + d\*x]])/(a^2 + b^2) + (2\*Csch[c + d\*x])/a - ((I\*a + b)\*(a^2 + 2\*b^2)\*Log[I - Sinh[c + d\*x]])/(a^2 + b^2)^2 + (2\*b\*Log[Sinh[c + d\*x]])/a^2 + ((I\*a - b)\*(a^2 + 2\*b^2)\*Log[I + Sinh[c + d\*x]])/(a^2 + b^2)^2 - (2\*b^5\*Log[a + b\*Sinh[c + d\*x]])/(a^2\*(a^2 + b^2)^2) + (b\*Sech[c + d\*x]^2)/(a^2 + b^2) + (a\*Sech[c + d\*x]\*Tanh[c + d\*x])/(a^2 + b^2)))/(2\*d\*(b + a\*Csch[c + d\*x]))





```

*x + c)^5 + (a^4*b + 2*a^2*b^3 + b^5)*sinh(d*x + c)^6 - a^4*b - 2*a^2*b^3 -
b^5 + (a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^4 + (a^4*b + 2*a^2*b^3 + b^5
+ 15*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(a^
4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^3 + (a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x
+ c))*sinh(d*x + c)^3 - (a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^2 - (a^4*b
+ 2*a^2*b^3 + b^5 - 15*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^4 - 6*(a^4*b
+ 2*a^2*b^3 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(a^4*b + 2*a^2
*b^3 + b^5)*cosh(d*x + c)^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c)^3 -
(a^4*b + 2*a^2*b^3 + b^5)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c
))/(cosh(d*x + c) - sinh(d*x + c)) + (3*a^5 + 5*a^3*b^2 + 2*a*b^4 + 5*(3*a^
5 + 5*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^4 + 8*(a^4*b + a^2*b^3)*cosh(d*x + c
)^3 + 6*(a^5 + 3*a^3*b^2 + 2*a*b^4)*cosh(d*x + c)^2 - 4*(a^4*b + a^2*b^3)*c
osh(d*x + c))*sinh(d*x + c))/((a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^6
+ 6*(a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^6 + 2
*a^4*b^2 + a^2*b^4)*d*sinh(d*x + c)^6 + (a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(
d*x + c)^4 + (15*(a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^2 + (a^6 + 2*a
^4*b^2 + a^2*b^4)*d)*sinh(d*x + c)^4 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d
*x + c)^2 + 4*(5*(a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^3 + (a^6 + 2*a
^4*b^2 + a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^6 + 2*a^4*b^2 +
a^2*b^4)*d*cosh(d*x + c)^4 + 6*(a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)
^2 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d)*sinh(d*x + c)^2 - (a^6 + 2*a^4*b^2 + a^
2*b^4)*d + 2*(3*(a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^5 + 2*(a^6 + 2*
a^4*b^2 + a^2*b^4)*d*cosh(d*x + c)^3 - (a^6 + 2*a^4*b^2 + a^2*b^4)*d*cosh(d
*x + c))*sinh(d*x + c))

```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)\*\*2\*sech(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [B]** time = 1.36711, size = 637, normalized size = 3.54

$$\frac{b^6 \log\left(\left|b\left(e^{(dx+c)} - e^{-(dx-c)}\right) + 2a\right|\right)}{a^6 b d + 2 a^4 b^3 d + a^2 b^5 d} - \frac{\left(\pi + 2 \arctan\left(\frac{1}{2}\left(e^{(2dx+2c)} - 1\right)e^{-(dx-c)}\right)\right)\left(3a^3 + 5ab^2\right)}{4\left(a^4 d + 2a^2 b^2 d + b^4 d\right)} + \frac{\left(a^2 b + 2b^3\right) \log\left(\left|e^{(dx+c)} - e^{-(dx-c)}\right|\right)}{2\left(a^4 d + 2a^2 b^2 d + b^4 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] b^6*log(abs(b*(e^(d*x + c) - e^(-d*x - c)) + 2*a))/(a^6*b*d + 2*a^4*b^3*d +
a^2*b^5*d) - 1/4*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(
3*a^3 + 5*a*b^2)/(a^4*d + 2*a^2*b^2*d + b^4*d) + 1/2*(a^2*b + 2*b^3)*log((e
^(d*x + c) - e^(-d*x - c))^2 + 4)/(a^4*d + 2*a^2*b^2*d + b^4*d) + 1/3*(b^5*
(e^(d*x + c) - e^(-d*x - c))^3 - 9*a^5*(e^(d*x + c) - e^(-d*x - c))^2 - 15*
a^3*b^2*(e^(d*x + c) - e^(-d*x - c))^2 - 6*a*b^4*(e^(d*x + c) - e^(-d*x - c
))^2 - 6*a^4*b*(e^(d*x + c) - e^(-d*x - c)) - 6*a^2*b^3*(e^(d*x + c) - e^(-
d*x - c)) + 4*b^5*(e^(d*x + c) - e^(-d*x - c)) - 24*a^5 - 48*a^3*b^2 - 24*a
*b^4)/((a^6*d + 2*a^4*b^2*d + a^2*b^4*d)*((e^(d*x + c) - e^(-d*x - c))^3 +
4*e^(d*x + c) - 4*e^(-d*x - c))) - b*log(abs(e^(d*x + c) - e^(-d*x - c)))/(
a^2*d)
```

$$3.475 \quad \int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Optimal.** Leaf size=38

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}, x\right)$$

[Out] Unintegrable[(Csch[c + d\*x]^2\*Sech[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.135229, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Csch[c + d\*x]^2\*Sech[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] Defer[Int] [(Csch[c + d\*x]^2\*Sech[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.002, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Csch[c + d\*x]^2\*Sech[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] \$Aborted

**Maple [A]** time = 3.345, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx+c))^2 (\operatorname{sech}(dx+c))^3}{(fx+e)(a+b \sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)`

[Out] `int(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `(a*b*f + (2*b^2*d*e*e^(5*c) + (3*d*e - f)*a^2*e^(5*c) + (3*a^2*d*f*e^(5*c) + 2*b^2*d*f*e^(5*c))*x)*e^(5*d*x) + (2*a*b*d*f*x*e^(4*c) + (2*d*e - f)*a*b*e^(4*c))*e^(4*d*x) + 2*(a^2*d*e*e^(3*c) + 2*b^2*d*e*e^(3*c) + (a^2*d*f*e^(3*c) + 2*b^2*d*f*e^(3*c))*x)*e^(3*d*x) - 2*(a*b*d*f*x*e^(2*c) + a*b*d*e*e^(2*c))*e^(2*d*x) + (2*b^2*d*e*e^c + (3*d*e + f)*a^2*e^c + (3*a^2*d*f*e^c + 2*b^2*d*f*e^c)*x)*e^(d*x)/(a^3*d^2*e^2 + a*b^2*d^2*e^2 + (a^3*d^2*f^2 + a*b^2*d^2*f^2)*x^2 + 2*(a^3*d^2*e*f + a*b^2*d^2*e*f)*x - (a^3*d^2*e^2*e^(6*c) + a*b^2*d^2*e^2*e^(6*c) + (a^3*d^2*f^2*e^(6*c) + a*b^2*d^2*f^2*e^(6*c))*x^2 + 2*(a^3*d^2*e*f*e^(6*c) + a*b^2*d^2*e*f*e^(6*c))*x)*e^(6*d*x) - (a^3*d^2*e^2*e^(4*c) + a*b^2*d^2*e^2*e^(4*c) + (a^3*d^2*f^2*e^(4*c) + a*b^2*d^2*f^2*e^(4*c))*x^2 + 2*(a^3*d^2*e*f*e^(4*c) + a*b^2*d^2*e*f*e^(4*c))*x)*e^(4*d*x) + (a^3*d^2*e^2*e^(2*c) + a*b^2*d^2*e^2*e^(2*c) + (a^3*d^2*f^2*e^(2*c) + a*b^2*d^2*f^2*e^(2*c))*x^2 + 2*(a^3*d^2*e*f*e^(2*c) + a*b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x) - 32*integrate(-1/16*(a*b^5*e^(d*x + c) - b^6)/(a^6*b*e + 2*a^4*b^3*e + a^2*b^5*e + (a^6*b*f + 2*a^4*b^3*f + a^2*b^5*f)*x - (a^6*b*e*e^(2*c) + 2*a^4*b^3*e*e^(2*c) + a^2*b^5*e*e^(2*c) + (a^6*b*f*e^(2*c) + 2*a^4*b^3*f*e^(2*c) + a^2*b^5*f*e^(2*c))*x)*e^(2*d*x) - 2*(a^7*e*e^c + 2*a^5*b^2*e`



```

e^c + a^3*b^4*e*e^c + (a^7*f*e^c + 2*a^5*b^2*f*e^c + a^3*b^4*f*e^c)*x)*e^(d
*x)), x) - 32*integrate(1/32*(2*(d^2*e^2 - f^2)*a^2*b + 2*(2*d^2*e^2 - f^2)
*b^3 + 2*(a^2*b*d^2*f^2 + 2*b^3*d^2*f^2)*x^2 + 4*(a^2*b*d^2*e*f + 2*b^3*d^2
*e*f)*x + ((3*d^2*e^2 - 2*f^2)*a^3*e^c + (5*d^2*e^2 - 2*f^2)*a*b^2*e^c + (3
*a^3*d^2*f^2*e^c + 5*a*b^2*d^2*f^2*e^c)*x^2 + 2*(3*a^3*d^2*e*f*e^c + 5*a*b^
2*d^2*e*f*e^c)*x)*e^(d*x))/(a^4*d^2*e^3 + 2*a^2*b^2*d^2*e^3 + b^4*d^2*e^3 +
(a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*e*f^2 + 2
*a^2*b^2*d^2*e*f^2 + b^4*d^2*e*f^2)*x^2 + 3*(a^4*d^2*e^2*f + 2*a^2*b^2*d^2*
e^2*f + b^4*d^2*e^2*f)*x + (a^4*d^2*e^3*e^(2*c) + 2*a^2*b^2*d^2*e^3*e^(2*c)
+ b^4*d^2*e^3*e^(2*c) + (a^4*d^2*f^3*e^(2*c) + 2*a^2*b^2*d^2*f^3*e^(2*c) +
b^4*d^2*f^3*e^(2*c))*x^3 + 3*(a^4*d^2*e*f^2*e^(2*c) + 2*a^2*b^2*d^2*e*f^2*
e^(2*c) + b^4*d^2*e*f^2*e^(2*c))*x^2 + 3*(a^4*d^2*e^2*f*e^(2*c) + 2*a^2*b^2
*d^2*e^2*f*e^(2*c) + b^4*d^2*e^2*f*e^(2*c))*x)*e^(2*d*x)), x) - 32*integrat
e(-1/32*(b*d*f*x + b*d*e + a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^2*d*e^2
- (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)), x) + 32
*integrate(1/32*(b*d*f*x + b*d*e - a*f)/(a^2*d*f^2*x^2 + 2*a^2*d*e*f*x + a^
2*d*e^2 + (a^2*d*f^2*x^2*e^c + 2*a^2*d*e*f*x*e^c + a^2*d*e^2*e^c)*e^(d*x)),
x)

```

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.476 \quad \int \frac{(e+fx)^3 \coth(c+dx) \operatorname{csch}^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=752

$$\frac{6b^2 f^2 (e+fx) \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3 d^3} + \frac{6b^2 f^2 (e+fx) \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3 d^3} - \frac{3b^2 f^2 (e+fx) \operatorname{PolyLog}\left(3, e^{2(c+dx)}\right)}{2a^3 d^3}$$

```
[Out] (-3*f*(e + f*x)^2)/(2*a*d^2) + (6*b*f*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d^2) - (3*f*(e + f*x)^2*Coth[c + d*x])/(2*a*d^2) + (b*(e + f*x)^3*Csch[c + d*x])/(a^2*d) - ((e + f*x)^3*Csch[c + d*x]^2)/(2*a*d) - (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(a^3*d) - (b^2*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(a^3*d) + (3*f^2*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d^3) + (b^2*(e + f*x)^3*Log[1 - E^(2*(c + d*x))])/(a^3*d) + (6*b*f^2*(e + f*x)*PolyLog[2, -E^(c + d*x)]/(a^2*d^3) - (6*b*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)]/(a^2*d^3) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (3*f^3*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^4) + (3*b^2*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*x))])/(2*a^3*d^2) - (6*b*f^3*PolyLog[3, -E^(c + d*x)]/(a^2*d^4) + (6*b*f^3*PolyLog[3, E^(c + d*x)]/(a^2*d^4) + (6*b^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^3) + (6*b^2*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^3) - (3*b^2*f^2*(e + f*x)*PolyLog[3, E^(2*(c + d*x))])/(2*a^3*d^3) - (6*b^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^4) - (6*b^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^4) + (3*b^2*f^3*PolyLog[4, E^(2*(c + d*x))])/(4*a^3*d^4)
```

**Rubi [A]** time = 1.35304, antiderivative size = 752, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 14, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {5587, 5452, 4184, 3716, 2190, 2279, 2391, 4182, 2531, 2282, 6589, 5569, 6609, 5561}

$$\frac{6b^2 f^2 (e+fx) \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3 d^3} + \frac{6b^2 f^2 (e+fx) \operatorname{PolyLog}\left(3, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3 d^3} - \frac{3b^2 f^2 (e+fx) \operatorname{PolyLog}\left(3, e^{2(c+dx)}\right)}{2a^3 d^3}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (-3*f*(e + f*x)^2)/(2*a*d^2) + (6*b*f*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d^2) - (3*f*(e + f*x)^2*Coth[c + d*x])/(2*a*d^2) + (b*(e + f*x)^3*Csch[c
```

```

+ d*x))/(a^2*d) - ((e + f*x)^3*Csch[c + d*x]^2)/(2*a*d) - (b^2*(e + f*x)^3*
Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*d) - (b^2*(e + f*x)^3*
Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*d) + (3*f^2*(e + f*x)*
Log[1 - E^(2*(c + d*x))])/(a*d^3) + (b^2*(e + f*x)^3*Log[1 - E^(2*(c + d*x)
)])/(a^3*d) + (6*b*f^2*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a^2*d^3) - (6*b
*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^2*d^3) - (3*b^2*f*(e + f*x)^2*Po
lyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) - (3*b^2*f*(e
+ f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) +
(3*f^3*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^4) + (3*b^2*f*(e + f*x)^2*PolyL
og[2, E^(2*(c + d*x))])/(2*a^3*d^2) - (6*b*f^3*PolyLog[3, -E^(c + d*x)])/(a
^2*d^4) + (6*b*f^3*PolyLog[3, E^(c + d*x)])/(a^2*d^4) + (6*b^2*f^2*(e + f*x
)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^3) + (6*b^2*
f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d
^3) - (3*b^2*f^2*(e + f*x)*PolyLog[3, E^(2*(c + d*x))])/(2*a^3*d^3) - (6*b^2
*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^4) - (6*b
^2*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^4) + (3
*b^2*f^3*PolyLog[4, E^(2*(c + d*x))])/(4*a^3*d^4)

```

#### Rule 5587

```

Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
(f_.)*(x_))^(m_.))/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Csch[c + d*x]^(p - 1)*Coth[c + d*x]^n)/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
IGtQ[p, 0]

```

#### Rule 5452

```

Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

```

#### Rule 4184

```

Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

```

#### Rule 3716

```

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*

```

$e) + f*Fz*x)/E^{(2*I*k*Pi)}), x], x] /; FreeQ[\{c, d, e, f, fz\}, x] \&\& Integ$   
 $erQ[4*k] \&\& IGtQ[m, 0]$

### Rule 2190

$Int[(((F_)^{((g_)*(e_) + (f_)*(x_))})^{(n_)*((c_) + (d_)*(x_))^{(m_)} /$   
 $((a_) + (b_)*((F_)^{((g_)*(e_) + (f_)*(x_))})^{(n_)}), x\_Symbol] \rightarrow Simp$   
 $[((c + d*x)^m * Log[1 + (b*(F^{(g*(e + f*x)))^n)/a]) / (b*f*g*n*Log[F]), x] - Di$   
 $st[(d*m) / (b*f*g*n*Log[F]), Int[(c + d*x)^{(m-1)} * Log[1 + (b*(F^{(g*(e + f*x)$   
 $)^n)/a], x], x] /; FreeQ[\{F, a, b, c, d, e, f, g, n\}, x] \&\& IGtQ[m, 0]$

### Rule 2279

$Int[Log[(a_) + (b_)*((F_)^{((e_)*((c_) + (d_)*(x_))})^{(n_)}], x\_Symbol]$   
 $\rightarrow Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^{(e*(c + d*x))$   
 $)^n], x] /; FreeQ[\{F, a, b, c, d, e, n\}, x] \&\& GtQ[a, 0]$

### Rule 2391

$Int[Log[(c_)*((d_) + (e_)*(x_)^{(n_)})]/(x_), x\_Symbol] \rightarrow -Simp[PolyLog[2$   
 $, -(c*e*x^n)]/n, x] /; FreeQ[\{c, d, e, n\}, x] \&\& EqQ[c*d, 1]$

### Rule 4182

$Int[csc[(e_) + (Complex[0, fz_])*(f_)*(x_)]*((c_) + (d_)*(x_))^{(m_)}, x$   
 $\_Symbol] \rightarrow Simp[(-2*(c + d*x)^m * ArcTanh[E^{-(I*e) + f*fz*x}]) / (f*fz*I), x]$   
 $+ (-Dist[(d*m) / (f*fz*I), Int[(c + d*x)^{(m-1)} * Log[1 - E^{-(I*e) + f*fz*x}$   
 $], x], x] + Dist[(d*m) / (f*fz*I), Int[(c + d*x)^{(m-1)} * Log[1 + E^{-(I*e) +$   
 $f*fz*x}], x], x] /; FreeQ[\{c, d, e, f, fz\}, x] \&\& IGtQ[m, 0]$

### Rule 2531

$Int[Log[1 + (e_)*((F_)^{((c_)*((a_) + (b_)*(x_))})^{(n_)}]*((f_) + (g_)$   
 $*(x_))^{(m_)}, x\_Symbol] \rightarrow -Simp[((f + g*x)^m * PolyLog[2, -(e*(F^{(c*(a + b*x)$   
 $))^n]) / (b*c*n*Log[F]), x] + Dist[(g*m) / (b*c*n*Log[F]), Int[(f + g*x)^{(m-1)}$   
 $* PolyLog[2, -(e*(F^{(c*(a + b*x)))^n}], x], x] /; FreeQ[\{F, a, b, c, e, f$   
 $, g, n\}, x] \&\& GtQ[m, 0]$

### Rule 2282

$Int[u_, x\_Symbol] \rightarrow With[\{v = FunctionOfExponential[u, x]\}, Dist[v/D[v, x]$   
 $, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi$   
 $onOfExponentialQ[u, x] \&\& !MatchQ[u, (w_)*((a_)*(v_)^{(n_)})^{(m_)} /; FreeQ[$   
 $\{a, m, n\}, x] \&\& IntegerQ[m*n] \&\& !MatchQ[u, E^{((c_)*((a_) + (b_)*x))*$

$(F\_)[v\_]$  /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 5569

Int[(Coth[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] :> Dist[1/a, Int[(e + f\*x)^m\*Coth[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Cosh[c + d\*x]\*Coth[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 6609

Int[((e\_.) + (f\_.)\*(x\_))^(m\_.)\*PolyLog[n\_, (d\_.)\*((F\_)^(c\_.)\*((a\_.) + (b\_.)\*(x\_)))^(p\_.)], x\_Symbol] :> Simp[((e + f\*x)^m\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p])/(b\*c\*p\*Log[F]), x] - Dist[(f\*m)/(b\*c\*p\*Log[F]), Int[(e + f\*x)^(m - 1)\*PolyLog[n + 1, d\*(F^(c\*(a + b\*x)))^p], x], x] /; FreeQ[{F, a, b, c, d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] :> -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \coth(c+dx) \operatorname{csch}^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \coth(c+dx) \operatorname{csch}^2(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^3 \coth(c+dx) \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= -\frac{(e+fx)^3 \operatorname{csch}^2(c+dx)}{2ad} - \frac{b \int (e+fx)^3 \coth(c+dx) \operatorname{csch}(c+dx) dx}{a^2} + \dots \\
&= -\frac{3f(e+fx)^2 \coth(c+dx)}{2ad^2} + \frac{b(e+fx)^3 \operatorname{csch}(c+dx)}{a^2 d} - \frac{(e+fx)^3 \operatorname{csch}^2(c+dx)}{2ad} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{6bf(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^2 d^2} - \frac{3f(e+fx)^2 \coth(c+dx)}{2ad^2} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{6bf(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^2 d^2} - \frac{3f(e+fx)^2 \coth(c+dx)}{2ad^2} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{6bf(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^2 d^2} - \frac{3f(e+fx)^2 \coth(c+dx)}{2ad^2} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{6bf(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^2 d^2} - \frac{3f(e+fx)^2 \coth(c+dx)}{2ad^2} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{6bf(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^2 d^2} - \frac{3f(e+fx)^2 \coth(c+dx)}{2ad^2} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{6bf(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^2 d^2} - \frac{3f(e+fx)^2 \coth(c+dx)}{2ad^2} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{6bf(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^2 d^2} - \frac{3f(e+fx)^2 \coth(c+dx)}{2ad^2}
\end{aligned}$$

**Mathematica [C]** time = 65.6056, size = 6441, normalized size = 8.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] Result too large to show
```

---

**Maple [F]** time = 0.936, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 \coth(dx + c) (\operatorname{csch}(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)`

[Out] `int((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `-e^3*(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c)))/((2*a^2*e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + b^2*log(-2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - b^2*log(e^(-d*x - c) + 1)/(a^3*d) - b^2*log(e^(-d*x - c) - 1)/(a^3*d) + (3*a*f^3*x^2 + 6*a*e*f^2*x + 3*a*e^2*f + 2*(b*d*f^3*x^3*e^(3*c) + 3*b*d*e*f^2*x^2*e^(3*c) + 3*b*d*e^2*f*x*e^(3*c)))*e^(3*d*x) - (2*a*d*f^3*x^3*e^(2*c) + 3*a*e^2*f*e^(2*c) + 3*(2*d*e*f^2 + f^3)*a*x^2*e^(2*c) + 6*(d*e^2*f + e*f^2)*a*x*e^(2*c))*e^(2*d*x) - 2*(b*d*f^3*x^3*e^c + 3*b*d*e*f^2*x^2*e^c + 3*b*d*e^2*f*x*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) + (d^3*x^3*log(e^(d*x + c) + 1) + 3*d^2*x^2*dilog(-e^(d*x + c)) - 6*d*x*polylog(3, -e^(d*x + c)) + 6*polylog(4, -e^(d*x + c)))*b^2*f^3/(a^3*d^4) + (d^3*x^3*log(-e^(d*x + c) + 1) + 3*d^2*x^2*dilog(e^(d*x + c)) - 6*d*x*polylog(3, e^(d*x + c)) + 6*polylog(4, e^(d*x + c)))*b^2*f^3/(a^3*d^4) - 3*(b*d*e^2*f + a*e*f^2)*x/(a^2*d^2) + 3*(b*d*e^2*f - a*e*f^2)*x/(a^2*d^2) + 3*(b*d*e^2*f + a*e*f^2)*log(e^(d*x + c) + 1)/(a^2*d^3) - 3*(b*d*e^2*f - a*e*f^2)*log(e^(d*x + c) - 1)/(a^2*d^3) + 3*(b^2*d*e*f^2 + a*b*f^3)*(d^2*x^2*log(e^(d*x + c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))/(a^3*d^4) + 3*(b^2*d*e*f^2 - a*b*f^3)*(d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x + c)) - 2*polylog(3, e^(d*x + c)))/(a^3*d^4) + 3*(b^2*d^2*e^2*f + 2*a*b*d*e*f^2 + a^2*f^3)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)))/(a^3*d^4) + 3*(b^2*d^2*e^2*f - 2*a*b*d*e*f^2 + a^2*f^3)*(d*x*log(-e^(d*x + c) + 1) + dilog(e^(d*x + c)`



$$\left. \right) / (a^3 d^4) - 1/4 * (b^2 d^4 f^3 x^4 + 4 * (b^2 d e f^2 + a b f^3) d^3 x^3 + 6 * (b^2 d^2 e^2 f + 2 a b d e f^2 + a^2 f^3) d^2 x^2) / (a^3 d^4) - 1/4 * (b^2 d^4 f^3 x^4 + 4 * (b^2 d e f^2 - a b f^3) d^3 x^3 + 6 * (b^2 d^2 e^2 f - 2 a b d e f^2 + a^2 f^3) d^2 x^2) / (a^3 d^4) + \text{integrate}(-2 * (b^3 f^3 x^3 + 3 b^3 e f^2 x^2 + 3 b^3 e^2 f x - (a b^2 f^3 x^3 e^c + 3 a b^2 e f^2 x^2 e^c + 3 a b^2 e^2 f x e^c) e^{(d x)}) / (a^3 b e^{(2 d x + 2 c)} + 2 a^4 e^{(d x + c)} - a^3 b), x)$$

**Fricas [C]** time = 4.92579, size = 24935, normalized size = 33.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out]  $(3 a^2 d^2 e^2 f - 6 a^2 c d e f^2 + 3 a^2 c^2 f^3 - 3 (a^2 d^2 f^3 x^2 + 2 a^2 d^2 e f^2 x + 2 a^2 c d e f^2 - a^2 c^2 f^3) \cosh(d x + c)^4 - 3 (a^2 d^2 f^3 x^2 + 2 a^2 d^2 e f^2 x + 2 a^2 c d e f^2 - a^2 c^2 f^3) \sinh(d x + c)^4 + 2 (a b d^3 f^3 x^3 + 3 a b d^3 e f^2 x^2 + 3 a b d^3 e^2 f x + a b d^3 e^3) \cosh(d x + c)^3 + 2 (a b d^3 f^3 x^3 + 3 a b d^3 e f^2 x^2 + 3 a b d^3 e^2 f x + a b d^3 e^3 - 6 (a^2 d^2 f^3 x^2 + 2 a^2 d^2 e f^2 x + 2 a^2 c d e f^2 - a^2 c^2 f^3) \cosh(d x + c)) \sinh(d x + c)^3 - (2 a^2 d^3 f^3 x^3 + 2 a^2 d^3 e^3 + 3 a^2 d^2 e^2 f - 12 a^2 c d e f^2 + 6 a^2 c^2 f^3 + 3 (2 a^2 d^3 e f^2 - a^2 d^2 f^3) x^2 + 6 (a^2 d^3 e^2 f - a^2 d^2 e f^2) x) \cosh(d x + c)^2 - (2 a^2 d^3 f^3 x^3 + 2 a^2 d^3 e^3 + 3 a^2 d^2 e^2 f - 12 a^2 c d e f^2 + 6 a^2 c^2 f^3 + 3 (2 a^2 d^3 e f^2 - a^2 d^2 f^3) x^2 + 18 (a^2 d^2 f^3 x^2 + 2 a^2 d^2 e f^2 x + 2 a^2 c d e f^2 - a^2 c^2 f^3) \cosh(d x + c)^2 + 6 (a^2 d^3 e^2 f - a^2 d^2 e f^2) x - 6 (a b d^3 f^3 x^3 + 3 a b d^3 e f^2 x^2 + 3 a b d^3 e^2 f x + a b d^3 e^3) \cosh(d x + c)) \sinh(d x + c)^2 - 2 (a b d^3 f^3 x^3 + 3 a b d^3 e f^2 x^2 + 3 a b d^3 e^2 f x + a b d^3 e^3) \cosh(d x + c) - 3 (b^2 d^2 f^3 x^2 + 2 b^2 d^2 e f^2 x + b^2 d^2 e^2 f + (b^2 d^2 f^3 x^2 + 2 b^2 d^2 e f^2 x + b^2 d^2 e^2 f) \cosh(d x + c)^4 + 4 (b^2 d^2 f^3 x^2 + 2 b^2 d^2 e f^2 x + b^2 d^2 e^2 f) \cosh(d x + c) \sinh(d x + c)^3 + (b^2 d^2 f^3 x^2 + 2 b^2 d^2 e f^2 x + b^2 d^2 e^2 f) \sinh(d x + c)^4 - 2 (b^2 d^2 f^3 x^2 + 2 b^2 d^2 e f^2 x + b^2 d^2 e^2 f) \cosh(d x + c)^2 - 2 (b^2 d^2 f^3 x^2 + 2 b^2 d^2 e f^2 x + b^2 d^2 e^2 f - 3 (b^2 d^2 f^3 x^2 + 2 b^2 d^2 e f^2 x + b^2 d^2 e^2 f) \cosh(d x + c)^2) \sinh(d x + c)^2 + 4 ((b^2 d^2 f^3 x^2 + 2 b^2 d^2 e f^2 x + b^2 d^2 e^2 f) \cosh(d x + c)^3 - (b^2 d^2 f^3 x^2 + 2 b^2 d^2 e f^2 x + b^2 d^2 e^2 f) \cosh(d x + c)) \sinh(d x + c)) \text{dilog}((a \cosh(d x + c) + a \sinh(d x + c) + (b \cosh(d x + c) + b \sinh(d x + c)))^2)$

$$\begin{aligned}
& d*x + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2) - b)/b + 1) - 3*(b^2*d^2* \\
& f^3*x^2 + 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f + (b^2*d^2*f^3*x^2 + 2*b^2*d^2* \\
& e*f^2*x + b^2*d^2*e^2*f)*\cosh(d*x + c)^4 + 4*(b^2*d^2*f^3*x^2 + 2*b^2*d^2* \\
& e*f^2*x + b^2*d^2*e^2*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^2*d^2*f^3*x^2 + \\
& 2*b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*\sinh(d*x + c)^4 - 2*(b^2*d^2*f^3*x^2 + 2 \\
& *b^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*\cosh(d*x + c)^2 - 2*(b^2*d^2*f^3*x^2 + 2* \\
& b^2*d^2*e*f^2*x + b^2*d^2*e^2*f - 3*(b^2*d^2*f^3*x^2 + 2*b^2*d^2*e*f^2*x + \\
& b^2*d^2*e^2*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((b^2*d^2*f^3*x^2 + 2*b \\
& ^2*d^2*e*f^2*x + b^2*d^2*e^2*f)*\cosh(d*x + c)^3 - (b^2*d^2*f^3*x^2 + 2*b^2* \\
& d^2*e*f^2*x + b^2*d^2*e^2*f)*\cosh(d*x + c))*\sinh(d*x + c))*\operatorname{dilog}((a*\cosh(d* \\
& x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + \\
& b^2)/b^2) - b)/b + 1) + 3*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*f^2 \\
& + a^2*f^3 + (b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*f^2 + a^2*f^3 + 2* \\
& (b^2*d^2*e*f^2 - a*b*d*f^3)*x)*\cosh(d*x + c)^4 + 4*(b^2*d^2*f^3*x^2 + b^2*d \\
& ^2*e^2*f - 2*a*b*d*e*f^2 + a^2*f^3 + 2*(b^2*d^2*e*f^2 - a*b*d*f^3)*x)*\cosh( \\
& d*x + c)*\sinh(d*x + c)^3 + (b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*f^2 \\
& + a^2*f^3 + 2*(b^2*d^2*e*f^2 - a*b*d*f^3)*x)*\sinh(d*x + c)^4 - 2*(b^2*d^2* \\
& f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*f^2 + a^2*f^3 + 2*(b^2*d^2*e*f^2 - a*b* \\
& d*f^3)*x)*\cosh(d*x + c)^2 - 2*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e* \\
& f^2 + a^2*f^3 - 3*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*f^2 + a^2*f^ \\
& 3 + 2*(b^2*d^2*e*f^2 - a*b*d*f^3)*x)*\cosh(d*x + c)^2 + 2*(b^2*d^2*e*f^2 - a \\
& *b*d*f^3)*x)*\sinh(d*x + c)^2 + 2*(b^2*d^2*e*f^2 - a*b*d*f^3)*x + 4*((b^2*d^ \\
& 2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e*f^2 + a^2*f^3 + 2*(b^2*d^2*e*f^2 - a* \\
& b*d*f^3)*x)*\cosh(d*x + c)^3 - (b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f - 2*a*b*d*e* \\
& f^2 + a^2*f^3 + 2*(b^2*d^2*e*f^2 - a*b*d*f^3)*x)*\cosh(d*x + c))*\sinh(d*x + \\
& c))*\operatorname{dilog}(\cosh(d*x + c) + \sinh(d*x + c)) + 3*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2 \\
& *f + 2*a*b*d*e*f^2 + a^2*f^3 + (b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f + 2*a*b*d*e \\
& *f^2 + a^2*f^3 + 2*(b^2*d^2*e*f^2 + a*b*d*f^3)*x)*\cosh(d*x + c)^4 + 4*(b^2* \\
& d^2*f^3*x^2 + b^2*d^2*e^2*f + 2*a*b*d*e*f^2 + a^2*f^3 + 2*(b^2*d^2*e*f^2 + \\
& a*b*d*f^3)*x)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^2*d^2*f^3*x^2 + b^2*d^2*e^ \\
& 2*f + 2*a*b*d*e*f^2 + a^2*f^3 + 2*(b^2*d^2*e*f^2 + a*b*d*f^3)*x)*\sinh(d*x + \\
& c)^4 - 2*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f + 2*a*b*d*e*f^2 + a^2*f^3 + 2*(b \\
& ^2*d^2*e*f^2 + a*b*d*f^3)*x)*\cosh(d*x + c)^2 - 2*(b^2*d^2*f^3*x^2 + b^2*d^2 \\
& *e^2*f + 2*a*b*d*e*f^2 + a^2*f^3 - 3*(b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f + 2*a \\
& *b*d*e*f^2 + a^2*f^3 + 2*(b^2*d^2*e*f^2 + a*b*d*f^3)*x)*\cosh(d*x + c)^2 + 2 \\
& *(b^2*d^2*e*f^2 + a*b*d*f^3)*x)*\sinh(d*x + c)^2 + 2*(b^2*d^2*e*f^2 + a*b*d* \\
& f^3)*x + 4*((b^2*d^2*f^3*x^2 + b^2*d^2*e^2*f + 2*a*b*d*e*f^2 + a^2*f^3 + 2* \\
& (b^2*d^2*e*f^2 + a*b*d*f^3)*x)*\cosh(d*x + c)^3 - (b^2*d^2*f^3*x^2 + b^2*d^2 \\
& *e^2*f + 2*a*b*d*e*f^2 + a^2*f^3 + 2*(b^2*d^2*e*f^2 + a*b*d*f^3)*x)*\cosh(d* \\
& x + c))*\sinh(d*x + c))*\operatorname{dilog}(-\cosh(d*x + c) - \sinh(d*x + c)) - (b^2*d^3*e^3 \\
& - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3 + (b^2*d^3*e^3 - 3*b \\
& ^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*\cosh(d*x + c)^4 + 4*(b^2* \\
& d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - b^2*c^3*f^3)*\cosh(d*x + c \\
& )*\sinh(d*x + c)^3 + (b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c^2*d*e*f^2 - \\
& b^2*c^3*f^3)*\sinh(d*x + c)^4 - 2*(b^2*d^3*e^3 - 3*b^2*c*d^2*e^2*f + 3*b^2*c
\end{aligned}$$





$$\begin{aligned}
& ^2e^{2f} + 3*(b^2*c^2 + 2*a*b*c + a^2)*d*e*f^2 - (b^2*c^3 + 3*a*b*c^2 + 3*a \\
& ^2*c)*f^3)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) \\
& - 1) + (b^2*d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 \\
& + (b^2*d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 + ( \\
& b^2*c^3 + 3*a*b*c^2 + 3*a^2*c)*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + \\
& 3*(b^2*d^3*e^2*f - 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x)*\cosh(d*x + c)^4 + 4*(b^2 \\
& *d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 + (b^2*c^3 \\
& + 3*a*b*c^2 + 3*a^2*c)*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(b^2* \\
& d^3*e^2*f - 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x)*\cosh(d*x + c)*\sinh(d*x + c)^3 + \\
& (b^2*d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 + (b^ \\
& 2*c^3 + 3*a*b*c^2 + 3*a^2*c)*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3* \\
& (b^2*d^3*e^2*f - 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x)*\sinh(d*x + c)^4 + (b^2*c^3 \\
& + 3*a*b*c^2 + 3*a^2*c)*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 - 2*(b^2* \\
& d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 + (b^2*c^3 \\
& + 3*a*b*c^2 + 3*a^2*c)*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(b^2*d \\
& ^3*e^2*f - 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x)*\cosh(d*x + c)^2 - 2*(b^2*d^3*f^3 \\
& *x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 + (b^2*c^3 + 3*a*b \\
& *c^2 + 3*a^2*c)*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 - 3*(b^2*d^3*f^3* \\
& x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b*c)*d*e*f^2 + (b^2*c^3 + 3*a*b \\
& c^2 + 3*a^2*c)*f^3 + 3*(b^2*d^3*e*f^2 - a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f \\
& - 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x)*\cosh(d*x + c)^2 + 3*(b^2*d^3*e^2*f - 2*a \\
& *b*d^2*e*f^2 + a^2*d*f^3)*x)*\sinh(d*x + c)^2 + 3*(b^2*d^3*e^2*f - 2*a*b*d^2 \\
& *e*f^2 + a^2*d*f^3)*x + 4*((b^2*d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^ \\
& 2 + 2*a*b*c)*d*e*f^2 + (b^2*c^3 + 3*a*b*c^2 + 3*a^2*c)*f^3 + 3*(b^2*d^3*e*f \\
& ^2 - a*b*d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f - 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x)* \\
& \cosh(d*x + c)^3 - (b^2*d^3*f^3*x^3 + 3*b^2*c*d^2*e^2*f - 3*(b^2*c^2 + 2*a*b \\
& *c)*d*e*f^2 + (b^2*c^3 + 3*a*b*c^2 + 3*a^2*c)*f^3 + 3*(b^2*d^3*e*f^2 - a*b \\
& d^2*f^3)*x^2 + 3*(b^2*d^3*e^2*f - 2*a*b*d^2*e*f^2 + a^2*d*f^3)*x)*\cosh(d*x \\
& + c))*\sinh(d*x + c))*\log(-\cosh(d*x + c) - \sinh(d*x + c) + 1) - 6*(b^2*f^3*c \\
& osh(d*x + c)^4 + 4*b^2*f^3*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^2*f^3*\sinh(d*x \\
& + c)^4 - 2*b^2*f^3*\cosh(d*x + c)^2 + b^2*f^3 + 2*(3*b^2*f^3*\cosh(d*x + c)^ \\
& 2 - b^2*f^3)*\sinh(d*x + c)^2 + 4*(b^2*f^3*\cosh(d*x + c)^3 - b^2*f^3*\cosh(d* \\
& x + c))*\sinh(d*x + c))*\text{polylog}(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*c \\
& osh(d*x + c) + b*\sinh(d*x + c))*\text{sqrt}((a^2 + b^2)/b^2))/b) - 6*(b^2*f^3*\cosh \\
& (d*x + c)^4 + 4*b^2*f^3*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^2*f^3*\sinh(d*x + \\
& c)^4 - 2*b^2*f^3*\cosh(d*x + c)^2 + b^2*f^3 + 2*(3*b^2*f^3*\cosh(d*x + c)^2 - \\
& b^2*f^3)*\sinh(d*x + c)^2 + 4*(b^2*f^3*\cosh(d*x + c)^3 - b^2*f^3*\cosh(d*x + \\
& c))*\sinh(d*x + c))*\text{polylog}(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*cosh \\
& (d*x + c) + b*\sinh(d*x + c))*\text{sqrt}((a^2 + b^2)/b^2))/b) + 6*(b^2*f^3*\cosh(d* \\
& x + c)^4 + 4*b^2*f^3*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^2*f^3*\sinh(d*x + c)^ \\
& 4 - 2*b^2*f^3*\cosh(d*x + c)^2 + b^2*f^3 + 2*(3*b^2*f^3*\cosh(d*x + c)^2 - b^ \\
& 2*f^3)*\sinh(d*x + c)^2 + 4*(b^2*f^3*\cosh(d*x + c)^3 - b^2*f^3*\cosh(d*x + c) \\
& )*\sinh(d*x + c))*\text{polylog}(4, \cosh(d*x + c) + \sinh(d*x + c)) + 6*(b^2*f^3*\cos \\
& h(d*x + c)^4 + 4*b^2*f^3*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^2*f^3*\sinh(d*x + \\
& c)^4 - 2*b^2*f^3*\cosh(d*x + c)^2 + b^2*f^3 + 2*(3*b^2*f^3*\cosh(d*x + c)^2
\end{aligned}$$

$$\begin{aligned}
& - b^2 f^3 \sinh(dx + c)^2 + 4(b^2 f^3 \cosh(dx + c)^3 - b^2 f^3 \cosh(dx + c)) \sinh(dx + c) \operatorname{polylog}(4, -\cosh(dx + c) - \sinh(dx + c)) + 6(b^2 d f^3 x + b^2 d e f^2 + (b^2 d f^3 x + b^2 d e f^2) \cosh(dx + c)^4 + 4(b^2 d f^3 x + b^2 d e f^2) \cosh(dx + c) \sinh(dx + c)^3 + (b^2 d f^3 x + b^2 d e f^2) \sinh(dx + c)^4 - 2(b^2 d f^3 x + b^2 d e f^2) \cosh(dx + c)^2 - 2(b^2 d f^3 x + b^2 d e f^2 - 3(b^2 d f^3 x + b^2 d e f^2) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4((b^2 d f^3 x + b^2 d e f^2) \cosh(dx + c)^3 - (b^2 d f^3 x + b^2 d e f^2) \cosh(dx + c)) \sinh(dx + c)) \operatorname{polylog}(3, (a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2})/b) + 6(b^2 d f^3 x + b^2 d e f^2 + (b^2 d f^3 x + b^2 d e f^2) \cosh(dx + c)^4 + 4(b^2 d f^3 x + b^2 d e f^2) \cosh(dx + c) \sinh(dx + c)^3 + (b^2 d f^3 x + b^2 d e f^2) \sinh(dx + c)^4 - 2(b^2 d f^3 x + b^2 d e f^2) \cosh(dx + c)^2 - 2(b^2 d f^3 x + b^2 d e f^2 - 3(b^2 d f^3 x + b^2 d e f^2) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4((b^2 d f^3 x + b^2 d e f^2) \cosh(dx + c)^3 - (b^2 d f^3 x + b^2 d e f^2) \cosh(dx + c)) \sinh(dx + c)) \operatorname{polylog}(3, (a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2})/b) - 6(b^2 d f^3 x + b^2 d e f^2 - a b f^3 + (b^2 d f^3 x + b^2 d e f^2 - a b f^3) \cosh(dx + c)^4 + 4(b^2 d f^3 x + b^2 d e f^2 - a b f^3) \cosh(dx + c) \sinh(dx + c)^3 + (b^2 d f^3 x + b^2 d e f^2 - a b f^3) \sinh(dx + c)^4 - 2(b^2 d f^3 x + b^2 d e f^2 - a b f^3) \cosh(dx + c)^2 - 2(b^2 d f^3 x + b^2 d e f^2 - a b f^3 - 3(b^2 d f^3 x + b^2 d e f^2 - a b f^3) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4((b^2 d f^3 x + b^2 d e f^2 - a b f^3) \cosh(dx + c)^3 - (b^2 d f^3 x + b^2 d e f^2 - a b f^3) \cosh(dx + c)) \sinh(dx + c)) \operatorname{polylog}(3, \cosh(dx + c) + \sinh(dx + c)) - 6(b^2 d f^3 x + b^2 d e f^2 + a b f^3 + (b^2 d f^3 x + b^2 d e f^2 + a b f^3) \cosh(dx + c)^4 + 4(b^2 d f^3 x + b^2 d e f^2 + a b f^3) \cosh(dx + c) \sinh(dx + c)^3 + (b^2 d f^3 x + b^2 d e f^2 + a b f^3) \sinh(dx + c)^4 - 2(b^2 d f^3 x + b^2 d e f^2 + a b f^3) \cosh(dx + c)^2 - 2(b^2 d f^3 x + b^2 d e f^2 + a b f^3 - 3(b^2 d f^3 x + b^2 d e f^2 + a b f^3) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4((b^2 d f^3 x + b^2 d e f^2 + a b f^3) \cosh(dx + c)^3 - (b^2 d f^3 x + b^2 d e f^2 + a b f^3) \cosh(dx + c)) \sinh(dx + c)) \operatorname{polylog}(3, -\cosh(dx + c) - \sinh(dx + c)) - 2(a^3 d^3 f^3 x^3 + 3 a^3 d^3 e f^2 x^2 + 3 a^3 d^3 e^2 f x + a^3 d^3 e^3 + 6(a^2 d^2 f^3 x^2 + 2 a^2 d^2 e f^2 x + 2 a^2 c d e f^2 - a^2 c^2 f^3) \cosh(dx + c)^3 - 3(a^3 d^3 f^3 x^3 + 3 a^3 d^3 e f^2 x^2 + 3 a^3 d^3 e^2 f x + a^3 d^3 e^3) \cosh(dx + c)^2 + (2 a^2 d^3 f^3 x^3 + 2 a^2 d^3 e^3 + 3 a^2 d^2 e^2 f - 12 a^2 c d e f^2 + 6 a^2 c^2 f^3 + 3(2 a^2 d^3 e f^2 - a^2 d^2 f^3) x^2 + 6(a^2 d^3 e^2 f - a^2 d^2 e f^2) x) \cosh(dx + c)) \sinh(dx + c)) / (a^3 d^4 \cosh(dx + c)^4 + 4 a^3 d^4 \cosh(dx + c) \sinh(dx + c)^3 + a^3 d^4 \sinh(dx + c)^4 - 2 a^3 d^4 \cosh(dx + c)^2 + a^3 d^4 + 2(3 a^3 d^4 \cosh(dx + c)^2 - a^3 d^4) \sinh(dx + c)^2 + 4(a^3 d^4 \cosh(dx + c)^3 - a^3 d^4 \cosh(dx + c)) \sinh(dx + c))
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*coth(d*x+c)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.477 \quad \int \frac{(e+fx)^2 \coth(c+dx) \operatorname{csch}^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=502

$$-\frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3 d^2} - \frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3 d^2} + \frac{b^2 f(e+fx) \operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{a^3 d^2} + \frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{a^3 d^2}$$

[Out] (4\*b\*f\*(e + f\*x)\*ArcTanh[E^(c + d\*x)]/(a^2\*d^2) - (f\*(e + f\*x)\*Coth[c + d\*x])/(a\*d^2) + (b\*(e + f\*x)^2\*Csch[c + d\*x])/(a^2\*d) - ((e + f\*x)^2\*Csch[c + d\*x]^2)/(2\*a\*d) - (b^2\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^3\*d) - (b^2\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^3\*d) + (b^2\*(e + f\*x)^2\*Log[1 - E^(2\*(c + d\*x))])/(a^3\*d) + (f^2\*Log[Sinh[c + d\*x]])/(a\*d^3) + (2\*b\*f^2\*PolyLog[2, -E^(c + d\*x)]/(a^2\*d^3) - (2\*b\*f^2\*PolyLog[2, E^(c + d\*x)]/(a^2\*d^3) - (2\*b^2\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^3\*d^2) - (2\*b^2\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^3\*d^2) + (b^2\*f\*(e + f\*x)\*PolyLog[2, E^(2\*(c + d\*x))])/(a^3\*d^2) + (2\*b^2\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^3\*d^3) + (2\*b^2\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^3\*d^3) - (b^2\*f^2\*PolyLog[3, E^(2\*(c + d\*x))])/(2\*a^3\*d^3)

**Rubi [A]** time = 1.00402, antiderivative size = 502, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 14, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {5587, 5452, 4184, 3475, 4182, 2279, 2391, 5569, 3716, 2190, 2531, 2282, 6589, 5561}

$$-\frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3 d^2} - \frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3 d^2} + \frac{b^2 f(e+fx) \operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{a^3 d^2} + \frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{a^3 d^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Coth[c + d\*x]\*Csch[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (4\*b\*f\*(e + f\*x)\*ArcTanh[E^(c + d\*x)]/(a^2\*d^2) - (f\*(e + f\*x)\*Coth[c + d\*x])/(a\*d^2) + (b\*(e + f\*x)^2\*Csch[c + d\*x])/(a^2\*d) - ((e + f\*x)^2\*Csch[c + d\*x]^2)/(2\*a\*d) - (b^2\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^3\*d) - (b^2\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^3\*d) + (b^2\*(e + f\*x)^2\*Log[1 - E^(2\*(c + d\*x))])/(a^3\*d) + (f^2\*Log[Sinh[c + d\*x]])/(a\*d^3) + (2\*b\*f^2\*PolyLog[2, -E^(c + d\*x)]/(a^2\*d^3) - (2\*b\*f^2\*PolyLog[2, E^(c + d\*x)]/(a^2\*d^3) - (2\*b^2\*f\*(e + f\*x)\*PolyLo



```
g[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a^3*d^2) - (2*b^2*f*(e + f
*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a^3*d^2) + (b^2*
f*(e + f*x)*PolyLog[2, E^(2*(c + d*x))]/(a^3*d^2) + (2*b^2*f^2*PolyLog[3,
-((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a^3*d^3) + (2*b^2*f^2*PolyLog[3,
-((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a^3*d^3) - (b^2*f^2*PolyLog[3,
E^(2*(c + d*x))]/(2*a^3*d^3)
```

### Rule 5587

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
(f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Csch[c + d*x]^(p - 1)*Coth[c + d*x]^n)/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
IGtQ[p, 0]
```

### Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

### Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/ (f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 5569

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[(e + f*x)^m*Coth[c
+ d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]
```

### Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
```

```

onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

### Rule 6589

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]

```

### Rule 5561

```

Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \coth(c+dx) \operatorname{csch}^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \coth(c+dx) \operatorname{csch}^2(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^2 \coth(c+dx) \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= -\frac{(e+fx)^2 \operatorname{csch}^2(c+dx)}{2ad} - \frac{b \int (e+fx)^2 \coth(c+dx) \operatorname{csch}(c+dx) dx}{a^2} + \frac{b^2}{a^2} \\
&= -\frac{f(e+fx) \coth(c+dx)}{ad^2} + \frac{b(e+fx)^2 \operatorname{csch}(c+dx)}{a^2 d} - \frac{(e+fx)^2 \operatorname{csch}^2(c+dx)}{2ad} \\
&= \frac{4bf(e+fx) \tanh^{-1}(e^{c+dx})}{a^2 d^2} - \frac{f(e+fx) \coth(c+dx)}{ad^2} + \frac{b(e+fx)^2 \operatorname{csch}(c+dx)}{a^2 d} \\
&= \frac{4bf(e+fx) \tanh^{-1}(e^{c+dx})}{a^2 d^2} - \frac{f(e+fx) \coth(c+dx)}{ad^2} + \frac{b(e+fx)^2 \operatorname{csch}(c+dx)}{a^2 d} \\
&= \frac{4bf(e+fx) \tanh^{-1}(e^{c+dx})}{a^2 d^2} - \frac{f(e+fx) \coth(c+dx)}{ad^2} + \frac{b(e+fx)^2 \operatorname{csch}(c+dx)}{a^2 d} \\
&= \frac{4bf(e+fx) \tanh^{-1}(e^{c+dx})}{a^2 d^2} - \frac{f(e+fx) \coth(c+dx)}{ad^2} + \frac{b(e+fx)^2 \operatorname{csch}(c+dx)}{a^2 d} \\
&= \frac{4bf(e+fx) \tanh^{-1}(e^{c+dx})}{a^2 d^2} - \frac{f(e+fx) \coth(c+dx)}{ad^2} + \frac{b(e+fx)^2 \operatorname{csch}(c+dx)}{a^2 d}
\end{aligned}$$

**Mathematica [B]** time = 30.4058, size = 1816, normalized size = 3.62

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*(e + f*x)^2*Csch[c])/(a^2*d) + ((-e^2 - 2*e*f*x - f^2*x^2)*Csch[c/2 + (d*x)/2]^2)/(8*a*d) - (12*d*E^(2*c)*(b^2*d^2*e^2 + a^2*f^2)*x - 12*d*(-1 + E^(2*c))*(b^2*d^2*e^2 + a^2*f^2)*x + 12*b^2*d^3*e*f*x^2 + 4*b^2*d^3*f^2*x^3 - 24*a*b*d*e*(-1 + E^(2*c))*f*ArcTanh[E^(c + d*x)] + 6*b^2*d^2*e^2*(-1 + E^(2*c))*(2*d*x - Log[1 - E^(2*(c + d*x))]) + 6*a^2*(-1 + E^(2*c))*f^2*(2*d*x - Log[1 - E^(2*(c + d*x))]) + 12*a*b*(-1 + E^(2*c))*f^2*(d*x*(Log[1 - E^(c + d*x)] - Log[1 + E^(c + d*x)]) - PolyLog[2, -E^(c + d*x)] + PolyLog[2, E^(c + d*x)]) + 6*b^2*d*e*(-1 + E^(2*c))*f*(2*d*x*(d*x - Log[1 - E^(2*(c + d*x))]) - PolyLog[2, E^(2*(c + d*x))]) + b^2*(-1 + E^(2*c))*f^2*(2*d^2*x^2*(2*
```

```

d*x - 3*Log[1 - E^(2*(c + d*x))] - 6*d*x*PolyLog[2, E^(2*(c + d*x))] + 3*P
olyLog[3, E^(2*(c + d*x))]/(6*a^3*d^3*(-1 + E^(2*c))) + (b^2*(6*e^2*E^(2*
c)*x + 6*e*E^(2*c)*f*x^2 + 2*E^(2*c)*f^2*x^3 + (6*a*Sqrt[a^2 + b^2]*e^2*Arc
Tan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2]])/(Sqrt[-(a^2 + b^2)^2]*d) + (6*a*
Sqrt[-(a^2 + b^2)^2]*e^2*E^(2*c)*ArcTan[(a + b*E^(c + d*x))/Sqrt[-a^2 - b^2
]])/((a^2 + b^2)^(3/2)*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^
(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^(3/2)*d) + (6*a*Sqrt[-(a^2 + b^2
)^2]*e^2*E^(2*c)*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]])/((-a^2 - b^2
)^(3/2)*d) + (3*e^2*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))]/d - (3
*e^2*E^(2*c)*Log[2*a*E^(c + d*x) + b*(-1 + E^(2*(c + d*x)))]/d + (6*e*f*x*
Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])]/d - (6*e*E^
(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])]/d
+ (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)])
])/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^
2)*E^(2*c)])]/d + (6*e*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 +
b^2)*E^(2*c)])]/d - (6*e*E^(2*c)*f*x*Log[1 + (b*E^(2*c + d*x))/(a*E^c + Sq
rt[(a^2 + b^2)*E^(2*c)])]/d + (3*f^2*x^2*Log[1 + (b*E^(2*c + d*x))/(a*E^c
+ Sqrt[(a^2 + b^2)*E^(2*c)])]/d - (3*E^(2*c)*f^2*x^2*Log[1 + (b*E^(2*c + d
*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)])]/d - (6*(-1 + E^(2*c))*f*(e + f*x
)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))]/d^2
- (6*(-1 + E^(2*c))*f*(e + f*x)*PolyLog[2, -((b*E^(2*c + d*x))/(a*E^c + Sq
rt[(a^2 + b^2)*E^(2*c)]))]/d^2 - (6*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*
E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))]/d^3 + (6*E^(2*c)*f^2*PolyLog[3, -((b*E^
(2*c + d*x))/(a*E^c - Sqrt[(a^2 + b^2)*E^(2*c)]))]/d^3 - (6*f^2*PolyLog[3,
-((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]/d^3 + (6*E^(2*c
)*f^2*PolyLog[3, -((b*E^(2*c + d*x))/(a*E^c + Sqrt[(a^2 + b^2)*E^(2*c)]))]
/d^3))/(3*a^3*(-1 + E^(2*c))) + ((e^2 + 2*e*f*x + f^2*x^2)*Sech[c/2 + (d*x
)/2]^2)/(8*a*d) + (Sech[c/2]*Sech[c/2 + (d*x)/2]*(-(b*d*e^2*Sinh[(d*x)/2]) -
a*e*f*Sinh[(d*x)/2] - 2*b*d*e*f*x*Sinh[(d*x)/2] - a*f^2*x*Sinh[(d*x)/2] -
b*d*f^2*x^2*Sinh[(d*x)/2]))/(2*a^2*d^2) + (Csch[c/2]*Csch[c/2 + (d*x)/2]*(-
(b*d*e^2*Sinh[(d*x)/2]) + a*e*f*Sinh[(d*x)/2] - 2*b*d*e*f*x*Sinh[(d*x)/2] +
a*f^2*x*Sinh[(d*x)/2] - b*d*f^2*x^2*Sinh[(d*x)/2]))/(2*a^2*d^2)

```

---

**Maple [F]** time = 0.615, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 \coth(dx + c) (\operatorname{csch}(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*coth(d\*x+c)\*csch(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out]  $\text{int}((f*x+e)^2*\text{coth}(d*x+c)*\text{csch}(d*x+c)^2/(a+b*\sinh(d*x+c)),x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x+e)^2*\text{coth}(d*x+c)*\text{csch}(d*x+c)^2/(a+b*\sinh(d*x+c)),x, \text{algorithm}="maxima")$

[Out] 
$$\begin{aligned} & -e^{2*(2*(b*e^{(-d*x - c)} - a*e^{(-2*d*x - 2*c)} - b*e^{(-3*d*x - 3*c)})/((2*a^2* \\ & e^{(-2*d*x - 2*c)} - a^2*e^{(-4*d*x - 4*c)} - a^2)*d) + b^2*\log(-2*a*e^{(-d*x - \\ & c) + b*e^{(-2*d*x - 2*c)} - b)/(a^3*d) - b^2*\log(e^{(-d*x - c)} + 1)/(a^3*d) - \\ & b^2*\log(e^{(-d*x - c)} - 1)/(a^3*d)) + 2*(a*f^2*x + a*e*f + (b*d*f^2*x^2*e^{(3 \\ & *c) + 2*b*d*e*f*x*e^{(3*c)})}*e^{(3*d*x)} - (a*d*f^2*x^2*e^{(2*c)} + a*e*f*e^{(2*c)} \\ & + (2*d*e*f + f^2)*a*x*e^{(2*c)})*e^{(2*d*x)} - (b*d*f^2*x^2*e^c + 2*b*d*e*f*x* \\ & e^c)*e^{(d*x)})/(a^2*d^2*e^{(4*d*x + 4*c)} - 2*a^2*d^2*e^{(2*d*x + 2*c)} + a^2*d^2 \\ & + (d^2*x^2*\log(e^{(d*x + c)} + 1) + 2*d*x*\text{dilog}(-e^{(d*x + c)}) - 2*\text{polylog}( \\ & 3, -e^{(d*x + c)}))*b^2*f^2/(a^3*d^3) + (d^2*x^2*\log(-e^{(d*x + c)} + 1) + 2*d* \\ & x*\text{dilog}(e^{(d*x + c)}) - 2*\text{polylog}(3, e^{(d*x + c)}))*b^2*f^2/(a^3*d^3) - (2*b* \\ & d*e*f + a*f^2)*x/(a^2*d^2) + (2*b*d*e*f - a*f^2)*x/(a^2*d^2) + (2*b*d*e*f + \\ & a*f^2)*\log(e^{(d*x + c)} + 1)/(a^2*d^3) - (2*b*d*e*f - a*f^2)*\log(e^{(d*x + c)} \\ & - 1)/(a^2*d^3) + 2*(b^2*d*e*f + a*b*f^2)*(d*x*\log(e^{(d*x + c)} + 1) + \text{dilo} \\ & \text{g}(-e^{(d*x + c)}))/(a^3*d^3) + 2*(b^2*d*e*f - a*b*f^2)*(d*x*\log(-e^{(d*x + c)} \\ & + 1) + \text{dilog}(e^{(d*x + c)}))/(a^3*d^3) - 1/3*(b^2*d^3*f^2*x^3 + 3*(b^2*d*e*f \\ & + a*b*f^2)*d^2*x^2)/(a^3*d^3) - 1/3*(b^2*d^3*f^2*x^3 + 3*(b^2*d*e*f - a*b*f^ \\ & ^2)*d^2*x^2)/(a^3*d^3) + \text{integrate}(-2*(b^3*f^2*x^2 + 2*b^3*e*f*x - (a*b^2*f \\ & ^2*x^2*e^c + 2*a*b^2*e*f*x*e^c)*e^{(d*x)})/(a^3*b*e^{(2*d*x + 2*c)} + 2*a^4*e^{( \\ & d*x + c)} - a^3*b), x) \end{aligned}$$

**Fricas [C]** time = 3.81377, size = 14549, normalized size = 28.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((f*x+e)^2*\text{coth}(d*x+c)*\text{csch}(d*x+c)^2/(a+b*\sinh(d*x+c)),x, \text{algorithm}="fricas")$

[Out]  $(2a^2d^2ef - 2a^2c^2f^2 - 2(a^2d^2f^2x + a^2c^2f^2)\cosh(dx + c))^4 - 2(a^2d^2f^2x + a^2c^2f^2)\sinh(dx + c)^4 + 2(a^2b^2d^2f^2x^2 + 2a^2b^2d^2e^2efx + a^2b^2d^2e^2)\cosh(dx + c)^3 + 2(a^2b^2d^2f^2x^2 + 2a^2b^2d^2e^2efx + a^2b^2d^2e^2 - 4(a^2d^2f^2x + a^2c^2f^2)\cosh(dx + c))\sinh(dx + c)^3 - 2(a^2d^2f^2x^2 + a^2d^2e^2 + a^2d^2ef - 2a^2c^2f^2 + (2a^2d^2e^2ef - a^2d^2f^2)x)\cosh(dx + c)^2 - 2(a^2d^2f^2x^2 + a^2d^2e^2 + a^2d^2ef - 2a^2c^2f^2 + 6(a^2d^2f^2x + a^2c^2f^2)\cosh(dx + c)^2 + (2a^2d^2e^2ef - a^2d^2f^2)x - 3(a^2b^2d^2f^2x^2 + 2a^2b^2d^2e^2efx + a^2b^2d^2e^2)\cosh(dx + c))\sinh(dx + c)^2 - 2(a^2b^2d^2f^2x^2 + 2a^2b^2d^2e^2efx + a^2b^2d^2e^2)\cosh(dx + c) - 2(b^2d^2f^2x + b^2d^2ef + (b^2d^2f^2x + b^2d^2ef)\cosh(dx + c))^4 + 4(b^2d^2f^2x + b^2d^2ef)\cosh(dx + c)\sinh(dx + c)^3 + (b^2d^2f^2x + b^2d^2ef)\sinh(dx + c)^4 - 2(b^2d^2f^2x + b^2d^2ef)\cosh(dx + c)^2 - 2(b^2d^2f^2x + b^2d^2ef - 3(b^2d^2f^2x + b^2d^2ef)\cosh(dx + c)^2)\sinh(dx + c)^2 + 4((b^2d^2f^2x + b^2d^2ef)\cosh(dx + c))^3 - (b^2d^2f^2x + b^2d^2ef)\cosh(dx + c)\sinh(dx + c)\operatorname{dilog}((a\cosh(dx + c) + a\sinh(dx + c) + (b\cosh(dx + c) + b\sinh(dx + c))\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 2(b^2d^2f^2x + b^2d^2ef + (b^2d^2f^2x + b^2d^2ef)\cosh(dx + c))^4 + 4(b^2d^2f^2x + b^2d^2ef)\cosh(dx + c)\sinh(dx + c)^3 + (b^2d^2f^2x + b^2d^2ef)\sinh(dx + c)^4 - 2(b^2d^2f^2x + b^2d^2ef)\cosh(dx + c)^2 - 2(b^2d^2f^2x + b^2d^2ef - 3(b^2d^2f^2x + b^2d^2ef)\cosh(dx + c)^2)\sinh(dx + c)^2 + 4((b^2d^2f^2x + b^2d^2ef)\cosh(dx + c))^3 - (b^2d^2f^2x + b^2d^2ef)\cosh(dx + c)\sinh(dx + c)\operatorname{dilog}((a\cosh(dx + c) + a\sinh(dx + c) - (b\cosh(dx + c) + b\sinh(dx + c))\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 2(b^2d^2f^2x + b^2d^2ef + (b^2d^2f^2x + b^2d^2ef - a^2b^2f^2)\cosh(dx + c))^4 + 4(b^2d^2f^2x + b^2d^2ef - a^2b^2f^2)\cosh(dx + c)\sinh(dx + c)^3 + (b^2d^2f^2x + b^2d^2ef - a^2b^2f^2)\sinh(dx + c)^4 - a^2b^2f^2 - 2(b^2d^2f^2x + b^2d^2ef - a^2b^2f^2)\cosh(dx + c)^2 - 2(b^2d^2f^2x + b^2d^2ef - a^2b^2f^2 - 3(b^2d^2f^2x + b^2d^2ef - a^2b^2f^2)\cosh(dx + c)^2)\sinh(dx + c)^2 + 4((b^2d^2f^2x + b^2d^2ef - a^2b^2f^2)\cosh(dx + c))^3 - (b^2d^2f^2x + b^2d^2ef - a^2b^2f^2)\cosh(dx + c)\sinh(dx + c)\operatorname{dilog}(\cosh(dx + c) + \sinh(dx + c)) + 2(b^2d^2f^2x + b^2d^2ef + (b^2d^2f^2x + b^2d^2ef + a^2b^2f^2)\cosh(dx + c))^4 + 4(b^2d^2f^2x + b^2d^2ef + a^2b^2f^2)\cosh(dx + c)\sinh(dx + c)^3 + (b^2d^2f^2x + b^2d^2ef + a^2b^2f^2)\sinh(dx + c)^4 + a^2b^2f^2 - 2(b^2d^2f^2x + b^2d^2ef + a^2b^2f^2)\cosh(dx + c)^2 - 2(b^2d^2f^2x + b^2d^2ef + a^2b^2f^2 - 3(b^2d^2f^2x + b^2d^2ef + a^2b^2f^2)\cosh(dx + c)^2)\sinh(dx + c)^2 + 4((b^2d^2f^2x + b^2d^2ef + a^2b^2f^2)\cosh(dx + c))^3 - (b^2d^2f^2x + b^2d^2ef + a^2b^2f^2)\cosh(dx + c)\sinh(dx + c)\operatorname{dilog}(-\cosh(dx + c) - \sinh(dx + c)) - (b^2d^2e^2 - 2b^2c^2d^2ef + b^2c^2f^2 + (b^2d^2e^2 - 2b^2c^2d^2ef + b^2c^2f^2)\cosh(dx + c))^4 + 4(b^2d^2e^2 - 2b^2c^2d^2ef + b^2c^2f^2)\cosh(dx + c)\sinh(dx + c)^3 + (b^2d^2e^2 - 2b^2c^2d^2ef + b^2c^2f^2)\sinh(dx + c)^4 - 2(b^2d^2e^2 - 2b^2c^2d^2ef + b^2c^2f^2)\cosh(dx + c)^2 - 2(b^2d^2e^2 - 2b^2c^2d^2ef + b^2c^2f^2 - 3(b^2d^2e^2 - 2b^2c^2d^2ef + b^2c^2f^2)\cosh(dx + c)^2)\sinh(dx + c)^2 + 4((b^2d^2e^2 - 2b^2c^2d^2ef + b^2c^2f^2)\cosh(dx + c))^3 - (b^2d^2e^2 - 2b^2c^2d^2ef + b^2c^2f^2)\cosh(dx + c)\sinh(dx + c)^2$

$$\begin{aligned}
& 2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2) * \cosh(d*x + c) * \sinh(d*x + c) * \log( \\
& 2*b * \cosh(d*x + c) + 2*b * \sinh(d*x + c) + 2*b * \sqrt{(a^2 + b^2)/b^2} + 2*a) - \\
& (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2 + (b^2*d^2*e^2 - 2*b^2*c*d*e*f + \\
& b^2*c^2*f^2) * \cosh(d*x + c)^4 + 4*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2 \\
& ) * \cosh(d*x + c) * \sinh(d*x + c)^3 + (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2 \\
& ) * \sinh(d*x + c)^4 - 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2) * \cosh(d* \\
& x + c)^2 - 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2 - 3*(b^2*d^2*e^2 - \\
& 2*b^2*c*d*e*f + b^2*c^2*f^2) * \cosh(d*x + c)^2) * \sinh(d*x + c)^2 + 4*((b^2*d^2 \\
& *e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2) * \cosh(d*x + c)^3 - (b^2*d^2*e^2 - 2*b^2* \\
& c*d*e*f + b^2*c^2*f^2) * \cosh(d*x + c)) * \sinh(d*x + c)) * \log(2*b * \cosh(d*x + c) \\
& + 2*b * \sinh(d*x + c) - 2*b * \sqrt{(a^2 + b^2)/b^2} + 2*a) - (b^2*d^2*f^2*x^2 + \\
& 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2 + (b^2*d^2*f^2*x^2 + 2*b^2*d^2 \\
& ^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2) * \cosh(d*x + c)^4 + 4*(b^2*d^2*f^2*x^ \\
& 2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2) * \cosh(d*x + c) * \sinh(d*x + \\
& c)^3 + (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2) * s \\
& \sinh(d*x + c)^4 - 2*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2 \\
& *c^2*f^2) * \cosh(d*x + c)^2 - 2*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c* \\
& d*e*f - b^2*c^2*f^2 - 3*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f \\
& - b^2*c^2*f^2) * \cosh(d*x + c)^2) * \sinh(d*x + c)^2 + 4*((b^2*d^2*f^2*x^2 + 2*b \\
& ^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2) * \cosh(d*x + c)^3 - (b^2*d^2*f^2* \\
& x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2) * \cosh(d*x + c)) * \sinh(d* \\
& x + c)) * \log(-(a * \cosh(d*x + c) + a * \sinh(d*x + c) + (b * \cosh(d*x + c) + b * \sinh \\
& (d*x + c)) * \sqrt{(a^2 + b^2)/b^2} - b)/b) - (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f \\
& *x + 2*b^2*c*d*e*f - b^2*c^2*f^2 + (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b \\
& ^2*c*d*e*f - b^2*c^2*f^2) * \cosh(d*x + c)^4 + 4*(b^2*d^2*f^2*x^2 + 2*b^2*d^2* \\
& e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2) * \cosh(d*x + c) * \sinh(d*x + c)^3 + (b^2*d \\
& ^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2) * \sinh(d*x + c)^4 \\
& - 2*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2) * \cosh \\
& (d*x + c)^2 - 2*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^ \\
& 2*f^2 - 3*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2) \\
& ) * \cosh(d*x + c)^2) * \sinh(d*x + c)^2 + 4*((b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + \\
& 2*b^2*c*d*e*f - b^2*c^2*f^2) * \cosh(d*x + c)^3 - (b^2*d^2*f^2*x^2 + 2*b^2*d^ \\
& 2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2) * \cosh(d*x + c)) * \sinh(d*x + c)) * \log(-( \\
& a * \cosh(d*x + c) + a * \sinh(d*x + c) - (b * \cosh(d*x + c) + b * \sinh(d*x + c)) * \sqrt{ \\
& (a^2 + b^2)/b^2} - b)/b) + (b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d*e*f + \\
& (b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 + 2*(b^2*d^2*e*f + \\
& a*b*d*f^2)*x) * \cosh(d*x + c)^4 + 4*(b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d* \\
& e*f + a^2*f^2 + 2*(b^2*d^2*e*f + a*b*d*f^2)*x) * \cosh(d*x + c) * \sinh(d*x + c)^ \\
& 3 + (b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 + 2*(b^2*d^2*e*f \\
& + a*b*d*f^2)*x) * \sinh(d*x + c)^4 + a^2*f^2 - 2*(b^2*d^2*f^2*x^2 + b^2*d^2*e \\
& ^2 + 2*a*b*d*e*f + a^2*f^2 + 2*(b^2*d^2*e*f + a*b*d*f^2)*x) * \cosh(d*x + c)^2 \\
& - 2*(b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 - 3*(b^2*d^2*f^ \\
& 2*x^2 + b^2*d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 + 2*(b^2*d^2*e*f + a*b*d*f^2)*x \\
& ) * \cosh(d*x + c)^2 + 2*(b^2*d^2*e*f + a*b*d*f^2)*x) * \sinh(d*x + c)^2 + 2*(b^2 \\
& *d^2*e*f + a*b*d*f^2)*x + 4*((b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + 2*a*b*d*e*f +
\end{aligned}$$





```
*cosh(d*x + c)^3 - b^2*f^2*cosh(d*x + c))*sinh(d*x + c))*polylog(3, -cosh(d
*x + c) - sinh(d*x + c)) - 2*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + a*b*d^2*e
^2 + 4*(a^2*d*f^2*x + a^2*c*f^2))*cosh(d*x + c)^3 - 3*(a*b*d^2*f^2*x^2 + 2*a
*b*d^2*e*f*x + a*b*d^2*e^2)*cosh(d*x + c)^2 + 2*(a^2*d^2*f^2*x^2 + a^2*d^2*
e^2 + a^2*d*e*f - 2*a^2*c*f^2 + (2*a^2*d^2*e*f - a^2*d*f^2)*x)*cosh(d*x + c
))*sinh(d*x + c))/(a^3*d^3*cosh(d*x + c)^4 + 4*a^3*d^3*cosh(d*x + c)*sinh(d
*x + c)^3 + a^3*d^3*sinh(d*x + c)^4 - 2*a^3*d^3*cosh(d*x + c)^2 + a^3*d^3 +
2*(3*a^3*d^3*cosh(d*x + c)^2 - a^3*d^3)*sinh(d*x + c)^2 + 4*(a^3*d^3*cosh(
d*x + c)^3 - a^3*d^3*cosh(d*x + c))*sinh(d*x + c))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*coth(d*x+c)*csch(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.478 \quad \int \frac{(e+fx) \coth(c+dx) \operatorname{csch}^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=298

$$\frac{b^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3 d^2} - \frac{b^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3 d^2} + \frac{b^2 f \operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{2a^3 d^2} - \frac{b^2(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{a^3 d}$$

```
[Out] (b*f*ArcTanh[Cosh[c + d*x]])/(a^2*d^2) - (f*Coth[c + d*x])/(2*a*d^2) + (b*(
e + f*x)*Csch[c + d*x])/(a^2*d) - ((e + f*x)*Csch[c + d*x]^2)/(2*a*d) - (b^
2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*d) - (b^2*
(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*d) + (b^2*(e
+ f*x)*Log[1 - E^(2*(c + d*x))])/(a^3*d) - (b^2*f*PolyLog[2, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) - (b^2*f*PolyLog[2, -((b*E^(c + d*x
)))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (b^2*f*PolyLog[2, E^(2*(c + d*x))])
/(2*a^3*d^2)
```

**Rubi [A]** time = 0.570451, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 11, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$ , Rules used = {5587, 5452, 3767, 8, 3770, 5569, 3716, 2190, 2279, 2391, 5561}

$$\frac{b^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3 d^2} - \frac{b^2 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3 d^2} + \frac{b^2 f \operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{2a^3 d^2} - \frac{b^2(e+fx) \log\left(\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1\right)}{a^3 d}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*f*ArcTanh[Cosh[c + d*x]])/(a^2*d^2) - (f*Coth[c + d*x])/(2*a*d^2) + (b*(
e + f*x)*Csch[c + d*x])/(a^2*d) - ((e + f*x)*Csch[c + d*x]^2)/(2*a*d) - (b^
2*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*d) - (b^2*
(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*d) + (b^2*(e
+ f*x)*Log[1 - E^(2*(c + d*x))])/(a^3*d) - (b^2*f*PolyLog[2, -((b*E^(c + d
*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) - (b^2*f*PolyLog[2, -((b*E^(c + d*x
)))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (b^2*f*PolyLog[2, E^(2*(c + d*x))])
/(2*a^3*d^2)
```

**Rule 5587**

```
Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
(f_.)*(x_))^(m_.)]/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := D
```

ist[1/a, Int[(e + f\*x)^m\*Csch[c + d\*x]^p\*Coth[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Csch[c + d\*x]^(p - 1)\*Coth[c + d\*x]^n)/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5452

Int[Coth[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*Csch[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Csch[a + b\*x]^n)/(b\*n), x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Csch[a + b\*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

### Rule 3767

Int[csc[(c\_.) + (d\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d\*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

### Rule 5569

Int[(Coth[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Coth[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Cosh[c + d\*x]\*Coth[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 3716

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)], x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x))/(E^(2\*I\*k\*Pi)\*(1 + E^(2\*(-I\*e) + f\*fz\*x)))/E^(2\*I\*k\*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

### Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*(c_) + (d_)*(x_)))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 5561

```
Int[(Cosh[(c_) + (d_)*(x_)]*((e_) + (f_)*(x_))^(m_))/((a_) + (b_)*Sin
h[(c_) + (d_)*(x_)], x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e + fx) \coth(c + dx) \operatorname{csch}^2(c + dx)}{a + b \sinh(c + dx)} dx &= \frac{\int (e + fx) \coth(c + dx) \operatorname{csch}^2(c + dx) dx}{a} - \frac{b \int \frac{(e + fx) \coth(c + dx) \operatorname{csch}(c + dx)}{a + b \sinh(c + dx)} dx}{a} \\
&= -\frac{(e + fx) \operatorname{csch}^2(c + dx)}{2ad} - \frac{b \int (e + fx) \coth(c + dx) \operatorname{csch}(c + dx) dx}{a^2} + \frac{b^2 \int \frac{(e + fx) \coth(c + dx)}{a + b \sinh(c + dx)} dx}{a^3} \\
&= \frac{b(e + fx) \operatorname{csch}(c + dx)}{a^2 d} - \frac{(e + fx) \operatorname{csch}^2(c + dx)}{2ad} + \frac{b^2 \int (e + fx) \coth(c + dx) dx}{a^3} \\
&= \frac{bf \tanh^{-1}(\cosh(c + dx))}{a^2 d^2} - \frac{f \coth(c + dx)}{2ad^2} + \frac{b(e + fx) \operatorname{csch}(c + dx)}{a^2 d} - \frac{(e + fx) \operatorname{csch}^2(c + dx)}{2ad} \\
&= \frac{bf \tanh^{-1}(\cosh(c + dx))}{a^2 d^2} - \frac{f \coth(c + dx)}{2ad^2} + \frac{b(e + fx) \operatorname{csch}(c + dx)}{a^2 d} - \frac{(e + fx) \operatorname{csch}^2(c + dx)}{2ad} \\
&= \frac{bf \tanh^{-1}(\cosh(c + dx))}{a^2 d^2} - \frac{f \coth(c + dx)}{2ad^2} + \frac{b(e + fx) \operatorname{csch}(c + dx)}{a^2 d} - \frac{(e + fx) \operatorname{csch}^2(c + dx)}{2ad} \\
&= \frac{bf \tanh^{-1}(\cosh(c + dx))}{a^2 d^2} - \frac{f \coth(c + dx)}{2ad^2} + \frac{b(e + fx) \operatorname{csch}(c + dx)}{a^2 d} - \frac{(e + fx) \operatorname{csch}^2(c + dx)}{2ad}
\end{aligned}$$

**Mathematica [A]** time = 6.8722, size = 376, normalized size = 1.26

$$-8b^2 \left( f \operatorname{PolyLog} \left( 2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a} \right) + f \operatorname{PolyLog} \left( 2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} \right) + f(c+dx) \log \left( \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1 \right) + f(c+dx) \log \left( \frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} + 1 \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)*Coth[c + d*x]*Csch[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*(-(a*f) + 2*b*d*(e + f*x))*Coth[(c + d*x)/2] - a^2*d*(e + f*x)*Csch[(c + d*x)/2]^2 + 8*b^2*d*e*Log[Sinh[c + d*x]] - 8*b^2*c*f*Log[Sinh[c + d*x]] - 8*a*b*f*Log[Tanh[(c + d*x)/2]] + 4*b^2*f*((c + d*x)*(c + d*x + 2*Log[1 - E^(-2*(c + d*x))]) - PolyLog[2, E^(-2*(c + d*x))]) - 8*b^2*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]) + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]) + a^2*d*(e + f*x)*Sech[(c + d*x)/2]^2 - 2*a*(a*f + 2*b*d*(e + f*x))*Tanh[(c + d*x)/2])/(8*a^3*d^2)
```

---

**Maple [B]** time = 0.143, size = 649, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x)`

[Out] 
$$\begin{aligned} & -(-2*b*d*f*x*\exp(3*d*x+3*c)+2*a*d*f*x*\exp(2*d*x+2*c)-2*b*d*e*\exp(3*d*x+3*c) \\ & +2*a*d*e*\exp(2*d*x+2*c)+2*b*d*f*x*\exp(d*x+c)+a*f*\exp(2*d*x+2*c)+2*b*d*e*\exp \\ & (d*x+c)-a*f)/d^2/a^2/(\exp(2*d*x+2*c)-1)^2-1/a^3/d^2*b^2*f*c*\ln(\exp(d*x+c)-1) \\ & )+1/a^3*b^2/d^2*f*c*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)+1/a^2/d^2*b*f*\ln \\ & (\exp(d*x+c)+1)-1/a^2/d^2*b*f*\ln(\exp(d*x+c)-1)-1/a^3/d^2*b^2*f*dilog(\exp(d*x+c) \\ & )-1/a^3*b^2/d^2*f*dilog((-b*\exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))) \\ & )-1/a^3*b^2/d^2*f*dilog((b*\exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))) \\ & )+1/a^3/d^2*b^2*f*dilog(\exp(d*x+c)+1)+1/a^3/d*b^2*e*\ln(\exp(d*x+c)-1)- \\ & 1/a^3*b^2/d*e*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)+1/a^3/d*b^2*e*\ln(\exp(d*x+c) \\ & +1)-1/a^3*b^2/d*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))) \\ & )*x-1/a^3*b^2/d^2*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))) \\ & )*c-1/a^3*b^2/d*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))) \\ & )*x-1/a^3*b^2/d^2*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))) \\ & )*c+1/a^3/d*b^2*f*\ln(\exp(d*x+c)+1)*x \end{aligned}$$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\left(4b^2d \int \frac{x}{4(a^3de^{dx+c} + a^3d)} dx - 4b^2d \int \frac{x}{4(a^3de^{dx+c} - a^3d)} dx + ab \left( \frac{dx+c}{a^3d^2} - \frac{\log(e^{dx+c} + 1)}{a^3d^2} \right) - ab \left( \frac{dx+c}{a^3d^2} - \frac{\log(e^{dx+c} - 1)}{a^3d^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)*coth(d*x+c)*csch(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & -(4*b^2*d*\integrate(1/4*x/(a^3*d*e^(d*x + c) + a^3*d), x) - 4*b^2*d*\integrate \\ & (1/4*x/(a^3*d*e^(d*x + c) - a^3*d), x) + a*b*((d*x + c)/(a^3*d^2) - \log(e \\ & ^{(d*x + c) + 1})/(a^3*d^2)) - a*b*((d*x + c)/(a^3*d^2) - \log(e^{(d*x + c) - 1}) \\ & )/(a^3*d^2)) - (2*b*d*x*e^(3*d*x + 3*c) - 2*b*d*x*e^(d*x + c) - (2*a*d*x*e^(2*c) \\ & + a*e^(2*c))*e^(2*d*x) + a)/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) \\ & + a^2*d^2) - 4*\integrate(1/2*(a*b^2*x*e^(d*x + c) - b^3*x)/(a^3*b*e^(2*d*x + 2*c) \\ & + 2*a^4*e^(d*x + c) - a^3*b), x)*f - e*(2*(b*e^(-d*x - \end{aligned}$$

$$c) - a*e^{(-2*d*x - 2*c)} - b*e^{(-3*d*x - 3*c)})/((2*a^2*e^{(-2*d*x - 2*c)} - a^2*e^{(-4*d*x - 4*c)} - a^2)*d) + b^2*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/(a^3*d) - b^2*\log(e^{(-d*x - c)} + 1)/(a^3*d) - b^2*\log(e^{(-d*x - c)} - 1)/(a^3*d))$$

**Fricas [B]** time = 2.60422, size = 7004, normalized size = 23.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*coth(d\*x+c)\*csch(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] (2\*(a\*b\*d\*f\*x + a\*b\*d\*e)\*cosh(d\*x + c)^3 + 2\*(a\*b\*d\*f\*x + a\*b\*d\*e)\*sinh(d\*x + c)^3 + a^2\*f - (2\*a^2\*d\*f\*x + 2\*a^2\*d\*e + a^2\*f)\*cosh(d\*x + c)^2 - (2\*a^2\*d\*f\*x + 2\*a^2\*d\*e + a^2\*f - 6\*(a\*b\*d\*f\*x + a\*b\*d\*e)\*cosh(d\*x + c))\*sinh(d\*x + c)^2 - 2\*(a\*b\*d\*f\*x + a\*b\*d\*e)\*cosh(d\*x + c) - (b^2\*f\*cosh(d\*x + c)^4 + 4\*b^2\*f\*cosh(d\*x + c)\*sinh(d\*x + c)^3 + b^2\*f\*sinh(d\*x + c)^4 - 2\*b^2\*f\*cosh(d\*x + c)^2 + b^2\*f + 2\*(3\*b^2\*f\*cosh(d\*x + c)^2 - b^2\*f)\*sinh(d\*x + c)^2 + 4\*(b^2\*f\*cosh(d\*x + c)^3 - b^2\*f\*cosh(d\*x + c))\*sinh(d\*x + c))\*dilog((a\*cosh(d\*x + c) + a\*sinh(d\*x + c) + (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^2\*f\*cosh(d\*x + c)^4 + 4\*b^2\*f\*cosh(d\*x + c)\*sinh(d\*x + c)^3 + b^2\*f\*sinh(d\*x + c)^4 - 2\*b^2\*f\*cosh(d\*x + c)^2 + b^2\*f + 2\*(3\*b^2\*f\*cosh(d\*x + c)^2 - b^2\*f)\*sinh(d\*x + c)^2 + 4\*(b^2\*f\*cosh(d\*x + c)^3 - b^2\*f\*cosh(d\*x + c))\*sinh(d\*x + c))\*dilog((a\*cosh(d\*x + c) + a\*sinh(d\*x + c) - (b\*cosh(d\*x + c) + b\*sinh(d\*x + c))\*sqrt((a^2 + b^2)/b^2) - b)/b + 1) + (b^2\*f\*cosh(d\*x + c)^4 + 4\*b^2\*f\*cosh(d\*x + c)\*sinh(d\*x + c)^3 + b^2\*f\*sinh(d\*x + c)^4 - 2\*b^2\*f\*cosh(d\*x + c)^2 + b^2\*f + 2\*(3\*b^2\*f\*cosh(d\*x + c)^2 - b^2\*f)\*sinh(d\*x + c)^2 + 4\*(b^2\*f\*cosh(d\*x + c)^3 - b^2\*f\*cosh(d\*x + c))\*sinh(d\*x + c))\*dilog(cosh(d\*x + c) + sinh(d\*x + c)) + (b^2\*f\*cosh(d\*x + c)^4 + 4\*b^2\*f\*cosh(d\*x + c)\*sinh(d\*x + c)^3 + b^2\*f\*sinh(d\*x + c)^4 - 2\*b^2\*f\*cosh(d\*x + c)^2 + b^2\*f + 2\*(3\*b^2\*f\*cosh(d\*x + c)^2 - b^2\*f)\*sinh(d\*x + c)^2 + 4\*(b^2\*f\*cosh(d\*x + c)^3 - b^2\*f\*cosh(d\*x + c))\*sinh(d\*x + c))\*dilog(-cosh(d\*x + c) - sinh(d\*x + c)) - ((b^2\*d\*e - b^2\*c\*f)\*cosh(d\*x + c)^4 + 4\*(b^2\*d\*e - b^2\*c\*f)\*cosh(d\*x + c)\*sinh(d\*x + c)^3 + (b^2\*d\*e - b^2\*c\*f)\*sinh(d\*x + c)^4 + b^2\*d\*e - b^2\*c\*f - 2\*(b^2\*d\*e - b^2\*c\*f)\*cosh(d\*x + c)^2 - 2\*(b^2\*d\*e - b^2\*c\*f - 3\*(b^2\*d\*e - b^2\*c\*f)\*cosh(d\*x + c)^2)\*sinh(d\*x + c)^2 + 4\*((b^2\*d\*e - b^2\*c\*f)\*cosh(d\*x + c)^3 - (b^2\*d\*e - b^2\*c\*f)\*cosh(d\*x + c))\*sinh(d\*x + c))\*log(2\*b\*cosh(d\*x + c) + 2\*b\*sinh(d\*x + c) + 2\*b\*sqrt((a^2 + b^2)/b^2) + 2\*a) - ((b^2\*d\*e - b^2\*c\*f)\*cosh(d\*x + c)^4 + 4\*(b^2\*d\*e - b^2\*c\*f)\*cosh(d\*x + c)\*sinh(d\*x + c)^3 + (b^2\*d\*e - b^2\*c\*f)\*



$$\begin{aligned}
& \sinh(dx + c)^4 + b^2 d e - b^2 c f - 2(b^2 d e - b^2 c f) \cosh(dx + c)^2 \\
& - 2(b^2 d e - b^2 c f - 3(b^2 d e - b^2 c f) \cosh(dx + c)^2) \sinh(dx + c)^2 \\
& + 4((b^2 d e - b^2 c f) \cosh(dx + c)^3 - (b^2 d e - b^2 c f) \cosh(dx + c)) \sinh(dx + c) \\
& * \log(2b \cosh(dx + c) + 2b \sinh(dx + c) - 2b \sqrt{(a^2 + b^2)/b^2} + 2a) \\
& - (b^2 d f x + (b^2 d f x + b^2 c f) \cosh(dx + c))^4 + 4(b^2 d f x + b^2 c f) \cosh(dx + c) \sinh(dx + c)^3 \\
& + (b^2 d f x + b^2 c f) \sinh(dx + c)^4 + b^2 c f - 2(b^2 d f x + b^2 c f) \cosh(dx + c)^2 \\
& - 2(b^2 d f x + b^2 c f - 3(b^2 d f x + b^2 c f) \cosh(dx + c)^2) \sinh(dx + c)^2 \\
& + 4((b^2 d f x + b^2 c f) \cosh(dx + c)^3 - (b^2 d f x + b^2 c f) \cosh(dx + c)) \sinh(dx + c) \\
& * \log(-a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b \\
& - (b^2 d f x + (b^2 d f x + b^2 c f) \cosh(dx + c))^4 + 4(b^2 d f x + b^2 c f) \cosh(dx + c) \sinh(dx + c)^3 \\
& + (b^2 d f x + b^2 c f) \sinh(dx + c)^4 + b^2 c f - 2(b^2 d f x + b^2 c f) \cosh(dx + c)^2 \\
& - 2(b^2 d f x + b^2 c f - 3(b^2 d f x + b^2 c f) \cosh(dx + c)^2) \sinh(dx + c)^2 \\
& + 4((b^2 d f x + b^2 c f) \cosh(dx + c)^3 - (b^2 d f x + b^2 c f) \cosh(dx + c)) \sinh(dx + c) \\
& * \log(-a \cosh(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b)/b \\
& + (b^2 d f x + (b^2 d f x + b^2 d e + a b f) \cosh(dx + c))^4 + 4(b^2 d f x + b^2 d e + a b f) \cosh(dx + c) \sinh(dx + c)^3 \\
& + (b^2 d f x + b^2 d e + a b f) \sinh(dx + c)^4 + b^2 d e + a b f - 2(b^2 d f x + b^2 d e + a b f) \cosh(dx + c)^2 \\
& - 2(b^2 d f x + b^2 d e + a b f - 3(b^2 d f x + b^2 d e + a b f) \cosh(dx + c)^2) \sinh(dx + c)^2 \\
& + 4((b^2 d f x + b^2 d e + a b f) \cosh(dx + c)^3 - (b^2 d f x + b^2 d e + a b f) \cosh(dx + c)) \sinh(dx + c) \\
& * \log(\cosh(dx + c) + \sinh(dx + c) + 1) + ((b^2 d e - (b^2 c + a b) f) \cosh(dx + c))^4 \\
& + 4(b^2 d e - (b^2 c + a b) f) \cosh(dx + c) \sinh(dx + c)^3 + (b^2 d e - (b^2 c + a b) f) \sinh(dx + c)^4 \\
& + b^2 d e - 2(b^2 d e - (b^2 c + a b) f) \cosh(dx + c)^2 - 2(b^2 d e - 3(b^2 d e - (b^2 c + a b) f) \cosh(dx + c)^2 \\
& - (b^2 c + a b) f) \sinh(dx + c)^2 - (b^2 c + a b) f + 4((b^2 d e - (b^2 c + a b) f) \cosh(dx + c))^3 \\
& - (b^2 d e - (b^2 c + a b) f) \cosh(dx + c) \sinh(dx + c) * \log(\cosh(dx + c) + \sinh(dx + c) - 1) \\
& + (b^2 d f x + (b^2 d f x + b^2 c f) \cosh(dx + c))^4 + 4(b^2 d f x + b^2 c f) \cosh(dx + c) \sinh(dx + c)^3 \\
& + (b^2 d f x + b^2 c f) \sinh(dx + c)^4 + b^2 c f - 2(b^2 d f x + b^2 c f) \cosh(dx + c)^2 \\
& - 2(b^2 d f x + b^2 c f - 3(b^2 d f x + b^2 c f) \cosh(dx + c)^2) \sinh(dx + c)^2 \\
& + 4((b^2 d f x + b^2 c f) \cosh(dx + c)^3 - (b^2 d f x + b^2 c f) \cosh(dx + c)) \sinh(dx + c) \\
& * \log(-\cosh(dx + c) - \sinh(dx + c) + 1) - 2(a b d f x + a b d e - 3(a b d f x + a b d e) \cosh(dx + c)^2 \\
& + (2a^2 d f x + 2a^2 d e + a^2 f) \cosh(dx + c)) \sinh(dx + c) / (a^3 d^2 \cosh(dx + c))^4 \\
& + 4a^3 d^2 \cosh(dx + c) \sinh(dx + c)^3 + a^3 d^2 \sinh(dx + c)^4 - 2a^3 d^2 \cosh(dx + c)^2 \\
& + a^3 d^2 + 2(3a^3 d^2 \cosh(dx + c)^2 - a^3 d^2) \sinh(dx + c)^2 + 4(a^3 d^2 \cosh(dx + c))^3 \\
& - a^3 d^2 \cosh(dx + c) \sinh(dx + c)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*coth(d\*x+c)\*csch(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \coth(dx + c) \operatorname{csch}(dx + c)^2}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*coth(d\*x+c)\*csch(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)\*coth(d\*x + c)\*csch(d\*x + c)^2/(b\*sinh(d\*x + c) + a), x)

$$3.479 \quad \int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=72

$$\frac{b^2 \log(\sinh(c+dx))}{a^3 d} - \frac{b^2 \log(a+b\sinh(c+dx))}{a^3 d} + \frac{b\operatorname{csch}(c+dx)}{a^2 d} - \frac{\operatorname{csch}^2(c+dx)}{2ad}$$

[Out] (b\*Csch[c + d\*x])/(a^2\*d) - Csch[c + d\*x]^2/(2\*a\*d) + (b^2\*Log[Sinh[c + d\*x]])/(a^3\*d) - (b^2\*Log[a + b\*Sinh[c + d\*x]])/(a^3\*d)

**Rubi [A]** time = 0.110946, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2833, 12, 44}

$$\frac{b^2 \log(\sinh(c+dx))}{a^3 d} - \frac{b^2 \log(a+b\sinh(c+dx))}{a^3 d} + \frac{b\operatorname{csch}(c+dx)}{a^2 d} - \frac{\operatorname{csch}^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

[In] Int[(Coth[c + d\*x]\*Csch[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (b\*Csch[c + d\*x])/(a^2\*d) - Csch[c + d\*x]^2/(2\*a\*d) + (b^2\*Log[Sinh[c + d\*x]])/(a^3\*d) - (b^2\*Log[a + b\*Sinh[c + d\*x]])/(a^3\*d)

### Rule 2833

Int[cos[(e\_.) + (f\_.)\*(x\_.)]\*((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(m\_.)\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)])^(n\_.), x\_Symbol] :> Dist[1/(b\*f), Subst[Int[(a + x)^m\*(c + (d\*x)/b)^n, x], x, b\*Sine[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x]

### Rule 12

Int[(a\_.)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_.)\*(v\_) /; FreeQ[b, x]]

### Rule 44

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m

+ n + 2, 0])

### Rubi steps

$$\begin{aligned}
 \int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{b^3}{x^3(a+x)} dx, x, b\sinh(c+dx)\right)}{bd} \\
 &= \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{x^3(a+x)} dx, x, b\sinh(c+dx)\right)}{d} \\
 &= \frac{b^2 \operatorname{Subst}\left(\int \left(\frac{1}{ax^3} - \frac{1}{a^2x^2} + \frac{1}{a^3x} - \frac{1}{a^3(a+x)}\right) dx, x, b\sinh(c+dx)\right)}{d} \\
 &= \frac{b\operatorname{csch}(c+dx)}{a^2d} - \frac{\operatorname{csch}^2(c+dx)}{2ad} + \frac{b^2 \log(\sinh(c+dx))}{a^3d} - \frac{b^2 \log(a+b\sinh(c+dx))}{a^3d}
 \end{aligned}$$

**Mathematica [A]** time = 0.100926, size = 60, normalized size = 0.83

$$\frac{-a^2\operatorname{csch}^2(c+dx) + 2b^2(\log(\sinh(c+dx)) - \log(a+b\sinh(c+dx))) + 2abc\operatorname{csch}(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

[In] Integrate[(Coth[c + d\*x]\*Csch[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]), x]

[Out] (2\*a\*b\*Csch[c + d\*x] - a^2\*Csch[c + d\*x]^2 + 2\*b^2\*(Log[Sinh[c + d\*x]] - Log[a + b\*Sinh[c + d\*x]]))/(2\*a^3\*d)

**Maple [A]** time = 0.001, size = 73, normalized size = 1.

$$-\frac{1}{2da(\sinh(dx+c))^2} + \frac{b^2 \ln(\sinh(dx+c))}{a^3d} + \frac{b}{da^2 \sinh(dx+c)} - \frac{b^2 \ln(a+b\sinh(dx+c))}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d\*x+c)\*csch(d\*x+c)^2/(a+b\*sinh(d\*x+c)), x)

[Out] -1/2/d/a/sinh(d\*x+c)^2+b^2\*ln(sinh(d\*x+c))/a^3/d+1/d/a^2\*b/sinh(d\*x+c)-b^2\*ln(a+b\*sinh(d\*x+c))/a^3/d

---

**Maxima [B]** time = 1.26917, size = 217, normalized size = 3.01

$$\frac{2 \left( b e^{(-dx-c)} - a e^{(-2dx-2c)} - b e^{(-3dx-3c)} \right)}{\left( 2 a^2 e^{(-2dx-2c)} - a^2 e^{(-4dx-4c)} - a^2 \right) d} - \frac{b^2 \log \left( -2 a e^{(-dx-c)} + b e^{(-2dx-2c)} - b \right)}{a^3 d} + \frac{b^2 \log \left( e^{(-dx-c)} + 1 \right)}{a^3 d} + \frac{b^2 \log \left( e^{(-dx-c)} - 1 \right)}{a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)\*csch(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] -2\*(b\*e^(-d\*x - c) - a\*e^(-2\*d\*x - 2\*c) - b\*e^(-3\*d\*x - 3\*c))/((2\*a^2\*e^(-2\*d\*x - 2\*c) - a^2\*e^(-4\*d\*x - 4\*c) - a^2)\*d) - b^2\*log(-2\*a\*e^(-d\*x - c) + b\*e^(-2\*d\*x - 2\*c) - b)/(a^3\*d) + b^2\*log(e^(-d\*x - c) + 1)/(a^3\*d) + b^2\*log(e^(-d\*x - c) - 1)/(a^3\*d)

---

**Fricas [B]** time = 2.19279, size = 1370, normalized size = 19.03

$$2 ab \cosh(dx + c)^3 + 2 ab \sinh(dx + c)^3 - 2 a^2 \cosh(dx + c)^2 - 2 ab \cosh(dx + c) + 2 \left( 3 ab \cosh(dx + c) - a^2 \right) \sinh(dx + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)\*csch(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] (2\*a\*b\*cosh(d\*x + c)^3 + 2\*a\*b\*sinh(d\*x + c)^3 - 2\*a^2\*cosh(d\*x + c)^2 - 2\*a\*b\*cosh(d\*x + c) + 2\*(3\*a\*b\*cosh(d\*x + c) - a^2)\*sinh(d\*x + c)^2 - (b^2\*cosh(d\*x + c)^4 + 4\*b^2\*cosh(d\*x + c)\*sinh(d\*x + c)^3 + b^2\*sinh(d\*x + c)^4 - 2\*b^2\*cosh(d\*x + c)^2 + 2\*(3\*b^2\*cosh(d\*x + c)^2 - b^2)\*sinh(d\*x + c)^2 + b^2 + 4\*(b^2\*cosh(d\*x + c)^3 - b^2\*cosh(d\*x + c))\*sinh(d\*x + c))\*log(2\*(b\*sinh(d\*x + c) + a)/(cosh(d\*x + c) - sinh(d\*x + c))) + (b^2\*cosh(d\*x + c)^4 + 4\*b^2\*cosh(d\*x + c)\*sinh(d\*x + c)^3 + b^2\*sinh(d\*x + c)^4 - 2\*b^2\*cosh(d\*x + c)^2 + 2\*(3\*b^2\*cosh(d\*x + c)^2 - b^2)\*sinh(d\*x + c)^2 + b^2 + 4\*(b^2\*cosh(d\*x + c)^3 - b^2\*cosh(d\*x + c))\*sinh(d\*x + c))\*log(2\*sinh(d\*x + c)/(cosh(d\*x + c) - sinh(d\*x + c))) + 2\*(3\*a\*b\*cosh(d\*x + c)^2 - 2\*a^2\*cosh(d\*x + c) - a\*b)\*sinh(d\*x + c))/(a^3\*d\*cosh(d\*x + c)^4 + 4\*a^3\*d\*cosh(d\*x + c)\*sinh(d\*x + c)^3 + a^3\*d\*sinh(d\*x + c)^4 - 2\*a^3\*d\*cosh(d\*x + c)^2 + a^3\*d + 2\*(3\*a^3\*d\*cosh(d\*x + c)^2 - a^3\*d)\*sinh(d\*x + c)^2 + 4\*(a^3\*d\*cosh(d\*x + c)^3

$$- a^3 d \cosh(dx + c) \sinh(dx + c)$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(c + dx) \operatorname{csch}^2(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)\*csch(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(coth(c + d\*x)\*csch(c + d\*x)\*\*2/(a + b\*sinh(c + d\*x)), x)

**Giac [A]** time = 1.43603, size = 181, normalized size = 2.51

$$\frac{\frac{b^2 \log(e^{(dx+c)+1})}{a^3} - \frac{b^2 \log(|be^{(2dx+2c)+2ae^{(dx+c)}-b|)}{a^3} + \frac{b^2 \log(|e^{(dx+c)}-1|)}{a^3} + \frac{2(ab e^{(3dx+3c)} - a^2 e^{(2dx+2c)} - a b e^{(dx+c)})}{a^3 (e^{(dx+c)+1})^2 (e^{(dx+c)}-1)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)\*csch(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] (b^2\*log(e^(d\*x + c) + 1)/a^3 - b^2\*log(abs(b\*e^(2\*d\*x + 2\*c) + 2\*a\*e^(d\*x + c) - b))/a^3 + b^2\*log(abs(e^(d\*x + c) - 1))/a^3 + 2\*(a\*b\*e^(3\*d\*x + 3\*c) - a^2\*e^(2\*d\*x + 2\*c) - a\*b\*e^(d\*x + c))/(a^3\*(e^(d\*x + c) + 1)^2\*(e^(d\*x + c) - 1)^2))/d

$$3.480 \quad \int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Optimal. Leaf size=36

$$\text{Unintegrable}\left(\frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}, x\right)$$

[Out] Unintegrable[(Coth[c + d\*x]\*Csch[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0920873, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Coth[c + d\*x]\*Csch[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int] [(Coth[c + d\*x]\*Csch[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\coth(c+dx)\operatorname{csch}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.001, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Coth[c + d\*x]\*Csch[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] \$Aborted

**Maple [A]** time = 1.971, size = 0, normalized size = 0.

$$\int \frac{\coth(dx+c) (\operatorname{csch}(dx+c))^2}{(fx+e)(a+b\sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)`

[Out] `int(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{af - 2(bdfxe^{(3c)} + bdee^{(3c)})e^{(3dx)} + (2adfxe^{(2c)} + (2de - f)ae^{(2c)})e^{(2dx)} + 2(bdfxe^c + bdee^c)e^{(dx)}}{a^2d^2f^2x^2 + 2a^2d^2efx + a^2d^2e^2 + (a^2d^2f^2x^2e^{(4c)} + 2a^2d^2efxe^{(4c)} + a^2d^2e^2e^{(4c)})e^{(4dx)} - 2(a^2d^2f^2x^2e^{(2c)} + 2a^2d^2efxe^{(2c)})e^{(2dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)*csch(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `-(a*f - 2*(b*d*f*x*e^(3*c) + b*d*e*e^(3*c))*e^(3*d*x) + (2*a*d*f*x*e^(2*c) + (2*d*e - f)*a*e^(2*c))*e^(2*d*x) + 2*(b*d*f*x*e^c + b*d*e*e^c)*e^(d*x))/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e^2 + (a^2*d^2*f^2*x^2*e^(4*c) + 2*a^2*d^2*e*f*x*e^(4*c) + a^2*d^2*e^2*e^(4*c))*e^(4*d*x) - 2*(a^2*d^2*f^2*x^2*e^(2*c) + 2*a^2*d^2*e*f*x*e^(2*c) + a^2*d^2*e^2*e^(2*c))*e^(2*d*x)) + 4*integrate(-1/4*(b^2*d^2*f^2*x^2 + b^2*d^2*e^2 + a*b*d*e*f + a^2*f^2 + (2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c))*e^(d*x), x) - 4*integrate(1/4*(b^2*d^2*f^2*x^2 + b^2*d^2*e^2 - a*b*d*e*f + a^2*f^2 + (2*b^2*d^2*e*f - a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c))*e^(d*x), x) + 4*integrate(-1/2*(a*b^2*e^(d*x + c) - b^3)/(a^3*b*f*x + a^3*b*e - (a^3*b*f*x*e^(2*c) + a^3*b*e*e^(2*c))*e^(2*d*x) - 2*(a^4*f*x*e^c + a^4*e*e^c))*e^(d*x), x)`



---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth(dx+c)\operatorname{csch}(dx+c)^2}{afx+ae+(bfx+be)\sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)\*csch(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] integral(coth(d\*x + c)\*csch(d\*x + c)^2/(a\*f\*x + a\*e + (b\*f\*x + b\*e)\*sinh(d\*x + c)), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)\*csch(d\*x+c)\*\*2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)\*csch(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.481 \quad \int \frac{(e+fx)^3 \coth^2(c+dx) \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=1038

result too large to display

```
[Out] (b*(e + f*x)^3)/(a^2*d) - (6*f^2*(e + f*x)*ArcTanh[E^(c + d*x)])/(a*d^3) -
((e + f*x)^3*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2*(e + f*x)^3*ArcTanh[E^(c
+ d*x)])/(a^3*d) + (b*(e + f*x)^3*Coth[c + d*x])/(a^2*d) - (3*f*(e + f*x)^2
*Csch[c + d*x])/(2*a*d^2) - ((e + f*x)^3*Coth[c + d*x]*Csch[c + d*x])/(2*a*
d) - (b*Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2]]))/(a^3*d) + (b*Sqrt[a^2 + b^2]*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(
a + Sqrt[a^2 + b^2]]))/(a^3*d) - (3*b*f*(e + f*x)^2*Log[1 - E^(2*(c + d*x))
])/(a^2*d^2) - (3*f^3*PolyLog[2, -E^(c + d*x)])/(a*d^4) - (3*f*(e + f*x)^2*
PolyLog[2, -E^(c + d*x)])/(2*a*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -E^(c
+ d*x)])/(a^3*d^2) + (3*f^3*PolyLog[2, E^(c + d*x)])/(a*d^4) + (3*f*(e + f
*x)^2*PolyLog[2, E^(c + d*x)])/(2*a*d^2) + (3*b^2*f*(e + f*x)^2*PolyLog[2,
E^(c + d*x)])/(a^3*d^2) - (3*b*Sqrt[a^2 + b^2]*f*(e + f*x)^2*PolyLog[2, -((
b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^2) + (3*b*Sqrt[a^2 + b^2]*f*
(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2)
- (3*b*f^2*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a^2*d^3) + (3*f^2*(e +
f*x)*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (6*b^2*f^2*(e + f*x)*PolyLog[3, -E
^(c + d*x)])/(a^3*d^3) - (3*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a*d^3)
- (6*b^2*f^2*(e + f*x)*PolyLog[3, E^(c + d*x)])/(a^3*d^3) + (6*b*Sqrt[a^2 +
b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(
a^3*d^3) - (6*b*Sqrt[a^2 + b^2]*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/(a^3*d^3) + (3*b*f^3*PolyLog[3, E^(2*(c + d*x))])/(
2*a^2*d^4) - (3*f^3*PolyLog[4, -E^(c + d*x)])/(a*d^4) - (6*b^2*f^3*PolyLog
[4, -E^(c + d*x)])/(a^3*d^4) + (3*f^3*PolyLog[4, E^(c + d*x)])/(a*d^4) + (6
*b^2*f^3*PolyLog[4, E^(c + d*x)])/(a^3*d^4) - (6*b*Sqrt[a^2 + b^2]*f^3*Poly
Log[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^4) + (6*b*Sqrt[a^2
+ b^2]*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^4)
```

**Rubi [A]** time = 2.24077, antiderivative size = 1038, normalized size of antiderivative = 1., number of steps used = 67, number of rules used = 22, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$ , Rules used = {5587, 5457, 4182, 2531, 6609, 2282, 6589, 4186, 2279, 2391, 5569, 3720, 3716, 2190, 32, 5585, 5450, 3296, 2637, 5565, 3322, 2264}

$$\frac{3\operatorname{PolyLog}\left(2, -e^{c+dx}\right) f^3}{ad^4} + \frac{3\operatorname{PolyLog}\left(2, e^{c+dx}\right) f^3}{ad^4} + \frac{3b\operatorname{PolyLog}\left(3, e^{2(c+dx)}\right) f^3}{2a^2d^4} - \frac{6b^2\operatorname{PolyLog}\left(4, -e^{c+dx}\right) f^3}{a^3d^4} - \frac{3\operatorname{PolyLog}\left(4, -e^{c+dx}\right) f^3}{a^3d^4}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Coth[c + d\*x]^2\*Csch[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (b\*(e + f\*x)^3)/(a^2\*d) - (6\*f^2\*(e + f\*x)\*ArcTanh[E^(c + d\*x)])/(a\*d^3) - ((e + f\*x)^3\*ArcTanh[E^(c + d\*x)])/(a\*d) - (2\*b^2\*(e + f\*x)^3\*ArcTanh[E^(c + d\*x)])/(a^3\*d) + (b\*(e + f\*x)^3\*Coth[c + d\*x])/(a^2\*d) - (3\*f\*(e + f\*x)^2\*Csch[c + d\*x])/(2\*a\*d^2) - ((e + f\*x)^3\*Coth[c + d\*x]\*Csch[c + d\*x])/(2\*a\*d) - (b\*Sqrt[a^2 + b^2]\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]]))/(a^3\*d) + (b\*Sqrt[a^2 + b^2]\*(e + f\*x)^3\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]]))/(a^3\*d) - (3\*b\*f\*(e + f\*x)^2\*Log[1 - E^(2\*(c + d\*x))])/(a^2\*d^2) - (3\*f^3\*PolyLog[2, -E^(c + d\*x)])/(a\*d^4) - (3\*f\*(e + f\*x)^2\*PolyLog[2, -E^(c + d\*x)])/(2\*a\*d^2) - (3\*b^2\*f\*(e + f\*x)^2\*PolyLog[2, -E^(c + d\*x)])/(a^3\*d^2) + (3\*f^3\*PolyLog[2, E^(c + d\*x)])/(a\*d^4) + (3\*f\*(e + f\*x)^2\*PolyLog[2, E^(c + d\*x)])/(2\*a\*d^2) + (3\*b^2\*f\*(e + f\*x)^2\*PolyLog[2, E^(c + d\*x)])/(a^3\*d^2) - (3\*b\*Sqrt[a^2 + b^2]\*f\*(e + f\*x)^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^3\*d^2) + (3\*b\*Sqrt[a^2 + b^2]\*f\*(e + f\*x)^2\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^3\*d^2) - (3\*b\*f^2\*(e + f\*x)\*PolyLog[2, E^(2\*(c + d\*x))])/(a^2\*d^3) + (3\*f^2\*(e + f\*x)\*PolyLog[3, -E^(c + d\*x)])/(a\*d^3) + (6\*b^2\*f^2\*(e + f\*x)\*PolyLog[3, -E^(c + d\*x)])/(a^3\*d^3) - (3\*f^2\*(e + f\*x)\*PolyLog[3, E^(c + d\*x)])/(a\*d^3) - (6\*b^2\*f^2\*(e + f\*x)\*PolyLog[3, E^(c + d\*x)])/(a^3\*d^3) + (6\*b\*Sqrt[a^2 + b^2]\*f^2\*(e + f\*x)\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^3\*d^3) - (6\*b\*Sqrt[a^2 + b^2]\*f^2\*(e + f\*x)\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^3\*d^3) + (3\*b\*f^3\*PolyLog[3, E^(2\*(c + d\*x))])/(2\*a^2\*d^4) - (3\*f^3\*PolyLog[4, -E^(c + d\*x)])/(a\*d^4) - (6\*b^2\*f^3\*PolyLog[4, -E^(c + d\*x)])/(a^3\*d^4) + (3\*f^3\*PolyLog[4, E^(c + d\*x)])/(a\*d^4) + (6\*b^2\*f^3\*PolyLog[4, E^(c + d\*x)])/(a^3\*d^4) - (6\*b\*Sqrt[a^2 + b^2]\*f^3\*PolyLog[4, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^3\*d^4) + (6\*b\*Sqrt[a^2 + b^2]\*f^3\*PolyLog[4, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^3\*d^4)

### Rule 5587

Int[(Coth[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*Csch[(c\_.) + (d\_.)\*(x\_)]^(p\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Csch[c + d\*x]^p\*Coth[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Csch[c + d\*x]^(p - 1)\*Coth[c + d\*x]^n)/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5457

Int[Coth[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*Csch[(a\_.) + (b\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[(c + d\*x)^m\*Csch[a + b\*x]\*Coth[a + b\*x]^(p - 2), x] + Int[(c + d\*x)^m\*Csch[a + b\*x]^3\*Coth[a + b\*x]^(p - 2), x] /; FreeQ[

{a, b, c, d, m}, x] && IGtQ[p/2, 0]

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)]/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x)] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

### Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
```

$(m - 2) \cdot (b \cdot \text{Csc}[e + f \cdot x])^{(n - 2)}, x, x] + \text{Dist}[(b^2 \cdot (n - 2)) / (n - 1), \text{Int}[(c + d \cdot x)^m \cdot (b \cdot \text{Csc}[e + f \cdot x])^{(n - 2)}, x, x] - \text{Simp}[(b^2 \cdot d \cdot m \cdot (c + d \cdot x)^{(m - 1)} \cdot (b \cdot \text{Csc}[e + f \cdot x])^{(n - 2)}) / (f^2 \cdot (n - 1) \cdot (n - 2)), x]] /; \text{FreeQ}\{b, c, d, e, f\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{NeQ}[n, 2] \&\& \text{GtQ}[m, 1]$

### Rule 2279

$\text{Int}[\text{Log}[(a_) + (b_) \cdot ((F_)^{((e_) \cdot ((c_) + (d_) \cdot (x_)))})^{(n_)}], x\_Symbol] \rightarrow \text{Dist}[1 / (d \cdot e \cdot n \cdot \text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b \cdot x] / x, x], x, (F^{(e \cdot (c + d \cdot x))})^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \&\& \text{GtQ}[a, 0]$

### Rule 2391

$\text{Int}[\text{Log}[(c_) \cdot ((d_) + (e_) \cdot (x_)^{(n_)})] / (x_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c \cdot e \cdot x^n)] / n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \&\& \text{EqQ}[c \cdot d, 1]$

### Rule 5569

$\text{Int}[(\text{Coth}[(c_) + (d_) \cdot (x_)^{(n_)}] \cdot ((e_) + (f_) \cdot (x_)^{(m_)})) / ((a_) + (b_) \cdot \text{Sinh}[(c_) + (d_) \cdot (x_)^{(n_)}]), x\_Symbol] \rightarrow \text{Dist}[1/a, \text{Int}[(e + f \cdot x)^m \cdot \text{Coth}[c + d \cdot x]^n, x], x] - \text{Dist}[b/a, \text{Int}[(e + f \cdot x)^m \cdot \text{Cosh}[c + d \cdot x] \cdot \text{Coth}[c + d \cdot x]^{(n - 1)} / (a + b \cdot \text{Sinh}[c + d \cdot x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{IGtQ}[n, 0]$

### Rule 3720

$\text{Int}[(c_) + (d_) \cdot (x_)^{(m_)}] \cdot ((b_) \cdot \tan[(e_) + (f_) \cdot (x_)^{(n_)}])^{(n_)}, x\_Symbol] \rightarrow \text{Simp}[(b \cdot (c + d \cdot x)^m \cdot (b \cdot \tan[e + f \cdot x])^{(n - 1)}) / (f \cdot (n - 1)), x] + (-\text{Dist}[(b \cdot d \cdot m) / (f \cdot (n - 1)), \text{Int}[(c + d \cdot x)^{(m - 1)} \cdot (b \cdot \tan[e + f \cdot x])^{(n - 1)}, x], x] - \text{Dist}[b^2, \text{Int}[(c + d \cdot x)^m \cdot (b \cdot \tan[e + f \cdot x])^{(n - 2)}, x], x]) /; \text{FreeQ}\{b, c, d, e, f\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{GtQ}[m, 0]$

### Rule 3716

$\text{Int}[(c_) + (d_) \cdot (x_)^{(m_)}] \cdot \tan[(e_) + \text{Pi} \cdot (k_) + (\text{Complex}[0, fz]) \cdot (f_) \cdot (x_)^{(n_)}], x\_Symbol] \rightarrow -\text{Simp}[(I \cdot (c + d \cdot x)^{(m + 1)}) / (d \cdot (m + 1)), x] + \text{Dist}[2 \cdot I, \text{Int}[(c + d \cdot x)^m \cdot E^{(2 \cdot (-I \cdot e) + f \cdot fz \cdot x))} / (E^{(2 \cdot I \cdot k \cdot \text{Pi})} \cdot (1 + E^{(2 \cdot (-I \cdot e) + f \cdot fz \cdot x))} / E^{(2 \cdot I \cdot k \cdot \text{Pi})})], x], x] /; \text{FreeQ}\{c, d, e, f, fz\}, x] \&\& \text{IntegerQ}[4 \cdot k] \&\& \text{IGtQ}[m, 0]$

### Rule 2190

$\text{Int}[(F_)^{((g_) \cdot ((e_) + (f_) \cdot (x_)))})^{(n_)} \cdot ((c_) + (d_) \cdot (x_)^{(m_)})) / ((a_) + (b_) \cdot (F_)^{((g_) \cdot ((e_) + (f_) \cdot (x_)))})^{(n_)}], x\_Symbol] \rightarrow \text{Simp}[(c + d \cdot x)^m \cdot \text{Log}[1 + (b \cdot (F^{(g \cdot (e + f \cdot x))})^n) / a] / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), x] - \text{Di}$

st[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

### Rule 5585

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]^(p\_.)\*Coth[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Cosh[c + d\*x]^p\*Coth[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(p + 1)\*Coth[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5450

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*Coth[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[(c + d\*x)^m\*Cosh[a + b\*x]^n\*Coth[a + b\*x]^(p - 2), x] + Int[(c + d\*x)^m\*Cosh[a + b\*x]^(n - 2)\*Coth[a + b\*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rule 5565

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Dist[a/b^2, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_.) + (b_.)*sin[(e_.) + (Complex[0, fz_]*)
(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :=> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \coth^2(c+dx) \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \coth^2(c+dx) \operatorname{csch}(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^3 \coth^2(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= \frac{\int (e+fx)^3 \operatorname{csch}(c+dx) dx}{a} + \frac{\int (e+fx)^3 \operatorname{csch}^3(c+dx) dx}{a} - \frac{b \int (e+fx)^3 \coth^2(c+dx) \operatorname{csch}(c+dx) dx}{a} \\
&= -\frac{2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} + \frac{b(e+fx)^3 \coth(c+dx)}{a^2d} - \frac{3f(e+fx)^2 \operatorname{csch}(c+dx)}{2ad^2} \\
&= \frac{b(e+fx)^3}{a^2d} - \frac{b(e+fx)^4}{4a^2f} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} - \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} \\
&= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} - \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2}{a^2d} \\
&= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} - \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2}{a^2d} \\
&= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} - \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2}{a^2d} \\
&= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} - \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2}{a^2d} \\
&= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} - \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2}{a^2d} \\
&= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} - \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2}{a^2d} \\
&= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} - \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2}{a^2d} \\
&= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} - \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2}{a^2d} \\
&= \frac{b(e+fx)^3}{a^2d} - \frac{6f^2(e+fx) \tanh^{-1}(e^{c+dx})}{ad^3} - \frac{(e+fx)^3 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2}{a^2d}
\end{aligned}$$

**Mathematica [C]** time = 43.6309, size = 2384, normalized size = 2.3

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)^3*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```



```

[Out] (e^3*Log[Tanh[(c + d*x)/2]])/(2*a*d) + (b^2*e^3*Log[Tanh[(c + d*x)/2]])/(a^
3*d) + (3*e*f^2*Log[Tanh[(c + d*x)/2]])/(a*d^3) + (3*e^2*f*(-(c*Log[Tanh[(c
+ d*x)/2]]) - I*((I*c + I*d*x)*(Log[1 - E^(I*(I*c + I*d*x))] - Log[1 + E^(
I*(I*c + I*d*x))]) + I*(PolyLog[2, -E^(I*(I*c + I*d*x))] - PolyLog[2, E^(I*
(I*c + I*d*x))])))/(2*a*d^2) + (3*b^2*e^2*f*(-(c*Log[Tanh[(c + d*x)/2]]) -
I*((I*c + I*d*x)*(Log[1 - E^(I*(I*c + I*d*x))] - Log[1 + E^(I*(I*c + I*d*x
))]) + I*(PolyLog[2, -E^(I*(I*c + I*d*x))] - PolyLog[2, E^(I*(I*c + I*d*x))
]])))/(a^3*d^2) + (3*f^3*(-(c*Log[Tanh[(c + d*x)/2]]) - I*((I*c + I*d*x)*(L
og[1 - E^(I*(I*c + I*d*x))] - Log[1 + E^(I*(I*c + I*d*x))]) + I*(PolyLog[2,
-E^(I*(I*c + I*d*x))] - PolyLog[2, E^(I*(I*c + I*d*x))])))/(a*d^4) + (b*E
^c*f^3*Csch[c]*(2*d^3*x^3)/E^(2*c) - 3*d^2*(1 - E^(-2*c))*x^2*Log[1 - E^(-
c - d*x)] - 3*d^2*(1 - E^(-2*c))*x^2*Log[1 + E^(-c - d*x)] + 6*(1 - E^(-2*c
))*d*x*PolyLog[2, -E^(-c - d*x)] + PolyLog[3, -E^(-c - d*x)] + 6*(1 - E^(-
2*c))*d*x*PolyLog[2, E^(-c - d*x)] + PolyLog[3, E^(-c - d*x)])))/(2*a^2*d
^4) - (3*e*f^2*(d^2*x^2*ArcTanh[Cosh[c + d*x] + Sinh[c + d*x]] + d*x*PolyLo
g[2, -Cosh[c + d*x] - Sinh[c + d*x]] - d*x*PolyLog[2, Cosh[c + d*x] + Sinh[
c + d*x]] - PolyLog[3, -Cosh[c + d*x] - Sinh[c + d*x]] + PolyLog[3, Cosh[c
+ d*x] + Sinh[c + d*x]])))/(a*d^3) - (6*b^2*e*f^2*(d^2*x^2*ArcTanh[Cosh[c +
d*x] + Sinh[c + d*x]] + d*x*PolyLog[2, -Cosh[c + d*x] - Sinh[c + d*x]] - d*
x*PolyLog[2, Cosh[c + d*x] + Sinh[c + d*x]] - PolyLog[3, -Cosh[c + d*x] - S
inh[c + d*x]] + PolyLog[3, Cosh[c + d*x] + Sinh[c + d*x]])))/(a^3*d^3) + (b*
Sqrt[a^2 + b^2]*(2*d^3*e^3*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 3
*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - 3*d^3*e*f^2*x
^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] - d^3*f^3*x^3*Log[1 + (b*
E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + 3*d^3*e^2*f*x*Log[1 + (b*E^(c + d*x))
/(a + Sqrt[a^2 + b^2])] + 3*d^3*e*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])] + d^3*f^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])]
- 3*d^2*f*(e + f*x)^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] +
3*d^2*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] +
6*d*e*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + 6*d*f^3*x*Po
lyLog[3, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] - 6*d*e*f^2*PolyLog[3, -((
b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))] - 6*d*f^3*x*PolyLog[3, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))] - 6*f^3*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[
a^2 + b^2])] + 6*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])
)/(a^3*d^4) + (f^3*(-2*d^3*x^3*ArcTanh[Cosh[c + d*x] + Sinh[c + d*x]] - 3*d^
2*x^2*PolyLog[2, -Cosh[c + d*x] - Sinh[c + d*x]] + 3*d^2*x^2*PolyLog[2, Cos
h[c + d*x] + Sinh[c + d*x]] + 6*d*x*PolyLog[3, -Cosh[c + d*x] - Sinh[c + d*
x]] - 6*d*x*PolyLog[3, Cosh[c + d*x] + Sinh[c + d*x]] - 6*PolyLog[4, -Cosh[
c + d*x] - Sinh[c + d*x]] + 6*PolyLog[4, Cosh[c + d*x] + Sinh[c + d*x]])))/(
2*a*d^4) + (b^2*f^3*(-2*d^3*x^3*ArcTanh[Cosh[c + d*x] + Sinh[c + d*x]] - 3*
d^2*x^2*PolyLog[2, -Cosh[c + d*x] - Sinh[c + d*x]] + 3*d^2*x^2*PolyLog[2, C
osh[c + d*x] + Sinh[c + d*x]] + 6*d*x*PolyLog[3, -Cosh[c + d*x] - Sinh[c +
d*x]] - 6*d*x*PolyLog[3, Cosh[c + d*x] + Sinh[c + d*x]] - 6*PolyLog[4, -Cos
h[c + d*x] - Sinh[c + d*x]] + 6*PolyLog[4, Cosh[c + d*x] + Sinh[c + d*x]]))
)/(a^3*d^4) + (3*b*e^2*f*Csch[c]*(-(d*x*Cosh[c]) + Log[Cosh[d*x]*Sinh[c] + C

```

```

osh[c]*Sinh[d*x]]*Sinh[c]))/(a^2*d^2*(-Cosh[c]^2 + Sinh[c]^2)) + (Csch[c]*C
sch[c + d*x]^2*(2*b*d*e^3*Cosh[c] + 6*b*d*e^2*f*x*Cosh[c] + 6*b*d*e*f^2*x^2
*Cosh[c] + 2*b*d*f^3*x^3*Cosh[c] + 3*a*e^2*f*Cosh[d*x] + 6*a*e*f^2*x*Cosh[d
*x] + 3*a*f^3*x^2*Cosh[d*x] - 3*a*e^2*f*Cosh[2*c + d*x] - 6*a*e*f^2*x*Cosh[
2*c + d*x] - 3*a*f^3*x^2*Cosh[2*c + d*x] - 2*b*d*e^3*Cosh[c + 2*d*x] - 6*b*
d*e^2*f*x*Cosh[c + 2*d*x] - 6*b*d*e*f^2*x^2*Cosh[c + 2*d*x] - 2*b*d*f^3*x^3
*Cosh[c + 2*d*x] + a*d*e^3*Sinh[d*x] + 3*a*d*e^2*f*x*Sinh[d*x] + 3*a*d*e*f^
2*x^2*Sinh[d*x] + a*d*f^3*x^3*Sinh[d*x] - a*d*e^3*Sinh[2*c + d*x] - 3*a*d*e
^2*f*x*Sinh[2*c + d*x] - 3*a*d*e*f^2*x^2*Sinh[2*c + d*x] - a*d*f^3*x^3*Sinh
[2*c + d*x]))/(4*a^2*d^2) - (3*b*e*f^2*Csch[c]*Sech[c]*(-(d^2*x^2)/E^ArcTan
h[Tanh[c]])) + (I*(-(d*x*(-Pi + (2*I)*ArcTanh[Tanh[c]]))) - Pi*Log[1 + E^(2*
d*x)] - 2*(I*d*x + I*ArcTanh[Tanh[c]])*Log[1 - E^((2*I)*(I*d*x + I*ArcTanh[
Tanh[c]])]) + Pi*Log[Cosh[d*x]] + (2*I)*ArcTanh[Tanh[c]]*Log[I*Sinh[d*x] + A
rcTanh[Tanh[c]]]]) + I*PolyLog[2, E^((2*I)*(I*d*x + I*ArcTanh[Tanh[c]])])]*T
anh[c])/Sqrt[1 - Tanh[c]^2]))/(a^2*d^3*Sqrt[Sech[c]^2*(Cosh[c]^2 - Sinh[c]^
2)])

```

**Maple [F]** time = 0.889, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 (\coth(dx + c))^2 \operatorname{csch}(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```





$$\begin{aligned}
&^2*d*e*f^2 + b^2*c^3*f^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} \\
&2)*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x \\
&+ c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 2*(b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2* \\
&x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f \\
&^3 + (b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d \\
&^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*\cosh(d*x + c)^4 + 4*(b^2*d^3*f^ \\
&3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2 \\
&*c^2*d*e*f^2 + b^2*c^3*f^3)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^2*d^3*f^3*x^ \\
&3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2 \\
&*d*e*f^2 + b^2*c^3*f^3)*\sinh(d*x + c)^4 - 2*(b^2*d^3*f^3*x^3 + 3*b^2*d^3*e* \\
&f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c \\
&^3*f^3)*\cosh(d*x + c)^2 - 2*(b^2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2* \\
&d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3 - 3*(b^2* \\
&d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - \\
&3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((b^ \\
&2*d^3*f^3*x^3 + 3*b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f \\
&- 3*b^2*c^2*d*e*f^2 + b^2*c^3*f^3)*\cosh(d*x + c)^3 - (b^2*d^3*f^3*x^3 + 3* \\
&b^2*d^3*e*f^2*x^2 + 3*b^2*d^3*e^2*f*x + 3*b^2*c*d^2*e^2*f - 3*b^2*c^2*d*e*f \\
&^2 + b^2*c^3*f^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}*\log(- \\
&(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sq \\
&rt((a^2 + b^2)/b^2) - b)/b) + 12*(b^2*f^3*\cosh(d*x + c)^4 + 4*b^2*f^3*\cosh( \\
&d*x + c)*\sinh(d*x + c)^3 + b^2*f^3*\sinh(d*x + c)^4 - 2*b^2*f^3*\cosh(d*x + c \\
&)^2 + b^2*f^3 + 2*(3*b^2*f^3*\cosh(d*x + c)^2 - b^2*f^3)*\sinh(d*x + c)^2 + 4 \\
&*(b^2*f^3*\cosh(d*x + c)^3 - b^2*f^3*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 \\
&+ b^2)/b^2}*polylog(4, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + \\
&c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) - 12*(b^2*f^3*\cosh(d*x + c) \\
&^4 + 4*b^2*f^3*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^2*f^3*\sinh(d*x + c)^4 - 2* \\
&b^2*f^3*\cosh(d*x + c)^2 + b^2*f^3 + 2*(3*b^2*f^3*\cosh(d*x + c)^2 - b^2*f^3) \\
&*\sinh(d*x + c)^2 + 4*(b^2*f^3*\cosh(d*x + c)^3 - b^2*f^3*\cosh(d*x + c))*\sinh \\
&(d*x + c))*\sqrt{(a^2 + b^2)/b^2}*polylog(4, (a*\cosh(d*x + c) + a*\sinh(d*x + \\
&c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) - 12*(b \\
&^2*d*f^3*x + b^2*d*e*f^2 + (b^2*d*f^3*x + b^2*d*e*f^2)*\cosh(d*x + c)^4 + 4* \\
&(b^2*d*f^3*x + b^2*d*e*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^2*d*f^3*x + \\
&b^2*d*e*f^2)*\sinh(d*x + c)^4 - 2*(b^2*d*f^3*x + b^2*d*e*f^2)*\cosh(d*x + c)^ \\
&2 - 2*(b^2*d*f^3*x + b^2*d*e*f^2 - 3*(b^2*d*f^3*x + b^2*d*e*f^2)*\cosh(d*x + \\
&c)^2)*\sinh(d*x + c)^2 + 4*((b^2*d*f^3*x + b^2*d*e*f^2)*\cosh(d*x + c)^3 - ( \\
&b^2*d*f^3*x + b^2*d*e*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b \\
&^2}*polylog(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*si \\
&nh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}))/b) + 12*(b^2*d*f^3*x + b^2*d*e*f^2 + (b \\
&^2*d*f^3*x + b^2*d*e*f^2)*\cosh(d*x + c)^4 + 4*(b^2*d*f^3*x + b^2*d*e*f^2)*c \\
&osh(d*x + c)*\sinh(d*x + c)^3 + (b^2*d*f^3*x + b^2*d*e*f^2)*\sinh(d*x + c)^4 \\
&- 2*(b^2*d*f^3*x + b^2*d*e*f^2)*\cosh(d*x + c)^2 - 2*(b^2*d*f^3*x + b^2*d*e* \\
&f^2 - 3*(b^2*d*f^3*x + b^2*d*e*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*(( \\
&b^2*d*f^3*x + b^2*d*e*f^2)*\cosh(d*x + c)^3 - (b^2*d*f^3*x + b^2*d*e*f^2)*co \\
&sh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}*polylog(3, (a*\cosh(d*x +
\end{aligned}$$

$$\begin{aligned}
& c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2) / b^2} \\
& + 2(a^2 d^3 f^3 x^3 + a^2 d^3 e^3 - 3a^2 d^2 e^2 f + 3(a^2 d^3 e f^2 - a^2 d^2 f^3) x^2 + 3(a^2 d^3 e^2 f - 2a^2 d^2 e f^2) x) \cosh(dx + c) \\
& - 3((a^2 + 2b^2) d^2 f^3 x^2 + (a^2 + 2b^2) d^2 e^2 f - 4a b d e f^2 + 2a^2 f^3 + ((a^2 + 2b^2) d^2 f^3 x^2 + (a^2 + 2b^2) d^2 e^2 f - 4a b d e f^2 + 2a^2 f^3 + 2((a^2 + 2b^2) d^2 e f^2 - 2a b d f^3) x) \cosh(dx + c)^4 \\
& + 4((a^2 + 2b^2) d^2 f^3 x^2 + (a^2 + 2b^2) d^2 e^2 f - 4a b d e f^2 + 2a^2 f^3 + 2((a^2 + 2b^2) d^2 e f^2 - 2a b d f^3) x) \cosh(dx + c) \sinh(dx + c)^3 + ((a^2 + 2b^2) d^2 f^3 x^2 + (a^2 + 2b^2) d^2 e^2 f - 4a b d e f^2 + 2a^2 f^3 + 2((a^2 + 2b^2) d^2 e f^2 - 2a b d f^3) x) \sinh(dx + c)^4 \\
& - 2((a^2 + 2b^2) d^2 f^3 x^2 + (a^2 + 2b^2) d^2 e^2 f - 4a b d e f^2 + 2a^2 f^3 + 2((a^2 + 2b^2) d^2 e f^2 - 2a b d f^3) x) \cosh(dx + c)^2 - 2((a^2 + 2b^2) d^2 f^3 x^2 + (a^2 + 2b^2) d^2 e^2 f - 4a b d e f^2 + 2a^2 f^3 - 3((a^2 + 2b^2) d^2 f^3 x^2 + (a^2 + 2b^2) d^2 e^2 f - 4a b d e f^2 + 2a^2 f^3 + 2((a^2 + 2b^2) d^2 e f^2 - 2a b d f^3) x) \cosh(dx + c)^2 + 2((a^2 + 2b^2) d^2 e f^2 - 2a b d f^3) x) \sinh(dx + c)^2 \\
& + 2((a^2 + 2b^2) d^2 e f^2 - 2a b d f^3) x + 4(((a^2 + 2b^2) d^2 f^3 x^2 + (a^2 + 2b^2) d^2 e^2 f - 4a b d e f^2 + 2a^2 f^3 + 2((a^2 + 2b^2) d^2 e f^2 - 2a b d f^3) x) \cosh(dx + c)^3 - ((a^2 + 2b^2) d^2 f^3 x^2 + (a^2 + 2b^2) d^2 e^2 f - 4a b d e f^2 + 2a^2 f^3 + 2((a^2 + 2b^2) d^2 e f^2 - 2a b d f^3) x) \cosh(dx + c)) \sinh(dx + c) \operatorname{dilog}(\cosh(dx + c) + \sinh(dx + c)) + 3((a^2 + 2b^2) d^2 f^3 x^2 + (a^2 + 2b^2) d^2 e^2 f + 4a b d e f^2 + 2a^2 f^3 + ((a^2 + 2b^2) d^2 f^3 x^2 + (a^2 + 2b^2) d^2 e^2 f + 4a b d e f^2 + 2a^2 f^3 + 2((a^2 + 2b^2) d^2 e f^2 + 2a b d f^3) x) \cosh(dx + c)^4 + 4((a^2 + 2b^2) d^2 f^3 x^2 + (a^2 + 2b^2) d^2 e^2 f + 4a b d e f^2 + 2a^2 f^3 + 2((a^2 + 2b^2) d^2 e f^2 + 2a b d f^3) x) \cosh(dx + c) \sinh(dx + c)^3 + ((a^2 + 2b^2) d^2 f^3 x^2 + (a^2 + 2b^2) d^2 e^2 f + 4a b d e f^2 + 2a^2 f^3 + 2((a^2 + 2b^2) d^2 e f^2 + 2a b d f^3) x) \sinh(dx + c)^4 - 2((a^2 + 2b^2) d^2 f^3 x^2 + (a^2 + 2b^2) d^2 e^2 f + 4a b d e f^2 + 2a^2 f^3 + 2((a^2 + 2b^2) d^2 e f^2 + 2a b d f^3) x) \cosh(dx + c)^2 - 2((a^2 + 2b^2) d^2 f^3 x^2 + (a^2 + 2b^2) d^2 e^2 f + 4a b d e f^2 + 2a^2 f^3 - 3((a^2 + 2b^2) d^2 f^3 x^2 + (a^2 + 2b^2) d^2 e^2 f + 4a b d e f^2 + 2a^2 f^3 + 2((a^2 + 2b^2) d^2 e f^2 + 2a b d f^3) x) \cosh(dx + c)^2 + 2((a^2 + 2b^2) d^2 e f^2 + 2a b d f^3) x) \sinh(dx + c)^2 + 2((a^2 + 2b^2) d^2 e f^2 + 2a b d f^3) x + 4(((a^2 + 2b^2) d^2 f^3 x^2 + (a^2 + 2b^2) d^2 e^2 f + 4a b d e f^2 + 2a^2 f^3 + 2((a^2 + 2b^2) d^2 e f^2 + 2a b d f^3) x) \cosh(dx + c)^3 - ((a^2 + 2b^2) d^2 f^3 x^2 + (a^2 + 2b^2) d^2 e^2 f + 4a b d e f^2 + 2a^2 f^3 + 2((a^2 + 2b^2) d^2 e f^2 + 2a b d f^3) x) \cosh(dx + c)) \sinh(dx + c) \operatorname{dilog}(-\cosh(dx + c) - \sinh(dx + c)) + ((a^2 + 2b^2) d^3 f^3 x^3 + (a^2 + 2b^2) d^3 e^3 + 6a b d^2 e^2 f + 6a^2 d e f^2 + ((a^2 + 2b^2) d^3 f^3 x^3 + (a^2 + 2b^2) d^3 e^3 + 6a b d^2 e^2 f + 6a^2 d e f^2 + 3((a^2 + 2b^2) d^3 e f^2 + 2a b d^2 f^3) x^2 + 3((a^2 + 2b^2) d^3 e^2 f + 4a b d^2 e f^2 + 2a^2 d f^3) x) \cosh(dx + c)^4 + 4((a^2 + 2b^2) d^3 f^3 x^3 + (a^2 + 2b^2) d^3 e^3 + 6a b d^2 e^2 f + 6a^2 d e f^2
\end{aligned}$$

$$\begin{aligned}
& + 3*((a^2 + 2*b^2)*d^3*e*f^2 + 2*a*b*d^2*f^3)*x^2 + 3*((a^2 + 2*b^2)*d^3*e \\
& ^2*f + 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (( \\
& a^2 + 2*b^2)*d^3*f^3*x^3 + (a^2 + 2*b^2)*d^3*e^3 + 6*a*b*d^2*e^2*f + 6*a^2* \\
& d*e*f^2 + 3*((a^2 + 2*b^2)*d^3*e*f^2 + 2*a*b*d^2*f^3)*x^2 + 3*((a^2 + 2*b^2 \\
& )*d^3*e^2*f + 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)*\sinh(d*x + c)^4 + 3*((a^2 + \\
& 2*b^2)*d^3*e*f^2 + 2*a*b*d^2*f^3)*x^2 - 2*((a^2 + 2*b^2)*d^3*f^3*x^3 + (a^ \\
& 2 + 2*b^2)*d^3*e^3 + 6*a*b*d^2*e^2*f + 6*a^2*d*e*f^2 + 3*((a^2 + 2*b^2)*d^3 \\
& *e*f^2 + 2*a*b*d^2*f^3)*x^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f + 4*a*b*d^2*e*f^2 \\
& + 2*a^2*d*f^3)*x)*\cosh(d*x + c)^2 - 2*((a^2 + 2*b^2)*d^3*f^3*x^3 + (a^2 + 2 \\
& *b^2)*d^3*e^3 + 6*a*b*d^2*e^2*f + 6*a^2*d*e*f^2 + 3*((a^2 + 2*b^2)*d^3*e*f^ \\
& ^2 + 2*a*b*d^2*f^3)*x^2 - 3*((a^2 + 2*b^2)*d^3*f^3*x^3 + (a^2 + 2*b^2)*d^3*e \\
& ^3 + 6*a*b*d^2*e^2*f + 6*a^2*d*e*f^2 + 3*((a^2 + 2*b^2)*d^3*e*f^2 + 2*a*b*d \\
& ^2*f^3)*x^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f + 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x \\
& )*\cosh(d*x + c)^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f + 4*a*b*d^2*e*f^2 + 2*a^2*d* \\
& f^3)*x)*\sinh(d*x + c)^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f + 4*a*b*d^2*e*f^2 + 2* \\
& a^2*d*f^3)*x + 4*((a^2 + 2*b^2)*d^3*f^3*x^3 + (a^2 + 2*b^2)*d^3*e^3 + 6*a* \\
& b*d^2*e^2*f + 6*a^2*d*e*f^2 + 3*((a^2 + 2*b^2)*d^3*e*f^2 + 2*a*b*d^2*f^3)*x \\
& ^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f + 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)*\cosh(d* \\
& x + c)^3 - ((a^2 + 2*b^2)*d^3*f^3*x^3 + (a^2 + 2*b^2)*d^3*e^3 + 6*a*b*d^2*e \\
& ^2*f + 6*a^2*d*e*f^2 + 3*((a^2 + 2*b^2)*d^3*e*f^2 + 2*a*b*d^2*f^3)*x^2 + 3* \\
& ((a^2 + 2*b^2)*d^3*e^2*f + 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)*\cosh(d*x + c)) \\
& *\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) - ((a^2 + 2*b^2)*d^3 \\
& *e^3 - 3*(2*a*b + (a^2 + 2*b^2)*c)*d^2*e^2*f + 3*(4*a*b*c + (a^2 + 2*b^2)*c \\
& ^2 + 2*a^2)*d*e*f^2 + ((a^2 + 2*b^2)*d^3*e^3 - 3*(2*a*b + (a^2 + 2*b^2)*c)* \\
& d^2*e^2*f + 3*(4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*d*e*f^2 - (6*a*b*c^2 + \\
& (a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3)*\cosh(d*x + c)^4 + 4*((a^2 + 2*b^2)*d^3*e^ \\
& 3 - 3*(2*a*b + (a^2 + 2*b^2)*c)*d^2*e^2*f + 3*(4*a*b*c + (a^2 + 2*b^2)*c^2 \\
& + 2*a^2)*d*e*f^2 - (6*a*b*c^2 + (a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3)*\cosh(d*x \\
& + c)*\sinh(d*x + c)^3 + ((a^2 + 2*b^2)*d^3*e^3 - 3*(2*a*b + (a^2 + 2*b^2)*c) \\
& *d^2*e^2*f + 3*(4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*d*e*f^2 - (6*a*b*c^2 + \\
& (a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3)*\sinh(d*x + c)^4 - (6*a*b*c^2 + (a^2 + 2* \\
& b^2)*c^3 + 6*a^2*c)*f^3 - 2*((a^2 + 2*b^2)*d^3*e^3 - 3*(2*a*b + (a^2 + 2*b^ \\
& 2)*c)*d^2*e^2*f + 3*(4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*d*e*f^2 - (6*a*b* \\
& c^2 + (a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3)*\cosh(d*x + c)^2 - 2*((a^2 + 2*b^2)* \\
& d^3*e^3 - 3*(2*a*b + (a^2 + 2*b^2)*c)*d^2*e^2*f + 3*(4*a*b*c + (a^2 + 2*b^2 \\
& )*c^2 + 2*a^2)*d*e*f^2 - (6*a*b*c^2 + (a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3 - 3* \\
& ((a^2 + 2*b^2)*d^3*e^3 - 3*(2*a*b + (a^2 + 2*b^2)*c)*d^2*e^2*f + 3*(4*a*b*c \\
& + (a^2 + 2*b^2)*c^2 + 2*a^2)*d*e*f^2 - (6*a*b*c^2 + (a^2 + 2*b^2)*c^3 + 6* \\
& a^2*c)*f^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*(((a^2 + 2*b^2)*d^3*e^3 - \\
& 3*(2*a*b + (a^2 + 2*b^2)*c)*d^2*e^2*f + 3*(4*a*b*c + (a^2 + 2*b^2)*c^2 + 2* \\
& a^2)*d*e*f^2 - (6*a*b*c^2 + (a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3)*\cosh(d*x + c) \\
& ^3 - ((a^2 + 2*b^2)*d^3*e^3 - 3*(2*a*b + (a^2 + 2*b^2)*c)*d^2*e^2*f + 3*(4* \\
& a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*d*e*f^2 - (6*a*b*c^2 + (a^2 + 2*b^2)*c^3 \\
& + 6*a^2*c)*f^3)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x \\
& + c) - 1) - ((a^2 + 2*b^2)*d^3*f^3*x^3 + 3*(a^2 + 2*b^2)*c*d^2*e^2*f - 3*(
\end{aligned}$$

$$\begin{aligned}
& 4*a*b*c + (a^2 + 2*b^2)*c^2)*d*e*f^2 + ((a^2 + 2*b^2)*d^3*f^3*x^3 + 3*(a^2 \\
& + 2*b^2)*c*d^2*e^2*f - 3*(4*a*b*c + (a^2 + 2*b^2)*c^2)*d*e*f^2 + (6*a*b*c^2 \\
& + (a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3 + 3*((a^2 + 2*b^2)*d^3*e*f^2 - 2*a*b*d^2 \\
& *f^3)*x^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f - 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x) \\
& *cosh(d*x + c)^4 + 4*((a^2 + 2*b^2)*d^3*f^3*x^3 + 3*(a^2 + 2*b^2)*c*d^2*e^2 \\
& *f - 3*(4*a*b*c + (a^2 + 2*b^2)*c^2)*d*e*f^2 + (6*a*b*c^2 + (a^2 + 2*b^2)*c \\
& ^3 + 6*a^2*c)*f^3 + 3*((a^2 + 2*b^2)*d^3*e*f^2 - 2*a*b*d^2*f^3)*x^2 + 3*((a \\
& ^2 + 2*b^2)*d^3*e^2*f - 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)*cosh(d*x + c)*sin \\
& h(d*x + c)^3 + ((a^2 + 2*b^2)*d^3*f^3*x^3 + 3*(a^2 + 2*b^2)*c*d^2*e^2*f - 3 \\
& *(4*a*b*c + (a^2 + 2*b^2)*c^2)*d*e*f^2 + (6*a*b*c^2 + (a^2 + 2*b^2)*c^3 + 6 \\
& *a^2*c)*f^3 + 3*((a^2 + 2*b^2)*d^3*e*f^2 - 2*a*b*d^2*f^3)*x^2 + 3*((a^2 + 2 \\
& *b^2)*d^3*e^2*f - 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)*sinh(d*x + c)^4 + (6*a* \\
& b*c^2 + (a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3 + 3*((a^2 + 2*b^2)*d^3*e*f^2 - 2*a \\
& *b*d^2*f^3)*x^2 - 2*((a^2 + 2*b^2)*d^3*f^3*x^3 + 3*(a^2 + 2*b^2)*c*d^2*e^2* \\
& f - 3*(4*a*b*c + (a^2 + 2*b^2)*c^2)*d*e*f^2 + (6*a*b*c^2 + (a^2 + 2*b^2)*c^ \\
& 3 + 6*a^2*c)*f^3 + 3*((a^2 + 2*b^2)*d^3*e*f^2 - 2*a*b*d^2*f^3)*x^2 + 3*((a^ \\
& 2 + 2*b^2)*d^3*e^2*f - 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)*cosh(d*x + c)^2 - \\
& 2*((a^2 + 2*b^2)*d^3*f^3*x^3 + 3*(a^2 + 2*b^2)*c*d^2*e^2*f - 3*(4*a*b*c + ( \\
& a^2 + 2*b^2)*c^2)*d*e*f^2 + (6*a*b*c^2 + (a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3 + \\
& 3*((a^2 + 2*b^2)*d^3*e*f^2 - 2*a*b*d^2*f^3)*x^2 - 3*((a^2 + 2*b^2)*d^3*f^3 \\
& *x^3 + 3*(a^2 + 2*b^2)*c*d^2*e^2*f - 3*(4*a*b*c + (a^2 + 2*b^2)*c^2)*d*e*f^ \\
& 2 + (6*a*b*c^2 + (a^2 + 2*b^2)*c^3 + 6*a^2*c)*f^3 + 3*((a^2 + 2*b^2)*d^3*e* \\
& f^2 - 2*a*b*d^2*f^3)*x^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f - 4*a*b*d^2*e*f^2 + 2 \\
& *a^2*d*f^3)*x)*cosh(d*x + c)^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f - 4*a*b*d^2*e*f \\
& ^2 + 2*a^2*d*f^3)*x)*sinh(d*x + c)^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f - 4*a*b*d \\
& ^2*e*f^2 + 2*a^2*d*f^3)*x + 4*((a^2 + 2*b^2)*d^3*f^3*x^3 + 3*(a^2 + 2*b^2) \\
& *c*d^2*e^2*f - 3*(4*a*b*c + (a^2 + 2*b^2)*c^2)*d*e*f^2 + (6*a*b*c^2 + (a^2 \\
& + 2*b^2)*c^3 + 6*a^2*c)*f^3 + 3*((a^2 + 2*b^2)*d^3*e*f^2 - 2*a*b*d^2*f^3)*x \\
& ^2 + 3*((a^2 + 2*b^2)*d^3*e^2*f - 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)*cosh(d* \\
& x + c)^3 - ((a^2 + 2*b^2)*d^3*f^3*x^3 + 3*(a^2 + 2*b^2)*c*d^2*e^2*f - 3*(4* \\
& a*b*c + (a^2 + 2*b^2)*c^2)*d*e*f^2 + (6*a*b*c^2 + (a^2 + 2*b^2)*c^3 + 6*a^2 \\
& *c)*f^3 + 3*((a^2 + 2*b^2)*d^3*e*f^2 - 2*a*b*d^2*f^3)*x^2 + 3*((a^2 + 2*b^2) \\
& )*d^3*e^2*f - 4*a*b*d^2*e*f^2 + 2*a^2*d*f^3)*x)*cosh(d*x + c))*sinh(d*x + c \\
& ))*log(-cosh(d*x + c) - sinh(d*x + c) + 1) - 6*((a^2 + 2*b^2)*f^3*cosh(d*x \\
& + c)^4 + 4*(a^2 + 2*b^2)*f^3*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*b^2)* \\
& f^3*sinh(d*x + c)^4 - 2*(a^2 + 2*b^2)*f^3*cosh(d*x + c)^2 + (a^2 + 2*b^2)*f \\
& ^3 + 2*(3*(a^2 + 2*b^2)*f^3*cosh(d*x + c)^2 - (a^2 + 2*b^2)*f^3)*sinh(d*x + \\
& c)^2 + 4*((a^2 + 2*b^2)*f^3*cosh(d*x + c)^3 - (a^2 + 2*b^2)*f^3*cosh(d*x + \\
& c))*sinh(d*x + c))*polylog(4, cosh(d*x + c) + sinh(d*x + c)) + 6*((a^2 + 2 \\
& *b^2)*f^3*cosh(d*x + c)^4 + 4*(a^2 + 2*b^2)*f^3*cosh(d*x + c)*sinh(d*x + c) \\
& ^3 + (a^2 + 2*b^2)*f^3*sinh(d*x + c)^4 - 2*(a^2 + 2*b^2)*f^3*cosh(d*x + c)^ \\
& 2 + (a^2 + 2*b^2)*f^3 + 2*(3*(a^2 + 2*b^2)*f^3*cosh(d*x + c)^2 - (a^2 + 2*b \\
& ^2)*f^3)*sinh(d*x + c)^2 + 4*((a^2 + 2*b^2)*f^3*cosh(d*x + c)^3 - (a^2 + 2* \\
& b^2)*f^3*cosh(d*x + c))*sinh(d*x + c))*polylog(4, -cosh(d*x + c) - sinh(d*x \\
& + c)) + 6*((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 - 2*a*b*f^3 + ((a
\end{aligned}$$



$$\begin{aligned}
&^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 - 2*a*b*f^3)*\cosh(d*x + c)^4 + \\
&4*((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 - 2*a*b*f^3)*\cosh(d*x + c) \\
&*\sinh(d*x + c)^3 + ((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 - 2*a*b*f \\
&^3)*\sinh(d*x + c)^4 - 2*((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 - 2* \\
&a*b*f^3)*\cosh(d*x + c)^2 - 2*((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 \\
&- 2*a*b*f^3 - 3*((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 - 2*a*b*f^3 \\
&)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*(((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b \\
&^2)*d*e*f^2 - 2*a*b*f^3)*\cosh(d*x + c))^3 - ((a^2 + 2*b^2)*d*f^3*x + (a^2 + \\
&2*b^2)*d*e*f^2 - 2*a*b*f^3)*\cosh(d*x + c))*\sinh(d*x + c))*\text{polylog}(3, \cosh(d \\
&*x + c) + \sinh(d*x + c)) - 6*((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 \\
&+ 2*a*b*f^3 + ((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 + 2*a*b*f^3)* \\
&\cosh(d*x + c)^4 + 4*((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 + 2*a*b* \\
&f^3)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2) \\
&*d*e*f^2 + 2*a*b*f^3)*\sinh(d*x + c)^4 - 2*((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2 \\
&*b^2)*d*e*f^2 + 2*a*b*f^3)*\cosh(d*x + c)^2 - 2*((a^2 + 2*b^2)*d*f^3*x + (a^ \\
&2 + 2*b^2)*d*e*f^2 + 2*a*b*f^3 - 3*((a^2 + 2*b^2)*d*f^3*x + (a^2 + 2*b^2)*d \\
&*e*f^2 + 2*a*b*f^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*(((a^2 + 2*b^2)*d* \\
&f^3*x + (a^2 + 2*b^2)*d*e*f^2 + 2*a*b*f^3)*\cosh(d*x + c))^3 - ((a^2 + 2*b^2) \\
&*d*f^3*x + (a^2 + 2*b^2)*d*e*f^2 + 2*a*b*f^3)*\cosh(d*x + c))*\sinh(d*x + c)) \\
&*\text{polylog}(3, -\cosh(d*x + c) - \sinh(d*x + c)) + 2*(a^2*d^3*f^3*x^3 + a^2*d^3* \\
&e^3 - 3*a^2*d^2*e^2*f - 8*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^ \\
&3*e^2*f*x + 3*a*b*c*d^2*e^2*f - 3*a*b*c^2*d*e*f^2 + a*b*c^3*f^3)*\cosh(d*x + \\
&c)^3 + 3*(a^2*d^3*e*f^2 - a^2*d^2*f^3)*x^2 + 3*(a^2*d^3*f^3*x^3 + a^2*d^3* \\
&e^3 + 3*a^2*d^2*e^2*f + 3*(a^2*d^3*e*f^2 + a^2*d^2*f^3)*x^2 + 3*(a^2*d^3*e^ \\
&2*f + 2*a^2*d^2*e*f^2)*x)*\cosh(d*x + c)^2 + 3*(a^2*d^3*e^2*f - 2*a^2*d^2*e* \\
&f^2)*x + 4*(a*b*d^3*f^3*x^3 + 3*a*b*d^3*e*f^2*x^2 + 3*a*b*d^3*e^2*f*x - a*b \\
&*d^3*e^3 + 6*a*b*c*d^2*e^2*f - 6*a*b*c^2*d*e*f^2 + 2*a*b*c^3*f^3)*\cosh(d*x \\
&+ c))*\sinh(d*x + c))/(a^3*d^4*\cosh(d*x + c)^4 + 4*a^3*d^4*\cosh(d*x + c)*\sin \\
&h(d*x + c)^3 + a^3*d^4*\sinh(d*x + c)^4 - 2*a^3*d^4*\cosh(d*x + c)^2 + a^3*d^ \\
&4 + 2*(3*a^3*d^4*\cosh(d*x + c)^2 - a^3*d^4)*\sinh(d*x + c)^2 + 4*(a^3*d^4*\co \\
&sh(d*x + c)^3 - a^3*d^4*\cosh(d*x + c))*\sinh(d*x + c))
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*3\*coth(d\*x+c)\*\*2\*csch(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.482 \quad \int \frac{(e+fx)^2 \coth^2(c+dx) \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=714

$$-\frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{a^3 d^2} + \frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, e^{c+dx}\right)}{a^3 d^2} - \frac{2bf\sqrt{a^2+b^2}(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3 d^2}$$

```
[Out] (b*(e + f*x)^2)/(a^2*d) - ((e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2
*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^3*d) - (f^2*ArcTanh[Cosh[c + d*x]])/(
a*d^3) + (b*(e + f*x)^2*Coth[c + d*x])/(a^2*d) - (f*(e + f*x)*Csch[c + d*x]
)/(a*d^2) - ((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x])/(2*a*d) - (b*Sqrt[a^2
+ b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]))/(a^3*d)
+ (b*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b
^2]]))/(a^3*d) - (2*b*f*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a^2*d^2) - (f*
(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2,
-E^(c + d*x)])/(a^3*d^2) + (f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) +
(2*b^2*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^3*d^2) - (2*b*Sqrt[a^2 + b^
2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d
^2) + (2*b*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/(a^3*d^2) - (b*f^2*PolyLog[2, E^(2*(c + d*x))])/(a^2*d^3)
+ (f^2*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (2*b^2*f^2*PolyLog[3, -E^(c + d
*x)])/(a^3*d^3) - (f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) - (2*b^2*f^2*PolyLo
g[3, E^(c + d*x)])/(a^3*d^3) + (2*b*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^(
c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^3) - (2*b*Sqrt[a^2 + b^2]*f^2*Pol
yLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^3)
```

**Rubi [A]** time = 1.728, antiderivative size = 714, normalized size of antiderivative = 1., number of steps used = 52, number of rules used = 22, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$ , Rules used = {5587, 5457, 4182, 2531, 2282, 6589, 4186, 3770, 5569, 3720, 3716, 2190, 2279, 2391, 32, 5585, 5450, 3296, 2638, 5565, 3322, 2264}

$$-\frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{a^3 d^2} + \frac{2b^2 f(e+fx) \operatorname{PolyLog}\left(2, e^{c+dx}\right)}{a^3 d^2} - \frac{2bf\sqrt{a^2+b^2}(e+fx) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3 d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*(e + f*x)^2)/(a^2*d) - ((e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2
*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^3*d) - (f^2*ArcTanh[Cosh[c + d*x]])/(
```

$$\begin{aligned}
& a*d^3) + (b*(e + f*x)^2*Coth[c + d*x])/(a^2*d) - (f*(e + f*x)*Csch[c + d*x]) \\
& )/(a*d^2) - ((e + f*x)^2*Coth[c + d*x]*Csch[c + d*x])/(2*a*d) - (b*Sqrt[a^2 \\
& + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*d) \\
& + (b*Sqrt[a^2 + b^2]*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b \\
& ^2])])/(a^3*d) - (2*b*f*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a^2*d^2) - (f* \\
& (e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, \\
& -E^(c + d*x)])/(a^3*d^2) + (f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) + \\
& (2*b^2*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^3*d^2) - (2*b*Sqrt[a^2 + b^ \\
& 2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d \\
& ^2) + (2*b*Sqrt[a^2 + b^2]*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sq \\
& rt[a^2 + b^2]))])/(a^3*d^2) - (b*f^2*PolyLog[2, E^(2*(c + d*x))])/(a^2*d^3) \\
& + (f^2*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (2*b^2*f^2*PolyLog[3, -E^(c + d \\
& *x)])/(a^3*d^3) - (f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) - (2*b^2*f^2*PolyLo \\
& g[3, E^(c + d*x)])/(a^3*d^3) + (2*b*Sqrt[a^2 + b^2]*f^2*PolyLog[3, -((b*E^( \\
& c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*d^3) - (2*b*Sqrt[a^2 + b^2]*f^2*Pol \\
& yLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^3)
\end{aligned}$$

### Rule 5587

```

Int[(Coth[(c_.) + (d_.)*(x_)]^(n_.)*Csch[(c_.) + (d_.)*(x_)]^(p_.)*((e_.) +
(f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> D
ist[1/a, Int[(e + f*x)^m*Csch[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Csch[c + d*x]^(p - 1)*Coth[c + d*x]^n)/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
IGtQ[p, 0]

```

### Rule 5457

```

Int[Coth[(a_.) + (b_.)*(x_)]^(p_)*Csch[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(
x_))^(m_.), x_Symbol] :> Int[(c + d*x)^m*Csch[a + b*x]*Coth[a + b*x]^(p - 2
), x] + Int[(c + d*x)^m*Csch[a + b*x]^3*Coth[a + b*x]^(p - 2), x] /; FreeQ[
{a, b, c, d, m}, x] && IGtQ[p/2, 0]

```

### Rule 4182

```

Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

```

### Rule 2531

```

Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))]^(n_.)*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x

```

```

)))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]

```

### Rule 2282

```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

### Rule 6589

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]

```

### Rule 4186

```

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] := -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

```

### Rule 3770

```

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]

```

### Rule 5569

```

Int[(Coth[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_))/((a_.) + (b_
.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Coth[c
+ d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Cosh[c + d*x]*Coth[c + d*x]^
(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && I
GtQ[m, 0] && IGtQ[n, 0]

```

### Rule 3720

```

Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di

```

```
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

### Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rule 5585

```
Int[(Cosh[(c_) + (d_)*(x_)]^(p_)*Coth[(c_) + (d_)*(x_)]^(n_)*((e_) +
(f_)*(x_))^(m_))/((a_) + (b_)*Sinh[(c_) + (d_)*(x_)]), x_Symbol] := D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
0] && IGtQ[p, 0]
```

Rule 5450

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*Coth[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[(c + d\*x)^m\*Cosh[a + b\*x]^n\*Coth[a + b\*x]^(p - 2), x] + Int[(c + d\*x)^m\*Cosh[a + b\*x]^(n - 2)\*Coth[a + b\*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

Rule 5565

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Dist[a/b^2, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

Rule 3322

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*sin[(e\_.) + (Complex[0, fz\_])\* (f\_.)\*(x\_)]), x\_Symbol] := Dist[2, Int[((c + d\*x)^m\*E^(-(I\*e) + f\*fz\*x))/(-(I\*b) + 2\*a\*E^(-(I\*e) + f\*fz\*x) + I\*b\*E^(2\*(-(I\*e) + f\*fz\*x))), x], x] /; FreeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]

Rule 2264

Int[((F\_)^(u\_.)\*((f\_.) + (g\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*(F\_)^(u\_.) + (c\_.)\*(F\_)^(v\_.)), x\_Symbol] := With[{q = Rt[b^2 - 4\*a\*c, 2]}, Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b - q + 2\*c\*F^u), x], x] - Dist[(2\*c)/q, Int[(f + g\*x)^m\*F^u/(b + q + 2\*c\*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v, 2\*u] && LinearQ[u, x] && NeQ[b^2 - 4\*a\*c, 0] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \coth^2(c+dx) \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \coth^2(c+dx) \operatorname{csch}(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^2 \coth^2(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= \frac{\int (e+fx)^2 \operatorname{csch}(c+dx) dx}{a} + \frac{\int (e+fx)^2 \operatorname{csch}^3(c+dx) dx}{a} - \frac{b \int (e+fx)^2 \coth^2(c+dx) \operatorname{csch}(c+dx) dx}{a} \\
&= -\frac{2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} + \frac{b(e+fx)^2 \coth(c+dx)}{a^2 d} - \frac{f(e+fx) \operatorname{csch}(c+dx)}{ad^2} \\
&= \frac{b(e+fx)^2}{a^2 d} - \frac{b(e+fx)^3}{3a^2 f} - \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} \\
&= \frac{b(e+fx)^2}{a^2 d} - \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3 d} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} \\
&= \frac{b(e+fx)^2}{a^2 d} - \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3 d} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} \\
&= \frac{b(e+fx)^2}{a^2 d} - \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3 d} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} \\
&= \frac{b(e+fx)^2}{a^2 d} - \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3 d} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} \\
&= \frac{b(e+fx)^2}{a^2 d} - \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3 d} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} \\
&= \frac{b(e+fx)^2}{a^2 d} - \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3 d} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} \\
&= \frac{b(e+fx)^2}{a^2 d} - \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3 d} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} \\
&= \frac{b(e+fx)^2}{a^2 d} - \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3 d} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3} \\
&= \frac{b(e+fx)^2}{a^2 d} - \frac{(e+fx)^2 \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3 d} - \frac{f^2 \tanh^{-1}(\cosh(c+dx))}{ad^3}
\end{aligned}$$

**Mathematica [B]** time = 26.4172, size = 1529, normalized size = 2.14

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[((e + f*x)^2*Coth[c + d*x]^2*Csch[c + d*x])/(a + b*Sinh[c + d*x]), x]
```

```
[Out] (8*a*b*d^2*e^E^(2*c)*f*x + 4*a*b*d^2*e^E^(2*c)*f^2*x^2 + 2*a^2*d^2*e^2*ArcTan
h[E^(c + d*x)] + 4*b^2*d^2*e^2*ArcTanh[E^(c + d*x)] - 2*a^2*d^2*e^2*E^(2*c)
*ArcTanh[E^(c + d*x)] - 4*b^2*d^2*e^2*E^(2*c)*ArcTanh[E^(c + d*x)] + 4*a^2*
```



```

f^2*ArcTanh[E^(c + d*x)] - 4*a^2*E^(2*c)*f^2*ArcTanh[E^(c + d*x)] - 2*a^2*d
^2*e*f*x*Log[1 - E^(c + d*x)] - 4*b^2*d^2*e*f*x*Log[1 - E^(c + d*x)] + 2*a^
2*d^2*e*E^(2*c)*f*x*Log[1 - E^(c + d*x)] + 4*b^2*d^2*e*E^(2*c)*f*x*Log[1 -
E^(c + d*x)] - a^2*d^2*f^2*x^2*Log[1 - E^(c + d*x)] - 2*b^2*d^2*f^2*x^2*Log
[1 - E^(c + d*x)] + a^2*d^2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + 2*b^2*d^
2*E^(2*c)*f^2*x^2*Log[1 - E^(c + d*x)] + 2*a^2*d^2*e*f*x*Log[1 + E^(c + d*x
)] + 4*b^2*d^2*e*f*x*Log[1 + E^(c + d*x)] - 2*a^2*d^2*e*E^(2*c)*f*x*Log[1 +
E^(c + d*x)] - 4*b^2*d^2*e*E^(2*c)*f*x*Log[1 + E^(c + d*x)] + a^2*d^2*f^2*x
^2*Log[1 + E^(c + d*x)] + 2*b^2*d^2*f^2*x^2*Log[1 + E^(c + d*x)] - a^2*d^2
*E^(2*c)*f^2*x^2*Log[1 + E^(c + d*x)] - 2*b^2*d^2*E^(2*c)*f^2*x^2*Log[1 + E
^(c + d*x)] + 4*a*b*d*e*f*Log[1 - E^(2*(c + d*x))] - 4*a*b*d*e*E^(2*c)*f*Lo
g[1 - E^(2*(c + d*x))] + 4*a*b*d*f^2*x*Log[1 - E^(2*(c + d*x))] - 4*a*b*d*E
^(2*c)*f^2*x*Log[1 - E^(2*(c + d*x))] - 2*(a^2 + 2*b^2)*d*(-1 + E^(2*c))*f*(
e + f*x)*PolyLog[2, -E^(c + d*x)] + 2*(a^2 + 2*b^2)*d*(-1 + E^(2*c))*f*(e
+ f*x)*PolyLog[2, E^(c + d*x)] + 2*a*b*f^2*PolyLog[2, E^(2*(c + d*x))] - 2*
a*b*E^(2*c)*f^2*PolyLog[2, E^(2*(c + d*x))] - 2*a^2*f^2*PolyLog[3, -E^(c +
d*x)] - 4*b^2*f^2*PolyLog[3, -E^(c + d*x)] + 2*a^2*E^(2*c)*f^2*PolyLog[3, -
E^(c + d*x)] + 4*b^2*E^(2*c)*f^2*PolyLog[3, -E^(c + d*x)] + 2*a^2*f^2*PolyL
og[3, E^(c + d*x)] + 4*b^2*f^2*PolyLog[3, E^(c + d*x)] - 2*a^2*E^(2*c)*f^2*
PolyLog[3, E^(c + d*x)] - 4*b^2*E^(2*c)*f^2*PolyLog[3, E^(c + d*x)]/(2*a^3
*d^3*(-1 + E^(2*c))) + (b*Sqrt[a^2 + b^2]*(2*d^2*e^2*ArcTanh[(a + b*E^(c +
d*x))/Sqrt[a^2 + b^2]] - 2*d^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2
+ b^2]]) - d^2*f^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]) + 2*d
^2*e*f*x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) + d^2*f^2*x^2*Log[1
+ (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) - 2*d*f*(e + f*x)*PolyLog[2, (b*E
^(c + d*x))/(-a + Sqrt[a^2 + b^2]]) + 2*d*f*(e + f*x)*PolyLog[2, -(b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]]) + 2*f^2*PolyLog[3, (b*E^(c + d*x))/(-a + Sq
rt[a^2 + b^2]]) - 2*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))
]))/(a^3*d^3) + (Csch[c]*Csch[c + d*x]^2*(2*b*d*e^2*Cosh[c] + 4*b*d*e*f*x*C
osh[c] + 2*b*d*f^2*x^2*Cosh[c] + 2*a*e*f*Cosh[d*x] + 2*a*f^2*x*Cosh[d*x] -
2*a*e*f*Cosh[2*c + d*x] - 2*a*f^2*x*Cosh[2*c + d*x] - 2*b*d*e^2*Cosh[c + 2*
d*x] - 4*b*d*e*f*x*Cosh[c + 2*d*x] - 2*b*d*f^2*x^2*Cosh[c + 2*d*x] + a*d*e^
2*Sinh[d*x] + 2*a*d*e*f*x*Sinh[d*x] + a*d*f^2*x^2*Sinh[d*x] - a*d*e^2*Sinh[
2*c + d*x] - 2*a*d*e*f*x*Sinh[2*c + d*x] - a*d*f^2*x^2*Sinh[2*c + d*x]))/(4
*a^2*d^2)

```

---

**Maple [F]** time = 0.684, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\coth(dx + c))^2 \operatorname{csch}(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [C]** time = 3.62596, size = 17416, normalized size = 24.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*b*d^2*e^2 - 8*a*b*c*d*e*f + 4*a*b*c^2*f^2 - 4*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2)*cosh(d*x + c)^4 - 4*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2)*sinh(d*x + c)^4 + 2*(a^2*d^2*f^2*x^2 + a^2*d^2*e^2 + 2*a^2*d*e*f + 2*(a^2*d^2*e*f + a^2*d*f^2)*x)*cosh(d*x + c)^3 + 2*(a^2*d^2*f^2*x^2 + a^2*d^2*e^2 + 2*a^2*d*e*f + 2*(a^2*d^2*e*f + a^2*d*f^2)*x - 8*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x - a*b*d^2*e^2 + 4*a*b*c*d*e*f - 2*a*b*c^2*f^2)*cosh(d*x + c)^2 + 2*(2*a*b*d^2*f^2*x^2 + 4*a*b*d^2*e*f*x - 2*a*b*d^2*e^2 + 8*a*b*c*d*e*f - 4*a*b*c^2*f^2 - 12*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*e*f*x + 2*a*b*c*d*e*f - a*b*c^2*f^2)*cosh(d*x + c)^2 + 3*(a^2*d^2*f^2*x^2 + a^2*d^2*e^2 + 2*a^2*d*e*f + 2*(a^2*d^2*e*f + a^2*d*f^2)*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 4*(b^2*d*f^2*x + b^2*d*e*f + (b^2*d*f^2*x + b^2*d*e*f)*cosh(d*x + c)^4 + 4*(b^2*d*f^2*x + b^2*d*e*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (b^2*d*f^2*x + b^2*d*e*f)*sinh(d*x + c)^4 - 2*(b^2*d*f^2*x + b^2*d*e*f)*cosh(d*x + c)^2 - 2*(b^
```

$$\begin{aligned}
& 2*d*f^2*x + b^2*d*e*f - 3*(b^2*d*f^2*x + b^2*d*e*f)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*((b^2*d*f^2*x + b^2*d*e*f)*\cosh(d*x + c))^3 - (b^2*d*f^2*x + b^2*d*e*f)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}*dilog((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 4*(b^2*d*f^2*x + b^2*d*e*f + (b^2*d*f^2*x + b^2*d*e*f)*\cosh(d*x + c)^4 + 4*(b^2*d*f^2*x + b^2*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^2*d*f^2*x + b^2*d*e*f)*\sinh(d*x + c)^4 - 2*(b^2*d*f^2*x + b^2*d*e*f)*\cosh(d*x + c)^2 - 2*(b^2*d*f^2*x + b^2*d*e*f - 3*(b^2*d*f^2*x + b^2*d*e*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((b^2*d*f^2*x + b^2*d*e*f)*\cosh(d*x + c)^3 - (b^2*d*f^2*x + b^2*d*e*f)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}*dilog((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2 + (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*\cosh(d*x + c)^4 + 4*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*\sinh(d*x + c)^4 - 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*\cosh(d*x + c)^2 - 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2 - 3*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*\cosh(d*x + c)^3 - (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}*log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2 + (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*\cosh(d*x + c)^4 + 4*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*\sinh(d*x + c)^4 - 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*\cosh(d*x + c)^2 - 2*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2 - 3*(b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*\cosh(d*x + c)^3 - (b^2*d^2*e^2 - 2*b^2*c*d*e*f + b^2*c^2*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}*log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + 2*a) + 2*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2 + (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*\cosh(d*x + c)^4 + 4*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*\sinh(d*x + c)^4 - 2*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*\cosh(d*x + c)^2 - 2*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*\cosh(d*x + c)^3 - (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2}*log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) - 2*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2 + (b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*\cosh(d*x + c)^4 + 4*(b^2*d^2*f^2*x^2 + 2*b^2*d^2*e*f*x + 2*b^2*c*d*e*f - b^2*c^2*f^2)*
\end{aligned}$$



$$\begin{aligned}
& f + 2*a*b*d*f^2)*x)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^2 + 2*b^2)*d^2*f^2*x^2 + (a^2 + 2*b^2)*d^2*e^2 + 4*a*b*d*e*f + 2*a^2*f^2 + 2*((a^2 + 2*b^2)*d^2*e*f + 2*a*b*d*f^2)*x)*\sinh(d*x + c)^4 + 2*a^2*f^2 - 2*((a^2 + 2*b^2)*d^2*f^2*x^2 + (a^2 + 2*b^2)*d^2*e^2 + 4*a*b*d*e*f + 2*a^2*f^2 + 2*((a^2 + 2*b^2)*d^2*e*f + 2*a*b*d*f^2)*x)*\cosh(d*x + c)^2 - 2*((a^2 + 2*b^2)*d^2*f^2*x^2 + (a^2 + 2*b^2)*d^2*e^2 + 4*a*b*d*e*f + 2*a^2*f^2 - 3*((a^2 + 2*b^2)*d^2*f^2*x^2 + (a^2 + 2*b^2)*d^2*e^2 + 4*a*b*d*e*f + 2*a^2*f^2 + 2*((a^2 + 2*b^2)*d^2*e*f + 2*a*b*d*f^2)*x)*\cosh(d*x + c)^2 + 2*((a^2 + 2*b^2)*d^2*e*f + 2*a*b*d*f^2)*x)*\sinh(d*x + c)^2 + 2*((a^2 + 2*b^2)*d^2*e*f + 2*a*b*d*f^2)*x + 4*((a^2 + 2*b^2)*d^2*f^2*x^2 + (a^2 + 2*b^2)*d^2*e^2 + 4*a*b*d*e*f + 2*a^2*f^2 + 2*((a^2 + 2*b^2)*d^2*e*f + 2*a*b*d*f^2)*x)*\cosh(d*x + c)^3 - ((a^2 + 2*b^2)*d^2*f^2*x^2 + (a^2 + 2*b^2)*d^2*e^2 + 4*a*b*d*e*f + 2*a^2*f^2 + 2*((a^2 + 2*b^2)*d^2*e*f + 2*a*b*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) - ((a^2 + 2*b^2)*d^2*e^2 + ((a^2 + 2*b^2)*d^2*e^2 - 2*(2*a*b + (a^2 + 2*b^2)*c)*d*e*f + (4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*f^2)*\cosh(d*x + c)^4 + 4*((a^2 + 2*b^2)*d^2*e^2 - 2*(2*a*b + (a^2 + 2*b^2)*c)*d*e*f + (4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^2 + 2*b^2)*d^2*e^2 - 2*(2*a*b + (a^2 + 2*b^2)*c)*d*e*f + (4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*f^2)*\sinh(d*x + c)^4 - 2*(2*a*b + (a^2 + 2*b^2)*c)*d*e*f + (4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*f^2 - 2*((a^2 + 2*b^2)*d^2*e^2 - 2*(2*a*b + (a^2 + 2*b^2)*c)*d*e*f + (4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*f^2)*\cosh(d*x + c)^2 - 2*((a^2 + 2*b^2)*d^2*e^2 - 2*(2*a*b + (a^2 + 2*b^2)*c)*d*e*f + (4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*f^2) - 3*((a^2 + 2*b^2)*d^2*e^2 - 2*(2*a*b + (a^2 + 2*b^2)*c)*d*e*f + (4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((a^2 + 2*b^2)*d^2*e^2 - 2*(2*a*b + (a^2 + 2*b^2)*c)*d*e*f + (4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*f^2)*\cosh(d*x + c)^3 - ((a^2 + 2*b^2)*d^2*e^2 - 2*(2*a*b + (a^2 + 2*b^2)*c)*d*e*f + (4*a*b*c + (a^2 + 2*b^2)*c^2 + 2*a^2)*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) - ((a^2 + 2*b^2)*d^2*f^2*x^2 + 2*(a^2 + 2*b^2)*c*d*e*f + ((a^2 + 2*b^2)*d^2*f^2*x^2 + 2*(a^2 + 2*b^2)*c*d*e*f - (4*a*b*c + (a^2 + 2*b^2)*c^2)*f^2 + 2*((a^2 + 2*b^2)*d^2*e*f - 2*a*b*d*f^2)*x)*\cosh(d*x + c)^4 + 4*((a^2 + 2*b^2)*d^2*f^2*x^2 + 2*(a^2 + 2*b^2)*c*d*e*f - (4*a*b*c + (a^2 + 2*b^2)*c^2)*f^2 + 2*((a^2 + 2*b^2)*d^2*e*f - 2*a*b*d*f^2)*x)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^2 + 2*b^2)*d^2*f^2*x^2 + 2*(a^2 + 2*b^2)*c*d*e*f - (4*a*b*c + (a^2 + 2*b^2)*c^2)*f^2 + 2*((a^2 + 2*b^2)*d^2*e*f - 2*a*b*d*f^2)*x)*\sinh(d*x + c)^4 - (4*a*b*c + (a^2 + 2*b^2)*c^2)*f^2 - 2*((a^2 + 2*b^2)*d^2*f^2*x^2 + 2*(a^2 + 2*b^2)*c*d*e*f - (4*a*b*c + (a^2 + 2*b^2)*c^2)*f^2 + 2*((a^2 + 2*b^2)*d^2*e*f - 2*a*b*d*f^2)*x)*\cosh(d*x + c)^2 - 2*((a^2 + 2*b^2)*d^2*f^2*x^2 + 2*(a^2 + 2*b^2)*c*d*e*f - (4*a*b*c + (a^2 + 2*b^2)*c^2)*f^2 - 3*((a^2 + 2*b^2)*d^2*f^2*x^2 + 2*(a^2 + 2*b^2)*c*d*e*f - (4*a*b*c + (a^2 + 2*b^2)*c^2)*f^2 + 2*((a^2 + 2*b^2)*d^2*e*f - 2*a*b*d*f^2)*x)*\cosh(d*x + c)^2 + 2*((a^2 + 2*b^2)*d^2*e*f - 2*a*b*d*f^2)*x)*\sinh(d*x + c)^2 + 2*((a^2 + 2*b^2)*d^2*e*f - 2*a*b*d*f^2)*x + 4*((a^2 + 2*b^2)*d^2*f^2*x^2 + 2*(a^2 + 2*b^2)*c*d*e*f - (4*a*b*c + (a^2 + 2*b^2)*c^2)*f^2 + 2*((a^2 + 2*b^2)*d^2*e*f - 2*a*b*d*f^2)*x)*
\end{aligned}$$

$$\begin{aligned} & \cosh(dx + c)^3 - ((a^2 + 2b^2)d^2f^2x^2 + 2(a^2 + 2b^2)cd*ef - (4 \\ & *a*b*c + (a^2 + 2b^2)c^2)f^2 + 2((a^2 + 2b^2)d^2*ef - 2*a*b*d*f^2)*x \\ & )*\cosh(dx + c))*\sinh(dx + c))*\log(-\cosh(dx + c) - \sinh(dx + c) + 1) + 2 \\ & *((a^2 + 2b^2)f^2*\cosh(dx + c)^4 + 4*(a^2 + 2b^2)f^2*\cosh(dx + c)*\sinh(dx + c)^3 + (a^2 + 2b^2)f^2*\sinh(dx + c)^4 - 2*(a^2 + 2b^2)f^2*\cosh(dx + c)^2 + (a^2 + 2b^2)f^2 + 2*(3*(a^2 + 2b^2)f^2*\cosh(dx + c)^2 - (a^2 + 2b^2)f^2)*\sinh(dx + c)^2 + 4*((a^2 + 2b^2)f^2*\cosh(dx + c)^3 - (a^2 + 2b^2)f^2*\cosh(dx + c))*\sinh(dx + c))*\text{polylog}(3, \cosh(dx + c) + \sinh(dx + c)) - 2*((a^2 + 2b^2)f^2*\cosh(dx + c)^4 + 4*(a^2 + 2b^2)f^2*2*\cosh(dx + c)*\sinh(dx + c)^3 + (a^2 + 2b^2)f^2*\sinh(dx + c)^4 - 2*(a^2 + 2b^2)f^2*\cosh(dx + c)^2 + (a^2 + 2b^2)f^2 + 2*(3*(a^2 + 2b^2)f^2*\cosh(dx + c)^2 - (a^2 + 2b^2)f^2)*\sinh(dx + c)^2 + 4*((a^2 + 2b^2)f^2*2*\cosh(dx + c)^3 - (a^2 + 2b^2)f^2*\cosh(dx + c))*\sinh(dx + c))*\text{polylog}(3, -\cosh(dx + c) - \sinh(dx + c)) + 2*(a^2*d^2*f^2*x^2 + a^2*d^2*e^2 - 2*a^2*d*ef - 8*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*ef*x + 2*a*b*c*d*ef - a*b*c^2*f^2)*\cosh(dx + c)^3 + 3*(a^2*d^2*f^2*x^2 + a^2*d^2*e^2 + 2*a^2*d*ef + 2*(a^2*d^2*ef + a^2*d*f^2)*x)*\cosh(dx + c)^2 + 2*(a^2*d^2*ef - a^2*d*f^2)*x + 4*(a*b*d^2*f^2*x^2 + 2*a*b*d^2*ef*x - a*b*d^2*e^2 + 4*a*b*c*d*ef - 2*a*b*c^2*f^2)*\cosh(dx + c))*\sinh(dx + c))/(a^3*d^3*\cosh(dx + c)^4 + 4*a^3*d^3*\cosh(dx + c)*\sinh(dx + c)^3 + a^3*d^3*\sinh(dx + c)^4 - 2*a^3*d^3*\cosh(dx + c)^2 + a^3*d^3 + 2*(3*a^3*d^3*\cosh(dx + c)^2 - a^3*d^3)*\sinh(dx + c)^2 + 4*(a^3*d^3*\cosh(dx + c)^3 - a^3*d^3*\cosh(dx + c))*\sinh(dx + c)) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*coth(dx+c)\*\*2\*csch(dx+c)/(a+b\*sinh(dx+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm  
m="giac")
```

```
[Out] Timed out
```

$$3.483 \quad \int \frac{(e+fx) \coth^2(c+dx) \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=413

$$-\frac{b^2 f \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{a^3 d^2} + \frac{b^2 f \operatorname{PolyLog}\left(2, e^{c+dx}\right)}{a^3 d^2} - \frac{b f \sqrt{a^2 + b^2} \operatorname{PolyLog}\left(2, -\frac{b e^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{a^3 d^2} + \frac{b f \sqrt{a^2 + b^2} \operatorname{PolyLog}\left(2, -\frac{b e^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{a^3 d^2}$$

[Out] -(((e + f\*x)\*ArcTanh[E^(c + d\*x)])/(a\*d)) - (2\*b^2\*(e + f\*x)\*ArcTanh[E^(c + d\*x)])/(a^3\*d) + (b\*(e + f\*x)\*Coth[c + d\*x])/(a^2\*d) - (f\*Csch[c + d\*x])/(2\*a\*d^2) - ((e + f\*x)\*Coth[c + d\*x]\*Csch[c + d\*x])/(2\*a\*d) - (b\*Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^3\*d) + (b\*Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^3\*d) - (b\*f\*Log[Sinh[c + d\*x]])/(a^2\*d^2) - (f\*PolyLog[2, -E^(c + d\*x)])/(2\*a\*d^2) - (b^2\*f\*PolyLog[2, -E^(c + d\*x)])/(a^3\*d^2) + (f\*PolyLog[2, E^(c + d\*x)])/(2\*a\*d^2) + (b^2\*f\*PolyLog[2, E^(c + d\*x)])/(a^3\*d^2) - (b\*Sqrt[a^2 + b^2]\*f\*PolyLog[2, -(b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^3\*d^2) + (b\*Sqrt[a^2 + b^2]\*f\*PolyLog[2, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^3\*d^2)

**Rubi [A]** time = 0.928764, antiderivative size = 413, normalized size of antiderivative = 1., number of steps used = 38, number of rules used = 17, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.531$ , Rules used = {5587, 5457, 4182, 2279, 2391, 4185, 5569, 3720, 3475, 5585, 5450, 3296, 2637, 5565, 3322, 2264, 2190}

$$-\frac{b^2 f \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{a^3 d^2} + \frac{b^2 f \operatorname{PolyLog}\left(2, e^{c+dx}\right)}{a^3 d^2} - \frac{b f \sqrt{a^2 + b^2} \operatorname{PolyLog}\left(2, -\frac{b e^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{a^3 d^2} + \frac{b f \sqrt{a^2 + b^2} \operatorname{PolyLog}\left(2, -\frac{b e^{c+dx}}{a + \sqrt{a^2 + b^2}}\right)}{a^3 d^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Coth[c + d\*x]^2\*Csch[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] -(((e + f\*x)\*ArcTanh[E^(c + d\*x)])/(a\*d)) - (2\*b^2\*(e + f\*x)\*ArcTanh[E^(c + d\*x)])/(a^3\*d) + (b\*(e + f\*x)\*Coth[c + d\*x])/(a^2\*d) - (f\*Csch[c + d\*x])/(2\*a\*d^2) - ((e + f\*x)\*Coth[c + d\*x]\*Csch[c + d\*x])/(2\*a\*d) - (b\*Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^3\*d) + (b\*Sqrt[a^2 + b^2]\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^3\*d) - (b\*f\*Log[Sinh[c + d\*x]])/(a^2\*d^2) - (f\*PolyLog[2, -E^(c + d\*x)])/(2\*a\*d^2) - (b^2\*f\*PolyLog[2, -E^(c + d\*x)])/(a^3\*d^2) + (f\*PolyLog[2, E^(c + d\*x)])/(2\*a\*d^2) + (b^2\*f\*PolyLog[2, E^(c + d\*x)])/(a^3\*d^2) - (b\*Sqrt[a^2 + b^2]\*f\*PolyLog[2, -(b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^3\*d^2) + (b\*Sqrt[a^2 + b^2]\*f\*PolyLog[2, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^3\*d^2)



) + (b\*Sqrt[a^2 + b^2]\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))]/(a^3\*d^2))

### Rule 5587

Int[(Coth[(c\_.) + (d\_.)\*(x\_.)]^(n\_.)\*Csch[(c\_.) + (d\_.)\*(x\_.)]^(p\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Csch[c + d\*x]^p\*Coth[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Csch[c + d\*x]^(p - 1)\*Coth[c + d\*x]^n)/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5457

Int[Coth[(a\_.) + (b\_.)\*(x\_.)]^(p\_.)\*Csch[(a\_.) + (b\_.)\*(x\_.)]\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] := Int[(c + d\*x)^m\*Csch[a + b\*x]\*Coth[a + b\*x]^(p - 2), x] + Int[(c + d\*x)^m\*Csch[a + b\*x]^3\*Coth[a + b\*x]^(p - 2), x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p/2, 0]

### Rule 4182

Int[csc[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_.)]\*((c\_.) + (d\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_.) + (b\_.)\*((F\_)^(e\_.)\*((c\_.) + (d\_.)\*(x\_.)))]^(n\_.), x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.)^(n\_.))]/(x\_.), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 4185

Int[(csc[(e\_.) + (f\_.)\*(x\_.)]\*(b\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_.)), x\_Symbol] := -Simp[(b^2\*(c + d\*x)\*Cot[e + f\*x]\*(b\*Csc[e + f\*x])^(n - 2))/(f\*(n - 1)), x] + (Dist[(b^2\*(n - 2))/(n - 1), Int[(c + d\*x)\*(b\*Csc[e + f\*x])^(n - 2), x], x] - Simp[(b^2\*d\*(b\*Csc[e + f\*x])^(n - 2))/(f^2\*(n - 1)\*(n - 2)), x]) /;

FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]

### Rule 5569

Int[(Coth[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Coth[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Cosh[c + d\*x]\*Coth[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 3720

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^(n\_)), x\_Symbol] := Simp[(b\*(c + d\*x)^m\*(b\*Tan[e + f\*x])^(n - 1))/(f\*(n - 1)), x] + (-Dist[(b\*d\*m)/(f\*(n - 1)), Int[(c + d\*x)^(m - 1)\*(b\*Tan[e + f\*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d\*x)^m\*(b\*Tan[e + f\*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

### Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

### Rule 5585

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]^(p\_.)\*Coth[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Cosh[c + d\*x]^p\*Coth[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(p + 1)\*Coth[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5450

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*Coth[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[(c + d\*x)^m\*Cosh[a + b\*x]^n\*Coth[a + b\*x]^(p - 2), x] + Int[(c + d\*x)^m\*Cosh[a + b\*x]^(n - 2)\*Coth[a + b\*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)])^(n_)*((e_.) + (f_.)*(x_))^(m_)/((a_) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x)) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_)/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)*((c_.) + (d_.)*(x_))^(m_))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx) \coth^2(c+dx) \operatorname{csch}(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx) \coth^2(c+dx) \operatorname{csch}(c+dx) dx}{a} - \frac{b \int \frac{(e+fx) \coth^2(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= \frac{\int (e+fx) \operatorname{csch}(c+dx) dx}{a} + \frac{\int (e+fx) \operatorname{csch}^3(c+dx) dx}{a} - \frac{b \int (e+fx) \coth^2(c+dx) dx}{a^2} \\
&= -\frac{2(e+fx) \tanh^{-1}(e^{c+dx})}{ad} + \frac{b(e+fx) \coth(c+dx)}{a^2 d} - \frac{f \operatorname{csch}(c+dx)}{2ad^2} - \frac{(e+fx) \coth(c+dx)}{a^2 d} \\
&= -\frac{bex}{a^2} - \frac{bfx^2}{2a^2} - \frac{(e+fx) \tanh^{-1}(e^{c+dx})}{ad} + \frac{b(e+fx) \coth(c+dx)}{a^2 d} - \frac{f \operatorname{csch}(c+dx)}{2ad^2} \\
&= -\frac{(e+fx) \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx) \tanh^{-1}(e^{c+dx})}{a^3 d} + \frac{b(e+fx) \coth(c+dx)}{a^2 d} \\
&= -\frac{(e+fx) \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx) \tanh^{-1}(e^{c+dx})}{a^3 d} + \frac{b(e+fx) \coth(c+dx)}{a^2 d} \\
&= -\frac{(e+fx) \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx) \tanh^{-1}(e^{c+dx})}{a^3 d} + \frac{b(e+fx) \coth(c+dx)}{a^2 d} \\
&= -\frac{(e+fx) \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx) \tanh^{-1}(e^{c+dx})}{a^3 d} + \frac{b(e+fx) \coth(c+dx)}{a^2 d} \\
&= -\frac{(e+fx) \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx) \tanh^{-1}(e^{c+dx})}{a^3 d} + \frac{b(e+fx) \coth(c+dx)}{a^2 d} \\
&= -\frac{(e+fx) \tanh^{-1}(e^{c+dx})}{ad} - \frac{2b^2(e+fx) \tanh^{-1}(e^{c+dx})}{a^3 d} + \frac{b(e+fx) \coth(c+dx)}{a^2 d}
\end{aligned}$$

**Mathematica [C]** time = 8.17093, size = 734, normalized size = 1.78

$$\frac{b\sqrt{a^2+b^2} \left( -f \operatorname{PolyLog} \left( 2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a} \right) + f \operatorname{PolyLog} \left( 2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} \right) + 2de \tanh^{-1} \left( \frac{a+be^{c+dx}}{\sqrt{a^2+b^2}} \right) - f(c+dx) \log \left( \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1 \right) \right)}{a^3 d^2}$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Coth[c + d\*x]^2\*Csch[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] ((2\*b\*d\*e\*Cosh[(c + d\*x)/2] - a\*f\*Cosh[(c + d\*x)/2] - 2\*b\*c\*f\*Cosh[(c + d\*x)/2] + 2\*b\*f\*(c + d\*x)\*Cosh[(c + d\*x)/2])\*Csch[(c + d\*x)/2])/(4\*a^2\*d^2) + ((-(d\*e) + c\*f - f\*(c + d\*x))\*Csch[(c + d\*x)/2]^2)/(8\*a\*d^2) - (b\*f\*Log[Sinh[c + d\*x]])/(a^2\*d^2) + (e\*Log[Tanh[(c + d\*x)/2]])/(2\*a\*d) + (b^2\*e\*Log[Ta

$$\begin{aligned} & \text{nh}[(c + d*x)/2]]/(a^3*d) - (c*f*\text{Log}[\text{Tanh}[(c + d*x)/2]])/(2*a*d^2) - (b^2*c \\ & *f*\text{Log}[\text{Tanh}[(c + d*x)/2]])/(a^3*d^2) - ((I/2)*f*(I*(c + d*x)*(\text{Log}[1 - E^{(-c} \\ & - d*x)] - \text{Log}[1 + E^{(-c - d*x)}]) + I*(\text{PolyLog}[2, -E^{(-c - d*x)}] - \text{PolyLog}[ \\ & 2, E^{(-c - d*x)}])))/(a*d^2) - (I*b^2*f*(I*(c + d*x)*(\text{Log}[1 - E^{(-c - d*x)}] \\ & - \text{Log}[1 + E^{(-c - d*x)}]) + I*(\text{PolyLog}[2, -E^{(-c - d*x)}] - \text{PolyLog}[2, E^{(-c} \\ & - d*x)])))/(a^3*d^2) + (b*\text{Sqrt}[a^2 + b^2]*(2*d*e*\text{ArcTanh}[(a + b*E^{(c + d*x)} \\ & )/\text{Sqrt}[a^2 + b^2]] - 2*c*f*\text{ArcTanh}[(a + b*E^{(c + d*x)})/\text{Sqrt}[a^2 + b^2]] - f \\ & *(c + d*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + b^2])] + f*(c + d*x)*\text{Log} \\ & [1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])] - f*\text{PolyLog}[2, (b*E^{(c + d*x)})/ \\ & (-a + \text{Sqrt}[a^2 + b^2])] + f*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^ \\ & 2])))]/(a^3*d^2) + ((-(d*e) + c*f - f*(c + d*x))*\text{Sech}[(c + d*x)/2]^2)/(8*a \\ & *d^2) + (\text{Sech}[(c + d*x)/2]*(2*b*d*e*\text{Sinh}[(c + d*x)/2] + a*f*\text{Sinh}[(c + d*x)/ \\ & 2] - 2*b*c*f*\text{Sinh}[(c + d*x)/2] + 2*b*f*(c + d*x)*\text{Sinh}[(c + d*x)/2]))/(4*a^2 \\ & *d^2) \end{aligned}$$


---

**Maple [B]** time = 0.233, size = 1284, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)*\text{coth}(d*x+c)^2*\text{csch}(d*x+c)/(a+b*\text{sinh}(d*x+c)),x)$

[Out] 
$$\begin{aligned} & -1/a^3/d^2*b^2*f*c*\ln(\exp(d*x+c)-1)-1/a^3/d^2*b^3*f/(a^2+b^2)^{(1/2)}*\text{dilog}(( \\ & -b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))+1/a^3/d^2*b^3*f/(a^2 \\ & +b^2)^{(1/2)}*\text{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))-1/a \\ & ^3/d*b^2*f*\ln(\exp(d*x+c)+1)*x+2/a^3/d*b^3*e/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2* \\ & b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}+1/a^3/d*b^3*f/(a^2+b^2)^{(1/2)}*\ln((b*\exp( \\ & d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*x-1/a^3/d^2*b^3*f/(a^2+b^2)^{(1/2)} \\ & *\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*c+1/a^3/d^2 \\ & *b^3*f/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) \\ & *c-2/a^3/d^2*b^3*f*c/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/ \\ & (a^2+b^2)^{(1/2)}-1/a^3/d*b^3*f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)} \\ & -a)/(-a+(a^2+b^2)^{(1/2)}))*x+2/d*e*b/a/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b \\ & *exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}-1/d^2*f*b/a/(a^2+b^2)^{(1/2)}*\text{dilog}((-b*\exp \\ & (d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))+1/d^2*f*b/a/(a^2+b^2)^{(1/2)} \\ & )*\text{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))-2/d^2*f*c*b/a \\ & /a^3/d^2*b^3*f/(a^2+b^2)^{(1/2)}*\text{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}-1/d*f*b/a \\ & /a^3/d^2*b^3*f/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) \\ & *x-1/d^2*f*b/a/(a^2+b^2)^{(1/2)}*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)})) \\ & *c+1/d*f*b/a/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)})) \\ & *x+1/d^2*f*b/a/(a^2+b^2)^{(1/2)}*\ln((b*\exp(d*x+c)+(a \end{aligned}$$

$$\begin{aligned} & \sqrt{a^2+b^2} + a) / (a + \sqrt{a^2+b^2}) * c - 1/a^3/d^2*b^2*f*dilog(\exp(d*x+c)) - 1 \\ & / a^3/d^2*b^2*f*dilog(\exp(d*x+c)+1) + 2/a^2/d^2*b*f*\ln(\exp(d*x+c)) - 1/a^2/d^2*b \\ & *f*\ln(\exp(d*x+c)-1) - 1/a^2/d^2*b*f*\ln(\exp(d*x+c)+1) + 1/a^3/d*b^2*e*\ln(\exp(d*x \\ & +c)-1) - 1/a^3/d*b^2*e*\ln(\exp(d*x+c)+1) - 1/2/d^2*f/a*dilog(\exp(d*x+c)) - 1/2/d^2 \\ & *f/a*dilog(\exp(d*x+c)+1) + 1/2/d/a*e*\ln(\exp(d*x+c)-1) - 1/2/d/a*e*\ln(\exp(d*x+c) \\ & +1) - 1/2/d/a*\ln(\exp(d*x+c)+1)*f*x - 1/2/d^2/a*f*c*\ln(\exp(d*x+c)-1) - (a*d*f*x*ex \\ & p(3*d*x+3*c) + a*d*e*\exp(3*d*x+3*c) - 2*b*d*f*x*\exp(2*d*x+2*c) + a*d*f*x*\exp(d*x+ \\ & c) + a*f*\exp(3*d*x+3*c) - 2*b*d*e*\exp(2*d*x+2*c) + a*d*e*\exp(d*x+c) + 2*b*d*f*x - a*f \\ & * \exp(d*x+c) + 2*b*d*e) / d^2/a^2/(\exp(2*d*x+2*c)-1)^2 \end{aligned}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [B]** time = 2.71947, size = 8606, normalized size = 20.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] 1/2*(4*(a*b*d*f*x + a*b*c*f)*cosh(d*x + c)^4 + 4*(a*b*d*f*x + a*b*c*f)*sinh
(d*x + c)^4 - 4*a*b*d*e + 4*a*b*c*f - 2*(a^2*d*f*x + a^2*d*e + a^2*f)*cosh(
d*x + c)^3 - 2*(a^2*d*f*x + a^2*d*e + a^2*f - 8*(a*b*d*f*x + a*b*c*f)*cosh(
d*x + c))*sinh(d*x + c)^3 - 4*(a*b*d*f*x - a*b*d*e + 2*a*b*c*f)*cosh(d*x +
c)^2 - 2*(2*a*b*d*f*x - 2*a*b*d*e + 4*a*b*c*f - 12*(a*b*d*f*x + a*b*c*f)*co
sh(d*x + c)^2 + 3*(a^2*d*f*x + a^2*d*e + a^2*f)*cosh(d*x + c))*sinh(d*x + c
)^2 - 2*(b^2*f*cosh(d*x + c)^4 + 4*b^2*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^
2*f*sinh(d*x + c)^4 - 2*b^2*f*cosh(d*x + c)^2 + b^2*f + 2*(3*b^2*f*cosh(d*x
+ c)^2 - b^2*f)*sinh(d*x + c)^2 + 4*(b^2*f*cosh(d*x + c)^3 - b^2*f*cosh(d*
x + c))*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2)*dilog((a*cosh(d*x + c) + a*sin
```

$$\begin{aligned}
& h(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} - b \\
& /b + 1 + 2(b^2 f \cosh(dx + c)^4 + 4b^2 f \cosh(dx + c) \sinh(dx + c)^3 \\
& + b^2 f \sinh(dx + c)^4 - 2b^2 f \cosh(dx + c)^2 + b^2 f + 2(3b^2 f \cosh \\
& (dx + c)^2 - b^2 f) \sinh(dx + c)^2 + 4(b^2 f \cosh(dx + c)^3 - b^2 f \cosh \\
& (dx + c)) \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} \operatorname{dilog}((a \cosh(dx + c) + a \\
& \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} \\
& - b)/b + 1) + 2((b^2 d e - b^2 c f) \cosh(dx + c)^4 + 4(b^2 d e - b^2 c f) \\
& ) \cosh(dx + c) \sinh(dx + c)^3 + (b^2 d e - b^2 c f) \sinh(dx + c)^4 + b^2 \\
& * d e - b^2 * c f - 2(b^2 d e - b^2 c f) \cosh(dx + c)^2 - 2(b^2 d e - b^2 c \\
& * f - 3(b^2 d e - b^2 c f) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4((b^2 d e - \\
& b^2 c f) \cosh(dx + c)^3 - (b^2 d e - b^2 c f) \cosh(dx + c)) \sinh(dx + c) \\
& )) \sqrt{(a^2 + b^2)/b^2} \log(2b \cosh(dx + c) + 2b \sinh(dx + c) + 2b \sqrt{ \\
& (a^2 + b^2)/b^2} + 2a) - 2((b^2 d e - b^2 c f) \cosh(dx + c)^4 + 4(b^2 \\
& d e - b^2 c f) \cosh(dx + c) \sinh(dx + c)^3 + (b^2 d e - b^2 c f) \sinh(dx \\
& * x + c)^4 + b^2 d e - b^2 c f - 2(b^2 d e - b^2 c f) \cosh(dx + c)^2 - 2( \\
& b^2 d e - b^2 c f - 3(b^2 d e - b^2 c f) \cosh(dx + c)^2) \sinh(dx + c)^2 \\
& + 4((b^2 d e - b^2 c f) \cosh(dx + c)^3 - (b^2 d e - b^2 c f) \cosh(dx + c) \\
& )) \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} \log(2b \cosh(dx + c) + 2b \sinh(dx \\
& x + c) - 2b \sqrt{(a^2 + b^2)/b^2} + 2a) - 2(b^2 d f x + (b^2 d f x + b^2 \\
& * c f) \cosh(dx + c)^4 + 4(b^2 d f x + b^2 c f) \cosh(dx + c) \sinh(dx + c) \\
& ^3 + (b^2 d f x + b^2 c f) \sinh(dx + c)^4 + b^2 c f - 2(b^2 d f x + b^2 c \\
& * f) \cosh(dx + c)^2 - 2(b^2 d f x + b^2 c f - 3(b^2 d f x + b^2 c f) \cosh \\
& (dx + c)^2) \sinh(dx + c)^2 + 4((b^2 d f x + b^2 c f) \cosh(dx + c)^3 - ( \\
& b^2 d f x + b^2 c f) \cosh(dx + c)) \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} \log(- \\
& (a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \\
& \sqrt{(a^2 + b^2)/b^2} - b)/b) + 2(b^2 d f x + (b^2 d f x + b^2 c f) \cosh(dx \\
& * x + c)^4 + 4(b^2 d f x + b^2 c f) \cosh(dx + c) \sinh(dx + c)^3 + (b^2 d \\
& * f x + b^2 c f) \sinh(dx + c)^4 + b^2 c f - 2(b^2 d f x + b^2 c f) \cosh(dx \\
& x + c)^2 - 2(b^2 d f x + b^2 c f - 3(b^2 d f x + b^2 c f) \cosh(dx + c)^2) \\
& ) \sinh(dx + c)^2 + 4((b^2 d f x + b^2 c f) \cosh(dx + c)^3 - (b^2 d f x + \\
& b^2 c f) \cosh(dx + c)) \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} \log(-(a \cosh( \\
& dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^2 \\
& + b^2)/b^2} - b)/b) - 2(a^2 d f x + a^2 d e - a^2 f) \cosh(dx + c) + ((a^2 \\
& + 2b^2) f \cosh(dx + c)^4 + 4(a^2 + 2b^2) f \cosh(dx + c) \sinh(dx + c) \\
& ^3 + (a^2 + 2b^2) f \sinh(dx + c)^4 - 2(a^2 + 2b^2) f \cosh(dx + c)^2 + \\
& 2(3(a^2 + 2b^2) f \cosh(dx + c)^2 - (a^2 + 2b^2) f) \sinh(dx + c)^2 + ( \\
& a^2 + 2b^2) f + 4((a^2 + 2b^2) f \cosh(dx + c)^3 - (a^2 + 2b^2) f \cosh( \\
& dx + c)) \sinh(dx + c)) \operatorname{dilog}(\cosh(dx + c) + \sinh(dx + c)) - ((a^2 + 2b \\
& ^2) f \cosh(dx + c)^4 + 4(a^2 + 2b^2) f \cosh(dx + c) \sinh(dx + c)^3 + ( \\
& a^2 + 2b^2) f \sinh(dx + c)^4 - 2(a^2 + 2b^2) f \cosh(dx + c)^2 + 2(3( \\
& a^2 + 2b^2) f \cosh(dx + c)^2 - (a^2 + 2b^2) f) \sinh(dx + c)^2 + (a^2 + \\
& 2b^2) f + 4((a^2 + 2b^2) f \cosh(dx + c)^3 - (a^2 + 2b^2) f \cosh(dx + \\
& c)) \sinh(dx + c)) \operatorname{dilog}(-\cosh(dx + c) - \sinh(dx + c)) - (((a^2 + 2b^2) * \\
& d f x + (a^2 + 2b^2) d e + 2a b f) \cosh(dx + c)^4 + 4((a^2 + 2b^2) d f \\
& * x + (a^2 + 2b^2) d e + 2a b f) \cosh(dx + c) \sinh(dx + c)^3 + ((a^2 + 2
\end{aligned}$$

```

*b^2)*d*f*x + (a^2 + 2*b^2)*d*e + 2*a*b*f)*sinh(d*x + c)^4 + (a^2 + 2*b^2)*
d*f*x + (a^2 + 2*b^2)*d*e + 2*a*b*f - 2*((a^2 + 2*b^2)*d*f*x + (a^2 + 2*b^2)
)*d*e + 2*a*b*f)*cosh(d*x + c)^2 - 2*((a^2 + 2*b^2)*d*f*x + (a^2 + 2*b^2)*d
*e + 2*a*b*f - 3*((a^2 + 2*b^2)*d*f*x + (a^2 + 2*b^2)*d*e + 2*a*b*f)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 + 4*((a^2 + 2*b^2)*d*f*x + (a^2 + 2*b^2)*d*e +
2*a*b*f)*cosh(d*x + c)^3 - ((a^2 + 2*b^2)*d*f*x + (a^2 + 2*b^2)*d*e + 2*a*b
*f)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) +
(((a^2 + 2*b^2)*d*e - (2*a*b + (a^2 + 2*b^2)*c)*f)*cosh(d*x + c)^4 + 4*((a^
2 + 2*b^2)*d*e - (2*a*b + (a^2 + 2*b^2)*c)*f)*cosh(d*x + c)*sinh(d*x + c)^3
+ ((a^2 + 2*b^2)*d*e - (2*a*b + (a^2 + 2*b^2)*c)*f)*sinh(d*x + c)^4 + (a^2
+ 2*b^2)*d*e - 2*((a^2 + 2*b^2)*d*e - (2*a*b + (a^2 + 2*b^2)*c)*f)*cosh(d*
x + c)^2 - 2*((a^2 + 2*b^2)*d*e - 3*((a^2 + 2*b^2)*d*e - (2*a*b + (a^2 + 2*
b^2)*c)*f)*cosh(d*x + c)^2 - (2*a*b + (a^2 + 2*b^2)*c)*f)*sinh(d*x + c)^2 -
(2*a*b + (a^2 + 2*b^2)*c)*f + 4*((a^2 + 2*b^2)*d*e - (2*a*b + (a^2 + 2*b^
2)*c)*f)*cosh(d*x + c)^3 - ((a^2 + 2*b^2)*d*e - (2*a*b + (a^2 + 2*b^2)*c)*f
)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + ((
(a^2 + 2*b^2)*d*f*x + (a^2 + 2*b^2)*c*f)*cosh(d*x + c)^4 + 4*((a^2 + 2*b^2)
*d*f*x + (a^2 + 2*b^2)*c*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + 2*b^2)*
d*f*x + (a^2 + 2*b^2)*c*f)*sinh(d*x + c)^4 + (a^2 + 2*b^2)*d*f*x + (a^2 + 2
*b^2)*c*f - 2*((a^2 + 2*b^2)*d*f*x + (a^2 + 2*b^2)*c*f)*cosh(d*x + c)^2 - 2
*((a^2 + 2*b^2)*d*f*x + (a^2 + 2*b^2)*c*f - 3*((a^2 + 2*b^2)*d*f*x + (a^2 +
2*b^2)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^2 + 2*b^2)*d*f*x + (
a^2 + 2*b^2)*c*f)*cosh(d*x + c)^3 - ((a^2 + 2*b^2)*d*f*x + (a^2 + 2*b^2)*c*
f)*cosh(d*x + c))*sinh(d*x + c))*log(-cosh(d*x + c) - sinh(d*x + c) + 1) -
2*(a^2*d*f*x + a^2*d*e - 8*(a*b*d*f*x + a*b*c*f)*cosh(d*x + c)^3 - a^2*f +
3*(a^2*d*f*x + a^2*d*e + a^2*f)*cosh(d*x + c)^2 + 4*(a*b*d*f*x - a*b*d*e +
2*a*b*c*f)*cosh(d*x + c))*sinh(d*x + c))/(a^3*d^2*cosh(d*x + c)^4 + 4*a^3*d
^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d^2*sinh(d*x + c)^4 - 2*a^3*d^2*cosh
(d*x + c)^2 + a^3*d^2 + 2*(3*a^3*d^2*cosh(d*x + c)^2 - a^3*d^2)*sinh(d*x +
c)^2 + 4*(a^3*d^2*cosh(d*x + c)^3 - a^3*d^2*cosh(d*x + c))*sinh(d*x + c))

```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*coth(d\*x+c)\*\*2\*csch(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

---



**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.484 \quad \int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=111

$$\frac{2b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d} - \frac{(a^2+2b^2)\tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

[Out]  $-\left((a^2+2b^2)\operatorname{ArcTanh}[\operatorname{Cosh}[c+d*x]]\right)/(2*a^3*d) + (2*b*\operatorname{Sqrt}[a^2+b^2]*\operatorname{ArcTanh}[(b-a*\operatorname{Tanh}[(c+d*x)/2])/ \operatorname{Sqrt}[a^2+b^2]])/(a^3*d) + (b*\operatorname{Coth}[c+d*x])/ (a^2*d) - (\operatorname{Coth}[c+d*x]*\operatorname{Csch}[c+d*x])/(2*a*d)$

**Rubi [A]** time = 0.569431, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$ , Rules used = {2889, 3056, 3055, 3001, 3770, 2660, 618, 204}

$$\frac{2b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d} - \frac{(a^2+2b^2)\tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Coth}[c+d*x]^2*\operatorname{Csch}[c+d*x])/(a+b*\operatorname{Sinh}[c+d*x]),x]$

[Out]  $-\left((a^2+2b^2)\operatorname{ArcTanh}[\operatorname{Cosh}[c+d*x]]\right)/(2*a^3*d) + (2*b*\operatorname{Sqrt}[a^2+b^2]*\operatorname{ArcTanh}[(b-a*\operatorname{Tanh}[(c+d*x)/2])/ \operatorname{Sqrt}[a^2+b^2]])/(a^3*d) + (b*\operatorname{Coth}[c+d*x])/ (a^2*d) - (\operatorname{Coth}[c+d*x]*\operatorname{Csch}[c+d*x])/(2*a*d)$

### Rule 2889

$\operatorname{Int}[\cos[(e_.) + (f_.)*(x_)]^2*((d_.)*\sin[(e_.) + (f_.)*(x_)]^{(n_)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^{(m_)}), x\_Symbol] :> \operatorname{Int}[(d*\sin[e + f*x])^n*(a + b*\sin[e + f*x])^m*(1 - \sin[e + f*x]^2), x] /; \operatorname{FreeQ}\{a, b, d, e, f, m, n\}, x] \&\& \operatorname{NeQ}[a^2 - b^2, 0] \&\& (\operatorname{IGtQ}[m, 0] \parallel \operatorname{IntegersQ}[2*m, 2*n])$

### Rule 3056

$\operatorname{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^{(m_)}*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]^{(n_)}*((A_.) + (C_.)*\sin[(e_.) + (f_.)*(x_)]^2), x\_Symbol] :> -\operatorname{Simp}[(A*b^2 + a^2*C)*\cos[e + f*x]*(a + b*\sin[e + f*x])^{(m+1)}*(c + d*\sin[e + f*x])^{(n+1)}], x]$

```
[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n + 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

### Rule 3055

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n + 1))/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] && !IntegerQ[n]) || !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] && !IntegerQ[m]) || EqQ[a, 0])))
```

### Rule 3001

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] := Dist[(A*b - a*B)/(b*c - a*d), Int[1/(a + b*Sin[e + f*x]), x], x] + Dist[(B*c - A*d)/(b*c - a*d), Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

### Rule 2660

```
Int[((a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

$a^2 - b^2, 0]$

### Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

### Rule 204

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])`

### Rubi steps

$$\begin{aligned}
 \int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{a+b\sinh(c+dx)} dx &= \int \frac{\operatorname{csch}^3(c+dx)(1+\sinh^2(c+dx))}{a+b\sinh(c+dx)} dx \\
 &= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} + \frac{i \int \frac{\operatorname{csch}^2(c+dx)(2ib-ia\sinh(c+dx)+ib\sinh^2(c+dx))}{a+b\sinh(c+dx)} dx}{2a} \\
 &= \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{\int \frac{\operatorname{csch}(c+dx)(-a^2-2b^2+ab\sinh(c+dx))}{a+b\sinh(c+dx)} dx}{2a^2} \\
 &= \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{(b(a^2+b^2)) \int \frac{1}{a+b\sinh(c+dx)} dx}{a^3} + \frac{(a^2}{2a^3d} \\
 &= -\frac{(a^2+2b^2)\tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} + \frac{(a^2}{2a^3d} \\
 &= -\frac{(a^2+2b^2)\tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{b\coth(c+dx)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{(a^2}{2a^3d} \\
 &= -\frac{(a^2+2b^2)\tanh^{-1}(\cosh(c+dx))}{2a^3d} + \frac{2b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3d} + \frac{b\coth(c+dx)}{a^2d}
 \end{aligned}$$

**Mathematica [A]** time = 1.28068, size = 145, normalized size = 1.31

$$\frac{4(a^2+2b^2)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + 16b\sqrt{-a^2-b^2}\tan^{-1}\left(\frac{b-a\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right) - a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right) - a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8a^3d}$$

Antiderivative was successfully verified.

[In] Integrate[(Coth[c + d\*x]^2\*Csch[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out]  $(16*b*\sqrt{-a^2 - b^2}*\text{ArcTan}[(b - a*\text{Tanh}[(c + d*x)/2])/ \sqrt{-a^2 - b^2}]] + 4*a*b*\text{Coth}[(c + d*x)/2] - a^2*\text{Csch}[(c + d*x)/2]^2 + 4*(a^2 + 2*b^2)*\text{Log}[\text{Tanh}[(c + d*x)/2]] - a^2*\text{Sech}[(c + d*x)/2]^2 + 4*a*b*\text{Tanh}[(c + d*x)/2])/(8*a^3*d)$

**Maple [A]** time = 0.003, size = 162, normalized size = 1.5

$$\frac{1}{8da} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2 + \frac{b}{2da^2} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{8da} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^{-2} + \frac{1}{2da} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \frac{b^2}{da^3} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d\*x+c)^2\*csch(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out]  $1/8/d/a*\tanh(1/2*d*x+1/2*c)^2+1/2/d/a^2*\tanh(1/2*d*x+1/2*c)*b-1/8/d/a/\tanh(1/2*d*x+1/2*c)^2+1/2/d/a*\ln(\tanh(1/2*d*x+1/2*c))+1/d/a^3*\ln(\tanh(1/2*d*x+1/2*c))*b^2+1/2/d*b/a^2/\tanh(1/2*d*x+1/2*c)-2/d*b*(a^2+b^2)^(1/2)/a^3*\text{arctanh}(1/2*(2*a*\tanh(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)^2\*csch(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 2.46252, size = 2272, normalized size = 20.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*csch(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*cosh(d*x + c)^3 + 2*a^2*sinh(d*x + c)^3 - 4*a*b*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c) + 2*(3*a^2*cosh(d*x + c) - 2*a*b)*sinh(d*x + c)^2 - 2*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(a^2 + b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 + b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 + b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) - b)) + 4*a*b + ((a^2 + 2*b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*b^2)*sinh(d*x + c)^4 - 2*(a^2 + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*b^2)*cosh(d*x + c)^2 - a^2 - 2*b^2)*sinh(d*x + c)^2 + a^2 + 2*b^2 + 4*((a^2 + 2*b^2)*cosh(d*x + c)^3 - (a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - ((a^2 + 2*b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*b^2)*sinh(d*x + c)^4 - 2*(a^2 + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*b^2)*cosh(d*x + c)^2 - a^2 - 2*b^2)*sinh(d*x + c)^2 + a^2 + 2*b^2 + 4*((a^2 + 2*b^2)*cosh(d*x + c)^3 - (a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*a^2*cosh(d*x + c)^2 - 4*a*b*cosh(d*x + c) + a^2)*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^4 + 4*a^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d*sinh(d*x + c)^4 - 2*a^3*d*cosh(d*x + c)^2 + a^3*d + 2*(3*a^3*d*cosh(d*x + c)^2 - a^3*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 - a^3*d*cosh(d*x + c))*sinh(d*x + c))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(c + dx) \operatorname{csch}(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*csch(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Integral(coth(c + d*x)**2*csch(c + d*x)/(a + b*sinh(c + d*x)), x)
```

---

**Giac [B]** time = 1.99255, size = 298, normalized size = 2.68

$$\frac{(a^2 e^c + 2 b^2 e^c) e^{(-c)} \log(e^{(dx+c)} + 1)}{a^3} - \frac{(a^2 e^c + 2 b^2 e^c) e^{(-c)} \log(|e^{(dx+c)} - 1|)}{a^3} + \frac{2(a^2 b e^c + b^3 e^c) e^{(-c)} \log\left(\frac{2 b e^{(dx+2c)} + 2 a e^c - 2 \sqrt{a^2 + b^2} e^c}{2 b e^{(dx+2c)} + 2 a e^c + 2 \sqrt{a^2 + b^2} e^c}\right)}{\sqrt{a^2 + b^2} a^3} + \frac{2(a e^{(3dx+3c)} - 2 b e^{(2dx+2c)})}{a^2 (e^{(2dx+2c)} - 1)^2} \cdot \frac{1}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)^2\*csc(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] 
$$-1/2*((a^2*e^c + 2*b^2*e^c)*e^{(-c)}*\log(e^{(d*x + c)} + 1)/a^3 - (a^2*e^c + 2*b^2*e^c)*e^{(-c)}*\log(\text{abs}(e^{(d*x + c)} - 1))/a^3 + 2*(a^2*b*e^c + b^3*e^c)*e^{(-c)}*\log(\text{abs}(2*b*e^{(d*x + 2*c)} + 2*a*e^c - 2*\text{sqrt}(a^2 + b^2)*e^c)/\text{abs}(2*b*e^{(d*x + 2*c)} + 2*a*e^c + 2*\text{sqrt}(a^2 + b^2)*e^c))/(\text{sqrt}(a^2 + b^2)*a^3) + 2*(a*e^{(3*d*x + 3*c)} - 2*b*e^{(2*d*x + 2*c)} + a*e^{(d*x + c)} + 2*b)/(a^2*(e^{(2*d*x + 2*c)} - 1)^2))/d$$

$$3.485 \quad \int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable}\left(\frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}, x\right)$$

[Out] Unintegrable[(Coth[c + d\*x]^2\*Csch[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x]))], x]

**Rubi [A]** time = 0.0877947, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Coth[c + d\*x]^2\*Csch[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] Defer[Int] [(Coth[c + d\*x]^2\*Csch[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x]))], x]

Rubi steps

$$\int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\coth^2(c+dx)\operatorname{csch}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.008, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Coth[c + d\*x]^2\*Csch[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]



[Out] \$Aborted

**Maple [A]** time = 2.181, size = 0, normalized size = 0.

$$\int \frac{(\coth(dx+c))^2 \operatorname{csch}(dx+c)}{(fx+e)(a+b\sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d\*x+c)^2\*csch(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(coth(d\*x+c)^2\*csch(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)^2\*csch(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-2*(a^2*b*e^c + b^3*e^c)*\operatorname{integrate}(-e^{(d*x)}/(a^3*b*f*x + a^3*b*e - (a^3*b*f*x*e^{(2*c)} + a^3*b*e*e^{(2*c)}))e^{(2*d*x)} - 2*(a^4*f*x*e^c + a^4*e*e^c)e^{(d*x)}, x) - (2*b*d*f*x + 2*b*d*e + (a*d*f*x*e^{(3*c)} + (d*e - f)*a*e^{(3*c)})e^{(3*d*x)} - 2*(b*d*f*x*e^{(2*c)} + b*d*e*e^{(2*c)})e^{(2*d*x)} + (a*d*f*x*e^c + (d*e + f)*a*e^c)e^{(d*x)})/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e^2 + (a^2*d^2*f^2*x^2*e^{(4*c)} + 2*a^2*d^2*e*f*x*e^{(4*c)} + a^2*d^2*e^2*e^{(4*c)})e^{(4*d*x)} - 2*(a^2*d^2*f^2*x^2*e^{(2*c)} + 2*a^2*d^2*e*f*x*e^{(2*c)} + a^2*d^2*e^2*e^{(2*c)})e^{(2*d*x)}) + 2*\operatorname{integrate}(-1/4*(2*b^2*d^2*e^2 + 2*a*b*d*e*f + (d^2*e^2 + 2*f^2)*a^2 + (a^2*d^2*f^2 + 2*b^2*d^2*f^2)*x^2 + 2*(a^2*d^2*e*f + 2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c))e^{(d*x)}, x) + 2*\operatorname{integrate}(1/4*(2*b^2*d^2*e^2 - 2*a*b*d*e*f + (d^2*e^2 + 2*f^2)*a^2 + (a^2*d^2*f^2 + 2*b^2*d^2*f^2)*x^2 + 2*(a^2*d^2*e*f + 2*b^2*d^2*e*f - a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c))e^{(d*x)}, x)$

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth(dx+c)^2 \operatorname{csch}(dx+c)}{afx+ae+(bfx+be)\sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)^2\*csc(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] integral(coth(d\*x + c)^2\*csc(d\*x + c)/(a\*f\*x + a\*e + (b\*f\*x + b\*e)\*sinh(d\*x + c)), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)\*\*2\*csc(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)^2\*csc(d\*x+c)/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.486 \quad \int \frac{(e+fx)^3 \coth^3(c+dx)}{a+b \sinh(c+dx)} dx$$

Optimal. Leaf size=972

result too large to display

```
[Out] (-3*f*(e + f*x)^2)/(2*a*d^2) + (e + f*x)^3/(2*a*d) - (e + f*x)^4/(4*a*f) -
(b^2*(e + f*x)^4)/(4*a^3*f) + ((a^2 + b^2)*(e + f*x)^4)/(4*a^3*f) + (6*b*f*
(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d^2) - (3*f*(e + f*x)^2*Coth[c + d*x
])/(2*a*d^2) - ((e + f*x)^3*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*x)^3*Csch[
c + d*x])/(a^2*d) - ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 + b^2]])/(a^3*d) - ((a^2 + b^2)*(e + f*x)^3*Log[1 + (b*E^(c + d*x)
))/(a + Sqrt[a^2 + b^2]])/(a^3*d) + (3*f^2*(e + f*x)*Log[1 - E^(2*(c + d*x)
)])/((a*d^3) + ((e + f*x)^3*Log[1 - E^(2*(c + d*x))])/(a*d) + (b^2*(e + f*x)
^3*Log[1 - E^(2*(c + d*x))])/(a^3*d) + (6*b*f^2*(e + f*x)*PolyLog[2, -E^(c
+ d*x)]/(a^2*d^3) - (6*b*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)]/(a^2*d^3)
- (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 +
b^2]))])/(a^3*d^2) - (3*(a^2 + b^2)*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d
*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (3*f^3*PolyLog[2, E^(2*(c + d*x)
)])/((2*a*d^4) + (3*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^2) + (3
*b^2*f*(e + f*x)^2*PolyLog[2, E^(2*(c + d*x))])/(2*a^3*d^2) - (6*b*f^3*Poly
Log[3, -E^(c + d*x)]/(a^2*d^4) + (6*b*f^3*PolyLog[3, E^(c + d*x)]/(a^2*d^
4) + (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^
2 + b^2]))])/(a^3*d^3) + (6*(a^2 + b^2)*f^2*(e + f*x)*PolyLog[3, -((b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^3) - (3*f^2*(e + f*x)*PolyLog[3, E^
(2*(c + d*x))])/(2*a*d^3) - (3*b^2*f^2*(e + f*x)*PolyLog[3, E^(2*(c + d*x)
)])/((2*a^3*d^3) - (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[
a^2 + b^2]))])/(a^3*d^4) - (6*(a^2 + b^2)*f^3*PolyLog[4, -((b*E^(c + d*x))/
(a + Sqrt[a^2 + b^2]))])/(a^3*d^4) + (3*f^3*PolyLog[4, E^(2*(c + d*x))])/(4
*a*d^4) + (3*b^2*f^3*PolyLog[4, E^(2*(c + d*x))])/(4*a^3*d^4)
```

**Rubi [A]** time = 2.20277, antiderivative size = 972, normalized size of antiderivative = 1., number of steps used = 62, number of rules used = 23, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.821$ , Rules used = {5569, 3720, 3716, 2190, 2279, 2391, 32, 2531, 6609, 2282, 6589, 5585, 5450, 3296, 2638, 5452, 4182, 5446, 3311, 2635, 8, 5565, 5561}

$$-\frac{b^2(e+fx)^4}{4a^3f} + \frac{(a^2+b^2)(e+fx)^4}{4a^3f} - \frac{(e+fx)^4}{4af} - \frac{\coth^2(c+dx)(e+fx)^3}{2ad} + \frac{b\text{csch}(c+dx)(e+fx)^3}{a^2d} - \frac{(a^2+b^2)\log\left(\frac{a^2+b^2}{a}\right)}{a^2d}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Coth[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] 
$$\begin{aligned} & (-3*f*(e + f*x)^2)/(2*a*d^2) + (e + f*x)^3/(2*a*d) - (e + f*x)^4/(4*a*f) - \\ & (b^2*(e + f*x)^4)/(4*a^3*f) + ((a^2 + b^2)*(e + f*x)^4)/(4*a^3*f) + (6*b*f* \\ & (e + f*x)^2*\text{ArcTanh}[E^{(c + d*x)}])/(a^2*d^2) - (3*f*(e + f*x)^2*\text{Coth}[c + d*x] \\ & )/(2*a*d^2) - ((e + f*x)^3*\text{Coth}[c + d*x]^2)/(2*a*d) + (b*(e + f*x)^3*\text{Csch}[ \\ & c + d*x])/(a^2*d) - ((a^2 + b^2)*(e + f*x)^3*\text{Log}[1 + (b*E^{(c + d*x)})/(a - \text{S} \\ & \text{qrt}[a^2 + b^2])])/(a^3*d) - ((a^2 + b^2)*(e + f*x)^3*\text{Log}[1 + (b*E^{(c + d*x)} \\ & )/(a + \text{Sqrt}[a^2 + b^2])])/(a^3*d) + (3*f^2*(e + f*x)*\text{Log}[1 - E^{(2*(c + d*x)} \\ & )])/(a*d^3) + ((e + f*x)^3*\text{Log}[1 - E^{(2*(c + d*x))}])/(a*d) + (b^2*(e + f*x) \\ & ^3*\text{Log}[1 - E^{(2*(c + d*x))}])/(a^3*d) + (6*b*f^2*(e + f*x)*\text{PolyLog}[2, -E^{(c \\ & + d*x)}])/(a^2*d^3) - (6*b*f^2*(e + f*x)*\text{PolyLog}[2, E^{(c + d*x)}])/(a^2*d^3) \\ & - (3*(a^2 + b^2)*f*(e + f*x)^2*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^2 + \\ & b^2])])/(a^3*d^2) - (3*(a^2 + b^2)*f*(e + f*x)^2*\text{PolyLog}[2, -((b*E^{(c + d \\ & *x)})/(a + \text{Sqrt}[a^2 + b^2])])/(a^3*d^2) + (3*f^3*\text{PolyLog}[2, E^{(2*(c + d*x)} \\ & )])/(2*a*d^4) + (3*f*(e + f*x)^2*\text{PolyLog}[2, E^{(2*(c + d*x))}])/(2*a*d^2) + (3 \\ & *b^2*f*(e + f*x)^2*\text{PolyLog}[2, E^{(2*(c + d*x))}])/(2*a^3*d^2) - (6*b*f^3*\text{Poly} \\ & \text{Log}[3, -E^{(c + d*x)}])/(a^2*d^4) + (6*b*f^3*\text{PolyLog}[3, E^{(c + d*x)}])/(a^2*d^ \\ & 4) + (6*(a^2 + b^2)*f^2*(e + f*x)*\text{PolyLog}[3, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[a^ \\ & 2 + b^2])])/(a^3*d^3) + (6*(a^2 + b^2)*f^2*(e + f*x)*\text{PolyLog}[3, -((b*E^{(c \\ & + d*x)})/(a + \text{Sqrt}[a^2 + b^2])])/(a^3*d^3) - (3*f^2*(e + f*x)*\text{PolyLog}[3, E^ \\ & (2*(c + d*x))])/(2*a*d^3) - (3*b^2*f^2*(e + f*x)*\text{PolyLog}[3, E^{(2*(c + d*x)} \\ & )])/(2*a^3*d^3) - (6*(a^2 + b^2)*f^3*\text{PolyLog}[4, -((b*E^{(c + d*x)})/(a - \text{Sqrt}[ \\ & a^2 + b^2])])/(a^3*d^4) - (6*(a^2 + b^2)*f^3*\text{PolyLog}[4, -((b*E^{(c + d*x)})/ \\ & (a + \text{Sqrt}[a^2 + b^2])])/(a^3*d^4) + (3*f^3*\text{PolyLog}[4, E^{(2*(c + d*x))}])/(4 \\ & *a*d^4) + (3*b^2*f^3*\text{PolyLog}[4, E^{(2*(c + d*x))}])/(4*a^3*d^4) \end{aligned}$$

### Rule 5569

Int[((Coth[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Coth[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Cosh[c + d\*x]\*Coth[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 3720

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*((b\_.)\*tan[(e\_.) + (f\_.)\*(x\_)]^(n\_.), x\_Symbol] := Simp[(b\*(c + d\*x)^m\*(b\*Tan[e + f\*x])^(n - 1))/(f\*(n - 1)), x] + (-Dist[(b\*d\*m)/(f\*(n - 1)), Int[(c + d\*x)^(m - 1)\*(b\*Tan[e + f\*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d\*x)^m\*(b\*Tan[e + f\*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

### Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-I*
e) + f*fz*x))/E^(2*I*k*Pi))], x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

### Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.)
*(x_))))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_)\*((a\_) + (b\_)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_)\*((a\_) + (b\_)\*(x\_))^(p\_)]/((d\_) + (e\_)\*(x\_)), x\_Symbol] :=> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 5585

Int[(Cosh[(c\_) + (d\_)\*(x\_)]^(p\_)\*Coth[(c\_) + (d\_)\*(x\_)]^(n\_)\*((e\_) + (f\_)\*(x\_))^(m\_)]/((a\_) + (b\_)\*Sinh[(c\_) + (d\_)\*(x\_)]), x\_Symbol] :=> Dist[1/a, Int[(e + f\*x)^m\*Cosh[c + d\*x]^p\*Coth[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(p + 1)\*Coth[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5450

Int[Cosh[(a\_) + (b\_)\*(x\_)]^(n\_)\*Coth[(a\_) + (b\_)\*(x\_)]^(p\_)\*((c\_) + (d\_)\*(x\_))^(m\_), x\_Symbol] :=> Int[(c + d\*x)^m\*Cosh[a + b\*x]^n\*Coth[a + b\*x]^(p - 2), x] + Int[(c + d\*x)^m\*Cosh[a + b\*x]^(n - 2)\*Coth[a + b\*x]^p, x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 3296

Int[((c\_) + (d\_)\*(x\_))^(m\_)\*sin[(e\_) + (f\_)\*(x\_)], x\_Symbol] :=> -Simp[(c + d\*x)^m\*Cos[e + f\*x]/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2638

Int[sin[(c\_) + (d\_)\*(x\_)], x\_Symbol] :=> -Simp[Cos[c + d\*x]/d, x] /; FreeQ[{c, d}, x]

### Rule 5452

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x))]/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 5446

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] := Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n +
1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

### Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbo
l] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Ssin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Ssin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Ssin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Ssin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

### Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] := -Simp[(b*Cos[c + d*x
]*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

### Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

### Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
```

```
) * Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m * Cosh[c + d
*x]^(n - 2))/(a + b * Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m * E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b * E^(c + d*x))
, x] + Int[((e + f*x)^m * E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b * E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(e+fx)^3 \coth^3(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \coth^3(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^3 \cosh(c+dx) \coth^2(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= -\frac{(e+fx)^3 \coth^2(c+dx)}{2ad} + \frac{\int (e+fx)^3 \coth(c+dx) dx}{a} - \frac{b \int (e+fx)^3 \cosh(c+dx) dx}{a^2} \\
&= -\frac{(e+fx)^4}{4af} - \frac{3f(e+fx)^2 \coth(c+dx)}{2ad^2} - \frac{(e+fx)^3 \coth^2(c+dx)}{2ad} - \frac{2 \int \frac{e^{2(c+dx)}(e+fx)^3}{1-e^{2(c+dx)}} dx}{a} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{3f(e+fx)^2 \coth(c+dx)}{2ad^2} - \frac{(e+fx)^3 \coth^2(c+dx)}{2ad} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{b^2(e+fx)^4}{4a^3f} + \frac{(a^2+b^2)(e+fx)^4}{4a^3f} + \frac{6bf(e+fx)^4}{4a^3f} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{b^2(e+fx)^4}{4a^3f} + \frac{(a^2+b^2)(e+fx)^4}{4a^3f} + \frac{6bf(e+fx)^4}{4a^3f} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{b^2(e+fx)^4}{4a^3f} + \frac{(a^2+b^2)(e+fx)^4}{4a^3f} + \frac{6bf(e+fx)^4}{4a^3f} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{b^2(e+fx)^4}{4a^3f} + \frac{(a^2+b^2)(e+fx)^4}{4a^3f} + \frac{6bf(e+fx)^4}{4a^3f} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{b^2(e+fx)^4}{4a^3f} + \frac{(a^2+b^2)(e+fx)^4}{4a^3f} + \frac{6bf(e+fx)^4}{4a^3f} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{b^2(e+fx)^4}{4a^3f} + \frac{(a^2+b^2)(e+fx)^4}{4a^3f} + \frac{6bf(e+fx)^4}{4a^3f} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{(e+fx)^4}{4af} - \frac{b^2(e+fx)^4}{4a^3f} + \frac{(a^2+b^2)(e+fx)^4}{4a^3f} + \frac{6bf(e+fx)^4}{4a^3f}
\end{aligned}$$

**Mathematica [C]** time = 73.7676, size = 11848, normalized size = 12.19

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^3\*Coth[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] Result too large to show

---

**Maple [F]** time = 0.872, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 (\coth(dx + c))^3}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*coth(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*coth(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*coth(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -e^3*(2*(b*e^{(-d*x - c)} - a*e^{(-2*d*x - 2*c)} - b*e^{(-3*d*x - 3*c)})/((2*a^2* \\ & e^{(-2*d*x - 2*c)} - a^2*e^{(-4*d*x - 4*c)} - a^2)*d) + (a^2 + b^2)*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/(a^3*d) - (a^2 + b^2)*\log(e^{(-d*x - c)} + 1)/(a^3*d) - (a^2 + b^2)*\log(e^{(-d*x - c)} - 1)/(a^3*d)) + (3*a*f^3*x^2 + 6*a*e*f^2*x + 3*a*e^2*f + 2*(b*d*f^3*x^3*e^{(3*c)} + 3*b*d*e*f^2*x^2*e^{(3*c)} + 3*b*d*e^2*f*x*e^{(3*c)})*e^{(3*d*x)} - (2*a*d*f^3*x^3*e^{(2*c)} + 3*a*e^2*f*e^{(2*c)} + 3*(2*d*e*f^2 + f^3)*a*x^2*e^{(2*c)} + 6*(d*e^2*f + e*f^2)*a*x*e^{(2*c)})*e^{(2*d*x)} - 2*(b*d*f^3*x^3*e^c + 3*b*d*e*f^2*x^2*e^c + 3*b*d*e^2*f*x*e^c)*e^{(d*x)})/(a^2*d^2*e^{(4*d*x + 4*c)} - 2*a^2*d^2*e^{(2*d*x + 2*c)} + a^2*d^2) - 3*(b*d*e^2*f + a*e*f^2)*x/(a^2*d^2) + 3*(b*d*e^2*f - a*e*f^2)*x/(a^2*d^2) + 3*(b*d*e^2*f + a*e*f^2)*\log(e^{(d*x + c)} + 1)/(a^2*d^3) - 3*(b*d*e^2*f - a*e*f^2)*\log(e^{(d*x + c)} - 1)/(a^2*d^3) + (d^3*x^3*\log(e^{(d*x + c)} + 1) + 3*d^2*x^2*dilog(-e^{(d*x + c)}) - 6*d*x*polylog(3, -e^{(d*x + c)}) + 6*polylog(4, -e^{(d*x + c)}))*(a^2*f^3 + b^2*f^3)/(a^3*d^4) + (d^3*x^3*\log(-e^{(d*x + c)} + 1) + 3*d^2*x^2*dilog(e^{(d*x + c)}) - 6*d*x*polylog(3, e^{(d*x + c)}) + 6*polylog(4, e^{(d*x + c)}))*(a^2*f^3 + b^2*f^3)/(a^3*d^4) + 3*(a^2*d*e*f^2 + b^2*d*e*f^2 + a*b*f^3)*(d^2*x^2*\log(e^{(d*x + c)} + 1) + 2*d*x*dilog(-e^{(d*x + c)}) - 2*polylog(3, -e^{(d*x + c)}))/(a^3*d^4) + 3*(a^2*d*e*f^2 + b^2*d*e*f^2 - a*b*f^3)*(d^2*x^2*\log(-e^{(d*x + c)} + 1) + 2*d*x*dilog(e^{(d*x + c)}) - 2*polylog(3, e^{(d*x + c)}))/(a^3*d^4) + 3*(b^2*d^2*e^2*f + 2*a*b*d*e*f^2 + (d^2*e^2*f + f^3)*a^2)*(d*x*\log(e^{(d*x + c)} + 1) + dilog(-e^{(d*x + c)}))/(a^3*d^4) + \end{aligned}$$

$$3*(b^2*d^2*e^2*f - 2*a*b*d*e*f^2 + (d^2*e^2*f + f^3)*a^2)*(d*x*\log(-e^(d*x + c) + 1) + \operatorname{dilog}(e^(d*x + c)))/(a^3*d^4) - 1/4*((a^2*f^3 + b^2*f^3)*d^4*x^4 + 4*(a^2*d*e*f^2 + b^2*d*e*f^2 + a*b*f^3)*d^3*x^3 + 6*(b^2*d^2*e^2*f + 2*a*b*d*e*f^2 + (d^2*e^2*f + f^3)*a^2)*d^2*x^2)/(a^3*d^4) - 1/4*((a^2*f^3 + b^2*f^3)*d^4*x^4 + 4*(a^2*d*e*f^2 + b^2*d*e*f^2 - a*b*f^3)*d^3*x^3 + 6*(b^2*d^2*e^2*f - 2*a*b*d*e*f^2 + (d^2*e^2*f + f^3)*a^2)*d^2*x^2)/(a^3*d^4) + \operatorname{integrate}(-2*((a^2*b*f^3 + b^3*f^3)*x^3 + 3*(a^2*b*e*f^2 + b^3*e*f^2)*x^2 + 3*(a^2*b*e^2*f + b^3*e^2*f)*x - ((a^3*f^3*e^c + a*b^2*f^3*e^c)*x^3 + 3*(a^3*e*f^2*e^c + a*b^2*e*f^2*e^c)*x^2 + 3*(a^3*e^2*f*e^c + a*b^2*e^2*f*e^c)*x)*e^(d*x))/(a^3*b*e^(2*d*x + 2*c) + 2*a^4*e^(d*x + c) - a^3*b), x)$$

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)**3*coth(d*x+c)**3/(a+b*sinh(d*x+c)),x)`

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.487 \quad \int \frac{(e+fx)^2 \coth^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=689

$$\frac{2f(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2} - \frac{2f(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^2} + \frac{b^2f(e+fx)\text{PolyLog}}{a^3d^2}$$

```
[Out] (e*f*x)/(a*d) + (f^2*x^2)/(2*a*d) - (e + f*x)^3/(3*a*f) - (b^2*(e + f*x)^3)
/(3*a^3*f) + ((a^2 + b^2)*(e + f*x)^3)/(3*a^3*f) + (4*b*f*(e + f*x)*ArcTanh
[E^(c + d*x)])/(a^2*d^2) - (f*(e + f*x)*Coth[c + d*x])/(a*d^2) - ((e + f*x)
^2*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*x)^2*Csch[c + d*x])/(a^2*d) - ((a^2
+ b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(a^3*d)
- ((a^2 + b^2)*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])
)/(a^3*d) + ((e + f*x)^2*Log[1 - E^(2*(c + d*x))])/(a*d) + (b^2*(e + f*x)^2*
Log[1 - E^(2*(c + d*x))])/(a^3*d) + (f^2*Log[Sinh[c + d*x]])/(a*d^3) + (2*b
*f^2*PolyLog[2, -E^(c + d*x)]/(a^2*d^3) - (2*b*f^2*PolyLog[2, E^(c + d*x)]
)/(a^2*d^3) - (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2]))])/(a^3*d^2) - (2*(a^2 + b^2)*f*(e + f*x)*PolyLog[2, -((b*
E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*d^2) + (f*(e + f*x)*PolyLog[2, E
^(2*(c + d*x))])/(a*d^2) + (b^2*f*(e + f*x)*PolyLog[2, E^(2*(c + d*x))])/(a
^3*d^2) + (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b
^2]))])/(a^3*d^3) + (2*(a^2 + b^2)*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sq
rt[a^2 + b^2]))])/(a^3*d^3) - (f^2*PolyLog[3, E^(2*(c + d*x))])/(2*a*d^3) -
(b^2*f^2*PolyLog[3, E^(2*(c + d*x))])/(2*a^3*d^3)
```

**Rubi [A]** time = 1.7085, antiderivative size = 689, normalized size of antiderivative = 1., number of steps used = 47, number of rules used = 20, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$ , Rules used = {5569, 3720, 3475, 3716, 2190, 2531, 2282, 6589, 5585, 5450, 3296, 2637, 5452, 4182, 2279, 2391, 5446, 3310, 5565, 5561}

$$\frac{2f(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3d^2} - \frac{2f(a^2+b^2)(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3d^2} + \frac{b^2f(e+fx)\text{PolyLog}}{a^3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[((e + f*x)^2*Coth[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (e*f*x)/(a*d) + (f^2*x^2)/(2*a*d) - (e + f*x)^3/(3*a*f) - (b^2*(e + f*x)^3)
/(3*a^3*f) + ((a^2 + b^2)*(e + f*x)^3)/(3*a^3*f) + (4*b*f*(e + f*x)*ArcTanh
[E^(c + d*x)])/(a^2*d^2) - (f*(e + f*x)*Coth[c + d*x])/(a*d^2) - ((e + f*x)
```

$$\begin{aligned} & ^2 \operatorname{Coth}[c + d*x]^2 / (2*a*d) + (b*(e + f*x)^2 \operatorname{Csch}[c + d*x]) / (a^2*d) - ((a^2 + b^2)*(e + f*x)^2 \operatorname{Log}[1 + (b*E^c + d*x)] / (a - \operatorname{Sqrt}[a^2 + b^2])) / (a^3*d) \\ & - ((a^2 + b^2)*(e + f*x)^2 \operatorname{Log}[1 + (b*E^c + d*x)] / (a + \operatorname{Sqrt}[a^2 + b^2])) / (a^3*d) + ((e + f*x)^2 \operatorname{Log}[1 - E^{2*(c + d*x)}]) / (a*d) + (b^2*(e + f*x)^2 \operatorname{Log}[1 - E^{2*(c + d*x)}]) / (a^3*d) \\ & + (f^2 \operatorname{Log}[\operatorname{Sinh}[c + d*x]]) / (a*d^3) + (2*b*f^2 \operatorname{PolyLog}[2, -E^{c + d*x}]) / (a^2*d^3) - (2*b*f^2 \operatorname{PolyLog}[2, E^{c + d*x}]) / (a^2*d^3) \\ & - (2*(a^2 + b^2)*f*(e + f*x)*\operatorname{PolyLog}[2, -(b*E^{c + d*x}) / (a - \operatorname{Sqrt}[a^2 + b^2])]) / (a^3*d^2) - (2*(a^2 + b^2)*f*(e + f*x)*\operatorname{PolyLog}[2, -(b*E^{c + d*x}) / (a + \operatorname{Sqrt}[a^2 + b^2])]) / (a^3*d^2) \\ & + (f*(e + f*x)*\operatorname{PolyLog}[2, E^{2*(c + d*x)}]) / (a*d^2) + (b^2*f*(e + f*x)*\operatorname{PolyLog}[2, E^{2*(c + d*x)}]) / (a^3*d^2) \\ & + (2*(a^2 + b^2)*f^2 \operatorname{PolyLog}[3, -(b*E^{c + d*x}) / (a - \operatorname{Sqrt}[a^2 + b^2])]) / (a^3*d^3) + (2*(a^2 + b^2)*f^2 \operatorname{PolyLog}[3, -(b*E^{c + d*x}) / (a + \operatorname{Sqrt}[a^2 + b^2])]) / (a^3*d^3) \\ & - (f^2 \operatorname{PolyLog}[3, E^{2*(c + d*x)}]) / (2*a*d^3) - (b^2*f^2 \operatorname{PolyLog}[3, E^{2*(c + d*x)}]) / (2*a^3*d^3) \end{aligned}$$

### Rule 5569

$$\operatorname{Int}[(\operatorname{Coth}[(c_.) + (d_.)*(x_)]^{(n_.)} * ((e_.) + (f_.)*(x_))^{(m_.)}) / ((a_.) + (b_.) * \operatorname{Sinh}[(c_.) + (d_.)*(x_)]), x\_Symbol] \rightarrow \operatorname{Dist}[1/a, \operatorname{Int}[(e + f*x)^m \operatorname{Coth}[c + d*x]^n, x], x] - \operatorname{Dist}[b/a, \operatorname{Int}[(e + f*x)^m \operatorname{Cosh}[c + d*x] \operatorname{Coth}[c + d*x]^{(n-1)} / (a + b * \operatorname{Sinh}[c + d*x]), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \operatorname{IGtQ}[m, 0] \&\& \operatorname{IGtQ}[n, 0]$$

### Rule 3720

$$\operatorname{Int}[(c_.) + (d_.)*(x_)]^{(m_.)} * ((b_.) * \tan[(e_.) + (f_.)*(x_)]^{(n_.)}), x\_Symbol] \rightarrow \operatorname{Simp}[(b*(c + d*x)^m * (b * \tan[e + f*x])^{(n-1)}) / (f*(n-1)), x] + (-\operatorname{Dist}[(b*d^m) / (f*(n-1)), \operatorname{Int}[(c + d*x)^{(m-1)} * (b * \tan[e + f*x])^{(n-1)}, x], x] - \operatorname{Dist}[b^2, \operatorname{Int}[(c + d*x)^m * (b * \tan[e + f*x])^{(n-2)}, x], x]) /; \operatorname{FreeQ}\{b, c, d, e, f\}, x] \&\& \operatorname{GtQ}[n, 1] \&\& \operatorname{GtQ}[m, 0]$$

### Rule 3475

$$\operatorname{Int}[\tan[(c_.) + (d_.)*(x_)], x\_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[\operatorname{Cos}[c + d*x], x]] / d, x] /; \operatorname{FreeQ}\{c, d\}, x]$$

### Rule 3716

$$\operatorname{Int}[(c_.) + (d_.)*(x_)]^{(m_.)} * \tan[(e_.) + \operatorname{Pi}*(k_.) + (\operatorname{Complex}[0, fz_])*(f_.)*(x_)], x\_Symbol] \rightarrow -\operatorname{Simp}[(I*(c + d*x)^{(m+1)}) / (d*(m+1)), x] + \operatorname{Dist}[2*I, \operatorname{Int}[(c + d*x)^m * E^{2*(-(I*e) + f*fz*x)}] / (E^{2*I*k*Pi} * (1 + E^{2*(-(I*e) + f*fz*x)}) / E^{2*I*k*Pi}), x], x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x] \&\& \operatorname{IntegerQ}[4*k] \&\& \operatorname{IGtQ}[m, 0]$$

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)
*(x_))^(m_), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]), x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_)*((a_) + (b_)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_)*((a_) + (b_)*(x_))^(p_)]/((d_) + (e_)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 5585

```
Int[(Cosh[(c_) + (d_)*(x_)]^(p_)*Coth[(c_) + (d_)*(x_)]^(n_))*((e_) +
(f_)*(x_))^(m_)]/((a_) + (b_)*Sinh[(c_) + (d_)*(x_)]), x_Symbol] := D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_) + (b_)*(x_)]^(n_)*Coth[(a_) + (b_)*(x_)]^(p_)*((c_) +
(d_)*(x_))^(m_), x_Symbol] := Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*x
]^p, x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
```

/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 3296

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*sin[(e\_.) + (f\_.)\*(x\_)], x\_Symbol] := -Simp[  
((c + d\*x)^m\*Cos[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cos[  
e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 2637

Int[sin[Pi/2 + (c\_.) + (d\_.)\*(x\_)], x\_Symbol] := Simp[Sin[c + d\*x]/d, x] /;  
FreeQ[{c, d}, x]

### Rule 5452

Int[Coth[(a\_.) + (b\_.)\*(x\_)]^(p\_.)\*Csch[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*((c\_.) +  
(d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Csch[a + b\*x]^n)/(b\*n),  
x] + Dist[(d\*m)/(b\*n), Int[(c + d\*x)^(m - 1)\*Csch[a + b\*x]^n, x], x] /; Fre  
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

### Rule 4182

Int[csc[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x  
\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)])/(f\*fz\*I), x]  
+ (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)  
], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) +  
f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol]  
:= Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x))  
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2  
, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 5446

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sinh[(a\_.) + (b\_.)\*  
(x\_)]^(n\_.), x\_Symbol] := Simp[((c + d\*x)^m\*Sinh[a + b\*x]^(n + 1))/(b\*(n +  
1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*Sinh[a + b\*x]^(n +



1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

### Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
  Simp[(d*(b*SIN[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*SIN[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*SIN[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

### Rule 5565

```
Int[(Cosh[(c_.) + (d_.)*(x_)]^(n_)*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.
)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[a/b^2, Int[(e + f*x)^m*Cosh
[c + d*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f*x)^m*Cosh[c + d*x]^(n - 2
)*Sinh[c + d*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f*x)^m*Cosh[c + d
*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]
```

### Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \coth^3(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \coth^3(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^2 \cosh(c+dx) \coth^2(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= -\frac{(e+fx)^2 \coth^2(c+dx)}{2ad} + \frac{\int (e+fx)^2 \coth(c+dx) dx}{a} - \frac{b \int (e+fx)^2 \cosh(c+dx) \cot}{a^2} \\
&= -\frac{(e+fx)^3}{3af} - \frac{f(e+fx) \coth(c+dx)}{ad^2} - \frac{(e+fx)^2 \coth^2(c+dx)}{2ad} - \frac{2 \int \frac{e^{2(c+dx)}(e+fx)^2}{1-e^{2(c+dx)}} dx}{a} \\
&= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{(e+fx)^3}{3af} - \frac{f(e+fx) \coth(c+dx)}{ad^2} - \frac{(e+fx)^2 \coth^2(c+dx)}{2ad} + \frac{b(e+fx)}{a^2} \\
&= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{(e+fx)^3}{3af} - \frac{b^2(e+fx)^3}{3a^3f} + \frac{(a^2+b^2)(e+fx)^3}{3a^3f} + \frac{4bf(e+fx) \tanh^{-1}(e^{c+dx})}{a^2d^2} \\
&= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{(e+fx)^3}{3af} - \frac{b^2(e+fx)^3}{3a^3f} + \frac{(a^2+b^2)(e+fx)^3}{3a^3f} + \frac{4bf(e+fx) \tanh^{-1}(e^{c+dx})}{a^2d^2} \\
&= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{(e+fx)^3}{3af} - \frac{b^2(e+fx)^3}{3a^3f} + \frac{(a^2+b^2)(e+fx)^3}{3a^3f} + \frac{4bf(e+fx) \tanh^{-1}(e^{c+dx})}{a^2d^2} \\
&= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{(e+fx)^3}{3af} - \frac{b^2(e+fx)^3}{3a^3f} + \frac{(a^2+b^2)(e+fx)^3}{3a^3f} + \frac{4bf(e+fx) \tanh^{-1}(e^{c+dx})}{a^2d^2} \\
&= \frac{efx}{ad} + \frac{f^2x^2}{2ad} - \frac{(e+fx)^3}{3af} - \frac{b^2(e+fx)^3}{3a^3f} + \frac{(a^2+b^2)(e+fx)^3}{3a^3f} + \frac{4bf(e+fx) \tanh^{-1}(e^{c+dx})}{a^2d^2}
\end{aligned}$$

**Mathematica [B]** time = 42.2086, size = 2277, normalized size = 3.3

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Coth[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] (b\*(e + f\*x)^2\*Csch[c])/(a^2\*d) - ((e + f\*x)^2\*Csch[(c + d\*x)/2]^2)/(8\*a\*d) - (12\*a^2\*d^3\*e^2\*E^(2\*c)\*x + 12\*b^2\*d^3\*e^2\*E^(2\*c)\*x + 12\*a^2\*d\*E^(2\*c)\*f^2\*x + 12\*a^2\*d^3\*e\*E^(2\*c)\*f\*x^2 + 12\*b^2\*d^3\*e\*E^(2\*c)\*f\*x^2 + 4\*a^2\*d^3\*E^(2\*c)\*f^2\*x^3 + 4\*b^2\*d^3\*E^(2\*c)\*f^2\*x^3 + 24\*a\*b\*d\*e\*f\*ArcTanh[E^(c + d\*x)] - 24\*a\*b\*d\*e\*E^(2\*c)\*f\*ArcTanh[E^(c + d\*x)] - 12\*a\*b\*d\*f^2\*x\*Log[1 -

$$\begin{aligned}
& E^c + dx] + 12abde^{2c}f^2x \log[1 - E^{c+dx}] + 12abd^2f^2x \\
& \log[1 + E^{c+dx}] - 12abd^2e^{2c}f^2x \log[1 + E^{c+dx}] + 6a^2 \\
& d^2e^{2c} \log[1 - E^{2(c+dx)}] + 6b^2d^2e^{2c} \log[1 - E^{2(c+dx)}] \\
& - 6a^2d^2e^{2c} \log[1 - E^{2(c+dx)}] - 6b^2d^2e^{2c} \log[1 - E^{2(c+dx)}] \\
& + 6a^2f^2 \log[1 - E^{2(c+dx)}] - 6a^2E^{2c}f^2 \log[1 - E^{2(c+dx)}] \\
& + 12a^2d^2efx \log[1 - E^{2(c+dx)}] + 12b^2d^2efx \log[1 - E^{2(c+dx)}] \\
& - 12a^2d^2eE^{2c}fx \log[1 - E^{2(c+dx)}] - 12b^2d^2eE^{2c}fx \log[1 - E^{2(c+dx)}] \\
& + 6a^2d^2f^2x^2 \log[1 - E^{2(c+dx)}] + 6b^2d^2f^2x^2 \log[1 - E^{2(c+dx)}] \\
& - 6a^2d^2E^{2c}f^2x^2 \log[1 - E^{2(c+dx)}] - 6b^2d^2E^{2c}f^2x^2 \log[1 - E^{2(c+dx)}] \\
& - 12ab(-1 + E^{2c})f^2 \text{PolyLog}[2, E^{c+dx}] + 6a^2d^2ef \text{PolyLog}[2, E^{2(c+dx)}] \\
& + 6b^2d^2ef \text{PolyLog}[2, E^{2(c+dx)}] - 6a^2d^2eE^{2c}f \text{PolyLog}[2, E^{2(c+dx)}] \\
& - 6b^2d^2eE^{2c}f \text{PolyLog}[2, E^{2(c+dx)}] + 6a^2d^2f^2x \text{PolyLog}[2, E^{2(c+dx)}] \\
& + 6b^2d^2f^2x \text{PolyLog}[2, E^{2(c+dx)}] - 6a^2d^2E^{2c}f^2x \text{PolyLog}[2, E^{2(c+dx)}] \\
& - 6b^2d^2E^{2c}f^2x \text{PolyLog}[2, E^{2(c+dx)}] - 3a^2f^2 \text{PolyLog}[3, E^{2(c+dx)}] \\
& - 3b^2f^2 \text{PolyLog}[3, E^{2(c+dx)}] + 3a^2E^{2c}f^2 \text{PolyLog}[3, E^{2(c+dx)}] \\
& + 3b^2E^{2c}f^2 \text{PolyLog}[3, E^{2(c+dx)}] / (6a^3d^3(-1 + E^{2c})) + ((a^2 + b^2)(6e^{2c} \\
& x + 6eE^{2c}fx^2 + 2E^{2c}f^2x^3 + (6a\sqrt{a^2 + b^2}e^{2c} \text{ArcTan}[(a + bE^{c+dx}) / \sqrt{-a^2 - b^2}] / (\sqrt{-(a^2 + b^2)^2}d) + \\
& (6a\sqrt{-(a^2 + b^2)^2}e^{2c} \text{ArcTan}[(a + bE^{c+dx}) / \sqrt{-a^2 - b^2}] / ((a^2 + b^2)^{3/2}d) - (6a\sqrt{-(a^2 + b^2)^2}e^{2c} \text{ArcTanh}[(a + \\
& bE^{c+dx}) / \sqrt{a^2 + b^2}] / ((-a^2 - b^2)^{3/2}d) + (6a\sqrt{-(a^2 + b^2)^2}e^{2c} \text{ArcTanh}[(a + bE^{c+dx}) / \sqrt{a^2 + b^2}] / ((-a^2 - \\
& b^2)^{3/2}d) + (3e^{2c} \log[2aE^{c+dx} + b(-1 + E^{2(c+dx)})])) / d \\
& - (3e^{2c}E^{2c} \log[2aE^{c+dx} + b(-1 + E^{2(c+dx)})]) / d + (6efx \log[1 + (bE^{2c+dx}) / (aE^c - \sqrt{(a^2 + b^2)E^{2c}})]) / d - (6 \\
& eE^{2c}fx \log[1 + (bE^{2c+dx}) / (aE^c - \sqrt{(a^2 + b^2)E^{2c}})]) / d + (3f^2x^2 \log[1 + (bE^{2c+dx}) / (aE^c - \sqrt{(a^2 + b^2)E^{2c}})]) / d - (3E^{2c}f^2x^2 \log[1 + (bE^{2c+dx}) / (aE^c - \sqrt{(a^2 + b^2)E^{2c}})]) / d + (6efx \log[1 + (bE^{2c+dx}) / (aE^c + \sqrt{(a^2 + b^2)E^{2c}})]) / d - (6eE^{2c}fx \log[1 + (bE^{2c+dx}) / (aE^c + \sqrt{(a^2 + b^2)E^{2c}})]) / d + (3f^2x^2 \log[1 + (bE^{2c+dx}) / (aE^c + \sqrt{(a^2 + b^2)E^{2c}})]) / d - (3E^{2c}f^2x^2 \log[1 + (bE^{2c+dx}) / (aE^c + \sqrt{(a^2 + b^2)E^{2c}})]) / d - (6(-1 + E^{2c})f(e + fx) \text{PolyLog}[2, -(bE^{2c+dx}) / (aE^c - \sqrt{(a^2 + b^2)E^{2c}})]) / d^2 - (6(-1 + E^{2c})f(e + fx) \text{PolyLog}[2, -(bE^{2c+dx}) / (aE^c + \sqrt{(a^2 + b^2)E^{2c}})]) / d^2 - (6f^2 \text{PolyLog}[3, -(bE^{2c+dx}) / (aE^c - \sqrt{(a^2 + b^2)E^{2c}})]) / d^3 + (6E^{2c}f^2 \text{PolyLog}[3, -(bE^{2c+dx}) / (aE^c - \sqrt{(a^2 + b^2)E^{2c}})]) / d^3 - (6f^2 \text{PolyLog}[3, -(bE^{2c+dx}) / (aE^c + \sqrt{(a^2 + b^2)E^{2c}})]) / d^3 + (6E^{2c}f^2 \text{PolyLog}[3, -(bE^{2c+dx}) / (aE^c + \sqrt{(a^2 + b^2)E^{2c}})]) / d^3 + (6E^{2c}f^2 \text{PolyLog}[3, -(bE^{2c+dx}) / (aE^c + \sqrt{(a^2 + b^2)E^{2c}})]) / d^3)) / (3a^3(-1 + E^{2c})) + ((e + fx)^2 \text{Sech}[(c + dx) / 2])^2 / (8a
\end{aligned}$$

```
*d) - ((e + f*x)*(-(a*f) + b*d*(e + f*x))*Csch[c/2]*Csch[(c + d*x)/2]*Sinh[
(d*x)/2])/(2*a^2*d^2) - ((e + f*x)*(a*f + b*d*(e + f*x))*Sech[c/2]*Sech[(c
+ d*x)/2]*Sinh[(d*x)/2])/(2*a^2*d^2)
```

**Maple [F]** time = 0.671, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\coth(dx + c))^3}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

```
[Out] int((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*coth(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] -e^2*(2*(b*e^(-d*x - c) - a*e^(-2*d*x - 2*c) - b*e^(-3*d*x - 3*c)))/((2*a^2*
e^(-2*d*x - 2*c) - a^2*e^(-4*d*x - 4*c) - a^2)*d) + (a^2 + b^2)*log(-2*a*e^
(-d*x - c) + b*e^(-2*d*x - 2*c) - b)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x - c)
+ 1)/(a^3*d) - (a^2 + b^2)*log(e^(-d*x - c) - 1)/(a^3*d) + 2*(a*f^2*x + a
*e*f + (b*d*f^2*x^2*e^(3*c) + 2*b*d*e*f*x*e^(3*c))*e^(3*d*x) - (a*d*f^2*x^2
*e^(2*c) + a*e*f*e^(2*c) + (2*d*e*f + f^2)*a*x*e^(2*c))*e^(2*d*x) - (b*d*f^
2*x^2*e^c + 2*b*d*e*f*x*e^c)*e^(d*x))/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*
e^(2*d*x + 2*c) + a^2*d^2) - (2*b*d*e*f + a*f^2)*x/(a^2*d^2) + (2*b*d*e*f -
a*f^2)*x/(a^2*d^2) + (2*b*d*e*f + a*f^2)*log(e^(d*x + c) + 1)/(a^2*d^3) -
(2*b*d*e*f - a*f^2)*log(e^(d*x + c) - 1)/(a^2*d^3) + (d^2*x^2*log(e^(d*x +
c) + 1) + 2*d*x*dilog(-e^(d*x + c)) - 2*polylog(3, -e^(d*x + c)))*(a^2*f^2
+ b^2*f^2)/(a^3*d^3) + (d^2*x^2*log(-e^(d*x + c) + 1) + 2*d*x*dilog(e^(d*x
+ c)) - 2*polylog(3, e^(d*x + c)))*(a^2*f^2 + b^2*f^2)/(a^3*d^3) + 2*(a^2*d
*e*f + b^2*d*e*f + a*b*f^2)*(d*x*log(e^(d*x + c) + 1) + dilog(-e^(d*x + c)
))/(a^3*d^3) + 2*(a^2*d*e*f + b^2*d*e*f - a*b*f^2)*(d*x*log(-e^(d*x + c) + 1
) + dilog(e^(d*x + c)))/(a^3*d^3) - 1/3*((a^2*f^2 + b^2*f^2)*d^3*x^3 + 3*(a
```

$$\begin{aligned} & \frac{2d^2ef + b^2d^2ef + a^2bf^2}{a^3d^3} - \frac{1}{3} \frac{(a^2f^2 + b^2f^2)d^3x^3 + 3(a^2d^2ef + b^2d^2ef - a^2bf^2)d^2x^2}{a^3d^3} + \text{integrate} \\ & (-2((a^2bf^2 + b^3f^2)x^2 + 2(a^2b^2ef + b^3e^2f)x - ((a^3f^2e^c + a^2b^2f^2e^c)x^2 + 2(a^3e^2f^2e^c + a^2b^2e^2f^2e^c)x))e^{(dx)}) / (a^3 \\ & * b^2e^{(2dx + 2c)} + 2a^4e^{(dx + c)} - a^3b), x \end{aligned}$$

**Fricas [C]** time = 2.99641, size = 18048, normalized size = 26.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*coth(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $(2a^2d^2ef - 2a^2c^2f^2 - 2(a^2d^2f^2x + a^2c^2f^2)\cosh(dx + c))^4 - 2(a^2d^2f^2x + a^2c^2f^2)\sinh(dx + c)^4 + 2(a^2b^2d^2f^2x^2 + 2a^2b^2d^2e^2f^2x + a^2b^2d^2e^2)\cosh(dx + c)^3 + 2(a^2b^2d^2f^2x^2 + 2a^2b^2d^2e^2f^2x + a^2b^2d^2e^2 - 4(a^2d^2f^2x + a^2c^2f^2)\cosh(dx + c))\sinh(dx + c)^3 - 2(a^2d^2f^2x^2 + a^2d^2e^2 + a^2d^2ef - 2a^2c^2f^2 + (2a^2d^2e^2f - a^2d^2f^2)x)\cosh(dx + c)^2 - 2(a^2d^2f^2x^2 + a^2d^2e^2 + a^2d^2ef - 2a^2c^2f^2 + 6(a^2d^2f^2x + a^2c^2f^2)\cosh(dx + c)^2 + (2a^2d^2e^2f - a^2d^2f^2)x - 3(a^2b^2d^2f^2x^2 + 2a^2b^2d^2e^2f^2x + a^2b^2d^2e^2)\cosh(dx + c))\sinh(dx + c)^2 - 2(a^2b^2d^2f^2x^2 + 2a^2b^2d^2e^2f^2x + a^2b^2d^2e^2)\cosh(dx + c) - 2((a^2 + b^2)d^2f^2x + ((a^2 + b^2)d^2f^2x + (a^2 + b^2)d^2ef)\cosh(dx + c))^4 + 4((a^2 + b^2)d^2f^2x + (a^2 + b^2)d^2ef)\cosh(dx + c)\sinh(dx + c)^3 + ((a^2 + b^2)d^2f^2x + (a^2 + b^2)d^2ef)\sinh(dx + c)^4 + (a^2 + b^2)d^2ef - 2((a^2 + b^2)d^2f^2x + (a^2 + b^2)d^2ef)\cosh(dx + c)^2 - 2((a^2 + b^2)d^2f^2x + (a^2 + b^2)d^2ef) - 3((a^2 + b^2)d^2f^2x + (a^2 + b^2)d^2ef)\cosh(dx + c)^2)\sinh(dx + c)^2 + 4(((a^2 + b^2)d^2f^2x + (a^2 + b^2)d^2ef)\cosh(dx + c))^3 - ((a^2 + b^2)d^2f^2x + (a^2 + b^2)d^2ef)\cosh(dx + c))\sinh(dx + c))*\text{dilog}((a\cosh(dx + c) + a\sinh(dx + c) + (b\cosh(dx + c) + b\sinh(dx + c))\sqrt{\text{rt}((a^2 + b^2)/b^2) - b}/b + 1) - 2((a^2 + b^2)d^2f^2x + ((a^2 + b^2)d^2f^2x + (a^2 + b^2)d^2ef)\cosh(dx + c))^4 + 4((a^2 + b^2)d^2f^2x + (a^2 + b^2)d^2ef)\cosh(dx + c)\sinh(dx + c))^3 + ((a^2 + b^2)d^2f^2x + (a^2 + b^2)d^2ef)\sinh(dx + c))^4 + (a^2 + b^2)d^2ef - 2((a^2 + b^2)d^2f^2x + (a^2 + b^2)d^2ef)\cosh(dx + c)^2 - 2((a^2 + b^2)d^2f^2x + (a^2 + b^2)d^2ef) - 3((a^2 + b^2)d^2f^2x + (a^2 + b^2)d^2ef)\cosh(dx + c)^2)\sinh(dx + c)^2 + 4(((a^2 + b^2)d^2f^2x + (a^2 + b^2)d^2ef)\cosh(dx + c))^3 - ((a^2 + b^2)d^2f^2x + (a^2 + b^2)d^2ef)\cosh(dx + c))\sinh(dx + c))*\text{dilog}((a\cosh(dx + c) + a\sinh(dx + c) - (b\cosh(dx + c) + b\sinh(dx + c))\sqrt{\text{rt}((a^2 + b^2)/b^2) - b}/b + 1) + 2((a^2 + b^2)d^2f^2x + ((a^2 + b^2)d^2$

$$\begin{aligned}
& *f^2*x + (a^2 + b^2)*d*e*f - a*b*f^2)*\cosh(d*x + c)^4 + 4*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f - a*b*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f - a*b*f^2)*\sinh(d*x + c)^4 + (a^2 + b^2)*d*e*f - a*b*f^2 - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f - a*b*f^2)*\cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f - a*b*f^2) - 3*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f - a*b*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f - a*b*f^2)*\cosh(d*x + c)^3 - ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f - a*b*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\operatorname{dilog}(\cosh(d*x + c) + \sinh(d*x + c)) + 2*((a^2 + b^2)*d*f^2*x + ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f + a*b*f^2)*\cosh(d*x + c)^4 + 4*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f + a*b*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f + a*b*f^2)*\sinh(d*x + c)^4 + (a^2 + b^2)*d*e*f + a*b*f^2 - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f + a*b*f^2)*\cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f + a*b*f^2) - 3*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f + a*b*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f + a*b*f^2)*\cosh(d*x + c)^3 - ((a^2 + b^2)*d*f^2*x + (a^2 + b^2)*d*e*f + a*b*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\operatorname{dilog}(-\cosh(d*x + c) - \sinh(d*x + c)) - ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2 + ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)^4 + 4*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\sinh(d*x + c)^4 - 2*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2) - 3*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)^3 - ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{((a^2 + b^2)/b^2) + 2*a} - ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2 + ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)^4 + 4*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\sinh(d*x + c)^4 - 2*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2) - 3*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)^3 - ((a^2 + b^2)*d^2*e^2 - 2*(a^2 + b^2)*c*d*e*f + (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{((a^2 + b^2)/b^2) + 2*a} - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2 + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)^4 + 4*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c
\end{aligned}$$

$$\begin{aligned}
& *d*e*f - (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^2 + b^2)* \\
& d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2) \\
& *c^2*f^2)*\sinh(d*x + c)^4 - 2*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2* \\
& e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)^2 - 2*(( \\
& a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - \\
& (a^2 + b^2)*c^2*f^2 - 3*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x \\
& + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + \\
& c)^2 + 4*(((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2) \\
& )*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)^3 - ((a^2 + b^2)*d^2*f^2*x^2 \\
& + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*c \\
& osh(d*x + c))*\sinh(d*x + c))*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*c \\
& osh(d*x + c) + b*\sinh(d*x + c))*\sqrt{((a^2 + b^2)/b^2) - b}/b) - ((a^2 + b^2) \\
& )*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2) \\
& )*c^2*f^2 + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + \\
& b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)^4 + 4*((a^2 + b^2)*d^2*f^ \\
& 2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f \\
& ^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2) \\
& )*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\sinh(d*x + c)^4 \\
& - 2*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d* \\
& e*f - (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*f^2*x^2 + 2 \\
& *(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2 - 3*(( \\
& a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - \\
& (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*(((a^2 + b^2)*d^2 \\
& *f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^ \\
& 2*f^2)*\cosh(d*x + c)^3 - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*d^2*e*f*x \\
& + 2*(a^2 + b^2)*c*d*e*f - (a^2 + b^2)*c^2*f^2)*\cosh(d*x + c))*\sinh(d*x + c \\
& ))*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x \\
& + c))*\sqrt{((a^2 + b^2)/b^2) - b}/b) + ((a^2 + b^2)*d^2*f^2*x^2 + (a^2 + b^2) \\
& )*d^2*e^2 + 2*a*b*d*e*f + ((a^2 + b^2)*d^2*f^2*x^2 + (a^2 + b^2)*d^2*e^2 + \\
& 2*a*b*d*e*f + a^2*f^2 + 2*((a^2 + b^2)*d^2*e*f + a*b*d*f^2)*x)*\cosh(d*x + c \\
& )^4 + 4*((a^2 + b^2)*d^2*f^2*x^2 + (a^2 + b^2)*d^2*e^2 + 2*a*b*d*e*f + a^2*f \\
& ^2 + 2*((a^2 + b^2)*d^2*e*f + a*b*d*f^2)*x)*\cosh(d*x + c)*\sinh(d*x + c)^3 \\
& + ((a^2 + b^2)*d^2*f^2*x^2 + (a^2 + b^2)*d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 + \\
& 2*((a^2 + b^2)*d^2*e*f + a*b*d*f^2)*x)*\sinh(d*x + c)^4 + a^2*f^2 - 2*((a^2 \\
& + b^2)*d^2*f^2*x^2 + (a^2 + b^2)*d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 + 2*((a^2 \\
& + b^2)*d^2*e*f + a*b*d*f^2)*x)*\cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*f^2*x^2 \\
& + (a^2 + b^2)*d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 - 3*((a^2 + b^2)*d^2*f^2*x^2 \\
& + (a^2 + b^2)*d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 + 2*((a^2 + b^2)*d^2*e*f + a \\
& *b*d*f^2)*x)*\cosh(d*x + c)^2 + 2*((a^2 + b^2)*d^2*e*f + a*b*d*f^2)*x)*\sinh( \\
& d*x + c)^2 + 2*((a^2 + b^2)*d^2*e*f + a*b*d*f^2)*x + 4*(((a^2 + b^2)*d^2*f^ \\
& 2*x^2 + (a^2 + b^2)*d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 + 2*((a^2 + b^2)*d^2*e* \\
& f + a*b*d*f^2)*x)*\cosh(d*x + c)^3 - ((a^2 + b^2)*d^2*f^2*x^2 + (a^2 + b^2)* \\
& d^2*e^2 + 2*a*b*d*e*f + a^2*f^2 + 2*((a^2 + b^2)*d^2*e*f + a*b*d*f^2)*x)*co \\
& sh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) + ((a^2 \\
& + b^2)*d^2*e^2 + ((a^2 + b^2)*d^2*e^2 - 2*(a*b + (a^2 + b^2)*c)*d*e*f + (2*
\end{aligned}$$

$$\begin{aligned}
& a*b*c + (a^2 + b^2)*c^2 + a^2*f^2)*\cosh(d*x + c)^4 + 4*((a^2 + b^2)*d^2*e^2 \\
& - 2*(a*b + (a^2 + b^2)*c)*d*e*f + (2*a*b*c + (a^2 + b^2)*c^2 + a^2*f^2)* \\
& \cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^2 + b^2)*d^2*e^2 - 2*(a*b + (a^2 + b^2) \\
& *c)*d*e*f + (2*a*b*c + (a^2 + b^2)*c^2 + a^2*f^2)*\sinh(d*x + c)^4 - 2*(a*b \\
& + (a^2 + b^2)*c)*d*e*f + (2*a*b*c + (a^2 + b^2)*c^2 + a^2*f^2 - 2*((a^2 + \\
& b^2)*d^2*e^2 - 2*(a*b + (a^2 + b^2)*c)*d*e*f + (2*a*b*c + (a^2 + b^2)*c^2 \\
& + a^2*f^2)*\cosh(d*x + c)^2 - 2*((a^2 + b^2)*d^2*e^2 - 2*(a*b + (a^2 + b^2) \\
& *c)*d*e*f + (2*a*b*c + (a^2 + b^2)*c^2 + a^2*f^2 - 3*((a^2 + b^2)*d^2*e^2 \\
& - 2*(a*b + (a^2 + b^2)*c)*d*e*f + (2*a*b*c + (a^2 + b^2)*c^2 + a^2*f^2)*\co \\
& sh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((a^2 + b^2)*d^2*e^2 - 2*(a*b + (a^2 + \\
& b^2)*c)*d*e*f + (2*a*b*c + (a^2 + b^2)*c^2 + a^2*f^2)*\cosh(d*x + c)^3 - (( \\
& a^2 + b^2)*d^2*e^2 - 2*(a*b + (a^2 + b^2)*c)*d*e*f + (2*a*b*c + (a^2 + b^2) \\
& *c^2 + a^2*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x \\
& + c) - 1) + ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*c*d*e*f + ((a^2 + b^2) \\
& )*d^2*f^2*x^2 + 2*(a^2 + b^2)*c*d*e*f - (2*a*b*c + (a^2 + b^2)*c^2)*f^2 + 2 \\
& *((a^2 + b^2)*d^2*e*f - a*b*d*f^2)*x)*\cosh(d*x + c)^4 + 4*((a^2 + b^2)*d^2* \\
& f^2*x^2 + 2*(a^2 + b^2)*c*d*e*f - (2*a*b*c + (a^2 + b^2)*c^2)*f^2 + 2*((a^2 \\
& + b^2)*d^2*e*f - a*b*d*f^2)*x)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^2 + b^2) \\
& )*d^2*f^2*x^2 + 2*(a^2 + b^2)*c*d*e*f - (2*a*b*c + (a^2 + b^2)*c^2)*f^2 + 2 \\
& *((a^2 + b^2)*d^2*e*f - a*b*d*f^2)*x)*\sinh(d*x + c)^4 - (2*a*b*c + (a^2 + b \\
& ^2)*c^2)*f^2 - 2*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*c*d*e*f - (2*a*b* \\
& c + (a^2 + b^2)*c^2)*f^2 + 2*((a^2 + b^2)*d^2*e*f - a*b*d*f^2)*x)*\cosh(d*x \\
& + c)^2 - 2*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*c*d*e*f - (2*a*b*c + (a \\
& ^2 + b^2)*c^2)*f^2 - 3*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*c*d*e*f - ( \\
& 2*a*b*c + (a^2 + b^2)*c^2)*f^2 + 2*((a^2 + b^2)*d^2*e*f - a*b*d*f^2)*x)*\cos \\
& h(d*x + c)^2 + 2*((a^2 + b^2)*d^2*e*f - a*b*d*f^2)*x)*\sinh(d*x + c)^2 + 2*( \\
& (a^2 + b^2)*d^2*e*f - a*b*d*f^2)*x + 4*((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + \\
& b^2)*c*d*e*f - (2*a*b*c + (a^2 + b^2)*c^2)*f^2 + 2*((a^2 + b^2)*d^2*e*f - \\
& a*b*d*f^2)*x)*\cosh(d*x + c)^3 - ((a^2 + b^2)*d^2*f^2*x^2 + 2*(a^2 + b^2)*c* \\
& d*e*f - (2*a*b*c + (a^2 + b^2)*c^2)*f^2 + 2*((a^2 + b^2)*d^2*e*f - a*b*d*f^2 \\
& )*x)*\cosh(d*x + c))*\sinh(d*x + c))*\log(-\cosh(d*x + c) - \sinh(d*x + c) + 1) \\
& + 2*((a^2 + b^2)*f^2*\cosh(d*x + c)^4 + 4*(a^2 + b^2)*f^2*\cosh(d*x + c)*\sin \\
& h(d*x + c)^3 + (a^2 + b^2)*f^2*\sinh(d*x + c)^4 - 2*(a^2 + b^2)*f^2*\cosh(d*x \\
& + c)^2 + (a^2 + b^2)*f^2 + 2*(3*(a^2 + b^2)*f^2*\cosh(d*x + c)^2 - (a^2 + b \\
& ^2)*f^2)*\sinh(d*x + c)^2 + 4*((a^2 + b^2)*f^2*\cosh(d*x + c)^3 - (a^2 + b^2) \\
& *f^2*\cosh(d*x + c))*\sinh(d*x + c))*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x \\
& + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\text{sqrt}((a^2 + b^2)/b^2))/b) + 2*( \\
& (a^2 + b^2)*f^2*\cosh(d*x + c)^4 + 4*(a^2 + b^2)*f^2*\cosh(d*x + c)*\sinh(d*x \\
& + c)^3 + (a^2 + b^2)*f^2*\sinh(d*x + c)^4 - 2*(a^2 + b^2)*f^2*\cosh(d*x + c)^ \\
& 2 + (a^2 + b^2)*f^2 + 2*(3*(a^2 + b^2)*f^2*\cosh(d*x + c)^2 - (a^2 + b^2)*f^ \\
& 2)*\sinh(d*x + c)^2 + 4*((a^2 + b^2)*f^2*\cosh(d*x + c)^3 - (a^2 + b^2)*f^2*c \\
& osh(d*x + c))*\sinh(d*x + c))*\text{polylog}(3, (a*\cosh(d*x + c) + a*\sinh(d*x + c) \\
& - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\text{sqrt}((a^2 + b^2)/b^2))/b) - 2*((a^2 + \\
& b^2)*f^2*\cosh(d*x + c)^4 + 4*(a^2 + b^2)*f^2*\cosh(d*x + c)*\sinh(d*x + c)^3 \\
& + (a^2 + b^2)*f^2*\sinh(d*x + c)^4 - 2*(a^2 + b^2)*f^2*\cosh(d*x + c)^2 + (a
\end{aligned}$$



$$\begin{aligned} &^2 + b^2) * f^2 + 2 * (3 * (a^2 + b^2) * f^2 * \cosh(dx + c)^2 - (a^2 + b^2) * f^2) * \sinh(dx + c)^2 + 4 * ((a^2 + b^2) * f^2 * \cosh(dx + c)^3 - (a^2 + b^2) * f^2 * \cosh(dx + c)) * \sinh(dx + c) * \text{polylog}(3, \cosh(dx + c) + \sinh(dx + c)) - 2 * ((a^2 + b^2) * f^2 * \cosh(dx + c)^4 + 4 * (a^2 + b^2) * f^2 * \cosh(dx + c) * \sinh(dx + c)^3 + (a^2 + b^2) * f^2 * \sinh(dx + c)^4 - 2 * (a^2 + b^2) * f^2 * \cosh(dx + c)^2 + (a^2 + b^2) * f^2 + 2 * (3 * (a^2 + b^2) * f^2 * \cosh(dx + c)^2 - (a^2 + b^2) * f^2) * \sinh(dx + c)^2 + 4 * ((a^2 + b^2) * f^2 * \cosh(dx + c)^3 - (a^2 + b^2) * f^2 * \cosh(dx + c)) * \sinh(dx + c)) * \text{polylog}(3, -\cosh(dx + c) - \sinh(dx + c)) - 2 * (a * b * d^2 * f^2 * x^2 + 2 * a * b * d^2 * e * f * x + a * b * d^2 * e^2 + 4 * (a^2 * d * f^2 * x + a^2 * c * f^2) * \cosh(dx + c)^3 - 3 * (a * b * d^2 * f^2 * x^2 + 2 * a * b * d^2 * e * f * x + a * b * d^2 * e^2) * \cosh(dx + c)^2 + 2 * (a^2 * d^2 * f^2 * x^2 + a^2 * d^2 * e^2 + a^2 * d * e * f - 2 * a^2 * c * f^2 + (2 * a^2 * d^2 * e * f - a^2 * d * f^2) * x) * \cosh(dx + c)) * \sinh(dx + c)) / (a^3 * d^3 * \cosh(dx + c)^4 + 4 * a^3 * d^3 * \cosh(dx + c) * \sinh(dx + c)^3 + a^3 * d^3 * \sinh(dx + c)^4 - 2 * a^3 * d^3 * \cosh(dx + c)^2 + a^3 * d^3 + 2 * (3 * a^3 * d^3 * \cosh(dx + c)^2 - a^3 * d^3) * \sinh(dx + c)^2 + 4 * (a^3 * d^3 * \cosh(dx + c)^3 - a^3 * d^3 * \cosh(dx + c)) * \sinh(dx + c)) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*coth(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*coth(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.488 \quad \int \frac{(e+fx) \coth^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=435

$$\frac{b^2 f \operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{2a^3 d^2} - \frac{f(a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{a^3 d^2} - \frac{f(a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right)}{a^3 d^2} + \frac{f \operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{2ad^2}$$

[Out] (f\*x)/(2\*a\*d) - (e + f\*x)^2/(2\*a\*f) - (b^2\*(e + f\*x)^2)/(2\*a^3\*f) + ((a^2 + b^2)\*(e + f\*x)^2)/(2\*a^3\*f) + (b\*f\*ArcTanh[Cosh[c + d\*x]])/(a^2\*d^2) - (f\*Coth[c + d\*x])/(2\*a\*d^2) - ((e + f\*x)\*Coth[c + d\*x]^2)/(2\*a\*d) + (b\*(e + f\*x)\*Csch[c + d\*x])/(a^2\*d) - ((a^2 + b^2)\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^3\*d) - ((a^2 + b^2)\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^3\*d) + ((e + f\*x)\*Log[1 - E^(2\*(c + d\*x))])/(a\*d) + (b^2\*(e + f\*x)\*Log[1 - E^(2\*(c + d\*x))])/(a^3\*d) - ((a^2 + b^2)\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^3\*d^2) - ((a^2 + b^2)\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^3\*d^2) + (f\*PolyLog[2, E^(2\*(c + d\*x))])/(2\*a\*d^2) + (b^2\*f\*PolyLog[2, E^(2\*(c + d\*x))])/(2\*a^3\*d^2)

**Rubi [A]** time = 0.976158, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 18, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$ , Rules used = {5569, 3720, 3473, 8, 3716, 2190, 2279, 2391, 5585, 5450, 3296, 2638, 5452, 3770, 5446, 2635, 5565, 5561}

$$\frac{b^2 f \operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{2a^3 d^2} - \frac{f(a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a - \sqrt{a^2 + b^2}}\right)}{a^3 d^2} - \frac{f(a^2 + b^2) \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2 + b^2} + a}\right)}{a^3 d^2} + \frac{f \operatorname{PolyLog}\left(2, e^{2(c+dx)}\right)}{2ad^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Coth[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] (f\*x)/(2\*a\*d) - (e + f\*x)^2/(2\*a\*f) - (b^2\*(e + f\*x)^2)/(2\*a^3\*f) + ((a^2 + b^2)\*(e + f\*x)^2)/(2\*a^3\*f) + (b\*f\*ArcTanh[Cosh[c + d\*x]])/(a^2\*d^2) - (f\*Coth[c + d\*x])/(2\*a\*d^2) - ((e + f\*x)\*Coth[c + d\*x]^2)/(2\*a\*d) + (b\*(e + f\*x)\*Csch[c + d\*x])/(a^2\*d) - ((a^2 + b^2)\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^3\*d) - ((a^2 + b^2)\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^3\*d) + ((e + f\*x)\*Log[1 - E^(2\*(c + d\*x))])/(a\*d) + (b^2\*(e + f\*x)\*Log[1 - E^(2\*(c + d\*x))])/(a^3\*d) - ((a^2 + b^2)\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^3\*d^2) - ((a^2 + b^2)\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^3\*d^2) + (f\*PolyLog[2, E^(2\*(c + d\*x))])/(2\*a\*d^2) + (b^2\*f\*PolyLog[2, E^(2\*(c + d\*x))])/(2\*a^3\*d^2)

\*PolyLog[2, E^(2\*(c + d\*x))]/(2\*a\*d^2) + (b^2\*f\*PolyLog[2, E^(2\*(c + d\*x))]/(2\*a^3\*d^2)

### Rule 5569

Int[(Coth[(c\_.) + (d\_.)\*(x\_.)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_.)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Coth[c + d\*x]^n, x], x] - Dist[b/a, Int[((e + f\*x)^m\*Cosh[c + d\*x]\*Coth[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0]

### Rule 3720

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*((b\_.)\*tan[(e\_.) + (f\_.)\*(x\_.)]^(n\_.), x\_Symbol] := Simp[(b\*(c + d\*x)^m\*(b\*Tan[e + f\*x])^(n - 1))/(f\*(n - 1)), x] + (-Dist[(b\*d\*m)/(f\*(n - 1)), Int[(c + d\*x)^(m - 1)\*(b\*Tan[e + f\*x])^(n - 1), x], x] - Dist[b^2, Int[(c + d\*x)^m\*(b\*Tan[e + f\*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]

### Rule 3473

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_.)]^(n\_.), x\_Symbol] := Simp[(b\*(b\*Tan[c + d\*x])^(n - 1))/(d\*(n - 1)), x] - Dist[b^2, Int[(b\*Tan[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rule 3716

Int[((c\_.) + (d\_.)\*(x\_.))^(m\_.)\*tan[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_.)], x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x)))/(E^(2\*I\*k\*Pi)\*(1 + E^(2\*(-I\*e) + f\*fz\*x))/E^(2\*I\*k\*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

### Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_.)))^(n\_.)\*((c\_.) + (d\_.)\*(x\_.))^(m\_.))/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_.)))^(n\_.), x\_Symbol] := Simp[(((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a)/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x))))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5585

```
Int[(Cosh[(c_) + (d_)*(x_)]^(p_)*Coth[(c_) + (d_)*(x_)]^(n_)*((e_) +
(f_)*(x_))^(m_))/((a_) + (b_)*Sinh[(c_) + (d_)*(x_)]), x_Symbol] := D
ist[1/a, Int[(e + f*x)^m*Cosh[c + d*x]^p*Coth[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Cosh[c + d*x]^(p + 1)*Coth[c + d*x]^(n - 1))/(a + b*Sinh
[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n,
0] && IGtQ[p, 0]
```

Rule 5450

```
Int[Cosh[(a_) + (b_)*(x_)]^(n_)*Coth[(a_) + (b_)*(x_)]^(p_)*((c_) +
(d_)*(x_))^(m_), x_Symbol] := Int[(c + d*x)^m*Cosh[a + b*x]^n*Coth[a + b*
x]^(p - 2), x] + Int[(c + d*x)^m*Cosh[a + b*x]^(n - 2)*Coth[a + b*x]^p, x]
/; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3296

```
Int[((c_) + (d_)*(x_))^(m_)*sin[(e_) + (f_)*(x_)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_) + (d_)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 5452

```
Int[Coth[(a_) + (b_)*(x_)]^(p_)*Csch[(a_) + (b_)*(x_)]^(n_)*((c_) +
(d_)*(x_))^(m_), x_Symbol] := -Simp[((c + d*x)^m*Csch[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x]  
 /; FreeQ[{c, d}, x]

Rule 5446

Int[Cosh[(a\_.) + (b\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sinh[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Simp[((c + d\*x)^m\*Sinh[a + b\*x]^(n + 1))/(b\*(n + 1)), x] - Dist[(d\*m)/(b\*(n + 1)), Int[(c + d\*x)^(m - 1)\*Sinh[a + b\*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 2635

Int[((b\_.)\*sin[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := -Simp[(b\*Cos[c + d\*x]\*(b\*Sinh[c + d\*x])^(n - 1))/(d\*n), x] + Dist[(b^2\*(n - 1))/n, Int[(b\*Sinh[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2\*n]

Rule 5565

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Dist[a/b^2, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2), x], x] + (Dist[1/b, Int[(e + f\*x)^m\*Cosh[c + d\*x]^(n - 2)\*Sinh[c + d\*x], x], x] + Dist[(a^2 + b^2)/b^2, Int[((e + f\*x)^m\*Cosh[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 1] && NeQ[a^2 + b^2, 0] && IGtQ[m, 0]

Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[((e + f\*x)^m\*E^(c + d\*x))/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[((e + f\*x)^m\*E^(c + d\*x))/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx) \coth^3(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx) \coth^3(c+dx) dx}{a} - \frac{b \int \frac{(e+fx) \cosh(c+dx) \coth^2(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= -\frac{(e+fx) \coth^2(c+dx)}{2ad} + \frac{\int (e+fx) \coth(c+dx) dx}{a} - \frac{b \int (e+fx) \cosh(c+dx) \coth^2(c+dx) dx}{a^2} \\
&= -\frac{(e+fx)^2}{2af} - \frac{f \coth(c+dx)}{2ad^2} - \frac{(e+fx) \coth^2(c+dx)}{2ad} - \frac{2 \int \frac{e^{2(c+dx)}(e+fx)}{1-e^{2(c+dx)}} dx}{a} - \frac{b \int (e+fx) dx}{a^2} \\
&= \frac{fx}{2ad} - \frac{(e+fx)^2}{2af} - \frac{f \coth(c+dx)}{2ad^2} - \frac{(e+fx) \coth^2(c+dx)}{2ad} + \frac{b(e+fx) \operatorname{csch}(c+dx)}{a^2d} + \dots \\
&= \frac{fx}{2ad} - \frac{(e+fx)^2}{2af} - \frac{b^2(e+fx)^2}{2a^3f} + \frac{(a^2+b^2)(e+fx)^2}{2a^3f} + \frac{bf \tanh^{-1}(\cosh(c+dx))}{a^2d^2} + \frac{bf \coth(c+dx)}{a^2d} \\
&= \frac{fx}{2ad} - \frac{(e+fx)^2}{2af} - \frac{b^2(e+fx)^2}{2a^3f} + \frac{(a^2+b^2)(e+fx)^2}{2a^3f} + \frac{bf \tanh^{-1}(\cosh(c+dx))}{a^2d^2} - \frac{f \coth(c+dx)}{a^2d} \\
&= \frac{fx}{2ad} - \frac{(e+fx)^2}{2af} - \frac{b^2(e+fx)^2}{2a^3f} + \frac{(a^2+b^2)(e+fx)^2}{2a^3f} + \frac{bf \tanh^{-1}(\cosh(c+dx))}{a^2d^2} - \frac{f \coth(c+dx)}{a^2d} \\
&= \frac{fx}{2ad} - \frac{(e+fx)^2}{2af} - \frac{b^2(e+fx)^2}{2a^3f} + \frac{(a^2+b^2)(e+fx)^2}{2a^3f} + \frac{bf \tanh^{-1}(\cosh(c+dx))}{a^2d^2} - \frac{f \coth(c+dx)}{a^2d}
\end{aligned}$$

**Mathematica [A]** time = 4.72672, size = 455, normalized size = 1.05

$$-8(a^2+b^2) \left( f \operatorname{PolyLog} \left( 2, \frac{be^{c+dx}}{\sqrt{a^2+b^2}-a} \right) + f \operatorname{PolyLog} \left( 2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a} \right) + f(c+dx) \log \left( \frac{be^{c+dx}}{a-\sqrt{a^2+b^2}} + 1 \right) + f(c+dx) \log \left( \frac{be^{c+dx}}{\sqrt{a^2+b^2}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((e + f\*x)\*Coth[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] (2\*a\*(-(a\*f) + 2\*b\*d\*(e + f\*x))\*Coth[(c + d\*x)/2] - a^2\*d\*(e + f\*x)\*Csch[(c + d\*x)/2]^2 + 8\*a^2\*d\*e\*Log[Sinh[c + d\*x]] + 8\*b^2\*d\*e\*Log[Sinh[c + d\*x]] - 8\*a^2\*c\*f\*Log[Sinh[c + d\*x]] - 8\*b^2\*c\*f\*Log[Sinh[c + d\*x]] - 8\*a\*b\*f\*Log[Tanh[(c + d\*x)/2]] + 4\*a^2\*f\*((c + d\*x)\*(c + d\*x + 2\*Log[1 - E^(-2\*(c + d\*x))])) - PolyLog[2, E^(-2\*(c + d\*x))] + 4\*b^2\*f\*((c + d\*x)\*(c + d\*x + 2\*Log[1 - E^(-2\*(c + d\*x))])) - PolyLog[2, E^(-2\*(c + d\*x))] - 8\*(a^2 + b^2)\*(-(f\*(c + d\*x)^2)/2 + f\*(c + d\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])

$$\begin{aligned} & )] + f*(c + d*x)*\text{Log}[1 + (b*E^{(c + d*x)})/(a + \text{Sqrt}[a^2 + b^2])] + d*e*\text{Log}[a \\ & + b*\text{Sinh}[c + d*x]] - c*f*\text{Log}[a + b*\text{Sinh}[c + d*x]] + f*\text{PolyLog}[2, (b*E^{(c + \\ & d*x)})/(-a + \text{Sqrt}[a^2 + b^2])] + f*\text{PolyLog}[2, -((b*E^{(c + d*x)})/(a + \text{Sqrt}[a \\ & ^2 + b^2]))] + a^2*d*(e + f*x)*\text{Sech}[(c + d*x)/2]^2 - 2*a*(a*f + 2*b*d*(e + \\ & f*x))*\text{Tanh}[(c + d*x)/2]/(8*a^3*d^2) \end{aligned}$$

**Maple [B]** time = 0.171, size = 1098, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((f*x+e)*\text{coth}(d*x+c)^3/(a+b*\text{sinh}(d*x+c)), x)$

[Out]  $\frac{1}{a^3 b^2 d^2 f} c \ln(b \exp(2 d x+2 c)+2 a \exp(d x+c)-b)-\frac{1}{a^3 b^2 d^2 f} \ln\left(\frac{-b \exp(d x+c)+\left(a^2+b^2\right)^{1 / 2}-a}{-a+\left(a^2+b^2\right)^{1 / 2}}\right) x-\frac{1}{a^3 b^2 d^2 f} \ln\left(\frac{-b \exp(d x+c)+\left(a^2+b^2\right)^{1 / 2}-a}{-a+\left(a^2+b^2\right)^{1 / 2}}\right) c-\frac{1}{a^3 b^2 d^2 f} \ln\left(\frac{b \exp(d x+c)+\left(a^2+b^2\right)^{1 / 2}+a}{a+\left(a^2+b^2\right)^{1 / 2}}\right) x-\frac{1}{a^3 b^2 d^2 f} \ln\left(\frac{b \exp(d x+c)+\left(a^2+b^2\right)^{1 / 2}+a}{a+\left(a^2+b^2\right)^{1 / 2}}\right) c-\frac{1}{a^3 d^2 b^2 f} c \ln\left(\frac{\exp(d x+c)-1}{\exp(d x+c)+1}\right) x-\frac{1}{d^2 f a} \ln\left(\frac{-b \exp(d x+c)+\left(a^2+b^2\right)^{1 / 2}-a}{-a+\left(a^2+b^2\right)^{1 / 2}}\right) x-\frac{1}{d^2 f a} \ln\left(\frac{-b \exp(d x+c)+\left(a^2+b^2\right)^{1 / 2}-a}{-a+\left(a^2+b^2\right)^{1 / 2}}\right) c-\frac{1}{d^2 f a} \ln\left(\frac{b \exp(d x+c)+\left(a^2+b^2\right)^{1 / 2}+a}{a+\left(a^2+b^2\right)^{1 / 2}}\right) x-\frac{1}{d^2 f a} \ln\left(\frac{b \exp(d x+c)+\left(a^2+b^2\right)^{1 / 2}+a}{a+\left(a^2+b^2\right)^{1 / 2}}\right) c+\frac{1}{d^2 f c a} \ln\left(\frac{b \exp(2 d x+2 c)+2 a \exp(d x+c)-b}{b \exp(d x+c)+\left(a^2+b^2\right)^{1 / 2}-a}\right)-\frac{1}{a^3 b^2 d^2 f} \text{dilog}\left(\frac{-b \exp(d x+c)+\left(a^2+b^2\right)^{1 / 2}-a}{-a+\left(a^2+b^2\right)^{1 / 2}}\right)-\frac{1}{a^3 b^2 d^2 f} \text{dilog}\left(\frac{b \exp(d x+c)+\left(a^2+b^2\right)^{1 / 2}+a}{a+\left(a^2+b^2\right)^{1 / 2}}\right)-\frac{1}{a^3 b^2 d^2 f} e \ln\left(\frac{b \exp(2 d x+2 c)+2 a \exp(d x+c)-b}{b \exp(d x+c)+\left(a^2+b^2\right)^{1 / 2}-a}\right)-\frac{1}{d^2 f a} \text{dilog}\left(\frac{-b \exp(d x+c)+\left(a^2+b^2\right)^{1 / 2}-a}{-a+\left(a^2+b^2\right)^{1 / 2}}\right)-\frac{1}{d^2 f a} \text{dilog}\left(\frac{b \exp(d x+c)+\left(a^2+b^2\right)^{1 / 2}+a}{a+\left(a^2+b^2\right)^{1 / 2}}\right)-\frac{1}{a^3 d^2 b^2 f} \text{dilog}\left(\frac{\exp(d x+c)+1}{\exp(d x+c)-1}\right)+\frac{1}{a^2 d^2 b^2 f} \ln\left(\frac{\exp(d x+c)+1}{\exp(d x+c)-1}\right)+\frac{1}{a^3 d^2 b^2 e} \ln\left(\frac{\exp(d x+c)-1}{\exp(d x+c)+1}\right)+\frac{1}{a^3 d^2 b^2 e} \ln\left(\frac{\exp(d x+c)+1}{\exp(d x+c)-1}\right)+\frac{1}{d^2 f a} \text{dilog}\left(\frac{\exp(d x+c)+1}{\exp(d x+c)-1}\right)+\frac{1}{d^2 f a} \text{dilog}\left(\frac{\exp(d x+c)+1}{\exp(d x+c)-1}\right)+\frac{1}{d a} e \ln\left(\frac{\exp(d x+c)+1}{\exp(d x+c)-1}\right)+\frac{1}{d a} e \ln\left(\frac{\exp(d x+c)+1}{\exp(d x+c)-1}\right)+\frac{1}{d a} \ln\left(\frac{\exp(d x+c)+1}{\exp(d x+c)-1}\right) f x-\frac{1}{d^2 a f c} \ln\left(\frac{\exp(d x+c)-1}{\exp(d x+c)+1}\right)-\frac{2 b d f x \exp(3 d x+3 c)+2 a d f x \exp(2 d x+2 c)-2 b d e \exp(3 d x+3 c)+2 a d e \exp(2 d x+2 c)+2 b d f x \exp(d x+c)+a f \exp(2 d x+2 c)+2 b d e \exp(d x+c)-a f}{d^2 a^2\left(\exp(2 d x+2 c)-1\right)^2}-\frac{1}{d e a} \ln\left(\frac{b \exp(2 d x+2 c)+2 a \exp(d x+c)-b}{b \exp(d x+c)+\left(a^2+b^2\right)^{1 / 2}-a}\right)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\left(a^2d \int \frac{x}{a^3de^{(dx+c)} + a^3d} dx + b^2d \int \frac{x}{a^3de^{(dx+c)} + a^3d} dx - a^2d \int \frac{x}{a^3de^{(dx+c)} - a^3d} dx - b^2d \int \frac{x}{a^3de^{(dx+c)} - a^3d} dx + ab \left( \frac{d}{a^3d} \int \frac{x}{a^3de^{(dx+c)} + a^3d} dx - \frac{d}{a^3d} \int \frac{x}{a^3de^{(dx+c)} - a^3d} dx \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*coth(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out]  $-(a^2d \int (x/(a^3d e^{(dx+c)} + a^3d), x) + b^2d \int (x/(a^3d e^{(dx+c)} + a^3d), x) - a^2d \int (x/(a^3d e^{(dx+c)} - a^3d), x) - b^2d \int (x/(a^3d e^{(dx+c)} - a^3d), x) + a*b*((dx+c)/(a^3d^2) - \log(e^{(dx+c)} + 1)/(a^3d^2)) - a*b*((dx+c)/(a^3d^2) - \log(e^{(dx+c)} - 1)/(a^3d^2)) - (2*b*d*x*e^{(3*d*x+3*c)} - 2*b*d*x*e^{(d*x+c)} - (2*a*d*x*e^{(2*c)} + a*e^{(2*c)})*e^{(2*d*x)} + a)/(a^2*d^2*e^{(4*d*x+4*c)} - 2*a^2*d^2*e^{(2*d*x+2*c)} + a^2*d^2) - \int (2*((a^3*e^c + a*b^2*e^c)*x*e^{(d*x)} - (a^2*b + b^3)*x)/(a^3*b*e^{(2*d*x+2*c)} + 2*a^4*e^{(d*x+c)} - a^3*b), x)) * f - e*(2*(b*e^{(-d*x-c)} - a*e^{(-2*d*x-2*c)} - b*e^{(-3*d*x-3*c)})/((2*a^2*e^{(-2*d*x-2*c)} - a^2*e^{(-4*d*x-4*c)} - a^2)*d) + (a^2 + b^2)*\log(-2*a*e^{(-d*x-c)} + b*e^{(-2*d*x-2*c)} - b)/(a^3*d) - (a^2 + b^2)*\log(e^{(-d*x-c)} + 1)/(a^3*d) - (a^2 + b^2)*\log(e^{(-d*x-c)} - 1)/(a^3*d))$

**Fricas [B]** time = 2.38878, size = 8753, normalized size = 20.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*coth(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $(2*(a*b*d*f*x + a*b*d*e)*\cosh(d*x+c)^3 + 2*(a*b*d*f*x + a*b*d*e)*\sinh(d*x+c)^3 + a^2*f - (2*a^2*d*f*x + 2*a^2*d*e + a^2*f)*\cosh(d*x+c)^2 - (2*a^2*d*f*x + 2*a^2*d*e + a^2*f - 6*(a*b*d*f*x + a*b*d*e)*\cosh(d*x+c))*\sinh(d*x+c)^2 - 2*(a*b*d*f*x + a*b*d*e)*\cosh(d*x+c) - ((a^2 + b^2)*f*\cosh(d*x+c)^4 + 4*(a^2 + b^2)*f*\cosh(d*x+c)*\sinh(d*x+c)^3 + (a^2 + b^2)*f*\sinh(d*x+c)^4 - 2*(a^2 + b^2)*f*\cosh(d*x+c)^2 + 2*(3*(a^2 + b^2)*f*\cosh(d*x+c)^2 - (a^2 + b^2)*f)*\sinh(d*x+c)^2 + (a^2 + b^2)*f + 4*((a^2 + b^2)*f*\cosh(d*x+c)^3 - (a^2 + b^2)*f*\cosh(d*x+c))*\sinh(d*x+c))*\operatorname{dilog}((a*\cosh(d*x+c) + a*\sinh(d*x+c) + (b*\cosh(d*x+c) + b*\sinh(d*x+c)))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) - ((a^2 + b^2)*f*\cosh(d*x+c)^4 + 4*(a^2 + b^2)*f*\cosh(d*x+c)*\sinh(d*x+c)^3 + (a^2 + b^2)*f*\sinh(d*x+c)^4 - 2*(a^2 +$



$$\begin{aligned}
& b^2 * f * \cosh(dx + c)^2 + 2 * (3 * (a^2 + b^2) * f * \cosh(dx + c)^2 - (a^2 + b^2) * \\
& f * \sinh(dx + c)^2 + (a^2 + b^2) * f + 4 * ((a^2 + b^2) * f * \cosh(dx + c)^3 - (a^2 + b^2) * f * \cosh(dx + c)) * \sinh(dx + c)) * \operatorname{dilog}((a * \cosh(dx + c) + a * \sinh(dx + c) - (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b + 1) + ((a^2 + b^2) * f * \cosh(dx + c)^4 + 4 * (a^2 + b^2) * f * \cosh(dx + c) * \sinh(dx + c)^3 + (a^2 + b^2) * f * \sinh(dx + c)^4 - 2 * (a^2 + b^2) * f * \cosh(dx + c)^2 + 2 * (3 * (a^2 + b^2) * f * \cosh(dx + c)^2 - (a^2 + b^2) * f) * \sinh(dx + c)^2 + (a^2 + b^2) * f + 4 * ((a^2 + b^2) * f * \cosh(dx + c)^3 - (a^2 + b^2) * f * \cosh(dx + c)) * \sinh(dx + c)) * \operatorname{dilog}(\cosh(dx + c) + \sinh(dx + c)) + ((a^2 + b^2) * f * \cosh(dx + c)^4 + 4 * (a^2 + b^2) * f * \cosh(dx + c) * \sinh(dx + c)^3 + (a^2 + b^2) * f * \sinh(dx + c)^4 - 2 * (a^2 + b^2) * f * \cosh(dx + c)^2 + 2 * (3 * (a^2 + b^2) * f * \cosh(dx + c)^2 - (a^2 + b^2) * f) * \sinh(dx + c)^2 + (a^2 + b^2) * f + 4 * ((a^2 + b^2) * f * \cosh(dx + c)^3 - (a^2 + b^2) * f * \cosh(dx + c)) * \sinh(dx + c)) * \operatorname{dilog}(-\cosh(dx + c) - \sinh(dx + c)) - (((a^2 + b^2) * d * e - (a^2 + b^2) * c * f) * \cosh(dx + c)^4 + 4 * ((a^2 + b^2) * d * e - (a^2 + b^2) * c * f) * \cosh(dx + c) * \sinh(dx + c)^3 + ((a^2 + b^2) * d * e - (a^2 + b^2) * c * f) * \sinh(dx + c)^4 + (a^2 + b^2) * d * e - (a^2 + b^2) * c * f - 2 * ((a^2 + b^2) * d * e - (a^2 + b^2) * c * f) * \cosh(dx + c)^2 - 2 * ((a^2 + b^2) * d * e - (a^2 + b^2) * c * f - 3 * ((a^2 + b^2) * d * e - (a^2 + b^2) * c * f) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 4 * (((a^2 + b^2) * d * e - (a^2 + b^2) * c * f) * \cosh(dx + c)^3 - ((a^2 + b^2) * d * e - (a^2 + b^2) * c * f) * \cosh(dx + c)) * \sinh(dx + c)) * \log(2 * b * \cosh(dx + c) + 2 * b * \sinh(dx + c) + 2 * b * \sqrt{(a^2 + b^2) / b^2} + 2 * a) - (((a^2 + b^2) * d * e - (a^2 + b^2) * c * f) * \cosh(dx + c)^4 + 4 * ((a^2 + b^2) * d * e - (a^2 + b^2) * c * f) * \cosh(dx + c) * \sinh(dx + c)^3 + ((a^2 + b^2) * d * e - (a^2 + b^2) * c * f) * \sinh(dx + c)^4 + (a^2 + b^2) * d * e - (a^2 + b^2) * c * f - 2 * ((a^2 + b^2) * d * e - (a^2 + b^2) * c * f) * \cosh(dx + c)^2 - 2 * ((a^2 + b^2) * d * e - (a^2 + b^2) * c * f - 3 * ((a^2 + b^2) * d * e - (a^2 + b^2) * c * f) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 4 * (((a^2 + b^2) * d * e - (a^2 + b^2) * c * f) * \cosh(dx + c)^3 - ((a^2 + b^2) * d * e - (a^2 + b^2) * c * f) * \cosh(dx + c)) * \sinh(dx + c)) * \log(2 * b * \cosh(dx + c) + 2 * b * \sinh(dx + c) - 2 * b * \sqrt{(a^2 + b^2) / b^2} + 2 * a) - (((a^2 + b^2) * d * f * x + (a^2 + b^2) * c * f) * \cosh(dx + c)^4 + 4 * ((a^2 + b^2) * d * f * x + (a^2 + b^2) * c * f) * \cosh(dx + c) * \sinh(dx + c)^3 + ((a^2 + b^2) * d * f * x + (a^2 + b^2) * c * f) * \sinh(dx + c)^4 + (a^2 + b^2) * d * f * x + (a^2 + b^2) * c * f - 2 * ((a^2 + b^2) * d * f * x + (a^2 + b^2) * c * f) * \cosh(dx + c)^2 - 2 * ((a^2 + b^2) * d * f * x + (a^2 + b^2) * c * f - 3 * ((a^2 + b^2) * d * f * x + (a^2 + b^2) * c * f) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 4 * (((a^2 + b^2) * d * f * x + (a^2 + b^2) * c * f) * \cosh(dx + c)^3 - ((a^2 + b^2) * d * f * x + (a^2 + b^2) * c * f) * \cosh(dx + c)) * \sinh(dx + c)) * \log(-(a * \cosh(dx + c) + a * \sinh(dx + c) + (b * \cosh(dx + c) + b * \sinh(dx + c)) * \sqrt{(a^2 + b^2) / b^2} - b) / b) - (((a^2 + b^2) * d * f * x + (a^2 + b^2) * c * f) * \cosh(dx + c)^4 + 4 * ((a^2 + b^2) * d * f * x + (a^2 + b^2) * c * f) * \cosh(dx + c) * \sinh(dx + c)^3 + ((a^2 + b^2) * d * f * x + (a^2 + b^2) * c * f) * \sinh(dx + c)^4 + (a^2 + b^2) * d * f * x + (a^2 + b^2) * c * f - 2 * ((a^2 + b^2) * d * f * x + (a^2 + b^2) * c * f) * \cosh(dx + c)^2 - 2 * ((a^2 + b^2) * d * f * x + (a^2 + b^2) * c * f - 3 * ((a^2 + b^2) * d * f * x + (a^2 + b^2) * c * f) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 4 * (((a^2 + b^2) * d * f * x + (a^2 + b^2) * c * f) * \cosh(dx + c)^3 - ((a^2 + b^2) * d * f * x + (a^2 + b^2) * c * f) * \cosh(dx + c)) * \sinh(dx + c)) * \log(-(a * \cosh(dx + c) + a * \sinh(dx + c)
\end{aligned}$$

```

) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b) + (((
a^2 + b^2)*d*f*x + (a^2 + b^2)*d*e + a*b*f)*cosh(d*x + c)^4 + 4*((a^2 + b^2
)*d*f*x + (a^2 + b^2)*d*e + a*b*f)*cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 +
b^2)*d*f*x + (a^2 + b^2)*d*e + a*b*f)*sinh(d*x + c)^4 + (a^2 + b^2)*d*f*x +
(a^2 + b^2)*d*e + a*b*f - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*d*e + a*b*f)*
cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*d*e + a*b*f - 3*((a^2
+ b^2)*d*f*x + (a^2 + b^2)*d*e + a*b*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
4*(((a^2 + b^2)*d*f*x + (a^2 + b^2)*d*e + a*b*f)*cosh(d*x + c)^3 - ((a^2 +
b^2)*d*f*x + (a^2 + b^2)*d*e + a*b*f)*cosh(d*x + c))*sinh(d*x + c))*log(cos
h(d*x + c) + sinh(d*x + c) + 1) + (((a^2 + b^2)*d*e - (a*b + (a^2 + b^2)*c)
*f)*cosh(d*x + c)^4 + 4*((a^2 + b^2)*d*e - (a*b + (a^2 + b^2)*c)*f)*cosh(d*
x + c)*sinh(d*x + c)^3 + ((a^2 + b^2)*d*e - (a*b + (a^2 + b^2)*c)*f)*sinh(d
*x + c)^4 + (a^2 + b^2)*d*e - 2*((a^2 + b^2)*d*e - (a*b + (a^2 + b^2)*c)*f)
*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*e - 3*((a^2 + b^2)*d*e - (a*b + (a^2 +
b^2)*c)*f)*cosh(d*x + c)^2 - (a*b + (a^2 + b^2)*c)*f)*sinh(d*x + c)^2 - (a*
b + (a^2 + b^2)*c)*f + 4*(((a^2 + b^2)*d*e - (a*b + (a^2 + b^2)*c)*f)*cosh(
d*x + c)^3 - ((a^2 + b^2)*d*e - (a*b + (a^2 + b^2)*c)*f)*cosh(d*x + c))*sin
h(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + (((a^2 + b^2)*d*f*x +
(a^2 + b^2)*c*f)*cosh(d*x + c)^4 + 4*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*
cosh(d*x + c)*sinh(d*x + c)^3 + ((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*sinh(
d*x + c)^4 + (a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f - 2*((a^2 + b^2)*d*f*x + (
a^2 + b^2)*c*f)*cosh(d*x + c)^2 - 2*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f -
3*((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
4*(((a^2 + b^2)*d*f*x + (a^2 + b^2)*c*f)*cosh(d*x + c)^3 - ((a^2 + b^2)*d*f
*x + (a^2 + b^2)*c*f)*cosh(d*x + c))*sinh(d*x + c))*log(-cosh(d*x + c) - si
nh(d*x + c) + 1) - 2*(a*b*d*f*x + a*b*d*e - 3*(a*b*d*f*x + a*b*d*e)*cosh(d*
x + c)^2 + (2*a^2*d*f*x + 2*a^2*d*e + a^2*f)*cosh(d*x + c))*sinh(d*x + c))/
(a^3*d^2*cosh(d*x + c)^4 + 4*a^3*d^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^3*d^
2*sinh(d*x + c)^4 - 2*a^3*d^2*cosh(d*x + c)^2 + a^3*d^2 + 2*(3*a^3*d^2*cosh
(d*x + c)^2 - a^3*d^2)*sinh(d*x + c)^2 + 4*(a^3*d^2*cosh(d*x + c)^3 - a^3*d
^2*cosh(d*x + c))*sinh(d*x + c))

```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx) \coth^3(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*coth(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Integral((e + f\*x)\*coth(c + d\*x)\*\*3/(a + b\*sinh(c + d\*x)), x)

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e) \coth(dx + c)^3}{b \sinh(dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*coth(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] integrate((f\*x + e)\*coth(d\*x + c)^3/(b\*sinh(d\*x + c) + a), x)

$$3.489 \quad \int \frac{\coth^3(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=80

$$\frac{(a^2 + b^2) \log(\sinh(c + dx))}{a^3 d} - \frac{(a^2 + b^2) \log(a + b \sinh(c + dx))}{a^3 d} + \frac{b \operatorname{csch}(c + dx)}{a^2 d} - \frac{\operatorname{csch}^2(c + dx)}{2ad}$$

[Out] (b\*Csch[c + d\*x])/(a^2\*d) - Csch[c + d\*x]^2/(2\*a\*d) + ((a^2 + b^2)\*Log[Sinh[c + d\*x]])/(a^3\*d) - ((a^2 + b^2)\*Log[a + b\*Sinh[c + d\*x]])/(a^3\*d)

**Rubi [A]** time = 0.105165, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2721, 894}

$$\frac{(a^2 + b^2) \log(\sinh(c + dx))}{a^3 d} - \frac{(a^2 + b^2) \log(a + b \sinh(c + dx))}{a^3 d} + \frac{b \operatorname{csch}(c + dx)}{a^2 d} - \frac{\operatorname{csch}^2(c + dx)}{2ad}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d\*x]^3/(a + b\*Sinh[c + d\*x]),x]

[Out] (b\*Csch[c + d\*x])/(a^2\*d) - Csch[c + d\*x]^2/(2\*a\*d) + ((a^2 + b^2)\*Log[Sinh[c + d\*x]])/(a^3\*d) - ((a^2 + b^2)\*Log[a + b\*Sinh[c + d\*x]])/(a^3\*d)

#### Rule 2721

Int[((a\_) + (b\_)\*sin[(e\_) + (f\_)\*(x\_)])^(m\_)\*tan[(e\_) + (f\_)\*(x\_)]^(p\_), x\_Symbol] :> Dist[1/f, Subst[Int[(x^p\*(a + x)^m)/(b^2 - x^2)^((p + 1)/2), x], x, b\*Sin[e + f\*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]

#### Rule 894

Int[((d\_) + (e\_)\*(x\_))^(m\_)\*((f\_) + (g\_)\*(x\_))^(n\_)\*((a\_) + (c\_)\*(x\_)^2)^(p\_), x\_Symbol] :> Int[ExpandIntegrand[(d + e\*x)^m\*(f + g\*x)^n\*(a + c\*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && NeQ[c\*d^2 + a\*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

#### Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(c+dx)}{a+b\sinh(c+dx)} dx &= -\frac{\text{Subst}\left(\int \frac{-b^2-x^2}{x^3(a+x)} dx, x, b\sinh(c+dx)\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \left(-\frac{b^2}{ax^3} + \frac{b^2}{a^2x^2} + \frac{-a^2-b^2}{a^3x} + \frac{a^2+b^2}{a^3(a+x)}\right) dx, x, b\sinh(c+dx)\right)}{d} \\
&= \frac{b\text{csch}(c+dx)}{a^2d} - \frac{\text{csch}^2(c+dx)}{2ad} + \frac{(a^2+b^2)\log(\sinh(c+dx))}{a^3d} - \frac{(a^2+b^2)\log(a+b\sinh(c+dx))}{a^3d}
\end{aligned}$$

**Mathematica [A]** time = 0.11834, size = 64, normalized size = 0.8

$$\frac{2(a^2+b^2)(\log(\sinh(c+dx))-\log(a+b\sinh(c+dx)))-a^2\text{csch}^2(c+dx)+2ab\text{csch}(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d\*x]^3/(a + b\*Sinh[c + d\*x]),x]

[Out] (2\*a\*b\*Csch[c + d\*x] - a^2\*Csch[c + d\*x]^2 + 2\*(a^2 + b^2)\*(Log[Sinh[c + d\*x]] - Log[a + b\*Sinh[c + d\*x]]))/(2\*a^3\*d)

**Maple [B]** time = 0.001, size = 194, normalized size = 2.4

$$-\frac{1}{8da}\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2 - \frac{b}{2da^2}\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{8da}\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-2} + \frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \frac{b^2}{da^3}\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x)

[Out] -1/8/d/a\*tanh(1/2\*d\*x+1/2\*c)^2-1/2/d/a^2\*tanh(1/2\*d\*x+1/2\*c)\*b-1/8/d/a/tanh(1/2\*d\*x+1/2\*c)^2+1/d/a\*ln(tanh(1/2\*d\*x+1/2\*c))+1/d/a^3\*ln(tanh(1/2\*d\*x+1/2\*c))\*b^2+1/2/d\*b/a^2/tanh(1/2\*d\*x+1/2\*c)-1/d/a\*ln(tanh(1/2\*d\*x+1/2\*c))^2\*a-2\*tanh(1/2\*d\*x+1/2\*c)\*b-a-1/d/a^3\*ln(tanh(1/2\*d\*x+1/2\*c))^2\*a-2\*tanh(1/2\*d\*x+1/2\*c)\*b-a)\*b^2

**Maxima [B]** time = 1.29418, size = 234, normalized size = 2.92

$$\frac{2 \left( b e^{(-dx-c)} - a e^{(-2dx-2c)} - b e^{(-3dx-3c)} \right)}{\left( 2 a^2 e^{(-2dx-2c)} - a^2 e^{(-4dx-4c)} - a^2 \right) d} - \frac{\left( a^2 + b^2 \right) \log \left( -2 a e^{(-dx-c)} + b e^{(-2dx-2c)} - b \right)}{a^3 d} + \frac{\left( a^2 + b^2 \right) \log \left( e^{(-dx-c)} + 1 \right)}{a^3 d} + \left( \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$\frac{-2 \left( b e^{(-dx-c)} - a e^{(-2dx-2c)} - b e^{(-3dx-3c)} \right)}{\left( 2 a^2 e^{(-2dx-2c)} - a^2 e^{(-4dx-4c)} - a^2 \right) d} - \frac{\left( a^2 + b^2 \right) \log \left( -2 a e^{(-dx-c)} + b e^{(-2dx-2c)} - b \right)}{a^3 d} + \frac{\left( a^2 + b^2 \right) \log \left( e^{(-dx-c)} + 1 \right)}{a^3 d} + \frac{\left( a^2 + b^2 \right) \log \left( e^{(-dx-c)} - 1 \right)}{a^3 d}$$

**Fricas [B]** time = 1.84773, size = 1554, normalized size = 19.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & \left( 2 a^2 b \cosh(d x + c)^3 + 2 a b^2 \sinh(d x + c)^3 - 2 a^2 \cosh(d x + c)^2 - 2 a b \cosh(d x + c) + 2 \left( 3 a b \cosh(d x + c) - a^2 \right) \sinh(d x + c)^2 - \left( \left( a^2 + b^2 \right) \cosh(d x + c)^4 + 4 \left( a^2 + b^2 \right) \cosh(d x + c) \sinh(d x + c)^3 + \left( a^2 + b^2 \right) \sinh(d x + c)^4 - 2 \left( a^2 + b^2 \right) \cosh(d x + c)^2 + 2 \left( 3 \left( a^2 + b^2 \right) \cosh(d x + c)^2 - a^2 - b^2 \right) \sinh(d x + c)^2 + a^2 + b^2 + 4 \left( \left( a^2 + b^2 \right) \cosh(d x + c)^3 - \left( a^2 + b^2 \right) \cosh(d x + c) \right) \sinh(d x + c) \right) \log \left( 2 \left( b \sinh(d x + c) + a \right) / \left( \cosh(d x + c) - \sinh(d x + c) \right) \right) + \left( \left( a^2 + b^2 \right) \cosh(d x + c)^4 + 4 \left( a^2 + b^2 \right) \cosh(d x + c) \sinh(d x + c)^3 + \left( a^2 + b^2 \right) \sinh(d x + c)^4 - 2 \left( a^2 + b^2 \right) \cosh(d x + c)^2 + 2 \left( 3 \left( a^2 + b^2 \right) \cosh(d x + c)^2 - a^2 - b^2 \right) \sinh(d x + c)^2 + a^2 + b^2 + 4 \left( \left( a^2 + b^2 \right) \cosh(d x + c)^3 - \left( a^2 + b^2 \right) \cosh(d x + c) \right) \sinh(d x + c) \right) \log \left( 2 \sinh(d x + c) / \left( \cosh(d x + c) - \sinh(d x + c) \right) \right) + 2 \left( 3 a b \cosh(d x + c)^2 - 2 a^2 \cosh(d x + c) - a b \right) \sinh(d x + c) \right) / \left( a^3 d \cosh(d x + c)^4 + 4 a^3 d \cosh(d x + c) \sinh(d x + c)^3 + a^3 d \sinh(d x + c)^4 - 2 a^3 d \cosh(d x + c)^2 + a^3 d + 2 \left( 3 a^3 d \cosh(d x + c)^2 - a^3 d \right) \sinh(d x + c)^2 + 4 \left( a^3 d \cosh(d x + c)^3 - a^3 d \cosh(d x + c) \right) \sinh(d x + c) \right) \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(c + dx)}{a + b \sinh(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)), x)

[Out] Integral(coth(c + d\*x)\*\*3/(a + b\*sinh(c + d\*x)), x)

**Giac [B]** time = 1.41714, size = 224, normalized size = 2.8

$$\frac{(a^2 e^c + b^2 e^c) e^{(-c)} \log(e^{(dx+c)} + 1)}{a^3} + \frac{(a^2 e^c + b^2 e^c) e^{(-c)} \log(|e^{(dx+c)} - 1|)}{a^3} - \frac{(a^2 + b^2) \log(|b e^{(2dx+2c)} + 2 a e^{(dx+c)} - b|)}{a^3} + \frac{2 (a b e^{(3dx+3c)} - a^2 e^{(2dx+2c)} - a b e^{(dx+c)})}{a^3 (e^{(dx+c)} + 1)^2 (e^{(dx+c)} - 1)^2}$$

$d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)^3/(a+b\*sinh(d\*x+c)), x, algorithm="giac")

[Out] ((a^2\*e^c + b^2\*e^c)\*e^(-c)\*log(e^(d\*x + c) + 1)/a^3 + (a^2\*e^c + b^2\*e^c)\*e^(-c)\*log(abs(e^(d\*x + c) - 1))/a^3 - (a^2 + b^2)\*log(abs(b\*e^(2\*d\*x + 2\*c) + 2\*a\*e^(d\*x + c) - b))/a^3 + 2\*(a\*b\*e^(3\*d\*x + 3\*c) - a^2\*e^(2\*d\*x + 2\*c) - a\*b\*e^(d\*x + c))/(a^3\*(e^(d\*x + c) + 1)^2\*(e^(d\*x + c) - 1)^2))/d

$$3.490 \quad \int \frac{\coth^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Optimal.** Leaf size=30

$$\text{Unintegrable} \left( \frac{\coth^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))}, x \right)$$

[Out] Unintegrable[Coth[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.0761779, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[Coth[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int][Coth[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\coth^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx = \int \frac{\coth^3(c+dx)}{(e+fx)(a+b \sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.001, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[Coth[c + d\*x]^3/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] \$Aborted



---

**Maple [A]** time = 0.499, size = 0, normalized size = 0.

$$\int \frac{(\coth(dx + c))^3}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)`

[Out] `int(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)`

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] `-(a*f - 2*(b*d*f*x*e^(3*c) + b*d*e*e^(3*c))*e^(3*d*x) + (2*a*d*f*x*e^(2*c) + (2*d*e - f)*a*e^(2*c))*e^(2*d*x) + 2*(b*d*f*x*e^c + b*d*e*e^c)*e^(d*x))/(a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e^2 + (a^2*d^2*f^2*x^2*e^(4*c) + 2*a^2*d^2*e*f*x*e^(4*c) + a^2*d^2*e^2*e^(4*c))*e^(4*d*x) - 2*(a^2*d^2*f^2*x^2*e^(2*c) + 2*a^2*d^2*e*f*x*e^(2*c) + a^2*d^2*e^2*e^(2*c))*e^(2*d*x)) + integrate(-(b^2*d^2*e^2 + a*b*d*e*f + (d^2*e^2 + f^2)*a^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + (2*a^2*d^2*e*f + 2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) - integrate((b^2*d^2*e^2 - a*b*d*e*f + (d^2*e^2 + f^2)*a^2 + (a^2*d^2*f^2 + b^2*d^2*f^2)*x^2 + (2*a^2*d^2*e*f + 2*b^2*d^2*e*f - a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) + integrate(2*(a^2*b + b^3 - (a^3*e^c + a*b^2*e^c)*e^(d*x))/(a^3*b*f*x + a^3*b*e - (a^3*b*f*x*e^(2*c) + a^3*b*e*e^(2*c))*e^(2*d*x) - 2*(a^4*f*x*e^c + a^4*e*e^c)*e^(d*x)), x)`

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\coth(dx+c)^3}{(afx+ae+(bfx+be)\sinh(dx+c))}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] integral(coth(d\*x + c)^3/(a\*f\*x + a\*e + (b\*f\*x + b\*e)\*sinh(d\*x + c)), x)

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(c+dx)}{(a+b\sinh(c+dx))(e+fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)\*\*3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] Integral(coth(c + d\*x)\*\*3/((a + b\*sinh(c + d\*x))\*(e + f\*x)), x)

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\text{sage}_0x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d\*x+c)^3/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] sage0\*x

$$3.491 \quad \int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx$$

Optimal. Leaf size=1795

result too large to display

```
[Out] (-3*f*(e + f*x)^2)/(2*a*d^2) + (e + f*x)^3/(2*a*d) + (2*b*(e + f*x)^3*ArcTan[E^(c + d*x)])/(a^2*d) - (2*b^3*(e + f*x)^3*ArcTan[E^(c + d*x)])/(a^2*(a^2 + b^2)*d) + (6*b*f*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d^2) + (2*(e + f*x)^3*ArcTanh[E^(2*c + 2*d*x)])/(a*d) - (2*b^2*(e + f*x)^3*ArcTanh[E^(2*c + 2*d*x)])/(a^3*d) - (3*f*(e + f*x)^2*Coth[c + d*x])/(2*a*d^2) - ((e + f*x)^3*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*x)^3*Csch[c + d*x])/(a^2*d) - (b^4*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)*d) - (b^4*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)*d) + (3*f^2*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d^3) + (b^4*(e + f*x)^3*Log[1 + E^(2*(c + d*x))])/(a^3*(a^2 + b^2)*d) + (6*b*f^2*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a^2*d^3) - ((3*I)*b*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*d^2) + ((3*I)*b^3*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) + ((3*I)*b*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(a^2*d^2) - ((3*I)*b^3*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) - (6*b*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^2*d^3) - (3*b^4*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^2) - (3*b^4*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^2) + (3*b^4*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*a^3*(a^2 + b^2)*d^2) + (3*f^3*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^4) + (3*f*(e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a^3*d^2) - (3*f*(e + f*x)^2*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2) + (3*b^2*f*(e + f*x)^2*PolyLog[2, E^(2*c + 2*d*x)])/(2*a^3*d^2) - (6*b*f^3*PolyLog[3, -E^(c + d*x)])/(a^2*d^4) + ((6*I)*b*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(a^2*d^3) - ((6*I)*b^3*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^3) - ((6*I)*b*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a^2*d^3) + ((6*I)*b^3*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^3) + (6*b*f^3*PolyLog[3, E^(c + d*x)])/(a^2*d^4) + (6*b^4*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^3) + (6*b^4*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^3) - (3*b^4*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))])/(2*a^3*(a^2 + b^2)*d^3) - (3*f^2*(e + f*x)*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a*d^3) + (3*b^2*f^2*(e + f*x)*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a^3*d^3) + (3*f^2*(e + f*x)*PolyLog[3, E^(2*c + 2*d*x)])/(2*a*d^3) - (3*b^2*f^2*(e + f*x)*PolyLog[3, E^(2*c + 2*d*x)])/(2*a^3*d^3) - ((6*I)*b*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(a^2*d^4) + ((6*I)*b^3*f^3*PolyLog[4, (-I)*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^4) + ((6*I)*b*f^3*PolyLog[4, I*E^(c + d*x)])/(a^2*d^4) - ((6
```

$$\begin{aligned} & *I)*b^3*f^3*PolyLog[4, I*E^(c + d*x)]/(a^2*(a^2 + b^2)*d^4) - (6*b^4*f^3*PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])))]/(a^3*(a^2 + b^2)*d^4) \\ & - (6*b^4*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])))]/(a^3*(a^2 + b^2)*d^4) + (3*b^4*f^3*PolyLog[4, -E^(2*(c + d*x)))]/(4*a^3*(a^2 + b^2)*d^4) \\ & + (3*f^3*PolyLog[4, -E^(2*c + 2*d*x)])/(4*a*d^4) - (3*b^2*f^3*PolyLog[4, -E^(2*c + 2*d*x)])/(4*a^3*d^4) - (3*f^3*PolyLog[4, E^(2*c + 2*d*x)])/(4*a*d^4) \\ & + (3*b^2*f^3*PolyLog[4, E^(2*c + 2*d*x)])/(4*a^3*d^4) \end{aligned}$$

**Rubi [A]** time = 3.27209, antiderivative size = 1795, normalized size of antiderivative = 1., number of steps used = 87, number of rules used = 28, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$ , Rules used = {5589, 2620, 14, 5462, 6741, 12, 6742, 3720, 3716, 2190, 2279, 2391, 32, 2551, 4182, 2531, 6609, 2282, 6589, 2621, 321, 207, 5205, 4180, 5461, 5573, 5561, 3718}

result too large to display

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^3\*Csch[c + d\*x]^3\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] 
$$\begin{aligned} & (-3*f*(e + f*x)^2)/(2*a*d^2) + (e + f*x)^3/(2*a*d) + (2*b*(e + f*x)^3*ArcTan[E^(c + d*x)])/(a^2*d) - (2*b^3*(e + f*x)^3*ArcTan[E^(c + d*x)])/(a^2*(a^2 + b^2)*d) \\ & + (6*b*f*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^2*d^2) + (2*(e + f*x)^3*ArcTanh[E^(2*c + 2*d*x)])/(a*d) - (2*b^2*(e + f*x)^3*ArcTanh[E^(2*c + 2*d*x)])/(a^3*d) \\ & - (3*f*(e + f*x)^2*Coth[c + d*x])/(2*a*d^2) - ((e + f*x)^3*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*x)^3*Csch[c + d*x])/(a^2*d) - (b^4*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)*d) \\ & - (b^4*(e + f*x)^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)*d) + (3*f^2*(e + f*x)*Log[1 - E^(2*(c + d*x))])/(a*d^3) + (b^4*(e + f*x)^3*Log[1 + E^(2*(c + d*x))])/(a^3*(a^2 + b^2)*d) + (6*b*f^2*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a^2*d^3) \\ & - ((3*I)*b*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*d^2) + ((3*I)*b^3*f*(e + f*x)^2*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) + ((3*I)*b*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(a^2*d^2) \\ & - ((3*I)*b^3*f*(e + f*x)^2*PolyLog[2, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) - (6*b*f^2*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^2*d^3) - (3*b^4*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])))]/(a^3*(a^2 + b^2)*d^2) \\ & - (3*b^4*f*(e + f*x)^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])))]/(a^3*(a^2 + b^2)*d^2) + (3*b^4*f*(e + f*x)^2*PolyLog[2, -E^(2*(c + d*x))])/(2*a^3*(a^2 + b^2)*d^2) + (3*f^3*PolyLog[2, E^(2*(c + d*x))])/(2*a*d^4) \\ & + (3*f*(e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a*d^2) - (3*b^2*f*(e + f*x)^2*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a^3*d^2) - (3*f*(e + f*x)^2*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2) + (3*b^2*f*(e + f*x)^2*PolyLog[2, E^(2*c + 2*d*x)])/(2*a^3*d^2) - (6*b*f^3*PolyLog[3, -E^(c \end{aligned}$$

$$\begin{aligned}
& + d*x)))/(a^2*d^4) + ((6*I)*b*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)]/ \\
& (a^2*d^3) - ((6*I)*b^3*f^2*(e + f*x)*PolyLog[3, (-I)*E^(c + d*x)]/(a^2*(a^ \\
& 2 + b^2)*d^3) - ((6*I)*b*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)]/(a^2*d^3) \\
& + ((6*I)*b^3*f^2*(e + f*x)*PolyLog[3, I*E^(c + d*x)]/(a^2*(a^2 + b^2)*d^3 \\
& ) + (6*b*f^3*PolyLog[3, E^(c + d*x)]/(a^2*d^4) + (6*b^4*f^2*(e + f*x)*Poly \\
& Log[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a^3*(a^2 + b^2)*d^3) + ( \\
& 6*b^4*f^2*(e + f*x)*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/( \\
& a^3*(a^2 + b^2)*d^3) - (3*b^4*f^2*(e + f*x)*PolyLog[3, -E^(2*(c + d*x))]/( \\
& 2*a^3*(a^2 + b^2)*d^3) - (3*f^2*(e + f*x)*PolyLog[3, -E^(2*c + 2*d*x)]/(2* \\
& a*d^3) + (3*b^2*f^2*(e + f*x)*PolyLog[3, -E^(2*c + 2*d*x)]/(2*a^3*d^3) + ( \\
& 3*f^2*(e + f*x)*PolyLog[3, E^(2*c + 2*d*x)]/(2*a*d^3) - (3*b^2*f^2*(e + f* \\
& x)*PolyLog[3, E^(2*c + 2*d*x)]/(2*a^3*d^3) - ((6*I)*b*f^3*PolyLog[4, (-I)* \\
& E^(c + d*x)]/(a^2*d^4) + ((6*I)*b^3*f^3*PolyLog[4, (-I)*E^(c + d*x)]/(a^2 \\
& *(a^2 + b^2)*d^4) + ((6*I)*b*f^3*PolyLog[4, I*E^(c + d*x)]/(a^2*d^4) - ((6 \\
& *I)*b^3*f^3*PolyLog[4, I*E^(c + d*x)]/(a^2*(a^2 + b^2)*d^4) - (6*b^4*f^3*P \\
& olyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))]/(a^3*(a^2 + b^2)*d^4) \\
& - (6*b^4*f^3*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))]/(a^3*(a^ \\
& 2 + b^2)*d^4) + (3*b^4*f^3*PolyLog[4, -E^(2*(c + d*x))]/(4*a^3*(a^2 + b^2) \\
& *d^4) + (3*f^3*PolyLog[4, -E^(2*c + 2*d*x)]/(4*a*d^4) - (3*b^2*f^3*PolyLog \\
& [4, -E^(2*c + 2*d*x)]/(4*a^3*d^4) - (3*f^3*PolyLog[4, E^(2*c + 2*d*x)]/(4 \\
& *a*d^4) + (3*b^2*f^3*PolyLog[4, E^(2*c + 2*d*x)]/(4*a^3*d^4)
\end{aligned}$$

### Rule 5589

```

Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
(d_.)*(x_)]^(p_.)]/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
IGtQ[p, 0]

```

### Rule 2620

```

Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:= Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]

```

### Rule 14

```

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_
+ (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

```

### Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)^(n_.)*((c_.) + (d_.)*(x_)^(m_.)*Sech[(a_.) +
(b_.)*(x_)^(p_.), x_Symbol] :=> With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

### Rule 6741

```
Int[u_, x_Symbol] :=> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v !=
= u]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 6742

```
Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 3720

```
Int[((c_.) + (d_.)*(x_)^(m_.))*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :=> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

### Rule 3716

```
Int[((c_.) + (d_.)*(x_)^(m_.))*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :=> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

### Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_)^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :=> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x))))^n]/a)]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n]/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 32

```
Int[((a_) + (b_)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2551

```
Int[Log[u]*((a_) + (b_)*(x_))^(m_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[(a + b*x)^(m + 1)*D[u, x])/u, x], x] /; FreeQ[{a, b, m}, x] && InverseFunctionFreeQ[u, x] && NeQ[m, -1]
```

Rule 4182

```
Int[csc[(e_) + (Complex[0, fz_])*(f_)*(x_)]*((c_) + (d_)*(x_))^(m_), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(n_))]*((f_) + (g_)*(x_))^(m_), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 6609

```
Int[((e_) + (f_)*(x_))^(m_)*PolyLog[n_, (d_)*((F_)^((c_)*((a_) + (b_)*(x_)))^(p_))], x_Symbol] :> Simp[(e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

d, e, f, n, p}, x] && GtQ[m, 0]

### Rule 2282

Int[u\_, x\_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w\_)\*((a\_.)\*(v\_)^(n\_))^(m\_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m\*n] && !MatchQ[u, E^((c\_.)\*((a\_.) + (b\_.)\*x))\*(F\_)[v\_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :=> Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

### Rule 2621

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*sec[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] :=> -Dist[(f\*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a\*Csc[e + f\*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_.) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :=> Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

### Rule 207

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :=> -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

### Rule 5205

Int[((a\_.) + ArcTan[u\_]\*(b\_.))\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] :=> Simp[p[((c + d\*x)^(m + 1)\*(a + b\*ArcTan[u]))/(d\*(m + 1)), x] - Dist[b/(d\*(m + 1)), Int[SimplifyIntegrand[((c + d\*x)^(m + 1)\*D[u, x])/(1 + u^2), x], x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c + d\*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m +



1, x]]

### Rule 4180

Int[csc[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)/E^(I\*k\*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 5461

Int[Csch[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sech[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Dist[2^n, Int[(c + d\*x)^m\*Csch[2\*a + 2\*b\*x]^(n, x), x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]

### Rule 5573

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[b^2/(a^2 + b^2), Int[(e + f\*x)^m\*Sech[c + d\*x]^(n - 2)/(a + b\*Sinh[c + d\*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f\*x)^m\*Sech[c + d\*x]^(n\*(a - b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]

### Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[(e + f\*x)^m\*E^(c + d\*x)/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[(e + f\*x)^m\*E^(c + d\*x)/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

### Rule 3718

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]), x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[(c + d\*x)^m\*E^(2\*(-(I\*e) + f\*fz\*x)))/(1 + E^(2\*(-(I\*e) + f\*fz\*x))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^3 \operatorname{csch}^3(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^3 \operatorname{csch}^3(c+dx) \operatorname{sech}(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^3 \operatorname{csch}^2(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= -\frac{(e+fx)^3 \operatorname{coth}^2(c+dx)}{2ad} - \frac{(e+fx)^3 \log(\tanh(c+dx))}{ad} - \frac{b \int (e+fx)^3 \operatorname{csch}^2(c+dx) \operatorname{sech}(c+dx) dx}{a^2d} \\
&= \frac{b(e+fx)^3 \tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{(e+fx)^3 \operatorname{coth}^2(c+dx)}{2ad} + \frac{b(e+fx)^3 \operatorname{csch}^2(c+dx) \operatorname{sech}(c+dx)}{a^2d} \\
&= \frac{b(e+fx)^3 \tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{(e+fx)^3 \operatorname{coth}^2(c+dx)}{2ad} + \frac{b(e+fx)^3 \operatorname{csch}^2(c+dx) \operatorname{sech}(c+dx)}{a^2d} \\
&= \frac{b^4(e+fx)^4}{4a^3(a^2+b^2)f} + \frac{b(e+fx)^3 \tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{2b^2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^3d} \\
&= \frac{b^4(e+fx)^4}{4a^3(a^2+b^2)f} + \frac{b(e+fx)^3 \tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{2b^2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^3d} \\
&= -\frac{2b^3(e+fx)^3 \tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)d} + \frac{b(e+fx)^3 \tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{2b^2(e+fx)^3 \tanh^{-1}(e^{c+dx})}{a^3d} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{2b^3(e+fx)^3 \tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)d} + \frac{6bf(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^2d^2} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} - \frac{2b^3(e+fx)^3 \tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)d} + \frac{6bf(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^2d^2} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} + \frac{2b(e+fx)^3 \tan^{-1}(e^{c+dx})}{a^2d} - \frac{2b^3(e+fx)^3 \tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} + \frac{2b(e+fx)^3 \tan^{-1}(e^{c+dx})}{a^2d} - \frac{2b^3(e+fx)^3 \tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} + \frac{2b(e+fx)^3 \tan^{-1}(e^{c+dx})}{a^2d} - \frac{2b^3(e+fx)^3 \tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} + \frac{2b(e+fx)^3 \tan^{-1}(e^{c+dx})}{a^2d} - \frac{2b^3(e+fx)^3 \tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)} \\
&= -\frac{3f(e+fx)^2}{2ad^2} + \frac{(e+fx)^3}{2ad} + \frac{2b(e+fx)^3 \tan^{-1}(e^{c+dx})}{a^2d} - \frac{2b^3(e+fx)^3 \tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)}
\end{aligned}$$

**Mathematica [B]** time = 87.9895, size = 9045, normalized size = 5.04

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^3\*Csch[c + d\*x]^3\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] Result too large to show

**Maple [F]** time = 2.194, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^3 (\operatorname{csch}(dx + c))^3 \operatorname{sech}(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^3\*csch(d\*x+c)^3\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^3\*csch(d\*x+c)^3\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^3\*csch(d\*x+c)^3\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$-(b^4 \log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/((a^5 + a^3*b^2)*d) + 2*b*\arctan(e^{(-d*x - c)})/((a^2 + b^2)*d) - a*\log(e^{(-2*d*x - 2*c)} + 1)/((a^2 + b^2)*d) + 2*(b*e^{(-d*x - c)} - a*e^{(-2*d*x - 2*c)} - b*e^{(-3*d*x - 3*c)})/((2*a^2*e^{(-2*d*x - 2*c)} - a^2*e^{(-4*d*x - 4*c)} - a^2)*d) + (a^2 - b^2)*\log(e^{(-d*x - c)} + 1)/(a^3*d) + (a^2 - b^2)*\log(e^{(-d*x - c)} - 1)/(a^3*d)*e^3 + (3*a*f^3*x^2 + 6*a*e*f^2*x + 3*a*e^2*f + 2*(b*d*f^3*x^3*e^{(3*c)} + 3*b*d*e*f^2*x^2*e^{(3*c)} + 3*b*d*e^2*f*x*e^{(3*c)}))*e^{(3*d*x)} - (2*a*d*f^3*x^3*e^{(2*c)} + 3*a*e^2*f*e^{(2*c)} + 3*(2*d*e*f^2 + f^3)*a*x^2*e^{(2*c)} + 6*(d*e^2*f +$$

$$\begin{aligned}
& e^{f^2} a x e^{(2c)} e^{(2dx)} - 2(b d f^3 x^3 e^c + 3 b d e f^2 x^2 e^c + 3 b d e^2 f x e^c) e^{(dx)} / (a^2 d^2 e^{(4dx + 4c)} - 2 a^2 d^2 e^{(2dx + 2c)} + a^2 d^2) - 3(b d e^2 f + a e f^2) x / (a^2 d^2) + 3(b d e^2 f - a e f^2) x / (a^2 d^2) + 3(b d e^2 f + a e f^2) \log(e^{(dx + c)} + 1) / (a^2 d^3) \\
& - 3(b d e^2 f - a e f^2) \log(e^{(dx + c)} - 1) / (a^2 d^3) - (d^3 x^3 \log(e^{(dx + c)} + 1) + 3 d^2 x^2 \operatorname{dilog}(-e^{(dx + c)}) - 6 d x \operatorname{polylog}(3, -e^{(dx + c)}) + 6 \operatorname{polylog}(4, -e^{(dx + c)})) (a^2 f^3 - b^2 f^3) / (a^3 d^4) - (d^3 x^3 \log(-e^{(dx + c)} + 1) + 3 d^2 x^2 \operatorname{dilog}(e^{(dx + c)}) - 6 d x \operatorname{polylog}(3, e^{(dx + c)}) + 6 \operatorname{polylog}(4, e^{(dx + c)})) (a^2 f^3 - b^2 f^3) / (a^3 d^4) - 3(a^2 d e f^2 - b^2 d e f^2 - a b f^3) (d^2 x^2 \log(e^{(dx + c)} + 1) + 2 d x \operatorname{dilog}(-e^{(dx + c)}) - 2 \operatorname{polylog}(3, -e^{(dx + c)})) / (a^3 d^4) - 3(a^2 d e f^2 - b^2 d e f^2 + a b f^3) (d^2 x^2 \log(-e^{(dx + c)} + 1) + 2 d x \operatorname{dilog}(e^{(dx + c)}) - 2 \operatorname{polylog}(3, e^{(dx + c)})) / (a^3 d^4) + 3(b^2 d^2 e^2 f + 2 a b d e f^2 - (d^2 e^2 f - f^3) a^2) (d x \log(e^{(dx + c)} + 1) + \operatorname{dilog}(-e^{(dx + c)})) / (a^3 d^4) + 3(b^2 d^2 e^2 f - 2 a b d e f^2 - (d^2 e^2 f - f^3) a^2) (d x \log(-e^{(dx + c)} + 1) + \operatorname{dilog}(e^{(dx + c)})) / (a^3 d^4) + 1/4((a^2 f^3 - b^2 f^3) d^4 x^4 + 4(a^2 d e f^2 - b^2 d e f^2 + a b f^3) d^3 x^3 - 6(b^2 d^2 e^2 f - 2 a b d e f^2 - (d^2 e^2 f - f^3) a^2) d^2 x^2) / (a^3 d^4) + 1/4((a^2 f^3 - b^2 f^3) d^4 x^4 + 4(a^2 d e f^2 - b^2 d e f^2 - a b f^3) d^3 x^3 - 6(b^2 d^2 e^2 f + 2 a b d e f^2 - (d^2 e^2 f - f^3) a^2) d^2 x^2) / (a^3 d^4) + \operatorname{integrate}(2(b^5 f^3 x^3 + 3 b^5 e f^2 x^2 + 3 b^5 e^2 f x - (a b^4 f^3 x^3 e^c + 3 a b^4 e f^2 x^2 e^c + 3 a b^4 e^2 f x e^c) e^{(dx)}) / (a^5 b + a^3 b^3 - (a^5 b e^{(2c)} + a^3 b^3 e^{(2c)}) e^{(2dx)} - 2(a^6 e^c + a^4 b^2 e^c) e^{(dx)}), x) + \operatorname{integrate}(-2(a f^3 x^3 + 3 a e f^2 x^2 + 3 a e^2 f x - (b f^3 x^3 e^c + 3 b e f^2 x^2 e^c + 3 b e^2 f x e^c) e^{(dx)}) / (a^2 + b^2 + (a^2 e^{(2c)} + b^2 e^{(2c)}) e^{(2dx)}), x)
\end{aligned}$$

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)^3*csh(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**3*csch(d*x+c)**3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^3*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm  
m="giac")
```

```
[Out] Timed out
```

$$3.492 \quad \int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=1219

result too large to display

```
[Out] (e*f*x)/(a*d) + (f^2*x^2)/(2*a*d) + (2*b*(e + f*x)^2*ArcTan[E^(c + d*x)])/(a^2*d) - (2*b^3*(e + f*x)^2*ArcTan[E^(c + d*x)])/(a^2*(a^2 + b^2)*d) + (4*b*f*(e + f*x)*ArcTanh[E^(c + d*x)])/(a^2*d^2) + (2*(e + f*x)^2*ArcTanh[E^(2*c + 2*d*x)])/(a*d) - (2*b^2*(e + f*x)^2*ArcTanh[E^(2*c + 2*d*x)])/(a^3*d) - (f*(e + f*x)*Coth[c + d*x])/(a*d^2) - ((e + f*x)^2*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*x)^2*Csch[c + d*x])/(a^2*d) - (b^4*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)*d) - (b^4*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)*d) + (b^4*(e + f*x)^2*Log[1 + E^(2*(c + d*x))])/(a^3*(a^2 + b^2)*d) + (f^2*Log[Sinh[c + d*x]])/(a*d^3) + (2*b*f^2*PolyLog[2, -E^(c + d*x)])/(a^2*d^3) - ((2*I)*b*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*d^2) + ((2*I)*b^3*f*(e + f*x)*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) + ((2*I)*b*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(a^2*d^2) - ((2*I)*b^3*f*(e + f*x)*PolyLog[2, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) - (2*b*f^2*PolyLog[2, E^(c + d*x)])/(a^2*d^3) - (2*b^4*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^2) - (2*b^4*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^2) + (b^4*f*(e + f*x)*PolyLog[2, -E^(2*(c + d*x))])/(a^3*(a^2 + b^2)*d^2) + (f*(e + f*x)*PolyLog[2, -E^(2*c + 2*d*x)])/(a*d^2) - (b^2*f*(e + f*x)*PolyLog[2, -E^(2*c + 2*d*x)])/(a^3*d^2) - (f*(e + f*x)*PolyLog[2, E^(2*c + 2*d*x)])/(a*d^2) + (b^2*f*(e + f*x)*PolyLog[2, E^(2*c + 2*d*x)])/(a^3*d^2) + ((2*I)*b*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(a^2*d^3) - ((2*I)*b^3*f^2*PolyLog[3, (-I)*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^3) - ((2*I)*b*f^2*PolyLog[3, I*E^(c + d*x)])/(a^2*d^3) + ((2*I)*b^3*f^2*PolyLog[3, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^3) + (2*b^4*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^3) + (2*b^4*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^3) - (b^4*f^2*PolyLog[3, -E^(2*(c + d*x))])/(2*a^3*(a^2 + b^2)*d^3) - (f^2*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a*d^3) + (b^2*f^2*PolyLog[3, -E^(2*c + 2*d*x)])/(2*a^3*d^3) + (f^2*PolyLog[3, E^(2*c + 2*d*x)])/(2*a*d^3) - (b^2*f^2*PolyLog[3, E^(2*c + 2*d*x)])/(2*a^3*d^3)
```

---

**Rubi [A]** time = 2.22243, antiderivative size = 1219, normalized size of antiderivative = 1., number of steps used = 71, number of rules used = 26, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}}$  = 0.765, Rules used = {5589, 2620, 14, 5462, 6741, 12, 6742, 3720, 3475, 2551, 4182, 2531,

2282, 6589, 2621, 321, 207, 5205, 4180, 2279, 2391, 5461, 5573, 5561, 2190, 3718]

$$\frac{(e+fx)^2 \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^4}{a^3(a^2+b^2)d} - \frac{(e+fx)^2 \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^4}{a^3(a^2+b^2)d} + \frac{(e+fx)^2 \log(1+e^{2(c+dx)})b^4}{a^3(a^2+b^2)d} - \frac{2f(e+fx)\text{PolyLog}}{a^3(a^2+b^2)d}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Csch[c + d\*x]^3\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (e\*f\*x)/(a\*d) + (f^2\*x^2)/(2\*a\*d) + (2\*b\*(e + f\*x)^2\*ArcTan[E^(c + d\*x)])/(a^2\*d) - (2\*b^3\*(e + f\*x)^2\*ArcTan[E^(c + d\*x)])/(a^2\*(a^2 + b^2)\*d) + (4\*b\*f\*(e + f\*x)\*ArcTanh[E^(c + d\*x)])/(a^2\*d^2) + (2\*(e + f\*x)^2\*ArcTanh[E^(2\*c + 2\*d\*x)])/(a\*d) - (2\*b^2\*(e + f\*x)^2\*ArcTanh[E^(2\*c + 2\*d\*x)])/(a^3\*d) - (f\*(e + f\*x)\*Coth[c + d\*x])/(a\*d^2) - ((e + f\*x)^2\*Coth[c + d\*x]^2)/(2\*a\*d) + (b\*(e + f\*x)^2\*Csch[c + d\*x])/(a^2\*d) - (b^4\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^3\*(a^2 + b^2)\*d) - (b^4\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^3\*(a^2 + b^2)\*d) + (b^4\*(e + f\*x)^2\*Log[1 + E^(2\*(c + d\*x))])/(a^3\*(a^2 + b^2)\*d) + (f^2\*Log[Sinh[c + d\*x]])/(a\*d^3) + (2\*b\*f^2\*PolyLog[2, -E^(c + d\*x)])/(a^2\*d^3) - ((2\*I)\*b\*f\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a^2\*d^2) + ((2\*I)\*b^3\*f\*(e + f\*x)\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a^2\*(a^2 + b^2)\*d^2) + ((2\*I)\*b\*f\*(e + f\*x)\*PolyLog[2, I\*E^(c + d\*x)])/(a^2\*d^2) - ((2\*I)\*b^3\*f\*(e + f\*x)\*PolyLog[2, I\*E^(c + d\*x)])/(a^2\*(a^2 + b^2)\*d^2) - (2\*b\*f^2\*PolyLog[2, E^(c + d\*x)])/(a^2\*d^3) - (2\*b^4\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^3\*(a^2 + b^2)\*d^2) - (2\*b^4\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^3\*(a^2 + b^2)\*d^2) + (b^4\*f\*(e + f\*x)\*PolyLog[2, -E^(2\*(c + d\*x))])/(a^3\*(a^2 + b^2)\*d^2) + (f\*(e + f\*x)\*PolyLog[2, -E^(2\*c + 2\*d\*x)])/(a\*d^2) - (b^2\*f\*(e + f\*x)\*PolyLog[2, -E^(2\*c + 2\*d\*x)])/(a^3\*d^2) - (f\*(e + f\*x)\*PolyLog[2, E^(2\*c + 2\*d\*x)])/(a\*d^2) + (b^2\*f\*(e + f\*x)\*PolyLog[2, E^(2\*c + 2\*d\*x)])/(a^3\*d^2) + ((2\*I)\*b\*f^2\*PolyLog[3, (-I)\*E^(c + d\*x)])/(a^2\*d^3) - ((2\*I)\*b^3\*f^2\*PolyLog[3, (-I)\*E^(c + d\*x)])/(a^2\*(a^2 + b^2)\*d^3) - ((2\*I)\*b\*f^2\*PolyLog[3, I\*E^(c + d\*x)])/(a^2\*d^3) + ((2\*I)\*b^3\*f^2\*PolyLog[3, I\*E^(c + d\*x)])/(a^2\*(a^2 + b^2)\*d^3) + (2\*b^4\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^3\*(a^2 + b^2)\*d^3) + (2\*b^4\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^3\*(a^2 + b^2)\*d^3) - (b^4\*f^2\*PolyLog[3, -E^(2\*(c + d\*x))])/(2\*a^3\*(a^2 + b^2)\*d^3) - (f^2\*PolyLog[3, -E^(2\*c + 2\*d\*x)])/(2\*a\*d^3) + (b^2\*f^2\*PolyLog[3, -E^(2\*c + 2\*d\*x)])/(2\*a^3\*d^3) + (f^2\*PolyLog[3, E^(2\*c + 2\*d\*x)])/(2\*a\*d^3) - (b^2\*f^2\*PolyLog[3, E^(2\*c + 2\*d\*x)])/(2\*a^3\*d^3)

### Rule 5589

Int[(Csch[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(p\_.)]/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := D

```
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
IGtQ[p, 0]
```

### Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

### Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

### Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] := With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

### Rule 6741

```
Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v !=
= u]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_.)*(v_)] /; FreeQ[b, x]
```

### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

### Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symb
ol] := Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
```



$x] - \text{Dist}[b^2, \text{Int}[(c + d*x)^m*(b*\text{Tan}[e + f*x])^{(n - 2)}, x], x] /; \text{FreeQ}[\{b, c, d, e, f\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{GtQ}[m, 0]$

### Rule 3475

$\text{Int}[\text{tan}[(c_.) + (d_.)*(x_.)], x\_Symbol] \ :> \ -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

### Rule 2551

$\text{Int}[\text{Log}[u_]*((a_.) + (b_.)*(x_.))^{(m_.)}, x\_Symbol] \ :> \ \text{Simp}[((a + b*x)^{(m + 1)}*\text{Log}[u])/ (b*(m + 1)), x] - \text{Dist}[1/(b*(m + 1)), \text{Int}[\text{SimplifyIntegrand}[(a + b*x)^{(m + 1)}*D[u, x])/u, x], x] /; \text{FreeQ}[\{a, b, m\}, x] \ \&\& \ \text{InverseFunctionFreeQ}[u, x] \ \&\& \ \text{NeQ}[m, -1]$

### Rule 4182

$\text{Int}[\text{csc}[(e_.) + (\text{Complex}[0, fz\_])*(f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^{(m_.)}, x\_Symbol] \ :> \ \text{Simp}[(-2*(c + d*x)^m*\text{ArcTanh}[E^{-(I*e) + f*fz*x}]/(f*fz*I), x] + (-\text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 - E^{-(I*e) + f*fz*x}], x], x] + \text{Dist}[(d*m)/(f*fz*I), \text{Int}[(c + d*x)^{(m - 1)}*\text{Log}[1 + E^{-(I*e) + f*fz*x}], x], x]) /; \text{FreeQ}[\{c, d, e, f, fz\}, x] \ \&\& \ \text{IGtQ}[m, 0]$

### Rule 2531

$\text{Int}[\text{Log}[1 + (e_.)*((F_)^{((c_.)*((a_.) + (b_.)*(x_.)))^{(n_.)}})]*((f_.) + (g_.)*(x_.))^{(m_.)}, x\_Symbol] \ :> \ -\text{Simp}[(f + g*x)^m*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n])/ (b*c*n*\text{Log}[F]), x] + \text{Dist}[(g*m)/(b*c*n*\text{Log}[F]), \text{Int}[(f + g*x)^{(m - 1)}*\text{PolyLog}[2, -(e*(F^{(c*(a + b*x))))^n}], x], x] /; \text{FreeQ}[\{F, a, b, c, e, f, g, n\}, x] \ \&\& \ \text{GtQ}[m, 0]$

### Rule 2282

$\text{Int}[u_, x\_Symbol] \ :> \ \text{With}[\{v = \text{FunctionOfExponential}[u, x]\}, \text{Dist}[v/D[v, x], \text{Subst}[\text{Int}[\text{FunctionOfExponentialFunction}[u, x]/x, x], x, v], x] /; \text{FunctionOfExponentialQ}[u, x] \ \&\& \ !\text{MatchQ}[u, (w_)*((a_.)*(v_)^{(n_)})^{(m_)}] /; \text{FreeQ}[\{a, m, n\}, x] \ \&\& \ \text{IntegerQ}[m*n] \ \&\& \ !\text{MatchQ}[u, E^{((c_.)*((a_.) + (b_.)*x))* (F_)}][v_] /; \text{FreeQ}[\{a, b, c\}, x] \ \&\& \ \text{InverseFunctionQ}[F[x]]]$

### Rule 6589

$\text{Int}[\text{PolyLog}[n_, (c_.)*((a_.) + (b_.)*(x_.))^{(p_.)}]/((d_.) + (e_.)*(x_.)), x\_Symbol] \ :> \ \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] /; \text{FreeQ}[\{a, b, c, d, e, n, p\}, x] \ \&\& \ \text{EqQ}[b*d, a*e]$

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(a_.))^(m_.)*sec[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol]
:> -Dist[(f*a^n)^(-1), Subst[Int[x^(m+n-1)/(-1+x^2/a^2)^((n+1)/2), x], x, a*Csc[e+f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n+1)/2] && !(IntegerQ[(m+1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol]
:> Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^n*(m-n+1))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol]
:> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 5205

```
Int[((a_.) + ArcTan[u_]*(b_.))*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[((c+d*x)^(m+1)*(a+b*ArcTan[u]))/(d*(m+1)), x] - Dist[b/(d*(m+1)), Int[SimplifyIntegrand[(c+d*x)^(m+1)*D[u, x]]/(1+u^2), x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x] && !FunctionOfQ[(c+d*x)^(m+1), u, x] && FalseQ[PowerVariableExpn[u, m+1, x]]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[(-2*(c+d*x)^m*ArcTanh[E^(-(I*e)+f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c+d*x)^(m-1)*Log[1-E^(-(I*e)+f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c+d*x)^(m-1)*Log[1+E^(-(I*e)+f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_.))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a+b*x]/x, x], x, (F^(e*(c+d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

Int[Log[(c\_.)\*(d\_) + (e\_.)\*(x\_)^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 5461

Int[Csch[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sech[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Dist[2^n, Int[(c + d\*x)^m\*Csch[2\*a + 2\*b\*x]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]

Rule 5573

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[b^2/(a^2 + b^2), Int[(e + f\*x)^m\*Sech[c + d\*x]^(n - 2)/(a + b\*Sinh[c + d\*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f\*x)^m\*Sech[c + d\*x]^n\*(a - b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]

Rule 5561

Int[(Cosh[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))^(m\_.))/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := -Simp[(e + f\*x)^(m + 1)/(b\*f\*(m + 1)), x] + (Int[(e + f\*x)^m\*E^(c + d\*x)/(a - Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x] + Int[(e + f\*x)^m\*E^(c + d\*x)/(a + Rt[a^2 + b^2, 2] + b\*E^(c + d\*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]

Rule 2190

Int[(((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)/((a\_.) + (b\_.)\*((F\_)^(g\_.)\*((e\_.) + (f\_.)\*(x\_))))^(n\_.), x\_Symbol] := Simp[((c + d\*x)^m\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a])/(b\*f\*g\*n\*Log[F]), x] - Dist[(d\*m)/(b\*f\*g\*n\*Log[F]), Int[(c + d\*x)^(m - 1)\*Log[1 + (b\*(F^(g\*(e + f\*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 3718

Int[(((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]), x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[(c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x))/(1 + E^(2\*(-I\*e) + f\*fz\*x)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \operatorname{csch}^3(c+dx) \operatorname{sech}(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx) \operatorname{sech}(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= -\frac{(e+fx)^2 \coth^2(c+dx)}{2ad} - \frac{(e+fx)^2 \log(\tanh(c+dx))}{ad} - \frac{b \int (e+fx)^2 \operatorname{csch}^2(c+dx) \operatorname{sech}(c+dx) dx}{a} \\
&= \frac{b(e+fx)^2 \tan^{-1}(\sinh(c+dx))}{a^2 d} - \frac{(e+fx)^2 \coth^2(c+dx)}{2ad} + \frac{b(e+fx)^2 \operatorname{csch}^2(c+dx) \operatorname{sech}(c+dx)}{a^2 d} \\
&= \frac{b(e+fx)^2 \tan^{-1}(\sinh(c+dx))}{a^2 d} - \frac{(e+fx)^2 \coth^2(c+dx)}{2ad} + \frac{b(e+fx)^2 \operatorname{csch}^2(c+dx) \operatorname{sech}(c+dx)}{a^2 d} \\
&= \frac{b^4(e+fx)^3}{3a^3(a^2+b^2)f} + \frac{b(e+fx)^2 \tan^{-1}(\sinh(c+dx))}{a^2 d} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3 d} \\
&= \frac{b^4(e+fx)^3}{3a^3(a^2+b^2)f} + \frac{b(e+fx)^2 \tan^{-1}(\sinh(c+dx))}{a^2 d} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3 d} \\
&= -\frac{2b^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)d} + \frac{b(e+fx)^2 \tan^{-1}(\sinh(c+dx))}{a^2 d} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3 d} \\
&= \frac{efx}{ad} + \frac{f^2 x^2}{2ad} - \frac{2b^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)d} + \frac{4bf(e+fx) \tanh^{-1}(e^{c+dx})}{a^2 d^2} - \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3 d} \\
&= \frac{efx}{ad} + \frac{f^2 x^2}{2ad} - \frac{2b^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)d} + \frac{4bf(e+fx) \tanh^{-1}(e^{c+dx})}{a^2 d^2} + \frac{2b^2(e+fx)^2 \tanh^{-1}(e^{c+dx})}{a^3 d} \\
&= \frac{efx}{ad} + \frac{f^2 x^2}{2ad} + \frac{2b(e+fx)^2 \tan^{-1}(e^{c+dx})}{a^2 d} - \frac{2b^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)d} + \frac{4bf(e+fx) \tanh^{-1}(e^{c+dx})}{a^2 d^2} \\
&= \frac{efx}{ad} + \frac{f^2 x^2}{2ad} + \frac{2b(e+fx)^2 \tan^{-1}(e^{c+dx})}{a^2 d} - \frac{2b^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)d} + \frac{4bf(e+fx) \tanh^{-1}(e^{c+dx})}{a^2 d^2} \\
&= \frac{efx}{ad} + \frac{f^2 x^2}{2ad} + \frac{2b(e+fx)^2 \tan^{-1}(e^{c+dx})}{a^2 d} - \frac{2b^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)d} + \frac{4bf(e+fx) \tanh^{-1}(e^{c+dx})}{a^2 d^2} \\
&= \frac{efx}{ad} + \frac{f^2 x^2}{2ad} + \frac{2b(e+fx)^2 \tan^{-1}(e^{c+dx})}{a^2 d} - \frac{2b^3(e+fx)^2 \tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)d} + \frac{4bf(e+fx) \tanh^{-1}(e^{c+dx})}{a^2 d^2}
\end{aligned}$$

**Mathematica [B]** time = 42.4488, size = 2784, normalized size = 2.28

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Csch[c + d\*x]^3\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (b\*(e + f\*x)^2\*Csch[c])/(a^2\*d) - ((e + f\*x)^2\*Csch[(c + d\*x)/2]^2)/(8\*a\*d) + (-12\*a\*d^3\*e^2\*E^(2\*c)\*x + 12\*a\*d^3\*e^2\*(1 + E^(2\*c))\*x + 12\*a\*d^3\*e\*f\*x^2 + 4\*a\*d^3\*f^2\*x^3 + 12\*b\*d^2\*e^2\*(1 + E^(2\*c))\*ArcTan[E^(c + d\*x)] - 6\*a\*d^2\*e^2\*(1 + E^(2\*c))\*(2\*d\*x - Log[1 + E^(2\*(c + d\*x))]) + (12\*I)\*b\*d\*e\*(1 + E^(2\*c))\*f\*(d\*x\*(Log[1 - I\*E^(c + d\*x)] - Log[1 + I\*E^(c + d\*x)]) - PolyLog[2, (-I)\*E^(c + d\*x)] + PolyLog[2, I\*E^(c + d\*x)]) - 6\*a\*d\*e\*(1 + E^(2\*c))\*f\*(2\*d\*x\*(d\*x - Log[1 + E^(2\*(c + d\*x))]) - PolyLog[2, -E^(2\*(c + d\*x))]) + (6\*I)\*b\*(1 + E^(2\*c))\*f^2\*(d^2\*x^2\*Log[1 - I\*E^(c + d\*x)] - d^2\*x^2\*Log[1 + I\*E^(c + d\*x)] - 2\*d\*x\*PolyLog[2, (-I)\*E^(c + d\*x)] + 2\*d\*x\*PolyLog[2, I\*E^(c + d\*x)] + 2\*PolyLog[3, (-I)\*E^(c + d\*x)] - 2\*PolyLog[3, I\*E^(c + d\*x)]) - a\*(1 + E^(2\*c))\*f^2\*(2\*d^2\*x^2\*(2\*d\*x - 3\*Log[1 + E^(2\*(c + d\*x))]) - 6\*d\*x\*PolyLog[2, -E^(2\*(c + d\*x))] + 3\*PolyLog[3, -E^(2\*(c + d\*x))])/(6\*(a^2 + b^2)\*d^3\*(1 + E^(2\*c))) - (-12\*a^2\*d^3\*e^2\*E^(2\*c)\*x + 12\*b^2\*d^3\*e^2\*E^(2\*c)\*x + 12\*a^2\*d^3\*e\*E^(2\*c)\*f\*x^2 + 12\*b^2\*d^3\*e\*E^(2\*c)\*f\*x^2 - 4\*a^2\*d^3\*E^(2\*c)\*f^2\*x^3 + 4\*b^2\*d^3\*E^(2\*c)\*f^2\*x^3 + 24\*a\*b\*d\*e\*f\*ArcTanh[E^(c + d\*x)] - 24\*a\*b\*d\*e\*E^(2\*c)\*f\*ArcTanh[E^(c + d\*x)] - 12\*a\*b\*d\*f^2\*x\*Log[1 - E^(c + d\*x)] + 12\*a\*b\*d\*E^(2\*c)\*f^2\*x\*Log[1 - E^(c + d\*x)] + 12\*a\*b\*d\*f^2\*x\*Log[1 + E^(c + d\*x)] - 12\*a\*b\*d\*E^(2\*c)\*f^2\*x\*Log[1 + E^(c + d\*x)] - 6\*a^2\*d^2\*e^2\*Log[1 - E^(2\*(c + d\*x))] + 6\*b^2\*d^2\*e^2\*Log[1 - E^(2\*(c + d\*x))] + 6\*a^2\*d^2\*e^2\*E^(2\*c)\*Log[1 - E^(2\*(c + d\*x))] - 6\*b^2\*d^2\*e^2\*E^(2\*c)\*Log[1 - E^(2\*(c + d\*x))] + 6\*a^2\*f^2\*Log[1 - E^(2\*(c + d\*x))] - 6\*a^2\*E^(2\*c)\*f^2\*Log[1 - E^(2\*(c + d\*x))] - 12\*a^2\*d^2\*e\*f\*x\*Log[1 - E^(2\*(c + d\*x))] + 12\*b^2\*d^2\*e\*f\*x\*Log[1 - E^(2\*(c + d\*x))] + 12\*a^2\*d^2\*e\*E^(2\*c)\*f\*x\*Log[1 - E^(2\*(c + d\*x))] - 12\*b^2\*d^2\*e\*E^(2\*c)\*f\*x\*Log[1 - E^(2\*(c + d\*x))] - 6\*a^2\*d^2\*f^2\*x^2\*Log[1 - E^(2\*(c + d\*x))] + 6\*b^2\*d^2\*f^2\*x^2\*Log[1 - E^(2\*(c + d\*x))] + 6\*a^2\*d^2\*E^(2\*c)\*f^2\*x^2\*Log[1 - E^(2\*(c + d\*x))] - 6\*b^2\*d^2\*E^(2\*c)\*f^2\*x^2\*Log[1 - E^(2\*(c + d\*x))] - 12\*a\*b\*(-1 + E^(2\*c))\*f^2\*PolyLog[2, -E^(c + d\*x)] + 12\*a\*b\*(-1 + E^(2\*c))\*f^2\*PolyLog[2, E^(c + d\*x)] - 6\*a^2\*d\*e\*f\*PolyLog[2, E^(2\*(c + d\*x))] + 6\*b^2\*d\*e\*f\*PolyLog[2, E^(2\*(c + d\*x))] + 6\*a^2\*d\*e\*E^(2\*c)\*f\*PolyLog[2, E^(2\*(c + d\*x))] - 6\*b^2\*d\*e\*E^(2\*c)\*f\*PolyLog[2, E^(2\*(c + d\*x))] - 6\*a^2\*d\*f^2\*x\*PolyLog[2, E^(2\*(c + d\*x))] + 6\*b^2\*d\*f^2\*x\*PolyLog[2, E^(2\*(c + d\*x))] + 6\*a^2\*d\*E^(2\*c)\*f^2\*x\*PolyLog[2, E^(2\*(c + d\*x))] - 6\*b^2\*d\*E^(2\*c)\*f^2\*x\*PolyLog[2, E^(2\*(c + d\*x))] + 3\*a^2\*f^2\*PolyLog[3, E^(2\*(c + d\*x))] - 3\*b^2\*f^2\*PolyLog[3, E^(2\*(c + d\*x))] - 3\*a^2\*E^(2\*c)\*f^2\*PolyLog[3, E^(2\*(c + d\*x))] - 3\*b^2\*E^(2\*c)\*f^2\*PolyLog[3, E^(2\*(c + d\*x))]

$$\begin{aligned}
& d*x)) + 3*b^2*E^{(2*c)}*f^2*PolyLog[3, E^{(2*(c + d*x))}]/(6*a^3*d^3*(-1 + E^{(2*c)})) + (b^4*(6*e^2*E^{(2*c)}*x + 6*e*E^{(2*c)}*f*x^2 + 2*E^{(2*c)}*f^2*x^3 + \\
& (6*a*Sqrt[a^2 + b^2]*e^2*ArcTan[(a + b*E^{(c + d*x)})/Sqrt[-a^2 - b^2]])/Sqrt[-(a^2 + b^2)^2]*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^{(2*c)}*ArcTan[(a + b* \\
& E^{(c + d*x)})/Sqrt[-a^2 - b^2]])/((a^2 + b^2)^{(3/2)}*d) - (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*ArcTanh[(a + b*E^{(c + d*x)})/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^{(3/2)} \\
& )*d) + (6*a*Sqrt[-(a^2 + b^2)^2]*e^2*E^{(2*c)}*ArcTanh[(a + b*E^{(c + d*x)})/Sqrt[a^2 + b^2]])/((-a^2 - b^2)^{(3/2)}*d) + (3*e^2*Log[2*a*E^{(c + d*x)} + b*(-1 \\
& + E^{(2*(c + d*x))})]/d - (3*e^2*E^{(2*c)}*Log[2*a*E^{(c + d*x)} + b*(-1 + E^{(2*(c + d*x))})])/d + (6*e*f*x*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c - Sqrt[(a^2 + \\
& b^2)*E^{(2*c)}])])/d - (6*e*E^{(2*c)}*f*x*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d + (3*f^2*x^2*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d - (3*E^{(2*c)}*f^2*x^2*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d + (6*e*f*x*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d - (6*e*E^{(2*c)}*f*x*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d + (3*f^2*x^2*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d - (3*E^{(2*c)}*f^2*x^2*Log[1 + (b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d - (6*(-1 + E^{(2*c)})*f*(e + f*x)*PolyLog[2, -((b*E^{(2*c + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^2 - (6*(-1 + E^{(2*c)})*f*(e + f*x)*PolyLog[2, -((b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^2 - (6*f^2*PolyLog[3, -((b*E^{(2*c + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^3 + (6*E^{(2*c)}*f^2*PolyLog[3, -((b*E^{(2*c + d*x)})/(a*E^c - Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^3 - (6*f^2*PolyLog[3, -((b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^3 + (6*E^{(2*c)}*f^2*PolyLog[3, -((b*E^{(2*c + d*x)})/(a*E^c + Sqrt[(a^2 + b^2)*E^{(2*c)}])])/d^3)/(3*a^3*(a^2 + b^2)*(-1 + E^{(2*c)})) - (a*x*(3*e^2 + 3*e*f*x + f^2*x^2)*Csch[c]*Sech[c])/(3*(a^2 + b^2)) + ((e + f*x)^2*Sech[(c + d*x)/2]^2)/(8*a*d) - ((e + f*x)*(-(a*f) + b*d*(e + f*x))*Csch[c/2]*Csch[(c + d*x)/2]*Sinh[(d*x)/2])/(2*a^2*d^2) - ((e + f*x)*(a*f + b*d*(e + f*x))*Sech[c/2]*Sech[(c + d*x)/2]*Sinh[(d*x)/2])/(2*a^2*d^2)
\end{aligned}$$

**Maple [F]** time = 1.398, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\operatorname{csch}(dx + c))^3 \operatorname{sech}(dx + c)}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*csch(d\*x+c)^3\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*csch(d\*x+c)^3\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*csh(d\*x+c)^3\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$-(b^4 \log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/((a^5 + a^3*b^2)*d) + 2*b*\arctan(e^{(-d*x - c)})/((a^2 + b^2)*d) - a*\log(e^{(-2*d*x - 2*c)} + 1)/((a^2 + b^2)*d) + 2*(b*e^{(-d*x - c)} - a*e^{(-2*d*x - 2*c)} - b*e^{(-3*d*x - 3*c)})/((2*a^2*e^{(-2*d*x - 2*c)} - a^2*e^{(-4*d*x - 4*c)} - a^2)*d) + (a^2 - b^2)*\log(e^{(-d*x - c)} + 1)/(a^3*d) + (a^2 - b^2)*\log(e^{(-d*x - c)} - 1)/(a^3*d))*e^{2 + 2*(a*f^2*x + a*e*f + (b*d*f^2*x^2*e^{(3*c)} + 2*b*d*e*f*x*e^{(3*c)}))*e^{(3*d*x)} - (a*d*f^2*x^2*e^{(2*c)} + a*e*f*e^{(2*c)} + (2*d*e*f + f^2)*a*x*e^{(2*c)})*e^{(2*d*x)} - (b*d*f^2*x^2*e^c + 2*b*d*e*f*x*e^c)*e^{(d*x)}/(a^2*d^2*e^{(4*d*x + 4*c)} - 2*a^2*d^2*e^{(2*d*x + 2*c)} + a^2*d^2) - (2*b*d*e*f + a*f^2)*x/(a^2*d^2) + (2*b*d*e*f - a*f^2)*x/(a^2*d^2) + (2*b*d*e*f + a*f^2)*\log(e^{(d*x + c)} + 1)/(a^2*d^3) - (2*b*d*e*f - a*f^2)*\log(e^{(d*x + c)} - 1)/(a^2*d^3) - (d^2*x^2*\log(e^{(d*x + c)} + 1) + 2*d*x*dilog(-e^{(d*x + c)}) - 2*polylog(3, -e^{(d*x + c)}))*(a^2*f^2 - b^2*f^2)/(a^3*d^3) - (d^2*x^2*\log(-e^{(d*x + c)} + 1) + 2*d*x*dilog(e^{(d*x + c)}) - 2*polylog(3, e^{(d*x + c)}))*(a^2*f^2 - b^2*f^2)/(a^3*d^3) - 2*(a^2*d*e*f - b^2*d*e*f - a*b*f^2)*(d*x*\log(e^{(d*x + c)} + 1) + dilog(-e^{(d*x + c)}))/(a^3*d^3) - 2*(a^2*d*e*f - b^2*d*e*f + a*b*f^2)*(d*x*\log(-e^{(d*x + c)} + 1) + dilog(e^{(d*x + c)}))/(a^3*d^3) + 1/3*((a^2*f^2 - b^2*f^2)*d^3*x^3 + 3*(a^2*d*e*f - b^2*d*e*f + a*b*f^2)*d^2*x^2)/(a^3*d^3) + 1/3*((a^2*f^2 - b^2*f^2)*d^3*x^3 + 3*(a^2*d*e*f - b^2*d*e*f - a*b*f^2)*d^2*x^2)/(a^3*d^3) + integrate(2*(b^5*f^2*x^2 + 2*b^5*e*f*x - (a*b^4*f^2*x^2*e^c + 2*a*b^4*e*f*x*e^c)*e^{(d*x)})/(a^5*b + a^3*b^3 - (a^5*b*e^{(2*c)} + a^3*b^3*e^{(2*c)}))*e^{(2*d*x)} - 2*(a^6*e^c + a^4*b^2*e^c)*e^{(d*x)}, x) + integrate(-2*(a*f^2*x^2 + 2*a*e*f*x - (b*f^2*x^2*e^c + 2*b*e*f*x*e^c)*e^{(d*x)})/(a^2 + b^2 + (a^2*e^{(2*c)} + b^2*e^{(2*c)}))*e^{(2*d*x)}, x)$$

---

**Fricas [C]** time = 6.1085, size = 29429, normalized size = 24.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)^2\*csch(d\*x+c)^3\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$-(2*((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*c*f^2)*\cosh(d*x + c)^4 + 2*((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*c*f^2)*\sinh(d*x + c)^4 - 2*(a^4 + a^2*b^2)*d*e*f + 2*(a^4 + a^2*b^2)*c*f^2 - 2*((a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x + (a^3*b + a*b^3)*d^2*e^2)*\cosh(d*x + c)^3 - 2*((a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x + (a^3*b + a*b^3)*d^2*e^2 - 4*((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*c*f^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*((a^4 + a^2*b^2)*d^2*f^2*x^2 + (a^4 + a^2*b^2)*d^2*e^2 + (a^4 + a^2*b^2)*d*e*f - 2*(a^4 + a^2*b^2)*c*f^2 + (2*(a^4 + a^2*b^2)*d^2*e*f - (a^4 + a^2*b^2)*d*f^2)*x)*\cosh(d*x + c)^2 + 2*((a^4 + a^2*b^2)*d^2*f^2*x^2 + (a^4 + a^2*b^2)*d^2*e^2 + (a^4 + a^2*b^2)*d*e*f - 2*(a^4 + a^2*b^2)*c*f^2 + 6*((a^4 + a^2*b^2)*d*f^2*x + (a^4 + a^2*b^2)*c*f^2)*\cosh(d*x + c)^2 + (2*(a^4 + a^2*b^2)*d^2*e*f - (a^4 + a^2*b^2)*d*f^2)*x - 3*((a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x + (a^3*b + a*b^3)*d^2*e^2)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 2*((a^3*b + a*b^3)*d^2*f^2*x^2 + 2*(a^3*b + a*b^3)*d^2*e*f*x + (a^3*b + a*b^3)*d^2*e^2)*\cosh(d*x + c) + 2*(b^4*d*f^2*x + b^4*d*e*f + (b^4*d*f^2*x + b^4*d*e*f)*\cosh(d*x + c))^4 + 4*(b^4*d*f^2*x + b^4*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^4*d*f^2*x + b^4*d*e*f)*\sinh(d*x + c)^4 - 2*(b^4*d*f^2*x + b^4*d*e*f)*\cosh(d*x + c)^2 - 2*(b^4*d*f^2*x + b^4*d*e*f - 3*(b^4*d*f^2*x + b^4*d*e*f)*\cosh(d*x + c))^2*\sinh(d*x + c)^2 + 4*((b^4*d*f^2*x + b^4*d*e*f)*\cosh(d*x + c))^3 - (b^4*d*f^2*x + b^4*d*e*f)*\cosh(d*x + c))*\sinh(d*x + c))*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) + (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 2*(b^4*d*f^2*x + b^4*d*e*f + (b^4*d*f^2*x + b^4*d*e*f)*\cosh(d*x + c))^4 + 4*(b^4*d*f^2*x + b^4*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^4*d*f^2*x + b^4*d*e*f)*\sinh(d*x + c)^4 - 2*(b^4*d*f^2*x + b^4*d*e*f)*\cosh(d*x + c)^2 - 2*(b^4*d*f^2*x + b^4*d*e*f - 3*(b^4*d*f^2*x + b^4*d*e*f)*\cosh(d*x + c))^2*\sinh(d*x + c)^2 + 4*((b^4*d*f^2*x + b^4*d*e*f)*\cosh(d*x + c))^3 - (b^4*d*f^2*x + b^4*d*e*f)*\cosh(d*x + c))*\sinh(d*x + c))*\operatorname{dilog}((a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b + 1) + 2*((a^4 - b^4)*d*f^2*x + ((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f + (a^3*b + a*b^3)*f^2)*\cosh(d*x + c))^4 + 4*((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f + (a^3*b + a*b^3)*f^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + ((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f + (a^3*b + a*b^3)*f^2)*\sinh(d*x + c)^4 + (a^4 - b^4)*d*e*f + (a^3*b + a*b^3)*f^2 - 2*((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f + (a^3*b + a*b^3)*f^2)*\cosh(d*x + c)^2 - 2*((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f + (a^3*b + a*b^3)*f^2 - 3*((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f + (a^3*b + a*b^3)*f^2)*\cosh(d*x + c))^2*\sinh(d*x + c)^2 + 4*((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f + (a^3*b + a*b^3)*f^2)*\cosh(d*x + c))^3 - ((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f + (a^3*b + a*b^3)*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\operatorname{dilog}(\cosh(d*x + c) + \sinh(d*x + c)) - (2*a^4*d*f^2*x + 2*I*a^3*b*d*f^2*x + 2*a^4*d*e*f + 2*I*a^3*b*d*e*f + (2*a^4*d*f^2*x + 2*I*a^3*b$$



$$\begin{aligned}
& b*d*f^2*x + 2*a^4*d*e*f + 2*I*a^3*b*d*e*f)*\cosh(d*x + c)^4 + (8*a^4*d*f^2*x \\
& + 8*I*a^3*b*d*f^2*x + 8*a^4*d*e*f + 8*I*a^3*b*d*e*f)*\cosh(d*x + c)*\sinh(d* \\
& x + c)^3 + (2*a^4*d*f^2*x + 2*I*a^3*b*d*f^2*x + 2*a^4*d*e*f + 2*I*a^3*b*d*e \\
& *f)*\sinh(d*x + c)^4 - (4*a^4*d*f^2*x + 4*I*a^3*b*d*f^2*x + 4*a^4*d*e*f + 4* \\
& I*a^3*b*d*e*f)*\cosh(d*x + c)^2 - (4*a^4*d*f^2*x + 4*I*a^3*b*d*f^2*x + 4*a^4 \\
& *d*e*f + 4*I*a^3*b*d*e*f - (12*a^4*d*f^2*x + 12*I*a^3*b*d*f^2*x + 12*a^4*d* \\
& e*f + 12*I*a^3*b*d*e*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + ((8*a^4*d*f^2*x \\
& + 8*I*a^3*b*d*f^2*x + 8*a^4*d*e*f + 8*I*a^3*b*d*e*f)*\cosh(d*x + c)^3 - (8*a \\
& ^4*d*f^2*x + 8*I*a^3*b*d*f^2*x + 8*a^4*d*e*f + 8*I*a^3*b*d*e*f)*\cosh(d*x + \\
& c))*\sinh(d*x + c))*\operatorname{dilog}(I*\cosh(d*x + c) + I*\sinh(d*x + c)) - (2*a^4*d*f^2* \\
& x - 2*I*a^3*b*d*f^2*x + 2*a^4*d*e*f - 2*I*a^3*b*d*e*f + (2*a^4*d*f^2*x - 2* \\
& I*a^3*b*d*f^2*x + 2*a^4*d*e*f - 2*I*a^3*b*d*e*f)*\cosh(d*x + c)^4 + (8*a^4*d \\
& *f^2*x - 8*I*a^3*b*d*f^2*x + 8*a^4*d*e*f - 8*I*a^3*b*d*e*f)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (2*a^4*d*f^2*x - 2*I*a^3*b*d*f^2*x + 2*a^4*d*e*f - 2*I*a^3 \\
& *b*d*e*f)*\sinh(d*x + c)^4 - (4*a^4*d*f^2*x - 4*I*a^3*b*d*f^2*x + 4*a^4*d*e*f \\
& - 4*I*a^3*b*d*e*f)*\cosh(d*x + c)^2 - (4*a^4*d*f^2*x - 4*I*a^3*b*d*f^2*x + \\
& 4*a^4*d*e*f - 4*I*a^3*b*d*e*f - (12*a^4*d*f^2*x - 12*I*a^3*b*d*f^2*x + 12* \\
& a^4*d*e*f - 12*I*a^3*b*d*e*f)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + ((8*a^4*d* \\
& f^2*x - 8*I*a^3*b*d*f^2*x + 8*a^4*d*e*f - 8*I*a^3*b*d*e*f)*\cosh(d*x + c)^3 \\
& - (8*a^4*d*f^2*x - 8*I*a^3*b*d*f^2*x + 8*a^4*d*e*f - 8*I*a^3*b*d*e*f)*\cosh( \\
& d*x + c))*\sinh(d*x + c))*\operatorname{dilog}(-I*\cosh(d*x + c) - I*\sinh(d*x + c)) + 2*((a^ \\
& 4 - b^4)*d*f^2*x + ((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f - (a^3*b + a*b^ \\
& 3)*f^2)*\cosh(d*x + c)^4 + 4*((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f - (a^3 \\
& *b + a*b^3)*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^4 - b^4)*d*f^2*x + (a^ \\
& 4 - b^4)*d*e*f - (a^3*b + a*b^3)*f^2)*\sinh(d*x + c)^4 + (a^4 - b^4)*d*e*f - \\
& (a^3*b + a*b^3)*f^2 - 2*((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f - (a^3*b \\
& + a*b^3)*f^2)*\cosh(d*x + c)^2 - 2*((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f \\
& - (a^3*b + a*b^3)*f^2 - 3*((a^4 - b^4)*d*f^2*x + (a^4 - b^4)*d*e*f - (a^3*b \\
& + a*b^3)*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*(((a^4 - b^4)*d*f^2*x + \\
& (a^4 - b^4)*d*e*f - (a^3*b + a*b^3)*f^2)*\cosh(d*x + c)^3 - ((a^4 - b^4)*d* \\
& f^2*x + (a^4 - b^4)*d*e*f - (a^3*b + a*b^3)*f^2)*\cosh(d*x + c))*\sinh(d*x + \\
& c))*\operatorname{dilog}(-\cosh(d*x + c) - \sinh(d*x + c)) + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + \\
& b^4*c^2*f^2 + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*\cosh(d*x + c)^4 + \\
& 4*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^ \\
& 3 + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*\sinh(d*x + c)^4 - 2*(b^4*d^ \\
& 2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*\cosh(d*x + c)^2 - 2*(b^4*d^2*e^2 - 2*b \\
& ^4*c*d*e*f + b^4*c^2*f^2 - 3*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*\co \\
& sh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2* \\
& f^2)*\cosh(d*x + c)^3 - (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*\cosh(d*x \\
& + c))*\sinh(d*x + c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) + 2*b*\sqrt{ \\
& (a^2 + b^2)/b^2} + 2*a) + (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2 + (b^4 \\
& *d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*\cosh(d*x + c)^4 + 4*(b^4*d^2*e^2 - \\
& 2*b^4*c*d*e*f + b^4*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^4*d^2*e^2 - \\
& 2*b^4*c*d*e*f + b^4*c^2*f^2)*\sinh(d*x + c)^4 - 2*(b^4*d^2*e^2 - 2*b^4*c*d* \\
& e*f + b^4*c^2*f^2)*\cosh(d*x + c)^2 - 2*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c
\end{aligned}$$

$$\begin{aligned}
& ^2*f^2 - 3*(b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*\cosh(d*x + c)^2*\sin \\
& h(d*x + c)^2 + 4*((b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*\cosh(d*x + c) \\
& ^3 - (b^4*d^2*e^2 - 2*b^4*c*d*e*f + b^4*c^2*f^2)*\cosh(d*x + c))*\sinh(d*x + \\
& c))*\log(2*b*\cosh(d*x + c) + 2*b*\sinh(d*x + c) - 2*b*\sqrt{(a^2 + b^2)/b^2} + \\
& 2*a) + (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2 + \\
& (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*\cosh(d*x \\
& + c)^4 + 4*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2) \\
& )*\cosh(d*x + c)*\sinh(d*x + c)^3 + (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^ \\
& 4*c*d*e*f - b^4*c^2*f^2)*\sinh(d*x + c)^4 - 2*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e \\
& *f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*\cosh(d*x + c)^2 - 2*(b^4*d^2*f^2*x^2 + \\
& 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2 - 3*(b^4*d^2*f^2*x^2 + 2*b^4*d \\
& ^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + \\
& 4*((b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*\cosh( \\
& d*x + c)^3 - (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f \\
& ^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) + \\
& (b*\cosh(d*x + c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + (b^4*d \\
& ^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2 + (b^4*d^2*f^2*x \\
& ^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*\cosh(d*x + c)^4 + 4*(b^ \\
& 4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*\cosh(d*x + c \\
& )*\sinh(d*x + c)^3 + (b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^ \\
& 4*c^2*f^2)*\sinh(d*x + c)^4 - 2*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c \\
& *d*e*f - b^4*c^2*f^2)*\cosh(d*x + c)^2 - 2*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f* \\
& x + 2*b^4*c*d*e*f - b^4*c^2*f^2 - 3*(b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2* \\
& b^4*c*d*e*f - b^4*c^2*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*((b^4*d^2*f \\
& ^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*\cosh(d*x + c)^3 - ( \\
& b^4*d^2*f^2*x^2 + 2*b^4*d^2*e*f*x + 2*b^4*c*d*e*f - b^4*c^2*f^2)*\cosh(d*x + \\
& c))*\sinh(d*x + c))*\log(-(a*\cosh(d*x + c) + a*\sinh(d*x + c) - (b*\cosh(d*x + \\
& c) + b*\sinh(d*x + c))*\sqrt{(a^2 + b^2)/b^2} - b)/b) + ((a^4 - b^4)*d^2*f^2 \\
& *x^2 + (a^4 - b^4)*d^2*e^2 + ((a^4 - b^4)*d^2*f^2*x^2 + (a^4 - b^4)*d^2*e^2 \\
& - 2*(a^3*b + a*b^3)*d*e*f - (a^4 + a^2*b^2)*f^2 + 2*((a^4 - b^4)*d^2*e*f - \\
& (a^3*b + a*b^3)*d*f^2)*x)*\cosh(d*x + c)^4 + 4*((a^4 - b^4)*d^2*f^2*x^2 + ( \\
& a^4 - b^4)*d^2*e^2 - 2*(a^3*b + a*b^3)*d*e*f - (a^4 + a^2*b^2)*f^2 + 2*((a^ \\
& 4 - b^4)*d^2*e*f - (a^3*b + a*b^3)*d*f^2)*x)*\cosh(d*x + c)*\sinh(d*x + c)^3 \\
& + ((a^4 - b^4)*d^2*f^2*x^2 + (a^4 - b^4)*d^2*e^2 - 2*(a^3*b + a*b^3)*d*e*f \\
& - (a^4 + a^2*b^2)*f^2 + 2*((a^4 - b^4)*d^2*e*f - (a^3*b + a*b^3)*d*f^2)*x)* \\
& \sinh(d*x + c)^4 - 2*(a^3*b + a*b^3)*d*e*f - (a^4 + a^2*b^2)*f^2 - 2*((a^4 - \\
& b^4)*d^2*f^2*x^2 + (a^4 - b^4)*d^2*e^2 - 2*(a^3*b + a*b^3)*d*e*f - (a^4 + \\
& a^2*b^2)*f^2 + 2*((a^4 - b^4)*d^2*e*f - (a^3*b + a*b^3)*d*f^2)*x)*\cosh(d*x \\
& + c)^2 - 2*((a^4 - b^4)*d^2*f^2*x^2 + (a^4 - b^4)*d^2*e^2 - 2*(a^3*b + a*b^ \\
& 3)*d*e*f - (a^4 + a^2*b^2)*f^2 - 3*((a^4 - b^4)*d^2*f^2*x^2 + (a^4 - b^4)*d \\
& ^2*e^2 - 2*(a^3*b + a*b^3)*d*e*f - (a^4 + a^2*b^2)*f^2 + 2*((a^4 - b^4)*d^2 \\
& *e*f - (a^3*b + a*b^3)*d*f^2)*x)*\cosh(d*x + c)^2 + 2*((a^4 - b^4)*d^2*e*f - \\
& (a^3*b + a*b^3)*d*f^2)*x)*\sinh(d*x + c)^2 + 2*((a^4 - b^4)*d^2*e*f - (a^3* \\
& b + a*b^3)*d*f^2)*x + 4*((a^4 - b^4)*d^2*f^2*x^2 + (a^4 - b^4)*d^2*e^2 - 2 \\
& *(a^3*b + a*b^3)*d*e*f - (a^4 + a^2*b^2)*f^2 + 2*((a^4 - b^4)*d^2*e*f - (a^
\end{aligned}$$

$$\begin{aligned}
& 3*b + a*b^3)*d*f^2)*x)*\cosh(d*x + c)^3 - ((a^4 - b^4)*d^2*f^2*x^2 + (a^4 - \\
& b^4)*d^2*e^2 - 2*(a^3*b + a*b^3)*d*e*f - (a^4 + a^2*b^2)*f^2 + 2*((a^4 - b^4)*d^2*e*f - (a^3*b + a*b^3)*d*f^2)*x)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\co \\
& sh(d*x + c) + \sinh(d*x + c) + 1) - (a^4*d^2*e^2 + I*a^3*b*d^2*e^2 - 2*a^4*c \\
& *d*e*f - 2*I*a^3*b*c*d*e*f + a^4*c^2*f^2 + I*a^3*b*c^2*f^2 + (a^4*d^2*e^2 + \\
& I*a^3*b*d^2*e^2 - 2*a^4*c*d*e*f - 2*I*a^3*b*c*d*e*f + a^4*c^2*f^2 + I*a^3* \\
& b*c^2*f^2)*\cosh(d*x + c)^4 + (4*a^4*d^2*e^2 + 4*I*a^3*b*d^2*e^2 - 8*a^4*c*d \\
& *e*f - 8*I*a^3*b*c*d*e*f + 4*a^4*c^2*f^2 + 4*I*a^3*b*c^2*f^2)*\cosh(d*x + c) \\
& *\sinh(d*x + c)^3 + (a^4*d^2*e^2 + I*a^3*b*d^2*e^2 - 2*a^4*c*d*e*f - 2*I*a^3 \\
& *b*c*d*e*f + a^4*c^2*f^2 + I*a^3*b*c^2*f^2)*\sinh(d*x + c)^4 - (2*a^4*d^2*e^ \\
& 2 + 2*I*a^3*b*d^2*e^2 - 4*a^4*c*d*e*f - 4*I*a^3*b*c*d*e*f + 2*a^4*c^2*f^2 + \\
& 2*I*a^3*b*c^2*f^2)*\cosh(d*x + c)^2 - (2*a^4*d^2*e^2 + 2*I*a^3*b*d^2*e^2 - \\
& 4*a^4*c*d*e*f - 4*I*a^3*b*c*d*e*f + 2*a^4*c^2*f^2 + 2*I*a^3*b*c^2*f^2 - (6* \\
& a^4*d^2*e^2 + 6*I*a^3*b*d^2*e^2 - 12*a^4*c*d*e*f - 12*I*a^3*b*c*d*e*f + 6*a \\
& ^4*c^2*f^2 + 6*I*a^3*b*c^2*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + ((4*a^4* \\
& d^2*e^2 + 4*I*a^3*b*d^2*e^2 - 8*a^4*c*d*e*f - 8*I*a^3*b*c*d*e*f + 4*a^4*c^2 \\
& *f^2 + 4*I*a^3*b*c^2*f^2)*\cosh(d*x + c)^3 - (4*a^4*d^2*e^2 + 4*I*a^3*b*d^2* \\
& e^2 - 8*a^4*c*d*e*f - 8*I*a^3*b*c*d*e*f + 4*a^4*c^2*f^2 + 4*I*a^3*b*c^2*f^2 \\
& )*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + I) - (a \\
& ^4*d^2*e^2 - I*a^3*b*d^2*e^2 - 2*a^4*c*d*e*f + 2*I*a^3*b*c*d*e*f + a^4*c^2* \\
& f^2 - I*a^3*b*c^2*f^2 + (a^4*d^2*e^2 - I*a^3*b*d^2*e^2 - 2*a^4*c*d*e*f + 2* \\
& I*a^3*b*c*d*e*f + a^4*c^2*f^2 - I*a^3*b*c^2*f^2)*\cosh(d*x + c)^4 + (4*a^4*d \\
& ^2*e^2 - 4*I*a^3*b*d^2*e^2 - 8*a^4*c*d*e*f + 8*I*a^3*b*c*d*e*f + 4*a^4*c^2* \\
& f^2 - 4*I*a^3*b*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^4*d^2*e^2 - I*a \\
& ^3*b*d^2*e^2 - 2*a^4*c*d*e*f + 2*I*a^3*b*c*d*e*f + a^4*c^2*f^2 - I*a^3*b*c^ \\
& 2*f^2)*\sinh(d*x + c)^4 - (2*a^4*d^2*e^2 - 2*I*a^3*b*d^2*e^2 - 4*a^4*c*d*e*f \\
& + 4*I*a^3*b*c*d*e*f + 2*a^4*c^2*f^2 - 2*I*a^3*b*c^2*f^2)*\cosh(d*x + c)^2 - \\
& (2*a^4*d^2*e^2 - 2*I*a^3*b*d^2*e^2 - 4*a^4*c*d*e*f + 4*I*a^3*b*c*d*e*f + 2 \\
& *a^4*c^2*f^2 - 2*I*a^3*b*c^2*f^2 - (6*a^4*d^2*e^2 - 6*I*a^3*b*d^2*e^2 - 12* \\
& a^4*c*d*e*f + 12*I*a^3*b*c*d*e*f + 6*a^4*c^2*f^2 - 6*I*a^3*b*c^2*f^2)*\cosh( \\
& d*x + c)^2)*\sinh(d*x + c)^2 + ((4*a^4*d^2*e^2 - 4*I*a^3*b*d^2*e^2 - 8*a^4*c \\
& *d*e*f + 8*I*a^3*b*c*d*e*f + 4*a^4*c^2*f^2 - 4*I*a^3*b*c^2*f^2)*\cosh(d*x + \\
& c)^3 - (4*a^4*d^2*e^2 - 4*I*a^3*b*d^2*e^2 - 8*a^4*c*d*e*f + 8*I*a^3*b*c*d*e \\
& *f + 4*a^4*c^2*f^2 - 4*I*a^3*b*c^2*f^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(c \\
& osh(d*x + c) + \sinh(d*x + c) - I) + ((a^4 - b^4)*d^2*e^2 + ((a^4 - b^4)*d^2 \\
& *e^2 + 2*(a^3*b + a*b^3 - (a^4 - b^4)*c)*d*e*f - (a^4 + a^2*b^2 - (a^4 - b^4) \\
& 4)*c^2 + 2*(a^3*b + a*b^3)*c)*f^2)*\cosh(d*x + c)^4 + 4*((a^4 - b^4)*d^2*e^2 \\
& + 2*(a^3*b + a*b^3 - (a^4 - b^4)*c)*d*e*f - (a^4 + a^2*b^2 - (a^4 - b^4)*c \\
& ^2 + 2*(a^3*b + a*b^3)*c)*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + ((a^4 - b^4) \\
& *d^2*e^2 + 2*(a^3*b + a*b^3 - (a^4 - b^4)*c)*d*e*f - (a^4 + a^2*b^2 - (a^4 - \\
& b^4)*c^2 + 2*(a^3*b + a*b^3)*c)*f^2)*\sinh(d*x + c)^4 + 2*(a^3*b + a*b^3 - \\
& (a^4 - b^4)*c)*d*e*f - (a^4 + a^2*b^2 - (a^4 - b^4)*c^2 + 2*(a^3*b + a*b^3 \\
& )*c)*f^2 - 2*((a^4 - b^4)*d^2*e^2 + 2*(a^3*b + a*b^3 - (a^4 - b^4)*c)*d*e*f \\
& - (a^4 + a^2*b^2 - (a^4 - b^4)*c^2 + 2*(a^3*b + a*b^3)*c)*f^2)*\cosh(d*x + \\
& c)^2 - 2*((a^4 - b^4)*d^2*e^2 + 2*(a^3*b + a*b^3 - (a^4 - b^4)*c)*d*e*f - (
\end{aligned}$$

$$\begin{aligned}
& a^4 + a^2b^2 - (a^4 - b^4)c^2 + 2*(a^3b + ab^3)*c)*f^2 - 3*((a^4 - b^4) \\
& *d^2e^2 + 2*(a^3b + ab^3 - (a^4 - b^4)*c)*d*ef - (a^4 + a^2b^2 - (a^4 \\
& - b^4)*c^2 + 2*(a^3b + ab^3)*c)*f^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4 \\
& *(((a^4 - b^4)*d^2e^2 + 2*(a^3b + ab^3 - (a^4 - b^4)*c)*d*ef - (a^4 + a \\
& ^2b^2 - (a^4 - b^4)*c^2 + 2*(a^3b + ab^3)*c)*f^2)*\cosh(d*x + c)^3 - ((a^4 \\
& - b^4)*d^2e^2 + 2*(a^3b + ab^3 - (a^4 - b^4)*c)*d*ef - (a^4 + a^2b^2 \\
& - (a^4 - b^4)*c^2 + 2*(a^3b + ab^3)*c)*f^2)*\cosh(d*x + c))*\sinh(d*x + c) \\
& )*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) - (a^4*d^2*f^2*x^2 - I*a^3*b*d^2*f \\
& ^2*x^2 + 2*a^4*d^2*ef*x - 2*I*a^3*b*d^2*ef*x + 2*a^4*c*d*ef - 2*I*a^3*b*c \\
& *d*ef - a^4*c^2*f^2 + I*a^3*b*c^2*f^2 + (a^4*d^2*f^2*x^2 - I*a^3*b*d^2*f^2 \\
& ^2*x^2 + 2*a^4*d^2*ef*x - 2*I*a^3*b*d^2*ef*x + 2*a^4*c*d*ef - 2*I*a^3*b*c \\
& *d*ef - a^4*c^2*f^2 + I*a^3*b*c^2*f^2)*\cosh(d*x + c)^4 + (4*a^4*d^2*f^2*x^2 \\
& - 4*I*a^3*b*d^2*f^2*x^2 + 8*a^4*d^2*ef*x - 8*I*a^3*b*d^2*ef*x + 8*a^4*c \\
& *d*ef - 8*I*a^3*b*c*d*ef - 4*a^4*c^2*f^2 + 4*I*a^3*b*c^2*f^2)*\cosh(d*x + \\
& c)*\sinh(d*x + c)^3 + (a^4*d^2*f^2*x^2 - I*a^3*b*d^2*f^2*x^2 + 2*a^4*d^2*ef \\
& *x - 2*I*a^3*b*d^2*ef*x + 2*a^4*c*d*ef - 2*I*a^3*b*c*d*ef - a^4*c^2*f^2 \\
& + I*a^3*b*c^2*f^2)*\sinh(d*x + c)^4 - (2*a^4*d^2*f^2*x^2 - 2*I*a^3*b*d^2*f^2 \\
& *x^2 + 4*a^4*d^2*ef*x - 4*I*a^3*b*d^2*ef*x + 4*a^4*c*d*ef - 4*I*a^3*b*c \\
& *d*ef - 2*a^4*c^2*f^2 + 2*I*a^3*b*c^2*f^2)*\cosh(d*x + c)^2 - (2*a^4*d^2*f^2 \\
& *x^2 - 2*I*a^3*b*d^2*f^2*x^2 + 4*a^4*d^2*ef*x - 4*I*a^3*b*d^2*ef*x + 4*a^4 \\
& *c*d*ef - 4*I*a^3*b*c*d*ef - 2*a^4*c^2*f^2 + 2*I*a^3*b*c^2*f^2 - (6*a^4*d^2 \\
& *f^2*x^2 - 6*I*a^3*b*d^2*f^2*x^2 + 12*a^4*d^2*ef*x - 12*I*a^3*b*d^2*ef \\
& *x + 12*a^4*c*d*ef - 12*I*a^3*b*c*d*ef - 6*a^4*c^2*f^2 + 6*I*a^3*b*c^2*f^2) \\
& )*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + (((a^4*d^2*f^2*x^2 - 4*I*a^3*b*d^2*f \\
& ^2*x^2 + 8*a^4*d^2*ef*x - 8*I*a^3*b*d^2*ef*x + 8*a^4*c*d*ef - 8*I*a^3*b \\
& *c*d*ef - 4*a^4*c^2*f^2 + 4*I*a^3*b*c^2*f^2)*\cosh(d*x + c)^3 - (4*a^4*d^2*f \\
& ^2*x^2 - 4*I*a^3*b*d^2*f^2*x^2 + 8*a^4*d^2*ef*x - 8*I*a^3*b*d^2*ef*x + 8 \\
& *a^4*c*d*ef - 8*I*a^3*b*c*d*ef - 4*a^4*c^2*f^2 + 4*I*a^3*b*c^2*f^2)*\cosh(d \\
& *x + c))*\sinh(d*x + c))*\log(I*\cosh(d*x + c) + I*\sinh(d*x + c) + 1) - (a^4*d \\
& ^2*f^2*x^2 + I*a^3*b*d^2*f^2*x^2 + 2*a^4*d^2*ef*x + 2*I*a^3*b*d^2*ef*x + \\
& 2*a^4*c*d*ef + 2*I*a^3*b*c*d*ef - a^4*c^2*f^2 - I*a^3*b*c^2*f^2 + (a^4*d^2 \\
& *f^2*x^2 + I*a^3*b*d^2*f^2*x^2 + 2*a^4*d^2*ef*x + 2*I*a^3*b*d^2*ef*x + 2 \\
& *a^4*c*d*ef + 2*I*a^3*b*c*d*ef - a^4*c^2*f^2 - I*a^3*b*c^2*f^2)*\cosh(d*x \\
& + c)^4 + (4*a^4*d^2*f^2*x^2 + 4*I*a^3*b*d^2*f^2*x^2 + 8*a^4*d^2*ef*x + 8*I \\
& *a^3*b*d^2*ef*x + 8*a^4*c*d*ef + 8*I*a^3*b*c*d*ef - 4*a^4*c^2*f^2 - 4*I \\
& *a^3*b*c^2*f^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^4*d^2*f^2*x^2 + I*a^3*b*d^2 \\
& *f^2*x^2 + 2*a^4*d^2*ef*x + 2*I*a^3*b*d^2*ef*x + 2*a^4*c*d*ef + 2*I*a^3 \\
& *b*c*d*ef - a^4*c^2*f^2 - I*a^3*b*c^2*f^2)*\sinh(d*x + c)^4 - (2*a^4*d^2*f \\
& ^2*x^2 + 2*I*a^3*b*d^2*f^2*x^2 + 4*a^4*d^2*ef*x + 4*I*a^3*b*d^2*ef*x + 4 \\
& *a^4*c*d*ef + 4*I*a^3*b*c*d*ef - 2*a^4*c^2*f^2 - 2*I*a^3*b*c^2*f^2)*\cosh(d \\
& *x + c)^2 - (2*a^4*d^2*f^2*x^2 + 2*I*a^3*b*d^2*f^2*x^2 + 4*a^4*d^2*ef*x + \\
& 4*I*a^3*b*d^2*ef*x + 4*a^4*c*d*ef + 4*I*a^3*b*c*d*ef - 2*a^4*c^2*f^2 - 2 \\
& *I*a^3*b*c^2*f^2 - (6*a^4*d^2*f^2*x^2 + 6*I*a^3*b*d^2*f^2*x^2 + 12*a^4*d^2* \\
& ef*x + 12*I*a^3*b*d^2*ef*x + 12*a^4*c*d*ef + 12*I*a^3*b*c*d*ef - 6*a^4*c \\
& ^2*f^2 - 6*I*a^3*b*c^2*f^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + ((4*a^4*d^2
\end{aligned}$$

$$\begin{aligned}
& f^2 x^2 + 4 I a^3 b d^2 f^2 x^2 + 8 a^4 d^2 e f x + 8 I a^3 b d^2 e f x + \\
& 8 a^4 c d e f + 8 I a^3 b c d e f - 4 a^4 c^2 f^2 - 4 I a^3 b c^2 f^2) \cosh \\
& (d x + c)^3 - (4 a^4 d^2 f^2 x^2 + 4 I a^3 b d^2 f^2 x^2 + 8 a^4 d^2 e f x + \\
& 8 I a^3 b d^2 e f x + 8 a^4 c d e f + 8 I a^3 b c d e f - 4 a^4 c^2 f^2 - \\
& 4 I a^3 b c^2 f^2) \cosh(d x + c) \sinh(d x + c) \log(-I \cosh(d x + c) - I \\
& \sinh(d x + c) + 1) + ((a^4 - b^4) d^2 f^2 x^2 + 2(a^4 - b^4) c d e f + ((a \\
& ^4 - b^4) d^2 f^2 x^2 + 2(a^4 - b^4) c d e f - ((a^4 - b^4) c^2 - 2(a^3 b \\
& + a b^3) c) f^2 + 2((a^4 - b^4) d^2 e f + (a^3 b + a b^3) d f^2) x) \cosh( \\
& d x + c)^4 + 4((a^4 - b^4) d^2 f^2 x^2 + 2(a^4 - b^4) c d e f - ((a^4 - b \\
& ^4) c^2 - 2(a^3 b + a b^3) c) f^2 + 2((a^4 - b^4) d^2 e f + (a^3 b + a b^ \\
& 3) d f^2) x) \cosh(d x + c) \sinh(d x + c)^3 + ((a^4 - b^4) d^2 f^2 x^2 + 2( \\
& a^4 - b^4) c d e f - ((a^4 - b^4) c^2 - 2(a^3 b + a b^3) c) f^2 + 2((a^4 \\
& - b^4) d^2 e f + (a^3 b + a b^3) d f^2) x) \sinh(d x + c)^4 - ((a^4 - b^4) c \\
& ^2 - 2(a^3 b + a b^3) c) f^2 - 2((a^4 - b^4) d^2 f^2 x^2 + 2(a^4 - b^4) c \\
& d e f - ((a^4 - b^4) c^2 - 2(a^3 b + a b^3) c) f^2 + 2((a^4 - b^4) d^2 e \\
& f + (a^3 b + a b^3) d f^2) x) \cosh(d x + c)^2 - 2((a^4 - b^4) d^2 f^2 x^ \\
& 2 + 2(a^4 - b^4) c d e f - ((a^4 - b^4) c^2 - 2(a^3 b + a b^3) c) f^2 - 3 \\
& *((a^4 - b^4) d^2 f^2 x^2 + 2(a^4 - b^4) c d e f - ((a^4 - b^4) c^2 - 2(a \\
& ^3 b + a b^3) c) f^2 + 2((a^4 - b^4) d^2 e f + (a^3 b + a b^3) d f^2) x) c \\
& osh(d x + c)^2 + 2((a^4 - b^4) d^2 e f + (a^3 b + a b^3) d f^2) x) \sinh(d x \\
& + c)^2 + 2((a^4 - b^4) d^2 e f + (a^3 b + a b^3) d f^2) x + 4(((a^4 - b \\
& ^4) d^2 f^2 x^2 + 2(a^4 - b^4) c d e f - ((a^4 - b^4) c^2 - 2(a^3 b + a b \\
& ^3) c) f^2 + 2((a^4 - b^4) d^2 e f + (a^3 b + a b^3) d f^2) x) \cosh(d x + \\
& c)^3 - ((a^4 - b^4) d^2 f^2 x^2 + 2(a^4 - b^4) c d e f - ((a^4 - b^4) c^2 \\
& - 2(a^3 b + a b^3) c) f^2 + 2((a^4 - b^4) d^2 e f + (a^3 b + a b^3) d f^2 \\
& ) x) \cosh(d x + c) \sinh(d x + c) \log(-\cosh(d x + c) - \sinh(d x + c) + 1) \\
& - 2(b^4 f^2 \cosh(d x + c)^4 + 4 b^4 f^2 \cosh(d x + c) \sinh(d x + c)^3 + b^ \\
& 4 f^2 \sinh(d x + c)^4 - 2 b^4 f^2 \cosh(d x + c)^2 + b^4 f^2 + 2(3 b^4 f^2 \cosh \\
& (d x + c)^2 - b^4 f^2) \sinh(d x + c)^2 + 4(b^4 f^2 \cosh(d x + c)^3 - b \\
& ^4 f^2 \cosh(d x + c) \sinh(d x + c)) \operatorname{polylog}(3, (a \cosh(d x + c) + a \sinh(d \\
& x + c) + (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2})/b) - 2 \\
& * (b^4 f^2 \cosh(d x + c)^4 + 4 b^4 f^2 \cosh(d x + c) \sinh(d x + c)^3 + b^4 f \\
& ^2 \sinh(d x + c)^4 - 2 b^4 f^2 \cosh(d x + c)^2 + b^4 f^2 + 2(3 b^4 f^2 \cos \\
& h(d x + c)^2 - b^4 f^2) \sinh(d x + c)^2 + 4(b^4 f^2 \cosh(d x + c)^3 - b^4 f \\
& ^2 \cosh(d x + c) \sinh(d x + c)) \operatorname{polylog}(3, (a \cosh(d x + c) + a \sinh(d x \\
& + c) - (b \cosh(d x + c) + b \sinh(d x + c)) \sqrt{(a^2 + b^2)/b^2})/b) - 2(( \\
& a^4 - b^4) f^2 \cosh(d x + c)^4 + 4(a^4 - b^4) f^2 \cosh(d x + c) \sinh(d x + \\
& c)^3 + (a^4 - b^4) f^2 \sinh(d x + c)^4 - 2(a^4 - b^4) f^2 \cosh(d x + c)^2 \\
& + (a^4 - b^4) f^2 + 2(3(a^4 - b^4) f^2 \cosh(d x + c)^2 - (a^4 - b^4) f^2 \\
& ) \sinh(d x + c)^2 + 4((a^4 - b^4) f^2 \cosh(d x + c)^3 - (a^4 - b^4) f^2 \co \\
& sh(d x + c) \sinh(d x + c)) \operatorname{polylog}(3, \cosh(d x + c) + \sinh(d x + c)) + (2 \\
& a^4 f^2 + 2 I a^3 b f^2 + (2 a^4 f^2 + 2 I a^3 b f^2) \cosh(d x + c)^4 + (8 \\
& a^4 f^2 + 8 I a^3 b f^2) \cosh(d x + c) \sinh(d x + c)^3 + (2 a^4 f^2 + 2 I a \\
& ^3 b f^2) \sinh(d x + c)^4 - (4 a^4 f^2 + 4 I a^3 b f^2) \cosh(d x + c)^2 - ( \\
& 4 a^4 f^2 + 4 I a^3 b f^2 - (12 a^4 f^2 + 12 I a^3 b f^2) \cosh(d x + c)^2) *
\end{aligned}$$

$$\begin{aligned} & \sinh(dx + c)^2 + ((8a^4f^2 + 8Ia^3b^2f^2)\cosh(dx + c)^3 - (8a^4f^2 \\ & + 8Ia^3b^2f^2)\cosh(dx + c))\sinh(dx + c))\text{polylog}(3, I\cosh(dx + c) \\ & + I\sinh(dx + c)) + (2a^4f^2 - 2Ia^3b^2f^2 + (2a^4f^2 - 2Ia^3b^2f^2) \\ & \cosh(dx + c)^4 + (8a^4f^2 - 8Ia^3b^2f^2)\cosh(dx + c)\sinh(dx + c) \\ & )^3 + (2a^4f^2 - 2Ia^3b^2f^2)\sinh(dx + c)^4 - (4a^4f^2 - 4Ia^3b^2f^2) \\ & \cosh(dx + c)^2 - (4a^4f^2 - 4Ia^3b^2f^2 - (12a^4f^2 - 12Ia^3b^2f^2) \\ & \cosh(dx + c)^2)\sinh(dx + c)^2 + ((8a^4f^2 - 8Ia^3b^2f^2)\cosh \\ & (dx + c)^3 - (8a^4f^2 - 8Ia^3b^2f^2)\cosh(dx + c))\sinh(dx + c))\text{pol} \\ & \text{ylog}(3, -I\cosh(dx + c) - I\sinh(dx + c)) - 2*((a^4 - b^4)f^2\cosh(dx + \\ & c)^4 + 4*(a^4 - b^4)f^2\cosh(dx + c)\sinh(dx + c)^3 + (a^4 - b^4)f^2s \\ & \text{inh}(dx + c)^4 - 2*(a^4 - b^4)f^2\cosh(dx + c)^2 + (a^4 - b^4)f^2 + 2*(3 \\ & *(a^4 - b^4)f^2\cosh(dx + c)^2 - (a^4 - b^4)f^2)\sinh(dx + c)^2 + 4*((a \\ & ^4 - b^4)f^2\cosh(dx + c)^3 - (a^4 - b^4)f^2\cosh(dx + c))\sinh(dx + c \\ & ))\text{polylog}(3, -\cosh(dx + c) - \sinh(dx + c)) + 2*((a^3b + a^2b^3)d^2f^2* \\ & x^2 + 2*(a^3b + a^2b^3)d^2e*f*x + (a^3b + a^2b^3)d^2e^2 + 4*((a^4 + a^2 \\ & *b^2)*d*f^2*x + (a^4 + a^2*b^2)*c*f^2)\cosh(dx + c)^3 - 3*((a^3b + a^2b^3) \\ & *d^2f^2*x^2 + 2*(a^3b + a^2b^3)d^2e*f*x + (a^3b + a^2b^3)d^2e^2)\cosh \\ & (dx + c)^2 + 2*((a^4 + a^2*b^2)d^2f^2*x^2 + (a^4 + a^2*b^2)d^2e^2 + (a^4 \\ & + a^2*b^2)d^2e*f - 2*(a^4 + a^2*b^2)*c*f^2 + (2*(a^4 + a^2*b^2)d^2e*f - \\ & (a^4 + a^2*b^2)d^2f^2)*x)\cosh(dx + c))\sinh(dx + c))/((a^5 + a^3*b^2)d \\ & ^3\cosh(dx + c)^4 + 4*(a^5 + a^3*b^2)d^3\cosh(dx + c)\sinh(dx + c)^3 + \\ & (a^5 + a^3*b^2)d^3\sinh(dx + c)^4 - 2*(a^5 + a^3*b^2)d^3\cosh(dx + c)^2 \\ & + (a^5 + a^3*b^2)d^3 + 2*(3*(a^5 + a^3*b^2)d^3\cosh(dx + c)^2 - (a^5 + \\ & a^3*b^2)d^3)\sinh(dx + c)^2 + 4*((a^5 + a^3*b^2)d^3\cosh(dx + c)^3 - (a \\ & ^5 + a^3*b^2)d^3\cosh(dx + c))\sinh(dx + c)) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*2\*csch(dx+c)\*\*3\*sech(dx+c)/(a+b\*sinh(dx+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm  
m="giac")
```

```
[Out] Timed out
```

$$3.493 \quad \int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=762

$$\frac{b^4 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3 d^2 (a^2 + b^2)} - \frac{b^4 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3 d^2 (a^2 + b^2)} + \frac{b^4 f \operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{2a^3 d^2 (a^2 + b^2)} + \frac{ib^3 f \operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{a^2 d^2 (a^2 + b^2)}$$

[Out] (f\*x)/(2\*a\*d) + (2\*b\*f\*x\*ArcTan[E^(c + d\*x)])/(a^2\*d) - (2\*b^3\*(e + f\*x)\*ArcTan[E^(c + d\*x)])/(a^2\*(a^2 + b^2)\*d) - (b\*f\*x\*ArcTan[Sinh[c + d\*x]])/(a^2\*d) + (b\*(e + f\*x)\*ArcTan[Sinh[c + d\*x]])/(a^2\*d) + (2\*f\*x\*ArcTanh[E^(2\*c + 2\*d\*x)])/(a\*d) - (2\*b^2\*(e + f\*x)\*ArcTanh[E^(2\*c + 2\*d\*x)])/(a^3\*d) + (b\*f\*ArcTanh[Cosh[c + d\*x]])/(a^2\*d^2) - (f\*Coth[c + d\*x])/(2\*a\*d^2) - ((e + f\*x)\*Coth[c + d\*x]^2)/(2\*a\*d) + (b\*(e + f\*x)\*Csch[c + d\*x])/(a^2\*d) - (b^4\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^3\*(a^2 + b^2)\*d) - (b^4\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^3\*(a^2 + b^2)\*d) + (b^4\*(e + f\*x)\*Log[1 + E^(2\*(c + d\*x))])/(a^3\*(a^2 + b^2)\*d) + (f\*x\*Log[Tanh[c + d\*x]])/(a\*d) - ((e + f\*x)\*Log[Tanh[c + d\*x]])/(a\*d) - (I\*b\*f\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a^2\*d^2) + (I\*b^3\*f\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a^2\*(a^2 + b^2)\*d^2) + (I\*b\*f\*PolyLog[2, I\*E^(c + d\*x)])/(a^2\*d^2) - (I\*b^3\*f\*PolyLog[2, I\*E^(c + d\*x)])/(a^2\*(a^2 + b^2)\*d^2) - (b^4\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^3\*(a^2 + b^2)\*d^2) - (b^4\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^3\*(a^2 + b^2)\*d^2) + (b^4\*f\*PolyLog[2, -E^(2\*(c + d\*x))])/(2\*a^3\*(a^2 + b^2)\*d^2) + (f\*PolyLog[2, -E^(2\*c + 2\*d\*x)])/(2\*a\*d^2) - (b^2\*f\*PolyLog[2, -E^(2\*c + 2\*d\*x)])/(2\*a^3\*d^2) - (f\*PolyLog[2, E^(2\*c + 2\*d\*x)])/(2\*a\*d^2) + (b^2\*f\*PolyLog[2, E^(2\*c + 2\*d\*x)])/(2\*a^3\*d^2)

**Rubi [A]** time = 1.13785, antiderivative size = 762, normalized size of antiderivative = 1., number of steps used = 49, number of rules used = 23, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.719$ , Rules used = {5589, 2620, 14, 5462, 3473, 8, 2548, 12, 4182, 2279, 2391, 2621, 321, 207, 5203, 4180, 3770, 5461, 5573, 5561, 2190, 6742, 3718}

$$\frac{b^4 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3 d^2 (a^2 + b^2)} - \frac{b^4 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3 d^2 (a^2 + b^2)} + \frac{b^4 f \operatorname{PolyLog}\left(2, -e^{2(c+dx)}\right)}{2a^3 d^2 (a^2 + b^2)} + \frac{ib^3 f \operatorname{PolyLog}\left(2, -ie^{c+dx}\right)}{a^2 d^2 (a^2 + b^2)}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Csch[c + d\*x]^3\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]), x]



```
[Out] (f*x)/(2*a*d) + (2*b*f*x*ArcTan[E^(c + d*x)])/(a^2*d) - (2*b^3*(e + f*x)*ArcTan[E^(c + d*x)])/(a^2*(a^2 + b^2)*d) - (b*f*x*ArcTan[Sinh[c + d*x]])/(a^2*d) + (b*(e + f*x)*ArcTan[Sinh[c + d*x]])/(a^2*d) + (2*f*x*ArcTanh[E^(2*c + 2*d*x)])/(a*d) - (2*b^2*(e + f*x)*ArcTanh[E^(2*c + 2*d*x)])/(a^3*d) + (b*f*ArcTanh[Cosh[c + d*x]])/(a^2*d^2) - (f*Coth[c + d*x])/(2*a*d^2) - ((e + f*x)*Coth[c + d*x]^2)/(2*a*d) + (b*(e + f*x)*Csch[c + d*x])/(a^2*d) - (b^4*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)*d) - (b^4*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)*d) + (b^4*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a^3*(a^2 + b^2)*d) + (f*x*Log[Tanh[c + d*x]])/(a*d) - ((e + f*x)*Log[Tanh[c + d*x]])/(a*d) - (I*b*f*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*d^2) + (I*b^3*f*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) + (I*b*f*PolyLog[2, I*E^(c + d*x)])/(a^2*d^2) - (I*b^3*f*PolyLog[2, I*E^(c + d*x)])/(a^2*(a^2 + b^2)*d^2) - (b^4*f*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^2) - (b^4*f*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)*d^2) + (b^4*f*PolyLog[2, -E^(2*(c + d*x))])/(2*a^3*(a^2 + b^2)*d^2) + (f*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a*d^2) - (b^2*f*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a^3*d^2) - (f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a*d^2) + (b^2*f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a^3*d^2)
```

#### Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(p_.)]/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

#### Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

#### Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

#### Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
```

$b*x]^p, x]$ , Dist[(c + d\*x)^m, u, x] - Dist[d\*m, Int[(c + d\*x)^(m - 1)\*u, x], x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]

### Rule 3473

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*(b\*Tan[c + d\*x])^(n - 1))/(d\*(n - 1)), x] - Dist[b^2, Int[(b\*Tan[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rule 2548

Int[Log[u\_], x\_Symbol] := Simp[x\*Log[u], x] - Int[SimplifyIntegrand[(x\*D[u, x])/u, x], x] /; InverseFunctionFreeQ[u, x]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 4182

Int[csc[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)]/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_) + (b\_.)\*((F\_)^((e\_.)\*((c\_.) + (d\_.)\*(x\_))))^(n\_.)], x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] := -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

### Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

### Rule 5203

```
Int[ArcTan[u_], x_Symbol] := Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

### Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_)*((c_.) + (d_.)*(x_))^(m_)*Sech[(a_.) + (b_.)*(x_)]^(n_), x_Symbol] := Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5573

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) + (d_.)*(x_)]^(n_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 5561

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rule 3718

```
Int[(((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]), x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))]/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx}{a} \\
&= -\frac{(e+fx)\coth^2(c+dx)}{2ad} - \frac{(e+fx)\log(\tanh(c+dx))}{ad} - \frac{b \int (e+fx)\operatorname{csch}^2(c+dx) dx}{a} \\
&= \frac{b(e+fx)\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{(e+fx)\coth^2(c+dx)}{2ad} + \frac{b(e+fx)\operatorname{csch}(c+dx)}{a^2d} \\
&= \frac{b(e+fx)\tan^{-1}(\sinh(c+dx))}{a^2d} - \frac{f\coth(c+dx)}{2ad^2} - \frac{(e+fx)\coth^2(c+dx)}{2ad} \\
&= \frac{fx}{2ad} + \frac{b^4(e+fx)^2}{2a^3(a^2+b^2)f} - \frac{bfx\tan^{-1}(\sinh(c+dx))}{a^2d} + \frac{b(e+fx)\tan^{-1}(\sinh(c+dx))}{a^2d} \\
&= \frac{fx}{2ad} + \frac{b^4(e+fx)^2}{2a^3(a^2+b^2)f} - \frac{bfx\tan^{-1}(\sinh(c+dx))}{a^2d} + \frac{b(e+fx)\tan^{-1}(\sinh(c+dx))}{a^2d} \\
&= \frac{fx}{2ad} + \frac{2bfx\tan^{-1}(e^{c+dx})}{a^2d} - \frac{2b^3(e+fx)\tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)d} - \frac{bfx\tan^{-1}(\sinh(c+dx))}{a^2d} \\
&= \frac{fx}{2ad} + \frac{2bfx\tan^{-1}(e^{c+dx})}{a^2d} - \frac{2b^3(e+fx)\tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)d} - \frac{bfx\tan^{-1}(\sinh(c+dx))}{a^2d} \\
&= \frac{fx}{2ad} + \frac{2bfx\tan^{-1}(e^{c+dx})}{a^2d} - \frac{2b^3(e+fx)\tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)d} - \frac{bfx\tan^{-1}(\sinh(c+dx))}{a^2d} \\
&= \frac{fx}{2ad} + \frac{2bfx\tan^{-1}(e^{c+dx})}{a^2d} - \frac{2b^3(e+fx)\tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)d} - \frac{bfx\tan^{-1}(\sinh(c+dx))}{a^2d}
\end{aligned}$$

**Mathematica [A]** time = 7.78415, size = 913, normalized size = 1.2

$$\frac{\left(-\frac{1}{2}f(c+dx)^2 + f \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}} + 1\right)(c+dx) + f \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}} + 1\right)(c+dx) + de \log(a+b\sinh(c+dx)) - cf \log(a-b\sinh(c+dx))\right)}{a^3(a^2+b^2)d^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)\*Csch[c + d\*x]^3\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]), x

]

```
[Out] ((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*x)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2) + ((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) - (e*Log[Sinh[c + d*x]])/(a*d) + (b^2*e*Log[Sinh[c + d*x]])/(a^3*d) + (c*f*Log[Sinh[c + d*x]])/(a*d^2) - (b^2*c*f*Log[Sinh[c + d*x]])/(a^3*d^2) - (b*f*Log[Tanh[(c + d*x)/2]])/(a^2*d^2) + (I*f*(I*(c + d*x)*Log[1 - E^(-2*(c + d*x))] - (I/2)*(-(c + d*x)^2 + PolyLog[2, E^(-2*(c + d*x))]])))/(a*d^2) - (I*b^2*f*(I*(c + d*x)*Log[1 - E^(-2*(c + d*x))] - (I/2)*(-(c + d*x)^2 + PolyLog[2, E^(-2*(c + d*x))]])))/(a^3*d^2) - (b^4*(-(f*(c + d*x)^2)/2 + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]]) + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]]) + d*e*Log[a + b*Sinh[c + d*x]] - c*f*Log[a + b*Sinh[c + d*x]] + f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2]]) + f*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])))/(a^3*(a^2 + b^2)*d^2) + (-(a*d*e*(c + d*x) + a*c*f*(c + d*x) - (a*f*(c + d*x)^2)/2 + 2*b*d*e*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] + 2*b*f*(c + d*x)*ArcTan[Cosh[c + d*x] + Sinh[c + d*x]] + a*d*e*Log[1 + Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] - a*c*f*Log[1 + Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] + a*f*(c + d*x)*Log[1 + Cosh[2*(c + d*x)] + Sinh[2*(c + d*x)]] - I*b*f*PolyLog[2, (-I)*(Cosh[c + d*x] + Sinh[c + d*x])] + I*b*f*PolyLog[2, I*(Cosh[c + d*x] + Sinh[c + d*x])] + (a*f*PolyLog[2, -Cosh[2*(c + d*x)] - Sinh[2*(c + d*x)]])/2)/((a^2 + b^2)*d^2) + ((d*e - c*f + f*(c + d*x))*Sech[(c + d*x)/2]^2)/(8*a*d^2) + (Sech[(c + d*x)/2]*(-2*b*d*e*Sinh[(c + d*x)/2] - a*f*Sinh[(c + d*x)/2] + 2*b*c*f*Sinh[(c + d*x)/2] - 2*b*f*(c + d*x)*Sinh[(c + d*x)/2]))/(4*a^2*d^2)
```

---

**Maple [B]** time = 0.204, size = 1478, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/a^2/d^2*f*b^4/(a^2+b^2)^(3/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/a^2/d^2*f*b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/a^3/d*b^4*e/(a^2+b^2)*ln(b*exp(2*d*x+2*c)+2*a*exp(d*x+c)-b)-1/a^3/d^2*b^2*f*c*ln(exp(d*x+c)-1)+1/a^3/d*b^2*f*ln(exp(d*x+c)+1)*x-1/a^3/d^2*f*b^4/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a)/(-a+(a^2+b^2)^(1/2))) *c-1/a^3/d^2*f*b^4/(a^2+b^2)*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2)+a)/(a+(a^2+b^2)^(1/2))) *c-1/a^3/d*f*b^4/(a^2+b^2)*ln((-b*exp(d*x+c)+(a^2+b^2)^(1/2)-a
```

$$\begin{aligned} &)/(-a+(a^2+b^2)^{(1/2)}) * x - 1/a^3/d*f*b^4/(a^2+b^2) * \ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)+a)/(a+(a^2+b^2)^{(1/2)})) * x + 1/a^3/d^2*f*b^4*c/(a^2+b^2) * \ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b) - 4*I/d^2*f/(4*a^2+4*b^2) * \ln(1+I*\exp(d*x+c)) * b*c + 4/d^2*f/(4*a^2+4*b^2) * \operatorname{dilog}(1+I*\exp(d*x+c)) * a + 4/d^2*f/(4*a^2+4*b^2) * \operatorname{dilog}(1-I*\exp(d*x+c)) * a + 8/d*e/(4*a^2+4*b^2) * b * \arctan(\exp(d*x+c)) + 4/d*e/(4*a^2+4*b^2) * a * \ln(1+\exp(2*d*x+2*c)) - 1/a^3/d^2*b^2*f * \operatorname{dilog}(\exp(d*x+c)) + 1/a^3/d^2*b^2*f * \operatorname{dilog}(\exp(d*x+c)+1) - 1/a^2/d^2*b*f * \ln(\exp(d*x+c)-1) + 1/a^2/d^2*b*f * \ln(\exp(d*x+c)+1) + 1/a^3/d*b^2*e * \ln(\exp(d*x+c)-1) + 1/a^3/d*b^2*e * \ln(\exp(d*x+c)+1) + 1/d^2*f/a * \operatorname{dilog}(\exp(d*x+c)) - 1/d^2*f/a * \operatorname{dilog}(\exp(d*x+c)+1) - 8/d^2*f*c/(4*a^2+4*b^2) * b * \arctan(\exp(d*x+c)) + 4/d*f/(4*a^2+4*b^2) * \ln(1+I*\exp(d*x+c)) * a * x + 4/d^2*f/(4*a^2+4*b^2) * \ln(1+I*\exp(d*x+c)) * a * c + 4/d*f/(4*a^2+4*b^2) * \ln(1-I*\exp(d*x+c)) * a * x + 4/d^2*f/(4*a^2+4*b^2) * \ln(1-I*\exp(d*x+c)) * a * c - 4/d^2*f*c/(4*a^2+4*b^2) * a * \ln(1+\exp(2*d*x+2*c)) - 1/d/a * e * \ln(\exp(d*x+c)-1) - 1/d/a * e * \ln(\exp(d*x+c)+1) - 1/a^3/d^2*f*b^4/(a^2+b^2) * \operatorname{dilog}((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)+a)/(a+(a^2+b^2)^{(1/2)})) - 1/a^3/d^2*f*b^4/(a^2+b^2) * \operatorname{dilog}((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)-a}/(-a+(a^2+b^2)^{(1/2)})) - 4*I/d^2*f/(4*a^2+4*b^2) * \operatorname{dilog}(1+I*\exp(d*x+c)) * b + 4*I/d^2*f/(4*a^2+4*b^2) * \operatorname{dilog}(1-I*\exp(d*x+c)) * b + 1/d^2*b^2*f/(a^2+b^2)^{(3/2)} * \operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}) - 1/d/a * \ln(\exp(d*x+c)+1) * f * x + 1/d^2/a * f * c * \ln(\exp(d*x+c)-1) - (-2*b*d*f*x*\exp(3*d*x+3*c) + 2*a*d*f*x*\exp(2*d*x+2*c) - 2*b*d*e*\exp(3*d*x+3*c) + 2*a*d*e*\exp(2*d*x+2*c) + 2*b*d*f*x*\exp(d*x+c) + a*f*\exp(2*d*x+2*c) + 2*b*d*e*\exp(d*x+c) - a*f)/d^2/a^2/(exp(2*d*x+2*c)-1)^2 + 4*I/d^2*f/(4*a^2+4*b^2) * \ln(1-I*\exp(d*x+c)) * b * c - 4*I/d*f/(4*a^2+4*b^2) * \ln(1+I*\exp(d*x+c)) * b * x + 4*I/d*f/(4*a^2+4*b^2) * \ln(1-I*\exp(d*x+c)) * b * x \end{aligned}$$


---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)^3\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned} &-(b^4*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/((a^5 + a^3*b^2)*d) + 2*b*\arctan(e^{(-d*x - c)})/((a^2 + b^2)*d) - a*\log(e^{(-2*d*x - 2*c)} + 1)/((a^2 + b^2)*d) + 2*(b*e^{(-d*x - c)} - a*e^{(-2*d*x - 2*c)} - b*e^{(-3*d*x - 3*c)})/((2*a^2*e^{(-2*d*x - 2*c)} - a^2*e^{(-4*d*x - 4*c)} - a^2)*d) + (a^2 - b^2)*\log(e^{(-d*x - c)} + 1)/(a^3*d) + (a^2 - b^2)*\log(e^{(-d*x - c)} - 1)/(a^3*d)*e + (16*a^2*d*\integrate(1/16*x/(a^3*d*e^{(d*x + c)} + a^3*d), x) - 16*b^2*d*\integrate(1/16*x/(a^3*d*e^{(d*x + c)} + a^3*d), x) - 16*a^2*d*\integrate(1/16*x/(a^3*d*e^{(d*x + c)} - a^3*d), x) + 16*b^2*d*\integrate(1/16*x/(a^3*d*e^{(d*x + c)} - a^3*d), x) - a*b*((d*x + c)/(a^3*d^2) - \log(e^{(d*x + c)} + 1)/(a^3*d^2) \end{aligned}$$

```
) + a*b*((d*x + c)/(a^3*d^2) - log(e^(d*x + c) - 1)/(a^3*d^2)) + (2*b*d*x*e
^(3*d*x + 3*c) - 2*b*d*x*e^(d*x + c) - (2*a*d*x*e^(2*c) + a*e^(2*c))*e^(2*d
*x) + a)/(a^2*d^2*e^(4*d*x + 4*c) - 2*a^2*d^2*e^(2*d*x + 2*c) + a^2*d^2) +
16*integrate(-1/8*(a*b^4*x*e^(d*x + c) - b^5*x)/(a^5*b + a^3*b^3 - (a^5*b*e
^(2*c) + a^3*b^3*e^(2*c))*e^(2*d*x) - 2*(a^6*e^c + a^4*b^2*e^c)*e^(d*x)), x
) + 16*integrate(1/8*(b*x*e^(d*x + c) - a*x)/(a^2 + b^2 + (a^2*e^(2*c) + b^
2*e^(2*c))*e^(2*d*x)), x))*f
```

**Fricas [B]** time = 3.73601, size = 13535, normalized size = 17.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

```
[Out] (2*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e)*cosh(d*x + c)^3 + 2*((a^3*
b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e)*sinh(d*x + c)^3 - (2*(a^4 + a^2*b^2
)*d*f*x + 2*(a^4 + a^2*b^2)*d*e + (a^4 + a^2*b^2)*f)*cosh(d*x + c)^2 - (2*(
a^4 + a^2*b^2)*d*f*x + 2*(a^4 + a^2*b^2)*d*e + (a^4 + a^2*b^2)*f - 6*((a^3*
b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e)*cosh(d*x + c))*sinh(d*x + c)^2 + (a
^4 + a^2*b^2)*f - 2*((a^3*b + a*b^3)*d*f*x + (a^3*b + a*b^3)*d*e)*cosh(d*x
+ c) - (b^4*f*cosh(d*x + c)^4 + 4*b^4*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^4
*f*sinh(d*x + c)^4 - 2*b^4*f*cosh(d*x + c)^2 + b^4*f + 2*(3*b^4*f*cosh(d*x
+ c)^2 - b^4*f)*sinh(d*x + c)^2 + 4*(b^4*f*cosh(d*x + c)^3 - b^4*f*cosh(d*x
+ c))*sinh(d*x + c))*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*
x + c) + b*sinh(d*x + c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - (b^4*f*cosh(d
*x + c)^4 + 4*b^4*f*cosh(d*x + c)*sinh(d*x + c)^3 + b^4*f*sinh(d*x + c)^4 -
2*b^4*f*cosh(d*x + c)^2 + b^4*f + 2*(3*b^4*f*cosh(d*x + c)^2 - b^4*f)*sinh
(d*x + c)^2 + 4*(b^4*f*cosh(d*x + c)^3 - b^4*f*cosh(d*x + c))*sinh(d*x + c)
)*dilog((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x
+ c))*sqrt((a^2 + b^2)/b^2) - b)/b + 1) - ((a^4 - b^4)*f*cosh(d*x + c)^4 +
4*(a^4 - b^4)*f*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 - b^4)*f*sinh(d*x + c)
^4 - 2*(a^4 - b^4)*f*cosh(d*x + c)^2 + 2*(3*(a^4 - b^4)*f*cosh(d*x + c)^2 -
(a^4 - b^4)*f)*sinh(d*x + c)^2 + (a^4 - b^4)*f + 4*((a^4 - b^4)*f*cosh(d*x
+ c)^3 - (a^4 - b^4)*f*cosh(d*x + c))*sinh(d*x + c))*dilog(cosh(d*x + c) +
sinh(d*x + c)) + (a^4*f + I*a^3*b*f + (a^4*f + I*a^3*b*f)*cosh(d*x + c)^4
+ (4*a^4*f + 4*I*a^3*b*f)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*f + I*a^3*b*
f)*sinh(d*x + c)^4 - (2*a^4*f + 2*I*a^3*b*f)*cosh(d*x + c)^2 - (2*a^4*f + 2
*I*a^3*b*f - (6*a^4*f + 6*I*a^3*b*f)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((4
*a^4*f + 4*I*a^3*b*f)*cosh(d*x + c)^3 - (4*a^4*f + 4*I*a^3*b*f)*cosh(d*x +
```





$$\begin{aligned}
& d*x + c)) * \sinh(d*x + c)) * \log(\cosh(d*x + c) + \sinh(d*x + c) + 1) + (a^4*d*e \\
& + I*a^3*b*d*e - a^4*c*f - I*a^3*b*c*f + (a^4*d*e + I*a^3*b*d*e - a^4*c*f - \\
& I*a^3*b*c*f) * \cosh(d*x + c)^4 + (4*a^4*d*e + 4*I*a^3*b*d*e - 4*a^4*c*f - 4*I \\
& *a^3*b*c*f) * \cosh(d*x + c) * \sinh(d*x + c)^3 + (a^4*d*e + I*a^3*b*d*e - a^4*c* \\
& f - I*a^3*b*c*f) * \sinh(d*x + c)^4 - (2*a^4*d*e + 2*I*a^3*b*d*e - 2*a^4*c*f - \\
& 2*I*a^3*b*c*f) * \cosh(d*x + c)^2 - (2*a^4*d*e + 2*I*a^3*b*d*e - 2*a^4*c*f - \\
& 2*I*a^3*b*c*f - (6*a^4*d*e + 6*I*a^3*b*d*e - 6*a^4*c*f - 6*I*a^3*b*c*f) * \cos \\
& h(d*x + c)^2) * \sinh(d*x + c)^2 + ((4*a^4*d*e + 4*I*a^3*b*d*e - 4*a^4*c*f - 4 \\
& *I*a^3*b*c*f) * \cosh(d*x + c)^3 - (4*a^4*d*e + 4*I*a^3*b*d*e - 4*a^4*c*f - 4* \\
& I*a^3*b*c*f) * \cosh(d*x + c)) * \sinh(d*x + c)) * \log(\cosh(d*x + c) + \sinh(d*x + c \\
& ) + I) + (a^4*d*e - I*a^3*b*d*e - a^4*c*f + I*a^3*b*c*f + (a^4*d*e - I*a^3* \\
& b*d*e - a^4*c*f + I*a^3*b*c*f) * \cosh(d*x + c)^4 + (4*a^4*d*e - 4*I*a^3*b*d*e \\
& - 4*a^4*c*f + 4*I*a^3*b*c*f) * \cosh(d*x + c) * \sinh(d*x + c)^3 + (a^4*d*e - I* \\
& a^3*b*d*e - a^4*c*f + I*a^3*b*c*f) * \sinh(d*x + c)^4 - (2*a^4*d*e - 2*I*a^3*b \\
& *d*e - 2*a^4*c*f + 2*I*a^3*b*c*f) * \cosh(d*x + c)^2 - (2*a^4*d*e - 2*I*a^3*b* \\
& d*e - 2*a^4*c*f + 2*I*a^3*b*c*f - (6*a^4*d*e - 6*I*a^3*b*d*e - 6*a^4*c*f + \\
& 6*I*a^3*b*c*f) * \cosh(d*x + c)^2) * \sinh(d*x + c)^2 + ((4*a^4*d*e - 4*I*a^3*b*d* \\
& e - 4*a^4*c*f + 4*I*a^3*b*c*f) * \cosh(d*x + c)^3 - (4*a^4*d*e - 4*I*a^3*b*d* \\
& e - 4*a^4*c*f + 4*I*a^3*b*c*f) * \cosh(d*x + c)) * \sinh(d*x + c)) * \log(\cosh(d*x + \\
& c) + \sinh(d*x + c) - I) - (((a^4 - b^4) * d*e + (a^3*b + a*b^3 - (a^4 - b^4) \\
& *c) * f) * \cosh(d*x + c)^4 + 4 * ((a^4 - b^4) * d*e + (a^3*b + a*b^3 - (a^4 - b^4) * \\
& c) * f) * \cosh(d*x + c) * \sinh(d*x + c)^3 + ((a^4 - b^4) * d*e + (a^3*b + a*b^3 - ( \\
& a^4 - b^4) * c) * f) * \sinh(d*x + c)^4 + (a^4 - b^4) * d*e - 2 * ((a^4 - b^4) * d*e + ( \\
& a^3*b + a*b^3 - (a^4 - b^4) * c) * f) * \cosh(d*x + c)^2 - 2 * ((a^4 - b^4) * d*e - 3 * \\
& ((a^4 - b^4) * d*e + (a^3*b + a*b^3 - (a^4 - b^4) * c) * f) * \cosh(d*x + c)^2 + (a^ \\
& 3*b + a*b^3 - (a^4 - b^4) * c) * f) * \sinh(d*x + c)^2 + (a^3*b + a*b^3 - (a^4 - b \\
& ^4) * c) * f + 4 * (((a^4 - b^4) * d*e + (a^3*b + a*b^3 - (a^4 - b^4) * c) * f) * \cosh(d* \\
& x + c)^3 - ((a^4 - b^4) * d*e + (a^3*b + a*b^3 - (a^4 - b^4) * c) * f) * \cosh(d*x + \\
& c)) * \sinh(d*x + c)) * \log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + (a^4*d*f*x - I \\
& *a^3*b*d*f*x + a^4*c*f - I*a^3*b*c*f + (a^4*d*f*x - I*a^3*b*d*f*x + a^4*c*f \\
& - I*a^3*b*c*f) * \cosh(d*x + c)^4 + (4*a^4*d*f*x - 4*I*a^3*b*d*f*x + 4*a^4*c* \\
& f - 4*I*a^3*b*c*f) * \cosh(d*x + c) * \sinh(d*x + c)^3 + (a^4*d*f*x - I*a^3*b*d*f \\
& *x + a^4*c*f - I*a^3*b*c*f) * \sinh(d*x + c)^4 - (2*a^4*d*f*x - 2*I*a^3*b*d*f* \\
& x + 2*a^4*c*f - 2*I*a^3*b*c*f) * \cosh(d*x + c)^2 - (2*a^4*d*f*x - 2*I*a^3*b*d \\
& *f*x + 2*a^4*c*f - 2*I*a^3*b*c*f - (6*a^4*d*f*x - 6*I*a^3*b*d*f*x + 6*a^4*c \\
& *f - 6*I*a^3*b*c*f) * \cosh(d*x + c)^2) * \sinh(d*x + c)^2 + ((4*a^4*d*f*x - 4*I* \\
& a^3*b*d*f*x + 4*a^4*c*f - 4*I*a^3*b*c*f) * \cosh(d*x + c)^3 - (4*a^4*d*f*x - 4 \\
& *I*a^3*b*d*f*x + 4*a^4*c*f - 4*I*a^3*b*c*f) * \cosh(d*x + c)) * \sinh(d*x + c)) * \log \\
& (I * \cosh(d*x + c) + I * \sinh(d*x + c) + 1) + (a^4*d*f*x + I*a^3*b*d*f*x + a^ \\
& 4*c*f + I*a^3*b*c*f + (a^4*d*f*x + I*a^3*b*d*f*x + a^4*c*f + I*a^3*b*c*f) * \c \\
& osh(d*x + c)^4 + (4*a^4*d*f*x + 4*I*a^3*b*d*f*x + 4*a^4*c*f + 4*I*a^3*b*c*f) \\
& ) * \cosh(d*x + c) * \sinh(d*x + c)^3 + (a^4*d*f*x + I*a^3*b*d*f*x + a^4*c*f + I* \\
& a^3*b*c*f) * \sinh(d*x + c)^4 - (2*a^4*d*f*x + 2*I*a^3*b*d*f*x + 2*a^4*c*f + 2 \\
& *I*a^3*b*c*f) * \cosh(d*x + c)^2 - (2*a^4*d*f*x + 2*I*a^3*b*d*f*x + 2*a^4*c*f \\
& + 2*I*a^3*b*c*f - (6*a^4*d*f*x + 6*I*a^3*b*d*f*x + 6*a^4*c*f + 6*I*a^3*b*c*
\end{aligned}$$

$$\begin{aligned}
& f) \cosh(dx + c)^2 \sinh(dx + c)^2 + ((4a^4 d f x + 4I a^3 b d f x + 4a^4 c f + 4I a^3 b c f) \cosh(dx + c)^3 - (4a^4 d f x + 4I a^3 b d f x + 4a^4 c f + 4I a^3 b c f) \cosh(dx + c)) \sinh(dx + c) \log(-I \cosh(dx + c) - I \sinh(dx + c) + 1) - ((a^4 - b^4) d f x + (a^4 - b^4) c f) \cosh(dx + c)^4 + 4((a^4 - b^4) d f x + (a^4 - b^4) c f) \cosh(dx + c) \sinh(dx + c)^3 + ((a^4 - b^4) d f x + (a^4 - b^4) c f) \sinh(dx + c)^4 + (a^4 - b^4) d f x + (a^4 - b^4) c f - 2((a^4 - b^4) d f x + (a^4 - b^4) c f) \cosh(dx + c)^2 - 2((a^4 - b^4) d f x + (a^4 - b^4) c f - 3((a^4 - b^4) d f x + (a^4 - b^4) c f) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4(((a^4 - b^4) d f x + (a^4 - b^4) c f) \cosh(dx + c)^3 - ((a^4 - b^4) d f x + (a^4 - b^4) c f) \cosh(dx + c)) \sinh(dx + c) \log(-\cosh(dx + c) - \sinh(dx + c) + 1) - 2((a^3 b + a b^3) d f x + (a^3 b + a b^3) d e - 3((a^3 b + a b^3) d f x + (a^3 b + a b^3) d e) \cosh(dx + c)^2 + (2(a^4 + a^2 b^2) d f x + 2(a^4 + a^2 b^2) d e + (a^4 + a^2 b^2) f) \cosh(dx + c)) \sinh(dx + c) / ((a^5 + a^3 b^2) d^2 \cosh(dx + c)^4 + 4(a^5 + a^3 b^2) d^2 \cosh(dx + c) \sinh(dx + c)^3 + (a^5 + a^3 b^2) d^2 \sinh(dx + c)^4 - 2(a^5 + a^3 b^2) d^2 \cosh(dx + c)^2 + (a^5 + a^3 b^2) d^2 + 2(3(a^5 + a^3 b^2) d^2 \cosh(dx + c)^2 - (a^5 + a^3 b^2) d^2) \sinh(dx + c)^2 + 4((a^5 + a^3 b^2) d^2 \cosh(dx + c)^3 - (a^5 + a^3 b^2) d^2 \cosh(dx + c)) \sinh(dx + c))
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(dx+c)\*\*3\*sech(dx+c)/(a+b\*sinh(dx+c)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(dx+c)^3\*sech(dx+c)/(a+b\*sinh(dx+c)),x, algorithm="giac")

[Out] Timed out

$$3.494 \quad \int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=130

$$-\frac{b^4 \log(a+b\sinh(c+dx))}{a^3 d (a^2+b^2)} - \frac{(a^2-b^2) \log(\sinh(c+dx))}{a^3 d} + \frac{b \tan^{-1}(\sinh(c+dx))}{d (a^2+b^2)} + \frac{a \log(\cosh(c+dx))}{d (a^2+b^2)} + \frac{b \operatorname{csch}(c+dx)}{a^2 d}$$

[Out] (b\*ArcTan[Sinh[c + d\*x]])/((a^2 + b^2)\*d) + (b\*Csch[c + d\*x])/(a^2\*d) - Csch[c + d\*x]^2/(2\*a\*d) + (a\*Log[Cosh[c + d\*x]])/((a^2 + b^2)\*d) - ((a^2 - b^2)\*Log[Sinh[c + d\*x]])/(a^3\*d) - (b^4\*Log[a + b\*Sinh[c + d\*x]])/(a^3\*(a^2 + b^2)\*d)

**Rubi [A]** time = 0.235471, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2837, 12, 894, 635, 203, 260}

$$-\frac{b^4 \log(a+b\sinh(c+dx))}{a^3 d (a^2+b^2)} - \frac{(a^2-b^2) \log(\sinh(c+dx))}{a^3 d} + \frac{b \tan^{-1}(\sinh(c+dx))}{d (a^2+b^2)} + \frac{a \log(\cosh(c+dx))}{d (a^2+b^2)} + \frac{b \operatorname{csch}(c+dx)}{a^2 d}$$

Antiderivative was successfully verified.

[In] Int[(Csch[c + d\*x]^3\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] (b\*ArcTan[Sinh[c + d\*x]])/((a^2 + b^2)\*d) + (b\*Csch[c + d\*x])/(a^2\*d) - Csch[c + d\*x]^2/(2\*a\*d) + (a\*Log[Cosh[c + d\*x]])/((a^2 + b^2)\*d) - ((a^2 - b^2)\*Log[Sinh[c + d\*x]])/(a^3\*d) - (b^4\*Log[a + b\*Sinh[c + d\*x]])/(a^3\*(a^2 + b^2)\*d)

### Rule 2837

Int[cos[(e\_.) + (f\_.)\*(x\_)]^(p\_)\*((a\_) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(m\_.))\*((c\_.) + (d\_.)\*sin[(e\_.) + (f\_.)\*(x\_)]^(n\_.), x\_Symbol] :> Dist[1/(b^p\*f), Subst[Int[(a + x)^m\*(c + (d\*x)/b)^n\*(b^2 - x^2)^((p - 1)/2), x], x, b\*Sin[e + f\*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{a+b\sinh(c+dx)} dx &= -\frac{b \operatorname{Subst}\left(\int \frac{b^3}{x^3(a+x)(-b^2-x^2)} dx, x, b\sinh(c+dx)\right)}{d} \\
&= -\frac{b^4 \operatorname{Subst}\left(\int \frac{1}{x^3(a+x)(-b^2-x^2)} dx, x, b\sinh(c+dx)\right)}{d} \\
&= -\frac{b^4 \operatorname{Subst}\left(\int \left(-\frac{1}{ab^2x^3} + \frac{1}{a^2b^2x^2} + \frac{a^2-b^2}{a^3b^4x} + \frac{1}{a^3(a^2+b^2)(a+x)} + \frac{-b^2-ax}{b^4(a^2+b^2)(b^2+x^2)}\right) dx, x, b\sinh(c+dx)\right)}{d} \\
&= \frac{b\operatorname{csch}(c+dx)}{a^2d} - \frac{\operatorname{csch}^2(c+dx)}{2ad} - \frac{(a^2-b^2)\log(\sinh(c+dx))}{a^3d} - \frac{b^4\log(a+b\sinh(c+dx))}{a^3(a^2+b^2)d} \\
&= \frac{b\operatorname{csch}(c+dx)}{a^2d} - \frac{\operatorname{csch}^2(c+dx)}{2ad} - \frac{(a^2-b^2)\log(\sinh(c+dx))}{a^3d} - \frac{b^4\log(a+b\sinh(c+dx))}{a^3(a^2+b^2)d} \\
&= \frac{b \tan^{-1}(\sinh(c+dx))}{(a^2+b^2)d} + \frac{b\operatorname{csch}(c+dx)}{a^2d} - \frac{\operatorname{csch}^2(c+dx)}{2ad} + \frac{a\log(\cosh(c+dx))}{(a^2+b^2)d} - \frac{b^4\log(a+b\sinh(c+dx))}{a^3(a^2+b^2)d}
\end{aligned}$$

**Mathematica [A]** time = 0.36427, size = 164, normalized size = 1.26

$$\frac{-\frac{2b^4\log(a+b\sinh(c+dx))}{a^3(a^2+b^2)} + \frac{(a-\sqrt{-b^2})\log(\sqrt{-b^2}-b\sinh(c+dx))}{a^2+b^2} + \frac{(a+\sqrt{-b^2})\log(\sqrt{-b^2}+b\sinh(c+dx))}{a^2+b^2} + \frac{2b\operatorname{csch}(c+dx)}{a^2} - \frac{2(a-b)(a+b)\log(\sinh(c+dx))}{a^3}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Csch[c + d\*x]^3\*Sech[c + d\*x])/(a + b\*Sinh[c + d\*x]),x]

[Out] ((2\*b\*Csch[c + d\*x])/a^2 - Csch[c + d\*x]^2/a - (2\*(a - b)\*(a + b)\*Log[Sinh[c + d\*x]])/a^3 + ((a - Sqrt[-b^2])\*Log[Sqrt[-b^2] - b\*Sinh[c + d\*x]])/(a^2 + b^2) - (2\*b^4\*Log[a + b\*Sinh[c + d\*x]])/(a^3\*(a^2 + b^2)) + ((a + Sqrt[-b^2])\*Log[Sqrt[-b^2] + b\*Sinh[c + d\*x]])/(a^2 + b^2))/(2\*d)

**Maple [A]** time = 0.003, size = 219, normalized size = 1.7

$$-\frac{1}{8da} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2 - \frac{b}{2da^2} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{8da} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^{-2} - \frac{1}{da} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \frac{b^2}{da^3} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x)`

[Out] 
$$-1/8/d/a*\tanh(1/2*d*x+1/2*c)^2-1/2/d/a^2*\tanh(1/2*d*x+1/2*c)*b-1/8/d/a/\tanh(1/2*d*x+1/2*c)^2-1/d/a*\ln(\tanh(1/2*d*x+1/2*c))+1/d/a^3*\ln(\tanh(1/2*d*x+1/2*c))*b^2+1/2/d*b/a^2/\tanh(1/2*d*x+1/2*c)-1/d*b^4/a^3/(a^2+b^2)*\ln(\tanh(1/2*d*x+1/2*c))^2*a-2*\tanh(1/2*d*x+1/2*c)*b-a)+1/d/(a^2+b^2)*a*\ln(\tanh(1/2*d*x+1/2*c)^2+1)+2/d/(a^2+b^2)*b*\arctan(\tanh(1/2*d*x+1/2*c))$$

**Maxima [A]** time = 1.69105, size = 319, normalized size = 2.45

$$\frac{b^4 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{(a^5 + a^3b^2)d} - \frac{2b \arctan(e^{(-dx-c)})}{(a^2 + b^2)d} + \frac{a \log(e^{(-2dx-2c)} + 1)}{(a^2 + b^2)d} - \frac{2(be^{(-dx-c)} - ae^{(-2dx-2c)} - be^{(-4dx-4c)})}{(2a^2e^{(-2dx-2c)} - a^2e^{(-4dx-4c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] 
$$-b^4*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/((a^5 + a^3*b^2)*d) - 2*b*\arctan(e^{(-d*x - c)})/((a^2 + b^2)*d) + a*\log(e^{(-2*d*x - 2*c)} + 1)/((a^2 + b^2)*d) - 2*(b*e^{(-d*x - c)} - a*e^{(-2*d*x - 2*c)} - b*e^{(-3*d*x - 3*c)})/((2*a^2*e^{(-2*d*x - 2*c)} - a^2*e^{(-4*d*x - 4*c)} - a^2)*d) - (a^2 - b^2)*\log(e^{(-d*x - c)} + 1)/(a^3*d) - (a^2 - b^2)*\log(e^{(-d*x - c)} - 1)/(a^3*d)$$

**Fricas [B]** time = 3.37535, size = 2503, normalized size = 19.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3*sech(d*x+c)/(a+b*sinh(d*x+c)),x, algorithm="fricas")`

[Out] 
$$(2*(a^3*b + a*b^3)*\cosh(d*x + c)^3 + 2*(a^3*b + a*b^3)*\sinh(d*x + c)^3 - 2*(a^4 + a^2*b^2)*\cosh(d*x + c)^2 - 2*(a^4 + a^2*b^2 - 3*(a^3*b + a*b^3))*\cosh(d*x + c)*\sinh(d*x + c)^2 + 2*(a^3*b*\cosh(d*x + c)^4 + 4*a^3*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + a^3*b*\sinh(d*x + c)^4 - 2*a^3*b*\cosh(d*x + c)^2 + a^3*$$

$$\begin{aligned}
& b + 2*(3*a^3*b*cosh(d*x + c)^2 - a^3*b)*sinh(d*x + c)^2 + 4*(a^3*b*cosh(d*x \\
& + c)^3 - a^3*b*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d \\
& *x + c)) - 2*(a^3*b + a*b^3)*cosh(d*x + c) - (b^4*cosh(d*x + c)^4 + 4*b^4*c \\
& osh(d*x + c)*sinh(d*x + c)^3 + b^4*sinh(d*x + c)^4 - 2*b^4*cosh(d*x + c)^2 \\
& + b^4 + 2*(3*b^4*cosh(d*x + c)^2 - b^4)*sinh(d*x + c)^2 + 4*(b^4*cosh(d*x + \\
& c)^3 - b^4*cosh(d*x + c))*sinh(d*x + c))*log(2*(b*sinh(d*x + c) + a)/(cosh \\
& (d*x + c) - sinh(d*x + c))) + (a^4*cosh(d*x + c)^4 + 4*a^4*cosh(d*x + c)*si \\
& nh(d*x + c)^3 + a^4*sinh(d*x + c)^4 - 2*a^4*cosh(d*x + c)^2 + a^4 + 2*(3*a^ \\
& 4*cosh(d*x + c)^2 - a^4)*sinh(d*x + c)^2 + 4*(a^4*cosh(d*x + c)^3 - a^4*cos \\
& h(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + \\
& c))) - ((a^4 - b^4)*cosh(d*x + c)^4 + 4*(a^4 - b^4)*cosh(d*x + c)*sinh(d*x \\
& + c)^3 + (a^4 - b^4)*sinh(d*x + c)^4 + a^4 - b^4 - 2*(a^4 - b^4)*cosh(d*x + \\
& c)^2 - 2*(a^4 - b^4 - 3*(a^4 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*( \\
& (a^4 - b^4)*cosh(d*x + c)^3 - (a^4 - b^4)*cosh(d*x + c))*sinh(d*x + c))*log \\
& (2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - 2*(a^3*b + a*b^3 - 3*(a \\
& ^3*b + a*b^3)*cosh(d*x + c)^2 + 2*(a^4 + a^2*b^2)*cosh(d*x + c))*sinh(d*x + \\
& c))/((a^5 + a^3*b^2)*d*cosh(d*x + c)^4 + 4*(a^5 + a^3*b^2)*d*cosh(d*x + c) \\
& *sinh(d*x + c)^3 + (a^5 + a^3*b^2)*d*sinh(d*x + c)^4 - 2*(a^5 + a^3*b^2)*d* \\
& cosh(d*x + c)^2 + 2*(3*(a^5 + a^3*b^2)*d*cosh(d*x + c)^2 - (a^5 + a^3*b^2)* \\
& d)*sinh(d*x + c)^2 + (a^5 + a^3*b^2)*d + 4*((a^5 + a^3*b^2)*d*cosh(d*x + c) \\
& ^3 - (a^5 + a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c))
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)\*\*3\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [B]** time = 1.346, size = 370, normalized size = 2.85

$$\frac{b^5 \log\left(\left|b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 2a\right|\right)}{a^5bd + a^3b^3d} + \frac{\left(\pi + 2 \arctan\left(\frac{1}{2}\left(e^{(2dx+2c)} - 1\right)e^{(-dx-c)}\right)\right)b}{2(a^2d + b^2d)} + \frac{a \log\left(\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4\right)}{2(a^2d + b^2d)} - (a$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate(csch(d\*x+c)^3\*sech(d\*x+c)/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] 
$$-b^5 \log(\operatorname{abs}(b(e^{d*x+c} - e^{-d*x-c}) + 2a)) / (a^5 b d + a^3 b^3 d) + 1/2(\pi + 2 \arctan(1/2(e^{2d*x+2c} - 1)e^{-d*x-c})) * b / (a^2 d + b^2 d) + 1/2 a \log((e^{d*x+c} - e^{-d*x-c})^2 + 4) / (a^2 d + b^2 d) - (a^2 - b^2) \log(\operatorname{abs}(e^{d*x+c} - e^{-d*x-c})) / (a^3 d) + 1/2(3a^2(e^{d*x+c} - e^{-d*x-c})^2 - 3b^2(e^{d*x+c} - e^{-d*x-c})^2 + 4ab(e^{d*x+c} - e^{-d*x-c}) - 4a^2) / (a^3 d(e^{d*x+c} - e^{-d*x-c})^2)$$

$$3.495 \quad \int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))}, x\right)$$

[Out] Unintegrable[(Csch[c + d\*x]^3\*Sech[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x]))], x]

**Rubi [A]** time = 0.09124, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Csch[c + d\*x]^3\*Sech[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

[Out] Defer[Int] [(Csch[c + d\*x]^3\*Sech[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x]))], x]

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Mathematica [A]** time = 158.407, size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csch[c + d\*x]^3\*Sech[c + d\*x])/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

```
[Out] Integrate[(Csch[c + d*x]^3*Sech[c + d*x])/((e + f*x)*(a + b*Sinh[c + d*x]))
, x]
```

**Maple [A]** time = 3.241, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx + c))^3 \operatorname{sech}(dx + c)}{(fx + e)(a + b \sinh(dx + c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] int(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"maxima")
```

```
[Out] -(a*f - 2*(b*d*f*x*e^(3*c) + b*d*e*e^(3*c))*e^(3*d*x) + (2*a*d*f*x*e^(2*c)
+ (2*d*e - f)*a*e^(2*c))*e^(2*d*x) + 2*(b*d*f*x*e^c + b*d*e*e^c)*e^(d*x))/(
a^2*d^2*f^2*x^2 + 2*a^2*d^2*e*f*x + a^2*d^2*e^2 + (a^2*d^2*f^2*x^2*e^(4*c)
+ 2*a^2*d^2*e*f*x*e^(4*c) + a^2*d^2*e^2*e^(4*c))*e^(4*d*x) - 2*(a^2*d^2*f^2
*x^2*e^(2*c) + 2*a^2*d^2*e*f*x*e^(2*c) + a^2*d^2*e^2*e^(2*c))*e^(2*d*x)) -
16*integrate(1/16*(b^2*d^2*e^2 + a*b*d*e*f - (d^2*e^2 - f^2)*a^2 - (a^2*d^2
*f^2 - b^2*d^2*f^2)*x^2 - (2*a^2*d^2*e*f - 2*b^2*d^2*e*f - a*b*d*f^2)*x)/(a
^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 - (a
^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*
d^2*e^3*e^c)*e^(d*x)), x) + 16*integrate(-1/16*(b^2*d^2*e^2 - a*b*d*e*f - (
d^2*e^2 - f^2)*a^2 - (a^2*d^2*f^2 - b^2*d^2*f^2)*x^2 - (2*a^2*d^2*e*f - 2*b
^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d
^2*e^2*f*x + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c +
3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) + 16*integrate(-1/8*
(a*b^4*e^(d*x + c) - b^5)/(a^5*b*e + a^3*b^3*e + (a^5*b*f + a^3*b^3*f)*x -
(a^5*b*e*e^(2*c) + a^3*b^3*e*e^(2*c) + (a^5*b*f*e^(2*c) + a^3*b^3*f*e^(2*c)
```

```
) * x) * e^(2*d*x) - 2*(a^6*e*e^c + a^4*b^2*e*e^c + (a^6*f*e^c + a^4*b^2*f*e^c)
*x) * e^(d*x)), x) + 16*integrate(1/8*(b*e^(d*x + c) - a)/(a^2*e + b^2*e + (a
^2*f + b^2*f)*x + (a^2*e*e^(2*c) + b^2*e*e^(2*c) + (a^2*f*e^(2*c) + b^2*f*e
^(2*c))*x) * e^(2*d*x)), x)
```

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"fricas")
```

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm=
"giac")
```

[Out] Timed out

$$3.496 \quad \int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx) \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx$$

**Optimal.** Leaf size=1245

result too large to display

```
[Out] (2*b*(e + f*x)^2)/(a^2*d) - (b^3*(e + f*x)^2)/(a^2*(a^2 + b^2)*d) + (4*f^2*x*ArcTan[E^(c + d*x)])/(a*d^2) - (4*b^2*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a^3*d^2) + (4*b^4*f*(e + f*x)*ArcTan[E^(c + d*x)])/(a^3*(a^2 + b^2)*d^2) + (2*e*f*ArcTan[Sinh[c + d*x]])/(a*d^2) + (3*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a*d) - (2*b^2*(e + f*x)^2*ArcTanh[E^(c + d*x)])/(a^3*d) - (f^2*ArcTanh[Cosh[c + d*x]])/(a*d^3) + (2*b*(e + f*x)^2*Coth[2*c + 2*d*x])/(a^2*d) - (e*f*Csch[c + d*x])/(a*d^2) - (f^2*x*Csch[c + d*x])/(a*d^2) - (b^5*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]])/(a^3*(a^2 + b^2)^(3/2)*d) + (b^5*(e + f*x)^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]])/(a^3*(a^2 + b^2)^(3/2)*d) + (2*b^3*f*(e + f*x)*Log[1 + E^(2*(c + d*x))])/(a^2*(a^2 + b^2)*d^2) - (2*b*f*(e + f*x)*Log[1 - E^(4*(c + d*x))])/(a^2*d^2) + (3*f*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a*d^2) - (2*b^2*f*(e + f*x)*PolyLog[2, -E^(c + d*x)])/(a^3*d^2) - ((2*I)*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a*d^3) + ((2*I)*b^2*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a^3*d^3) - ((2*I)*b^4*f^2*PolyLog[2, (-I)*E^(c + d*x)])/(a^3*(a^2 + b^2)*d^3) + ((2*I)*f^2*PolyLog[2, I*E^(c + d*x)])/(a*d^3) - ((2*I)*b^2*f^2*PolyLog[2, I*E^(c + d*x)])/(a^3*d^3) + ((2*I)*b^4*f^2*PolyLog[2, I*E^(c + d*x)])/(a^3*(a^2 + b^2)*d^3) - (3*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a*d^2) + (2*b^2*f*(e + f*x)*PolyLog[2, E^(c + d*x)])/(a^3*d^2) - (2*b^5*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)^(3/2)*d^2) + (2*b^5*f*(e + f*x)*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)^(3/2)*d^2) + (b^3*f^2*PolyLog[2, -E^(2*(c + d*x))])/(a^2*(a^2 + b^2)*d^3) - (b*f^2*PolyLog[2, E^(4*(c + d*x))])/(2*a^2*d^3) - (3*f^2*PolyLog[3, -E^(c + d*x)])/(a*d^3) + (2*b^2*f^2*PolyLog[3, -E^(c + d*x)])/(a^3*d^3) + (3*f^2*PolyLog[3, E^(c + d*x)])/(a*d^3) - (2*b^2*f^2*PolyLog[3, E^(c + d*x)])/(a^3*d^3) + (2*b^5*f^2*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)^(3/2)*d^3) - (2*b^5*f^2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 + b^2]))])/(a^3*(a^2 + b^2)^(3/2)*d^3) - (3*(e + f*x)^2*Sech[c + d*x])/(2*a*d) + (b^2*(e + f*x)^2*Sech[c + d*x])/(a^3*d) - (b^4*(e + f*x)^2*Sech[c + d*x])/(a^3*(a^2 + b^2)*d) - ((e + f*x)^2*Csch[c + d*x]^2*Sech[c + d*x])/(2*a*d) - (b^3*(e + f*x)^2*Tanh[c + d*x])/(a^2*(a^2 + b^2)*d)
```

---

**Rubi [A]** time = 3.43733, antiderivative size = 1245, normalized size of antiderivative = 1., number of steps used = 88, number of rules used = 33, integrand size = 36,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$ , Rules used = {5589, 2622, 288, 321, 207, 5462, 6688, 12, 6742, 6273, 4182, 2531,

2282, 6589, 4133, 453, 203, 4180, 2279, 2391, 2621, 5203, 3770, 5461, 4184, 3716, 2190, 6741, 5573, 3322, 2264, 3718, 5451}

$$-\frac{(e+fx)^2 \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^5}{a^3(a^2+b^2)^{3/2}d} + \frac{(e+fx)^2 \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^5}{a^3(a^2+b^2)^{3/2}d} - \frac{2f(e+fx)\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)b^5}{a^3(a^2+b^2)^{3/2}d^2} + \frac{2f(e+fx)}{a}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)^2\*Csch[c + d\*x]^3\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] (2\*b\*(e + f\*x)^2)/(a^2\*d) - (b^3\*(e + f\*x)^2)/(a^2\*(a^2 + b^2)\*d) + (4\*f^2\*x\*ArcTan[E^(c + d\*x)])/(a\*d^2) - (4\*b^2\*f\*(e + f\*x)\*ArcTan[E^(c + d\*x)])/(a^3\*d^2) + (4\*b^4\*f\*(e + f\*x)\*ArcTan[E^(c + d\*x)])/(a^3\*(a^2 + b^2)\*d^2) + (2\*e\*f\*ArcTan[Sinh[c + d\*x]])/(a\*d^2) + (3\*(e + f\*x)^2\*ArcTanh[E^(c + d\*x)])/(a\*d) - (2\*b^2\*(e + f\*x)^2\*ArcTanh[E^(c + d\*x)])/(a^3\*d) - (f^2\*ArcTanh[Cosh[c + d\*x]])/(a\*d^3) + (2\*b\*(e + f\*x)^2\*Coth[2\*c + 2\*d\*x])/(a^2\*d) - (e\*f\*Csch[c + d\*x])/(a\*d^2) - (f^2\*x\*Csch[c + d\*x])/(a\*d^2) - (b^5\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^3\*(a^2 + b^2)^(3/2)\*d) + (b^5\*(e + f\*x)^2\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^3\*(a^2 + b^2)^(3/2)\*d) + (2\*b^3\*f\*(e + f\*x)\*Log[1 + E^(2\*(c + d\*x))])/(a^2\*(a^2 + b^2)\*d^2) - (2\*b\*f\*(e + f\*x)\*Log[1 - E^(4\*(c + d\*x))])/(a^2\*d^2) + (3\*f\*(e + f\*x)\*PolyLog[2, -E^(c + d\*x)])/(a\*d^2) - (2\*b^2\*f\*(e + f\*x)\*PolyLog[2, -E^(c + d\*x)])/(a^3\*d^2) - ((2\*I)\*f^2\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a\*d^3) + ((2\*I)\*b^2\*f^2\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a^3\*d^3) - ((2\*I)\*b^4\*f^2\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a^3\*(a^2 + b^2)\*d^3) + ((2\*I)\*f^2\*PolyLog[2, I\*E^(c + d\*x)])/(a\*d^3) - ((2\*I)\*b^2\*f^2\*PolyLog[2, I\*E^(c + d\*x)])/(a^3\*d^3) + ((2\*I)\*b^4\*f^2\*PolyLog[2, I\*E^(c + d\*x)])/(a^3\*(a^2 + b^2)\*d^3) - (3\*f\*(e + f\*x)\*PolyLog[2, E^(c + d\*x)])/(a\*d^2) + (2\*b^2\*f\*(e + f\*x)\*PolyLog[2, E^(c + d\*x)])/(a^3\*d^2) - (2\*b^5\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^3\*(a^2 + b^2)^(3/2)\*d^2) + (2\*b^5\*f\*(e + f\*x)\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^3\*(a^2 + b^2)^(3/2)\*d^2) + (b^3\*f^2\*PolyLog[2, -E^(2\*(c + d\*x))])/(a^2\*(a^2 + b^2)\*d^3) - (b\*f^2\*PolyLog[2, E^(4\*(c + d\*x))])/(2\*a^2\*d^3) - (3\*f^2\*PolyLog[3, -E^(c + d\*x)])/(a\*d^3) + (2\*b^2\*f^2\*PolyLog[3, -E^(c + d\*x)])/(a^3\*d^3) + (3\*f^2\*PolyLog[3, E^(c + d\*x)])/(a\*d^3) - (2\*b^2\*f^2\*PolyLog[3, E^(c + d\*x)])/(a^3\*d^3) + (2\*b^5\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^3\*(a^2 + b^2)^(3/2)\*d^3) - (2\*b^5\*f^2\*PolyLog[3, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^3\*(a^2 + b^2)^(3/2)\*d^3) - (3\*(e + f\*x)^2\*Sech[c + d\*x])/(2\*a\*d) + (b^2\*(e + f\*x)^2\*Sech[c + d\*x])/(a^3\*d) - (b^4\*(e + f\*x)^2\*Sech[c + d\*x])/(a^3\*(a^2 + b^2)\*d) - ((e + f\*x)^2\*Csch[c + d\*x]^2\*Sech[c + d\*x])/(2\*a\*d) - (b^3\*(e + f\*x)^2\*Tanh[c + d\*x])/(a^2\*(a^2 + b^2)\*d)

Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
(d_.)*(x_)]^(p_.))/((a_.) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
IGtQ[p, 0]
```

### Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_.), x_S
ymbol] := Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

### Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 207

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

### Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] := With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

Rule 6688

```
Int[u_, x_Symbol] :=> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :=> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x]
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&& !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :=> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x)) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :=> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
)))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```



```
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p]/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

### Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_))^(n_.))^(p_.)*((c_) + (d_.)*(x_
)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

### Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt
[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

)<sup>n</sup>], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2621

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_)\*sec[(e\_.) + (f\_.)\*(x\_)]^(n\_), x\_Symbol] := -Dist[(f\*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a\*Csc[e + f\*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

### Rule 5203

Int[ArcTan[u\_], x\_Symbol] := Simp[x\*ArcTan[u], x] - Int[SimplifyIntegrand[(x\*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]

### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

### Rule 5461

Int[Csch[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sech[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Dist[2^n, Int[(c + d\*x)^m\*Csch[2\*a + 2\*b\*x]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]

### Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)]^2\*((c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

### Rule 3716

Int[((c\_.) + (d\_.)\*(x\_))^(m\_.)\*tan[(e\_.) + Pi\*(k\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)], x\_Symbol] := -Simp[(I\*(c + d\*x)^(m + 1))/(d\*(m + 1)), x] + Dist[2\*I, Int[((c + d\*x)^m\*E^(2\*(-I\*e) + f\*fz\*x)))/(E^(2\*I\*k\*Pi)\*(1 + E^(2\*(-I\*e) + f\*fz\*x)))/E^(2\*I\*k\*Pi)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[4\*k] && IGtQ[m, 0]

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v !=
= u]
```

Rule 5573

```
Int[(((e_) + (f_)*(x_))^(m_)*Sech[(c_) + (d_)*(x_)]^(n_))/((a_) + (b_
)*Sinh[(c_) + (d_)*(x_)]), x_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e +
f*x)^m*Sech[c + d*x]^(n - 2))/(a + b*Sinh[c + d*x]), x], x] + Dist[1/(a^2 +
b^2), Int[(e + f*x)^m*Sech[c + d*x]^n*(a - b*Sinh[c + d*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]
```

Rule 3322

```
Int[(((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*sin[(e_) + (Complex[0, fz_])*
(f_)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2264

```
Int[((F_)^(u_)*((f_) + (g_)*(x_))^(m_))/((a_) + (b_)*(F_)^(u_) + (c_)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 3718

```
Int[(((c_) + (d_)*(x_))^(m_)*tan[(e_) + (Complex[0, fz_])*(f_)*(x_)]), x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5451

```

Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)^2 \operatorname{csch}^3(c+dx) \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx &= \frac{\int (e+fx)^2 \operatorname{csch}^3(c+dx) \operatorname{sech}^2(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)^2 \operatorname{csch}^2(c+dx) \operatorname{sech}^2(c+dx)}{a+b \sinh(c+dx)} dx}{a} \\
&= \frac{3(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{3(e+fx)^2 \operatorname{sech}(c+dx)}{2ad} - \frac{(e+fx)^2 \operatorname{csch}^2(c+dx)}{a} \\
&= \frac{3(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{3(e+fx)^2 \operatorname{sech}(c+dx)}{2ad} - \frac{(e+fx)^2 \operatorname{csch}^2(c+dx)}{a} \\
&= \frac{3(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{b^2(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{a^3 d} + \frac{b^2(e+fx)^2 \operatorname{csch}^2(c+dx)}{a^3 d} \\
&= \frac{2b(e+fx)^2}{a^2 d} + \frac{3(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{b^2(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{a^3 d} + \frac{b^2(e+fx)^2 \operatorname{csch}^2(c+dx)}{a^3 d} \\
&= \frac{2b(e+fx)^2}{a^2 d} + \frac{3(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{b^2(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{a^3 d} + \frac{b^2(e+fx)^2 \operatorname{csch}^2(c+dx)}{a^3 d} \\
&= \frac{2b(e+fx)^2}{a^2 d} - \frac{b^2(e+fx)^2 \tanh^{-1}(\cosh(c+dx))}{a^3 d} + \frac{2b(e+fx)^2 \operatorname{coth}(2c+2dx)}{a^2 d} \\
&= \frac{2b(e+fx)^2}{a^2 d} - \frac{b^3(e+fx)^2}{a^2(a^2+b^2)d} + \frac{4b^4 f(e+fx) \tan^{-1}(e^{c+dx})}{a^3(a^2+b^2)d^2} - \frac{b^2(e+fx)^2 \operatorname{csch}^2(c+dx)}{a^3 d} \\
&= \frac{2b(e+fx)^2}{a^2 d} - \frac{b^3(e+fx)^2}{a^2(a^2+b^2)d} - \frac{4b^2 f(e+fx) \tan^{-1}(e^{c+dx})}{a^3 d^2} + \frac{4b^4 f(e+fx) \tan^{-1}(e^{c+dx})}{a^3(a^2+b^2)d^2} \\
&= \frac{2b(e+fx)^2}{a^2 d} - \frac{b^3(e+fx)^2}{a^2(a^2+b^2)d} - \frac{4b^2 f(e+fx) \tan^{-1}(e^{c+dx})}{a^3 d^2} + \frac{4b^4 f(e+fx) \tan^{-1}(e^{c+dx})}{a^3(a^2+b^2)d^2} \\
&= \frac{2b(e+fx)^2}{a^2 d} - \frac{b^3(e+fx)^2}{a^2(a^2+b^2)d} + \frac{6f^2 x \tan^{-1}(e^{c+dx})}{ad^2} - \frac{4b^2 f(e+fx) \tan^{-1}(e^{c+dx})}{a^3 d^2} \\
&= \frac{2b(e+fx)^2}{a^2 d} - \frac{b^3(e+fx)^2}{a^2(a^2+b^2)d} + \frac{6f^2 x \tan^{-1}(e^{c+dx})}{ad^2} - \frac{4b^2 f(e+fx) \tan^{-1}(e^{c+dx})}{a^3 d^2} \\
&= \frac{2b(e+fx)^2}{a^2 d} - \frac{b^3(e+fx)^2}{a^2(a^2+b^2)d} + \frac{6f^2 x \tan^{-1}(e^{c+dx})}{ad^2} - \frac{4b^2 f(e+fx) \tan^{-1}(e^{c+dx})}{a^3 d^2} \\
&= \frac{2b(e+fx)^2}{a^2 d} - \frac{b^3(e+fx)^2}{a^2(a^2+b^2)d} + \frac{6f^2 x \tan^{-1}(e^{c+dx})}{ad^2} - \frac{4b^2 f(e+fx) \tan^{-1}(e^{c+dx})}{a^3 d^2}
\end{aligned}$$

**Mathematica [B]** time = 27.5585, size = 2574, normalized size = 2.07

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((e + f\*x)^2\*Csch[c + d\*x]^3\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out]  $(8*a*b*d^2*e^E^{(2*c)}*f*x + 4*a*b*d^2*E^{(2*c)}*f^2*x^2 - 6*a^2*d^2*e^2*ArcTan h[E^{(c + d*x)}] + 4*b^2*d^2*e^2*ArcTanh[E^{(c + d*x)}] + 6*a^2*d^2*e^2*E^{(2*c)}*ArcTanh[E^{(c + d*x)}] - 4*b^2*d^2*e^2*E^{(2*c)}*ArcTanh[E^{(c + d*x)}] + 4*a^2*f^2*ArcTanh[E^{(c + d*x)}] - 4*a^2*E^{(2*c)}*f^2*ArcTanh[E^{(c + d*x)}] + 6*a^2*d^2*e*f*x*Log[1 - E^{(c + d*x)}] - 4*b^2*d^2*e*f*x*Log[1 - E^{(c + d*x)}] - 6*a^2*d^2*e*E^{(2*c)}*f*x*Log[1 - E^{(c + d*x)}] + 4*b^2*d^2*e*E^{(2*c)}*f*x*Log[1 - E^{(c + d*x)}] + 3*a^2*d^2*f^2*x^2*Log[1 - E^{(c + d*x)}] - 2*b^2*d^2*f^2*x^2*Log[1 - E^{(c + d*x)}] - 3*a^2*d^2*E^{(2*c)}*f^2*x^2*Log[1 - E^{(c + d*x)}] + 2*b^2*d^2*E^{(2*c)}*f^2*x^2*Log[1 - E^{(c + d*x)}] - 6*a^2*d^2*e*f*x*Log[1 + E^{(c + d*x)}] + 4*b^2*d^2*e*f*x*Log[1 + E^{(c + d*x)}] + 6*a^2*d^2*e*E^{(2*c)}*f*x*Log[1 + E^{(c + d*x)}] - 4*b^2*d^2*e*E^{(2*c)}*f*x*Log[1 + E^{(c + d*x)}] - 3*a^2*d^2*f^2*x^2*Log[1 + E^{(c + d*x)}] + 2*b^2*d^2*f^2*x^2*Log[1 + E^{(c + d*x)}] + 3*a^2*d^2*E^{(2*c)}*f^2*x^2*Log[1 + E^{(c + d*x)}] - 2*b^2*d^2*E^{(2*c)}*f^2*x^2*Log[1 + E^{(c + d*x)}] + 4*a*b*d*e*f*Log[1 - E^{(2*(c + d*x))}] - 4*a*b*d*e*E^{(2*c)}*f*Log[1 - E^{(2*(c + d*x))}] + 4*a*b*d*f^2*x*Log[1 - E^{(2*(c + d*x))}] - 4*a*b*d*E^{(2*c)}*f^2*x*Log[1 - E^{(2*(c + d*x))}] + 2*(3*a^2 - 2*b^2)*d*(-1 + E^{(2*c)})*f*(e + f*x)*PolyLog[2, -E^{(c + d*x)}] - 2*(3*a^2 - 2*b^2)*d*(-1 + E^{(2*c)})*f*(e + f*x)*PolyLog[2, E^{(c + d*x)}] + 2*a*b*f^2*PolyLog[2, E^{(2*(c + d*x))}] - 2*a*b*E^{(2*c)}*f^2*PolyLog[2, E^{(2*(c + d*x))}] + 6*a^2*f^2*PolyLog[3, -E^{(c + d*x)}] - 4*b^2*f^2*PolyLog[3, -E^{(c + d*x)}] - 6*a^2*E^{(2*c)}*f^2*PolyLog[3, -E^{(c + d*x)}] + 4*b^2*E^{(2*c)}*f^2*PolyLog[3, -E^{(c + d*x)}] - 6*a^2*f^2*PolyLog[3, E^{(c + d*x)}] + 4*b^2*f^2*PolyLog[3, E^{(c + d*x)}] + 6*a^2*E^{(2*c)}*f^2*PolyLog[3, E^{(c + d*x)}] - 4*b^2*E^{(2*c)}*f^2*PolyLog[3, E^{(c + d*x)}])/(2*a^3*d^3*(-1 + E^{(2*c)})) + (b^5*(2*d^2*e^2*ArcTanh[(a + b*E^{(c + d*x)})]/Sqrt[a^2 + b^2]) - 2*d^2*e*f*x*Log[1 + (b*E^{(c + d*x)})/(a - Sqrt[a^2 + b^2])] - d^2*f^2*x^2*Log[1 + (b*E^{(c + d*x)})/(a - Sqrt[a^2 + b^2])] + 2*d^2*e*f*x*Log[1 + (b*E^{(c + d*x)})/(a + Sqrt[a^2 + b^2])] + d^2*f^2*x^2*Log[1 + (b*E^{(c + d*x)})/(a + Sqrt[a^2 + b^2])] - 2*d*f*(e + f*x)*PolyLog[2, (b*E^{(c + d*x)})/(-a + Sqrt[a^2 + b^2])] + 2*d*f*(e + f*x)*PolyLog[2, -(b*E^{(c + d*x)})/(a + Sqrt[a^2 + b^2])] + 2*f^2*PolyLog[3, (b*E^{(c + d*x)})/(-a + Sqrt[a^2 + b^2])] - 2*f^2*PolyLog[3, -(b*E^{(c + d*x)})/(a + Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)^(3/2)*d^3) - (2*b*e*f*Sech[c]*(Cosh[c]*Log[Cosh[c]*Cosh[d*x] + Sinh[c]*Sinh[d*x]] - d*x*Sinh[c]))/((a^2 + b^2)*d^2*(Cosh[c]^2 - Sinh[c]^2)) + (4*a*e*f*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2]])/((a^2 + b^2)*d^2*Sqrt[Cosh[c]^2 - Sinh[c]^2]) + (b*f^2*Csch$

```

[c]*(-(d^2*x^2)/E^ArcTanh[Coth[c]]) + (I*Coth[c]*(-(d*x*(-Pi + (2*I)*ArcTan
h[Coth[c]])) - Pi*Log[1 + E^(2*d*x)] - 2*(I*d*x + I*ArcTanh[Coth[c]])*Log[
1 - E^((2*I)*(I*d*x + I*ArcTanh[Coth[c]]))]) + Pi*Log[Cosh[d*x]] + (2*I)*Arc
Tanh[Coth[c]]*Log[I*Sinh[d*x + ArcTanh[Coth[c]]]]) + I*PolyLog[2, E^((2*I)*(
I*d*x + I*ArcTanh[Coth[c]])))]/Sqrt[1 - Coth[c]^2])*Sech[c])/((a^2 + b^2)*
d^3*Sqrt[Csch[c]^2*(-Cosh[c]^2 + Sinh[c]^2)]) + (2*a*f^2*((-I)*Csch[c]*(I*
(d*x + ArcTanh[Coth[c]])*(Log[1 - E^(-(d*x) - ArcTanh[Coth[c]])) - Log[1 +
E^(-(d*x) - ArcTanh[Coth[c]]))]) + I*(PolyLog[2, -E^(-(d*x) - ArcTanh[Coth[c
]])] - PolyLog[2, E^(-(d*x) - ArcTanh[Coth[c]])))])/Sqrt[1 - Coth[c]^2] - (
2*ArcTan[(Sinh[c] + Cosh[c]*Tanh[(d*x)/2])/Sqrt[Cosh[c]^2 - Sinh[c]^2])*Arc
Tanh[Coth[c]]/Sqrt[Cosh[c]^2 - Sinh[c]^2])/((a^2 + b^2)*d^3) + (Csch[c]*C
sch[c + d*x]^2*Sech[c]*Sech[c + d*x]*(2*a^3*e*f*Cosh[2*d*x] + 2*a*b^2*e*f*C
osh[2*d*x] + 2*a^3*f^2*x*Cosh[2*d*x] + 2*a*b^2*f^2*x*Cosh[2*d*x] + 4*a^2*b*
d*e^2*Cosh[c - d*x] + 8*a^2*b*d*e*f*x*Cosh[c - d*x] + 4*a^2*b*d*f^2*x^2*Cos
h[c - d*x] + 2*b^3*d*e^2*Cosh[c + d*x] + 4*b^3*d*e*f*x*Cosh[c + d*x] + 2*b^
3*d*f^2*x^2*Cosh[c + d*x] + 2*b^3*d*e^2*Cosh[3*c + d*x] + 4*b^3*d*e*f*x*Cos
h[3*c + d*x] + 2*b^3*d*f^2*x^2*Cosh[3*c + d*x] - 2*a^3*e*f*Cosh[4*c + 2*d*x
] - 2*a*b^2*e*f*Cosh[4*c + 2*d*x] - 2*a^3*f^2*x*Cosh[4*c + 2*d*x] - 2*a*b^2
*f^2*x*Cosh[4*c + 2*d*x] - 4*a^2*b*d*e^2*Cosh[c + 3*d*x] - 2*b^3*d*e^2*Cosh
[c + 3*d*x] - 8*a^2*b*d*e*f*x*Cosh[c + 3*d*x] - 4*b^3*d*e*f*x*Cosh[c + 3*d*
x] - 4*a^2*b*d*f^2*x^2*Cosh[c + 3*d*x] - 2*b^3*d*f^2*x^2*Cosh[c + 3*d*x] -
2*b^3*d*e^2*Cosh[3*c + 3*d*x] - 4*b^3*d*e*f*x*Cosh[3*c + 3*d*x] - 2*b^3*d*f
^2*x^2*Cosh[3*c + 3*d*x] + 2*a^3*d*e^2*Sinh[2*c] - 2*a*b^2*d*e^2*Sinh[2*c]
+ 4*a^3*d*e*f*x*Sinh[2*c] - 4*a*b^2*d*e*f*x*Sinh[2*c] + 2*a^3*d*f^2*x^2*Sin
h[2*c] - 2*a*b^2*d*f^2*x^2*Sinh[2*c] + 3*a^3*d*e^2*Sinh[2*d*x] + a*b^2*d*e^
2*Sinh[2*d*x] + 6*a^3*d*e*f*x*Sinh[2*d*x] + 2*a*b^2*d*e*f*x*Sinh[2*d*x] + 3
*a^3*d*f^2*x^2*Sinh[2*d*x] + a*b^2*d*f^2*x^2*Sinh[2*d*x] - 3*a^3*d*e^2*Sinh
[4*c + 2*d*x] - a*b^2*d*e^2*Sinh[4*c + 2*d*x] - 6*a^3*d*e*f*x*Sinh[4*c + 2*
d*x] - 2*a*b^2*d*e*f*x*Sinh[4*c + 2*d*x] - 3*a^3*d*f^2*x^2*Sinh[4*c + 2*d*x
] - a*b^2*d*f^2*x^2*Sinh[4*c + 2*d*x]))/(16*a^2*(a^2 + b^2)*d^2)

```

---

**Maple [F]** time = 1.557, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^2 (\operatorname{csch}(dx + c))^3 (\operatorname{sech}(dx + c))^2}{a + b \sinh(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^2\*csch(d\*x+c)^3\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] int((f\*x+e)^2\*csch(d\*x+c)^3\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)**2*csch(d*x+c)**3*sech(d*x+c)**2/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)^2*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.497 \quad \int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=699

$$-\frac{b^5 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3 d^2 (a^2+b^2)^{3/2}} + \frac{b^5 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3 d^2 (a^2+b^2)^{3/2}} - \frac{b^2 f \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{a^3 d^2} + \frac{b^2 f \operatorname{PolyLog}\left(2, e^{c+dx}\right)}{a^3 d^2} + \frac{3fP}{a^3 d^2}$$

[Out] (f\*ArcTan[Sinh[c + d\*x]])/(a\*d^2) - (b^2\*f\*ArcTan[Sinh[c + d\*x]])/(a^3\*d^2) + (b^4\*f\*ArcTan[Sinh[c + d\*x]])/(a^3\*(a^2 + b^2)\*d^2) + (3\*f\*x\*ArcTanh[E^(c + d\*x)])/(a\*d) - (2\*b^2\*f\*x\*ArcTanh[E^(c + d\*x)])/(a^3\*d) - (3\*f\*x\*ArcTanh[Cosh[c + d\*x]])/(2\*a\*d) + (b^2\*f\*x\*ArcTanh[Cosh[c + d\*x]])/(a^3\*d) + (3\*(e + f\*x)\*ArcTanh[Cosh[c + d\*x]])/(2\*a\*d) - (b^2\*(e + f\*x)\*ArcTanh[Cosh[c + d\*x]])/(a^3\*d) + (2\*b\*(e + f\*x)\*Coth[2\*c + 2\*d\*x])/(a^2\*d) - (f\*Csch[c + d\*x])/(2\*a\*d^2) - (b^5\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^3\*(a^2 + b^2)^(3/2)\*d) + (b^5\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^3\*(a^2 + b^2)^(3/2)\*d) + (b^3\*f\*Log[Cosh[c + d\*x]])/(a^2\*(a^2 + b^2)\*d^2) - (b\*f\*Log[Sinh[2\*c + 2\*d\*x]])/(a^2\*d^2) + (3\*f\*PolyLog[2, -E^(c + d\*x)])/(2\*a\*d^2) - (b^2\*f\*PolyLog[2, -E^(c + d\*x)])/(a^3\*d^2) - (3\*f\*PolyLog[2, E^(c + d\*x)])/(2\*a\*d^2) + (b^2\*f\*PolyLog[2, E^(c + d\*x)])/(a^3\*d^2) - (b^5\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2]))])/(a^3\*(a^2 + b^2)^(3/2)\*d^2) + (b^5\*f\*PolyLog[2, -((b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2]))])/(a^3\*(a^2 + b^2)^(3/2)\*d^2) - (3\*(e + f\*x)\*Sech[c + d\*x])/(2\*a\*d) + (b^2\*(e + f\*x)\*Sech[c + d\*x])/(a^3\*d) - (b^4\*(e + f\*x)\*Sech[c + d\*x])/(a^3\*(a^2 + b^2)\*d) - ((e + f\*x)\*Csch[c + d\*x]^2\*Sech[c + d\*x])/(2\*a\*d) - (b^3\*(e + f\*x)\*Tanh[c + d\*x])/(a^2\*(a^2 + b^2)\*d)

**Rubi [A]** time = 1.35635, antiderivative size = 699, normalized size of antiderivative = 1., number of steps used = 44, number of rules used = 22, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$ , Rules used = {5589, 2622, 288, 321, 207, 5462, 6271, 12, 4182, 2279, 2391, 3770, 2621, 5461, 4184, 3475, 5573, 3322, 2264, 2190, 6742, 5451}

$$-\frac{b^5 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3 d^2 (a^2+b^2)^{3/2}} + \frac{b^5 f \operatorname{PolyLog}\left(2, -\frac{be^{c+dx}}{\sqrt{a^2+b^2}+a}\right)}{a^3 d^2 (a^2+b^2)^{3/2}} - \frac{b^2 f \operatorname{PolyLog}\left(2, -e^{c+dx}\right)}{a^3 d^2} + \frac{b^2 f \operatorname{PolyLog}\left(2, e^{c+dx}\right)}{a^3 d^2} + \frac{3fP}{a^3 d^2}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Csch[c + d\*x]^3\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

```
[Out] (f*ArcTan[Sinh[c + d*x]])/(a*d^2) - (b^2*f*ArcTan[Sinh[c + d*x]])/(a^3*d^2)
+ (b^4*f*ArcTan[Sinh[c + d*x]])/(a^3*(a^2 + b^2)*d^2) + (3*f*x*ArcTanh[E^(
c + d*x)]/(a*d) - (2*b^2*f*x*ArcTanh[E^(c + d*x)]/(a^3*d) - (3*f*x*ArcTan
h[Cosh[c + d*x]]/(2*a*d) + (b^2*f*x*ArcTanh[Cosh[c + d*x]]/(a^3*d) + (3*(
e + f*x)*ArcTanh[Cosh[c + d*x]]/(2*a*d) - (b^2*(e + f*x)*ArcTanh[Cosh[c +
d*x]]/(a^3*d) + (2*b*(e + f*x)*Coth[2*c + 2*d*x]]/(a^2*d) - (f*Csch[c + d*
x]]/(2*a*d^2) - (b^5*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2]
)]/(a^3*(a^2 + b^2)^(3/2)*d) + (b^5*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a +
Sqrt[a^2 + b^2]])/(a^3*(a^2 + b^2)^(3/2)*d) + (b^3*f*Log[Cosh[c + d*x]]/
(a^2*(a^2 + b^2)*d^2) - (b*f*Log[Sinh[2*c + 2*d*x]]/(a^2*d^2) + (3*f*PolyL
og[2, -E^(c + d*x)]/(2*a*d^2) - (b^2*f*PolyLog[2, -E^(c + d*x)]/(a^3*d^2)
- (3*f*PolyLog[2, E^(c + d*x)]/(2*a*d^2) + (b^2*f*PolyLog[2, E^(c + d*x)]
)/(a^3*d^2) - (b^5*f*PolyLog[2, -(b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])])/(
a^3*(a^2 + b^2)^(3/2)*d^2) + (b^5*f*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt
[a^2 + b^2])])/(a^3*(a^2 + b^2)^(3/2)*d^2) - (3*(e + f*x)*Sech[c + d*x])/(
2*a*d) + (b^2*(e + f*x)*Sech[c + d*x])/(a^3*d) - (b^4*(e + f*x)*Sech[c + d*
x])/(a^3*(a^2 + b^2)*d) - ((e + f*x)*Csch[c + d*x]^2*Sech[c + d*x])/(2*a*d)
- (b^3*(e + f*x)*Tanh[c + d*x])/(a^2*(a^2 + b^2)*d)
```

### Rule 5589

```
Int[(Csch[(c_.) + (d_.)*(x_)]^(n_.)*((e_.) + (f_.)*(x_))^(m_.)*Sech[(c_.) +
(d_.)*(x_)]^(p_.)]/((a_) + (b_.)*Sinh[(c_.) + (d_.)*(x_)]), x_Symbol] := D
ist[1/a, Int[(e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^n, x], x] - Dist[b/a
, Int[((e + f*x)^m*Sech[c + d*x]^p*Csch[c + d*x]^(n - 1))/(a + b*Sinh[c + d
*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] &&
IGtQ[p, 0]
```

### Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)]^(m_.), x_S
ymbol] := Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2
], x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

### Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 321

```
Int[((c_.)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

### Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)^(n_.)*((c_.) + (d_.)*(x_)^(m_.))*Sech[(a_.) +
(b_.)*(x_)^(p_.)], x_Symbol] := With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

### Rule 6271

```
Int[ArcTanh[u_], x_Symbol] := Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_)^(m_.), x
_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x))]/(f*fz*I), x]
+ (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

### Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 2621

Int[(csc[(e\_.) + (f\_.)\*(x\_)])\*(a\_.)^(m\_.)\*sec[(e\_.) + (f\_.)\*(x\_)^(n\_.), x\_Symbol] := -Dist[(f\*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a\*Csc[e + f\*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

Rule 5461

Int[Csch[(a\_.) + (b\_.)\*(x\_)^(n\_.)]\*((c\_.) + (d\_.)\*(x\_)^(m\_.))\*Sech[(a\_.) + (b\_.)\*(x\_)^(n\_.), x\_Symbol] := Dist[2^n, Int[(c + d\*x)^m\*Csch[2\*a + 2\*b\*x]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]

Rule 4184

Int[csc[(e\_.) + (f\_.)\*(x\_)^2]\*((c\_.) + (d\_.)\*(x\_)^(m\_.), x\_Symbol] := -Simp[((c + d\*x)^m\*Cot[e + f\*x])/f, x] + Dist[(d\*m)/f, Int[(c + d\*x)^(m - 1)\*Cot[e + f\*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3475

Int[tan[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[Log[RemoveContent[Cos[c + d\*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 5573

Int[(((e\_.) + (f\_.)\*(x\_)^(m\_.))\*Sech[(c\_.) + (d\_.)\*(x\_)^(n\_.)])/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e + f\*x)^m\*Sech[c + d\*x]^(n - 2))/(a + b\*Sinh[c + d\*x]), x], x] + Dist[1/(a^2 + b^2), Int[(e + f\*x)^m\*Sech[c + d\*x]^n\*(a - b\*Sinh[c + d\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0] && IGtQ[n, 0]

Rule 3322

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (Complex[0, fz_])*
(f_.)*(x_)]), x_Symbol] := Dist[2, Int[((c + d*x)^m*E^(-(I*e) + f*fz*x))/(-
(I*b) + 2*a*E^(-(I*e) + f*fz*x) + I*b*E^(2*(-(I*e) + f*fz*x))), x], x] /; F
reeQ[{a, b, c, d, e, f, fz}, x] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

#### Rule 2264

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[
((f + g*x)^m*F^u)/(b - q + 2*c*F^u), x], x] - Dist[(2*c)/q, Int[((f + g*x)^
m*F^u)/(b + q + 2*c*F^u), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

#### Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] := -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx}{a} \\
&= \frac{3(e+fx)\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{3(e+fx)\operatorname{sech}(c+dx)}{2ad} - \frac{(e+fx)\operatorname{csch}^2(c+dx)}{2ad} \\
&= \frac{3(e+fx)\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{3(e+fx)\operatorname{sech}(c+dx)}{2ad} - \frac{(e+fx)\operatorname{csch}^2(c+dx)}{2ad} \\
&= \frac{3f\tanh^{-1}(\sinh(c+dx))}{2ad^2} - \frac{3fx\tanh^{-1}(\cosh(c+dx))}{2ad} + \frac{3(e+fx)\tanh^{-1}(\cosh(c+dx))}{2ad} \\
&= \frac{3f\tanh^{-1}(\sinh(c+dx))}{2ad^2} - \frac{3fx\tanh^{-1}(\cosh(c+dx))}{2ad} + \frac{3(e+fx)\tanh^{-1}(\cosh(c+dx))}{2ad} \\
&= \frac{f\tanh^{-1}(\sinh(c+dx))}{ad^2} - \frac{b^2f\tanh^{-1}(\sinh(c+dx))}{a^3d^2} + \frac{3fx\tanh^{-1}(e^{c+dx})}{ad} \\
&= \frac{f\tanh^{-1}(\sinh(c+dx))}{ad^2} - \frac{b^2f\tanh^{-1}(\sinh(c+dx))}{a^3d^2} + \frac{3fx\tanh^{-1}(e^{c+dx})}{ad} \\
&= \frac{f\tanh^{-1}(\sinh(c+dx))}{ad^2} - \frac{b^2f\tanh^{-1}(\sinh(c+dx))}{a^3d^2} + \frac{b^4f\tanh^{-1}(\sinh(c+dx))}{a^3(a^2+b^2)d^2} \\
&= \frac{f\tanh^{-1}(\sinh(c+dx))}{ad^2} - \frac{b^2f\tanh^{-1}(\sinh(c+dx))}{a^3d^2} + \frac{b^4f\tanh^{-1}(\sinh(c+dx))}{a^3(a^2+b^2)d^2} \\
&= \frac{f\tanh^{-1}(\sinh(c+dx))}{ad^2} - \frac{b^2f\tanh^{-1}(\sinh(c+dx))}{a^3d^2} + \frac{b^4f\tanh^{-1}(\sinh(c+dx))}{a^3(a^2+b^2)d^2}
\end{aligned}$$

**Mathematica [C]** time = 8.4167, size = 863, normalized size = 1.23

$$\frac{\left(2de \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) - 2cf \tanh^{-1}\left(\frac{a+be^{c+dx}}{\sqrt{a^2+b^2}}\right) - f(c+dx) \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}} + 1\right) + f(c+dx) \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}} + 1\right) - f\operatorname{PolyLog}\left(2, \frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}\right) + f\operatorname{PolyLog}\left(2, \frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}\right)\right)}{a^3(a^2+b^2)^{3/2}d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((e + f*x)*Csch[c + d*x]^3*Sech[c + d*x]^2)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (2*a*f*ArcTan[Tanh[(c + d*x)/2]])/(d*(a^2*d + b^2*d)) + ((2*b*d*e*Cosh[(c + d*x)/2] - a*f*Cosh[(c + d*x)/2] - 2*b*c*f*Cosh[(c + d*x)/2] + 2*b*f*(c + d*x)*Cosh[(c + d*x)/2])*Csch[(c + d*x)/2])/(4*a^2*d^2) + ((-(d*e) + c*f - f*(c + d*x))*Csch[(c + d*x)/2]^2)/(8*a*d^2) - (b*f*Log[Cosh[c + d*x]])/((a^2 + b^2)*d^2) - (b*f*Log[Sinh[c + d*x]])/(a^2*d^2) - (3*e*Log[Tanh[(c + d*x)/2]])/(2*a*d) + (b^2*e*Log[Tanh[(c + d*x)/2]])/(a^3*d) + (3*c*f*Log[Tanh[(c + d*x)/2]])/(2*a*d^2) - (b^2*c*f*Log[Tanh[(c + d*x)/2]])/(a^3*d^2) + (((3*I)/2)*f*(I*(c + d*x)*(Log[1 - E^(-c - d*x)] - Log[1 + E^(-c - d*x)]) + I*(PolyLog[2, -E^(-c - d*x)] - PolyLog[2, E^(-c - d*x)])))/(a*d^2) - (I*b^2*f*(I*(c + d*x)*(Log[1 - E^(-c - d*x)] - Log[1 + E^(-c - d*x)]) + I*(PolyLog[2, -E^(-c - d*x)] - PolyLog[2, E^(-c - d*x)])))/(a^3*d^2) + (b^5*(2*d*e*ArcTan h[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - 2*c*f*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 + b^2]] - f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^2])] + f*(c + d*x)*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])] - f*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 + b^2])] + f*PolyLog[2, -(b*E^(c + d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)^(3/2)*d^2) + ((-(d*e) + c*f - f*(c + d*x))*Sech[(c + d*x)/2]^2)/(8*a*d^2) + (Sech[(c + d*x)/2]*(2*b*d*e*Sinh[(c + d*x)/2] + a*f*Sinh[(c + d*x)/2] - 2*b*c*f*Sinh[(c + d*x)/2] + 2*b*f*(c + d*x)*Sinh[(c + d*x)/2]))/(4*a^2*d^2) + (Sech[c + d*x]*(-(a*d*e) + a*c*f - a*f*(c + d*x) + b*d*e*Sinh[c + d*x] - b*c*f*Sinh[c + d*x] + b*f*(c + d*x)*Sinh[c + d*x]))/((a^2 + b^2)*d^2)
```

**Maple [B]** time = 0.276, size = 2767, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^2/(a+b*sinh(d*x+c)),x)
```

```
[Out] 1/(a^2+b^2)^(5/2)/d^2*f*b*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a^3+2/(a^2+b^2)^(5/2)/d^2*f*b^3*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))*a-3/2/(a^2+b^2)/d*a*e*ln(exp(d*x+c)-1)+3/2/(a^2+b^2)/d*a*e*ln(exp(d*x+c)+1)+3/2/(a^2+b^2)/d^2*a*f*dilog(exp(d*x+c))+3/2/(a^2+b^2)/d^2*a*f*dilog(exp(d*x+c)+1)+1/(a^2+b^2)^(5/2)/d*f*b^5/a*ln((b*exp(d*x+c)+(a^2+b^2)^(1/2))+a)/(a+(a^2+b^2)^(1/2)))*x+1/2/(a^2+b^2)^(3/2)/d^2*b^3*f*c/a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-3/2/(a^2+b^2)^(5/2)/d^2*f*a^3*b*c*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-3/2/(a^2+b^2)^(5/2)/d^2*b^5*f*c/a*arctanh(1/2*(2*b*exp(d*x+c)+2*a)/(a^2+b^2)^(1/2))-1/(a^2+b^2)^(5/2)/d*
```



$$\begin{aligned}
& f*b^5/a*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*x+1/(a^2+b^2)^{(5/2)}/d^2*f*b^5/a*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*c-1/(a^2+b^2)^{(5/2)}/d^2*f*b^5/a*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*c-1/a^3/(a^2+b^2)^{(5/2)}/d^2*b^7*f*dilog((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))+1/a^3/(a^2+b^2)^{(5/2)}/d^2*b^7*f*dilog((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))-1/a^3/(a^2+b^2)^{(5/2)}/d^2*b^4*f*c*\ln(\exp(d*x+c)-1)+1/a^3/(a^2+b^2)^{(3/2)}/d*b^5*e*arctanh(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))+1/a^3/(a^2+b^2)^{(5/2)}/d*b^7*e*arctanh(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))-1/a^3/(a^2+b^2)^{(5/2)}/d*b^4*f*\ln(\exp(d*x+c)+1)*x-1/a^3/(a^2+b^2)^{(5/2)}/d*b^4*e*\ln(\exp(d*x+c)+1)+1/a^3/(a^2+b^2)^{(5/2)}/d*b^4*e*\ln(\exp(d*x+c)-1)-1/a^3/(a^2+b^2)^{(5/2)}/d^2*b^4*f*dilog(\exp(d*x+c))-1/a^3/(a^2+b^2)^{(5/2)}/d^2*b^4*f*dilog(\exp(d*x+c)+1)-2*a/d^2/(a^2+b^2)^{(5/2)}/b^3*f*c*arctanh(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))+4/(a^2+b^2)^{(5/2)}/d^2*b*f*\ln(\exp(d*x+c))+3/2/(a^2+b^2)^{(5/2)}/d^2*a*f*c*\ln(\exp(d*x+c)-1)+1/2/(a^2+b^2)^{(5/2)}/d^2*b^2*f/a*dilog(\exp(d*x+c))+1/2/(a^2+b^2)^{(5/2)}/d^2*b^2*f/a*dilog(\exp(d*x+c)+1)-1/2/(a^2+b^2)^{(5/2)}/d*b^2*e/a*\ln(\exp(d*x+c)-1)+1/2/(a^2+b^2)^{(5/2)}/d*b^2*e/a*\ln(\exp(d*x+c)+1)-3/2*b/d*e/(a^2+b^2)^{(3/2)}/d^2*b^7*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*c-1/a^3/(a^2+b^2)^{(5/2)}/d^2*b^7*f*c*arctanh(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))-1/a^3/(a^2+b^2)^{(5/2)}/d*b^7*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*x+1/a^3/(a^2+b^2)^{(5/2)}/d*b^7*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*x-1/a^3/(a^2+b^2)^{(3/2)}/d^2*b^5*f*c*arctanh(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))-1/a^3/(a^2+b^2)^{(5/2)}/d^2*b^7*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*c+1/2/(a^2+b^2)^{(5/2)}/d^2*b^2*f*c/a*\ln(\exp(d*x+c)-1)-4/(a^2+b^2)^{(5/2)}/d^2*a^2*b*f/(4*a^2+4*b^2)*\ln(1+\exp(2*d*x+2*c))+8/(a^2+b^2)^{(5/2)}/d^2*a*b^2*f/(4*a^2+4*b^2)*arctan(\exp(d*x+c))-1/(a^2+b^2)^{(3/2)}/d^2*f*b^3/a*arctanh(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))-1/(a^2+b^2)^{(5/2)}/d^2*f*b^5/a*dilog((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))+1/(a^2+b^2)^{(5/2)}/d^2*f*b^5/a*dilog((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))+1/(a^2+b^2)^{(5/2)}/d^2*f*b^5/a*arctanh(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))-1/(a^2+b^2)^{(3/2)}/d^2*a*f*b*arctanh(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))-1/2/(a^2+b^2)^{(3/2)}/d*b^3*e/a*arctanh(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))+3/2/(a^2+b^2)^{(5/2)}/d*a^3*b*e*arctanh(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))+3/2/(a^2+b^2)^{(5/2)}/d*b^5*e/a*arctanh(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))+1/2/(a^2+b^2)^{(5/2)}/d*b^2*f/a*\ln(\exp(d*x+c)+1)*x-(3*a^3*d*f*x*\exp(5*d*x+5*c)+a*b^2*d*f*x*\exp(5*d*x+5*c)+3*a^3*d*e*\exp(5*d*x+5*c)+a*b^2*d*e*\exp(5*d*x+5*c)-2*b^3*d*f*x*\exp(4*d*x+4*c)-2*a^3*d*f*x*\exp(3*d*x+3*c)+a^3*f*\exp(5*d*x+5*c)+2*a*b^2*d*f*x*\exp(3*d*x+3*c)+a*b^2*f*\exp(5*d*x+5*c)-2*b^3*d*e*\exp(4*d*x+4*c)-2*a^3*d*e*\exp(3*d*x+3*c)-4*a^2*b*d*f*x*\exp(2*d*x+2*c)+2*a*b^2*d*e*\exp(3*d*x+3*c)+3*a^3*d*f*x*\exp(d*x+c)-4*a^2*b*d*e*\exp(2*d*x+2*c)+a*b^2*d*f*x*\exp(d*x+c)+3*a^3*d*e*\exp(d*x+c)+4*a^2*b*d*f*x+a*b^2*d*e*\exp(d*x+c)+2*b^3*d*f*x-a^3*f*\exp(d*x+c)+4*a^2*b*d*e-a*b^2*f*\exp(d*x+c)+2*b^3*d*e)/d^2/a^2/(\exp(2*d*x+2*c)-1)^
\end{aligned}$$

$$\frac{2}{(a^2+b^2)/(1+\exp(2dx+2c))+3/2*b/d^2*f*c/(a^2+b^2)^{(3/2)*\operatorname{arctanh}(1/2*(2*b*\exp(dx+c)+2*a)/(a^2+b^2)^{(1/2)})+2/a^2/(a^2+b^2)/d^2*b^3*f*\ln(\exp(dx+c))-1/a^2/(a^2+b^2)/d^2*b^3*f*\ln(\exp(dx+c)-1)-1/a^2/(a^2+b^2)/d^2*b^3*f*\ln(\exp(dx+c)+1)+2*a/d/(a^2+b^2)^{(5/2)*b^3*e*\operatorname{arctanh}(1/2*(2*b*\exp(dx+c)+2*a)/(a^2+b^2)^{(1/2)})-1/(a^2+b^2)/d^2*b*f*\ln(\exp(dx+c)-1)-1/(a^2+b^2)/d^2*b*f*\ln(\exp(dx+c)+1)}$$

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)^3\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [B]** time = 4.71445, size = 24195, normalized size = 34.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)^3\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out]  $\frac{1}{2}*(4*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*\cosh(d*x + c)^6 + 4*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*\sinh(d*x + c)^6 - 2*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*f*x + (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*e + (a^6 + 2*a^4*b^2 + a^2*b^4)*f)*\cosh(d*x + c)^5 - 2*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*f*x + (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*e + (a^6 + 2*a^4*b^2 + a^2*b^4)*f) - 12*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*\cosh(d*x + c))^5 - 4*(2*(a^5*b + a^3*b^3)*d*f*x - (a^3*b^3 + a*b^5)*d*e + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*\cosh(d*x + c)^4 - 2*(4*(a^5*b + a^3*b^3)*d*f*x - 2*(a^3*b^3 + a*b^5)*d*e + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*c*f - 30*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*\cosh(d*x + c)^2 + 5*((3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*f*x + (3*a^6 + 4*a^4*b^2 + a^2*b^4)*d*e + (a^6 + 2*a^4*b^2 + a^2*b^4)*f)*\cosh(d*x + c))^4 + 4*((a^6 -$



$$\begin{aligned}
& \sinh(dx + c)^3 - (b^6 d e - b^6 c f) \cosh(dx + c)^2 - (b^6 d e - b^6 c f \\
& - 15(b^6 d e - b^6 c f) \cosh(dx + c)^4 + 6(b^6 d e - b^6 c f) \cosh(dx + \\
& c)^2) \sinh(dx + c)^2 + 2(3(b^6 d e - b^6 c f) \cosh(dx + c)^5 - 2(b^6 d e \\
& - b^6 c f) \cosh(dx + c)^3 - (b^6 d e - b^6 c f) \cosh(dx + c)) \sinh(dx \\
& x + c)) \sqrt{(a^2 + b^2)/b^2} \log(2b \cosh(dx + c) + 2b \sinh(dx + c) - 2 \\
& b \sqrt{(a^2 + b^2)/b^2} + 2a) - 2(b^6 d f x + b^6 c f + (b^6 d f x + b^6 \\
& c f) \cosh(dx + c)^6 + 6(b^6 d f x + b^6 c f) \cosh(dx + c) \sinh(dx + c) \\
& ^5 + (b^6 d f x + b^6 c f) \sinh(dx + c)^6 - (b^6 d f x + b^6 c f) \cosh(dx \\
& + c)^4 - (b^6 d f x + b^6 c f - 15(b^6 d f x + b^6 c f) \cosh(dx + c)^2) * \\
& \sinh(dx + c)^4 + 4(5(b^6 d f x + b^6 c f) \cosh(dx + c)^3 - (b^6 d f x + \\
& b^6 c f) \cosh(dx + c)) \sinh(dx + c)^3 - (b^6 d f x + b^6 c f) \cosh(dx + \\
& c)^2 - (b^6 d f x + b^6 c f - 15(b^6 d f x + b^6 c f) \cosh(dx + c)^4 + 6 \\
& (b^6 d f x + b^6 c f) \cosh(dx + c)^2) \sinh(dx + c)^2 + 2(3(b^6 d f x + \\
& b^6 c f) \cosh(dx + c)^5 - 2(b^6 d f x + b^6 c f) \cosh(dx + c)^3 - (b^6 d \\
& f x + b^6 c f) \cosh(dx + c)) \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} \log(- \\
& a \cosh(dx + c) + a \sinh(dx + c) + (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{ \\
& t((a^2 + b^2)/b^2) - b)/b) + 2(b^6 d f x + b^6 c f + (b^6 d f x + b^6 c f) \\
& * \cosh(dx + c)^6 + 6(b^6 d f x + b^6 c f) \cosh(dx + c) \sinh(dx + c)^5 + \\
& (b^6 d f x + b^6 c f) \sinh(dx + c)^6 - (b^6 d f x + b^6 c f) \cosh(dx + c) \\
& ^4 - (b^6 d f x + b^6 c f - 15(b^6 d f x + b^6 c f) \cosh(dx + c)^2) \sinh( \\
& dx + c)^4 + 4(5(b^6 d f x + b^6 c f) \cosh(dx + c)^3 - (b^6 d f x + b^6 c \\
& f) \cosh(dx + c)) \sinh(dx + c)^3 - (b^6 d f x + b^6 c f) \cosh(dx + c)^2 \\
& - (b^6 d f x + b^6 c f - 15(b^6 d f x + b^6 c f) \cosh(dx + c)^4 + 6(b^6 \\
& d f x + b^6 c f) \cosh(dx + c)^2) \sinh(dx + c)^2 + 2(3(b^6 d f x + b^6 c \\
& f) \cosh(dx + c)^5 - 2(b^6 d f x + b^6 c f) \cosh(dx + c)^3 - (b^6 d f x \\
& + b^6 c f) \cosh(dx + c)) \sinh(dx + c)) \sqrt{(a^2 + b^2)/b^2} \log(-(a \cos \\
& h(dx + c) + a \sinh(dx + c) - (b \cosh(dx + c) + b \sinh(dx + c)) \sqrt{(a^ \\
& 2 + b^2)/b^2) - b)/b) + 4((a^6 + a^4 b^2) f \cosh(dx + c)^6 + 6(a^6 + a^4 \\
& b^2) f \cosh(dx + c) \sinh(dx + c)^5 + (a^6 + a^4 b^2) f \sinh(dx + c)^6 - \\
& (a^6 + a^4 b^2) f \cosh(dx + c)^4 + (15(a^6 + a^4 b^2) f \cosh(dx + c)^2 \\
& - (a^6 + a^4 b^2) f) \sinh(dx + c)^4 - (a^6 + a^4 b^2) f \cosh(dx + c)^2 + \\
& 4(5(a^6 + a^4 b^2) f \cosh(dx + c)^3 - (a^6 + a^4 b^2) f \cosh(dx + c)) \sinh \\
& (dx + c)^3 + (15(a^6 + a^4 b^2) f \cosh(dx + c)^4 - 6(a^6 + a^4 b^2) f \\
& \cosh(dx + c)^2 - (a^6 + a^4 b^2) f) \sinh(dx + c)^2 + (a^6 + a^4 b^2) f \\
& + 2(3(a^6 + a^4 b^2) f \cosh(dx + c)^5 - 2(a^6 + a^4 b^2) f \cosh(dx + c) \\
& )^3 - (a^6 + a^4 b^2) f \cosh(dx + c)) \sinh(dx + c)) \arctan(\cosh(dx + c) \\
& + \sinh(dx + c)) - 2(((3a^6 + 4a^4 b^2 + a^2 b^4) d f x + (3a^6 + 4a^4 b^2 \\
& + 4a^4 b^2 - a^2 b^4) d e - (a^6 + 2a^4 b^2 + a^2 b^4) f) \cosh(dx + c) - ((3a^6 \\
& + 4a^4 b^2 - a^2 b^4 - 2b^6) f \cosh(dx + c)^6 + 6(3a^6 + 4a^4 b^2 - \\
& a^2 b^4 - 2b^6) f \cosh(dx + c) \sinh(dx + c)^5 + (3a^6 + 4a^4 b^2 - a^2 \\
& b^4 - 2b^6) f \sinh(dx + c)^6 - (3a^6 + 4a^4 b^2 - a^2 b^4 - 2b^6) f \c \\
& osh(dx + c)^4 + (15(3a^6 + 4a^4 b^2 - a^2 b^4 - 2b^6) f \cosh(dx + c)^2 \\
& - (3a^6 + 4a^4 b^2 - a^2 b^4 - 2b^6) f) \sinh(dx + c)^4 - (3a^6 + 4a \\
& ^4 b^2 - a^2 b^4 - 2b^6) f \cosh(dx + c)^2 + 4(5(3a^6 + 4a^4 b^2 - a^2 \\
& b^4 - 2b^6) f \cosh(dx + c)^3 - (3a^6 + 4a^4 b^2 - a^2 b^4 - 2b^6) f * c
\end{aligned}$$



$$\begin{aligned}
& *e - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*f)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*f)*\cosh(d*x + c)^2 + (15*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*f)*\cosh(d*x + c)^4 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 6*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*f)*\cosh(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*f)*\sinh(d*x + c)^2 - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*f + 2*(3*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*f)*\cosh(d*x + c)^5 - 2*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*f)*\cosh(d*x + c)^3 - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 2*(a^5*b + 2*a^3*b^3 + a*b^5)*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) - (((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*\cosh(d*x + c)^6 + 6*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*\cosh(d*x + c)*\sinh(d*x + c)^5 + ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*\sinh(d*x + c)^6 - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*\cosh(d*x + c)^4 - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*\cosh(d*x + c)^2 + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*\sinh(d*x + c)^4 + 4*(5*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*\cosh(d*x + c)^3 - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*\cosh(d*x + c)^2 + (15*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*\cosh(d*x + c)^4 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e - 6*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*\cosh(d*x + c)^2 - (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*\sinh(d*x + c)^2 + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f + 2*(3*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*\cosh(d*x + c)^5 - 2*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c)*f)*\cosh(d*x + c)^3 - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2
\end{aligned}$$

$$\begin{aligned}
& *b^6)*d*e + (2*a^5*b + 4*a^3*b^3 + 2*a*b^5 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - \\
& 2*b^6)*c)*f)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + \\
& c) - 1) - (((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - \\
& 2*b^6)*c*f)*\cosh(d*x + c))^6 + 6*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - \\
& 2*b^6)*c*f)*\cosh(d*x + c))*\sinh(d*x + c))^5 + ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - \\
& 2*b^6)*c*f)*\sinh(d*x + c))^6 - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f)*\cosh(d*x \\
& + c))^4 - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f) - 15*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + \\
& (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f)*\cosh(d*x + c))^2*\sinh(d*x + c))^4 \\
& + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + 4*(5*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f)*\cosh(d \\
& *x + c))^3 - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f)*\sinh(d*x + c))^3 + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + \\
& (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f)*\cosh(d*x + c))^2 + (15*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f)*\cosh(d*x + c))^4 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f - 6*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f)*\cosh(d*x + c))^2*\sinh(d*x + c))^2 + 2*(3*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f)*\cosh(d*x + c))^5 - 2*((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f)*\cosh(d*x + c))^3 - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c*f)*\sinh(d*x + c))*\log(-\cosh(d*x + c) - \sinh(d*x + c) + 1) + 2*(12*((2*a^5*b + 3*a^3*b^3 + a*b^5)*d*f*x + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*\cosh(d*x + c))^5 - 5*((3*a^6 + 4*a^4*b^2 - a^2*b^4)*d*f*x + (3*a^6 + 4*a^4*b^2 - a^2*b^4)*d*e + (a^6 + 2*a^4*b^2 + a^2*b^4)*f)*\cosh(d*x + c))^4 - (3*a^6 + 4*a^4*b^2 - a^2*b^4)*d*f*x - 8*(2*(a^5*b + a^3*b^3)*d*f*x - (a^3*b^3 + a*b^5)*d*e + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*\cosh(d*x + c))^3 - (3*a^6 + 4*a^4*b^2 - a^2*b^4)*d*e + 6*((a^6 - a^2*b^4)*d*f*x + (a^6 - a^2*b^4)*d*e)*\cosh(d*x + c))^2 + (a^6 + 2*a^4*b^2 + a^2*b^4)*f - 4*((a^3*b^3 + a*b^5)*d*f*x - 2*(a^5*b + a^3*b^3)*d*e + (a^5*b + 2*a^3*b^3 + a*b^5)*c*f)*\cosh(d*x + c))*\sinh(d*x + c))/((a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*\cosh(d*x + c))^6 + 6*(a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*\cosh(d*x + c))*\sinh(d*x + c))^5 + (a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*\sinh(d*x + c))^6 - (a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*\cosh(d*x + c))^4 - (a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*\cosh(d*x + c))^2 + (15*(a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*\cosh(d*x + c))^2 - (a^7 + 2*a^5*b^2 + a^3*b^4)*d^2)*\sinh(d*x + c))^4 + 4*(5*(a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*\cosh(d*x + c))^3 - (a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*\cosh(d*x + c))*\sinh(d*x + c))^3 + (a^7 + 2*a^5*b^2 + a^3*b^4)*d^2 + (15*(a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*\cosh(d*x + c))^4 - 6*(a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*\cosh(d*x + c))^2 - (a^7 + 2*a^5*b^2 + a^3*b^4)*d^2)*\sinh(d*x + c))^2 + 2*(3*(a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*\cosh(d*x + c))^5 - 2*(a^7 + 2*a^5*b^2 + a^3*b^4)*d^2*\cosh(d*x
\end{aligned}$$

$$+ c)^3 - (a^7 + 2a^5b^2 + a^3b^4)d^2 \cosh(dx + c) \sinh(dx + c)$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)\*\*3\*sech(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)^3\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out



$$3.498 \quad \int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=206

$$\frac{2b^5 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3 d (a^2+b^2)^{3/2}} + \frac{b^2 \operatorname{sech}(c+dx)}{a^3 d} - \frac{b^2 \tanh^{-1}(\cosh(c+dx))}{a^3 d} - \frac{b^3 \operatorname{sech}(c+dx)(a \sinh(c+dx)+b)}{a^3 d (a^2+b^2)} + \frac{b^4 \operatorname{sech}(c+dx)}{a^3 d (a^2+b^2)}$$

[Out] (3\*ArcTanh[Cosh[c + d\*x]])/(2\*a\*d) - (b^2\*ArcTanh[Cosh[c + d\*x]])/(a^3\*d) + (2\*b^5\*ArcTanh[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[a^2 + b^2]])/(a^3\*(a^2 + b^2)^(3/2)\*d) + (b\*Coth[c + d\*x])/(a^2\*d) - (3\*Sech[c + d\*x])/(2\*a\*d) + (b^2\*Sech[c + d\*x])/(a^3\*d) - (Csch[c + d\*x]^2\*Sech[c + d\*x])/(2\*a\*d) - (b^3\*Sech[c + d\*x]\*(b + a\*Sinh[c + d\*x]))/(a^3\*(a^2 + b^2)\*d) + (b\*Tanh[c + d\*x])/(a^2\*d)

**Rubi [A]** time = 0.420027, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$ , Rules used = {2898, 2622, 321, 207, 2620, 14, 288, 2696, 12, 2660, 618, 204}

$$\frac{2b^5 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3 d (a^2+b^2)^{3/2}} + \frac{b^2 \operatorname{sech}(c+dx)}{a^3 d} - \frac{b^2 \tanh^{-1}(\cosh(c+dx))}{a^3 d} - \frac{b^3 \operatorname{sech}(c+dx)(a \sinh(c+dx)+b)}{a^3 d (a^2+b^2)} + \frac{b^4 \operatorname{sech}(c+dx)}{a^3 d (a^2+b^2)}$$

Antiderivative was successfully verified.

[In] Int[(Csch[c + d\*x]^3\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]), x]

[Out] (3\*ArcTanh[Cosh[c + d\*x]])/(2\*a\*d) - (b^2\*ArcTanh[Cosh[c + d\*x]])/(a^3\*d) + (2\*b^5\*ArcTanh[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[a^2 + b^2]])/(a^3\*(a^2 + b^2)^(3/2)\*d) + (b\*Coth[c + d\*x])/(a^2\*d) - (3\*Sech[c + d\*x])/(2\*a\*d) + (b^2\*Sech[c + d\*x])/(a^3\*d) - (Csch[c + d\*x]^2\*Sech[c + d\*x])/(2\*a\*d) - (b^3\*Sech[c + d\*x]\*(b + a\*Sinh[c + d\*x]))/(a^3\*(a^2 + b^2)\*d) + (b\*Tanh[c + d\*x])/(a^2\*d)

**Rule 2898**

Int[((cos[(e\_.) + (f\_.)\*(x\_.)]\*(g\_.))^p)\*sin[(e\_.) + (f\_.)\*(x\_.)]^(n\_)]/((a\_.) + (b\_.)\*sin[(e\_.) + (f\_.)\*(x\_.)]), x\_Symbol] :> Int[ExpandTrig[(g\*cos[e + f\*x])^p, sin[e + f\*x]^n/(a + b\*sin[e + f\*x]), x], x] /; FreeQ[{a, b, e, f,

$g, p\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{IntegerQ}[n] \&\& (\text{LtQ}[n, 0] \parallel \text{IGtQ}[p + 1/2, 0])$

### Rule 2622

$\text{Int}[\text{csc}[(e_.) + (f_.)(x_.)]^{(n_.)} * ((a_.) * \text{sec}[(e_.) + (f_.)(x_.)])^{(m_.)}, x\_Symbol] \rightarrow \text{Dist}[1/(f*a^n), \text{Subst}[\text{Int}[x^{(m+n-1)}/(-1+x^2/a^2)^{((n+1)/2)}, x], x, a*\text{Sec}[e+f*x]], x] /; \text{FreeQ}[\{a, e, f, m\}, x] \&\& \text{IntegerQ}[(n+1)/2] \&\& !(\text{IntegerQ}[(m+1)/2] \&\& \text{LtQ}[0, m, n])$

### Rule 321

$\text{Int}[((c_.)(x_.))^{(m_.)} * ((a_.) + (b_.)(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(c^{(n-1)} * (c*x)^{(m-n+1)} * (a + b*x^n)^{(p+1)}) / (b*(m+n*p+1)), x] - \text{Dist}[(a*c^{(n-m)} * (m-n+1)) / (b*(m+n*p+1)), \text{Int}[(c*x)^{(m-n)} * (a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[m, n-1] \&\& \text{NeQ}[m+n*p+1, 0] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

### Rule 207

$\text{Int}[((a_.) + (b_.)(x_.)^2)^{(-1)}, x\_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Rt}[b, 2]*x]/\text{Rt}[-a, 2]] / (\text{Rt}[-a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{LtQ}[a, 0] \parallel \text{GtQ}[b, 0])$

### Rule 2620

$\text{Int}[\text{csc}[(e_.) + (f_.)(x_.)]^{(m_.)} * \text{sec}[(e_.) + (f_.)(x_.)]^{(n_.)}, x\_Symbol] \rightarrow \text{Dist}[1/f, \text{Subst}[\text{Int}[(1+x^2)^{((m+n)/2-1)}/x^m, x], x, \text{Tan}[e+f*x]], x] /; \text{FreeQ}[\{e, f\}, x] \&\& \text{IntegersQ}[m, n, (m+n)/2]$

### Rule 14

$\text{Int}[(u_.) * ((c_.)(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m * u, x], x] /; \text{FreeQ}[\{c, m\}, x] \&\& \text{SumQ}[u] \&\& !\text{LinearQ}[u, x] \&\& !\text{MatchQ}[u, (a_.) + (b_.)(v_.)] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{InverseFunctionQ}[v]$

### Rule 288

$\text{Int}[((c_.)(x_.))^{(m_.)} * ((a_.) + (b_.)(x_.)^{(n_.)})^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(c^{(n-1)} * (c*x)^{(m-n+1)} * (a + b*x^n)^{(p+1)}) / (b*n*(p+1)), x] - \text{Dist}[(c^{(n-m)} * (m-n+1)) / (b*n*(p+1)), \text{Int}[(c*x)^{(m-n)} * (a + b*x^n)^{(p+1)}, x], x] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& \text{GtQ}[m+1, n] \&\& !\text{LtQ}[(m+n*(p+1)+1)/n, 0] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_), x_Symbol] :> Simp[((g*cos[e + f*x])^(p + 1)*(a + b*sin[e + f*x])^(m + 1)*(b - a*sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p + 1)), Int[(g*cos[e + f*x])^(p + 2)*(a + b*sin[e + f*x])^m*(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[2*m, 2*p]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{a+b\sinh(c+dx)} dx &= -\left( i \int \left( \frac{ib^2\operatorname{csch}(c+dx)\operatorname{sech}^2(c+dx)}{a^3} - \frac{ib\operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx)}{a^2} + \frac{i\operatorname{csch}^3(c+dx)}{a} \right) dx \right. \\
&= \frac{\int \operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx) dx}{a} - \frac{b \int \operatorname{csch}^2(c+dx)\operatorname{sech}^2(c+dx) dx}{a^2} + \frac{b^2 \int \operatorname{csch}(c+dx) dx}{a^3} \\
&= -\frac{b^3\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a^3(a^2+b^2)d} - \frac{b^3 \int \frac{b^2}{a+b\sinh(c+dx)} dx}{a^3(a^2+b^2)} - \frac{\operatorname{Subst}\left(\int \frac{x^4}{(-1+x^2)^2} dx\right)}{ad} \\
&= \frac{b^2\operatorname{sech}(c+dx)}{a^3d} - \frac{\operatorname{csch}^2(c+dx)\operatorname{sech}(c+dx)}{2ad} - \frac{b^3\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a^3(a^2+b^2)d} \\
&= -\frac{b^2 \tanh^{-1}(\cosh(c+dx))}{a^3d} + \frac{b \coth(c+dx)}{a^2d} - \frac{3\operatorname{sech}(c+dx)}{2ad} + \frac{b^2\operatorname{sech}(c+dx)}{a^3d} - \frac{b^3\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a^3(a^2+b^2)d} \\
&= \frac{3 \tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{b^2 \tanh^{-1}(\cosh(c+dx))}{a^3d} + \frac{b \coth(c+dx)}{a^2d} - \frac{3\operatorname{sech}(c+dx)}{2ad} - \frac{b^3\operatorname{sech}(c+dx)(b+a\sinh(c+dx))}{a^3(a^2+b^2)d} \\
&= \frac{3 \tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{b^2 \tanh^{-1}(\cosh(c+dx))}{a^3d} + \frac{2b^5 \tanh^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)^{3/2}d}
\end{aligned}$$

**Mathematica [A]** time = 2.47148, size = 185, normalized size = 0.9

$$\frac{-\frac{4(3a^2-2b^2)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{a^3} + \frac{16b^5 \tan^{-1}\left(\frac{b-a \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2-b^2}}\right)}{a^3(-a^2-b^2)^{3/2}} + \frac{8\operatorname{sech}(c+dx)(b\sinh(c+dx)-a)}{a^2+b^2} + \frac{4b \tanh\left(\frac{1}{2}(c+dx)\right)}{a^2} + \frac{4b \coth\left(\frac{1}{2}(c+dx)\right)}{a^2} - \frac{\operatorname{csch}^3(c+dx)}{a}}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[(Csch[c + d\*x]^3\*Sech[c + d\*x]^2)/(a + b\*Sinh[c + d\*x]),x]

[Out] ((16\*b^5\*ArcTan[(b - a\*Tanh[(c + d\*x)/2])/Sqrt[-a^2 - b^2]])/Sqrt[-a^2 - b^2])^(3/2) + (4\*b\*Coth[(c + d\*x)/2])/a^2 - Csch[(c + d\*x)/2]^2/a - (4\*(3\*a^2 - 2\*b^2)\*Log[Tanh[(c + d\*x)/2]])/a^3 - Sech[(c + d\*x)/2]^2/a + (8\*Sech[c + d\*x]\*(-a + b\*Sinh[c + d\*x]))/(a^2 + b^2) + (4\*b\*Tanh[(c + d\*x)/2])/a^2)/(8\*d)

---

**Maple [A]** time = 0.002, size = 233, normalized size = 1.1

$$\frac{1}{8da} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2 + \frac{b}{2da^2} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{8da} \left( \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^{-2} - \frac{3}{2da} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \frac{b^2}{da^3} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d\*x+c)^3\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x)

[Out] 1/8/d/a\*tanh(1/2\*d\*x+1/2\*c)^2+1/2/d/a^2\*tanh(1/2\*d\*x+1/2\*c)\*b-1/8/d/a/tanh(1/2\*d\*x+1/2\*c)^2-3/2/d/a\*ln(tanh(1/2\*d\*x+1/2\*c))+1/d/a^3\*ln(tanh(1/2\*d\*x+1/2\*c))\*b^2+1/2/d\*b/a^2/tanh(1/2\*d\*x+1/2\*c)-2/d/a^3\*b^5/(a^2+b^2)^(3/2)\*arctanh(1/2\*(2\*a\*tanh(1/2\*d\*x+1/2\*c)-2\*b)/(a^2+b^2)^(1/2))+2/d/(a^2+b^2)/(tanh(1/2\*d\*x+1/2\*c)^2+1)\*tanh(1/2\*d\*x+1/2\*c)\*b-2/d/(a^2+b^2)/(tanh(1/2\*d\*x+1/2\*c)^2+1)\*a

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^3\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [B]** time = 4.7177, size = 5921, normalized size = 28.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^3\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

```
[Out] -1/2*(8*a^5*b + 12*a^3*b^3 + 4*a*b^5 + 2*(3*a^6 + 4*a^4*b^2 + a^2*b^4))*cosh
(dx + c)^5 + 2*(3*a^6 + 4*a^4*b^2 + a^2*b^4)*sinh(dx + c)^5 - 4*(a^3*b^3
+ a*b^5)*cosh(dx + c)^4 - 2*(2*a^3*b^3 + 2*a*b^5 - 5*(3*a^6 + 4*a^4*b^2 +
a^2*b^4))*cosh(dx + c)*sinh(dx + c)^4 - 4*(a^6 - a^2*b^4)*cosh(dx + c)^3
- 4*(a^6 - a^2*b^4 - 5*(3*a^6 + 4*a^4*b^2 + a^2*b^4))*cosh(dx + c)^2 + 4*(
a^3*b^3 + a*b^5)*cosh(dx + c)*sinh(dx + c)^3 - 8*(a^5*b + a^3*b^3)*cosh(
dx + c)^2 - 4*(2*a^5*b + 2*a^3*b^3 - 5*(3*a^6 + 4*a^4*b^2 + a^2*b^4))*cosh(
dx + c)^3 + 6*(a^3*b^3 + a*b^5)*cosh(dx + c)^2 + 3*(a^6 - a^2*b^4)*cosh(d
*x + c)*sinh(dx + c)^2 - 2*(b^5*cosh(dx + c)^6 + 6*b^5*cosh(dx + c)*sin
h(dx + c)^5 + b^5*sinh(dx + c)^6 - b^5*cosh(dx + c)^4 - b^5*cosh(dx + c
)^2 + b^5 + (15*b^5*cosh(dx + c)^2 - b^5)*sinh(dx + c)^4 + 4*(5*b^5*cosh(
dx + c)^3 - b^5*cosh(dx + c))*sinh(dx + c)^3 + (15*b^5*cosh(dx + c)^4 -
6*b^5*cosh(dx + c)^2 - b^5)*sinh(dx + c)^2 + 2*(3*b^5*cosh(dx + c)^5 -
2*b^5*cosh(dx + c)^3 - b^5*cosh(dx + c))*sinh(dx + c))*sqrt(a^2 + b^2)*l
og((b^2*cosh(dx + c)^2 + b^2*sinh(dx + c)^2 + 2*a*b*cosh(dx + c) + 2*a^2
+ b^2 + 2*(b^2*cosh(dx + c) + a*b)*sinh(dx + c) + 2*sqrt(a^2 + b^2)*(b*c
osh(dx + c) + b*sinh(dx + c) + a))/(b*cosh(dx + c)^2 + b*sinh(dx + c)^2
+ 2*a*cosh(dx + c) + 2*(b*cosh(dx + c) + a)*sinh(dx + c) - b)) + 2*(3*a
^6 + 4*a^4*b^2 + a^2*b^4)*cosh(dx + c) - ((3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2
*b^6)*cosh(dx + c)^6 + 6*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(dx +
c)*sinh(dx + c)^5 + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*sinh(dx + c)^6
+ 3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b
^6)*cosh(dx + c)^4 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6 - 15*(3*a^6 + 4*a
^4*b^2 - a^2*b^4 - 2*b^6)*cosh(dx + c)^2)*sinh(dx + c)^4 + 4*(5*(3*a^6 +
4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(dx + c)^3 - (3*a^6 + 4*a^4*b^2 - a^2*b^4
- 2*b^6)*cosh(dx + c))*sinh(dx + c)^3 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2
*b^6)*cosh(dx + c)^2 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6 - 15*(3*a^6 +
4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(dx + c)^4 + 6*(3*a^6 + 4*a^4*b^2 - a^2*b
^4 - 2*b^6)*cosh(dx + c)^2)*sinh(dx + c)^2 + 2*(3*(3*a^6 + 4*a^4*b^2 - a
^2*b^4 - 2*b^6)*cosh(dx + c)^5 - 2*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*c
osh(dx + c)^3 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(dx + c))*sinh(d
*x + c))*log(cosh(dx + c) + sinh(dx + c) + 1) + ((3*a^6 + 4*a^4*b^2 - a^2
*b^4 - 2*b^6)*cosh(dx + c)^6 + 6*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cos
h(dx + c)*sinh(dx + c)^5 + (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*sinh(dx
+ c)^6 + 3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6 - (3*a^6 + 4*a^4*b^2 - a^2*b
^4 - 2*b^6)*cosh(dx + c)^4 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6 - 15*(3*a
^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(dx + c)^2)*sinh(dx + c)^4 + 4*(5*(
3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(dx + c)^3 - (3*a^6 + 4*a^4*b^2 -
a^2*b^4 - 2*b^6)*cosh(dx + c))*sinh(dx + c)^3 - (3*a^6 + 4*a^4*b^2 - a^2
*b^4 - 2*b^6)*cosh(dx + c)^2 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6 - 15*(
3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(dx + c)^4 + 6*(3*a^6 + 4*a^4*b^2
- a^2*b^4 - 2*b^6)*cosh(dx + c)^2)*sinh(dx + c)^2 + 2*(3*(3*a^6 + 4*a^4*
b^2 - a^2*b^4 - 2*b^6)*cosh(dx + c)^5 - 2*(3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2
*b^6)*cosh(dx + c)^3 - (3*a^6 + 4*a^4*b^2 - a^2*b^4 - 2*b^6)*cosh(dx + c)
)*sinh(dx + c))*log(cosh(dx + c) + sinh(dx + c) - 1) + 2*(3*a^6 + 4*a^4*
```

$$b^2 + a^2 b^4 + 5(3a^6 + 4a^4 b^2 + a^2 b^4) \cosh(dx + c)^4 - 8(a^3 b^3 + a b^5) \cosh(dx + c)^3 - 6(a^6 - a^2 b^4) \cosh(dx + c)^2 - 8(a^5 b + a^3 b^3) \cosh(dx + c) \sinh(dx + c) / ((a^7 + 2a^5 b^2 + a^3 b^4) d \cosh(dx + c)^6 + 6(a^7 + 2a^5 b^2 + a^3 b^4) d \cosh(dx + c) \sinh(dx + c)^5 + (a^7 + 2a^5 b^2 + a^3 b^4) d \sinh(dx + c)^6 - (a^7 + 2a^5 b^2 + a^3 b^4) d \cosh(dx + c)^4 + (15(a^7 + 2a^5 b^2 + a^3 b^4) d \cosh(dx + c)^2 - (a^7 + 2a^5 b^2 + a^3 b^4) d) \sinh(dx + c)^4 - (a^7 + 2a^5 b^2 + a^3 b^4) d \cosh(dx + c)^2 + 4(5(a^7 + 2a^5 b^2 + a^3 b^4) d \cosh(dx + c)^3 - (a^7 + 2a^5 b^2 + a^3 b^4) d \cosh(dx + c)) \sinh(dx + c)^3 + (15(a^7 + 2a^5 b^2 + a^3 b^4) d \cosh(dx + c)^4 - 6(a^7 + 2a^5 b^2 + a^3 b^4) d \cosh(dx + c)^2 - (a^7 + 2a^5 b^2 + a^3 b^4) d) \sinh(dx + c)^2 + (a^7 + 2a^5 b^2 + a^3 b^4) d + 2(3(a^7 + 2a^5 b^2 + a^3 b^4) d \cosh(dx + c)^5 - 2(a^7 + 2a^5 b^2 + a^3 b^4) d \cosh(dx + c)^3 - (a^7 + 2a^5 b^2 + a^3 b^4) d \cosh(dx + c)) \sinh(dx + c))$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)\*\*3\*sech(d\*x+c)\*\*2/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [A]** time = 1.77048, size = 317, normalized size = 1.54

$$\frac{b^5 \log\left(\frac{-2be^{(dx+c)} - 2a - 2\sqrt{a^2+b^2}}{-2be^{(dx+c)} - 2a + 2\sqrt{a^2+b^2}}\right)}{(a^5d + a^3b^2d)\sqrt{a^2+b^2}} - \frac{2(ae^{(dx+c)} + b)}{(a^2d + b^2d)(e^{(2dx+2c)} + 1)} + \frac{(3a^2 - 2b^2) \log(e^{(dx+c)} + 1)}{2a^3d} - \frac{(3a^2 - 2b^2) \log(|e^{(dx+c)} - 1|)}{2a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^3\*sech(d\*x+c)^2/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] b^5\*log(abs(-2\*b\*e^(d\*x + c) - 2\*a - 2\*sqrt(a^2 + b^2))/abs(-2\*b\*e^(d\*x + c) - 2\*a + 2\*sqrt(a^2 + b^2)))/((a^5\*d + a^3\*b^2\*d)\*sqrt(a^2 + b^2)) - 2\*(a\*e^(d\*x + c) + b)/((a^2\*d + b^2\*d)\*(e^(2\*d\*x + 2\*c) + 1)) + 1/2\*(3\*a^2 - 2\*b

$$\begin{aligned} &^2) \cdot \log(e^{(d \cdot x + c)} + 1) / (a^{3 \cdot d}) - 1/2 \cdot (3 \cdot a^2 - 2 \cdot b^2) \cdot \log(\text{abs}(e^{(d \cdot x + c)} \\ &- 1)) / (a^{3 \cdot d}) - (a \cdot e^{(3 \cdot d \cdot x + 3 \cdot c)} - 2 \cdot b \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + a \cdot e^{(d \cdot x + c)} + \\ &2 \cdot b) / (a^{2 \cdot d} \cdot (e^{(2 \cdot d \cdot x + 2 \cdot c)} - 1)^2) \end{aligned}$$



$$3.499 \quad \int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Optimal.** Leaf size=38

$$\operatorname{Unintegrable}\left(\frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))}, x\right)$$

[Out] Unintegrable[(Csch[c + d\*x]^3\*Sech[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.134132, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Csch[c + d\*x]^3\*Sech[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] Defer[Int] [(Csch[c + d\*x]^3\*Sech[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^2(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.001, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Csch[c + d\*x]^3\*Sech[c + d\*x]^2)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] \$Aborted

**Maple [A]** time = 3.536, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx+c))^3 (\operatorname{sech}(dx+c))^2}{(fx+e)(a+b \sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d\*x+c)^3\*sech(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

[Out] int(csch(d\*x+c)^3\*sech(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^3\*sech(d\*x+c)^2/(f\*x+e)/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -32*b^5*\integrate(-1/16*e^{(d*x+c)}/(a^5*b*e+a^3*b^3*e+(a^5*b*f+a^3*b^3*f)*x - (a^5*b*e*e^{(2*c)}+a^3*b^3*e*e^{(2*c)}+(a^5*b*f*e^{(2*c)}+a^3*b^3*f*e^{(2*c)})*x)*e^{(2*d*x)} - 2*(a^6*e*e^c+a^4*b^2*e*e^c+(a^6*f*e^c+a^4*b^2*f*e^c)*x)*e^{(d*x)}, x) - (4*a^2*b*d*e+2*b^3*d*e+2*(2*a^2*b*d*f+b^3*d*f)*x + ((3*d*e-f)*a^3*e^{(5*c)}+(d*e-f)*a*b^2*e^{(5*c)}+(3*a^3*d*f*e^{(5*c)}+a*b^2*d*f*e^{(5*c)})*x)*e^{(5*d*x)} - 2*(b^3*d*f*x*e^{(4*c)}+b^3*d*e*e^{(4*c)})*e^{(4*d*x)} - 2*(a^3*d*e*e^{(3*c)}-a*b^2*d*e*e^{(3*c)}+(a^3*d*f*e^{(3*c)}-a*b^2*d*f*e^{(3*c)})*x)*e^{(3*d*x)} - 4*(a^2*b*d*f*x*e^{(2*c)}+a^2*b*d*e*e^{(2*c)})*e^{(2*d*x)} + ((3*d*e+f)*a^3*e^c+(d*e+f)*a*b^2*e^c+(3*a^3*d*f*e^c+a*b^2*d*f*e^c)*x)*e^{(d*x)})/(a^4*d^2*e^2+a^2*b^2*d^2*e^2+(a^4*d^2*f^2+a^2*b^2*d^2*f^2)*x^2+2*(a^4*d^2*e*f+a^2*b^2*d^2*e*f)*x+(a^4*d^2*e^2*e^{(6*c)}+a^2*b^2*d^2*e^2*e^{(6*c)}+(a^4*d^2*f^2*e^{(6*c)}+a^2*b^2*d^2*f^2*e^{(6*c)})*x^2+2*(a^4*d^2*e*f*e^{(6*c)}+a^2*b^2*d^2*e*f*e^{(6*c)})*x)*e^{(6*d*x)} - (a^4*d^2*e^2*e^{(4*c)}+a^2*b^2*d^2*e^2*e^{(4*c)}+(a^4*d^2*f^2*e^{(4*c)}+a^2*b^2*d^2*f^2*e^{(4*c)})*x^2+2*(a^4*d^2*e*f*e^{(4*c)}+a^2*b^2*d^2*e*f*e^{(4*c)})*x)*e^{(4*d*x)} - (a^4*d^2*e^2*e^{(2*c)}+a^2*b^2*d^2*e^2*e^{(2*c)}+(a^4*d^2*f^2*e^{(2*c)}+a^2*b^2*d^2*f^2*e^{(2*c)})*x^2+2*(a^4*d^2*e*f*e^{(2*c)}+a^2*b^2*d^2*e*f*e^{(2*c)})*x)*e^{(2*d*x)} \end{aligned}$$

```
(2*c) + a^2*b^2*d^2*e*f*e^(2*c))*x)*e^(2*d*x)) - 32*integrate(1/64*(2*b^2*d^2*e^2 + 2*a*b*d*e*f - (3*d^2*e^2 - 2*f^2)*a^2 - (3*a^2*d^2*f^2 - 2*b^2*d^2*f^2)*x^2 - 2*(3*a^2*d^2*e*f - 2*b^2*d^2*e*f - a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) - 32*integrate(-1/64*(2*b^2*d^2*e^2 - 2*a*b*d*e*f - (3*d^2*e^2 - 2*f^2)*a^2 - (3*a^2*d^2*f^2 - 2*b^2*d^2*f^2)*x^2 - 2*(3*a^2*d^2*e*f - 2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^(d*x)), x) - 32*integrate(1/16*(a*f*e^(d*x + c) + b*f)/(a^2*d*e^2 + b^2*d*e^2 + (a^2*d*f^2 + b^2*d*f^2)*x^2 + 2*(a^2*d*e*f + b^2*d*e*f)*x + (a^2*d*e^2*e^(2*c) + b^2*d*e^2*e^(2*c) + (a^2*d*f^2*e^(2*c) + b^2*d*f^2*e^(2*c))*x^2 + 2*(a^2*d*e*f*e^(2*c) + b^2*d*e*f*e^(2*c))*x)*e^(2*d*x)), x)
```

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\text{csch}(dx+c)^3 \text{sech}(dx+c)^2}{afx+ae+(bfx+be)\sinh(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(csch(d*x + c)^3*sech(d*x + c)^2/(a*f*x + a*e + (b*f*x + b*e)*sinh(d*x + c)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*sech(d*x+c)**2/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^2/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.500 \quad \int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=1122

result too large to display

```
[Out] (b^2*f*x)/(2*a^3*d) + (3*b*f*x*ArcTan[E^(c + d*x)])/(a^2*d) - (2*b^5*(e + f
*x)*ArcTan[E^(c + d*x)])/(a^2*(a^2 + b^2)^2*d) - (b^3*(e + f*x)*ArcTan[E^(c
+ d*x)])/(a^2*(a^2 + b^2)*d) - (3*b*f*x*ArcTan[Sinh[c + d*x]])/(2*a^2*d) +
(3*b*(e + f*x)*ArcTan[Sinh[c + d*x]])/(2*a^2*d) - (2*b^2*f*x*ArcTanh[E^(2*
c + 2*d*x)])/(a^3*d) + (4*(e + f*x)*ArcTanh[E^(2*c + 2*d*x)])/(a*d) + (b*f*
ArcTanh[Cosh[c + d*x]])/(a^2*d^2) + (3*b*(e + f*x)*Csch[c + d*x])/(2*a^2*d)
- (f*Csch[2*c + 2*d*x])/(a*d^2) - (2*(e + f*x)*Coth[2*c + 2*d*x]*Csch[2*c
+ 2*d*x])/(a*d) - (b^6*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 + b^
2])])/(a^3*(a^2 + b^2)^2*d) - (b^6*(e + f*x)*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 + b^2])])/(a^3*(a^2 + b^2)^2*d) + (b^6*(e + f*x)*Log[1 + E^(2*(c +
d*x))])/(a^3*(a^2 + b^2)^2*d) - (b^2*f*x*Log[Tanh[c + d*x]])/(a^3*d) + (b^2
*(e + f*x)*Log[Tanh[c + d*x]])/(a^3*d) - (((3*I)/2)*b*f*PolyLog[2, (-I)*E^(
c + d*x)])/(a^2*d^2) + (I*b^5*f*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*(a^2 + b
^2)^2*d^2) + ((I/2)*b^3*f*PolyLog[2, (-I)*E^(c + d*x)])/(a^2*(a^2 + b^2)*d
^2) + (((3*I)/2)*b*f*PolyLog[2, I*E^(c + d*x)])/(a^2*d^2) - (I*b^5*f*PolyLog
[2, I*E^(c + d*x)])/(a^2*(a^2 + b^2)^2*d^2) - ((I/2)*b^3*f*PolyLog[2, I*E^(
c + d*x)])/(a^2*(a^2 + b^2)*d^2) - (b^6*f*PolyLog[2, -(b*E^(c + d*x))/(a -
Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)^2*d^2) - (b^6*f*PolyLog[2, -(b*E^(c
+ d*x))/(a + Sqrt[a^2 + b^2])])/(a^3*(a^2 + b^2)^2*d^2) + (b^6*f*PolyLog[2
, -E^(2*(c + d*x))])/(2*a^3*(a^2 + b^2)^2*d^2) + (f*PolyLog[2, -E^(2*c + 2*
d*x)])/(a*d^2) - (b^2*f*PolyLog[2, -E^(2*c + 2*d*x)])/(2*a^3*d^2) - (f*Poly
Log[2, E^(2*c + 2*d*x)])/(a*d^2) + (b^2*f*PolyLog[2, E^(2*c + 2*d*x)])/(2*a
^3*d^2) + (b*f*Sech[c + d*x])/(2*a^2*d^2) - (b^3*f*Sech[c + d*x])/(2*a^2*(a
^2 + b^2)*d^2) - (b^4*(e + f*x)*Sech[c + d*x]^2)/(2*a^3*(a^2 + b^2)*d) - (b
*(e + f*x)*Csch[c + d*x]*Sech[c + d*x]^2)/(2*a^2*d) - (b^2*f*Tanh[c + d*x])
/(2*a^3*d^2) + (b^4*f*Tanh[c + d*x])/(2*a^3*(a^2 + b^2)*d^2) - (b^3*(e + f*
x)*Sech[c + d*x]*Tanh[c + d*x])/(2*a^2*(a^2 + b^2)*d) - (b^2*(e + f*x)*Tanh
[c + d*x]^2)/(2*a^3*d)
```

---

**Rubi [A]** time = 1.80698, antiderivative size = 1122, normalized size of antiderivative = 1., number of steps used = 65, number of rules used = 28, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.824$ , Rules used = {5589, 5461, 4185, 4182, 2279, 2391, 2621, 288, 321, 207, 5462, 5203,

12, 4180, 3770, 2622, 2620, 14, 2548, 3473, 8, 5573, 5561, 2190, 6742, 3718, 5451, 3767}

$$\frac{(e+fx)\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2+b^2}}+1\right)b^6}{a^3(a^2+b^2)^2d} - \frac{(e+fx)\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2+b^2}}+1\right)b^6}{a^3(a^2+b^2)^2d} + \frac{(e+fx)\log(1+e^{2(c+dx)})b^6}{a^3(a^2+b^2)^2d} - \frac{f\text{PolyLog}\left(2, -\frac{be^{c+dx}}{a-\sqrt{a^2+b^2}}\right)}{a^3(a^2+b^2)^2d}$$

Antiderivative was successfully verified.

[In] Int[((e + f\*x)\*Csch[c + d\*x]^3\*Sech[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]), x]

[Out] (b^2\*f\*x)/(2\*a^3\*d) + (3\*b\*f\*x\*ArcTan[E^(c + d\*x)])/(a^2\*d) - (2\*b^5\*(e + f\*x)\*ArcTan[E^(c + d\*x)])/(a^2\*(a^2 + b^2)^2\*d) - (b^3\*(e + f\*x)\*ArcTan[E^(c + d\*x)])/(a^2\*(a^2 + b^2)\*d) - (3\*b\*f\*x\*ArcTan[Sinh[c + d\*x]])/(2\*a^2\*d) + (3\*b\*(e + f\*x)\*ArcTan[Sinh[c + d\*x]])/(2\*a^2\*d) - (2\*b^2\*f\*x\*ArcTanh[E^(2\*c + 2\*d\*x)])/(a^3\*d) + (4\*(e + f\*x)\*ArcTanh[E^(2\*c + 2\*d\*x)])/(a\*d) + (b\*f\*ArcTanh[Cosh[c + d\*x]])/(a^2\*d^2) + (3\*b\*(e + f\*x)\*Csch[c + d\*x])/(2\*a^2\*d) - (f\*Csch[2\*c + 2\*d\*x])/(a\*d^2) - (2\*(e + f\*x)\*Coth[2\*c + 2\*d\*x]\*Csch[2\*c + 2\*d\*x])/(a\*d) - (b^6\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^3\*(a^2 + b^2)^2\*d) - (b^6\*(e + f\*x)\*Log[1 + (b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^3\*(a^2 + b^2)^2\*d) + (b^6\*(e + f\*x)\*Log[1 + E^(2\*(c + d\*x))])/(a^3\*(a^2 + b^2)^2\*d) - (b^2\*f\*x\*Log[Tanh[c + d\*x]])/(a^3\*d) + (b^2\*(e + f\*x)\*Log[Tanh[c + d\*x]])/(a^3\*d) - (((3\*I)/2)\*b\*f\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a^2\*d^2) + (I\*b^5\*f\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a^2\*(a^2 + b^2)^2\*d^2) + ((I/2)\*b^3\*f\*PolyLog[2, (-I)\*E^(c + d\*x)])/(a^2\*(a^2 + b^2)\*d^2) + (((3\*I)/2)\*b\*f\*PolyLog[2, I\*E^(c + d\*x)])/(a^2\*d^2) - (I\*b^5\*f\*PolyLog[2, I\*E^(c + d\*x)])/(a^2\*(a^2 + b^2)^2\*d^2) - ((I/2)\*b^3\*f\*PolyLog[2, I\*E^(c + d\*x)])/(a^2\*(a^2 + b^2)\*d^2) - (b^6\*f\*PolyLog[2, -(b\*E^(c + d\*x))/(a - Sqrt[a^2 + b^2])])/(a^3\*(a^2 + b^2)^2\*d^2) - (b^6\*f\*PolyLog[2, -(b\*E^(c + d\*x))/(a + Sqrt[a^2 + b^2])])/(a^3\*(a^2 + b^2)^2\*d^2) + (b^6\*f\*PolyLog[2, -E^(2\*(c + d\*x))])/(2\*a^3\*(a^2 + b^2)^2\*d^2) + (f\*PolyLog[2, -E^(2\*c + 2\*d\*x)])/(a\*d^2) - (b^2\*f\*PolyLog[2, -E^(2\*c + 2\*d\*x)])/(2\*a^3\*d^2) - (f\*PolyLog[2, E^(2\*c + 2\*d\*x)])/(a\*d^2) + (b^2\*f\*PolyLog[2, E^(2\*c + 2\*d\*x)])/(2\*a^3\*d^2) + (b\*f\*Sech[c + d\*x])/(2\*a^2\*d^2) - (b^3\*f\*Sech[c + d\*x])/(2\*a^2\*(a^2 + b^2)\*d^2) - (b^4\*(e + f\*x)\*Sech[c + d\*x]^2)/(2\*a^3\*(a^2 + b^2)\*d) - (b\*(e + f\*x)\*Csch[c + d\*x]\*Sech[c + d\*x]^2)/(2\*a^2\*d) - (b^2\*f\*Tanh[c + d\*x])/(2\*a^3\*d^2) + (b^4\*f\*Tanh[c + d\*x])/(2\*a^3\*(a^2 + b^2)\*d^2) - (b^3\*(e + f\*x)\*Sech[c + d\*x]\*Tanh[c + d\*x])/(2\*a^2\*(a^2 + b^2)\*d) - (b^2\*(e + f\*x)\*Tanh[c + d\*x]^2)/(2\*a^3\*d)

### Rule 5589

Int[(Csch[(c\_.) + (d\_.)\*(x\_)]^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(p\_.)]/((a\_.) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[1/a, Int[(e + f\*x)^m\*Sech[c + d\*x]^p\*Csch[c + d\*x]^n, x] - Dist[b/a, Int[((e + f\*x)^m\*Sech[c + d\*x]^p\*Csch[c + d\*x]^(n - 1))/(a + b\*Sinh[c + d

\*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]

### Rule 5461

Int[Csch[(a\_.) + (b\_.)\*(x\_)]^(n\_.)\*((c\_.) + (d\_.)\*(x\_))^(m\_.)\*Sech[(a\_.) + (b\_.)\*(x\_)]^(n\_.), x\_Symbol] := Dist[2^n, Int[(c + d\*x)^m\*Csch[2\*a + 2\*b\*x]^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]

### Rule 4185

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(b\_.))^(n\_.)\*((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := -Simp[(b^2\*(c + d\*x)\*Cot[e + f\*x]\*(b\*Csc[e + f\*x])^(n - 2))/(f\*(n - 1)), x] + (Dist[(b^2\*(n - 2))/(n - 1), Int[(c + d\*x)\*(b\*Csc[e + f\*x])^(n - 2), x], x] - Simp[(b^2\*d\*(b\*Csc[e + f\*x])^(n - 2))/(f^2\*(n - 1)\*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]

### Rule 4182

Int[csc[(e\_.) + (Complex[0, fz\_])\*(f\_.)\*(x\_)]\*(c\_.) + (d\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(-2\*(c + d\*x)^m\*ArcTanh[E^(-(I\*e) + f\*fz\*x)])/(f\*fz\*I), x] + (-Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 - E^(-(I\*e) + f\*fz\*x)], x], x] + Dist[(d\*m)/(f\*fz\*I), Int[(c + d\*x)^(m - 1)\*Log[1 + E^(-(I\*e) + f\*fz\*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

### Rule 2279

Int[Log[(a\_.) + (b\_.)\*((F\_)^(e\_.)\*((c\_.) + (d\_.)\*(x\_)))]^(n\_.), x\_Symbol] := Dist[1/(d\*e\*n\*Log[F]), Subst[Int[Log[a + b\*x]/x, x], x, (F^(e\*(c + d\*x)))^n, x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^n)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2621

Int[(csc[(e\_.) + (f\_.)\*(x\_)]\*(a\_.))^(m\_.)\*sec[(e\_.) + (f\_.)\*(x\_)]^(n\_.), x\_Symbol] := -Dist[(f\*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a\*Csc[e + f\*x], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

### Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

### Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

### Rule 5462

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(p_.), x_Symbol] := With[{u = IntHide[Csch[a + b*x]^n*Sech[a +
b*x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x
], x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n,
p]
```

### Rule 5203

```
Int[ArcTan[u_], x_Symbol] := Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)]]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] /; FreeQ[{c,
```



d, e, f, fz}, x] && IntegerQ[2\*k] && IGtQ[m, 0]

### Rule 3770

Int[csc[(c\_.) + (d\_.)\*(x\_)], x\_Symbol] := -Simp[ArcTanh[Cos[c + d\*x]]/d, x] /; FreeQ[{c, d}, x]

### Rule 2622

Int[csc[(e\_.) + (f\_.)\*(x\_)]^(n\_.)\*((a\_.)\*sec[(e\_.) + (f\_.)\*(x\_)])^(m\_), x\_Symbol] := Dist[1/(f\*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2), x], x, a\*Sec[e + f\*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

### Rule 2620

Int[csc[(e\_.) + (f\_.)\*(x\_)]^(m\_.)\*sec[(e\_.) + (f\_.)\*(x\_)]^(n\_.), x\_Symbol] := Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f\*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]

### Rule 14

Int[(u\_)\*((c\_.)\*(x\_))^(m\_.), x\_Symbol] := Int[ExpandIntegrand[(c\*x)^m\*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a\_ + (b\_.)\*(v\_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

### Rule 2548

Int[Log[u\_], x\_Symbol] := Simp[x\*Log[u], x] - Int[SimplifyIntegrand[(x\*D[u, x])/u, x], x] /; InverseFunctionFreeQ[u, x]

### Rule 3473

Int[((b\_.)\*tan[(c\_.) + (d\_.)\*(x\_)])^(n\_), x\_Symbol] := Simp[(b\*(b\*Tan[c + d\*x])^(n - 1))/(d\*(n - 1)), x] - Dist[b^2, Int[(b\*Tan[c + d\*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

### Rule 8

Int[a\_, x\_Symbol] := Simp[a\*x, x] /; FreeQ[a, x]

### Rule 5573

Int[(((e\_.) + (f\_.)\*(x\_))^(m\_.)\*Sech[(c\_.) + (d\_.)\*(x\_)]^(n\_.))/((a\_) + (b\_.)\*Sinh[(c\_.) + (d\_.)\*(x\_)]), x\_Symbol] := Dist[b^2/(a^2 + b^2), Int[((e +

$f*x)^m*\text{Sech}[c + d*x]^{(n - 2)}/(a + b*\text{Sinh}[c + d*x]), x], x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(e + f*x)^m*\text{Sech}[c + d*x]^n*(a - b*\text{Sinh}[c + d*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[a^2 + b^2, 0] \&\& \text{IGtQ}[n, 0]$

### Rule 5561

$\text{Int}[(\text{Cosh}[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))^{(m_.)})/((a_.) + (b_.)*\text{Sinh}[(c_.) + (d_.)*(x_.)]), x\_Symbol] :> -\text{Simp}[(e + f*x)^{(m + 1)}/(b*f*(m + 1)), x] + (\text{Int}[(e + f*x)^m*\text{E}^{(c + d*x)}/(a - \text{Rt}[a^2 + b^2, 2] + b*\text{E}^{(c + d*x)}), x] + \text{Int}[(e + f*x)^m*\text{E}^{(c + d*x)}/(a + \text{Rt}[a^2 + b^2, 2] + b*\text{E}^{(c + d*x)}), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[m, 0] \&\& \text{NeQ}[a^2 + b^2, 0]$

### Rule 2190

$\text{Int}[(\text{F}^{((g_.)*((e_.) + (f_.)*(x_.)))^{(n_.)*((c_.) + (d_.)*(x_.))^{(m_.)})}/((a_.) + (b_.)*(\text{F}^{((g_.)*((e_.) + (f_.)*(x_.)))^{(n_.)}), x\_Symbol] :> \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(\text{F}^{(g*(e + f*x)))^n)/a}]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m - 1)*\text{Log}[1 + (b*(\text{F}^{(g*(e + f*x)))^n)/a}], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \&\& \text{IGtQ}[m, 0]$

### Rule 6742

$\text{Int}[u_, x\_Symbol] :> \text{With}\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]$

### Rule 3718

$\text{Int}[(c_.) + (d_.)*(x_.))^{(m_.)*\text{tan}[(e_.) + (\text{Complex}[0, fz\_])*(f_.)*(x_.)], x\_Symbol] :> -\text{Simp}[(I*(c + d*x)^{(m + 1)})/(d*(m + 1)), x] + \text{Dist}[2*I, \text{Int}[(c + d*x)^m*\text{E}^{(2*(-I*e) + f*fz*x))}/(1 + \text{E}^{(2*(-I*e) + f*fz*x))}, x], x] /; \text{FreeQ}\{c, d, e, f, fz\}, x] \&\& \text{IGtQ}[m, 0]$

### Rule 5451

$\text{Int}[(c_.) + (d_.)*(x_.))^{(m_.)*\text{Sech}[(a_.) + (b_.)*(x_.)]^{(n_.)*\text{Tanh}[(a_.) + (b_.)*(x_.)]^{(p_.)}, x\_Symbol] :> -\text{Simp}[(c + d*x)^m*\text{Sech}[a + b*x]^n/(b*n), x] + \text{Dist}[(d*m)/(b*n), \text{Int}[(c + d*x)^{(m - 1)*\text{Sech}[a + b*x]^n}, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[p, 1] \&\& \text{GtQ}[m, 0]$

### Rule 3767

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x\_Symbol] :> -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /; \text{FreeQ}\{c, d\}, x] \&\& \text{IGtQ}[n/2, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{(e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{\int (e+fx)\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx) dx}{a} - \frac{b \int \frac{(e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx}{a} \\
&= \frac{8 \int (e+fx)\operatorname{csch}^3(2c+2dx) dx}{a} - \frac{b \int (e+fx)\operatorname{csch}^2(c+dx)\operatorname{sech}^3(c+dx) dx}{a^2} \\
&= \frac{3b(e+fx)\tan^{-1}(\sinh(c+dx))}{2a^2d} + \frac{3b(e+fx)\operatorname{csch}(c+dx)}{2a^2d} - \frac{f\operatorname{csch}(2c+2dx)}{ad^2} \\
&= \frac{3b(e+fx)\tan^{-1}(\sinh(c+dx))}{2a^2d} + \frac{4(e+fx)\tanh^{-1}(e^{2c+2dx})}{ad} + \frac{3b(e+fx)}{2a^2d} \\
&= -\frac{3bfx\tan^{-1}(\sinh(c+dx))}{2a^2d} + \frac{3b(e+fx)\tan^{-1}(\sinh(c+dx))}{2a^2d} + \frac{4(e+fx)}{2a^2d} \\
&= \frac{b^6(e+fx)^2}{2a^3(a^2+b^2)^2f} - \frac{3bfx\tan^{-1}(\sinh(c+dx))}{2a^2d} + \frac{3b(e+fx)\tan^{-1}(\sinh(c+dx))}{2a^2d} \\
&= \frac{b^2fx}{2a^3d} + \frac{b^6(e+fx)^2}{2a^3(a^2+b^2)^2f} + \frac{3bfx\tan^{-1}(e^{c+dx})}{a^2d} - \frac{3bfx\tan^{-1}(\sinh(c+dx))}{2a^2d} \\
&= \frac{b^2fx}{2a^3d} + \frac{3bfx\tan^{-1}(e^{c+dx})}{a^2d} - \frac{2b^5(e+fx)\tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)^2d} - \frac{b^3(e+fx)\tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)} \\
&= \frac{b^2fx}{2a^3d} + \frac{3bfx\tan^{-1}(e^{c+dx})}{a^2d} - \frac{2b^5(e+fx)\tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)^2d} - \frac{b^3(e+fx)\tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)} \\
&= \frac{b^2fx}{2a^3d} + \frac{3bfx\tan^{-1}(e^{c+dx})}{a^2d} - \frac{2b^5(e+fx)\tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)^2d} - \frac{b^3(e+fx)\tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)} \\
&= \frac{b^2fx}{2a^3d} + \frac{3bfx\tan^{-1}(e^{c+dx})}{a^2d} - \frac{2b^5(e+fx)\tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)^2d} - \frac{b^3(e+fx)\tan^{-1}(e^{c+dx})}{a^2(a^2+b^2)}
\end{aligned}$$

**Mathematica [B]** time = 10.1996, size = 2868, normalized size = 2.56

Result too large to show



$$\begin{aligned} & \cos\left[\frac{\pi + (2I)(c + dx)}{4}\right] - 4\pi \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{c + dx}{2}\right]\right] - (4I) \operatorname{PolyLog}\left[2, (-I)E^{-c - dx}\right] / 4 - (I/2) \left(-\frac{(c + dx)^2}{2} + 2(c + dx) \operatorname{Log}\left[1 - E^{c + dx}\right] + 2 \operatorname{PolyLog}\left[2, E^{c + dx}\right]\right) / (a^3(a^2 + b^2)^2 d^2) - (b^6 \left(-\frac{f(c + dx)^2}{2} + f(c + dx) \operatorname{Log}\left[1 + \frac{bE^{c + dx}}{a - \sqrt{a^2 + b^2}}\right]\right) + f(c + dx) \operatorname{Log}\left[1 + \frac{bE^{c + dx}}{a + \sqrt{a^2 + b^2}}\right]) + d e \operatorname{Log}\left[a + b \operatorname{Sinh}[c + dx]\right] - c f \operatorname{Log}\left[a + b \operatorname{Sinh}[c + dx]\right] + f \operatorname{PolyLog}\left[2, \frac{bE^{c + dx}}{-a + \sqrt{a^2 + b^2}}\right] + f \operatorname{PolyLog}\left[2, -\left(\frac{bE^{c + dx}}{a + \sqrt{a^2 + b^2}}\right)\right] \right) / (8a^3(a^2 + b^2)^2 d^2) + \left(\frac{(I/2)a^3 f (-E^{((I/4)\pi})(c + dx)^2)}{4} + \left(\frac{\pi(c + dx)}{4} - \pi \operatorname{Log}\left[1 + E^{c + dx}\right] - 2\left(\frac{\pi}{4} + \frac{I}{2}\right)(c + dx) \operatorname{Log}\left[1 - E^{((2I)(\pi/4 + (I/2)(c + dx))}\right] + \pi \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{c + dx}{2}\right]\right] + \left(\frac{\pi \operatorname{Log}\left[\sin\left[\frac{\pi}{4} + \frac{I}{2}(c + dx)\right]\right]}{2} + I \operatorname{PolyLog}\left[2, E^{((2I)(\pi/4 + (I/2)(c + dx))}\right) / \sqrt{2}\right] \right) / (\sqrt{2}(a^2 + b^2)^2 d^2) + \left(\frac{(3I)}{4} a b^2 f (-E^{((I/4)\pi})(c + dx)^2)}{4} + \left(\frac{\pi(c + dx)}{4} - \pi \operatorname{Log}\left[1 + E^{c + dx}\right] - 2\left(\frac{\pi}{4} + \frac{I}{2}\right)(c + dx) \operatorname{Log}\left[1 - E^{((2I)(\pi/4 + (I/2)(c + dx))}\right] + \pi \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{c + dx}{2}\right]\right] + \left(\frac{\pi \operatorname{Log}\left[\sin\left[\frac{\pi}{4} + \frac{I}{2}(c + dx)\right]\right]}{2} + I \operatorname{PolyLog}\left[2, E^{((2I)(\pi/4 + (I/2)(c + dx))}\right) / \sqrt{2}\right] \right) / (\sqrt{2}(a^2 + b^2)^2 d^2) + (b(3a^2 + 5b^2)(2(d e - c f + f(c + dx)) \operatorname{ArcTan}\left[\operatorname{Cosh}[c + dx] + \operatorname{Sinh}[c + dx]\right] - I f \operatorname{PolyLog}\left[2, (-I)(\operatorname{Cosh}[c + dx] + \operatorname{Sinh}[c + dx])\right] + I f \operatorname{PolyLog}\left[2, I(\operatorname{Cosh}[c + dx] + \operatorname{Sinh}[c + dx])\right]) / (16(a^2 + b^2)^2 d^2) + (\operatorname{Csch}[c + dx]^2 \operatorname{Sech}[c + dx]^2 (-4a b^2 d e + 4a b^2 c f - 4a b^2 f(c + dx) - 2a^2 b f \operatorname{Cosh}[c + dx] - 8a^3 d e \operatorname{Cosh}[2(c + dx)] - 4a b^2 d e \operatorname{Cosh}[2(c + dx)] + 8a^3 c f \operatorname{Cosh}[2(c + dx)] + 4a b^2 c f \operatorname{Cosh}[2(c + dx)] - 8a^3 f(c + dx) \operatorname{Cosh}[2(c + dx)] - 4a b^2 f(c + dx) \operatorname{Cosh}[2(c + dx)] + 2a^2 b f \operatorname{Cosh}[3(c + dx)] - 2a^2 b d e \operatorname{Sinh}[c + dx] + 4b^3 d e \operatorname{Sinh}[c + dx] + 2a^2 b c f \operatorname{Sinh}[c + dx] - 4b^3 c f \operatorname{Sinh}[c + dx] - 2a^2 b f(c + dx) \operatorname{Sinh}[c + dx] + 4b^3 f(c + dx) \operatorname{Sinh}[c + dx] - 4a^3 f \operatorname{Sinh}[2(c + dx)] - 2a b^2 f \operatorname{Sinh}[2(c + dx)] + 6a^2 b d e \operatorname{Sinh}[3(c + dx)] + 4b^3 d e \operatorname{Sinh}[3(c + dx)] - 6a^2 b c f \operatorname{Sinh}[3(c + dx)] - 4b^3 c f \operatorname{Sinh}[3(c + dx)] + 6a^2 b f(c + dx) \operatorname{Sinh}[3(c + dx)] + 4b^3 f(c + dx) \operatorname{Sinh}[3(c + dx)] - a b^2 f \operatorname{Sinh}[4(c + dx)]) / (128a^2(a^2 + b^2)d^2) \end{aligned}$$


---

**Maple [B]** time = 0.289, size = 3563, normalized size = 3.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (f*x+e)*\operatorname{csch}(d*x+c)^3*\operatorname{sech}(d*x+c)^3/(a+b*\operatorname{sinh}(d*x+c)),x$

[Out]  $-1/a^2/d^2*f*b^4/(a^2+b^2)^{(3/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+1/a^2/d^2/(a^2+b^2)^{(5/2)}*b^6*f*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a$

$$\begin{aligned}
& ^2+b^2)^{(1/2)}-2/(a^2+b^2)/d*a*e*\ln(\exp(d*x+c)-1)-2/(a^2+b^2)/d*a*e*\ln(\exp(d*x+c)+1)+2/(a^2+b^2)/d^2*a*f*dilog(\exp(d*x+c))-2/(a^2+b^2)/d^2*a*f*dilog(\exp(d*x+c)+1)+1/d^2/(a^2+b^2)^{(5/2)}*a^2*b^2*f*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-1/a^3/d/(a^2+b^2)^2*b^6*e*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d*x+c)-b)-1/a^3/(a^2+b^2)/d^2*b^4*f*c*\ln(\exp(d*x+c)-1)+1/a^3/(a^2+b^2)/d*b^4*f*\ln(\exp(d*x+c)+1)*x+1/a^3/(a^2+b^2)/d*b^4*e*\ln(\exp(d*x+c)+1)+1/a^3/(a^2+b^2)/d*b^4*e*\ln(\exp(d*x+c)-1)-1/a^3/(a^2+b^2)/d^2*b^4*f*dilog(\exp(d*x+c))+1/a^3/(a^2+b^2)/d^2*b^4*f*dilog(\exp(d*x+c)+1)+12/(a^2+b^2)/d*b^2*f/(4*a^2+4*b^2)*\ln(1-I*\exp(d*x+c))*a*x+12/(a^2+b^2)/d^2*b^2*f/(4*a^2+4*b^2)*\ln(1-I*\exp(d*x+c))*a*c-12/(a^2+b^2)/d^2*b^2*f*c/(4*a^2+4*b^2)*a*\ln(1+\exp(2*d*x+2*c))+1/2/(a^2+b^2)^{(5/2)}/d^2*a^2*b^2*f*c*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-12/(a^2+b^2)/d^2*a^2*f*c/(4*a^2+4*b^2)*b*\operatorname{arctan}(\exp(d*x+c))+12/(a^2+b^2)/d*b^2*f/(4*a^2+4*b^2)*\ln(1+I*\exp(d*x+c))*a*x+12/(a^2+b^2)/d^2*b^2*f/(4*a^2+4*b^2)*\ln(1+I*\exp(d*x+c))*a*c+6*I*a^2/d^2/(a^2+b^2)*f/(4*a^2+4*b^2)*dilog(1-I*\exp(d*x+c))*b-6*I*a^2/d^2/(a^2+b^2)*f/(4*a^2+4*b^2)*dilog(1+I*\exp(d*x+c))*b+10*I/d^2/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*\ln(1-I*\exp(d*x+c))*c-10*I/d^2/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*\ln(1+I*\exp(d*x+c))*c+10*I/d/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*\ln(1-I*\exp(d*x+c))*x-10*I/d/(a^2+b^2)*b^3*f/(4*a^2+4*b^2)*\ln(1+I*\exp(d*x+c))*x-2/(a^2+b^2)/d*\ln(\exp(d*x+c)+1)*a*f*x+2/(a^2+b^2)/d^2*a*f*c*\ln(\exp(d*x+c)-1)+1/(a^2+b^2)/d^2*b^2*f/a*dilog(\exp(d*x+c))-1/(a^2+b^2)/d^2*b^2*f/a*dilog(\exp(d*x+c)+1)-1/(a^2+b^2)/d*b^2*e/a*\ln(\exp(d*x+c)-1)-1/(a^2+b^2)/d*b^2*e/a*\ln(\exp(d*x+c)+1)-1/d^2*b^2*f/(a^2+b^2)^{(3/2)}*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})+2/d^2/(a^2+b^2)^{(5/2)}*b^4*f*\operatorname{arctan}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)})-1/a^3/d^2/(a^2+b^2)^2*b^6*f*dilog((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))-1/a^3/d^2/(a^2+b^2)^2*b^6*f*dilog((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))+1/(a^2+b^2)/d^2*b^2*f*c/a*\ln(\exp(d*x+c)-1)-1/(a^2+b^2)/d*b^2*f/a*\ln(\exp(d*x+c)+1)*x-(2*a*b^2*d*e*\exp(6*d*x+6*c)-2*b^3*d*f*x*\exp(5*d*x+5*c)+a^2*b*d*e*\exp(5*d*x+5*c)+4*a*b^2*d*e*\exp(4*d*x+4*c)+3*a^2*b*d*f*x*\exp(d*x+c)-a^2*b*d*f*x*\exp(3*d*x+3*c)-2*b^3*d*f*x*\exp(7*d*x+7*c)+4*a^3*d*f*x*\exp(6*d*x+6*c)-3*a^2*b*d*e*\exp(7*d*x+7*c)+4*a^3*d*f*x*\exp(2*d*x+2*c)-a^2*b*d*e*\exp(3*d*x+3*c)+2*a*b^2*d*e*\exp(2*d*x+2*c)+2*b^3*d*e*\exp(d*x+c)-a^2*b*f*\exp(d*x+c)+2*b^3*d*e*\exp(3*d*x+3*c)+4*a^3*d*e*\exp(2*d*x+2*c)+a^2*b*f*\exp(3*d*x+3*c)-a*b^2*f*\exp(2*d*x+2*c)+2*a^3*f*\exp(6*d*x+6*c)-a*b^2*f-2*a^3*f*\exp(2*d*x+2*c)+2*a*b^2*d*f*x*\exp(2*d*x+2*c)+2*b^3*d*f*x*\exp(d*x+c)+3*a^2*b*d*e*\exp(d*x+c)+2*b^3*d*f*x*\exp(3*d*x+3*c)+a^2*b*f*\exp(5*d*x+5*c)+a*b^2*f*\exp(4*d*x+4*c)-2*b^3*d*e*\exp(7*d*x+7*c)+4*a^3*d*e*\exp(6*d*x+6*c)-a^2*b*f*\exp(7*d*x+7*c)+a*b^2*f*\exp(6*d*x+6*c)-2*b^3*d*e*\exp(5*d*x+5*c)-3*a^2*b*d*f*x*\exp(7*d*x+7*c)+2*a*b^2*d*f*x*\exp(6*d*x+6*c)+a^2*b*d*f*x*\exp(5*d*x+5*c)+4*a*b^2*d*f*x*\exp(4*d*x+4*c))/d^2/(a^2+b^2)/(1+\exp(2*d*x+2*c))^2/a^2/(\exp(2*d*x+2*c)-1)^2-6*I*a^2/d^2/(a^2+b^2)*f/(4*a^2+4*b^2)*\ln(1+I*\exp(d*x+c))*b*c+6*I*a^2/d^2/(a^2+b^2)*f/(4*a^2+4*b^2)*\ln(1-I*\exp(d*x+c))*b*c-6*I*a^2/d/(a^2+b^2)*f/(4*a^2+4*b^2)*\ln(1+I*\exp(d*x+c))*b*x+6*I*a^2/d/(a^2+b^2)*f/(4*a^2+4*b^2)*\ln(1-I*\exp(d*x+c))*b*x-1/a^2/(a^2+b^2)/d^2*b^3*f*\ln(\exp(d*x+c)-1)+1/a^2/(a^2+b^2)/d^2*b^3*f*\ln(\exp(d*x+c)+1)+1/2/(a^2+b^2)^{(3/2)}/d*b^2*e*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2
\end{aligned}$$

$$\begin{aligned}
& +b^2)^{(1/2)}+20/(a^2+b^2)/d*b^3*e/(4*a^2+4*b^2)*\arctan(\exp(d*x+c))-1/2/(a^2 \\
& +b^2)^{(5/2)}/d*b^4*e*\operatorname{arctanh}(1/2*(2*b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))+8/(a^ \\
& 2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*\operatorname{dilog}(1+I*\exp(d*x+c))+8/(a^2+b^2)/d^2*a^3*f/ \\
& (4*a^2+4*b^2)*\operatorname{dilog}(1-I*\exp(d*x+c))+8/(a^2+b^2)/d*a^3*e/(4*a^2+4*b^2)*\ln(1+ \\
& \exp(2*d*x+2*c))+1/a^3/d^2/(a^2+b^2)^2*b^6*f*c*\ln(b*\exp(2*d*x+2*c)+2*a*\exp(d \\
& *x+c)-b)-1/a^3/d/(a^2+b^2)^2*b^6*f*\ln((-b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}-a)/(-a \\
& +(a^2+b^2)^{(1/2)}))*x-1/a^3/d/(a^2+b^2)^2*b^6*f*\ln((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)} \\
& +a)/(a+(a^2+b^2)^{(1/2)}))*x-1/a^3/d^2/(a^2+b^2)^2*b^6*f*\ln((-b*\exp(d*x+c) \\
& +(a^2+b^2)^{(1/2)}-a)/(-a+(a^2+b^2)^{(1/2)}))*c-1/a^3/d^2/(a^2+b^2)^2*b^6*f*\ln \\
& ((b*\exp(d*x+c)+(a^2+b^2)^{(1/2)}+a)/(a+(a^2+b^2)^{(1/2)}))*c+10*I/d^2/(a^2+b^2) \\
& *b^3*f/(4*a^2+4*b^2)*\operatorname{dilog}(1-I*\exp(d*x+c))-10*I/d^2/(a^2+b^2)*b^3*f/(4*a^2+ \\
& 4*b^2)*\operatorname{dilog}(1+I*\exp(d*x+c))-1/2/(a^2+b^2)^{(3/2)}/d^2*b^2*f*c*\operatorname{arctanh}(1/2*(2 \\
& *b*\exp(d*x+c)+2*a)/(a^2+b^2)^{(1/2)}))-20/(a^2+b^2)/d^2*b^3*f*c/(4*a^2+4*b^2)* \\
& \arctan(\exp(d*x+c))+1/2/(a^2+b^2)^{(5/2)}/d^2*b^4*f*c*\operatorname{arctanh}(1/2*(2*b*\exp(d*x \\
& +c)+2*a)/(a^2+b^2)^{(1/2)}))+12/(a^2+b^2)/d^2*b^2*f/(4*a^2+4*b^2)*\operatorname{dilog}(1+I*\exp \\
& (d*x+c))*a+12/(a^2+b^2)/d^2*b^2*f/(4*a^2+4*b^2)*\operatorname{dilog}(1-I*\exp(d*x+c))*a-8/ \\
& (a^2+b^2)/d^2*a^3*f*c/(4*a^2+4*b^2)*\ln(1+\exp(2*d*x+2*c))+12/(a^2+b^2)/d*a^2 \\
& *e/(4*a^2+4*b^2)*b*\arctan(\exp(d*x+c))+12/(a^2+b^2)/d*b^2*e/(4*a^2+4*b^2)*a* \\
& \ln(1+\exp(2*d*x+2*c))-1/2/(a^2+b^2)^{(5/2)}/d*a^2*b^2*e*\operatorname{arctanh}(1/2*(2*b*\exp(d \\
& *x+c)+2*a)/(a^2+b^2)^{(1/2)}))+8/(a^2+b^2)/d*a^3*f/(4*a^2+4*b^2)*\ln(1+I*\exp(d* \\
& x+c))*x+8/(a^2+b^2)/d^2*a^3*f/(4*a^2+4*b^2)*\ln(1+I*\exp(d*x+c))*c+8/(a^2+b^2) \\
& )/d*a^3*f/(4*a^2+4*b^2)*\ln(1-I*\exp(d*x+c))*x+8/(a^2+b^2)/d^2*a^3*f/(4*a^2+4 \\
& *b^2)*\ln(1-I*\exp(d*x+c))*c-1/(a^2+b^2)/d^2*b*f*\ln(\exp(d*x+c)-1)+1/(a^2+b^2) \\
& /d^2*b*f*\ln(\exp(d*x+c)+1)
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)^3\*sech(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="maxima")

[Out] 
$$\begin{aligned}
& -(b^6*\log(-2*a*e^{-d*x-c}) + b*e^{-2*d*x-2*c} - b)/((a^7 + 2*a^5*b^2 + a \\
& ^3*b^4)*d) + (3*a^2*b + 5*b^3)*\arctan(e^{-d*x-c})/((a^4 + 2*a^2*b^2 + b^4) \\
& )*d) - (2*a^3 + 3*a*b^2)*\log(e^{-2*d*x-2*c} + 1)/((a^4 + 2*a^2*b^2 + b^4) \\
& )*d) + (4*a*b^2*e^{-4*d*x-4*c} - (3*a^2*b + 2*b^3)*e^{-d*x-c} + 2*(2*a^3 \\
& + a*b^2)*e^{-2*d*x-2*c} + (a^2*b - 2*b^3)*e^{-3*d*x-3*c} - (a^2*b - 2* \\
& b^3)*e^{-5*d*x-5*c} + 2*(2*a^3 + a*b^2)*e^{-6*d*x-6*c} + (3*a^2*b + 2*b \\
& ^3)*e^{-7*d*x-7*c})/((a^4 + a^2*b^2 - 2*(a^4 + a^2*b^2)*e^{-4*d*x-4*c} \\
& + (a^4 + a^2*b^2)*e^{-8*d*x-8*c}))*d) + (2*a^2 - b^2)*\log(e^{-d*x-c}) + 1
\end{aligned}$$

$$\begin{aligned} &)/(a^3d) + (2a^2 - b^2) \log(e^{-dx - c} - 1)/(a^3d))e + (128a^2d \int \text{egrate}(1/64x/(a^3de^{dx+c} + a^3d), x) - 64b^2d \int \text{egrate}(1/64x/(a^3de^{dx+c} + a^3d), x) - 128a^2d \int \text{egrate}(1/64x/(a^3de^{dx+c} - a^3d), x) + 64b^2d \int \text{egrate}(1/64x/(a^3de^{dx+c} - a^3d), x) - a*b*((dx+c)/(a^3d^2) - \log(e^{dx+c} + 1)/(a^3d^2)) + a*b*((dx+c)/(a^3d^2) - \log(e^{dx+c} - 1)/(a^3d^2)) + (a*b^2 + (a^2*b*e^{7c} + (3a^2*b*d*e^{7c} + 2*b^3*d*e^{7c}))*x)*e^{7dx} - (2a^3*e^{6c} + a*b^2*e^{6c} + 2*(2a^3*d*e^{6c} + a*b^2*d*e^{6c}))*x)*e^{6dx} - (a^2*b*e^{5c} + (a^2*b*d*e^{5c} - 2*b^3*d*e^{5c}))*x)*e^{5dx} - (4a*b^2*d*x*e^{4c} + a*b^2*e^{4c})*e^{4dx} - (a^2*b*e^{3c} - (a^2*b*d*e^{3c} - 2*b^3*d*e^{3c}))*x)*e^{3dx} + (2a^3*e^{2c} + a*b^2*e^{2c} - 2*(2a^3*d*e^{2c} + a*b^2*d*e^{2c}))*x)*e^{2dx} + (a^2*b*e^c - (3a^2*b*d*e^c + 2*b^3*d*e^c))*x)*e^{dx})/(a^4d^2 + a^2b^2d^2 + (a^4d^2*e^{8c} + a^2b^2d^2*e^{8c}))*e^{8dx} - 2*(a^4d^2*e^{4c} + a^2b^2d^2*e^{4c}))*e^{4dx}) + 64 \int \text{egrate}(-1/32*(a*b^6*x*e^{dx+c} - b^7*x)/(a^7*b + 2a^5*b^3 + a^3*b^5 - (a^7*b*e^{2c} + 2a^5*b^3*e^{2c} + a^3*b^5*e^{2c}))*e^{2dx} - 2*(a^8*e^c + 2a^6*b^2*e^c + a^4*b^4*e^c)*e^{dx}), x) + 64 \int \text{egrate}(1/64*((3a^2*b*e^c + 5b^3*e^c)*x*e^{dx} - 2*(2a^3 + 3a*b^2)*x)/(a^4 + 2a^2*b^2 + b^4 + (a^4*e^{2c} + 2a^2*b^2*e^{2c} + b^4*e^{2c}))*e^{2dx}), x))*f \end{aligned}$$

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*csch(d*x+c)**3*sech(d*x+c)**3/(a+b*sinh(d*x+c)),x)
```



[Out] Timed out

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*csch(d\*x+c)^3\*sech(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] Timed out

$$3.501 \quad \int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx$$

**Optimal.** Leaf size=211

$$-\frac{b^6 \log(a+b\sinh(c+dx))}{a^3 d (a^2+b^2)^2} - \frac{(2a^2-b^2) \log(\sinh(c+dx))}{a^3 d} + \frac{b(a^2+2b^2) \tan^{-1}(\sinh(c+dx))}{d(a^2+b^2)^2} + \frac{b \tan^{-1}(\sinh(c+dx))}{2d(a^2+b^2)}$$

```
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*(a^2 + b^2)*d) + (b*(a^2 + 2*b^2)*ArcTan[Sinh[
c + d*x]])/((a^2 + b^2)^2*d) + (b*Csch[c + d*x])/(a^2*d) - Csch[c + d*x]^2/
(2*a*d) + (a*(2*a^2 + 3*b^2)*Log[Cosh[c + d*x]])/((a^2 + b^2)^2*d) - ((2*a^
2 - b^2)*Log[Sinh[c + d*x]])/(a^3*d) - (b^6*Log[a + b*Sinh[c + d*x]])/(a^3*
(a^2 + b^2)^2*d) - (Sech[c + d*x]^2*(a - b*Sinh[c + d*x]))/(2*(a^2 + b^2)*d
)
```

**Rubi [A]** time = 0.366715, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$ , Rules used = {2837, 12, 894, 639, 203, 635, 260}

$$-\frac{b^6 \log(a+b\sinh(c+dx))}{a^3 d (a^2+b^2)^2} - \frac{(2a^2-b^2) \log(\sinh(c+dx))}{a^3 d} + \frac{b(a^2+2b^2) \tan^{-1}(\sinh(c+dx))}{d(a^2+b^2)^2} + \frac{b \tan^{-1}(\sinh(c+dx))}{2d(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csch[c + d*x]^3*Sech[c + d*x]^3)/(a + b*Sinh[c + d*x]),x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*(a^2 + b^2)*d) + (b*(a^2 + 2*b^2)*ArcTan[Sinh[
c + d*x]])/((a^2 + b^2)^2*d) + (b*Csch[c + d*x])/(a^2*d) - Csch[c + d*x]^2/
(2*a*d) + (a*(2*a^2 + 3*b^2)*Log[Cosh[c + d*x]])/((a^2 + b^2)^2*d) - ((2*a^
2 - b^2)*Log[Sinh[c + d*x]])/(a^3*d) - (b^6*Log[a + b*Sinh[c + d*x]])/(a^3*
(a^2 + b^2)^2*d) - (Sech[c + d*x]^2*(a - b*Sinh[c + d*x]))/(2*(a^2 + b^2)*d
)
```

**Rule 2837**

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_
.))*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

$\text{Int}[(a_*)(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{Match} \\ \text{Q}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

Rule 894

$\text{Int}[(d_.) + (e_.)*(x_)]^{(m_)}*((f_.) + (g_.)*(x_)]^{(n_)}*((a_.) + (c_.)*(x_)]^{(p_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; \text{FreeQ}\{a, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[e*f - d*g, 0] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ ((\text{EqQ}[p, 1] \ \&\& \ \text{IntegersQ}[m, n]) \ || \ (\text{ILtQ}[m, 0] \ \&\& \ \text{ILtQ}[n, 0]))$

Rule 639

$\text{Int}[(d_.) + (e_.)*(x_)]*((a_.) + (c_.)*(x_)]^{(p_.)}, x\_Symbol] \rightarrow \text{Simp}[(a*e - c*d*x)*(a + c*x^2)^{(p+1)} / (2*a*c*(p+1)), x] + \text{Dist}[(d*(2*p+3)) / (2*a*(p+1)), \text{Int}[(a + c*x^2)^{(p+1)}, x], x] /; \text{FreeQ}\{a, c, d, e\}, x] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{NeQ}[p, -3/2]$

Rule 203

$\text{Int}[(a_.) + (b_.)*(x_)]^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]]) / (\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 635

$\text{Int}[(d_.) + (e_.)*(x_)] / ((a_.) + (c_.)*(x_)]^2, x\_Symbol] \rightarrow \text{Dist}[d, \text{Int}[1/(a + c*x^2), x], x] + \text{Dist}[e, \text{Int}[x/(a + c*x^2), x], x] /; \text{FreeQ}\{a, c, d, e\}, x] \ \&\& \ !\text{NiceSqrtQ}[-(a*c)]$

Rule 260

$\text{Int}[(x_)]^{(m_.)} / ((a_.) + (b_.)*(x_)]^{(n_.)}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x^n, x]] / (b*n), x] /; \text{FreeQ}\{a, b, m, n\}, x] \ \&\& \ \text{EqQ}[m, n - 1]$

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{a+b\sinh(c+dx)} dx &= \frac{b^3 \operatorname{Subst}\left(\int \frac{b^3}{x^3(a+x)(-b^2-x^2)^2} dx, x, b\sinh(c+dx)\right)}{d} \\
&= \frac{b^6 \operatorname{Subst}\left(\int \frac{1}{x^3(a+x)(-b^2-x^2)^2} dx, x, b\sinh(c+dx)\right)}{d} \\
&= \frac{b^6 \operatorname{Subst}\left(\int \left(\frac{1}{ab^4x^3} - \frac{1}{a^2b^4x^2} + \frac{-2a^2+b^2}{a^3b^6x} - \frac{1}{a^3(a^2+b^2)^2(a+x)} + \frac{b^2+ax}{b^4(a^2+b^2)(b^2+x^2)^2} + \frac{b^2(a^2+2b^2)+}{b^6(a^2+b^2)}\right) dx, x, b\sinh(c+dx)\right)}{d} \\
&= \frac{b\operatorname{csch}(c+dx)}{a^2d} - \frac{\operatorname{csch}^2(c+dx)}{2ad} - \frac{(2a^2-b^2)\log(\sinh(c+dx))}{a^3d} - \frac{b^6\log(a+b\sinh(c+dx))}{a^3(a^2+b^2)^2} \\
&= \frac{b\operatorname{csch}(c+dx)}{a^2d} - \frac{\operatorname{csch}^2(c+dx)}{2ad} - \frac{(2a^2-b^2)\log(\sinh(c+dx))}{a^3d} - \frac{b^6\log(a+b\sinh(c+dx))}{a^3(a^2+b^2)^2} \\
&= \frac{b\tan^{-1}(\sinh(c+dx))}{2(a^2+b^2)d} + \frac{b(a^2+2b^2)\tan^{-1}(\sinh(c+dx))}{(a^2+b^2)^2d} + \frac{b\operatorname{csch}(c+dx)}{a^2d} - \frac{\operatorname{csch}^2(c+dx)}{2ad}
\end{aligned}$$

**Mathematica [C]** time = 0.807566, size = 237, normalized size = 1.12

$$\frac{-\frac{a\operatorname{sech}^2(c+dx)}{a^2+b^2} - \frac{2b^6\log(a+b\sinh(c+dx))}{a^3(a^2+b^2)^2} + \frac{(a-ib)(2a^2+iab+2b^2)\log(-\sinh(c+dx)+i)}{(a^2+b^2)^2} - \frac{2(2a^2-b^2)\log(\sinh(c+dx))}{a^3} + \frac{(a+ib)(2a^2-iab+2b^2)\log(\sinh(c+dx))}{(a^2+b^2)^2}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(Csch[c + d\*x]^3\*Sech[c + d\*x]^3)/(a + b\*Sinh[c + d\*x]),x]

[Out] ((b\*ArcTan[Sinh[c + d\*x]])/(a^2 + b^2) + (2\*b\*Csch[c + d\*x])/a^2 - Csch[c + d\*x]^2/a + ((a - I\*b)\*(2\*a^2 + I\*a\*b + 2\*b^2)\*Log[I - Sinh[c + d\*x]])/(a^2 + b^2)^2 - (2\*(2\*a^2 - b^2)\*Log[Sinh[c + d\*x]])/a^3 + ((a + I\*b)\*(2\*a^2 - I\*a\*b + 2\*b^2)\*Log[I + Sinh[c + d\*x]])/(a^2 + b^2)^2 - (2\*b^6\*Log[a + b\*Sinh[c + d\*x]])/(a^3\*(a^2 + b^2)^2) - (a\*Sech[c + d\*x]^2)/(a^2 + b^2) + (b\*Sech[c + d\*x]\*Tanh[c + d\*x])/(a^2 + b^2))/(2\*d)

**Maple [B]** time = 0.003, size = 539, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x)`

[Out] 
$$-1/8/d/a*\tanh(1/2*d*x+1/2*c)^2-1/2/d/a^2*\tanh(1/2*d*x+1/2*c)*b-1/8/d/a/\tanh(1/2*d*x+1/2*c)^2-2/d/a*\ln(\tanh(1/2*d*x+1/2*c))+1/d/a^3*\ln(\tanh(1/2*d*x+1/2*c))*b^2+1/2/d*b/a^2/\tanh(1/2*d*x+1/2*c)-1/d*b^6/(a^2+b^2)^2/a^3*\ln(\tanh(1/2*d*x+1/2*c))^2*a-2*\tanh(1/2*d*x+1/2*c)*b-a)-1/d/(a^2+b^2)^2/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)^3*a^2*b-1/d/(a^2+b^2)^2/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)^3*b^3+2/d/(a^2+b^2)^2/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)^2*a^3+2/d/(a^2+b^2)^2/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)^2*a*b^2+1/d/(a^2+b^2)^2/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)*a^2*b+1/d/(a^2+b^2)^2/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)*b^3+2/d/(a^2+b^2)^2*\ln(\tanh(1/2*d*x+1/2*c)^2+1)*a^3+3/d/(a^2+b^2)^2*\ln(\tanh(1/2*d*x+1/2*c)^2+1)*a*b^2+3/d/(a^2+b^2)^2*\arctan(\tanh(1/2*d*x+1/2*c))*a^2*b+5/d/(a^2+b^2)^2*\arctan(\tanh(1/2*d*x+1/2*c))*b^3$$

**Maxima [B]** time = 1.7642, size = 564, normalized size = 2.67

$$\frac{b^6 \log(-2ae^{(-dx-c)} + be^{(-2dx-2c)} - b)}{(a^7 + 2a^5b^2 + a^3b^4)d} - \frac{(3a^2b + 5b^3) \arctan(e^{(-dx-c)})}{(a^4 + 2a^2b^2 + b^4)d} + \frac{(2a^3 + 3ab^2) \log(e^{(-2dx-2c)} + 1)}{(a^4 + 2a^2b^2 + b^4)d} - \frac{4ab^2e^{(-2dx-2c)}}{(a^4 + 2a^2b^2 + b^4)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3*sech(d*x+c)^3/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] 
$$-b^6*\log(-2*a*e^{(-d*x - c)} + b*e^{(-2*d*x - 2*c)} - b)/((a^7 + 2*a^5*b^2 + a^3*b^4)*d) - (3*a^2*b + 5*b^3)*\arctan(e^{(-d*x - c)})/((a^4 + 2*a^2*b^2 + b^4)*d) + (2*a^3 + 3*a*b^2)*\log(e^{(-2*d*x - 2*c)} + 1)/((a^4 + 2*a^2*b^2 + b^4)*d) - (4*a*b^2*e^{(-4*d*x - 4*c)} - (3*a^2*b + 2*b^3)*e^{(-d*x - c)} + 2*(2*a^3 + a*b^2)*e^{(-2*d*x - 2*c)} + (a^2*b - 2*b^3)*e^{(-3*d*x - 3*c)} - (a^2*b - 2*b^3)*e^{(-5*d*x - 5*c)} + 2*(2*a^3 + a*b^2)*e^{(-6*d*x - 6*c)} + (3*a^2*b + 2*b^3)*e^{(-7*d*x - 7*c)})/((a^4 + a^2*b^2 - 2*(a^4 + a^2*b^2)*e^{(-4*d*x - 4*c)} + (a^4 + a^2*b^2)*e^{(-8*d*x - 8*c)})*d) - (2*a^2 - b^2)*\log(e^{(-d*x - c)} + 1)/(a^3*d) - (2*a^2 - b^2)*\log(e^{(-d*x - c)} - 1)/(a^3*d)$$

**Fricas [B]** time = 7.08528, size = 7170, normalized size = 33.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^3\*sech(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & ((3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*\cosh(d*x + c)^7 + (3*a^5*b + 5*a^3*b^3 + 2* \\ & *a*b^5)*\sinh(d*x + c)^7 - 2*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*\cosh(d*x + c)^6 - \\ & (4*a^6 + 6*a^4*b^2 + 2*a^2*b^4 - 7*(3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*\cosh(d* \\ & x + c))*\sinh(d*x + c)^6 - (a^5*b - a^3*b^3 - 2*a*b^5)*\cosh(d*x + c)^5 - (a^ \\ & 5*b - a^3*b^3 - 2*a*b^5 - 21*(3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*\cosh(d*x + c)^ \\ & 2 + 12*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 4*(a^ \\ & 4*b^2 + a^2*b^4)*\cosh(d*x + c)^4 - (4*a^4*b^2 + 4*a^2*b^4 - 35*(3*a^5*b + 5 \\ & *a^3*b^3 + 2*a*b^5)*\cosh(d*x + c))^3 + 30*(2*a^6 + 3*a^4*b^2 + a^2*b^4)*\cosh \\ & (d*x + c)^2 + 5*(a^5*b - a^3*b^3 - 2*a*b^5)*\cosh(d*x + c))*\sinh(d*x + c)^4 \\ & + (a^5*b - a^3*b^3 - 2*a*b^5)*\cosh(d*x + c)^3 + (a^5*b - a^3*b^3 - 2*a*b^5 \\ & + 35*(3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*\cosh(d*x + c))^4 - 40*(2*a^6 + 3*a^4*b^ \\ & 2 + a^2*b^4)*\cosh(d*x + c)^3 - 10*(a^5*b - a^3*b^3 - 2*a*b^5)*\cosh(d*x + c) \\ & ^2 - 16*(a^4*b^2 + a^2*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 2*(2*a^6 + 3*a \\ & ^4*b^2 + a^2*b^4)*\cosh(d*x + c)^2 - (4*a^6 + 6*a^4*b^2 + 2*a^2*b^4 - 21*(3* \\ & a^5*b + 5*a^3*b^3 + 2*a*b^5)*\cosh(d*x + c))^5 + 30*(2*a^6 + 3*a^4*b^2 + a^2* \\ & b^4)*\cosh(d*x + c)^4 + 10*(a^5*b - a^3*b^3 - 2*a*b^5)*\cosh(d*x + c)^3 + 24* \\ & (a^4*b^2 + a^2*b^4)*\cosh(d*x + c)^2 - 3*(a^5*b - a^3*b^3 - 2*a*b^5)*\cosh(d* \\ & x + c))*\sinh(d*x + c)^2 + ((3*a^5*b + 5*a^3*b^3)*\cosh(d*x + c))^8 + 56*(3*a^ \\ & 5*b + 5*a^3*b^3)*\cosh(d*x + c)^3*\sinh(d*x + c)^5 + 28*(3*a^5*b + 5*a^3*b^3) \\ & *\cosh(d*x + c)^2*\sinh(d*x + c)^6 + 8*(3*a^5*b + 5*a^3*b^3)*\cosh(d*x + c)*\si \\ & nh(d*x + c)^7 + (3*a^5*b + 5*a^3*b^3)*\sinh(d*x + c)^8 + 3*a^5*b + 5*a^3*b^3 \\ & - 2*(3*a^5*b + 5*a^3*b^3)*\cosh(d*x + c)^4 - 2*(3*a^5*b + 5*a^3*b^3 - 35*(3 \\ & *a^5*b + 5*a^3*b^3)*\cosh(d*x + c))^4)*\sinh(d*x + c)^4 + 8*(7*(3*a^5*b + 5*a^ \\ & 3*b^3)*\cosh(d*x + c)^5 - (3*a^5*b + 5*a^3*b^3)*\cosh(d*x + c))*\sinh(d*x + c) \\ & ^3 + 4*(7*(3*a^5*b + 5*a^3*b^3)*\cosh(d*x + c)^6 - 3*(3*a^5*b + 5*a^3*b^3)*\c \\ & osh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((3*a^5*b + 5*a^3*b^3)*\cosh(d*x + c))^7 \\ & - (3*a^5*b + 5*a^3*b^3)*\cosh(d*x + c)^3)*\sinh(d*x + c))*\arctan(\cosh(d*x + c \\ & ) + \sinh(d*x + c)) - (3*a^5*b + 5*a^3*b^3 + 2*a*b^5)*\cosh(d*x + c) - (b^6*c \\ & osh(d*x + c))^8 + 56*b^6*\cosh(d*x + c)^3*\sinh(d*x + c)^5 + 28*b^6*\cosh(d*x + \\ & c)^2*\sinh(d*x + c)^6 + 8*b^6*\cosh(d*x + c)*\sinh(d*x + c)^7 + b^6*\sinh(d*x \\ & + c)^8 - 2*b^6*\cosh(d*x + c)^4 + b^6 + 2*(35*b^6*\cosh(d*x + c)^4 - b^6)*\sin \\ & h(d*x + c)^4 + 8*(7*b^6*\cosh(d*x + c)^5 - b^6*\cosh(d*x + c))*\sinh(d*x + c)^ \\ & 3 + 4*(7*b^6*\cosh(d*x + c)^6 - 3*b^6*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*( \\ & b^6*\cosh(d*x + c)^7 - b^6*\cosh(d*x + c)^3)*\sinh(d*x + c))*\log(2*(b*\sinh(d*x \end{aligned}$$

$$\begin{aligned}
& + c) + a)/(\cosh(dx + c) - \sinh(dx + c))) + ((2a^6 + 3a^4b^2)\cosh(dx \\
& + c)^8 + 56(2a^6 + 3a^4b^2)\cosh(dx + c)^3\sinh(dx + c)^5 + 28(2a^6 \\
& + 3a^4b^2)\cosh(dx + c)^2\sinh(dx + c)^6 + 8(2a^6 + 3a^4b^2)\cosh \\
& (dx + c)\sinh(dx + c)^7 + (2a^6 + 3a^4b^2)\sinh(dx + c)^8 + 2a^6 + 3 \\
& *a^4b^2 - 2(2a^6 + 3a^4b^2)\cosh(dx + c)^4 - 2(2a^6 + 3a^4b^2 - 3 \\
& 5(2a^6 + 3a^4b^2)\cosh(dx + c)^4)\sinh(dx + c)^4 + 8(7(2a^6 + 3a^4 \\
& 4b^2)\cosh(dx + c)^5 - (2a^6 + 3a^4b^2)\cosh(dx + c))\sinh(dx + c)^3 \\
& + 4(7(2a^6 + 3a^4b^2)\cosh(dx + c)^6 - 3(2a^6 + 3a^4b^2)\cosh(dx \\
& x + c)^2)\sinh(dx + c)^2 + 8(((2a^6 + 3a^4b^2)\cosh(dx + c)^7 - (2a^6 \\
& + 3a^4b^2)\cosh(dx + c)^3)\sinh(dx + c))\log(2\cosh(dx + c)/(\cosh(dx \\
& + c) - \sinh(dx + c))) - ((2a^6 + 3a^4b^2 - b^6)\cosh(dx + c)^8 + 56( \\
& 2a^6 + 3a^4b^2 - b^6)\cosh(dx + c)^3\sinh(dx + c)^5 + 28(2a^6 + 3a^4 \\
& 4b^2 - b^6)\cosh(dx + c)^2\sinh(dx + c)^6 + 8(2a^6 + 3a^4b^2 - b^6)* \\
& \cosh(dx + c)\sinh(dx + c)^7 + (2a^6 + 3a^4b^2 - b^6)\sinh(dx + c)^8 + \\
& 2a^6 + 3a^4b^2 - b^6 - 2(2a^6 + 3a^4b^2 - b^6)\cosh(dx + c)^4 - 2* \\
& (2a^6 + 3a^4b^2 - b^6 - 35(2a^6 + 3a^4b^2 - b^6)\cosh(dx + c)^4)*\si \\
& nh(dx + c)^4 + 8(7(2a^6 + 3a^4b^2 - b^6)\cosh(dx + c)^5 - (2a^6 + 3 \\
& *a^4b^2 - b^6)\cosh(dx + c))\sinh(dx + c)^3 + 4(7(2a^6 + 3a^4b^2 - \\
& b^6)\cosh(dx + c)^6 - 3(2a^6 + 3a^4b^2 - b^6)\cosh(dx + c)^2)\sinh(dx \\
& x + c)^2 + 8(((2a^6 + 3a^4b^2 - b^6)\cosh(dx + c)^7 - (2a^6 + 3a^4b^ \\
& 2 - b^6)\cosh(dx + c)^3)\sinh(dx + c))\log(2\sinh(dx + c)/(\cosh(dx + c) \\
& - \sinh(dx + c))) + (7(3a^5b + 5a^3b^3 + 2a*b^5)\cosh(dx + c)^6 - 3 \\
& *a^5b - 5a^3b^3 - 2a*b^5 - 12(2a^6 + 3a^4b^2 + a^2b^4)\cosh(dx + \\
& c)^5 - 5(a^5b - a^3b^3 - 2a*b^5)\cosh(dx + c)^4 - 16(a^4b^2 + a^2b^ \\
& 4)\cosh(dx + c)^3 + 3(a^5b - a^3b^3 - 2a*b^5)\cosh(dx + c)^2 - 4(2a \\
& ^6 + 3a^4b^2 + a^2b^4)\cosh(dx + c))\sinh(dx + c))/((a^7 + 2a^5b^2 + \\
& a^3b^4)*d*\cosh(dx + c)^8 + 56(a^7 + 2a^5b^2 + a^3b^4)*d*\cosh(dx + c \\
& )^3\sinh(dx + c)^5 + 28(a^7 + 2a^5b^2 + a^3b^4)*d*\cosh(dx + c)^2\sinh \\
& (dx + c)^6 + 8(a^7 + 2a^5b^2 + a^3b^4)*d*\cosh(dx + c)\sinh(dx + c)^7 \\
& + (a^7 + 2a^5b^2 + a^3b^4)*d*\sinh(dx + c)^8 - 2(a^7 + 2a^5b^2 + a^3 \\
& *b^4)*d*\cosh(dx + c)^4 + 2(35(a^7 + 2a^5b^2 + a^3b^4)*d*\cosh(dx + c) \\
& ^4 - (a^7 + 2a^5b^2 + a^3b^4)*d)\sinh(dx + c)^4 + 8(7(a^7 + 2a^5b^2 \\
& + a^3b^4)*d*\cosh(dx + c)^5 - (a^7 + 2a^5b^2 + a^3b^4)*d*\cosh(dx + c) \\
& )\sinh(dx + c)^3 + 4(7(a^7 + 2a^5b^2 + a^3b^4)*d*\cosh(dx + c)^6 - 3* \\
& (a^7 + 2a^5b^2 + a^3b^4)*d*\cosh(dx + c)^2)\sinh(dx + c)^2 + (a^7 + 2a \\
& ^5b^2 + a^3b^4)*d + 8((a^7 + 2a^5b^2 + a^3b^4)*d*\cosh(dx + c)^7 - (a \\
& ^7 + 2a^5b^2 + a^3b^4)*d*\cosh(dx + c)^3)\sinh(dx + c))
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)\*\*3\*sech(d\*x+c)\*\*3/(a+b\*sinh(d\*x+c)),x)

[Out] Timed out

**Giac [B]** time = 1.77478, size = 652, normalized size = 3.09

$$-\frac{b^7 \log\left(\left|b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 2a\right|\right)}{a^7bd + 2a^5b^3d + a^3b^5d} + \frac{\left(\pi + 2 \arctan\left(\frac{1}{2}\left(e^{(2dx+2c)} - 1\right)e^{(-dx-c)}\right)\right)\left(3a^2b + 5b^3\right)}{4\left(a^4d + 2a^2b^2d + b^4d\right)} + \frac{\left(2a^3 + 3ab^2\right) \log\left(\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4\right)}{2\left(a^4d + 2a^2b^2d + b^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d\*x+c)^3\*sech(d\*x+c)^3/(a+b\*sinh(d\*x+c)),x, algorithm="giac")

[Out] 
$$-b^7 \log(\text{abs}(b \cdot (e^{(d \cdot x + c)} - e^{(-d \cdot x - c)}) + 2 \cdot a)) / (a^7 \cdot b \cdot d + 2 \cdot a^5 \cdot b^3 \cdot d + a^3 \cdot b^5 \cdot d) + 1/4 \cdot (\pi + 2 \cdot \arctan(1/2 \cdot (e^{(2 \cdot d \cdot x + 2 \cdot c)} - 1) \cdot e^{(-d \cdot x - c)})) \cdot (3 \cdot a^2 \cdot b + 5 \cdot b^3) / (a^4 \cdot d + 2 \cdot a^2 \cdot b^2 \cdot d + b^4 \cdot d) + 1/2 \cdot (2 \cdot a^3 + 3 \cdot a \cdot b^2) \cdot \log((e^{(d \cdot x + c)} - e^{(-d \cdot x - c)})^2 + 4) / (a^4 \cdot d + 2 \cdot a^2 \cdot b^2 \cdot d + b^4 \cdot d) - 1/2 \cdot (2 \cdot a^3 \cdot (e^{(d \cdot x + c)} - e^{(-d \cdot x - c)})^2 + 3 \cdot a \cdot b^2 \cdot (e^{(d \cdot x + c)} - e^{(-d \cdot x - c)})^2 - 2 \cdot a^2 \cdot b \cdot (e^{(d \cdot x + c)} - e^{(-d \cdot x - c)}) - 2 \cdot b^3 \cdot (e^{(d \cdot x + c)} - e^{(-d \cdot x - c)})) + 12 \cdot a^3 + 16 \cdot a \cdot b^2) / ((a^4 \cdot d + 2 \cdot a^2 \cdot b^2 \cdot d + b^4 \cdot d) \cdot ((e^{(d \cdot x + c)} - e^{(-d \cdot x - c)})^2 + 4)) - (2 \cdot a^2 - b^2) \cdot \log(\text{abs}(e^{(d \cdot x + c)} - e^{(-d \cdot x - c)})) / (a^3 \cdot d) + 1/2 \cdot (6 \cdot a^2 \cdot (e^{(d \cdot x + c)} - e^{(-d \cdot x - c)})^2 - 3 \cdot b^2 \cdot (e^{(d \cdot x + c)} - e^{(-d \cdot x - c)})^2 + 4 \cdot a \cdot b \cdot (e^{(d \cdot x + c)} - e^{(-d \cdot x - c)}) - 4 \cdot a^2) / (a^3 \cdot d \cdot (e^{(d \cdot x + c)} - e^{(-d \cdot x - c)})^2)$$



$$3.502 \quad \int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Optimal.** Leaf size=38

$$\text{Unintegrable}\left(\frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))}, x\right)$$

[Out] Unintegrable[(Csch[c + d\*x]^3\*Sech[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

**Rubi [A]** time = 0.133629, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

Verification is Not applicable to the result.

[In] Int[(Csch[c + d\*x]^3\*Sech[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] Defer[Int] [(Csch[c + d\*x]^3\*Sech[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])), x]

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx = \int \frac{\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{(e+fx)(a+b\sinh(c+dx))} dx$$

**Mathematica [F]** time = 180.001, size = 0, normalized size = 0.

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(Csch[c + d\*x]^3\*Sech[c + d\*x]^3)/((e + f\*x)\*(a + b\*Sinh[c + d\*x])),x]

[Out] \$Aborted

**Maple [A]** time = 3.319, size = 0, normalized size = 0.

$$\int \frac{(\operatorname{csch}(dx+c))^3 (\operatorname{sech}(dx+c))^3}{(fx+e)(a+b \sinh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)`

[Out] `int(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & -(a*b^2*f - (2*b^3*d*e*e^{(7*c)} + (3*d*e - f)*a^2*b*e^{(7*c)} + (3*a^2*b*d*f*e^{(7*c)} + 2*b^3*d*f*e^{(7*c)})) * x) * e^{(7*d*x)} + (2*(2*d*e - f)*a^3*e^{(6*c)} + (2*d*e - f)*a*b^2*e^{(6*c)} + 2*(2*a^3*d*f*e^{(6*c)} + a*b^2*d*f*e^{(6*c)}) * x) * e^{(6*d*x)} \\ & - (2*b^3*d*e*e^{(5*c)} - (d*e - f)*a^2*b*e^{(5*c)} - (a^2*b*d*f*e^{(5*c)} - 2*b^3*d*f*e^{(5*c)}) * x) * e^{(5*d*x)} + (4*a*b^2*d*f*x*e^{(4*c)} + (4*d*e - f)*a*b^2*e^{(4*c)}) * e^{(4*d*x)} \\ & + (2*b^3*d*e*e^{(3*c)} - (d*e + f)*a^2*b*e^{(3*c)} - (a^2*b*d*f*e^{(3*c)} - 2*b^3*d*f*e^{(3*c)}) * x) * e^{(3*d*x)} + (2*(2*d*e + f)*a^3*e^{(2*c)} + (2*d*e + f)*a*b^2*e^{(2*c)} + 2*(2*a^3*d*f*e^{(2*c)} + a*b^2*d*f*e^{(2*c)}) * x) * e^{(2*d*x)} \\ & + (2*b^3*d*e*e^c + (3*d*e + f)*a^2*b*e^c + (3*a^2*b*d*f*e^c + 2*b^3*d*f*e^c) * x) * e^{(d*x)} / (a^4*d^2*e^2 + a^2*b^2*d^2*e^2 + (a^4*d^2*f^2 + a^2*b^2*d^2*f^2) * x^2 + 2*(a^4*d^2*e*f + a^2*b^2*d^2*e*f) * x + (a^4*d^2*e^2*e^{(8*c)} + a^2*b^2*d^2*e^2*e^{(8*c)} + (a^4*d^2*f^2*e^{(8*c)} + a^2*b^2*d^2*f^2*e^{(8*c)}) * x^2 + 2*(a^4*d^2*e*f*e^{(8*c)} + a^2*b^2*d^2*e*f*e^{(8*c)}) * x) * e^{(8*d*x)} \\ & - 2*(a^4*d^2*e^2*e^{(4*c)} + a^2*b^2*d^2*e^2*e^{(4*c)} + (a^4*d^2*f^2*e^{(4*c)} + a^2*b^2*d^2*f^2*e^{(4*c)}) * x^2 + 2*(a^4*d^2*e*f*e^{(4*c)} + a^2*b^2*d^2*e*f*e^{(4*c)}) * x) * e^{(4*d*x)} + 64*integrate(-1/32*(a*b^6*e^{(d*x+c)} - b^7)/(a^7*b * e + 2*a^5*b^3*e + a^3*b^5*e + (a^7*b*f + 2*a^5*b^3*f + a^3*b^5*f) * x - (a^7 \end{aligned}$$

$$\begin{aligned}
& *b*e^e^{(2*c)} + 2*a^5*b^3*e^e^{(2*c)} + a^3*b^5*e^e^{(2*c)} + (a^7*b*f*e^e^{(2*c)} + \\
& 2*a^5*b^3*f*e^e^{(2*c)} + a^3*b^5*f*e^e^{(2*c)})*x)*e^{(2*d*x)} - 2*(a^8*e^e^c + 2*a \\
& ^6*b^2*e^e^c + a^4*b^4*e^e^c + (a^8*f*e^c + 2*a^6*b^2*f*e^c + a^4*b^4*f*e^c \\
& )*x)*e^{(d*x)}, x) - 64*\integrate(1/64*(b^2*d^2*e^2 + a*b*d*e*f - (2*d^2*e^2 \\
& - f^2)*a^2 - (2*a^2*d^2*f^2 - b^2*d^2*f^2)*x^2 - (4*a^2*d^2*e*f - 2*b^2*d^2 \\
& ^2*e*f - a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3*a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^ \\
& 2*f*x + a^3*d^2*e^3 - (a^3*d^2*f^3*x^3*e^c + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^ \\
& 3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^{(d*x)}, x) + 64*\integrate(-1/64*(b^2 \\
& *d^2*e^2 - a*b*d*e*f - (2*d^2*e^2 - f^2)*a^2 - (2*a^2*d^2*f^2 - b^2*d^2*f^2 \\
& )*x^2 - (4*a^2*d^2*e*f - 2*b^2*d^2*e*f + a*b*d*f^2)*x)/(a^3*d^2*f^3*x^3 + 3 \\
& *a^3*d^2*e*f^2*x^2 + 3*a^3*d^2*e^2*f*x + a^3*d^2*e^3 + (a^3*d^2*f^3*x^3*e^c \\
& + 3*a^3*d^2*e*f^2*x^2*e^c + 3*a^3*d^2*e^2*f*x*e^c + a^3*d^2*e^3*e^c)*e^{(d* \\
& x)}, x) + 64*\integrate(-1/64*(2*(2*d^2*e^2 - f^2)*a^3 + 2*(3*d^2*e^2 - f^2) \\
& *a*b^2 + 2*(2*a^3*d^2*f^2 + 3*a*b^2*d^2*f^2)*x^2 + 4*(2*a^3*d^2*e*f + 3*a*b \\
& ^2*d^2*e*f)*x - ((3*d^2*e^2 - 2*f^2)*a^2*b*e^c + (5*d^2*e^2 - 2*f^2)*b^3*e^ \\
& c + (3*a^2*b*d^2*f^2*e^c + 5*b^3*d^2*f^2*e^c)*x^2 + 2*(3*a^2*b*d^2*e*f*e^c \\
& + 5*b^3*d^2*e*f*e^c)*x)*e^{(d*x)})/(a^4*d^2*e^3 + 2*a^2*b^2*d^2*e^3 + b^4*d^2 \\
& *e^3 + (a^4*d^2*f^3 + 2*a^2*b^2*d^2*f^3 + b^4*d^2*f^3)*x^3 + 3*(a^4*d^2*e*f \\
& ^2 + 2*a^2*b^2*d^2*e*f^2 + b^4*d^2*e*f^2)*x^2 + 3*(a^4*d^2*e^2*f + 2*a^2*b^ \\
& 2*d^2*e^2*f + b^4*d^2*e^2*f)*x + (a^4*d^2*e^3*e^{(2*c)} + 2*a^2*b^2*d^2*e^3*e \\
& ^{(2*c)} + b^4*d^2*e^3*e^{(2*c)} + (a^4*d^2*f^3*e^{(2*c)} + 2*a^2*b^2*d^2*f^3*e^{( \\
& 2*c)} + b^4*d^2*f^3*e^{(2*c)})*x^3 + 3*(a^4*d^2*e*f^2*e^{(2*c)} + 2*a^2*b^2*d^2 \\
& *e*f^2*e^{(2*c)} + b^4*d^2*e*f^2*e^{(2*c)})*x^2 + 3*(a^4*d^2*e^2*f*e^{(2*c)} + 2*a \\
& ^2*b^2*d^2*e^2*f*e^{(2*c)} + b^4*d^2*e^2*f*e^{(2*c)})*x)*e^{(2*d*x)}, x)
\end{aligned}$$

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="fricas")
```

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*sech(d*x+c)**3/(f*x+e)/(a+b*sinh(d*x+c)),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*sech(d*x+c)^3/(f*x+e)/(a+b*sinh(d*x+c)),x, algorithm="giac")
```

```
[Out] Timed out
```

# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*      is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*      antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```

22     If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25     "C"],
26 If[FreeQ[result,Integrate] && FreeQ[result,Int],
27     "C",
28     "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46     If[AtomQ[expn],
47         1,
48     If[ListQ[expn],
49         Max[Map[ExpnType,expn]],
50     If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52             ExpnType[expn[[1]]],
53         If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55                 1,
56                 Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58     If[Head[expn]===Plus || Head[expn]===Times,
59         Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60     If[ElementaryFunctionQ[Head[expn]],
61         Max[3,ExpnType[expn[[1]]]],
62     If[SpecialFunctionQ[Head[expn]],
63         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64     If[HypergeometricFunctionQ[Head[expn]],
65         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66     If[AppellFunctionQ[Head[expn]],
67         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68     If[Head[expn]===RootSum,

```

```

69   Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
70   If[Head[expn]===Integrate || Head[expn]===Int,
71     Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
72   9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp, Log,
78     Sin, Cos, Tan, Cot, Sec, Csc,
79     ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
80     Sinh, Cosh, Tanh, Coth, Sech, Csch,
81     ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
82   }, func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   }, func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
99
100
101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1}, func]

```

## 4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 #
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 #
11 see problem 156, file Apostol_Problems

```

```

11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;
14
15     leaf_count_result:=leafcount(result);
16     #do NOT call ExpnType() if leaf size is too large. Recursion problem
17     if leaf_count_result > 500000 then
18         return "B";
19     fi;
20
21     leaf_count_optimal:=leafcount(optimal);
22
23     ExpnType_result:=ExpnType(result);
24     ExpnType_optimal:=ExpnType(optimal);
25
26     if debug then
27         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
28     fi;
29
30 # If result and optimal are mathematical expressions,
31 # GradeAntiderivative[result,optimal] returns
32 #   "F" if the result fails to integrate an expression that
33 #     is integrable
34 #   "C" if result involves higher level functions than necessary
35 #   "B" if result is more than twice the size of the optimal
36 #     antiderivative
37 #   "A" if result can be considered optimal
38
39 #This check below actually is not needed, since I only
40 #call this grading only for passed integrals. i.e. I check
41 #for "F" before calling this. But no harm of keeping it here.
42 #just in case.
43
44
45 if not type(result,freeof('int')) then
46     return "F";
47 end if;
48
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then

```



```
56     if debug then
57         print("both result and optimal complex");
58     fi;
59     #both result and optimal complex
60     if leaf_count_result<=2*leaf_count_optimal then
61         return "A";
62     else
63         return "B";
64     end if
65 else #result contains complex but optimal is not
66     if debug then
67         print("result contains complex but optimal is not");
68     fi;
69     return "C";
70 end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do not
as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
```

```

102 is_contains_complex:= proc(expression)
103   return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)
119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'^+') or type(expn,'*') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))

```

```

149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157 ElementaryFunctionQ := proc(func)
158     member(func,[
159         exp,log,ln,
160         sin,cos,tan,cot,sec,csc,
161         arcsin,arccos,arctan,arccot,arcsec,arccsc,
162         sinh,cosh,tanh,coth,sech,csch,
163         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
164 end proc:
165
166 SpecialFunctionQ := proc(func)
167     member(func,[
168         erf,erfc,erfi,
169         FresnelS,FresnelC,
170         Ei,Ei,Li,Si,Ci,Shi,Chi,
171         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
172         EllipticF,EllipticE,EllipticPi])
173 end proc:
174
175 HypergeometricFunctionQ := proc(func)
176     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
177 end proc:
178
179 AppellFunctionQ := proc(func)
180     member(func,[AppellF1])
181 end proc:
182
183
184 # u is a sum or product.  rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple

```

```

196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

### 4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:

```

```

42     if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43         return True
44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,'^^')
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
72 ))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type
77 (expn,'*')
78         m1 = expnType(expn.args[0])
79         m2 = expnType(list(expn.args[1:]))
80         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82         return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84         m1 = max(map(expnType, list(expn.args)))
85         return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88         m1 = max(map(expnType, list(expn.args)))

```

```

85     return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
86 elif is_appell_function(expn.func):
87     m1 = max(map(expnType, list(expn.args)))
88     return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
89 elif isinstance(expn,RootSum):
90     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
91     return max(7,m1)
92 elif str(expn).find("Integral") != -1:
93     m1 = max(map(expnType, list(expn.args)))
94     return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
95 else:
96     return 9
97
98 #main function
99 def grade_antiderivative(result,optimal):
100
101     leaf_count_result  = leaf_count(result)
102     leaf_count_optimal = leaf_count(optimal)
103
104     expnType_result  = expnType(result)
105     expnType_optimal = expnType(optimal)
106
107     if str(result).find("Integral") != -1:
108         return "F"
109
110     if expnType_result <= expnType_optimal:
111         if result.has(I):
112             if optimal.has(I): #both result and optimal complex
113                 if leaf_count_result <= 2*leaf_count_optimal:
114                     return "A"
115                 else:
116                     return "B"
117             else: #result contains complex but optimal is not
118                 return "C"
119         else: # result do not contain complex, this assumes optimal do not as
well
120             if leaf_count_result <= 2*leaf_count_optimal:
121                 return "A"
122             else:
123                 return "B"
124     else:
125         return "C"

```

## 4.0.4 SageMath grading function

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by

```

2 #           Albert Rich to use with Sagemath. This is used to
3 #           grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #           'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
33         flatten(tree(anti))))
34         return round(1.35*len(flatten(tree(anti)))) #fudge factor
35         #since this estimate of leaf count is bit lower than
36         #what it should be compared to Mathematica's
37
38 def is_sqrt(expr):
39     debug=False;
40     if expr.operator() == operator.pow: #isinstance(expr,Pow):
41         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
42             if debug: print ("expr is sqrt")
43             return True
44         else:
45             return False
46     else:
47         return False

```

```

48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func ," is special_function")
83         else:
84             print ("func ", func ," is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
91     ']
92
93 def is_appell_function(func):

```



```

93     return func.name() in ['hypergeometric']    #[appellf1] can't find this in
          sagemath
94
95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
104             return expn in expn.parent().base_ring() or expn in expn.parent().
          gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list:    #isinstance(expn,list):
121         return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
          Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0]))    #max(2,expnType(expn.
          args[0]))
127     elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer:    #isinstance(expn.args[1],Integer)
129             return expnType(expn.operands()[0])    #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational:    #isinstance(expn.args[1],
          Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
          Rational)
132                 return 1

```

```

133         else:
134             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137         elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138             m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139             m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140             return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
141         elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
142             return max(3,expnType(expn.operands()[0]))
143         elif is_special_function(expn.operator()): #is_special_function(expn.func)
144             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
145             return max(4,m1) #max(4,m1)
146         elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
148             return max(5,m1) #max(5,m1)
149         elif is_appell_function(expn.operator()):
150             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
151             return max(6,m1) #max(6,m1)
152         elif str(expn).find("Integral") != -1: #this will never happen, since it
153             #is checked before calling the grading function that is passed.
154             #but kept it here.
155             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
156             return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
157         else:
158             return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
176                     expnType_optimal)
177
178     if expnType_result <= expnType_optimal:
179         if result.has(I):
180             if optimal.has(I): #both result and optimal complex
181                 if leaf_count_result <= 2*leaf_count_optimal:
182                     return "A"
183             else:
184                 return "B"
185         else: #result contains complex but optimal is not
186             return "C"
187     else: # result do not contain complex, this assumes optimal do not as
188         well
189         if leaf_count_result <= 2*leaf_count_optimal:
190             return "A"
191         else:
192             return "B"
193     else:
194         return "C"
```